Title: Relationalism

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Abstract: I shall describe Relationalism, especially in the Leibniz-Mach-Barbour sense of the word and my variations on that theme. My presentation shall give five extensions to Barbour's work: (more or less) phase space, categorization, subsystems analysis, quantization, and physics as a propositional logic (`questions about physical systems'). I shall also briefly explain how some of Crane and Rovelli's ideas do fit within this scheme, whilst others are at odds with the LMB scheme, leaving one choosing options rather thanjust considering unions.   I shall also present how scale-invariant and scaled relational particle models (the latter originally discovered by Barbour and Bertotti in 1982) can, in dimension 1 and 2, which suffice to toy-model many midisuperspace aspects of GR, be very generally solved at the following levels. 1) configuration space geometry following my fortuitous connection with Kendall's work in the statistical theory of shape involving the self-same space of shapes, and then the cone over this in the scaled case. 2) Conserved quantities and classical equations of motion. 3) Quantum equations of motion and their solutions. 4) Parallels of many Problem of Time strategies. I view this second paragraph as relevant not only by 4) but more widely by how it is a model of quantum background independence (BI), with BI being argued to be the other half to 'relativistic gravitation' in that gestalt entity known as General Relativity.

10/05/12
PI Conference

Relationalism

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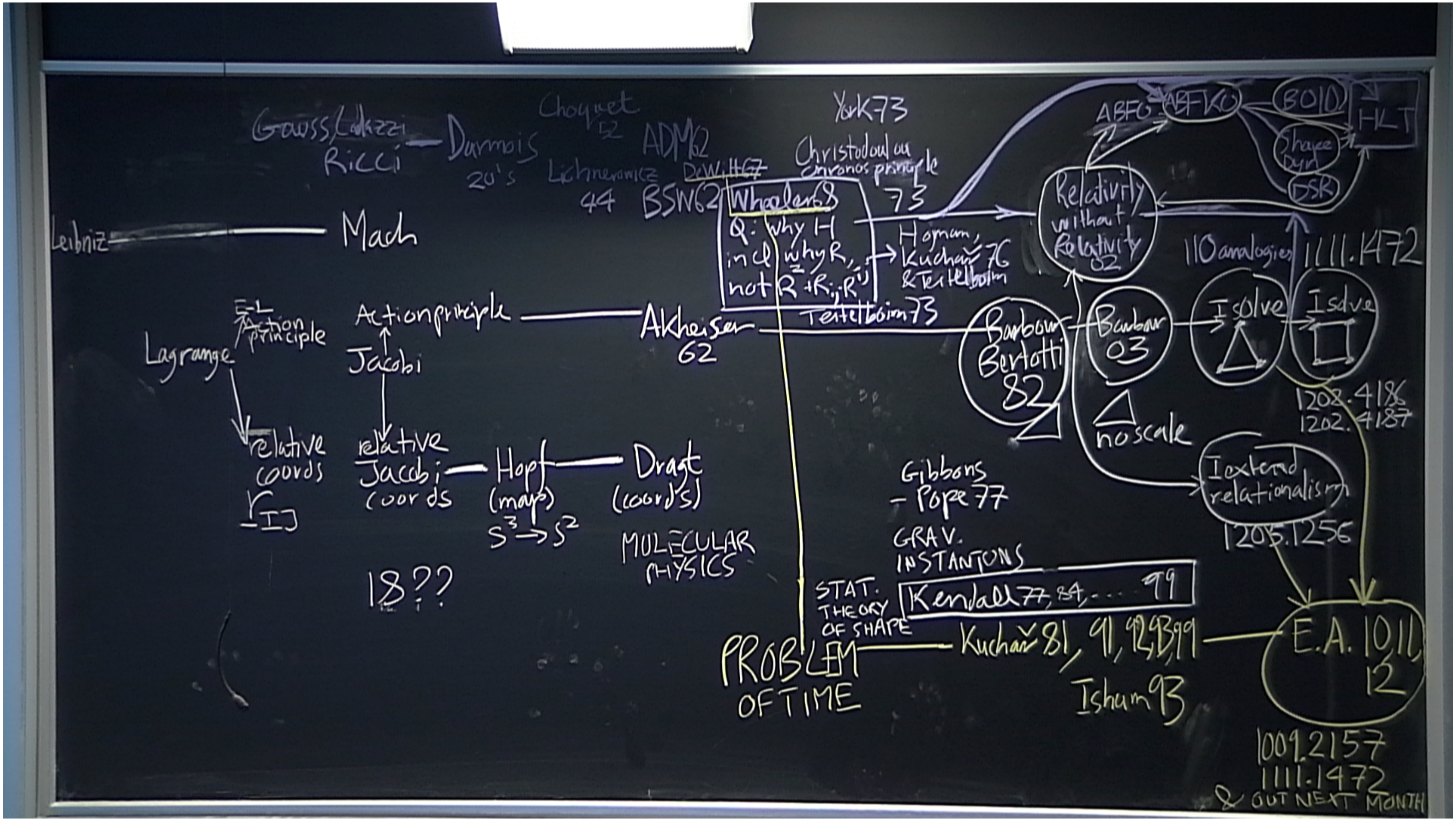
fqxi → Silicon Valley Foundation → Theiss Research → CNRS → APC.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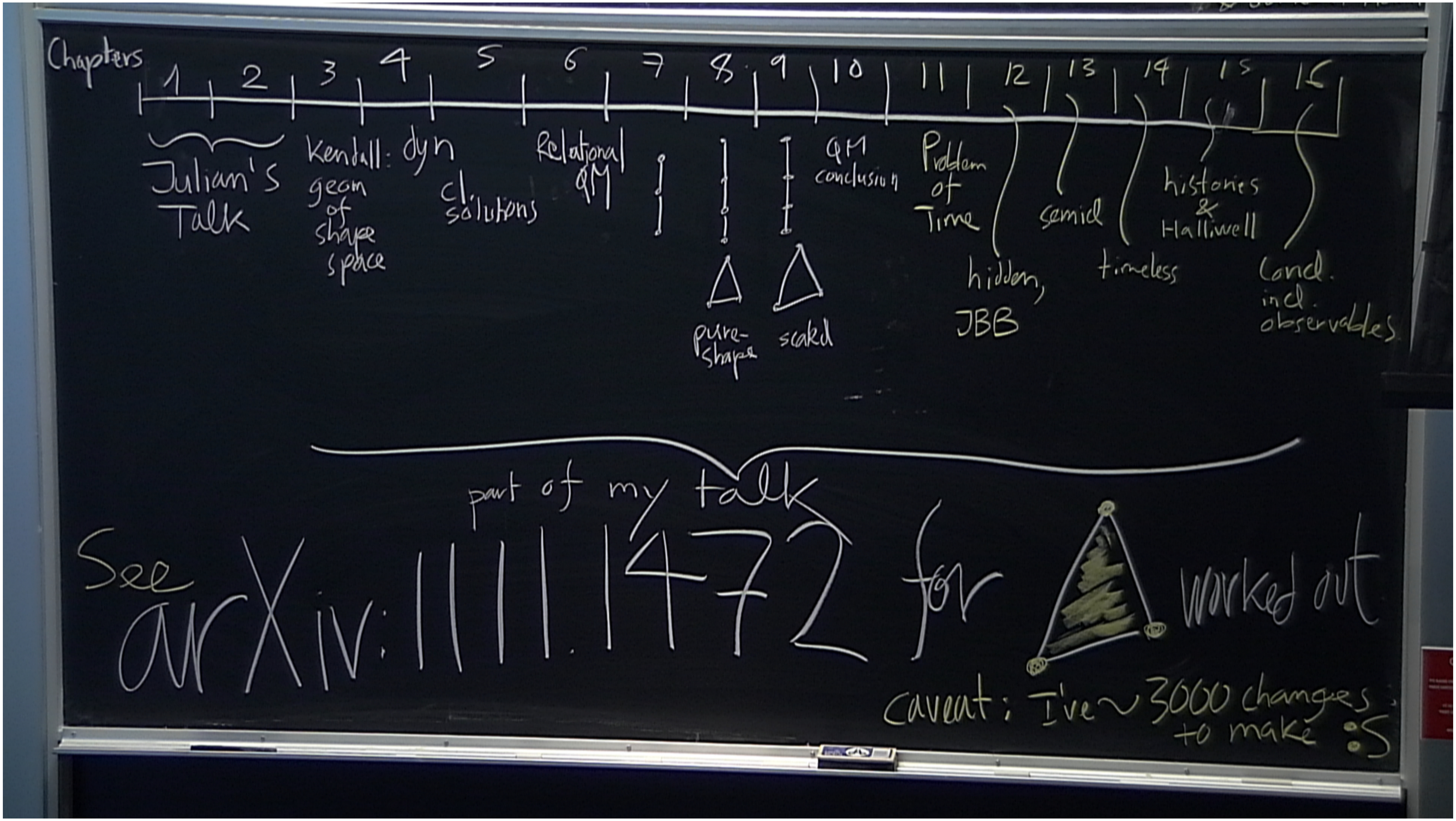
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Problem of Time
hidden, JBB
semicl
timeless
histories & Halliwell
concl. incl. observables
part of my talk
See arXiv:1111.1472 for worked out
caveat: I've ~ 3000 changes to make :S

