

Title: Molecular rotation in doped superfluid clusters

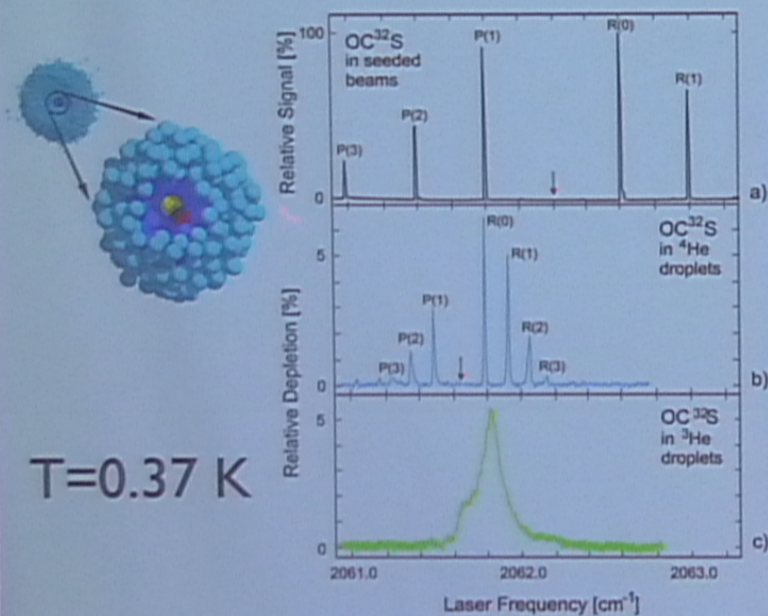
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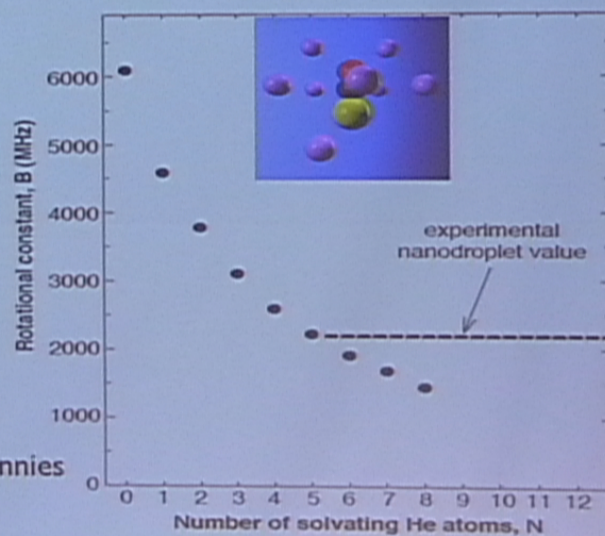
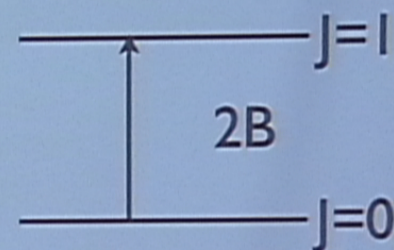
Abstract: Experiments where impurities were incorporated into helium nanodroplets have shown that the impurity freely rotates, and this has been attributed to the superfluidity of the nanodroplet [1]. Results from experiments with smaller helium clusters suggest that the onset of superfluidity is linked to system size and bosonic exchange effects [2]. We have used path integral techniques to investigate these systems and predict their spectroscopic behaviour in the microwave and the infrared regions of the spectrum. We are particularly interested in observing the superfluid response in clusters where the helium atoms have been substituted with parahydrogen molecules. Molecular hydrogen has been suggested as a potential candidate for the observation of superfluid response but this substance crystallizes before reaching a temperature low enough for superfluidity to appear. We will show theoretical and experimental results of a molecular superfluid response at the nanoscale via the formation of doped hydrogen clusters with a carbon dioxide probe molecule [3]. Properties such as density distributions, spectroscopic features, and effective rotational inertia can be extracted from the simulations. We will show new results for the case of asymmetric top molecules embedded in superfluid para-hydrogen clusters. A perspective on the current challenges of the field will be presented. [1] Grebenev, Toennies, and Vilesov, *Science* 279, 2083 (1998); Toennies and Vilesov, *Angew. Chem.-Int. Edit.* 43, 2622-2648 (2004). [2] Tang, Xu, McKellar and J  rger, *Science* 297, 2030 (2002) [3] Li, Le Roy, Roy, and McKellar, *Phys. Rev. Lett.* 105, 133401 (2010)

# Motivation

- Experiments on helium droplets and clusters
- Important observable: B constant



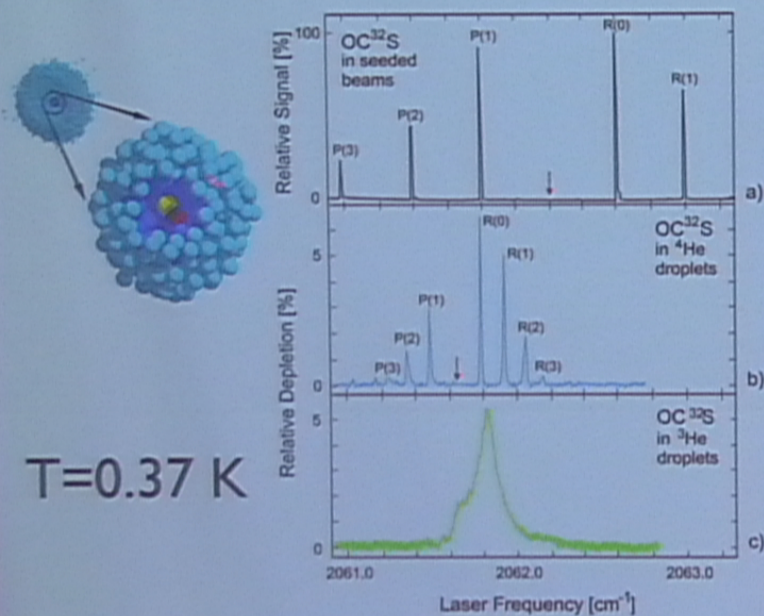
Grebenev, Toennies, and Vilesov, *Science* **279**, 2083 (1998); Toennies and Vilesov, *Angew. Chem.-Int. Edit.* **43**, 2622-2648 (2004).



J. Tang, Y. Xu, A. R. W. McKellar and W. Jäger, *Science* **297**, 2030 (2002)

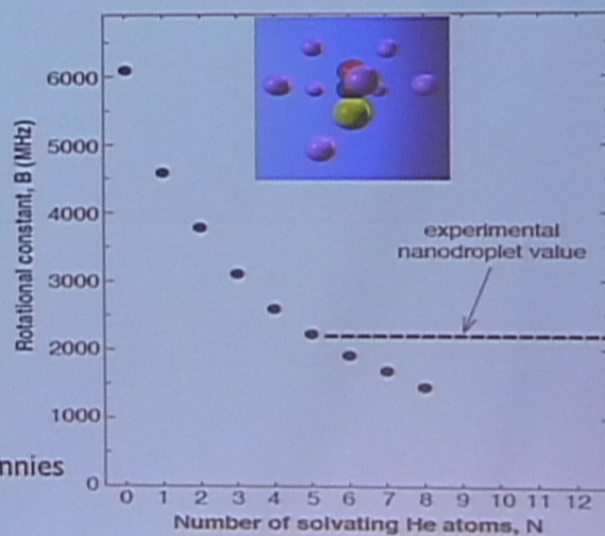
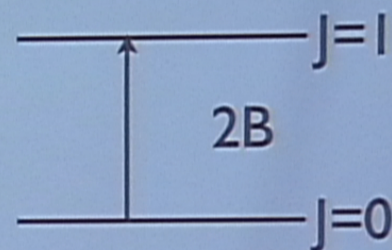
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T=0.37 K

