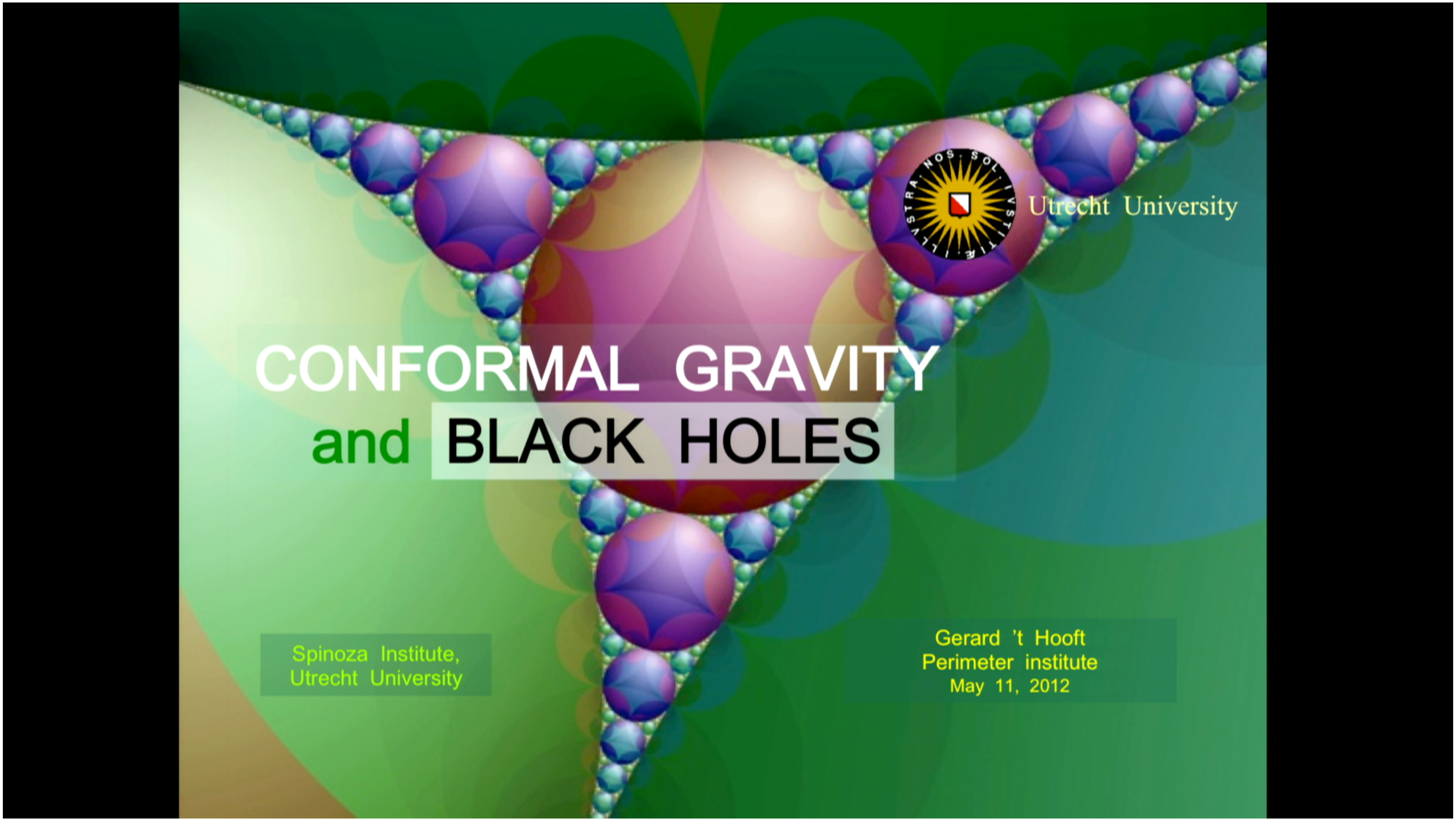
Title: Conformal Gravity and Black Hole Complementarity

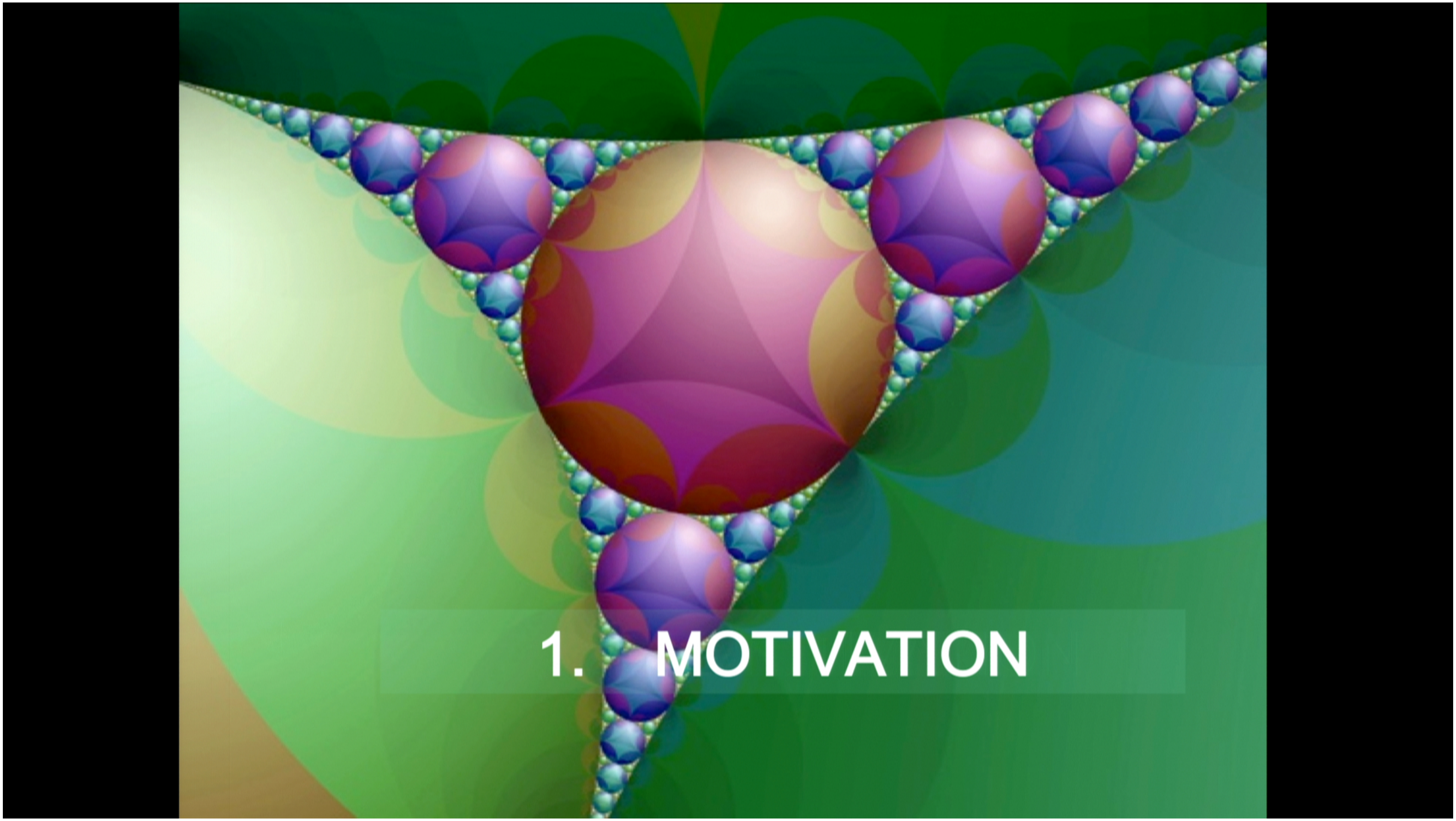
Date: May 11, 2012 04:40 PM

URL: http://pirsa.org/12050061

Abstract:


HOW ?


Utrecht University
CONFORMAL GRAVITY
and BLACK HOLES
Spinoza Institute,
Utrecht University
Gerard 't Hooft
Perimeter institute
May 11, 2012


1. MOTIVATION

Small distance structure in Quantum Gravity:

Fundamental cut-off generated by black hole formation.

$$\Delta x \geq \hbar / \Delta p \geq \hbar / E \geq \hbar / (\Delta x / 2G) = 2\hbar G / \Delta x$$

$$\Delta x \geq \sqrt{2\hbar G}$$ (for smallest “single state” black holes)

This does NOT happen “automatically” in most quantum gravity approaches (except: Superstring Theory … ?)

In a more viable theory, black hole microstates and Planck distance cut-off must arise inevitably.

HOW ?

Small distance structure in Quantum Gravity:

Fundamental cut-off generated by black hole formation.

$$\Delta x \geq \hbar / \Delta p \geq \hbar / E \geq \hbar / (\Delta x / 2G) = 2\hbar G / \Delta x$$

$$\Delta x \geq \sqrt{2\hbar G}$$

(for smallest "single state" black holes)

This does NOT happen "automatically" in most quantum gravity approaches (except: Superstring Theory … ?)

In a more viable theory, black hole microstates and Planck distance cut-off must arise inevitably.

HOW ?

The black hole horizon must be naturally described such that the microstates appear.

We must begin trying to rephrase perturbative gravity in such a way that the horizons are part of the theory.

Black holes form (part of) the spectrum of elementary particle excitations.

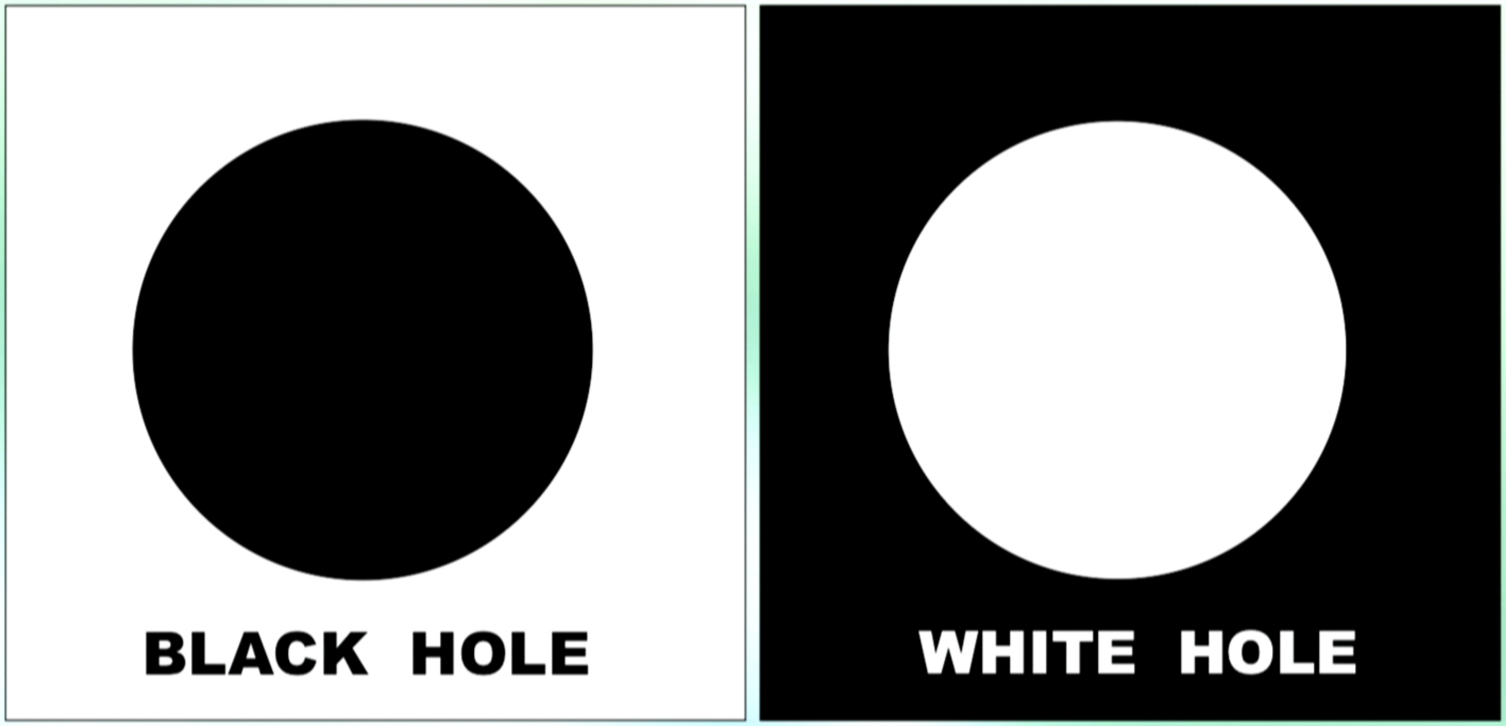
They are not merely solitons …

Particles going in and out of a horizon must be described by the “black hole channel” of an *S-matrix*

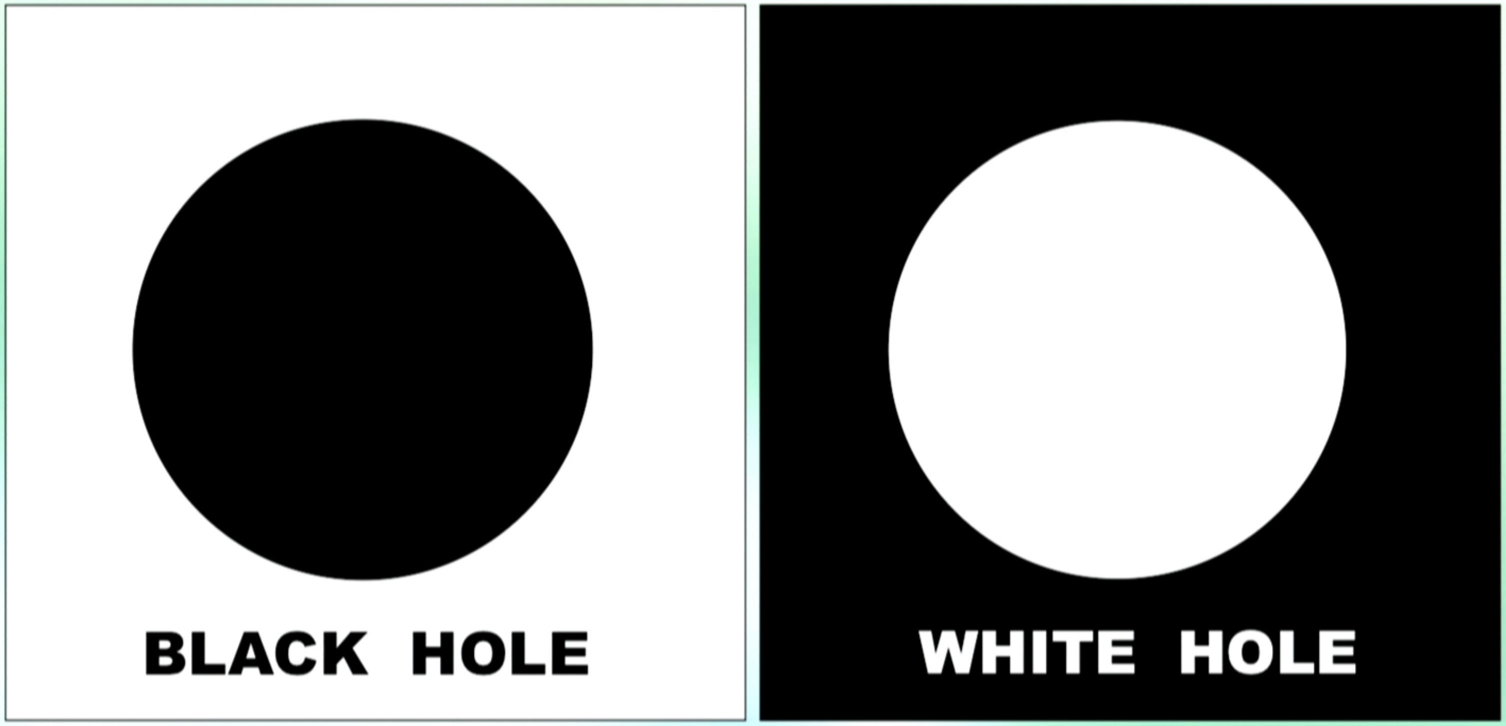
One natural conclusion of earlier work:

The Difference between

The Difference between



The Difference between



A black hole is a quantum superposition of white holes and *vice versa* !!

Would it be possible to create a theory with locality built in describing a black hole including its formation and its complete decay ?

Demand: ***unitarity*** and ***causality*** …

In principle, this should be possible:
just stretch spacetime so that singularities move to $t \to \infty$

Would it be possible to create a theory with locality built in describing a black hole including its formation and its complete decay ?

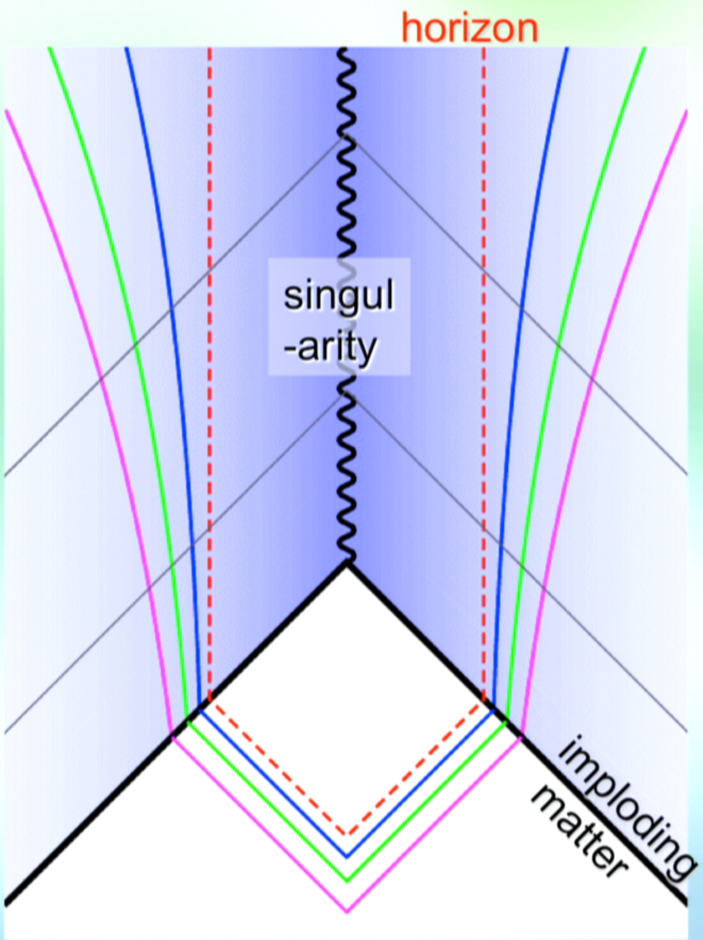
Demand: ***unitarity*** and ***causality*** …

In principle, this should be possible:
just stretch spacetime so that singularities move to $t \longrightarrow \infty$

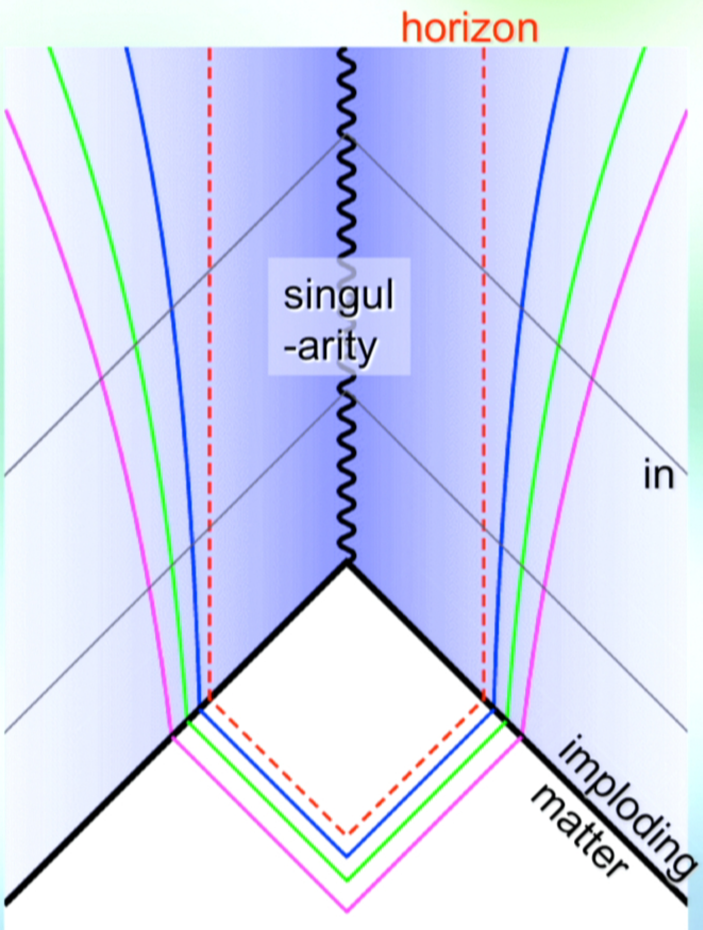
Would it be possible to create a theory with locality built in describing a black hole including its formation and its complete decay ?

