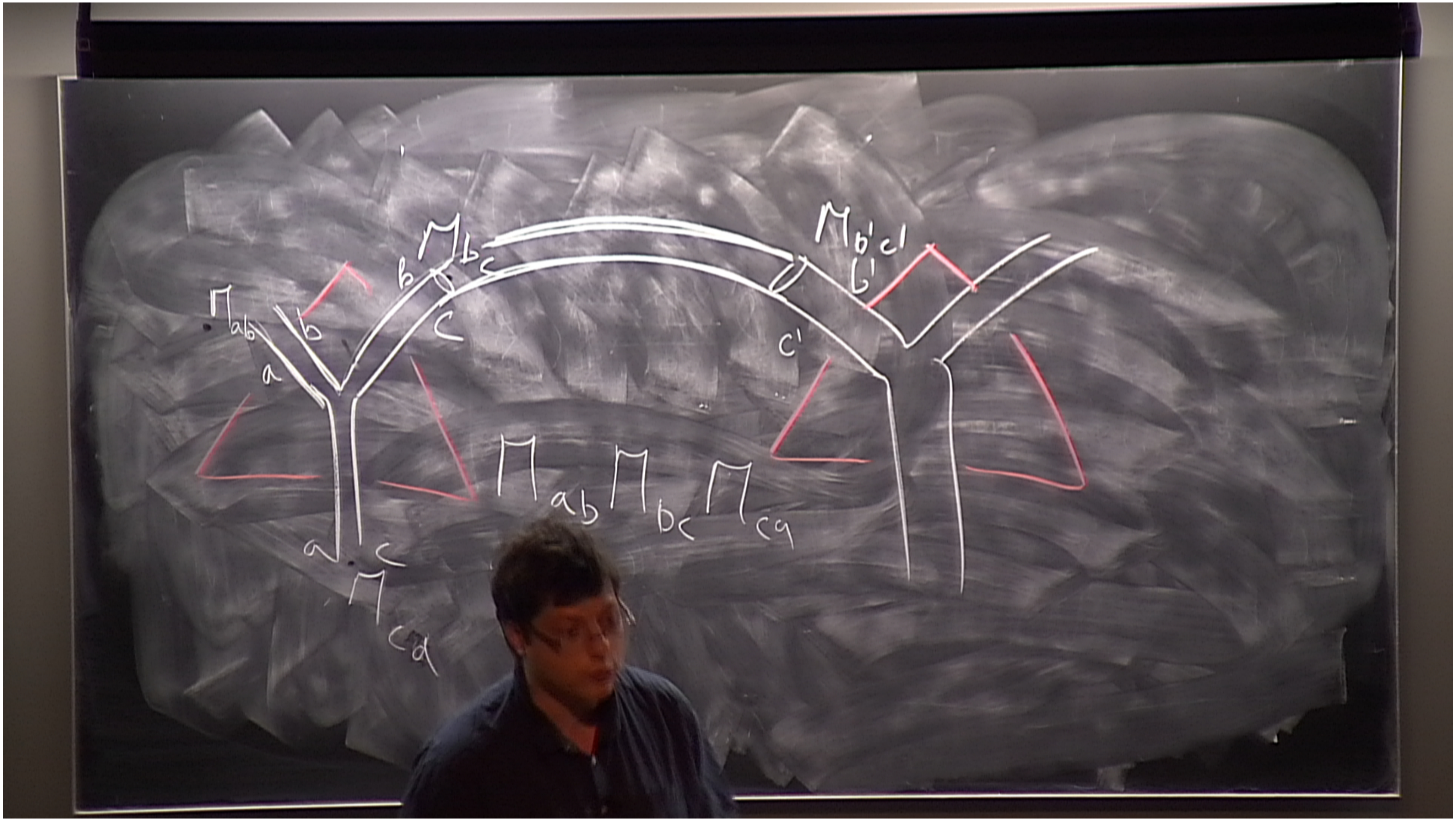
Title: The 1/N Expansion in Random Tensor Models

Date: May 07, 2012 11:15 AM

URL: http://pirsa.org/12050055

Abstract: Matrix models yield a theory of random two dimensional surfaces. They support a 1/N expansion dominated by planar graphs (corresponding to planar surfaces) and undergo a phase transition to a continuum theory. In higher dimensions matrix models generalize to tensor models. In the absence of a viable 1=N expansion, tensor models have for a long time been less successful in providing a theory of random higher dimensional spaces. This situation has drastically changed recently. Models for a non-symmetric complex tensor have been shown to admit a 1/N expansion dominated by graphs of spherical topology in arbitrary dimensions and to undergo a phase transition to a continuum theory. I will present an overview of these results and discuss their implications.


Π_ab
Π_bc
Π_b'c'
Π_ab Π_bc Π_ca
Π_ca
a
b
c
b'
c'

The large N limit of Tensor Models

Răzvan Gurău

GAP, 2012

Matrix Models

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A success story: Matrix Models and random two dimensional surfaces.

The theory of strong interactions, string theory, quantum gravity in $D = 2$, conformal field theory, invariants of algebraic curves, free probability theory, knot theory, the Riemann hypothesis, etc.

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- Generate ribbon graphs $\leftrightarrow$ discretized surfaces.
- Have built in scales: the size of the matrix, N.
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- support a "$1/N$ expansion": effective behavior dominated by planar graphs.

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Tensor models for a general complex tensor

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- Graphs encode (the full cellular structure of) D-dimensional spaces: canonical theory of random geometry in arbitrary dimensions.
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Tensor models provide today a consistent framework for the analytic study of random geometries in arbitrary dimensions relevant for conformal field theory, quantum gravity, integrability, statistical physics, etc. .

Matrix models and random surfaces

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Let M a $N \times N$ hermitian matrix, $Z(\lambda) = \int [dM] \, e^{-N\left(\frac{1}{2}\mathrm{Tr}M^2 + \lambda \mathrm{Tr}M^3\right)}$

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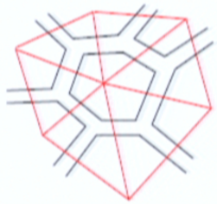

Ribbon vertices (M has two indices) $\leftrightarrow$ triangles.
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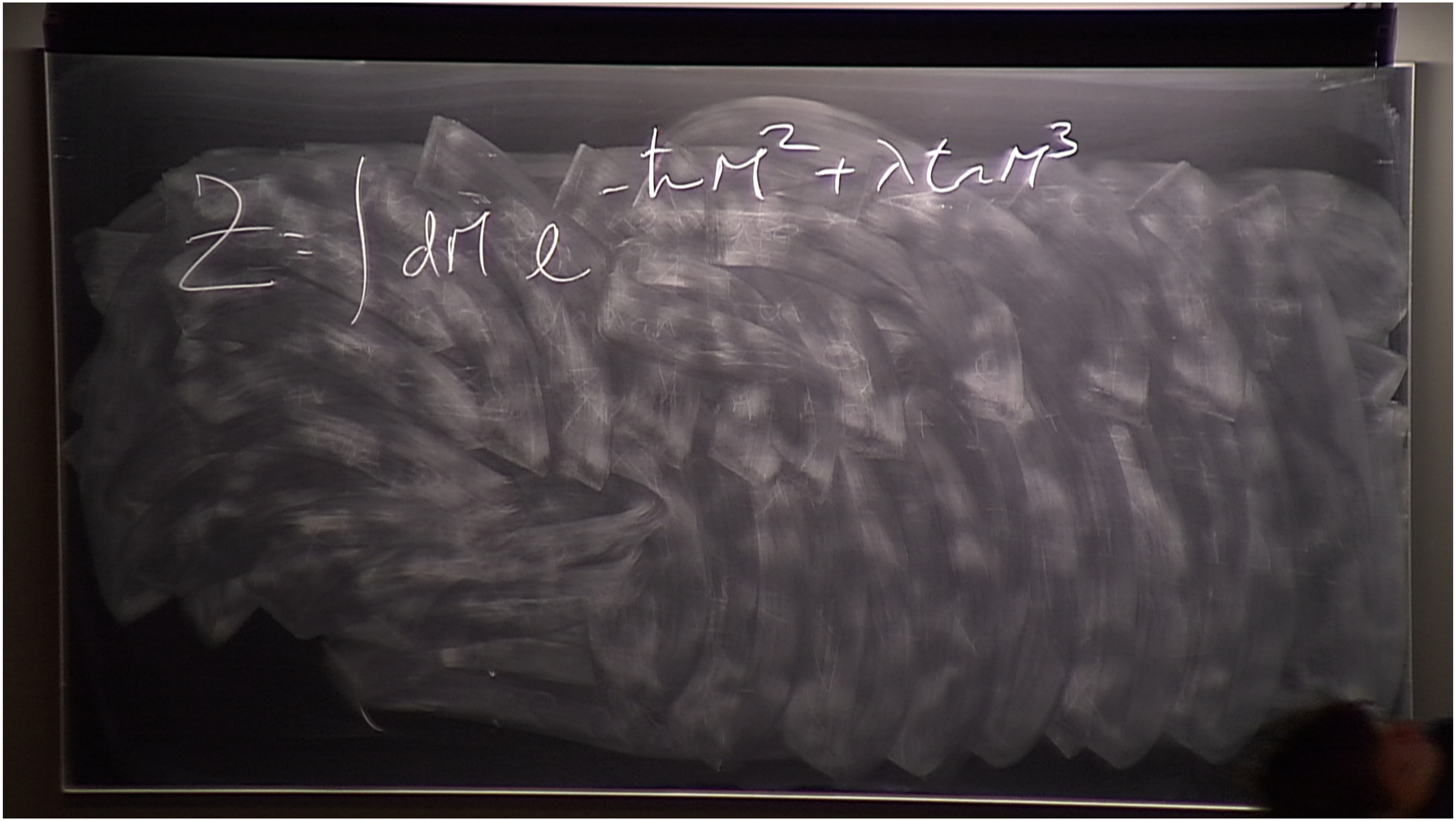
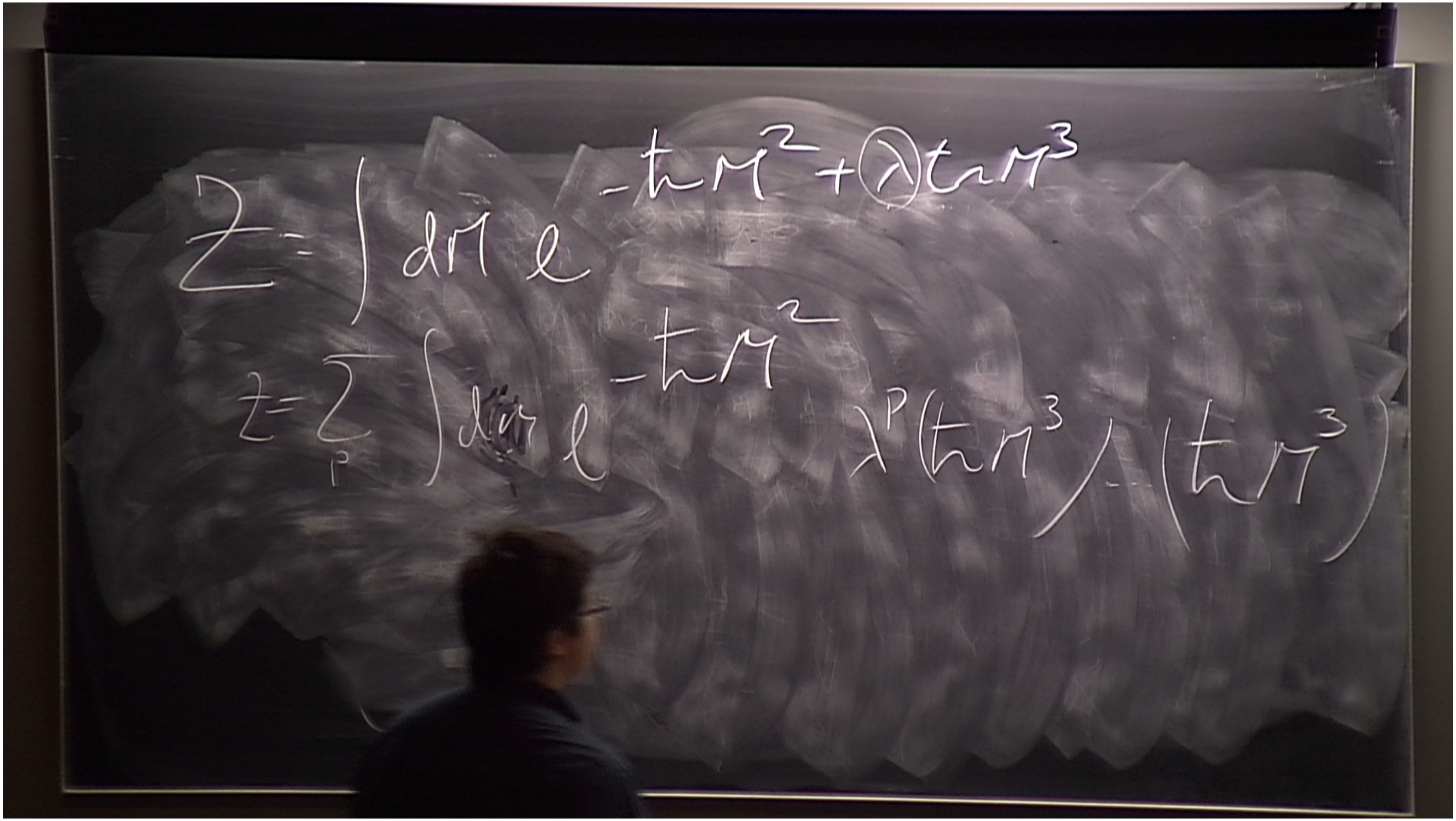
$Z(\lambda)$ is a sum over ribbon Feynman graphs $\leftrightarrow$ discrete surfaces.