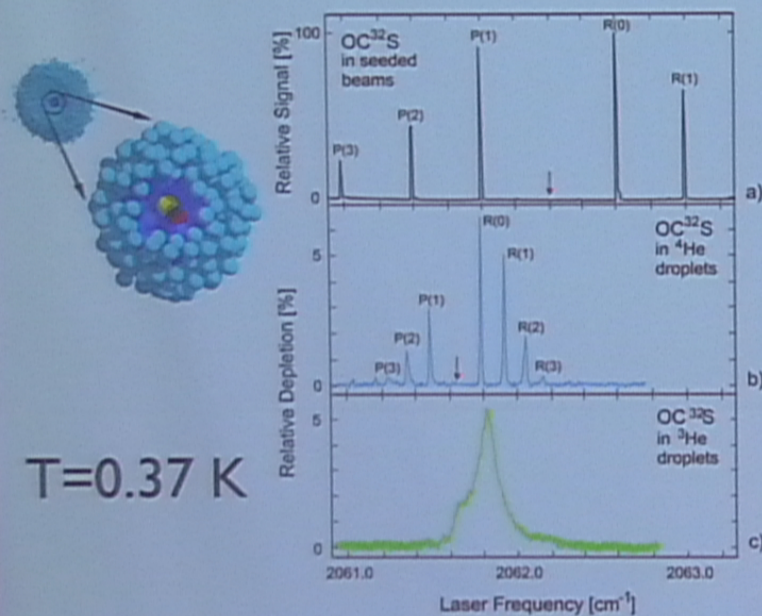
Grebenev, Toennies, and Vilesov, *Science* **279**, 2083 (1998); Toennies and Vilesov, *Angew. Chem.-Int. Edit.* **43**, 2622-2648 (2004).



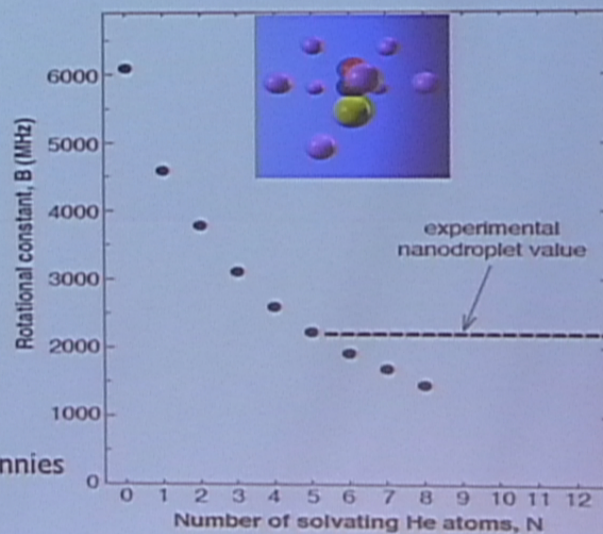
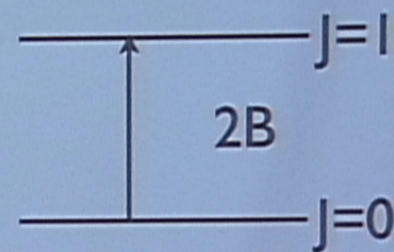
J. Tang, Y. Xu, A. R. W. McKellar and W. Jäger, *Science* **297**, 2030 (2002)

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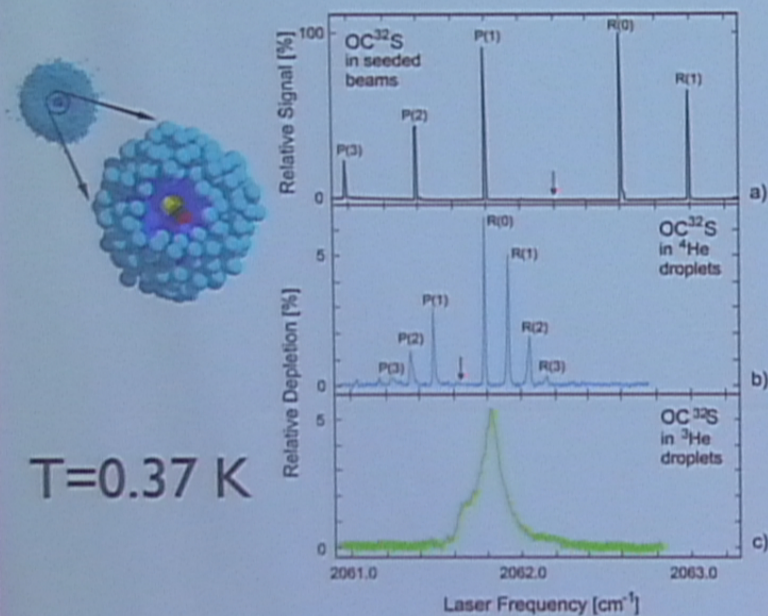


Grebenev, Toennies, and Vilesov, *Science* **279**, 2083 (1998); Toennies and Vilesov, *Angew. Chem.-Int. Edit.* **43**, 2622-2648 (2004).

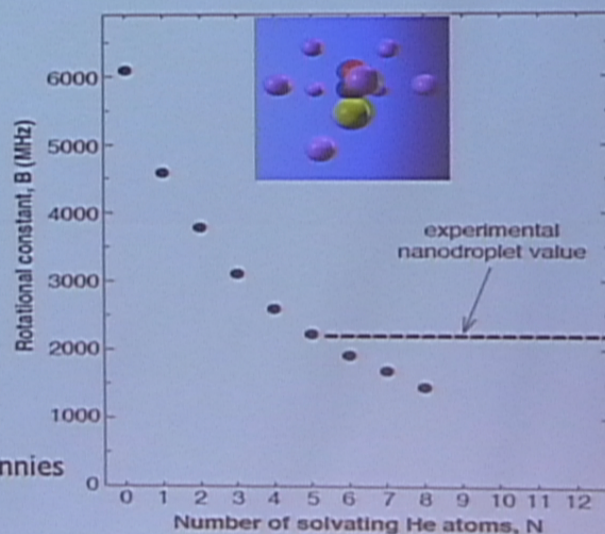
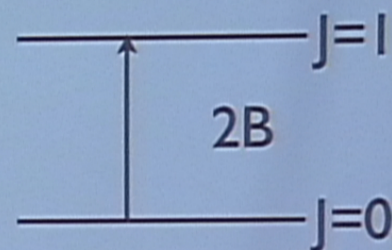
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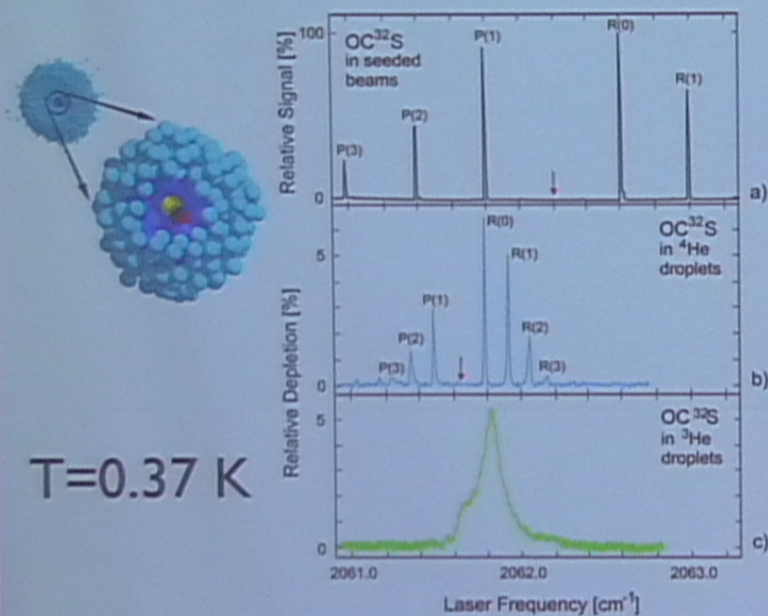
Grebenev, Toennies, and Vilesov, *Science* **279**, 2083 (1998); Toennies and Vilesov, *Angew. Chem.-Int. Edit.* **43**, 2622-2648 (2004).

