Title: Classical Wigner Crystals on Flat and Curved Surfaces, Topological Defects, `Pleats â€˜ and Particle Fractionalization

Date: May 03, 2012 01:30 PM

URL: http://pirsa.org/12050037

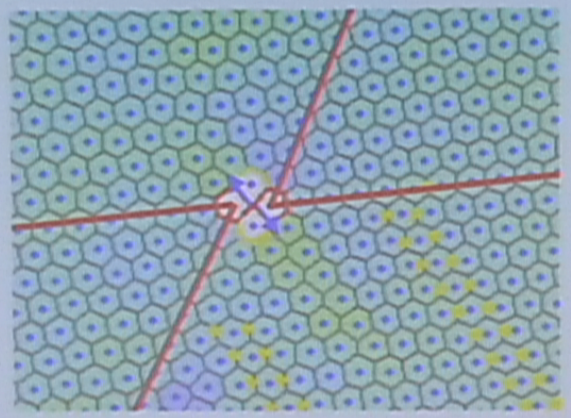
Abstract: Charged colloidal particles present a controllable system for study a host of condensed matter/many body problems such as crystallization. 2D crystals are invariably hexagonal. Hexagons perfectly tile a flat plane but a soccer ball requires exactly 12 pentagons dispersed among the hexagons on its curved surface. Pentagons and hexagons are positive and negative topological charges, disclinations, sources for positive and negative curvature. But we have discovered that â€œPleatsâ€•, grain boundaries which vanish on the surface (and play a similar role to fabric pleats) can provide a finer control of curvature. We experimentally investigate the generation of topological charge as flat surfaces are curved. For positive curvature, domes and barrels, there is one pentagon added for every 1/12 of a sphere. Negative curvature is different! For capillary bridges forming catenoids, pleats relieve the stress before heptagons appear on the surface. Pleats are important for controlling curvature from crystals on surfaces, to the shape of the spiked crown of the Chrysler building. Adding a particle to a flat surface produces an interstitial - usually an innocuous point defect. On a curved surface interstitials are remarkable, forming pairs or triplets of dislocations which can fission dividing the added particles into fractions which migrate to disclinations.   Work done with William Irvine, e.g. Nature 468, 947 (2010).

Classical Wigner Crystals on Flat and Curved Surfaces

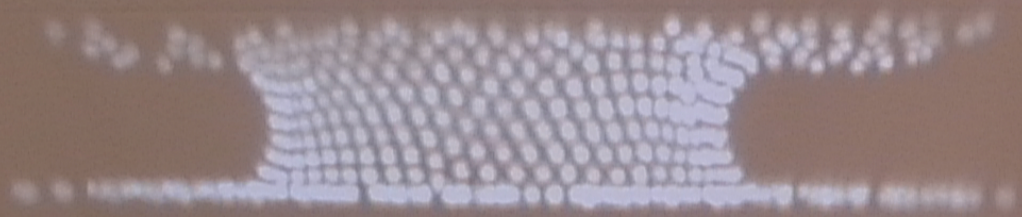
topological defects, "Pleats" and classical particle fractionalization

4 Corner Southwest Ontario Condensed Matter Symposium 2012
Perimeter Institute Waterloo Ont. 3 May 2012

Creating dislocation pairs



Dislocations and Disclinations on A Capilary Bridge



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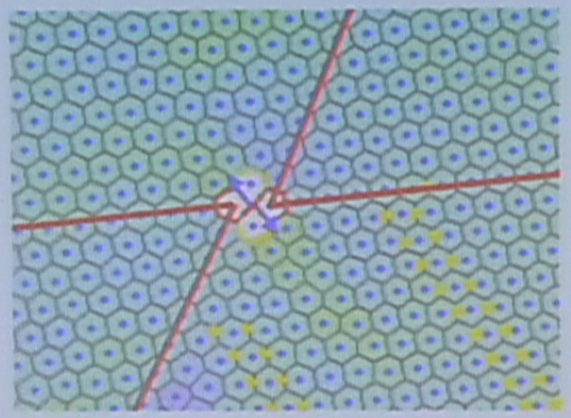

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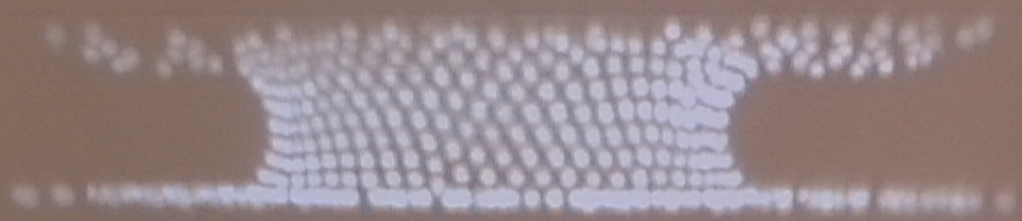
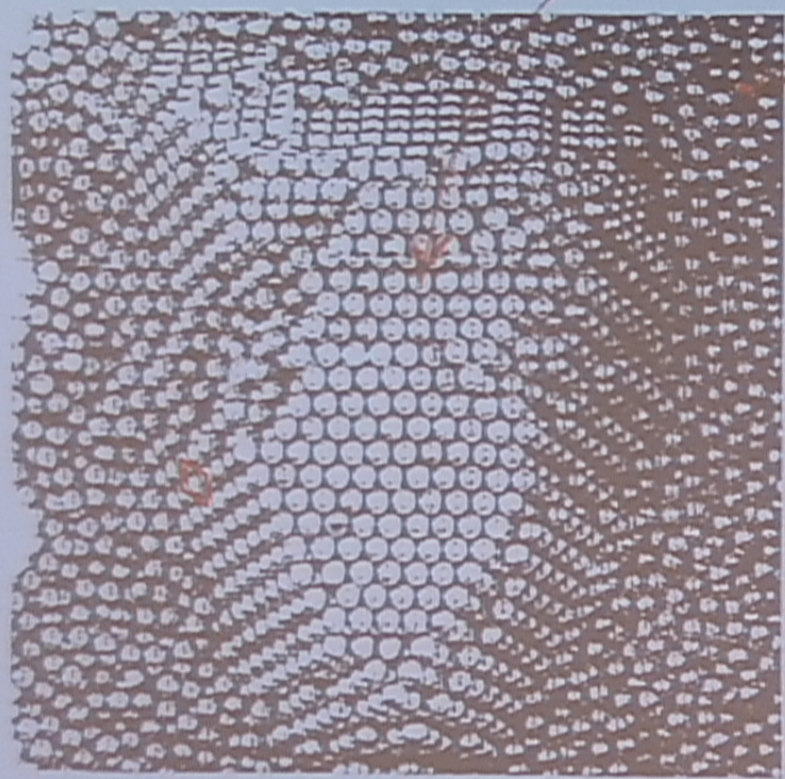


Packing Densities for Spheres

$\phi_{FCC}=0.74$

$\phi_{RCP}=0.64$

MRJ
maximally random jammed



Ball Bearings

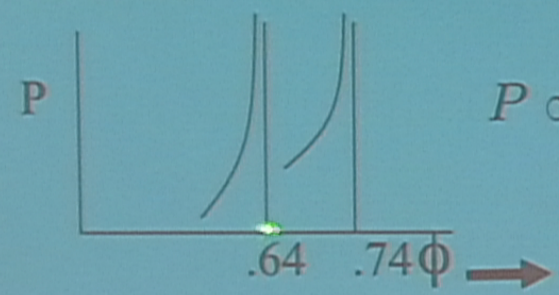
Figure 14. Face-centred cubic 'crystal' surrounded by 'liquid' caused by shearing ball-bearing mass. 111 face is shown at the top surface.

van der Waals and excluded volume

$$S = Nk_B \ln(V - Nb) = Nk_B \ln(V(1-\phi/\phi_C)) \qquad P = \frac{Nk_BT}{V - Nb} = \frac{Nk_BT}{V(1-\phi/\phi_C)}$$

Exact in 1D

Exact asymptotic form in any dimension



$$P \propto \frac{dnk_BT}{(1-\phi/\phi_c)}$$

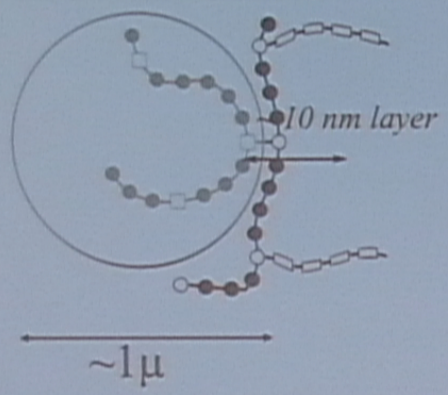
⇒ ***Entropy drives liquid to crystal***

$S_{liquid} \to 0$ as $\phi \to .64$ $\quad S_{crystal} \to 0$ as $\phi \to .74$

Highest Packing Fraction determines Stable High Density Phase

Polymer Hard Spheres - Colloids in Oil

PMMA-PHSA



Steric stabilization
in decalin - tetralin

methyl methacrylate (•); methacrylic acid (▫); glycidyl methacrylate (◦); 12-hydroxystearic acid (▭)

Originally Ron Ottewill - Bristol
ours were from Andy Schofield - Edinburgh
now home grown by **Andy Hollingsworth**

“Hard Sphere” Colloidal Sample

60% volume fraction

Crystallized in microgravity in space

Remelted in gravity forms “glass phase”

remains glass after ~ 1 year

7

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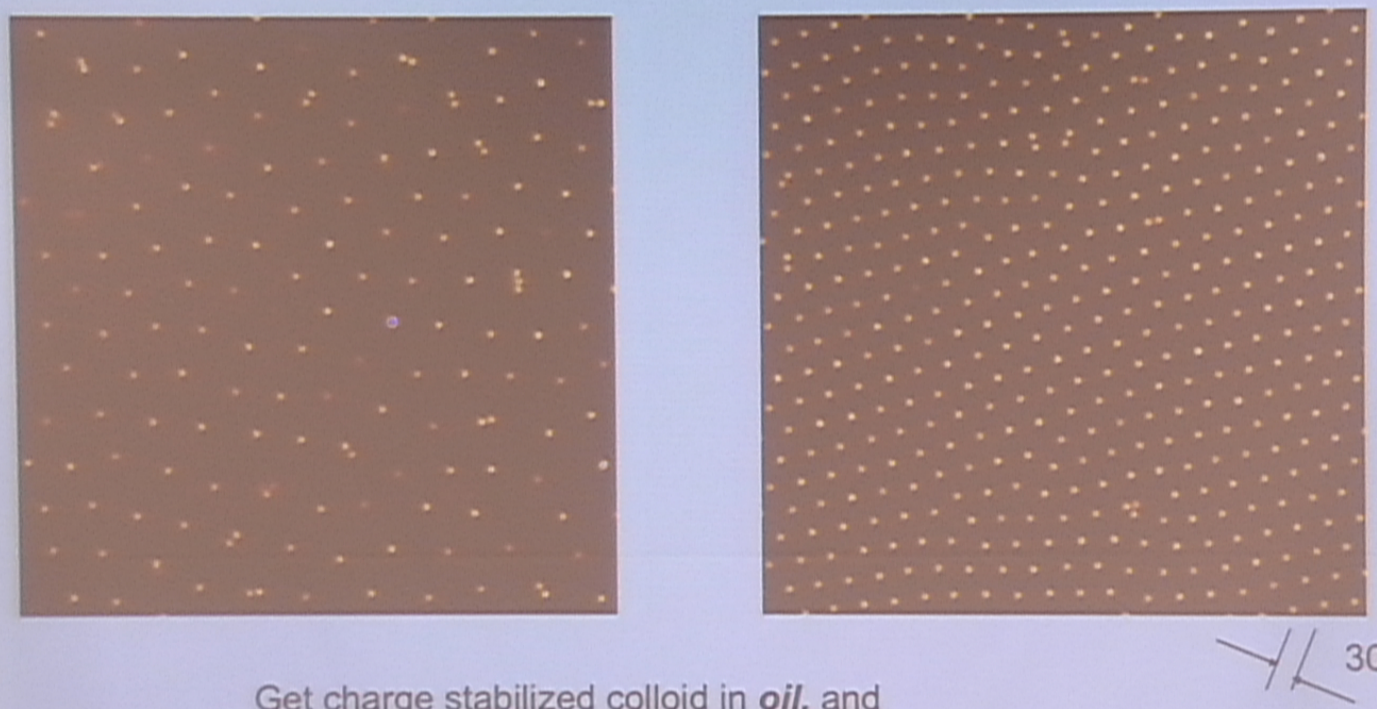
Try to density match in decalin – CHB (cyclohexylbromide)

Coulomb crystals

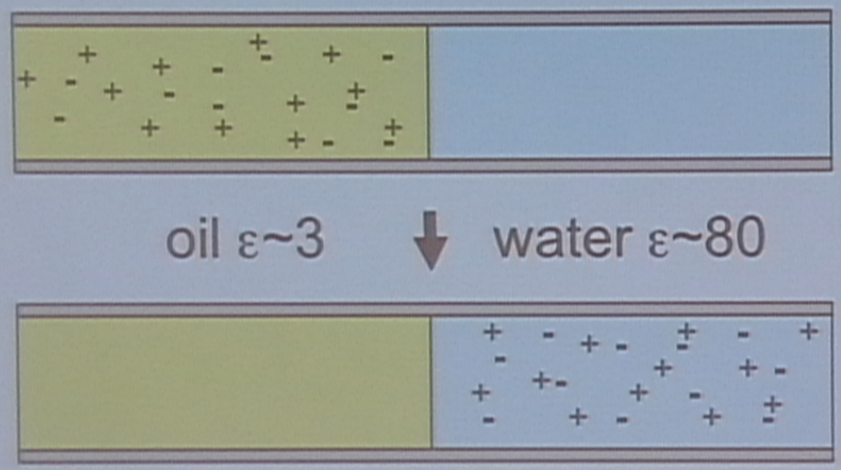


Get charge stabilized colloid in ***oil,*** and screening length is enormous, $\lambda > 30\mu$

Van Blaaderen - Utrecht

Only get colossal crystals in some samples.