$Z = \int dM \, e^{-\mathrm{tr} M^2 + \lambda \, \mathrm{tr} M^3}$


$Z = \int dM \, e^{-\mathrm{tr} M^2 + \lambda \mathrm{tr} M^3}$
$Z = \sum_p \int dM \, e^{-\mathrm{tr} M^2} \lambda^p (\mathrm{tr} M^3) \ldots (\mathrm{tr} M^3)$

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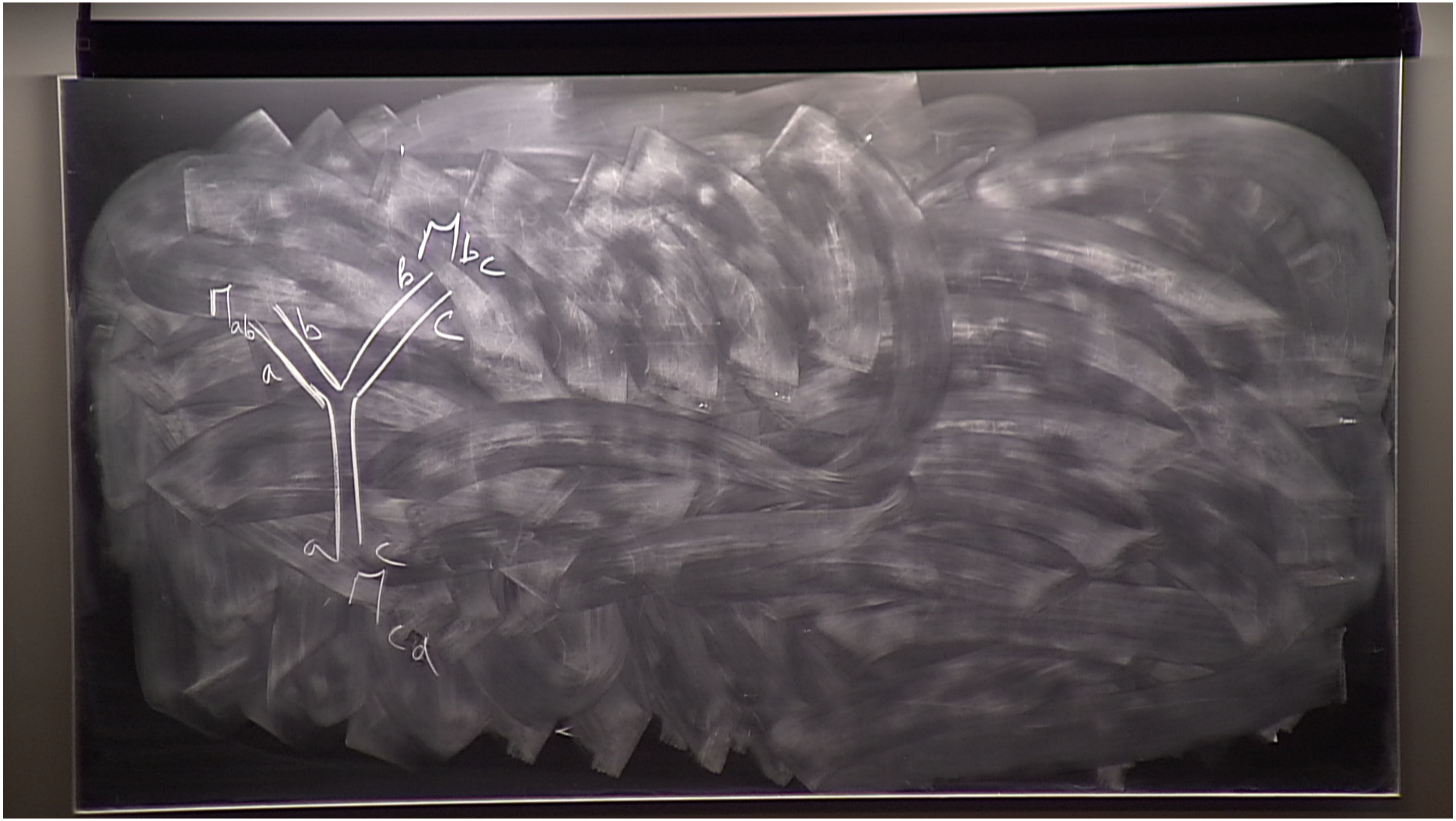
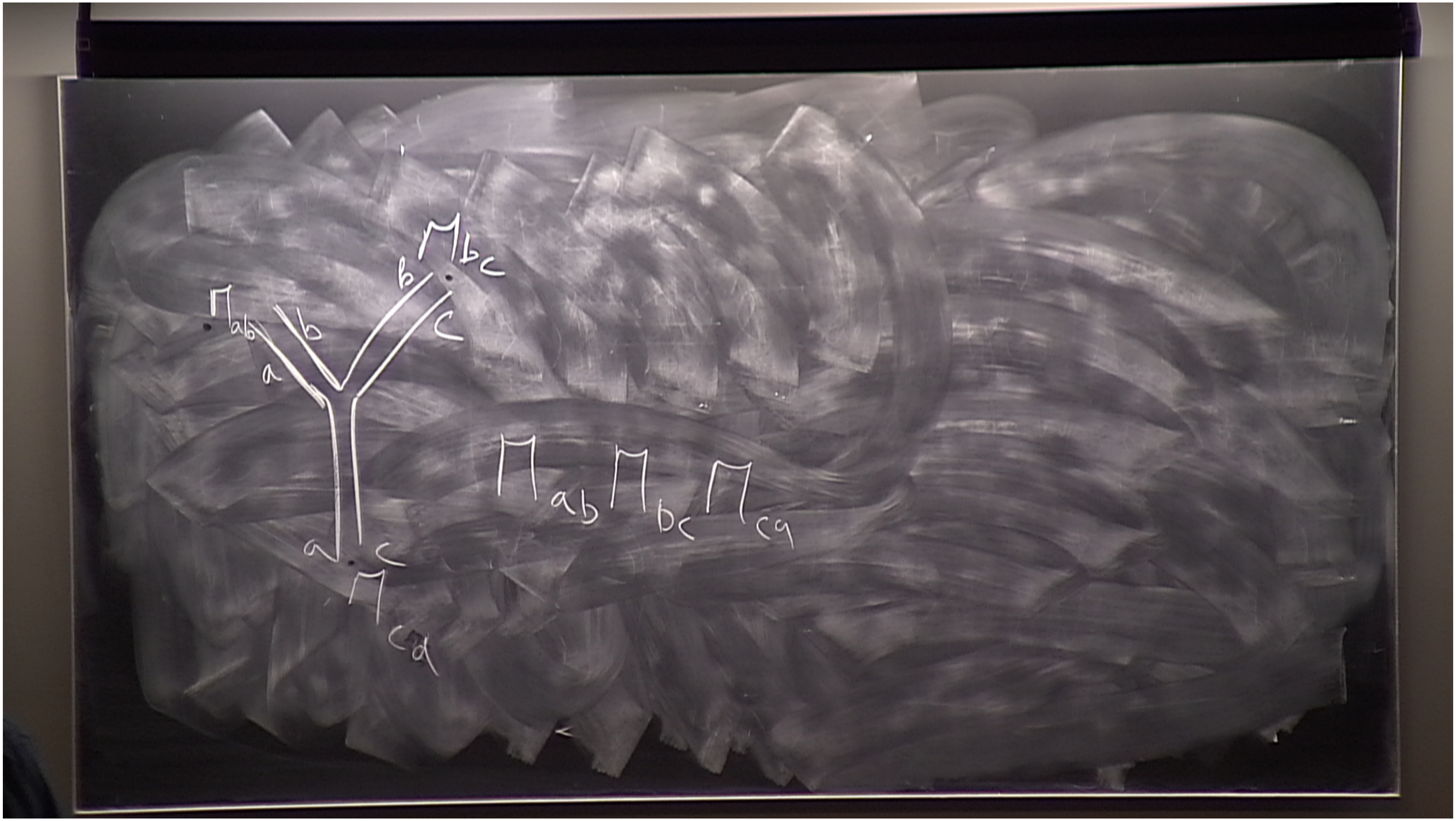
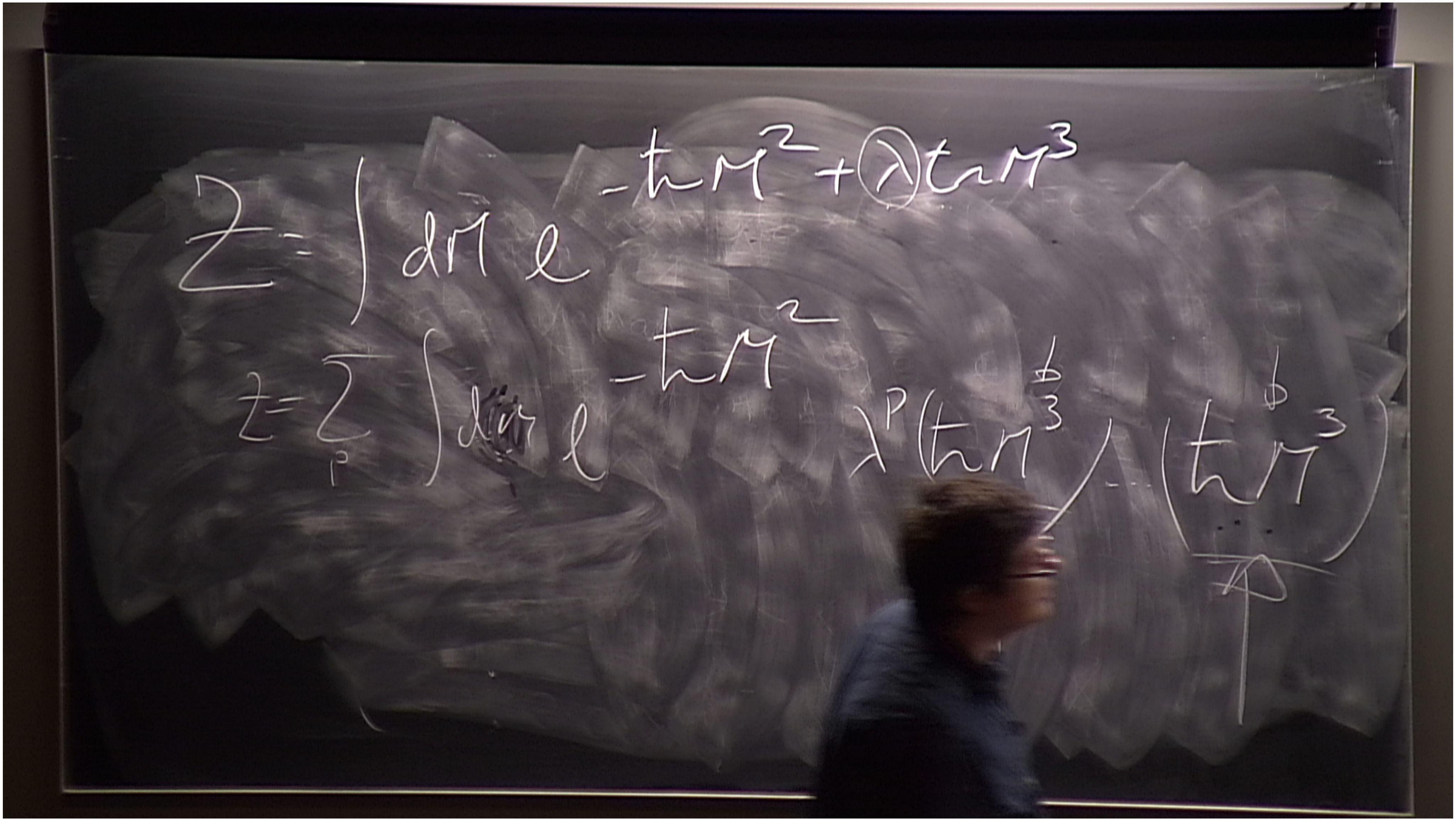
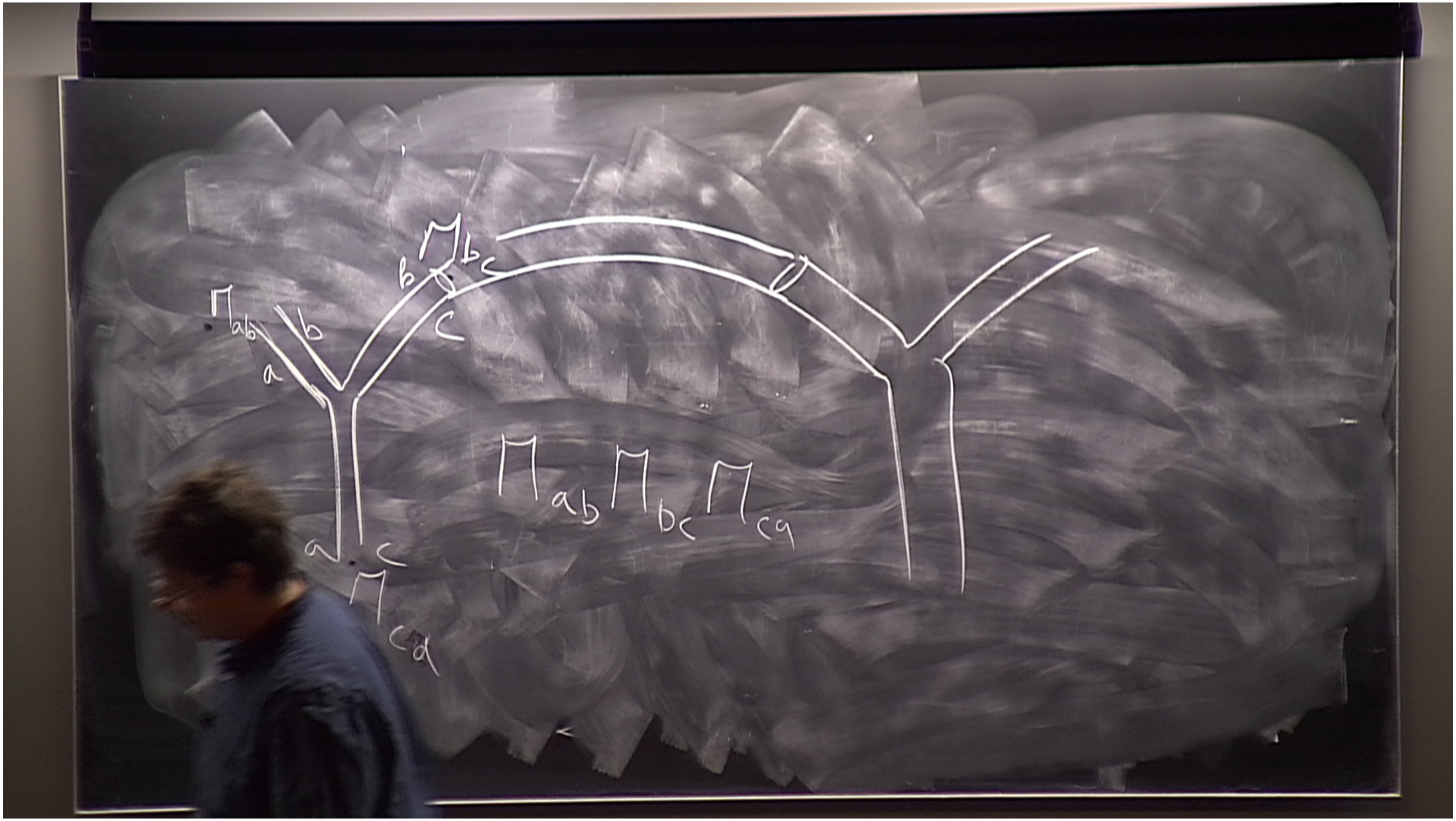