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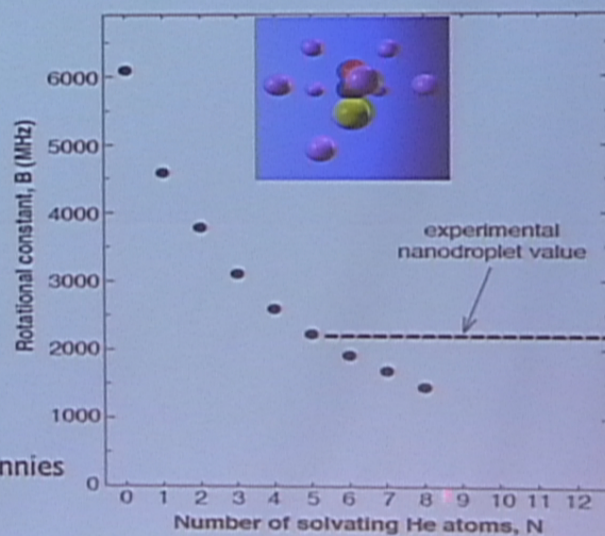
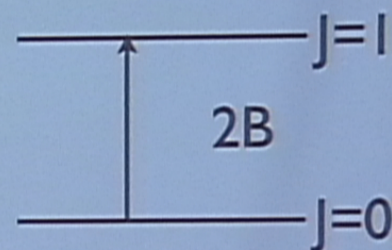


# Motivation

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Grebenev, Toennies, and Vilesov, *Science* **279**, 2083 (1998); Toennies and Vilesov, *Angew. Chem.-Int. Edit.* **43**, 2622-2648 (2004).

J. Tang, Y. Xu, A. R. W. McKellar and W. Jäger, *Science* **297**, 2030 (2002)

# Theoretical Tools

$$\hat{H} = \underbrace{B\hat{L}^2}_{\text{dopant rotation}} + \underbrace{\frac{\hat{P}^2}{2M}}_{\text{dopant translation}} + \sum_{i=1}^N \underbrace{\frac{\hat{p}_i^2}{2m_i}}_{\text{bosons: helium atoms}} + \sum_{i<j} v(|r_i - r_j|) + \sum_i V(r_i, R, \Omega)$$

dopant helium coupling  
**Anisotropic**

$$Z = \text{Tr} e^{-\beta\hat{H}}$$

Path integrals with exchange (Bose-Einstein statistics)  
Sampling using Monte Carlo (PIMC)

## Properties:

Energy, Structure (densities)

Correlation functions (imaginary time): dipole-dipole

Response properties: effective inertia

**Using worm algorithm:** M. Boninsegni, N.V. Prokof'ev and B.V. Svistunov, Phys. Rev. E 74, 036701 (2006). M. Boninsegni, N.V. Prokof'ev and B.V. Svistunov, Phys. Rev. Lett. 96, 070601 (2006).

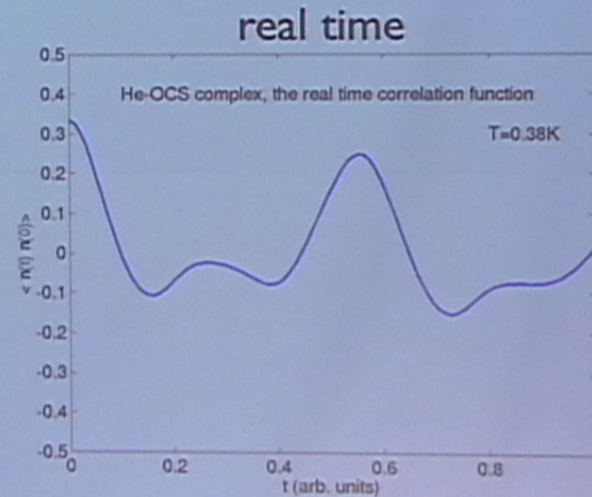
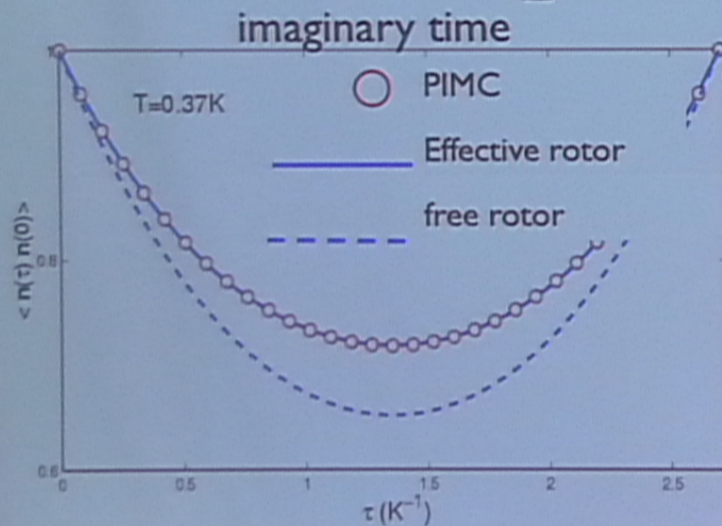


# Can we obtain spectral features?

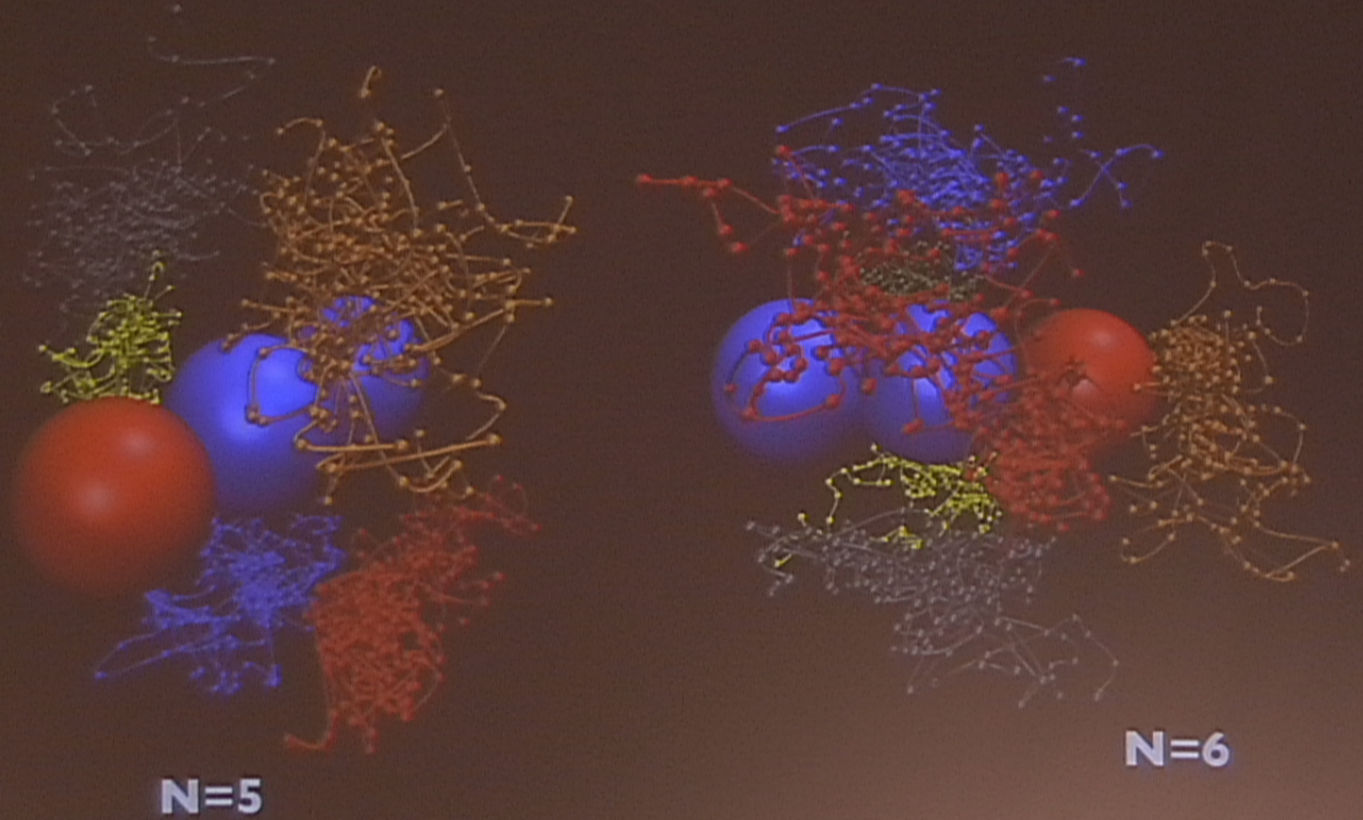
Rotational spectrum: Fourier transform of the real time dipole auto-correlation function (difficult to calculate for large systems)

