Title: Towards Conformal Degrees of Freedom in CDT

Date: May 09, 2012 09:10 AM

URL: http://pirsa.org/12050062

Abstract: TBA

(conformal aspects of (quantum) gravity, and CDT

- set the stage
- (generalized) Wheeler-DeWitt (WdW) metric, "little λ"
- potential pitfalls
- effective actions for QG ($\Leftrightarrow$ phase diagram CDT)
- beyond the conformal mode

Collabs: J. Ambjørn, J. Jurkiewicz, A. Görlich, T. Budd, S. Jordan, L. Pires

→ new Physics Report on CDT (arXiv 1203.3591)

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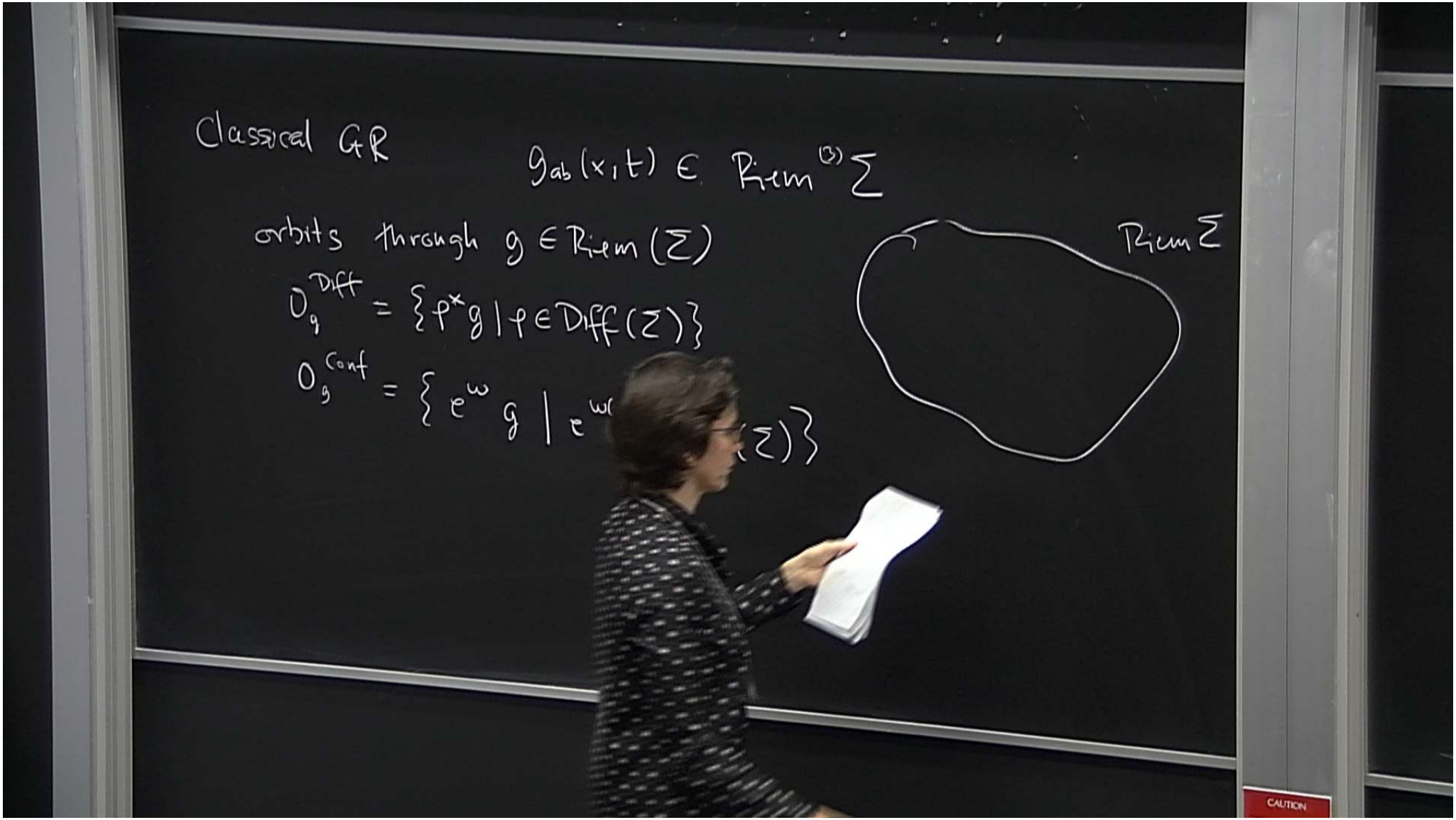
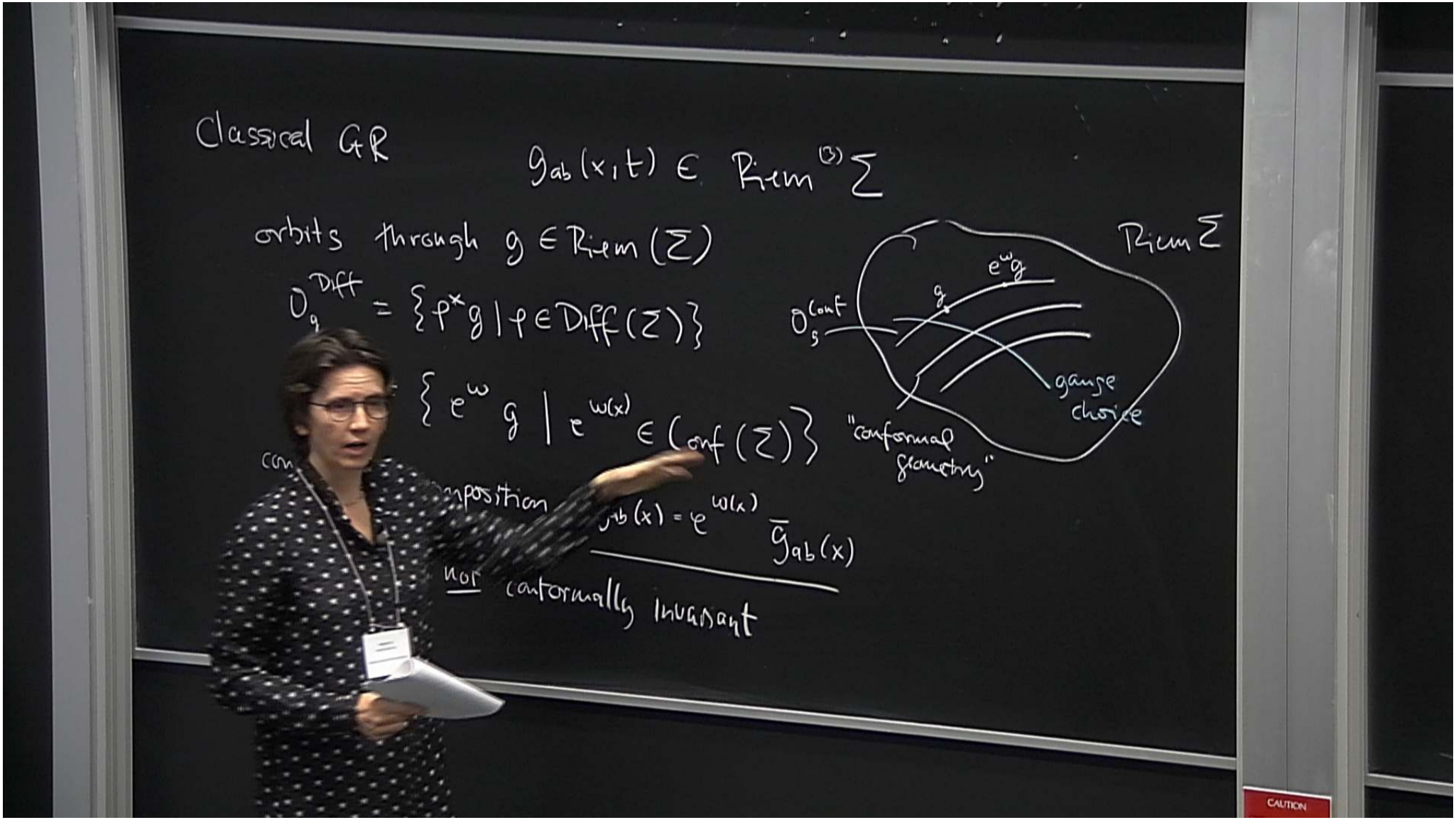