Γ_{ab}
Γ_{bc}
Γ_{cd}
a
b
b
c
a
c



Π_{bc}
Π_{ab}
b
b
c
a
a
c
Π_{ca}
$\Pi_{ab}\Pi_{bc}\Pi_{ca}$


Z = ∫ dM e^{-tr M^2 + λ tr M^3}
Z = Σ_p ∫ dM e^{-tr M^2} λ^p (tr M^3)^{p/3} ... (tr M^3)^p



Γab
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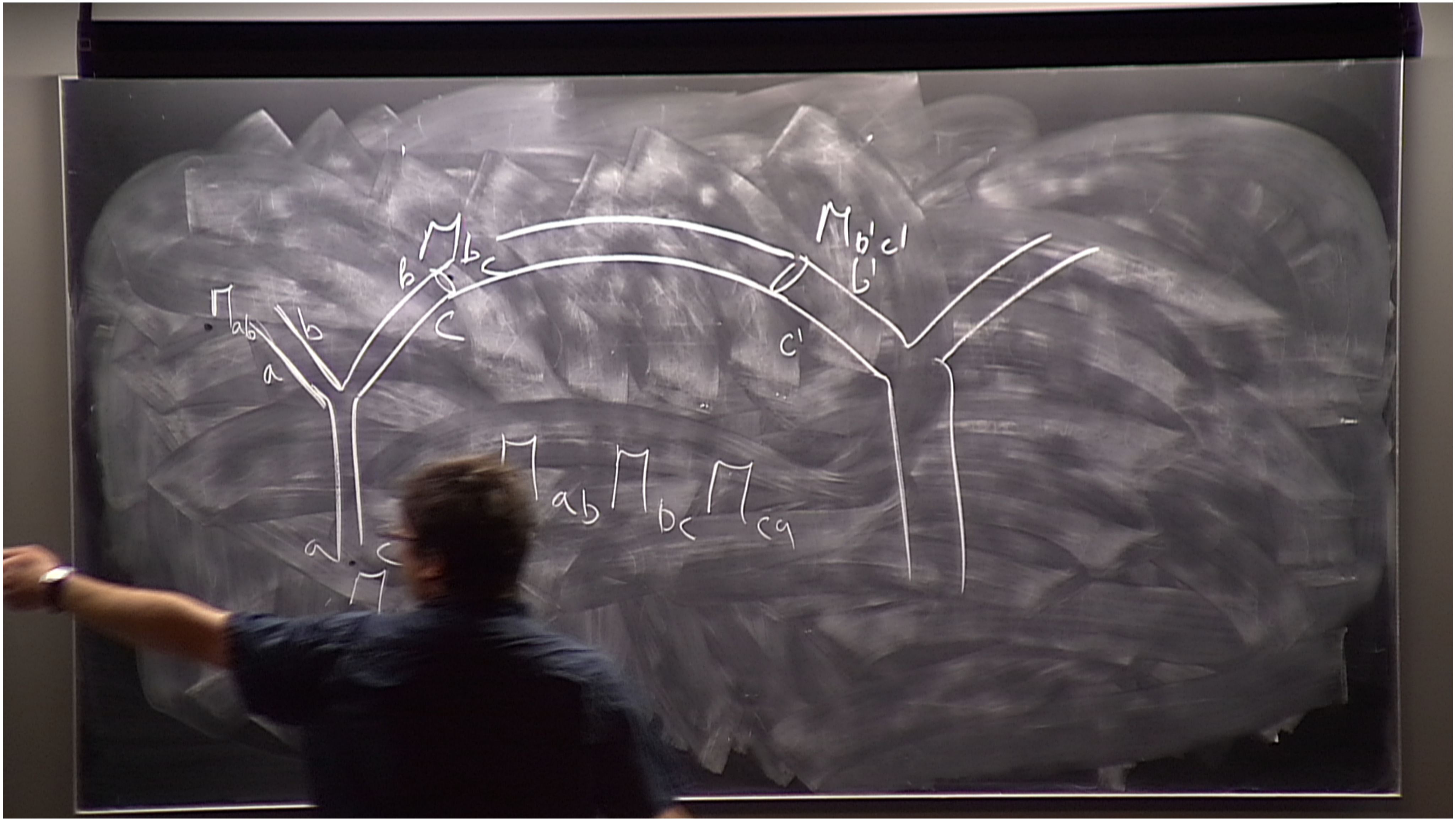
ΓabΓbcΓca
a
c
Γca


Π_ab
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Π_b'c'
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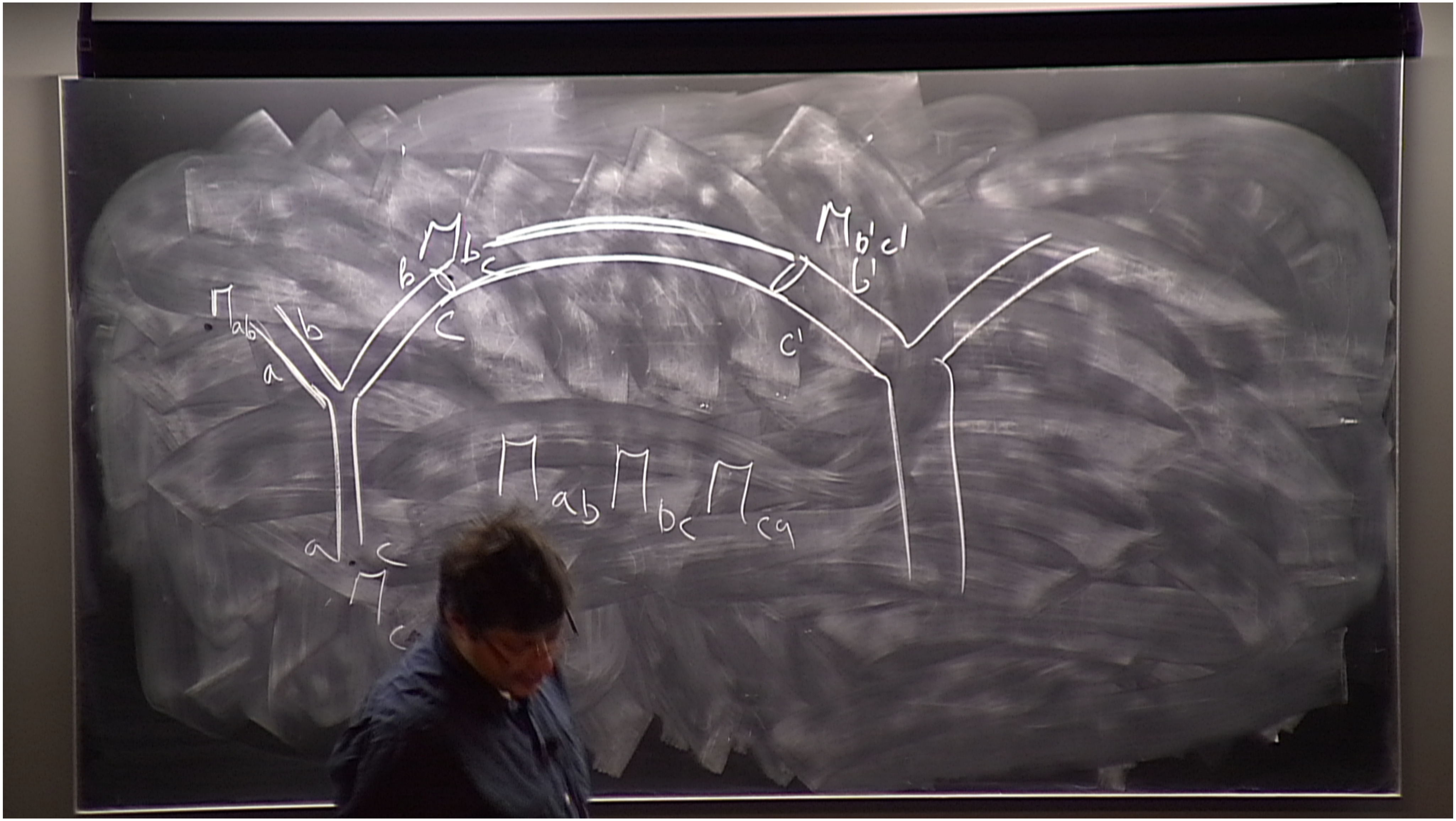
c'
Π_ab Π_bc Π_ca
a
c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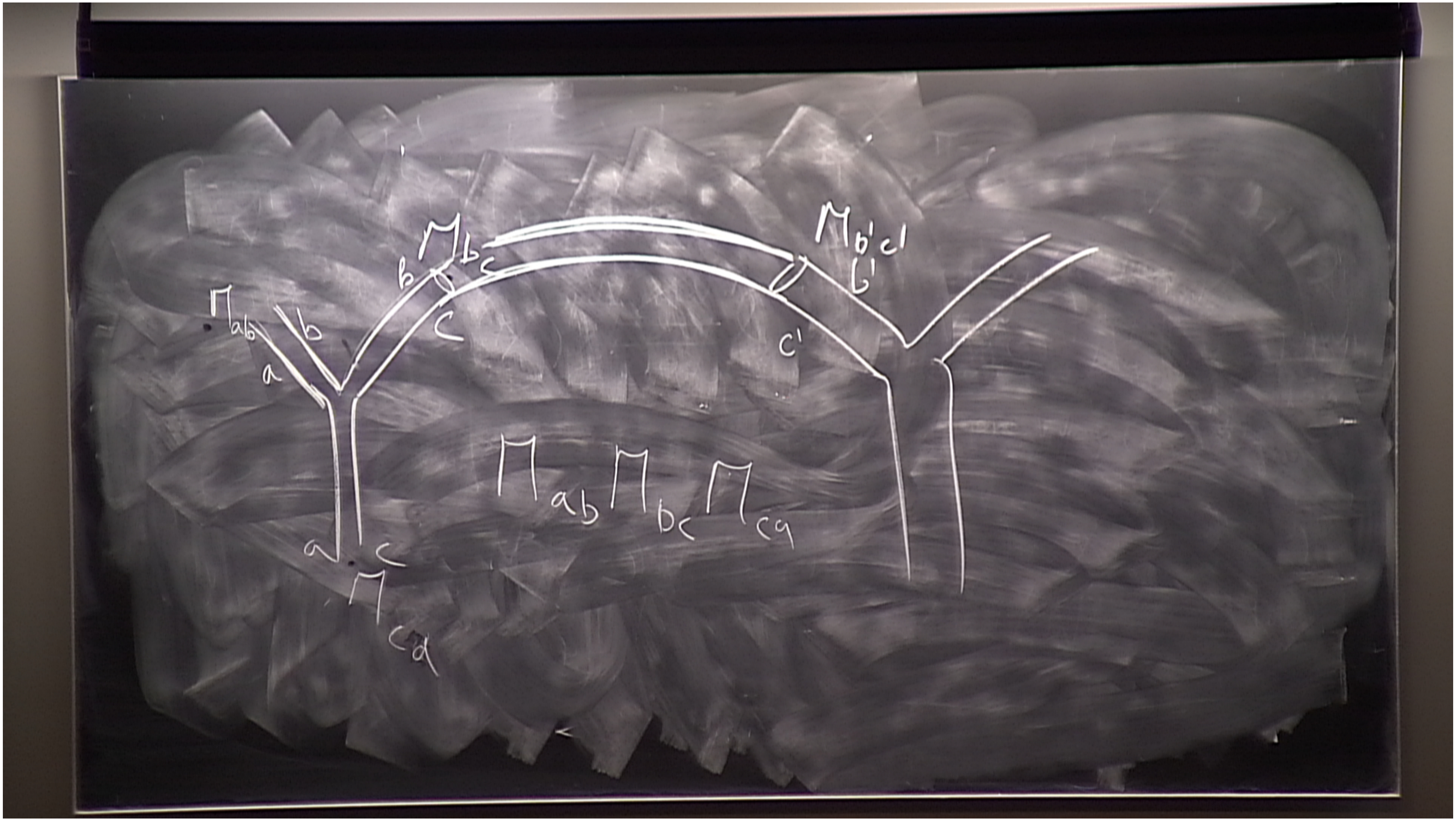
Mab
a
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Mb'c'
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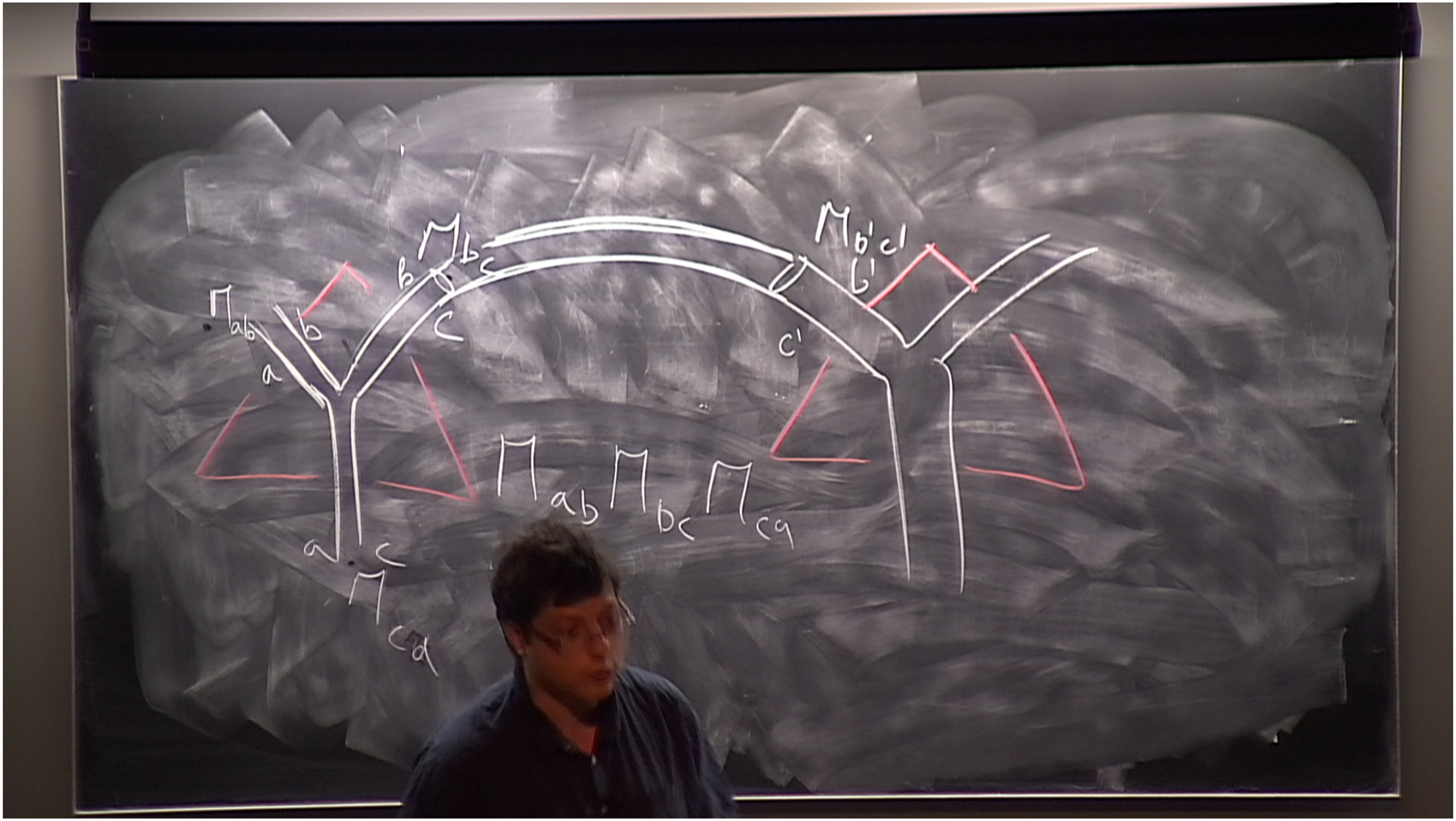
a
c
Mca