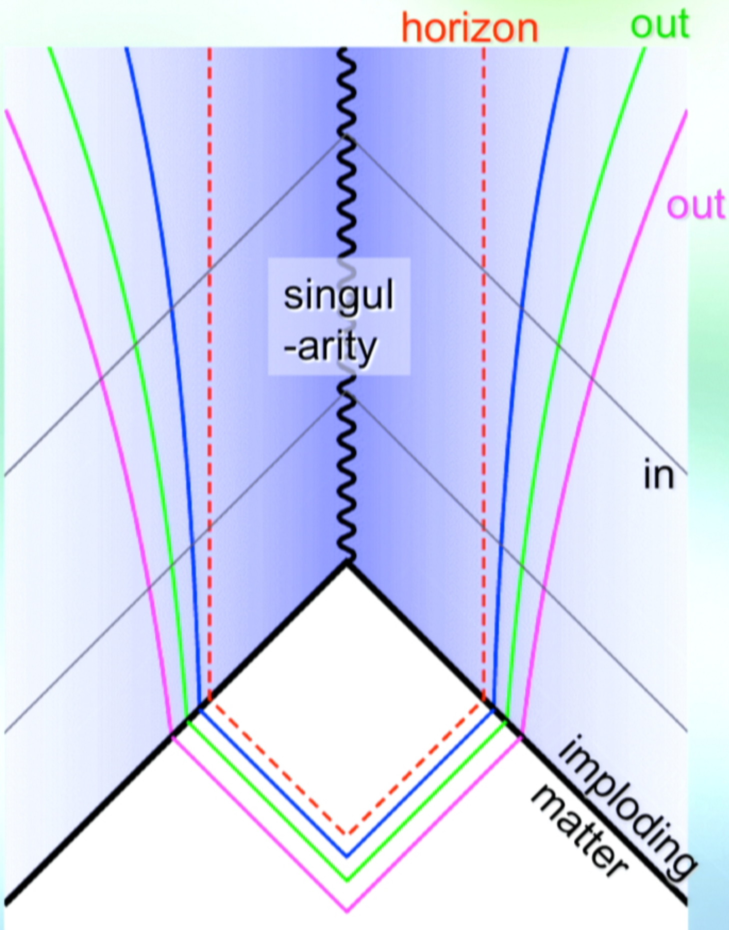
Demand: ***unitarity*** and ***causality*** …

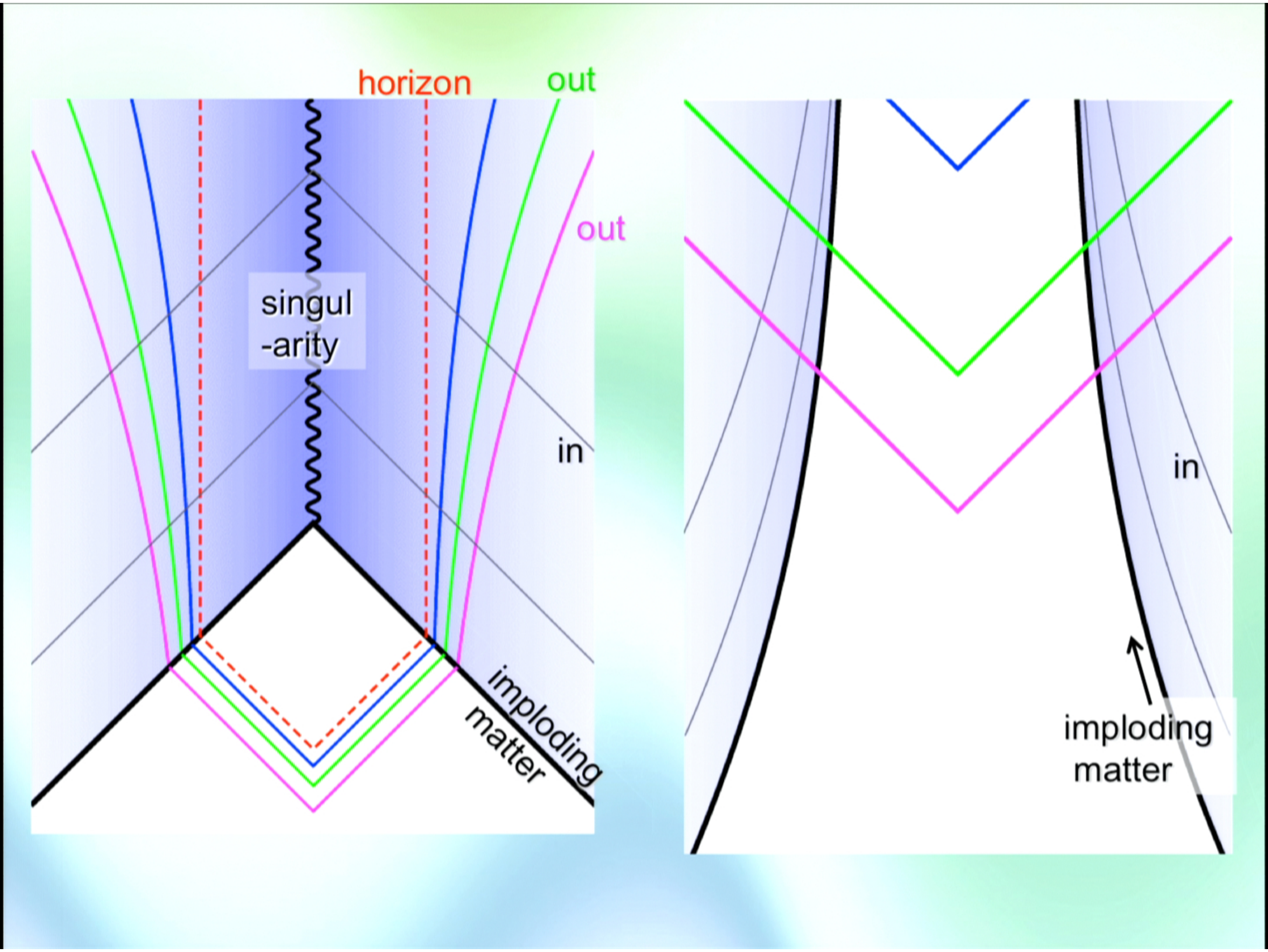
In principle, this should be possible:
just stretch spacetime so that singularities move to $t \to \infty$

