Title: Intrinsic, Anomalous Hall Effect in a Chiral Multiband Superconductor

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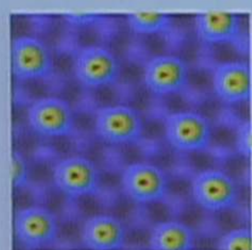
Abstract: Chiral superconducting states have attracted an enormous amount of interest in recent years, due both to their intrinsic novelty as well as their potential for quantum information processing. They break both parity and time-reversal sym-metries and have been predicted to harbour Majorana fermions in vortex cores and along their edges. A crucial challenge in the quest to find such states is identifying robust experimental probes of chirality. In this talk, I will discuss an intrinsic, anomalous Hall effect that arises in multiband chiral superconductors. This effect arises from interband transitions involving time-reversal symmetry breaking chiral Cooper pairs. I will discuss the implications of this effect for the putative chiral p-wave superconductor, Sr2RuO4, and show that it can contribute significantly to Kerr rotation experiments. Since the magnitude of the effect depends on the structure of the order parameter across the bands, this result may also be used to distinguish between different models proposed for the superconducting state of Sr2RuO4.

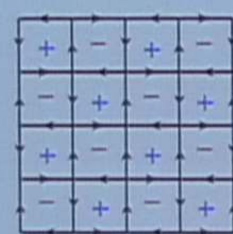
A more appropriate title for this talk

How to get an anomalous Hall effect (zero B-field) in a clean, non-ferromagnetic material.

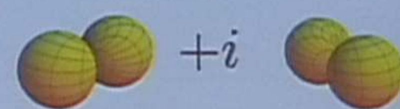
- Need currents (with "magnetic moment") for starters. Some places this happens:



Vortex matter
(sans *B*-field)