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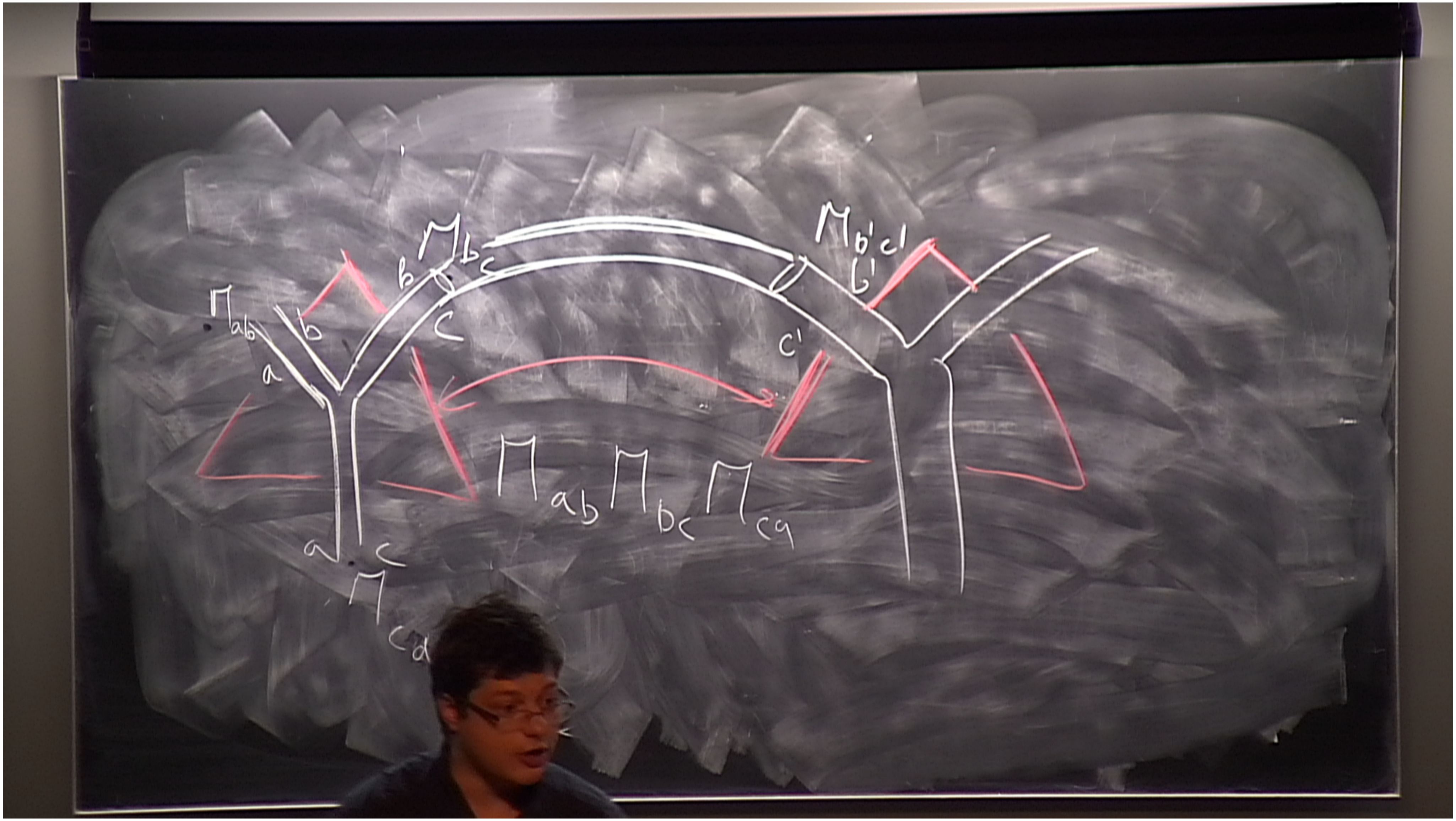

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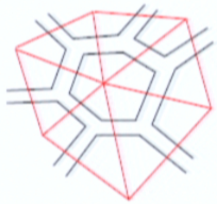

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$$A = \lambda^{\mathcal{N}} N^{2-2g(\mathcal{G})}$$

$N \to \infty$ planar graphs dominate. Summable family with a finite radius of convergence λ_c.

$\lambda \to \lambda_c$: phase transition to a theory of continuous surfaces. (Liouville gravity coupled with conformal matter).

From Matrix to Tensor Models

Matrix models	Tensor models
Invariant action for a matrix M_{ab}	Invariant action for a complex, generic tensor $T_{a^1 \dots a^D}$
surfaces $\leftrightarrow$ ribbon graphs	D dimensional spaces $\leftrightarrow$ colored graphs
	
$g(\mathcal{G}) \geq 0$ genus	$\omega(\mathcal{G}) \geq 0$ degree
$1/N$ expansion $A(\mathcal{G}) = N^{2-2g(\mathcal{G})}$	$1/N$ expansion $A(\mathcal{G}) = N^{D - \frac{2}{(D-1)!}\omega(\mathcal{G})}$
leading order: $g(\mathcal{G}) = 0$, spheres.	leading order: $\omega(\mathcal{G}) = 0$, spheres.
Phase transition	Phase transition

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Rank D complex tensor, no symmetry transforming under the external tensor product of D fundamental representations of $U(N)$

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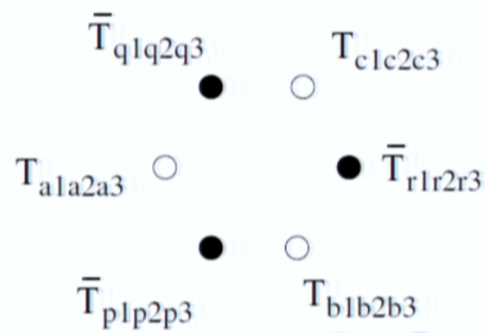
$$D=3\,, \qquad \sum \delta_{a^1p^1}\delta_{a^2q^2}\delta_{a^3r^3}\quad \delta_{b^1r^1}\delta_{b^2p^2}\delta_{b^3q^3}\quad \delta_{c^1q^1}\delta_{c^2r^2}\delta_{c^3p^3}$$
$$T_{a^1a^2a^3}\,T_{b^1b^2b^3}\,T_{c^1c^2c^3}\,\bar{T}_{p^1p^2p^3}\,\bar{T}_{q^1q^2q^3}\,\bar{T}_{r^1r^2r^3}$$



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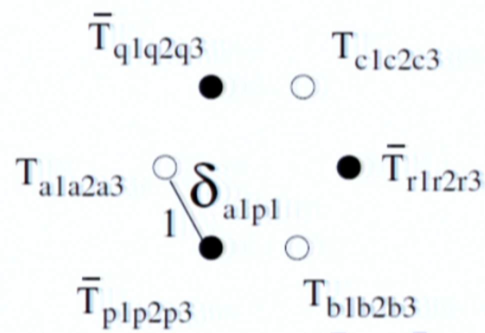
White (black) vertices for T $(\bar{T})$.

Lines for $\delta_{a^iq^i}$



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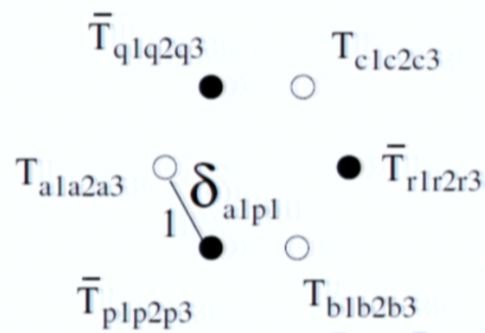

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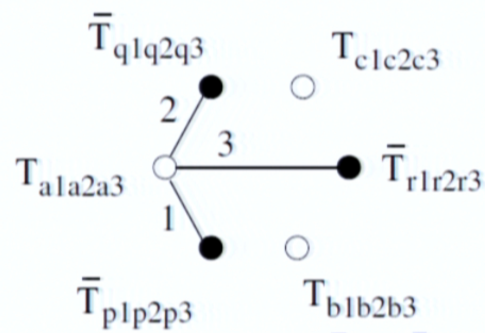

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$$\mathrm{Tr}_{\mathcal{B}}(T, \bar{T}) = \sum \prod_{v} T_{a^1_v \dots a^D_v} \prod_{\bar{v}} \bar{T}_{q^1_{\bar{v}} \dots q^D_{\bar{v}}} \prod_{i=1}^{D} \prod_{l^i=(w,\bar{w})} \delta_{a^i_w q^i_{\bar{w}}}$$

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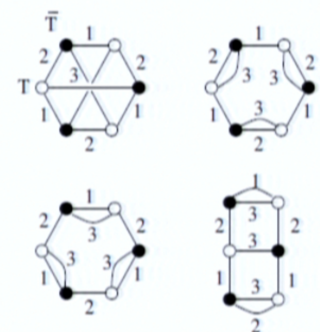


