Title: Chiral Mott Insulator of Bosons in a Fully Frustrated Bose Hubbard Model

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Abstract: Recent experiments have demonstrated that it is possible to create a synthetic magnetic field for neutral atoms in optical lattices using two-photon (Raman) processes. Motivated by exploring the interplay of such artificial magnetic fields and strong correlations for bosons, we have studied the Bose Hubbard model in the presence of pi-flux per plaquette. Using a variety of techniques, this model is shown to support a remarkable chiral Mott insulator phase on a 2-leg ladder. This state is a fully gapped insulator with staggered loop currents. We discuss physical insights as well experimental signatures of such a state for cold atoms as well as for Josephson junction arrays in a magnetic field.

Bosons and Strong Correlation Effects

Boson Hubbard Model

$$\hat{H} = -\sum_{\langle i,j \rangle} \hat{a}_i^\dagger t_{ij} \hat{a}_j - \mu \sum_i \hat{a}_i^\dagger \hat{a}_i + \frac{U}{2} \sum_i \hat{a}_i^\dagger \hat{a}_i^\dagger \hat{a}_i \hat{a}_i$$

Hopping of bosons

Local Repulsion

Fisher, Weichman, Grinstein, Fisher (PRB 1989)

Rokhsar, Kotliar (PRB 1991): Gutzwiller wfn
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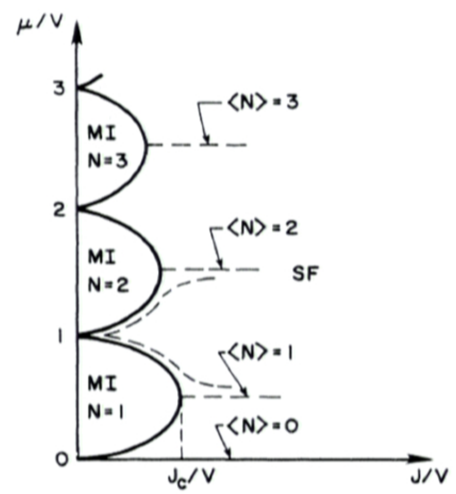
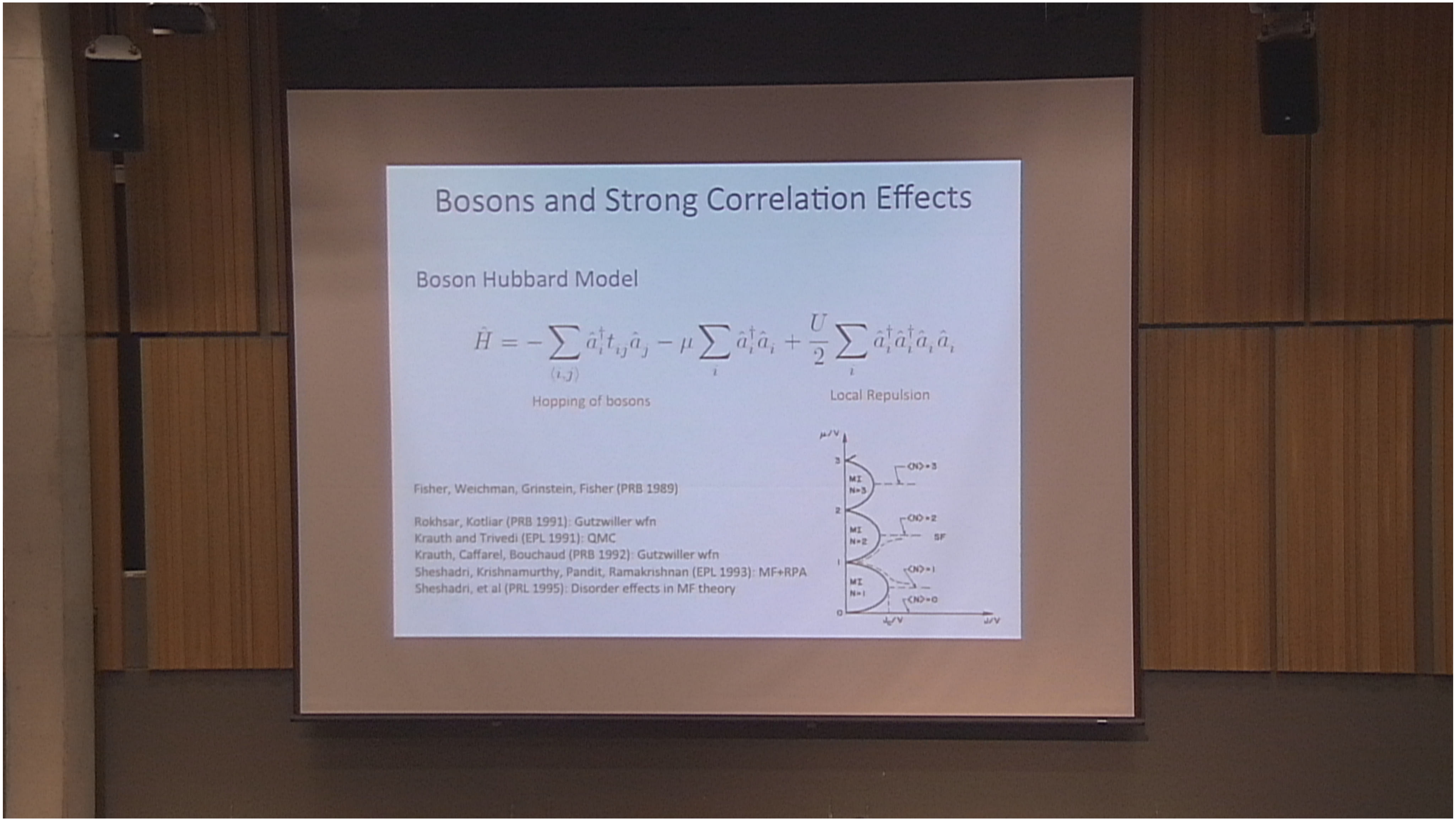
Sheshadri, et al (PRL 1995): Disorder effects in MF theory