Classical GR
$g_{ab}(x,t) \in \mathrm{Riem}^{(3)}\Sigma$
orbits through $g \in \mathrm{Riem}(\Sigma)$
$\mathcal{O}_g^{\mathrm{Diff}} = \{\varphi^* g \mid \varphi \in \mathrm{Diff}(\Sigma)\}$
$\mathcal{O}_g^{\mathrm{Conf}} = \{e^{\omega} g \mid e^{\omega($
$(\Sigma)\}$
$\mathrm{Riem}\,\Sigma$
CAUTION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orbits through $g \in \mathrm{Riem}(\Sigma)$
$O_g^{\mathrm{Diff}} = \{\varphi^* g \mid \varphi \in \mathrm{Diff}(\Sigma)\}$
$\{e^{\omega} g \mid e^{\omega(x)} \in \mathrm{Conf}(\Sigma)\}$
"conformal geometry"
$\mathrm{Riem}\,\Sigma$
$e^{\omega}g$
g
O_g^{Conf}
gauge choice
$\bar{g}_{ab}(x) = e^{\omega(x)}\, \bar{g}_{ab}(x)$
not conformally invariant

Classical GR $\quad g_{ab}(x,t) \in \mathrm{Riem}^{(3)}\Sigma$

orbits through $g \in \mathrm{Riem}(\Sigma)$

$$\mathcal{O}_g^{\mathrm{Diff}} = \{\varphi^* g \mid \varphi \in \mathrm{Diff}(\Sigma)\}$$