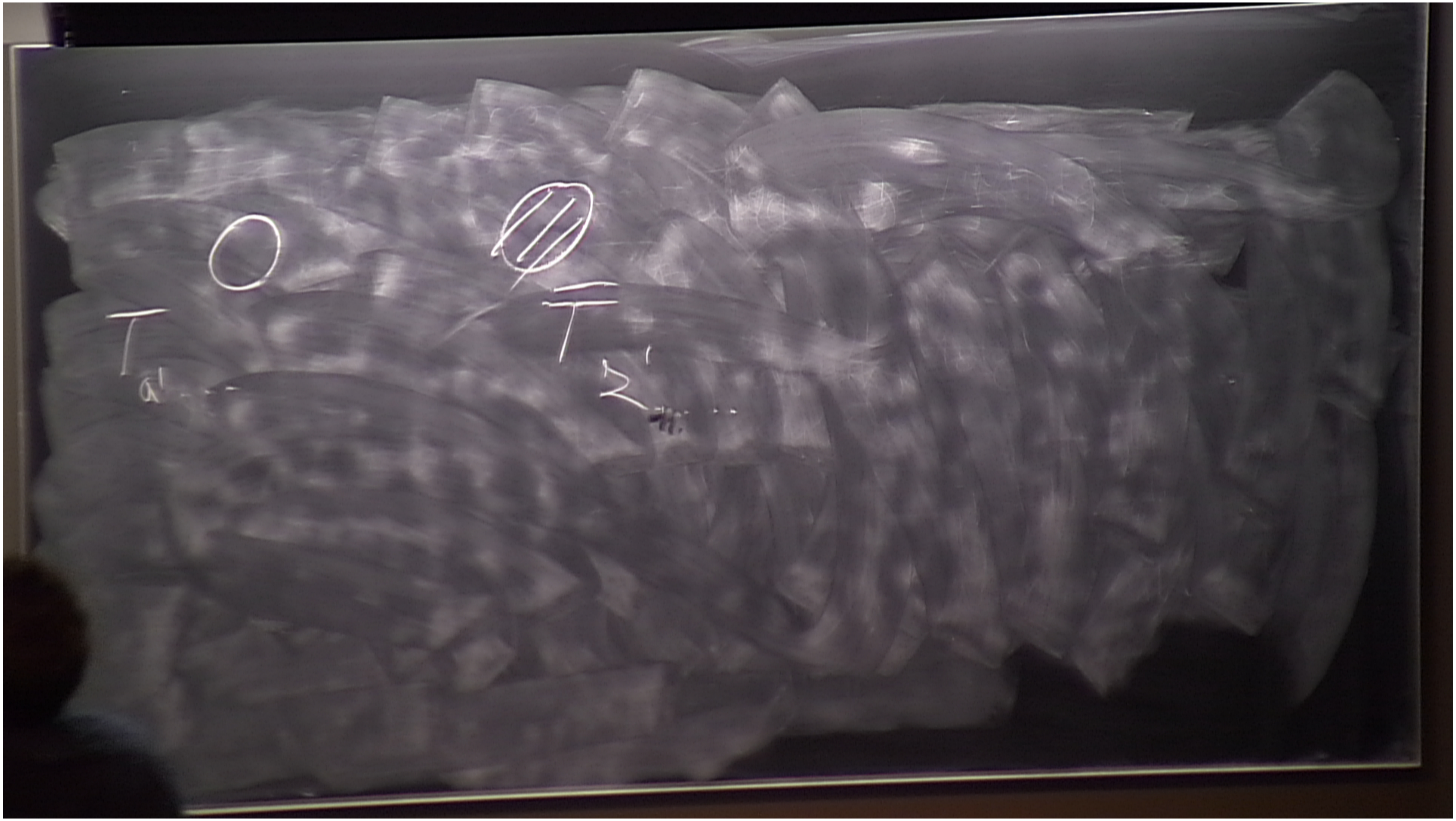
Invariant Actions for Tensor Models

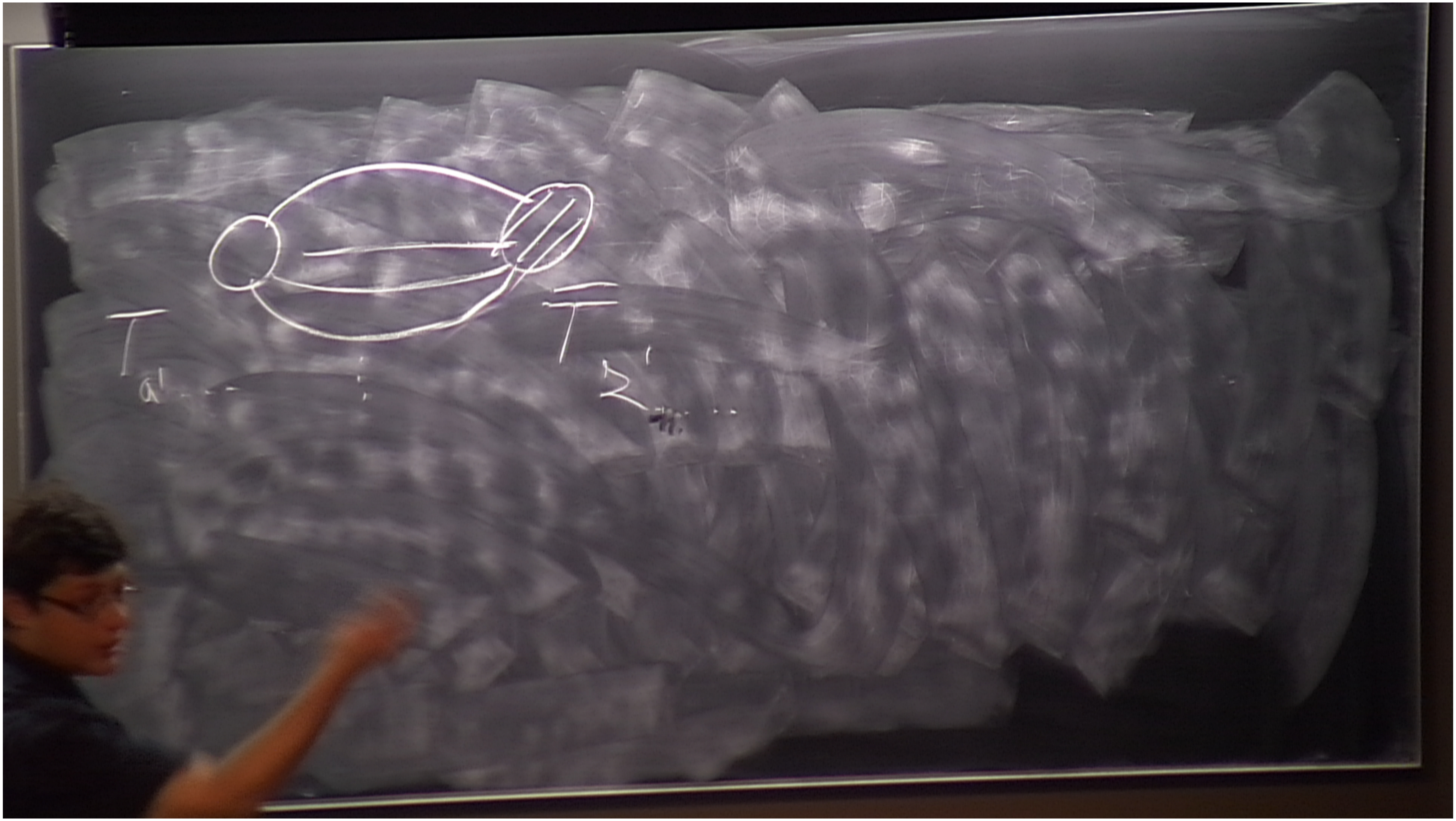
Invariant Actions for Tensor Models

The most general single trace tensor model

$$S(T,\bar{T}) = \sum T_{a^1\dots a^D}\bar{T}_{q^1\dots q^D}\prod_{i=1}^{D}\delta_{a^i q^i} + \sum_{\mathcal{B}} N^{?} t_{\mathcal{B}} \mathrm{Tr}_{\mathcal{B}}(\bar{T},T)$$

$$Z(g,t_{\mathcal{B}}) = e^{-N^D f(g,t_{\mathcal{B}})} = \int [d\bar{T}dT]\, e^{-\frac{N^{D-1}}{g}S(T,\bar{T})}$$





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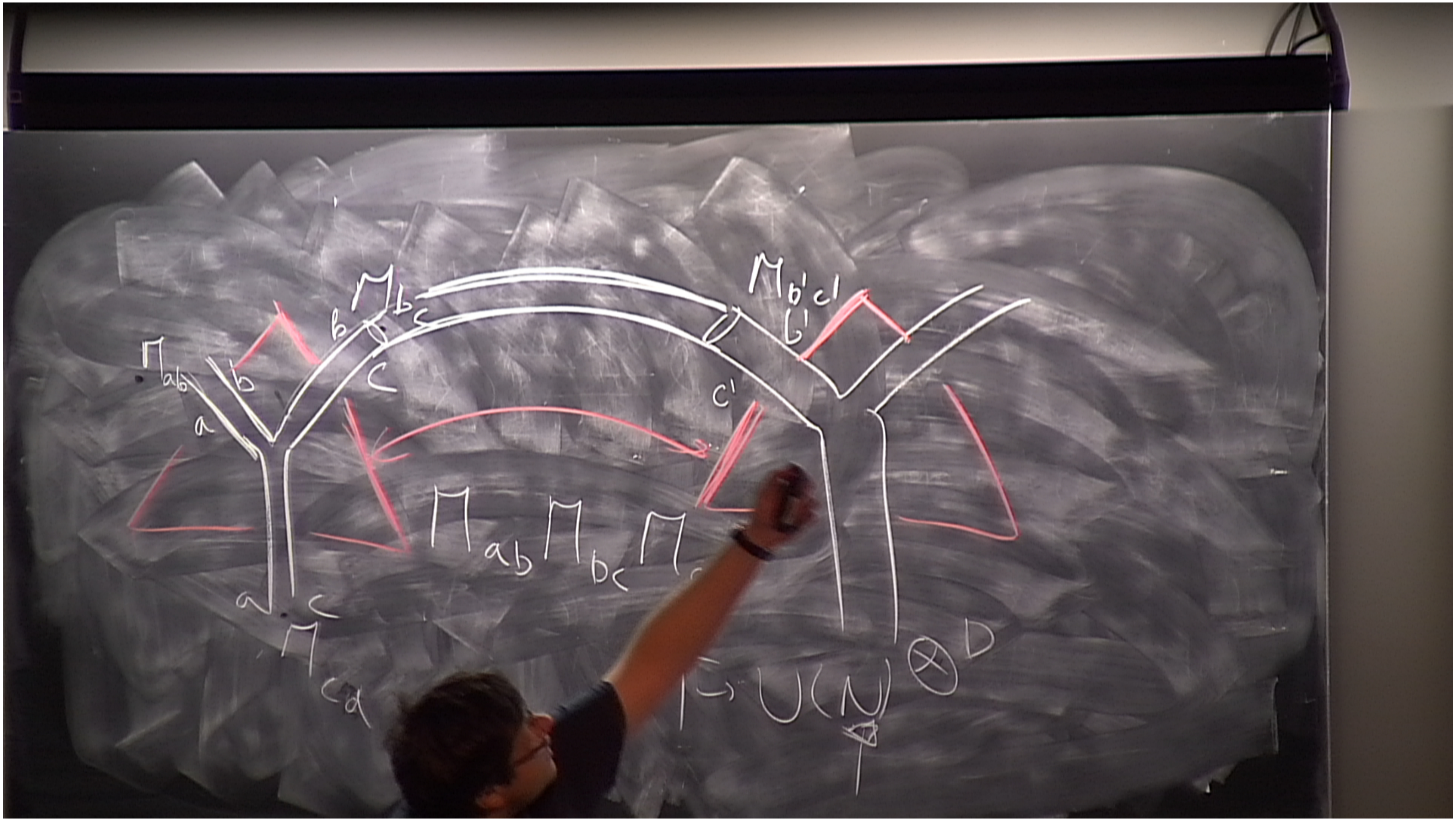
M ab
M bc
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M ab M bc M c
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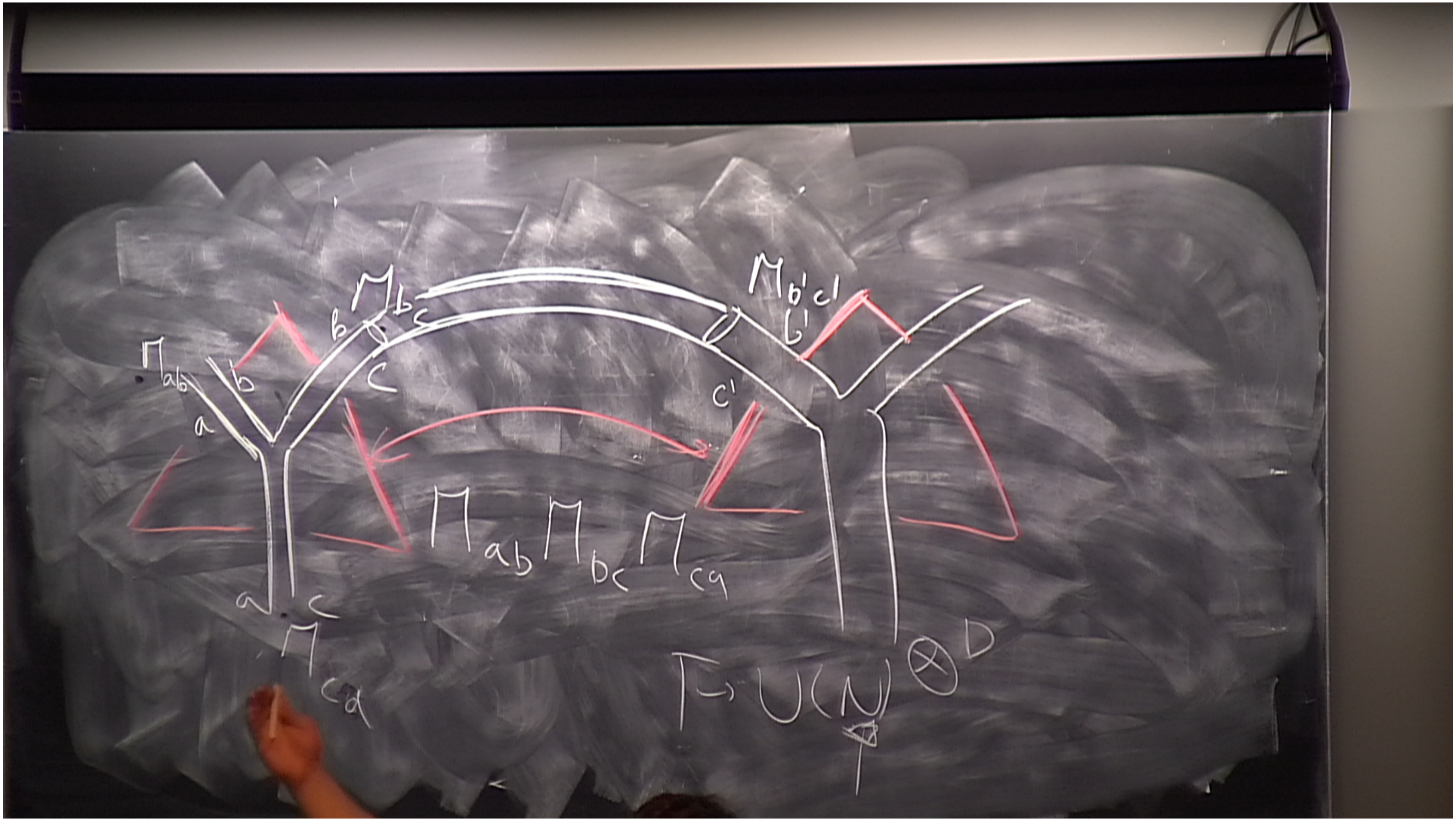
⊗ D



$M_{ab}\,M_{bc}\,M_{ca}$
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M_{ca}
$\Gamma \to U(N)^{\otimes D}$

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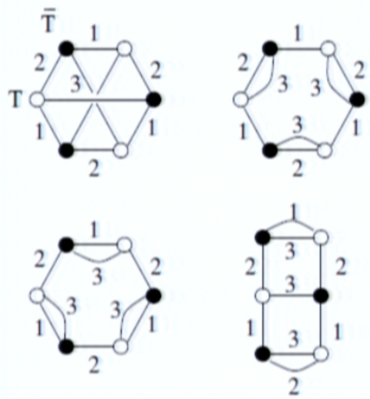
Feynman graphs: effective interactions $\mathcal{B}$.



$$\int_{\bar{T},T} \quad e^{-\frac{N^{D-1}}{g}\left(\sum T_{a^1\dots a^D} \bar{T}_{q^1\dots q^D} \prod_{i=1}^{D} \delta_{a^i q^i}\right)}$$

$$\mathrm{Tr}_{\mathcal{B}_1}(\bar{T},T)\mathrm{Tr}_{\mathcal{B}_2}(\bar{T},T)\dots$$

Invariant Actions for Tensor Models

The most general single trace tensor model

$$S(T,\bar{T}) = \sum T_{a^1\dots a^D}\bar{T}_{q^1\dots q^D}\prod_{i=1}^{D}\delta_{a^iq^i} + \sum_{\mathcal{B}} N^{?} t_{\mathcal{B}}\mathrm{Tr}_{\mathcal{B}}(\bar{T},T)$$

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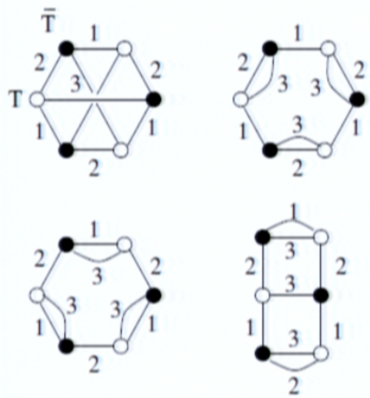

$$\int_{\bar{T},T} e^{-\frac{N^{D-1}}{g}\left(\sum T_{a^1\dots a^D}\bar{T}_{q^1\dots q^D}\prod_{i=1}^{D}\delta_{a^iq^i}\right)} \mathrm{Tr}_{\mathcal{B}_1}(\bar{T},T)\mathrm{Tr}_{\mathcal{B}_2}(\bar{T},T)\dots$$

