

Title: On a partially Reduced Phase Space Quantisation of General Relativity Conformally Coupled to a Scalar Field

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Abstract: We comment on a certain partially reduced phase space quantisation of general relativity conformally coupled to a scalar field, and its extension to standard model matter fields. The partially reduced phase space is reached by trading the Hamiltonian constraint for the generator of local conformal transformations on all phase space variables, inspired by the ideas of shape dynamics, and constructing conformally invariant connection variables. Furthermore, we review this trading of symmetries from the gauge fixing/unfixing perspective, which is dual to the concept of a linking theory. Finally we point out possible applications and open problems.

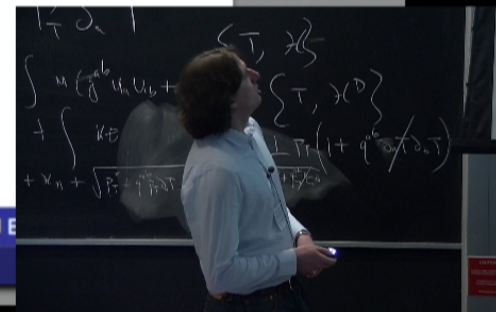
## Content of the talk

### Central topics

- Gauge fixing/unfixing [Mitra & Rajaraman, 1989, Anishetty & Vytheeswaran, 1993]  
as a dual picture to linking theories [Gomes & Koslowski, 2011]
- Application of the former to general relativity + matter  
using ideas from shape dynamics [Barbour, Gomes, Gryb, Mercati, Koslowski, 2011]
- Variables with simple conformal transformation behaviour
  - ↳ Construction of a **partially reduced phase space** quantisation
  - ↳ Extension to standard model matter
- Possible applications of the quantisation & open problems

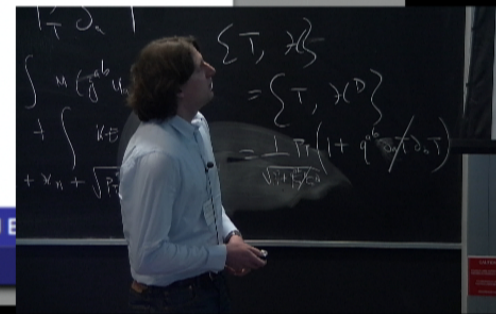
## Outline

- 1 Gauge fixing/unfixing
  - Gauge unfixing one part of the constraints
  - The role of observables
  - Trading dynamics
  
- 2 Gravity conformally coupled to a scalar field
  - Motivation & setup
  - Connection formulation & solution of the conformal constraints
  - Quantisation & observables
  
- 3 Extensions to standard model matter fields
  - Conformally invariant matter field actions
  - Scalar field potentials
  - Cosmological constant
  
- 4 Possible applications and open problems



## Section outline

- 1 Gauge fixing/unfixing
  - Gauge unfixing one part of the constraints
  - The role of observables
  - Trading dynamics
- 2 Gravity conformally coupled to a scalar field
- 3 Extensions to standard model matter fields
- 4 Possible applications and open problems



## General setup for partial gauge fixing

### Totally constrained, first class, irreducible system

- **Constraints**  $\{\mathcal{H}[N, \vec{N}]\}$  → **Section 2: Dirac algebra**

$$\{\mathcal{H}[N, \vec{N}], \mathcal{H}[N', \vec{N}']\} = \mathcal{H}(f^{\mathcal{H}}(p, q; N, N', \vec{N}, \vec{N}')) \quad (1)$$

- **Splitting** of the constraint  $\{\mathcal{H}[N, \vec{N}]\} = \{\mathcal{H}[N], \mathcal{H}_a[N^a]\}$

- **Gauge conditions**  $\{\mathcal{D}[\rho]\}$  for  $\{\mathcal{H}[N]\}$

$$\{\mathcal{H}[N], \mathcal{D}[\rho]\} = (N, \underbrace{M}_{\text{Dirac matrix}} \rho), \quad M \text{ invertible on "gauge cut"} \quad (2)$$

- **Invariance** of the gauge conditions under  $\{\mathcal{H}_a[N^a]\}$

$$\{\mathcal{H}_a[N^a], \mathcal{H}[N]\} = \mathcal{H}[N^a \triangleright N], \quad \{\mathcal{H}_a[N^a], \mathcal{D}[\rho]\} = \mathcal{D}[N^a \triangleright \rho]$$