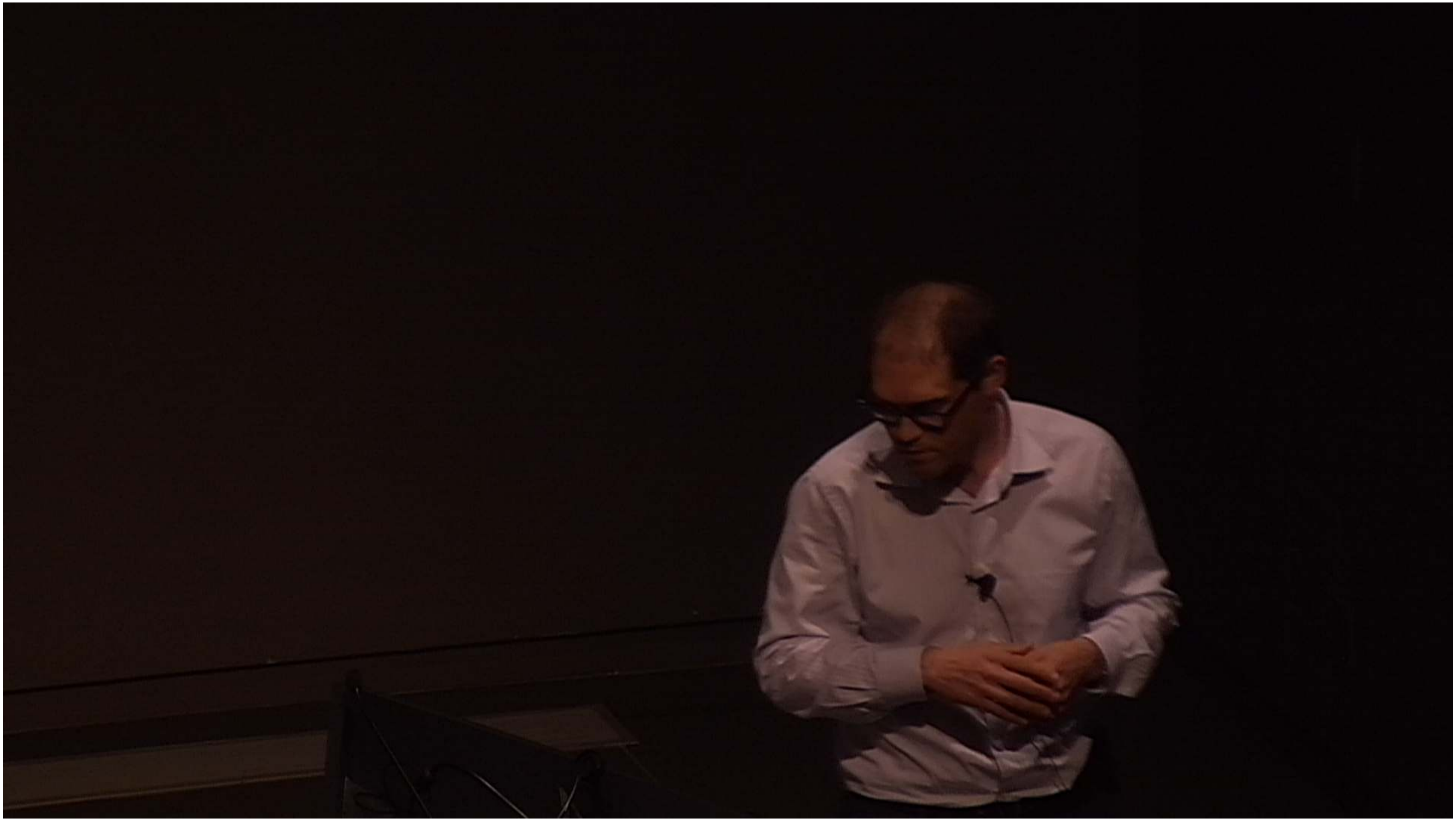


d+id density wave/
staggered flux



Chiral p_x+ip_y
superconductivity

- **A (ferromagnetic-like) current is not sufficient, however.**



What I'm going to tell you in this talk

- Anomalous, "intrinsic" (one-loop/clean limit) Hall effect only arises when there are *inter-orbital* currents.

- For chiral superconductors, need multiple *complex* (chiral) order parameters with different phases on these orbitals.
- In the putative chiral superconductor Sr_2RuO_4, Hall effect requires superconductivity on *Ru* d_{xz} & d_{yz} orbitals.

- Single-orbital + vertex corrections can produce nonzero Hall conductivity, but this vanishes at large frequencies.

- Multi-orbital physics likely dominant at large energies (~1eV) relevant to *Kerr rotation exp'ts*, a powerful probe of time-reversal symmetry-breaking states.

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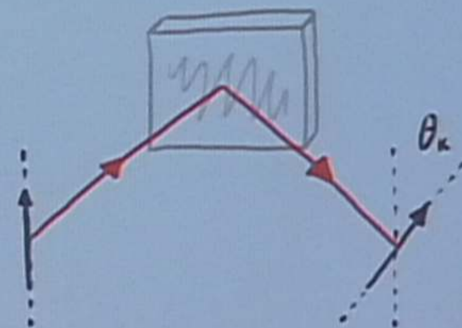
Kerr rotation experiments

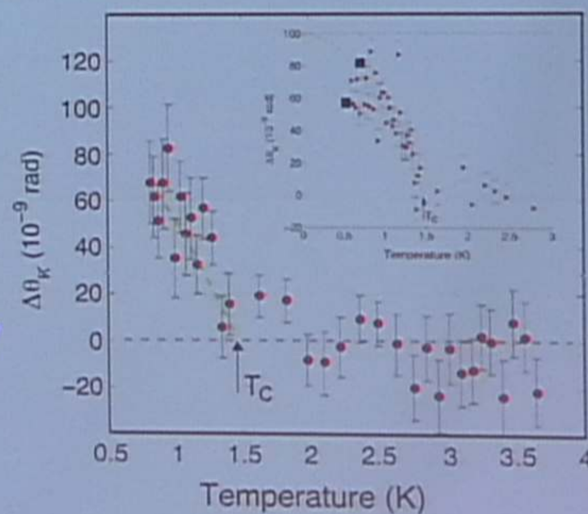
A. Kapitulnik

- Rotation in polarization of reflected light in e.g., ferromagnets.

$$\theta_K(\omega) \propto \mathrm{Im}\left[\frac{\sigma_H(\omega)}{n(\omega)(n^2(\omega)-1)}\right]$$

- Sensitive to time-reversal symmetry breaking (Hall conductivity σ_H).
- Kapitulnik group at Stanford has carried out measurements on Sr_2RuO_4, YBCO, LBCO,...
- Nonzero Kerr angle below $T_c \sim 1.5K$ in ultra-clean Sr_2RuO_4 & ω=0.8eV.





