Title: Emergent Gauge Fields and their Condensation in Quantum Spin Liquids

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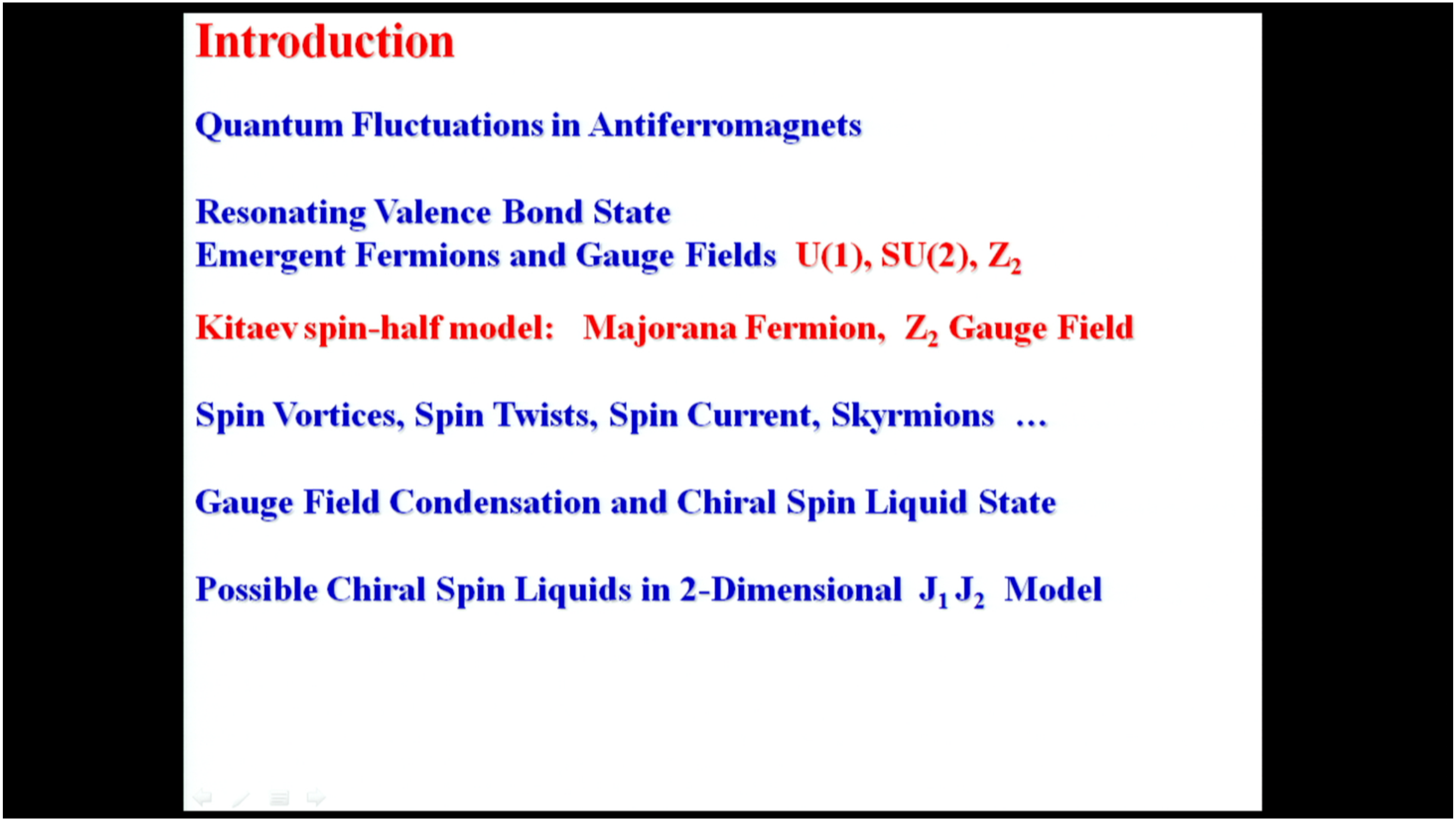
Abstract: Large quantum fluctuations in certain quantum spin systems destroy long range magnetic order such as antiferromagnetism. Resulting paramagnetic states are called a quantum spin liquids. These states support emergent gauge fields [1]. Under certain conditions, emergent gauge fields condense in the ground state, leading to a chiral spin liquid state [2]. A condensed `magnetic field' for example, correspond to presence of spontaneous circulating spin current or spin `chirality'[3]. In the light of recent search for spin liquids in frustrated quantum antiferromagnets, we revive our earlier approach and show that a variety of chiral spin liquid states are possible in 2D lattices.   [1] G. Baskaran and P.W. Anderson, Phys. Rev., B 37, 580 (1988) [2] V. Kalmeyer and R. B. Laughlin, Phys. Rev. Lett., 59, 2095 (1987) [3] X.-G. Wen, F. Wilczek and A. Zee, Phys. Rev., B 39, 11413 (1989) [4] G Baskaran, Phys. Rev. Lett., 63, 2524 (1989)

P W Anderson

2011


Introduction
Quantum Fluctuations in Antiferromagnets
Resonating Valence Bond State
Emergent Fermions and Gauge Fields U(1), SU(2), Z2
Kitaev spin-half model: Majorana Fermion, Z2 Gauge Field
Spin Vortices, Spin Twists, Spin Current, Skyrmions …
Gauge Field Condensation and Chiral Spin Liquid State
Possible Chiral Spin Liquids in 2-Dimensional J1 J2 Model

$$\sum J_{ij} \vec{S}_i \cdot \vec{S}_j \longleftrightarrow \sum \bar{\psi}_i U_{ij} \psi_j + H_G[U_{ij}]$$

emergent gauge field (Link) ← U_{ij}

emergent fermions (Lattice) ← $\bar{\psi}_i$

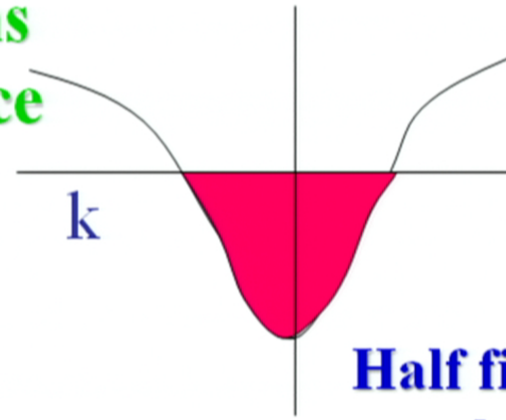
$$\vec{S} = \frac{1}{2} \vec{\sigma}_i$$

↑ Pauli Spin operators

Mott Insulators

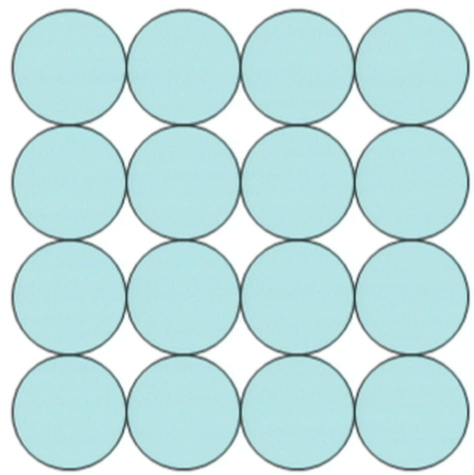
are seats of
Quantum Spin Liquids

A collection of hydrogen atoms forming a hypothetical 3D lattice



Half filled band

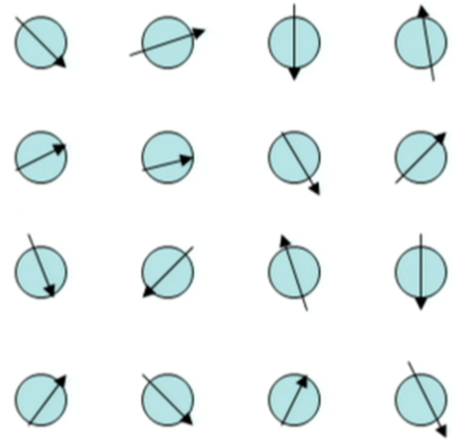
1s atomic orbitals
of individual H-atoms strongly overlap and form a tight binding half filled band



It is a metal

a, lattice constant ~ a_B Bohr radius

Let us expand the lattice



$a >> a_B$

For $a >> a_B$

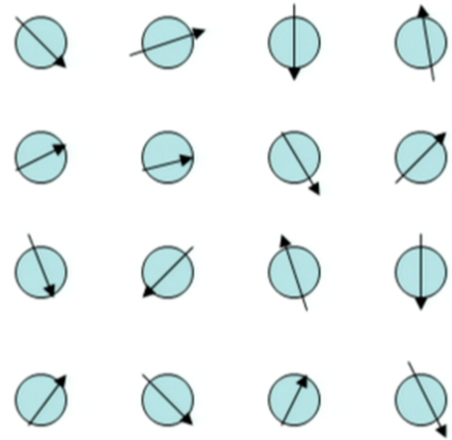
we get a Mott insulator

upper Hubbard band

lower Hubbard band

Spins are soft degrees of freedom while charges are frozen

Let us expand the lattice



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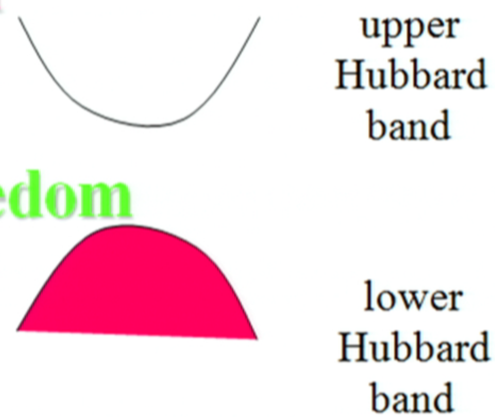
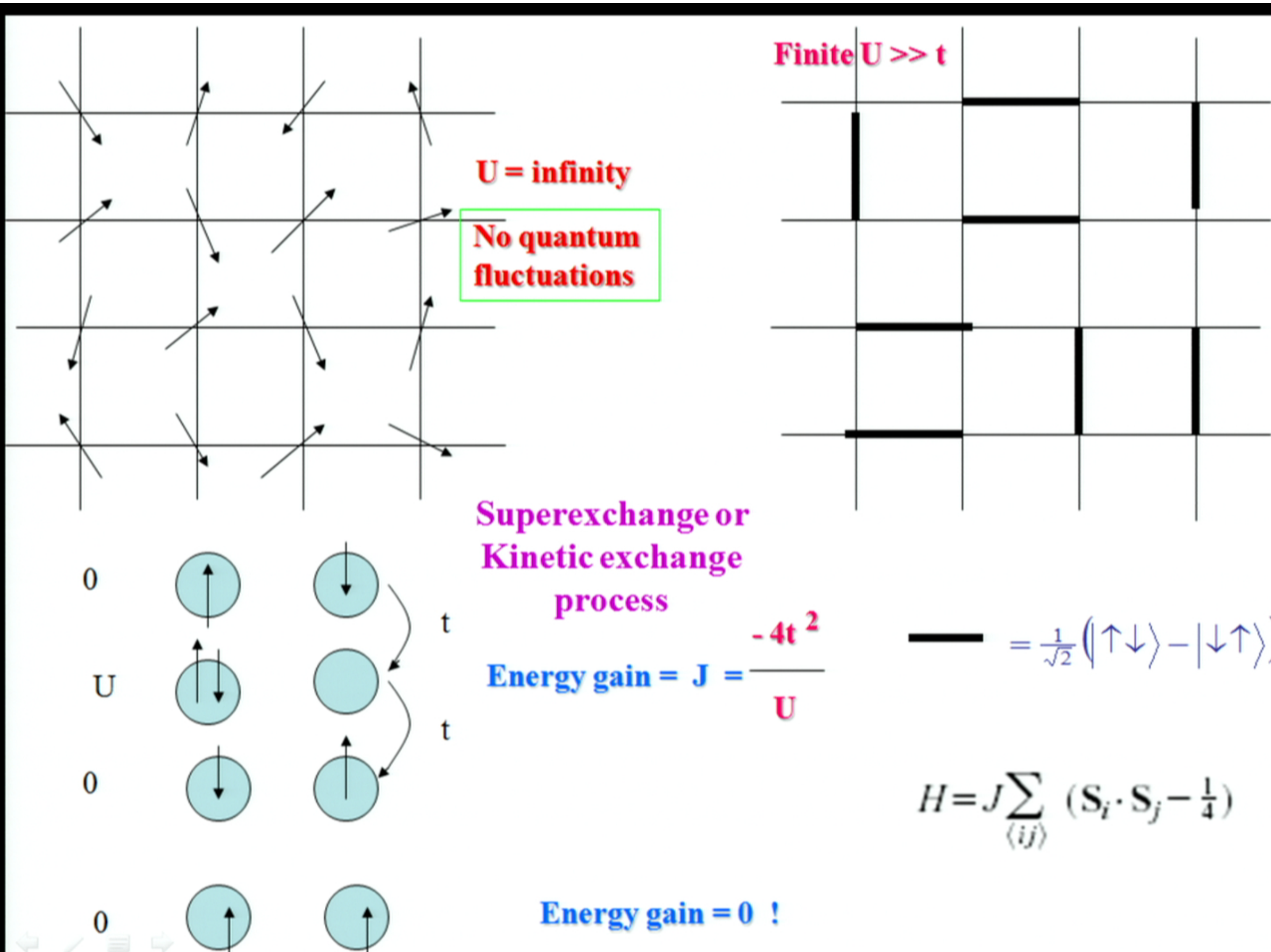
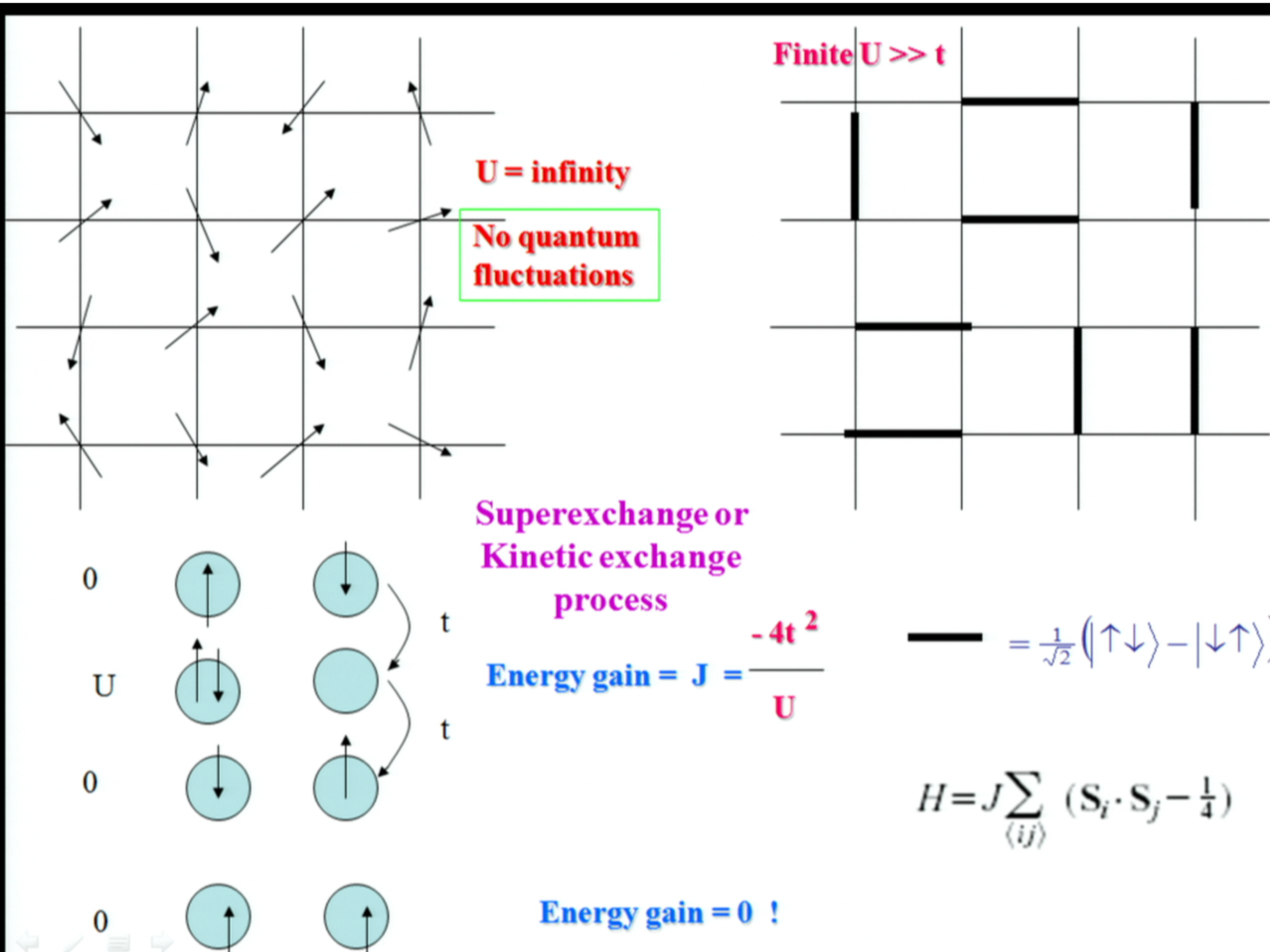