What's different? ---------- Water?



water has much higher ε

should suck ions out of oil

$$E = -\frac{\varepsilon E^2}{2}$$

Try to density match in decalin – CHB (cyclohexylbromide)

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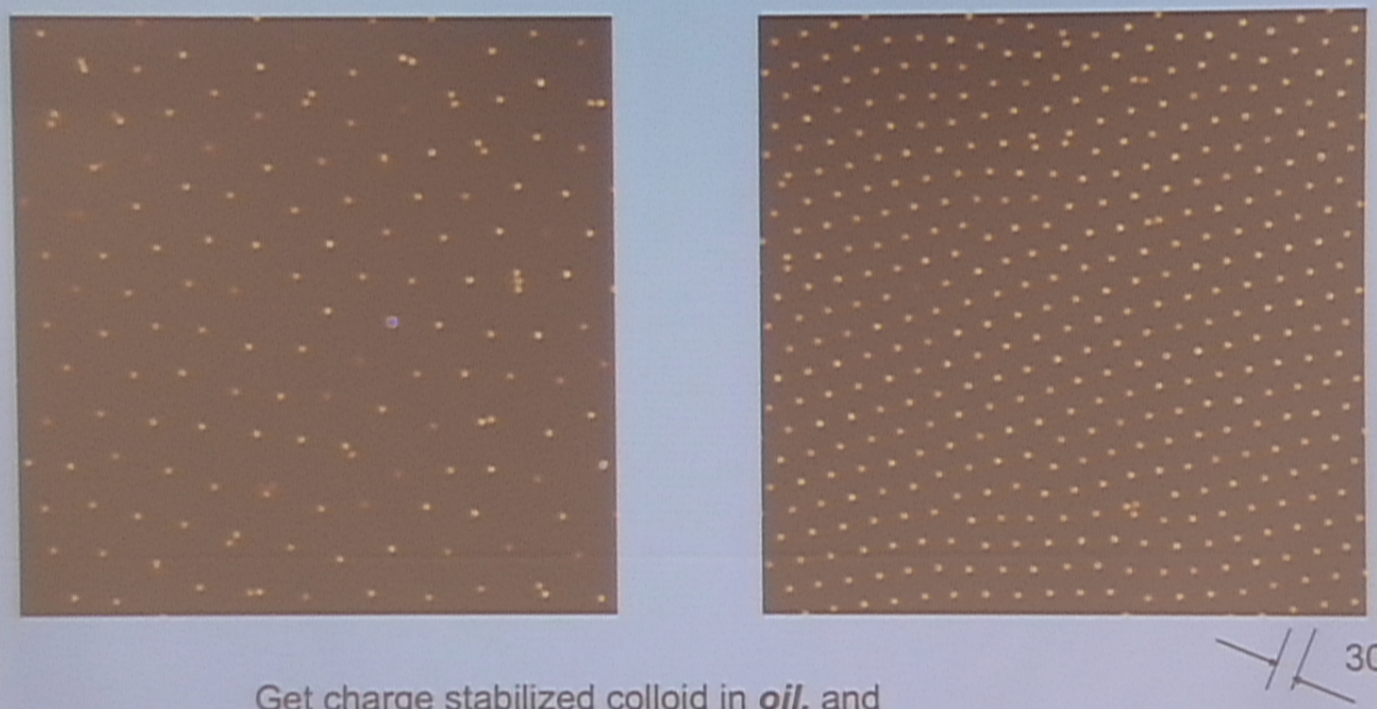

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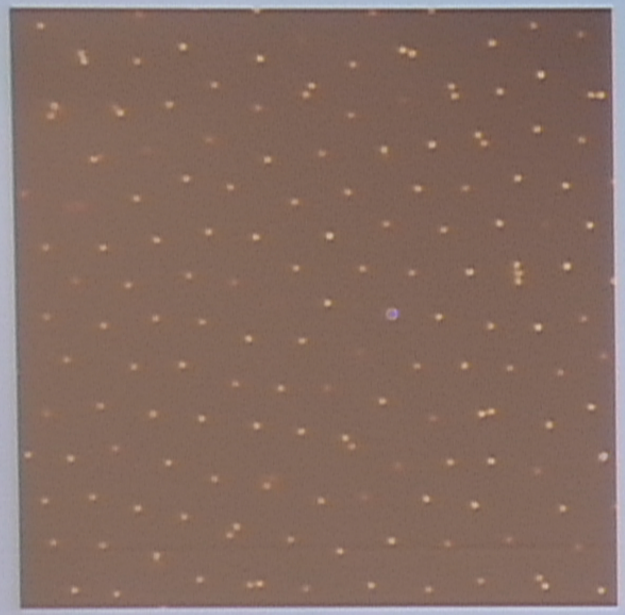

30μ

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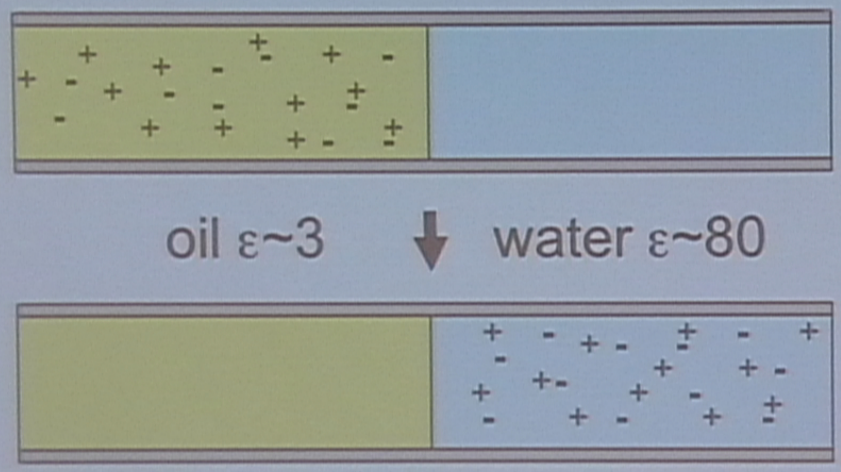

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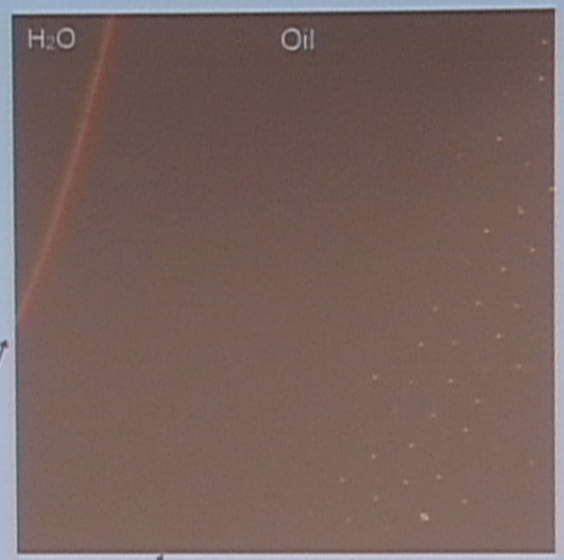
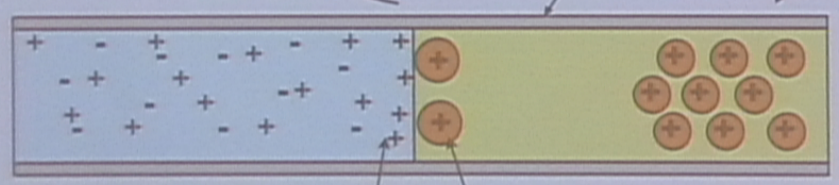
Add water to half of cell

Confocal veiw of a layer in the cell





Depleted zone

Colossal crystal

Interface is probably charged – surface chemistry?

Monolayer held by image charge

If water Droplets differentially pump ions from oil then we should see charging effects without colloids present

Two samples of clean cc CHB/decalin, with a drop of water added and sonicated:

- Wigner Crystal of 2-3 micron water droplets
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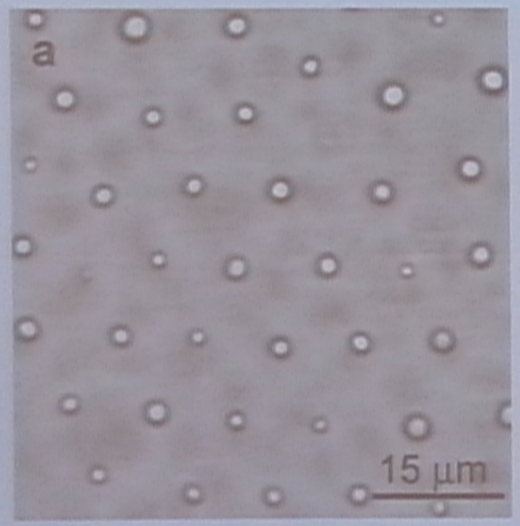
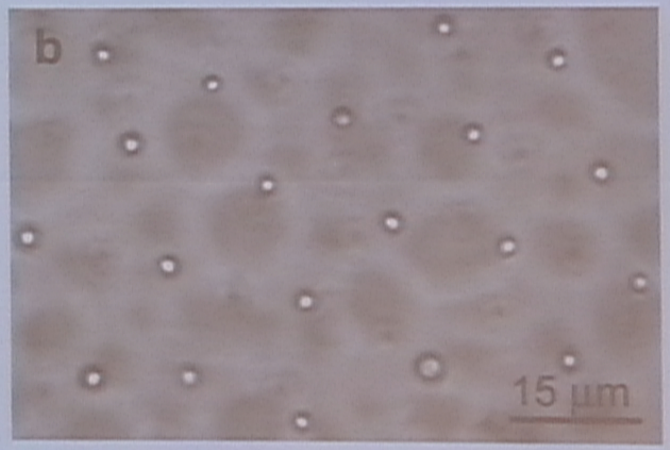
- Shake instead of sonicate, big, polydisperse, charge stabilized emulsion

New form of crystallized water
- Ice 10?



H2O - CHB



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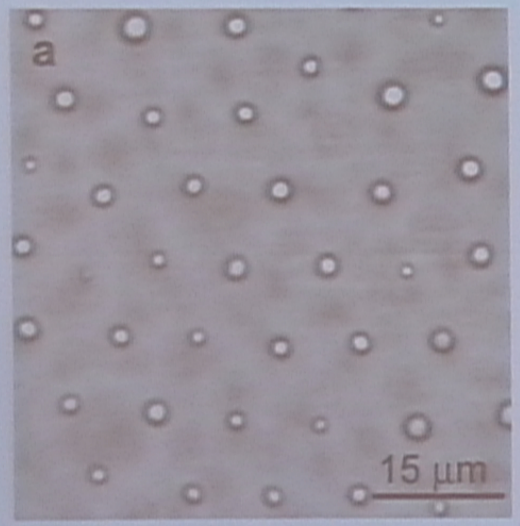

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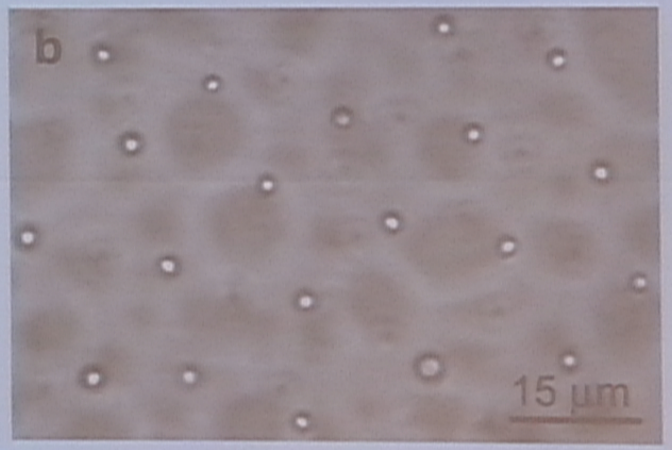