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μ/V
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2
1
0
MI
N=3
MI
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SF
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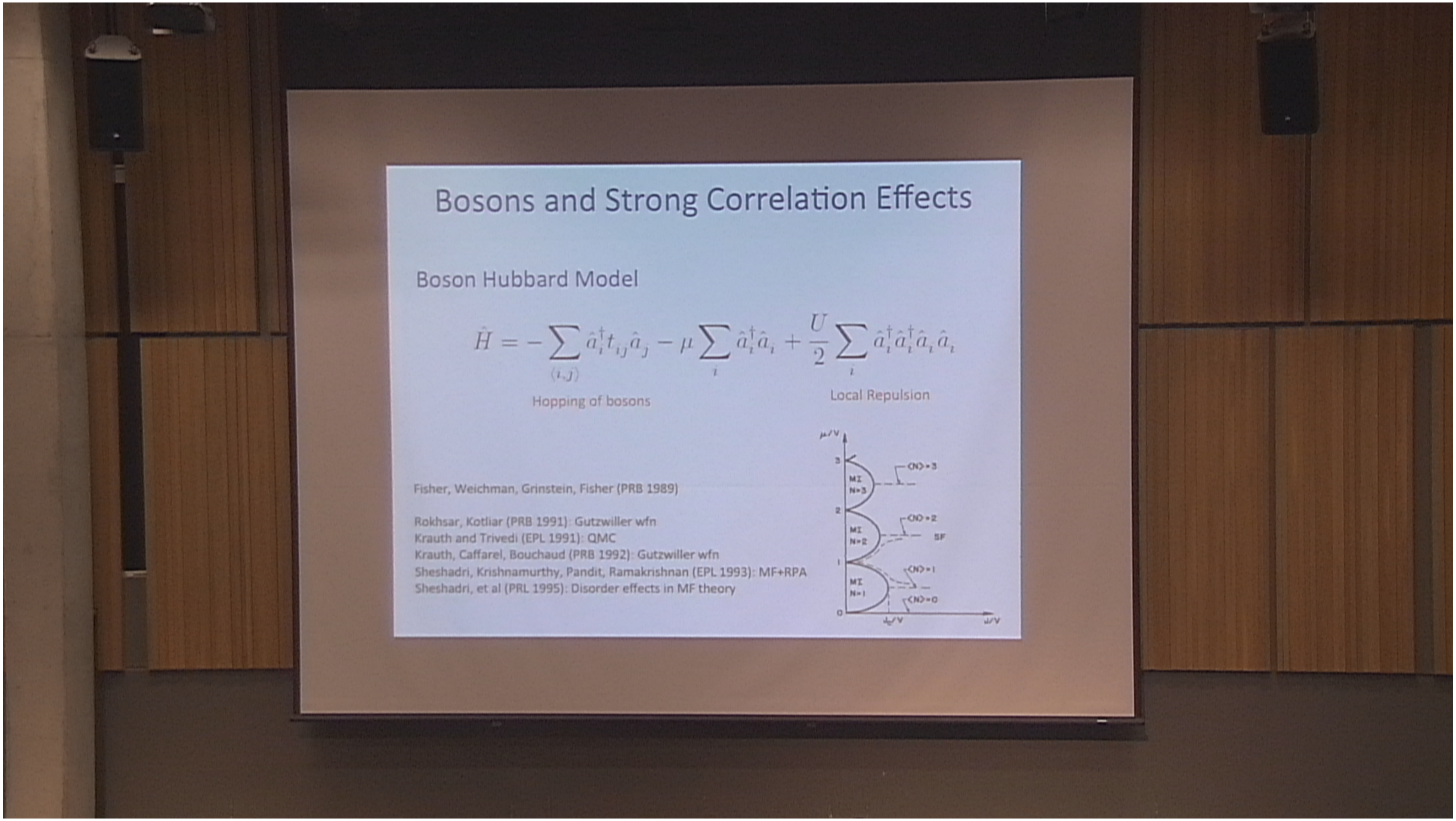
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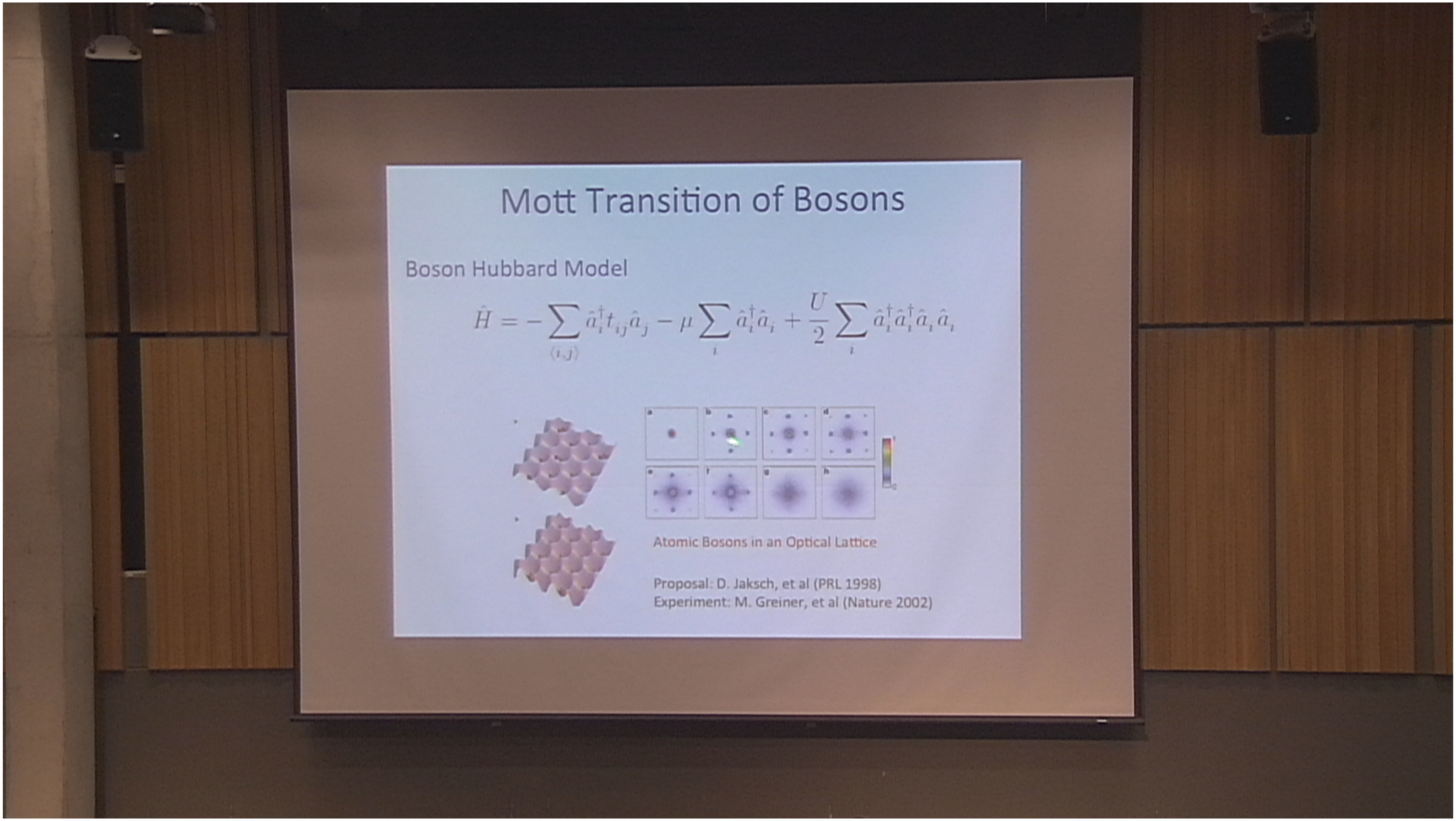
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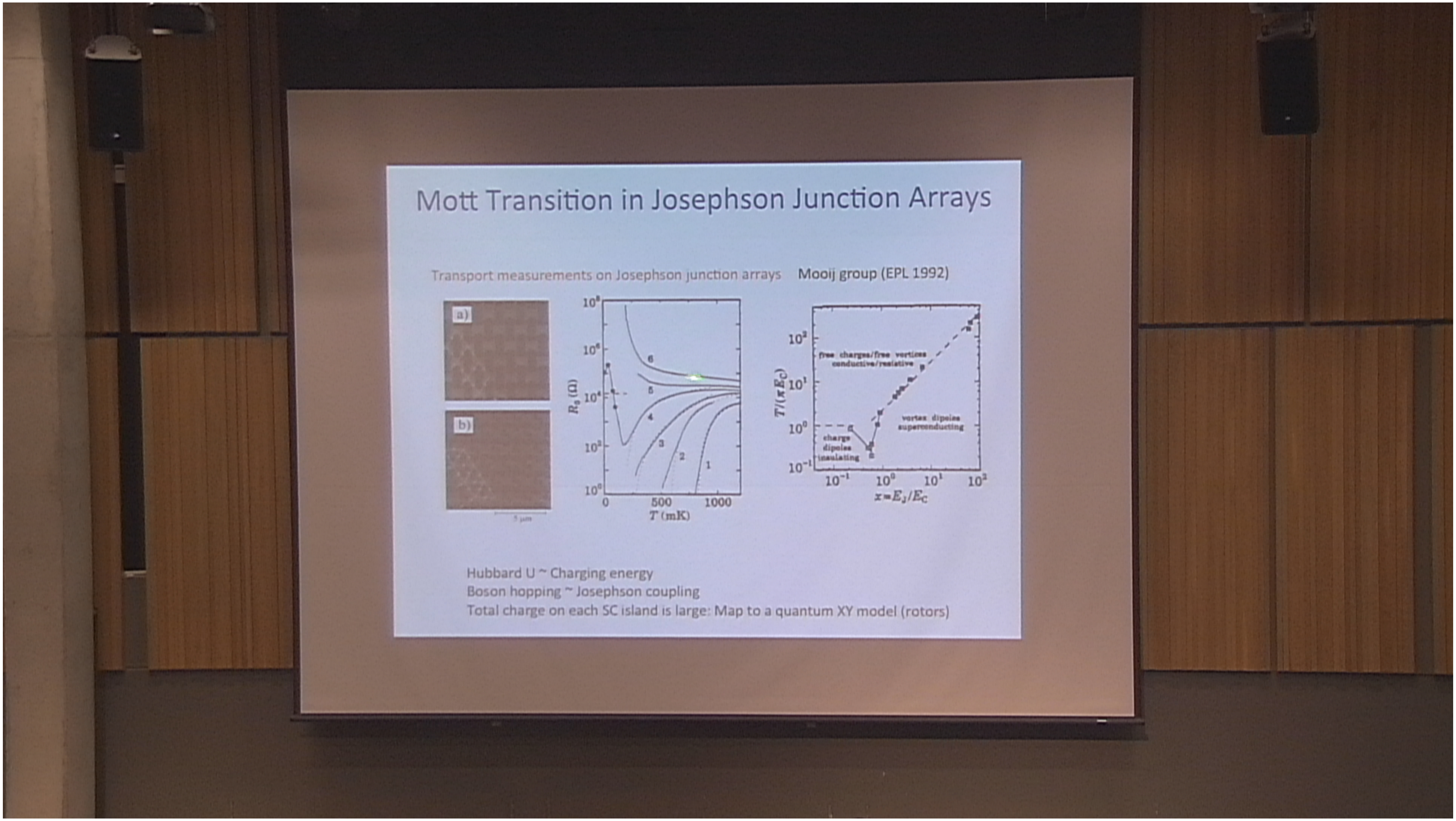
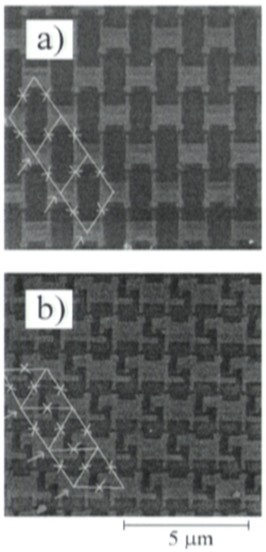
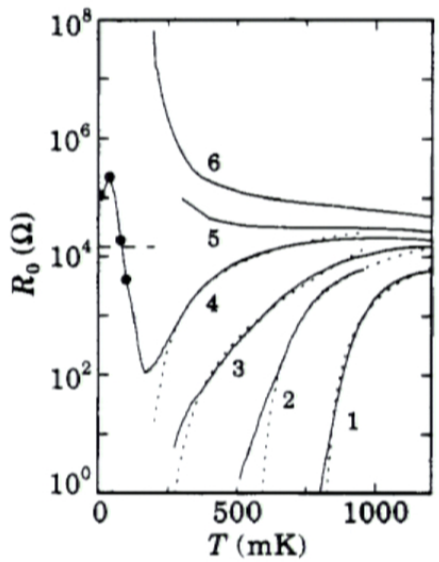
Atomic Bosons in an Optical Lattice
Proposal: D. Jaksch, et al (PRL 1998)
Experiment: M. Greiner, et al (Nature 2002)