§1 Introduction

⓪ **G.R.** = relativistic theory (⌐ Efe's.) of gravity AND a freeing from background structure.

① **Quantum G.R.** " "

but if field eq's (or replacement for) aren't (quite) E.f.e.'s,

② **Quantum Gravity** is biased toward 1st aspect

③ **Background Independent Quantum Gravity** implies splitting the 2 aspects.

④ $\boxed{\text{Quantum Gestalt}}$ —— 2 Toy models → Quantum Gravity; → Quantum Bckd Indep ⑤

• ② >> ⑤ [though QGdyn, LQG, some ideas of M-Theory are ④'s. BB82 RPM is a ⑤]

• Quantum Bckd Indep $\overset{\text{KQG?}}{\Rightarrow}$ Problem of Time.

- **Relationalism:** 1 word, many meanings.

"relative to" (but don't confuse w. relativity)

"relations 1·1 things"

- **opposite of absolutism** or of **unnec. background structures.**

so this needs to be "physical things" (better: "at least part-tangible things")

~ gauge theory is useful!

- **Leibniz-Mach-Barbour** sense
  - **act but can't be acted upon.**
  - **identity of indiscernibles**

v. different Rovelli-Crane sense

- **LMB-A sense**

LMB — **LMB-A** — RC

RC >> LMB(-A) in QG Lit.

but in fact LMB-(A) holds v. widely in Theoretical Physics.

LMB approach

gauge theory

• Time and Configuration (space + internal space)
are **heterogeneous** — each has its own relationalism.

• **Temporal relationalism**

1) Leibniz : No time for the universe as a whole
~ Not Primary.

2) Mach : Time is to be abstracted from Change.
~ Emergent

What change is quite a pre-occupation in this talk!
~ Details of how time is emergent, what form it takes.

• Implement Leibniz : MRI or MPI actions.

$\int d\lambda\, L$ (hom. of deg 1 in $\frac{dq}{d\lambda}$) so $\cancel{\frac{d}{d\lambda} d\lambda}$ or just $\int ds$

No primary t ⇒ No primary v and p defined ito ds.

$d(\text{all } q)$ "all change" in LMB.

$dq / d(\text{other } q)$

• Then JBB time emerges as simplifier.
Machian

2

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dq/d(other q) d(all q's)

• Then JBB time emerges as simplifier. "all change" in LMB.
Machian

Configurational Relationalism

③

- Configuration space q
- Group $\mathfrak{g}$ acting on q so as to make no physical difference.
- $q \to P(q, \mathfrak{g}) \to q/\mathfrak{g} = \tilde{q}$

reduced config. space.

is locally $q \times \mathfrak{g}$

wrong direction?

No. Varying wrt g's
→ 1st class constraints
→ kill $2|\mathfrak{g}|$ dofs.

could get it directly: relational config space.

at level of actions, is Barbour's Best Matching

- I did both for 1 & 2d RPM's

indirect

reduce (Barbour)

||

Jacobi-Synge (Kendall)

direct

turns a geometry into a mechanics

relationalspace approach

DAVID GEORGE KENDALL 1918–2007
Cambridge's 1st Professor of Statistics
"Founding father of British Probability"

Composing temporal & configurational relationalisms (5)

- TR, CR are logically independent. Dynamical tradition.
- want both, need to be slightly careful.

$\dot{\underline{q}} \longrightarrow \dot{\underline{q}} - \underline{\Omega} \times \underline{q}$ breaks MRI

use $\dot{\underline{q}} \longrightarrow \dot{\underline{q}} - \dot{\underline{B}} \times \underline{q}$ MRI

or $d\underline{q} \longrightarrow d\underline{q} - d\underline{B} \times \underline{q}$ MPI

- **Free end point variation ensures eq's unaffected.**

This means electric potential Φ is really a $\dot{\Phi}$
and the GR shift β^{μ} is really a $\dot{F}^{\mu}$ (velocity of the frame ~ here, grid.)

- **+ generally relationalism has a colourfully different set of Principles of dynamics**

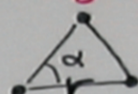
– append by cyclic vels, almost-Hamiltonian
($\dot{g}$'s ok)
almost-Dirac procedure
almost-Phase space ...

Last slide of Introduction

warning! trouser-shaped talk.

Examples : RPM's

- $q(N,d) \to r(N,d) \to P(N,d) \to S(N,d) \to \mathcal{R}(N,d)$ again; $r(N,d) \to \mathcal{R}(N,d) \to S(N,d)$
- $\mathfrak{g}$ = Eucl or Sim
  Tr, Rot   Dil
- Jacobi action



relative only

$$2\int \| dq - da - db \times q \|_M \sqrt{E - V(q^i \cdot q^j \text{ alone})}$$

flat mass metric

$$\mathcal{E} := \tfrac{1}{2}\|p\|_N^2 + V = E \quad \text{energy constraint}$$

$$\boxed{\mathcal{L} = \Sigma_I\, q^I \times p_I = 0 \quad \text{linear}}$$

$[\mathcal{P} = \Sigma_I\, p_I = 0$; $D = \Sigma_I\, q^I \cdot p_I = 0\ ]$

t^{JBB} emergent time.

Notion of clumping, inhomogeneity, structure

Geometrodynamics

$\text{Riem}(\Sigma) \to \text{CRiem}(\Sigma) \to \text{CS}(\Sigma) \to (\text{CS+V})(\Sigma)$; $\text{Riem}(\Sigma) \to \text{Superspace}(\Sigma) \to \text{CS}(\Sigma)$

$\mathfrak{g} = \text{Diff}(\Sigma)$ [and perhaps (VP) $\text{Conf}(\Sigma)$]

Baierlein-Sharp-Wheeler type (BFÓ-A) action.

$$\int \| dh - £_{dF}\, h \|_{\mathcal{M}} \sqrt{2\Lambda + \text{Ric}(h)}$$

DeWitt supermetric

$\mathcal{H} = 0$ GR Hamiltonian constraint.

$\mathcal{H}_\mu = 0$ GR momentum constraint (linear).

[$\pi = 0$ or $\pi/\sqrt{h} = 0$

t^{JBB} also

But : indefinite Kin term, GR-specific potential, field theory, Diff...