Finite U >> t
U = infinity
No quantum fluctuations
Superexchange or Kinetic exchange process
0
U
0
0
t
t
Energy gain = J = -4t 2 / U
= 1/√2 (|↑↓⟩ − |↓↑⟩)
H = J Σ⟨ij⟩ (Si · Sj − 1/4)
Energy gain = 0 !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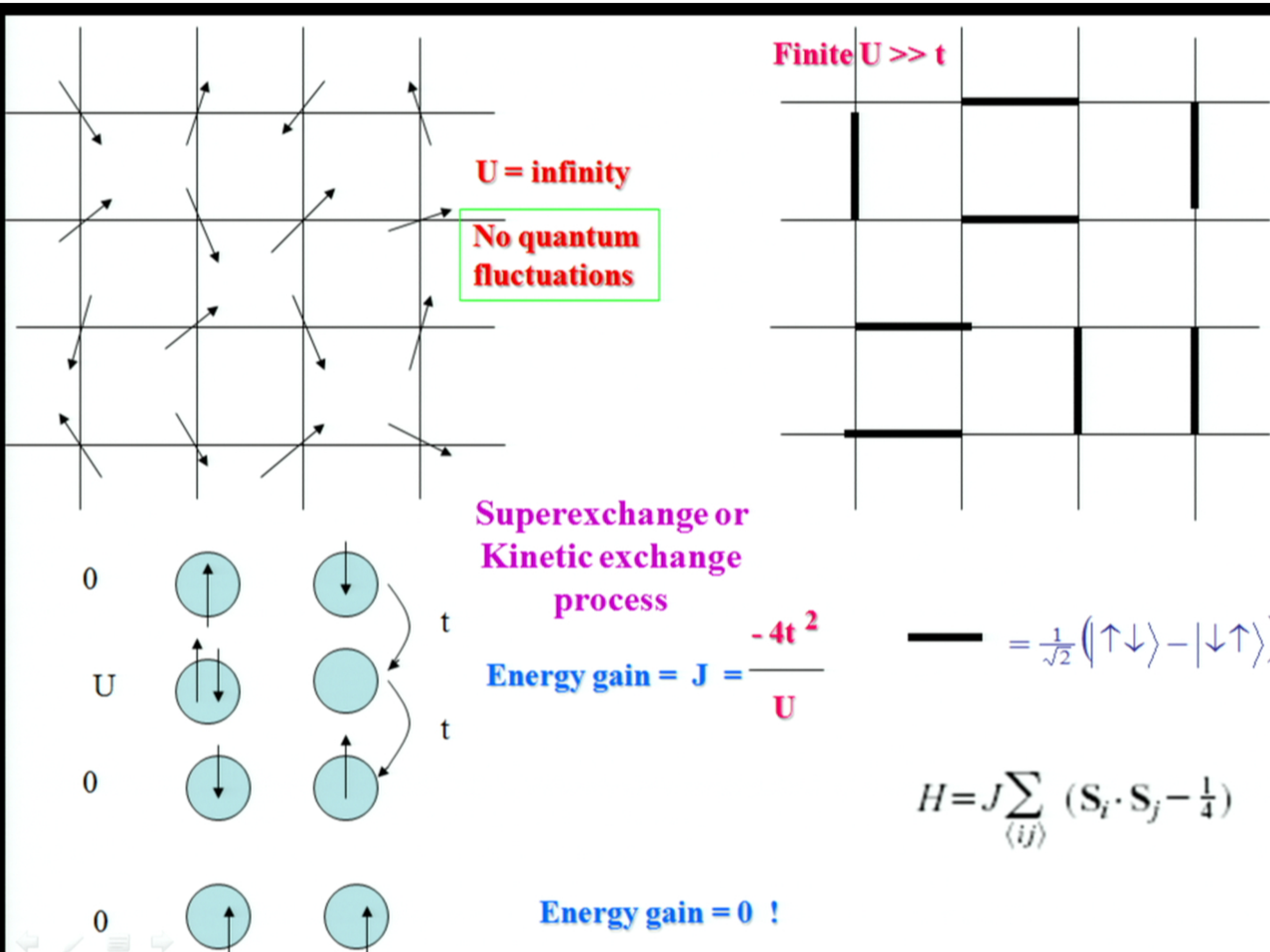

Finite U >> t
U = infinity
No quantum fluctuations
Superexchange or Kinetic exchange process
0
U
0
0
t
t
Energy gain = J = $\frac{-4t^2}{U}$
$= \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$
$H = J\sum_{\langle ij\rangle} (\mathbf{S}_i \cdot \mathbf{S}_j - \frac{1}{4})$
Energy gain = 0 !

Quantum Fluctuations in spin systems

Heisenberg Interaction among localized spins

$H = J\ \Sigma\, S_i \,.\, S_j$ Global SU(2), Spin Rotation Symmetry

$J < 0$ (Ferromagnetic interaction)

$|\uparrow\uparrow\uparrow\uparrow\ldots\rangle$ eigen state

Absence of zero point fluctuation

Quadratic spin wave (Boson) spectrum

$$|q\rangle = \sum_i e^{i\vec{q}\cdot\vec{R}_i} S_i^- |\uparrow\uparrow\ldots\uparrow\rangle$$

$$\omega_q \approx a q^2$$

Zero point or Quantum Fluctuations are important in antiferromagnets

$H = J \; \Sigma \; S_i \, . \, S_j$

$J > 0$ (Antiferromagnetic)

$|\uparrow\downarrow\uparrow\downarrow \cdots \uparrow\downarrow\rangle$ not an eigen state

Two spins $|\uparrow\downarrow\rangle \Rightarrow$ not an eigen state

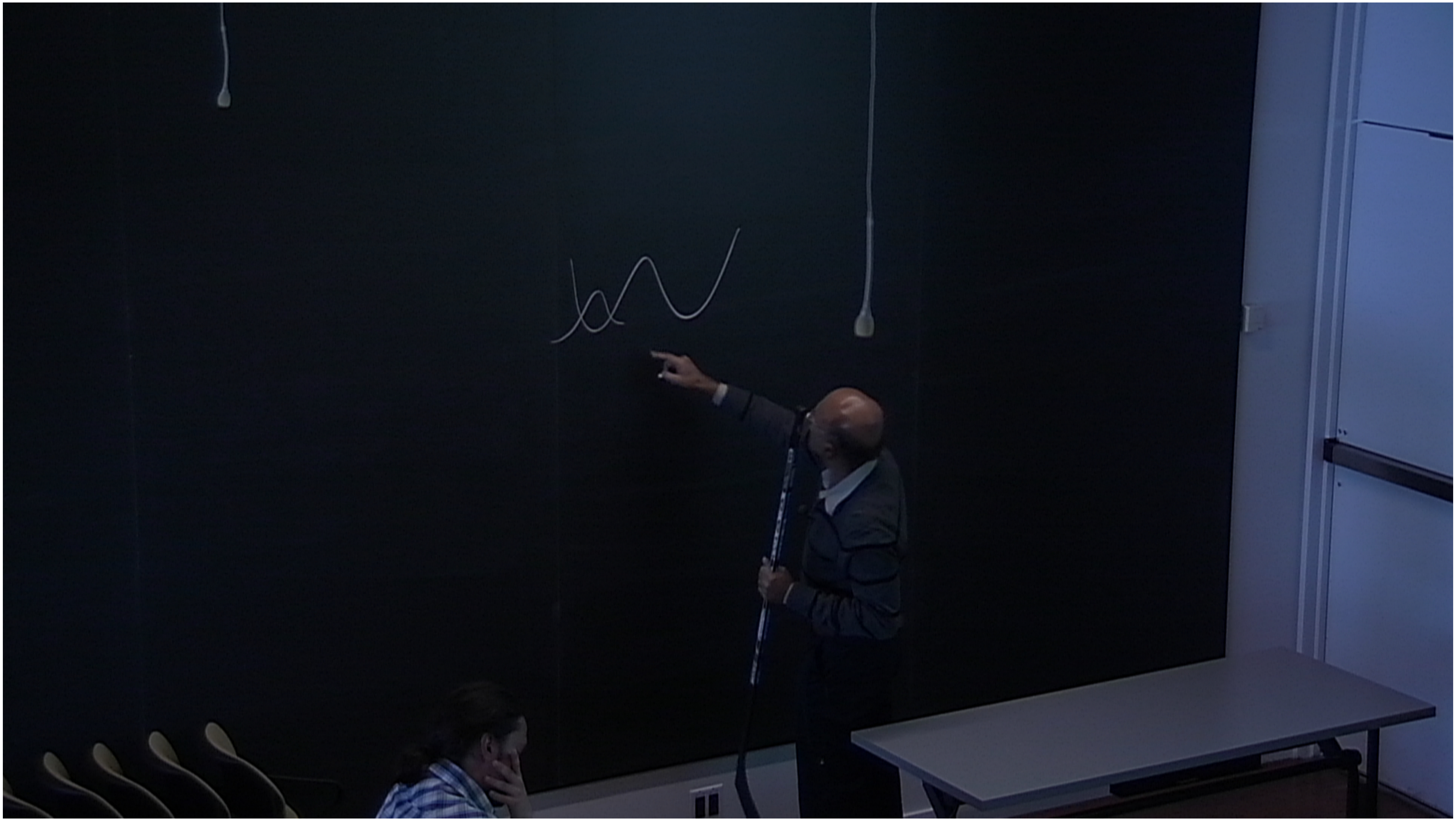
$|G\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$ ← zero point fluctuations

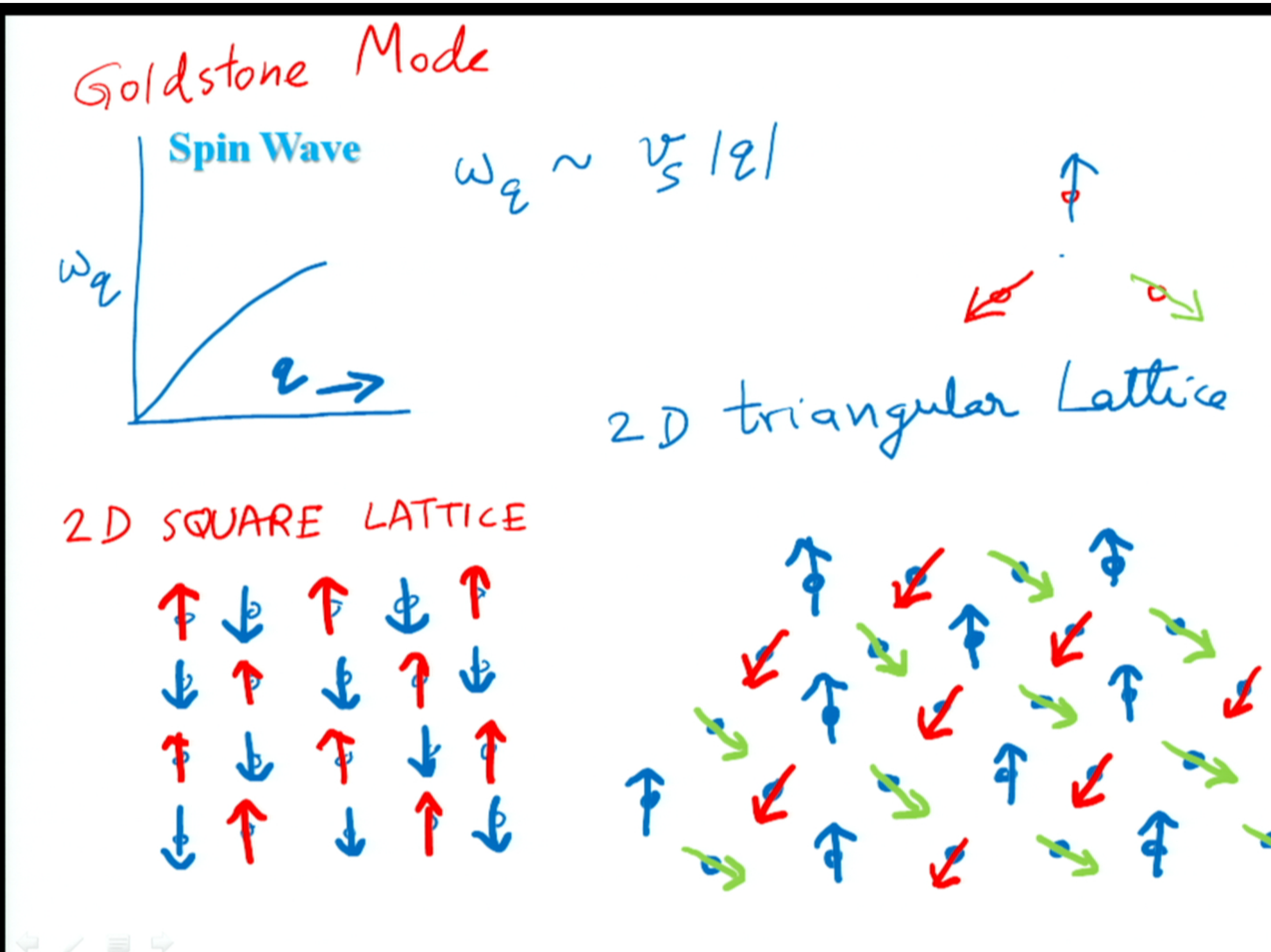
$$|G\rangle \sim e^{-\sum \varphi_{ij} S_i^+ S_j^-} |\uparrow\downarrow\uparrow \cdots \downarrow\uparrow\rangle$$

