

Title: Conformal Cosmology and Physics at Gravitational and Electroweak Scales

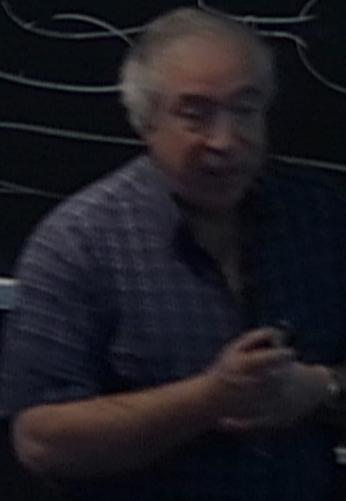
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Abstract: New techniques for obtaining the complete set of analytic solutions of the usual cosmological equations continue to shed new light on various aspects of cosmology. This approach, which was developed with a locally Weyl invariant formulation of gravity in 3+1 dimensions, was inspired by the 2T-physics formulation of all physics in 4+2 dimensions. I will first give a review of the general aspects of the analytic cosmological solutions, including recent work with Shi-Hung Chen, Paul Steinhardt and Neil Turok, on geodesic completeness through the singularity and the nature of the Big Crunch/Big Bang transition. Then I will include the full Standard Model, and discuss how the instability of the Higgs potential and the conditions for the electroweak phase transition develop slowly during cosmological evolution of the universe, thus understanding that the electroweak phase transition is not "spontaneous" and the Standard Model is not decoupled from gravity or cosmology. This scenario, which is currently being developed further, is the natural cosmological consequence of coupling the Standard Model to Gravity by imposing Weyl symmetry, a feature that is required if 2T-physics in 4+2 dimensions is considered as the starting point. There are no dimensionful parameters, not even the Newton constant. Emergent dimensionful scales are dynamical during the evolution of the universe.

1)
2)
3)
4)

$$R = K^2 - \cancel{(K^2)} \quad \Sigma^3 = T^3 \# T^3$$



CAUTION:
CONTAINS HIGHLY INFLAMMABLE
AND EXPLOSIVE MATERIALS.
KEEP AWAY FROM HEAT,
SPARKS AND FLAME.

Conformal Cosmology and Physics at the Gravitational and Electroweak Scales

Itzhak Bars
University of Southern California

Conformal Nature of the Universe
Perimeter Institute, May 2012

- 1) I.B. and S.H. Chen, **1004.0752**
- 2) I.B., and S.H. Chen and Neil Turok, **1105.3606**
- 3) I.B. + Chen + Steinhardt + Turok, **1112.2470** and to appear
- 4) I.B. in progress

Outline

- Gravity coupled to a scalar field. Complete set of analytic solutions of cosmological equations. (including curvature & radiation)
 - Tricks with conformal symmetry.
 - Several models solved exactly (i.e. several $V(\sigma)$'s)
 - No restrictions on values of parameters in $V(\sigma)$ or boundary conditions.

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 - Bounces at zero size and finite sizes.

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 - Big Crunch / Big Bang transition and antigravity
- Cosmological evolution to a hierarchy of energy scales
(e.g. electroweak/Planck , in Standard Model + Gravity, role of dark energy)

Cosmology with a scalar coupled to gravity

Could be low energy string theory dilaton gravity. Or not.

$$S = \int d^4x \sqrt{-g} \left\{ \frac{1}{2\kappa^2} R(g) - \frac{1}{2} g^{\mu\nu} \partial_\mu \sigma \partial_\nu \sigma - V(\sigma) \right\} + \text{radiation} + \text{matter}$$

Including relativistic matter, curvature, anisotropy.

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FRW

$$ds_E^2 = -dt^2 + a_E^2(t) ds_3^2 = a^2(\tau) (-d\tau^2 + ds_3^2), \quad dt = a(\tau) d\tau$$

$$ds_3^2 = \frac{dr^2}{1 - kr^2/r_0^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2); \quad k = 0, \pm 1. \quad K = \frac{k}{r_0^2}$$

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Friedmann equations

Analytically solved with this V
found ALL solutions

(also $V(\sigma) = \lambda e^{-2\sigma} + \zeta e^{-4\sigma}$, $V(\sigma) = \lambda e^{2k\sigma}$)

$$\begin{aligned} \frac{\dot{a}_E^2}{a_E^4} &= \frac{\kappa^2}{3} \left[\frac{\dot{\sigma}^2}{2a_E^2} + V(\sigma) \right] - \frac{K}{a_E^2} + \frac{\rho_0}{a_E^4} \\ \frac{\ddot{a}_E}{a_E^3} - \frac{\dot{a}_E^2}{a_E^4} &= -\frac{\kappa^2}{3} \left[\frac{\dot{\sigma}^2}{a_E^2} - V(\sigma) \right] - \frac{\rho_0}{3a_E^4} \\ \frac{\ddot{\sigma}}{a_E^2} + 2\frac{\dot{a}_E \dot{\sigma}}{a_E^3} + V'(\sigma) &= 0. \end{aligned}$$

6 parameters
4=b,c,K, ρ_0
2=(4-2) initial values

$$V(\sigma) = \left(\frac{\sqrt{6}}{\kappa} \right)^4 \left[b \cosh^4 \left(\frac{\kappa\sigma}{\sqrt{6}} \right) + c \sinh^4 \left(\frac{\kappa\sigma}{\sqrt{6}} \right) \right]$$



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Near Big Bang, very important,
must include anisotropic metrics :
Kasner (flat), Bianchi IX, VIII (curved)
Two more fields in metric.

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For geodesic completeness: a slight extension of Einstein gravity (with gauge degrees of freedom) [4/19](#)

Local scaling symmetry (Weyl): allows conformally coupled scalars (generalization possible)

(Plus gauge bosons, fermions , more conformal scalars , in complete Weyl invariant theory.)

$$(\phi, s) \rightarrow (\phi, s) e^{\lambda(x)}, \quad g_{\mu\nu} \rightarrow g_{\mu\nu} e^{-2\lambda(x)}$$

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A prediction of **2T-gravity in 4+2 dims.**

Fundamental approach: **Gauge symmetry in phase space**

Relation to 1T-physics: only extra gauge degrees of freedom

2T-gravity: I.B. 0804.1585, I.B.+Chen 0811.2510

Also motivated by colliding branes scenario.

Khury + Seiberg +Steinhardt + Turok

McFadden + Turok 0409122

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This is not the whole story: Einstein gauge is valid only in a patch of spacetime, where the gauge invariant quantity $\chi \equiv (-g)^{\frac{1}{4}}(\phi^2 - s^2)$ is positive in any gauge.

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Can dynamics push this factor to negative values? ANTIGRAVITY in some regions of spacetime?

γ -gauge

$$\phi_\gamma, s_\gamma, g_\gamma^{\mu\nu}$$

Conformal factor of metric = 1 for any metric. \rightarrow

$$a_\gamma = 1$$

For all t,x dependence.

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$$L = \frac{1}{2} \left(-\dot{\phi}_\gamma^2 + \dot{s}_\gamma^2 \right) - \frac{K}{2} \left(-\phi_\gamma^2 + s_\gamma^2 \right) - \phi^4 f \left(\frac{s}{\phi} \right)$$

Plus the energy constraint: $H=0$ which compensates for the ghost. \leftarrow This is equivalent to the 00 Einstein eq. $G_{00}=T_{00}$

γ -gauge

$$\phi_\gamma, s_\gamma, g_\gamma^{\mu\nu}$$

Conformal factor of metric = 1 for any metric. \rightarrow

$$a_\gamma = 1$$

For all t,x dependence.

case of only time dependent fields

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connection between the γ -gauge and the Einstein gauge

BCRT transform
Bars Chen
Steinhardt Turok

$$a_E^2 = \frac{\kappa^2}{6} (\phi_\gamma^2 - s_\gamma^2) \quad \sigma = \frac{\sqrt{6}}{\kappa} \frac{1}{2} \ln \left(\left| \frac{\phi_\gamma + s_\gamma}{\phi_\gamma - s_\gamma} \right| \right) \quad \text{Positive region}$$

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Nothing singular in γ -gauge

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$$\chi \equiv \frac{\kappa^2}{6} (-g)^{\frac{1}{4}} (\phi^2 - s^2)$$

BB singularity at $a_E=0$ in E-gauge : gauge invariant factor vanishes in γ -gauge, or any gauge!!

Analytic solutions – all of them!!

