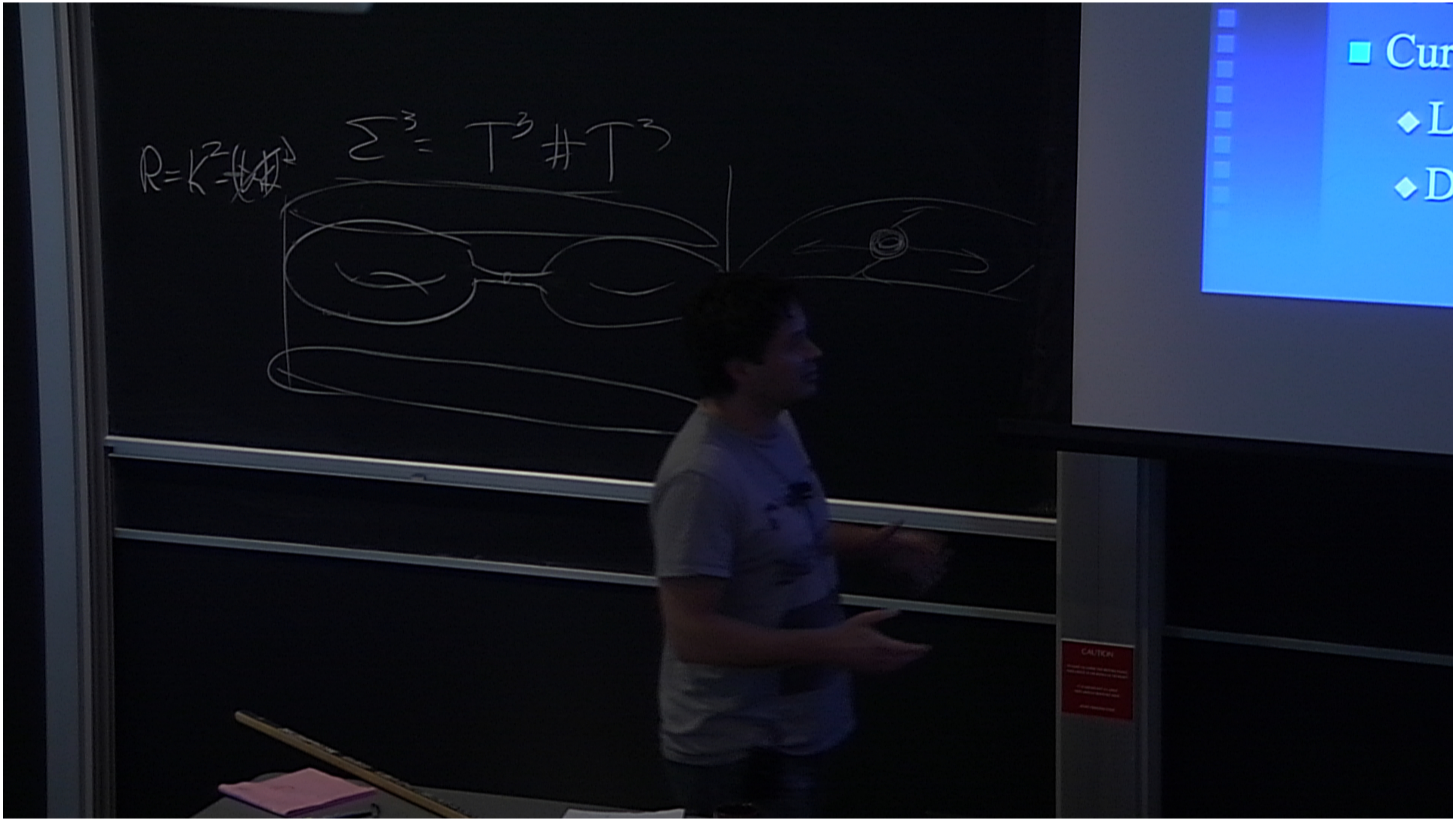
Title: Is Nothing Sacred? The Cosmological Pay Off from Breaking Lorentz and Diffeormorphism Invariance

Date: May 11, 2012 02:30 PM

URL: http://pirsa.org/12050074

Abstract: I show how the local Lorentz and/or diffeomorphism invariances may be broken by a varying c, softly or harshly, depending on taste. Regardless of the fundamental implications of such dramas, these symmetry breakings may be of great practical use in cosmology. They may solve the horizon and flatness problems. A near scale-invariant spectrum of fluctuation may arise, even without inflation. Distinct imprints may be left, teaching us an important lesson: our foundations may be flimsier than we like to think.


$\Sigma^3 = T^3 \# T^3$

The cosmological payoff of breaking Lorentz and/or diffeomorphism invariance

João Magueijo
2012
Imperial College, London

Let's not get tangled…

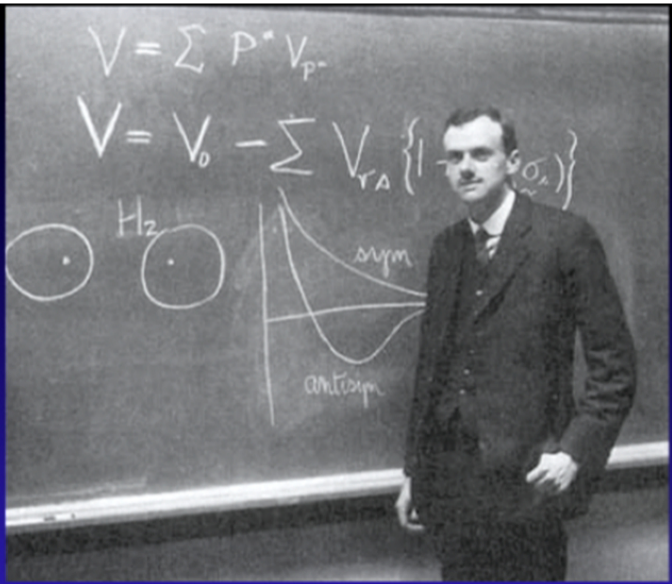
- Lorentz invariance (flat space):
  - No preferred frames
  - Constancy of speed of light
- Curved space-time:
  - Local Lorentz invariance
  - Diffeomorphism invariance

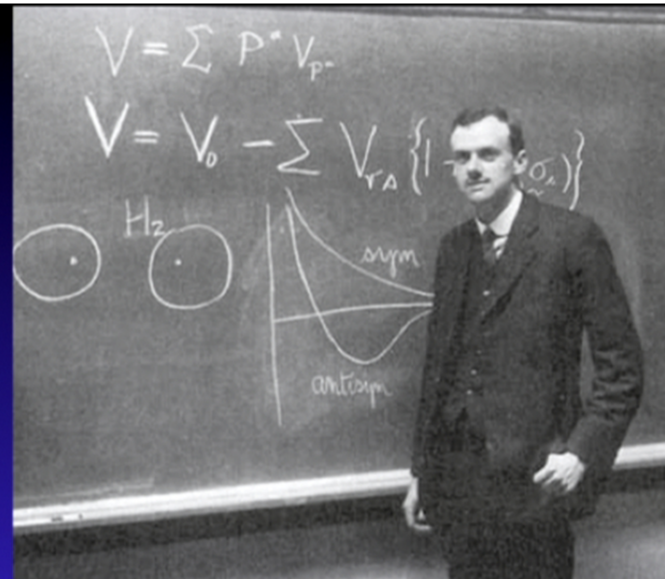
In destroying Lorentz invariance, we can mix and match at will

- Lorentz invariance (flat space):
  - No preferred frames ✗ ✓ ✗
  - Constancy of speed of light ✓ ✗ ✗
- Curved space-time:
  - Local Lorentz invariance ✗ ✓ ✗
  - Diffeomorphism invariance ✓ ✗ ✗

Emphasis of this talk:

- Varying speed of light theories
  - With/out preferred local Lorentz frames
  - With/out diffeomorphism invariance
- Focus on applications to cosmology:
  - Horizon and flatness problem
  - Primordial fluctuations

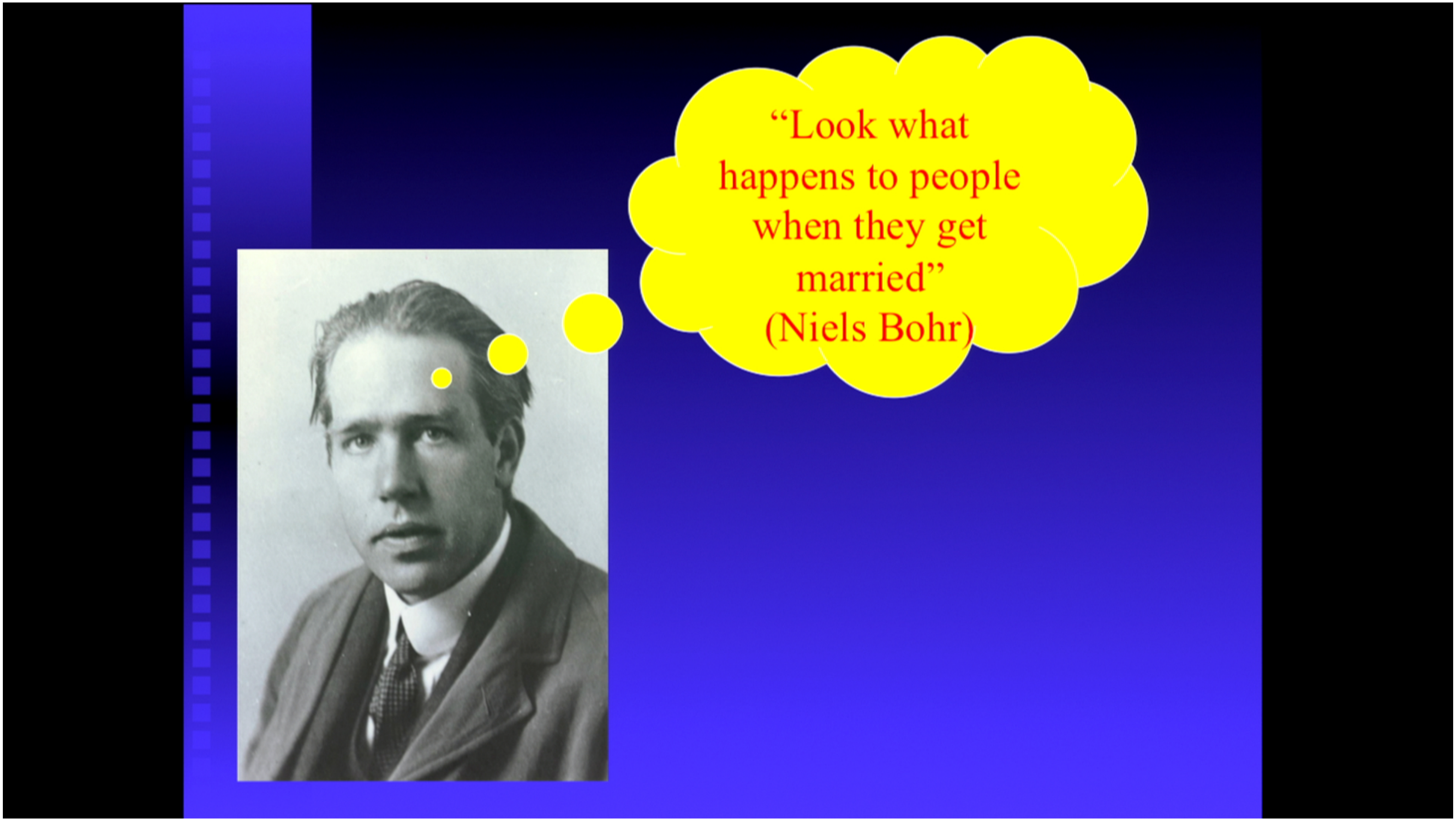

H2
sym
antisym



One field of work in which there has been too much speculation is cosmology. (…) [Its] models are probably all wrong. It is usually assumed that the laws of nature have always been the same as they are now. There is no justification for this. (…) in particular quantities which are considered to be constants of nature may be varying with cosmological time.

