

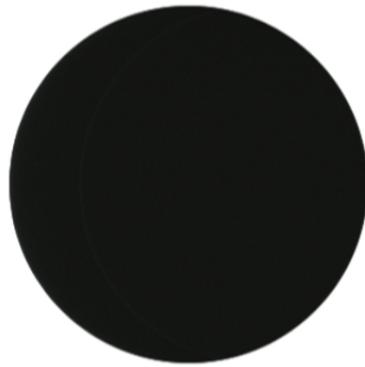
Title: Black Hole Entropy from Loop Quantum Gravity

Date: May 30, 2012 02:00 PM

URL: <http://pirsa.org/12050053>

Abstract: There is strong theoretical evidence that black holes have a finite thermodynamic entropy equal to one quarter the area A of the horizon. Providing a microscopic derivation of the entropy of the horizon is a major task for a candidate theory of quantum gravity. Loop quantum gravity has been shown to provide a geometric explanation of the finiteness of the entropy and of the proportionality to the area of the horizon. The microstates are quantum geometries of the horizon. What has been missing until recently is the identification of the near-horizon quantum dynamics and a derivation of the universal form of the Bekenstein-Hawking entropy with its $1/4$ prefactor. I report recent progress in this direction. In particular, I discuss the covariant spin foam dynamics and show that the entropy of the quantum horizon reproduces the Bekenstein-Hawking entropy $S=A/4$ with the proper one-fourth coefficient for all values of the Immirzi parameter.

Bekenstein-Hawking entropy
of a black hole horizon (1975)



$$S = k \frac{A}{4G\hbar} c^3$$



Bekenstein-Hawking entropy
of a black hole horizon (1975)

$$S = \frac{A}{4 G \hbar}$$

thermodynamics geometry

statistical mechanics relativity

gravity quantum mechanics

Loop quantum gravity:
 - microstates, quantum geometries
 - proportionality to the area
 - finiteness of the entropy

Rovelli, Smolin, Krasnov '95
 Ashtekar, Baez, Corichi, Krasnov '98
 Engle, Perez, Noui '10
 Ghosh, Perez '11

E. Bianchi black hole entropy from loop quantum gravity

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Missing until recently:

- dynamics of the quantum horizon d.o.f.
- derivation of the 1/4 prefactor in S

[I204.5122](#) : entropy density per horizon facet $s_f = 2\pi\gamma j_f$
from near-horizon spin-foam dynamics

Plan of the talk

- Near-horizon geometry of non-extremal black holes
- Loop quantum gravity: covariant “spin-foam” formulation
- The quantum horizon in loop quantum gravity:
energy, temperature and entropy

Near-horizon geometry of non-extremal black holes

Schwarzschild black hole of mass M

$$ds^2 = -(1 - \frac{2GM}{r})dt^2 + (1 - \frac{2GM}{r})^{-1}dr^2 + r^2d\Omega^2$$

Near-horizon metric in stationary coordinates: Rindler metric

$$ds^2 = -\frac{\ell^2}{(4GM)^2}dt^2 + d\ell^2 + (2GM)^2d\Omega^2 \quad \ell \ll 2GM$$

Stationary observer at proper distance ℓ from the horizon

4-velocity $u^\mu = \frac{4GM}{\ell}(\frac{\partial}{\partial t})^\mu$

4-acceleration $a^\mu = u^\nu \nabla_\nu u^\mu = a (\frac{\partial}{\partial \ell})^\mu$

acceleration $a = \ell^{-1}$



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Non-extremal Kerr-Newman black hole,

mass M , angular momentum J , charge Q

Stationary observer at proper distance ℓ from the horizon

4-velocity $u^\mu = \frac{\chi^\mu}{\|\chi\|} \quad \chi = \frac{\partial}{\partial t} + \Omega \frac{\partial}{\partial \phi}$

acceleration $a = \ell^{-1}$

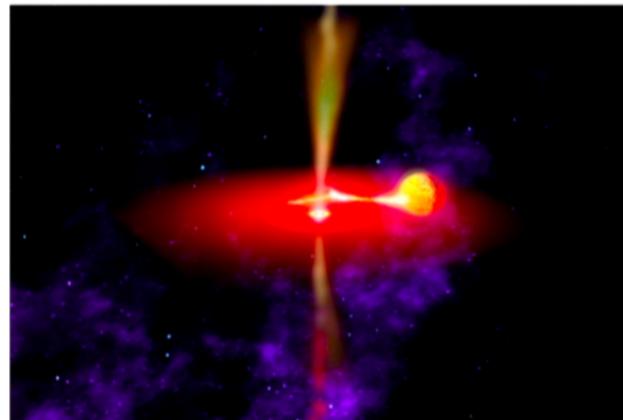
Near-horizon geometry of non-extremal black holes

[McClintock et al. astro-ph/0606076]

binary system GRS1915+105

$14 M_{\odot}$ black hole with very large angular momentum

$$a = J/M^2 \gtrsim 0.98$$



artist's conception of a rotating black hole with accretion disk
credit: Dana Berry (CfA/NASA)

Theoretical limit $a = 1$, extremal b.h.