$$\langle S_i^z \rangle = \frac{1}{2} - \delta m$$

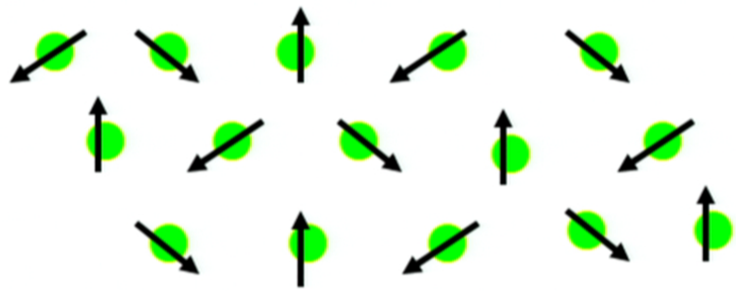
(zero point reduction → δm)

$\delta m \sim \ln L$ in 1-d



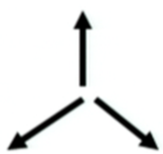

Goldstone Mode
Spin Wave
$\omega_q \sim v_s |q|$
ω_q
$q \rightarrow$
2D triangular Lattice
2D SQUARE LATTICE

Spins have
120 degree 3 sublattice pattern



Frustration

Triangular lattice



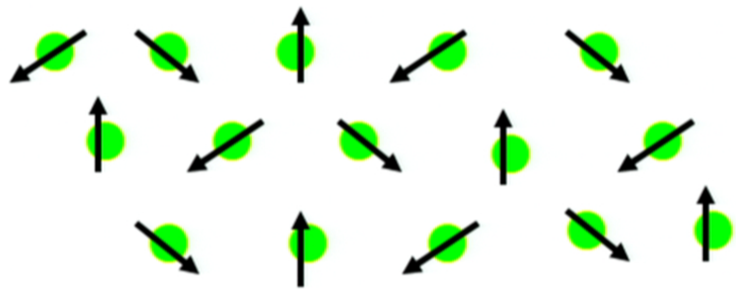
There is magnetic tension in the bonds
Not all bond energies can be
minimized simultaneously in the ground state

$$S_i . S_j = \tfrac{1}{2}(S_i^+ S_j^- + S_i^- S_j^+) + S_i^z S_j^z$$

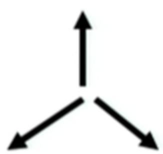
Kinetic energy potential energy

Spins have
120 degree 3 sublattice pattern

Frustration



Triangular lattice



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Not all bond energies can be minimized simultaneously in the ground state

$$\mathbf{S_i} \cdot \mathbf{S_j} = \tfrac{1}{2}(S_i^+ S_j^- + S_i^- S_j^+) + S_i^z S_j^z$$

Kinetic energy — potential energy

$$S_i . S_j = \frac{1}{2}(S_i^+ S_j^- + S_i^- S_j^+) + S_i^z S_j^z$$

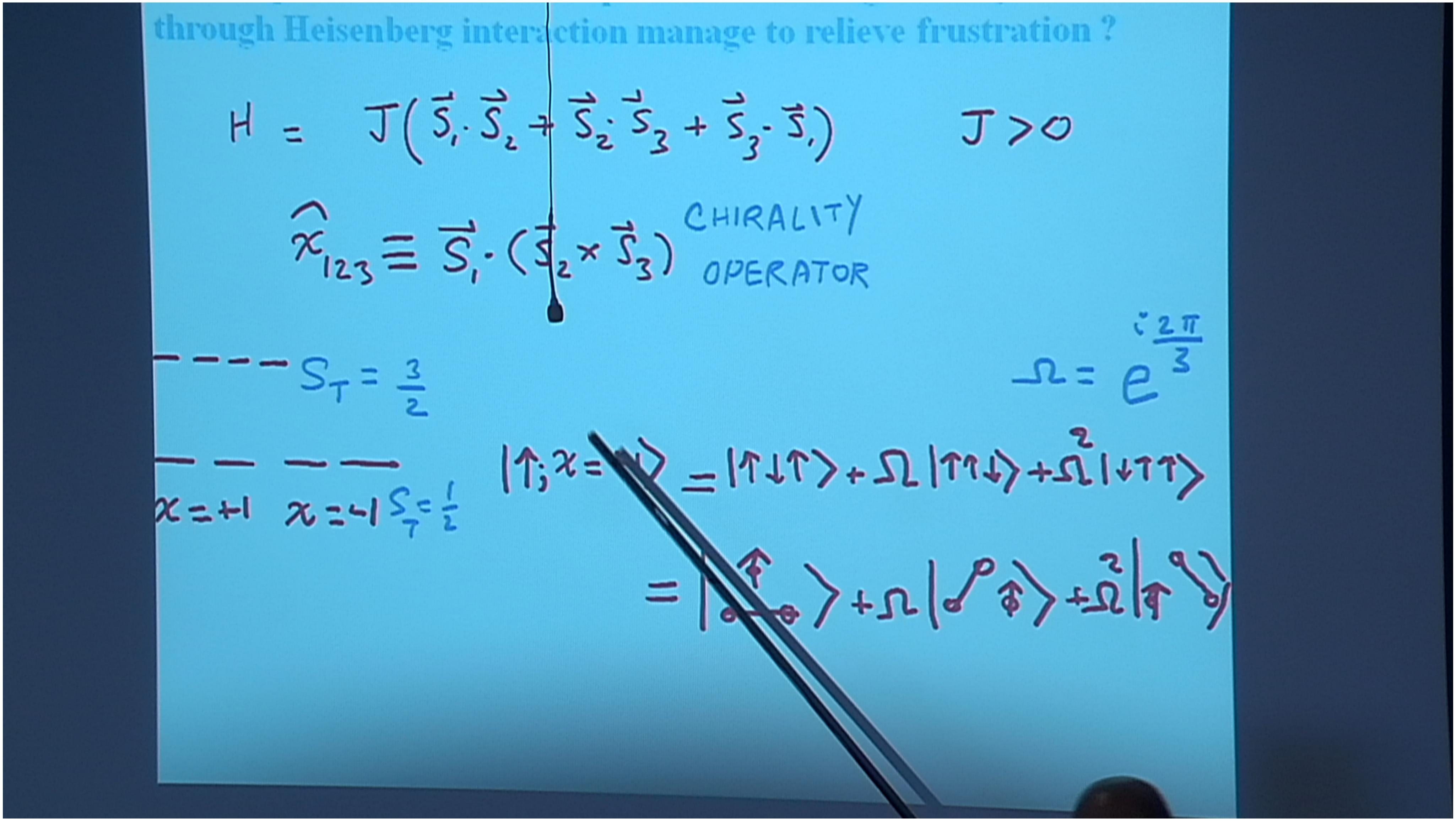
Kinetic energy potential energy

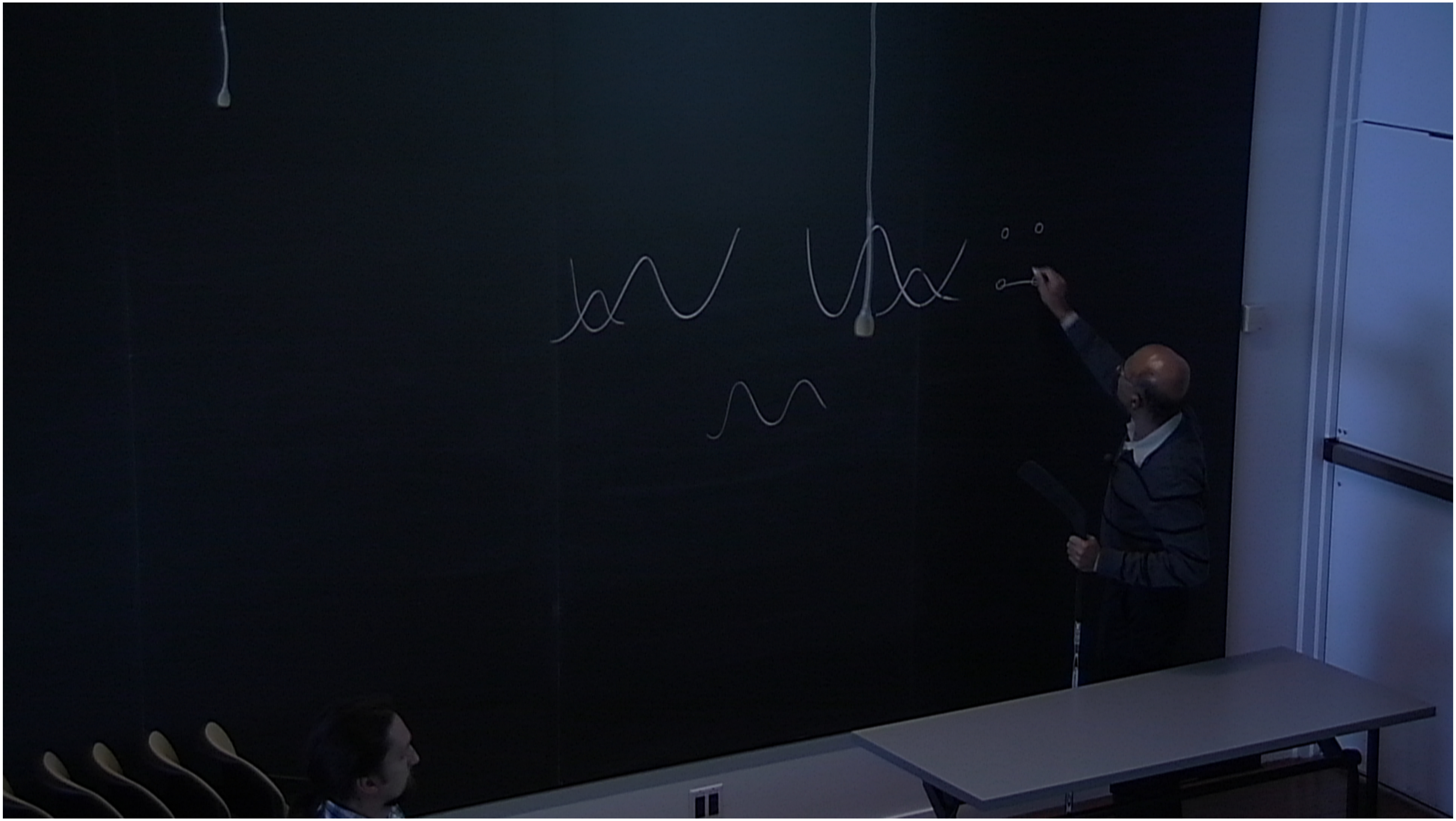
Interesting physics and phases arises because of competition between kinetic and potential energies

When kinetic energy wins we have Quantum Spin Liquids

Anisotropic interaction λ U(1) symmetry

$$\frac{1}{2}(S_i^+ S_j^- + S_i^- S_j^+) + \lambda S_i^z S_j^z$$


through Heisenberg interaction manage to relieve frustration ?
$H = J(\vec{S}_1 \cdot \vec{S}_2 + \vec{S}_2 \cdot \vec{S}_3 + \vec{S}_3 \cdot \vec{S}_1)$
$J > 0$
$\hat{\chi}_{123} \equiv \vec{S}_1 \cdot (\vec{S}_2 \times \vec{S}_3)$
CHIRALITY OPERATOR
$S_T = \frac{3}{2}$
$\Omega = e^{i\frac{2\pi}{3}}$
$\chi = +1$
$\chi = -1$
$S_T = \frac{1}{2}$
$|\uparrow; \chi = +1\rangle = |\uparrow\downarrow\uparrow\rangle + \Omega|\uparrow\uparrow\downarrow\rangle + \Omega^2|\downarrow\uparrow\uparrow\rangle$

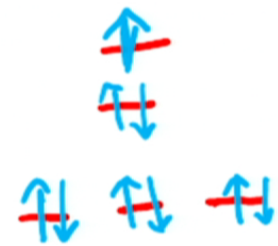


Classical	Quantum
Liquid	Superfluid liquid ^{4}He
Liquid Crystal	2D Vortex liquid ^{4}He, vortex glass
	Fermi Liquid, Nematic Liquid
	Superconductor
Ising AFM (Spin Crystal)	Solid ^{4}He, Super Solid ^{4}He spin-half Quantum Antiferromagnet
	Valence Bond Solid
Paramagnet (a Classical Spin Liquid)	Quantum Spin Liquid (Quantum Melted Spin Crystal)
Classical Spin Glass	Valence Bond Glass
	Chiral Glass