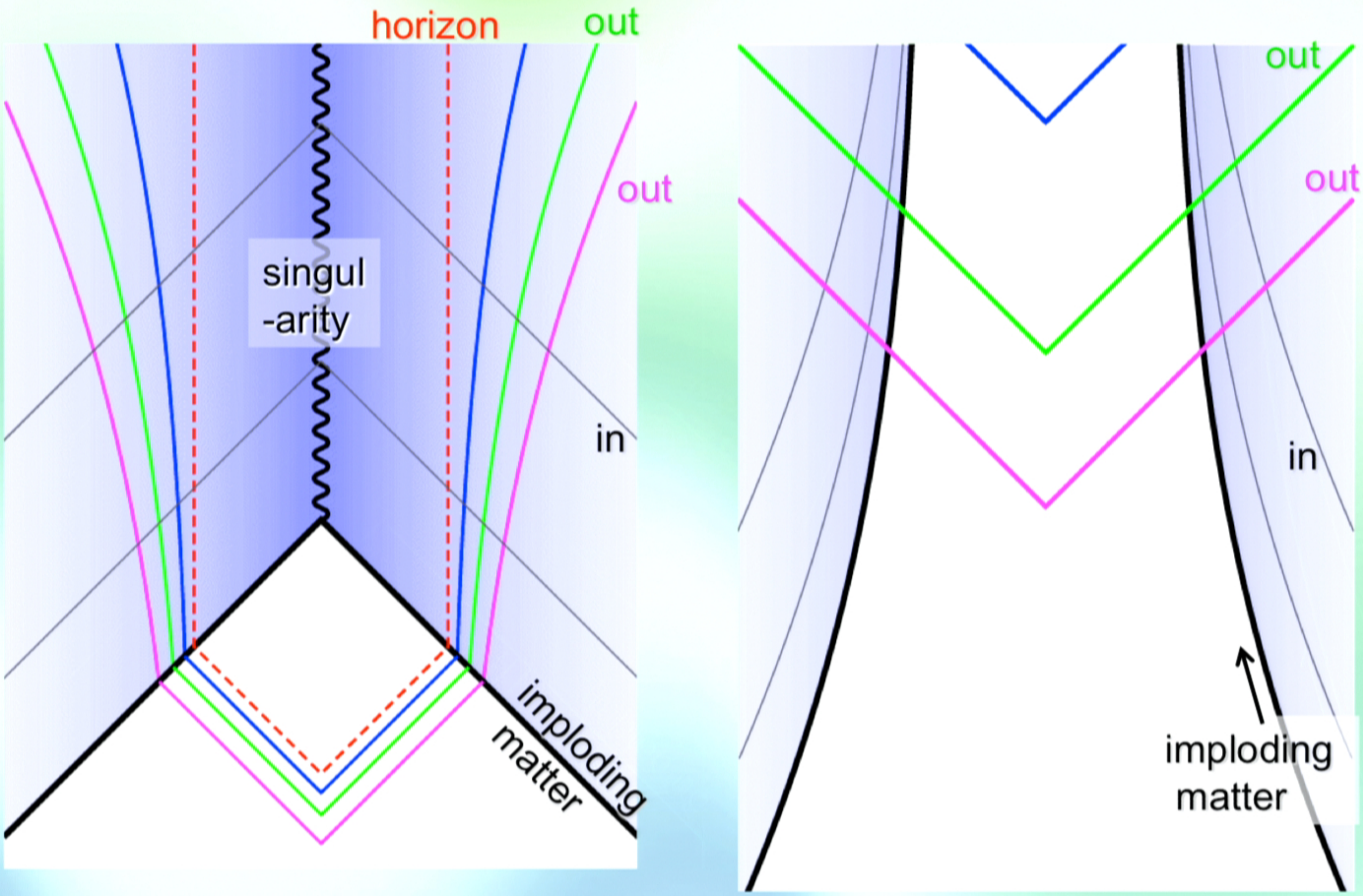


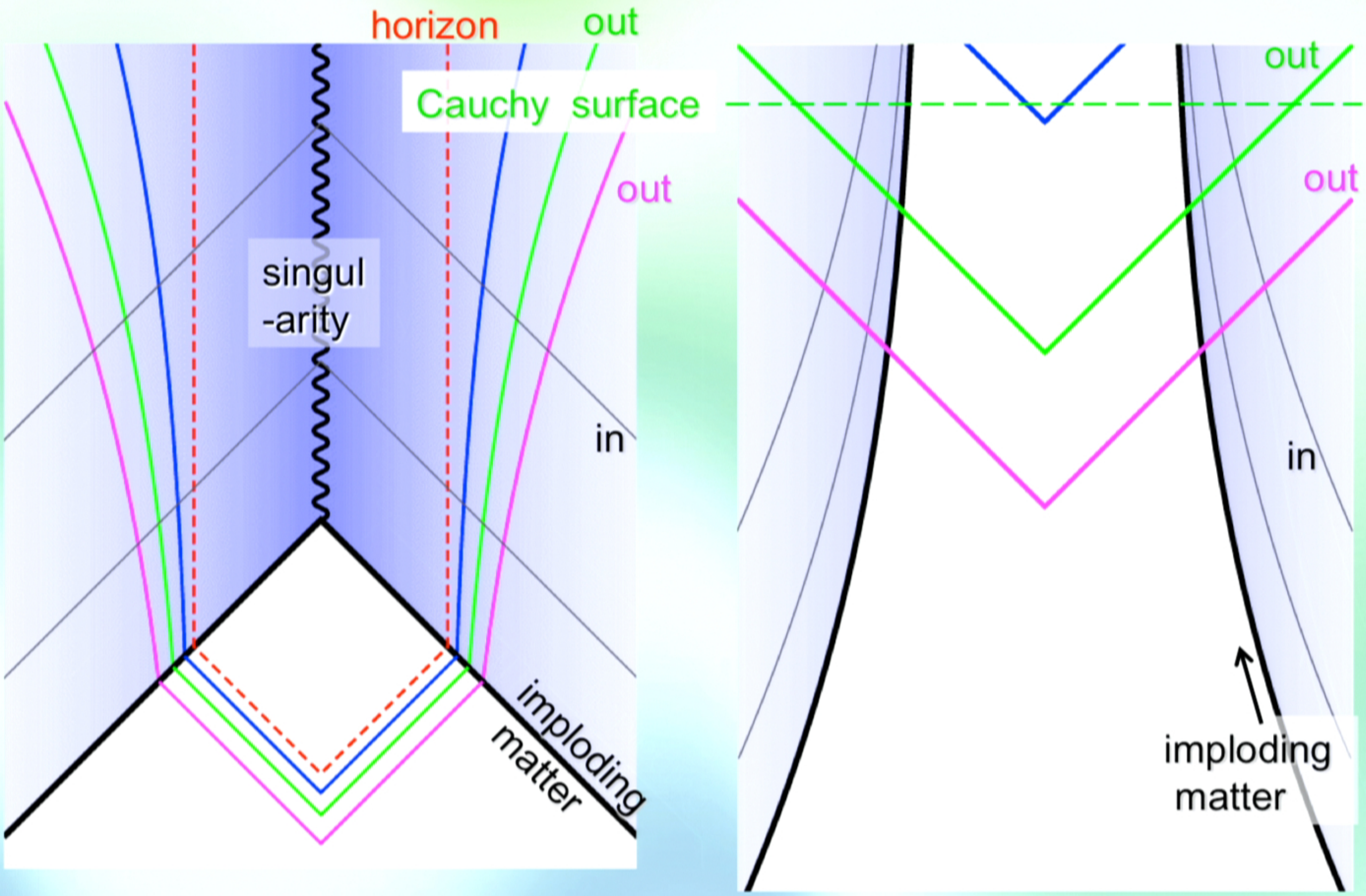
horizon
singul
-arity
imploding
matter

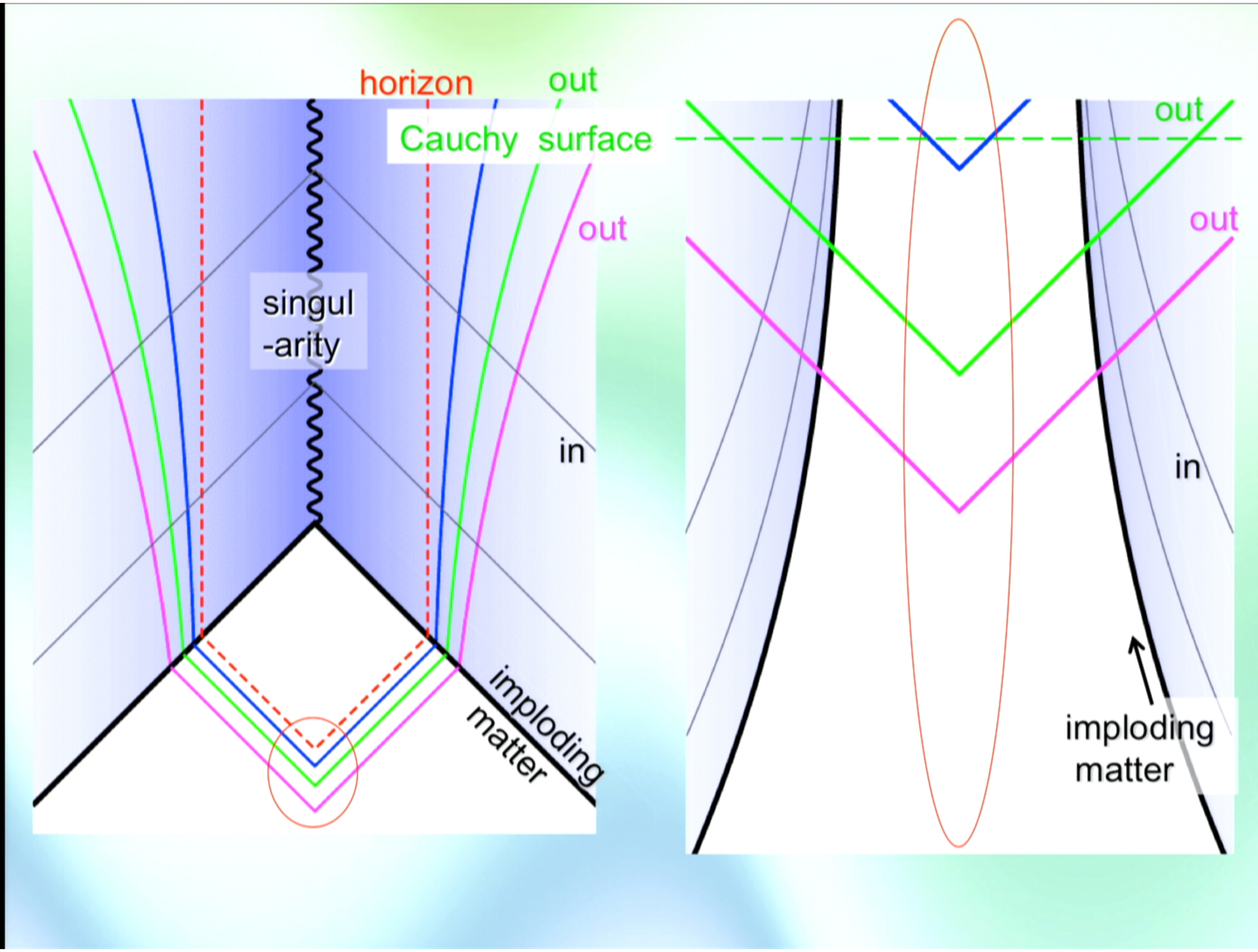

horizon
singul
-arity
in
imploding
matter

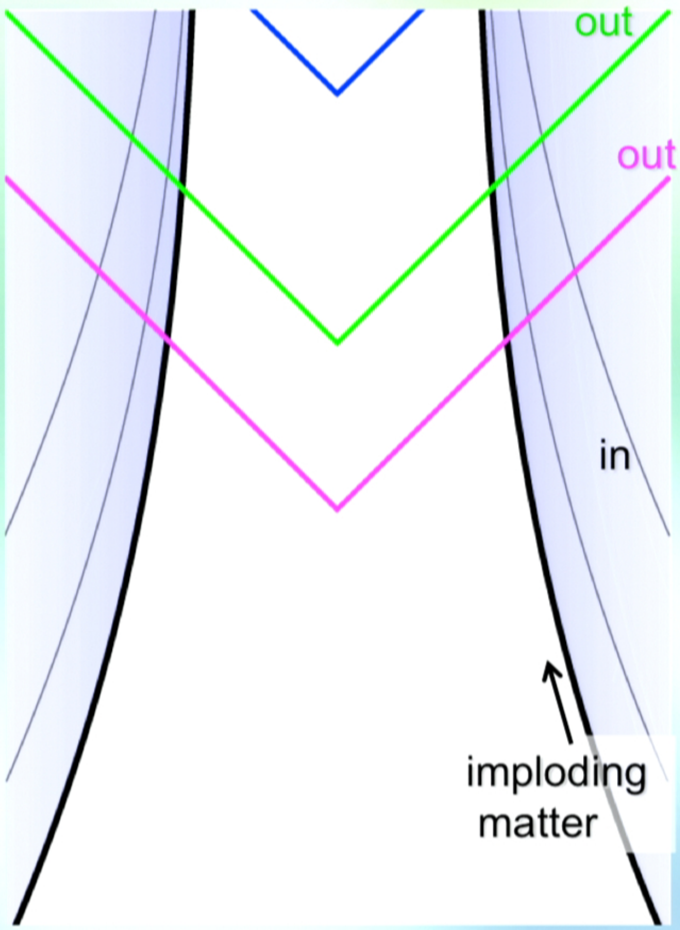

horizon
out
out
singul
-arity
in
imploding
matter

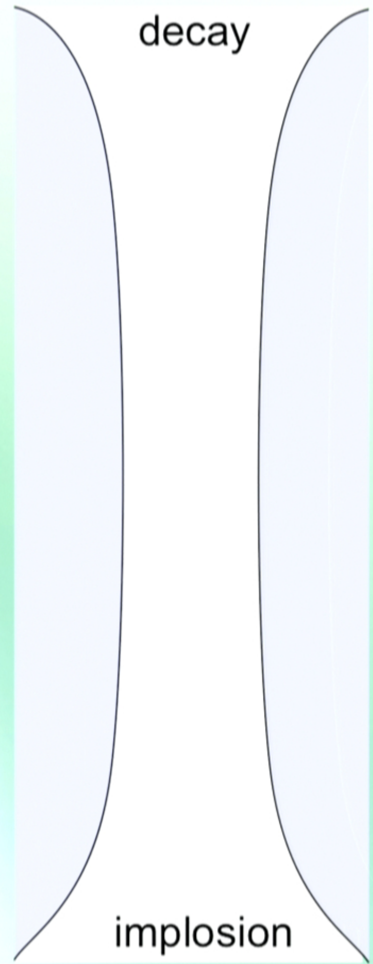

horizon
out
out
singul
-arity
in
imploding
matter
in
imploding
matter

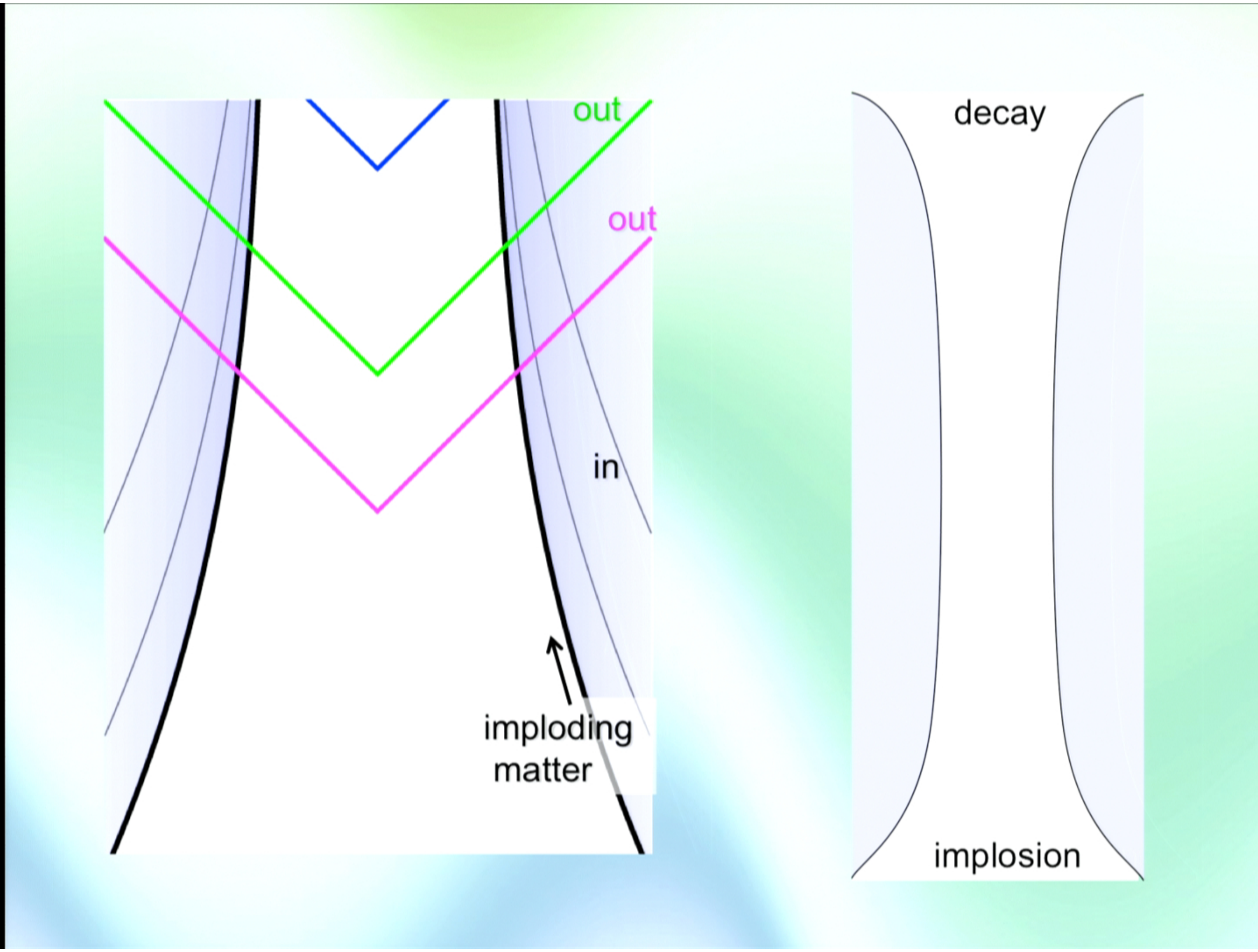

horizon
out
out
singul
-arity
in
imploding
matter
out
out
in
imploding
matter

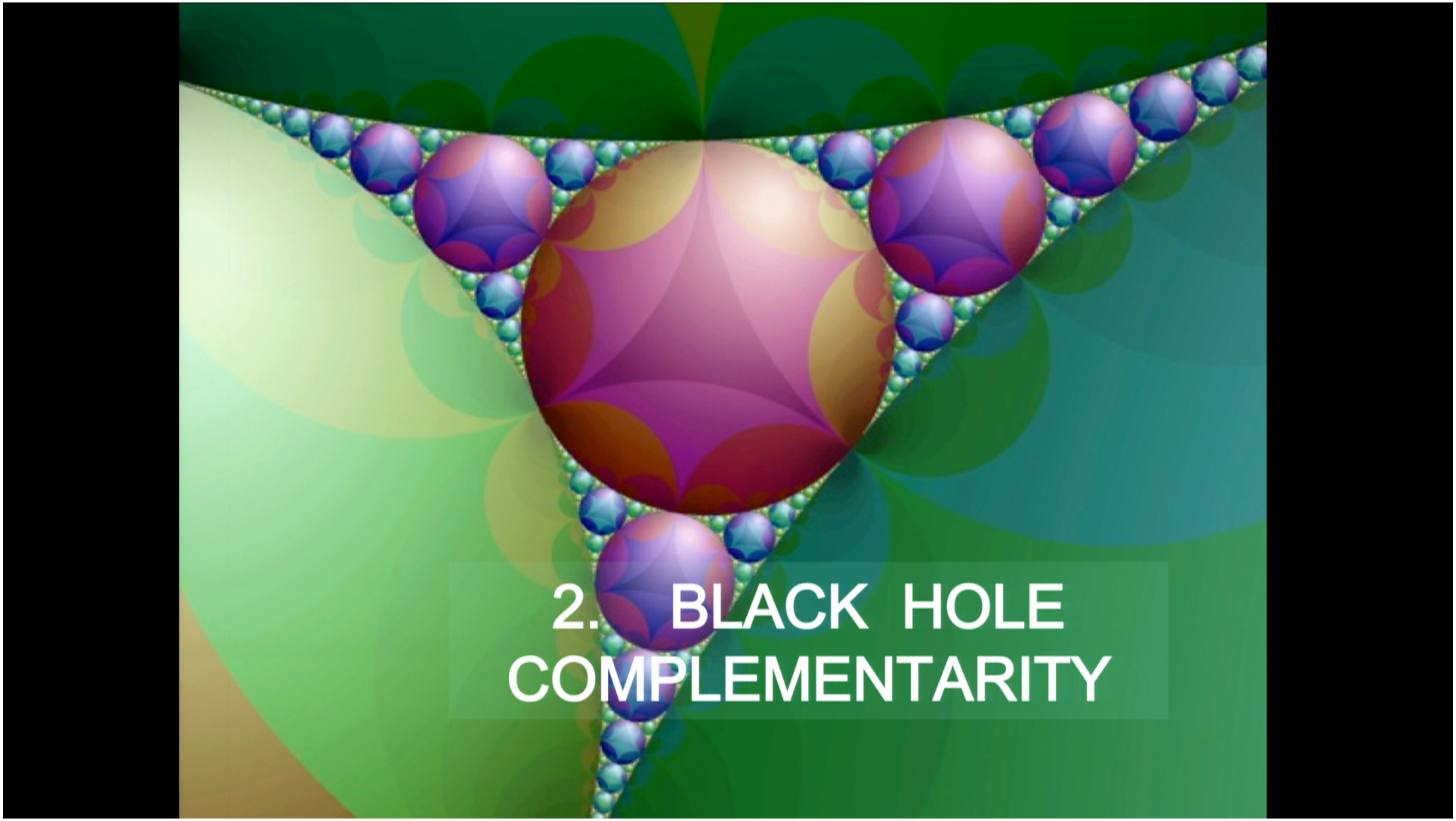

horizon
out
Cauchy surface
out
singul
-arity
in
imploding
matter
out
out
in
imploding
matter

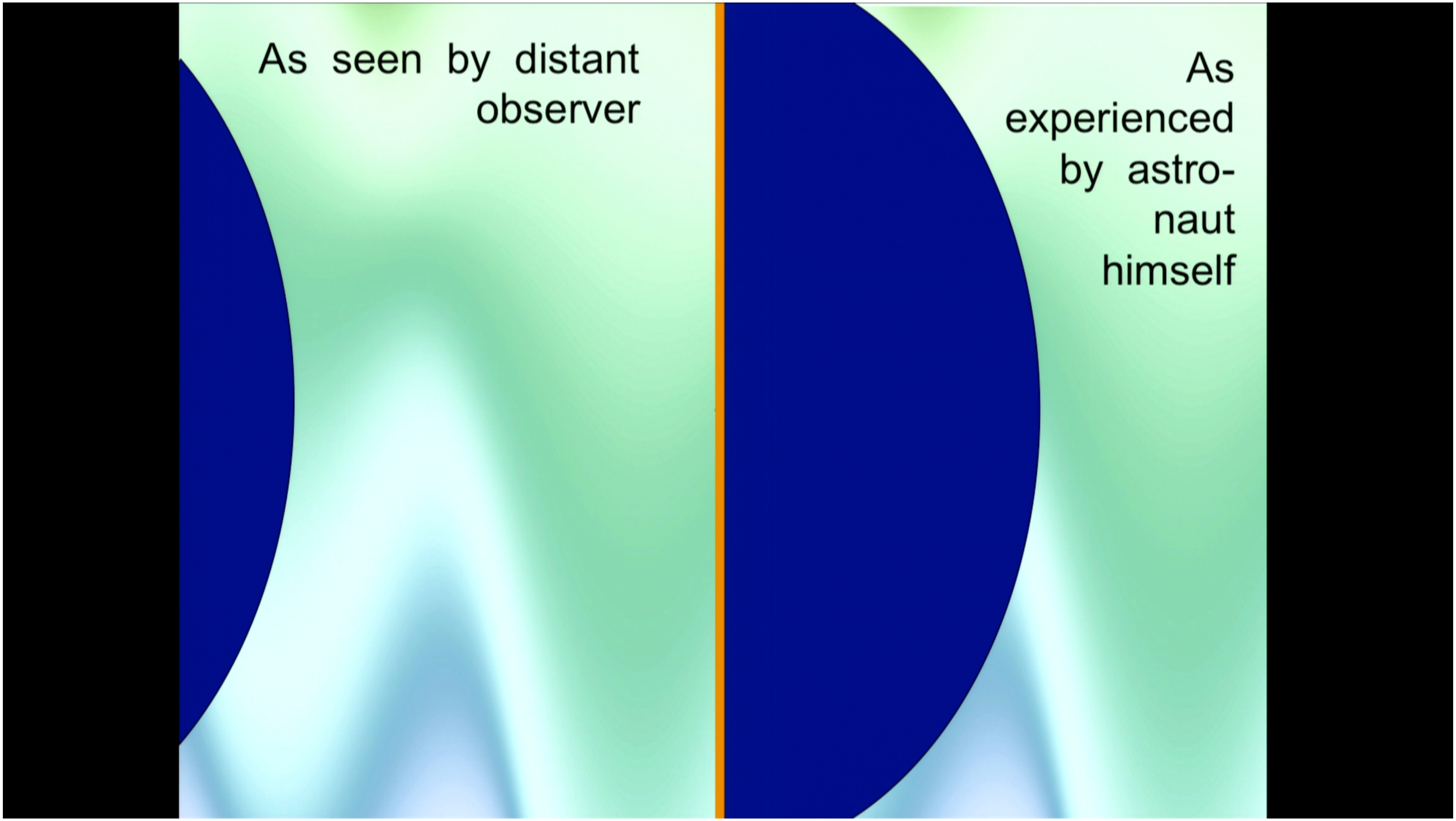

horizon
out
Cauchy surface
out
singul
-arity
out
in
imploding
matter
out
out
in
imploding
matter