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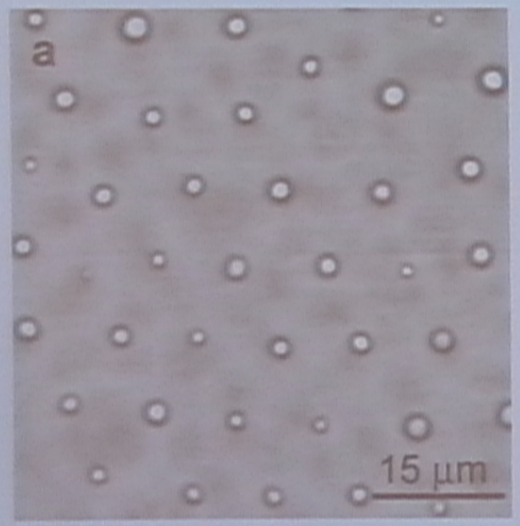

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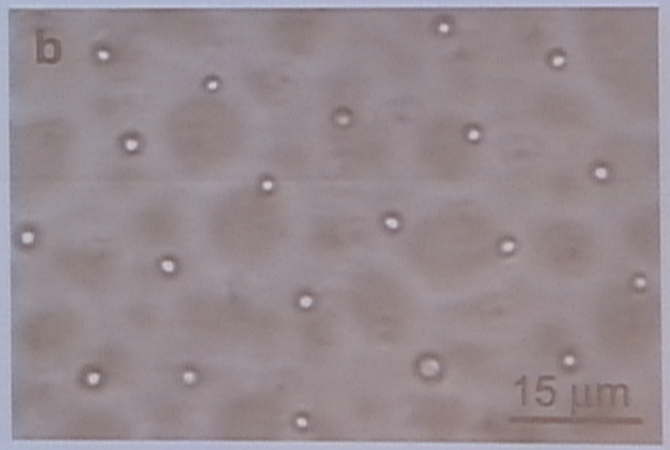

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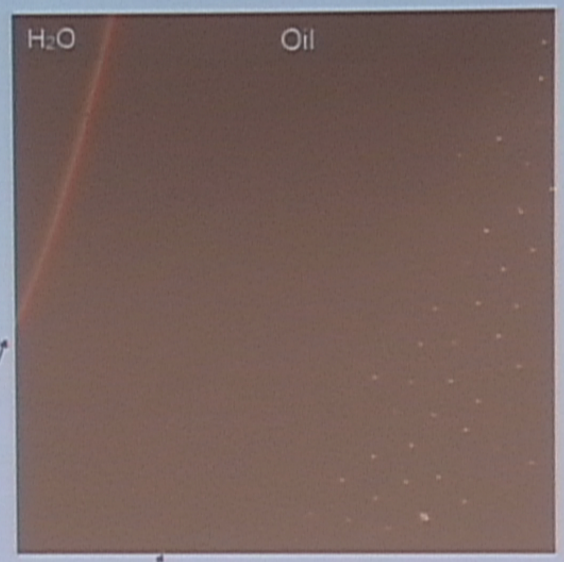

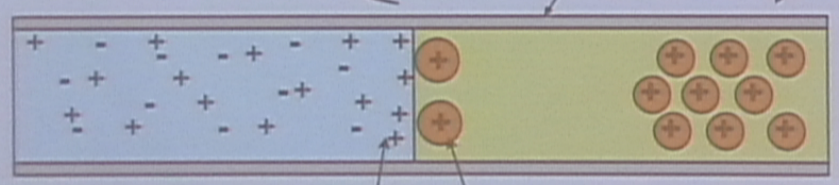

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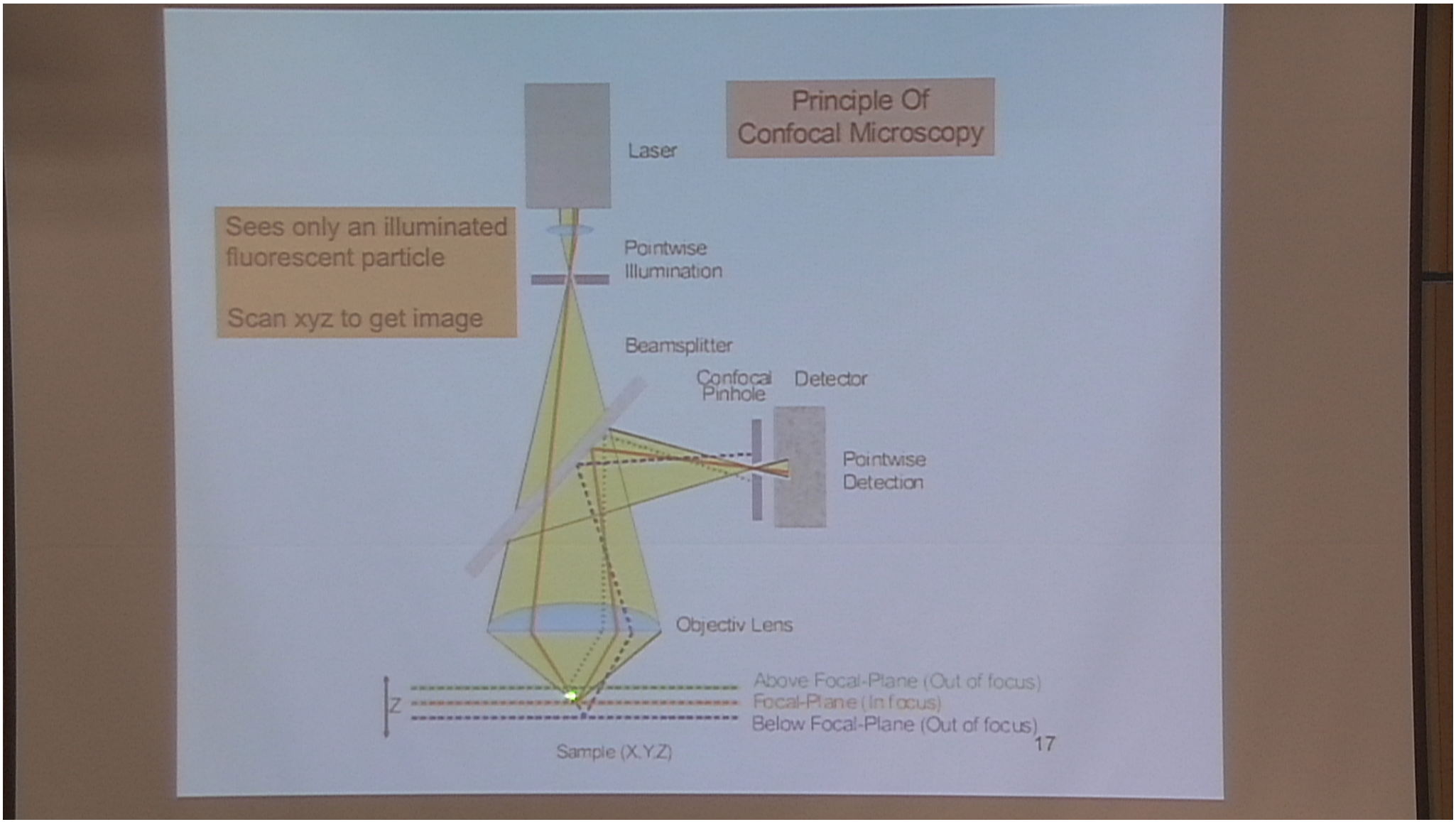
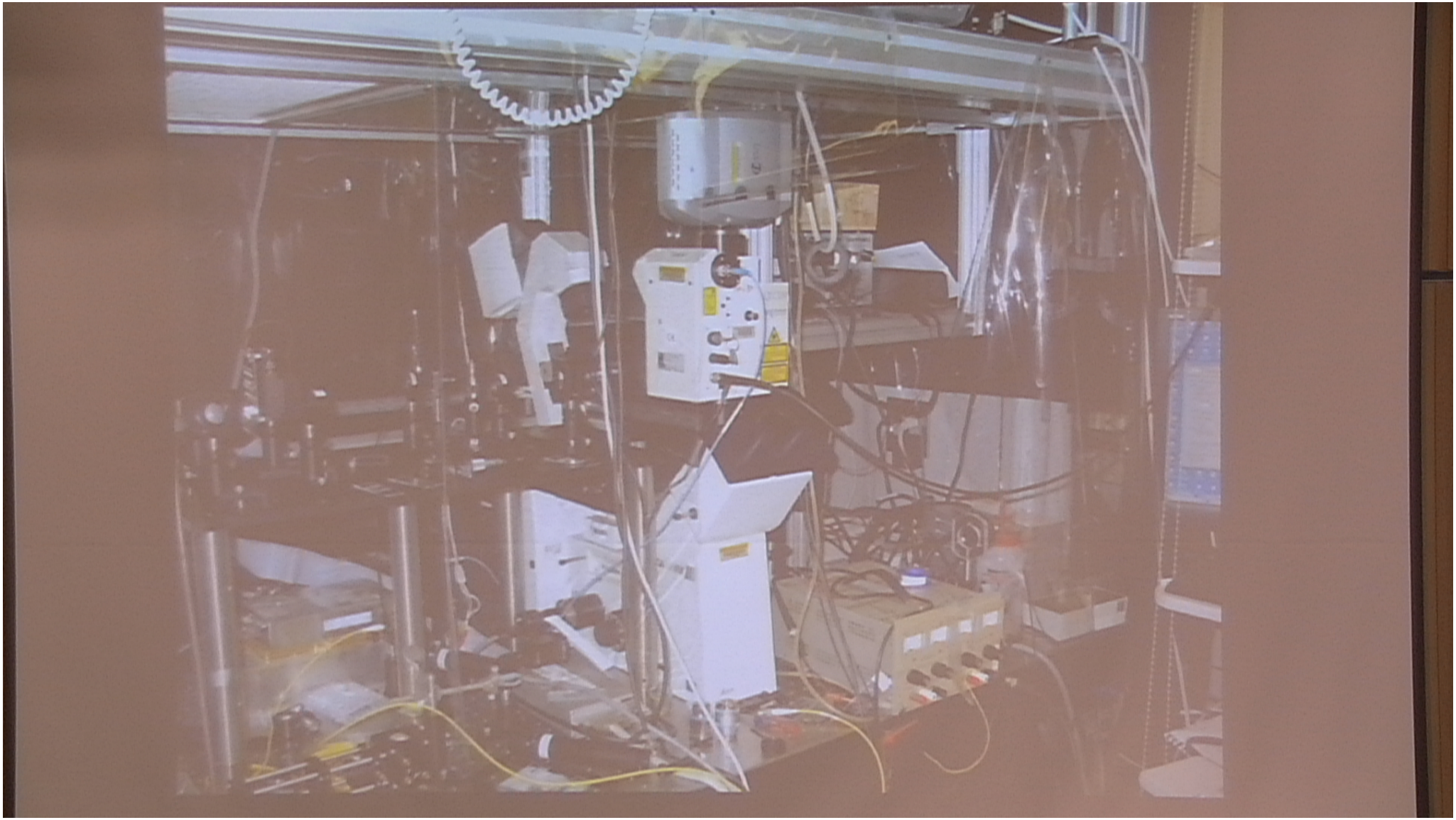
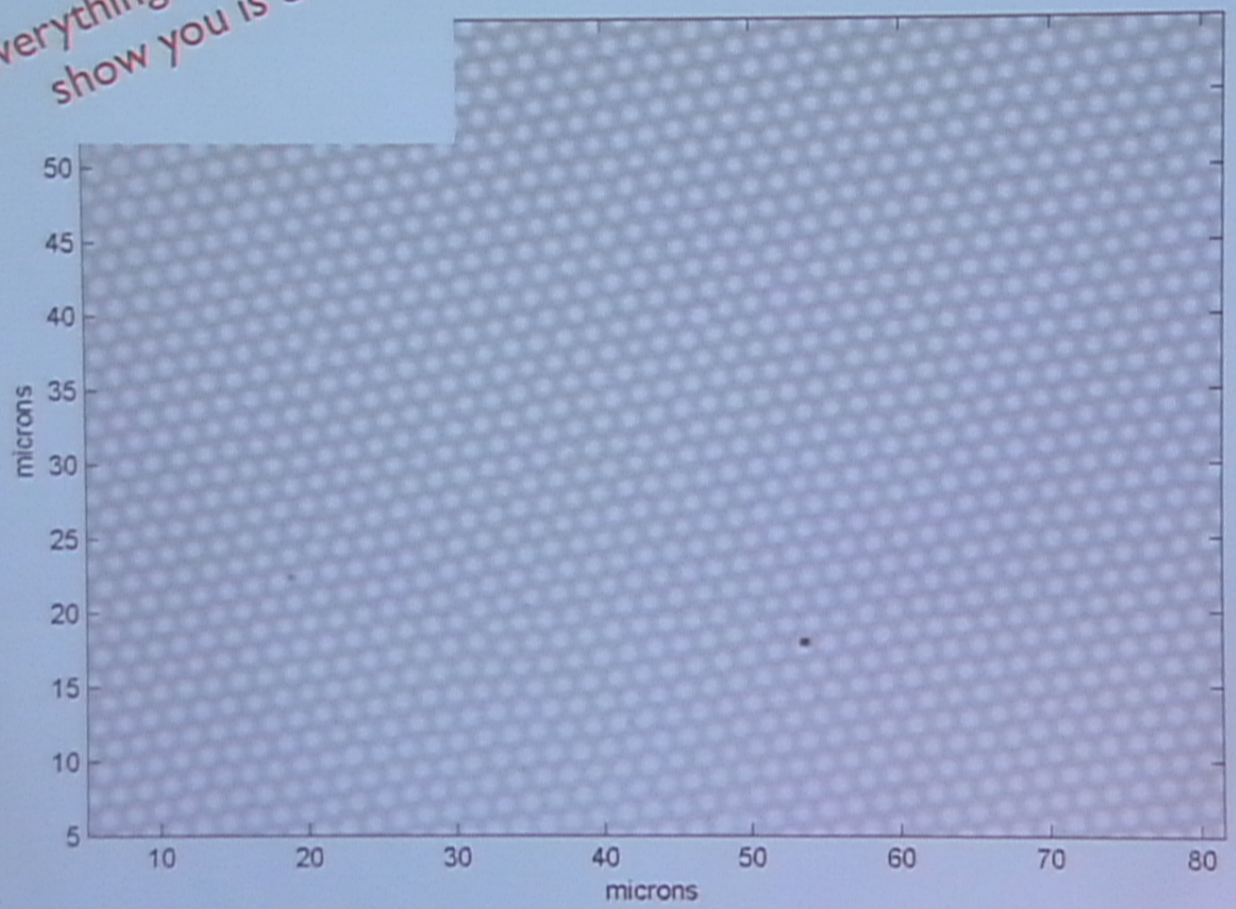

