Title: 2+1 gravity as a conformal gauge theory and some frontiers for Shape Dynamics

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Abstract: I will start by showing that gravity, with positive cosmological constant in 2+1 dimensions, can be formulated as a theory of dynamic conformal spatial geometry. Exploiting the isomorphism between the isometry group of de Sitter space in D+1 dimensions and the conformal group in D dimensions, I will reinterpret the Chern--Simons formulation of 2+1 gravity as a gauge theory of a conformal connection. In Cartan's generalization of geometry, this connection represents an evolving spatial geometry locally modeled off the conformal sphere. After a suitable phase space reduction, we obtain shape dynamics. This remodeling explains, in 2+1 dimensions, the remarkable success of the York procedure for solving the initial value problem of general relativity and the uniqueness of the shape dynamics Hamiltonian. I will finish by speculating about possible connections between this work and the general shape dynamics program with holographic renormalization, AdS/CFT, and Horava gravity.

2+1 gravity as a conformal gauge theory
and some frontiers for Shape Dynamics

Sean Gryb

Universiteit Utrecht

Institute for Theoretical Physics, Utrecht University

Conformal Nature of the Univserve
Perimeter Institute, Waterloo
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Collaborators

- $2+1$: F. Mercati. (paper to appear soon)
- Large Volume: H. Gomes, T. Koslowski, F. Mercati. (arxiv:1105.0938)
- General: J. Barbour, L. Smolin.

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A well–known isomorphism

Shape Dynamics:

Spacetime symmetry is *traded* for spatial scale invariance.

$\therefore$ many fingered *time* + global *scale* $\Leftrightarrow$ many fingered *scale* + global *time*

Relation between $g_{ab} \to e^{\phi} g_{ab}$ and $Conf(D)$?

Consider:

Isomorphism:

- Isometry group of $\mathrm{dS}^{D,1} = SO(D+1,1) = Conf(D,0)$.
- Isometry group of $\mathrm{AdS}^{D,1} = SO(D,2) = Conf(D-1,1)$.

Can this be used to get SD?

$2+1$: Yes!

- Isomorphism $\to$ spacetime geometry $\sim$ spatial "conformal" geometry.
- Key: relation between different *Cartan geometries*.
- Dynamic "conformal" geometry $\Rightarrow$ SD constraints.

Higher dimensions? Gauge/gravity duality?

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HJ equation in large volume limit

The large volume expansion (Gomes, Koslowski, Mercati, sg '11)

The SD Ham can be expanded in terms of $1/V^{1/D}$:

$$H_{sd} = 2\Lambda - \frac{D}{4(D-1)}\langle\pi\rangle^2 - R_{\text{Yamabe}}\left(\frac{V}{V_0}\right)^{-2/D} + \left\langle\pi^{TT}\cdot\pi^{TT}\right\rangle\left(\frac{V}{V_0}\right)^{-2} + \dots$$

for trajectories that reach $V \gg V_0$.

$\Rightarrow$ diff–invariant def'n of dS boundary + boundary is **conformal**.

HJ eq'n (in 3 + 1, asymptoticly homogeneous)

The HJ functional $\rightarrow$ solved for *all* constraints: (Yamabe trick)

$$S = \pm\left(\sqrt{\frac{16}{3}\Lambda}V - \sqrt{\frac{3}{\Lambda}}R_{\text{Yamabe}}V^{1/3} + \dots\right).$$

Structure of holographic RG counter–terms (Skenderis '00) but in V–expansion.

HJ = RG flow equation?

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AdS vs dS

Can translate results to AdS.

How?

- Use isomorphism $\mathrm{AdS}^{(D,1)}$ $Conf(D-1,1)$.
- Go to Euclidean $\Rightarrow$ radial hypersurfaces $=$ Cauchy surfaces.
- Use **radial** Ham constraint (minus sign between V and T).

Arguments *should* go through for $2+1$.

Boundaries:

- dS $\rightarrow$ spacelike infinite past.
- AdS $\rightarrow$ "timelike" spatial infinity.

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Cartan geometry

Hamster ball: (source: D. Wise, gr-qc/0611154)

Roll homogeneous space over manifold.
⇒ point of contact stabilizes symmetries of hamster ball.