For astrophysical black holes:

$$a < 0.998$$

due to accretion processes

[K. S. Thorne, "Disk Accretion Onto A Black Hole" *Astrophys. J.* 191, 507 (1974)]

E. Bianchi

black hole entropy from loop quantum gravity

Near-horizon geometry of non-extremal black holes

Horizon energy as seen by a stationary observer [Frodden-Ghosh-Perez [JHEP10\(2011\)055](#)]

$$E = \frac{1}{8\pi G} \int \nabla^\mu u^\nu dS_{\mu\nu} = \frac{1}{8\pi G} \int_S \sqrt{h} n_\mu a^\mu d^2x$$

$$n^\mu = (\frac{\partial}{\partial t})^\mu$$

$$E = \frac{A}{8\pi G} a$$



E. Bianchi

black hole entropy from loop quantum gravity

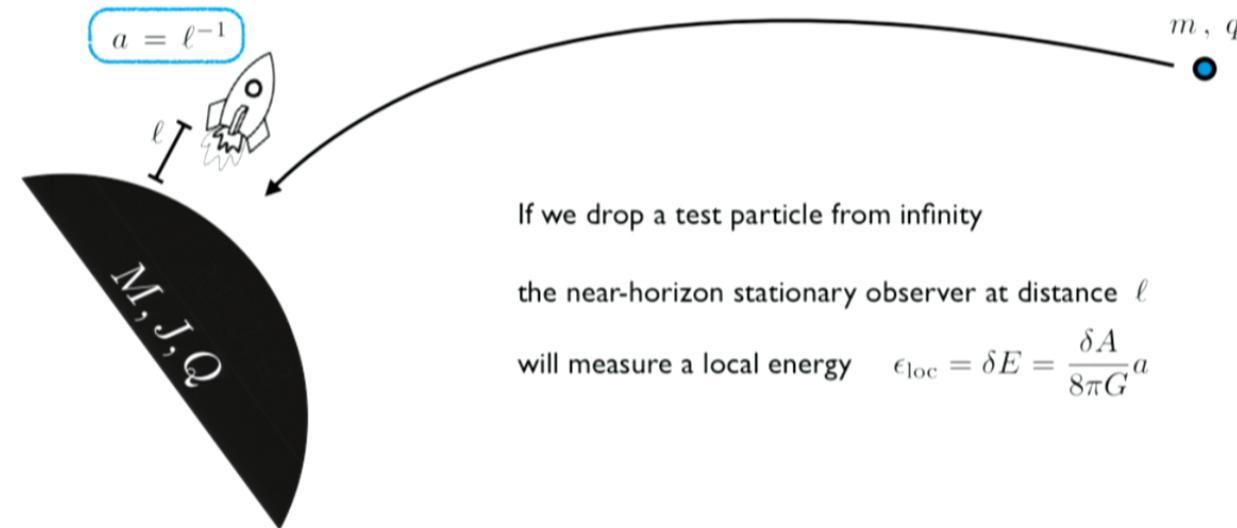
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If we drop a test particle from infinity

the near-horizon stationary observer at distance ℓ

will measure a local energy $\epsilon_{\text{loc}} = \delta E = \frac{\delta A}{8\pi G} a$

Loop Quantum Gravity and Spin Foams: key steps

-
- 1986 Yang Mills-like gauge variables (A, E) for General Relativity [Ashtekar]
1987 The Loop assumption: Wilson loop states $\Psi_\gamma[A] = \text{Tr } P \exp \int_\gamma A$ [Rovelli-Smolin]

Loop Quantum Gravity

- 1994 Hilbert Space of states: functional measure $d\mu[A]$ [Ashtekar-Lewandowski]
spin-network o.n. basis [Rovelli-Smolin]
- 1995 Quantum Geometry: discrete spectra [Rovelli-Smolin, Ashtekar-Lewandowski]
- 2000 Coherent spin-network states [Thiemann]

Dynamics: problem of the Hamiltonian constraint

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Dynamics: Spin Foam path-integral

- 2007 The partition function [Engle-Pereira-Rovelli-Livine, Freidel-Krasnov] (Barrett-Crane 1998)
2009- Semiclassical limit and General Relativity [Barrett-Dowdall-Fairbairn-Gomes-Hellmann]
Truncations. Coarse graining. Perturbative schemes? [Dittrich et al.]
2010- Study of the cosmological regime, of the perturbative graviton regime,
... quantum horizons

Spin-foam path integral

- Aim:

provide a realization of the path-integral over geometries
for 4d Lorentzian gravity

$$Z = \int \mathcal{D}g \ e^{iS[g]}$$

- * Action?
- * Measure?

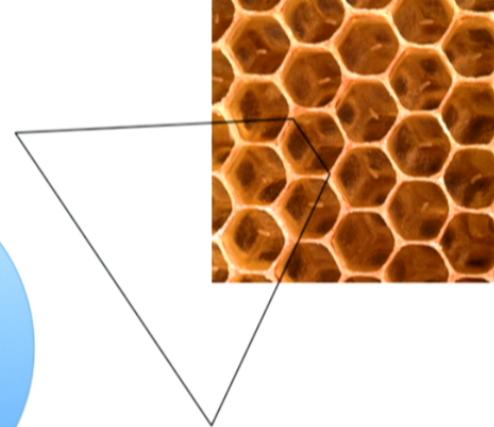
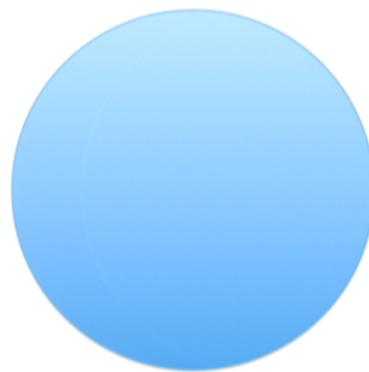
Spacetime manifold and the notion of 2d-foam

- M = Spacetime = 4d Manifold of trivial topology
- Δ = Topological decomposition of M in cells

