Title: Towards Conformal Degrees of Freedom in CDT

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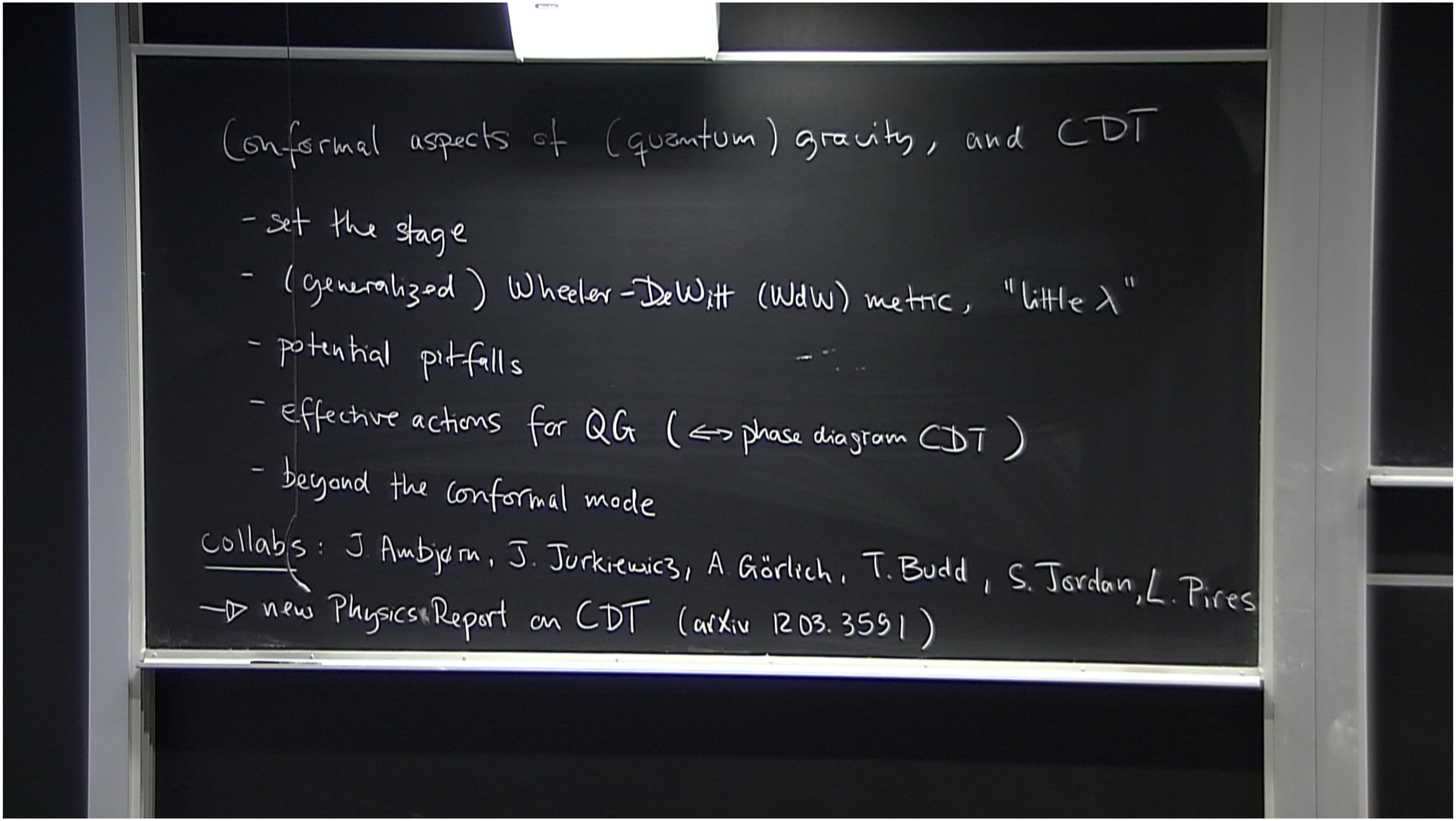
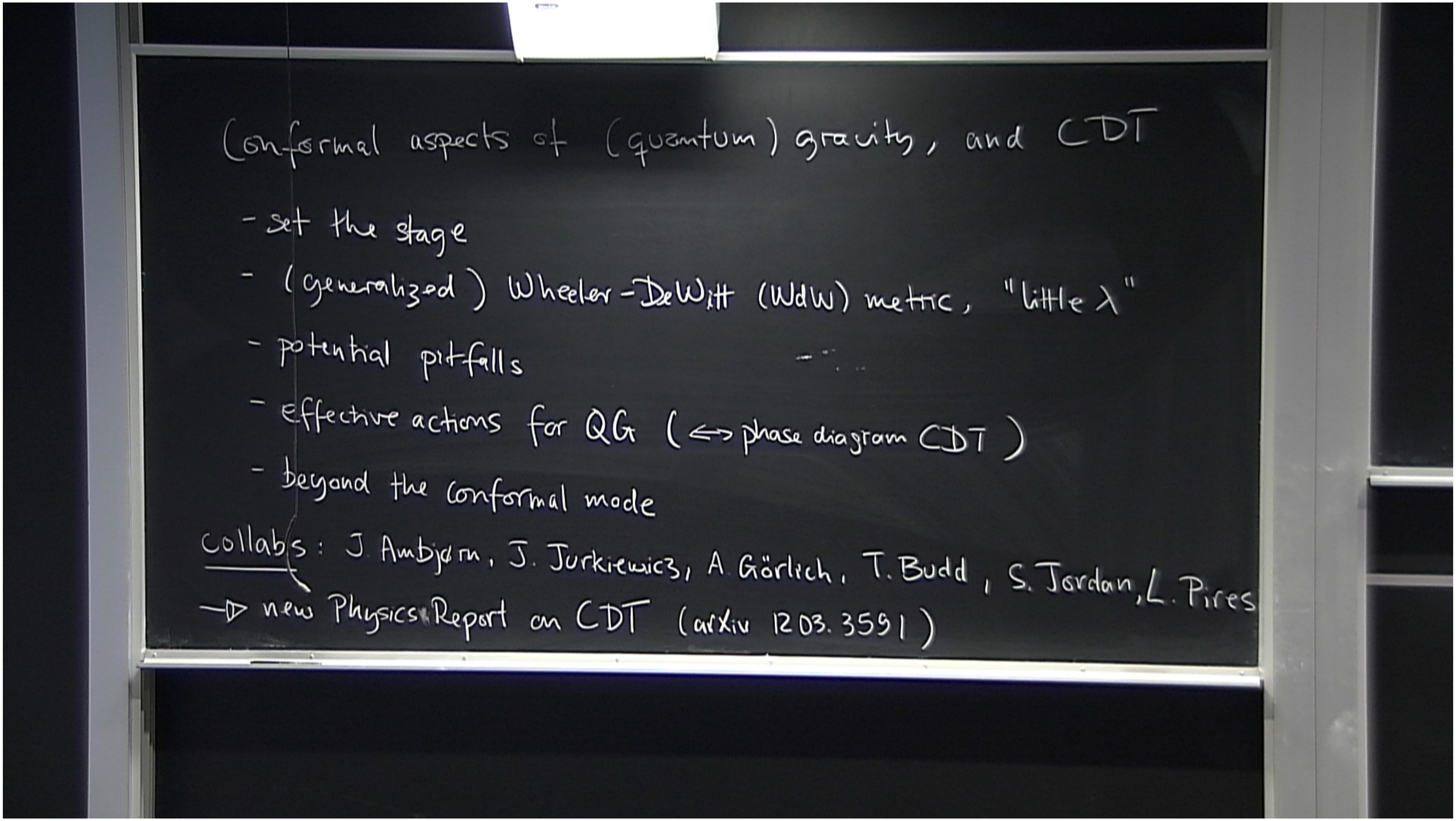
URL: http://pirsa.org/12050062