## Gauge unfixing one part of the constraints

### Additional structure of the gauge conditions

The gauge conditions are themselves first class functions

$$\{\mathcal{D}[\rho], \mathcal{D}[\rho']\} = \mathcal{D}(f^{\mathcal{D}}(p, q; \rho, \rho')). \quad (4)$$

**Section 2:**  $\mathcal{D}$  = generator of local conformal transformations



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**Section 2:**  $\mathcal{D}$  = generator of local conformal transformations

### Crucial Observation

The two sets of phase space functions

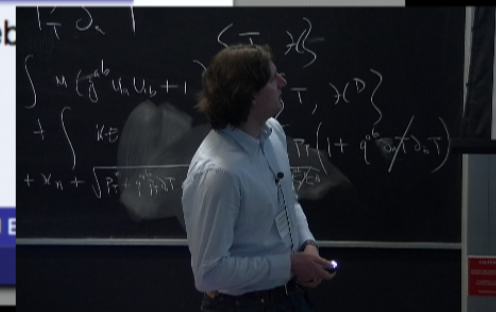
$$\{\mathcal{H}[N, \vec{N}]\} \text{ and } \{\mathcal{D}[\rho], H_a[N^a]\}$$

define the **same reduced phase space**, at least locally.

### Conclusion?

$\mathcal{H}$ -invariance can be traded for  $\mathcal{D}$ -invariance  $\rightarrow$  **simplify** the gauge algebra

Use  $\{\mathcal{D}[\rho], H_a[N^a]\}$  as constraints!



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## Dirac observables

Observables are defined as gauge invariant functions on the resp. constraint surfaces

$$\{\mathcal{H}[N], \mathcal{O}\} \approx 0, \quad \{\mathcal{H}_a[N^a], \mathcal{O}\} \approx 0 \quad \mathcal{H}\text{-observables} \quad (5)$$

$$\{\mathcal{D}[\rho], \mathcal{O}\} \approx 0, \quad \{\mathcal{H}_a[N^a], \mathcal{O}\} \approx 0 \quad \mathcal{D}\text{-observables} \quad (6)$$

## Trading observables

Interpretation & dynamical content is tied to the observables



Construct a map between  $\mathcal{H}$ - &  $\mathcal{D}$ -observables

The role of observables

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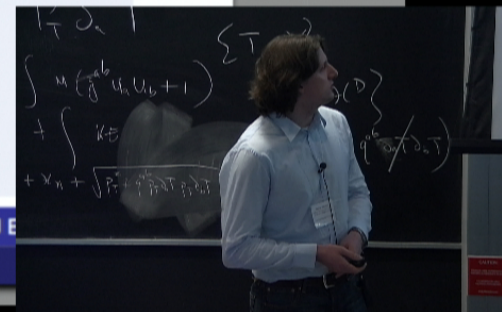
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### Partial observable projectors

[Mitra & Rajaraman, 1989, Henneaux & Teitelboim, 1994, Anishetty & Vytheeswaran, 1993, Rovelli, 2002, Dittrich, 2004, Thiemann, 2004]

functions on the reduced phase space  $\xrightarrow{1:1}$  gauge invariant extension  
provides the maps

$$\mathcal{H}\text{-observables} \xleftrightarrow{\mathbb{P}_{\mathcal{H}}^{\mathcal{D}}} \mathcal{F}(\Gamma_{\text{red}}) \xleftrightarrow{\mathbb{P}_{\mathcal{D}}^{\mathcal{H}}} \mathcal{D}\text{-observables}$$



## Explicit construction of $\mathbb{P}$

[Mitra & Rajaraman, 1989, Henneaux & Teitelboim, 1994, Anishetty & Vytheeswaran, 1993, Rovelli, 2002, Dittrich, 2004, Thiemann, 2004]

The projectors  $\mathbb{P}$  can be given in terms of a **power series**. In short hand notation:

$$\mathbb{P}_{\mathcal{D}}^{\mathcal{H}}(f) = e^{-\{\mathcal{D}[\rho], \cdot\}}(f)|_{\rho=M^{-1}\mathcal{H}}, \quad (8)$$

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- $\mathbb{P}$  is a (weak) **Poisson isomorphism**:  $\{\mathbb{P}(\cdot), \mathbb{P}(\cdot)\} \approx \mathbb{P}(\{\cdot, \cdot\}^*)!$

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## Possible caveat

Only applicable if the constraints used for extension are first class!