Xia et al. PRL '06

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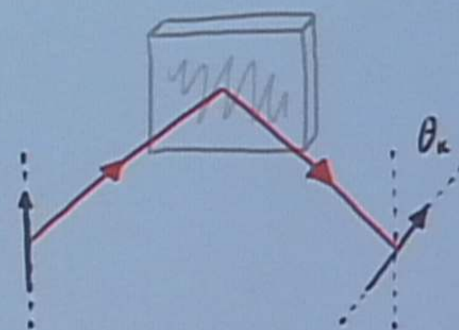
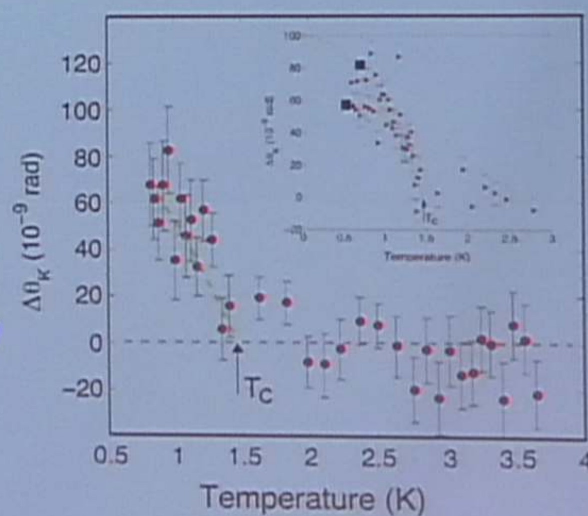


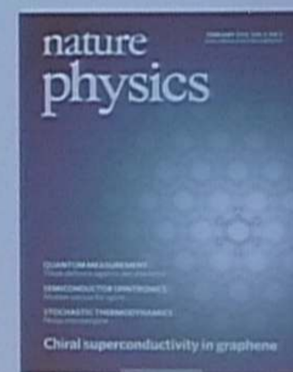


Xia et al. PRL '06

Chiral superconductivity

- Origin of Kerr angle in Sr_2RuO_4 : chiral (p_x+ip_y) superconductivity?

- Breaks time-reversal symmetry: Cooper pairs with **nonzero L_z (magnetic)**, as opposed to **only orbital angular momentum**.



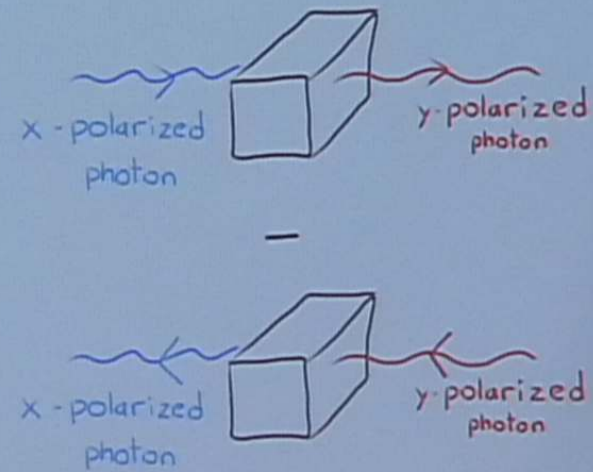
- Many proposals (high-T_c, *graphene*, Sr_2RuO_4,...) *but has never unambiguously been spotted in the wild.*

- Sr_2RuO_4 is likely chiral *p*-wave. Conflicting experimental results, though (Kallin & Berlinsky, J. Phys.: Cond. Mat. 21 '09).