We opt for its imaginary time counterpart and **model** fit:

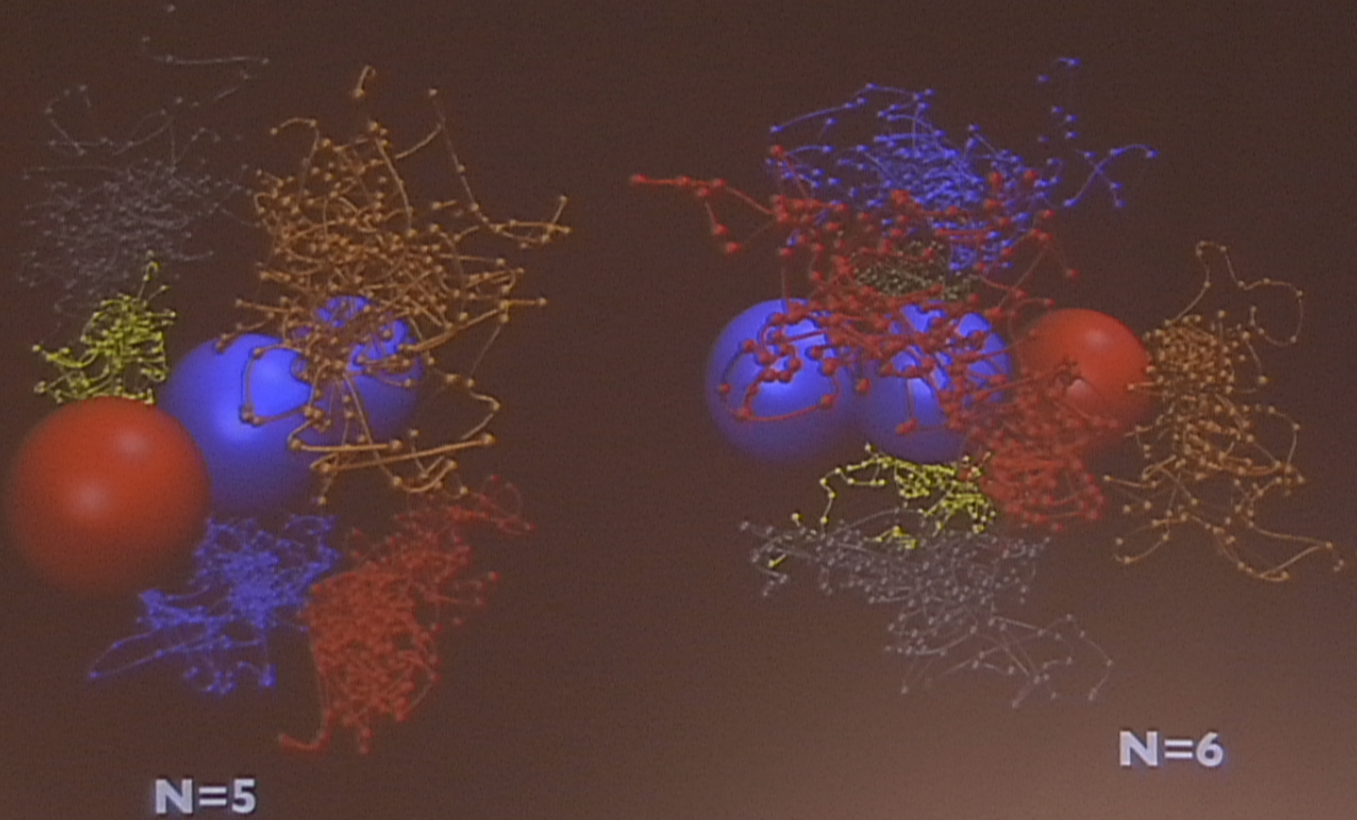
$$\langle \hat{\mathbf{n}}(\tau) \cdot \hat{\mathbf{n}}(0) \rangle = \frac{1}{Z} \text{Tr} \left\{ e^{-\beta \hat{H}} e^{\tau \hat{H}} \hat{\mathbf{n}} e^{-\tau \hat{H}} \cdot \hat{\mathbf{n}} \right\}$$