Last slide of Introduction

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Examples : RPM's

$$\begin{tikzcd}[column sep=small, row sep=small]
 & & \mathfrak{P}(N,d) \arrow[dr] & & \\
\bullet\; q(N,d) \arrow[r] & r(N,d) \arrow[ur] \arrow[dr] & & \mathfrak{S}(N,d) \arrow[r] & \mathfrak{R}(N,d) \\
 & & \mathfrak{R}(N,d) \arrow[ur] & &
\end{tikzcd}$$

again

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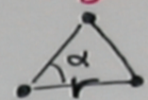

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\mathrm{Riem}(\Sigma) \arrow[ur] \arrow[dr] & & \mathrm{CS}(\Sigma) \arrow[r] & \{\mathrm{CS{+}V}\}(\Sigma) \\
 & \mathrm{Superspace}(\Sigma) \arrow[ur] & &
\end{tikzcd}$$

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t^{JBB} also

But : indefinite Kin term
GR-specific potential,
field theory, Diff ...

$$T = \frac{1}{Y} G^{abcd}_{\lambda} (\partial_F h_{ab}) \partial_F h_{cd}$$

$$V = A + BR$$

$$\mathcal{H} \approx \underbrace{(\lambda - 1)}_{\text{GR}} \underbrace{B}_{\text{strong Carroll}} \underbrace{Y}_{\text{Galileo}} D_i (\underbrace{\dot{M}^2 D^i \pi}_{\text{CMC slice.}})$$

$c = 1$ $\quad$ $c = 0$ $\quad$ $c = \infty$

e.g. arXiv 0711.0285.

§2 Relationalism Further Developed

<u>Relationalism can't pair any $\mathfrak{q}$ & $\mathfrak{g}$.</u> (2.1)

* $\dim(\mathfrak{g}) \geq \dim(\mathfrak{q}) - 1$ is trivial
* need structural compatibility
    - $\mathrm{Riem}(\Sigma)$, $\mathrm{Diff}(\Sigma)$
    - $\mathfrak{g}$ to have a natural group action on $\mathfrak{q}$
* Dirac procedure's input:
  $\mathrm{Riem}(\Sigma)$, id <u>fails</u>

but there's still <u>some</u> ambiguity in $\mathfrak{g}$ given a $\mathfrak{q}$

$$L = x \quad \text{Dirac}$$

$$0 = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) = \frac{\partial L}{\partial x} = 1$$

Barbour: q primality, at classical level.

5 Expansions

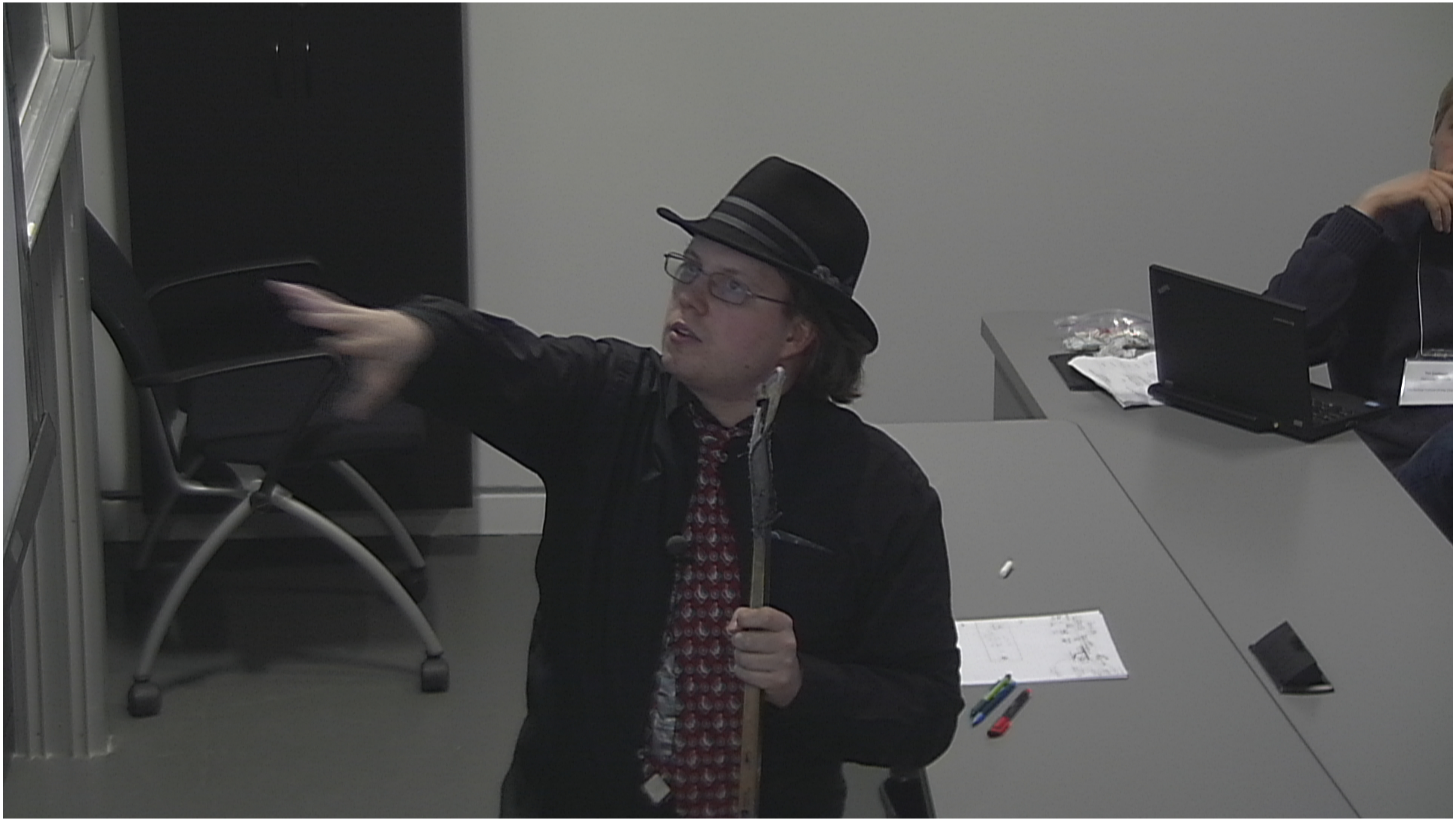
2.2

(q, point) Phase Set of Sub q's Prop( ) Quantize

And these are combineable

$(\text{Phase}, \text{Can}) \xrightarrow[\text{Quantize}]{}$,

Prop(Sub q)



(2·3)

Relational Theories

give

Frozen QM's.

$\mathcal{H}|\Psi\rangle = 0$

rather than

$\mathcal{H}|\Psi\rangle = i\frac{\partial}{\partial t}|\Psi\rangle$

$\mathcal{C} = 0 \longrightarrow \hat{\mathcal{C}}|\Psi\rangle = 0$

etc.

Both reduced & Dirac are relational.

A lot of std-relational difference wiped out

But TR, CR re-applicable.

$$\sqrt{TV} \quad \Big| \quad T - V$$

$$\mathcal{J}_H \qquad\qquad \mathcal{L}_H$$

$$L = x \qquad \text{Dirac}$$

$$0 = \frac{d}{d\lambda}\left(\frac{\partial L}{\partial \dot{x}}\right) = \frac{\partial L}{\partial x} = 1$$

Problem of Time Facets

- Frozen Formalism $\hookrightarrow$ i.p. problem.
- Best Matching Problem (formerly thin sandwich)
- Foliation dependence Problem 
- Functional Evolution problem $\{\mathcal{C}, \mathcal{C}'\} \approx 0 \not\Rightarrow [\hat{\mathcal{C}}, \hat{\mathcal{C}}'] \approx 0$
- Multiple Choice problem $QM(t_1) \underset{\text{Unitary}}{\not\longleftrightarrow} QM(t_2)$
- Global Problem of Time t_1 not valid everywhere.
- Problem of Beables (usually of observables) $\{\mathcal{L}in, O\} = 0$, $\{\mathcal{H}, O\} = 0$
- Spacetime Reconstruction (or Replacement) Problem.

Problem of Time Facets

2.4

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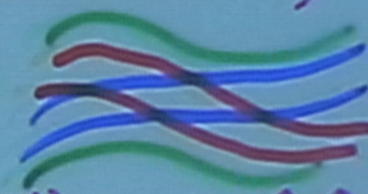
- Foliation dependence Problem 
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- Spacetime Reconstruction Problem

Problem of Time Strategies

(2.5)

Usually centred about FFP.

t^{York}

t^{Matter}

No time because ...