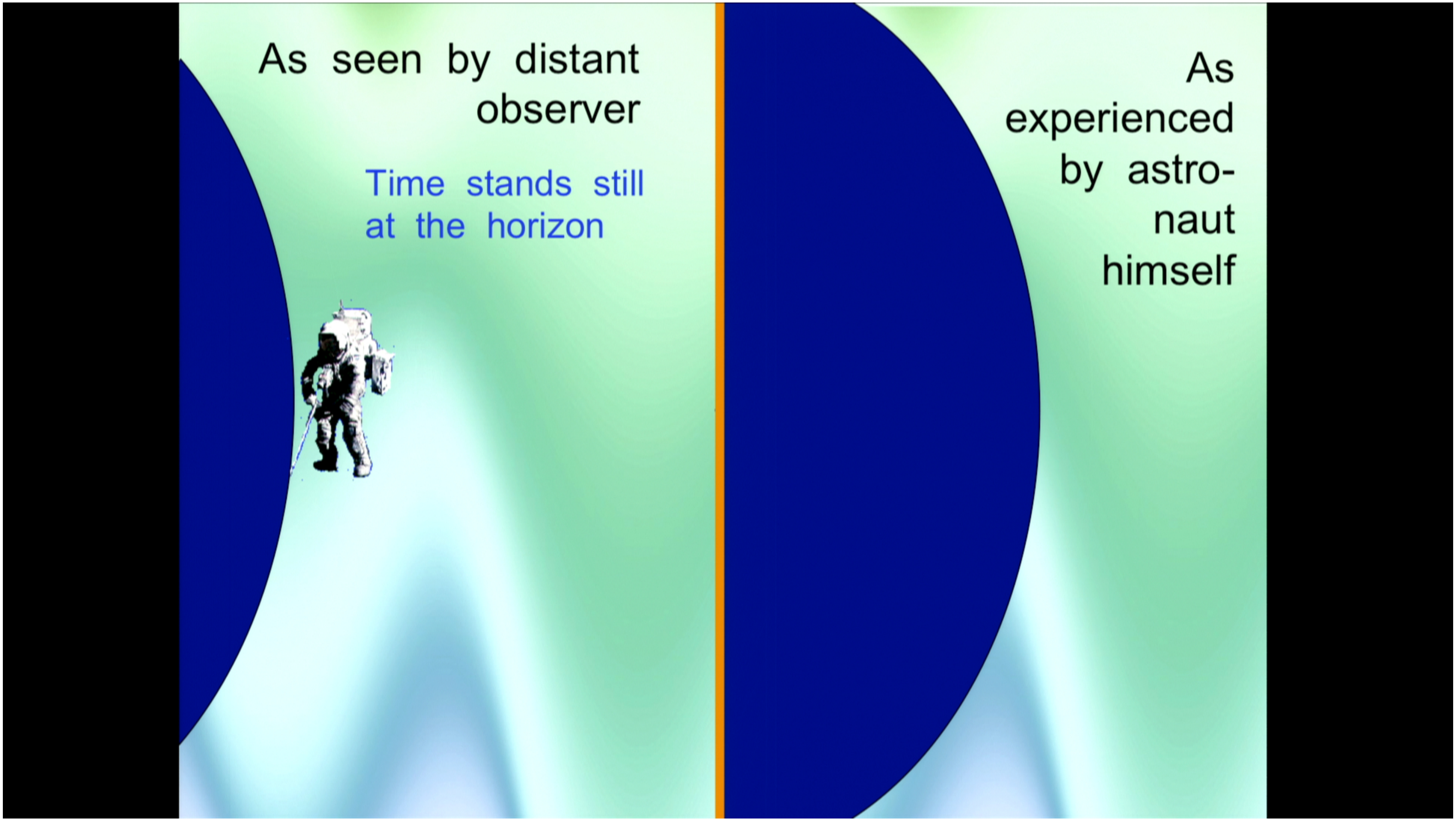

out
out
in
imploding
matter

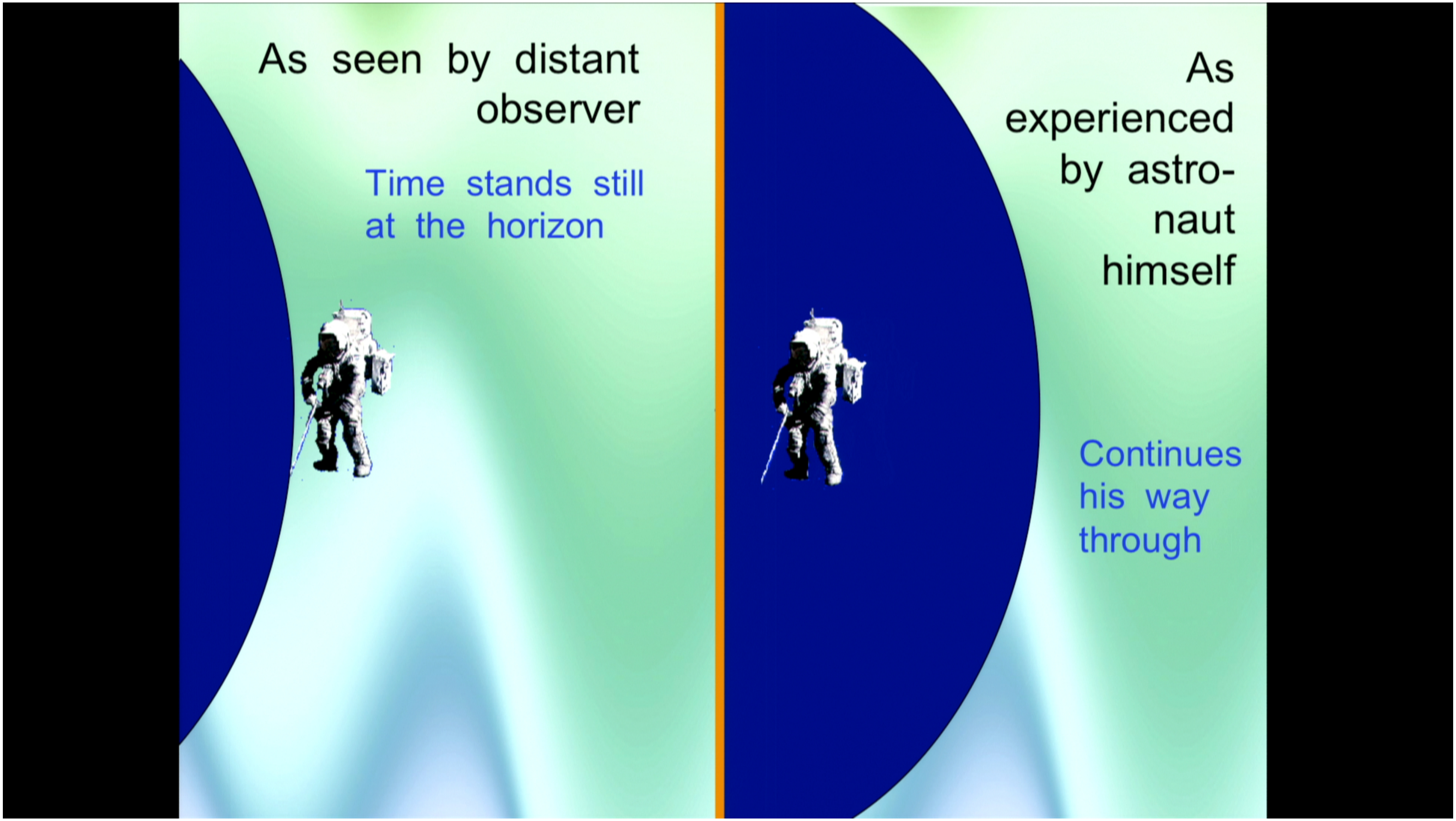

decay
implosion

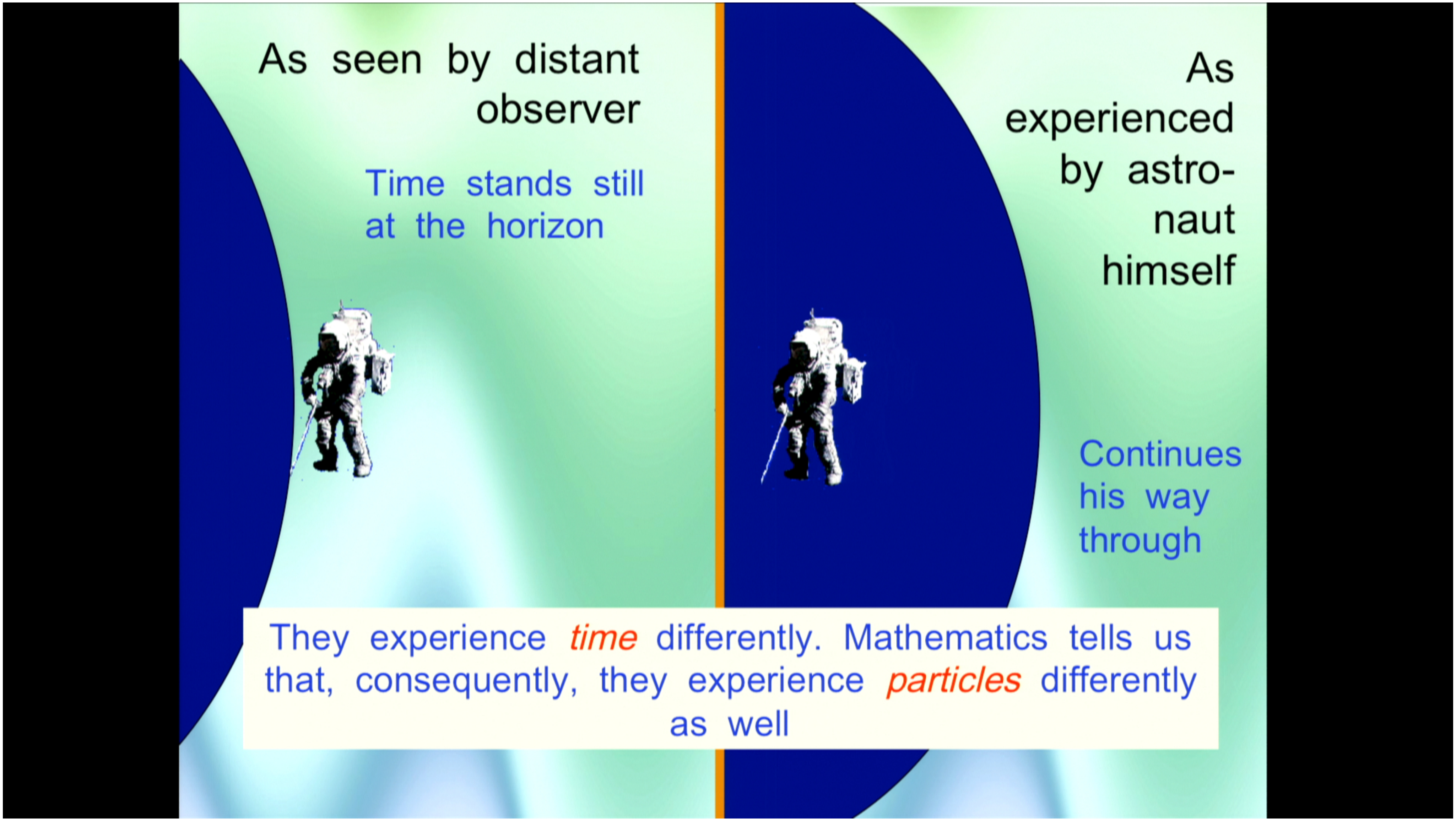

out
out
in
imploding
matter
decay
implosion

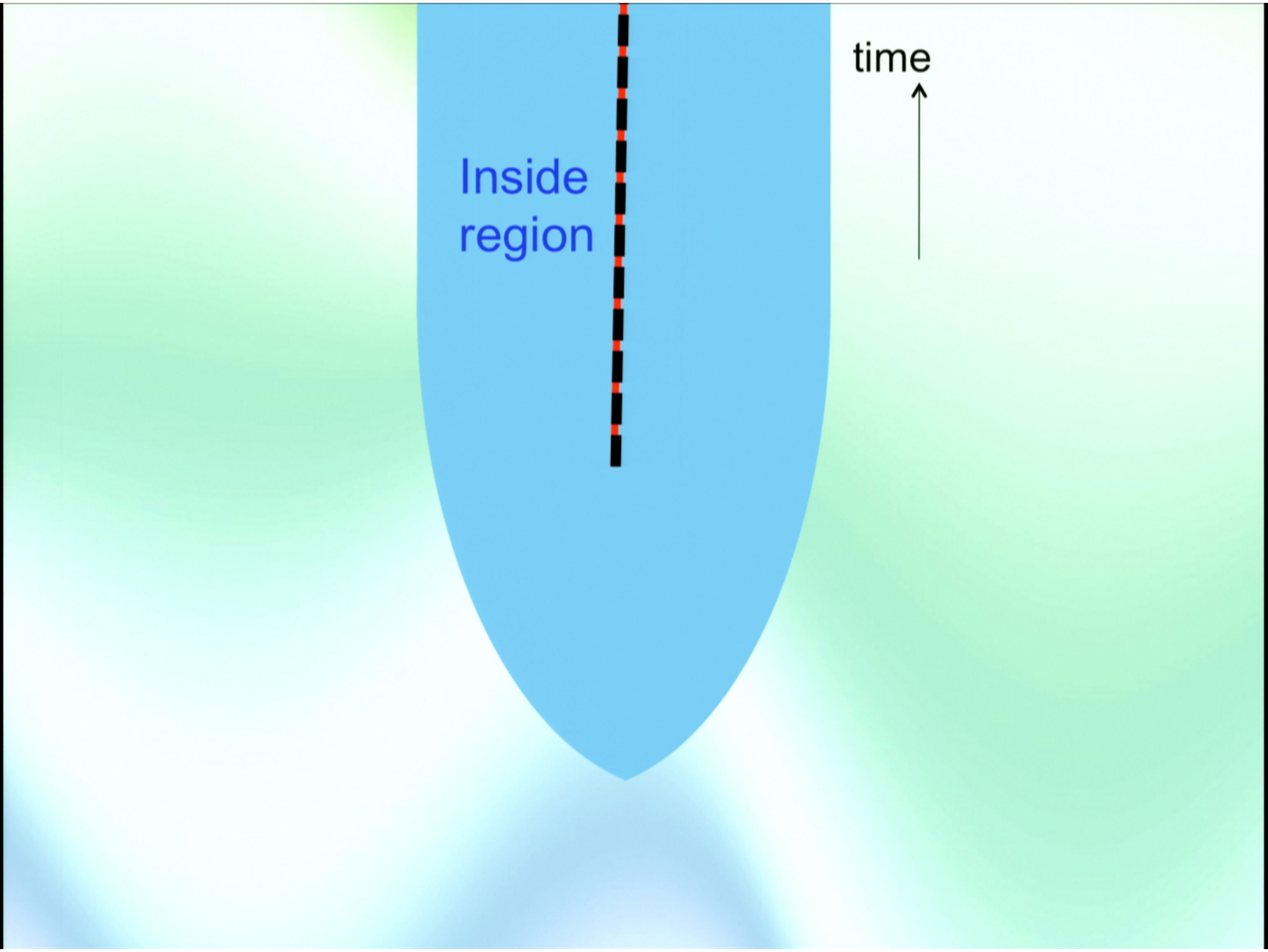

2. BLACK HOLE
COMPLEMENTARITY

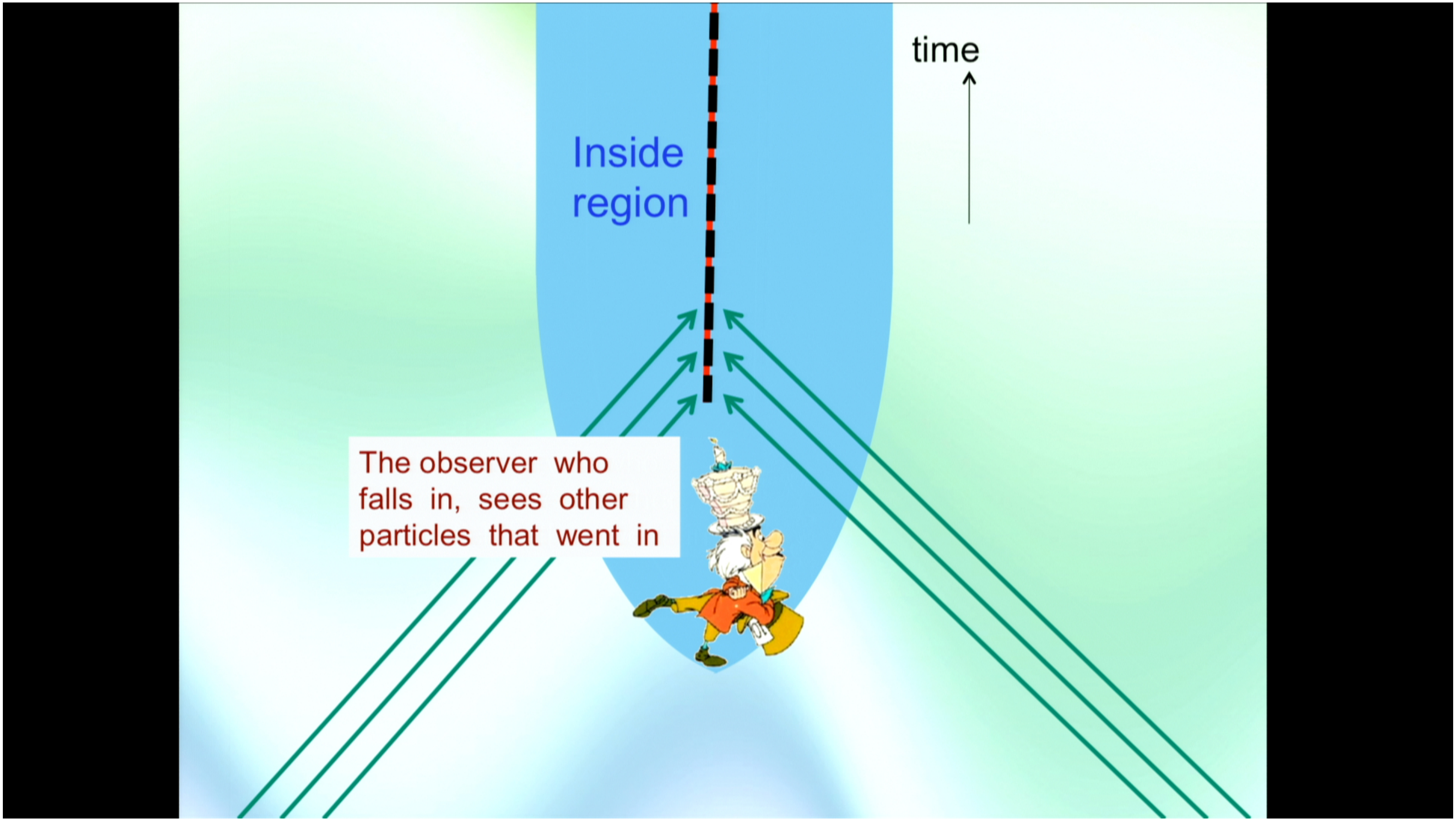

As seen by distant observer
As experienced by astro- naut himself


As seen by distant observer
Time stands still at the horizon
As experienced by astro-naut himself

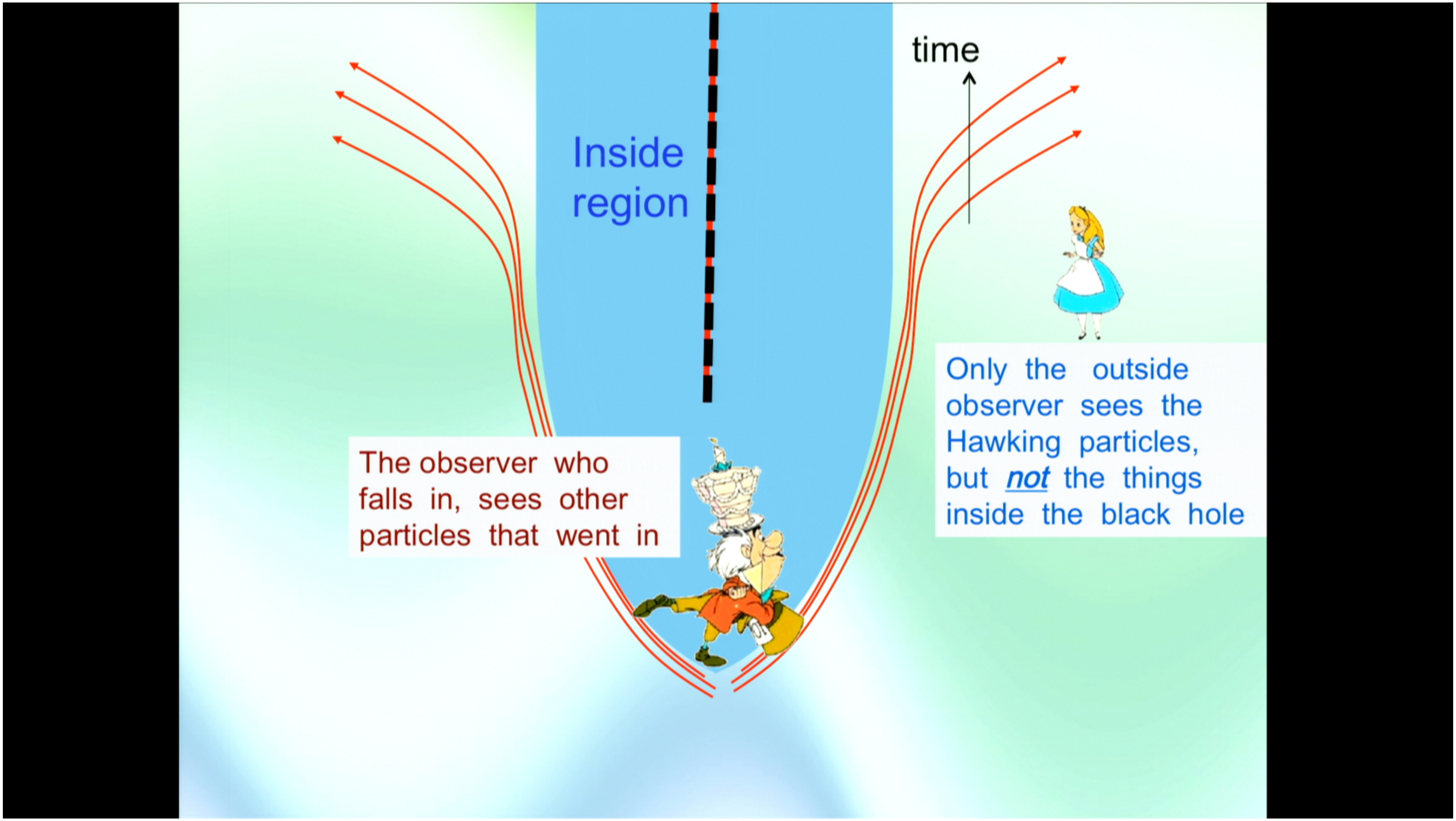

As seen by distant observer
Time stands still at the horizon
As experienced by astro-naut himself
Continues his way through


As seen by distant observer
Time stands still at the horizon
As experienced by astro-naut himself
Continues his way through
They experience *time* differently. Mathematics tells us that, consequently, they experience *particles* differently as well

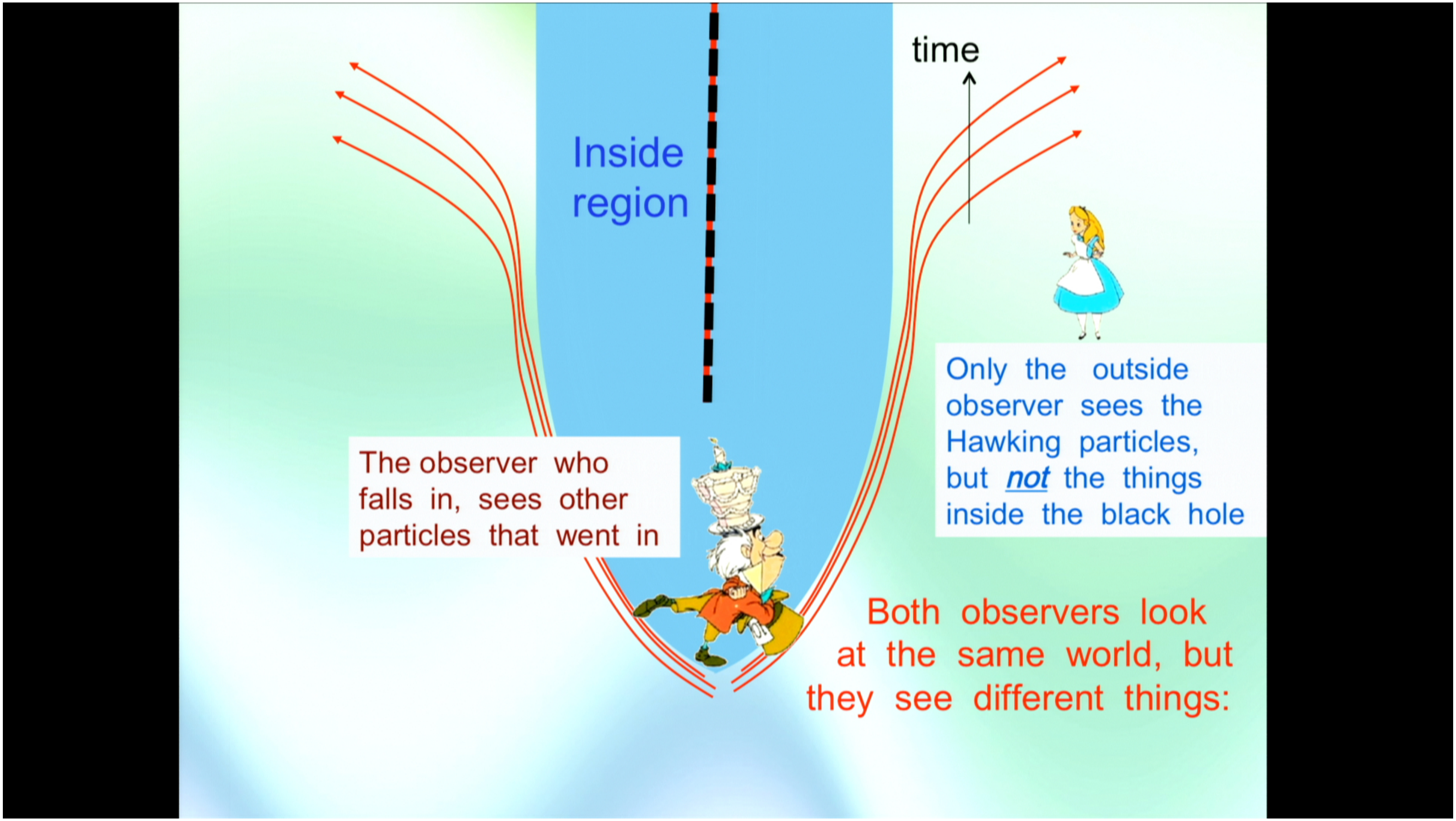

time
Inside
region

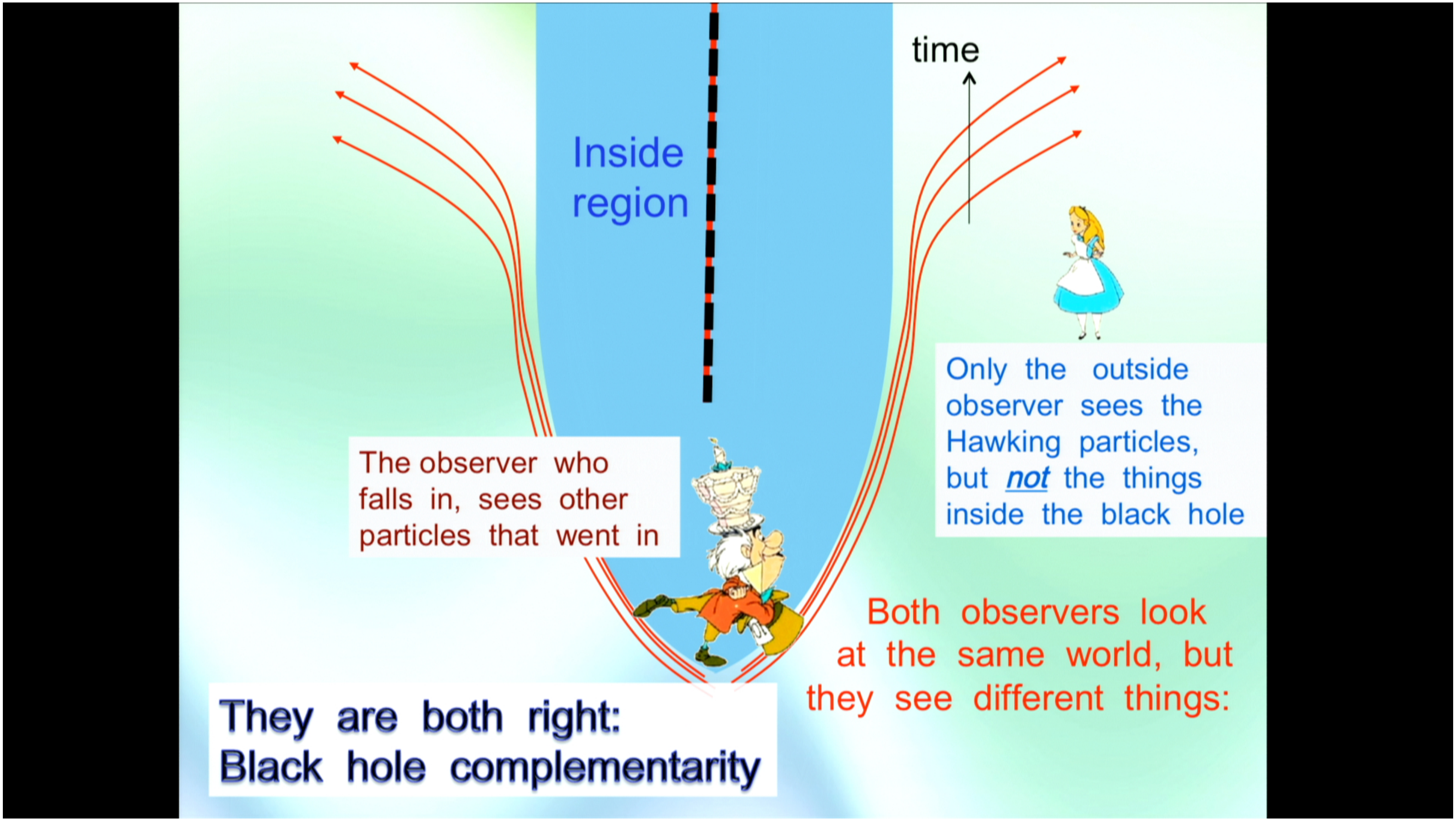


time
Inside region
The observer who falls in, sees other particles that went in



time
Inside region
The observer who falls in, sees other particles that went in
Only the outside observer sees the Hawking particles, but *not* the things inside the black hole


time
Inside region
The observer who falls in, sees other particles that went in
Only the outside observer sees the Hawking particles, but *not* the things inside the black hole
Both observers look at the same world, but they see different things:

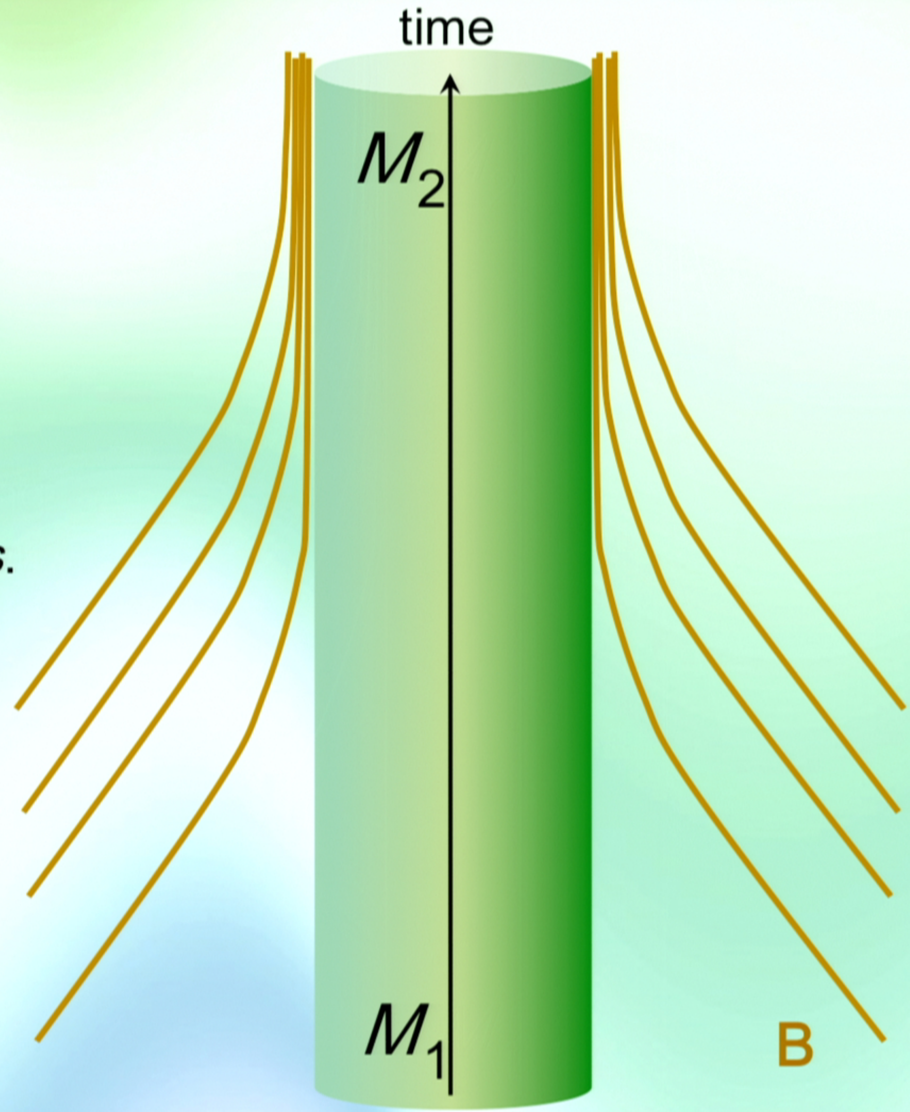

time
Inside region
The observer who falls in, sees other particles that went in
Only the outside observer sees the Hawking particles, but *not* the things inside the black hole
Both observers look at the same world, but they see different things:
They are both right: Black hole complementarity


3. The Need for EXACT LOCAL
CONFORMAL INVARIANCE

Consider a large Black Hole, slowly absorbing matter, and/or slowly decaying.

Observer B sees only particles going in: *black hole grows*.

Observer W sees only particles going out: “white hole” shrinks !

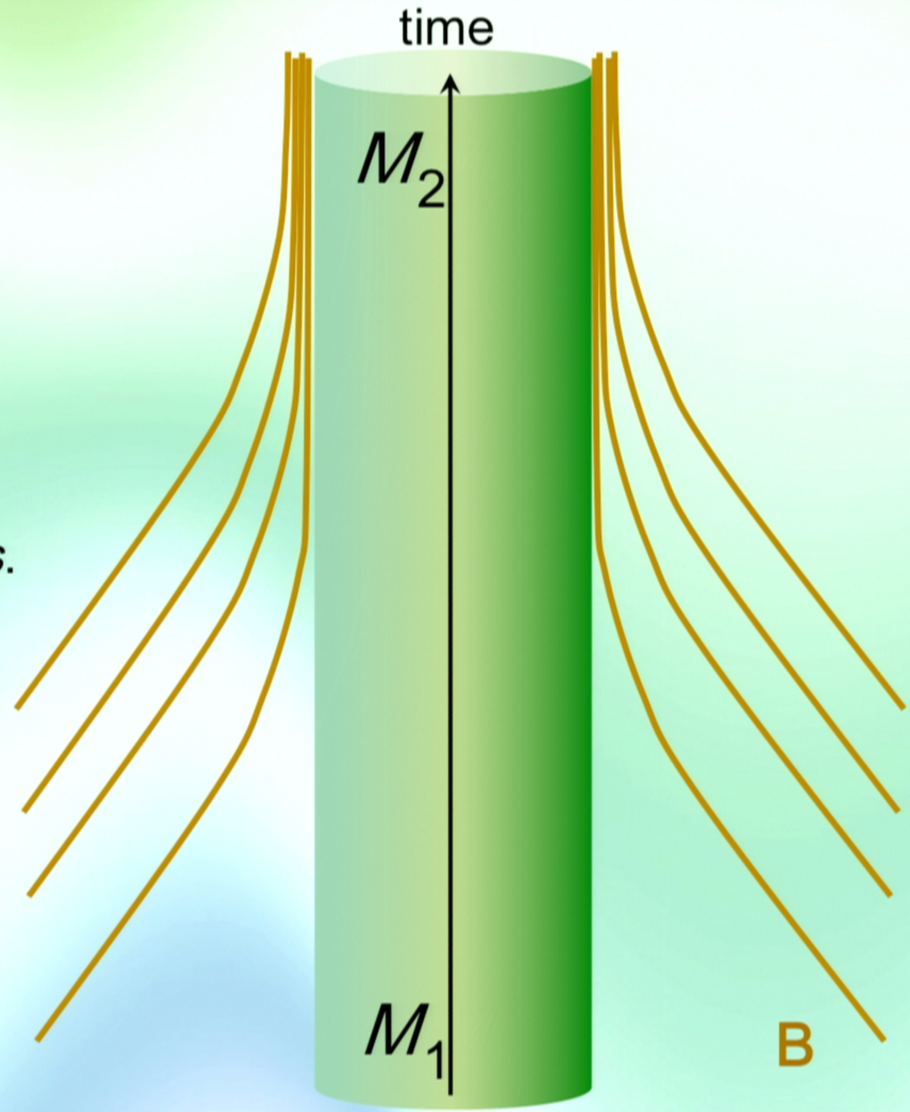


Consider a large Black Hole, slowly absorbing matter, and/or slowly decaying.

Observer B sees only particles going in: *black hole grows*.

$$M_2 > M_1$$

Observer W sees only particles going out: “white hole” shrinks !

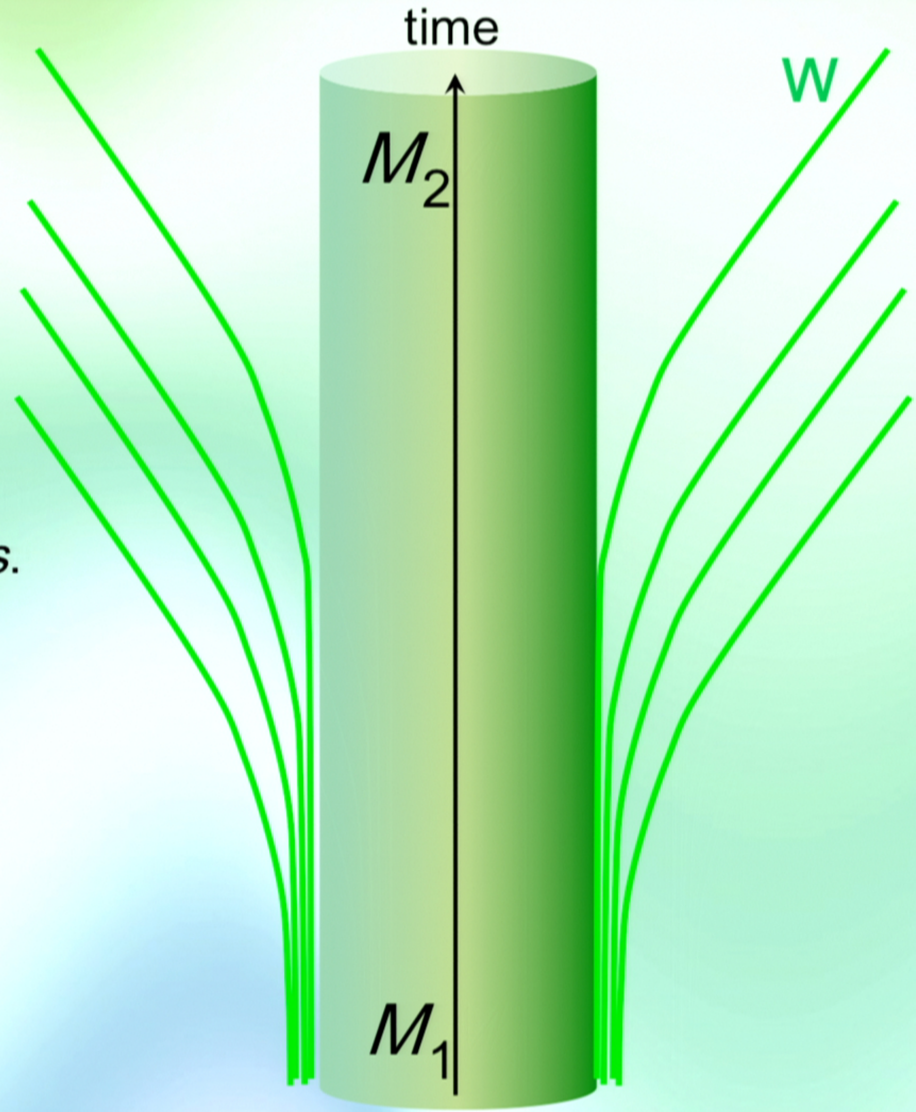


Consider a large Black Hole, slowly absorbing matter, and/or slowly decaying.

Observer B sees only particles going in: *black hole grows*.

$$M_2 > M_1$$

Observer W sees only particles going out: “white hole” shrinks !



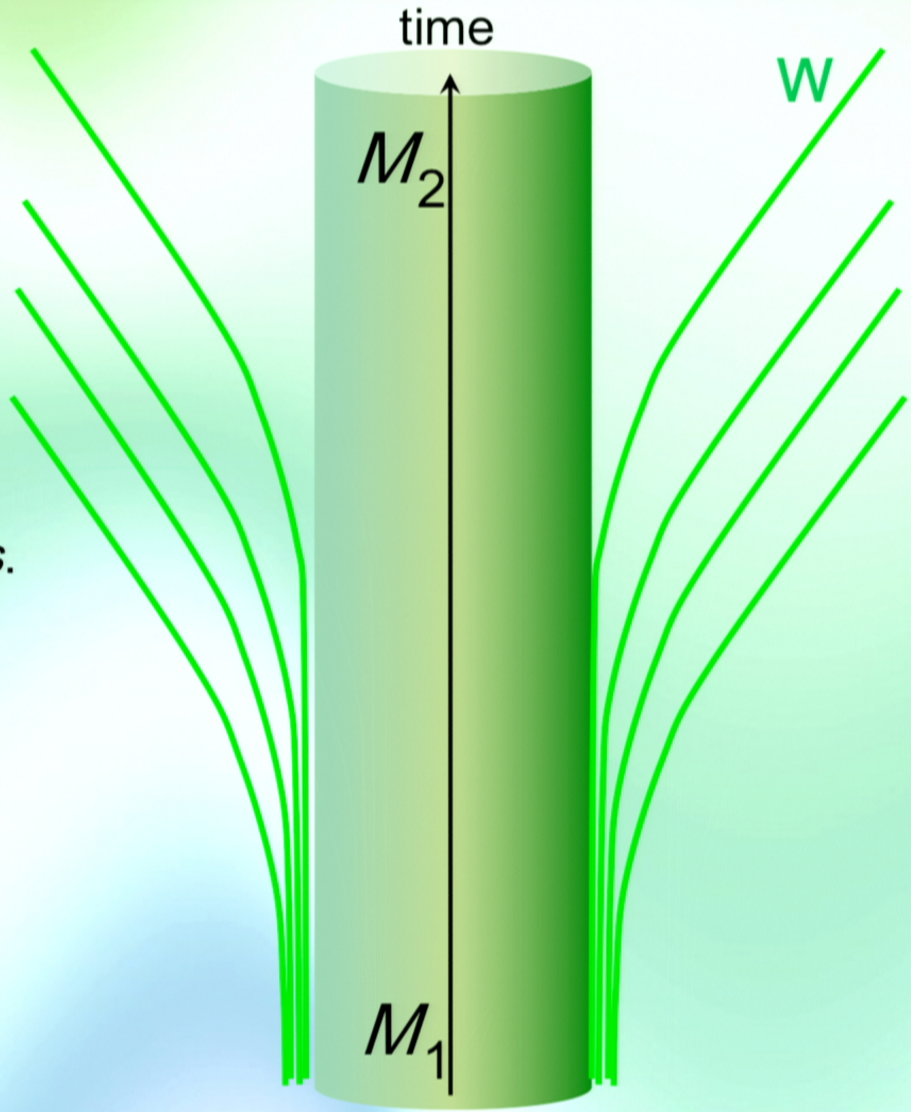
Consider a large Black Hole, slowly absorbing matter, and/or slowly decaying.

Observer B sees only particles going in: *black hole grows*.

$$M_2 > M_1$$

Observer W sees only particles going out: “white hole” shrinks !

$$M_1 > M_2 \quad ?$$



Both observers see the metric in the same Kruskal coordinates:

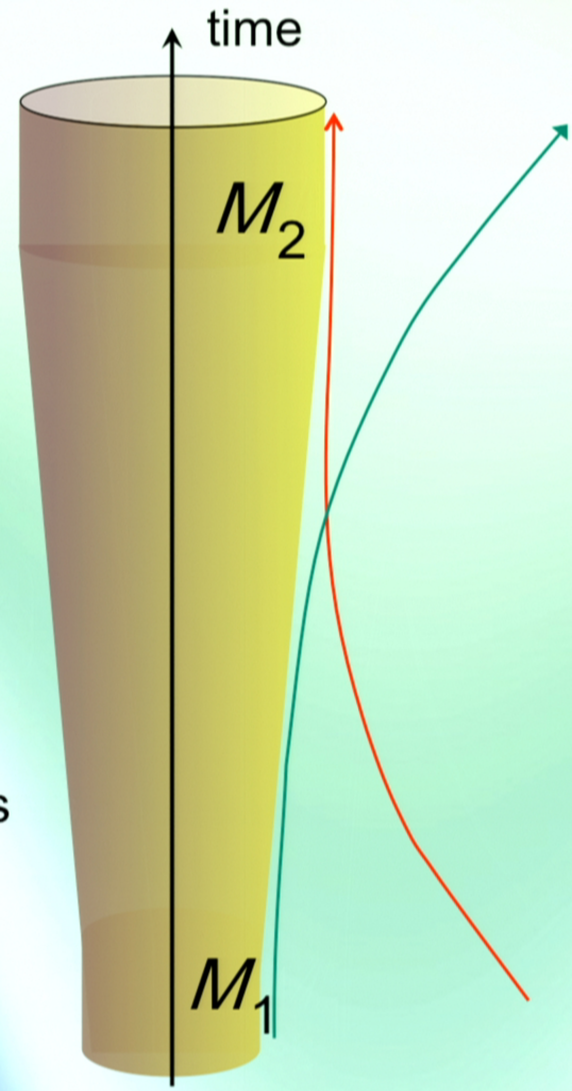
Let $r = \lambda(t)\rho$, $M(t) = \lambda(t)M$,

$$ds^2 = \frac{32\lambda^2(t)M^3}{\rho} e^{-\rho/2M}\, dx\, dy + \lambda^2(t)\rho^2 d\Omega^2$$

$$= \lambda^2(t)\left(\frac{32M^3}{\rho} e^{-\rho/2M}\, dx\, dy + \rho^2 d\Omega^2\right)$$

In these coordinates, both observers *see the same light cones* ! (causality)

This is a *local conformal transformation.*



Both observers see the metric in the same Kruskal coordinates:

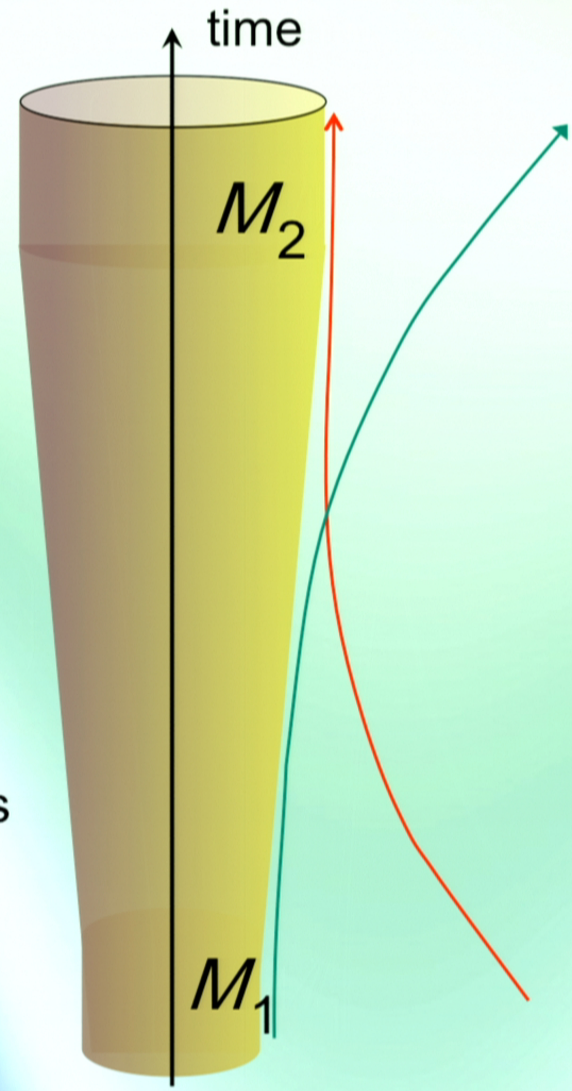
Let $r = \lambda(t)\rho \ , \ \ M(t) = \lambda(t)M \ ,$

$$ds^2 = \frac{32\lambda^2(t)M^3}{\rho} e^{-\rho/2M} \, dx\, dy + \lambda^2(t)\rho^2 d\Omega^2$$

$$= \lambda^2(t)\left(\frac{32M^3}{\rho} e^{-\rho/2M} \, dx\, dy + \rho^2 d\Omega^2 \right)$$

In these coordinates, both observers *see the same light cones* ! (causality)

This is a *local conformal transformation.*



Local conformal transformation: $g_{\mu\nu}(x) \to \lambda(x) g_{\mu\nu}(x)$
This modifies the curvature of space-time –
hence also the energy-momentum tensor:

$$T_{\mu\nu}(x) \to T_{\mu\nu}(x) - \left(\tfrac{1}{8\pi G}\right)(D_\mu \partial_\nu \lambda - g_{\mu\nu} D^2 \lambda) + \ldots$$

In a theory with exact local conformal invariance, one must fix the gauge, which means that ***one*** component of $T_{\mu\nu}(x)$ can be fixed – at *all* space-time points x.

Local conformal transformation: $g_{\mu\nu}(x) \rightarrow \lambda(x)\, g_{\mu\nu}(x)$

This modifies the curvature of space-time –

hence also the energy-momentum tensor:

$$T_{\mu\nu}(x) \rightarrow T_{\mu\nu}(x) - \left(\tfrac{1}{8\pi G}\right)(D_\mu \partial_\nu \lambda - g_{\mu\nu} D^2 \lambda) + \ldots$$

In a theory with exact local conformal invariance, one must fix the gauge, which means that ***one*** component of $T_{\mu\nu}(x)$ can be fixed – at *all* space-time points x.