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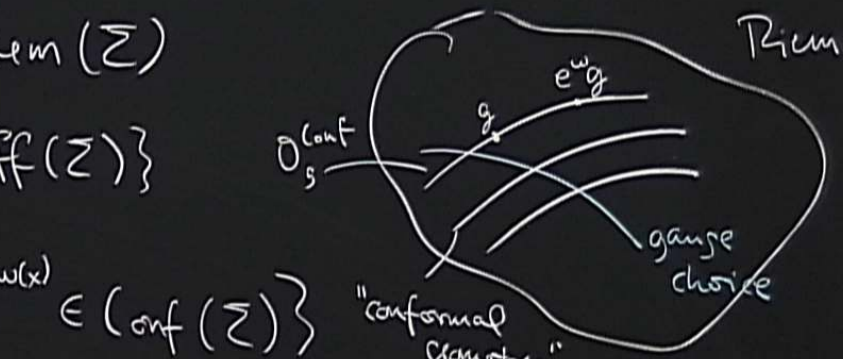


conformal decomposition $\quad g_{ab}(x) = e^{\omega(x)}\,\bar{g}_{ab}(x)$ $\quad$ conformal mode

N.B.: GR is <u>not</u> conformally invariant

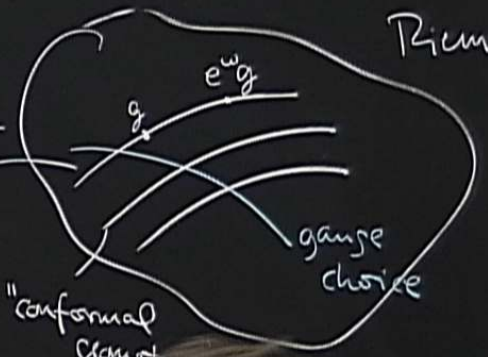
Classical GR

$g_{ab}(x,t) \in \mathrm{Riem}^{(2)}\Sigma$

orbits through $g \in \mathrm{Riem}(\Sigma)$

$O_g^{\mathrm{Diff}} = \{\varphi^* g \mid \varphi \in \mathrm{Diff}(\Sigma)\}$

$O_g^{\mathrm{Conf}} = \{e^{\omega} g \mid e^{\omega(x)} \in \mathrm{Conf}(\Sigma)\}$ "conformal group"