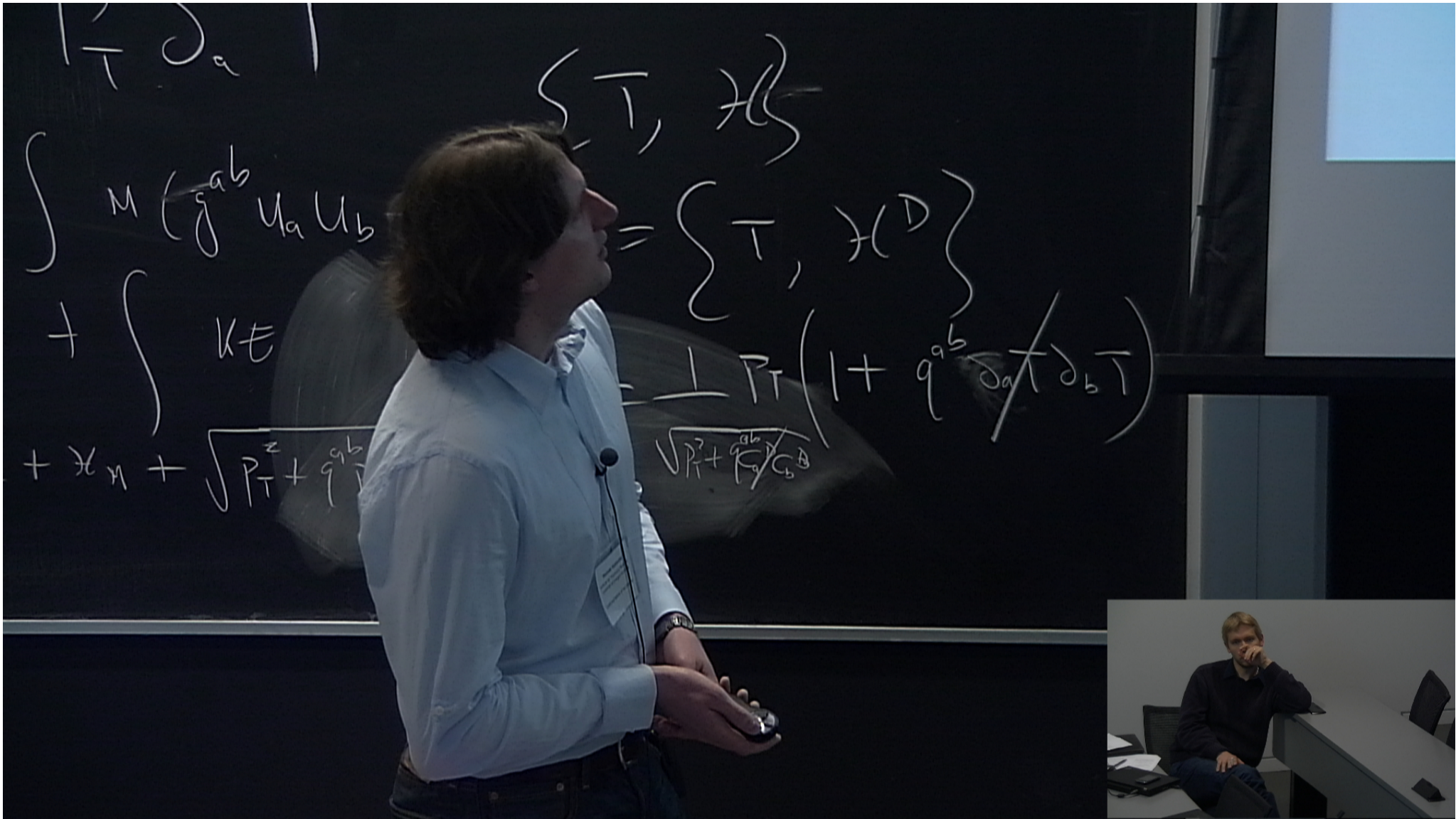
↓

Hamiltonian constraints in general relativity **do not** have this property.