- Hidden? Needs Appending?
- Cl. emergent from totality of change? t^{JBB} !!????
- semicl. emergent at least in some regions? t^{WKB}
- no time at all
  what Q's can be answered
  NSI, CPI
  records ⟶ semblance of dynamics
- not time but History?

and combos!

Time is to be abstracted from what change?

Any : Rovelli

All : Leibniz–Barbour

Enough locally-relevant change : E.A.

~ generalized local ephemeris procedure

Democracy, esp in generic case.

some times are better than others.

[current need to insert "bosonic"!]

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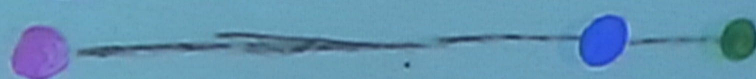

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37

- t^{York} ~ matter change can't contribute ✗
- t^{Matter} ~ tends to be unphysical
  unobservable ↔ intangible
  & rest of changes can't contribute.
- t^{scale} is <u>very bad</u> – only 1 d.o.f.

declined

- t^{JBB} has all (bosonic) change
  <u>But Doesn't unfreeze</u>
- $t^{\text{[illegible]}}$ is in fact usually scale based
  <u>but</u> that is only the 1st eq
  of an iterative procedure – all relevant
  can contribute ✓

select

(in fact t^{JBB} is //lar in practise)

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is also clarifiable by all, any etc (... suggests new scheme)

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3.8

Relationalism and POT Facets

• TR → $H = 0$ → $\hat{H}\Psi = 0$ FROZEN

• CR → $\mathcal{L}in = 0$ → $\widehat{\mathcal{L}in}\,\Psi = 0$

solving Lagrangian form of $\mathcal{L}in$ for g
is Best Matching ⊃ solving $\mathcal{M}_{\mu}$ for β^{μ}
sandwich!

• $\mathcal{L}in$, H algebra of commutators → Functional Ev. ←

• Foliation-dep is a background structure.

• RWR is classical counterpart of spacetime reconstruction

cl. Dirac Algebra kills all 3

• Kuchař or split-Dirac

3.8

Relationalism and POT Facets

• TR $\longrightarrow$ $H = 0$ $\longrightarrow$ $\hat{H}\Psi = 0$ FROZEN

• GR $\longrightarrow$ $\mathcal{L}in = 0$ $\longrightarrow$ $\hat{\mathcal{L}in}\,\Psi = 0$

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cf. Dirac Alg.
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Relationalism and POT Facets

(3.8)

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Relationalism and POT

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whilst P.O.'s are Rovelli.

Relationalism and PoT facets

• CR $\longrightarrow \mathcal{H}_{in} = 0 \longrightarrow \hat{\mathcal{H}}_{in}\Psi = 0$

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Rovelli-Crane Relationalism

(Perspectivalism)

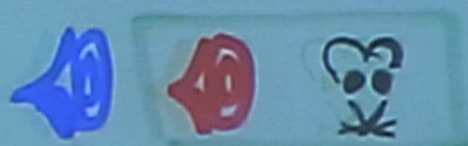
3.9

1) QM only makes sense for subsystems

(I dispute this)

Crane & my {Sub(Q)} differ slightly.

2) QM of observers observing other observers observing subsystems



3) Partial observables O_1, O_2 unphysical (no commuting) but O_1, O_2 correlations physical

4) Use anything as a time for anything else.

Rovelli-Crane Relationalism

(Perspectivalism)

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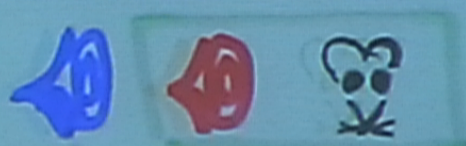
Crane & my $\{Sub(q_l)\}$ differ slightly.

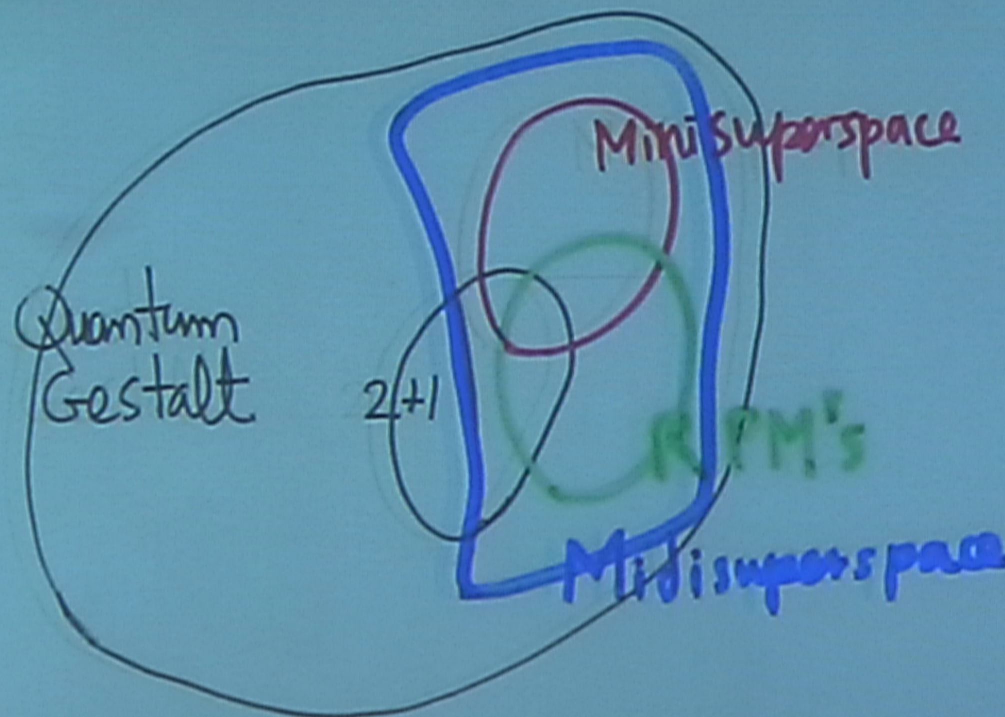
2) QM of observers observing other observers observing subsystems



3) Partial observables O_1, O_2 unphysical (no commuting) but O_1, O_2 correlations physical

4) Use anything as a time for anything else.

§3 Relational Particle Models

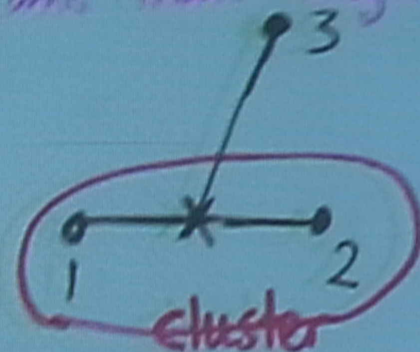


3.1

RPMs resolved

Key 1) $\forall$ models : use **Relative Jacobi Coords**

[and mass-weight them $\sim \rho_i$]