- Only previous theory of Kerr angle and chiral Sr_2RuO_4 invokes *extrinsic* (impurity) mechanism (Lutchyn et al., PRB 80, '09).

Intrinsic Hall conductivity & time reversal symmetry breaking

$$\sigma_H(\omega) = -\frac{1}{i\omega}\lim_{\mathbf{q}\to 0}\left[\pi_{xy}(\mathbf{q},\omega) - \pi_{yx}(\mathbf{q},\omega)\right]$$



- Picks up time-reversal antisymmetric part of *x*-*y* current correlator π_{xy}.

- one-loop approximation for π_{xy}:

$$\pi_{\alpha\beta} = \mathrm{Tr}[G v_\alpha G v_\beta]$$

- Need multiple orbitals (Green's fnc. G & velocity vertices v_α are non-commuting matrices) to have $\pi_{xy} \neq \pi_{yx}$.

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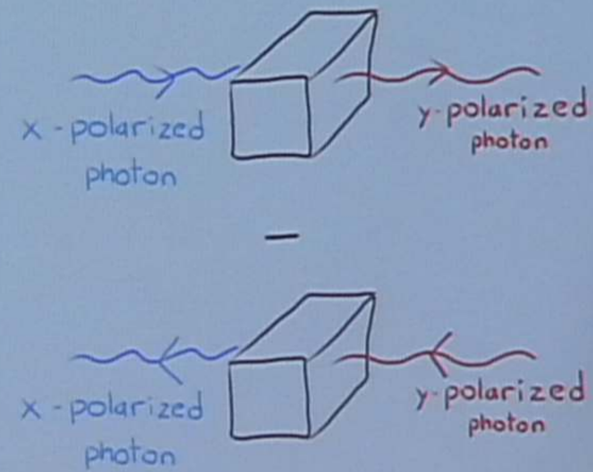

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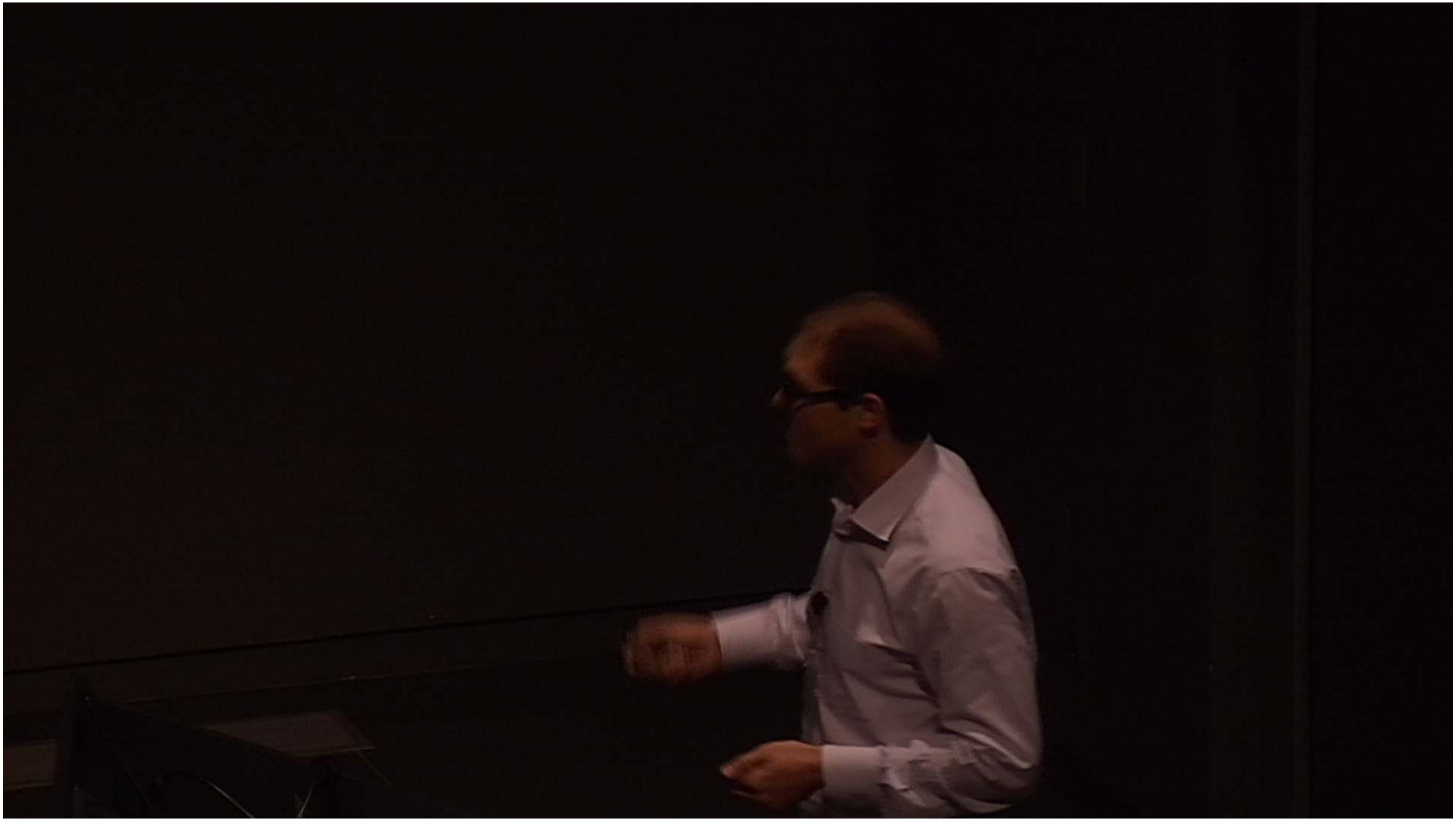