Abstract: TBA


Conformal aspects of (quantum) gravity, and CDT
- set the stage
- (generalized) Wheeler-DeWitt (WdW) metric, "little λ"
- potential pitfalls
- effective actions for QG (⟺ phase diagram CDT)
- beyond the conformal mode
Collabs: J. Ambjørn, J. Jurkiewicz, A. Görlich, T. Budd, S. Jordan, L. Pires
→ new Physics Report on CDT (arXiv 1203.3591)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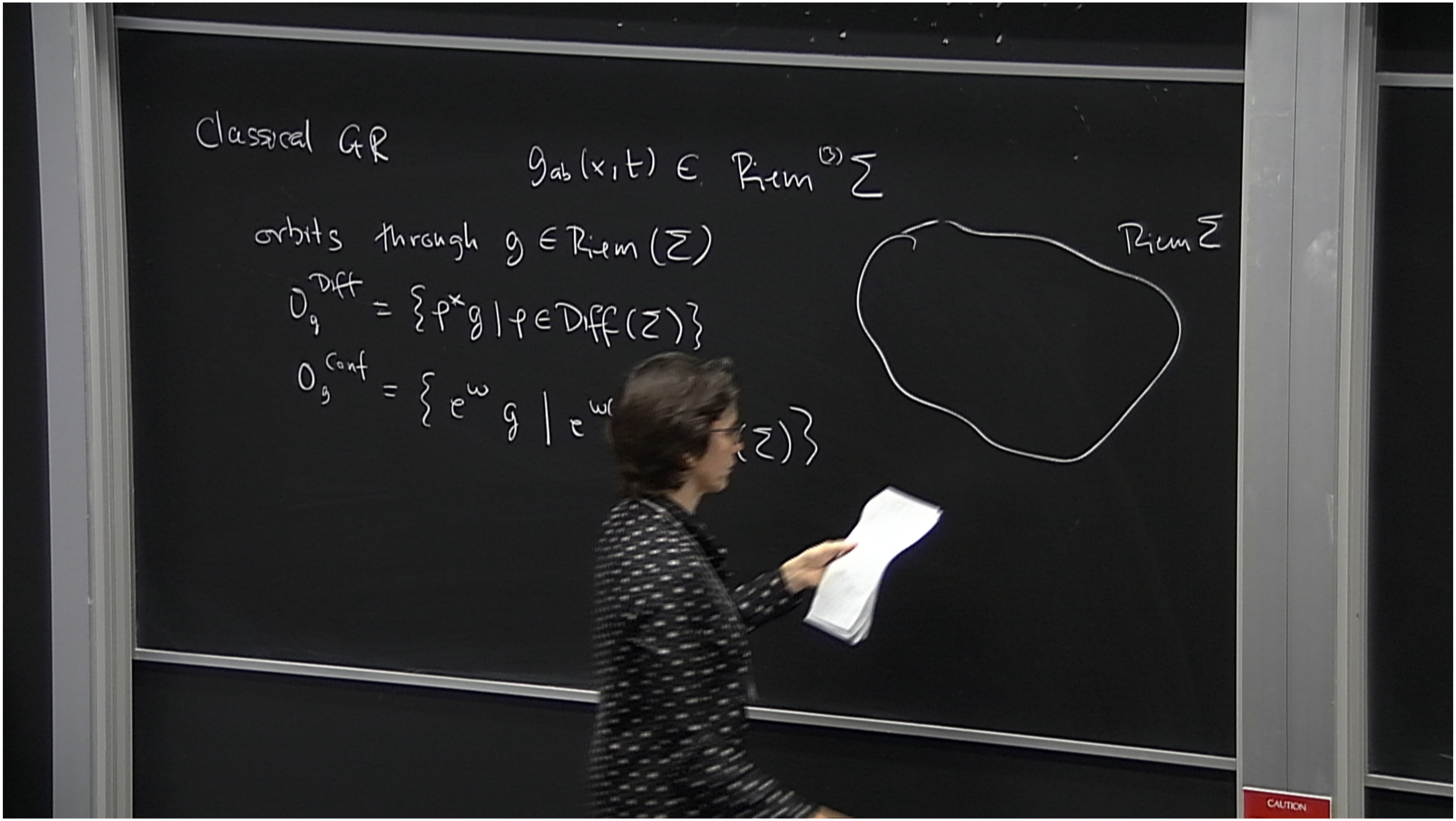
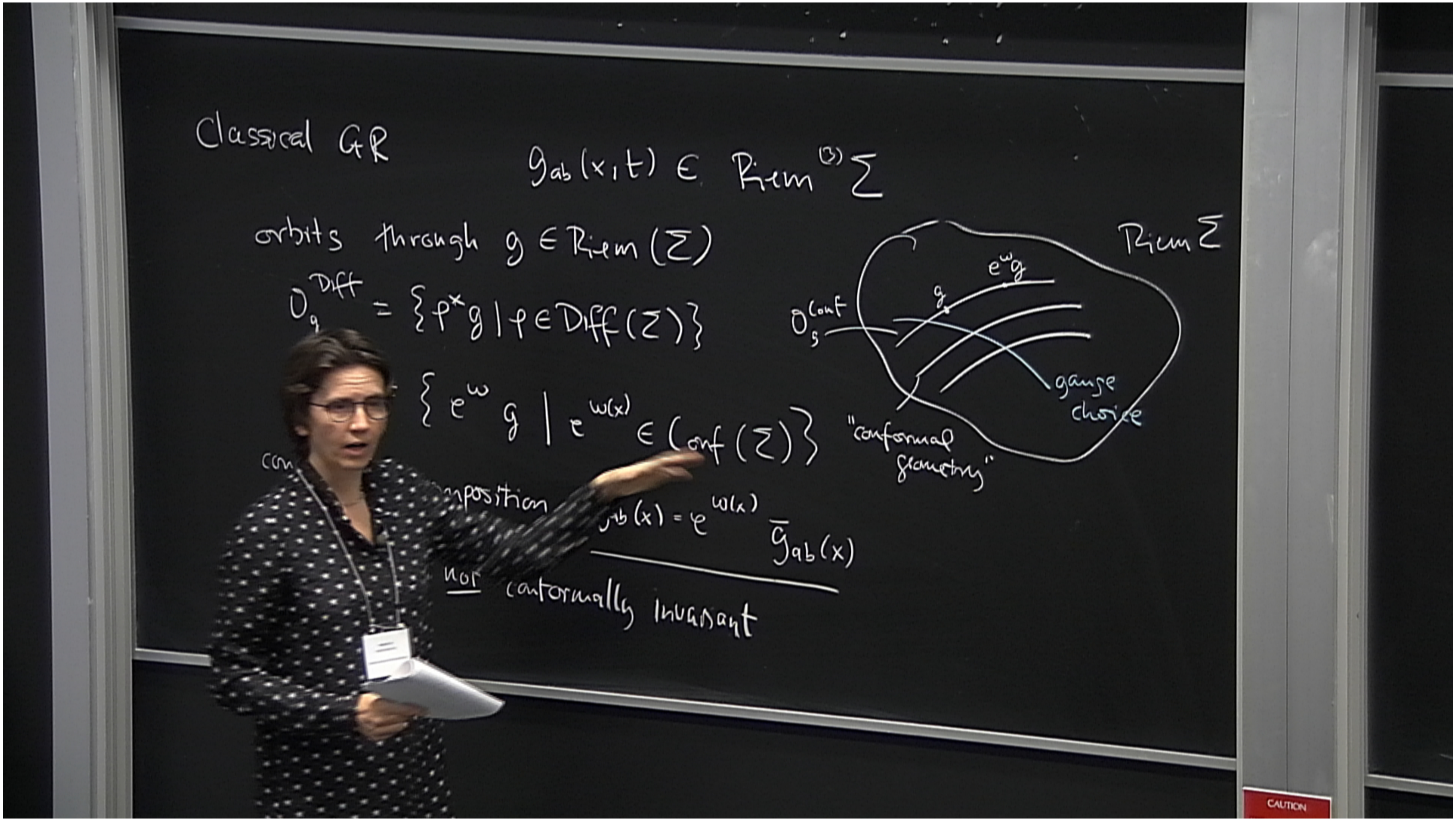