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Special case: $\phi^4 f(s/\phi) = b\phi^4 + cs^4$

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Completely decoupled equations,
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Solutions are **Jacobi elliptic functions**,
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Radiation (all massless particles) in dust approximation

Particle in a potential problem, intuitively solved by looking at the plot of the potential.

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Generic solution has 6 parameters $(b, c, K, \rho_0, E, \phi(\tau_0))$

Analytic expressions given in 25 different regions.

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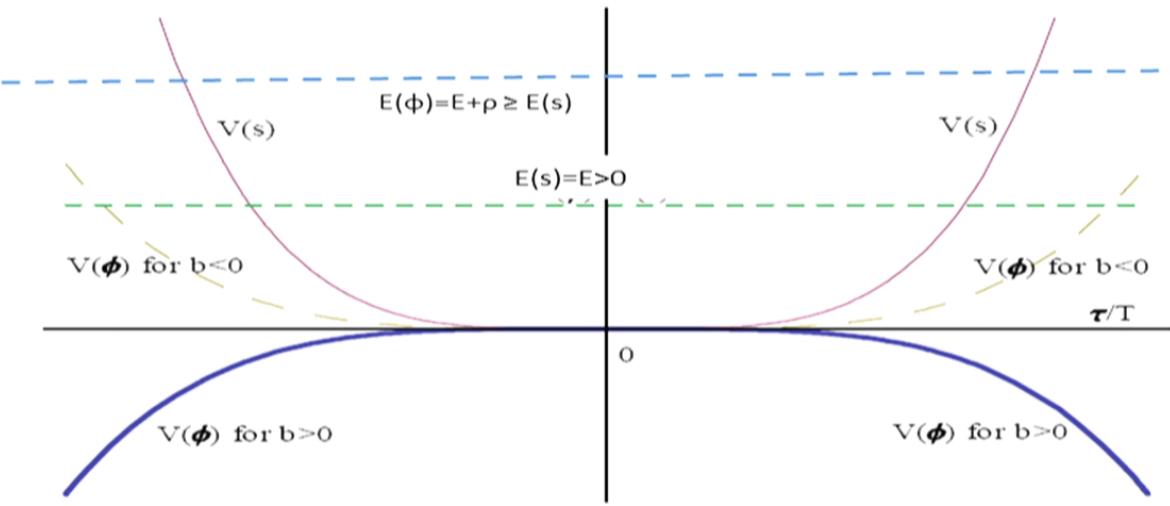
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Example,
 $K=0$ case.
**Pure quartic
potentials**
 $\phi(\tau)$, $s(\tau)$
Jacobi elliptic functions

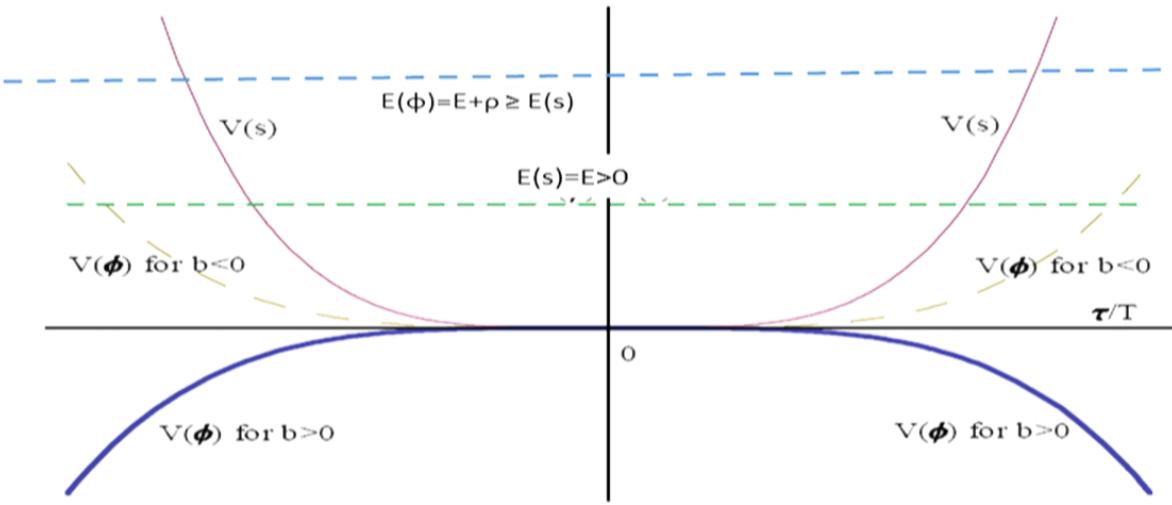
$$V(s) = \frac{1}{2}K\phi^2 + cs^4 \quad V(\phi) = \frac{1}{2}K\phi^2 - b\phi^4,$$



$\phi(\tau), s(\tau)$ perform independent oscillations

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There are special solutions that are geodesically complete in the restricted Einstein frame. Bounces at zero-size a_E , or finite-size a_E .
But must constrain parameter space and/or initial conditions.

Example,
K=0 case.

Pure quartic
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Jacobi elliptic functions

Example with
K=0 and $\rho=0$ and $b,c>0$
 $b=4c/n^2$, $n=1,2,3 \dots$
Zero-size bounce

$\phi(\tau), s(\tau)$ vanish together

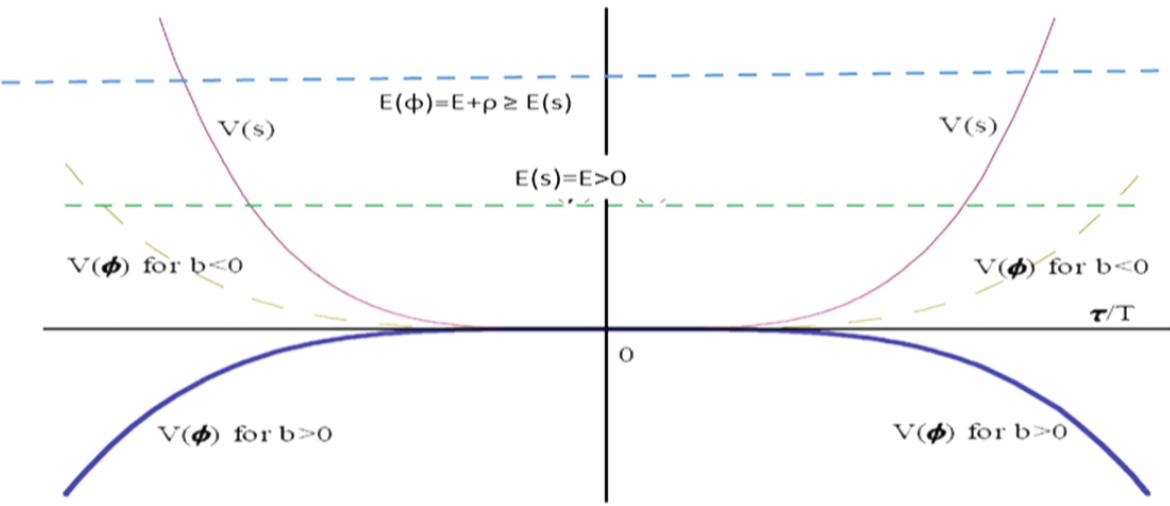
$$s_\gamma(\tau) = \frac{\kappa}{\sqrt{48cT}} \frac{sn\left(\frac{\tau}{T} | \frac{1}{2}\right)}{dn\left(\frac{\tau}{T} | \frac{1}{2}\right)}$$

$$\phi_\gamma(\tau) = \frac{\kappa n}{\sqrt{48cT}} \frac{sn\left(\frac{2\tau}{nT} | \frac{1}{2}\right)}{1 + cn\left(\frac{2\tau}{nT} | \frac{1}{2}\right)}$$

$$a_E^2 = \frac{\kappa^2}{6} (\phi_\gamma^2 - s_\gamma^2)$$

a_E(τ) always positive
Cyclic universe.

$$V(s) = \frac{1}{2}K\phi^2 + cs^4 \quad V(\phi) = \frac{1}{2}K\phi^2 - b\phi^4,$$



