Title: Doubly General Relativity

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Abstract:

Doubly General Relativity

Conformal Nature of the Universe

Tim A. Koslowski
tkoslowski@perimeterinstitute.ca

Perimeter Institute for Theoretical Physics

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Machian Picture

Two Machian Principles

1. **relativity of clocks**: local time reparametrization invariance
   $\Rightarrow$ local Hamilton constraints
2. **relativity of rods**: local spatial conformal invariance
   $\Rightarrow$ local spatial conformal constraints

Generically, second class constraint system
$\Rightarrow$ can **not** simultaneously realized as phase space symmetries.

On BRST-extended phase space:

Can simultaneously be realized as nilpotent transformations if on-shell Hamiltonian is doubly invariant.
$\Rightarrow$ hidden BRST-symmetry in gravity
$\Rightarrow$ Doubly General Relativity due to Shape Dynamics

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Heuristic

Constrained Hamiltonian path integral

$$
\begin{aligned}
Z &= \int Dq\, Dp\, \delta[\chi]\, \delta[\sigma]\, |\{\chi, \sigma\}|\, \exp(i \int dt\, p.\dot{q}) \\
&= \int Dq\, Dp\, D\eta\, DP\, \exp(i \int dt (p.\dot{q} + P.\dot{\eta} - \{\Omega, \Psi\}))
\end{aligned}
$$

with BRST-generator Ω, gauge-fixing Ψ.

Wish list

1. both Machian invariances implemented as invariances under $s_1. = \{\Omega, .\}$ and $s_2. = \{\Psi, .\}$
2. locality of $L = p.\dot{q} + P.\dot{\eta} - \{\Omega, \Psi\}$
3. linearly realized symmetries (not yet)
4. intuitive interpretation

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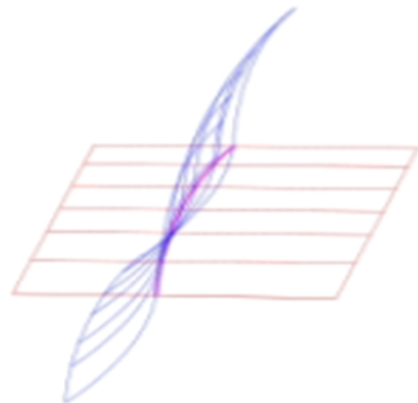
Duality between ADM and Shape Dynamics



ADM Gravity

$$S(N) = \int N \left(\frac{G(\pi,\pi)}{\sqrt{|g|}} - (R - 2\Lambda)\sqrt{|g|} \right)$$
$$H(v) = \int \pi^{ab} \mathcal{L}_v g_{ab}$$

Shape Dynamics

$$H_{SD} = V - V_o$$
$$Q(\rho) = \int (\pi - \langle \pi \rangle \sqrt{|g|})$$
$$H(v) = \int \pi^{ab} \mathcal{L}_v g_{ab}$$

BRST-Formalism (quick and dirty)

Abelian constraints χ_α

- BRST-generator $\Omega = \eta^\alpha \chi_\alpha$ satisfies $\{\Omega, \Omega\} = 0$ (nontrivial: $gh(\Omega) = 1$)

$\Rightarrow$ defines graded differential $s : f \to \{\Omega, f\}$, i.e. $s^2 = 0$

- Observables as cohomology of s at $gh(.) = 0$:
  - gauge invariance: $\{\Omega, f(p,q)\} = 0 \Rightarrow f$ (strong observable)
  - equivalence: $\tilde{f} = f + \{\Omega, \Psi\} = f + \sigma^\alpha \chi_\alpha + \mathcal{O}(\eta)$ (weak observable) for gauge fixing $\Psi = \sigma^\alpha P_\alpha + \mathcal{O}(\eta)$ with $gh(\Psi) = -1$

$\Rightarrow$ always strong equations on extended phase space

- gauge fixed Hamiltonian $H_{BRS} = H_o + \{\Omega, \Psi\}$ when $\{H_o, \Omega\} = 0$.