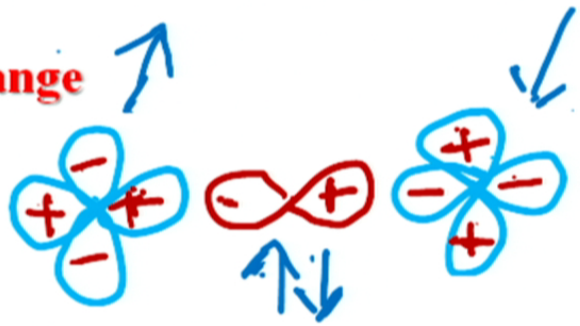
CuO - An elementary Mott Insulator

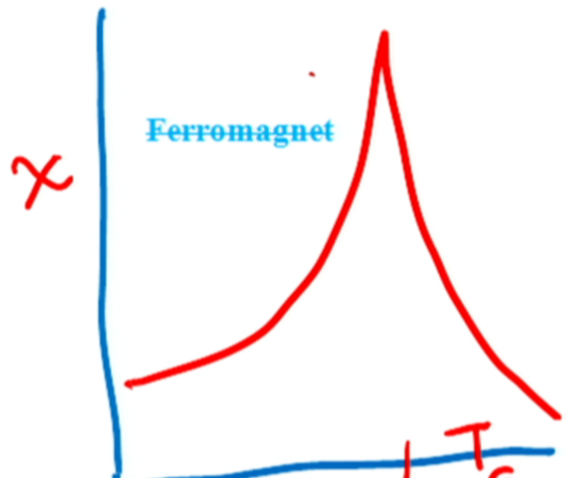
Cu: $3d^{10}\ 4s^{1}$ **$Cu^{2+}\ O^{2-}$** **Cu^{2+} : $3d^{9}$** **O^{2-} : $2s^{2}\ 3p^{6}$**

O: $2s^{2}\ 3p^{4}$



Superexchange





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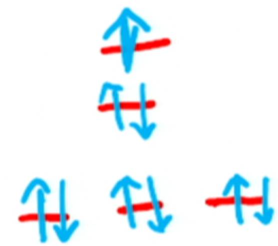

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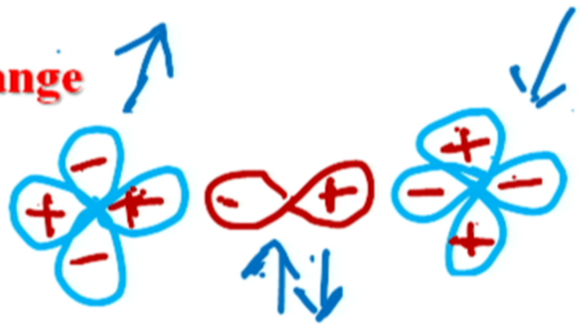
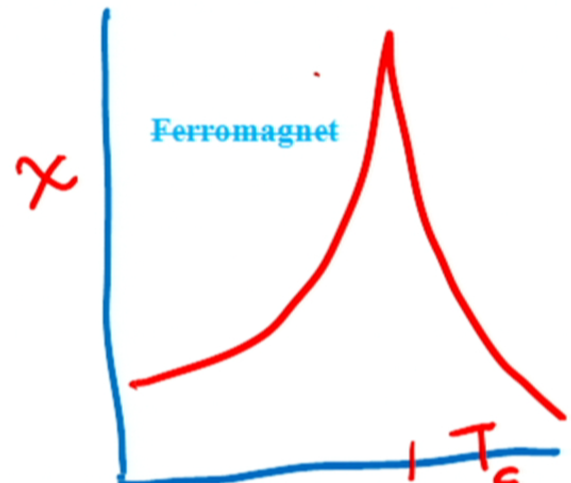
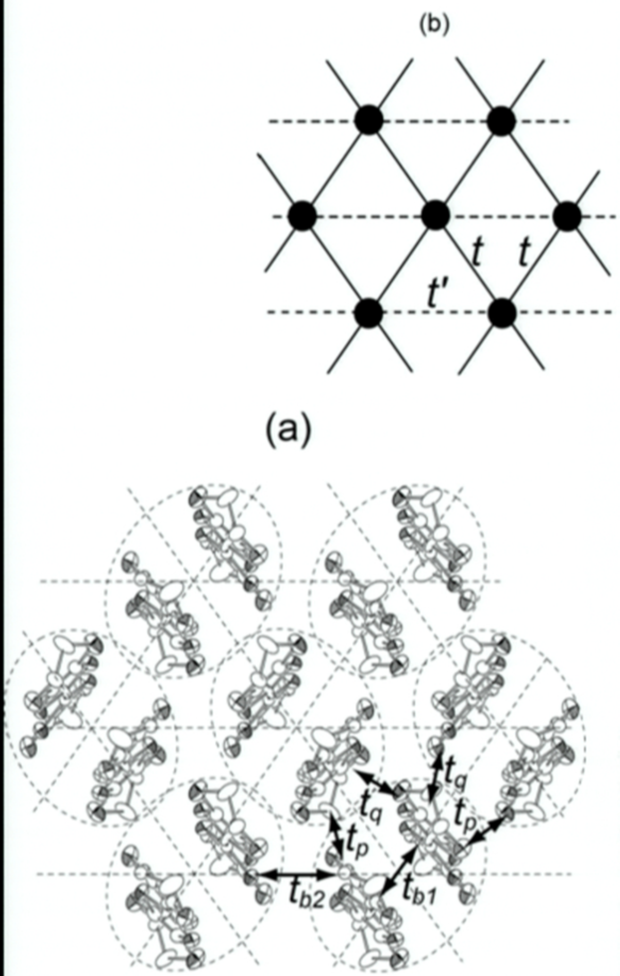






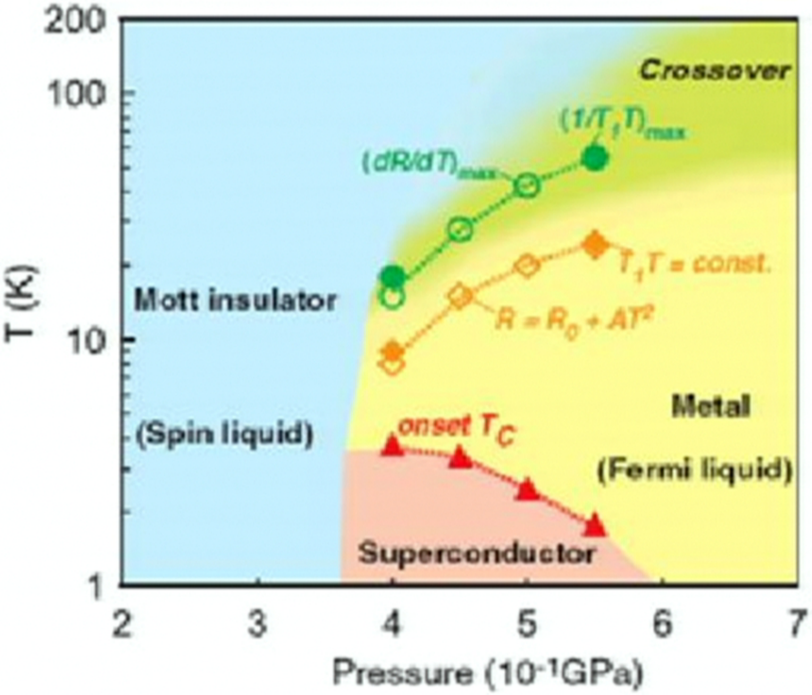
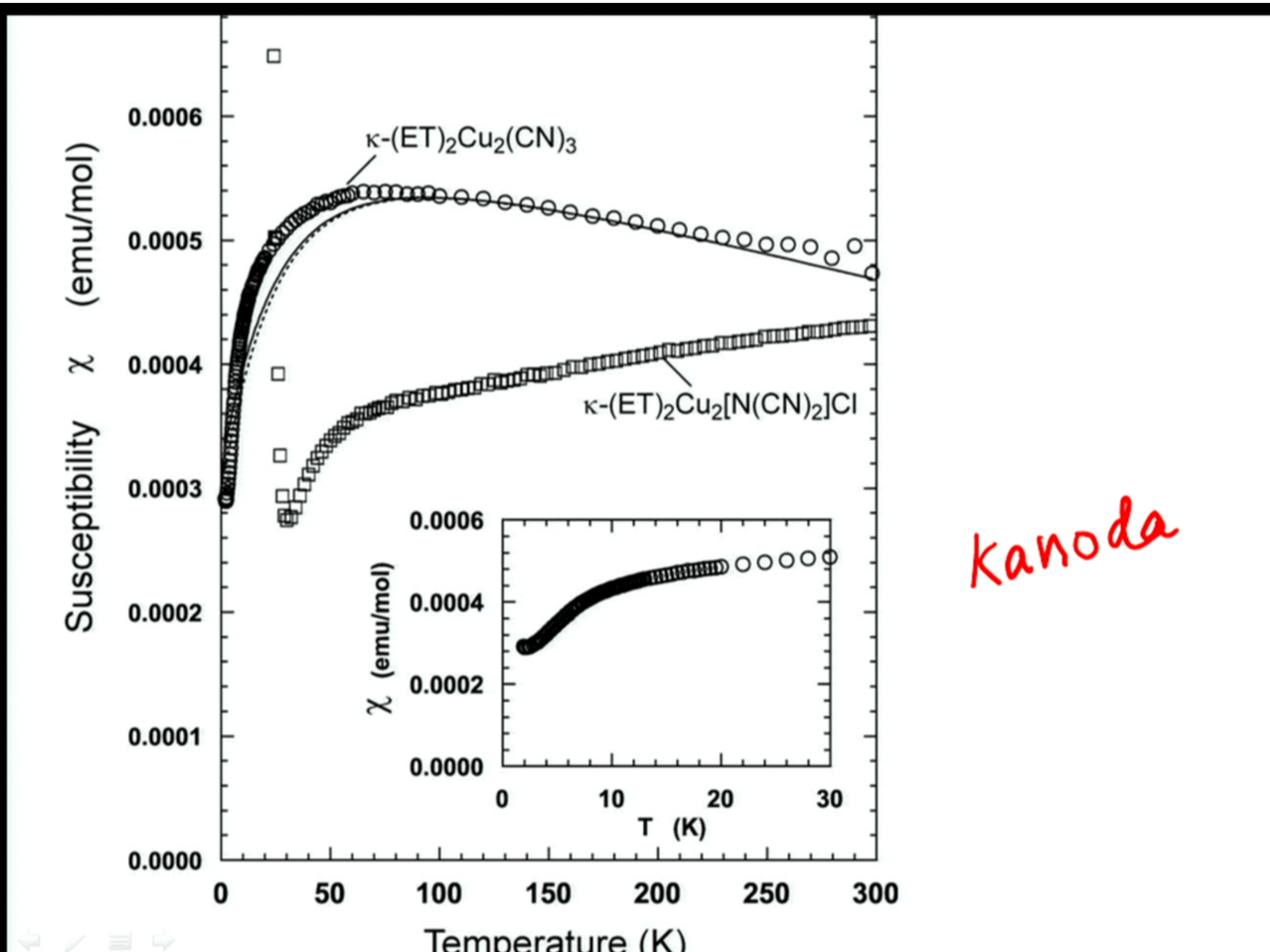


FIG. 1 (color online). The pressure-temperature phase diagram of κ-$(ET)_2Cu_2(CN)_3$, constructed on the basis of the resistance and NMR measurements under hydrostatic pressures. The Mott transition or crossover lines were identified as the temperature where $1/T_1T$ and dR/dT show the maximum as described in the text. The upper limit of the Fermi-liquid region was defined by the temperatures where $1/T_1T$ and R deviate from the Korringa's relation and $R_0 + AT^2$, respectively. The onset superconducting transition temperature was determined from the in-plane resistance measurements.

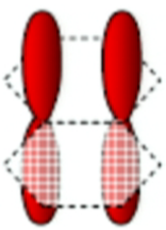
Kanoda


κ-(ET)2Cu2(CN)3
κ-(ET)2Cu2[N(CN)2]Cl
Susceptibility χ (emu/mol)
0.0006
0.0005
0.0004
0.0003
0.0002
0.0001
0.0000
0
50
100
150
200
250
300
Temperature (K)
χ (emu/mol)
T (K)
10
20
30
Kanoda

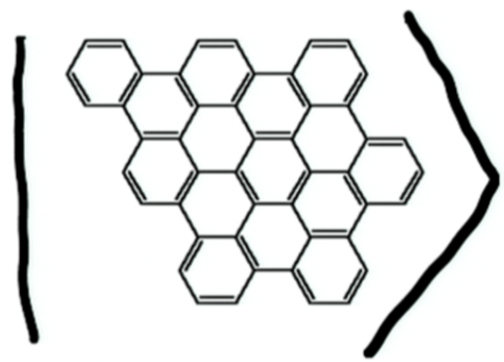
Pauling's Idea of Resonating Valence Bond for Graphene

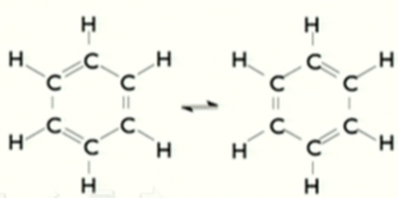
Dominance of **Neutral (C^0) configurations** **C: $2s^2$ $2p^2$**

As a first approximation **polar (C^{1+}, C^{1-}) configurations** are ignored
(there by approximating it as a Mott insulator !)



$$|RVB\rangle = \sum$$

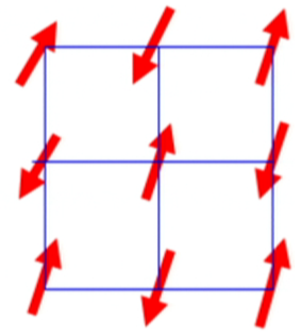




Resonating Valence Bond (RVB) States

Pauling
Anderson 1973
Fazekas

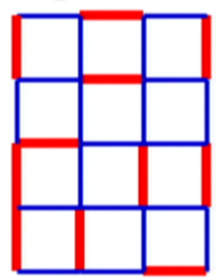
Quantum Heisenberg Antiferromagnets (spin half)



Quantum fluctuations
Encouraged by lattice frustration and lower dimensionality
may destroy long range AFM order (spin crystal)

Resulting in a Quantum Spin Liquid, a RVB state

$$\left|RVB\right\rangle = \sum_C \left|C\right\rangle$$



$$|RVB;\phi\rangle \equiv P_G(\sum_{ij}\phi_{ij}b^{\dagger}_{ij})^{\frac{N}{2}}|0\rangle$$

Short range spin correlations

$$= \frac{1}{\sqrt{2}}[\left|\uparrow\downarrow\right\rangle - \left|\downarrow\uparrow\right\rangle]$$