↳ Only in combination with diffeomorphism constraints

⇓

Inversion of the trading seems difficult ← **no partial reduction** through  $\mathbb{P}_{\mathcal{H}}^{\mathcal{D}}$



## Motivation & Setup $\mathcal{H}[N], \mathcal{H}_a[N^a]$ - Dirac algebra, $\mathcal{D}[\rho]$ - local conformal generator

### Quantisation of shape dynamics

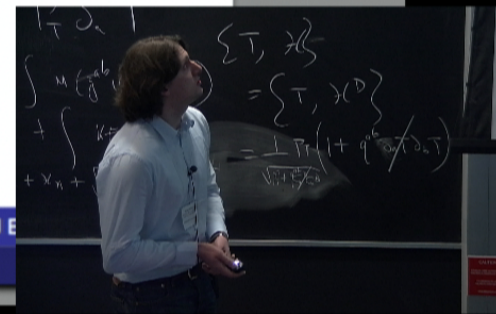
**Problem:** No connection formulation with simple conformal transformation behaviour

[pointed out to us by Sean Gryb at QGC 6]

↳ This is due to the **affine part** of the connection



Find conformally invariant variables



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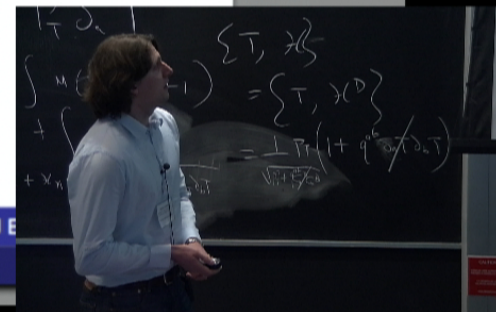


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### Elimination of the Hamiltonian constraint

**Problem:** Effective treatment of the Hamiltonian constraint in loop quantum gravity

↳ Use the ideas from shape dynamics to eliminate it **completely**.





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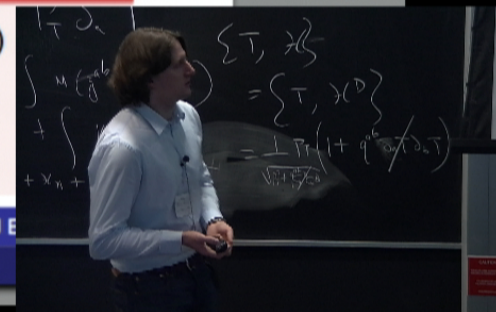
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### Important Observation

The metric variables  $(P^{ab}, q_{ab})$  possess non-trivial conformal weights  $(\Delta^P, \Delta^q)$

**Idea:** Couple a matter field  $\Phi$  that also has non-trivial weight  $\Delta^\Phi$ .

↳ Check, if the conformal generator still acts as gauge fixing.



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## $D + 1$ -split of the action

One gets

$$S = \int_{\mathbb{R}} dt \int_{\sigma} d^D x [P^{ab} \dot{q}_{ab} + \pi_{\Phi} \dot{\Phi} - N^a \mathcal{H}_a - N \mathcal{H}], \quad (15)$$

where  $\mathcal{H}$ ,  $\mathcal{H}_a$  generate the usual **hypersurface deformation algebra**.

## Gauge fixing

[Dirac, 1959], [Lichnerowicz 1944, Choquet-Bruhat 1962, Brill, York 1971, York & O'Murchadha, 1973, York & O'Murchadha & Isenberg 1976]

As in shape dynamics, we introduce the **gauge conditions**

$$\mathcal{D} := \Delta^q q_{ab} P^{ab} + \Delta^{\Phi} \Phi \pi_{\Phi} = \Delta^q \frac{1-D}{\kappa} \sqrt{q} q^{ab} K_{ab}, \quad (16)$$

which is the generator of **local conformal transformations** on the canonical variables.