Not relative Lagrange coords

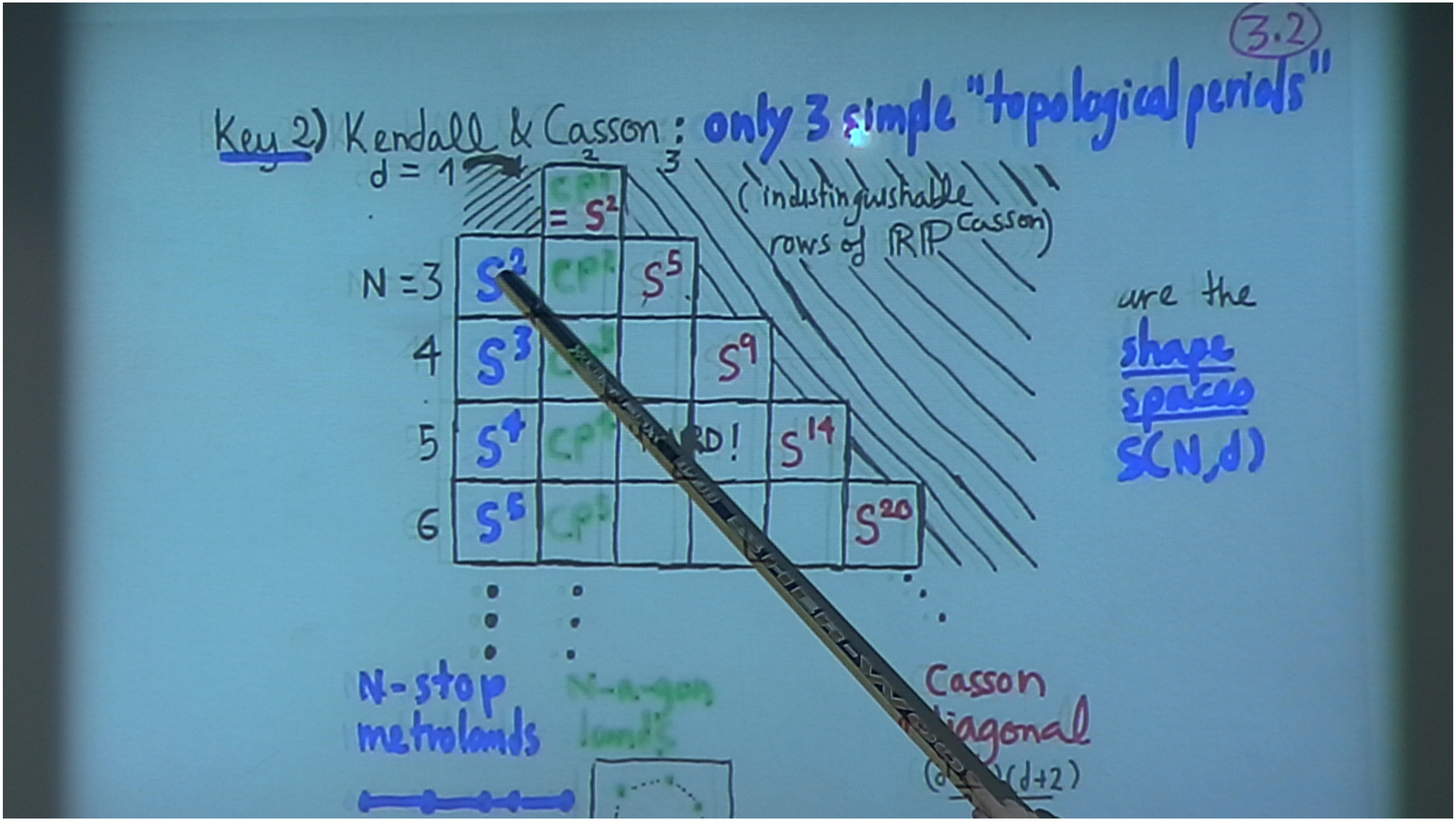
$$r_{IJ} = q_J - q_I$$

relative particle separation vectors

relative ... parti... cluster
separ... vec...

• These ... ize ... term

• ma...



3.2
Key 2) Kendall & Casson: only 3 simple "topological periods"
d = 1
2
3
(indistinguishable rows of $\mathbb{RP}^{\text{casson}}$)
$\mathbb{CP}^1 = S^2$
N = 3
S^2
S^5
4
S^3
S^9
5
S^4
S^{14}
6
S^5
S^{20}
are the shape spaces S(N,d)
N-stop metrolands
Casson diagonal

(ultra)(spherical) polars
diagonalize
shape space.

$\mathbb{S}^n$ have much
easy geometry
& linear methods...

Key 3) : at metric level

3.3

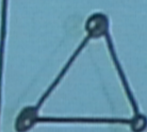
Everything else has **Preshape space**
$\mathbb{S}^{Nd-1}$

But then need to remove Rot(d)

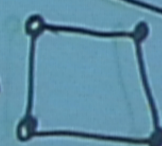
~~d ≥ 3 is far harder~~

N-agonlands: easy geom
Inhomogeneous coords,
Fubini–Study metric.

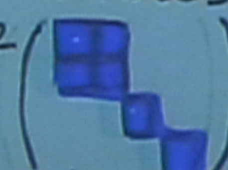
~~Casson diagonal no longer simple~~



$\mathbb{CP}^1 = \mathbb{S}^2$,
so **diagonal**



Gibbons–Pope
coordinates
are



& can't go f...

N ≥ 5 : ...
rather less
known...

Key 3): at metric level

(3.3)

o–o–o– ⋯ –o

(ultra)(spherical) polars **diagonalize** shape space.

$\mathbb{S}^n$ have much easy geometry & linear methods...

Everything else has **Preshape space** $\mathbb{S}^{Nd-1}$

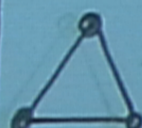
But then need to remove Rot(d)

~~$d \geq 3$ is far harder~~

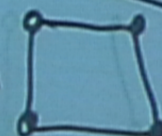
N-a-gonlands: Inhomogeneous coords, **Fubini–Study metric**.

easy geom

~~Casson diagonal no longer simple~~



$\mathbb{CP}^1 = \mathbb{S}^2$, so **diagonal**



Gibbons–Pope coordinates are (block diagonal)

$N \geq 5$: ... rather less known...

Key 3) : at metric level

(3.3)

(ultra)(spherical) polars **diagonalize** shape space.

$\boxed{\text{$\mathbb{S}^n$ have much easy geometry \& linear methods}\ldots}$ ✓

Everything else has **Preshape space** $\mathbb{S}^{Nd-1}$

But then need to remove Rot(d)

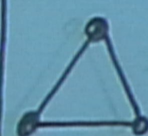
~~$d \geq 3$ is far harder~~

N-agonlands: $\boxed{\text{easy geom}}$ ✓

Inhomogeneous coords,

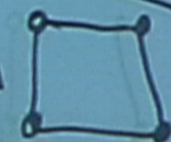
Fubini–Study metric.

~~Casson diagonal no longer simple~~



$\mathbb{CP}^1 = \mathbb{S}^2_{1/2}$,

so **diagonal** ✓



Gibbons–Pope coordinates are ( ) ✓

$\boxed{N \geq 5 : \text{rather less known}\ldots}$

Key 4) (3.4)

the cone over

Scaled Config Space = C(Shape space)

both at metric and at topological levels.

$C(S^{n-1}) = \mathbb{R}^n$

$C(\mathbb{CP}^1) = C(\mathbb{S}^2) = \mathbb{R}^3$

but $C(\mathbb{CP}^{N-2})$ $N > 3$

is sui generis.

3.5

Key 5) Coords for scaled RPM's

- N-stop:
  ρ^i magnitudes as coords

just ρ_1 here!
ρ_2
ρ_3

- also serve as subsystem-adapted

Δ : $\mathbb{R}^4$ is natural

$(\rho^1_x, \rho^1_y, \rho^2_x, \rho^2_y)$

so not easy to see how to coordinatize $\mathbb{R}^3$

In fact,

$$\mathbb{R}^4 \to \mathbb{S}^3 \to \mathbb{S}^2 \to \mathbb{R}^3$$

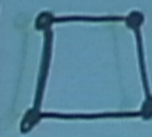
Hopf map.