4-cells Δ_4 = 4-ball

$\partial\Delta_4$ = 3-cells Δ_3

$\partial\Delta_3$ = 2-cells Δ_2



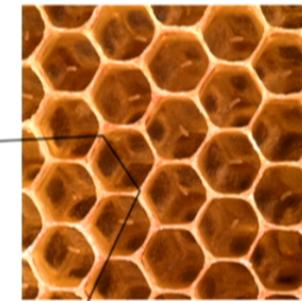
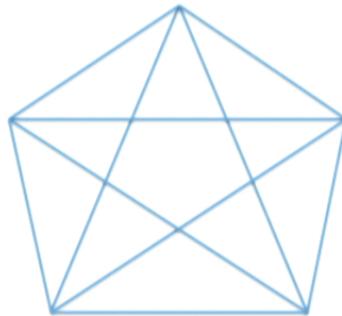
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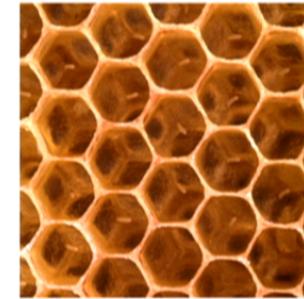
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- Set $\{\Delta_2\}$ = 2-skeleton of (M, Δ) = 2d-foam

* The manifold $M' = M - \{\Delta_2\}$ is non simply-connected, non-trivial π_1

non-contractible loops around Δ_2

Spacetime manifold and the notion of 2d-foam

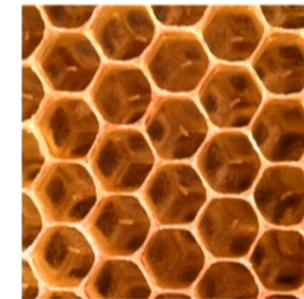
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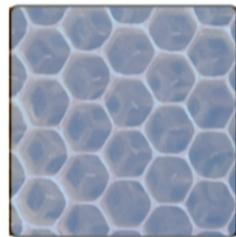


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Cellular decomposition of M



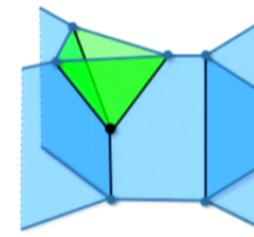
4-cells Δ_4

3-cells Δ_3

2-cells Δ_2



Dual 2-complex



vertex v

edge e

face f

E. Bianchi

black hole entropy from loop quantum gravity

The Spin Foam action: main idea

- Gravity: Einstein-Cartan action + Holst term

$$S[e, \omega] = \frac{1}{16\pi G} \int \frac{1}{2} \epsilon_{IJKL} e^I \wedge e^J \wedge F^{KL}(\omega) + \frac{1}{\gamma} e_I \wedge e_J \wedge F^{IJ}(\omega)$$

e^I = co-tetrad one-form

ω^{IJ} = Lorentz connection

$\gamma \in \mathbb{R}$ = Immirzi parameter

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- Topological Field Theory: BF action (Horowitz 1989) B^{IJ} = two-form field

$$S[B, \omega] = \int \frac{1}{2} \epsilon_{IJKL} B^{IJ} \wedge F^{KL}(\omega) + \frac{1}{\gamma} B_{IJ} \wedge F^{IJ}(\omega) \Rightarrow F^{IJ}(\omega) = 0$$

- * Gravity as a Topological Theory with constrained B -field: unfreezing $F^{IJ}(\omega)$ (Plebanski 1977)

Constraint imposed on

$$B^{IJ} = \frac{1}{16\pi G} e^I \wedge e^J$$



- everywhere on M

General Relativity

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- everywhere on M \rightarrow General Relativity
- on a 2d-foam in M \rightarrow Spin Foam action

$$B_{IJ} t^J = 0$$

2d-foam allows to unfreeze a finite number of gravitational degrees of freedom:

- quantization straightforward $\hat{K}_i \simeq \gamma \hat{L}_i$

- perspective: General Relativity as Effective description

The microscopic degrees of freedom

The 2d-foam unfreezes a finite number of local gravitational degrees of freedom:

$F^{IJ}(\omega) = 0$ everywhere, except on the 2d-foam

Finite number of gravitational d.o.f., completely captured by Wilson loops

Diff-invariant truncation of General Relativity. Full quantum theory, completion à la Fock.



The microscopic degrees of freedom

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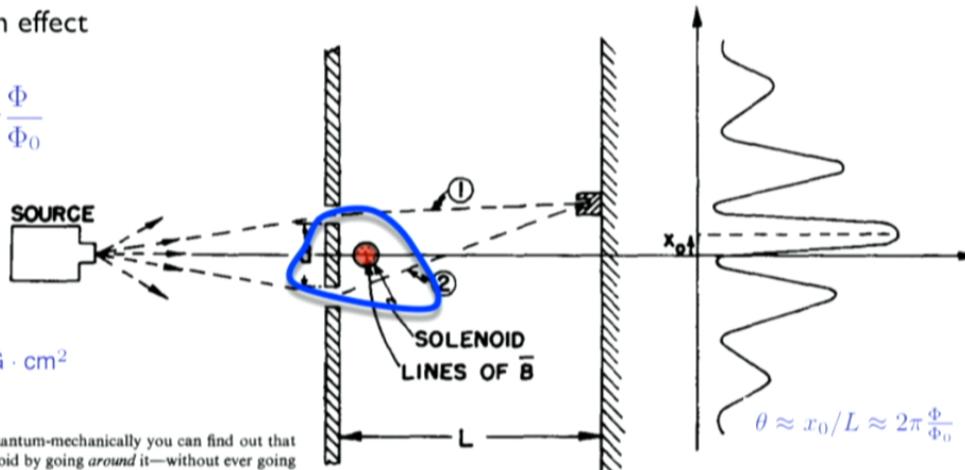
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cf. Aharonov-Bohm effect

$$e \frac{i}{\hbar} e \int_{\gamma} A dx = e^{i 2\pi \frac{\Phi}{\Phi_0}}$$



$\Phi = \text{flux of } B$
 $\Phi_0 = \frac{2\pi\hbar}{e} \approx 4 \times 10^{-7} \text{ G} \cdot \text{cm}^2$

From Feynman Lectures: But quantum-mechanically you can find out that

there is a magnetic field inside the solenoid by going *around* it—without ever going close to it!