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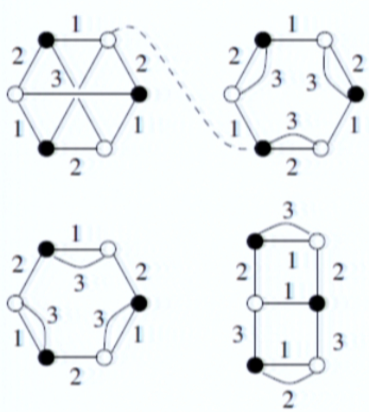
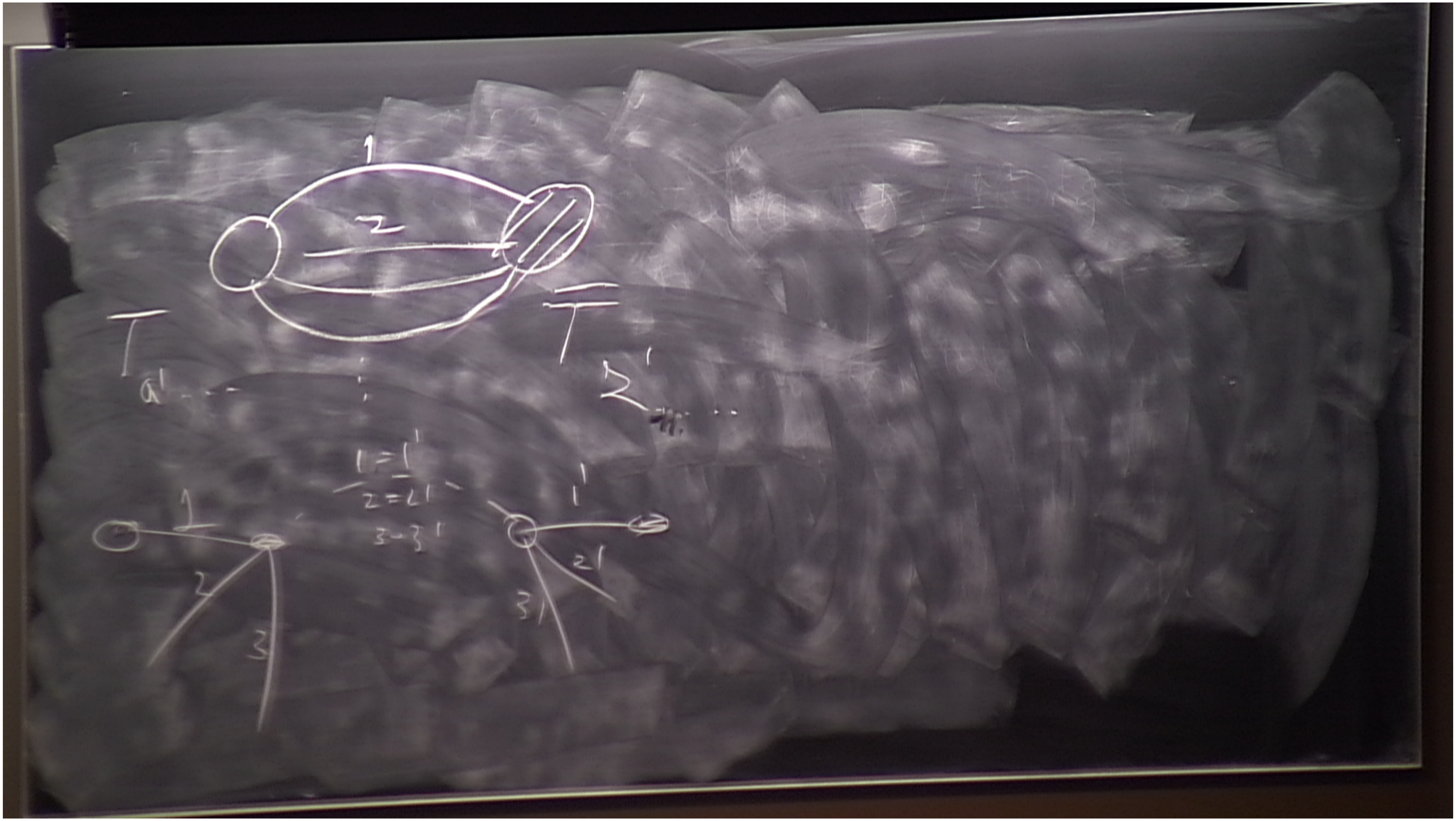


$$\int_{\bar{T},T} \quad e^{-\frac{N^{D-1}}{g}\left(\sum T_{a^1\dots a^D}\bar{T}_{q^1\dots q^D}\prod_{i=1}^{D}\delta_{a^iq^i}\right)}$$

$$\sum\left(\prod\delta_{\dots}\right)\underbrace{T_{a^1a^2a^3}\bar{T}_{p^1p^2p^3}}_{\sim\,\delta_{a^1p^1}\delta_{a^2p^2}\delta_{a^3p^3}}\dots$$


1
2
1 = 1'
2 = 2'
3 = 3'
2
3
1'
2'
3'

Invariant Actions for Tensor Models

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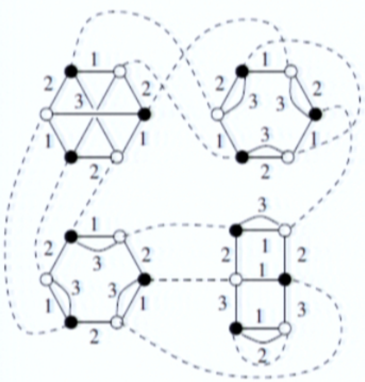

Graphs $\mathcal{G}$ with $D+1$ colors.

Invariant Actions for Tensor Models

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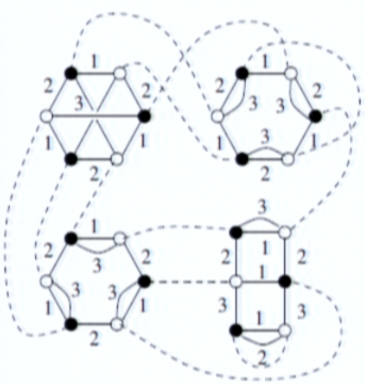

Graphs $\mathcal{G}$ with $D+1$ colors.

- Geometrical interpretation

Invariant Actions for Tensor Models

The most general single trace tensor model

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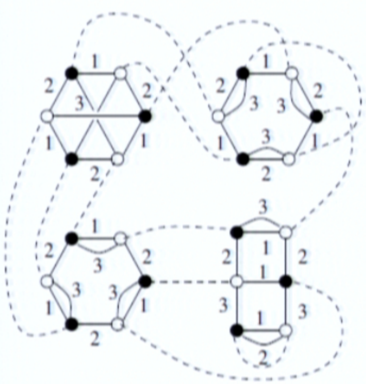
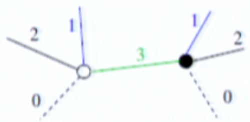


Graphs $\mathcal G$ with $D+1$ colors.

- Geometrical interpretation
- Amplitude

Colored Graphs as gluings of colored simplices

White and black $D+1$ valent vertices connected by lines with colors $0, 1 \dots D$.



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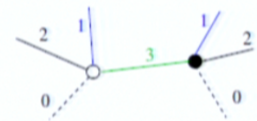
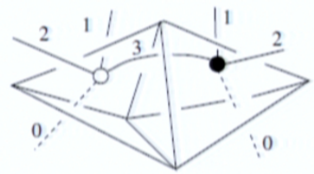
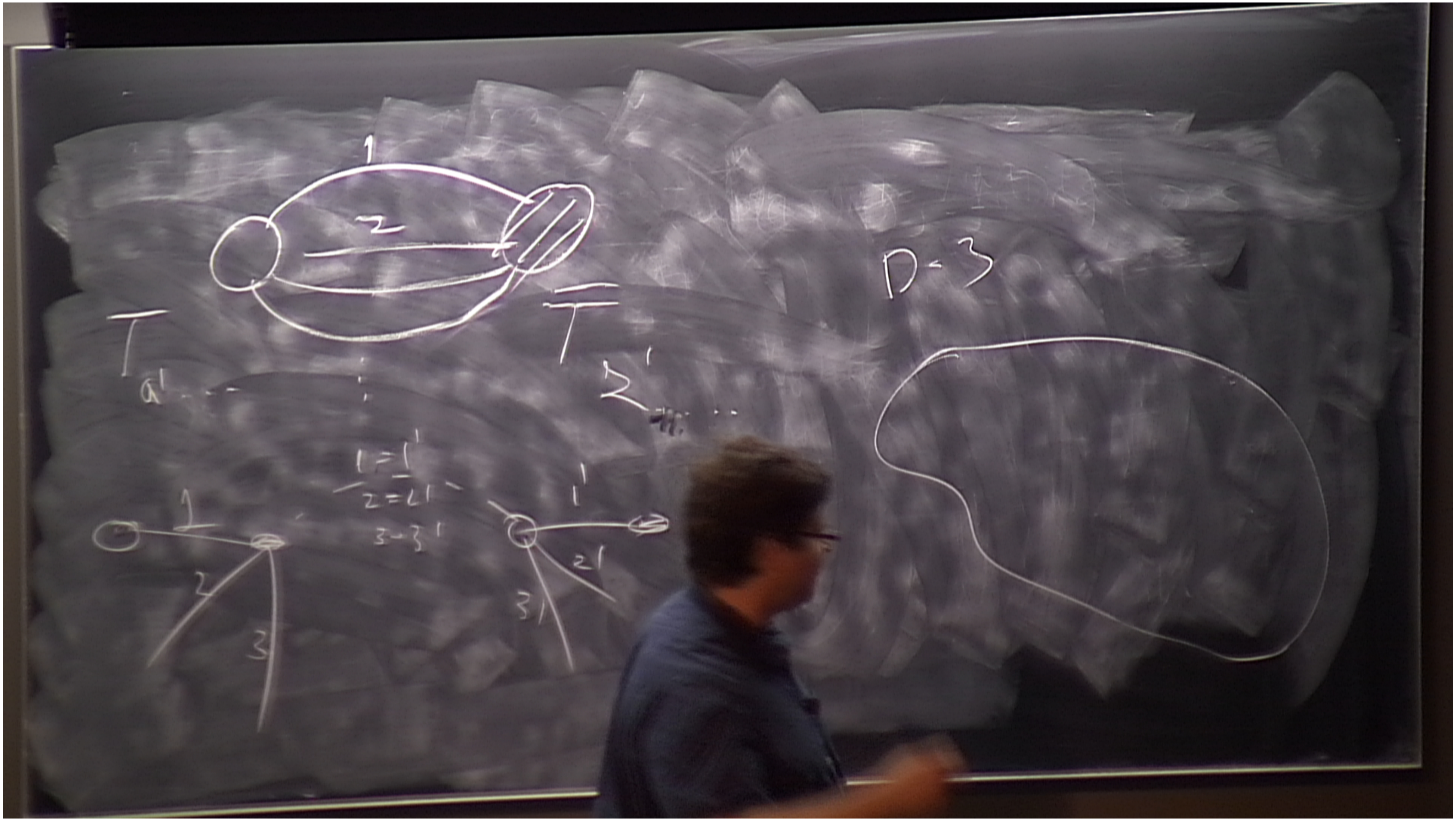
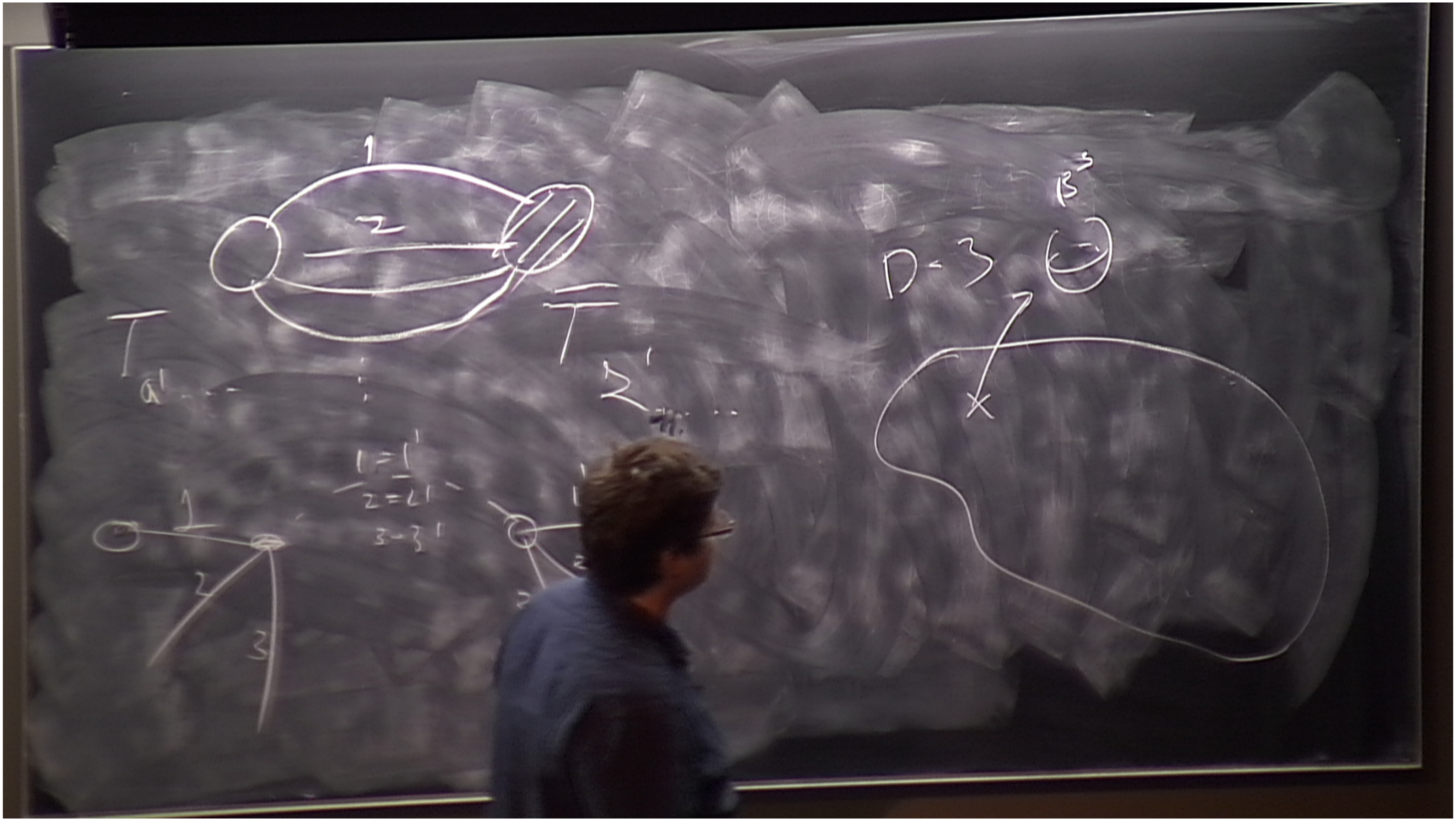