Nonabelian constraints $\tilde{\chi}_\alpha = M_\alpha^\beta \chi_\beta$

apply canonical transform $\exp(\{\eta_\alpha c_\beta^\alpha P^\beta, .\})$ to Abelian case

$\tilde{\Omega} = \eta^\alpha M_\alpha^\beta \chi_\beta + \mathcal{O}(\eta^2)$ defines $\tilde{s}$, cohomology same as of s at $gh(.) = 0$.

From Symmetry Trading to Symmetry Doubling

Symmetry Trading requires

two first class surfaces (original and equivalent gauge symmetry) that **gauge fix** one another

BRST-gauge-fixing

- Ω is nilpotent because orig. system is first class
- Ψ can be chosen nilpotent because equiv. system is first class
- if H_o (on shell) Poisson commutes with Ω and Ψ then gauge fixed

$$H_{BRS} = H_o + \{\Omega, \Psi\}$$

  is annihilated by both s_Ω and s_Ψ

Symmetry Doubling:

Canonical action $S = \int dt(p_i \dot{q}^i + P_\alpha \dot{\eta}^\alpha - H_{BRS})$ is invariant under two BRST-transformations (up to a boundary term).

Different from Anti-BRST

Abelian constraints: $\chi_\alpha \approx 0$

- double constraints $\chi^1_\alpha = \chi^2_\alpha = \chi_\alpha$
- reducibility condition $\chi^1_\alpha - \chi^2_\alpha = 0$

$\Rightarrow$ BRST operator $\Omega = \eta^\alpha_1 \chi^1_\alpha + \eta^\alpha_2 \chi^2_\alpha + \lambda^\alpha (P^1_\alpha - P^2_\alpha)$
bi-degree expansion yields two **commuting** generators
$\Omega = \eta^\alpha_1 \chi^1_\alpha + \lambda^\alpha P^2_\alpha$ and $\bar{\Omega} = \eta^\alpha_2 \chi^2_\alpha - \lambda^\alpha P^1_\alpha$.

Superalgebras

Anti-BRST: $\{\Omega, \Omega\} = 0 = \{\bar{\Omega}, \bar{\Omega}\}, \{\Omega, H\} = 0 = \{\bar{\Omega}, H\}$
$\{\Omega, \bar{\Omega}\} = 0$.
Symmetry Doubling: $\{\Omega, \Omega\} = 0 = \{\Psi, \Psi\}, \{\Omega, H\} = 0 = \{\Psi, H\}$
$\{\Omega, \Psi\} = H$.

Construction of Doubly General Relativity (I)

Extending Shape Dynamics

- fixed CMC condition $Q(x) = \pi(x) + \lambda\sqrt{|g|}$
- conformal spatial harmonic gauge $F^k(x) = (g^{ab}\delta^k_c + \frac{1}{3}g^{ak}\delta^b_c)e^c_\alpha(\nabla_a - \hat{\nabla}_a)e^\alpha_b$
- First class system: $\{Q(x), Q(y)\} = 0 = \{F^i(x), F^j(y)\}$ as well as $\{Q(x), F^i(y)\} = F^i(y)\delta(x, y)$

Interpretation as "local conformal system"

Q generates spatial dilatations and Poisson brackets resemble $C(3)$ at each point

Gauge fixing ADM

- gauge fixing operator is elliptic and invertible in a region R
- out side R: meager set with finite dimensional kernel

Construction of Doubly General Relativity (II)

BRST-charges

$$\Omega_{ADM} = \int d^3x \left(\eta S + \eta^a g_{ac} \pi^{cd}_{;d} + \eta^b \eta^a_{,b} P_a + \frac{1}{2} \eta^a \eta_{,a} P + \eta \eta_{,c} P_b g^{bc} \right)$$

$$\Omega_{ESD} = \int d^3x \left(P \frac{\pi}{\sqrt{g}} + P_a F^a + \frac{1}{2} \frac{P}{\sqrt{g}} P_a \eta^a \right)$$