Principle Of
Confocal Microscopy
Sees only an illuminated fluorescent particle
Scan xyz to get image
Laser
Pointwise
Illumination
Beamsplitter
Confocal
Pinhole
Detector
Pointwise
Detection
Objectiv Lens
Z
Above Focal-Plane (Out of focus)
Focal-Plane (In focus)
Below Focal-Plane (Out of focus)
Sample (X,Y,Z)
17



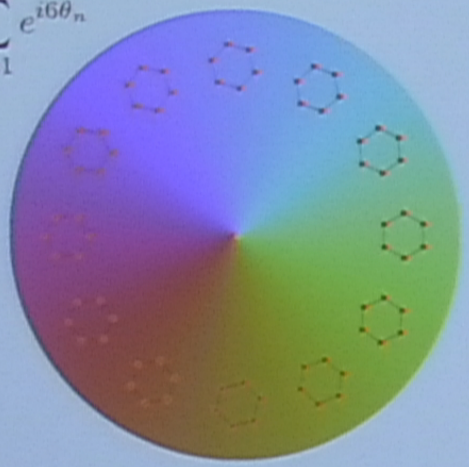
An :ourse on topological defects

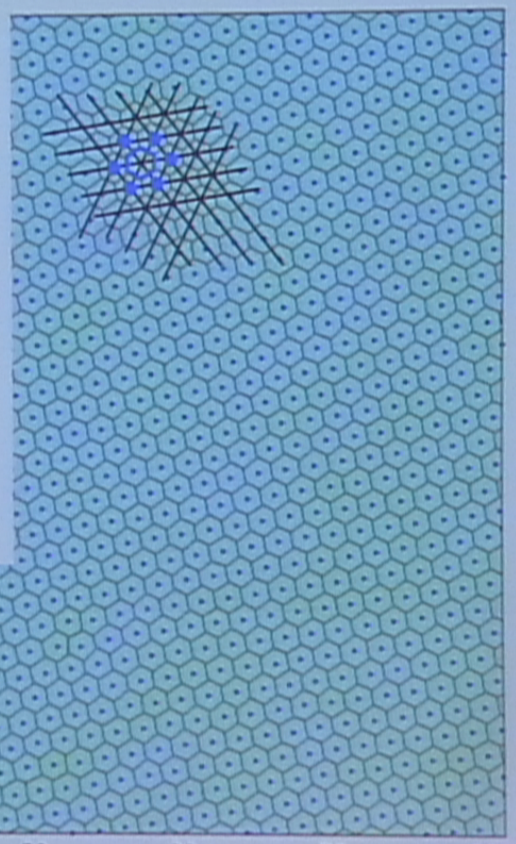
Everything I'm going to show you is data

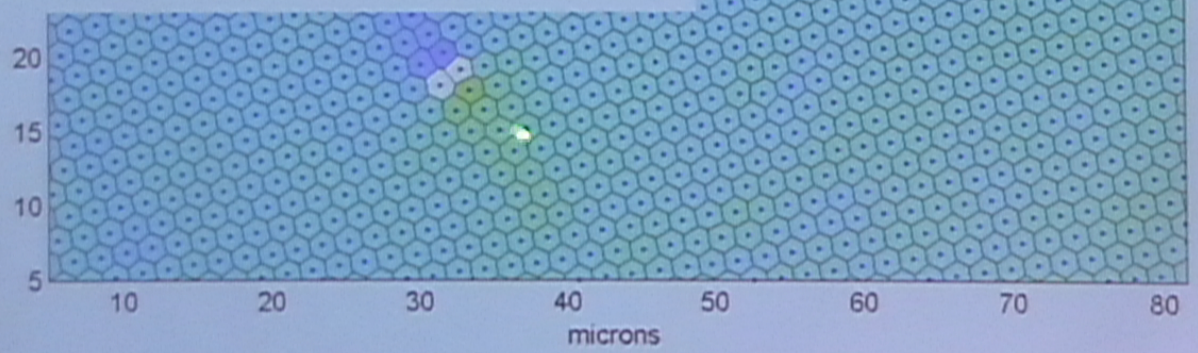


An experiment based course on topological defects

$$\psi = \frac{1}{6}\sum_{n=1}^{6} e^{i6\theta_n}$$







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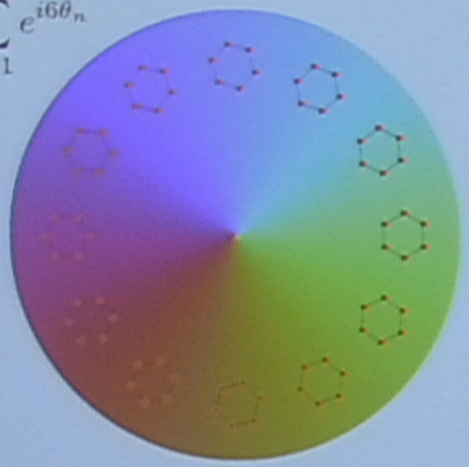

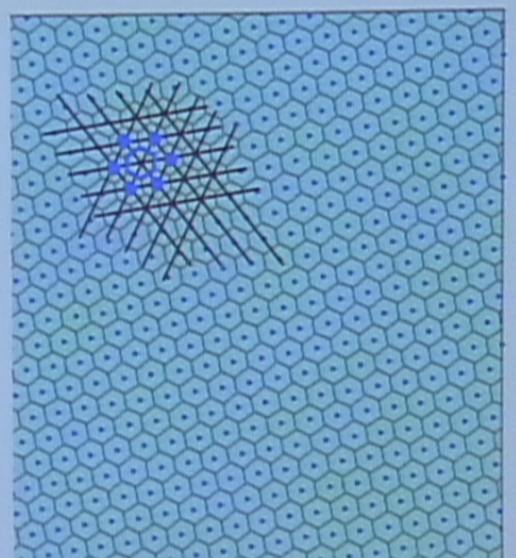

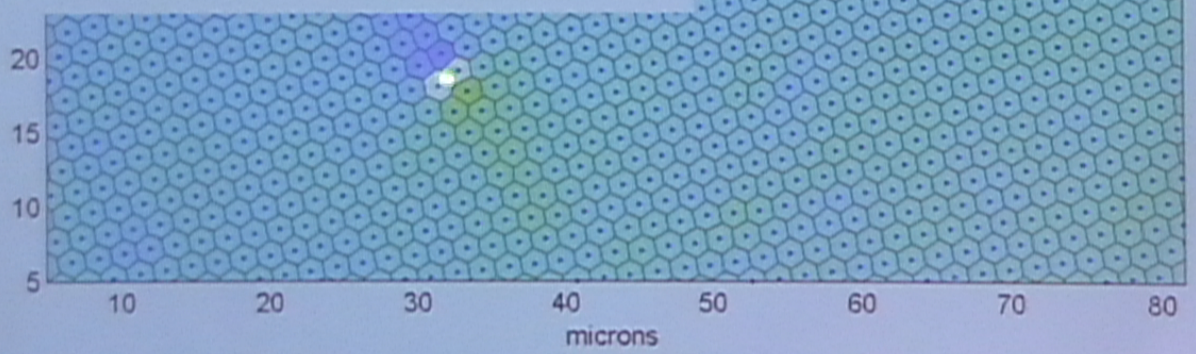

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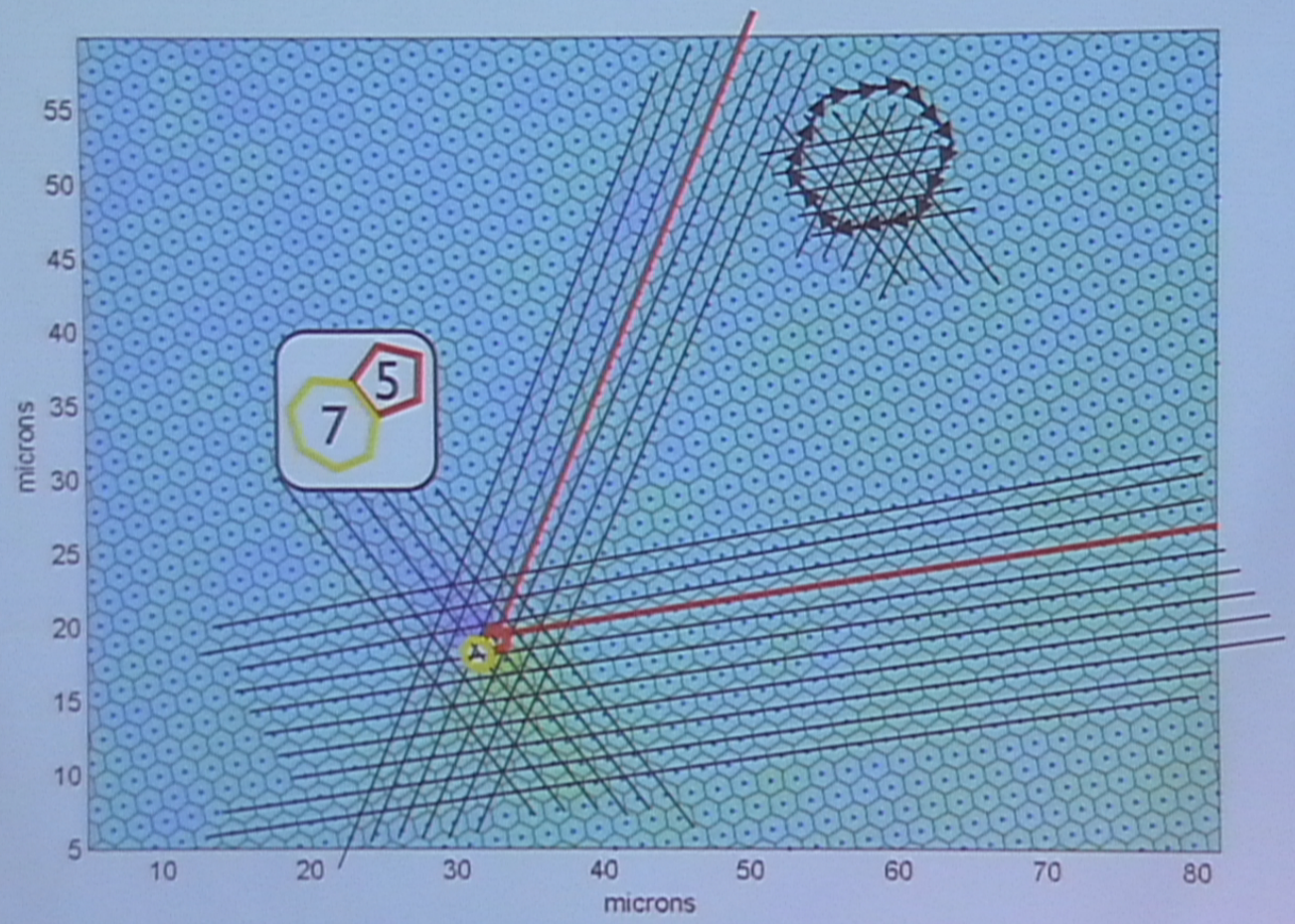
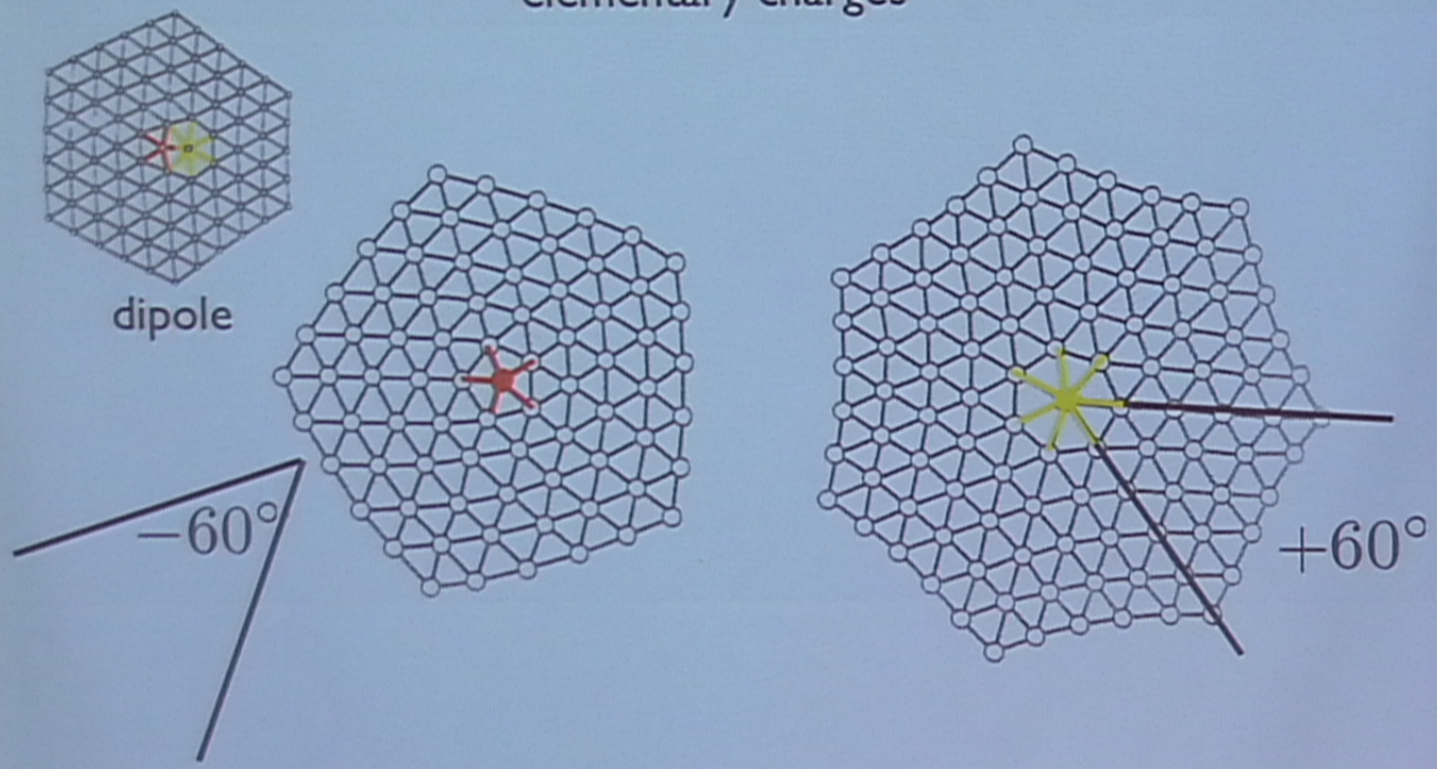