Anomalous Hall effect in a two-orbital chiral superconductor

Model

$$H = \sum_{\mathbf{k}} \left(c^\dagger_{\mathbf{k}1} \;\; c^\dagger_{\mathbf{k}2} \right) \begin{pmatrix} \xi_1(\mathbf{k}) & \epsilon_{12}(\mathbf{k}) \\ \epsilon_{12}(\mathbf{k}) & \xi_2(\mathbf{k}) \end{pmatrix} \begin{pmatrix} c_{\mathbf{k}1} \\ c_{\mathbf{k}2} \end{pmatrix} + H_{\text{int}}.$$

$\Delta_\alpha = \Delta'_\alpha + i\Delta''_\alpha$: Intra-orbital chiral (complex) Cooper pairs (α=1,2); BCS treatment.

relative phase: $\theta_\alpha \equiv \tan^{-1}(\Delta''_\alpha/\Delta'_\alpha)$ 2 BCS quasiparticle branches: $E_\pm$

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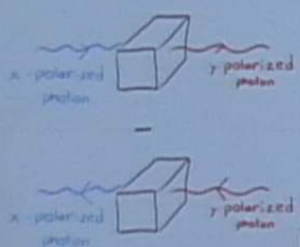
1-loop Hall conductivity



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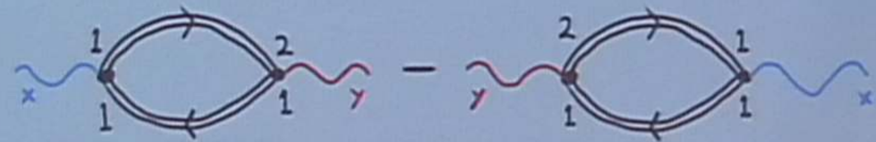

$$\mathrm{Re}\,\sigma_H(\omega) = 2e^2 \sum_{\mathbf{k}} \epsilon_{12} \mathrm{Im}(\Delta_1^* \Delta_2) \frac{[\partial_{k_x}(\epsilon_2 - \epsilon_1)\partial_{k_y}\epsilon_{12} - \partial_{k_y}(\epsilon_2 - \epsilon_1)\partial_{k_x}\epsilon_{12}]}{E_- E_+ (E_- + E_+)((E_- + E_+)^2 - \omega^2)}$$

$\sigma_H \neq 0$ i.f.f. *orbital* OPs with diff't rel. phases, $\theta_1 \neq \theta_2$