Paul Dirac

Dirac and Manci on their honeymoon, Brighton, January 1937


"Look what happens to people when they get married"
(Niels Bohr)

"Look what happens to people when they get married"
(Niels Bohr)

"The pseudo-science of invertebrate cosmythology"
Dingle, Nature, 1937

By now some varying-constants have become run of the mill

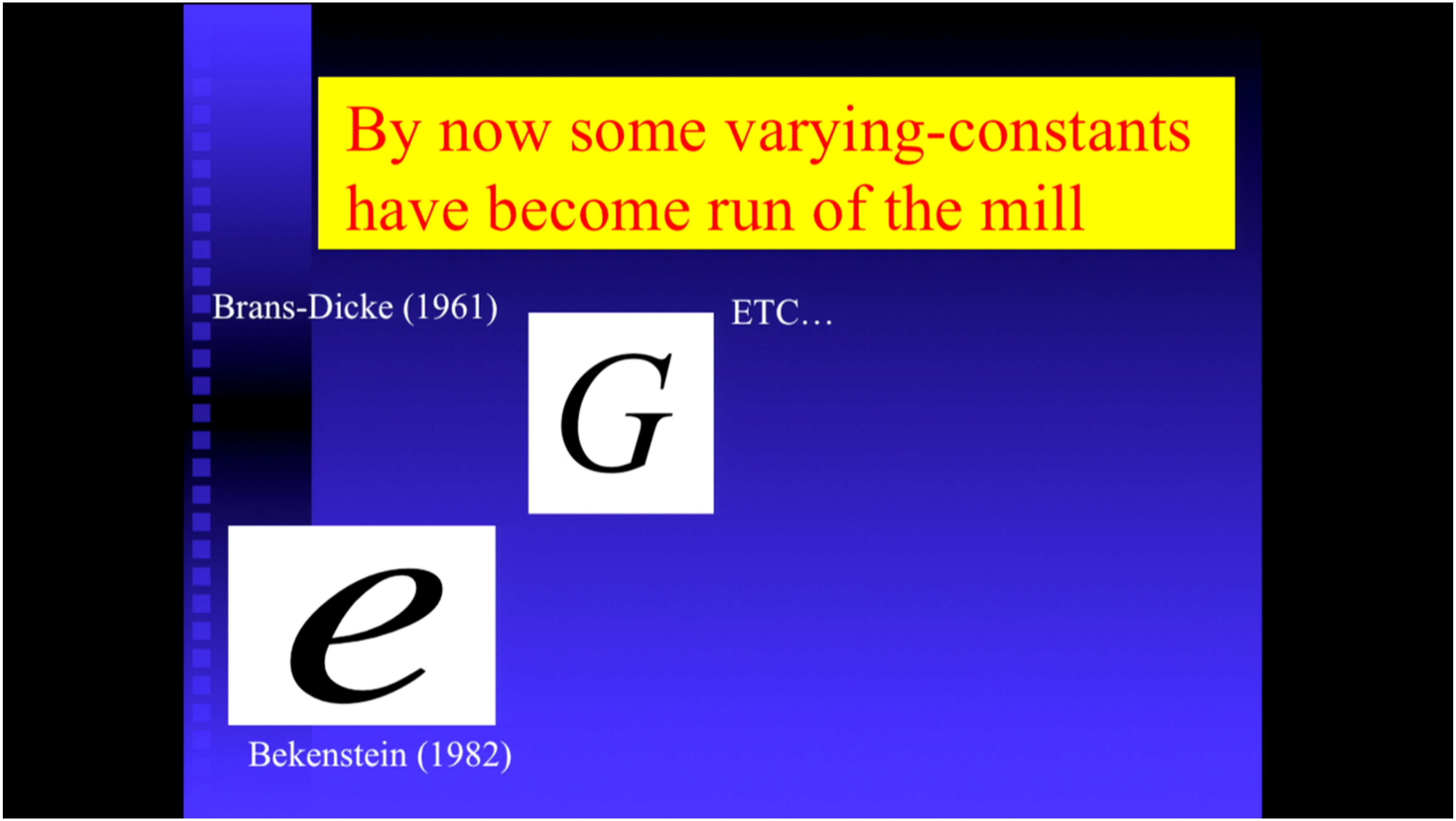
Brans-Dicke (1961)

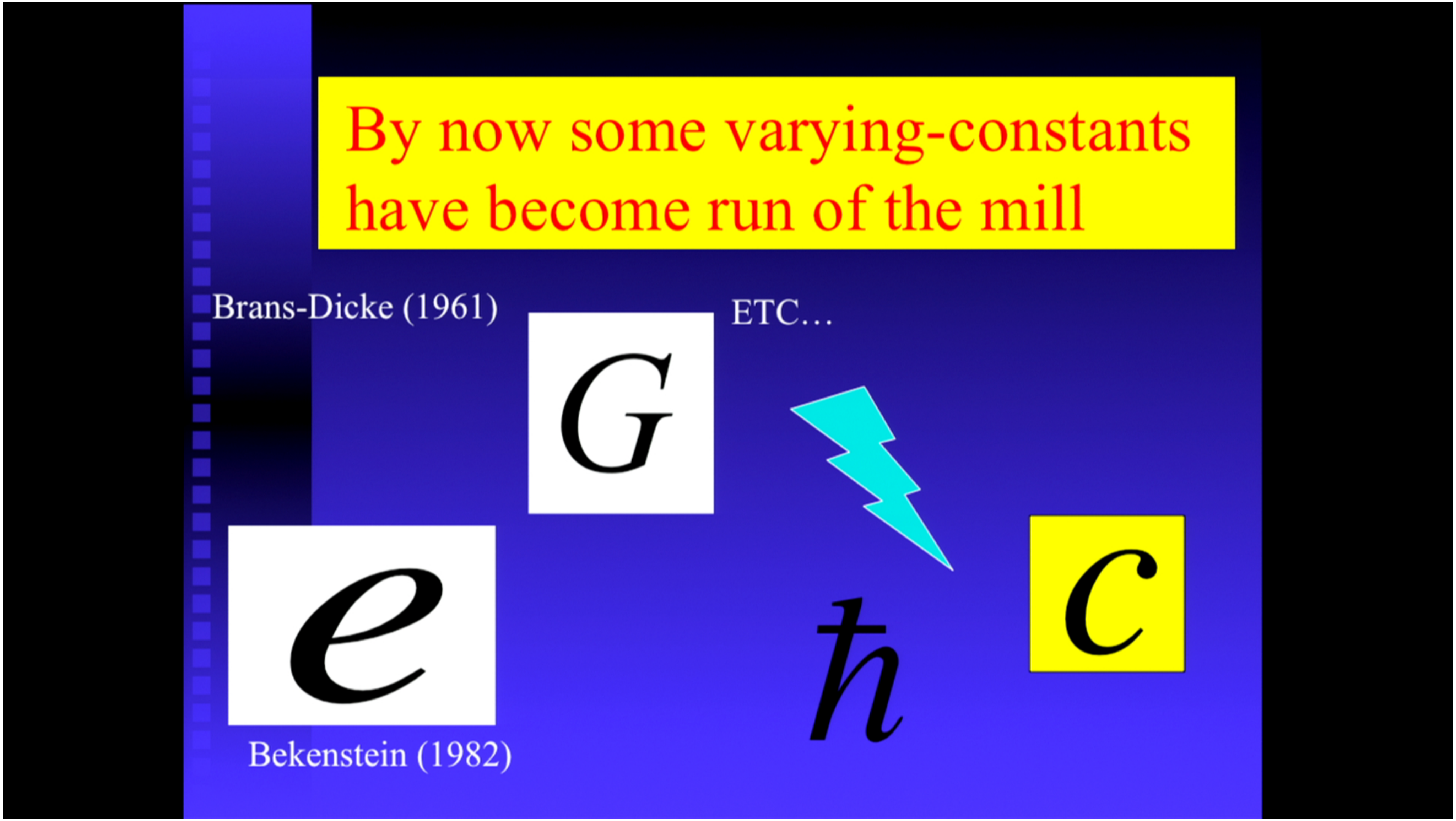
G

ETC…

e

Bekenstein (1982)


By now some varying-constants have become run of the mill
Brans-Dicke (1961)
G
ETC…
e
Bekenstein (1982)


By now some varying-constants have become run of the mill
Brans-Dicke (1961)
ETC…
G
e
ħ
c
Bekenstein (1982)


By now some varying-constants have become run of the mill
Brans-Dicke (1961)
ETC…
G
e
Bekenstein (1982)
ħ
c
Potential damage to Lorentz invariance guaranteed

Varying c theories

[JM, Rept. Prog. Phys. 66, 2025]

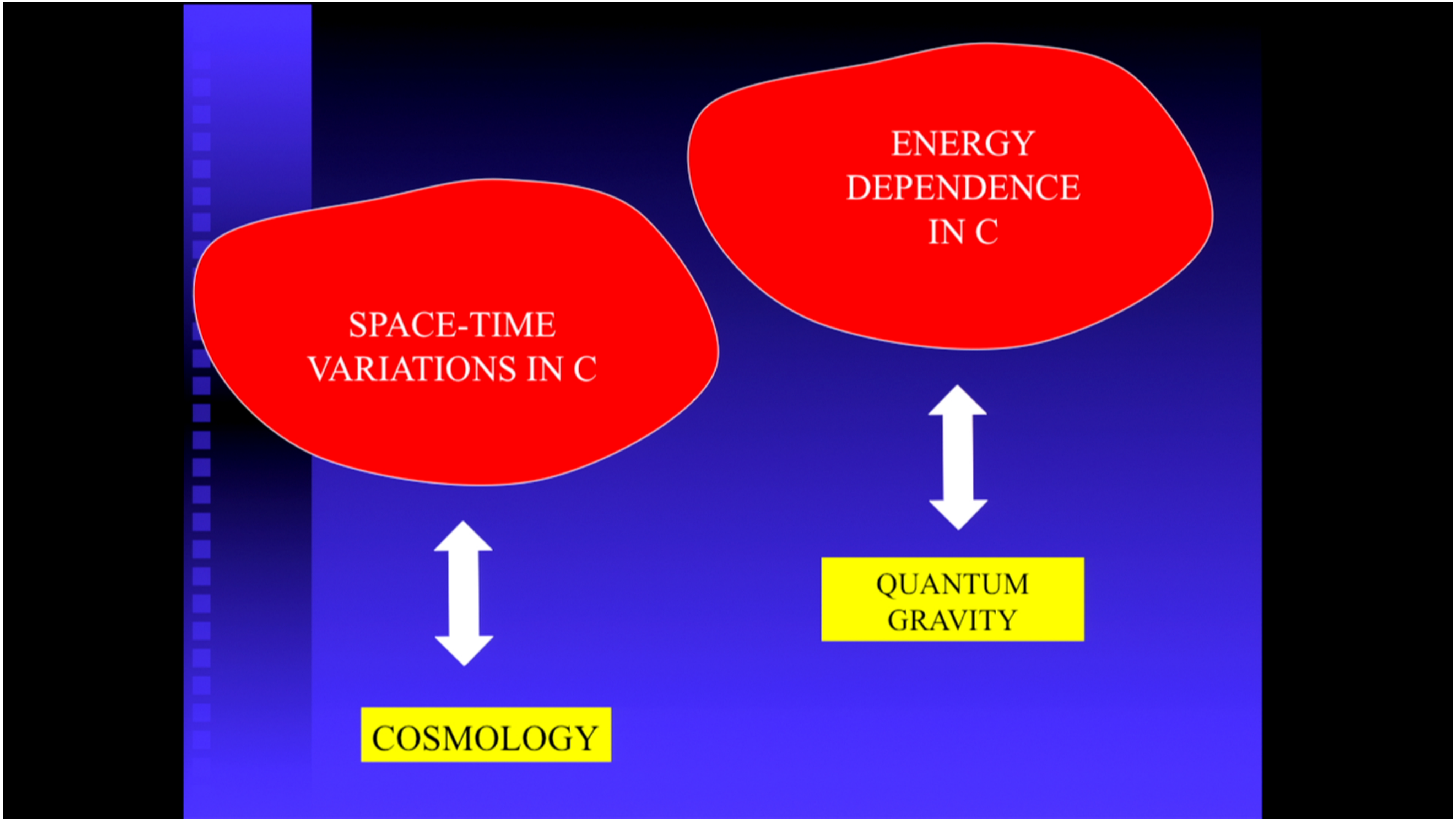
c(x,t)

- Covariant and Lorentz invariant
  [Moffat,Magueijo, etc, etc]
- Bimetric theories [Moffat, Clayton, Drummond, etc, etc]
- Preferred frame [Albrecht, Magueijo,Barrow,etc,etc]

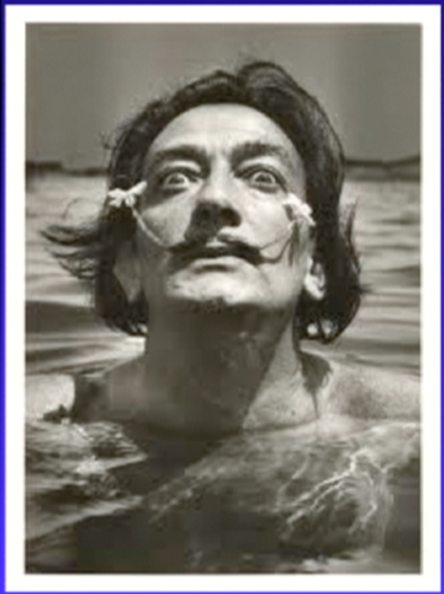
c(E)

- Deformed dispersion relations

  [Amelino-Camelia, Mavromatos, Magueijo & Smolin, etc, etc]


ENERGY
DEPENDENCE
IN C
SPACE-TIME
VARIATIONS IN C
QUANTUM
GRAVITY
COSMOLOGY

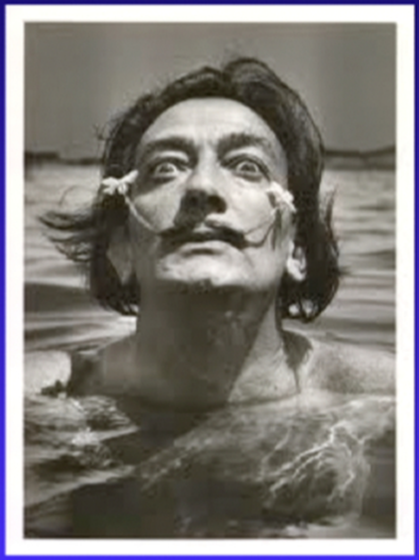
A whole range of options between total lunacy and suburban boredom



THE LUNACY CORNER:

- Hard breaking of diffeomorphisms and local Lorentz invariance

- Idea: while we're at it, let's break anything that moves…



The general recipe:

- Consider a preferred foliation (e.g. the cosmological frame)
- Apply a 3+1 ADM decomposition
- Mess it up in such a way that a diff invariant 4D theory cannot be re-constituted

THERE IS A PREFERRED FRAME IN THE UNIVERSE!