The microscopic degrees of freedom

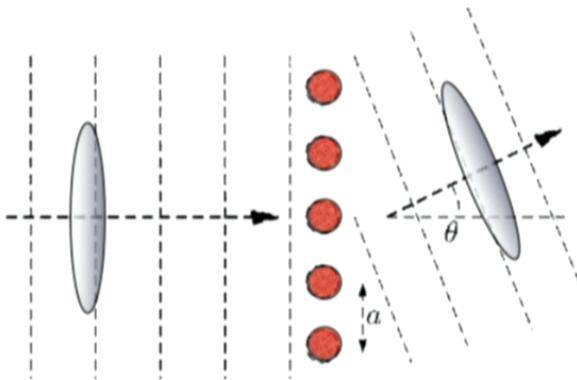
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Can we tell the difference from a smooth geometry?



A beam of particles passes through a grid of tubes, with a flux through each tube.

From X.G.Wen's book: **Problem 7.1.1. Deflecting without touching**

Plan of the talk

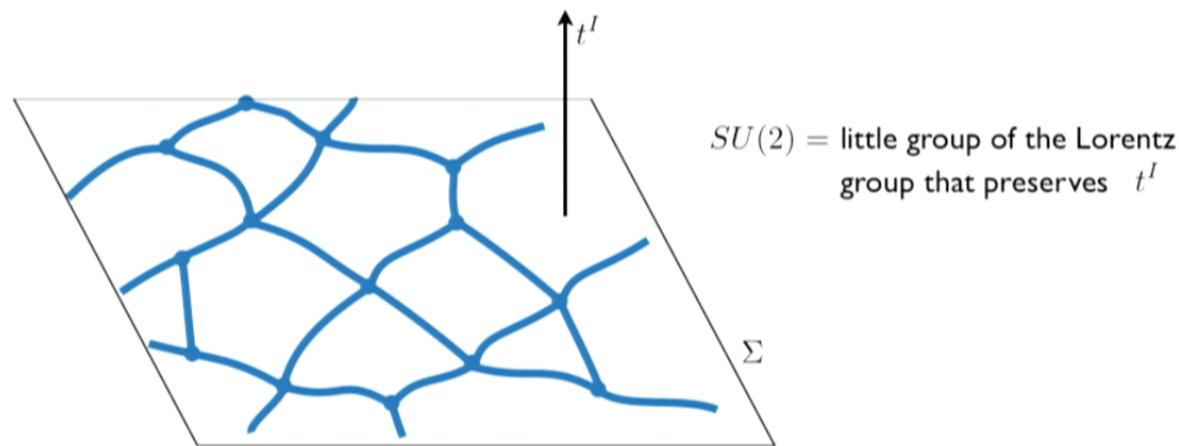
- i. Basics of loop quantum gravity
- ii. The quantum Rindler horizon
- iii. Energy of the quantum horizon
- iv. Temperature of the quantum horizon
- v. Entropy of the quantum horizon

i. Loop quantum gravity: states and dynamics

* $SU(2)$ spin-network states = quantum space

$$\Downarrow Y_\gamma$$

* $SL(2, \mathbb{C})$ spin-networks + local Lorentz invariance



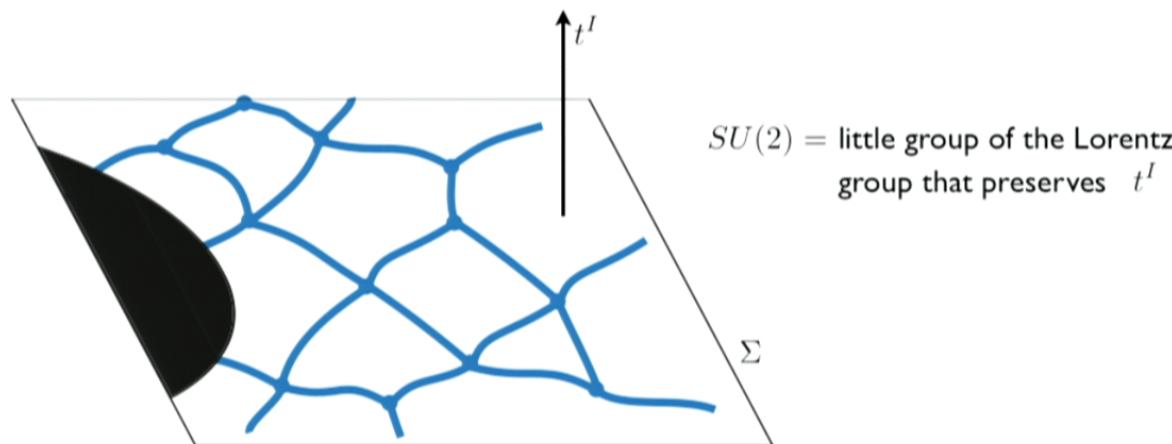
$SU(2)$ = little group of the Lorentz group that preserves t^I

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$SU(2) =$ little group of the Lorentz group that preserves t^I