Cartan geometry

Generalized hamster "ball"! (local best matching)

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Cartan geometry II

Ingredients

- Gauge group G (eg., $SO(3)$)
- Subgroup $H \subset G$ (eg., $SO(2)$)
  $\Rightarrow$ Homogeneous space $Y = G/H$ (eg., $\mathbf{S}^2$).
- Manifold $\mathcal{M}$ st $\dim(\mathcal{M}) = \dim(Y)$.

Geometry: given by 'rolling' Y over $\mathcal{M}$.
$\Rightarrow$ $\mathfrak{g}$–valued connection, A, on a principle right H–bundle over $\mathcal{M}$.
$\Rightarrow$ metric: projection of A on $\mathfrak{g}/\mathfrak{h}$. (non–degenerate)

Riemannian geometry

$G = ISO(p,q)$; $H = SO(p,q)$; $Y = R^{p,q}$.

$$A = e^a P_a + \omega^a J_a$$

$$F(A) = T^a P_a + R^a J_a$$

e^a - coframe field; ω^a - spin connection; T^a - Torsion; R^a - Curvature.

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$2+1$ Chern–Simons gravity

Define a Cartan geometry

- $G = ISO(2,1), SO(2,2), SO(3,1), \quad k = 0, -1, +1$
- $H = SO(2,1), \therefore \dim(\mathcal{M}) = 2+1.$

$$\Rightarrow A = e^{\alpha} P_{\alpha} + \omega^{\alpha} J_{\alpha} \qquad\qquad F = T^{\alpha} P_{\alpha} + \Omega^{\alpha} J_{\alpha}$$

$$\Omega^{\alpha} = R^{\alpha} + \frac{k}{l^2} \epsilon^{\alpha\beta\gamma}(e_{\beta} \wedge e_{\gamma})$$

Action (Witten '89)

Palatini action $\rightarrow$ Chern–Simons for A:

$$S_{\text{Palatini}}(e,\omega) = S_{\text{CS}}(A) = \frac{k'}{4\pi} \int \text{Tr}\left(A \wedge dA + \frac{2}{3} A^3\right)$$

EOMs: Vanishing curvature

$$T^{\alpha} = 0 \quad \text{(Torsionless)} \qquad\qquad R^{\alpha} = \frac{k}{l^2} \epsilon^{\alpha\beta\gamma}(e_{\beta} \wedge e_{\gamma}) \quad \text{(EEs)}$$

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$2+1$ split and ADM

Decompose action:

$$S_{CS} = \frac{k'}{4\pi}\int dt \int d^2x\,[2\dot{e}^{\alpha} \wedge \omega_{\alpha} + e_0^{\alpha}\Omega_{\alpha} + \omega_0^{\alpha} T_{\alpha}] \tag{1}$$

Read off:

- Symplectic structure: $\{e_{\mu}^{\alpha}(x), \omega_{\nu}^{\beta}(y)\} \propto \epsilon_{\mu\nu}\eta^{\alpha\beta}\delta(x,y)$
- Lagrange multipliers: $A_0^{\alpha} = (e_0^{\alpha}, \omega_0^{\alpha})$.
- Constraints: $T^{\alpha} = 0$ and $\Omega^{\alpha} = 0$
- Symmetries:
  - $\Omega \to$ spacetime translations (diffs on–shell).
  - $T \to SO(2,1)$ rotations.

ADM

Phase space reduction:

- Gauge fix $T^a = 0$ with $e_i^0 = 0$.
- $\omega_i^0 \to$ metric compatible $SO(2)$ connection.
- Variables left: e_i^a, ω_i^a (spatial)
- Constraints left: Ham + 2–diffs (Ω^{α}), $SO(2)$ Gauss constraint (T^0).