σ_H *small.* $\sim (\Delta/t)^2 e^2/\hbar$

"van Hove" singularity at ~ $2\epsilon_{12}$

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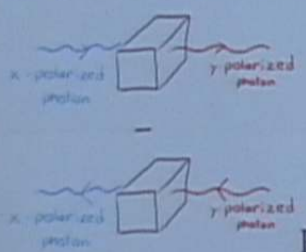

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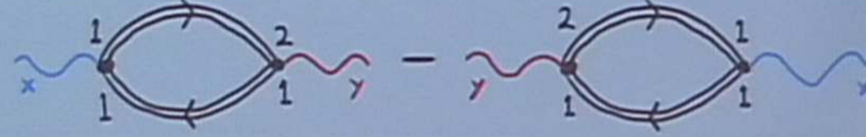

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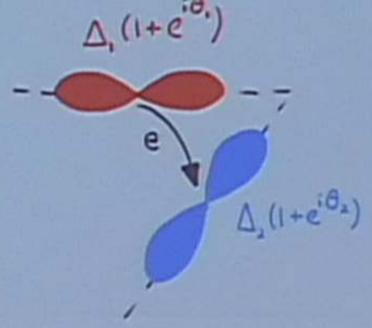
Inter-orbital currents in chiral superconductors

$$\mathrm{Re}\,\sigma_H(\omega) = 2e^2 \sum_{\mathbf{k}} \epsilon_{12} \mathrm{Im}(\Delta_1^* \Delta_2) \frac{[\partial_{k_x}(\epsilon_2 - \epsilon_1)\partial_{k_y}\epsilon_{12} - \partial_{k_y}(\epsilon_2 - \epsilon_1)\partial_{k_x}\epsilon_{12}]}{E_- E_+ (E_- + E_+)((E_- + E_+)^2 - \omega^2)}$$

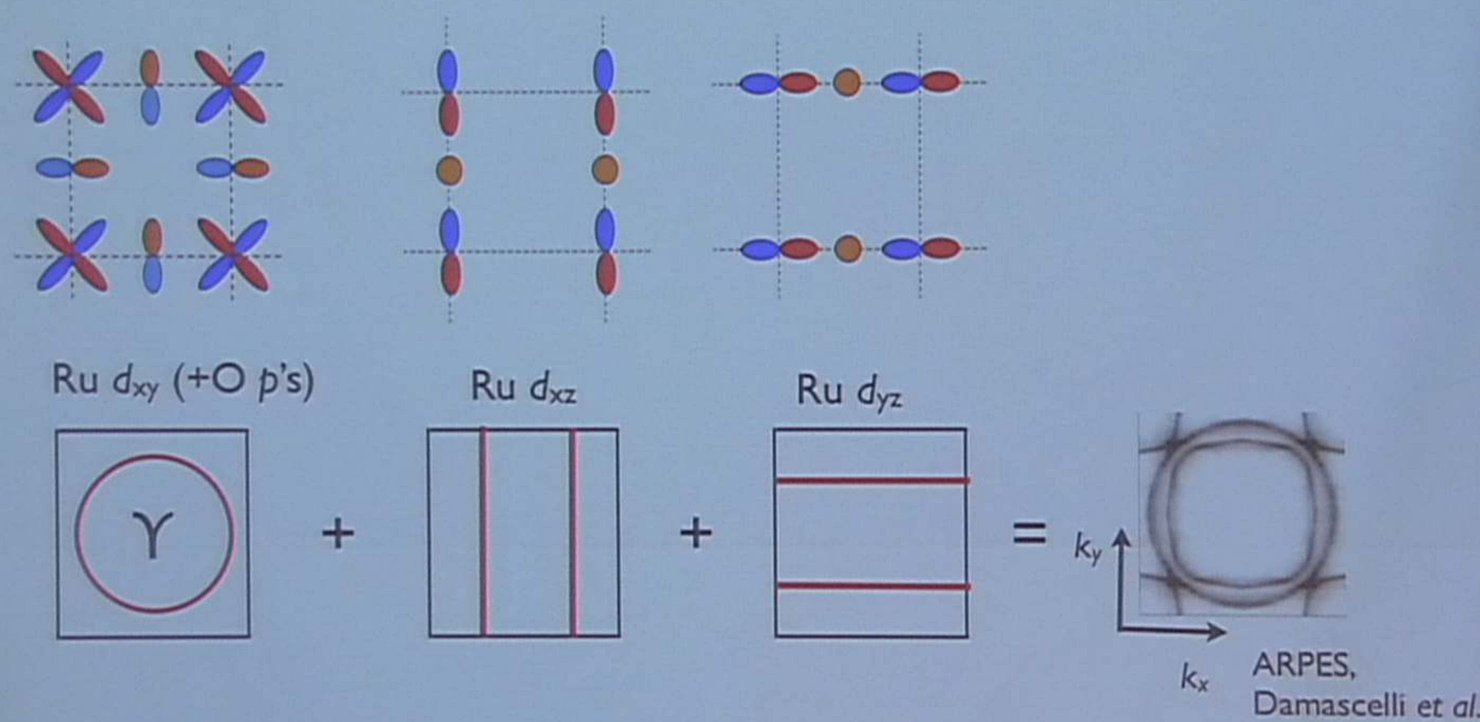
Interorbital coherence (current):