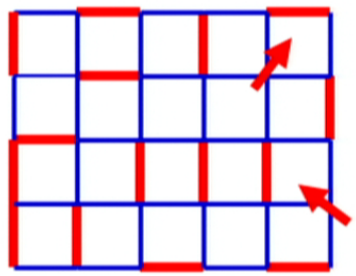
$$\langle S_{iz}S_{jz}\rangle \approx e^{\frac{-|i-j|}{\xi}} \quad or \quad \frac{1}{|i-j|^{\alpha}}$$

After a long gap came Cuprates (Bednorz Muller 1986)

It is a realisation of RVB Quantum spin liquid in 2D (Anderson 1987)

Spin Susceptibility of undoped La2CuO4 was Pauli like (Ganguly, Rao 1986)

Neutral Fermion excitations and Pseudo Fermi Surface
spinon



Spinon as a Topological object (Kivelson, Rokhsar, Sethna 1987)

A Quantum Dimer Model & Topological Order (Kivelson, Rokhsar)
RVB Mean field solutions & Quantum Order (Wen)

Emergent Fermions from Localized Spins

RVB Meanfield Theory (GB, Zou, Anderson 1987)

$$J(S_i . S_j - \frac{1}{4} n_i n_j) \equiv -J \quad b_{ij}^{+} b_{ij}$$

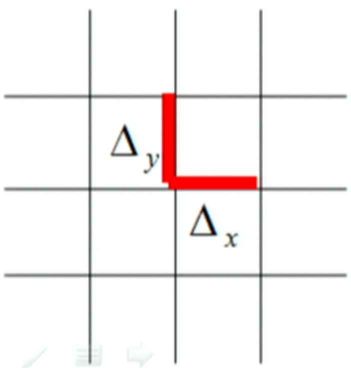
$$\vec{S}_i \equiv C_{i\alpha}^{+} \vec{\sigma}_{\alpha\beta} C_{i\beta} \qquad n_{i\uparrow} + n_{i\downarrow} = 1$$

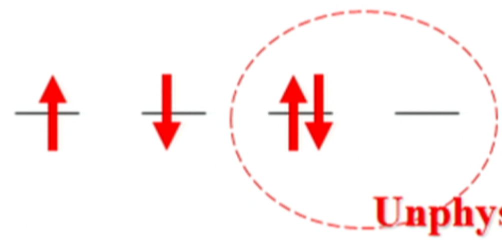
$$b_{ij}^{+} = \frac{1}{\sqrt{2}} (C_{i\uparrow}^{+} C_{j\downarrow}^{+} - C_{i\downarrow}^{+} C_{i\uparrow}^{+})$$

Two complex fermion Hilbert space $\quad 2^2 = 4$

$$b_{ij}^{+} b_{ij} \rightarrow \langle b_{ij}^{+} \rangle b_{ij} + b_{ij}^{+} \langle b_{ij} \rangle$$

Δ_{ij}^{*}





Extended - S $\longrightarrow$ $\Delta_x = \Delta_y$ BZA

Flux Phase $\longrightarrow$ $\Delta_x = -\Delta_y$ Kotliar, Affleck, Marston

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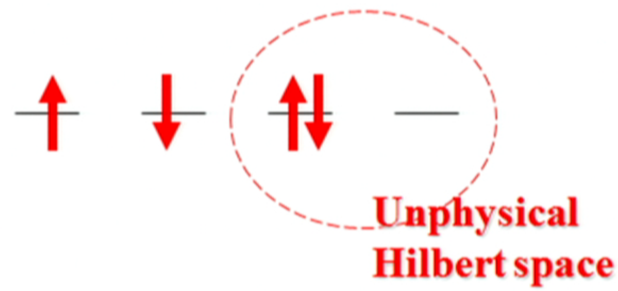
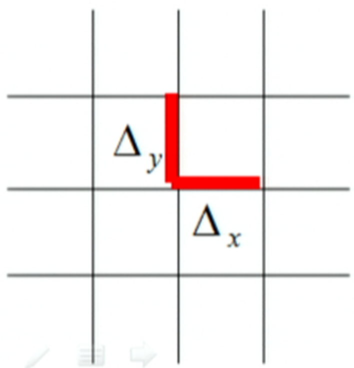




Extended - S $\longrightarrow$ $\Delta_x = \Delta_y$ **BZA**

Flux Phase $\longrightarrow$ $\Delta_x = -\Delta_y$ **Kotliar, Affleck, Marston**

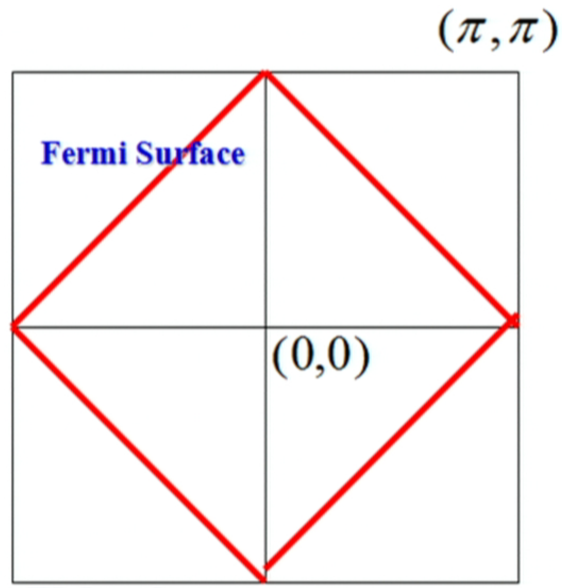
Extended - S $\longrightarrow \quad \Delta_x = \Delta_y$

$$H_s = J\sum_{\langle ij\rangle}(S_i \cdot S_j - \frac{1}{4}) = -J\sum_{\langle ij\rangle} b_{ij}^{\dagger} b_{ij}$$

$$b_{ij}^{+} b_{ij} \rightarrow \langle b_{ij}^{+}\rangle b_{ij} + b_{ij}^{+}\langle b_{ij}\rangle$$

$$H_{pair} = -J\sum_{k,k'} \gamma(\mathbf{k}-\mathbf{k}') c_{-k'\downarrow}^{\dagger} c_{k'\uparrow}^{\dagger} c_{k\uparrow} c_{-k\downarrow}$$

$$H_{mF} \sim J\sum_{k\alpha} |\cos k_x + \cos k_y| \; \alpha_{k\sigma}^{\dagger}\alpha_{k\sigma}$$



$$|2D\,RVB\rangle = P_G \prod_k (u_k + v_k c_{k\uparrow}^{\dagger} c_{-k\downarrow}^{\dagger})|0\rangle$$

GB, Zou, Anderson 1987

Recent work by
Tao Li and Vishwanath et al. (2011) find long range AFM order

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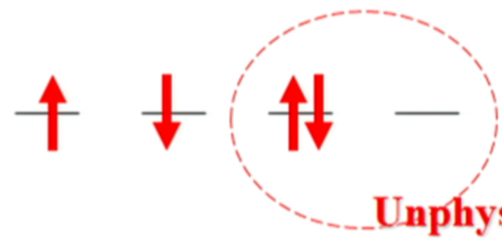
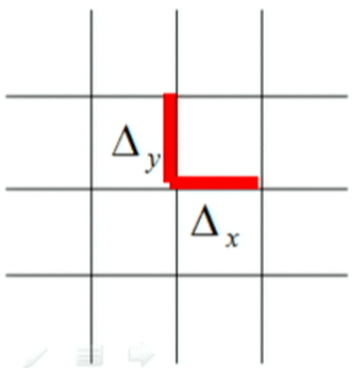
Δ_{ij}^{*}





Unphysical Hilbert space

Extended - S $\longrightarrow$ $\Delta_x = \Delta_y$ **BZA**

Flux Phase $\longrightarrow$ $\Delta_x = -\Delta_y$ **Kotliar, Affleck, Marston**

Dynamically generated Gauge Fields

GB, Anderson 1987

$$2^N \to 4^N$$

Spin waves (Goldstone modes) are elementary excitations
In magnetically ordered systems

Hilbert space enlargement helped us to see
presence of dynamically generated gauge field degree of freedom
in addition to topological excitations such as a spinon

Dynamically generated Gauge Fields

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$$2^N \rightarrow 4^N$$

Spin waves (Goldstone modes) are elementary excitations
In magnetically ordered systems

Hilbert space enlargement helped us to see
presence of dynamically generated gauge field degree of freedom
in addition to topological excitations such as a spinon

Local U(1) gauge symmetry

$$H_s = J\sum_{\langle ij\rangle}(\mathrm{S}_i \cdot \mathrm{S}_j - \frac{1}{4}) = -J\sum_{\langle ij\rangle} b_{ij}^{\dagger} b_{ij}$$

$$C_{i\alpha}^{+} \rightarrow e^{i\theta_i} C_{i\alpha}^{+} \qquad b_{ij}^{+} \rightarrow e^{i\theta_i} b_{ij}^{+} e^{i\theta_j} \qquad \Delta_{ij}^{*} \rightarrow e^{i\theta_i} \Delta_{ij}^{*} e^{i\theta_j}$$

GB, Anderson

U(1) RVB magnetic field

$$\Re \; e^{i\oint \mathbf{A}(\mathbf{r}).d\mathbf{l}} \sim \mathbf{S}_i \times (\mathbf{S}_j \times \mathbf{S}_k)$$

Wen Wilczek Zee

Local SU(2) symmetry

$$c_{i\uparrow} \rightarrow u_i c_{i\uparrow} + v_i c_{i\downarrow}^{\dagger} \qquad |u_i|^2 + |v_i|^2 = 1$$

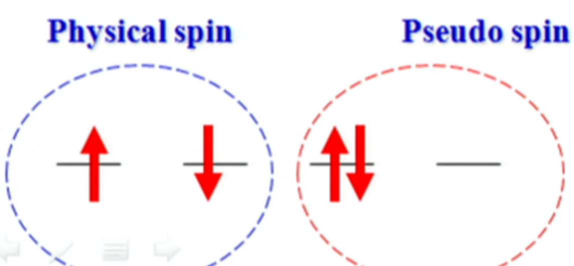
Affleck Anderson Zou Hsu

Two complex fermion

Hilbert space $2^2 = 4$

$$2^{2N} = 2^N \times \ldots\ldots \times 2^N$$

$$\leftarrow 2^N \mathit{times} \rightarrow$$



When a physical spin and a Pseudo spin get identified
the above 2^N sectors become gauge copies of a Z_2 gauge theory

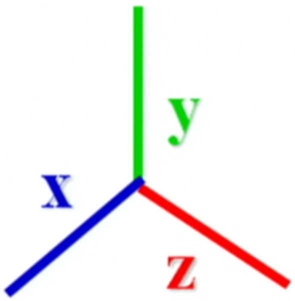
Key aspects of RVB theory

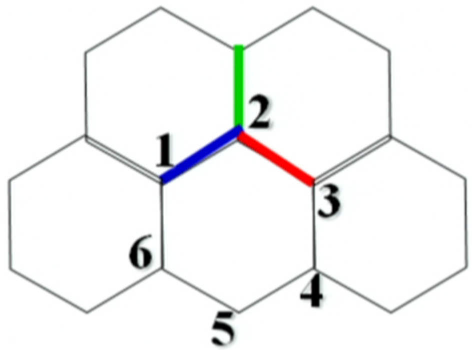
Enlargement of Hilbert space
Mean field theory – freezing link variables
Dynamically generated Gauge fields (Z_2, U(1), SU(2))

Topological order, Chern Simons Theory
Quantum number fractionization
Pseudo Fermi surface

Kitaev Model on a Honeycomb lattice

Kitaev 2001, 2003





Frustration in Spin Space

$$H = -J_x \sum_{\langle ij \rangle_x} \boxed{\sigma_i^x \sigma_j^x} - J_y \sum_{\langle ij \rangle_y} \boxed{\sigma_i^y \sigma_j^y} - J_z \sum_{\langle ij \rangle_z} \boxed{\sigma_i^z \sigma_j^z}$$

Flux Operator

$$\widehat{B}_P \equiv \sigma_1^y \sigma_2^z \sigma_3^x \sigma_4^y \sigma_5^z \sigma_6^x$$

$[\widehat{B}_P, H] = 0$ $\quad \widehat{B}_P^{\ 2} = 1$

Eigen value B_P = ± 1

$[\widehat{B}_P, \widehat{B}_{P'}] = 0$

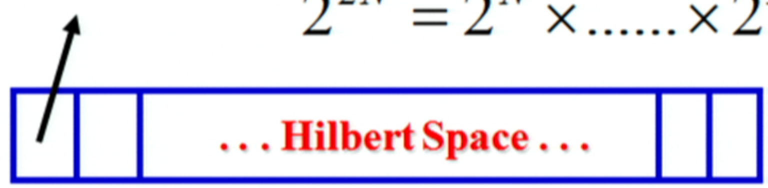
for every plaquette P

2^N possible configurations of B_P

2N sites

2^N states

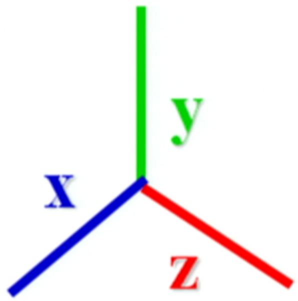
$$2^{2N} = 2^N \times \ldots\ldots \times 2^N$$

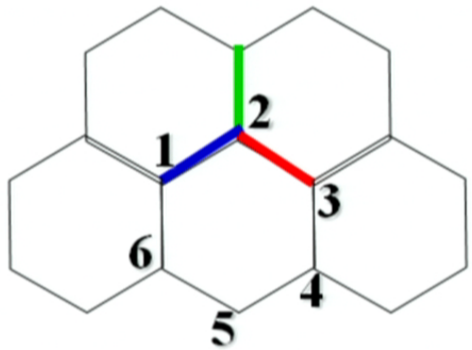


2^N different B_P Sectors

Kitaev Model on a Honeycomb lattice

Kitaev 2001, 2003





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2N sites

$[\widehat{B}_P, H] = 0$ $\quad \widehat{B}_P{}^2 = 1$

Eigen value $B_P = \pm 1$

2^N states

$$2^{2N} = 2^N \times \ldots\ldots \times 2^N$$

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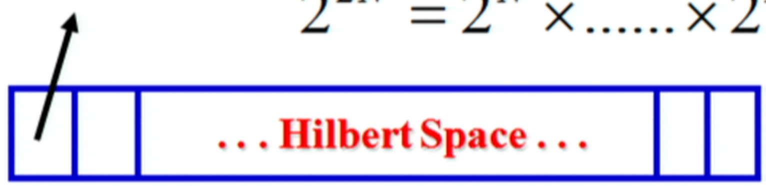