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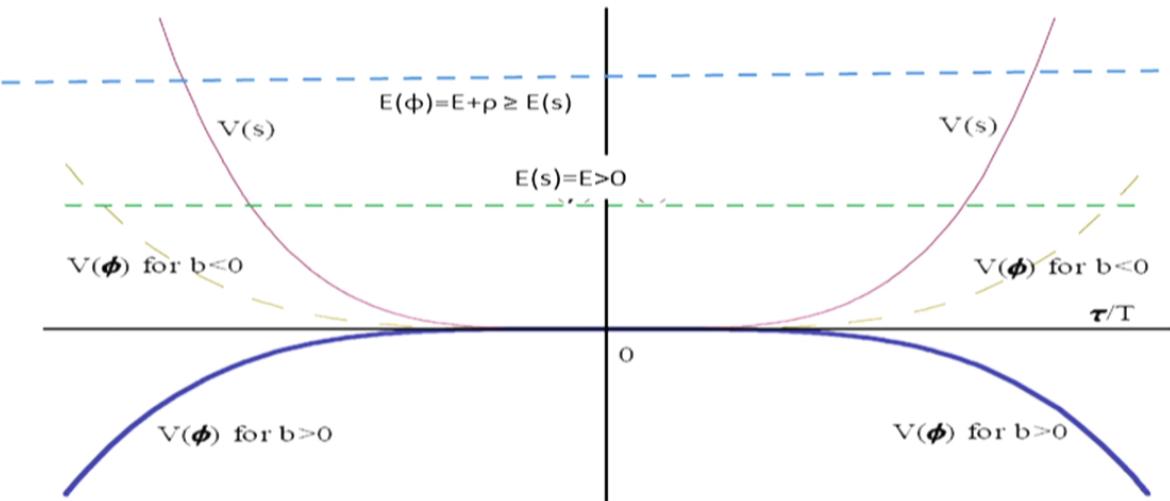
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There are special solutions that are geodesically complete in the restricted Einstein frame. Bounces at zero-size a_E , or finite-size a_E . But must constrain parameter space and/or initial conditions.

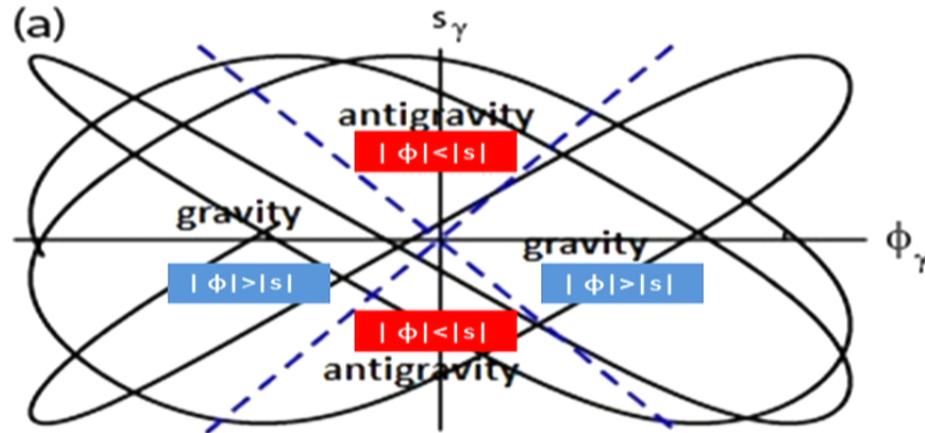
However, for **generic** initial conditions, the sign of $(\phi^2 - s^2)(\tau)$ changes over time.
Generic solution is geodesically incomplete in the Einstein gauge.
Becomes geodesically complete with the natural extension in ϕ, s space.

Geodesically complete larger space: ϕ_γ, s_γ plane

Generic solution
 $\phi(\tau), s(\tau)$

parametric plot
 (eliminate τ) is a
 smooth curve that
 spans all quadrants

Analytic expressions
 given in 25 different
 regions of parameter
 space without
 restricting parameters
 or boundary
 conditions.

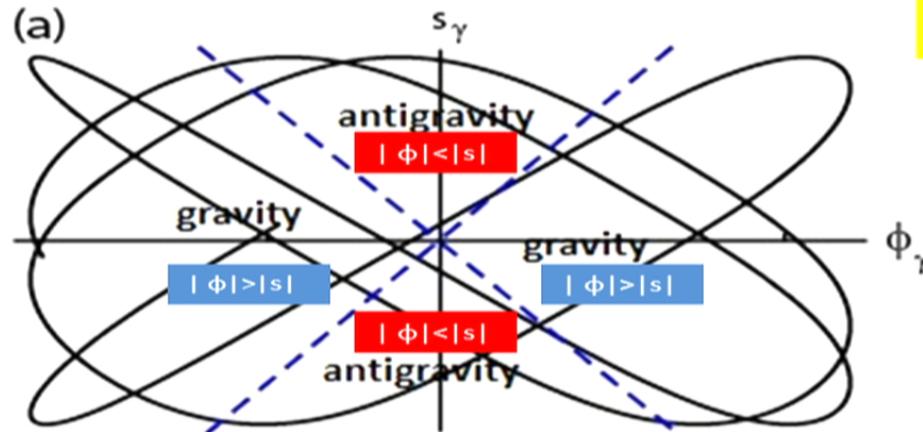


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Recall Kruskal-Szekeres versus
 Schwarzschild; now in field space.

No signature change in metric

$$a_E^2 = |\chi| , \chi \equiv \frac{\kappa^2}{6} (\phi_\gamma^2 - s_\gamma^2) , \sigma = \frac{\sqrt{6}}{\kappa} \frac{1}{2} \ln \left(\left| \frac{\phi_\gamma + s_\gamma}{\phi_\gamma - s_\gamma} \right| \right)$$

BCST

Transform
 E-gauge to
 γ -gauge

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Analytic solutions – all of them!!

$$L = \frac{1}{2} \left(-\dot{\phi}_\gamma^2 + \dot{s}_\gamma^2 \right) - \frac{K}{2} (-\phi_\gamma^2 + s_\gamma^2) - \phi^4 f \left(\frac{s}{\phi} \right)$$

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Friedmann equations become:

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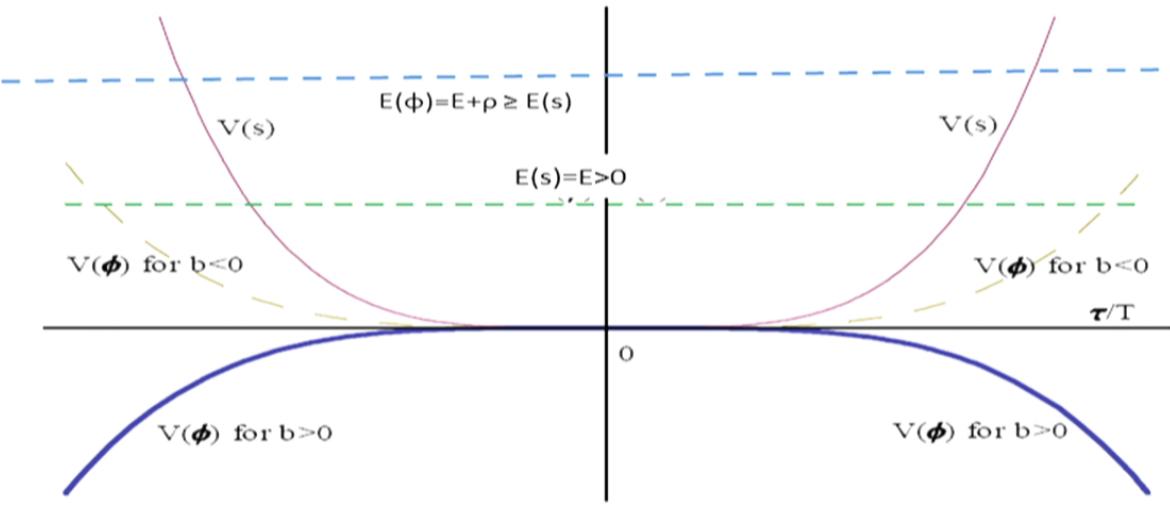
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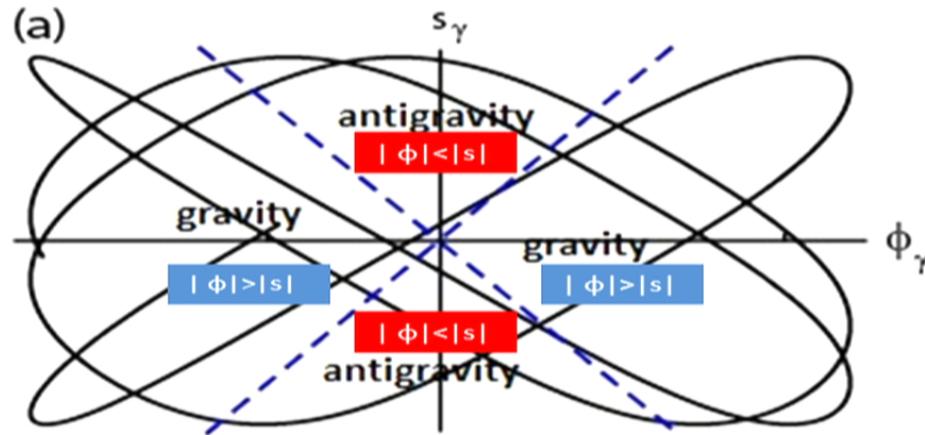
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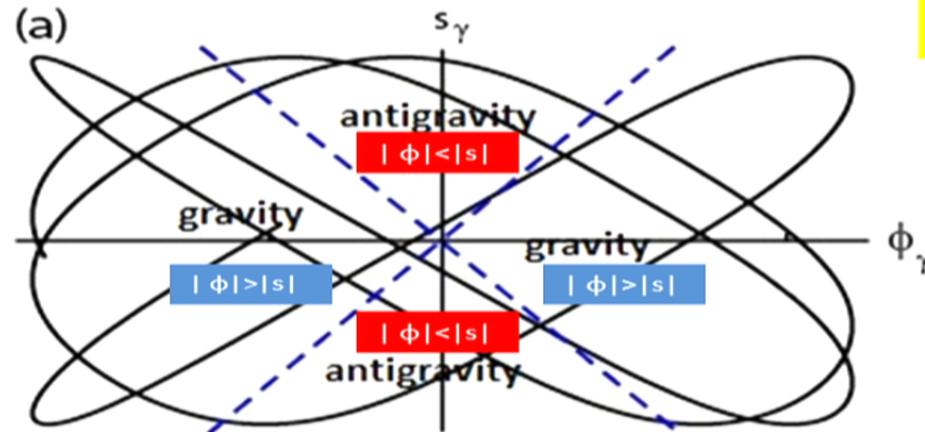