Radially symmetric metric for black / white hole:
start with Kruskal coordinates,

$$ds^2 = 4M^2 e^{\mu(x,y)} \left(\frac{4}{\rho(x,y)} dx\, dy + \rho^2(x,y) d\Omega^2 \right)$$

$$x\, y = (\rho(x,y) - 1) e^{\rho(x,y)} \quad ; \qquad \rho = \frac{r}{2M} \ , \quad \lambda = e^{\frac{\mu(x,y)}{2}}$$

Einstein tensor:

$$G_{xx} = \left(1 - \frac{1}{\rho^2}\right) \frac{\mu_x}{x} - \frac{1}{2} \mu_x^2 + \mu_{x\,x}$$

$$G_{yy} = \left(1 - \frac{1}{\rho^2}\right) \frac{\mu_y}{y} - \frac{1}{2} \mu_y^2 + \mu_{y\,y}$$

$$G_{xy} = 2 \frac{1-\rho}{\rho^2} \left(\frac{\mu_x}{y} + \frac{\mu_y}{x}\right) - \mu_x \mu_y - \mu_{x\,y}$$

$$G_{\theta\theta} = -\rho^3 e^{\rho} \left(\mu_{x\,y} + \frac{\mu_x \mu_y}{4}\right) - \frac{1}{2} \rho (x \mu_x + y \mu_y)$$

Black hole gauge:

$$\frac{\partial \mu}{\partial y} \equiv \mu_y = 0$$

White hole gauge:

$$\frac{\partial \mu}{\partial x} \equiv \mu_x = 0$$

Einstein tensor is then given by

$$G_{xx} = \left(1 - \tfrac{1}{\rho^2}\right)\frac{\mu_x}{x} - \tfrac{1}{2}\mu_x^2 + \mu_{xx}$$

$$G_{yy} = 0 \qquad \boxed{T_{yy} = 0}$$

$$G_{xy} = 2\,\frac{1-\rho}{\rho^2}\,\frac{\mu_x}{y}$$

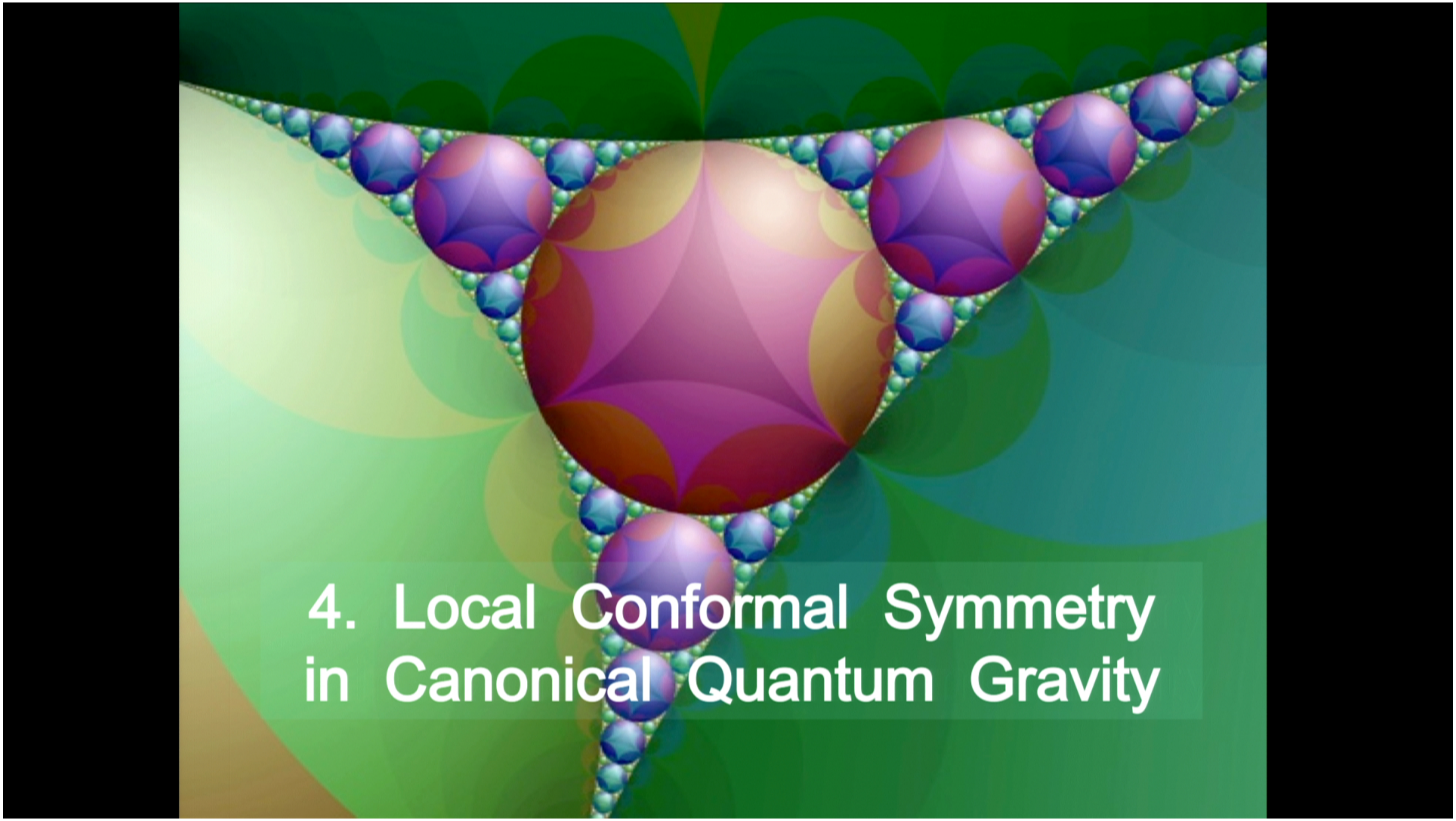
$$G_{\theta\theta} = -\tfrac{1}{2}\rho x \mu_x$$

$$G_{xx} = 0 \qquad \boxed{T_{xx} = 0}$$

$$G_{yy} = \left(1 - \tfrac{1}{\rho^2}\right)\frac{\mu_y}{y} - \tfrac{1}{2}\mu_y^2 + \mu_{yy}$$

$$G_{xy} = 2\,\frac{1-\rho}{\rho^2}\,\frac{\mu_y}{x}$$

$$G_{\theta\theta} = -\tfrac{1}{2}\rho y \mu_y$$


4. Local Conformal Symmetry
in Canonical Quantum Gravity

Invariance under scale transformations may serve as an essential new ingredient to quantize gravity

$$g_{\mu\nu} = \omega^2 \hat{g}_{\mu\nu} \ , \quad \det(\hat{g}_{\mu\nu}) = -1 \ ,$$

$$\omega = (-\det(g_{\mu\nu}))^{1/8}$$

$\hat{g}_{\mu\nu}$ describes light cones

ω describes scales

The outside, macroscopic world also has the scale factor:

$$\omega(x)\,; \quad g_{\mu\nu}(x) = \omega^2(x)\,\hat{g}_{\mu\nu}(x)$$

$$|\,x\,| \to \infty : \qquad \omega \to 1$$

Thus, the *vacuum states breaks Local Conformal Symmetry.*

What are the equations for $\hat{g}_{\mu\nu}(x)\,,\ \omega(x)$?

The transformations that keep the equation $\hat{g}_{\mu\nu} = \eta_{\mu\nu}$ unchanged are the *global conformal* transformations.

Exact local conformal invariance emerges *formally* in canonical quantum gravity:

$$\int D g_{\mu\nu}\, e^{i(S^{EH}+S^{M})} = \int D\hat{g}_{\mu\nu} \int D\omega\;\, e^{i(S^{EH}(\hat{g},\omega)+S^{M}(\hat{g},\omega))}\,;$$

$$\int D\omega\;\, e^{i(S^{EH}(\hat{g},\omega)+S^{M}(\hat{g},\omega))} = e^{iS^{eff}(\hat{g})}\;;$$

$S^{eff}(\hat{g}_{\mu\nu})$ does not depend on ω, therefore is *locally conformally invariant*!

Exact local conformal invariance emerges *formally* in canonical quantum gravity:

$$\int D g_{\mu\nu}\; e^{i(S^{EH}+S^{M})} = \int D\hat{g}_{\mu\nu}\int D\omega\;\; e^{i(S^{EH}(\hat{g},\omega)+S^{M}(\hat{g},\omega))}\,;$$

$$\int D\omega\;\; e^{i(S^{EH}(\hat{g},\omega)+S^{M}(\hat{g},\omega))} = e^{iS^{eff}(\hat{g})}\;;$$

$S^{eff}(\hat{g}_{\mu\nu})$ does not depend on ω, therefore is *locally conformally invariant* !

Exact local conformal invariance emerges *formally* in canonical quantum gravity:

$$\int D g_{\mu\nu}\ e^{i(S^{EH}+S^{M})} = \int D\hat{g}_{\mu\nu} \int D\omega\ e^{i(S^{EH}(\hat{g},\omega)+S^{M}(\hat{g},\omega))}\,;$$

$$\int D\omega\ e^{i(S^{EH}(\hat{g},\omega)+S^{M}(\hat{g},\omega))} = e^{iS^{eff}(\hat{g})}\,;$$

$S^{eff}(\hat{g}_{\mu\nu})$ does not depend on ω, therefore is *locally conformally invariant* !

Would this make $S^{eff}(\hat{g}_{\mu\nu})$ "renormalizable" !!?

Exact local conformal invariance emerges *formally* in canonical quantum gravity:

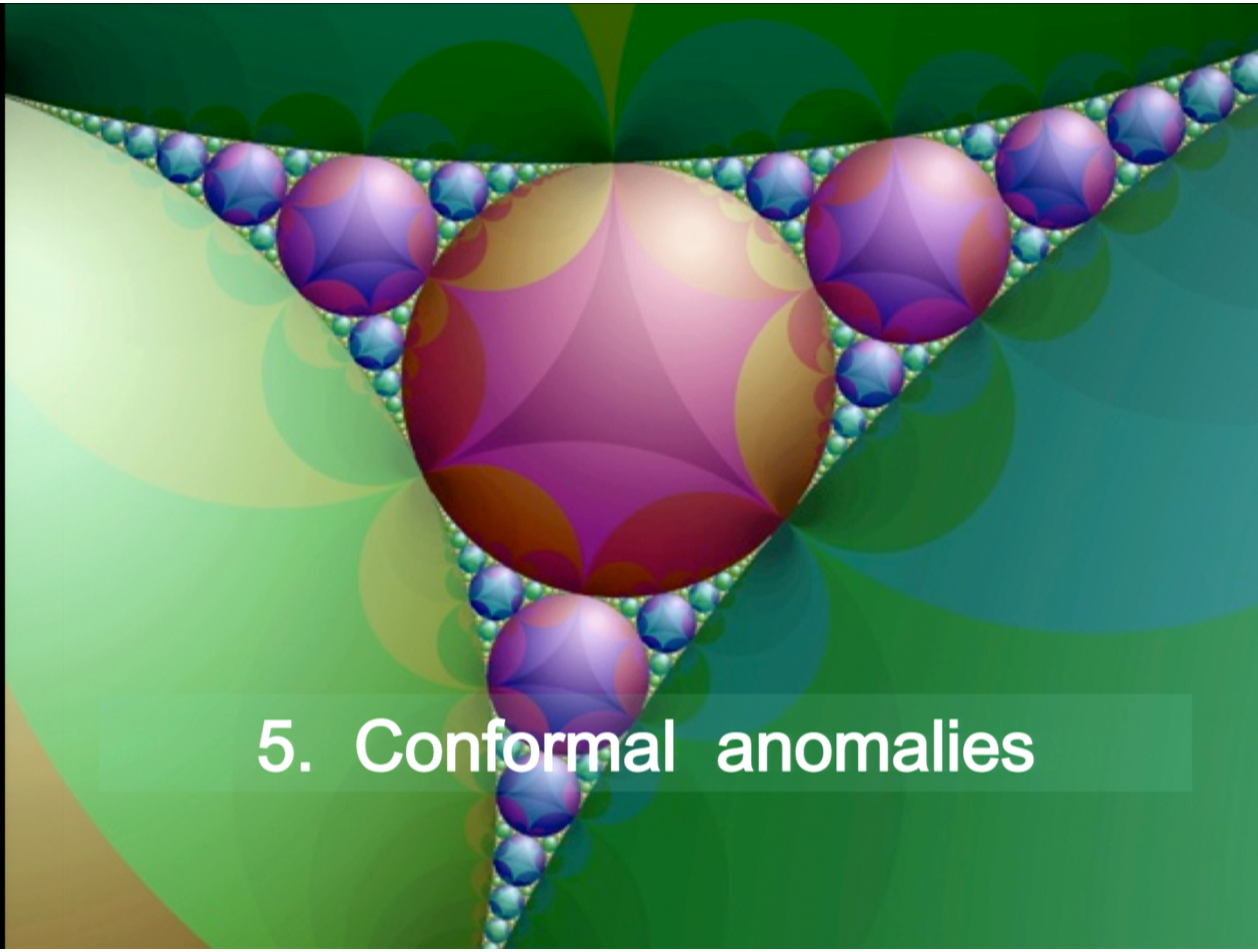
$$\int D\, g_{\mu\nu}\; e^{i(S^{EH}+S^{M})} = \int D\,\hat{g}_{\mu\nu} \int D\omega\;\; e^{i(S^{EH}(\hat{g},\omega)+S^{M}(\hat{g},\omega))}\,;$$

$$\int D\omega\;\; e^{i(S^{EH}(\hat{g},\omega)+S^{M}(\hat{g},\omega))} = e^{iS^{eff}(\hat{g})}\,;$$

$S^{eff}(\hat{g}_{\mu\nu})$ does not depend on ω, therefore is *locally conformally invariant* !

Would this make $S^{eff}(\hat{g}_{\mu\nu})$ "renormalizable" !!?

Unfortunately **no** !? *The conformal anomaly*


5. Conformal anomalies

$$\int D\omega \;\; e^{i(S^{EH}(\hat{g},\omega)+S^{M}(\hat{g},\omega))} = e^{iS^{\mathit{eff}}(\hat{g})} \; ;$$

$$S^{M}(\hat{g},\omega) = -\tfrac{1}{2}\hat{g}^{\mu\nu}\partial_{\mu}\phi\,\partial_{\nu}\phi - \tfrac{1}{2}m^{2}\omega^{2}\phi^{2};$$

$$\mathfrak{L}^{\text{eff, div}} = \frac{\sqrt{-\hat{g}}}{8\pi^{2}(4-n)}\Big(\tfrac{1}{120}(\hat{R}_{\mu\nu}\hat{R}^{\mu\nu} - \tfrac{1}{3}\hat{R}^{2}) +$$

$$+\tfrac{4}{9}\pi^{2}(G_{N}m^{2})^{2}\phi^{4}\Big)$$

is conformally invariant in 4 dimensions !

But $\frac{1}{(4-n)} \rightarrow \tfrac{1}{2}\log(\Lambda^{2}/k^{2})$

At $n \neq 4$ this “local term” is ***not*** conformally invariant.

Can one *cancel* that infinity in front of

$$\hat{R}_{\mu\nu}\hat{R}^{\mu\nu} - \tfrac{1}{3}\hat{R}^2 \quad ?$$

No !?

Matter fields (spin 0, ½, or 1) all contribute to the same anomaly, with the same sign !

Can one *cancel* that infinity in front of

$$\hat{R}_{\mu\nu}\hat{R}^{\mu\nu} - \tfrac{1}{3}\hat{R}^2 \ ?$$ No !?

Matter fields (spin 0, ½, or 1) all contribute to the same anomaly, with the same sign !

Coefficients:

grav. ω field: $+\frac{1}{120}$

N_0 Scalars: $+\frac{1}{120}N_0$

$N_{1/2}$ Dirac Spinors: $+\frac{1}{20}N_{1/2}$

N_1 Vector fields: $+\frac{1}{10}N_1$

$N_{3/2}$ Gravitinos : $-{}^{233}\!/_{720}\,N_{3/2}$

N_2 Tensor fields : $+{}^{53}\!/_{45}\,N_2$

If $R_{\mu\nu} \neq 0$ there are 2 kinds of conformal anomalies:

1) The scaling anomaly in a flat background;
2) The conformal anomalies when

$$R_{\mu\nu}^{\text{(background)}} \neq 0$$

If $R_{\mu\nu} \neq 0$ there are 2 kinds of conformal anomalies:

1) The scaling anomaly in a flat background;
2) The conformal anomalies when

$$R_{\mu\nu}^{\text{(background)}} \neq 0$$

Misc. remark:

$$\mathfrak{L} = \mathfrak{L}^{\hat{g}_{\mu\nu}} + \mathfrak{L}^{\omega} + \mathfrak{L}^{\text{matter}}$$

$$\mathfrak{L}^{\hat{g}_{\mu\nu}} = 0$$

$$T^{\omega}_{\mu\nu} = -\tfrac{1}{8\pi G}\hat{G}_{\mu\nu}$$

$$\frac{\partial \mathfrak{L}}{\partial \hat{g}^{\mu\nu}} = -\tfrac{\sqrt{-\hat{g}}}{2}(T^{\omega}_{\mu\nu} + T^{\text{matter}}_{\mu\nu})$$

$$= \tfrac{\sqrt{-\hat{g}}}{2}(-\tfrac{1}{8\pi G}\hat{G}_{\mu\nu} + T^{\text{matter}}_{\mu\nu}) = 0$$

Misc. remark:

$$\mathfrak{L} = \mathfrak{L}^{\hat{g}_{\mu\nu}} + \mathfrak{L}^{\omega} + \mathfrak{L}^{\text{matter}}$$