- The cosmological frame is an obvious preferred frame in the universe.
- It doesn't need to partake in the formulation of the physical laws.
- But it can be folded into theories which do break foliation invariance.
- [Exercise for the student: find the flaw in this nice argument.]

THERE IS A PREFERRED FRAME IN THE UNIVERSE!

- The cosmological frame is an obvious preferred frame in the universe.
- It doesn't need to partake in the formulation of the physical laws.
- But it can be folded into theories which do break foliation invariance.
- [Exercise for the student: find the flaw in this nice argument.]

THERE IS A PREFERRED FRAME IN THE UNIVERSE!

- The cosmological frame is an obvious preferred frame in the universe.
- It doesn't need to partake in the formulation of the physical laws.
- But it can be folded into theories which do break foliation invariance.
- [Exercise for the student: find the flaw in this nice argument.]

The general recipe:

- Consider a preferred foliation (e.g. the cosmological frame)
- Apply a 3+1 ADM decomposition
- Mess it up in such a way that a diff invariant 4D theory cannot be re-constituted

Some concrete examples of this (poisonous) recipe in action:

- Horava-Liftschiz theory
- "Brute force" Varying Speed of Light
  (Albrecht, JM, 1999, Barrow, JM, 1999,2000)

$$S = \int \left[\mathscr{T}(K) - \mathscr{V}(g) \right] \sqrt{g}\, N \, \mathrm{d}^3x \, \mathrm{d}t$$

$$S_K = \frac{2}{\kappa^2} \int dt\, d^D\mathbf{x}\, \sqrt{g} N \left(K_{ij}K^{ij} - \lambda K^2 \right)$$

$$\begin{aligned} \mathscr{V}(g) = {} & g_0\, \zeta^6 + g_1\, \zeta^4\, R + g_2\, \zeta^2\, R^2 + g_3\, \zeta^2\, R_{ij}R^{ij} \\ & + g_4\, R^3 + g_5\, R(R_{ij}R^{ij}) + g_6\, R^i{}_j R^j{}_k R^k{}_i \\ & + g_7\, R\nabla^2 R + g_8\, \nabla_i R_{jk}\, \nabla^i R^{jk}, \end{aligned}$$

$$\begin{aligned} \frac{1}{\sqrt{g}}\, \partial_t(\sqrt{g}\, \pi^{ij}) = {} & -2N \left\{ (K^2)^{ij} - K K^{ij} + \xi K K^{ij} \right\} \\ & + \frac{N}{2} \mathscr{T}(K)\, g^{ij} + (\nabla_m N^m)\, \pi^{ij} + [\mathscr{L}_{\vec{N}} \pi]^{ij} \\ & + \frac{N}{\sqrt{g}} \frac{\delta S_{\mathscr{V}}}{\delta g_{ij}}. \end{aligned}$$

$$\mathcal{H} = h^{1/2}\left[-{}^{(3)}R + h^{-1}\left(\Pi^{ij}\Pi_{ij} - \frac{1}{2}\Pi^2\right)\right]$$

$$\kappa_{ij} = \frac{1}{c}\frac{\dot{h}_{ij}}{2}$$

$$\Pi_{ij} = \frac{\partial \mathcal{L}}{\partial \dot{h}^{ij}} = h^{1/2}(\kappa_{ij} - \kappa h_{ij})$$

$$\frac{1}{c}\dot{h}_{ij} = 2h^{-1/2}\left(\Pi_{ij} - \frac{1}{2}h_{ij}\Pi\right)$$

$$\begin{aligned}\frac{1}{c^2}\frac{\sqrt{h}}{2}[\dot{h}_{ij} - \dot{h}h_{ij}]^{\cdot} \;\;=\;\; & -h^{1/2}\left({}^{(3)}R_{ij} - \frac{1}{2}h_{ij}\,{}^{(3)}R\right) \\ & +\frac{1}{2}h^{-1/2}h_{ij}\left(\Pi_{kl}\Pi^{kl} - \frac{1}{2}\Pi^2\right) \\ & -2h^{-1/2}\left(\Pi_i^k\Pi_{kj} - \frac{1}{2}\Pi\Pi_{ij}\right)\end{aligned}$$

Lessons to be drawn:

- Equations would look different in a different frame.
- Violation/modification of:
  - Bianchi identities
  - Local Stress-Energy conservation
  - (Time) diffeomorphism invariance

So what?....

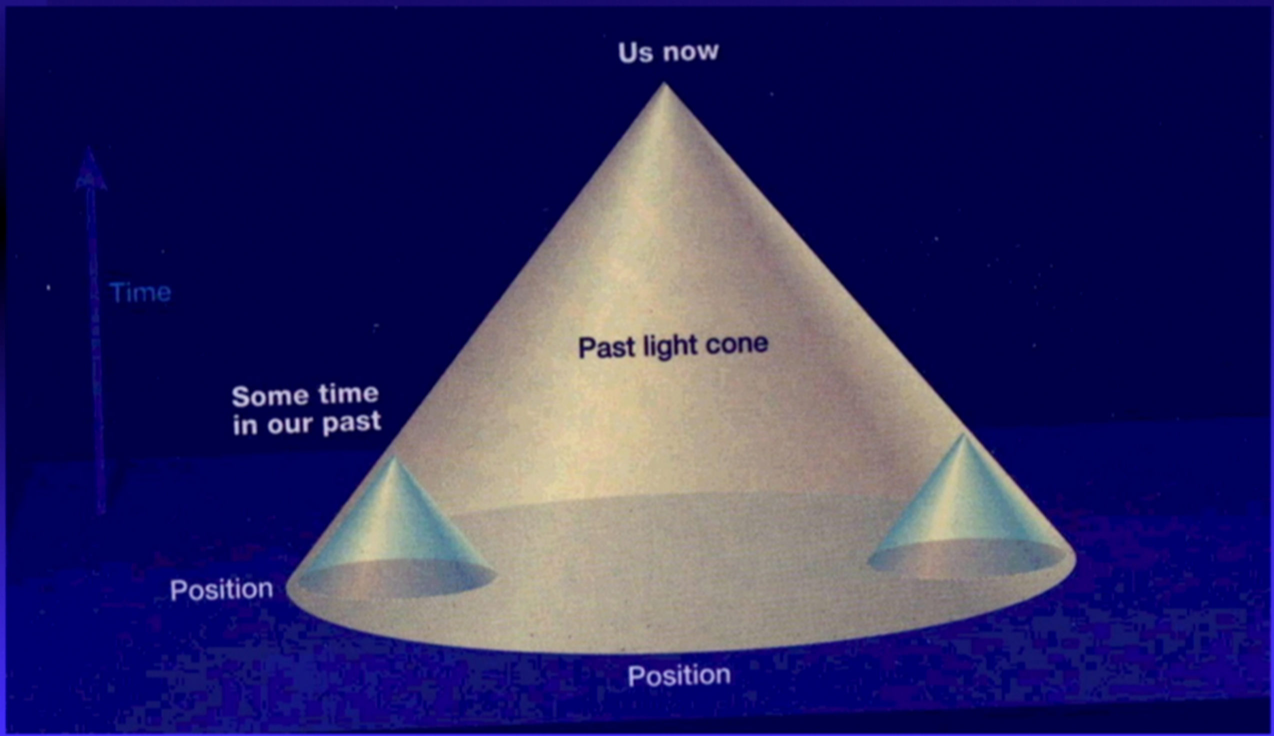
- No problem if it only affects phenomena at energy scales above the Planck scale (and it may help quantizing gravity)
- No problem if it all happens in the very early Universe (and it may help solve the cosmological problems of Big Bang theory)

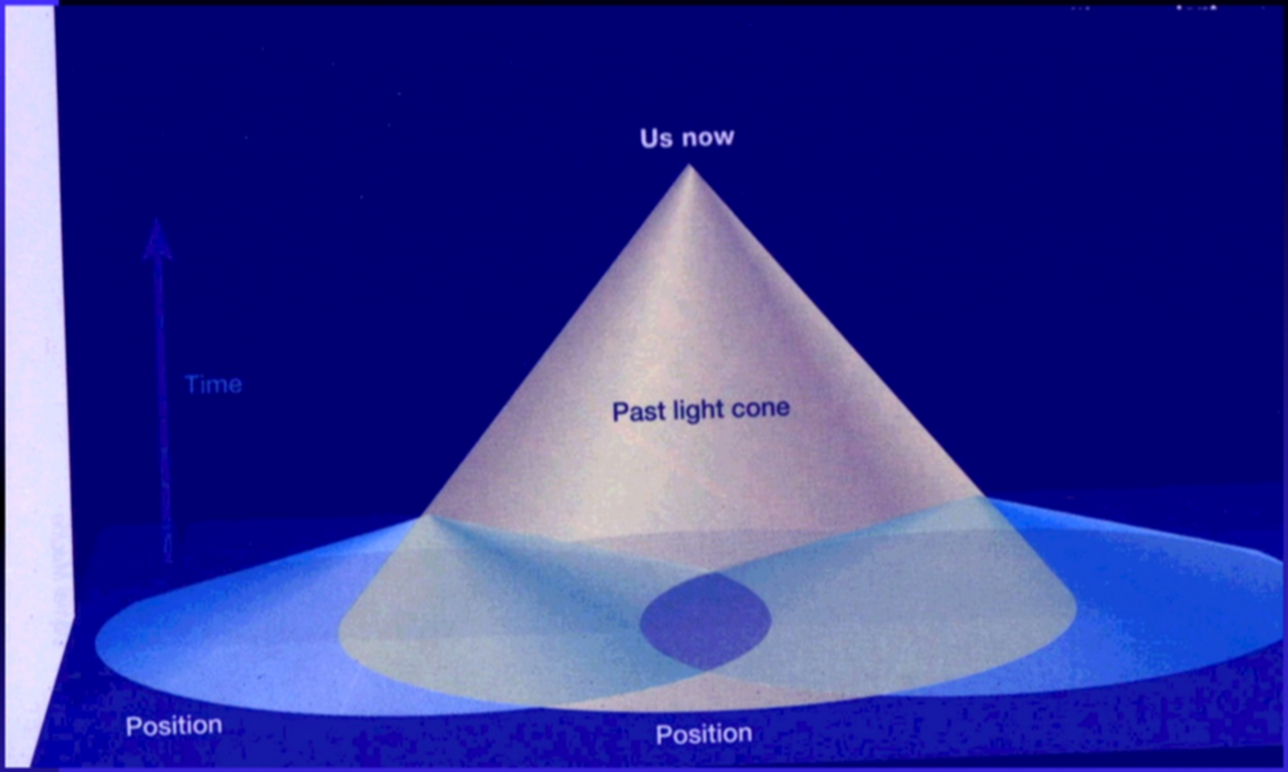
The cosmological problems

- Horizon problem
- Homogeneity problem
- Flatness problem
- Cosmological constant problem
- Singularity problem
- Structure formation
- ….

The tip of the iceberg

A non-inflationary solution to the horizon problem





Need a drop of at least 32 orders of magnitude in c, if this happened as a phase transition at the Planck time.