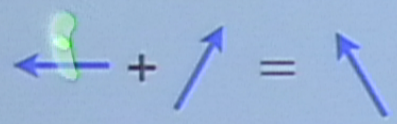
Disclinations

`elementary charges'

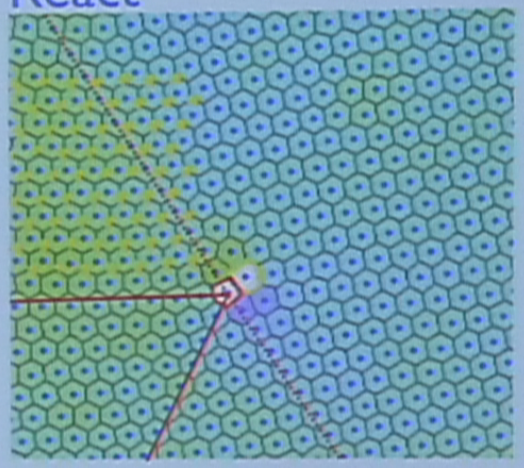


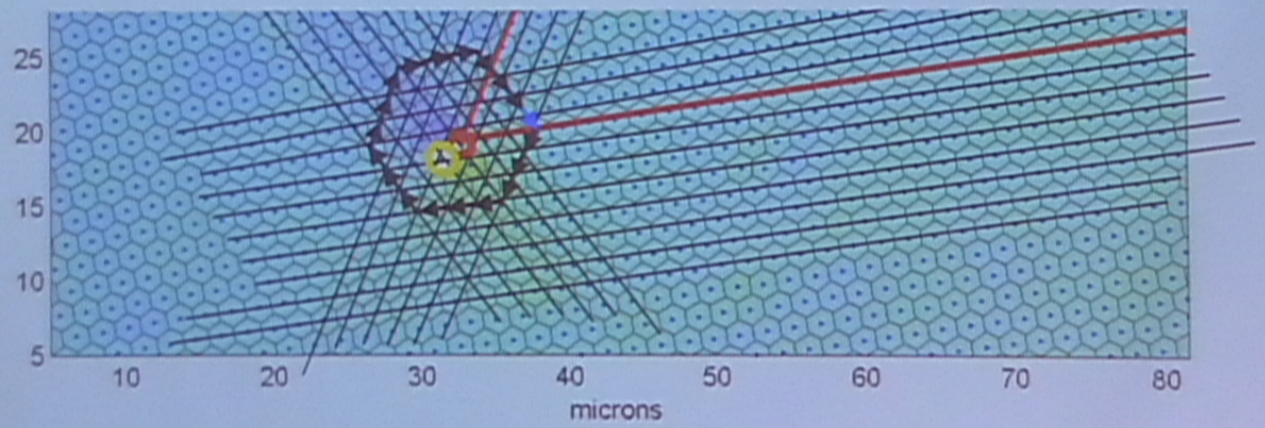
Very expensive in the plane

An experiment based course on topological defects

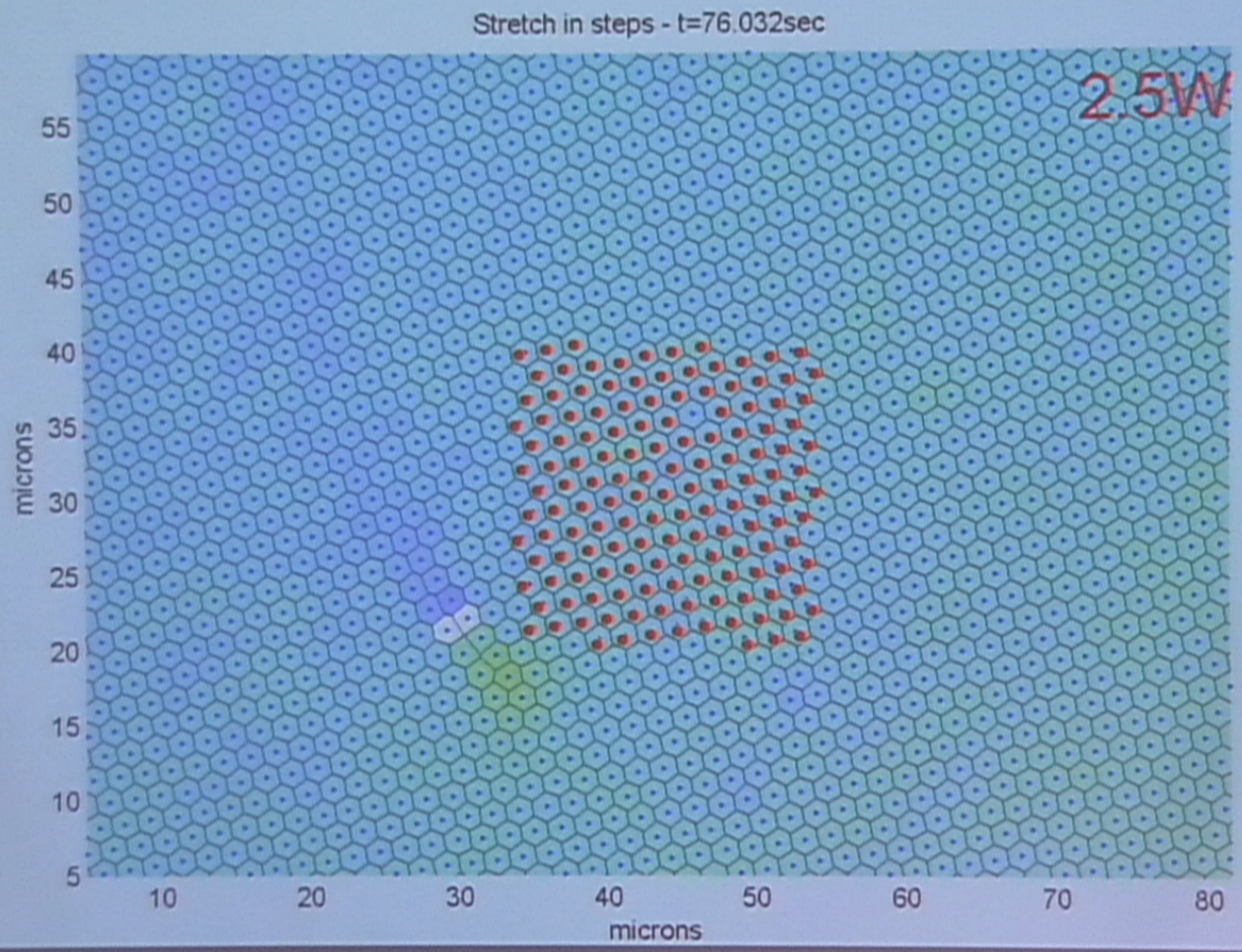


React

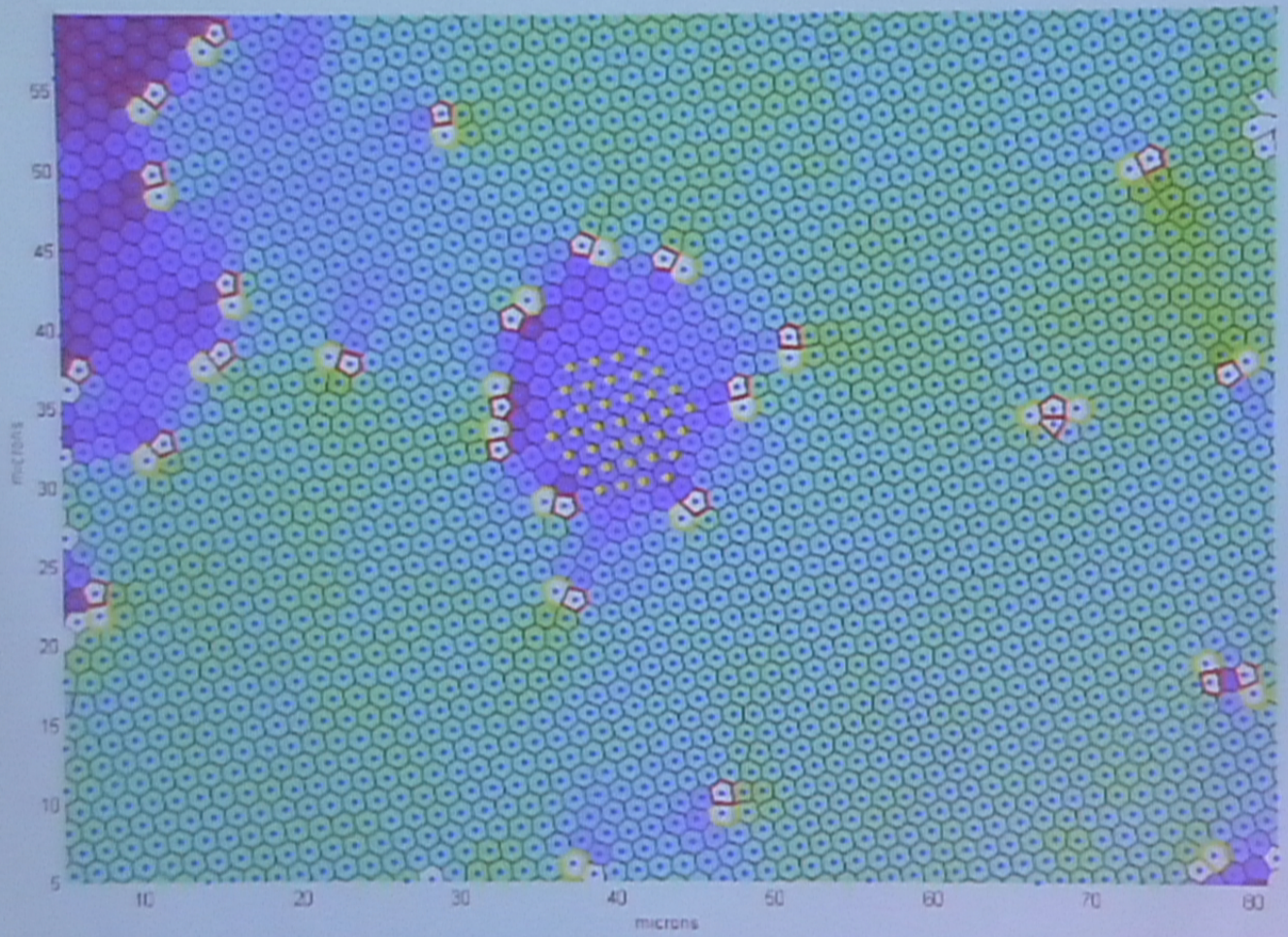




Stretching: From commensurate to incommensurate



Organization under dynamical potentials



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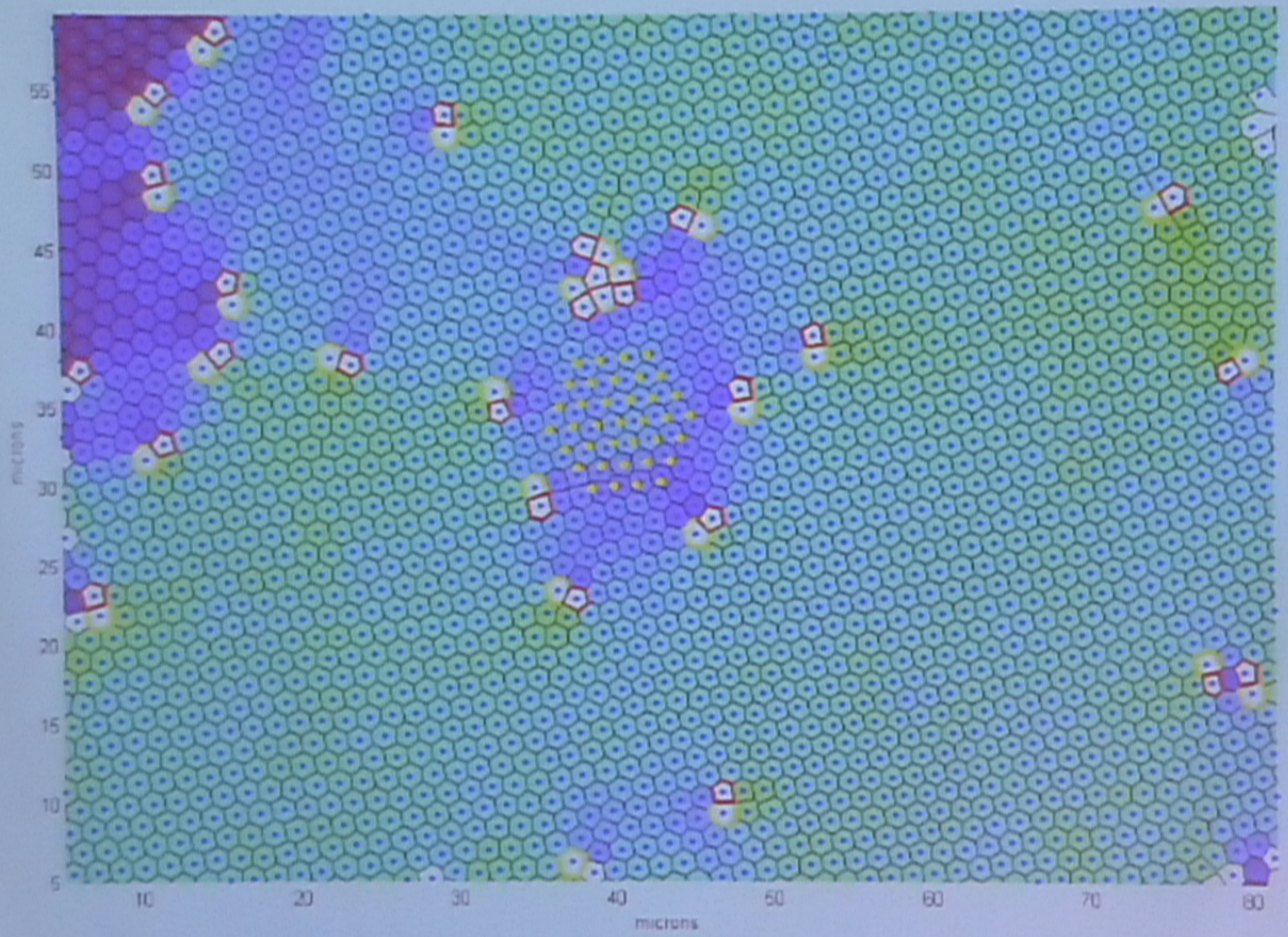


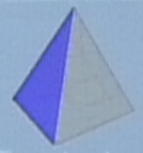