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BCST

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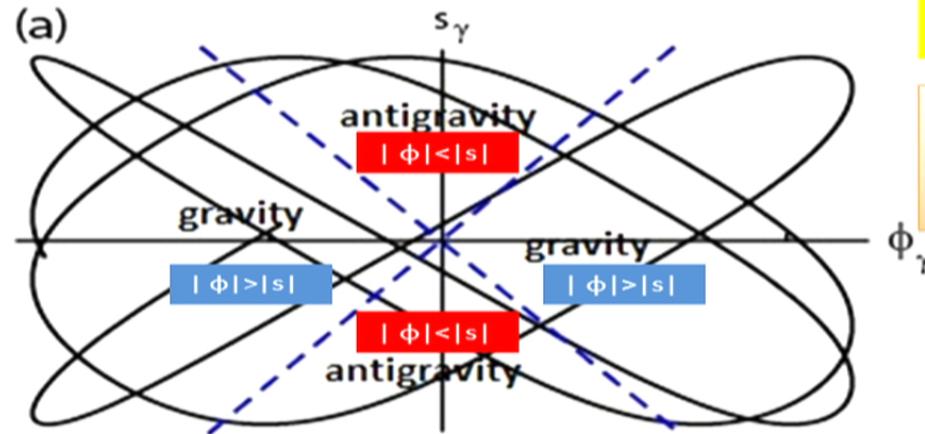
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**big bangs or big crunches
 in spacetime \leftrightarrow at the
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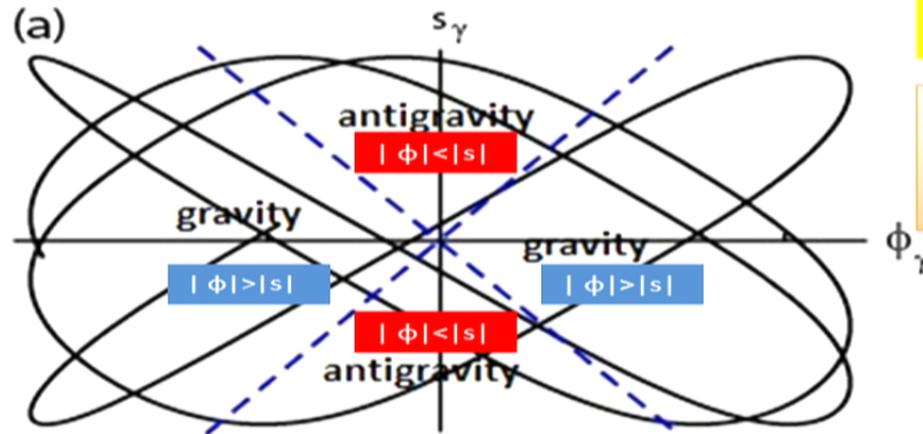
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**Generic solution is a
 cyclic universe with
 antigravity stuck
 between crunch and
 bang! Probably true
 for all $V(\sigma)$.**

No signature change in metric

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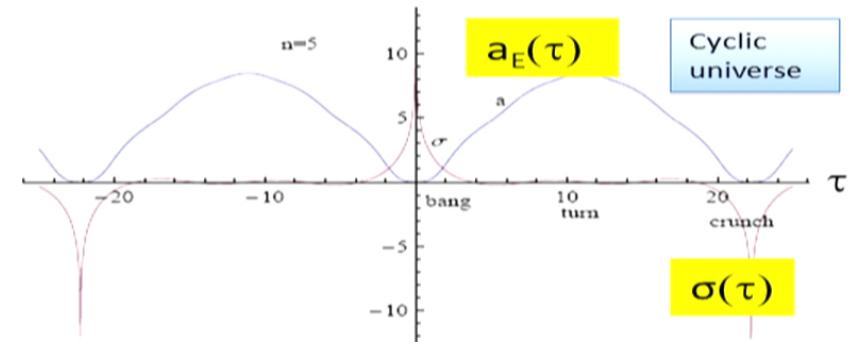
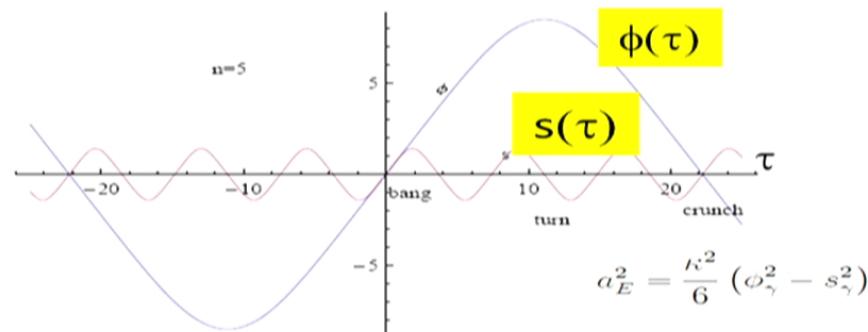
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Geodesically complete solutions in the Einstein gauge, **without antigravity**