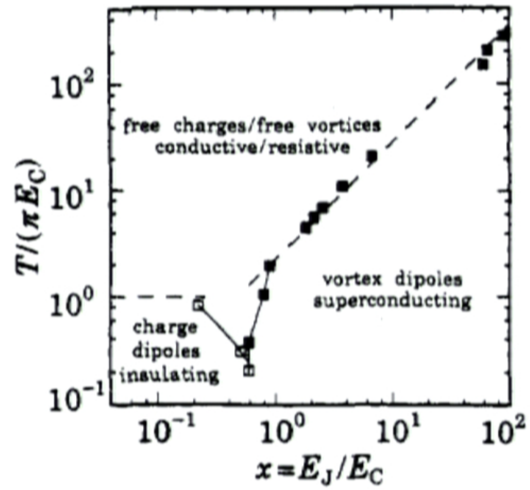

Mott Transition in Josephson Junction Arrays
Transport measurements on Josephson junction arrays
Mooij group (EPL 1992)
a)
b)
T (mK)
free charges/free vortices
conductive/resistive
vortex dipoles
superconducting
charge
dipoles
insulating
Hubbard U ~ Charging energy
Boson hopping ~ Josephson coupling
Total charge on each SC island is large: Map to a quantum XY model (rotors)

Mott Transition in Josephson Junction Arrays

Transport measurements on Josephson junction arrays   Mooij group (EPL 1992)







