Title: Two-dimensional Conformal Symmetry of Short-distance Spacetime

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Abstract: Evidence from several approaches to quantum gravity hints at the possibility that spacetime undergoes a "spontaneous dimensional reduction" at very short distances. If this is the case, the small scale universe might be described by a theory with two-dimensional conformal symmetry. I will summarize the evidence for dimensional reduction and indicate a tentative path towards using this conformal invariance to explore quantum gravity.


But there is accumulating evidence for "spontaneous dimensional reduction" to two-dimensional spacetime at short distance
Is conformal invariance restored near the Planck scale?
Some evidence from
• Causal dynamical triangulations
• Renormalization group/"asymptotic safety"
• High temperature string theory

Two-Dimensional Conformal Symmetry of Short Distance Spacetime?

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Conformal Nature of the Universe
Perimeter Institute
May 2012

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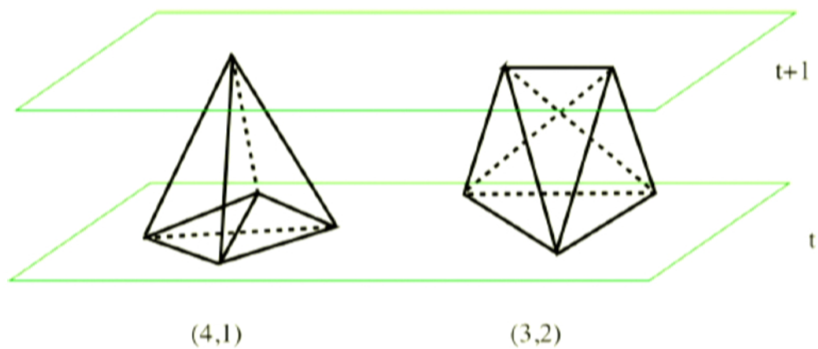
Causal dynamical triangulations

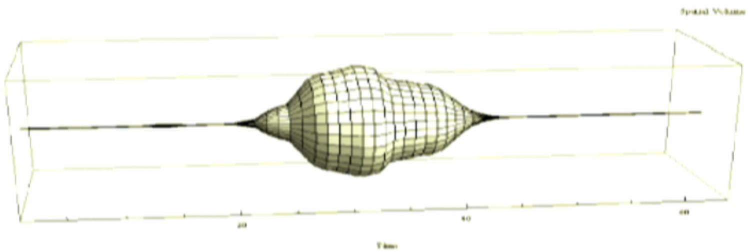
Approximate path integral by sum over discrete triangulated manifolds

$$\int [dg] e^{iI_{EH}[g]} \Rightarrow \sum e^{iI_{Regge}[\Delta]}$$

Fix causal structure (⇒ no topology change)