This amounts to the choice of a **maximal hypersurface**

$$\text{tr}_q K = 0, \quad (17)$$

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Leads to the **Dirac matrix**

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$$+ \int_{\sigma} d^D x (D-1) \sqrt{q} \Delta^q \rho [D_{\alpha} D^{\alpha} - \mathcal{P}(x)] N. \quad (19)$$

**On the gauge cut:** Quadratic form of an **elliptic PDO**.

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## Invertibility of the Dirac matrix

(Local) admissibility of the gauge is determined by the elliptic PDE

$$[D_\alpha D^\alpha - \mathcal{P}(x)]N(x) = 0. \quad (20)$$

It is **invertible** if we assume the **dominant energy condition** for  $\Phi$ .

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$$\mathcal{P} > 0 \quad (\text{strict positivity})$$

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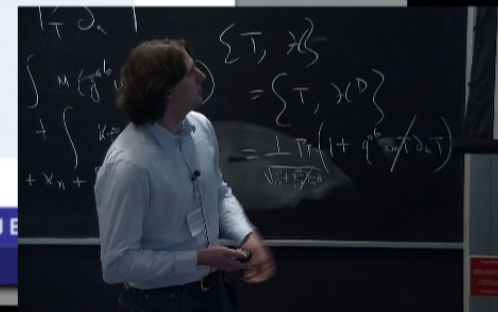
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## Connection formulation & solution of the conformal constraints

### Invariant metric variables

Restricting to a  $\Phi = e^\phi$  **Dilaton-type** field & rescaling the metric variables by the field

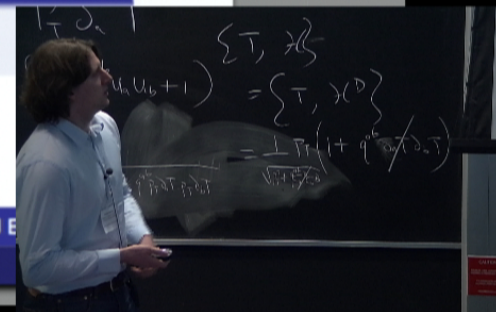
$$\{q_{ab}, P^{cd}, \Phi, \pi_\Phi\} \longrightarrow \{q_{ab}, P^{cd}, \phi := \ln \Phi, \pi_\phi := \Phi \pi_\Phi\} \quad (21)$$

$$\{q_{ab}, P^{ab}, \phi, \pi_\phi\} \longrightarrow \{\tilde{q}_{ab} := e^{\frac{4}{D-1}\phi} q_{ab}, \tilde{P}^{ab} := e^{-\frac{4}{D-1}\phi} P^{ab}, \tilde{\phi} := \phi, \tilde{\pi}_{\tilde{\phi}} := \frac{1}{\Delta \Phi} \mathcal{D}\} \quad (22)$$

we end up with invariant metric variables &  $\mathcal{D}$  as the momentum of the scalar field  $\tilde{\phi}$ .

### Remark

Opposite conformal weights of  $(q_{ab}, P^{ab})$  require a Dilaton-type field





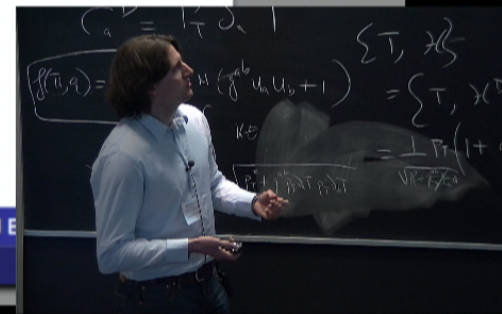
## Quantisation & observables

### Loop quantisation

Employ loop quantum gravity methods → physical Hilbert space

$$\mathfrak{H}_{\text{phys}} := \{\text{gauge \& diffeomorphism invariant spin networks}\} \quad (24)$$

↳ Obtain physical spectra of observables, solve non-dynamical problems



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### Interpretation

Interpretation of a quantum  $\mathcal{D}$ -observable  $\hat{\mathcal{O}}$



Spectrum gives the measurement values for  $\mathcal{O}$  at "time"  $\mathcal{D} = 0$

### Dynamics

Implementation of dynamics on the  $\mathcal{D}$ -observables so far only through  $\mathbb{P}_{\mathcal{H}}^{\mathcal{D}}$

$\hookrightarrow$  But there is a **clear geometric picture** of the time  $\mathcal{D}$ .