Hubbard U ~ Charging energy
Boson hopping ~ Josephson coupling
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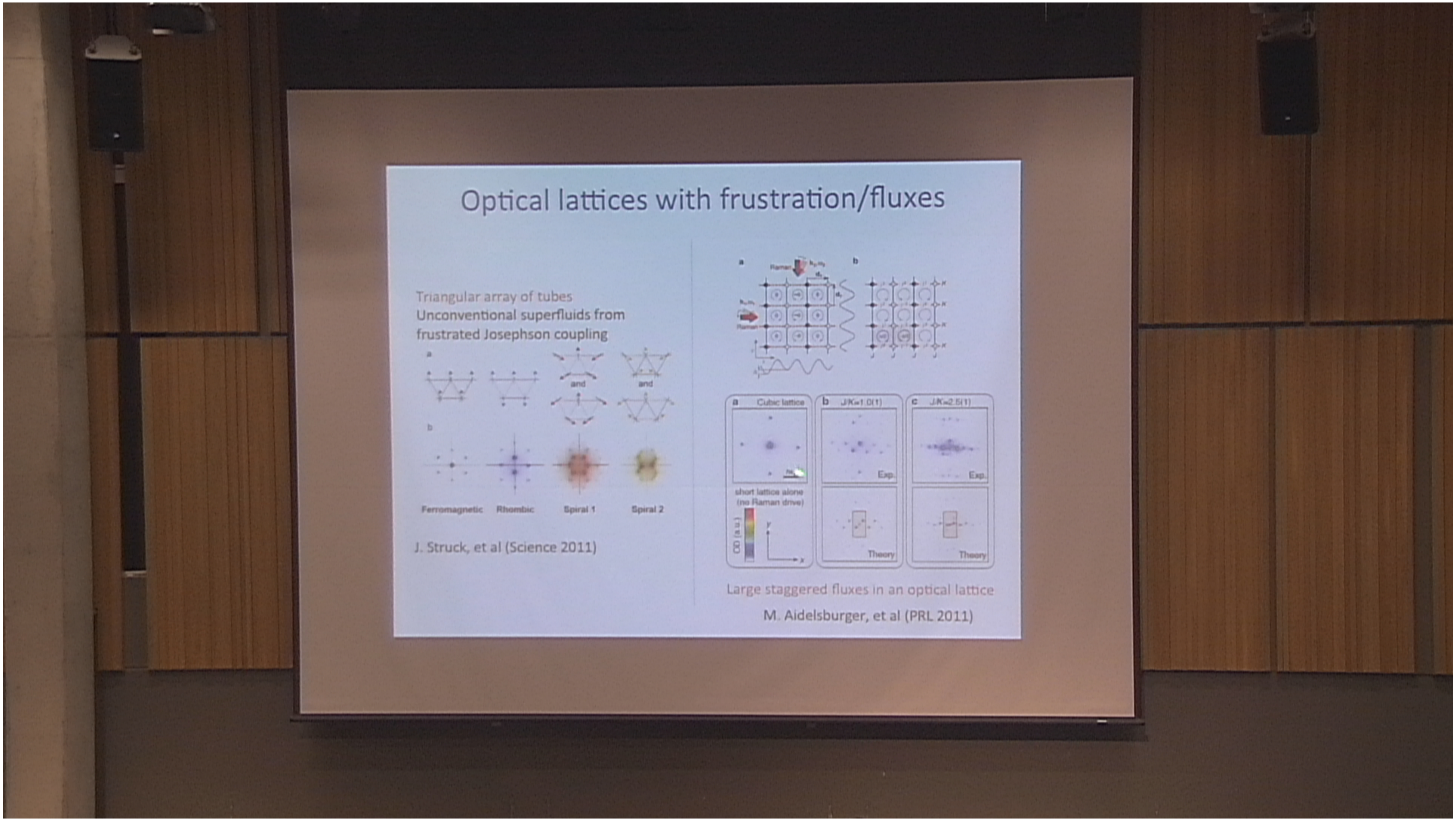
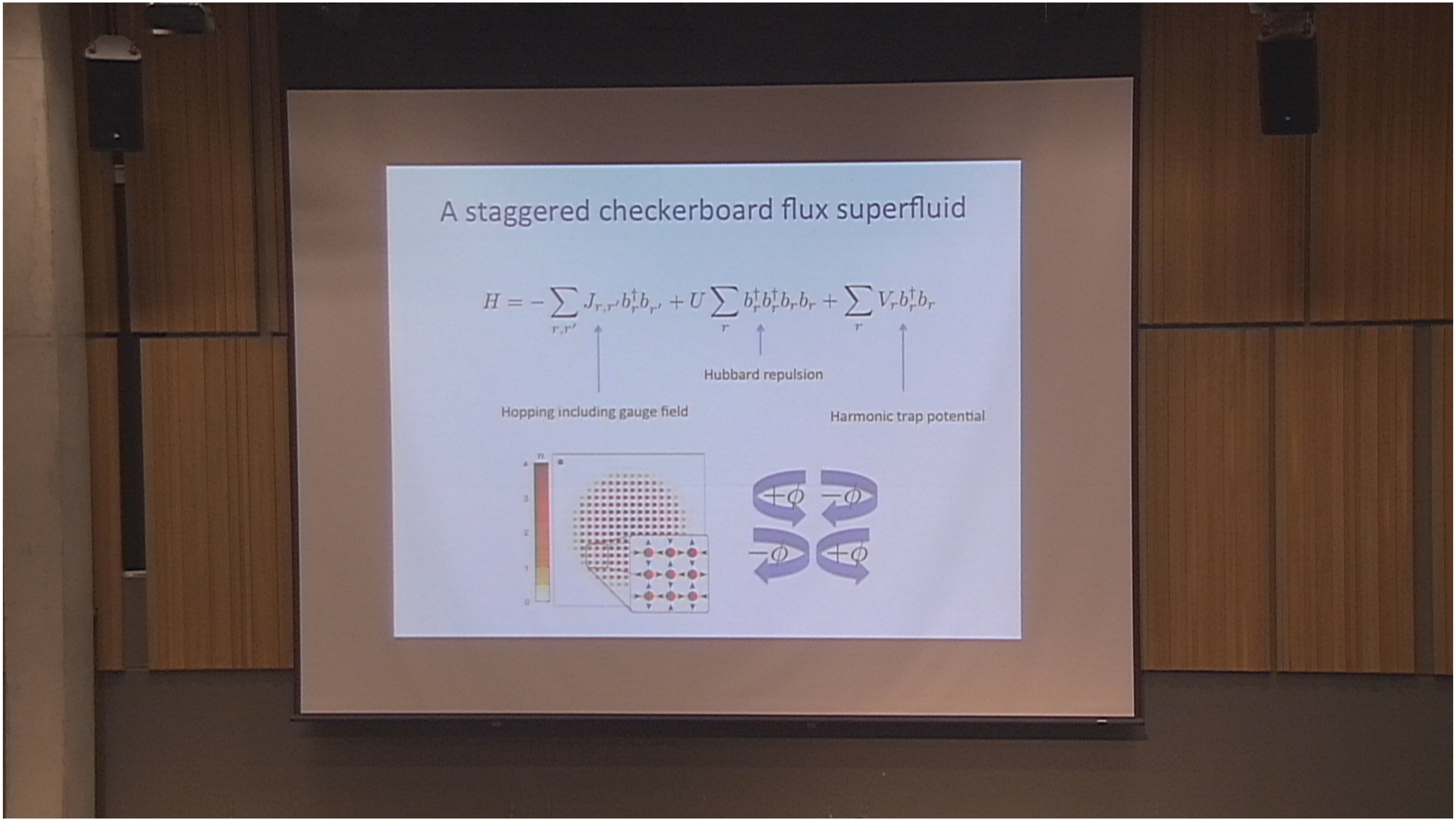
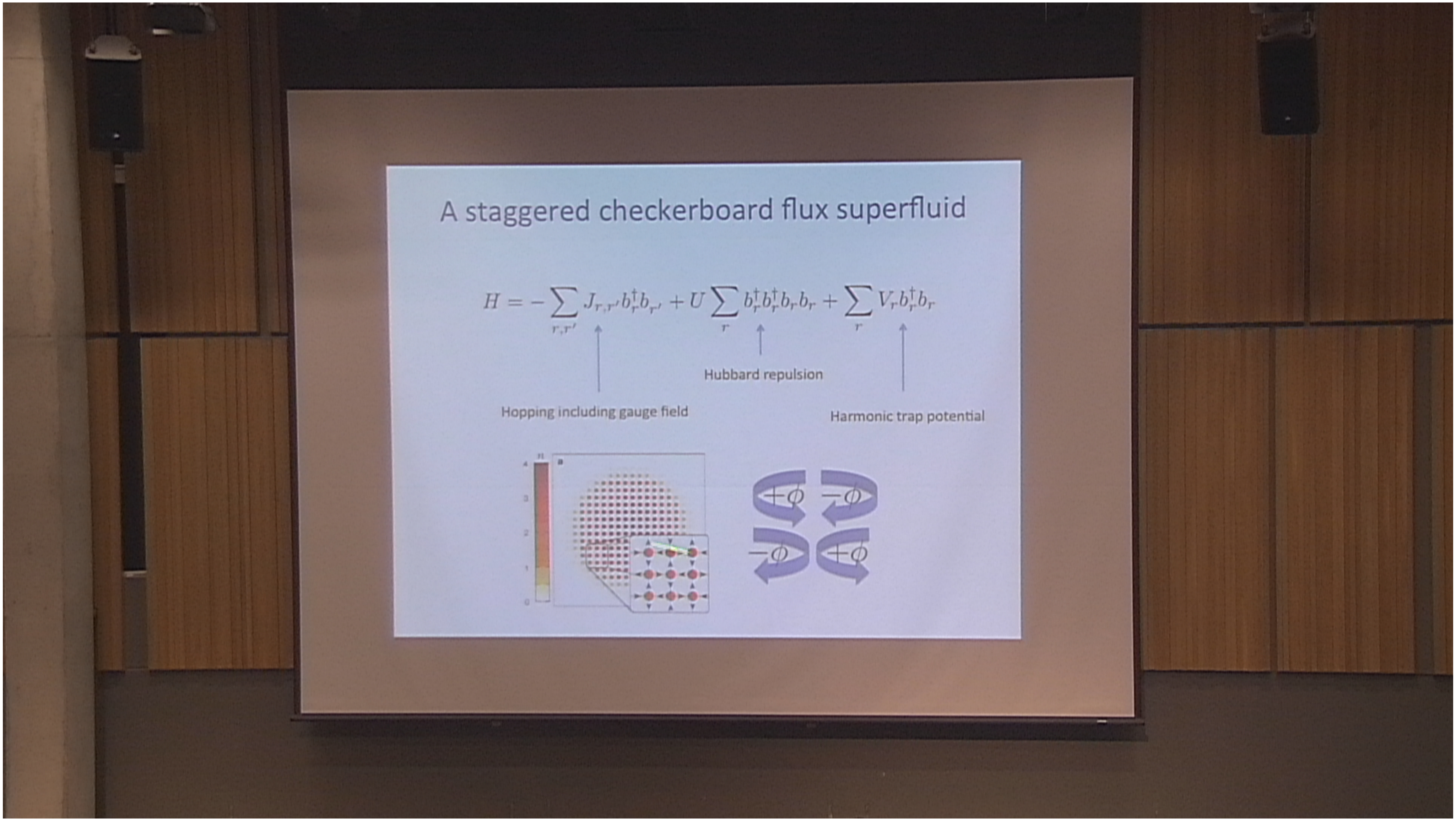