$$\mathrm{Im}\langle c_1^\dagger c_2 \rangle \propto \sin(\theta_1 - \theta_2)$$



- Very generally, Hall effect comes from 1-2 inter-orbital current.
- For a superconductor: complex order parameters on different orbitals with different phases (e.g., chiral) gives this.
- No intrinsic, anomalous Hall effect in a clean, single-orbital chiral superconductor.

Sr_2RuO_4: orbital structure



- Only d_{xz} and d_{yz} orbitals hybridize*. Optically, Sr_2RuO_4 is two-orbital.

* Absent spin-orbit coupling. SOC does not give rise to optical transitions 1sr order.

Anomalous Hall conductivity in Sr_2RuO_4

- Intrinsic Hall response in Sr_2RuO_4 only from Cooper pairs on d_{xz}, d_{yz} orbitals (e.g., models of Takimoto, PRB'00; Annett *et al*, PRB'02; Raghu *et al*, PRL'10):

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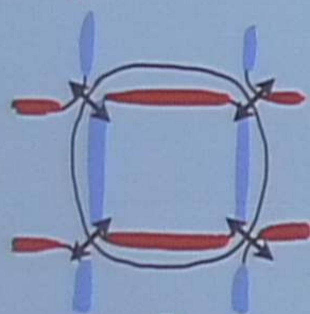

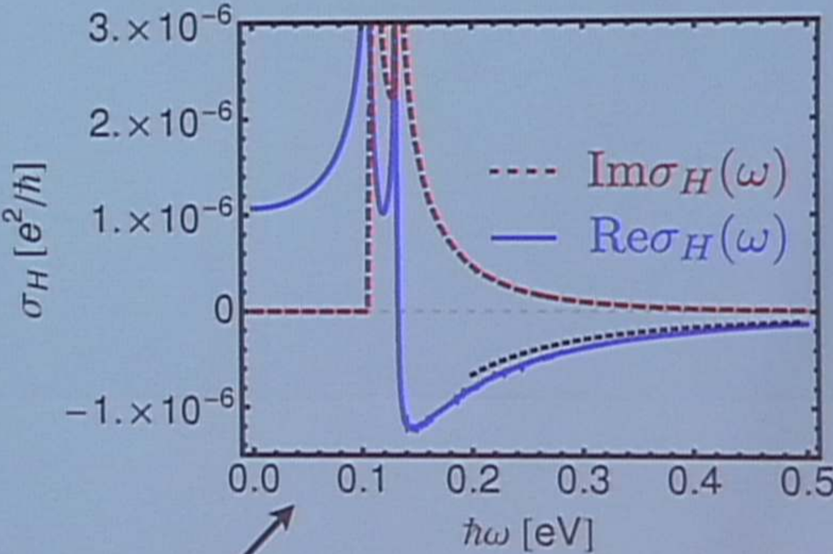

Hall conductivity in Sr_2RuO_4 using this model + typical parameters (d_{xz}, d_{yz} hopping ~ 0.04eV)