# Paths for helium atoms

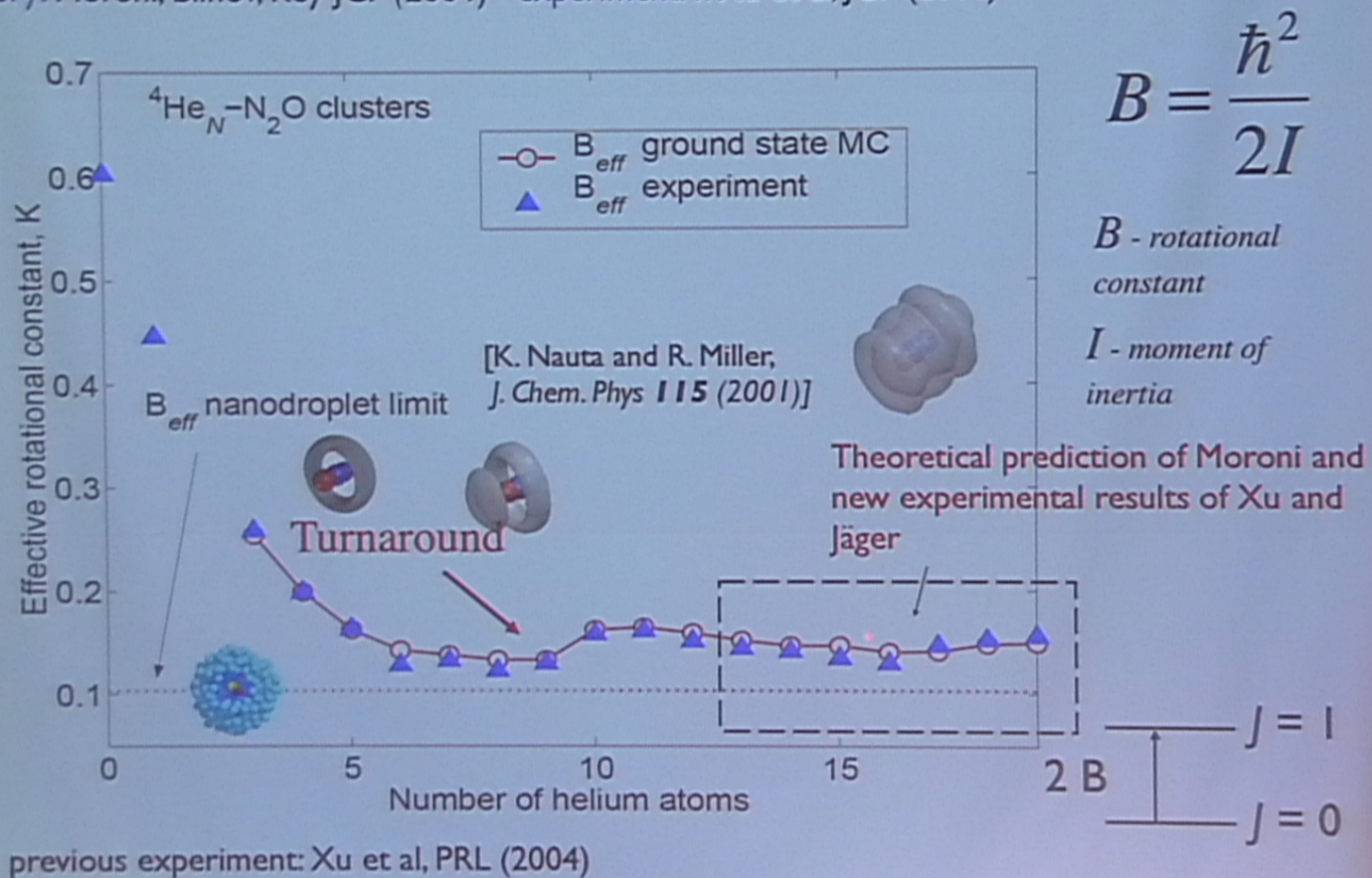


# Paths for helium atoms



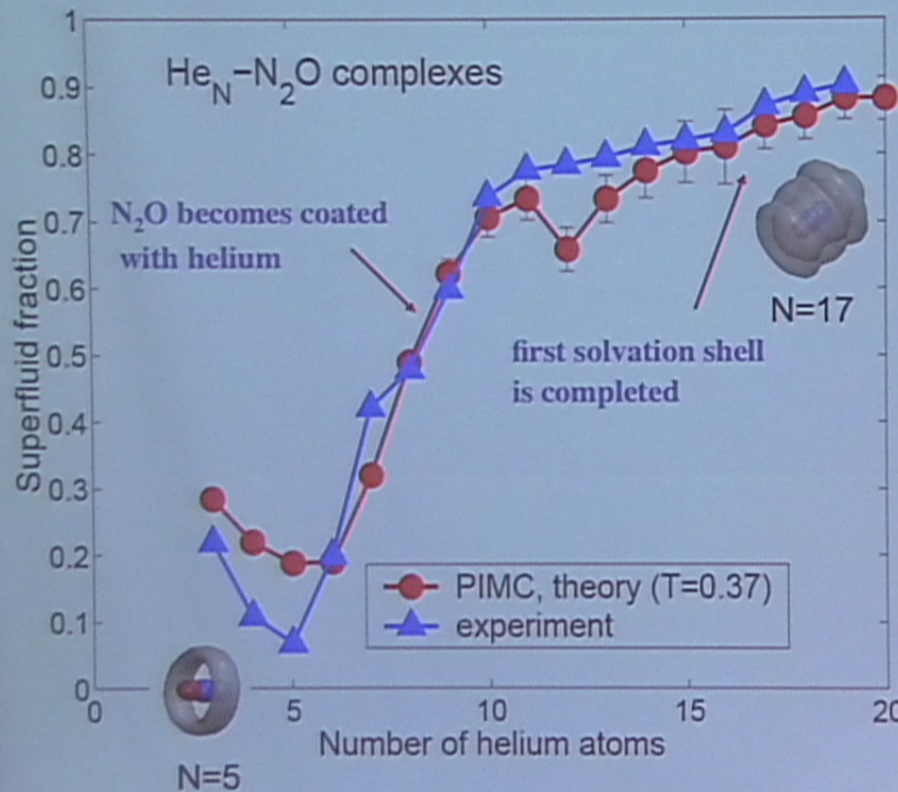
# Recurrences in rotational dynamics

theory: Moroni, Blinov, Roy JCP (2004) experiment: Y. Xu et al, JCP (2006)



previous experiment: Xu et al, PRL (2004)

# Proposal: dopant molecule as a probe of superfluidity at the microscopic level



Superfluid fraction:

$$f_s = 1 - \frac{I_n}{I_o}$$

with  $I_n$  deduced from spectroscopic experiments:

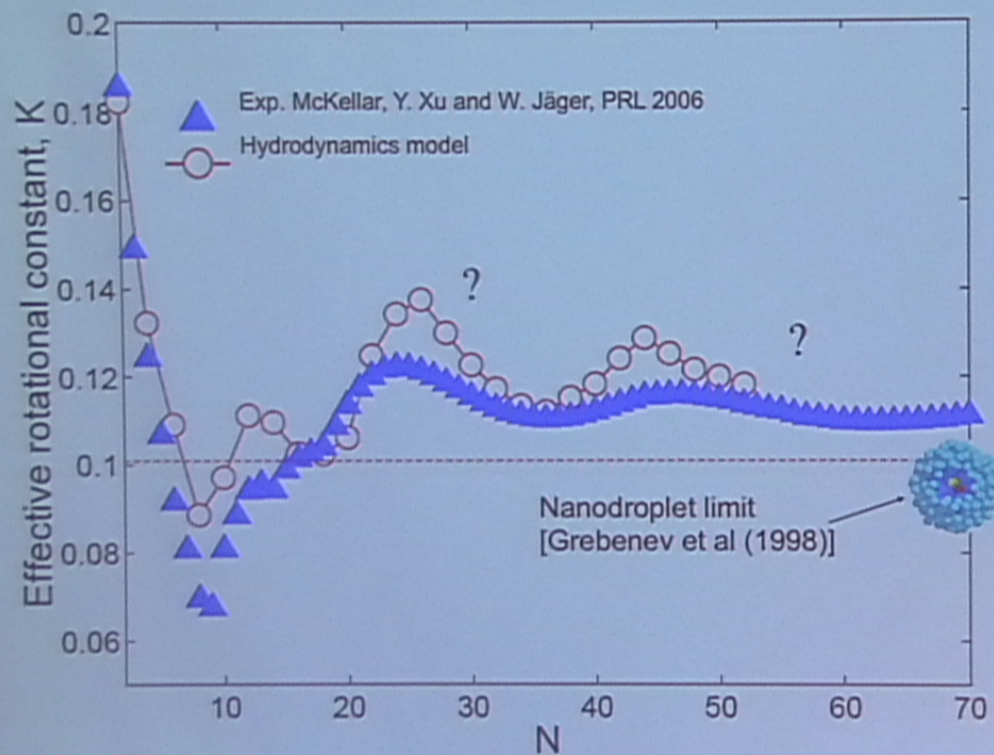
$$I_n = \frac{\hbar^2}{2B} - I^{imp}$$

and  $I_o$  obtained from MC calculations:

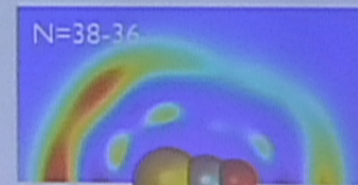
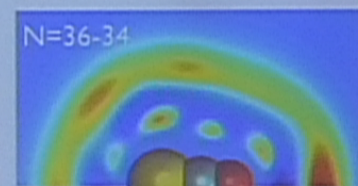
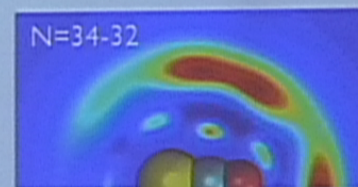
$$I_o = \int \rho_{total} r_{\perp}^2 dV$$

Xu, Blinov, Jager, Roy JCP (2006)

# He<sub>N</sub>-OCS complexes: oscillations in B



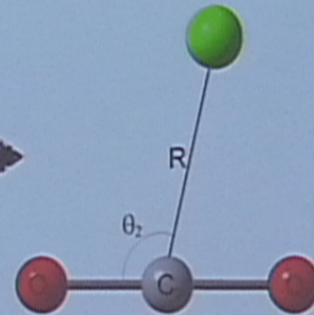
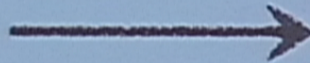
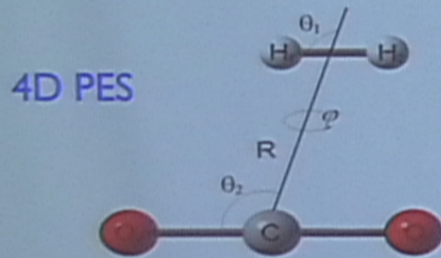
Differential densities