conformal decomposition $g_{ab}(x) = e^{\omega(x)}\,\bar{g}_{ab}(x)$ conformal

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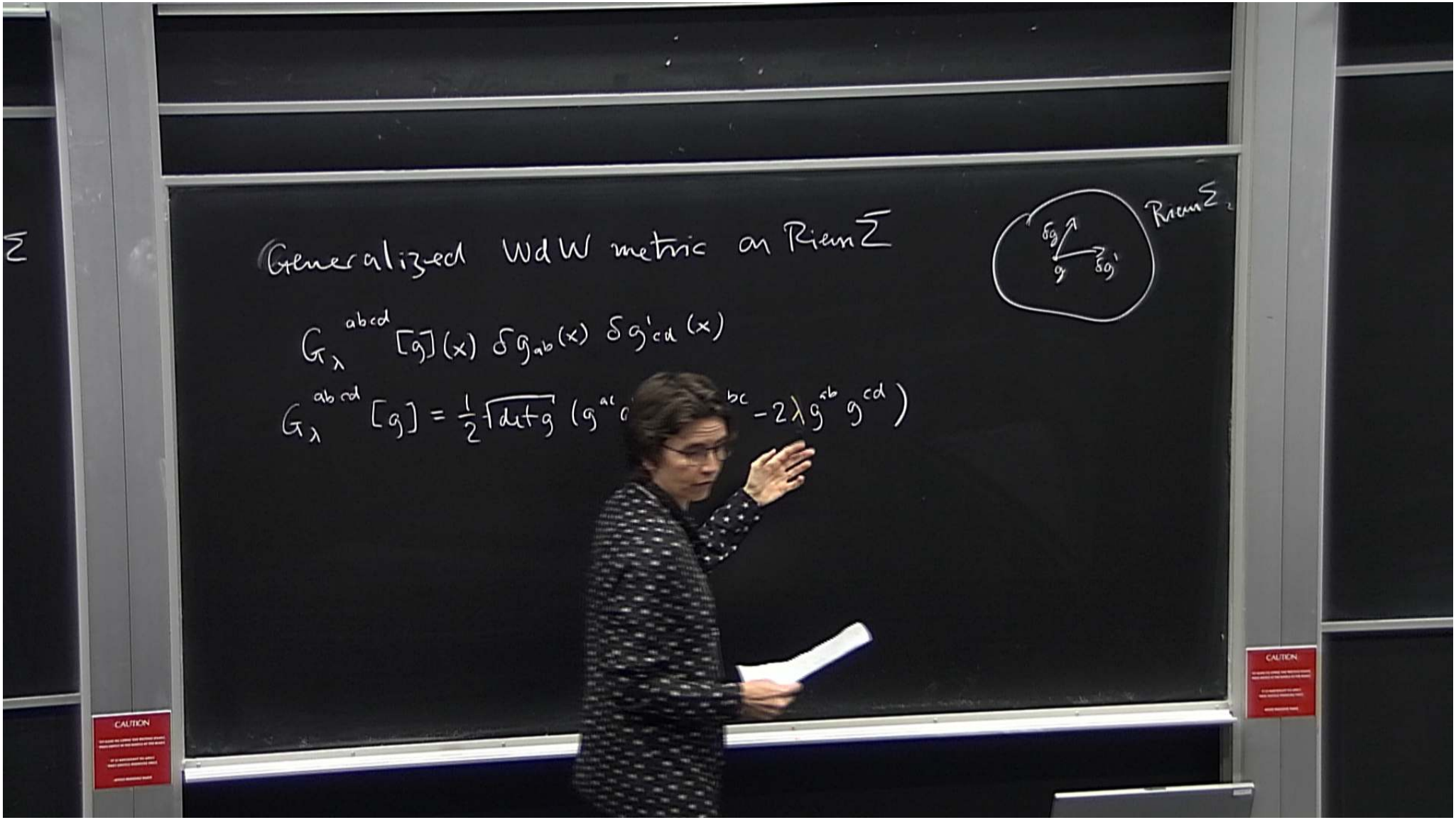
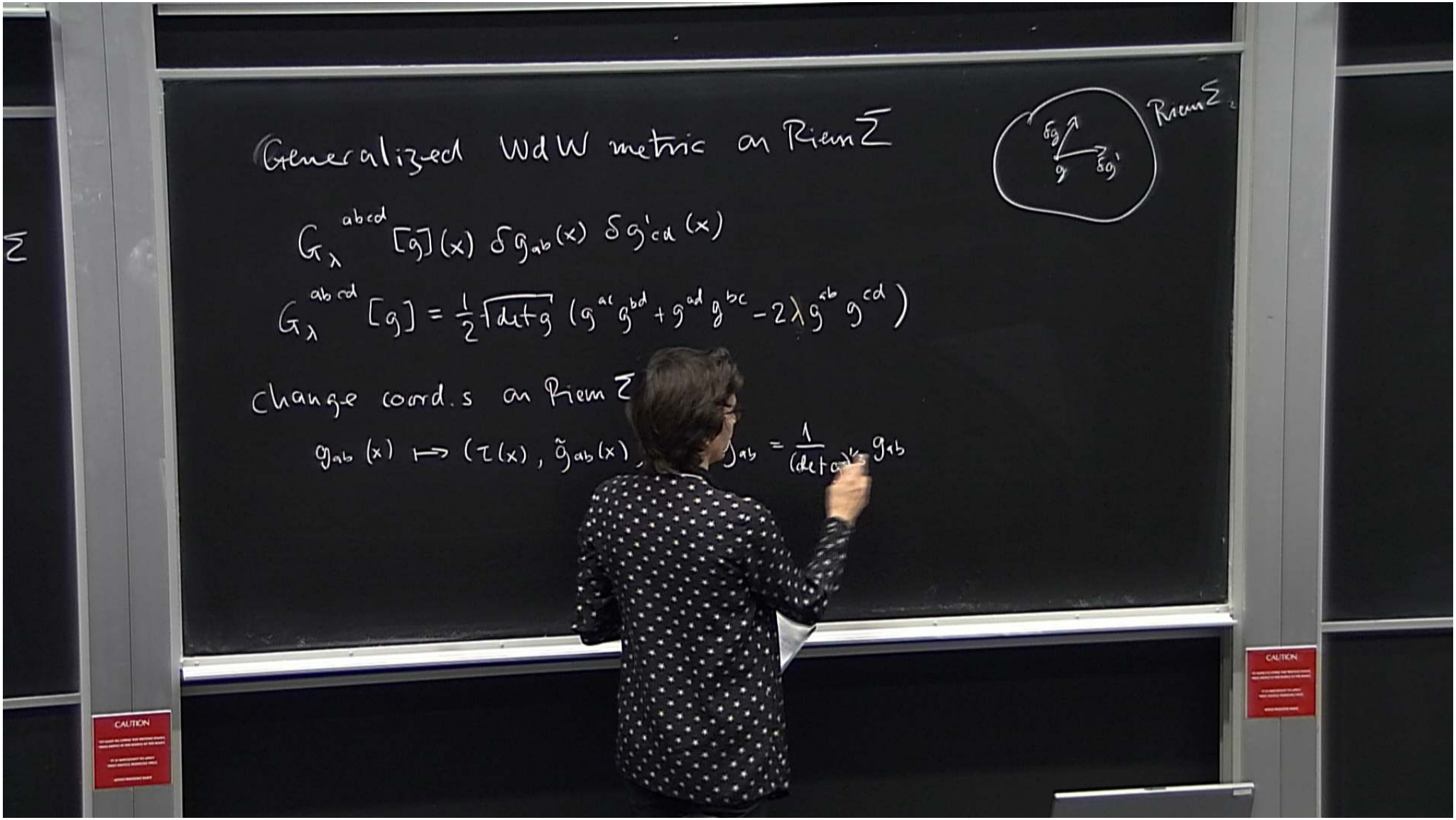
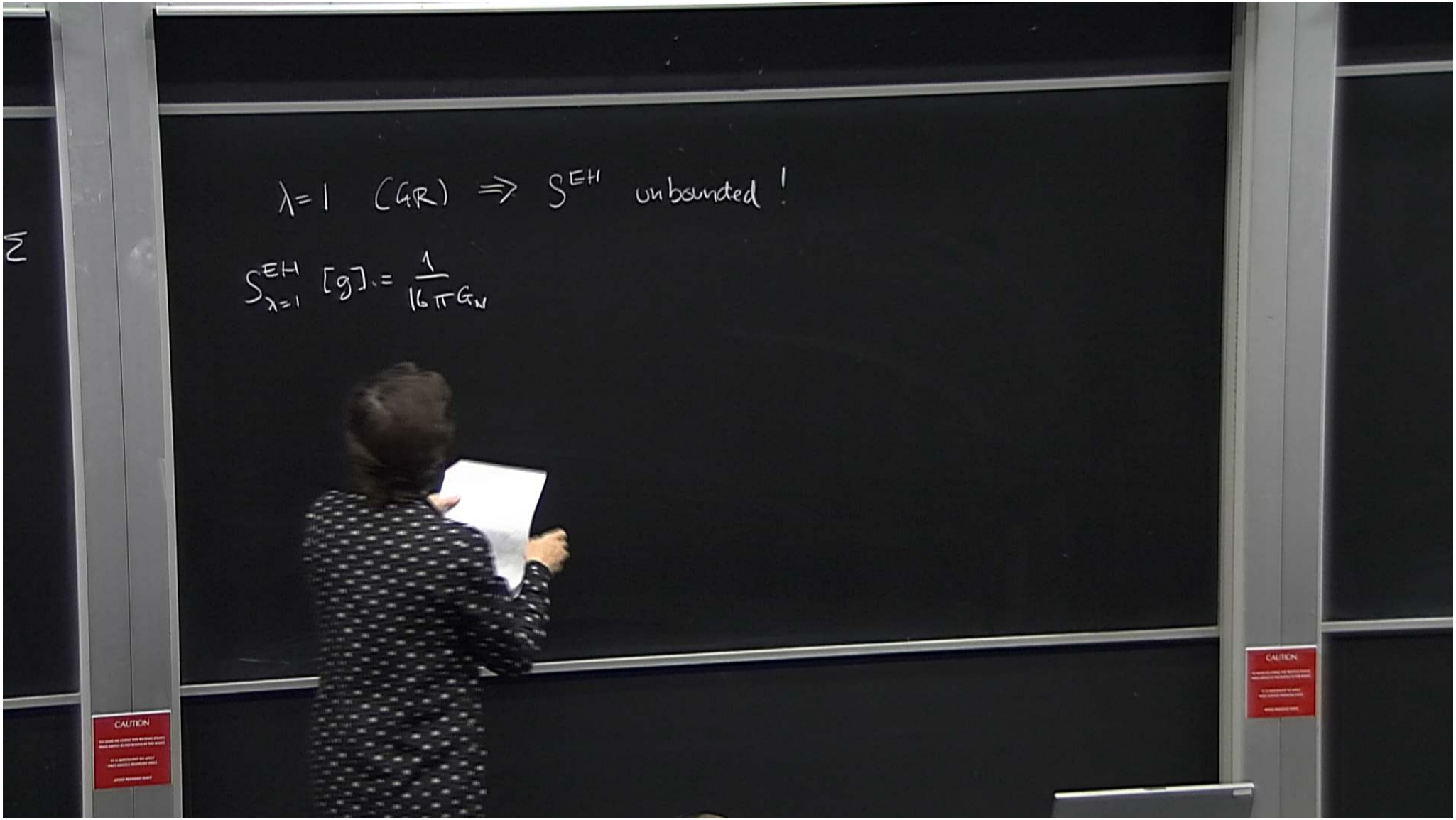
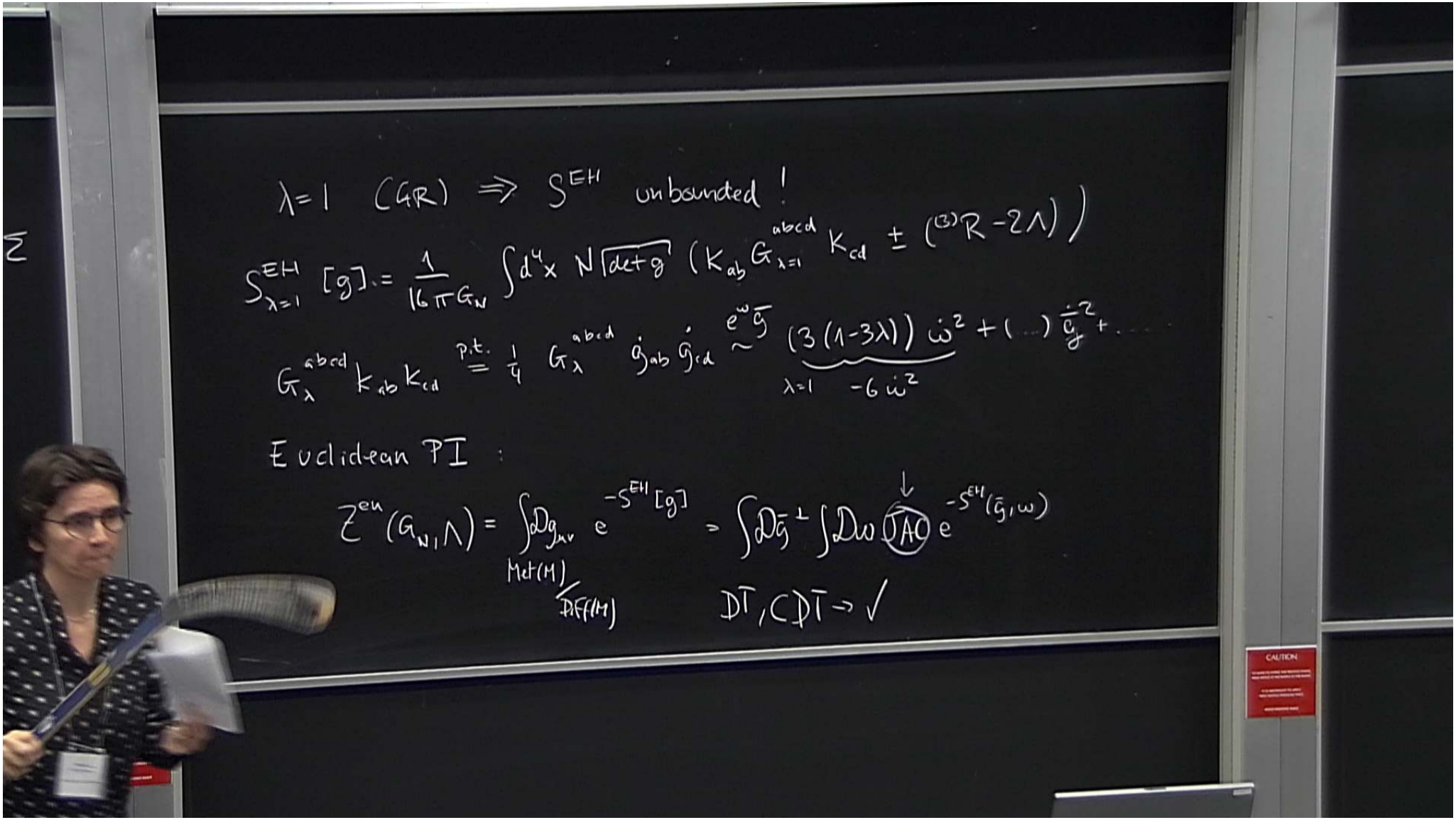