2^N possible configurations of B_P

⟵ 2^N different B_P Sectors ⟶

Kitaev's method of solution

2 Majorana fermions make one complex or Dirac fermion

$$\psi^{+} = \xi + i\zeta$$

$$2 = \sqrt{2} \times \sqrt{2}$$

$$\{\psi, \psi^{+}\} = 1$$

$$\{\xi, \zeta\} = 0$$

$$\xi^2 = \zeta^2 = 1$$

Introduce 4 Majorana fermions at each site:

$$c^{\alpha}, \ \alpha = 0, x, y, z \qquad \{c^{\alpha}, c^{\beta}\} = 2\delta_{\alpha\beta}$$

$$D_i |\Psi\rangle_{\rm phys} = |\Psi\rangle_{\rm phys}$$

$$D_i \equiv c_i \, c_i^x c_i^y c_i^z$$

Dimension of Physical Hilbert Space 2^{2N}

Dimension of Enlarged Hilbert Space 4^{2N}

$$\boxed{\sigma_i^a = i c_i c_i^a,} \qquad a = x, y, z$$

$$[\sigma_i^a, \sigma_j^b] = i\epsilon_{abc}\sigma^c\delta_{ij}$$

$$H = - \sum_{a=x,y,z} J_a \sum_{\langle ij \rangle_a} i c_i \hat{u}_{\langle ij \rangle_a} c_j$$

$$\hat{u}_{\langle ij \rangle_a} \equiv i c_i^a c_j^a \qquad \boxed{[H, \hat{u}_{\langle ij \rangle_a}] = 0}$$

$$\hat{u}^2_{\langle ij \rangle_a} = 1$$ **eigen value** $$u_{\langle ij \rangle_a} = \pm 1$$

$u_{\langle ij \rangle_a}$ (Ising) Z_2 gauge fields on the bonds

Local Z_2 gauge symmetry $u_{\langle ij \rangle a} \rightarrow \tau_i u_{\langle ij \rangle a} \tau_j$

with $\tau_i \pm 1$

$$H = -\sum_{a=x,y,z} J_a \sum_{\langle ij\rangle_a} i c_i \hat{u}_{\langle ij\rangle_a} c_j$$

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Local Z_2 gauge symmetry $u_{\langle ij \rangle a} \longrightarrow \tau_i u_{\langle ij \rangle a} \tau_j$

with $\tau_i \pm 1$

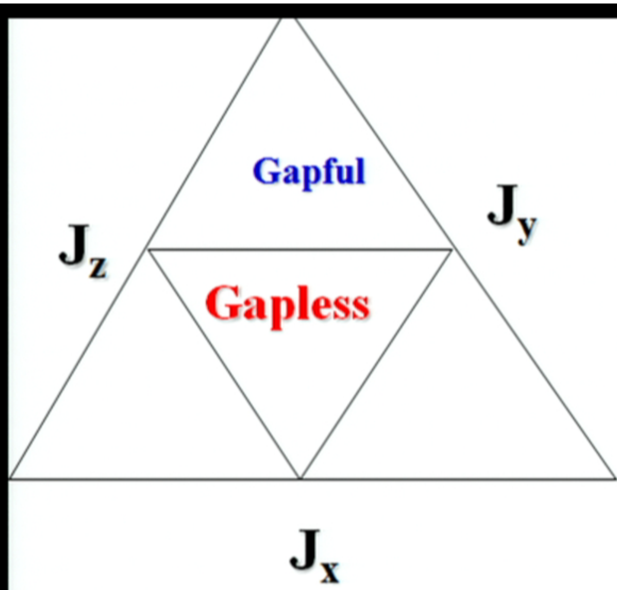
Zero flux sector $B_p = 1$ for all plaquettes

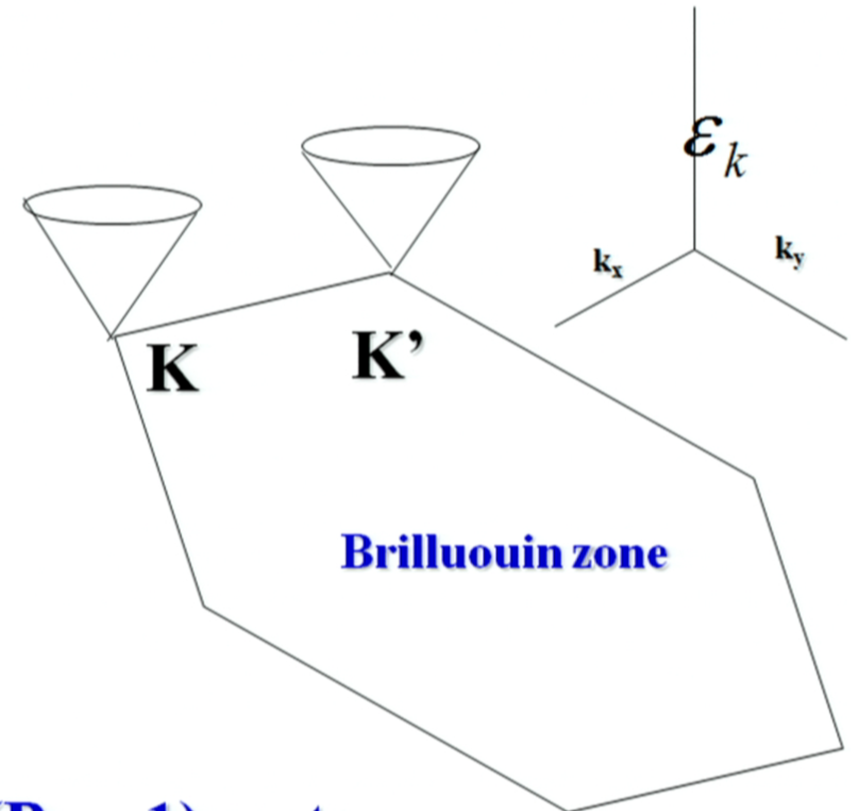
$$H = -\sum_{a=x,y,z} J_a \sum_{\langle ij\rangle_a} i c_i \, c_j$$

The problem reduces to freely propagating Majorana fermion on the honeycomb lattice

This can be solved by Bogoliubov transformation combined with fourier transformation

Only particle like fermionic excitations are present





For zero flux ($B_p = 1$) sector

Gauge Field Condensation and Chiral Symmetry Breaking

Is it possible to have non zero

$$\left\langle S_i \cdot (S_j \times S_k) \right\rangle \sim \left\langle e^{i \oint A \cdot dl} \right\rangle \neq 0$$

in the ground state ?

$$\oint \vec{A} \cdot d\vec{l} = a B_{RVB}$$

i.e. **Interaction among emergent gauge fields favor presence of finite magnetic fields in the ground state**

Hard core boson analogy

$$\mathbf{S_i \cdot S_j} = \tfrac{1}{2}(S_i^+ S_j^- + S_i^- S_j^+) + S_i^z S_j^z$$

$|\uparrow\rangle = |0\rangle$

$|\downarrow\rangle = |1\rangle$

~~$|2\rangle$~~

~~$|3\rangle$~~

$\vdots$

Hard core constraint

$$\vec{S}_i \cdot \vec{S}_j = \tfrac{1}{2}(b_i^{\dagger} b_j + h.c.) + b_i^{\dagger} b_i b_j^{\dagger} b_j - \tfrac{1}{2}(b_i^{\dagger} b_i + b_j^{\dagger} b_j)$$

hopping

2 body repulsion

chem. pot.

$$[b_i^{\dagger}, b_j] = 0 \quad i \neq j$$

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$$\mathbf{S_i . S_j} = \tfrac{1}{2}(S_i^+ S_j^- + S_i^- S_j^+) + S_i^z S_j^z$$

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Hard core constraint

$$\vec{S}_i \cdot \vec{S}_j = \tfrac{1}{2}\left(b_i^\dagger b_j + h.c.\right) \quad \text{hopping}$$

$$+ \; b_i^\dagger b_i \, b_j^\dagger b_j \quad \text{2 body repulsion}$$

$$- \tfrac{1}{2}\left(b_i^\dagger b_i + b_j^\dagger b_j\right)$$

chem. pot.

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$$-\frac{1}{2}\left(b_i^+ b_i + b_j^+ b_j\right) \quad \text{chem. pot.}$$

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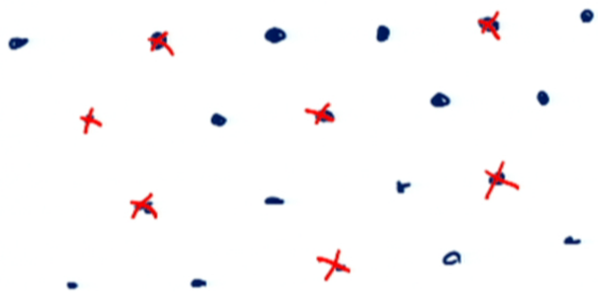
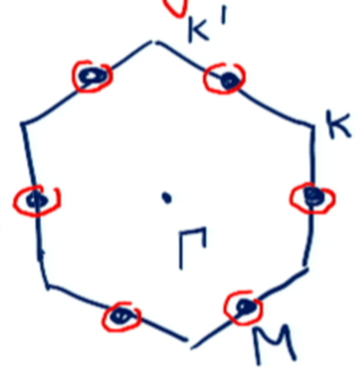
Hard core bosons hopping on a Δ^{lr} lattice
(Kalmeyer Laughlin)





3 Minima

Bosons — Bose condensation — M_1, M_2, M_3 ?

Hard core and nn repulsions

Bose condensation in 3 sublattices with phases $\theta_1 = 0$, $\theta_2 = \frac{2\pi}{3}$, $\theta_3 = \frac{4\pi}{3}$

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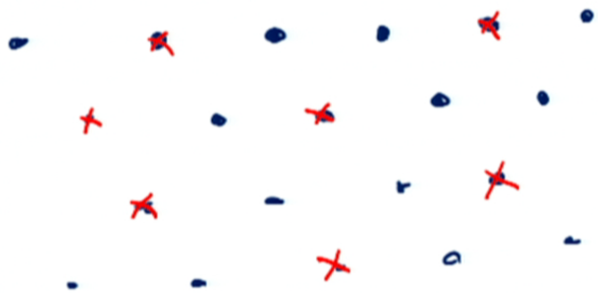

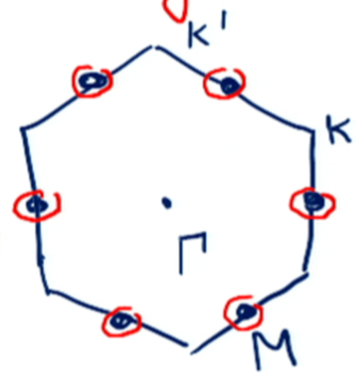

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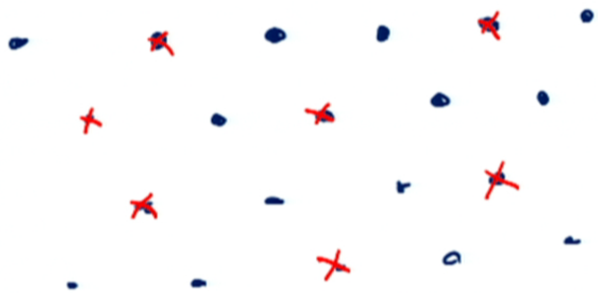

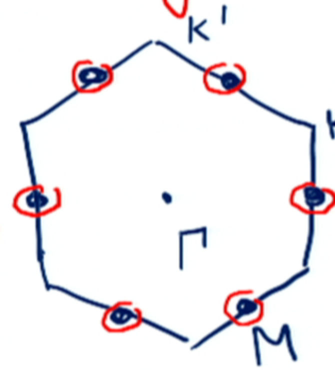

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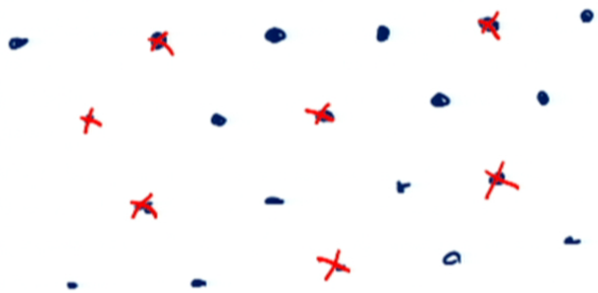

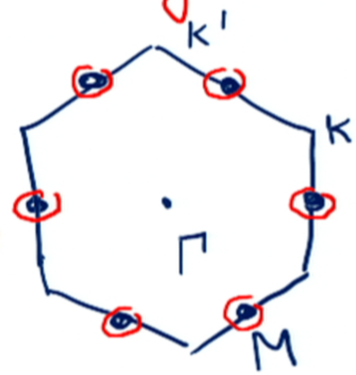

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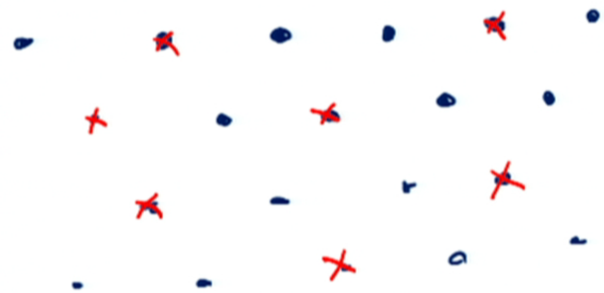

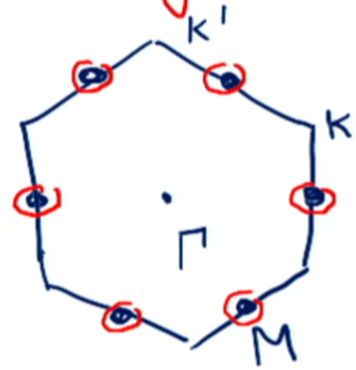

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chem. pot.