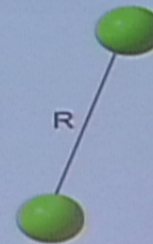
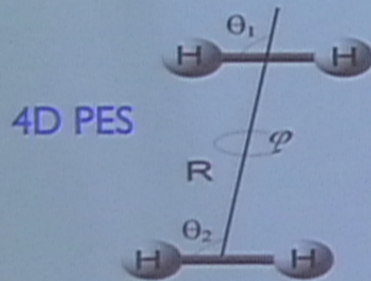
# Reduced Dimension PES

Rotational constant  $\text{CO}_2$  vs.  $\text{H}_2$ :

$$\frac{B_{\text{H}_2}}{B_{\text{CO}_2}} = \frac{59.322}{0.390} \approx 152$$



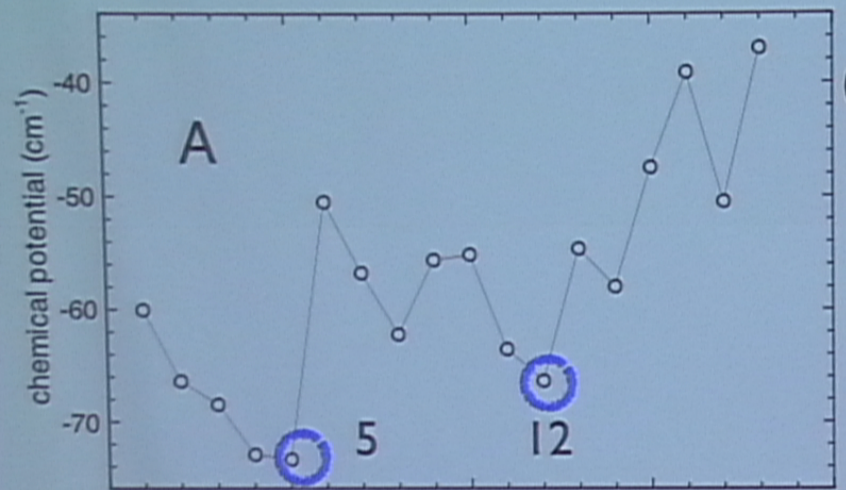
2D PES



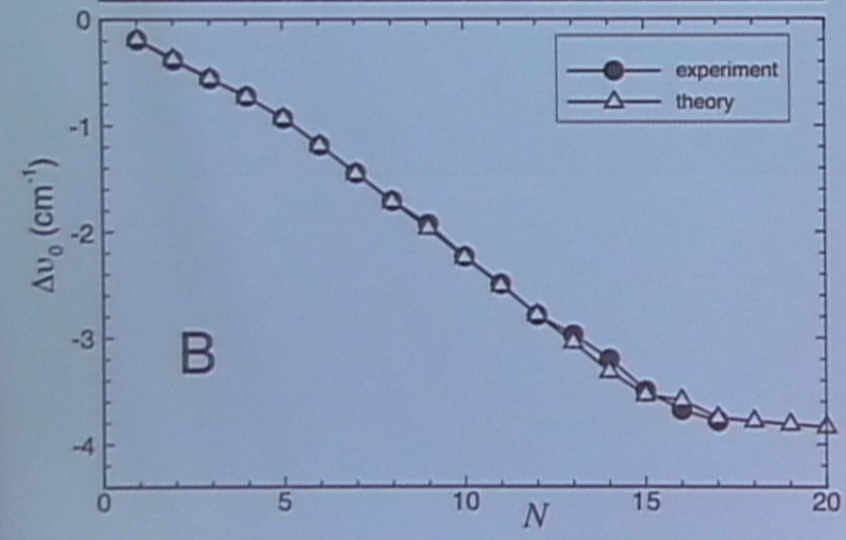
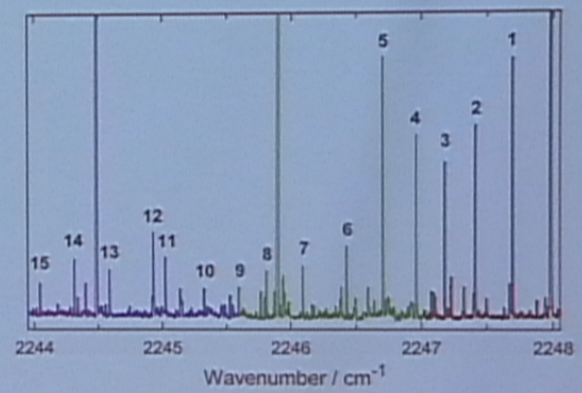
1D PES

Li, Roy, and Le Roy, *J. Chem. Phys.* **132**, 214309 (2010); *ibid* **133**, 104305 (2010).

# CO<sub>2</sub>-(para-H<sub>2</sub>)<sub>N</sub>



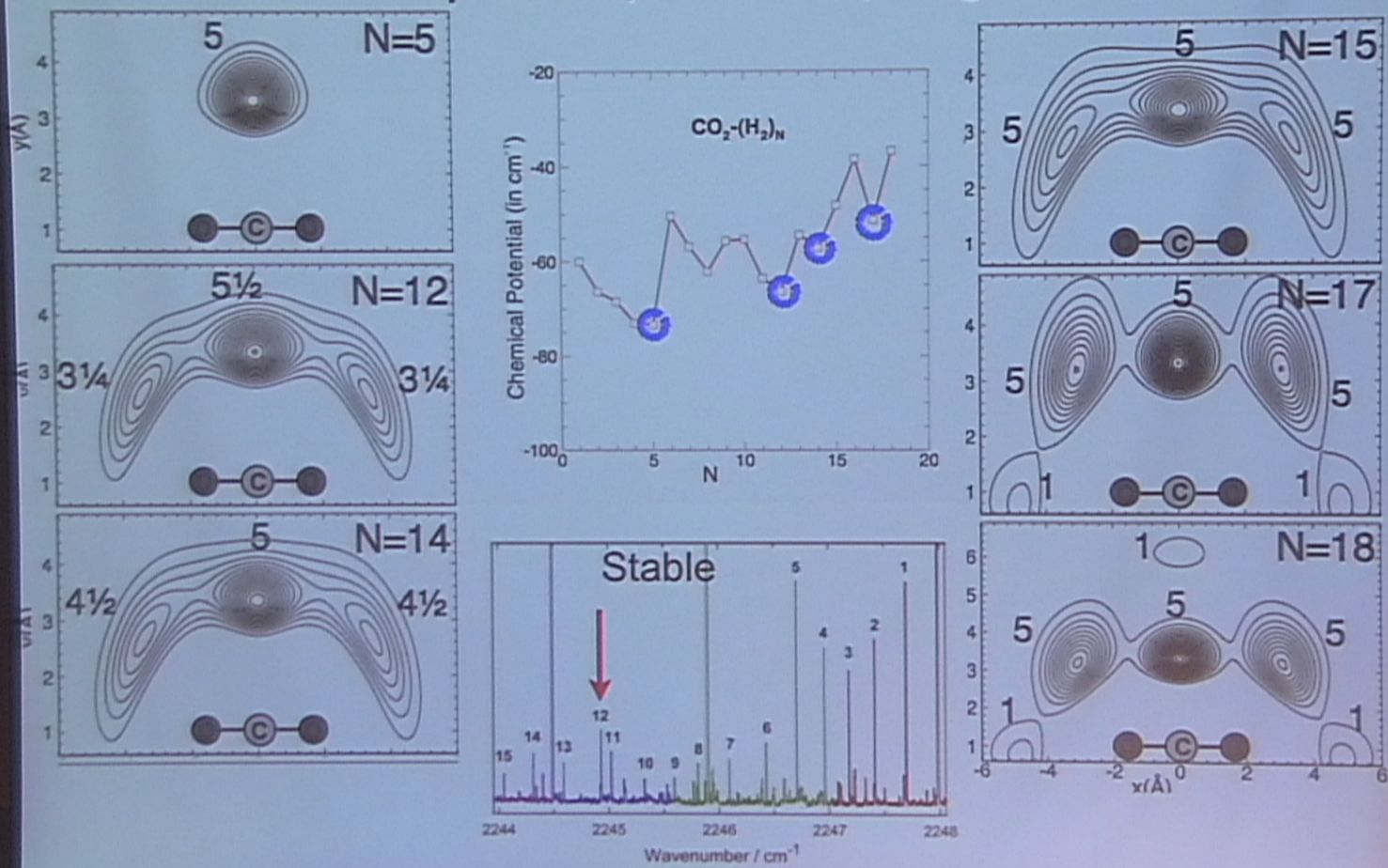
Experimental spectrum



McKellar, Ottawa



# Structure and magic numbers: *partial, fractional filling*



$$B_{\text{classical}} = \frac{\hbar^2}{2(I_{\text{CO}_2} + I_{\text{H}_2}^{\text{classical}})}$$