Classical GR
$g_{ab}(x,t) \in \mathrm{Riem}^{(3)}\Sigma$
orbits through $g \in \mathrm{Riem}(\Sigma)$
$O_g^{Diff} = \{\varphi^* g \mid \varphi \in Diff(\Sigma)\}$
$O_g^{Conf} = \{e^{\omega} g \mid e^{\omega}$
$\Sigma)\}$
Riem Σ
CAUTION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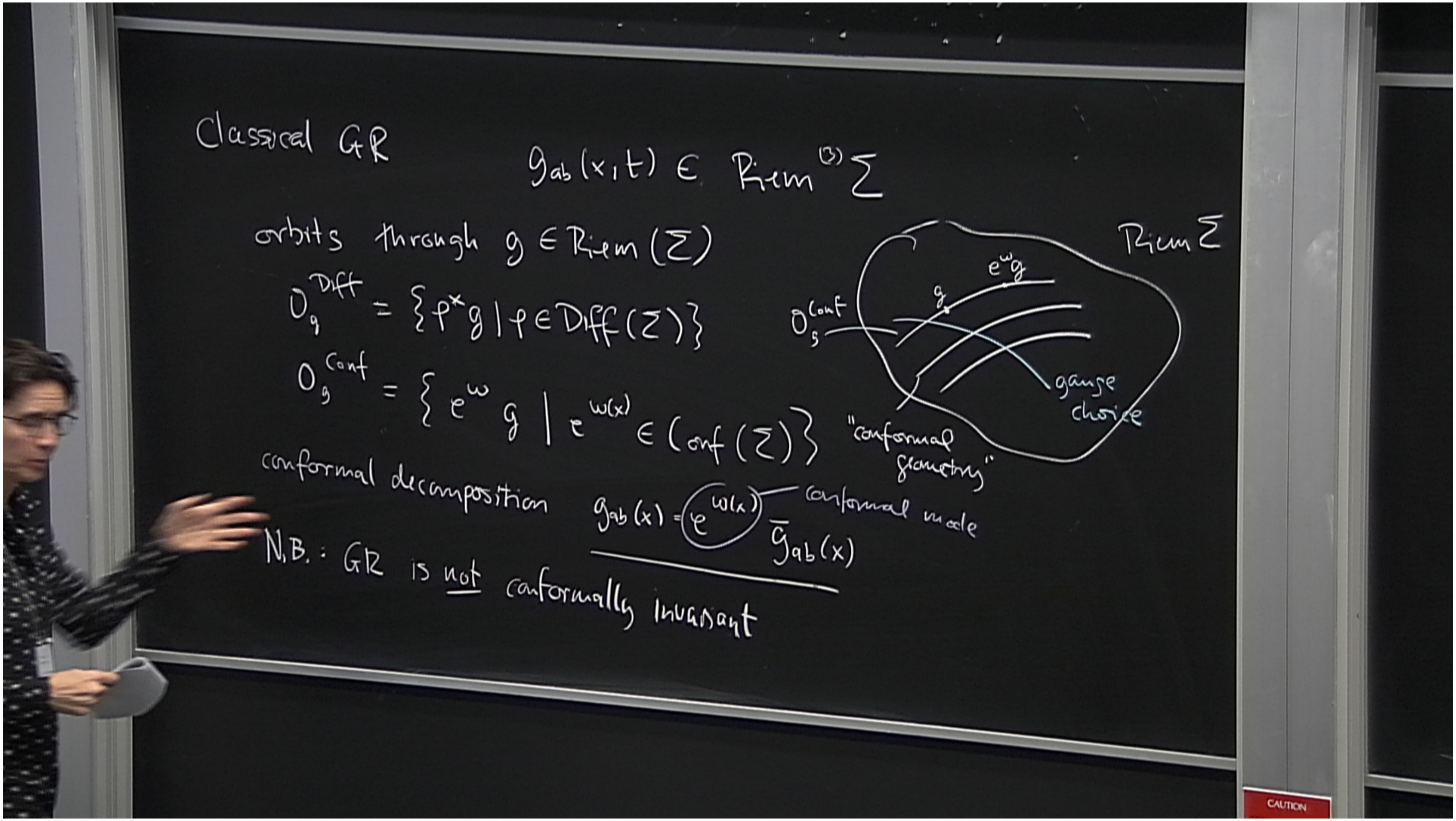
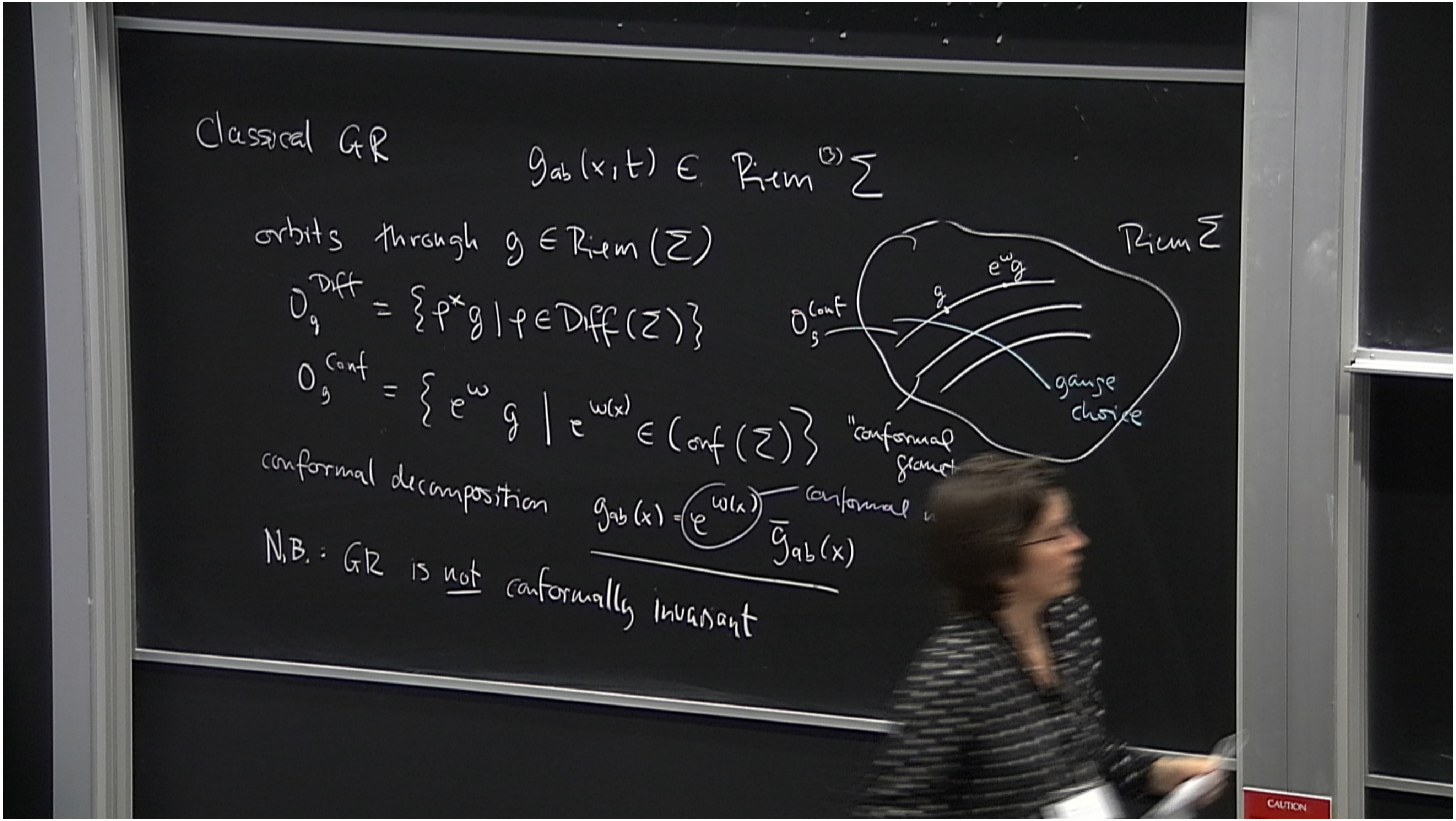
$O_g^{Diff} = \{\varphi^* g \mid \varphi \in \mathrm{Diff}(\Sigma)\}$
$\{e^{\omega} g \mid e^{\omega(x)} \in \mathrm{Conf}(\Sigma)\}$
"conformal geometry"
Riem Σ
$e^{\omega} g$
g
O_g^{Conf}
gauge choice
$= e^{\omega(x)} \bar{g}_{ab}(x)$
not conformally invariant