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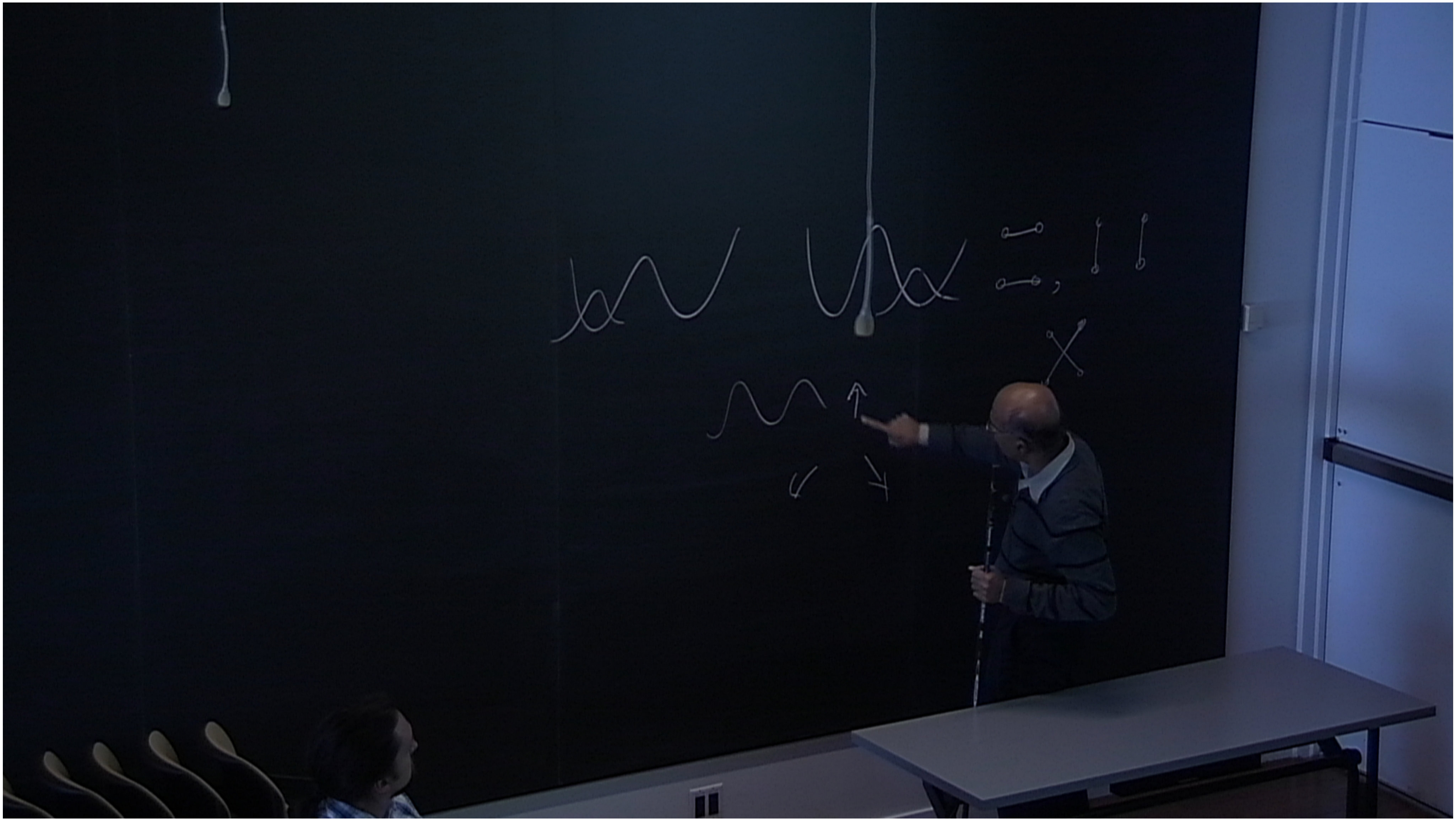
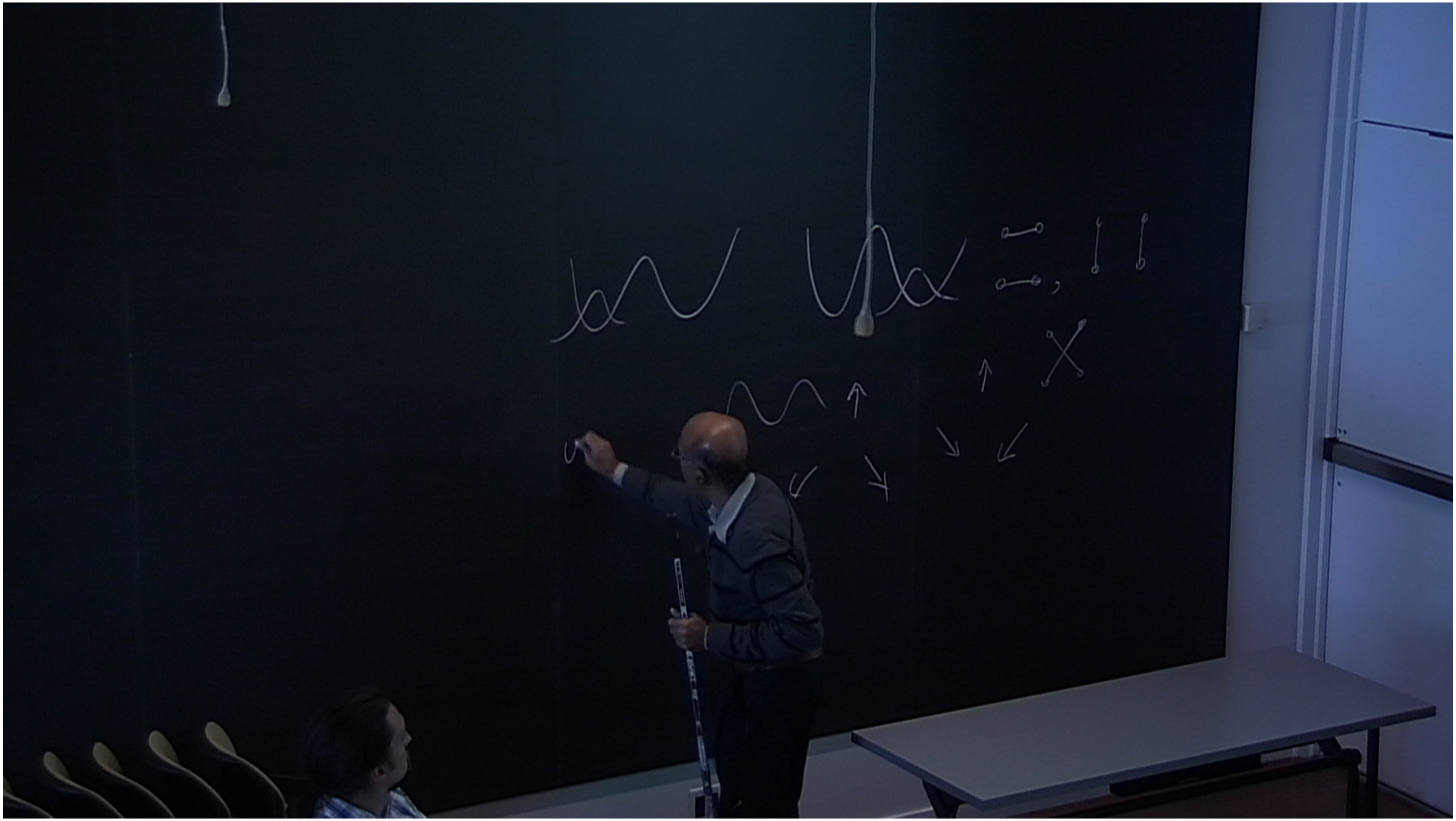
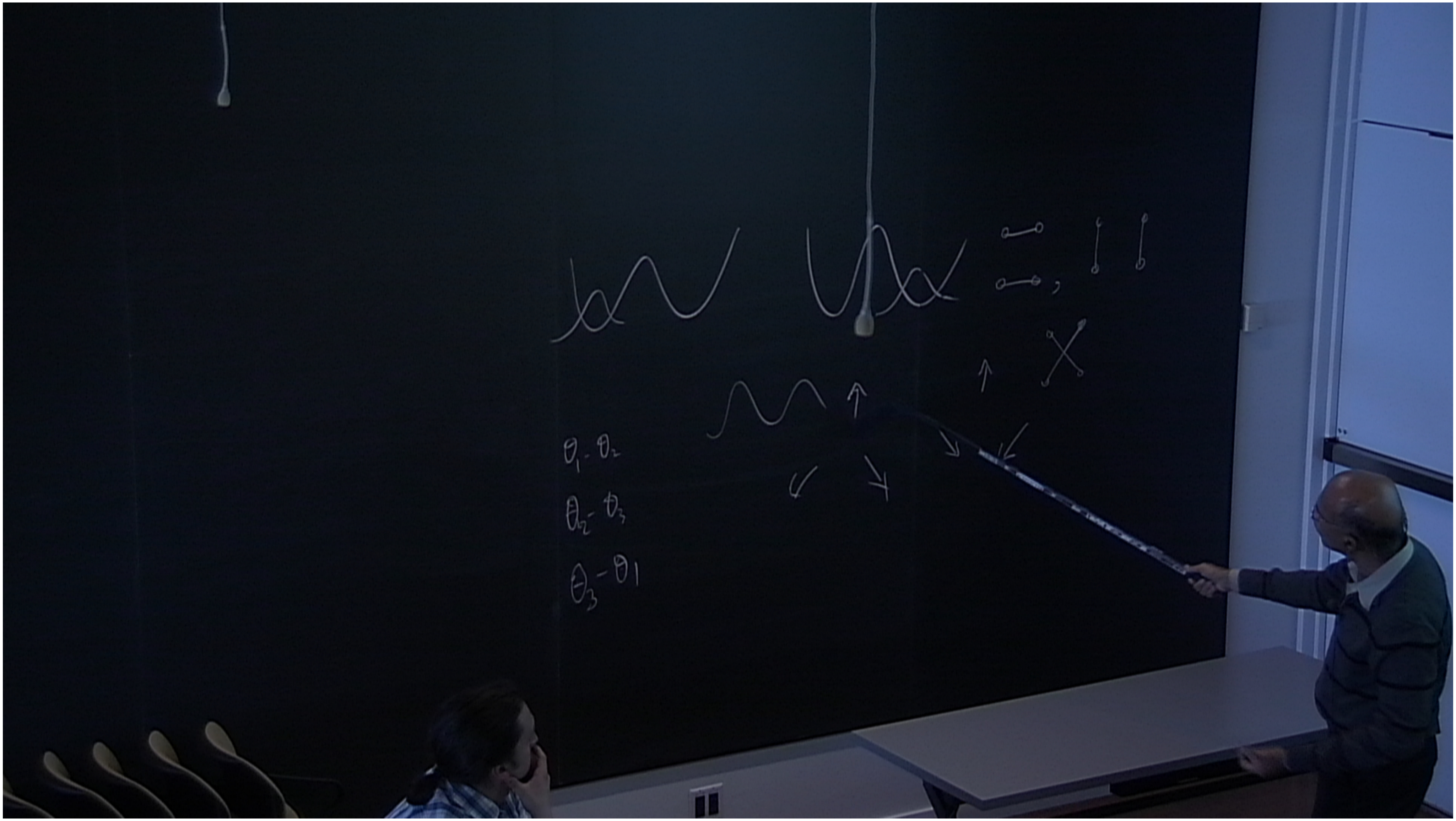
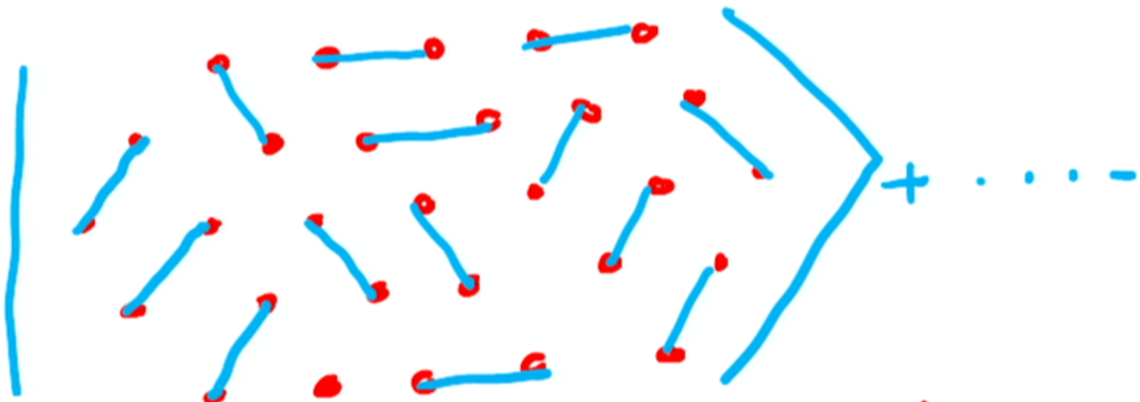





$\theta_1 - \theta_2$
$\theta_2 - \theta_3$
$\theta_3 - \theta_1$

Anderson (1973)



Kalmeyer-Laughlin (1987)

Fractional quantum Hall like wave function

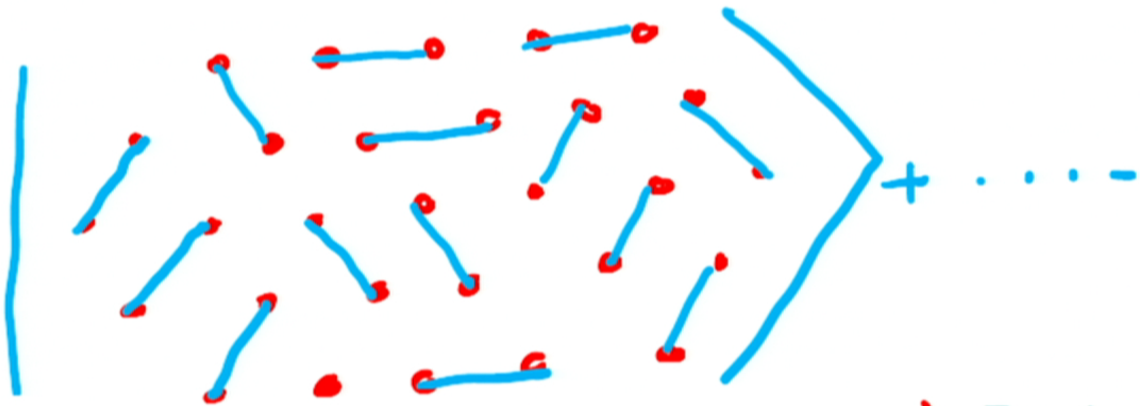
$$\psi(z_1 \cdots z_N) \sim \prod_i g(z_i) \prod_{i<j} (z_i - z_j)^2 \, e^{-\frac{1}{4}\sum |z_i|^2}$$

$$H_i = \frac{1}{2}\sum_{j(\neq i)} |w_{ij}|^2 - \frac{2i}{3}\sum_{j\neq k(\neq i)} w_{ij}^* w_{ik}\, \mathbf{S}_i\cdot(\mathbf{S}_j\times\mathbf{S}_k) + \frac{2}{3}\sum_{j(\neq i)} |w_{ij}|^2 \mathbf{S}_i\cdot\mathbf{S}_j + \frac{2}{3}\sum_{j\neq k(\neq i)} w_{ij}^* w_{ik}\mathbf{S}_j\cdot\mathbf{S}_k,$$

arXiv:1201.3096

Anne E. B. Nielsen,[1] J. Ignacio Cirac,[1] and Germán Sierra

Anderson (1973)