Optical lattices with frustration/fluxes
Triangular array of tubes
Unconventional superfluids from
frustrated Josephson coupling
Ferromagnetic
Rhombic
Spiral 1
Spiral 2
J. Struck, et al (Science 2011)
Cubic lattice
short lattice alone
(no Raman drive)
Exp.
Theory
Large staggered fluxes in an optical lattice
M. Aidelsburger, et al (PRL 2011)


A staggered checkerboard flux superfluid
$H = -\sum_{r,r'} J_{r,r'} b_r^\dagger b_{r'} + U \sum_r b_r^\dagger b_r^\dagger b_r b_r + \sum_r V_r b_r^\dagger b_r$
Hubbard repulsion
Hopping including gauge field
Harmonic trap potential
$+\phi$
$-\phi$
$-\phi$
$+\phi$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Hubbard repulsion
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Harmonic trap potential
$+\phi$ $-\phi$
$-\phi$ $+\phi$

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Hopping including gauge field

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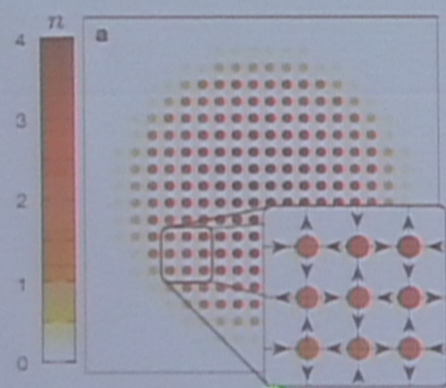