Generalized WdW metric on Riem Σ
$G_\lambda^{abcd}[g](x)\,\delta g_{ab}(x)\,\delta g'_{cd}(x)$
$G_\lambda^{abcd}[g] = \frac{1}{2}\sqrt{\det g}\,(g^{ac}$
$-2\lambda g^{ab}g^{cd})$
Riem Σ
δg
g
$\delta g'$
CAUTION


Generalized WdW metric on Riem Σ
$G_\lambda^{abcd}[g](x)\,\delta g_{ab}(x)\,\delta g'_{cd}(x)$
$G_\lambda^{abcd}[g] = \frac{1}{2}\sqrt{\det g}\,(g^{ac}g^{bd} + g^{ad}g^{bc} - 2\lambda g^{ab}g^{cd})$
change coord.s on Riem Σ
$g_{ab}(x) \mapsto (\tau(x), \tilde{g}_{ab}(x))$
$\tilde{g}_{ab} = \frac{1}{(\det g)^{1/4}} g_{ab}$
Riem Σ
δg
g
$\delta g'$
CAUTION
CAUTION


$\lambda = 1$ (GR) $\Rightarrow$ S^{EH} unbounded !
$S^{EH}_{\lambda=1}[g] = \frac{1}{16\pi G_N}$
CAUTION
CAUTION