Gauge-fixed gravity action

$S_{gf} = \int dt \left(\dot{z}_A P(z_A) - \{\Omega_{ADM}, \Omega_{ESD}\} \right)$ is invariant under usual ADM-BRST transformations **and**

a hidden BRST-invariance of S_{gf} under

$$\begin{aligned} s_{ESD} g_{ab} &= \frac{P}{\sqrt{g}} g_{ab} & s_{ESD} \pi^{ab} &= \text{“}long\text{''} \\ s_{ESD} \eta &= -\frac{1}{\sqrt{g}} (\pi + \frac{1}{2} P_c \eta^c) & s_{ESD} P &= 0 \\ s_{ESD} \eta^a &= -F^a + \frac{P}{2\sqrt{g}} \eta^a & s_{ESD} P_a &= \frac{P}{2\sqrt{g}} P_a \end{aligned}$$

due to extended Shape Dynamics.

Construction of Doubly General Relativity (III)

Interpretation

The Hamiltonian of Doubly General Relativity is

$$H_{DGR} = S(\frac{\pi}{\sqrt{|g|}} + \lambda) + H(F^a) + \mathcal{O}(\eta)$$

The ghost-free part is neither the "frozen Hamiltonain" $H = 0$ nor the CMC-Hamiltonian $H = S(N_{CMC}[g, \pi])$, but a generator of dynamics for $\lambda + \frac{\pi}{\sqrt{|g|}} > 0$.

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Consequences 1: Classical Theory and Observations

Refined definition of a gravity theory (eff. field th. reasoning):

Gravity = local action for $g_{ab}, \pi^{ab}, \eta, P, \eta^a, P_a$ at gh. number 0
that is invariant under ADM- and ESD- BRST-transformations s_{ADM}, s_{ESD}
also: dimensional analysis in IR
$\Rightarrow$ construction ppl. for classical Doubly General Relativity

Possible Observable Consequences

- Effective field theory for GR: all higher derivative curvature invariants are allowed (just suppressed at low energies)
- these are generally not compatible with Extended Shape Dynamics
  $\Rightarrow$ DGR can be experimentally distinguished from usual GR
  (but only beyond Einstein-Hilbert)

This theory space has **not** been explored!

What about Quantum Gravity?

Slavnov-Taylor Identities (assuming no annom.):

- assuming an invariant path integral measure $\Rightarrow$ BRST-variations yield:

$$\langle s_{ADM}\phi_A\rangle \frac{\delta_L \Gamma}{\delta \phi_A} = 0 \text{ and } \langle s_{ESD}\phi_A\rangle \frac{\delta_L \Gamma}{\delta \phi_A} = 0$$

BRST-variations are nonlinear $\Rightarrow$ difficult Legendre transform

- Nonlinearity obstructs use of two Zinn-Justin equations

$(\Gamma, \Gamma)_1|_{\hat{\phi}_2=0} = 0 = (\Gamma, \Gamma)_2|_{\hat{\phi}_1=0}$

$\Rightarrow$ seems unfeasible in metric formulation.

Current Directions:

- Find a formulation of DGR where enough transformations are linearly realized:
  This makes prediction about counter terms very feasible
- Find a gauge fixing with improved power counting.
  (Problem: Observables need Dictionary)

Conclusions

1. Two Machian principles as foundations of gravity theory
2. Symmetry trading is generic and gives equivalent gauge theories
3. Symmetry trading implies symmetry doubling in BRST formalism
4. Equivalence of Shape Dynamics and GR $\Rightarrow$ Doubly General Relativity
5. DGR implies a new theory space for gravity. To explore:
   - are there semiclassical predictions (beyond E-H-action)?
   - universality classes on this revised theory space (FRGE methods)?
   - new view on dualities?

"Doubly General Relativity" in one line:

There is a hidden BRST-invariance in gravity due to Shape Dynamics.

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