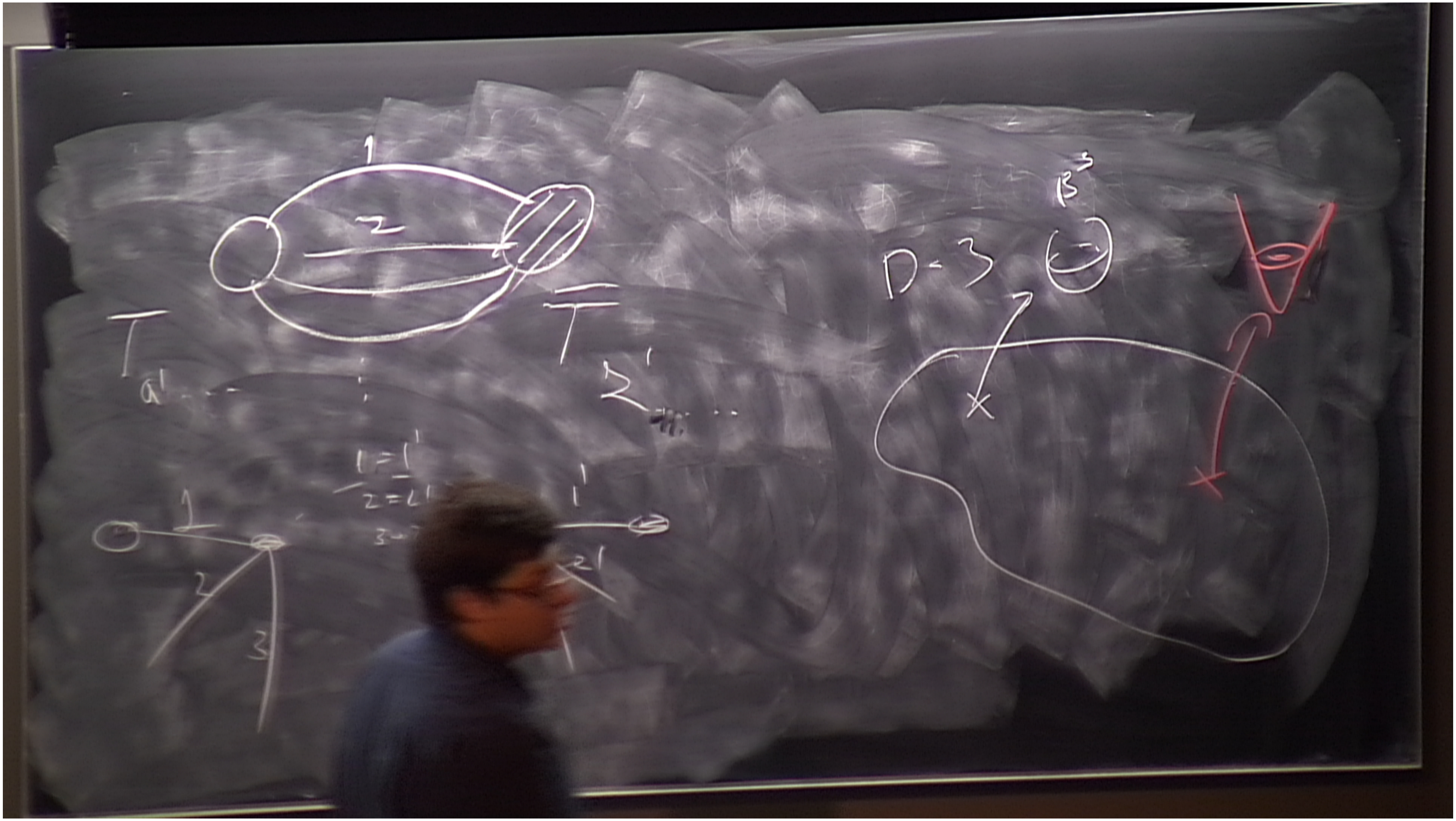
Theorem: Connected $D+1$ colored graphs are closed connected (orientable) D dimensional simplicial pseudomanifolds.



R.G. CQG 27 (2010) 235023






D = 3
B^3

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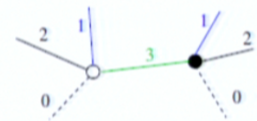

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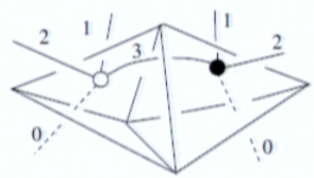

Vertex $\leftrightarrow$ colored D simplex .

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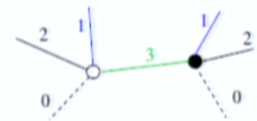

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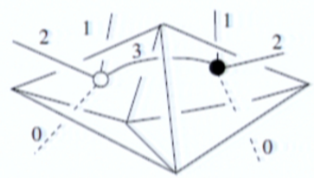

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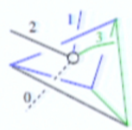

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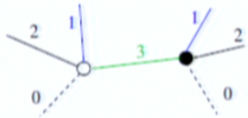

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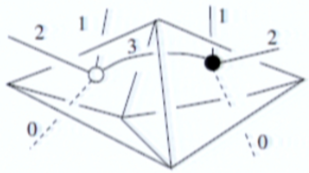

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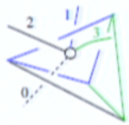

Lines $\leftrightarrow$ gluings along $D-1$ simplices respecting all the colorings

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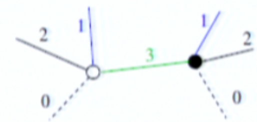

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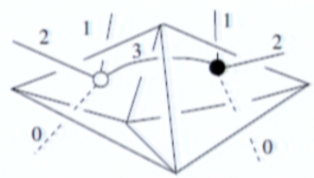

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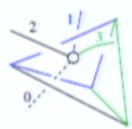

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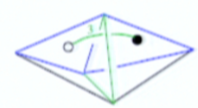

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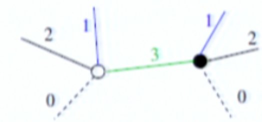

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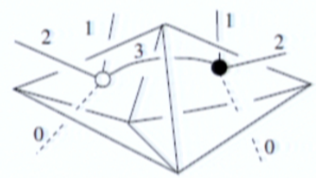

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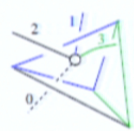

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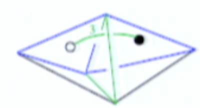

Faces (subgraphs with 2 colors)$\leftrightarrow$ $D-2$ simplices.

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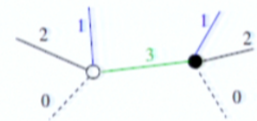
White and black $D+1$ valent vertices connected by lines with colors $0, 1 \dots D$.



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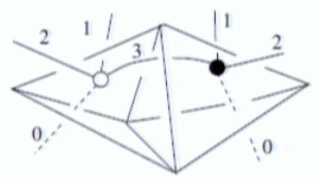

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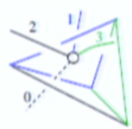

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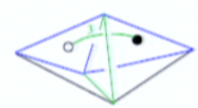

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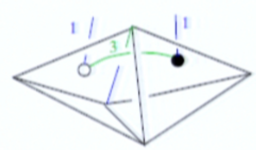

R.G. CQG 27 (2010) 235023

Two remarks

THEOREM: [M. Pezzana '74, M. Ferri and C. Gagliardi '82] Any D-dimensional close connected (orientable) manifold can be represented by a $D+1$ colored graph.

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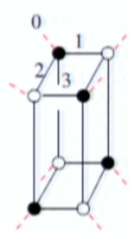

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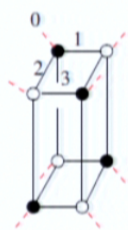
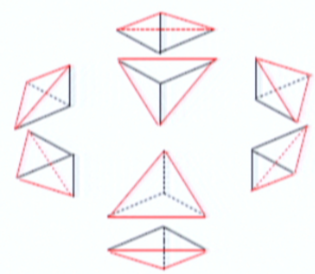
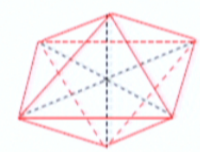
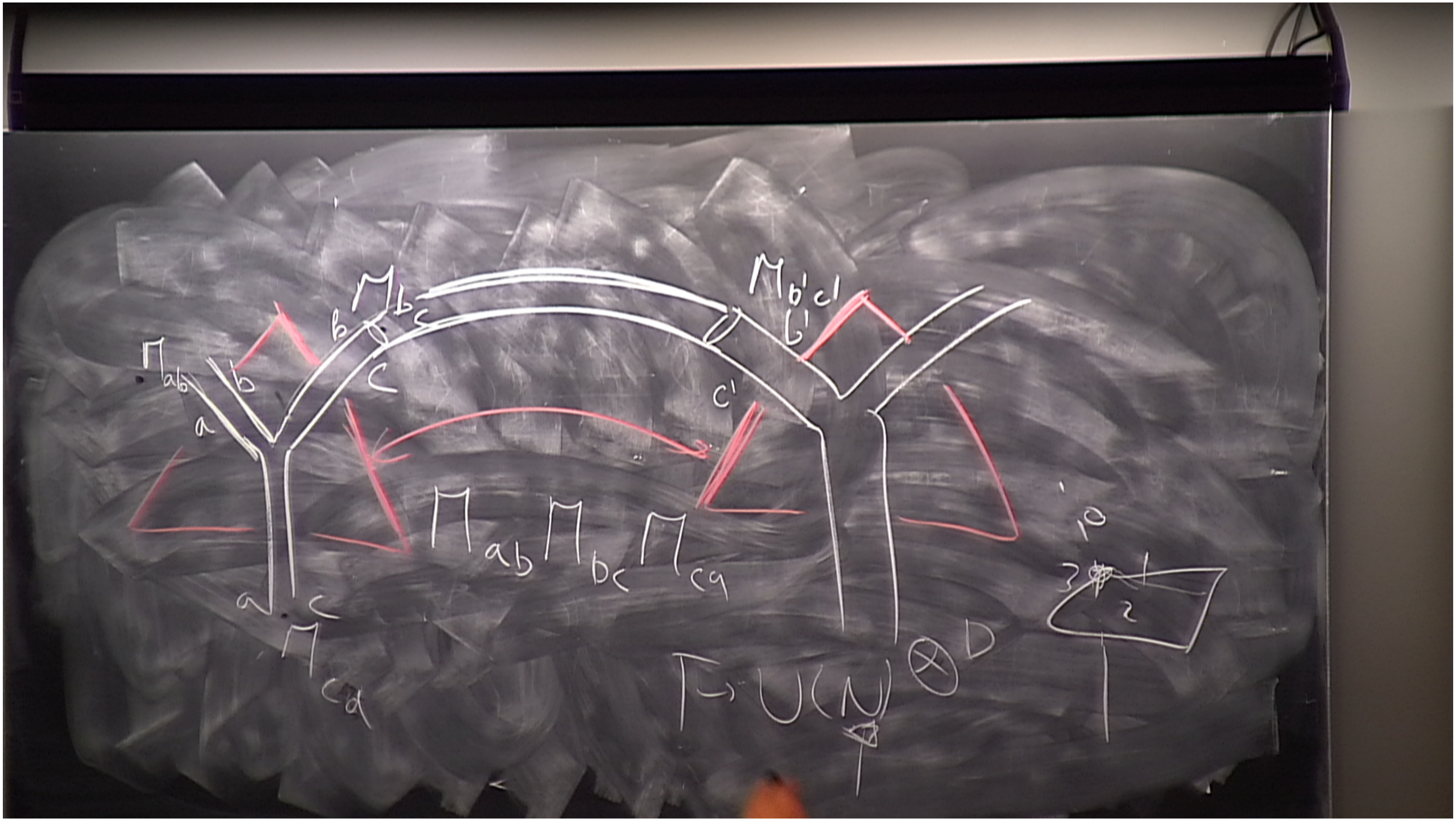


Every vertex is a D simplex



Gluing along all $D-1$ simplices except 0: "chunk" in D dimensions.





Mab
Mbc
Mb'c'
b'
c'
MabMbcMca
Mca
U(N)

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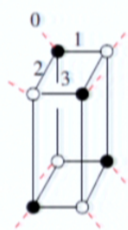

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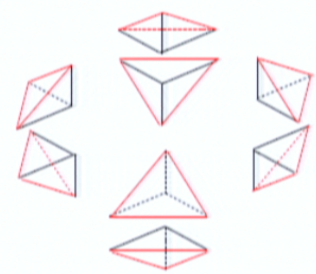

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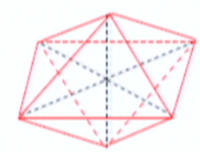

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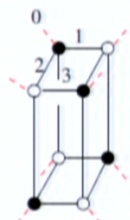

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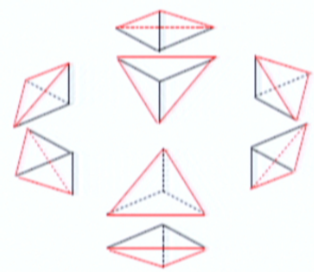

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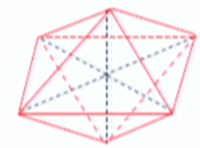

$\mathcal{B}$ is a D colored graph $\leftrightarrow$ $D-1$ simplicial pseudomanifold.

Adding the color $0 \leftrightarrow$ taking the topological cone over this pseudomanifold.

What replaces the genus?

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Count the faces $F(\mathcal{G})$ (subgraphs with two colors).

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Proposition: $\mathcal{G}$, $D+1$ colors, $2p$ vertices

$$F(\mathcal{G}) = \frac{1}{2}D(D-1)p + D - \frac{2}{(D-1)!}\omega(\mathcal{G}) \qquad \omega(\mathcal{G}) \geq 0$$

R.G., AHP 12, 829 (2011) R.G., V. Rivasseau, Europhys.Lett. 95 (2011) 50004 R.G., AHP 13, 399 (2012)

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Idea: Identify ribbon graphs embedded into the colored graph, the jackets $\mathcal{J}$ and count their faces.

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R.G., AHP 12, 829 (2011) R.G., V. Rivasseau, Europhys.Lett. 95 (2011) 50004 R.G., AHP 13, 399 (2012)

What replaces the genus?

Count the faces $F(\mathcal{G})$ (subgraphs with two colors).