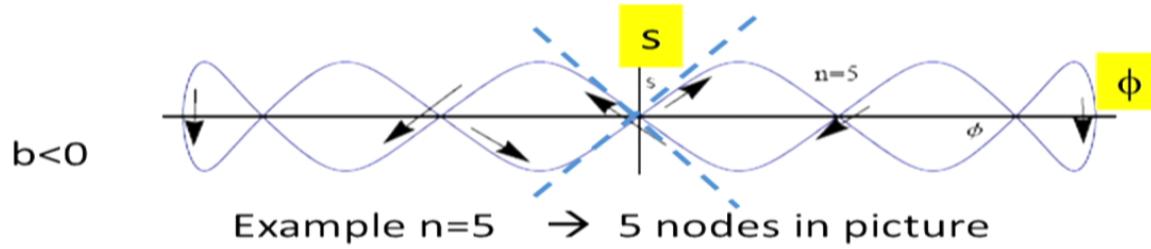
$b < 0$

Example $n=5 \rightarrow 5$ nodes in picture

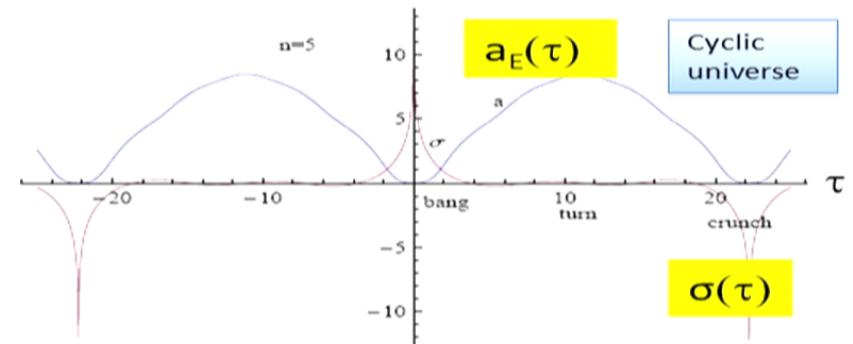
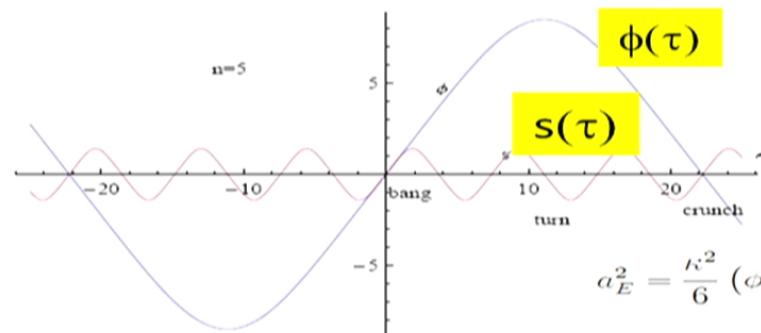
Conditions on 6 parameter space:
 (1) Synchronized initial values
 $\phi(0)=s(0)=0$
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 $P_\phi(\text{5 parameters}) = n P_s(\text{5 parameters})$



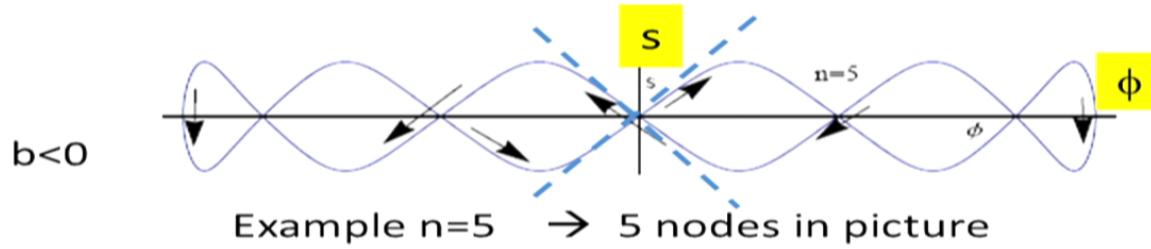
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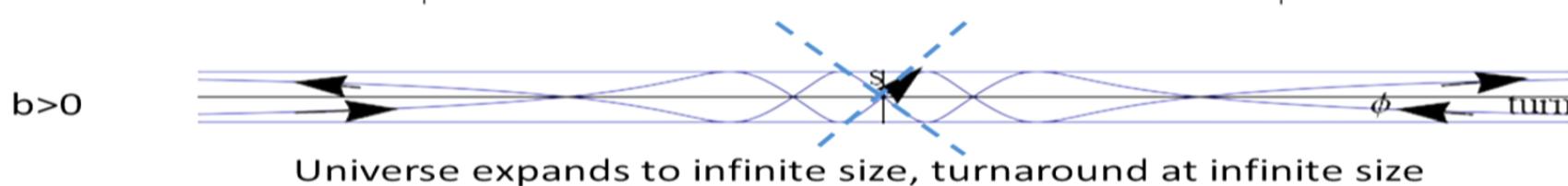
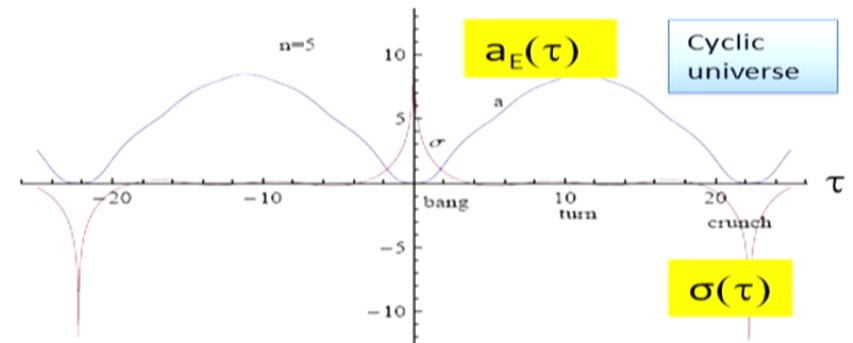
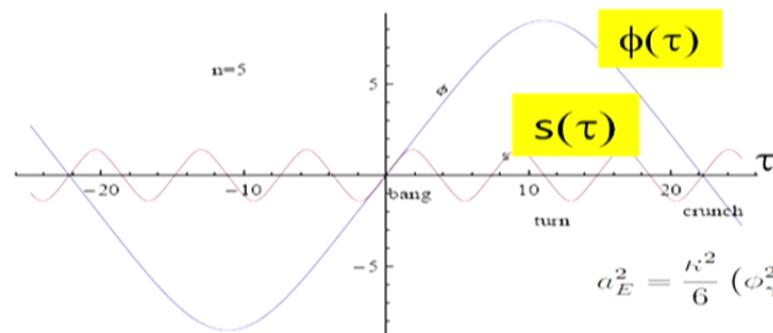
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very important
near the singularity

10/19

Anisotropy

$K > 0$

$K < 0$

$$ds^2 = a^2(\tau) (-d\tau^2 + ds_3^2) \quad \text{If } K \neq 0, \quad (ds_3)^2 = \text{Bianchi IX or VIII}, \\ d\sigma_{x,y,z}$$

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In Friedmann equations, 2 more fields $\alpha_1(\tau), \alpha_2(\tau)$, just like the $\sigma(\tau)$

$$\frac{\ddot{\alpha}_1}{a_E^2} + 2\frac{\dot{a}_E \dot{\alpha}_1}{a_E^3} + \frac{3}{\kappa^2 a_E^2} \partial_{\alpha_1} v(\alpha_1, \alpha_2) = 0, \quad \frac{\ddot{\alpha}_2}{a_E^2} + 2\frac{\dot{a}_E \dot{\alpha}_2}{a_E^3} + \frac{3}{\kappa^2 a_E^2} \partial_{\alpha_2} v(\alpha_1, \alpha_2) = 0$$

$$v(\alpha_1, \alpha_2) = \frac{K}{1 - 4 \operatorname{sign}(K)} \left[\begin{array}{l} \left(e^{-4\sqrt{2/3}\kappa\alpha_1} + 4e^{2\sqrt{2/3}\kappa\alpha_1} \sinh^2(\sqrt{2}\kappa\alpha_2) \right) \\ -4 \operatorname{sign}(K) e^{-\sqrt{2/3}\kappa\alpha_1} \cosh(\sqrt{2}\kappa\alpha_2) \end{array} \right]$$

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Rewrite equations in terms of momenta.

Near singularity $a_E=0$ kinetic terms dominate like $\pi^2/(a_E)^2$. So neglect all potentials.
The momenta $\pi_{1,2,\sigma}$ are then conserved while universe is evolving near singularity.

Transform to γ -gauge and solve equations near singularity:

$$\int d\tau \left(\frac{1}{2e} \left[-\dot{\phi}_\gamma^2 + \dot{s}_\gamma^2 + \frac{\kappa^2}{6} (\phi_\gamma^2 - s_\gamma^2) (\dot{\alpha}_1^2 + \dot{\alpha}_2^2) \right] - e\rho_r \right)$$

The solution is unique !!!

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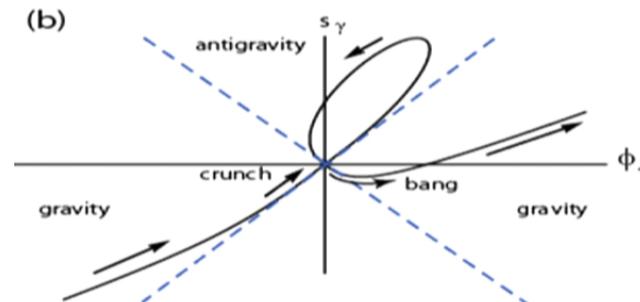
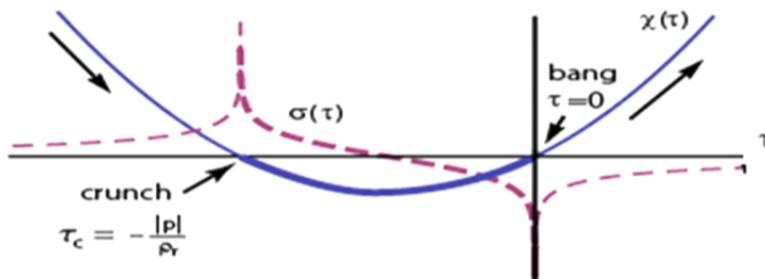
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 For all initial conditions!
Shape of loop controlled by anisotropy, kinetic energy and radiation.