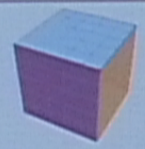
The Euler Characteristic

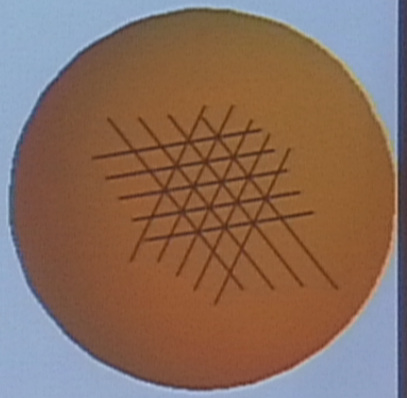
Vertices Faces

$$V - E + F = 2$$

Edges

$$1 - \frac{6}{2} + \frac{6}{3} = 0$$

Polyhedron	V	E	F	V-E+F
 Tetrahedron				
 Cube				
 Icosahedron	12	30	20	2



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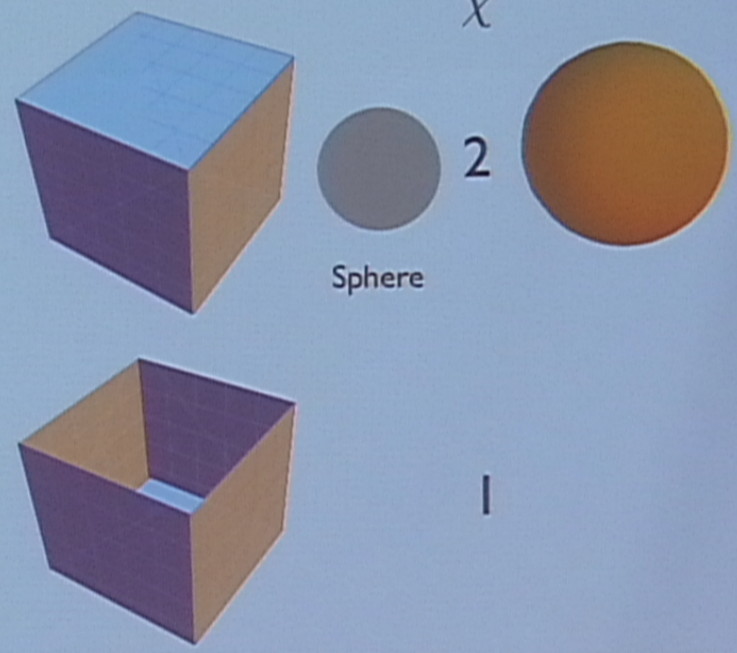
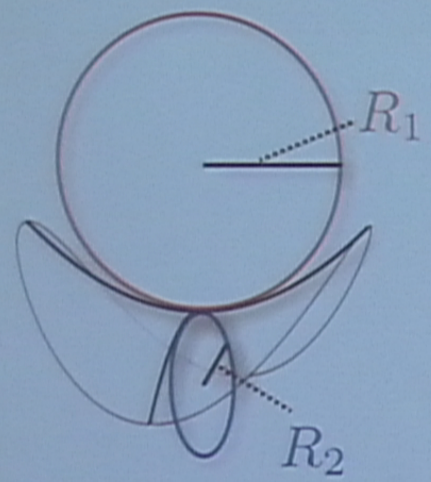


Curvature and non-euclidean geometry



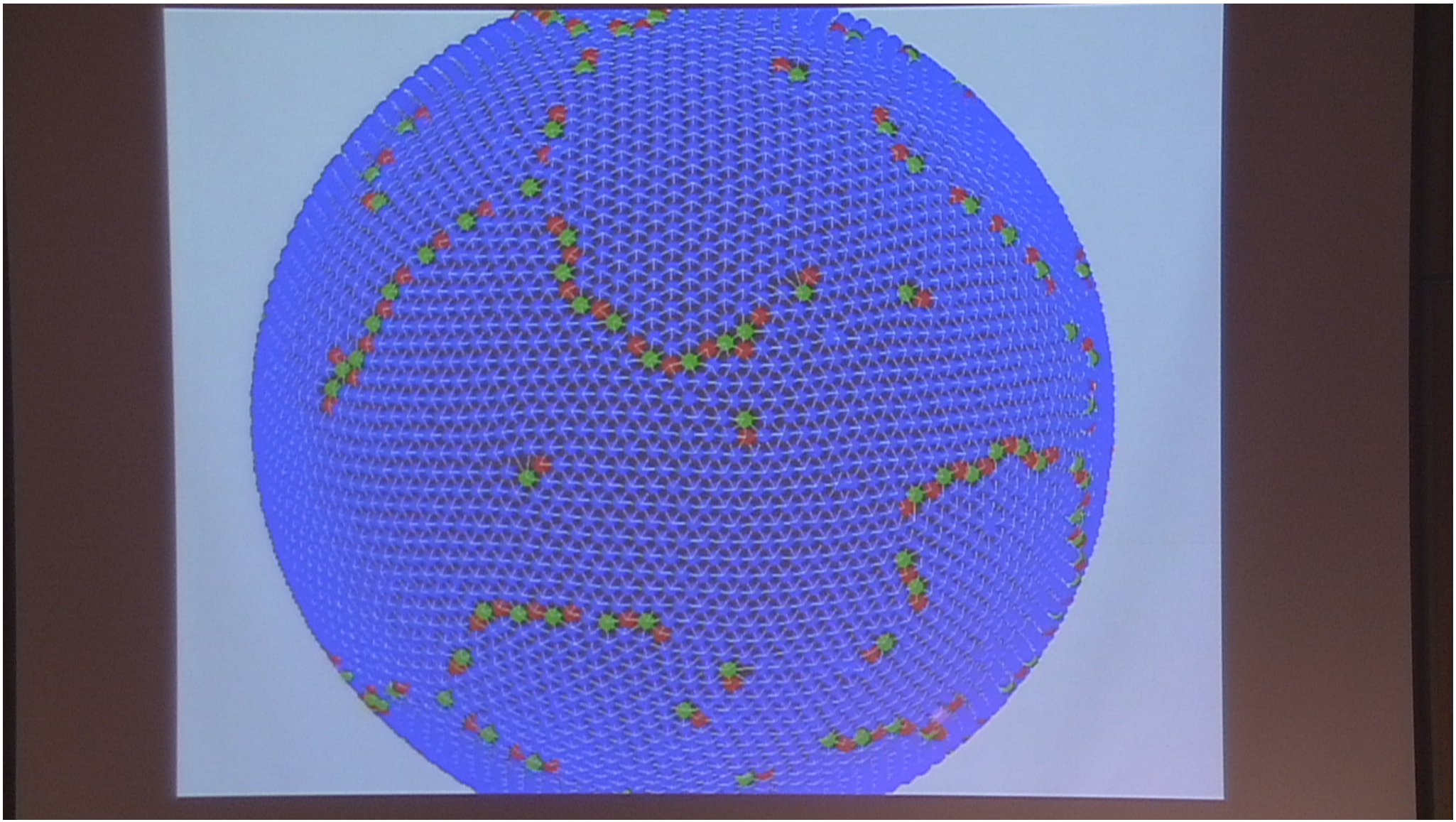
Gaussian curvature $\boxed{K = \frac{1}{R_1}\frac{1}{R_2}}$