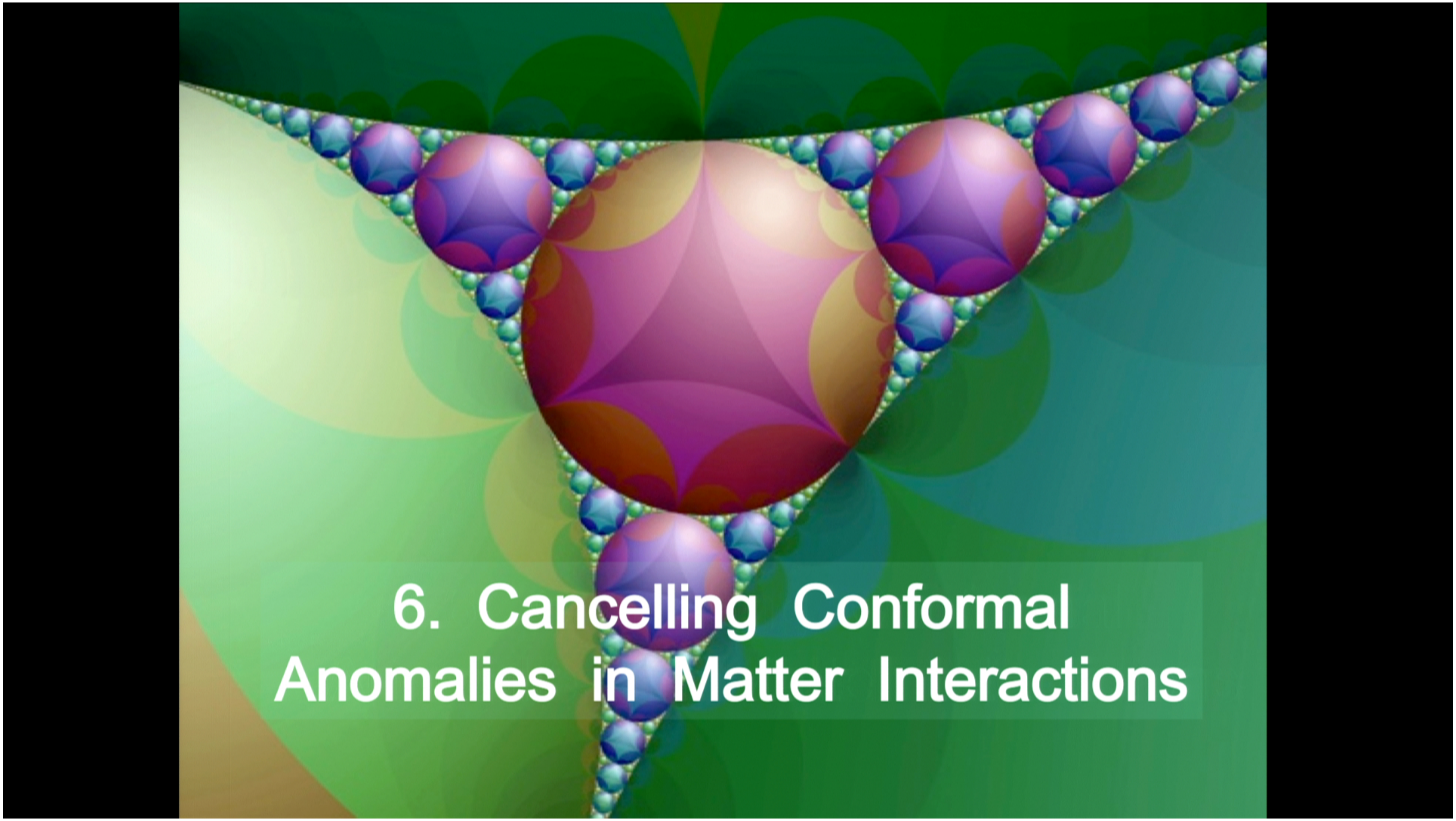
$$\mathfrak{L}^{\hat{g}_{\mu\nu}} = 0$$

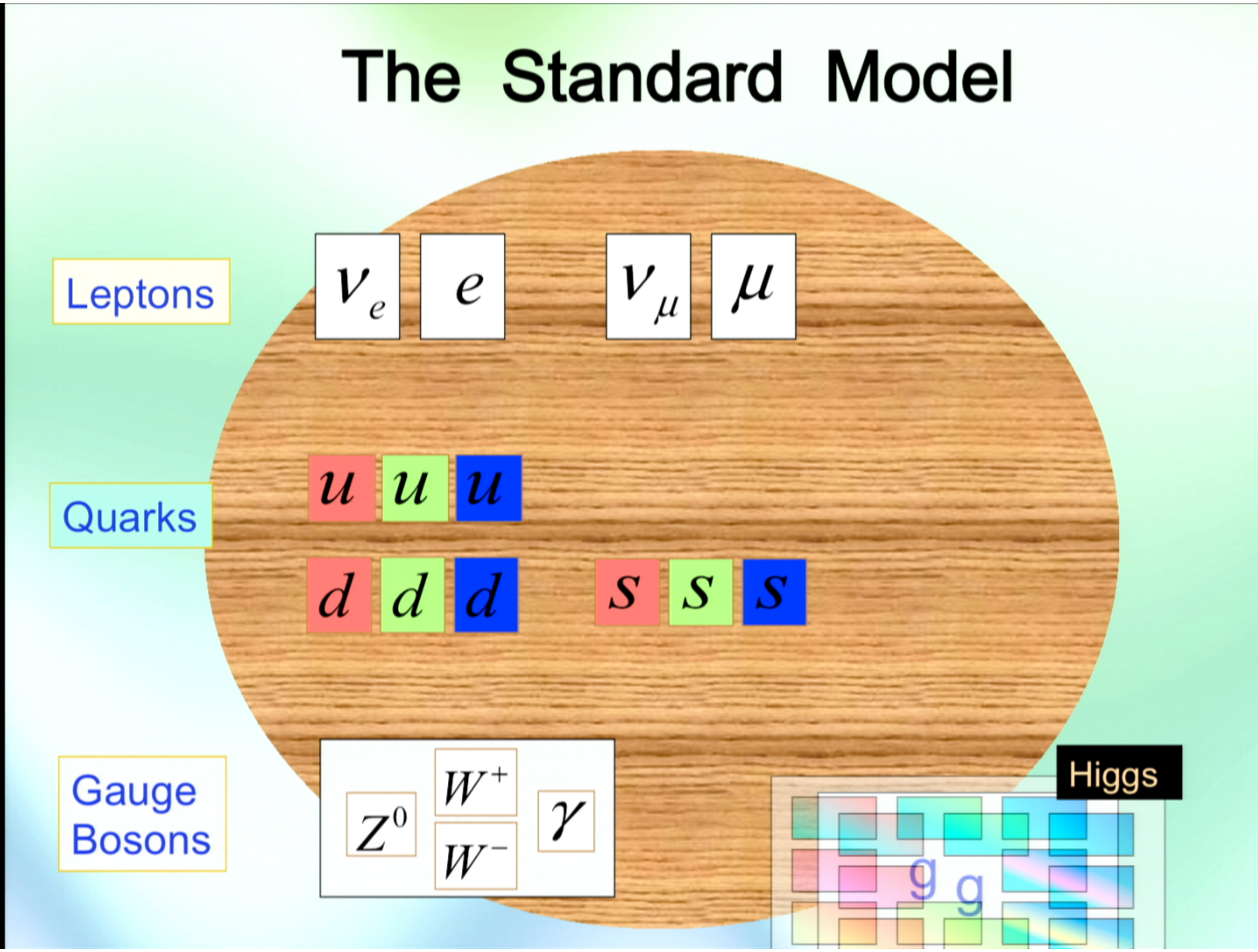
$$T^{\omega}_{\mu\nu} = -\frac{1}{8\pi G}\hat{G}_{\mu\nu}$$

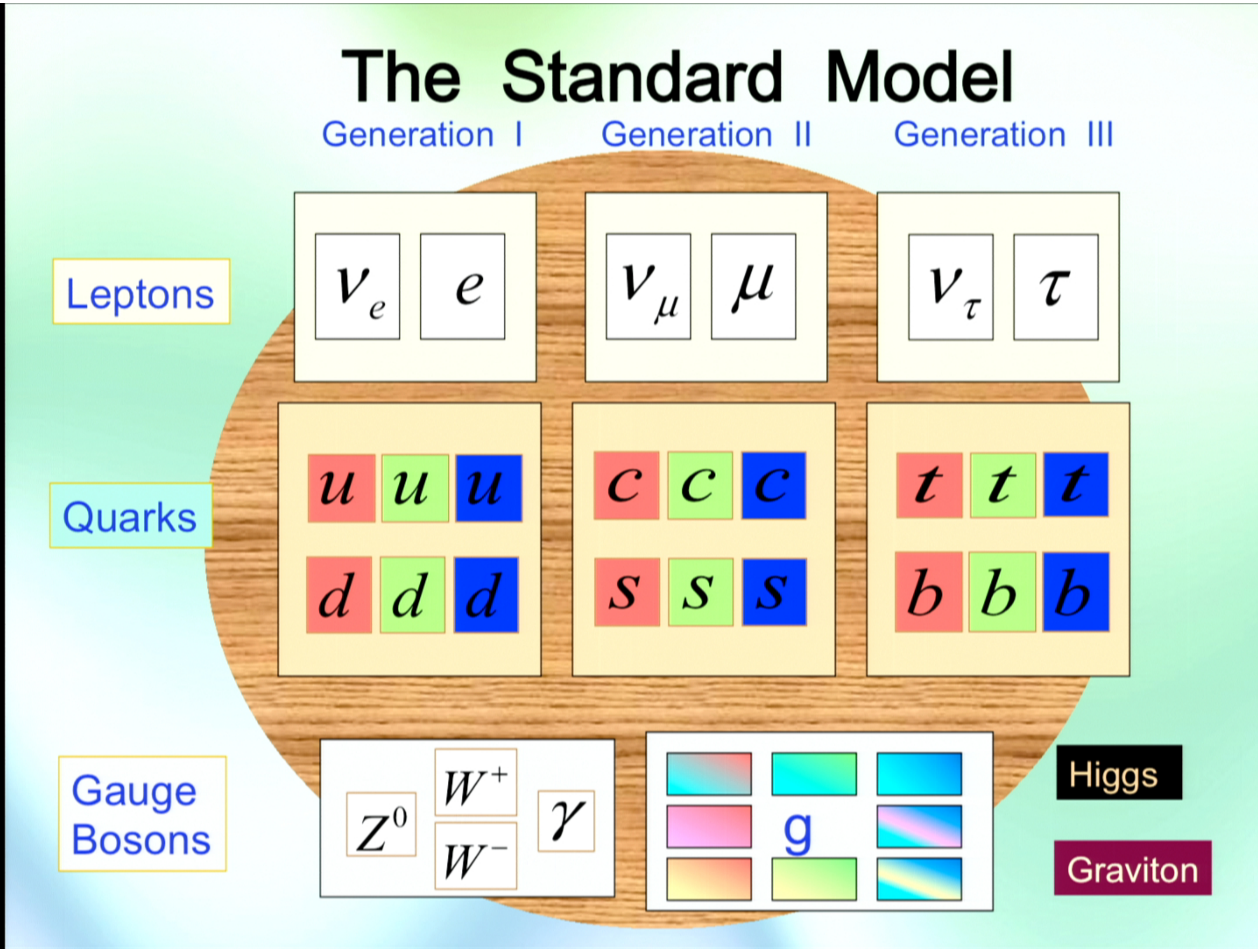
$$\frac{\partial \mathfrak{L}}{\partial \hat{g}^{\mu\nu}} = -\frac{\sqrt{-\hat{g}}}{2}(T^{\omega}_{\mu\nu} + T^{\text{matter}}_{\mu\nu})$$

$$= \frac{\sqrt{-\hat{g}}}{2}(-\frac{1}{8\pi G}\hat{G}_{\mu\nu} + T^{\text{matter}}_{\mu\nu}) = 0$$

You may either use $g_{\mu\nu}$ or $\hat{g}_{\mu\nu}$ to define these !


6. Cancelling Conformal
Anomalies in Matter Interactions


The Standard Model
Leptons
ν_e e ν_μ μ
Quarks
u u u
d d d s s s
Gauge Bosons
Z^0 W^+ W^- γ
Higgs
g g



The Standard Model
Generation I
Generation II
Generation III
Leptons
ν_e
e
ν_μ
μ
ν_τ
τ
Quarks
u u u
d d d
c c c
s s s
t t t
b b b
Gauge Bosons
Z^0
W^+
W^-
γ
g
Higgs
Graviton

The scale anomaly.

Take a conformally flat background spacetime.
Consider a complete EH + matter Lagrangian:

$$\mathfrak{L} = -\tfrac{1}{4} G_{\mu\nu} G_{\mu\nu} + \tfrac{1}{2} D_\mu \omega^2 - \tfrac{1}{6} \tilde{\kappa}^2 \Lambda \omega^4 - \tfrac{1}{2} D_\mu \varphi^2 - \tfrac{1}{8} \lambda \varphi^4$$

$$-\tfrac{1}{2}(\tilde{\kappa}^2 m_s^2)\omega^2 \varphi^2 - \tfrac{1}{6}(\tilde{\kappa} g_3)\omega\varphi^3 - \bar{\psi}(\gamma D + \tilde{\kappa} m_d \omega + y\varphi)\psi$$

$$\tilde{\kappa}^2 = \tfrac{1}{6}\kappa^2 = \tfrac{4}{3}\pi G_N$$

The scale anomaly.

Yang-Mills

Take a **conformally flat background spacetime.**
Consider a complete EH + matter Lagrangian:

$$\mathfrak{L} = -\tfrac{1}{4} G_{\mu\nu} G_{\mu\nu} + \tfrac{1}{2} D_\mu \omega^2 - \tfrac{1}{6}\tilde{\kappa}^2 \Lambda \omega^4 - \tfrac{1}{2} D_\mu \varphi^2 - \tfrac{1}{8}\lambda \varphi^4$$
$$- \tfrac{1}{2}(\tilde{\kappa}^2 m_s^2)\omega^2\varphi^2 - \tfrac{1}{6}(\tilde{\kappa} g_3)\omega\varphi^3 - \bar{\psi}(\gamma D + \tilde{\kappa} m_d \omega + y\varphi)\psi$$

$$\tilde{\kappa}^2 = \tfrac{1}{6}\kappa^2 = \tfrac{4}{3}\pi G_N$$

The scale anomaly.

grav. EH action

Take a conformally flat background spacetime.
Consider a complete EH + matter Lagrangian:

$$\mathfrak{L} = -\tfrac{1}{4} G_{\mu\nu} G_{\mu\nu} + \tfrac{1}{2} D_\mu \omega^2 - \tfrac{1}{6} \tilde{\kappa}^2 \Lambda \omega^4 - \tfrac{1}{2} D_\mu \varphi^2 - \tfrac{1}{8} \lambda \varphi^4$$

$$- \tfrac{1}{2} (\tilde{\kappa}^2 m_s^2) \omega^2 \varphi^2 - \tfrac{1}{6} (\tilde{\kappa} g_3) \omega \varphi^3 - \bar{\psi} (\gamma D + \tilde{\kappa} m_d \omega + y \varphi) \psi$$

$$\tilde{\kappa}^2 = \tfrac{1}{6} \kappa^2 = \tfrac{4}{3} \pi G_N$$

The scale anomaly.

Take a conformally flat background spacetime.
Consider a complete EH + matter Lagrangian:

$$\mathfrak{L} = -\tfrac{1}{4} G_{\mu\nu} G_{\mu\nu} + \tfrac{1}{2} D_\mu \omega^2 - \tfrac{1}{6} \tilde{\kappa}^2 \Lambda \omega^4 - \tfrac{1}{2} D_\mu \varphi^2 - \tfrac{1}{8} \lambda \varphi^4$$
$$- \tfrac{1}{2} (\tilde{\kappa}^2 m_s^2) \omega^2 \varphi^2 - \tfrac{1}{6} (\tilde{\kappa} g_3) \omega \varphi^3 - \bar{\psi} (\gamma D + \tilde{\kappa} m_d \omega + y \varphi) \psi$$

scalar field (J=0)

scalar masses

$$\tilde{\kappa}^2 = \tfrac{1}{6} \kappa^2 = \tfrac{4}{3} \pi G_N$$

The scale anomaly.

Take a conformally flat background spacetime.
Consider a complete EH + matter Lagrangian:

$$\mathfrak{L} = -\tfrac{1}{4} G_{\mu\nu} G_{\mu\nu} + \tfrac{1}{2} D_\mu \omega^2 - \tfrac{1}{6} \tilde{\kappa}^2 \Lambda \omega^4 - \tfrac{1}{2} D_\mu \varphi^2 - \tfrac{1}{8} \lambda \varphi^4$$

$$- \tfrac{1}{2} (\tilde{\kappa}^2 m_s^2) \omega^2 \varphi^2 - \tfrac{1}{6} (\tilde{\kappa} g_3) \omega \varphi^3 - \bar{\psi} (\gamma D + \tilde{\kappa} m_d \omega + y \varphi) \psi$$

fermionic fields (J=½)

$$\tilde{\kappa}^2 = \tfrac{1}{6} \kappa^2 = \tfrac{4}{3} \pi G_N$$

The scale anomaly.

Take a conformally flat background spacetime.
Consider a complete EH + matter Lagrangian:

$$\mathfrak{L} = -\tfrac{1}{4}G_{\mu\nu}G_{\mu\nu} + \tfrac{1}{2}D_\mu\omega^2 - \tfrac{1}{6}\tilde{\kappa}^2\Lambda\omega^4 - \tfrac{1}{2}D_\mu\varphi^2 - \tfrac{1}{8}\lambda\varphi^4$$
$$-\tfrac{1}{2}(\tilde{\kappa}^2 m_s^2)\omega^2\varphi^2 - \tfrac{1}{6}(\tilde{\kappa}g_3)\omega\varphi^3 - \bar{\psi}(\gamma D + \tilde{\kappa}m_d\omega + y\varphi)\psi$$

All parameters here are *dimensionless*, and, taking ω and φ to scale the same way, this lagrangian is *entirely conformally invariant* .....

$$\tilde{\kappa}^2 = \tfrac{1}{6}\kappa^2 = \tfrac{4}{3}\pi G_N$$

The scale anomaly.

Take a conformally flat background spacetime.
Consider a complete EH + matter Lagrangian:

$$\mathfrak{L} = -\tfrac{1}{4} G_{\mu\nu} G_{\mu\nu} + \tfrac{1}{2} D_\mu \omega^2 - \tfrac{1}{6}\tilde{\kappa}^2 \Lambda \omega^4 - \tfrac{1}{2} D_\mu \varphi^2 - \tfrac{1}{8}\lambda \varphi^4$$
$$-\tfrac{1}{2}(\tilde{\kappa}^2 m_s^2)\omega^2\varphi^2 - \tfrac{1}{6}(\tilde{\kappa} g_3)\omega\varphi^3 - \overline{\psi}(\gamma D + \tilde{\kappa} m_d \omega + y\varphi)\psi$$

All parameters here are *dimensionless*, and, taking ω and φ to scale the same way, this lagrangian is *entirely conformally invariant* .....

$$\tilde{\kappa}^2 = \tfrac{1}{6}\kappa^2 = \tfrac{4}{3}\pi G_N$$

… classically ! Quantum mechanically ? only if the beta function(s) vanish !

The scale anomaly.

Take a conformally flat background spacetime.
Consider a complete EH + matter Lagrangian:

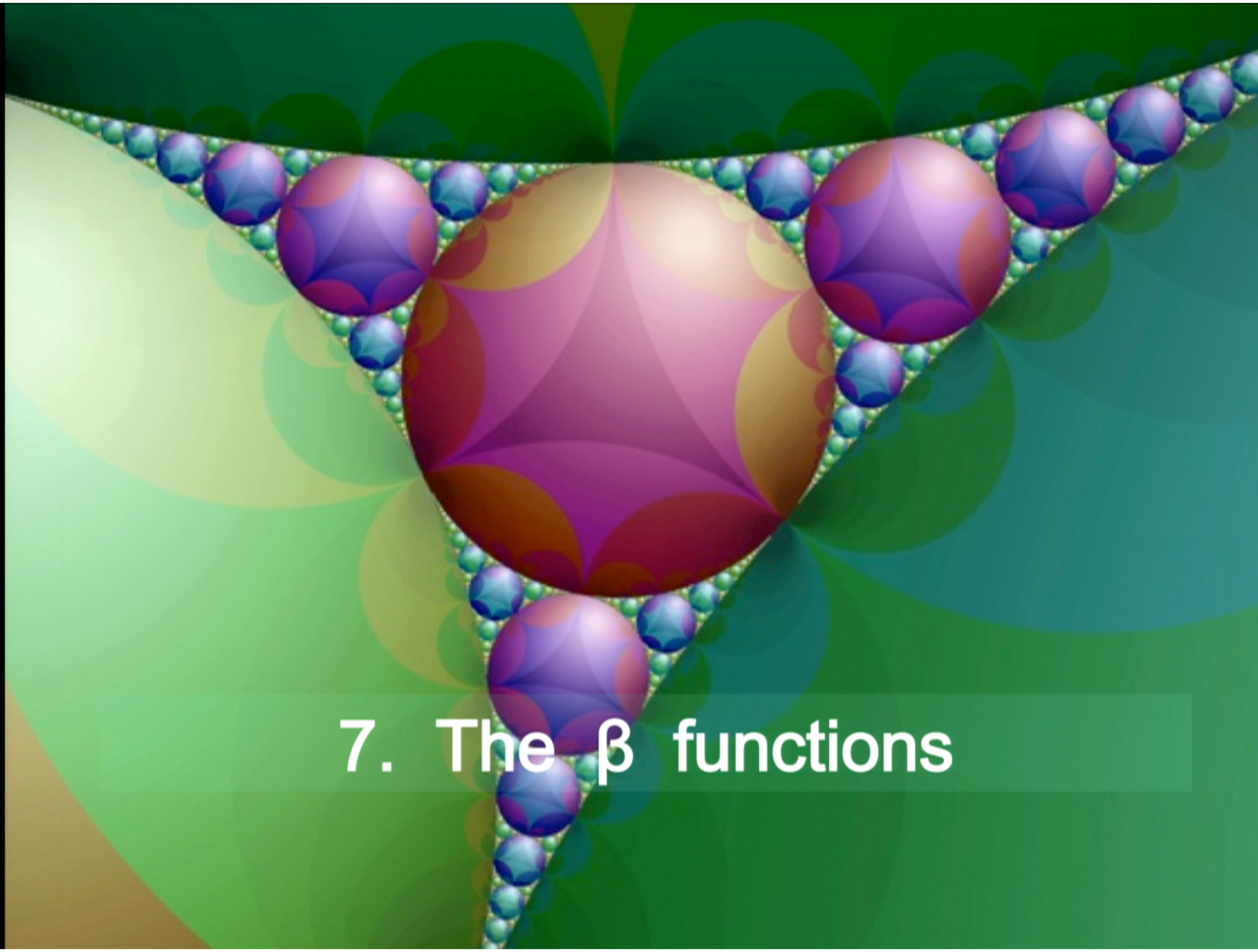
$$\mathfrak{L} = -\tfrac{1}{4} G_{\mu\nu} G_{\mu\nu} + \tfrac{1}{2} D_\mu \omega^2 - \tfrac{1}{6} \tilde{\kappa}^2 \Lambda \omega^4 - \tfrac{1}{2} D_\mu \varphi^2 - \tfrac{1}{8} \lambda \varphi^4$$

$$-\tfrac{1}{2} (\tilde{\kappa}^2 m_s^2) \omega^2 \varphi^2 - \tfrac{1}{6} (\tilde{\kappa} g_3) \omega \varphi^3 - \bar{\psi} (\gamma D + \tilde{\kappa} m_d \omega + y \varphi) \psi$$

All parameters here are *dimensionless*, and, taking ω and φ to scale the same way, this lagrangian is *entirely conformally invariant* …..

$$\tilde{\kappa}^2 = \tfrac{1}{6} \kappa^2 = \tfrac{4}{3} \pi G_N$$

… classically ! Quantum mechanically ? only if the beta function(s) vanish !