Isomorphism

Note:

$$\boxed{A_0 = \text{Lagrange multiplier}}$$

New decomposition:

$$\begin{aligned} A_i &= e_i^\alpha P_\alpha + \omega_i^\alpha J_\alpha \\ &= E_i^a p_a + B_i^a k_a + \omega_i j + \phi_i D \end{aligned}$$

Use isomorphism

$$M = \begin{pmatrix} 0 & j & -\frac{1}{\sqrt{2}}(p_1 + k_1) & \frac{1}{\sqrt{2}}(p_1 - k_1) \\ -j & 0 & -\frac{1}{\sqrt{2}}(p_2 + k_2) & \frac{1}{\sqrt{2}}(p_1 - k_1) \\ \frac{1}{\sqrt{2}}(p_1 + k_1) & \frac{1}{\sqrt{2}}(p_2 + k_2) & 0 & D \\ -\frac{1}{\sqrt{2}}(p_1 - k_1) & \frac{1}{\sqrt{2}}(p_2 - k_2) & -D & 0 \end{pmatrix}.$$

Then

$$E_i^a = \frac{1}{\sqrt{2}}\left(\epsilon^{ab}\omega_i^b + e_i^a\right) \qquad \phi_i = e_i^3$$

$$B_i^a = \frac{1}{\sqrt{2}}\left(\epsilon^{ab}\omega_i^b - e_i^a\right) \qquad \omega_i = \omega_i^3.$$

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Shape dynamics

$$F = T^a p_a + S^a k_a + \Omega j + \Theta D$$

New structure

- Symplectic structure:

$$\left\{E_i^a(x), B_j^b(y)\right\} \propto \frac{1}{2}\epsilon^{ab}\epsilon_{ij}\delta(x,y),\ \{\phi_i(x), \omega_j(y)\} \propto \epsilon_{ij}\delta(x,y).$$

- Symmetries:
  - $T \to$ special conformal transformations.
  - $S \to$ translations (2 diffs on–shell).
  - $\Omega \to SO(2)$ rotations.
  - $\Theta \to$ dilatations.

SD

Gauge fixing (inspired by Witten '89)

- Gauge fix T^a with $\phi_i = 0$.
- $\omega \to SO(2)$ spin connection.
- Variables left: E_i^a, B_i^a (dynamic spatial conformal geometry)
- Constraints left: S, Ω, Θ (SD!!)

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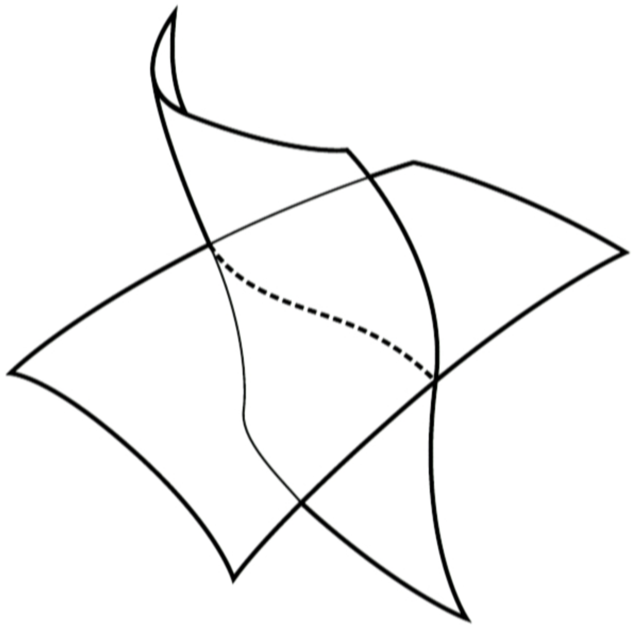
Picture on ADM phase space

2 intersecting surfaces (full symmetry trading)



Outlook

- Extend to AdS.
- Extend to $\Lambda = 0$. ($2+1$: same trace)
- Extend to $D+1$. (Stabilizer, action, etc...)
- gauge/gravity: pullback into bulk, a nice set of variables?

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Extensions: Construction principle for SD?

2 Requirements:

The SD Hamiltonian is constructed from:

1. Local scale invariance.
2. Dynamical equivalence to GR (or locality).

But:

- Observables are non–local $\rightarrow$ locality in bulk?
- SD should stand *on its own*.

Idea

Local description at conformal fixed point (boundary).
$\Rightarrow$ Time evolution given by holographic RG flow.

Interesting observation: (particle) shape space is holographic.

Can we use ideas from entropic gravity or Jacobson's argument?

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