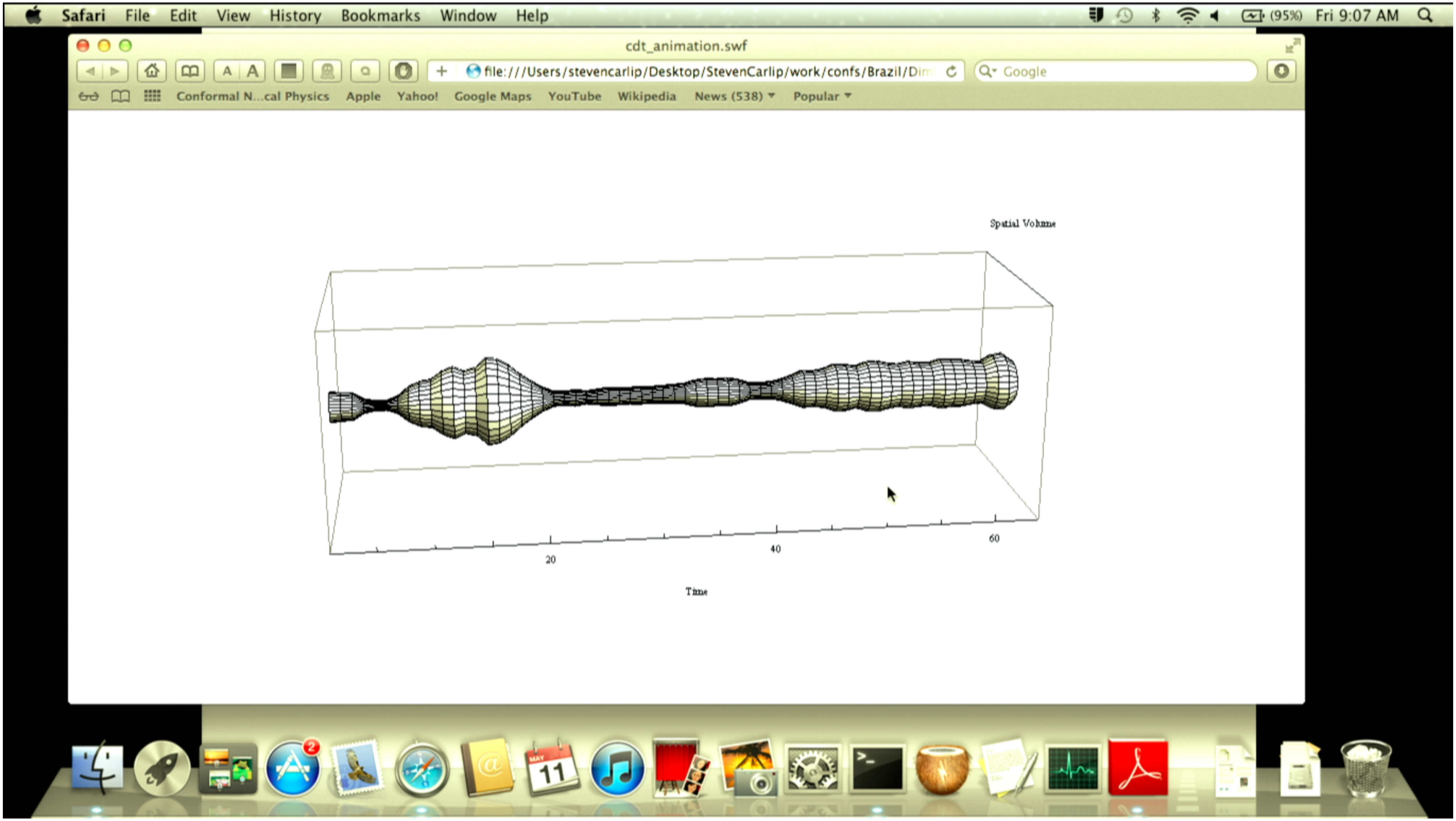
Nice “de Sitter” phase



- Volume profile fits (Euclidean) de Sitter
- Volume fluctuations fit Wheeler-DeWitt equation

But what about small scale structure?