Unitary Irreducible Reps of the Lorentz group and the map Y_γ

$SL(2, \mathbb{C})$ Lorentz group

⇒ quantum space-time

$SU(2)$ little group that preserves the vector t^I

⇒ quantum space



- $\mathcal{V}^{(p,k)}$ = U.I.R. of $SL(2, \mathbb{C})$; $p \in \mathbb{R}$, $k \in \mathbb{N}/2$. (principal series)

$$C_1 = \vec{K}^2 - \vec{L}^2 = p^2 - k^2 + 1$$

$$\vec{L}^2 \quad L_z$$

 $\downarrow \quad \downarrow$

$$C_2 = \vec{K} \cdot \vec{L} = p k$$

basis: $| (p, k); j, m \rangle$

Decomposition in U.I.R. of $SU(2)$: $\mathcal{V}^{(p,k)} = \bigoplus_{j=k}^{\infty} V^{(j)} = V^{(k)} \oplus V^{(k+1)} \oplus \dots$

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- γ -simple representations and the injection map Y_γ , $\gamma \in \mathbb{R}$

$$Y_\gamma : V^{(j)} \rightarrow \mathcal{V}^{(\gamma(j+1), j)}$$

$$| j, m \rangle \mapsto | (\gamma(j+1), j); j, m \rangle$$

$$p = \gamma(j+1)$$

$$k = j$$

$$\vec{K} \simeq \gamma \vec{L}$$

as matrix elements
on $\text{Im}Y_\gamma$

*

Unitary Irreducible Reps of the Lorentz group and the map Y_γ

$SL(2, \mathbb{C})$ Lorentz group

⇒ quantum space-time

$SU(2)$ little group that preserves the vector t^I

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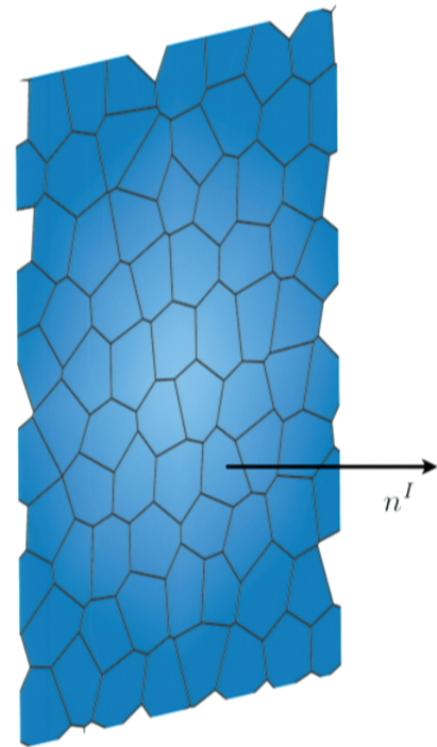
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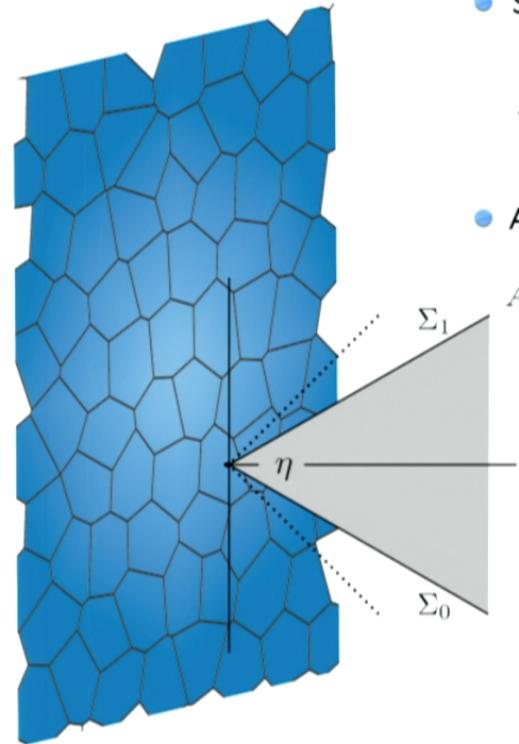
ii. The quantum Rindler horizon



- **States:** $|s\rangle = \bigotimes_f |j_f\rangle$ in $\bigotimes_f \mathcal{V}^{(\gamma(j_f+1), j_f)}$
 $j = 0, \frac{1}{2}, 1, \frac{3}{2}, \dots$ $|j\rangle \equiv |(\gamma(j+1), j); j, +j\rangle$
- **Area:** $A = 8\pi G\hbar\gamma \sum_f |\vec{L}_f|$
 $A = \sum_f A_f$, area density (per facet) $A_f = 8\pi G\hbar\gamma j_f$
- **Hamiltonian: boost operator**
$$H = \sum_f \hbar K_f^z a$$

$$K_z = \vec{K} \cdot \vec{n}$$
$$a > 0$$

ii. The quantum Rindler horizon



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- Area: $A = 8\pi G \hbar \gamma \sum_f |\vec{L}_f|$

$$A = \sum_f A_f, \text{area density (per facet)} \quad A_f = 8\pi G \hbar \gamma j_f$$

- Hamiltonian: boost operator

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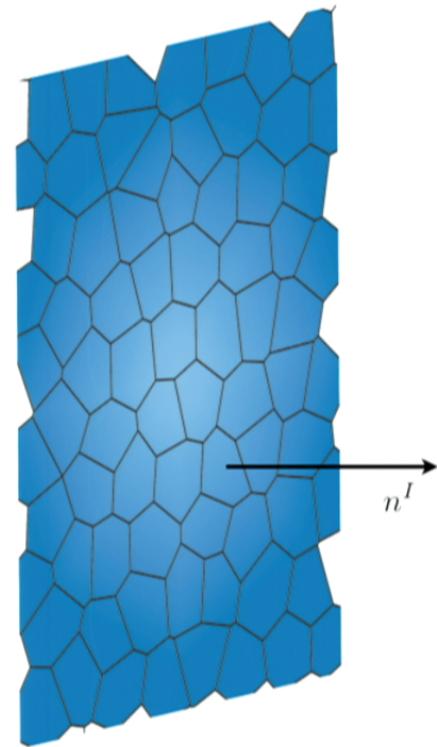
$$a > 0$$