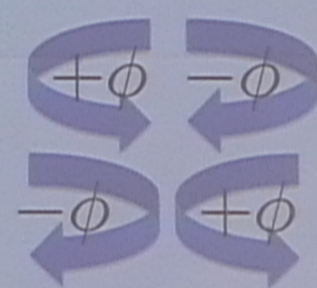

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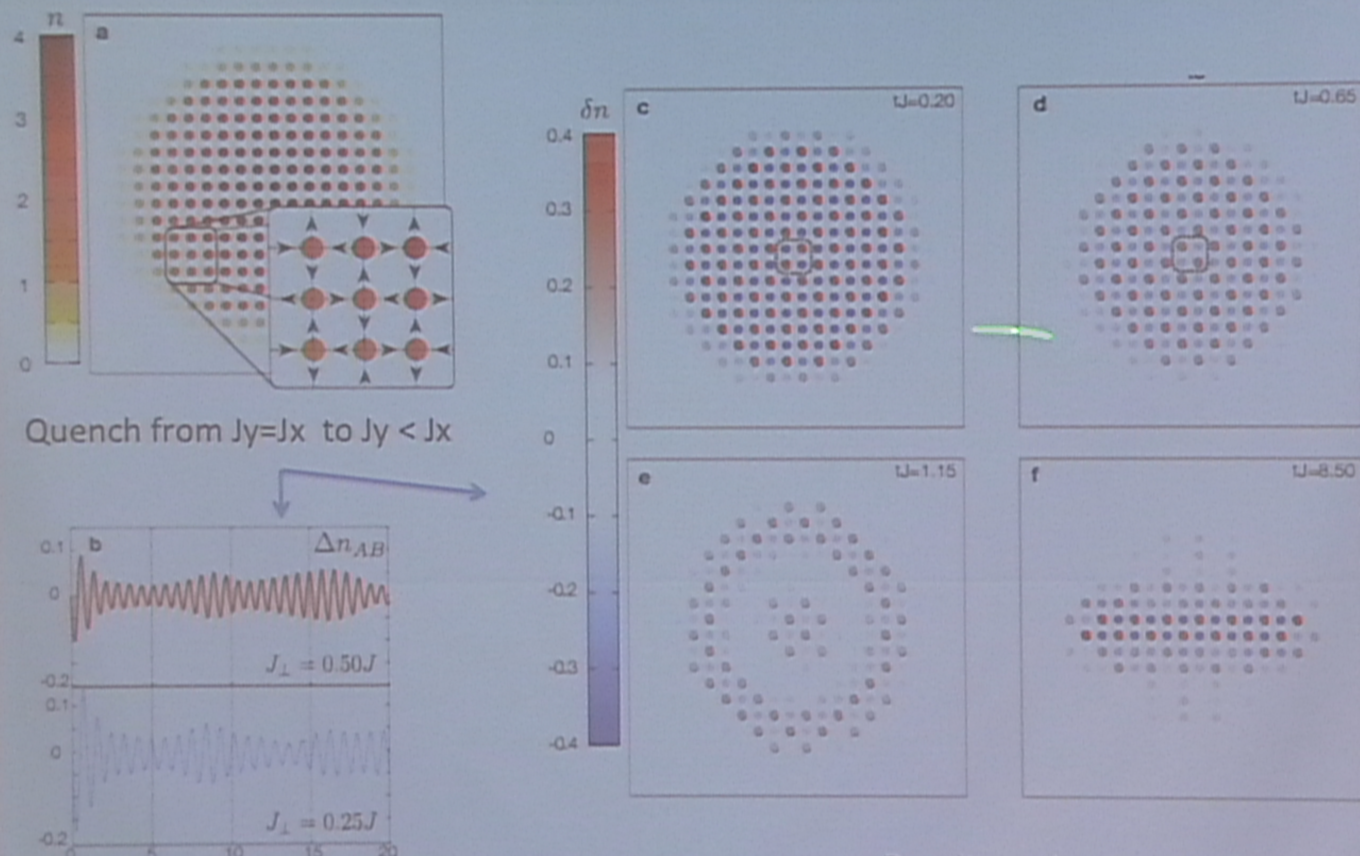
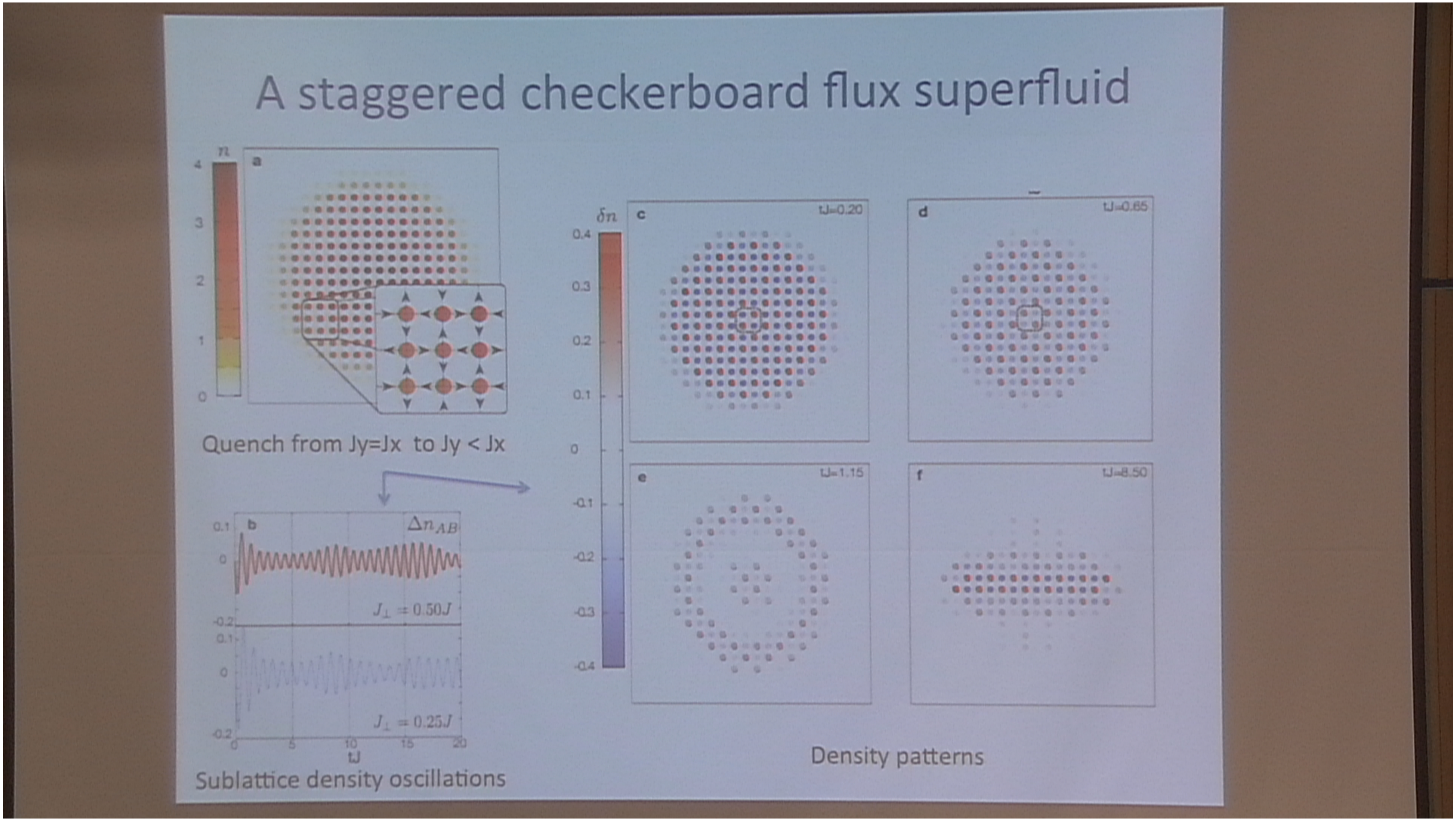