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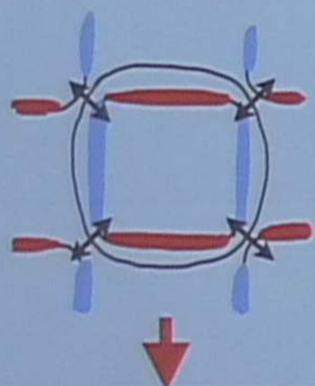

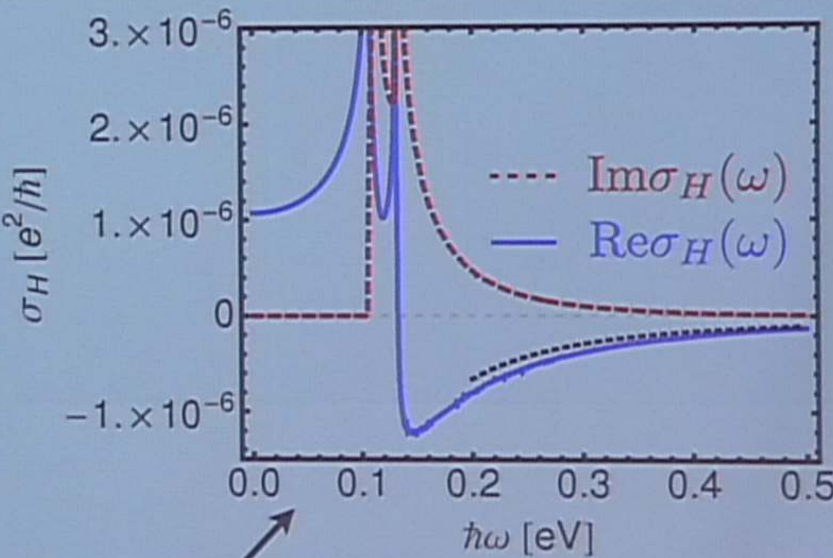

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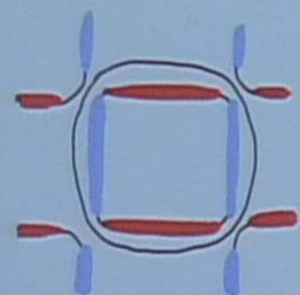
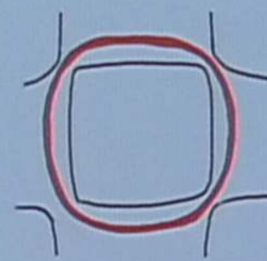


on d_{xz},d_{yz} orbitals

- Intrinsic Hall conductivity of the right order for Kerr rotation.
- Also extrinsic, impurity Hall conductivity, but not calculated.
- Chiral but not topological (Raghu *et al.*, PRL '10). No edge currents?



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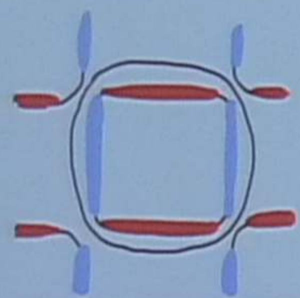

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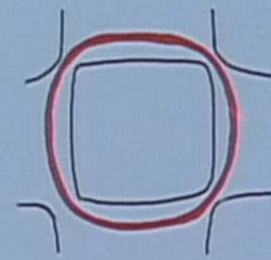

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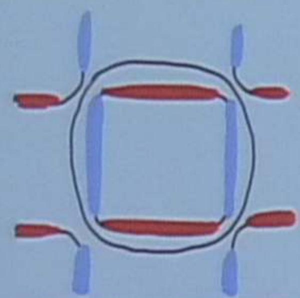

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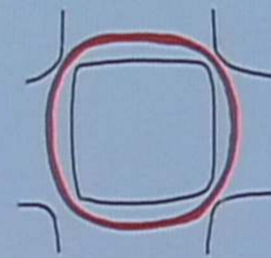

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E. Taylor & C. Kallin, PRL 108, 157001 (2012)