Classical GR
g_ab(x,t) ∈ Riem^(3) Σ
orbits through g ∈ Riem(Σ)
O_g^Diff = {φ*g | φ ∈ Diff(Σ)}
O_g^Conf = {e^ω g | e^ω(x) ∈ Conf(Σ)} "conformal geometry"
conformal decomposition g_ab(x) = e^ω(x) ḡ_ab(x)
conformal mode
N.B.: GR is not conformally invariant
Riem Σ
e^ω g
g
O_g^Conf
gauge choice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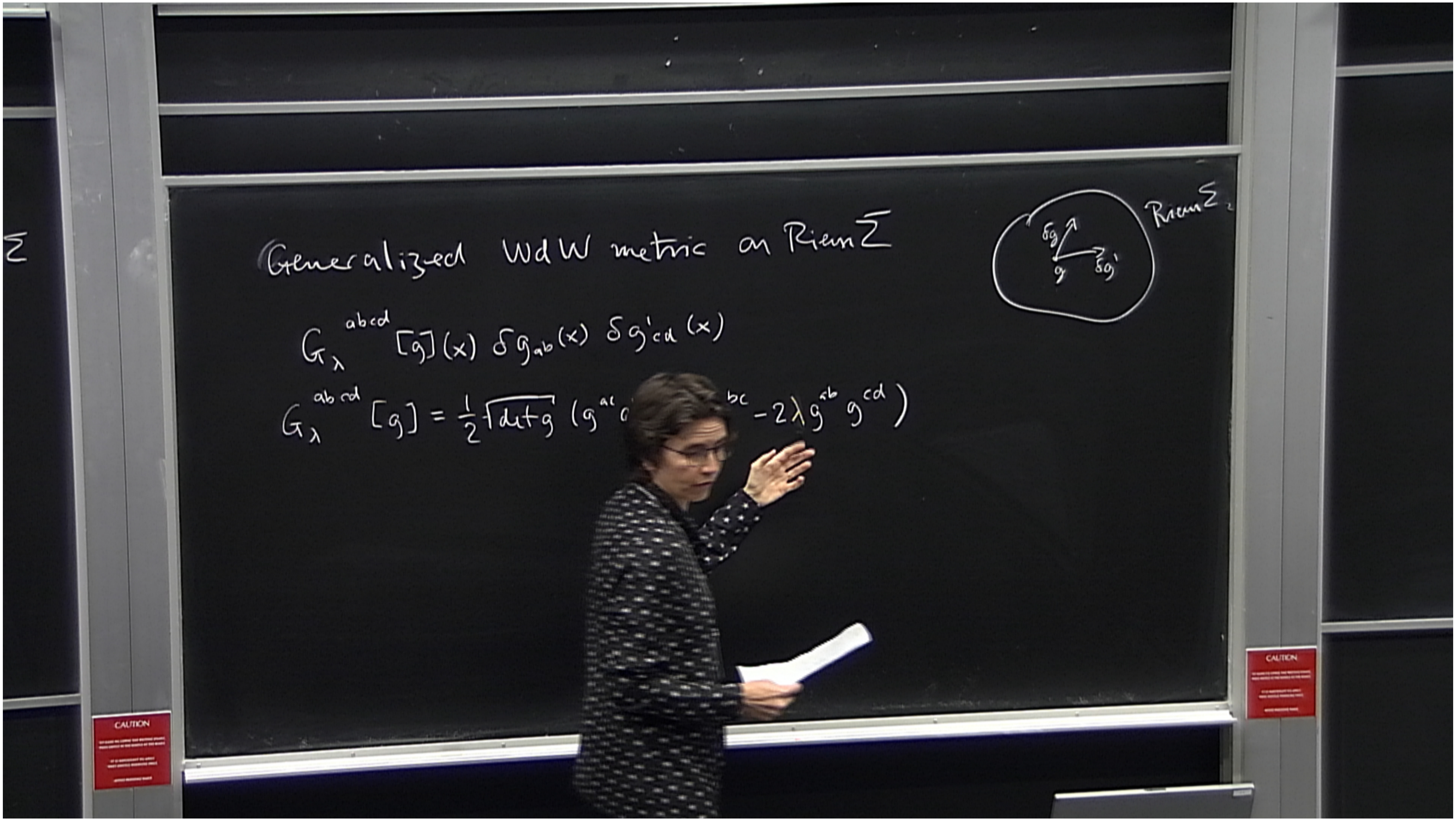