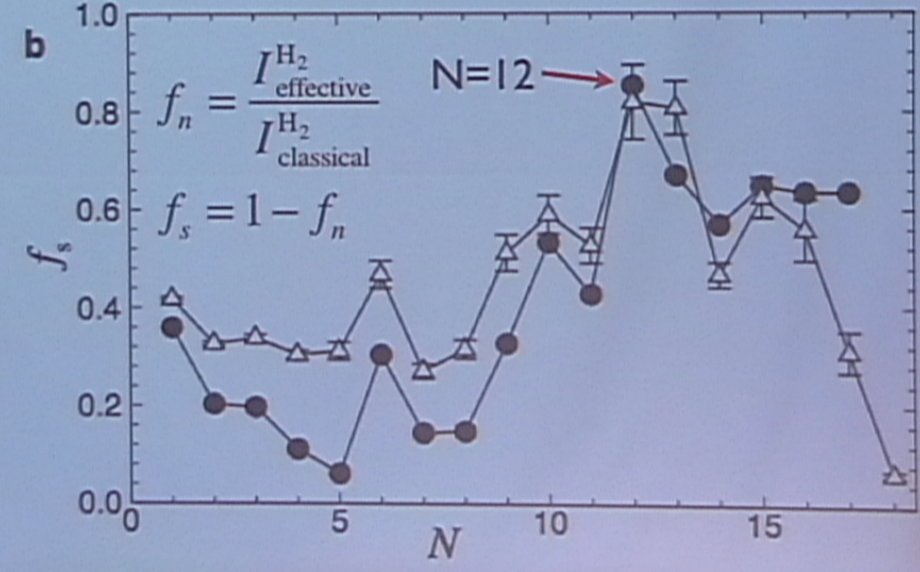
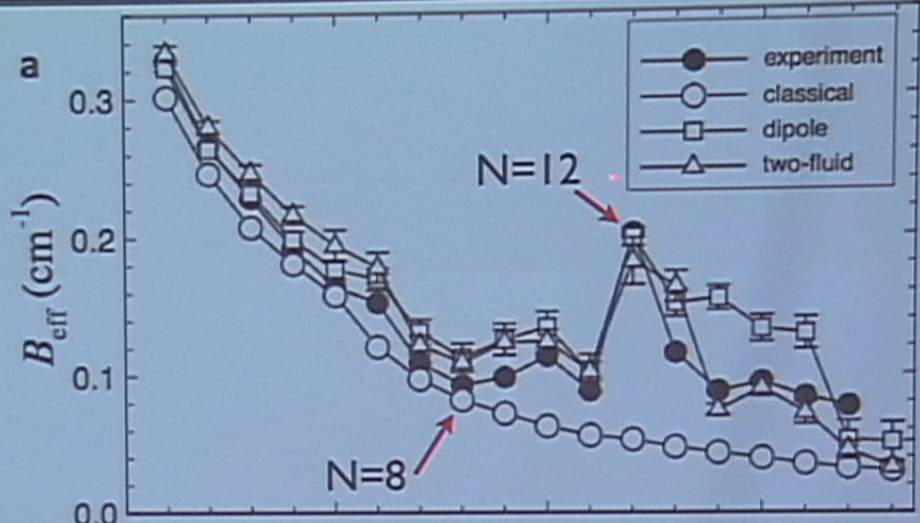
$$B_{\text{two fluid}} = \frac{\hbar^2}{2(I_{\text{CO}_2} + I_{\text{H}_2}^{\text{effective}})}$$

$$\frac{I_{\text{H}_2}^{\text{effective}} \omega^2}{2} = F_{\omega}$$

$$I_{\text{H}_2}^{\text{effective}} = \frac{\partial^2 F_{\omega}}{\partial \omega^2}$$

$$F_{\omega} = -k_B T \ln Z_{\omega}$$

N=12 Maximum  
corresponds to global  
fractional filling



# Conclusions

- Confirmed superfluid response to probe rotation in doped para-H<sub>2</sub> clusters
- Turnaround in  $B$  constants is a direct sign of the onset of decoupling
- First experimental determination of superfluid fraction in a **molecular** superfluid. Theory was necessary to interpret this experiment.
- But superfluid response seems to die out as  $N$  increases because of solidification

PRL 105, 133401 (2010)

PHYSICAL REVIEW LETTERS

week ending  
24 SEPTEMBER 2010

## Molecular Superfluid: Nonclassical Rotations in Doped Para-Hydrogen Clusters

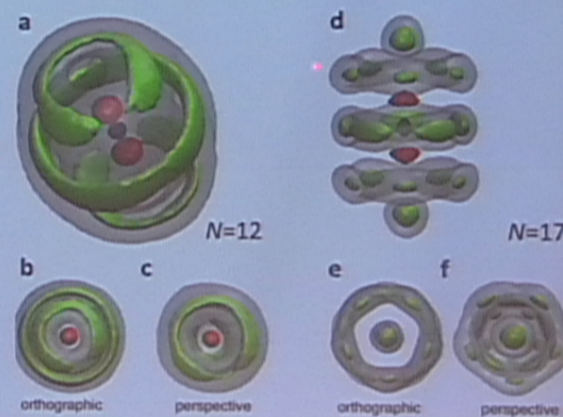
Hui Li,<sup>1,2</sup> Robert J. Le Roy,<sup>1</sup> Pierre-Nicholas Roy,<sup>1,\*</sup> and A. R. W. McKellar<sup>3,†</sup>

<sup>1</sup>Department of Chemistry, University of Waterloo, Waterloo, Ontario, N2L 3G1, Canada

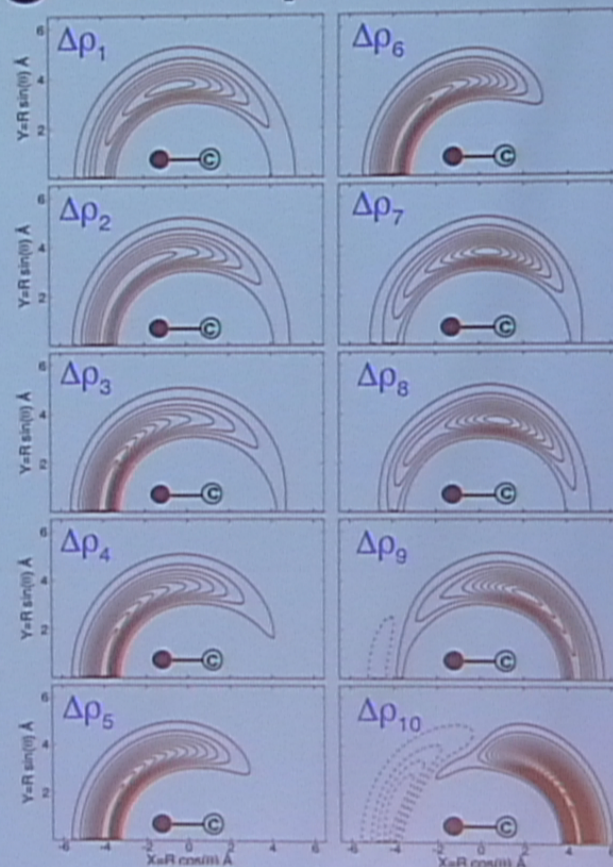
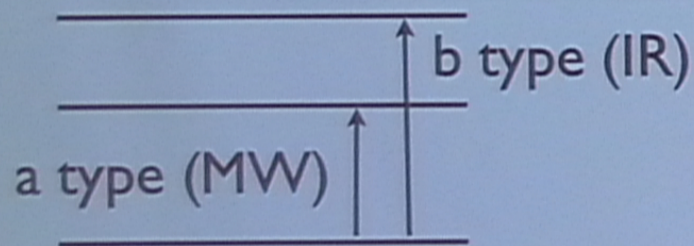
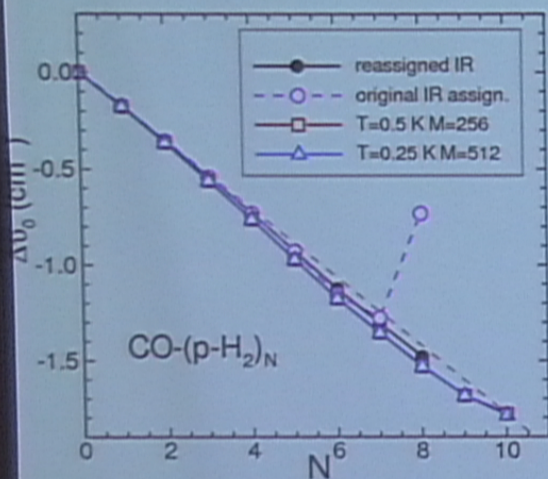
<sup>2</sup>Institute of Theoretical Chemistry, State Key Laboratory of Theoretical and Computational Chemistry, Jilin University, 2519 Jiefang Road, Changchun 130023, People's Republic of China