## Conformally invariant matter field actions

### General structure

Given a conformally invariant matter field action

$$S = \int_{\mathbb{R}} dt \int_{\sigma} d^D x \left( P^{ab} \dot{q}_{ab} + \pi_{\varphi}^I \dot{\varphi}_I - N \mathcal{H}_{\varphi} - N^a \mathcal{H}_a - \lambda_i C^i \right), \quad (25)$$

where the  $C^i$  denote gauge constraints of the field  $\varphi_I$ . We observe, that

- the conformal generator acquires new contributions

$$\mathcal{D} \longrightarrow \mathcal{D} + \Delta^{\varphi} \pi_{\varphi}^I \varphi_I \quad (26)$$

- the form of the Dirac matrix remains unchanged,
- the invertibility of the Dirac matrix follows along the same lines.

↳ Inclusion of **Yang-Mills fields**, fermions ( $\Delta = 0$ ) & conf. coupled **scalar**  
 spatial half densities

**multiplets**

### Observation

The model is **flexible** enough to include various matter fields.

## Extension of the gauge fixing

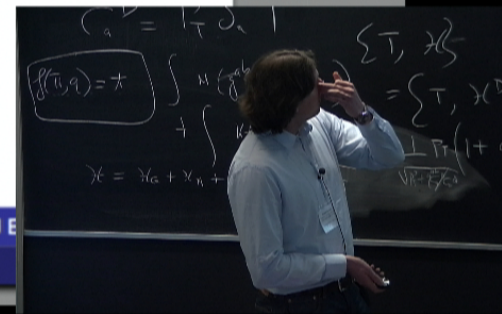
Replacing

$$\mathcal{D} \longrightarrow \mathcal{D} - \sqrt{q}\delta \quad (31)$$

allows to include arbitrary cosmological constant  $\longrightarrow$  **CMC-gauge** ( $q^{ab}K_{ab} = \text{const.}$ ).

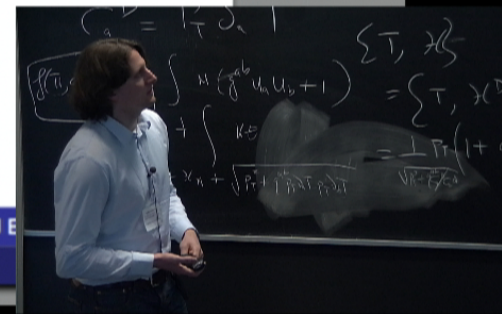
## Application to non-dynamical problems

- Spectra of observables → Geometric operators acquire different interpretation
  - ↳ **Symbolically:**  $\hat{A}^{\text{inv}} = \Phi^2 \hat{A}^{\text{LQG}}$ ,  $\hat{V}^{\text{inv}} = \Phi^{2D/(D-1)} \hat{V}^{\text{LQG}}$
  - ↳ Non-vanishing vacuum expectation  $\langle \Phi \rangle$  might increase fundamental scale
- Black hole state counting → isolated horizons framework extended to conformally coupled scalar field
  - [Ashtekar et al., 2003, Ashtekar & Corichi, 2003]
  - ↳ Interesting black hole solutions with non-trivial horizon topologies in locally asymptotically adS space-times [Nadalini et al., 2008]



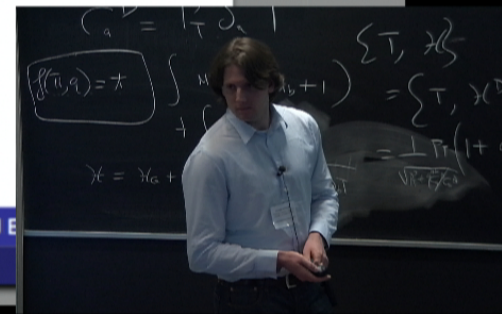
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## Open problems

- Implementation of dynamics → deparametrisation not suitable
  - ↳ **Problem:** Clock variable  $\mathcal{D}$  is not a space-time scalar [Kuchař, 1992]
- Construction of spatially diffeomorphism invariant observables
- Extension to asymptotically flat situations → **canonical maximal time function**
  - ↳ **Problem:** Gauge fixing of Supertranslations
  - ↳ ADM-Energy as natural generator of time-evolution
- Extension to supergravity?