THE FLATNESS PROBLEM:

$\rho > \rho_c \Leftrightarrow K = 1$

BIG CRUNCH

$\rho < \rho_c \Leftrightarrow K = -1$

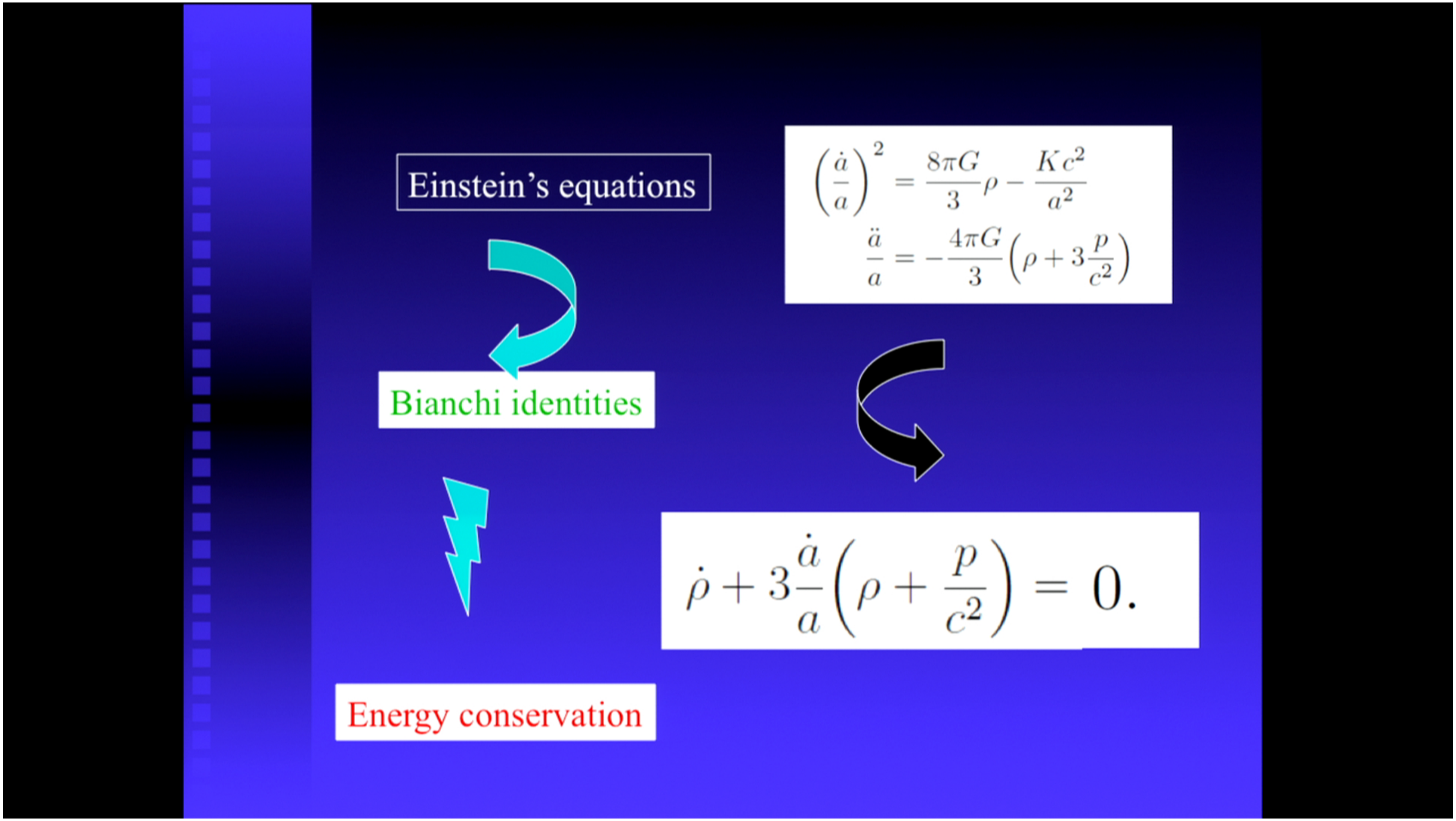
EMPTY

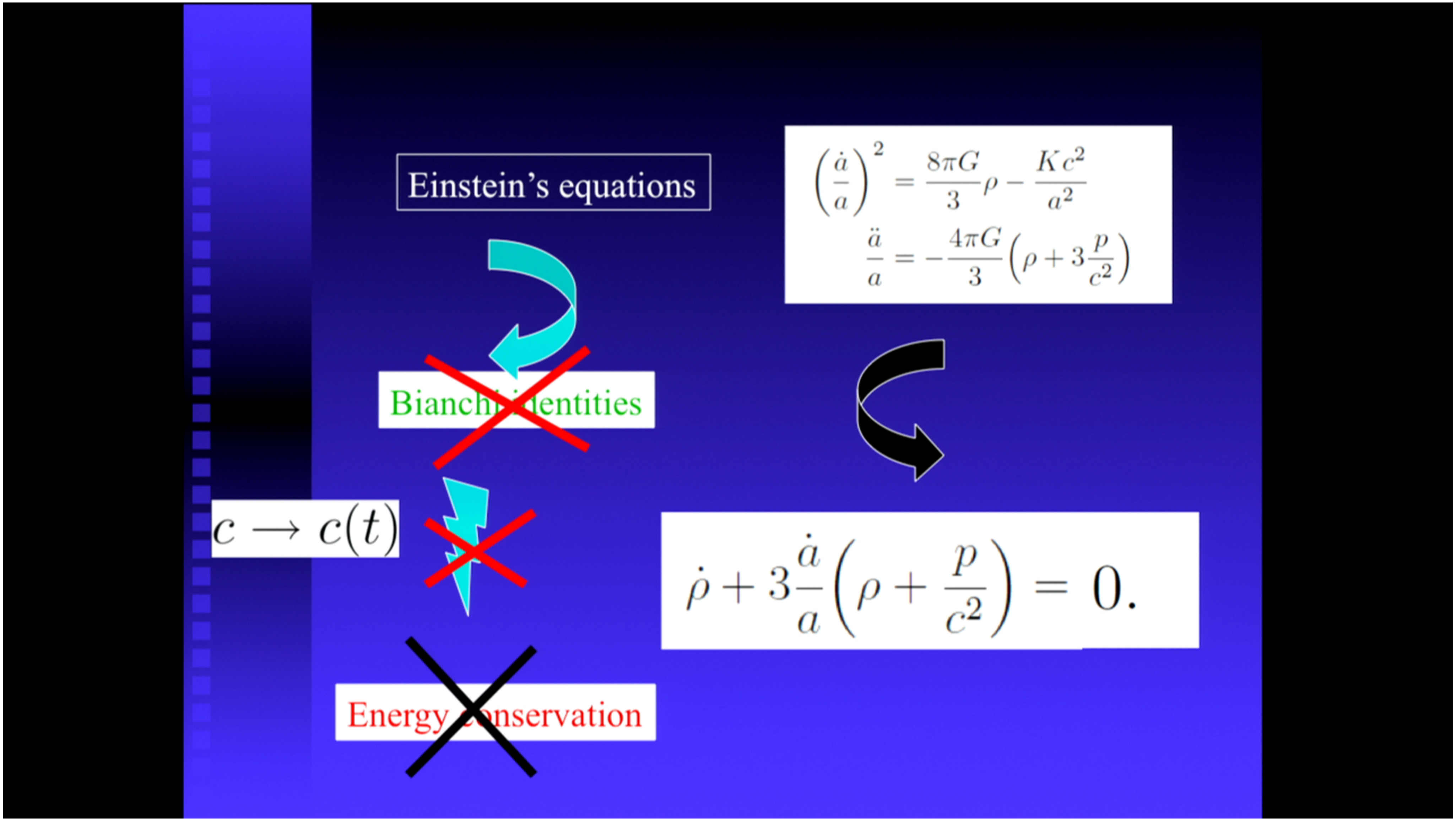
$\rho = \rho_c \Leftrightarrow K = 0$

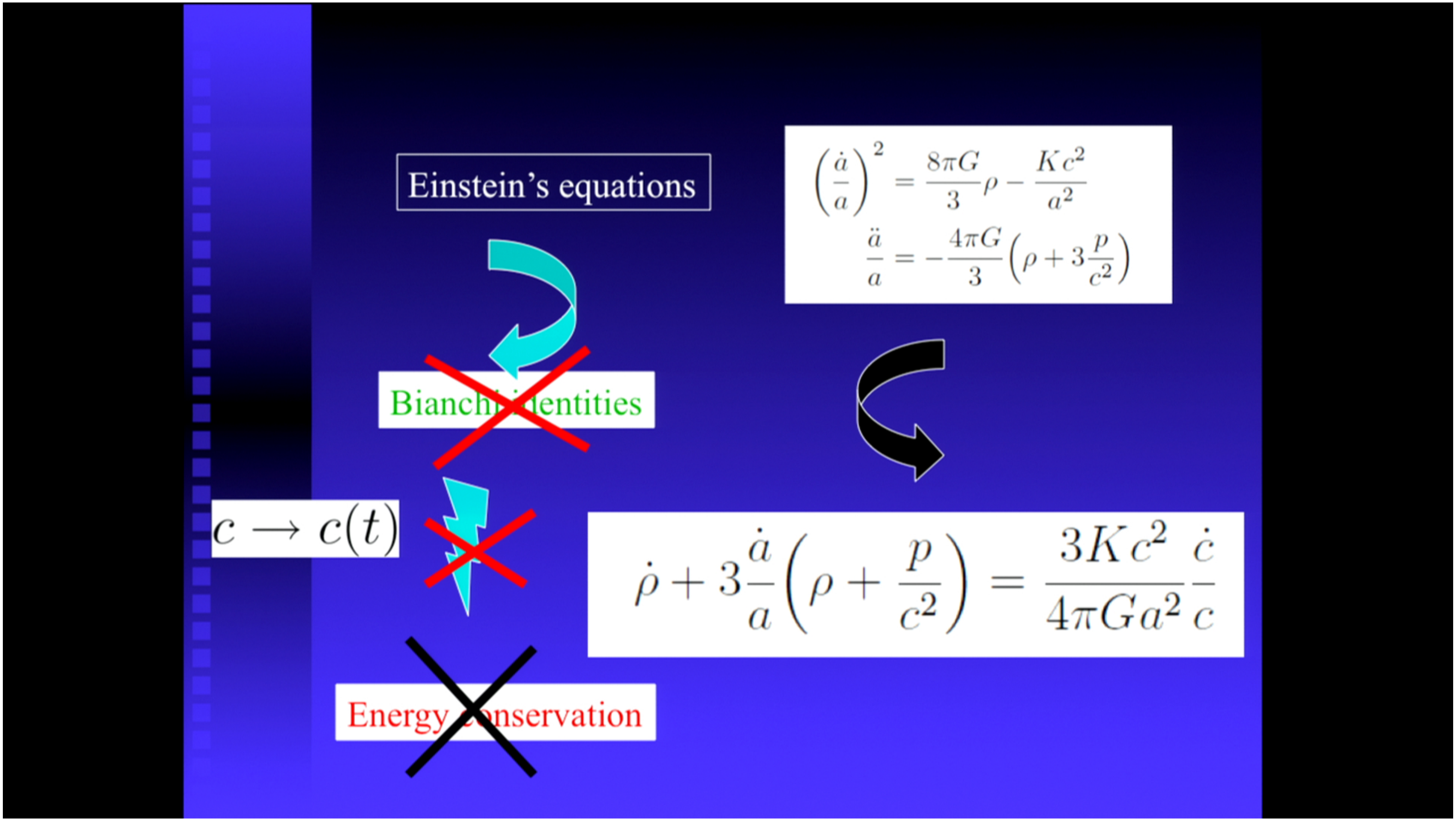
FLAT

Healthy

Flatness is highly unstable


Einstein's equations
$$\left(\frac{\dot a}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{Kc^2}{a^2}$$
$$\frac{\ddot a}{a} = -\frac{4\pi G}{3}\left(\rho + 3\frac{p}{c^2}\right)$$
Bianchi identities
$$\dot\rho + 3\frac{\dot a}{a}\left(\rho + \frac{p}{c^2}\right) = 0.$$
Energy conservation


Einstein's equations
$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{Kc^2}{a^2}$
$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + 3\frac{p}{c^2}\right)$
Bianchi identities
$c \to c(t)$
$\dot{\rho} + 3\frac{\dot{a}}{a}\left(\rho + \frac{p}{c^2}\right) = 0.$
Energy conservation

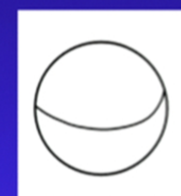

Einstein's equations
$$\left(\frac{\dot a}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{Kc^2}{a^2}$$
$$\frac{\ddot a}{a} = -\frac{4\pi G}{3}\left(\rho + 3\frac{p}{c^2}\right)$$
Bianchi identities
$c \to c(t)$
$$\dot\rho + 3\frac{\dot a}{a}\left(\rho + \frac{p}{c^2}\right) = \frac{3Kc^2}{4\pi G a^2}\frac{\dot c}{c}$$
Energy conservation

A non-inflationary solution to the horizon problem

$$\dot{\rho} + 3\frac{\dot{a}}{a}\left(\rho + \frac{p}{c^2}\right) = \frac{3Kc^2}{4\pi G a^2}\frac{\dot{c}}{c}$$

$$\frac{\dot{c}}{c} < 0$$

$$[\rho > \rho_c \Leftrightarrow K = 1] \Rightarrow \dot{\rho} < 0$$



$$[\rho < \rho_c \Leftrightarrow K = -1] \Rightarrow \dot{\rho} > 0$$

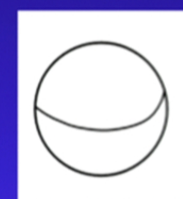
Need a drop of at least 32 orders of magnitude in c, if this happened as a phase transition at the Planck time.