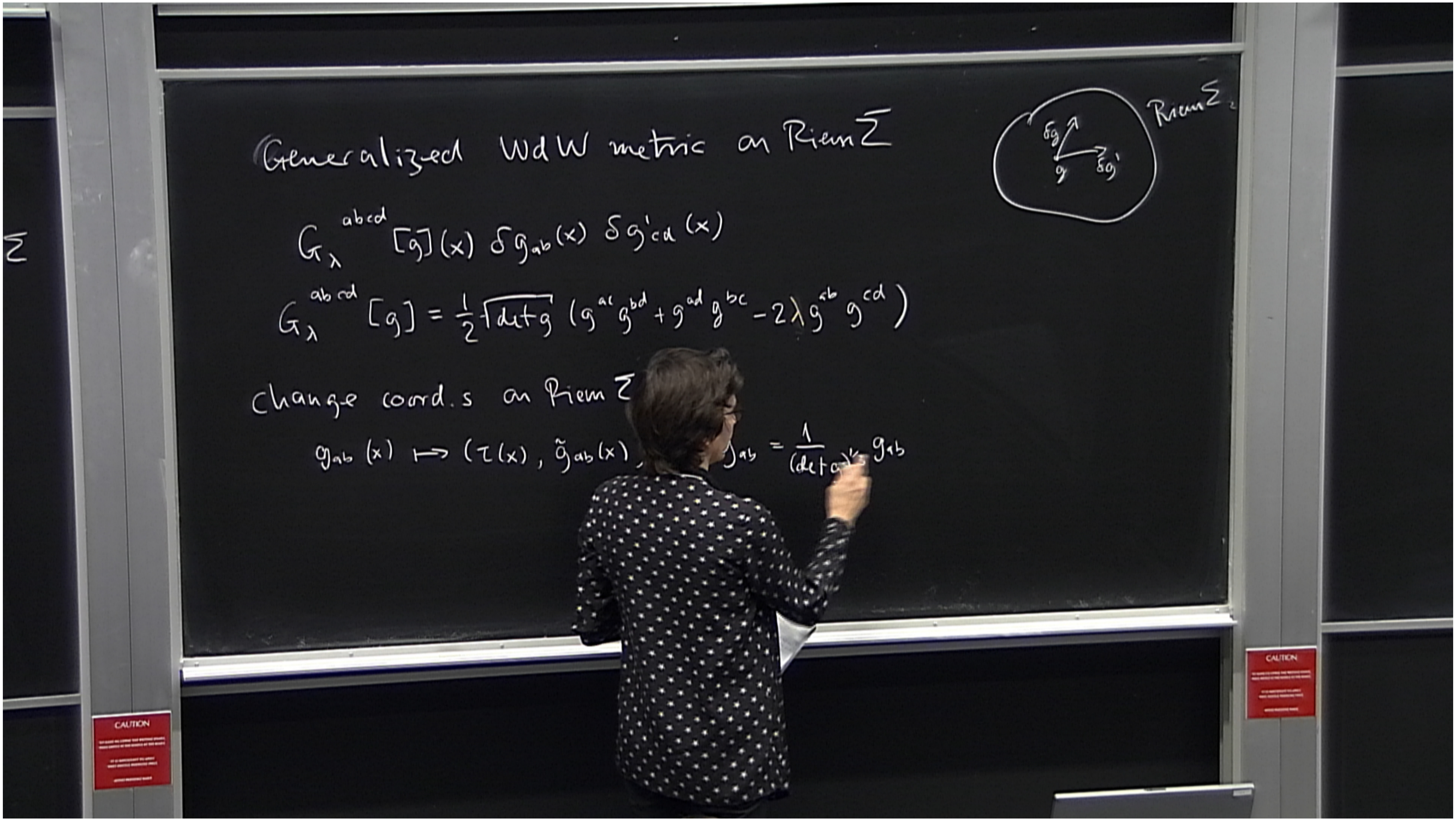
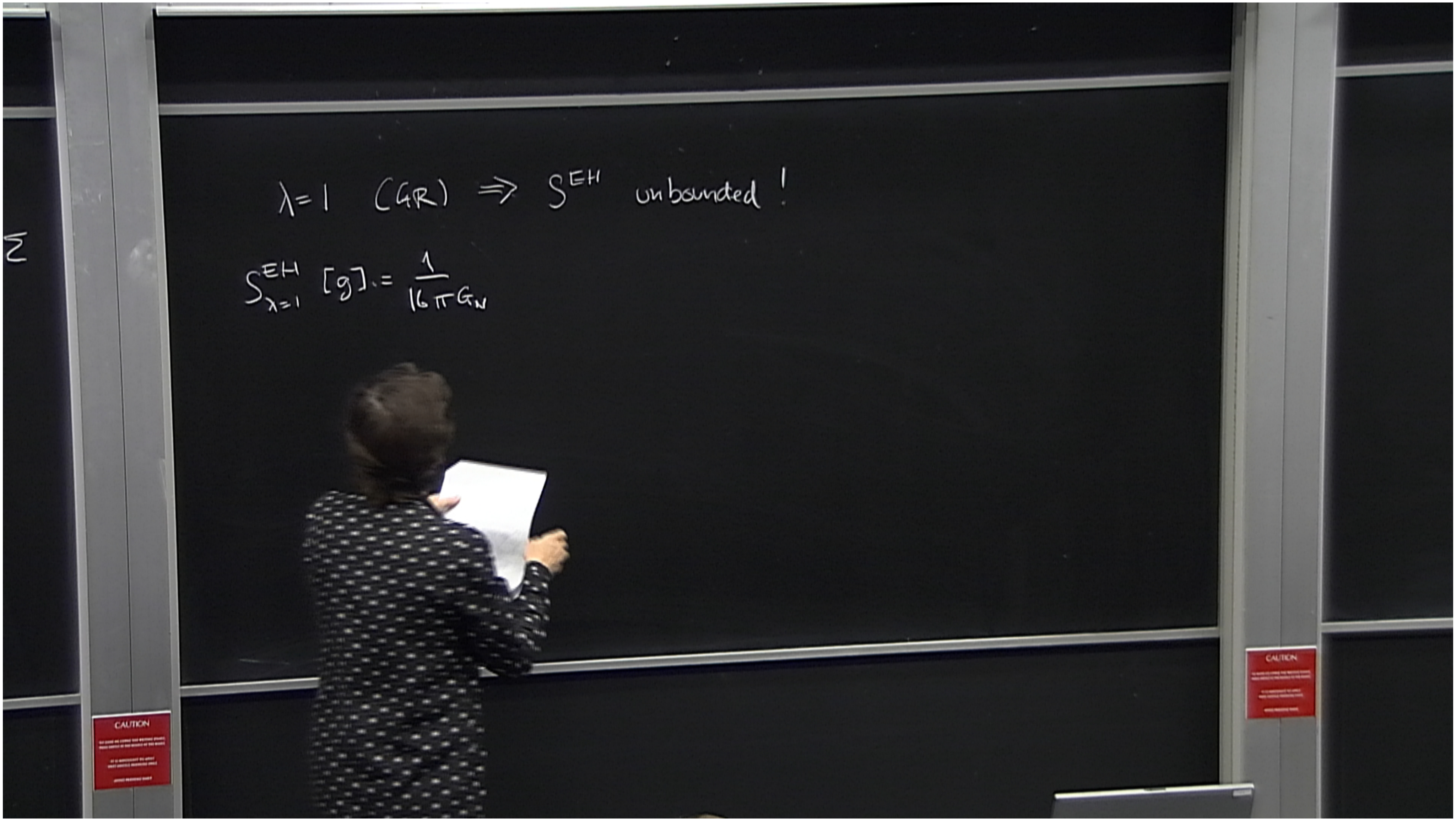
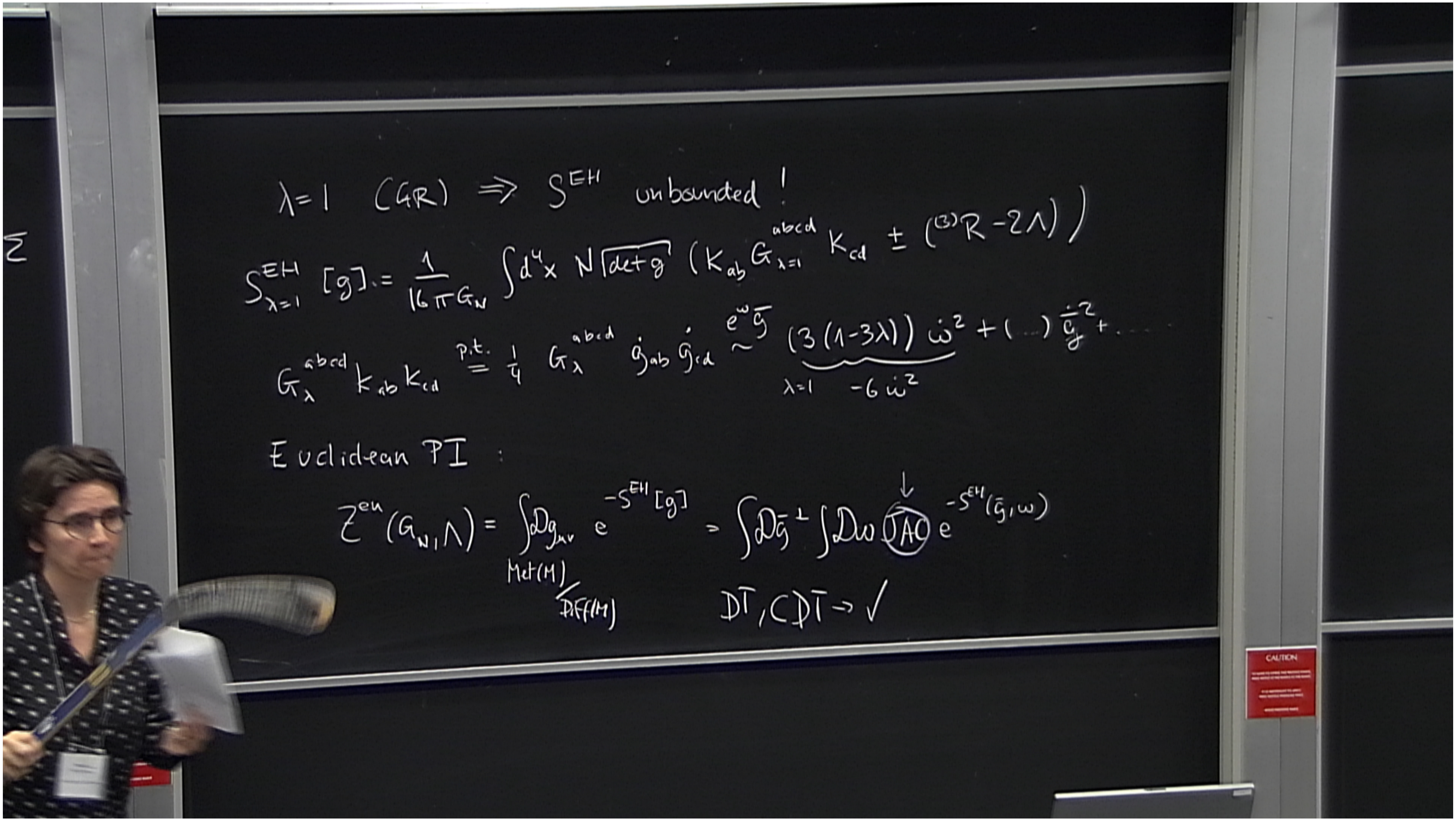
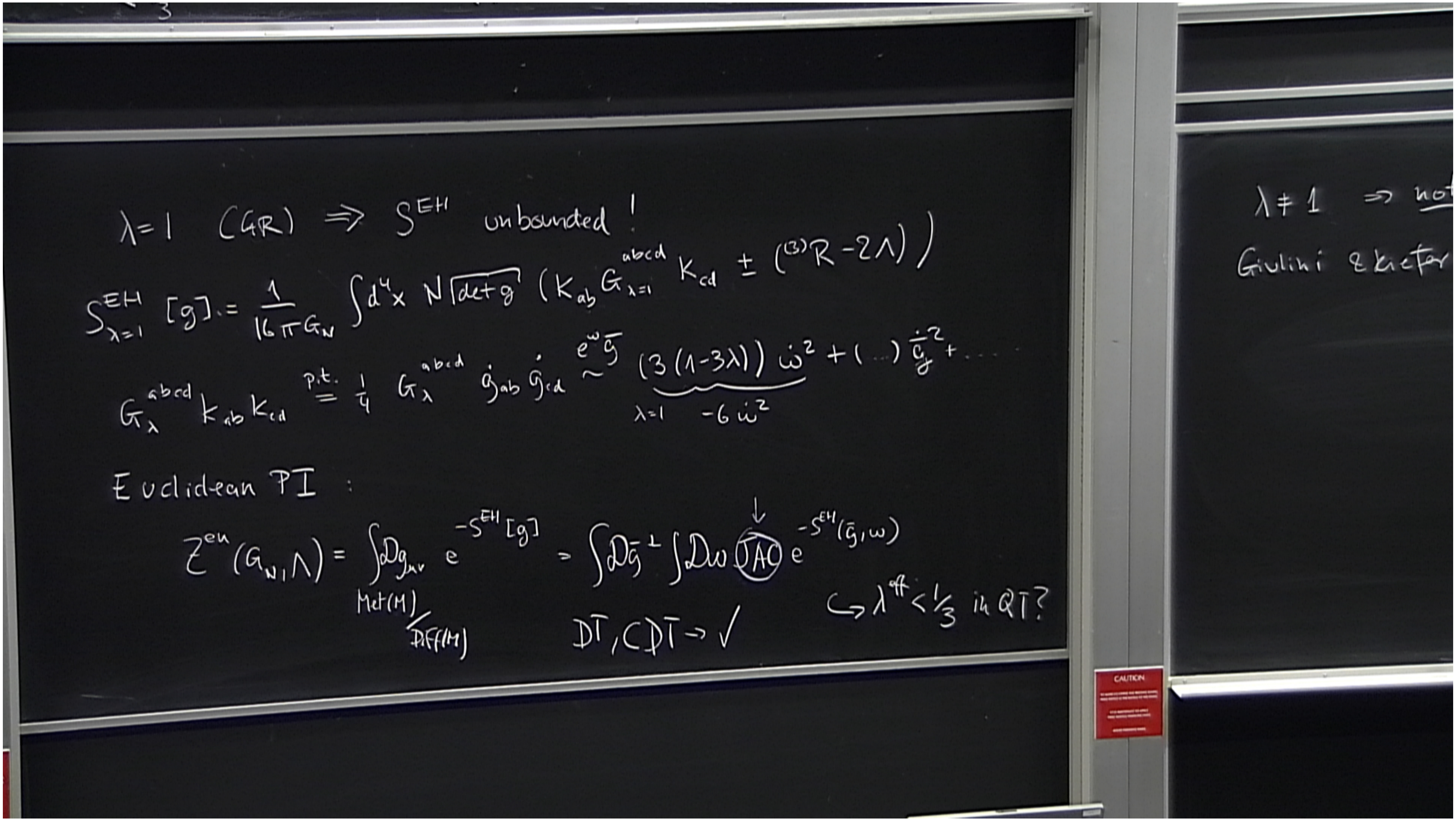
CAUTION