Demo$_3$

[illegible] = Aniso
[illegible] = 4 x Area
[illegible] = Ellip

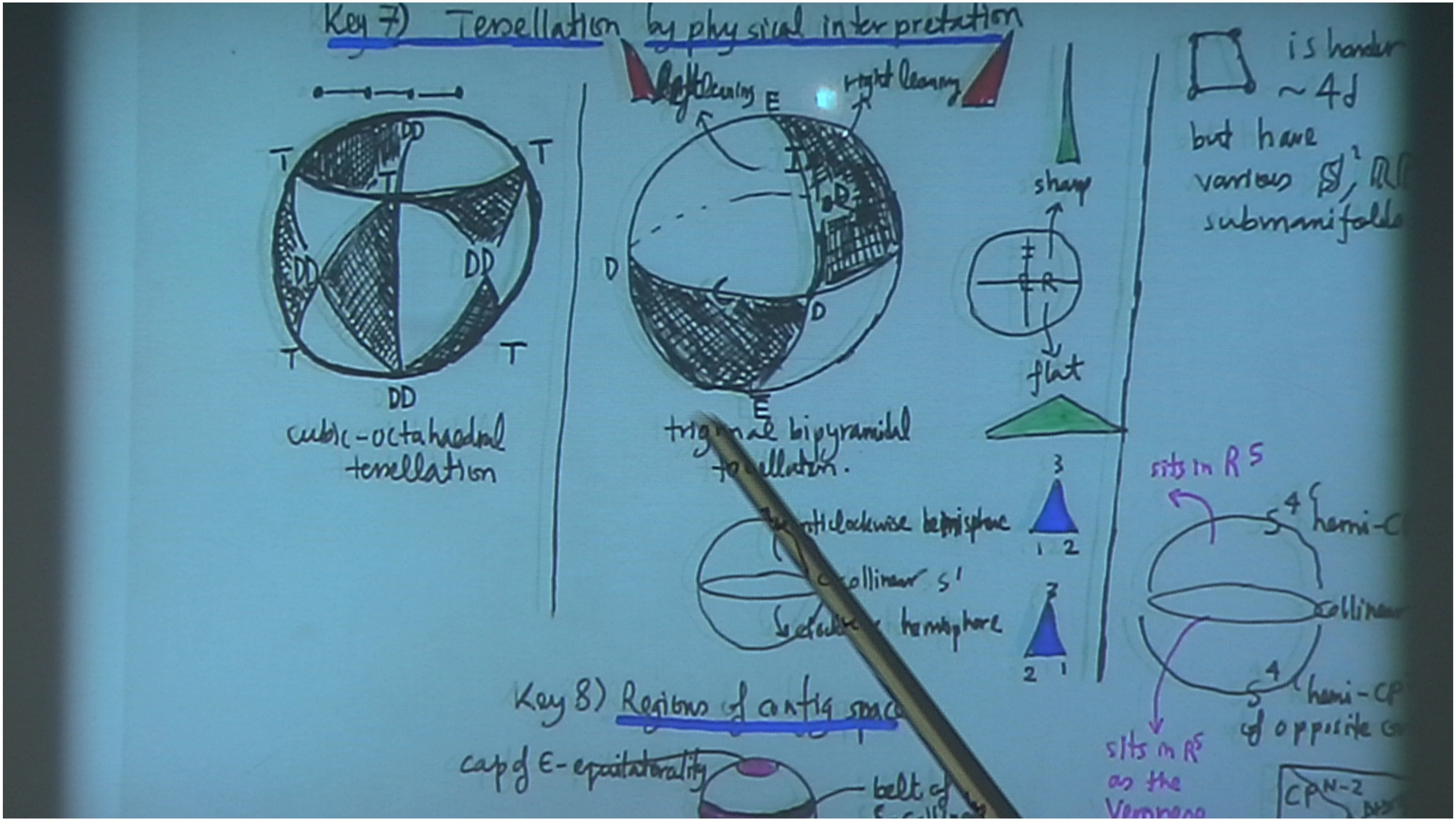
I_1, I_2, ϕ

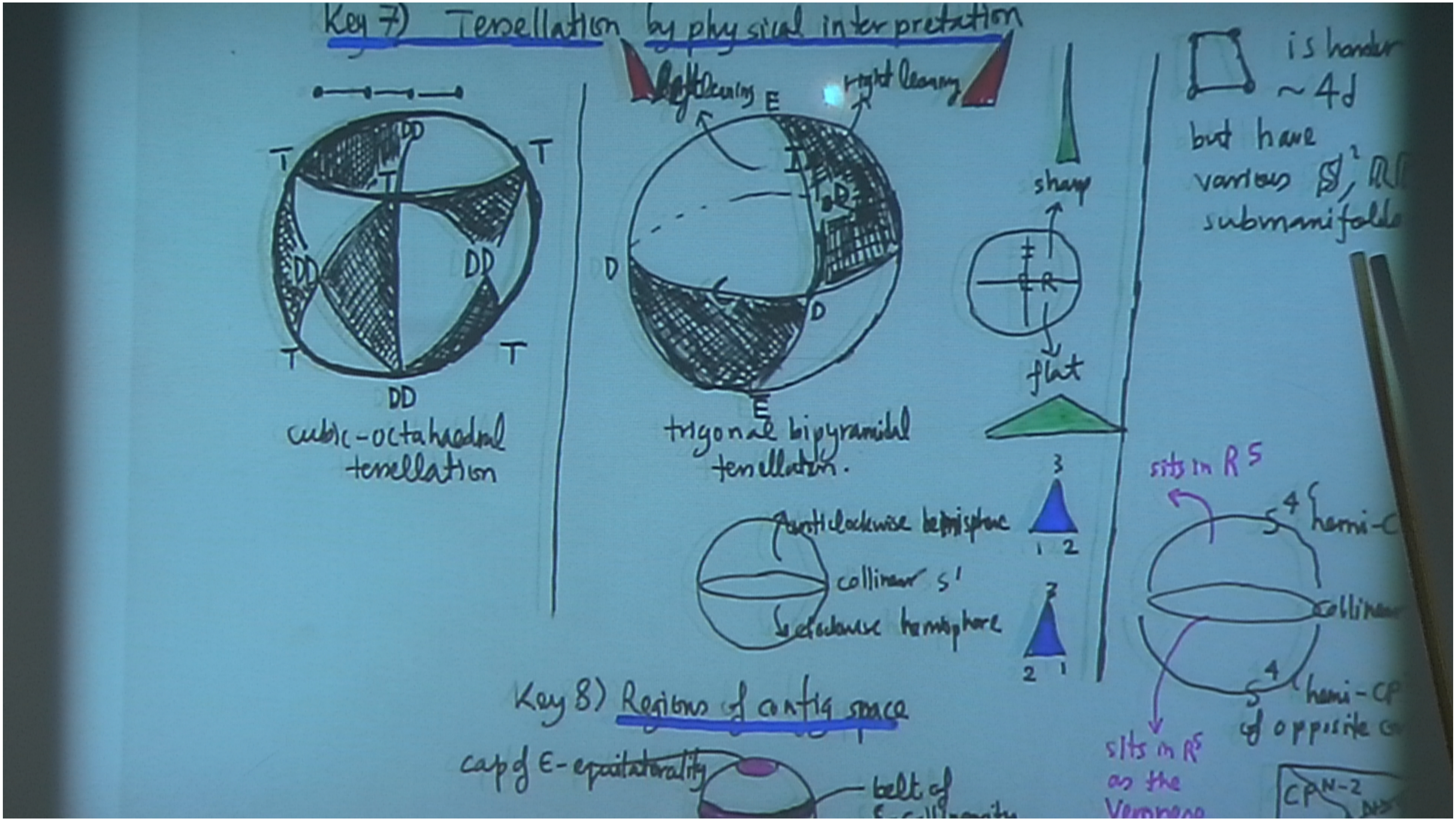
I_2

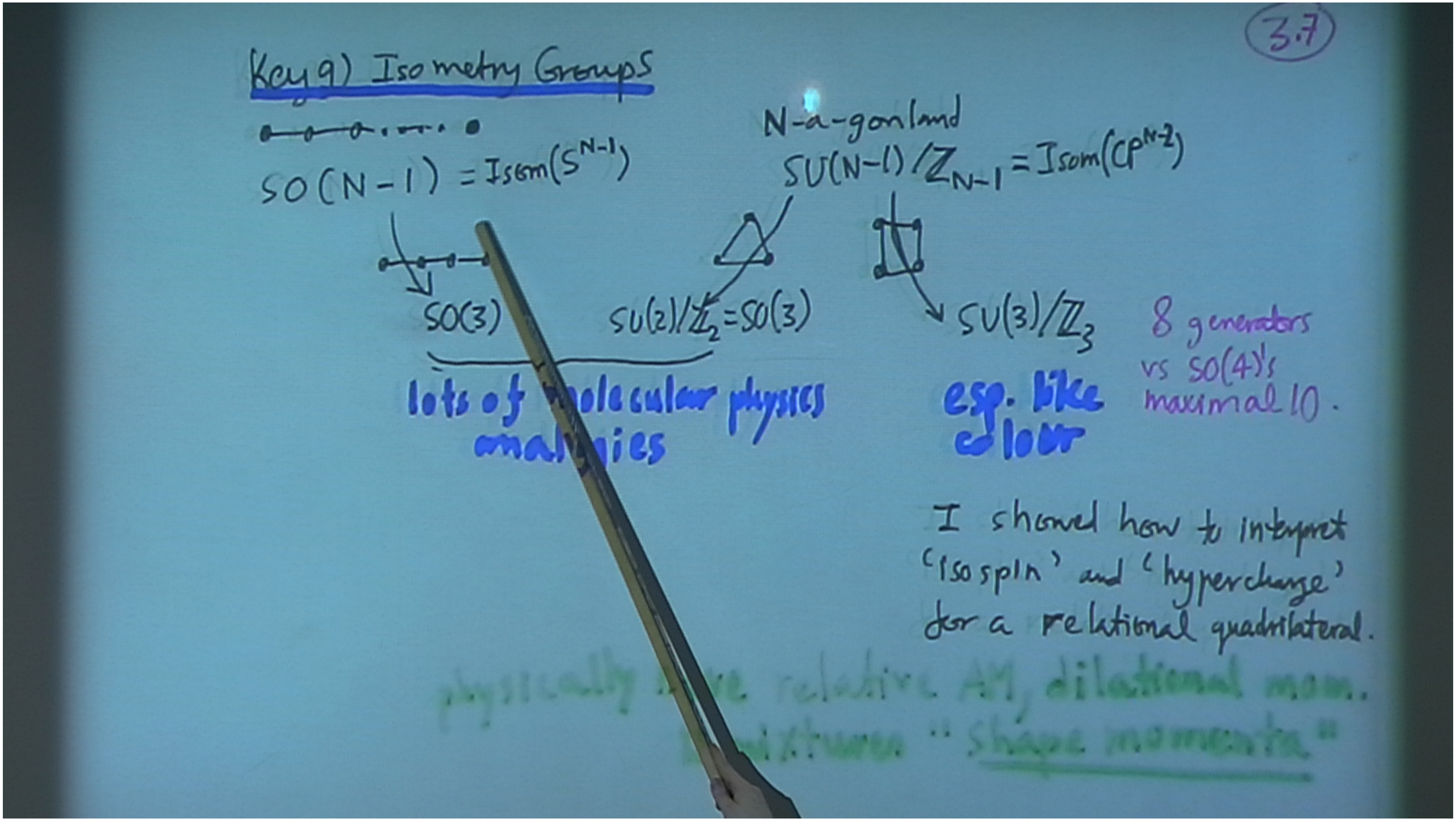


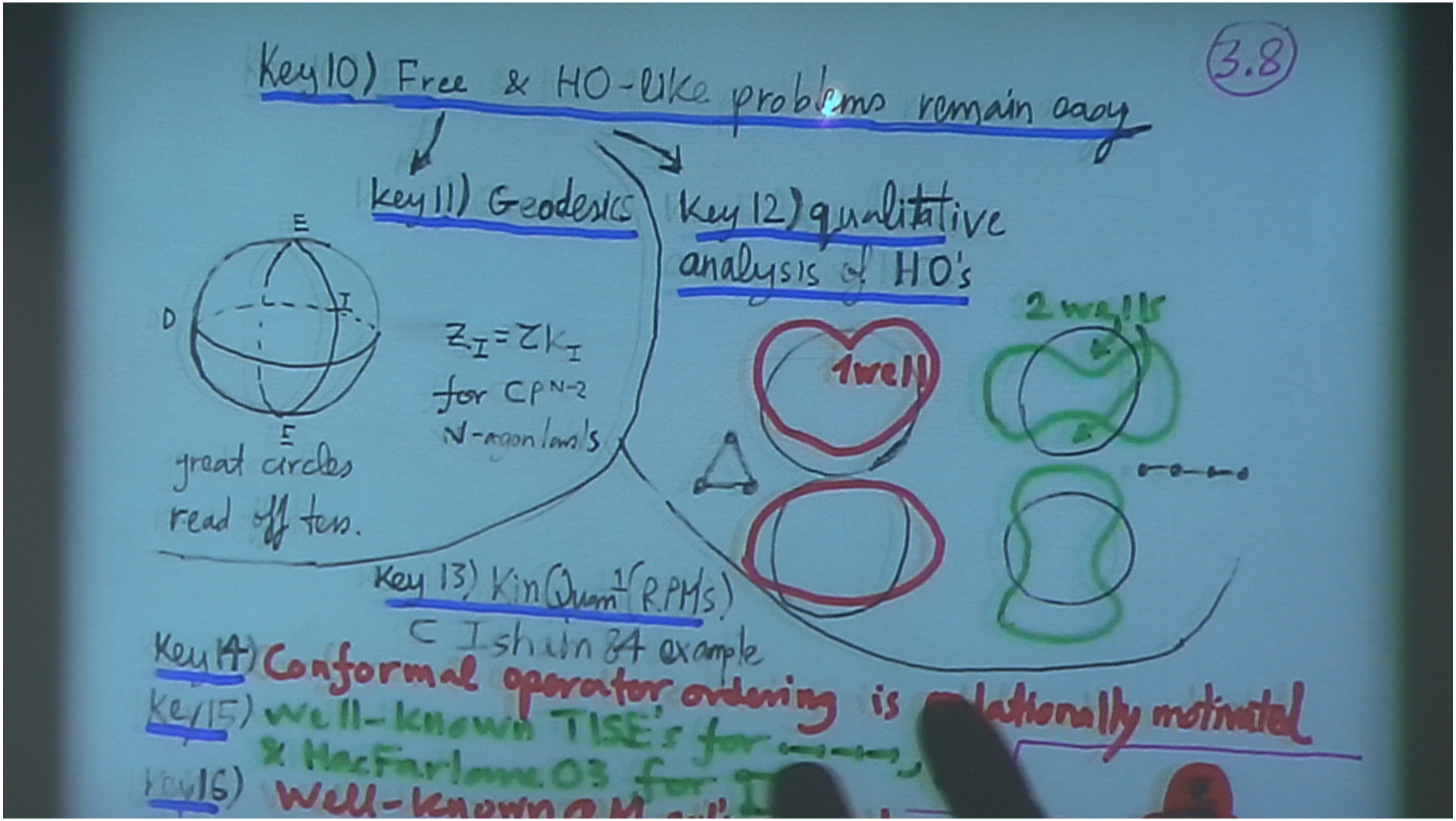
has a set of 6 such shape coords, now excluding Demo$_4$ $\propto \sqrt{\Sigma(\Delta_{area})^2}$

The nicest analogue:


Key 7) Tessellation by physical interpretation
DD
T
T
T
DD
DD
T
T
DD
cubic-octahedral tessellation
left leaning
right leaning
E
D
D
E
trigonal bipyramidal tessellation.
sharp
flat
anticlockwise hemisphere
collinear S¹
clockwise hemisphere
3
1 2
3
2 1
is harder
~4d
but have
various S², RP²
submanifolds
sits in R⁵
S⁴ hemi-CP
collinear
S⁴ hemi-CP of opposite c
sits in R⁵ as the Veronese
CP^{N-2}
Key 8) Regions of config space
cap of ε-equilateralality
belt of ε-collinear


Key 7) Tessellation by physical interpretation
left leaning
right leaning
E
D
cubic-octahaedral tessellation
trigonal bipyramidal tessellation.
sharp
flat
anticlockwise hemisphere
collinear S^1
clockwise hemisphere
is harder
~4d
but have
various S^1, $\mathbb{R}P$
submanifolds
sits in R^5
hemi-CP
collinear
sits in R^5
as the
of opposite
Key 8) Regions of config space
cap of ε-equilaterality
belt of