A non-inflationary solution to the horizon problem

$$\dot{\rho} + 3\frac{\dot{a}}{a}\left(\rho + \frac{p}{c^2}\right) = \frac{3Kc^2}{4\pi G a^2}\frac{\dot{c}}{c}$$

$$\frac{\dot{c}}{c} < 0$$

$$[\rho > \rho_c \Leftrightarrow K = 1] \Rightarrow \dot{\rho} < 0$$



$$[\rho < \rho_c \Leftrightarrow K = -1] \Rightarrow \dot{\rho} > 0$$

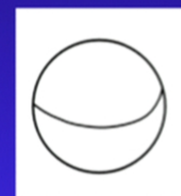
Need a drop of at least 32 orders of magnitude in c, if this happened as a phase transition at the Planck time.

A non-inflationary solution to the horizon problem

$$\dot{\rho} + 3\frac{\dot{a}}{a}\left(\rho + \frac{p}{c^2}\right) = \frac{3Kc^2}{4\pi G a^2}\frac{\dot{c}}{c}$$

$$\frac{\dot{c}}{c} < 0$$

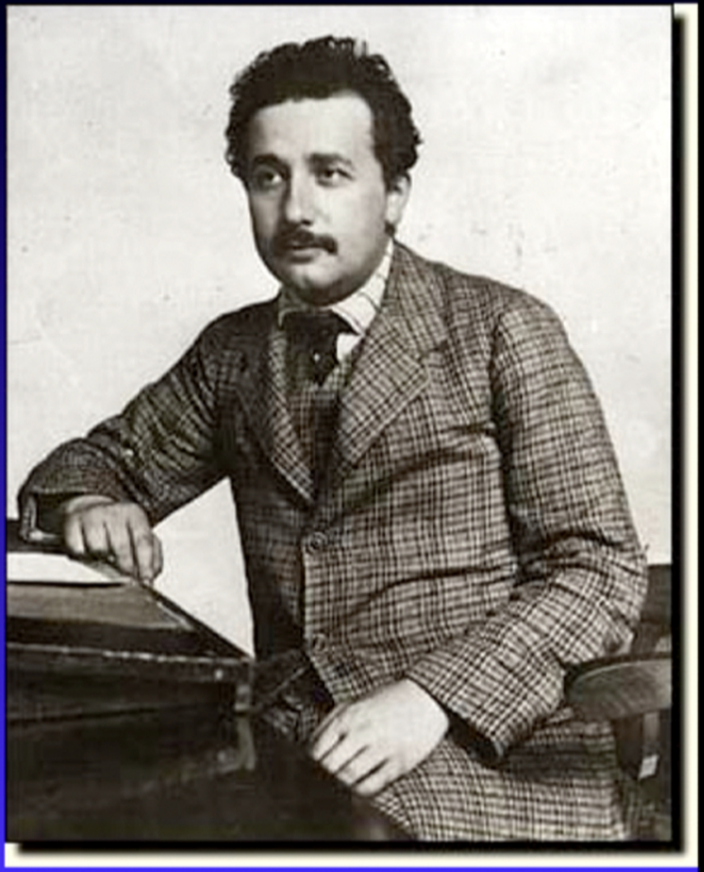
$$[\rho > \rho_c \Leftrightarrow K = 1] \Rightarrow \dot{\rho} < 0$$



$$[\rho < \rho_c \Leftrightarrow K = -1] \Rightarrow \dot{\rho} > 0$$

Need a drop of at least 32 orders of magnitude in c, if this happened as a phase transition at the Planck time.


Budweiser
Budvar
Czech Premium Lager
BUDWEISER



Weaknesses of this approach

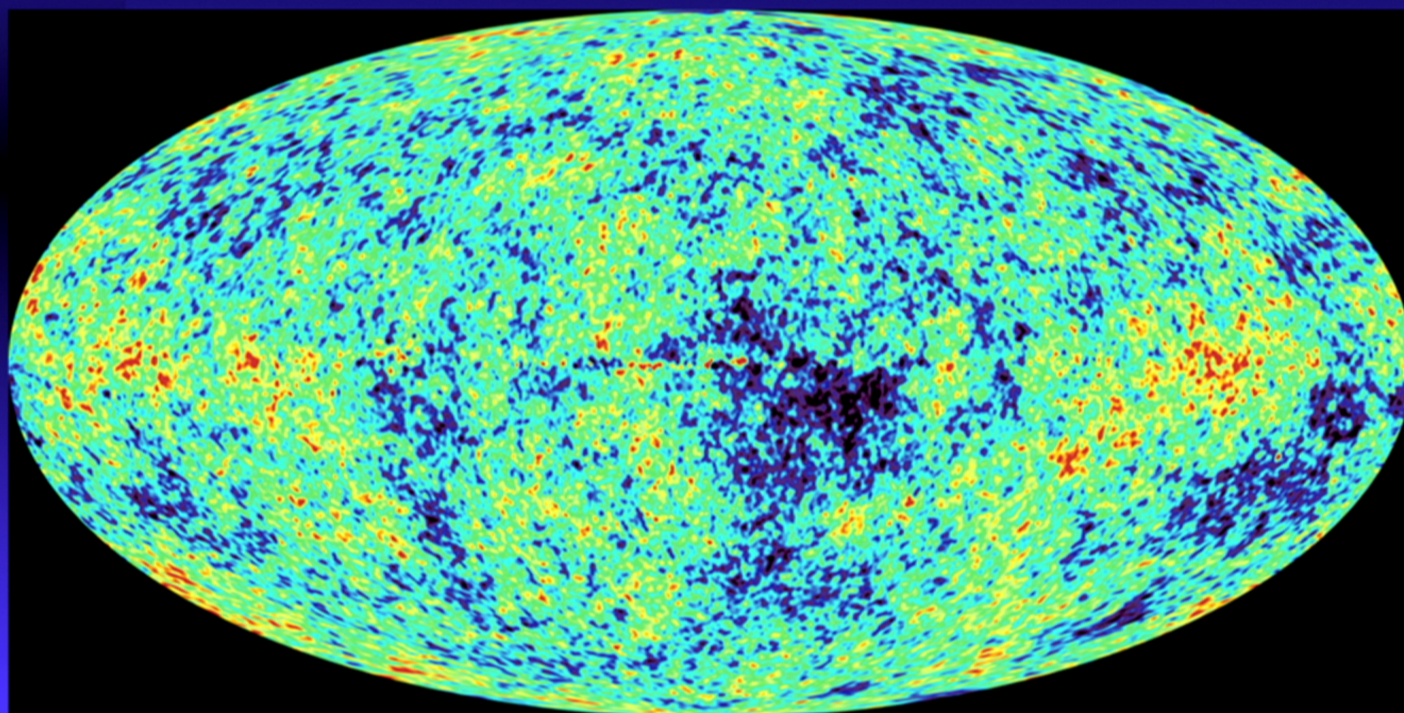
- Cosmology doesn't really fix the c(t) profile (could be phase transition or a power-law scenario).
- As soon as we go into perturbation theory the ambiguity in defining the preferred frame creeps in.

Let’s turn to the other end of the spectrum

- It is possible to have a VSL without extreme damage to Lorentz invariance
- Soft breaking, with a varying c, is a possibility

PRL 100, 231302, 2008;
CQG 25, 202002, 2008
PRD 79, 043525, 2009;
PRD 82, 043521, 2010.

And really: who cares about the flatness and horizon/homogeneity problem… Here's the real problem



The zero-th order "holy grail" of cosmology:

$$k^3|\zeta(k)|^2 = A^2\left(\frac{k}{k_c}\right)^{n_S-1}$$

- Near scale-invariance

$$n_S \sim 1$$

- Amplitude

$$A \sim 10^{-5}$$

Bimetric theories

A metric for gravity (Einstein frame):

$$g_{\mu\nu} \xrightarrow{\text{gravity}} S = \int dx^4 \sqrt{-g}\, R$$

A metric for matter (matter frame):

$$\hat{g}_{\mu\nu} \xrightarrow{\text{matter}} S_m = \int dx^4 \sqrt{-\hat{g}}\, L(\hat{g}_{\mu\nu}, \Psi, etc)$$

This is a rather conservative thing to do…

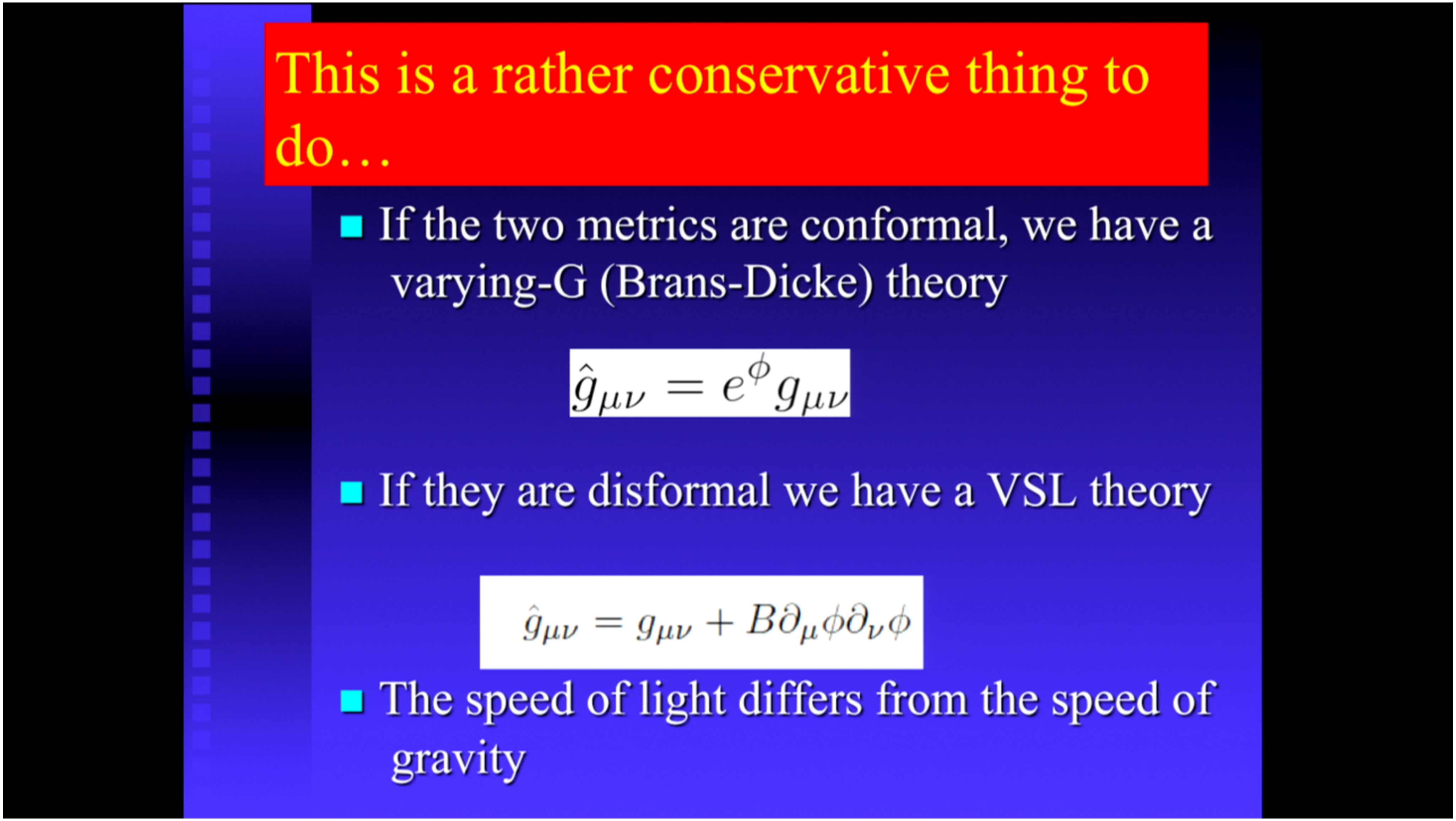
- If the two metrics are conformal, we have a varying-G (Brans-Dicke) theory

$$\hat{g}_{\mu\nu} = e^{\phi} g_{\mu\nu}$$

- If they are disformal we have a VSL theory

$$\hat{g}_{\mu\nu} = g_{\mu\nu} + B\partial_\mu\phi\partial_\nu\phi$$

- The speed of light differs from the speed of gravity


This is a rather conservative thing to do…
If the two metrics are conformal, we have a varying-G (Brans-Dicke) theory
$\hat{g}_{\mu\nu} = e^{\phi} g_{\mu\nu}$
If they are disformal we have a VSL theory
$\hat{g}_{\mu\nu} = g_{\mu\nu} + B\partial_\mu\phi\partial_\nu\phi$
The speed of light differs from the speed of gravity

The minimal bimetric VSL theory

$$\hat{g}_{\mu\nu} = g_{\mu\nu} + B\partial_\mu\phi\partial_\nu\phi$$

$$B = B(\phi) = const$$

$$S = \frac{M_{Pl}^2}{2}\int d^4x\sqrt{-g}\,R[g_{\mu\nu}] + \int d^4x\sqrt{-\hat{g}}\,\mathcal{L}_m[\hat{g}_{\mu\nu},\Phi_{Matt}] + S_\phi$$

$$S_\phi = ???$$

What sort of fluctuations come out in these theories?