How do you measure the “dimension” of a space that is not a nice manifold?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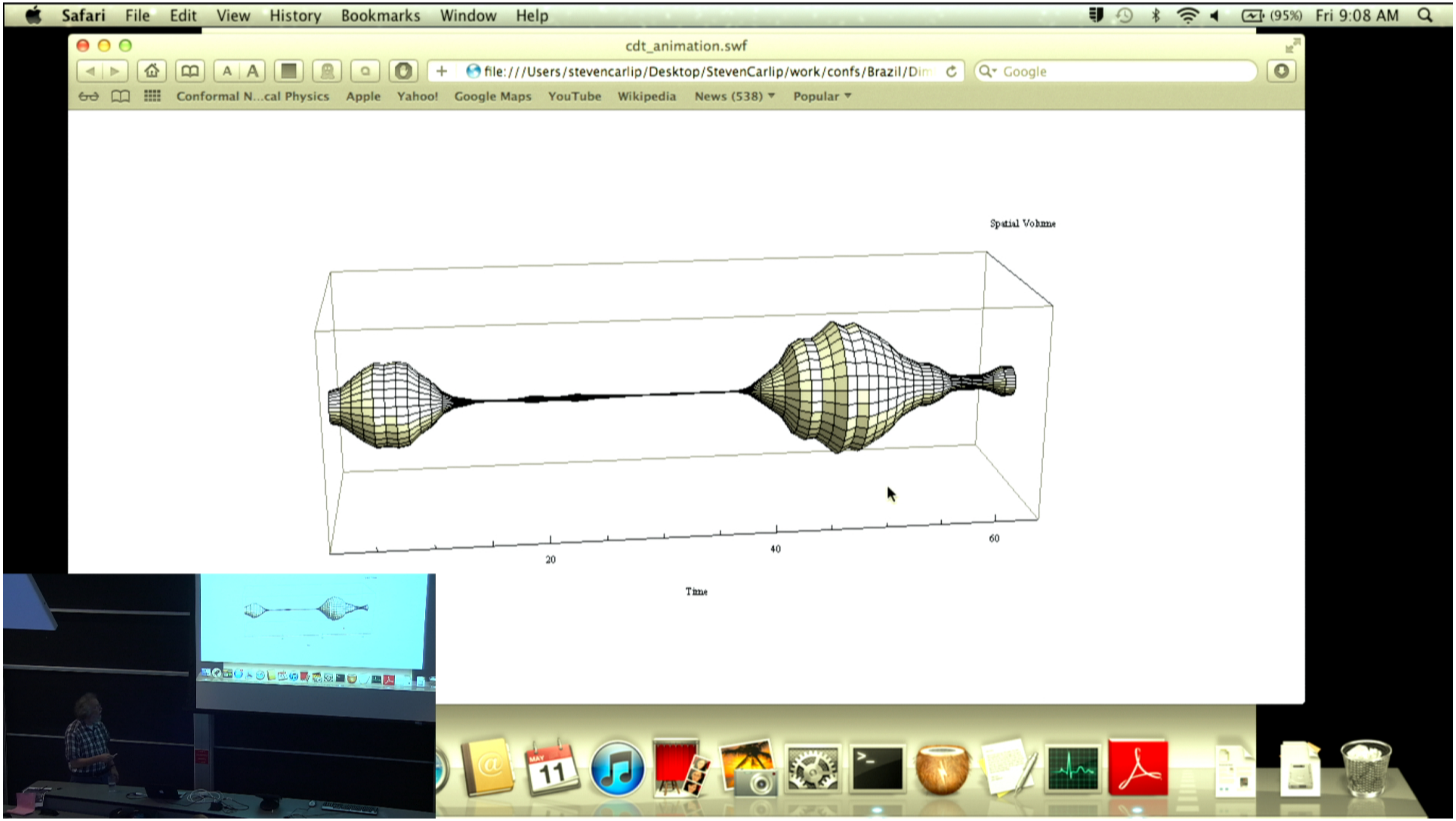
60
Time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Spectral dimension d_S: dimension of spacetime seen by random walker

Basic idea: more dimensions $\Rightarrow$ slower diffusion

Heat kernel $K(x,x';s)$: $\left(\frac{\partial}{\partial s} - \Delta_x\right) K(x,x';s) = 0$

$$K(x,x';s) \sim (4\pi s)^{-d_S/2} e^{-\sigma(x,x')/2s}\,(1+\ldots)$$

Ambjørn, Jurkiewicz, and Loll:

- $d_S(\sigma \to \infty) = 4,$
- $d_S(\sigma \to 0) \approx 2$

Propagator $G(x,x') \sim \int_0^\infty ds\, K(x,x';s) \sim \begin{cases} \sigma^{-1}(x,x') & \sigma \text{ large} \\ \log|\sigma(x,x')| & \sigma \text{ small} \end{cases}$

Short distances: characteristic behavior of a propagator for two-dimensional CFT

(Cooperman: physical scale for reduction $\sim 15\ell_p$)

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Renormalization group

Lauscher, Reuter, Niedermaier, etc.:

Look at renormalization group flow for Einstein gravity plus higher derivative terms

- Truncate effective action
- Use "exact renormalization group" methods
- Find evidence for non-Gaussian fixed point, "asymptotic safety"

At fixed point:

- anomalous dimensions ⇔ two-dimensional field theory
- propagators $\sim \log|x - x'|$ (two-dimensional CFT)
- spectral dimension $d_S \sim 2$

General argument (Percacci and Perini):

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High temperature string theory (Atick&Witten)

At high temperatures, free energy of a gas of strings is

$$F/VT \sim T \sim \textit{free energy of a 2D QFT}$$

"...a lattice theory with a (1+1)-dimensional field theory on each lattice site" (1988)

Short distance approximation

Wheeler-DeWitt equation:

$$\left\{16\pi\ell_p^2 G_{ijkl}\frac{\delta}{\delta g_{ij}}\frac{\delta}{\delta g_{kl}} - \frac{1}{16\pi\ell_p^2}\sqrt{g}\,{}^{(3)}R\right\}\Psi[g] = 0$$