- Quantum Rindler horizon

$$|s_t\rangle = U(t) |s\rangle$$

$$U(t) = e^{\frac{i}{\hbar} H t} = \bigotimes_f \exp(i K_f^z a t)$$

ii. The quantum Rindler horizon



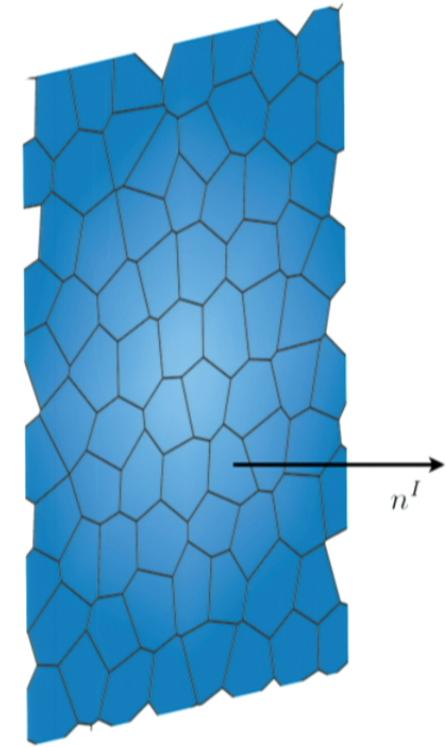
- Expectation value of $H = \sum_f \hbar K_f^z a$

$$E \equiv \langle s | H | s \rangle = \sum_f \hbar \gamma j_f a = \frac{\sum_f A_f}{8\pi G} a$$

use $\langle (\gamma(j+1), j); j, m' | K_z | (\gamma(j+1), j); j, m \rangle = \gamma m \delta_{mm'}$

and $A_f = 8\pi G \hbar \gamma j_f$

iii. Energy of the quantum horizon



- Expectation value of $H = \sum_f \hbar K_f^z a$

$$E \equiv \langle s | H | s \rangle = \sum_f \hbar \gamma j_f a = \frac{\sum_f A_f}{8\pi G} a$$

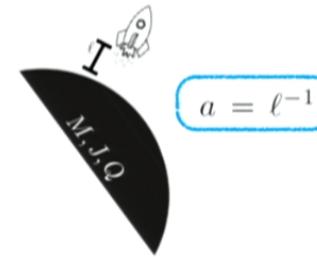
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and $A_f = 8\pi G \hbar \gamma j_f$

- Reproduces the near-horizon energy of FGP



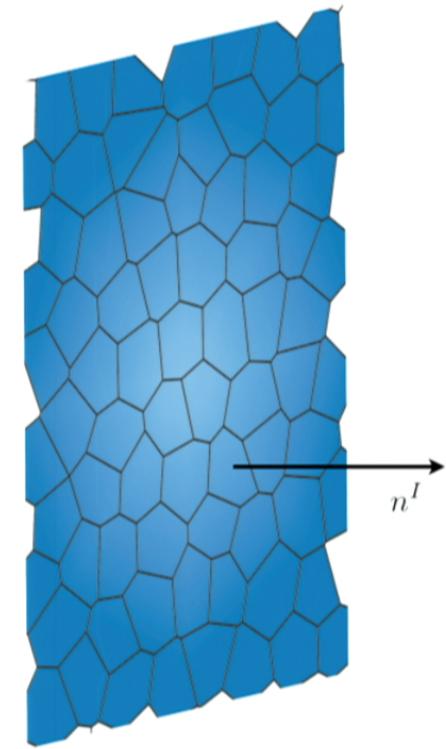
$$E = \frac{A}{8\pi G} a$$



E. Bianchi

black hole entropy from loop quantum gravity

iii. Energy of the quantum horizon



- Expectation value of $H = \sum_f \hbar K_f^z a$

$$E \equiv \langle s | H | s \rangle = \sum_f \hbar \gamma j_f a = \frac{\sum_f A_f}{8\pi G} a$$

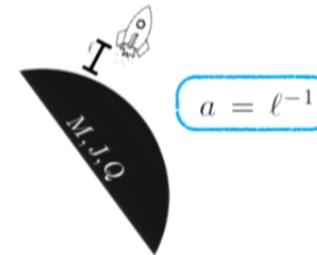
use $\langle (\gamma(j+1), j); j, m' | K_z | (\gamma(j+1), j); j, m \rangle = \gamma m \delta_{mm'}$

and $A_f = 8\pi G \hbar \gamma j_f$

- Reproduces the near-horizon energy of FGP



$$E = \frac{A}{8\pi G} a$$



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iv. Temperature of the quantum horizon

- 2-level system $|0\rangle, |1\rangle$ [Unruh '76]

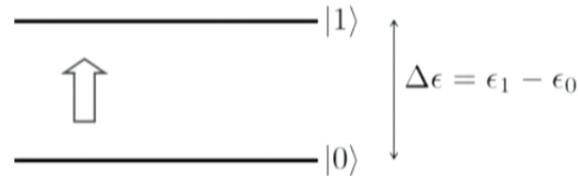
$$H_D = \epsilon_0|0\rangle\langle 0| + \epsilon_1|1\rangle\langle 1|$$