- If we project onto the Einstein frame, we end up with the same formalism usually used for inflation, but…
- including a varying speed of sound.
- This is the so-called

  K-essence inflation (an inflaton with non-quadratic kinetic terms).

The tools of (K-essence) varying speed of sound:

$$\mathcal{L} = K(X) - V(\phi)$$

$$X = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi$$

$$\begin{aligned} p &= K - V \\ \rho &= 2XK_{,X} - K + V \end{aligned}$$

$$c_s^2 = \frac{K_{,X}}{K_{,X} + 2XK_{,XX}}$$

Check formulae with inflation, cuscaton, etc…

The tools of (K-essence) varying speed of sound:

$$\mathcal{L} = K(X) - V(\phi)$$

$$X = \frac{1}{2}\partial_\mu \phi \partial^\mu \phi$$

$$\begin{aligned} p &= K - V \\ \rho &= 2XK_{,X} - K + V \end{aligned}$$

$$c_s^2 = \frac{K_{,X}}{K_{,X} + 2XK_{,XX}}$$

Check formulae with inflation, cuscaton, etc…

How to compute fluctuations:

$$\zeta = \frac{v}{z} \qquad z = \frac{a}{c_s}$$

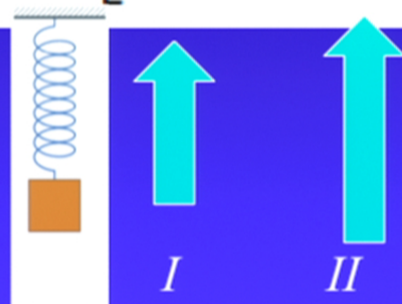
$$v'' + \left[c_s^2 k^2 - \frac{z''}{z}\right] v = 0$$

I *II*

How to compute fluctuations:

$$\zeta = \frac{v}{z} \qquad z = \frac{a}{c_s}$$

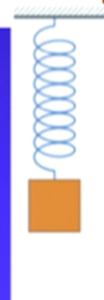
$$v'' + \left[c_s^2 k^2 - \frac{z''}{z}\right] v = 0$$



How to compute fluctuations:

$$\zeta = \frac{v}{z} \qquad z = \frac{a}{c_s}$$

$$v'' + \left[c_s^2 k^2 - \frac{z''}{z}\right] v = 0$$



I

II

Why the horizon problem leads to a real problem:

- If $c_s = const$
- If $1 + 3w > 0$ (with $w = \frac{p}{\rho}$)

$$v'' + \left[c_s^2 k^2 - \frac{z''}{z} \right] v = 0$$

$$\propto \frac{1}{\eta^2}$$

$$z = \frac{a}{c_s}$$

Dominates at late times

How inflation solves the problem:

- With $1 + 3w < 0$ $\quad \eta < 0$

$$v'' + \left[c_s^2 k^2 - \frac{z''}{z} \right] v = 0$$

Dominates earlier

Dominates later

$$\propto \frac{1}{\eta^2}$$

- But why do we get scale invariance?

How inflation solves the problem:

- With $1 + 3w < 0$ $\quad \eta < 0$

$$v'' + \left[c_s^2 k^2 - \frac{z''}{z}\right] v = 0$$

Dominates earlier ($c_s^2 k^2$)

Dominates later ($\frac{z''}{z}$) $\propto \frac{1}{\eta^2}$

- But why do we get scale invariance?

Follow up vacuum quantum fluctuations

- Consider first the regime $k|\eta| \gg 1$

$$v = \frac{e^{-ik\eta}}{\sqrt{2k}}$$

- With this normalization when we second quantize the amplitudes become creation /annihilation operators

$$v = \frac{e^{-ik\eta}}{\sqrt{2k}} a$$

- A miracle happens near deSitter (w=-1)

$$v = \frac{e^{-ik\eta}}{\sqrt{2k}}\left(1 - \frac{i}{k\eta}\right)a$$

- Compute the vacuum expectation value

$$< 0|\hat{v}^2|0 >= v^2 < 0|a^\dagger a + \frac{1}{2}|0 >$$

- In the limit $k|\eta| \ll 1$ we get:

$$< 0|\hat{v}^2|0 > \propto k^{-3}$$

- A miracle happens near deSitter (w=-1)

$$v = \frac{e^{-ik\eta}}{\sqrt{2k}}\left(1 - \frac{i}{k\eta}\right)a$$

- Compute the vacuum expectation value

$$< 0|\hat{v}^2|0 >= v^2 < 0|a^\dagger a + \frac{1}{2}|0 >$$

- In the limit $k|\eta| \ll 1$ we get:

$$< 0|\hat{v}^2|0 >\propto k^{-3}$$

How a varying speed of light solves the problem:

- With $1 + 3w > 0$ but $c_s \propto \eta^\beta$ with $\beta < -1$ we still get:

$$v'' + \left[c_s^2 k^2 - \frac{z''}{z} \right] v = 0$$

Dominates earlier

Dominates later $\propto \frac{1}{\eta^2}$

But could this lead to scale-invariance?

- Consider first the regime $k|\eta| \gg 1$

$$v = \frac{e^{-ik\eta}}{\sqrt{2k}} \quad \Rightarrow \quad v \sim \frac{e^{ik\int c_s d\eta}}{\sqrt{2c_s k}}$$

- With this normalization when we second quantize the amplitudes become creation /annihilation operators

- Can solve for a generic w and c_s

$$v = \sqrt{\beta\eta}(AJ_\nu(\beta c_s k\eta) + BJ_{-\nu}(\beta c_s k\eta))$$

- Compute the vacuum expectation value

$$< 0|\hat{v}^2|0 >= v^2 < 0|a^\dagger a + \frac{1}{2}|0 >$$

- take the limit $k|\eta| \ll 1$ and see when we get:

$$< 0|\hat{v}^2|0 > \propto k^{-3}$$

A remarkable result (!!!!!!!!!!!!!!)

- For ALL equations of state

$$c_s \propto \rho \implies n_s = 1$$

This scaling seems to be uniquely associated with scale invariance.

A remarkable result (!!!!!!!!!!!!!!)

- For ALL equations of state

$$c_s \propto \rho \implies n_s = 1$$

This scaling seems to be uniquely associated with scale invariance.

(For experts only; cf. k-essence)

- This can be understood:

$$k^3\zeta^2 \sim \frac{(5+3w)^2}{1+w}\frac{\rho}{M_{Pl}^4 c_s}$$

(For experts only; cf. k-essence)

- This can be understood:

$$k^3\zeta^2 \sim \frac{(5+3w)^2}{1+w}\frac{\rho}{M_{Pl}^4 c_s}$$

Where does the amplitude come from?

- Obviously the variations in c must be cut off at low energies:

$$c_s = c\left(1 + \frac{\rho}{\rho_\star}\right)$$

- The cut-off scale fixes the amplitude:

$$k^3\zeta^2 \sim \frac{(5+3w)^2}{1+w}\frac{\rho_\star}{M_{Pl}^4} \sim 10^{-10}$$

Where does the amplitude come from?

- Obviously the variations in c must be cut off at low energies:

$$c_s = c\left(1 + \frac{\rho}{\rho_\star}\right)$$

- The cut-off scale fixes the amplitude:

$$k^3\zeta^2 \sim \frac{(5+3w)^2}{1+w}\frac{\rho_\star}{M_{Pl}^4} \sim 10^{-10}$$

The minimal bimetric VSL theory

$$\hat{g}_{\mu\nu} = g_{\mu\nu} + B\partial_\mu\phi\partial_\nu\phi$$

$$B = B(\phi) = const$$

$$S = \frac{M_{Pl}^2}{2}\int d^4x\sqrt{-g}\,R[g_{\mu\nu}] + \int d^4x\sqrt{-\hat{g}}\,\mathcal{L}_m[\hat{g}_{\mu\nu}, \Phi_{Matt}] + S_\phi$$

$$S_\phi = ???$$

The tools of (K-essence) varying speed of sound:

$$\mathcal{L} = K(X) - V(\phi)$$

$$X = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi$$

$$\begin{aligned} p &= K - V \\ \rho &= 2XK_{,X} - K + V \end{aligned}$$

$$c_s^2 = \frac{K_{,X}}{K_{,X} + 2XK_{,XX}}$$

Check formulae with inflation, cuscaton, etc…

Something truly cool…

$$S^1_\phi = \int d^4x\sqrt{-\hat{g}}(-2\hat{\Lambda})$$

$$\hat{g} = g(1+2BX) \qquad X = \tfrac{1}{2}\partial_\mu\phi\partial^\mu\phi$$

$$S^1_\phi = \int d^4x\sqrt{-g}\sqrt{1+2BX}(-2\hat{\Lambda})$$

Something truly cool…

$$S_\phi^1 = \int d^4x \sqrt{-\hat{g}}(-2\hat{\Lambda})$$

$$\hat{g} = g(1 + 2BX)$$

$$X = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi$$

$$S_\phi^1 = \int d^4x \sqrt{-g}\sqrt{1 + 2BX}(-2\hat{\Lambda})$$

A cosmological constant in the matter frame leads to the (anti)DBI action

A positive Lambda in the Einstein frame balanced by a negative lambda in the matter frame:

$$+ - - -$$

$$S_\phi = \int d^4x \sqrt{-\hat{g}}\frac{1}{B} - \int d^4x \sqrt{-g}\frac{1}{B}$$

is equivalent to (with $f<0$ for $B>0$):

$$\mathcal{L} = -\frac{1}{f(\phi)}\sqrt{1-2f(\phi)X} + \frac{1}{f(\phi)} - V(\phi)$$

Recall our K-essence toolbox

$$\mathcal{L} = K(X) - V(\phi)$$

$$X = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi$$

$$\begin{aligned} p &= K - V \\ \rho &= 2XK_{,X} - K + V \end{aligned}$$

$$c_s^2 = \frac{K_{,X}}{K_{,X} + 2XK_{,XX}}$$

Recall our K-essence toolbox

$$\mathcal{L} = K(X) - V(\phi)$$

$$X = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi$$

$$\begin{aligned} p &= K - V \\ \rho &= 2XK_{,X} - K + V \end{aligned}$$

Constant w solutions for mass potentials

$$c_s^2 = \frac{K_{,X}}{K_{,X} + 2XK_{,XX}}$$

Apply to (anti)DBI to find that…

$$c_s = c\left(1 + \frac{\rho}{\rho_\star}\right)$$

So our remarkable result is even more remarkable

- It's possible to identify a universal varying speed of sound law associated with scale invariance
- This law can be realized by an anti-DBI model (in the Einstein frame)
- This turns out to be the minimal dynamics associated with a bimetric VSL

So our remarkable result is even more remarkable

- It's possible to identify a universal varying speed of sound law associated with scale invariance
- This law can be realized by an anti-DBI model (in the Einstein frame)
- This turns out to be the minimal dynamics associated with a bimetric VSL

Finish with a more technical discussion (for experts, only)

- For cosmologists: Beyond the zeroth order: beyond scale-invariance; beyond the power spectrum.
- For (some) quantum gravity enthusiasts: could this be done with DSR?

A cosmological constant in the matter frame leads to the (anti)DBI action

A positive Lambda in the Einstein frame balanced by a negative lambda in the matter frame:

$+ - - -$

$$S_\phi = \int d^4x \sqrt{-\hat{g}} \frac{1}{B} - \int d^4x \sqrt{-g} \frac{1}{B}$$

is equivalent to (with $f<0$ for $B>0$):

$$\mathcal{L} = -\frac{1}{f(\phi)} \sqrt{1 - 2f(\phi)X} + \frac{1}{f(\phi)} - V(\phi)$$

So our remarkable result is even more remarkable

- It's possible to identify a universal varying speed of sound law associated with scale invariance
- This law can be realized by an anti-DBI model (in the Einstein frame)
- This turns out to be the minimal dynamics associated with a bimetric VSL

A cosmological constant in the matter frame leads to the (anti)DBI action

A positive Lambda in the Einstein frame balanced by a negative lambda in the matter frame:

$+ - - -$

$$S_\phi = \int d^4x\sqrt{-\hat{g}}\frac{1}{B} - \int d^4x\sqrt{-g}\frac{1}{B}$$

is equivalent to (with f<0 for B>0):

$$\mathcal{L} = -\frac{1}{f(\phi)}\sqrt{1-2f(\phi)X} + \frac{1}{f(\phi)} - V(\phi)$$

Recall our K-essence toolbox

$$\mathcal{L} = K(X) - V(\phi)$$

$$X = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi$$

$$\begin{aligned} p &= K - V \\ \rho &= 2XK_{,X} - K + V \end{aligned}$$

Constant w solutions for mass potentials

$$c_s^2 = \frac{K_{,X}}{K_{,X} + 2XK_{,XX}}$$

Finish with a more technical discussion (for experts, only)

- For cosmologists: Beyond the zeroth order: beyond scale-invariance; beyond the power spectrum.
- For (some) quantum gravity enthusiasts: could this be done with DSR?