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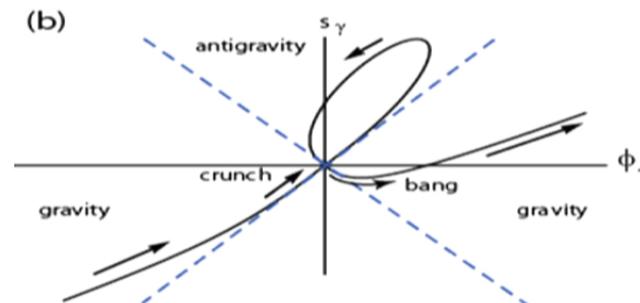
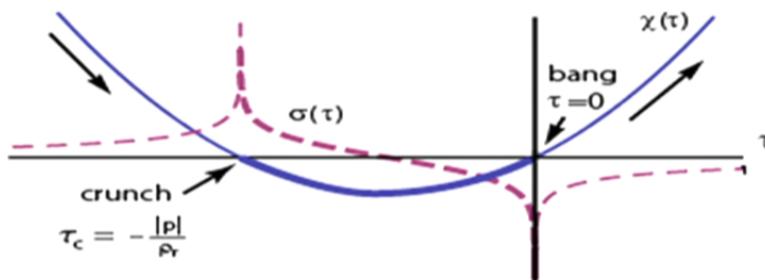
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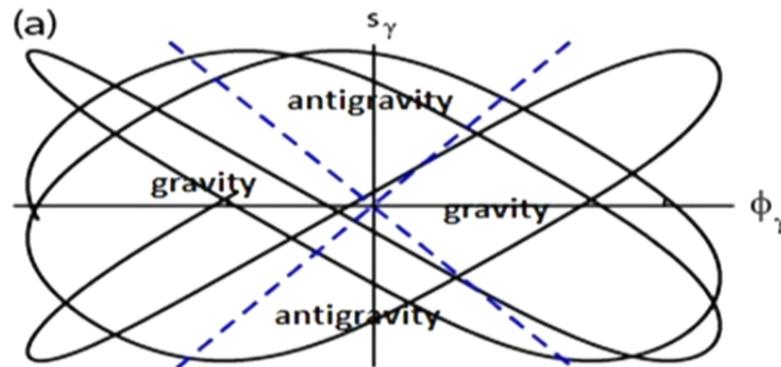
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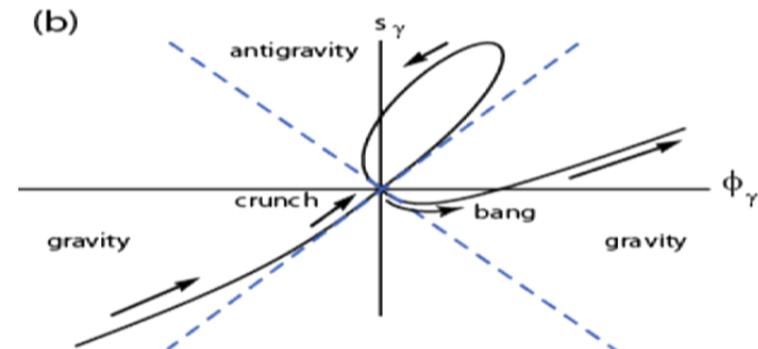
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Antigravity Loop



Without anisotropy



With ANY infinitesimal amount of anisotropy !!

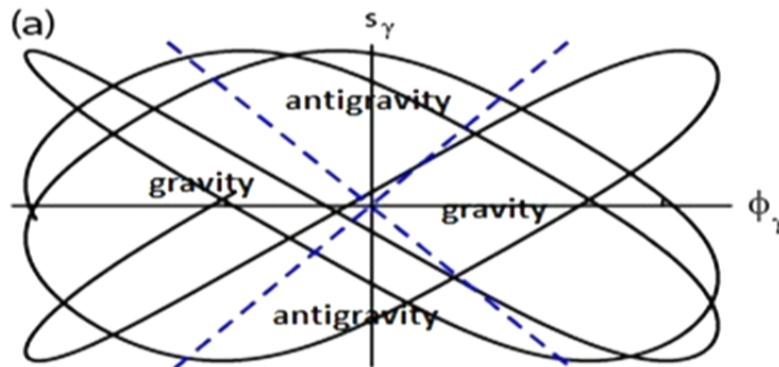
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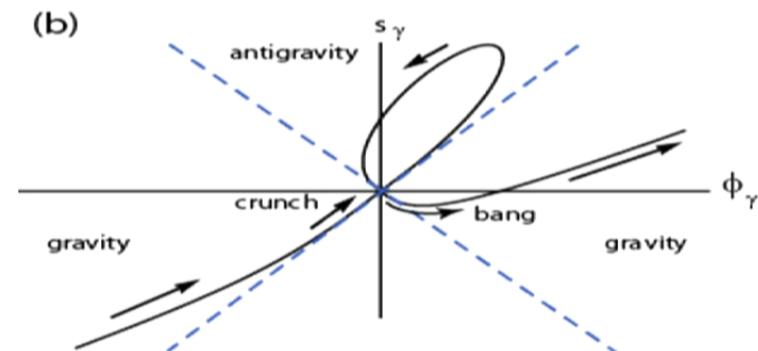
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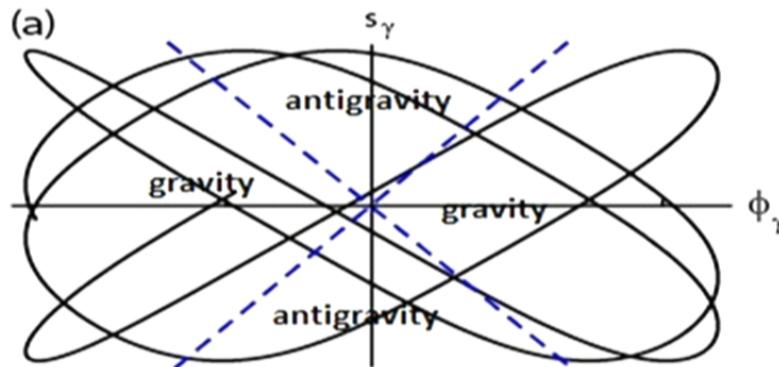


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UNAVOIDABLE CONCLUSION IN CLASSICAL GRAVITY

Generic, unique, model independent behavior:
 The universe contracts, passes briefly through
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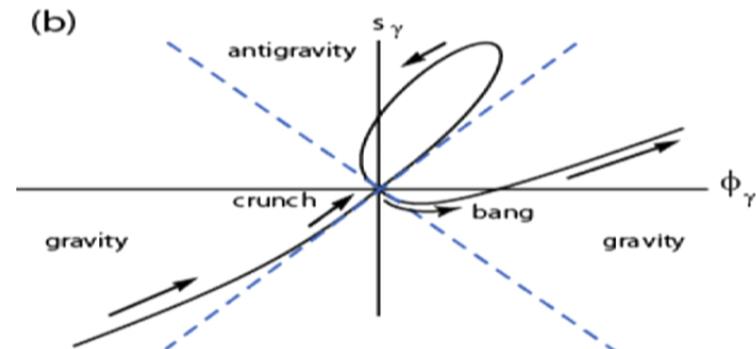


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Similar phenomena should be studied in string theory, including full quantum effects.
For starters, these solutions can be interpreted as solutions for string backgrounds,
with worldsheet conformal conditions (perturbative beta function=0).

What have we learned so far?

13/19

- Formulated techniques to solve cosmological equations analytically.
Found all solutions for several special potentials $V(\phi)$. (without anisotropy)
Several models in the problem general results (with anisotropy): completeness and an **attractor mechanism to the origin, $\phi \rightarrow 0$, for any initial values.**

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- Open: What are the observational effects today of a past antigravity period? This is an important project. Study of small fluctuations and fitting to current observations of the CMB (under investigation).