Kalmeyer-Laughlin (1987)

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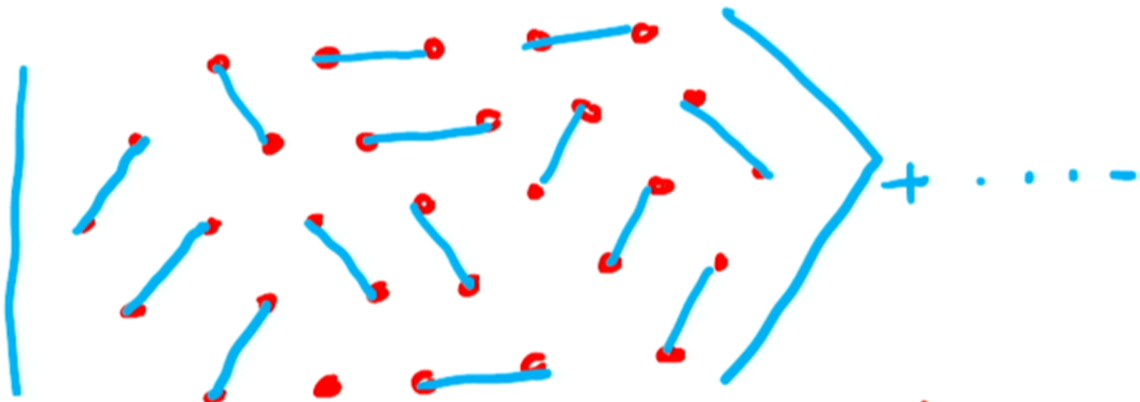

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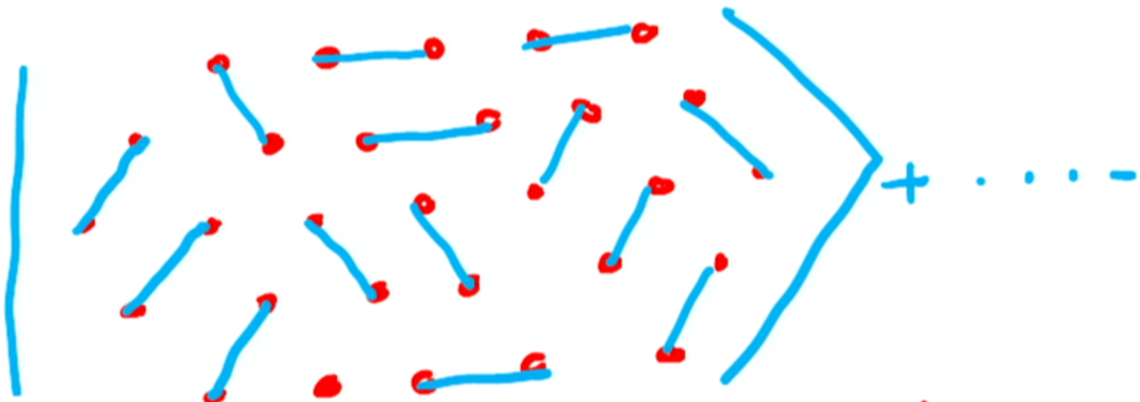

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$$\hat{\chi}_{ijk} \equiv \mathbf{S}_i \cdot (\mathbf{S}_j \times \mathbf{S}_k)$$

The eigenvalues of $\hat{\chi}$ are $\pm\sqrt{3}/4$ and 0

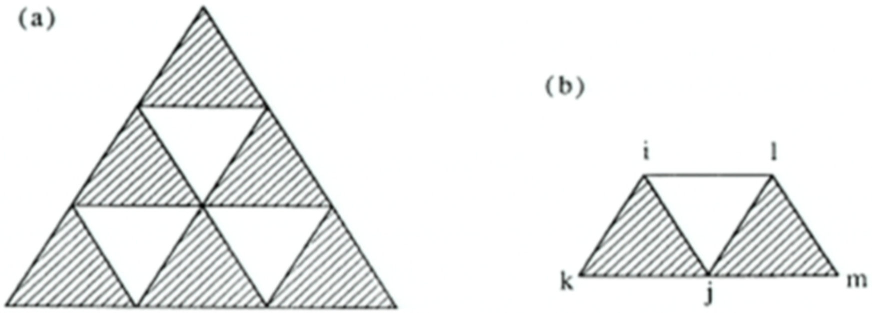
Wen Wilczek Zee PRB 1989

$$\hat{\chi}_{ijk}^2 = -\frac{1}{16}(\mathbf{S}_i + \mathbf{S}_j + \mathbf{S}_k)^2 + \frac{15}{64}$$

GB PRL 1989 Application to J_1 J_2 Model on a Triangular Lattice

$$H = J \sum_{\text{nn}} \mathbf{S}_i \cdot \mathbf{S}_j = \frac{1}{2} J \sum_{\langle ijk \rangle} (\mathbf{S}_i + \mathbf{S}_j + \mathbf{S}_k)^2$$

$$= -8J \sum_{\langle ijk \rangle} \hat{\chi}_{ijk}^2$$



$$H = J\sum_{\text{nn}} \mathbf{S}_i \cdot \mathbf{S}_j + \alpha J \sum_{\text{nnn}} \mathbf{S}_i \cdot \mathbf{S}_j \;\; = -8J \sum_{\langle ijk \rangle} \hat{\chi}_{ijk}^2 + \alpha J \sum_{\text{nnn}} \mathbf{S}_i \cdot \mathbf{S}_j$$

$$Z(\beta) = \int \mathcal{D}m \,\mathrm{Tr}\left\{ T \exp\left[16J \sum_{\Delta} \int_0^{\beta} \chi_{ijk}(\tau) m_{ijk}(\tau) d\tau \right] \exp\left[-8J \sum \int_0^{\beta} m_{ijk}^2(\tau) d\tau \right] \right\}$$

$$\beta F[m] \;\; = \frac{(16J)^4}{4!} \sum \mathrm{Tr}(T\chi\chi\chi\chi) m(\tau) m(\tau_2) m(\tau_3) m(\tau_4) d\tau_1 d\tau_2 d\tau_3 d\tau_4$$

$$- \tfrac{1}{24} (16\beta J)^2 g_1^2(\alpha) \sum m_{ijk} m_{jlm}$$

First term, independent of nnn coupling has a local Z_2 symmetry

$$m_{ijk}(\tau) \longrightarrow \sigma_{ijk} m_{ijk}(\tau)$$

Second term is an Ising term, which has a global Z_2 symmetry

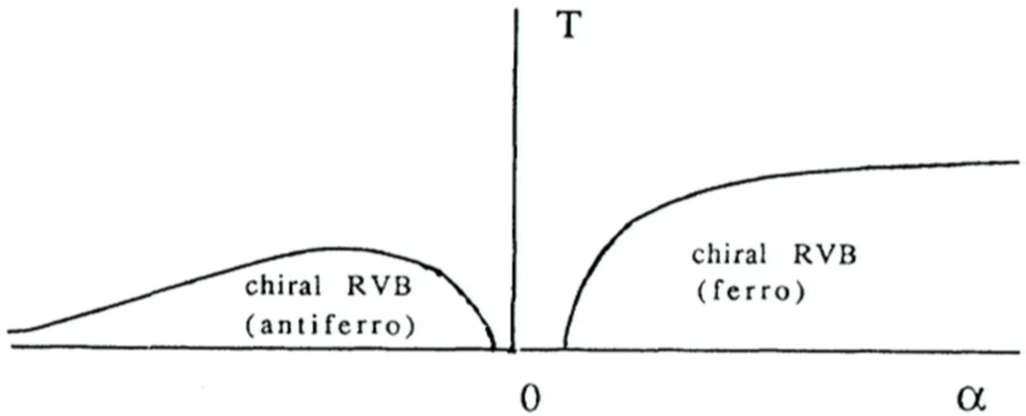


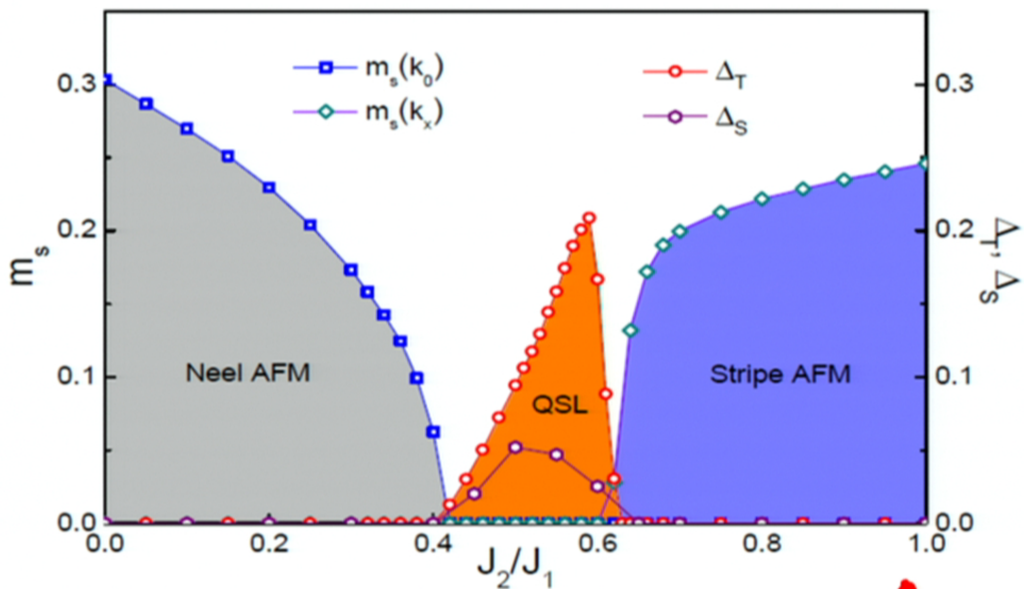
FIG. 2. Phase diagram as a function of the strength α of the next-nearest-neighbor coupling.

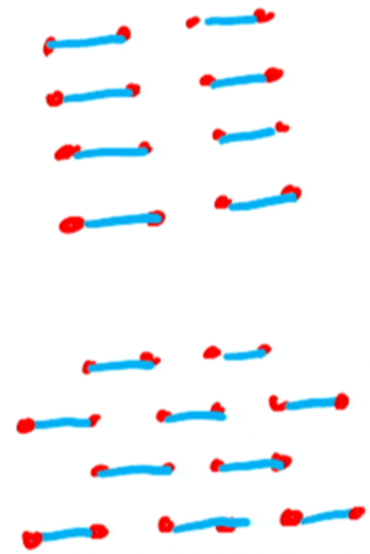
GB 1989

Recent excitement in J_1 J_2 Model on a Square lattice

$$H = J_1 \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j + J_2 \sum_{\langle\langle ij \rangle\rangle} \mathbf{S}_i \cdot \mathbf{S}_j$$

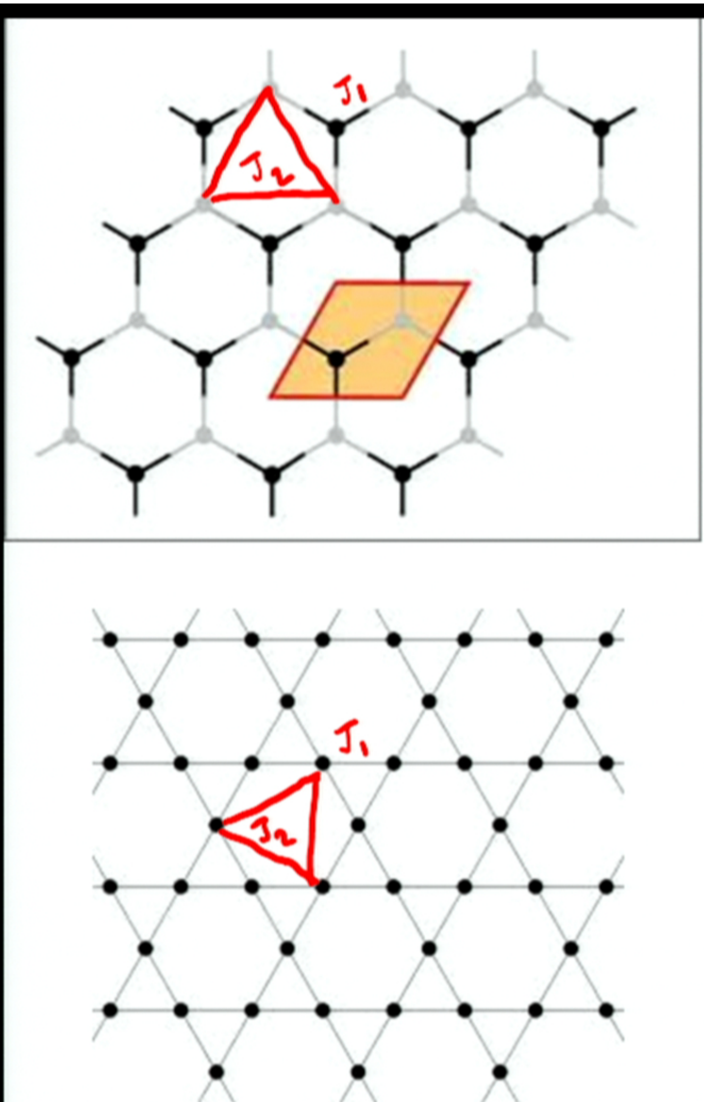
Valence bond order vs Spin liquid





at $\frac{J_2}{J_1} = 0.5$ $\quad H = -J \sum \hat{\chi}^2_{ijk}$

arXiv:1112.2241 Hong-Chen Jiang[1,2], Hong Yao[3,4] & Leon Balents



J_1 J_2

Special value $\frac{J_2}{J_1}$

$$H = -J \sum x_{ijk}^2$$