Beyond the "zeroth order" holy grail

- If the relation between the two metrics is

$$\hat{g}_{\mu\nu} = g_{\mu\nu} + B\partial_\mu\phi\partial_\nu\phi$$

$$B = B(\phi) \propto \phi^\alpha$$

then we obtain a tilted spectrum

$$n_S = f(\alpha)$$

Is this then another "theory of anything"? No!

- A possibility for a "consistency relation" is to look into the bispectrum (3-point function), by evaluating the cubic action:

$$\langle \zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3)\rangle = (2\pi)^7 \delta^3(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3) {P_\zeta}^2 \frac{1}{\Pi_j k_j^3} \mathcal{A} \,.$$

JM, Noller and Piazza
Phys.Rev. D82 (2010) 043521)

For scale-invariant varying c_s we obtain an equilateral bispectrum

Just like in DBI inflation we get a form factor:

$$\mathcal{A}_{c_s \to \infty} = -\frac{1}{8}\sum_i k_i^3 + \frac{1}{K}\sum_{i<j} k_i^2 k_j^2 - \frac{1}{2K^2}\sum_{i \neq j} k_i^2 k_j^3 .$$

(where $K \equiv k_1 + k_2 + k_3$) but now we find for the amplitude:

$$\mathcal{A}_{\epsilon \to 0} = \left(1 - \frac{1}{c_s^2}\right)\mathcal{A}_{c_s \to \infty} + \mathcal{O}(n_s - 1) ,$$

Summary in terms of f_{NL} (if you really must!)

$$f_{\rm NL} = 30 \frac{\mathcal{A}_{k_1=k_2=k_3}}{K^3}$$

$$k_1 = k_2 = k_3 = K/3$$

Standard inflation $f_{NL} \sim \epsilon \sim 0.1$

VSL $f_{NL} \sim 1 > 0$

DBI inflation $f_{NL} \sim -100$

However if we depart from scale-invariance with varying c_s we obtain:

$$\mathcal{A} = \left(\frac{k_1 k_2 k_3}{2K^3}\right)^{n_s-1}\left[-\frac{1}{8}\sum_i k_i^3 + \frac{1}{K}\sum_{i<j} k_i^2 k_j^2 - \frac{1}{2K^2}\sum_{i\neq j} k_i^2 k_j^3\right.$$
$$+ (n_s - 1)\left(-\frac{1}{8}\sum_i k_i^3 - \frac{1}{8}\sum_{i\neq j} k_i k_j^2 + \frac{1}{8} k_1 k_2 k_3 + \frac{1}{2K}\sum_{i<j} k_i^2 k_j^2 - \frac{1}{2K^2}\sum_{i\neq j} k_i^2 k_j^3\right)$$
$$\left. + \mathcal{O}\left(\frac{1}{c_s^2}\right)\right], \quad (4.$$

Specifically note the small "collapsed component":

$$\mathcal{A}_{k_1 \ll k_2, k_3} \approx -\frac{1}{2}(n_s - 1)\left(\frac{k_1}{k_2}\right)^{n_s-1}$$

For scale-invariant varying c_s we obtain an equilateral bispectrum

Just like in DBI inflation we get a form factor:

$$\mathcal{A}_{c_s \to \infty} = -\frac{1}{8}\sum_i k_i^3 + \frac{1}{K}\sum_{i<j} k_i^2 k_j^2 - \frac{1}{2K^2}\sum_{i \neq j} k_i^2 k_j^3 \,.$$

(where $K \equiv k_1 + k_2 + k_3$) but now we find for the amplitude:

$$\mathcal{A}_{\epsilon \to 0} = \left(1 - \frac{1}{c_s^2}\right)\mathcal{A}_{c_s \to \infty} \; + \; \mathcal{O}(n_s - 1) \; ,$$

However if we depart from scale-invariance with varying c_s we obtain:

$$
\begin{aligned}
\mathcal{A} \;=\; \left(\frac{k_1 k_2 k_3}{2K^3}\right)^{n_s-1} \Bigg[& -\frac{1}{8}\sum_i k_i^3 + \frac{1}{K}\sum_{i<j} k_i^2 k_j^2 - \frac{1}{2K^2}\sum_{i\neq j} k_i^2 k_j^3 \\
& + (n_s - 1)\left(-\frac{1}{8}\sum_i k_i^3 - \frac{1}{8}\sum_{i\neq j} k_i k_j^2 + \frac{1}{8} k_1 k_2 k_3 + \frac{1}{2K}\sum_{i<j} k_i^2 k_j^2 - \frac{1}{2K^2}\sum_{i\neq j} k_i^2 k_j^3 \right) \\
& + \mathcal{O}\left(\frac{1}{c_s^2}\right) \Bigg] \,, \qquad (4.
\end{aligned}
$$

Specifically note the small "collapsed component":

$$
\mathcal{A}_{k_1 \ll k_2, k_3} \;\approx\; -\frac{1}{2}(n_s - 1)\left(\frac{k_1}{k_2}\right)^{n_s-1}
$$

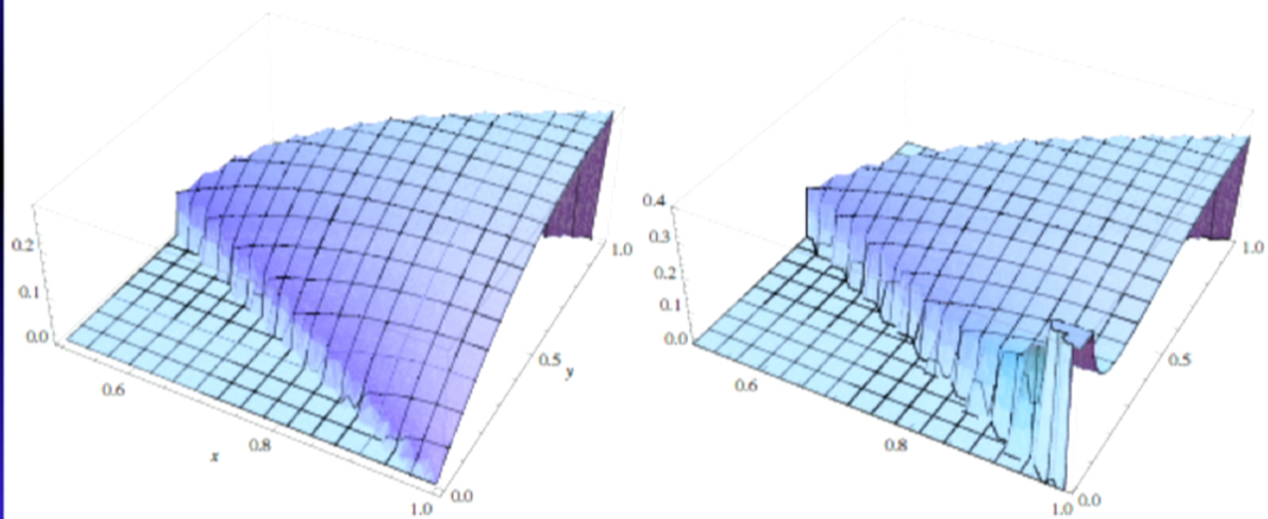
However if we depart from scale-invariance with varying c_s we obtain:

$$\mathcal{A} = \left(\frac{k_1k_2k_3}{2K^3}\right)^{n_s-1}\left[-\frac{1}{8}\sum_i k_i^3 + \frac{1}{K}\sum_{i<j}k_i^2k_j^2 - \frac{1}{2K^2}\sum_{i\neq j}k_i^2k_j^3\right.$$
$$+(n_s-1)\left(-\frac{1}{8}\sum_i k_i^3 - \frac{1}{8}\sum_{i\neq j}k_ik_j^2 + \frac{1}{8}k_1k_2k_3 + \frac{1}{2K}\sum_{i<j}k_i^2k_j^2 - \frac{1}{2K^2}\sum_{i\neq j}k_i^2k_j^3\right)$$
$$\left.+\mathcal{O}\left(\frac{1}{c_s^2}\right)\right], \qquad (4.$$

Specifically note the small "collapsed component":

$$\mathcal{A}_{k_1\ll k_2,k_3} \approx -\frac{1}{2}(n_s-1)\left(\frac{k_1}{k_2}\right)^{n_s-1}$$

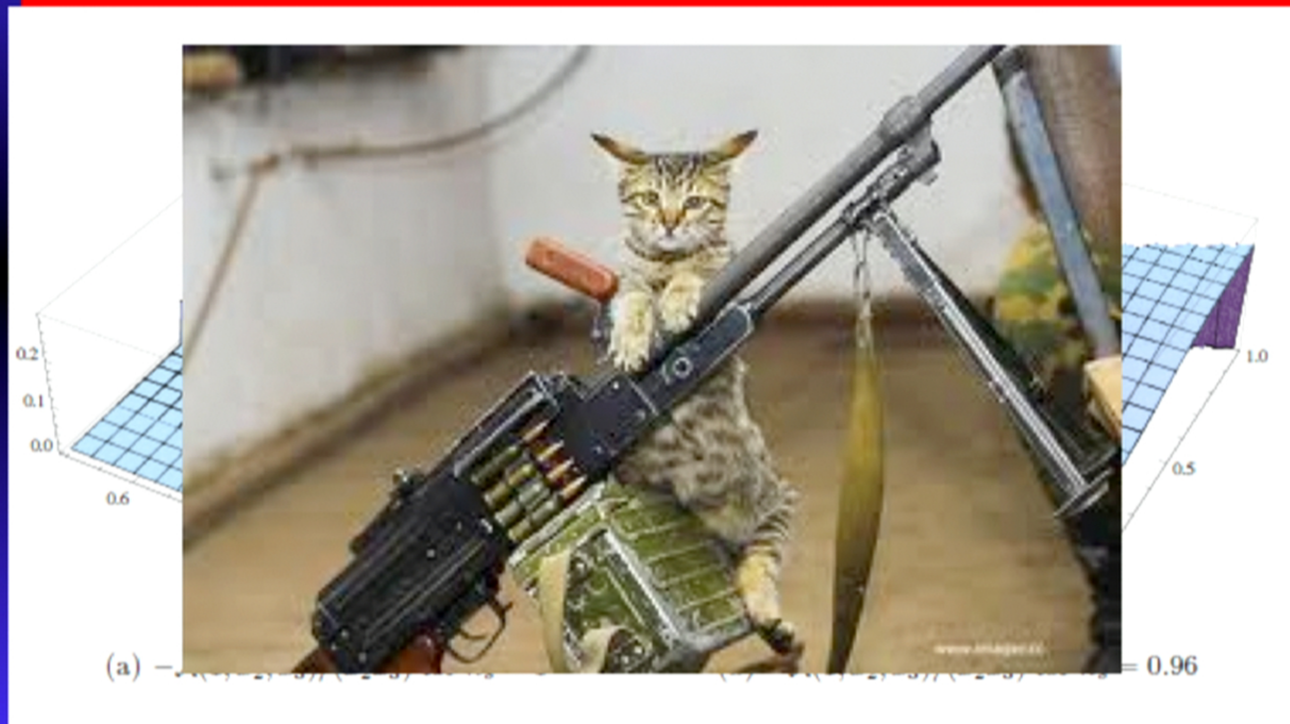
Is this then another “theory of anything”? No.



(a) $-\mathcal{A}(1, x_2, x_3)/(x_2 x_3)$ for $n_s = 1$ (b) $-\mathcal{A}(1, x_2, x_3)/(x_2 x_3)$ for $n_s = 0.96$

(see arXiv:1006:3216, Magueijo, Noller and Piazza)

Is this then another “theory of anything”? No.