Generalized WdW metric on Riem Σ
$G_\lambda^{abcd}[g](x)\,\delta g_{ab}(x)\,\delta g'_{cd}(x)$
$G_\lambda^{abcd}[g] = \frac{1}{2}\sqrt{\det g}\,(g^{ac}$ … $^{bc} - 2\lambda g^{ab} g^{cd})$
Riem Σ
δg
g
δg'


Generalized WdW metric on Riem Σ
$G_\lambda^{abcd}[g](x)\, \delta g_{ab}(x)\, \delta g'_{cd}(x)$
$G_\lambda^{abcd}[g] = \frac{1}{2}\sqrt{\det g}\,(g^{ac}g^{bd} + g^{ad}g^{bc} - 2\lambda g^{ab}g^{cd})$
change coord.s on Riem Σ
$g_{ab}(x) \mapsto (\tau(x), \tilde{g}_{ab}(x))$
$g_{ab} = \frac{1}{(\det g)}\, g_{ab}$
Riem Σ
δg
g
$\delta g'$


λ=1 (GR) ⇒ S^EH unbounded !
S^EH_λ=1 [g] := 1/(16πG_N)
CAUTION
CAUTION


$\lambda=1$ (GR) $\Rightarrow S^{EH}$ unbounded!
$S^{EH}_{\lambda=1}[g] := \frac{1}{16\pi G_N}\int d^4x\, N\sqrt{\det g}\,\left(K_{ab}G^{abcd}_{\lambda=1}K_{cd} \pm ({}^{(3)}R - 2\Lambda)\right)$
$G_\lambda^{abcd}K_{ab}K_{cd} \overset{p.t.}{=} \frac{1}{4} G_\lambda^{abcd}\dot g_{ab}\dot g_{cd} \overset{e^\omega \bar g}{\sim} \underbrace{(3(1-3\lambda))\dot\omega^2}_{\lambda=1:\ -6\dot\omega^2} + (\dots)\dot{\bar g}^2 + \dots$
Euclidean PI :
$Z^{eu}(G_N,\Lambda) = \int_{Met(M)/Diff(M)} \mathcal{D}g_{\mu\nu}\, e^{-S^{EH}[g]} = \int \mathcal{D}\bar g^\perp \int \mathcal{D}\omega\, JAC\, e^{-S^{EH}(\bar g,\omega)}$
DT, CDT $\rightarrow$ ✓

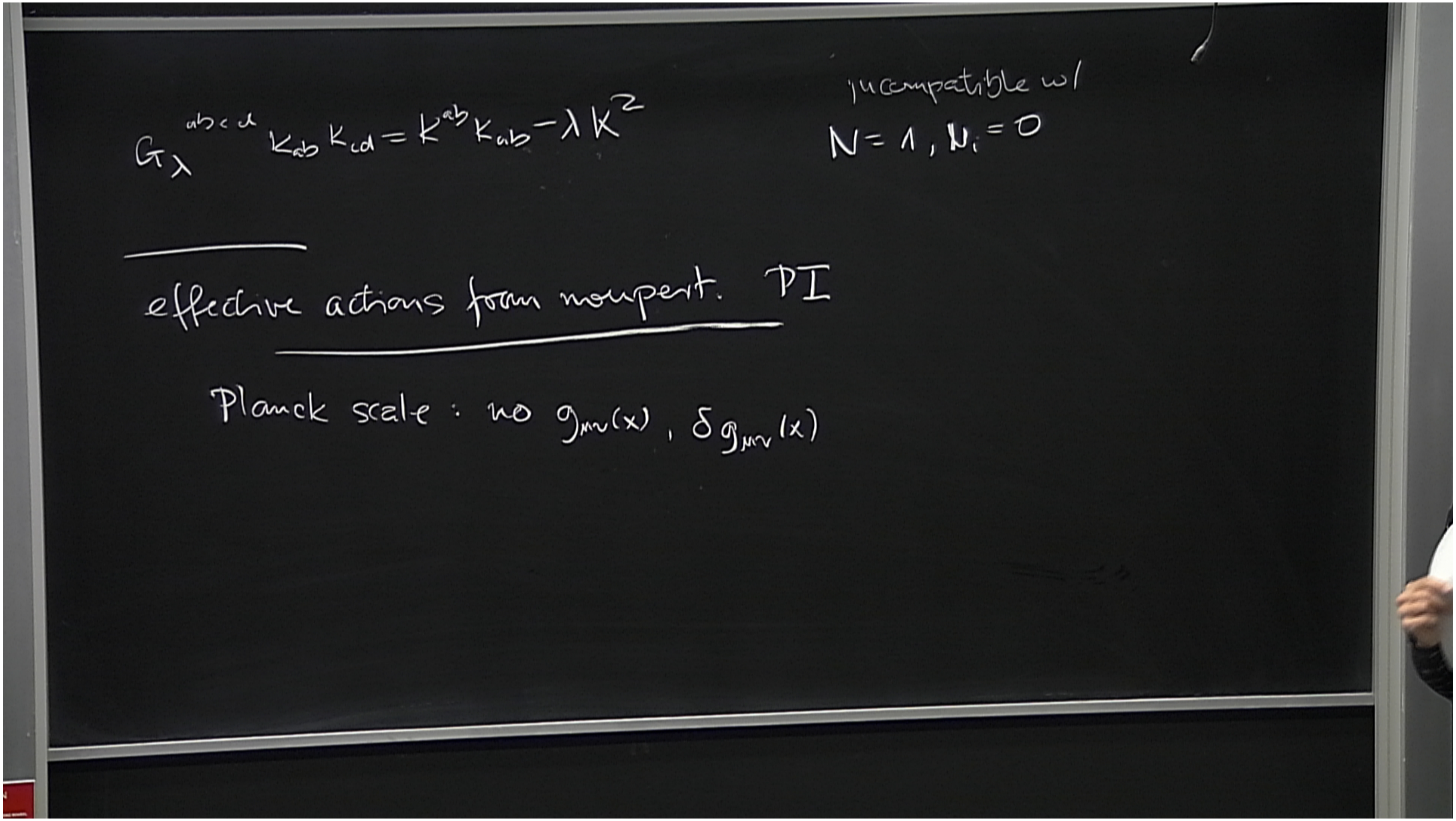

λ=1 (GR) ⇒ S^EH unbounded !
S^EH_λ=1 [g] := 1/(16π G_N) ∫d^4x N √(det g) (K_ab G^abcd_λ=1 K_cd ± (^(3)R − 2Λ))
G_λ^abcd K_ab K_cd =(p.t.) 1/4 G_λ^abcd ġ_ab ġ_cd ~ (3(1−3λ)) ω̇² + (...) ḡ̇² + ...
e^ω ḡ
λ=1 −6ω̇²
Euclidean PI :
Z^eu(G_N, Λ) = ∫ Dg_μν e^(−S^EH[g]) = ∫Dḡ ∫Dω JAC e^(−S^EH(ḡ,ω))
Met(M)/Diff(M)
DT, CDT → ✓
↪ λ^eff < 1/3 in QT?
λ ≠ 1 ⇒ no
Giulini & Kiefe

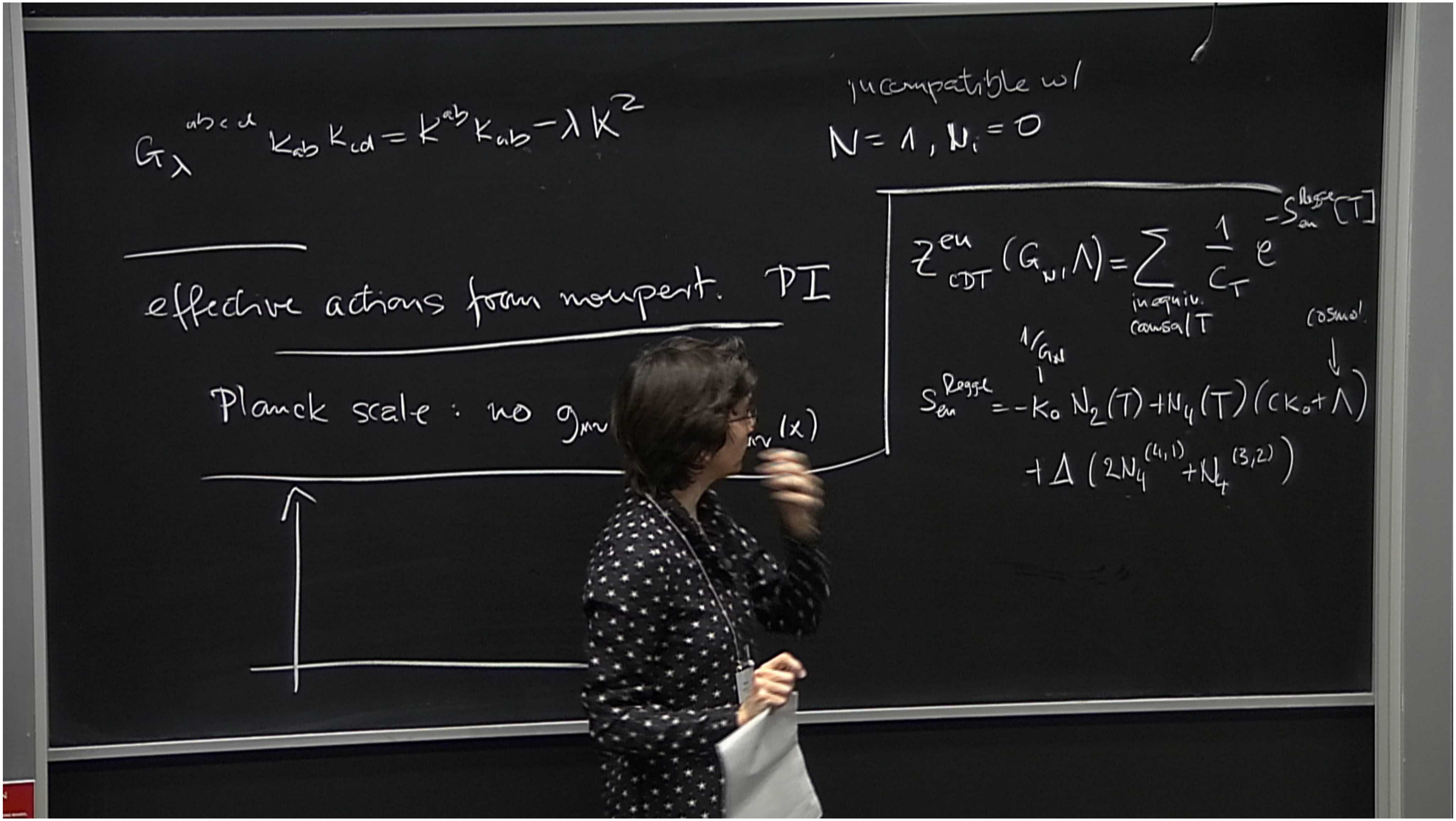
$\lambda \neq 1 \Rightarrow$ <u>not</u> GR ?