- Interaction with the quantum horizon

$$V = g (|0\rangle\langle 1| + |1\rangle\langle 0|) \psi \phi$$

$$\psi = |E_0\rangle\langle E_1| + |E_1\rangle\langle E_0|$$

$$\phi |\Omega\rangle = |j\rangle$$



$$\hbar a \ll \Delta\epsilon \ll E_1 - E_0$$

induces excitation of a facet

from the vacuum state $|\Omega\rangle = U(G)|\Omega\rangle$

to the state of definite area $|j\rangle$

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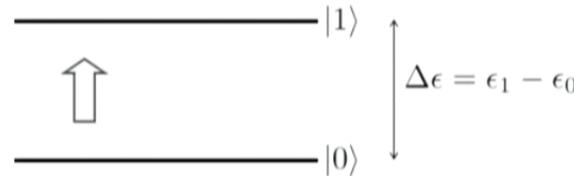
$$\phi |\Omega\rangle = |j\rangle$$

- Transition rates $\Gamma_{\pm} = \frac{g^2}{\hbar^2} \int_{-\infty}^{+\infty} d\tau e^{-\frac{\hbar}{\hbar}(E_1 - E_0 \mp \Delta\epsilon)\tau} \langle \Omega | \phi e^{iK_z a\tau} \phi | \Omega \rangle$

- Population at equilibrium

$$\dot{p}_1 = p_0\Gamma_+ - p_1\Gamma_- = 0$$

\Rightarrow Boltzmann distribution with temperature
 coincides with Unruh temperature
 measured by an accelerated observer



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$$T = \frac{\hbar a}{2\pi}$$

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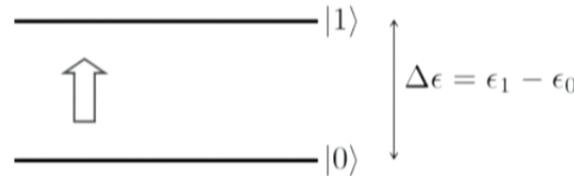
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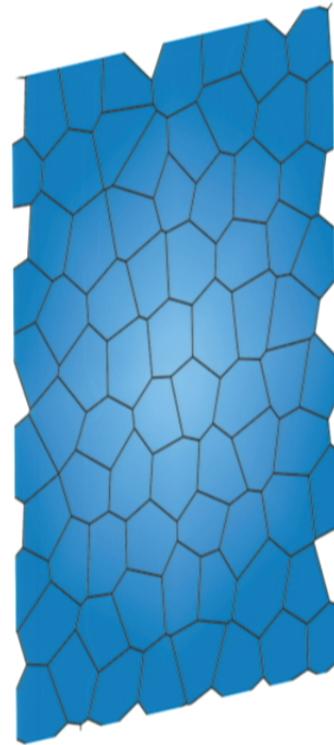


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v. Entropy of the quantum horizon

- Use Clausius relation to compute the entropy

$$\delta S = \frac{\delta E}{T}$$



- Process: new facet of spin j_f created $|\Omega\rangle \rightarrow |j_f\rangle$

energy $\delta E_f = \hbar\gamma j_f a$

temperature $T = \frac{\hbar a}{2\pi}$

entropy $\delta S_f = \frac{\delta E_f}{T} = \frac{\hbar\gamma j_f a}{\hbar a/2\pi} = 2\pi\gamma j_f$

Entropy of the quantum horizon

$$S = \sum_f 2\pi\gamma j_f$$

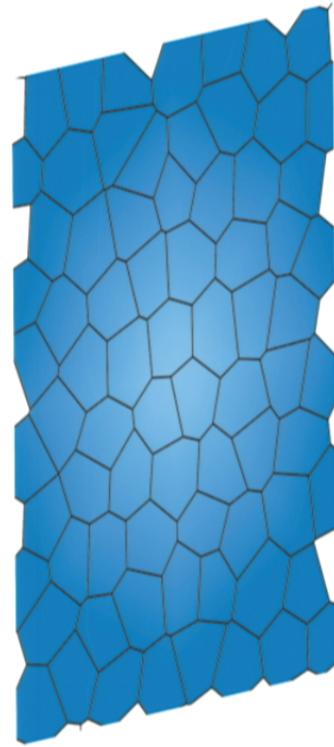
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Entropy of the quantum horizon = Bekenstein-Hawking entropy

$$S = \sum_f 2\pi\gamma j_f = 2\pi \frac{\sum_f 8\pi G\hbar\gamma j_f}{8\pi G\hbar} = 2\pi \frac{\sum_f A_f}{8\pi G\hbar} = \frac{A}{4G\hbar}$$

- Entropy density per facet

$$s_f = 2\pi\gamma j_f$$

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Plan of the talk

- i. Basics of loop quantum gravity
 - ii. The quantum Rindler horizon
 - iii. Energy of the quantum horizon
 - iv. Temperature of the quantum horizon
 - v. Entropy of the quantum horizon
- 
- some more recent
developments

iii'. Energy of the horizon as the boost boundary term in general relativity

$$I_2 = - \int_{M_2} \left(\frac{1}{2} \epsilon_{IJKL} B^{KL} + \frac{1}{\gamma} B_{IJ} \right) n^I z^J \eta = - H \eta$$

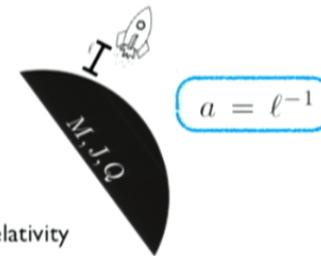
$$H = \int_{M_2} K_I z^I$$

- Carlip-Teitelboim: horizon Schrödinger eq.

$$i\hbar \frac{\partial}{\partial \eta} \psi = \hat{H} \psi$$

- Reproduces the near-horizon energy of FGP

$$E = \frac{A}{8\pi G} a$$



$$a = \ell^{-1}$$

[Carlip & Teitelboim gr-qc/9312002](#)

[Massar & Parentani gr-qc/9903027](#)