Quench from Jy=Jx to Jy < Jx

Sublattice density oscillations

Density patterns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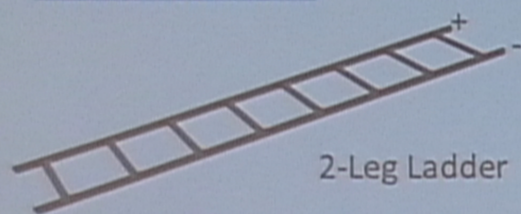
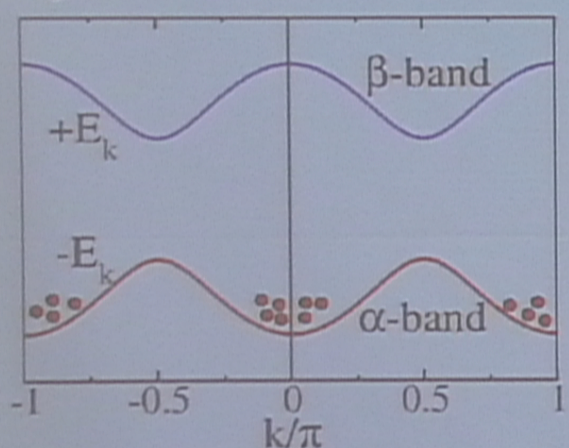
Density patterns
n
δn
Δn_{AB}
$J_\perp = 0.50J$
$J_\perp = 0.25J$
tJ
tJ=0.20
tJ=0.65
tJ=1.15
tJ=8.50

What happens with half-flux-quantum and Hubbard repulsion for checkerboard superfluid?

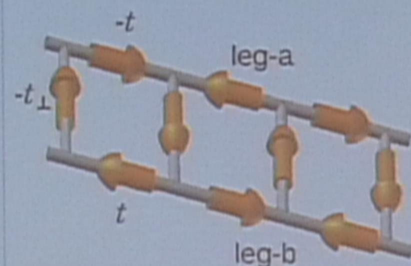
Weak Correlations



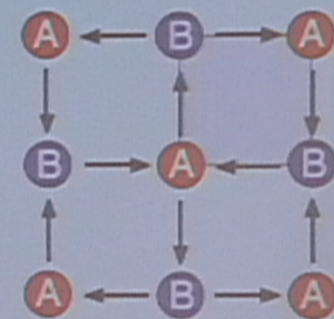


Boson dispersion has multiple minima (Kinetic Frustration)

2-Leg Ladder



2D square



SF state spontaneously breaks time-reversal and translational symmetry: "Chiral Superfluid"

Produces staggered loop currents on plaquettes

M. Polini, et al (PRL 2005)
Lim, et al (PRL 2008)
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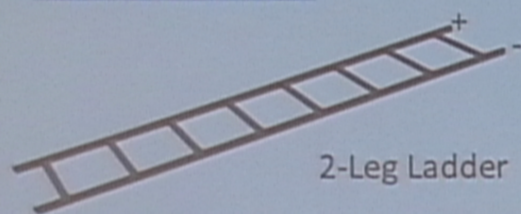

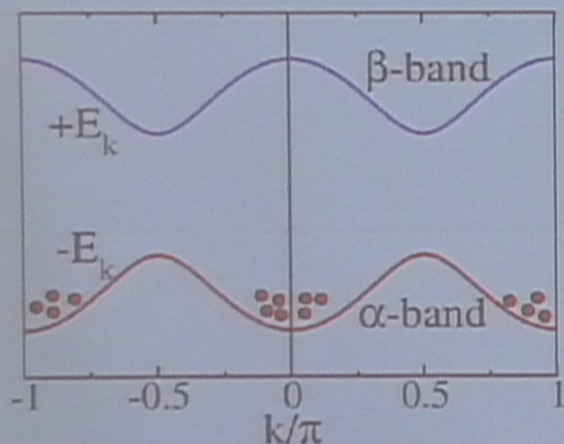

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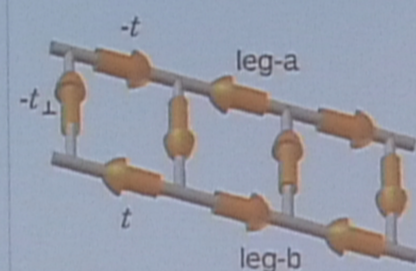

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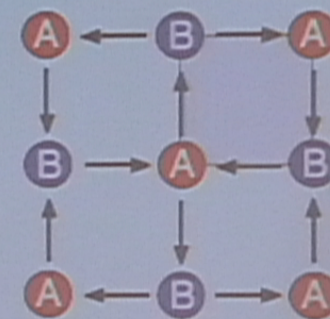

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Mean field theory

Generalize usual superfluid-Mott transition mean field theory
(effective single-site approximation)

1. Continuous Chiral Superfluid to Mott transition
2. Uc for the Mott transition suppressed

f=0 Superfluid Mott insulator U/t

f=1/2 Chiral superfluid Mott insulator U/t

Recent example: Lim, et al (PRL 2008)
More general point: Critical theory involves 2 bosonic modes (Eg: Sinha, Sengupta: EPL 2011)

BUT
How can two broken symmetries (SF and time-reversal) vanish continuously at a single transition? [Would be an example of a Landau-forbidden transition]

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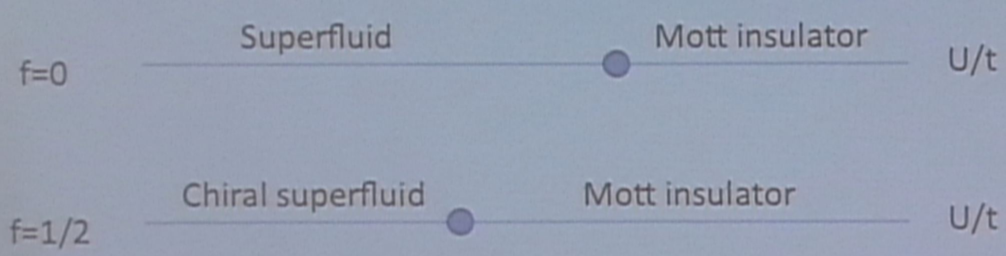

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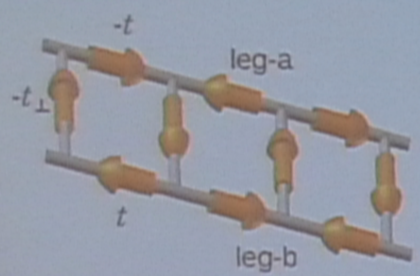
What happens as we increase U/t at fixed hoppings?