(see arXiv:1006:3216, Magueijo, Noller and Piazza)

What we did with bimetric VSL can also be done with DSR

- Deformed dispersion relations can give a frequency dependent speed of light

$$E^2 - g^2 p^2 = m^2$$

- The speed of sound would then also vary in time, by proxy, via expansion:

$$\omega = kg(\lambda k/a)$$

$$c = \frac{d\omega}{dk} = (\gamma + 1)\frac{\omega}{k} \propto \left(\frac{\lambda k}{a}\right)^{\gamma}$$

What we did with bimetric VSL can also be done with DSR

- Deformed dispersion relations can give a frequency dependent speed of light

$$E^2 - g^2 p^2 = m^2$$

- The speed of sound would then also vary in time, by proxy, via expansion:

$$\omega = kg(\lambda k/a)$$

$$c = \frac{d\omega}{dk} = (\gamma + 1)\frac{\omega}{k} \propto \left(\frac{\lambda k}{a}\right)^{\gamma}$$

Also in this context scale-invariance is associated with an universal law

$$v'' + \left[\omega^2 - \frac{z''}{z}\right] v = 0$$

$$\omega^2 - k^2(1 + (\lambda k)^2)^2 = m^2$$

$$\lambda \sim 10^5 L_{Pl}.$$

- Cf. Horava-Lifschitz.

Breaking Lorentz invariance is good for you… if you're a cosmologist

- An alternative to inflation for solving the cosmological problems
- Observationally distinct from inflation

Breaking Lorentz invariance is good for you… if you're a cosmologist

- An alternative to inflation for solving the cosmological
- Observationa n inflation

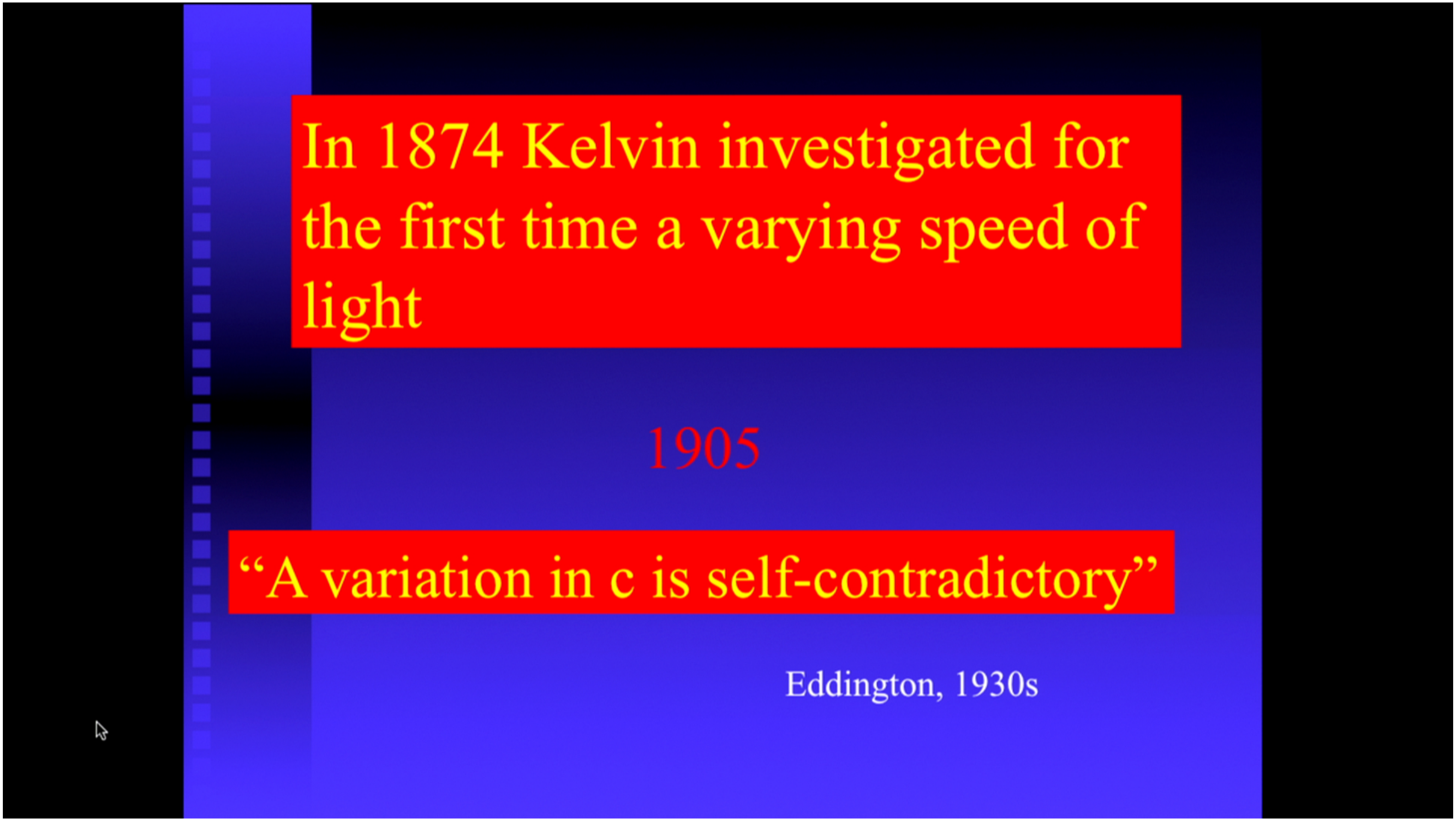
In 1874 Kelvin investigated for the first time a varying speed of light

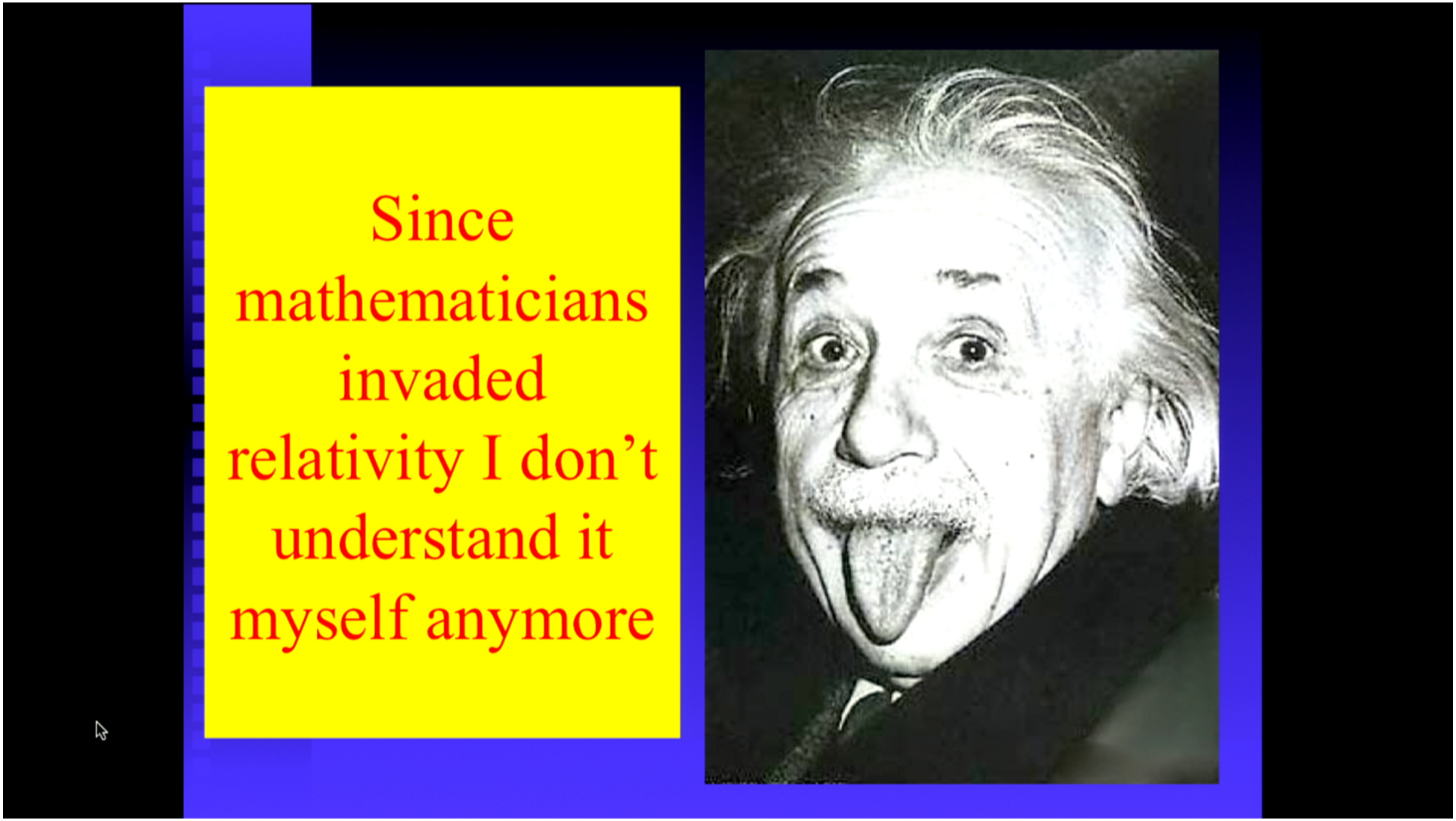
In 1874 Kelvin investigated for the first time a varying speed of light

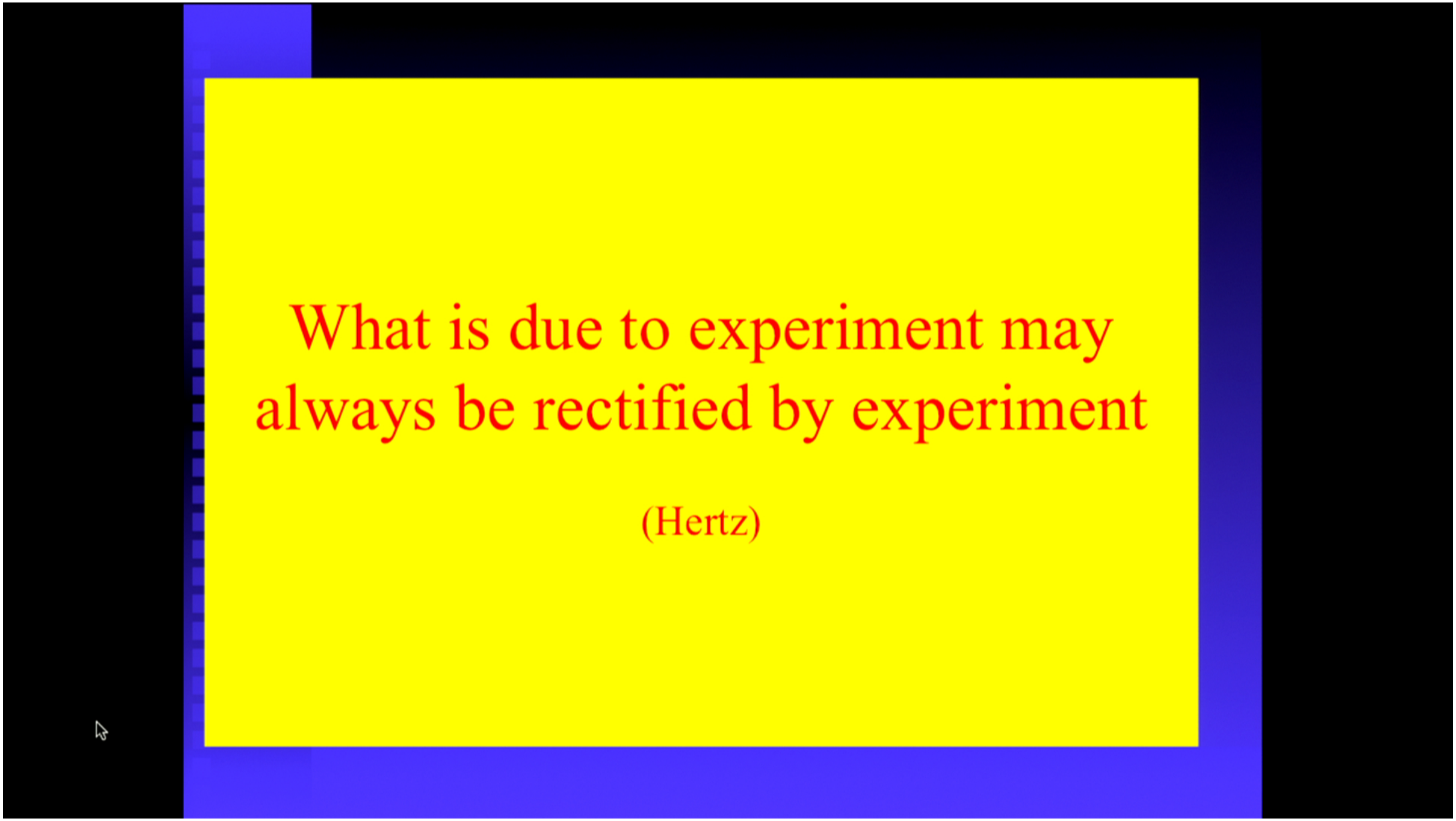
1905

"A variation in c is self-contradictory"

Eddington, 1930s


In 1874 Kelvin investigated for the first time a varying speed of light
1905
“A variation in c is self-contradictory”
Eddington, 1930s


Since mathematicians invaded relativity I don't understand it myself anymore


What is due to experiment may always be rectified by experiment
(Hertz)