<sup>3</sup>Steele Institute for Molecular Sciences, National Research Council of Canada, Ottawa, Ontario K1A 0R6, Canada  
(Received 21 April 2010; published 23 September 2010)

Clusters of para-hydrogen ( $p\text{H}_2$ ) have been predicted to exhibit superfluid behavior, but direct observation of this phenomenon has been elusive. Combining experiments and theoretical simulations, we have determined the size evolution of the superfluid response of  $p\text{H}_2$  clusters doped with carbon dioxide ( $\text{CO}_2$ ). Reduction of the effective inertia is observed when the dopant is surrounded by the  $p\text{H}_2$  solvent. This marks the onset of molecular superfluidity in  $p\text{H}_2$ . The fractional occupation of solvation rings around  $\text{CO}_2$  correlates with enhanced superfluid response for certain cluster sizes.

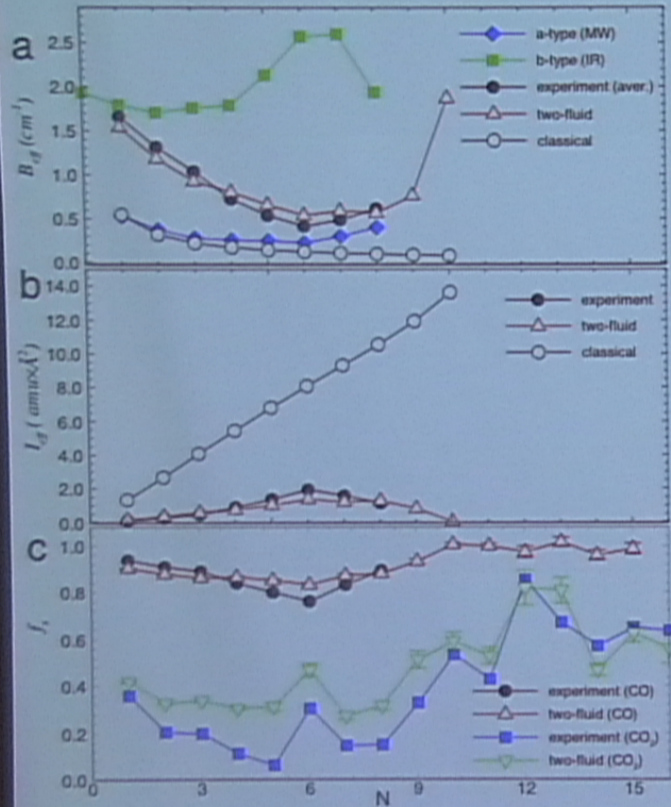


# CO, a lighter and gentler probe



Raston, Jager, Li, Le Roy, and Roy, *Phys. Rev. Lett.*, accepted

# A persistent molecular superfluid response



$$H = H_0 - \mu_z E_z \cos \alpha$$

perturbation theory

$$\Delta E_0^{(2)} = -\sum_n \frac{|\langle 0 | \cos \alpha | n \rangle|^2 \mu_z^2 E_z^2}{E_n - E_0}$$

$$\Delta E_0^{(2)} = \frac{-\mu_z^2 E_z^2}{6B} \quad \text{linear rotor model}$$

$$I_{eff} = \frac{-3\hbar^2}{\mu_z^2 E_z^2} \Delta E_0^{(2)} \quad \text{total effective inertia}$$

$$I_{eff} = \sum_n \frac{3|\langle 0 | \cos \alpha | n \rangle|^2 \hbar^2}{E_n - E_0}$$

$$I_{H_2} = I_{eff} - I_{CO}$$

New experimental estimator of  $f_s$ !

Raston, Jager, Li, Le Roy, and Roy, *Phys. Rev. Lett.*, accepted

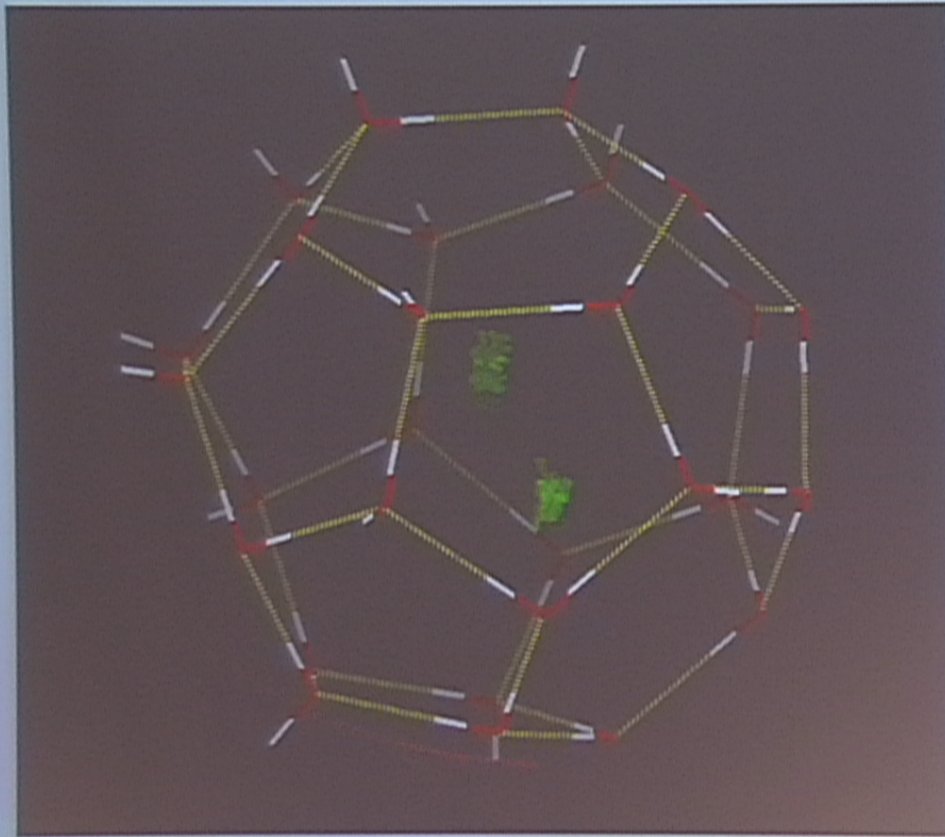
# Beyond linear dopants?

## **Water**

- a lighter and faster rotor
- asymmetric top molecule
- theoretical challenge and important coding work
- first PIMC simulation of a solvated asymmetric top with bosonic exchange for the environment.

Zeng, Li, and Roy, *Phys. Rev. Lett.*, submitted

# Many waters, a few hydrogens: clathrate hydrates



2 parahydrogen  
molecules in a  
water cage at 100K