Ladder mean field theory

$$\boxed{\frac{1}{\sqrt{4t^2+t_\perp^2}} = \frac{n}{\mu - U(n-1)} + \frac{n+1}{Un-\mu}}$$

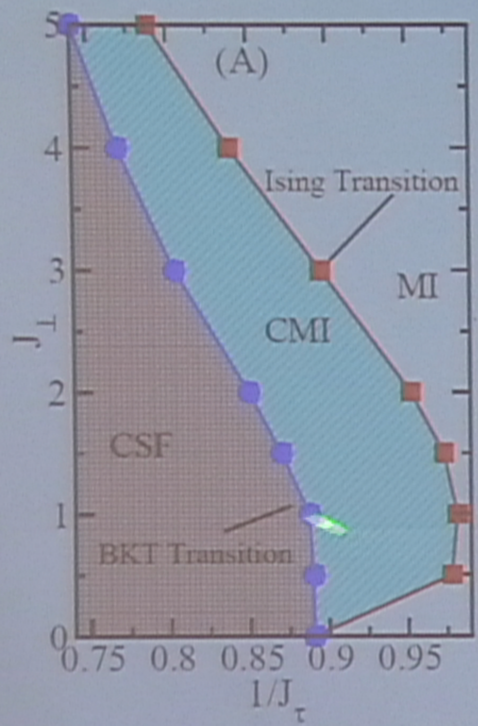
$$\boxed{\frac{U^{\pi-\mathrm{flux}}_{c,ladder}}{U^{0-\mathrm{flux}}_{c,ladder}} = \frac{\sqrt{4t^2+t_\perp^2}}{2t+t_\perp} < 1}$$

We will go **beyond mean field theory** using two different numerical approaches

1. Map to a 1+1=2 dimensional frustrated classical XY model (on a bilayer)
   Study using classical Monte Carlo methods

2. Directly simulate the ladder Hubbard model using DMRG methods

Classical XY model phase diagram



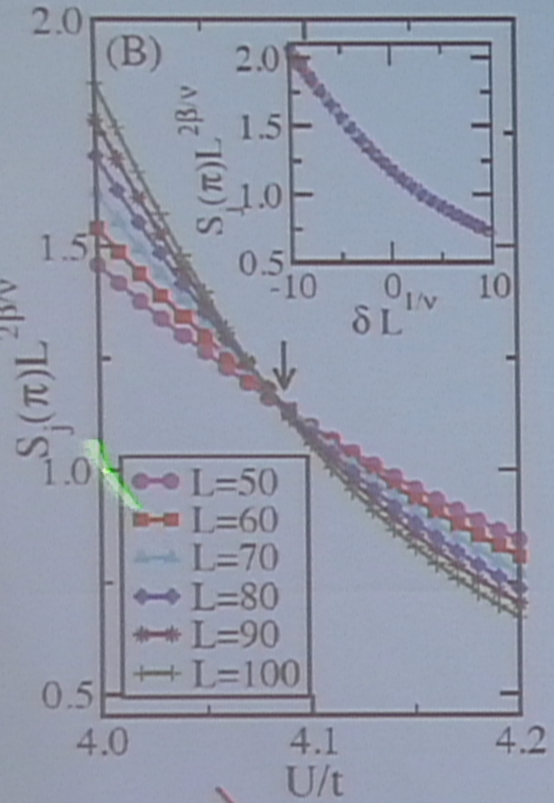
Three Phases

Chiral Superfluid
- power law phase order
- long range loop current order

Conventional Mott Insulator
- exponentially decaying phase (spin) correlations
- exponentially decaying current correlations

Chiral Mott Insulator
- exponentially decaying phase (spin) correlations
- long range loop current order

DMRG study of the Bose Hubbard Ladder



To detect the staggered current order, look at the scaling of the structure factor

Using 2D Ising exponents, find the transition point where loop current order vanishes

Observe scaling collapse of the data for various system sizes

Find consistent crossing point and data collapse for 2D Ising exponents

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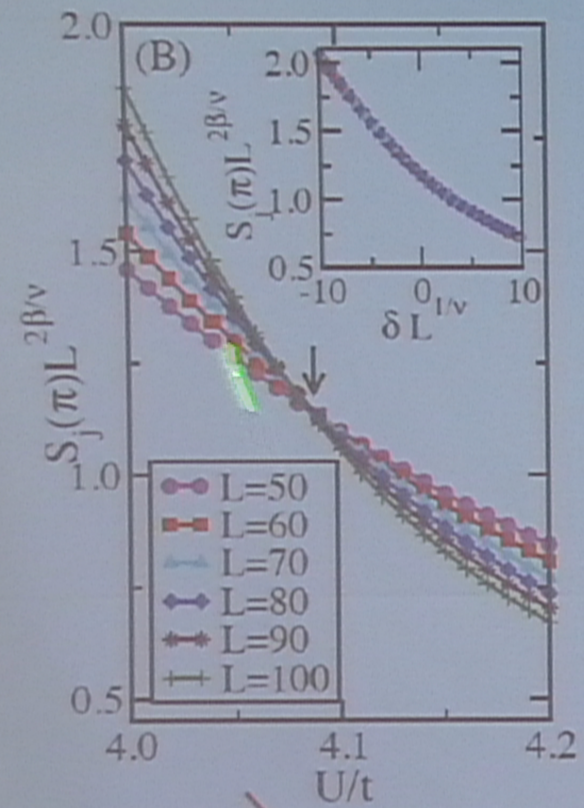

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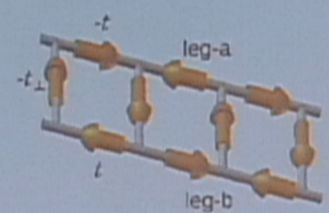
Physical Pictures for the Chiral Mott Insulator

Chiral superfluid = Vortex Crystal
. Flux nucleates vortex or antivortex
. Vortex-vortex interaction is repulsive
. Equal number of V/AV
. "Antiferromagnetic" crystal



Regular Mott insulator = Vortex Superfluid (*D. Haldane; Halperin/Dasgupta; Fisher/Lee*)
. Dual - proliferated quantum phase slips

Chiral Mott insulator = Vortex Supersolid
. Defect in crystal: Extra vacancy/interstitial vortex/antivortex
. Proliferating and condensing dilute defects: Vortex superfluid
. Background current pattern preserved: Vortex crystal

Physical Pictures for the Chiral Mott Insulator

Excitations of a Conventional Mott insulator

. Gapped Particles: “double occupancy”
. Gapped Holes: “vacancy”
. Dispersing particles/holes: Like a “semiconductor”

Excitations of a Conventional Mott insulator with flux

. Gapped Particles: “double occupancy”
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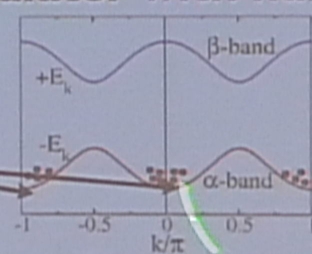
. Dispersing particles/holes **with multiple minima**
Like a “semiconductor” with multiple valleys



Semiconductors can have excitons and
exciton condensation: Halperin-Rice

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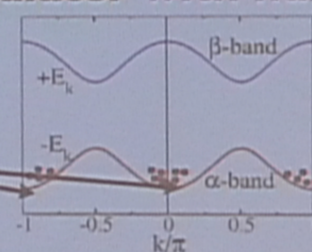

Chiral Mott insulator: Indirect Exciton condensate!

Experimental signatures

1. Josephson junction array realization:

. Insulator in transport
. Spontaneous staggered fields (SQUID microscopy)
$J \sim 1K$ coupling; $1\mu m \times 1\mu m$ cell; $1nT$ fields

2. Cold atom realization:

. No sharp peaks in n(k)
. Look for residual interference between the two peaks
. Do quench measurements