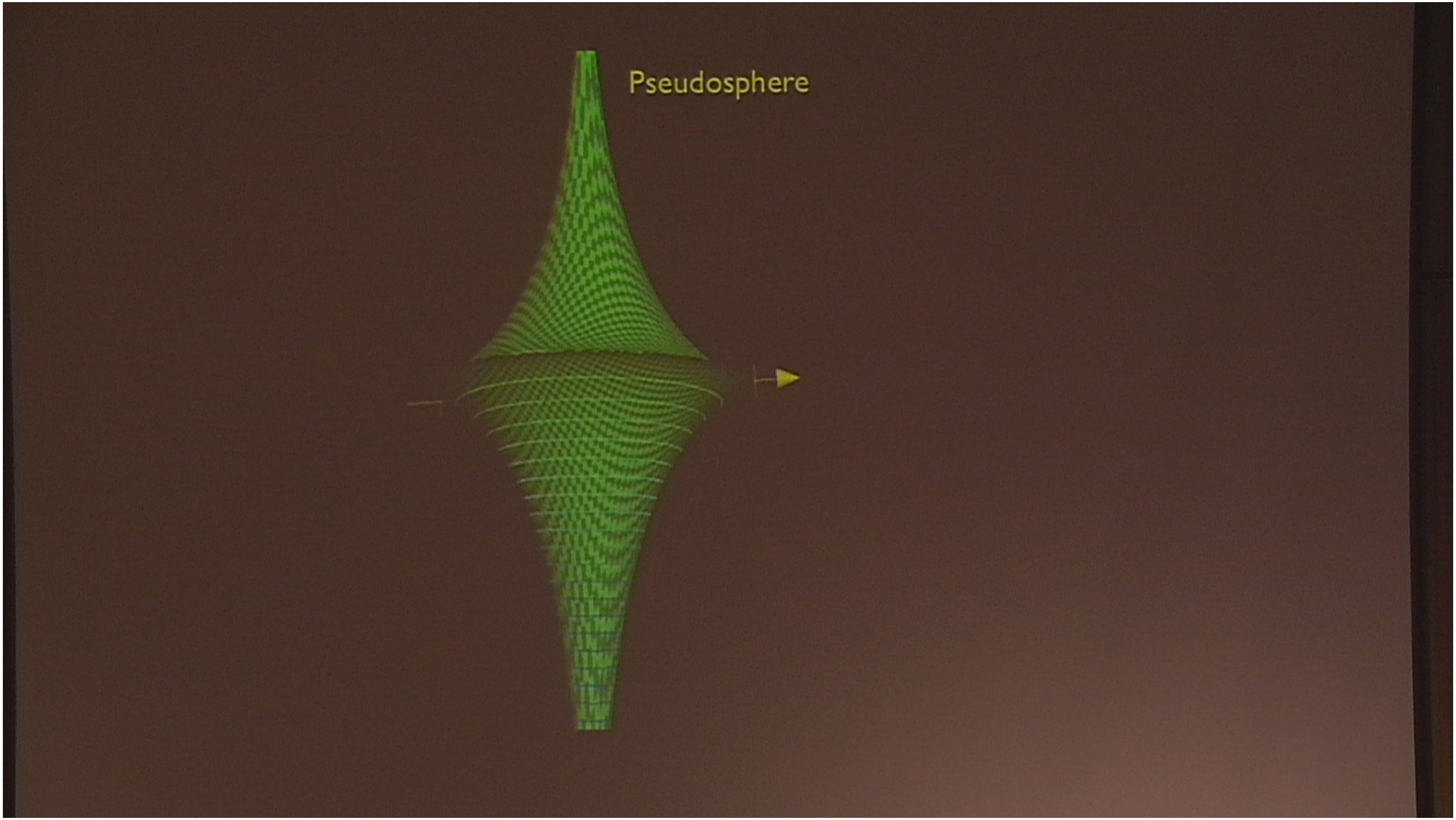
Positive Gaussian curvature

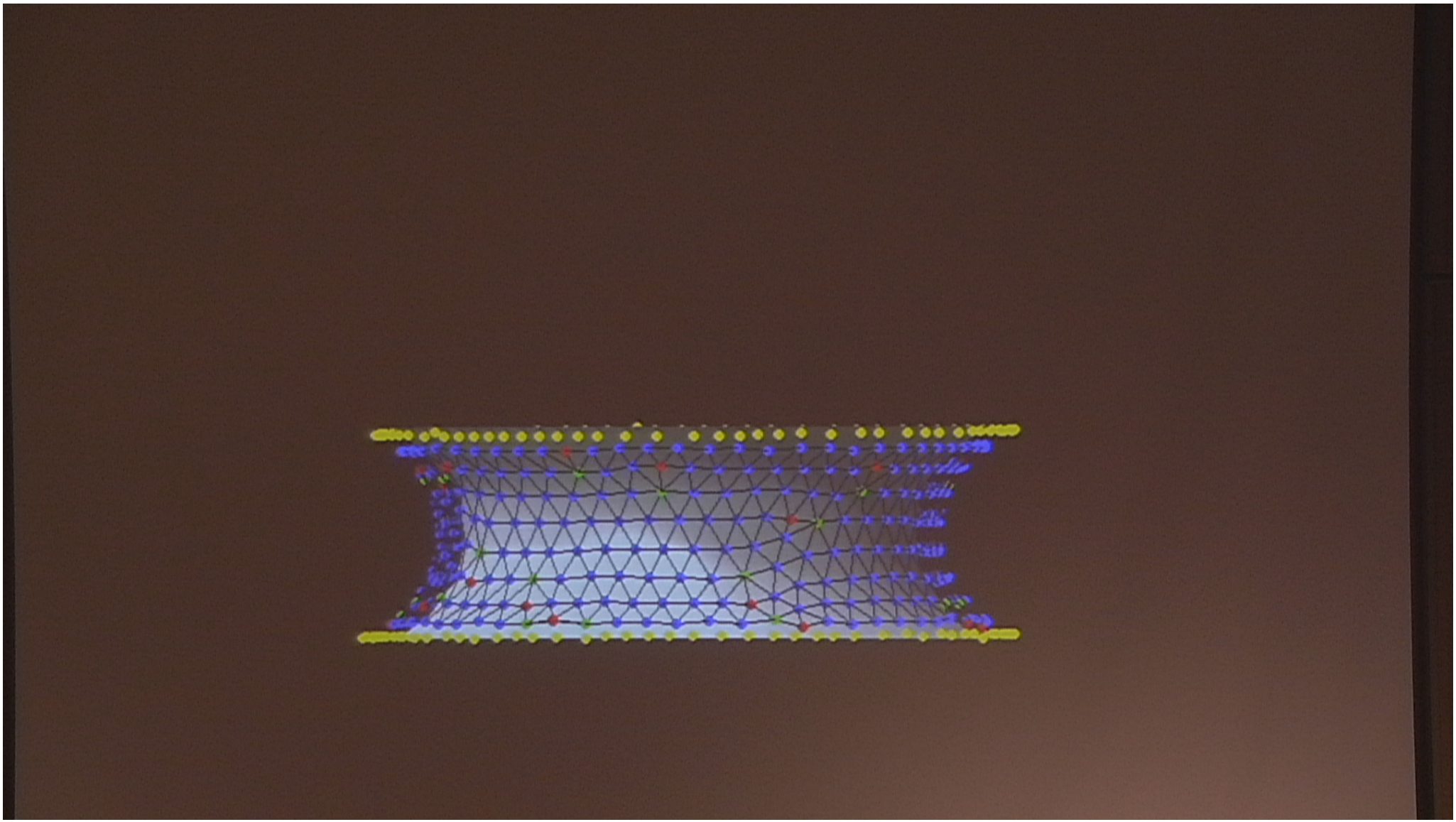
Expect one when $\int K \mathrm{d}A = \pi/3$

Negative Gaussian curvature

Expect one when $\int K \mathrm{d}A = -\pi/3$

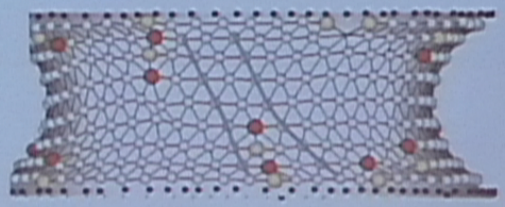


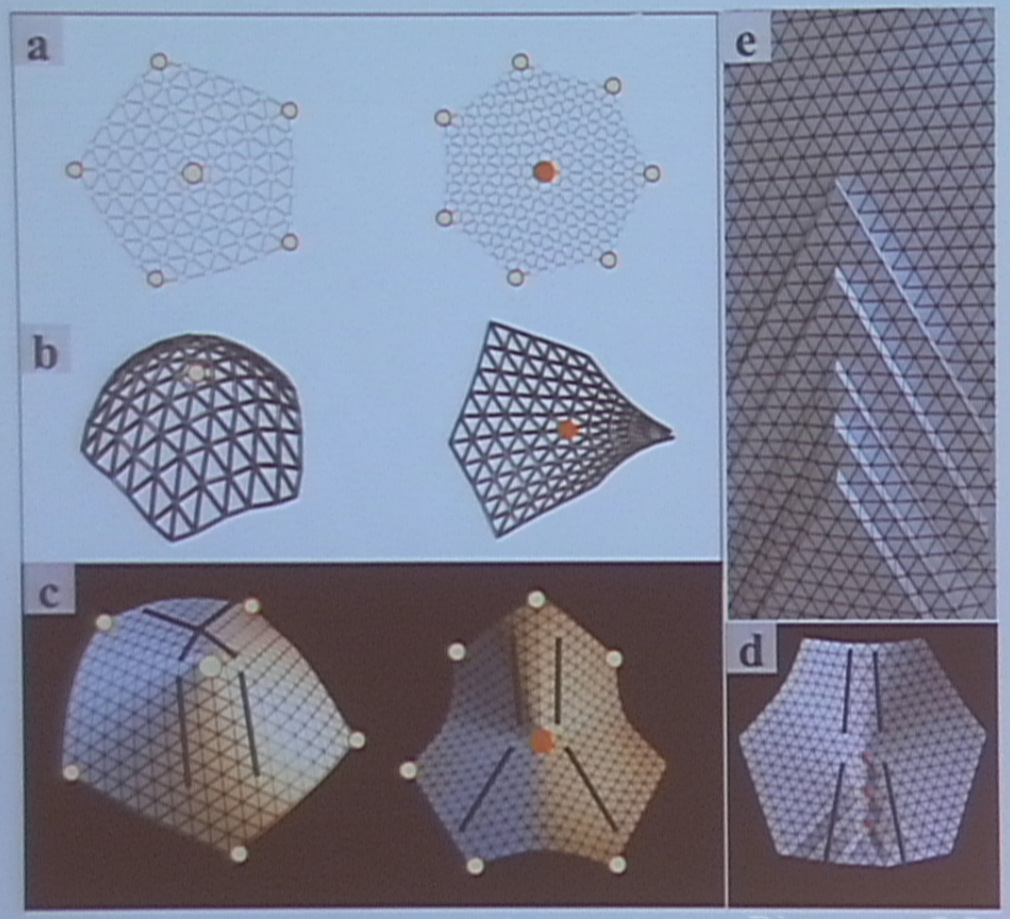

Pseudosphere



Pleats on Crystals on Curved Surfaces

Pleats
- Add width as one traverses their length
- As do aligned strings of *dislocations*
  grain boundaries which vanish on the surface
- Add an angular wedge from 0 to 30°
  (disclinations add 60°)
- Pleats produce "coneyness"
- Pleat gradients produce curvature