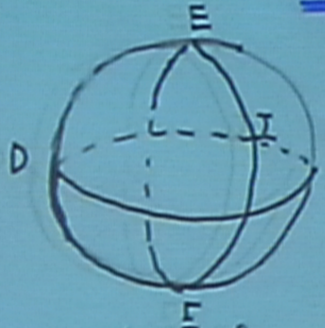

3.7
Key 9) Isometry Groups
N-a-gonland
SO(N-1) = Isom(S^{N-1})
SU(N-1)/Z_{N-1} = Isom(CP^{N-2})
SO(3)
SU(2)/Z_2 = SO(3)
SU(3)/Z_3
8 generators
vs SO(4)'s
maximal 10.
lots of molecular physics analogies
esp. like colour
I showed how to interpret 'isospin' and 'hypercharge' for a relational quadrilateral.
physically have relative AM, dilational mom.
mixtures "shape momenta"


3.8
Key 10) Free & HO-like problems remain easy
Key 11) Geodesics
E
D
I
E
great circles
read off tens.
$Z_I = Z K_I$
for CP^{N-2}
N-agon lands
Key 12) qualitative
analysis of HO's
1 well
2 wells
Key 13) Kin Quan^{1} (RPMs)
⊂ Ishan 84 example
Key 14) Conformal operator ordering is
lationally motivated
Key 15) Well-known TISE's for
& MacFarlane 03 for
Key 16) Well-known

3.8

Key 10) Free & HO-like problems remain easy

Key 11) Geodesics



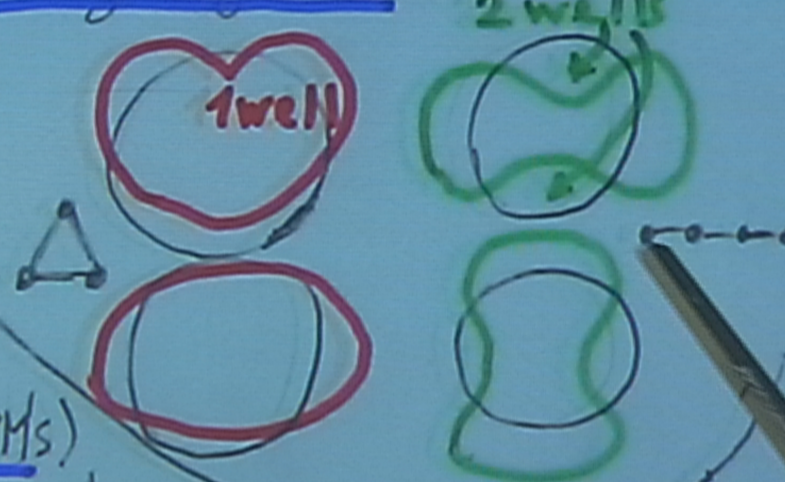
$Z_I = Z K_I$

for CP^{N-2}

N-agonland's

great circles read off tess.

Key 12) qualitative analysis of HO's



Key 13) Kin Quan^1 (RPMs)

⊂ Ishain 84 example

Key 14) Conformal operator ordering is relationally motivated

Key 15) well-known TISE's for —•—•—•, Δ

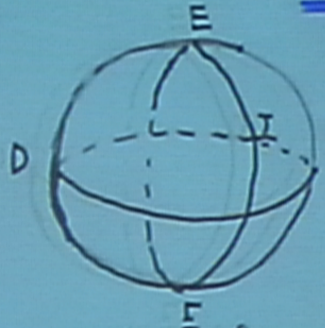
& MacFarlane 03 for □

Key 16) Well-known ...

3.8

Key 10) Free & HO-like problems remain easy

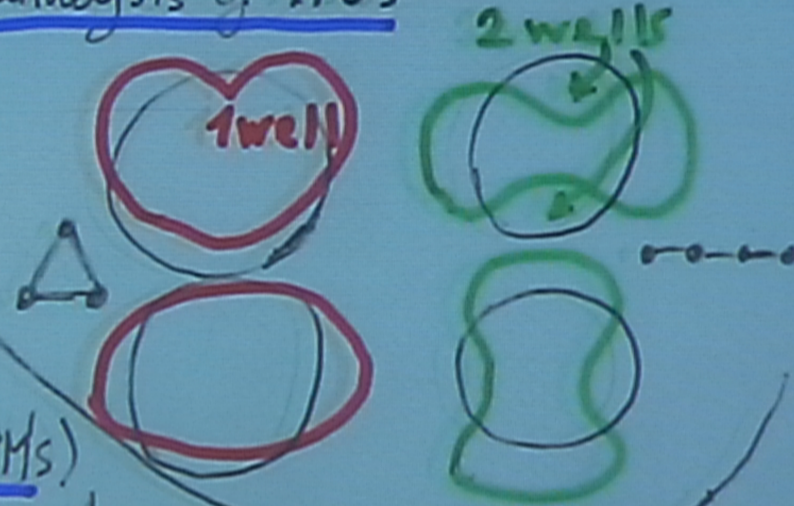
Key 11) Geodesics



great circles read off tens.

$Z_I = Z K_I$ for $\mathbb{C}P^{N-2}$ N-agonland's

Key 12) qualitative analysis of HO's



Key 13) Kin Quan^{-1} (RPM's) ⊂ Isham 84 example

Key 14) Conformal operator ordering is relationally motivated

Key 15) Well-known TISE's for , Δ & MacFarlane 03 for

Key 16) Well-known

Problem of Time Facets in RPM's

(3.9)

- Frozen formalism ✓ i.p. × (via def)
- ~~Best matching problem~~ ✓ and solved (1-; 2-d)
- Foliation Dependence problem ×
- Functional Evolution problem ×
- Spacetime reconstruction problem ×
- ~~Problem of Observables~~ ✓ and solved (1-, 2-d)
- Multiple Choice problem ✓
- Global Problem of time ✓

} at least some of this can be taken out by patching?

RPM Problem of Time Strategies

(3.10)

GR	RPM
$t^{York} \propto \frac{h_{ij}\pi^{ij}}{\sqrt{h}}$ ≈	$t^{Euler} = \sum_i \rho^i \pi_i$ ✓
reference fluid	reference particles ?
unimodular 'resolution'	fails to work for RPM ✗ (no criss-cross)
superspace time	" " ✗ (+ve def T)
[t^{JBB}	t^{JBB}]
semiclassical	✓
NSI	✓
CPI	✓
Records	✓
Histories	✓
Partial Observables	✓
Combinations	✓

Why I put effort into the model.

I do GR anyway

RPM Problem of Time Strategies (3.10)

GR	RPM	
t^{York} or $\frac{h_{ij}\pi^{ij}}{\sqrt{h}}$ $\approx$	$t^{\text{Euler}} = \sum_i \rho^i \pi_i$	✓
reference fluid	reference particles	?
unimodular 'resolution'	fails to work for RPM	✗ (no cross-cross)
superspace time	" "	✗ (+ve def T)
[t^{JBB}	t^{JBB}	]
semiclassical		✓
NSI		✓
CPI		✓
Records		✓
Histories		✓
Partial Observables		✓
[Combinations		✓

Why I put effort into the model.

RPM Problem of Time Strategies

(3.10)

GR	RPM	
t^{York} or $\frac{h_{ij}\pi^{ij}}{\sqrt{h}}$ $\approx$	$t^{Euler} = \sum_i \rho^i \pi_i$	✓
reference fluid	reference particles	?
unimodular 'resolution'	fails to work for RPM	× (no cross-cross)
superspace time	" "	× (+ve def T)
[t^{JBB}	t^{JBB}	]
Semiclassical		✓
NSI		✓
CPI		✓
Records		✓
Histories		✓
Partial Observables		✓
[Combinations		✓

I disagree anyway

Why I put effort into the model.

RPM Problem of Time Strategies (3.10)

GR	RPM	
t^{York} or $\frac{h_{ij}\pi^{ij}}{\sqrt{h}}$ $\approx$	$t^{\text{Euler}} = \sum_i \rho^i \pi_i$	✓
reference fluid	reference particles	?
unimodular 'resolution'	fails to work for RPM	✗ (no criss-cross)
superspace time	" "	✗ (+ve def T)
[t^{JBB}	t^{JBB}	]
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NSI		✓
CPI		✓
Records		✓
Histories		✓
Partial Observables		✓
[Combinations		✓

Why I put effort into the model.