$\lambda=1$ (GR) $\Rightarrow$ S^{EH} unbounded !
$S^{EH}_{\lambda=1}[g] = \frac{1}{16\pi G_N}\int d^4x\, N\sqrt{\det g}\,(K_{ab}\,G^{abcd}_{\lambda=1}\,K_{cd} \pm ({}^{(3)}R - 2\Lambda))$
$G^{abcd}_{\lambda}\,k_{ab}\,k_{cd} \overset{p.t.}{=} \frac{1}{4}\,G^{abcd}_{\lambda}\,\dot{g}_{ab}\,\dot{g}_{cd} \overset{e^{\omega}\bar{g}}{\sim} \underbrace{(3(1-3\lambda))}_{\lambda=1\quad -6}\,\dot{\omega}^2 + (\ldots)\,\dot{\bar{g}}^2 + \ldots$
Euclidean PI :
$Z^{eu}(G_N,\Lambda) = \int_{\mathrm{Met}(M)/\mathrm{Diff}(M)} \mathcal{D}g_{\mu\nu}\, e^{-S^{EH}[g]} = \int \mathcal{D}\bar{g} \int \mathcal{D}\omega\, \mathrm{JAC}\, e^{-S^{EH}(\bar{g},\omega)}$
$\mathcal{D}T, C\mathcal{D}T \to \checkmark$

$\lambda = 1$ (GR) $\Rightarrow$ S^{EH} unbounded !

$$S^{EH}_{\lambda=1}[g] := \frac{1}{16\pi G_N}\int d^4x\, N\sqrt{\det g}\,\left(K_{ab}\,G^{abcd}_{\lambda=1}\,K_{cd} \pm ({}^{(3)}R - 2\Lambda)\right)$$

$$G^{abcd}_{\lambda}\,K_{ab}K_{cd} \overset{\text{p.t.}}{=} \frac{1}{4}\,G^{abcd}_{\lambda}\,\dot{g}_{ab}\dot{g}_{cd} \overset{e^{\omega}\bar{g}}{\sim} \underbrace{(3(1-3\lambda))}_{\lambda=1 \quad -6}\,\dot{\omega}^2 + (\dots)\,\dot{\bar{g}}^2_{\bar{g}} + \dots$$

Euclidean PI :

$$Z^{eu}(G_N,\Lambda) = \int_{\mathrm{Met}(M)/\mathrm{Diff}(M)} \mathcal{D}g_{\mu\nu}\, e^{-S^{EH}[g]} = \int \mathcal{D}\bar{g}^{-1}\int \mathcal{D}\omega\, \underbrace{\mathrm{JAC}}\, e^{-S^{EH}(\bar{g},\omega)}$$

$D\bar{T}$, $C\bar{T} \to \checkmark$ $\quad\hookrightarrow \lambda^{\mathrm{eff}} < 1/3$ in QT?

$\lambda \neq 1 \Rightarrow$ not

Giulini & Kiefer

$\lambda \neq 1 \Rightarrow$ <u>not</u> GR ?

Giulini & Kiefer (1994) : "gravity" S_λ^{EH} for $\lambda < \frac{1}{3}$ is not attractive!

sign of acceleration of $V = \int d^3x \sqrt{g}$

$$\frac{d^2}{dt^2} V = -\frac{2}{3\lambda - 1} \int d^3x \left({}^{(3)}R - 3\Lambda \right) \quad \Rightarrow \lambda > \frac{1}{3} \; ?$$