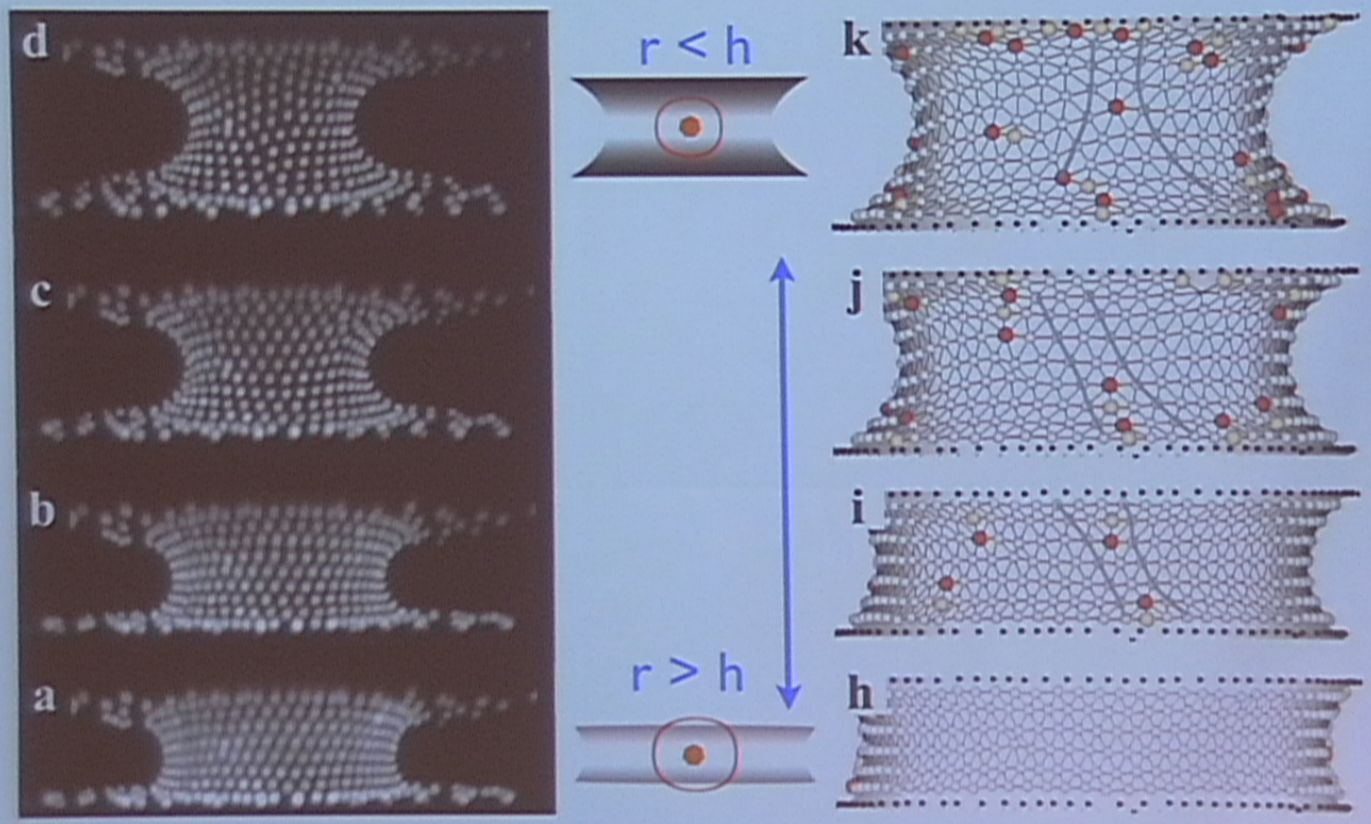


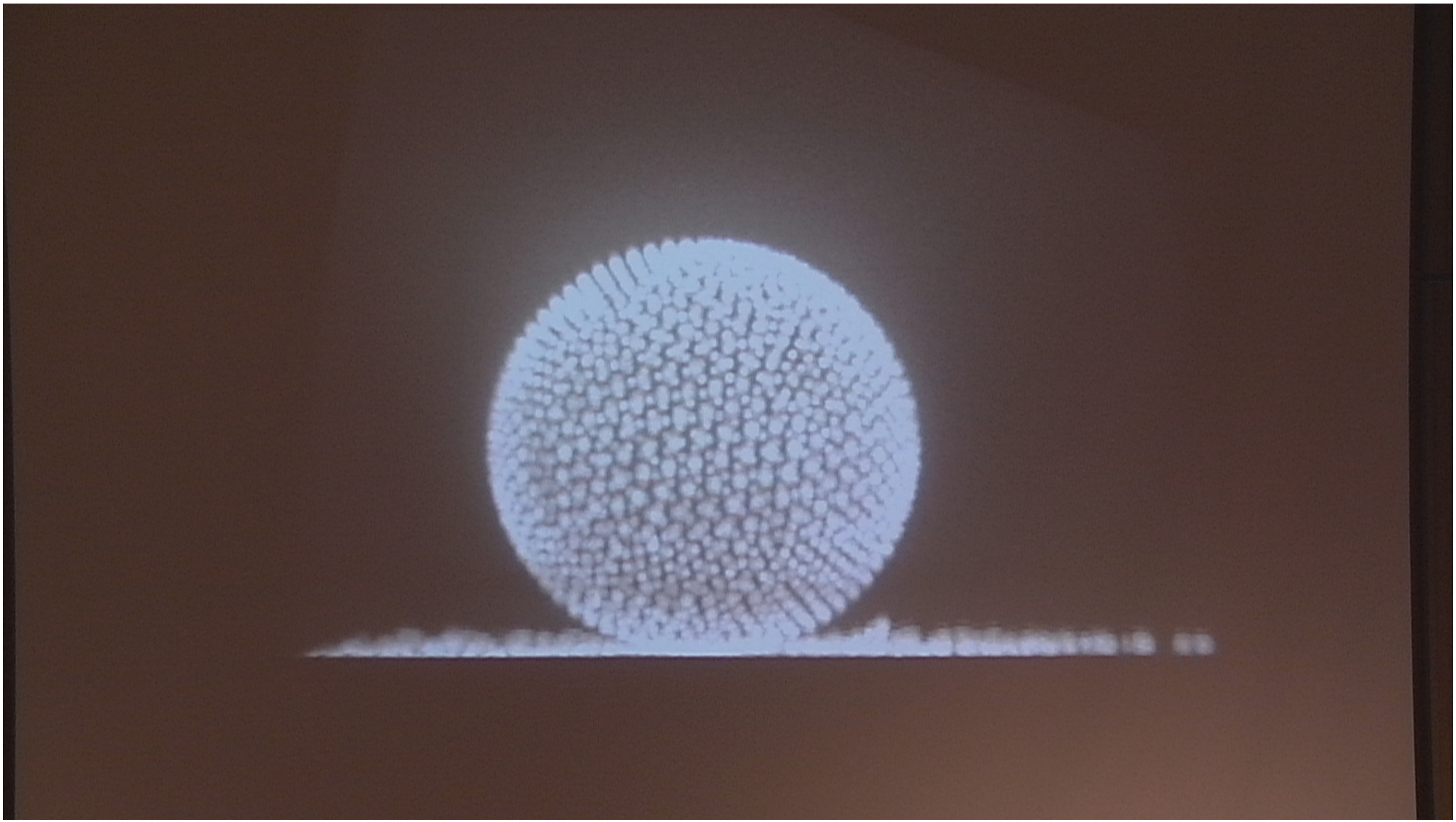
Disclinations 60°

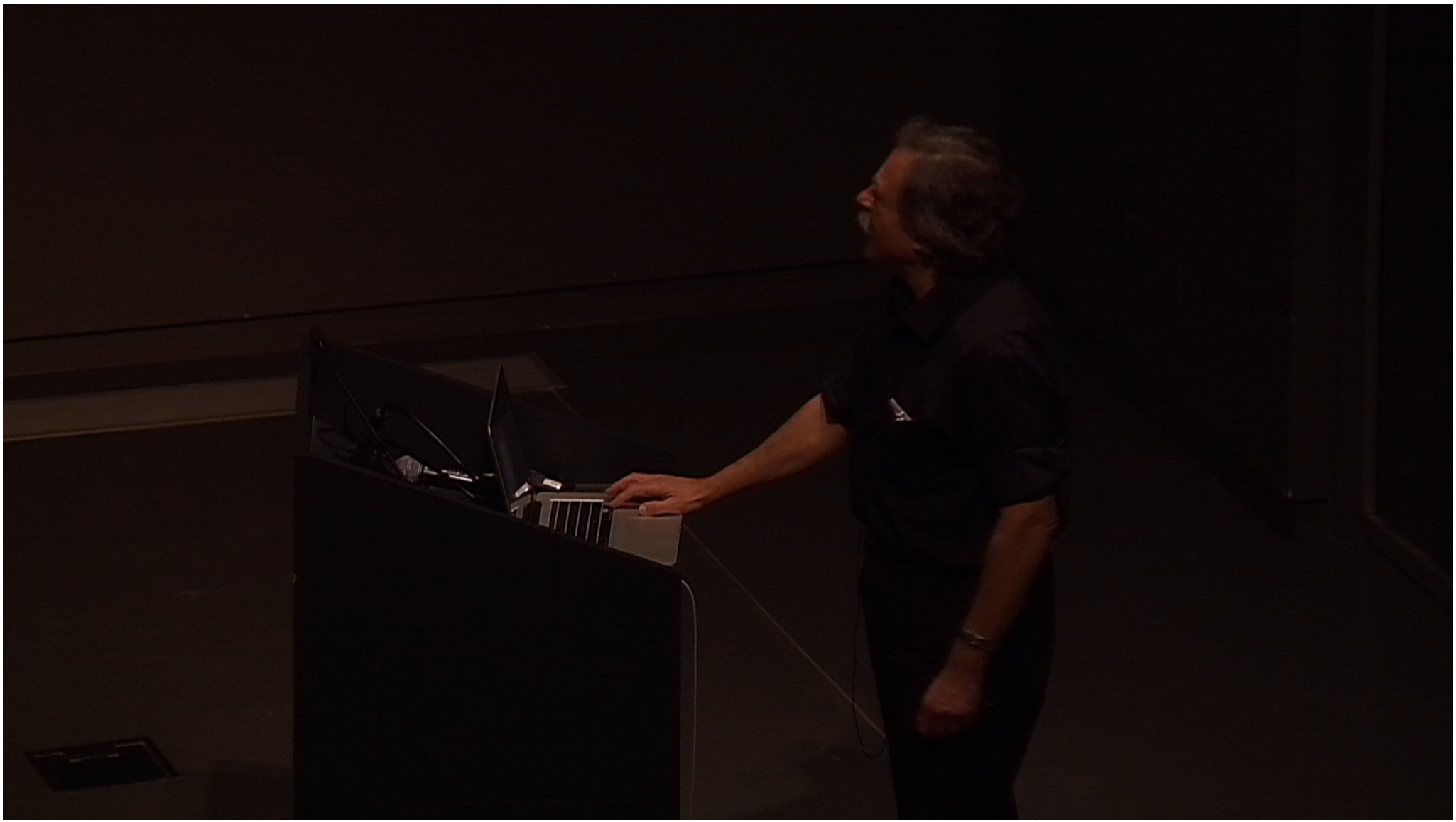
Pleats ~20°

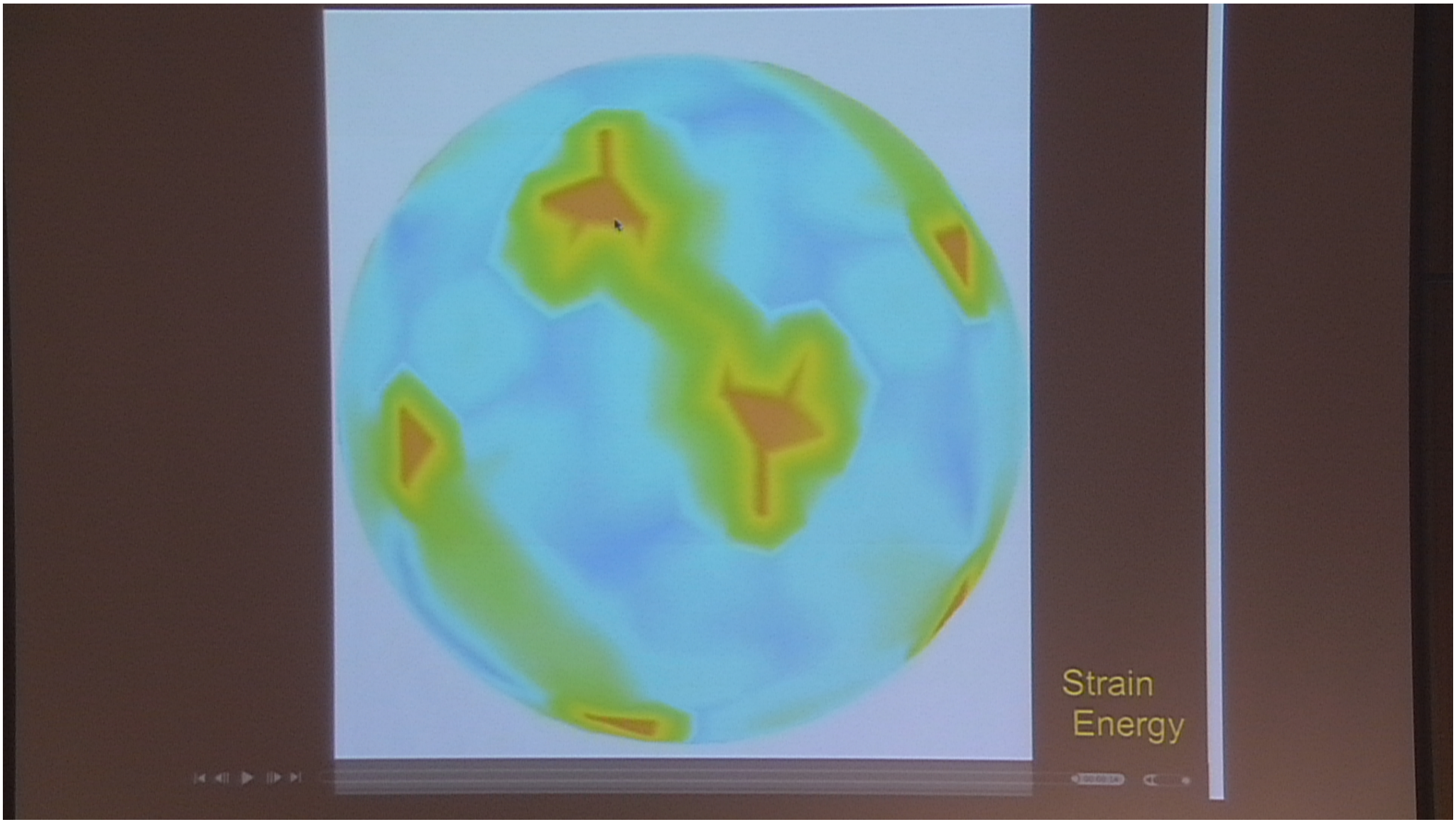
When do you pleat instead of making disclinations?

Area relieved by disclination $\int \kappa dA = \frac{\pi}{3}$ q $\frac{\pi r^2}{R_1 R_2} = \frac{\pi}{3}$ $\boxed{r = \sqrt{\frac{R_1 R_2}{3}}}$








Strain
Energy