Proposition: $\mathcal{G}$, $D+1$ colors, $2p$ vertices

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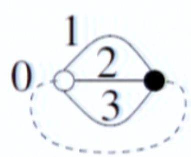
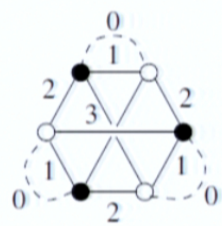
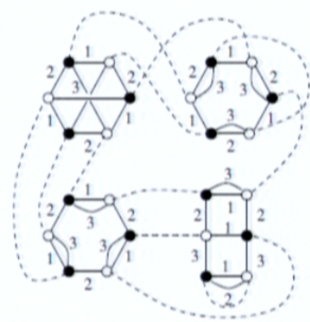


$\omega(\mathcal{G}) = 0$
$0+0+0$



$\omega(\mathcal{G}) = 4$
$1+1+2$



$\omega(\mathcal{G}) = 10$
$2+4+4$

The $1/N$ expansion

The $1/N$ expansion

$$Z = \int e^{-\frac{N^{D-1}}{g}\left[\sum T_{a^1\dots a^D}\bar{T}_{q^1\dots q^D}\prod_{i=1}^{D}\delta_{a^i q^i} + \sum_{\mathcal{B}} N^{-\frac{2}{(D-2)!}\omega(\mathcal{B})} t_{\mathcal{B}} \mathrm{Tr}_{\mathcal{B}}(\bar{T},T)\right]}$$

$$\mathrm{Tr}_{\mathcal{B}}(T,\bar{T}) = \sum \prod_{v} T_{a^1_v\dots a^D_v} \prod_{\bar{v}} \bar{T}_{q^1_{\bar{v}}\dots q^D_{\bar{v}}} \prod_{i=1}^{D} \prod_{l^i=(w,\bar{w})} \delta_{a^i_w q^i_{\bar{w}}}$$

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Theorem: Let $\mathcal{G}$ with $|l^0|$ lines of color 0 and $|\mathcal{B}|$ effective interactions

$$A(\mathcal{G}) = g^{|l^0|-|\mathcal{B}|}\left(\prod_{\mathcal{B}} t_{\mathcal{B}}\right) N^{D-\frac{2}{(D-1)!}\omega(\mathcal{G})}$$

R.G., Nucl.Phys. B852 (2011) 592-614

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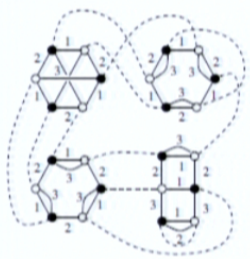
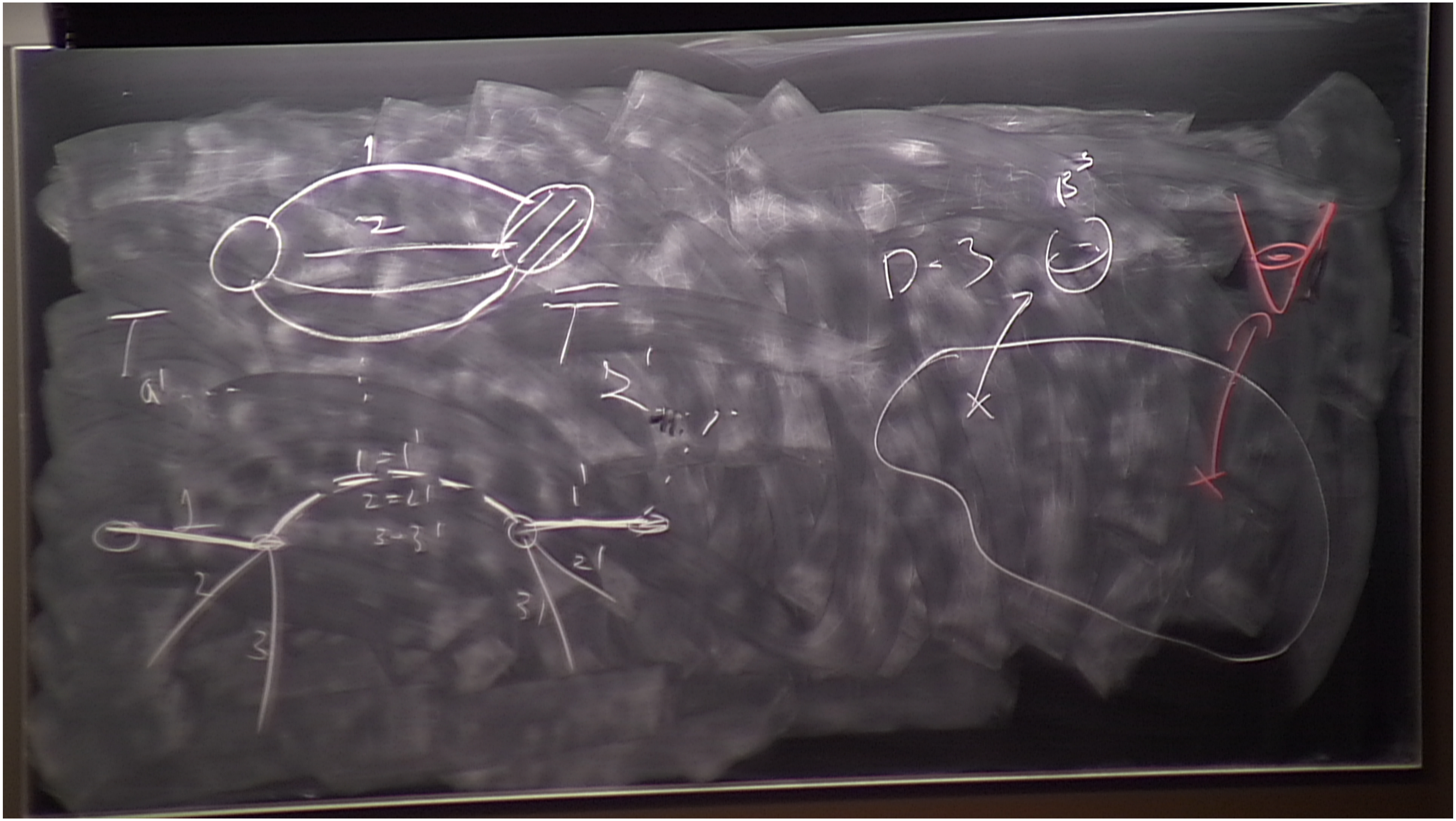


Idea:

$$\left(\prod_{i=1}^{D}\prod_{l^i=(w,\bar{w})}\delta_{a_w^i q_{\bar{w}}^i}\right) \quad \left(\prod_{l^0=(v,\bar{v})}\prod_{i=1}^{D}\delta_{a_v^i q_{\bar{v}}^i}\right)$$

δ's compose along the faces $0i \to N^{F^{0i}}$

R.G., Nucl.Phys. B852 (2011) 592-614



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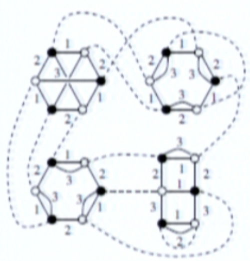

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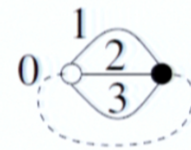
What replaces planar graphs?

Simplest graph: $\mathcal{G}_1$ with two vertices and $D+1$ lines



V. Bonzom, R.G, A. Riello, V. Rivasseau, Nucl.Phys. B852 (2011) 592-614

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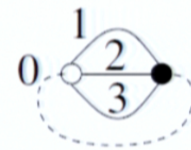
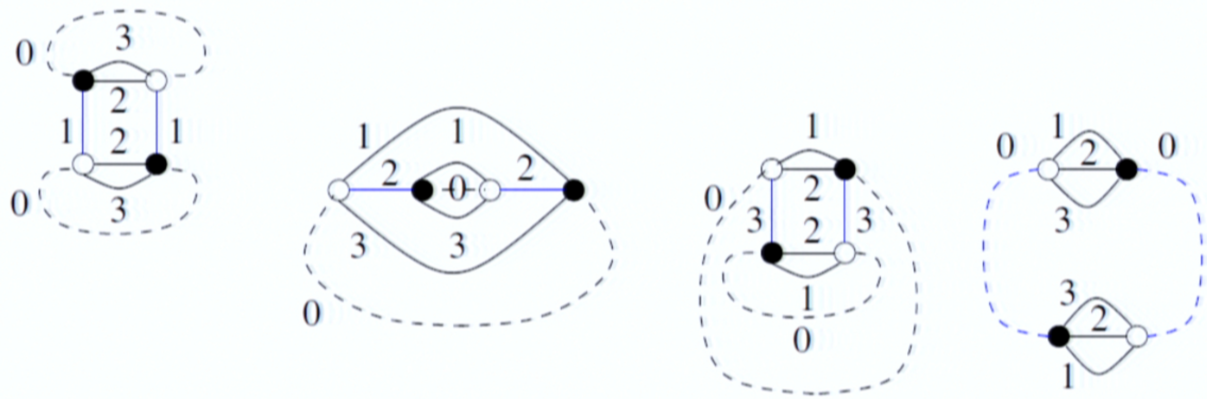


All others are obtained by inserting pairs of vertices connected by D lines on $\mathcal{G}_1$



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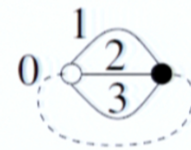

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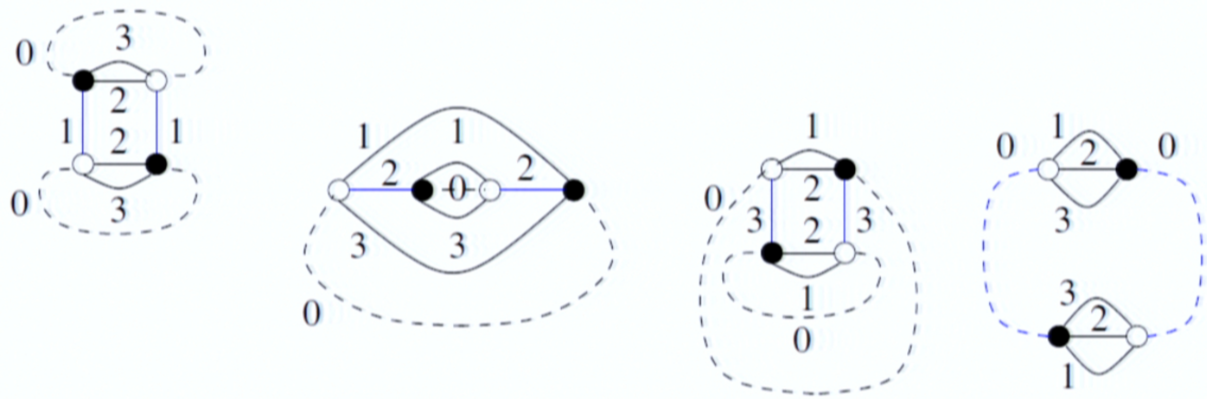

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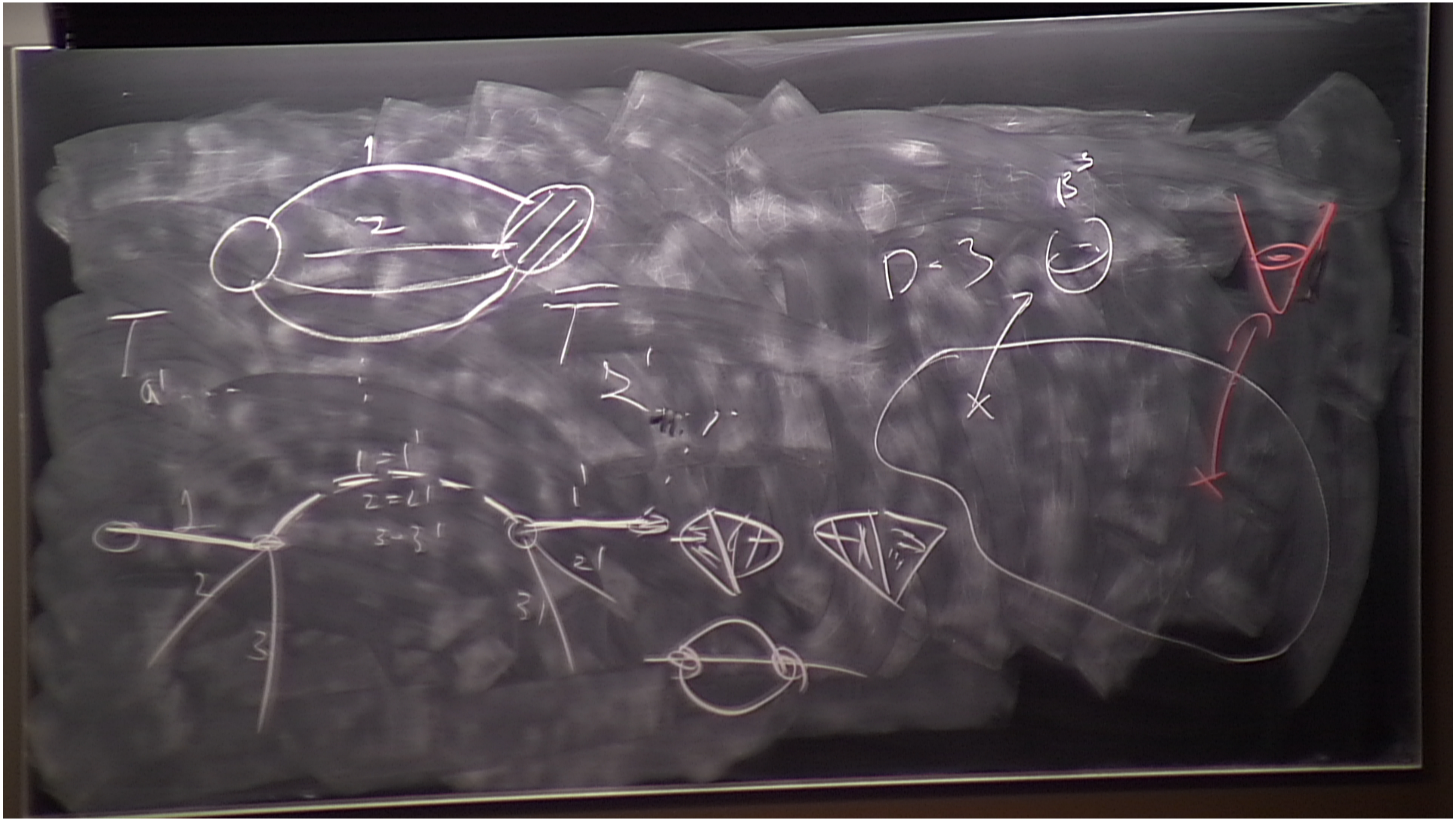

Conclusions

Results

- Phase transition to a continuum theory
- Trace invariant tensor measures exhibit universality
- Multicritical points ($\gamma_m = 1 - \frac{1}{m}$)
- Ising model, dually weighted models, hard dimer models on random lattices.
- Double scaling limit for a simplified model (only spherical topologies)
- Algebra of constrains: D-ary tree algebra
- Non invariant renormalizable models
- Symmetries
- Topological interpretation of the jackets

A very short list of open questions

- Representations and anomalies of the D-ary tree algebra
- Full double scaling limit
- Diffusion processes, Hausdorff and spectral dimensions for melons
- Renormalization group flows

Study the continuum theory: conformal field theory, statistical physics, KPZ, quantum gravity, etc. in higher dimensions