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14/19

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I will suggest a ROUGH idea that addresses these questions.
It is based on some general features of our exact solutions.

Standard Model coupled to Gravity in 3+1 dims.

The Weyl invariant master action

Weyl is an outcome of 2T-gravity:
an underlying spacetime of 4+2 dims,
Weyl is part of Diffs in extra 1+1.

$$\frac{L}{\sqrt{-g}} = \left[\begin{array}{c} \frac{1}{12} (\phi^2 - 2H^\dagger H) R(g) \\ + g^{\mu\nu} \left(\frac{1}{2} \partial_\mu \phi \partial_\nu \phi - D_\mu H^\dagger D_\nu H \right) \\ - \left(b\phi^4 + 2c (H^\dagger H - \xi^2 \phi^2)^2 \right) \\ + L_{\text{SM}} \left(\begin{array}{c} \text{quarks, leptons , gauge bosons,} \\ \text{Yukawa couplings to } H \end{array} \right) \end{array} \right]$$

$$\begin{aligned} g_{\mu\nu} &\rightarrow \Omega^{-2} g_{\mu\nu}, \quad \phi \rightarrow \Omega \phi, \quad H \rightarrow \Omega H, \\ \psi_{q,l} &\rightarrow \Omega^{3/2} \psi_{q,l}, \quad A_\mu^{\gamma,W,Z,g} \rightarrow \Omega^0 A_\mu^{\gamma,W,Z,g} \end{aligned}$$

not rescale x^μ or ∂_μ or $A_\mu(x)$

no dimensionful constants

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not rescale x^μ or ∂_μ or $A_\mu(x)$

no dimensionful constants

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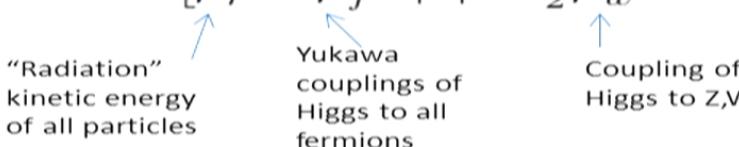
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So gravity, dark energy, and electroweak scale at
low energies could come from the same origin.
But why are the energy scales so different?
Let's examine the history of the universe
including the Higgs!

The effective cosmological action, with only homogeneous fields $\phi(\tau)$, $s(\tau)$, $a(\tau)$, takes the form

Invariant under
local Weyl
gauge symm.

$$L(\tau) = \begin{bmatrix} \frac{1}{e} \left[\frac{1}{12} a^4 (\phi^2 - s^2) R(a) + \frac{1}{2} a^2 (-\dot{\phi}^2 + \dot{s}^2) \right] \\ -\frac{1}{4} e a^4 [b\phi^4 + c(s^2 - \xi^2 \phi^2)^2] \\ -e [\rho_r + \rho_f a |s| + \frac{1}{2} \rho_w a^2 s^2] \end{bmatrix}$$



 “Radiation” kinetic energy of all particles Yukawa couplings of Higgs to all fermions Coupling of Higgs to Z,W

$$H(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ |s(x)| \end{pmatrix}$$

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In the γ -gauge, the master action reads

$$L = -e \left[\begin{array}{l} \frac{1}{2e} \left[-\dot{\phi}_\gamma^2 + \dot{s}_\gamma^2 + \frac{\kappa^2}{6} (\phi_\gamma^2 - s_\gamma^2) (\dot{\alpha}_1^2 + \dot{\alpha}_2^2) \right] \\ \left(b\phi_\gamma^4 + c(s_\gamma^2 - \xi^2 \phi_\gamma^2)^2 \right) + \frac{1}{2} (\phi_\gamma^2 - s_\gamma^2) v(\alpha_1, \alpha_2) \\ + \rho_r + \rho_f |s_\gamma| + \frac{1}{2} \rho_w s_\gamma^2 \end{array} \right]$$

$$a_E^2 = \frac{\kappa^2}{6} (\phi_\gamma^2 - s_\gamma^2)$$

Scale factor in the Einstein gauge

According to our attractor theorem all these fields must start at 0 at the Big Bang, and then evolve. So they must start with comparable energy scales.

We have ALL the analytic solutions. We see that the late time evolution of ϕ and s is controlled by the quartic parameters b and c

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$$V(s) = \frac{1}{2}K\phi^2 + cs^4 \quad V(\phi) = \frac{1}{2}K\phi^2 - b\phi^4,$$

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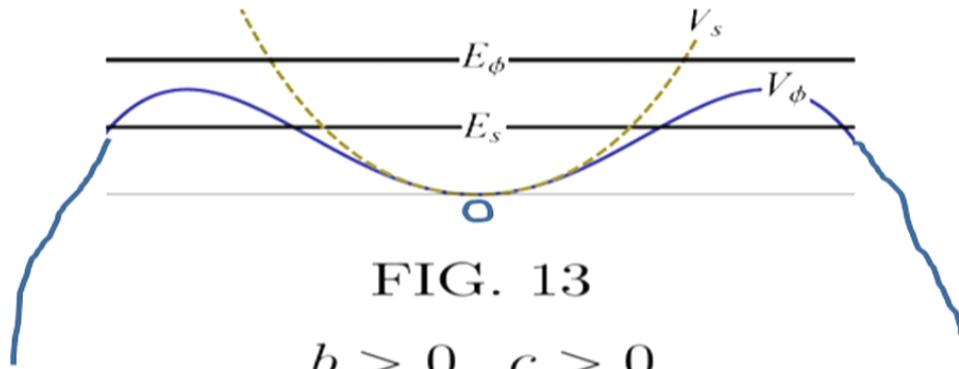


FIG. 13

$$b > 0, \quad c > 0,$$

$$\phi(\tau) = \left(\frac{(E + \rho_0)}{b} \right)^{1/4} \frac{\operatorname{sn}\left(\frac{\tau+\delta}{T_\phi} | m_\phi\right)}{1 + \operatorname{cn}\left(\frac{\tau+\delta}{T_\phi} | m_\phi\right)}, \quad m_\phi = \frac{1}{2} + KT_\phi^2$$

$$s(\tau) = \sqrt{\frac{1 - K^2 T_s^4}{8c T_s^2}} \frac{\operatorname{sn}(\frac{\tau}{T_s} | m_s)}{\operatorname{dn}(\frac{\tau}{T_s} | m_s)}, \quad m_s = \frac{1}{2} (1 - KT_s^2)$$

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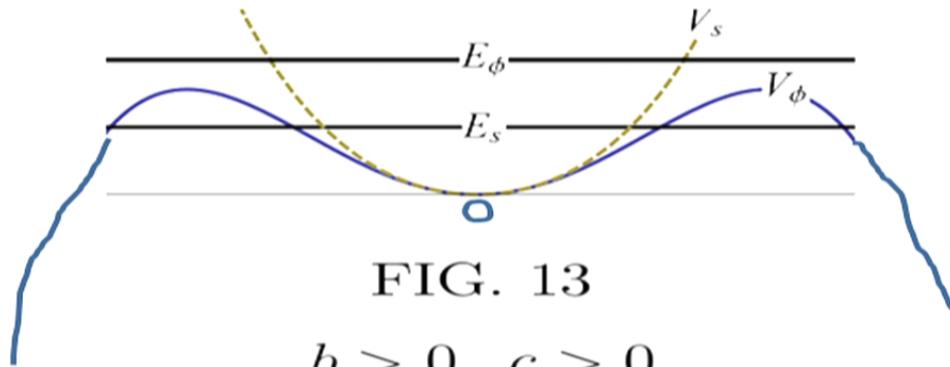