$G_\lambda^{ab\,cd} K_{ab} K_{cd} = K^{ab} K_{ab} - \lambda K^2$

incompatible w/ $N = 1, N_i = 0$

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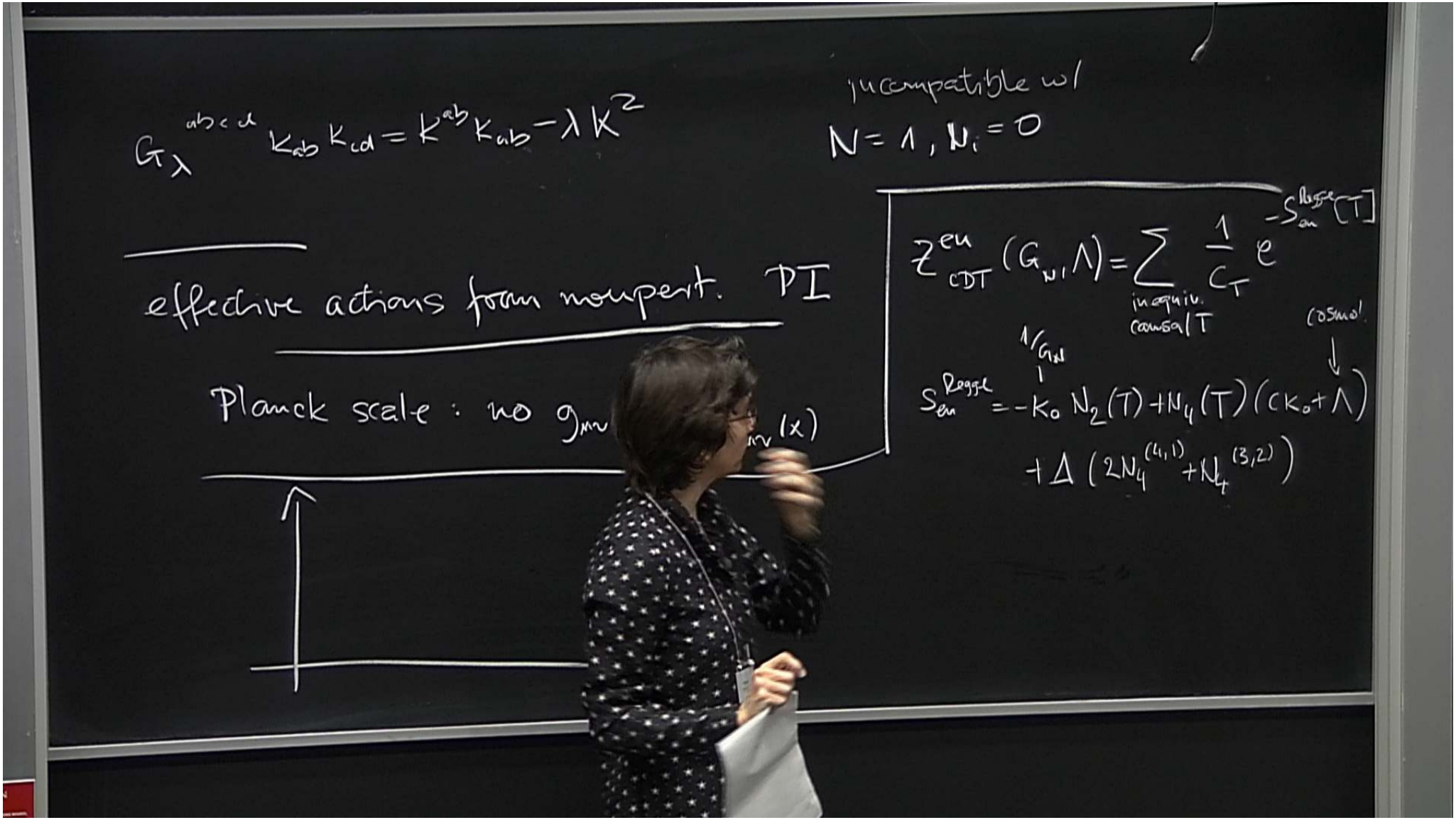
effective actions from nonpert. QI

Planck scale : no $g_{\mu\nu}(x)$, $\delta g_{\mu\nu}(x)$


$G_\lambda^{abcd} K_{ab} K_{cd} = K^{ab} K_{ab} - \lambda K^2$
incompatible w/
$N = 1, N_i = 0$
effective actions from nonpert. QI
Planck scale : no $g_{\mu\nu}$
$Z_{CDT}^{eu}(G_N, \Lambda) = \sum_{\substack{\text{inequiv.} \\ \text{comp}/T}} \frac{1}{C_T} e^{-S_{eu}^{Regge}[T]}$
cosmol.
$1/G_N$
$S_{eu}^{Regge} = -k_0 N_2(T) + N_4(T)(ck_0 + \Lambda)$
$+ \Delta (2N_4^{(4,1)} + N_4^{(3,2)})$

$G_\lambda^{abcd} K_{ab} K_{cd} = K^{ab} K_{ab} - \lambda K^2$

incompatible w/ $N = 1, N_i = 0$

effective actions from nonpert. QI

Planck scale: no $g_{\mu\nu}(x)$, $\delta g_{\mu\nu}(x)$

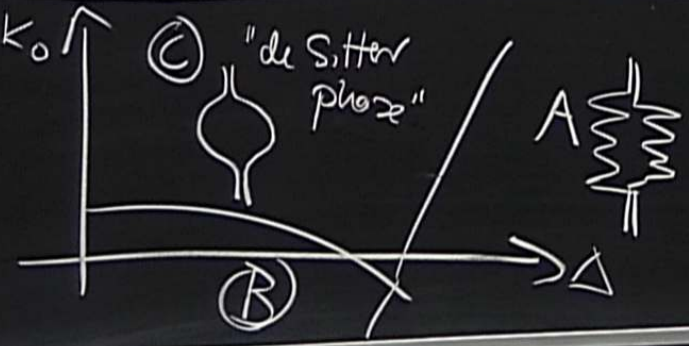
$$Z^{eq}_{CDT}(G_N, \Lambda) = \sum_{\substack{\text{inequiv.} \\ \text{config's}}} \frac{1}{C_T} e^{-S^{Regge}_{eq}[T]}$$

$$S^{Regge}_{eq} = -(k_0) N_2(T) + N_4(T)\,(c k_0 + (\Lambda))$$

cosmo!

$1/G_N$

$$+ (\Delta)(2N_4^{(4,1)} + N_4^{(3,2)})$$



$V_3(t) \approx a^3(t)$

$$e^{-S^{eff}_{eq}}, \quad S^{eff}_{eq} = k \int dt \left(\frac{\dot{V}_3^2}{V_3} + V_3^{1/3} + \right.$$

$$S^{min}_{eq} = k' \int dt \left(\left(\tfrac{1}{3} - \lambda\right) \frac{\dot{V}_3^2}{} + \ldots \right)$$