[Jacobson & Parentani gr-qc/0302099](#)

Bianchi & Wieland [1205.5325](#) Horizon energy as the boost boundary term in general relativity

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Padmanabhan [1205.5683](#) Noether energy and boundary term in gravitational action

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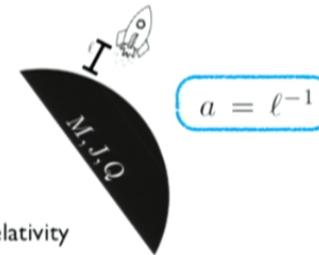
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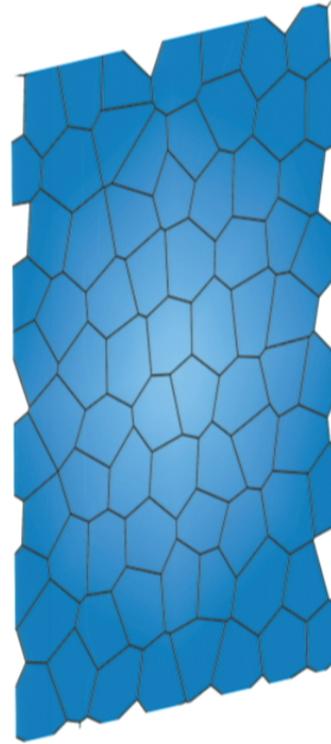
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v'. Horizon density matrix and the Clausius relation



density matrix $\rho_0 = \frac{1}{Z} \prod_{f=1}^N e^{-\beta_0 K_z^f}$

stationary $[\rho_0, H] = 0$ $\beta_0 = 2\pi$

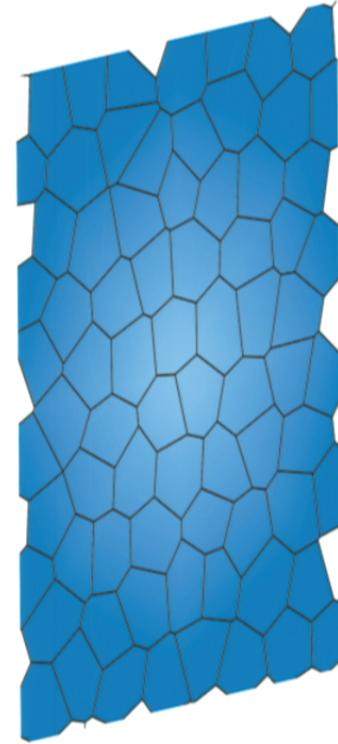
quasi-stationary process

$$|\Omega\rangle \rightarrow |j_f\rangle$$

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quasi-stationary process $\rho_* = \frac{1}{Z} \prod_{f=1}^{N-1} e^{-\beta_0 K_z^f} \otimes |j_f\rangle\langle j_f|$
 $|\Omega\rangle \rightarrow |j_f\rangle$

$$\Rightarrow \delta\rho = |j\rangle\langle j| - |\Omega\rangle\langle\Omega|$$

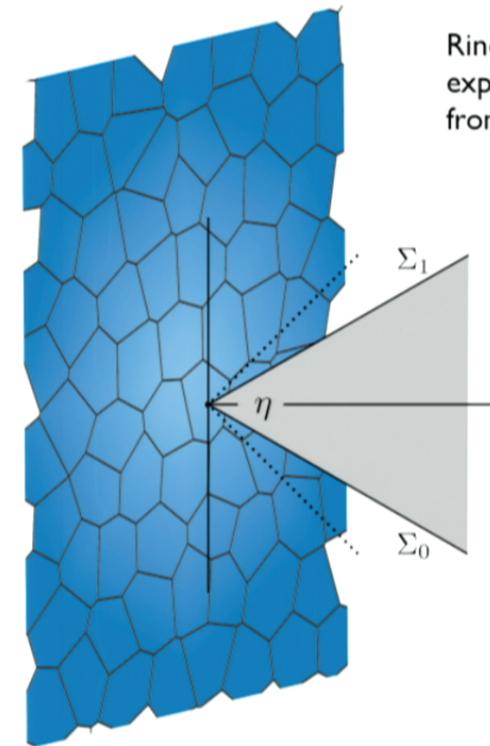
energy variation in the process

$$\begin{aligned} \delta E &= \delta \text{Tr}(\rho H) = \text{Tr}(\delta\rho H) + \text{Tr}(\rho_0 \delta H) \\ &= \delta Q + \cancel{\delta W} \\ &= \langle j | H | j \rangle - \langle \Omega | H | \Omega \rangle = \hbar\gamma j \end{aligned}$$

entropy variation in the process

$$\begin{aligned} \delta S &= -\delta(\text{Tr}\rho \log \rho) = -\text{Tr}(\delta\rho \log \rho_0) = \frac{\delta Q}{T} \\ &= \beta_0 \text{Tr}(K_z \delta\rho) = 2\pi \langle j | K_z | j \rangle = 2\pi\gamma j = \frac{\delta A}{4G\hbar} \quad \blacksquare \end{aligned}$$

Summary: horizon entropy from loop quantum gravity



Rindler near-horizon geometry of non-extremal black holes
exploited to derive the Bekenstein-Hawking entropy
from loop quantum gravity.

- ii. The quantum Rindler horizon
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- iv. Temperature $T = \frac{\hbar a}{2\pi}$
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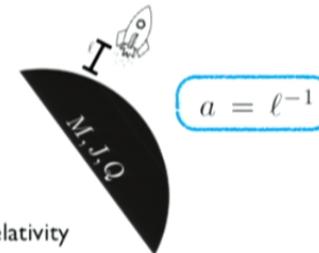
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