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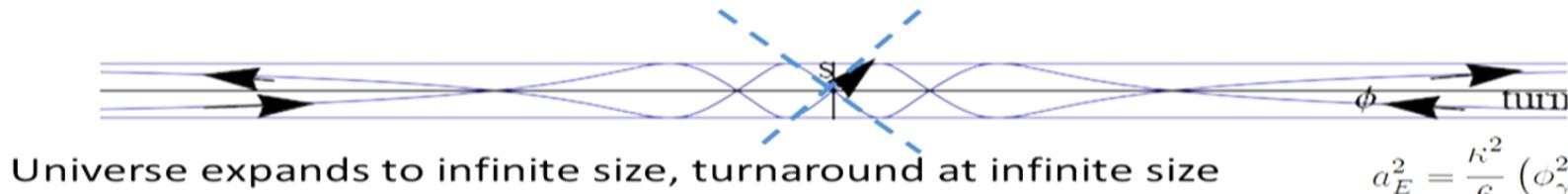
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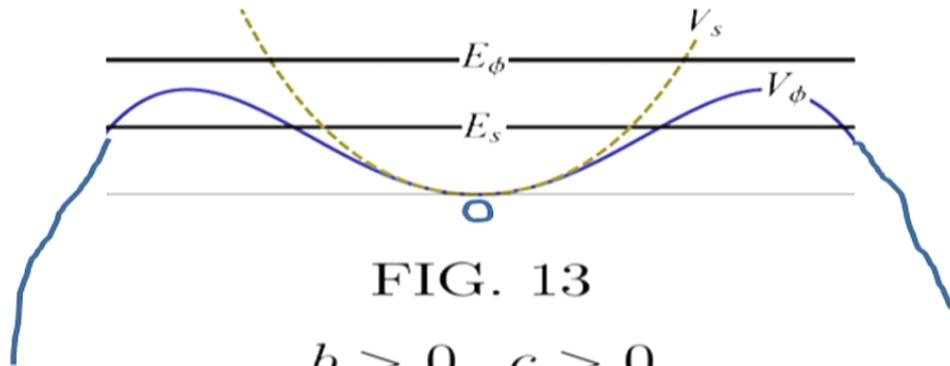


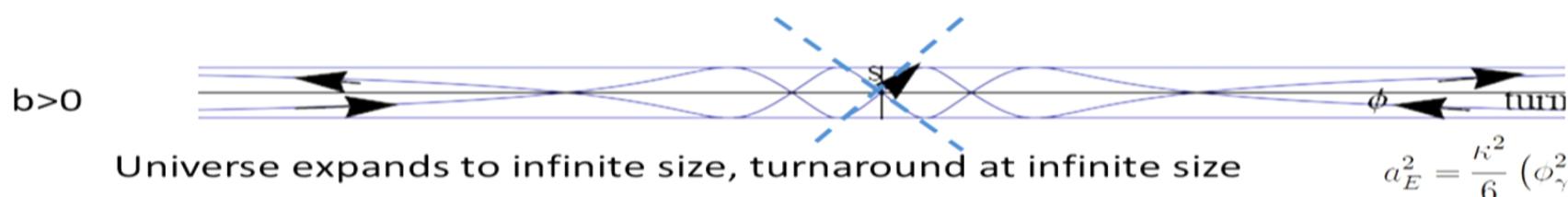
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$$b > 0, \quad c > 0,$$

$$\left| \frac{s_{amplitude}}{\phi_{amplitude}} \right| \stackrel{K \approx 0}{\sim} \left(\frac{b}{c} \frac{E}{E + \rho_r} \right)^{1/4} \sim 10^{-\frac{120}{4}} \sim 10^{-30}$$

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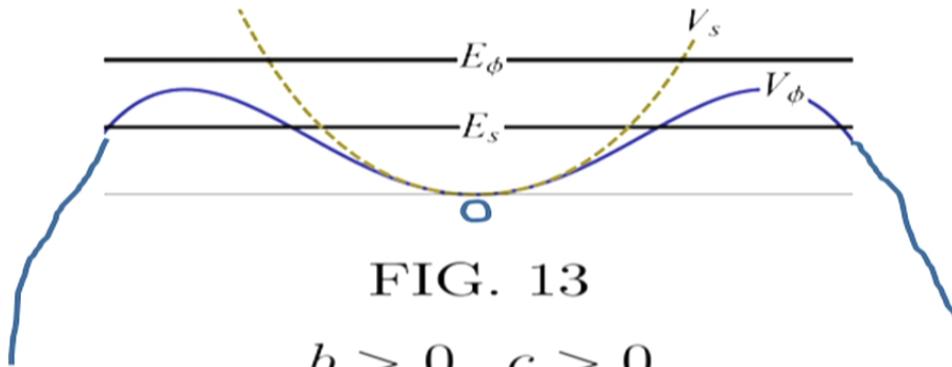


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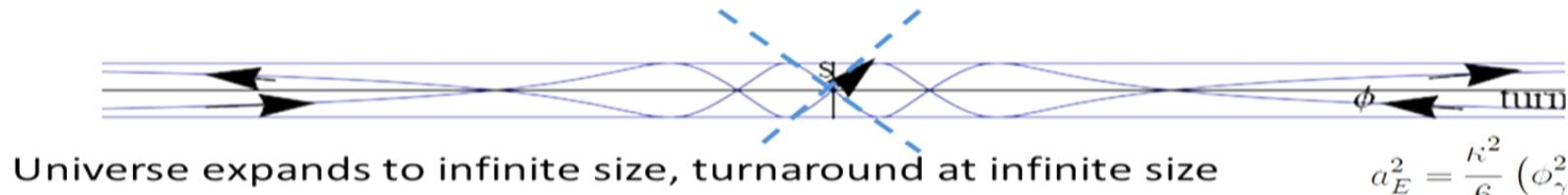
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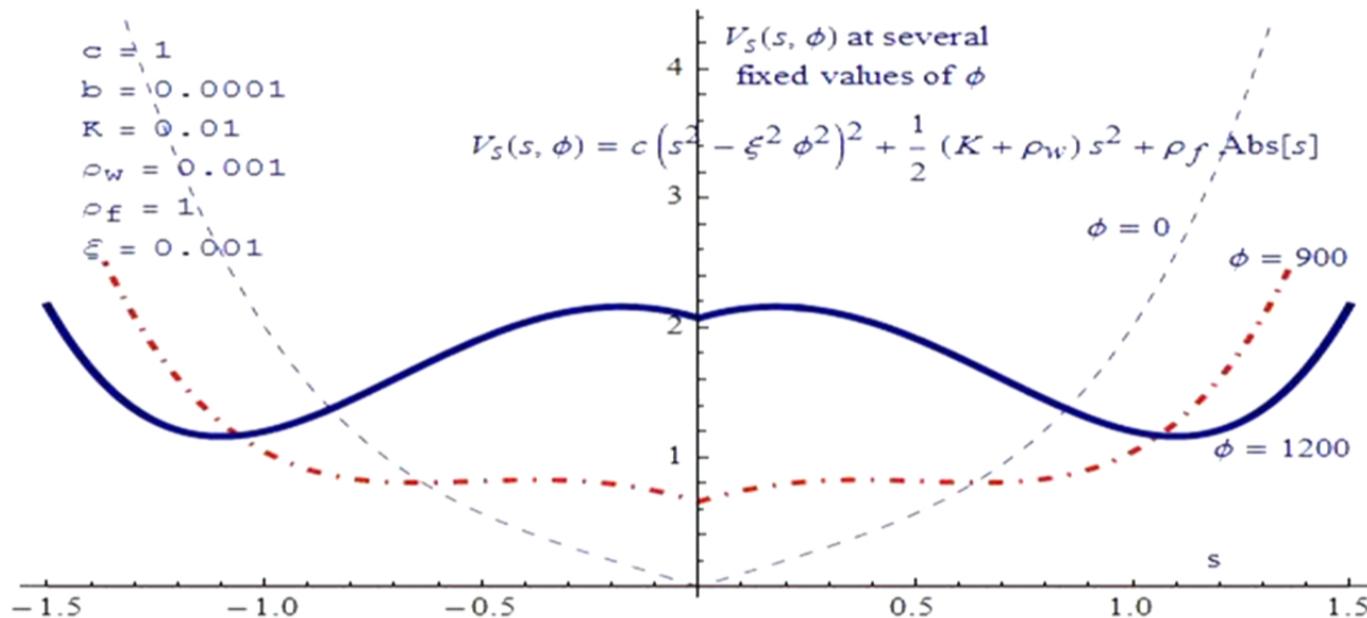
So, the hierarchy of electroweak/ Planck scales seems to be connected to the cosmological evolution of the Higgs, and related to the smallness of dark energy.

$$b > 0$$



$$a_E^2 = \frac{\kappa^2}{6} (\phi_\gamma^2 - s_\gamma^2)$$

The evolution of the effective potential for the Higgs, assuming a slowly varying ϕ



At late times the Mexican hat develops and the Higgs freezes at its lowest energy state to a non-zero value. The maximum amplitude of the Higgs is still limited by the previous argument (DE)