"strong coupling" ($G \to \infty$) ⇔ "small distance" ($\ell_p \to \infty$) ⇔ "ultralocal"

Classical solution:

– Kasner at each point if $\ell_p \to \infty$

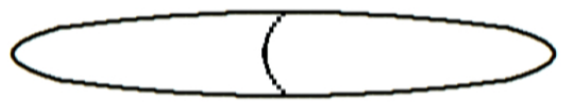
– normally BKL/Mixmaster if ℓ_p large but finite

Kasner Space is effectively (1+1)-dimensional

$$ds^2 = dt^2 - t^{2p_1}dx^2 - t^{2p_2}dy^2 - t^{2p_3}dz^2$$

Start timelike geodesic at $t = t_0$, $\mathrm{x} = 0$ with random initial velocity

Look at proper distance along each axis:



Particle horizon shrinks to line as $t \to 0$

Geodesics explore a nearly one-dimensional space!

Various approximations of heat kernel (Futamase, Berkin):

$$K(x, x; s) \sim \frac{1}{(4\pi s)^2}(1 + Qs) \quad \text{with } Q \sim \frac{1}{t^2}$$

Small t: Q term dominates, $d_S \sim 2$

[Hu and O'Connor (1986): "effective infrared dimension"]

For BKL behavior, "preferred" dimension changes chaotically in space and time

"Nonsystematic" breaking of Lorentz invariance

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Asymptotic silence?

Cosmology near generic spacelike singularity:

- Asymptotic silence: light cones shrink to timelike lines
- Asymptotic locality: inhomogeneities fall outside shrinking horizons faster than they grow

$\Rightarrow$ "anti-Newtonian" limit (as if $c \to 0$)

$\Rightarrow$ spatial points decouple; BKL behavior

Underlying physics: extreme focusing near initial singularity

Is this also true at very short distances?

In progress: investigating shape of light cones in causal dynamical triangulations (will also test Hořava-Lifshitz/anisotropc scaling model as an alternative)

Vacuum fluctuations and the Raychaudhuri equation

Expansion of a bundle of null geodesics: $\theta = \frac{1}{A}\frac{dA}{d\lambda}$

Raychaudhuri equation:

$$\frac{d\theta}{d\lambda} = -\frac{1}{2}\theta^2 - \sigma_a{}^b\sigma_b{}^a + \omega_{ab}\omega^{ab} - 16\pi G T_{ab}k^a k^b$$

Semiclassically:

– Expansion and shear focus geodesics

– Vorticity remains zero if it starts zero

– What about stress-energy tensor?

Fewster, Ford, and Roman:

Vacuum fluctuations of $T_{ab}k^a k^b$ are usually negative (defocusing)

But lower bound, long positive tail (focusing)

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– Frequent negative fluctuations will defocus geodesics, but their effect is limited
– Rare large positive fluctuations will strongly focus geodesics
– Once the focusing is strong enough, nonlinearities take over

"Gambler's ruin":

Whatever the odds, if you bet long enough against a House with unlimited resources, you always lose in the end.

Back-of-the envelope estimate:

Let $\min(T_{ab}k^a k^b) = -\mathcal{T}$
Let "smearing time" be Δt
Let ρ be the probability of a positive vacuum fluctuation with a value $> 2\mathcal{T}$

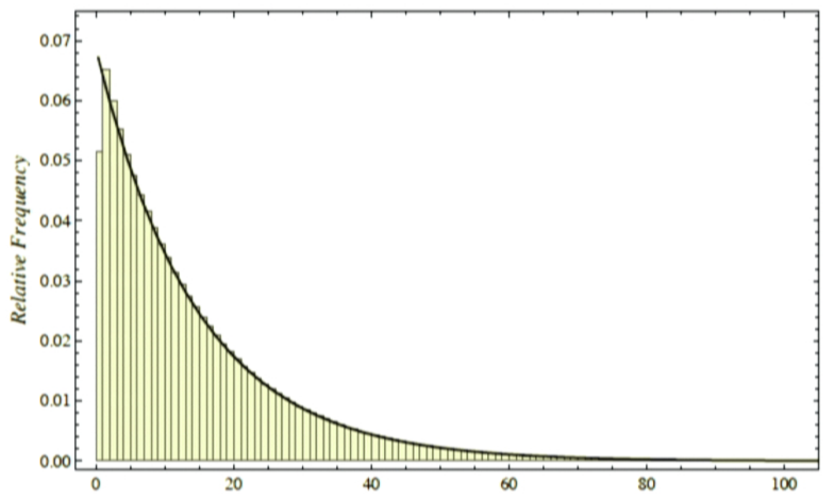
Then the time for θ to be driven to $-\infty$
is approximately described by an exponential distribution