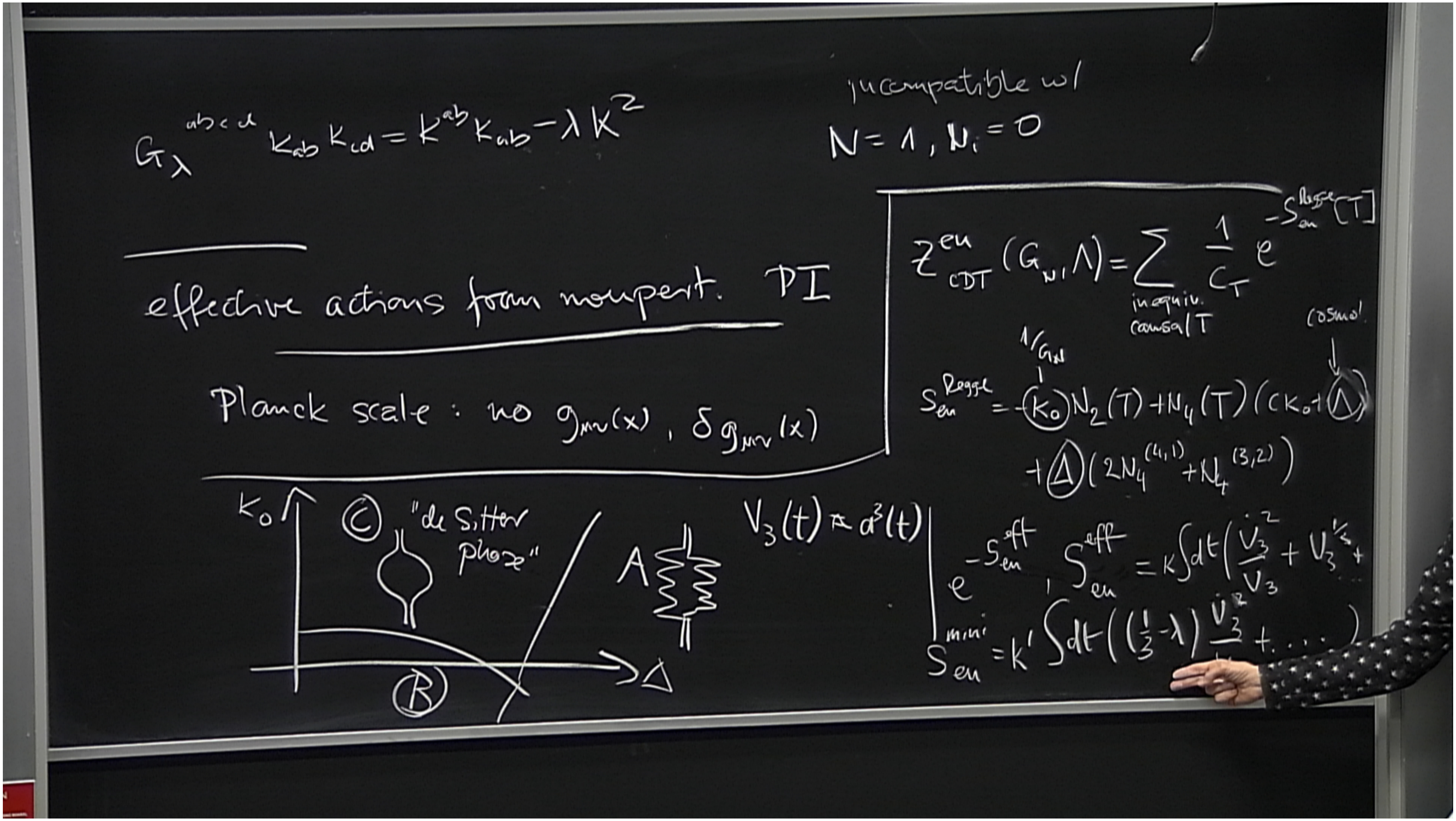
Giulini & Kiefer (1994) : "gravity" S_λ^{EH} for $\lambda < \frac{1}{3}$ is not attractive!

sign of acceleration of $V = \int d^3x \sqrt{g}$

$$\frac{d^2}{dt^2} V = -\frac{2}{3\lambda - 1} \int d^3x \left({}^{(3)}R - 3\Lambda \right) \Rightarrow \lambda > \frac{1}{3} \ ?$$


$G_\lambda^{abcd} K_{ab} K_{cd} = K^{ab} K_{ab} - \lambda K^2$
incompatible w/
$N = 1, N_i = 0$
effective actions from nonpert. PI
Planck scale: no $g_{\mu\nu}(x)$, $\delta g_{\mu\nu}(x)$


G_λ^abcd K_ab K_cd = K^ab K_ab − λ K^2
incompatible w/
N = 1, N_i = 0
effective actions from nonpert. PI
Planck scale: no g_μν
Z^eu_CDT (G_N, Λ) = Σ 1/C_T e^(−S^Regge_eu [T])
inequiv. causal T
cosmol.
1/G_N
S^Regge_eu = −k_0 N_2(T) + N_4(T)(ck_0 + Λ)
+ Δ (2N_4^(4,1) + N_4^(3,2))



G_λ^{abcd} K_ab K_cd = K^ab K_ab − λ K²
incompatible w/
N = 1, N_i = 0
effective actions from nonpert. PI
Planck scale: no g_μν(x), δg_μν(x)
Z^eu_CDT (G_N, Λ) = Σ_{inequiv. causal T} 1/C_T e^{−S^Regge_eu [T]}
1/G_N
cosmol.
S^Regge_eu = −(k_0) N_2(T) + N_4(T)(c k_0 + (Δ))
+ (Δ)(2N_4^(4,1) + N_4^(3,2))
K_0
C
"de Sitter phase"
A
B
Δ
V_3(t) ≈ a³(t)
e^{−S^eff_eu}, S^eff_eu = k ∫dt (V̇_3²/V_3 + V_3^{1/3} + …
S^mini_eu = k′ ∫dt ((1/3 − λ) V̇_3²/ … + …)