7. The β functions

The equations

$$\frac{\mu d}{d\mu}(\tilde{\Lambda}, g, \lambda, y, y^5, \tilde{\kappa} g_3, \tilde{\kappa} m_s, \tilde{\kappa} m_d, \ldots) =$$

$$\vec{\beta}(\tilde{\Lambda}, g, \lambda, y, y^5, \tilde{\kappa} g_3, \tilde{\kappa} m_s, \tilde{\kappa} m_d, \ldots) = 0$$

Have only isolated solutions ! All coupling parameters, including Λ, are completely fixed by these equations, which can be worked out.

The only choices we have are discrete parameters: the group structures and symmetry patterns.

The equations

$$\frac{\mu d}{d\mu}(\tilde{\Lambda}, g, \lambda, y, y^5, \tilde{\kappa} g_3, \tilde{\kappa} m_s, \tilde{\kappa} m_d, \ldots) =$$
$$\vec{\beta}(\tilde{\Lambda}, g, \lambda, y, y^5, \tilde{\kappa} g_3, \tilde{\kappa} m_s, \tilde{\kappa} m_d, \ldots) = 0$$

Have only isolated solutions ! All coupling parameters, including Λ, are completely fixed by these equations, which can be worked out.

The only choices we have are discrete parameters: the group structures and symmetry patterns.

a LANDSCAPE of “Standard Models”

The only difference with conventional quantum gravity:

Demanding that the ω field behaves regularly at the origin, just as the other scalars φ .

This is a statement about the ***small distance limit***

The only difference with conventional quantum gravity:

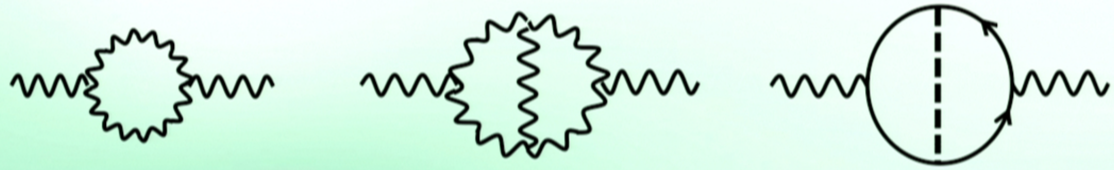
Demanding that the ω field behaves regularly at the origin, just as the other scalars φ .

This is a statement about the ***small distance limit***

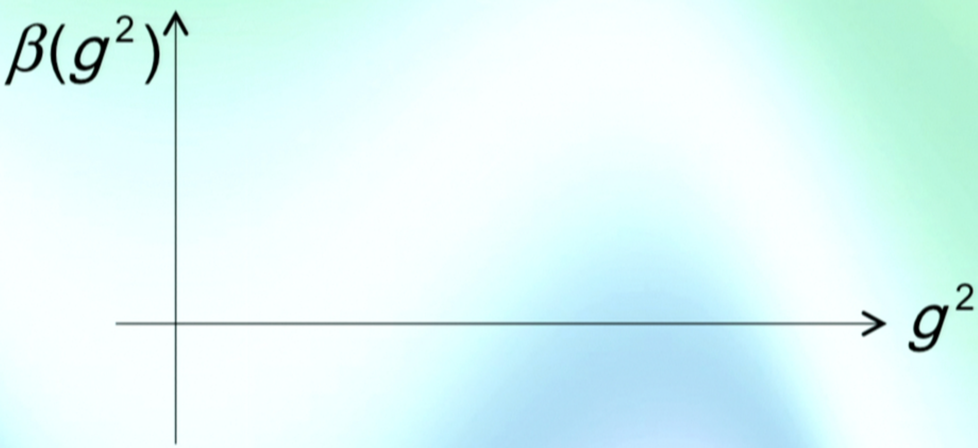
but such a postulate may well be permissible after having made the theory conformally invariant.

Solving the equations β = 0

First fix the gauge coupling constants:

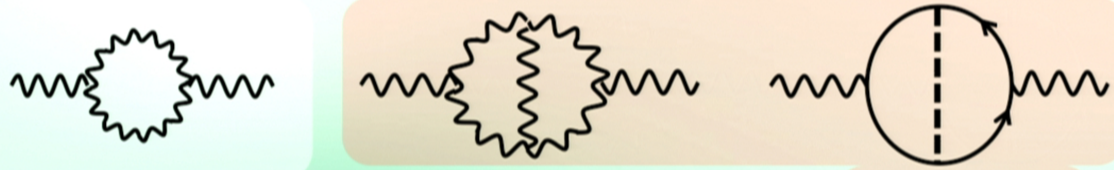


$$\beta(g^2) = \frac{1}{24\pi^2}\left(\tfrac{1}{2}C_S + 2C_F - 11C_g\right)g^4 \; + \; K\left(\tfrac{y}{g}\right)g^6$$

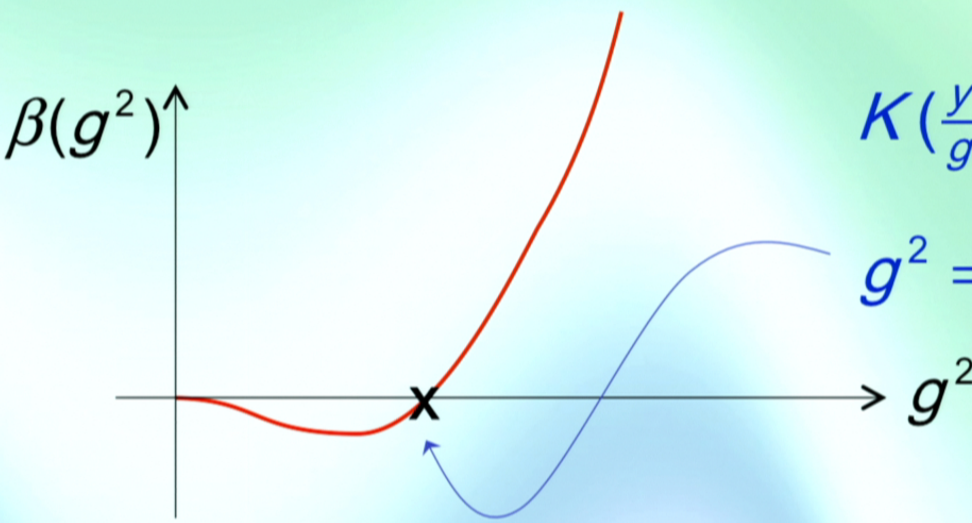


Solving the equations $\beta = 0$

First fix the gauge coupling constants:



$$\beta(g^2) = \frac{1}{24\pi^2}\left(\frac{1}{2}C_S + 2C_F - 11C_g\right)g^4 + K\left(\frac{y}{g}\right)g^6$$

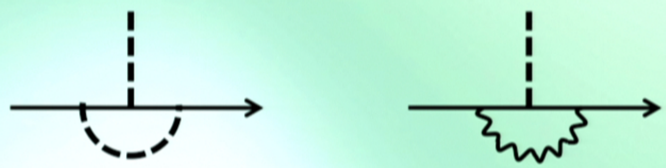


$$K\left(\frac{y}{g}\right) = O(N_g{}^2)$$

$$g^2 = O\left(\frac{1}{N_g{}^2}\right)$$

Then fix the Yukawa coupling constants:

$$\bar{\psi}_i \left(Y_{ijk}(\varphi_k,\omega) + i\gamma^5 Y^5_{ijk}(\varphi_k,\omega) \right) \psi_j$$

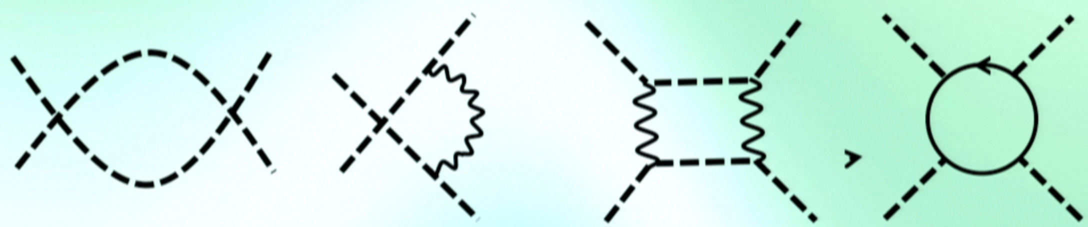


$$\beta(Y,Y^5) \propto c_3\left(Y^3, Y(Y^5)^2\right) - g^2 c_g(Y,Y^5) \quad \rightarrow$$

$$Y, Y^5 = \mathcal{O}(g)$$

Finally, the equations for the scalar couplings are more complex:

$$\beta(V(\varphi,\omega)) \propto$$
$$(\frac{\partial^2}{\partial\varphi_i \partial\varphi_j}V)^2 + V.(-c\, g^2 + c\,(Y^2, (Y^5)^2))$$
$$+(c\, g^4 - c\,(Y^4, Y^2(Y^5)^2, (Y^5)^4))\varphi^4$$



Often no real solutions, but many solutions are expected.

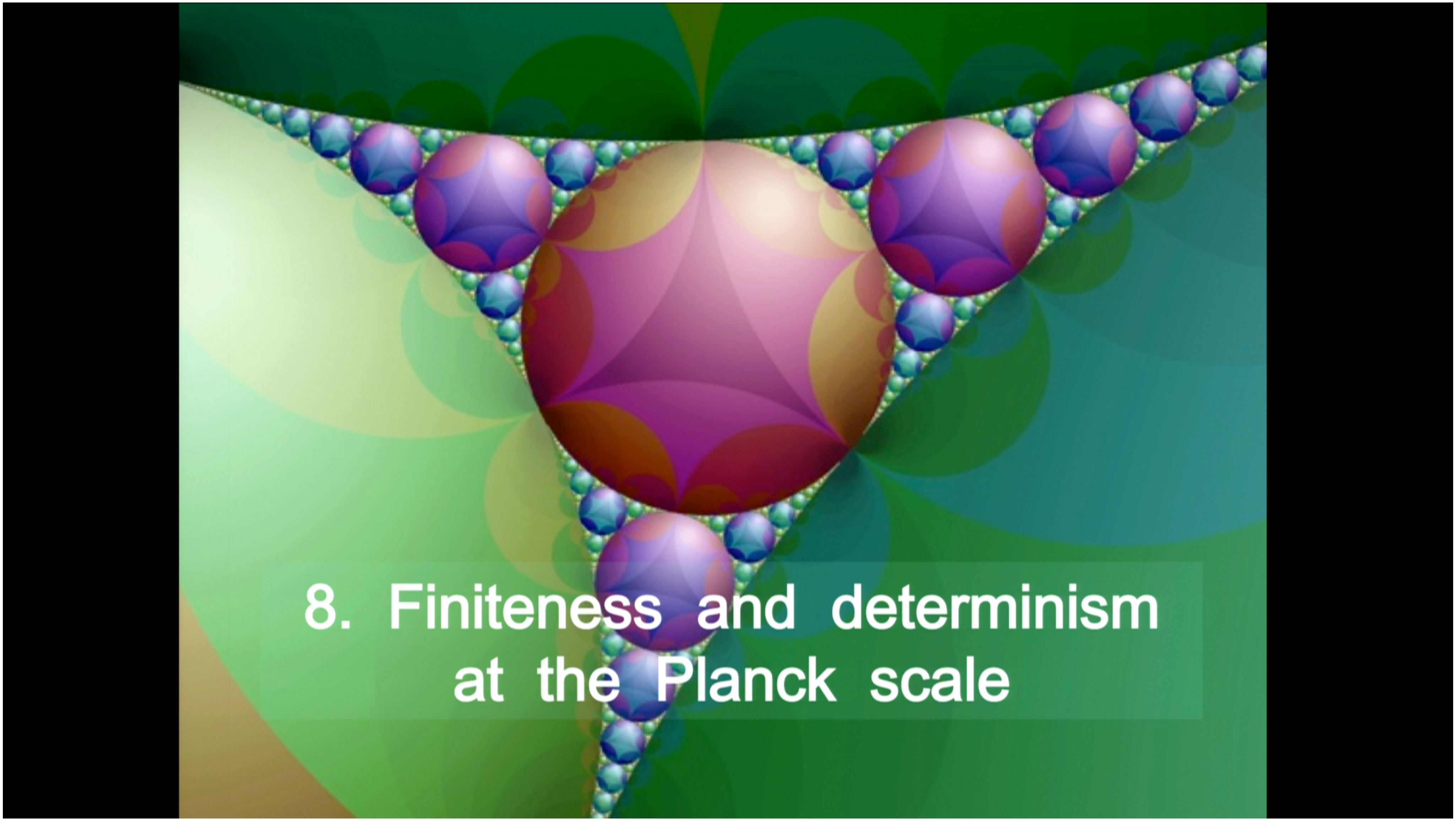
But all couplings will be of the same order, $O(1/N^2)$, which includes all masses and the cosmological coupling constant.

Thus, the ***hierarchy problem*** is not solved: why are dimensionless ratios of many physical parameters so extreme?

This theory is a proposal to handle the functional integral over the ω field.
The $\hat{g}_{\mu\nu}$ fields are not yet considered at all. They contribute to an other set of anomalies that must be cancel as well.
But the functional integral over these fields affect the light cones ➔ difficulties with causality and locality (incl. UV divergences)
The problem of *time* in quantum gravity…

Option: reconsider what quantum mechanics means here.

According to holography: in a small domain of space only a finite number of quantum states !!


8. Finiteness and determinism
at the Planck scale

Observation:

quantum field theories are cellular automata in disguise.

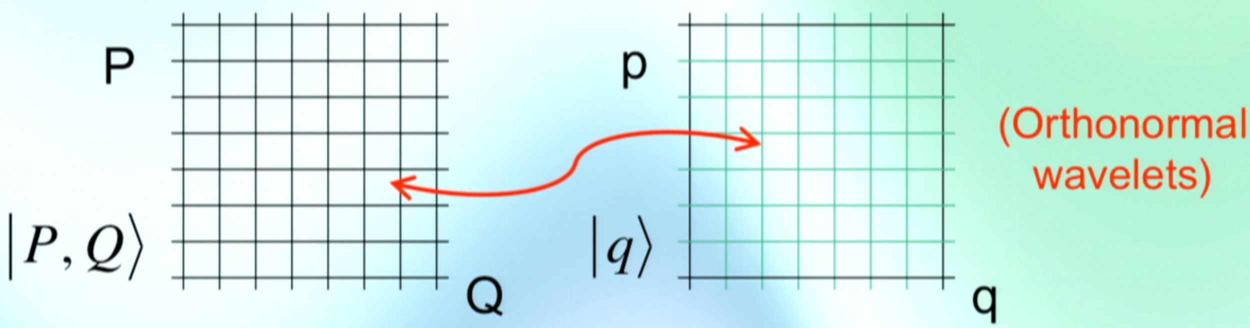
a mapping exists that maps the states of a cellular automaton onto those of an interacting quantum field theory.

First:

Consider integers $Z = P + i\,Q$. Write states in a Hilbert space: $|P, Q\rangle$.

Consider a quantum pair: (p, q) with $[q, p] = \frac{i}{2\pi}$

A mapping exists between states $|P, Q\rangle$ and states $|q\rangle$



Observation:

quantum field theories are cellular automata in disguise.

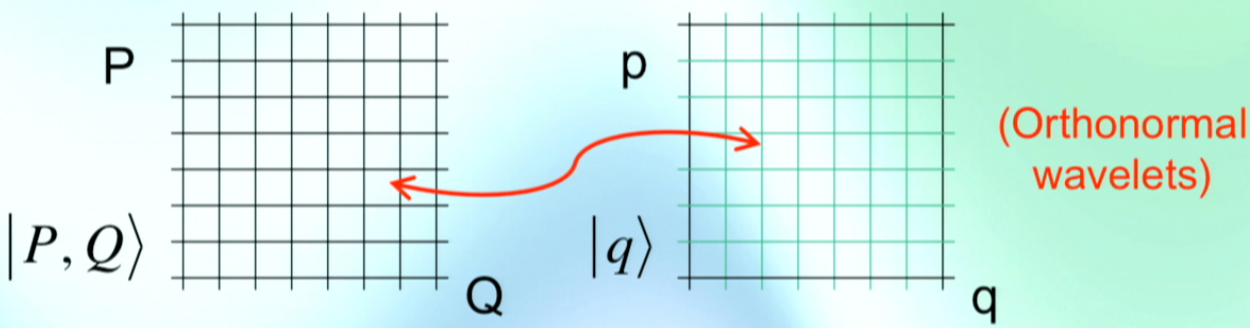
a mapping exists that maps the states of a cellular automaton onto those of an interacting quantum field theory.

First:

Consider integers $Z = P + i\,Q$. Write states in a Hilbert space: $|P, Q\rangle$.

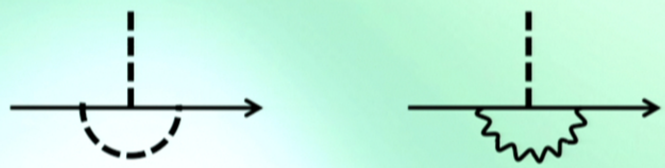
Consider a quantum pair: (p, q) with $[q, p] = \frac{i}{2\pi}$

A mapping exists between states $|P, Q\rangle$ and states $|q\rangle$



Then fix the Yukawa coupling constants:

$$\bar{\psi}_i \left(Y_{ijk}(\varphi_k,\omega) + i\gamma^5 Y^5_{ijk}(\varphi_k,\omega) \right) \psi_j$$

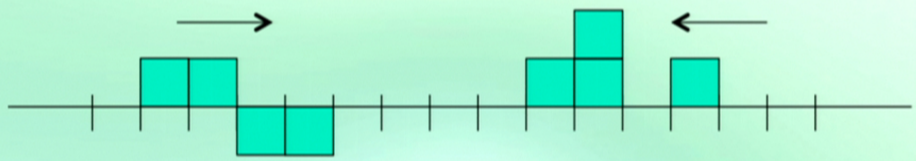


$$\beta(Y,Y^5) \propto c_3\left(Y^3, Y(Y^5)^2\right) - g^2 c_g(Y,Y^5) \quad \rightarrow$$

$$Y, Y^5 = \mathcal{O}(g)$$

Next: do this for a bosonic quantum field theory on a lattice (1+1 dimensions)

Cellular automaton: $P(x, t)$, $Q(x, t)$

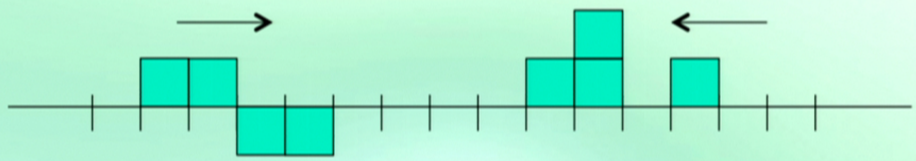


Map the Hilbert space of this system on the wavelets of the field theory: $|\phi(x,t)\rangle$

Can be done *explicitly* if you have massless left-movers and right-movers

Next: do this for a bosonic quantum field theory on a lattice (1+1 dimensions)

Cellular automaton: $P(x, t)$, $Q(x, t)$

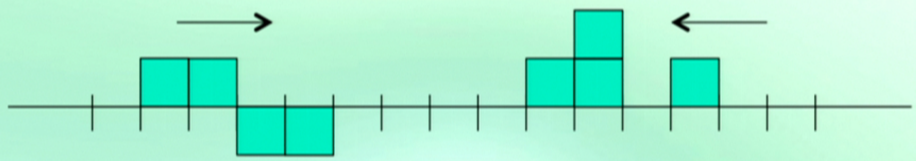


Map the Hilbert space of this system on the wavelets of the field theory: $|\phi(x,t)\rangle$

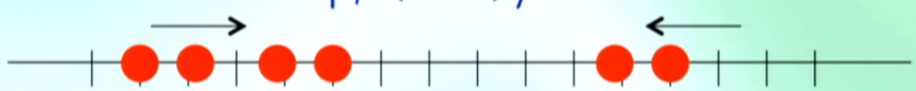
Can be done *explicitly* if you have massless left-movers and right-movers

Next: do this for a bosonic quantum field theory on a lattice (1+1 dimensions)

Cellular automaton: $P(x, t)$, $Q(x, t)$



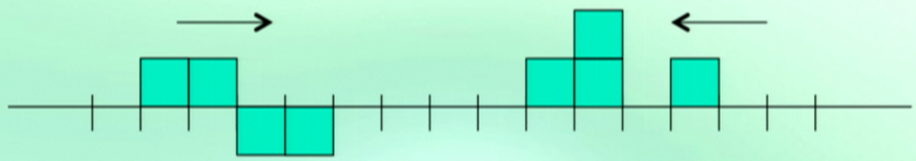
Map the Hilbert space of this system on the wavelets of the field theory: $|\phi(x,t)\rangle$



Can be done *explicitly* if you have massless left-movers and right-movers

Next: do this for a bosonic quantum field theory on a lattice (1+1 dimensions)

Cellular automaton: $P(x, t)$, $Q(x, t)$



Map the Hilbert space of this system on the wavelets of the field theory: $|\phi(x,t)\rangle$

Can be done *explicitly* if you have massless left-movers and right-movers