$$\frac{\rho}{\Delta t} e^{-\rho t/\Delta t}$$

with a mean value $\sim 15.4\Delta t$

Simulation for dilaton gravity (Mosna, Pitelli, S.C.):

- Dimensionally reduce to two dimensions
- For matter: massless scalar field (central charge $c = 1$)
- Take $\Delta t = t_p$
- Assume fluctuations are independent (not quite right. . .)
- Run simulation 10 million times, measure time to $\theta \to -\infty$



Probability of the expansion diverging to $-\infty$ as a function of Planck time steps. The solid line is the exponential distribution.

Short-distance picture (at perhaps $\sim 15\ell_P$):

- short distance asymptotic silence
- "random" direction at each point in space
  - not changing too rapidly in space: regions of size $\gg \ell_p$ fairly independent
  - evolving in time; "bouncing," axes rotating, etc.
- effective two-dimensional behavior:
  dynamics concentrated along preferred direction
- Lorentz violation near Planck scale, but "nonsystematic"

But is it conformal invariant?

't Hooft, Verlinde and Verlinde, Kabat and Ortiz: eikonal approximation

$$ds^2 = \ell_{\|}^2 g_{\mu\nu} dx^\mu dx^\nu + \ell_\perp^2 h_{ij} dy^i dy^j$$

with different natural scales for the two directions.

$$I = \frac{\ell_{\|}^2}{\ell_p^2} \int d^2x d^2y \sqrt{g} \left(\sqrt{h} R_h + \frac{1}{4} \sqrt{h} h^{ij} \partial_i g_{\mu\nu} \partial_j g_{\sigma\tau} \epsilon^{\mu\sigma} \epsilon^{\nu\tau} \right)$$

$$+ \frac{\ell_\perp^2}{\ell_p^2} \int d^2x d^2y \sqrt{h} \left(\sqrt{g} R_g + \frac{1}{4} \sqrt{g} g^{\mu\nu} \partial_\mu h_{ij} \partial_\nu h_{kl} \epsilon^{ik} \epsilon^{jl} \right)$$

Suppose "transverse" derivatives ∂_i negligible;
then action looks like two-dimensional action for $h_{ij}(x)$ in background metric g

Action for h not quite conformal:

$$T \sim \Box\sqrt{h}$$

But deviation may be small.

E.g.: Kasner space near $\tau = 0$ —

$$ds^2 = \tau^{2q}(d\tau^2 - dx^2) - h_{ij}(\tau)dy^i dy^j$$

with $q = \frac{p_1}{(1-p_1)} < 0$.

Find

$$h_{11} = \tau^{2r_1}, \quad h_{22} = \tau^{2r_2}, \quad r_1 + r_2 = 1, \quad r_1 r_2 = -q$$

$$\sqrt{h} = \tau \to 0$$

Set $h_{ij} = \Omega\sigma_{ij}$ with $\det\sigma = 1$; then

$$I \sim \frac{\ell_\perp^2 A_\perp}{\ell_p^2} \int d^2x \left(\sqrt{g} R_g \Omega + \frac{1}{4} \frac{\eta^{\mu\nu} \partial_\mu \Omega \partial_\nu \Omega}{\Omega} + \frac{1}{4} \Omega \eta^{\mu\nu} \partial_\mu \sigma_{ij} \partial_\nu \sigma^{ij} \right)$$

Set $\Omega = \bar{\Omega}(1 + \varphi)$;

Then φ is approximately a Liouville field with central charge

$$c \sim \frac{A_\perp}{\ell_p^2} \qquad \text{(relation to black holes?)}$$

Question: how to deal with different “domains”?

More general picture: BKL; Yoon

Ideas from condensed matter physics?

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