

Title: Quantum Spin Liquids: Life After the Drought

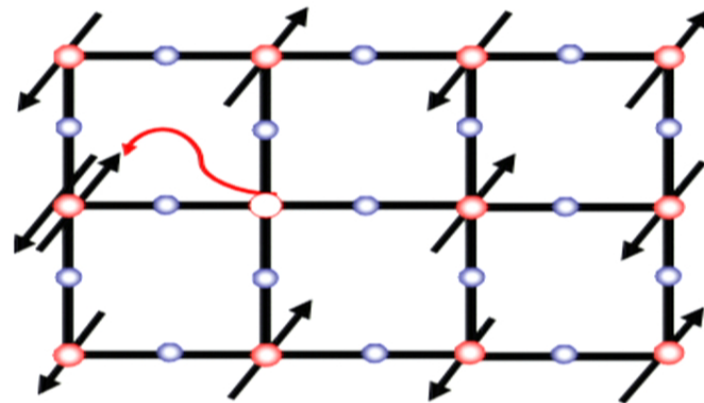
Date: May 03, 2012 10:00 AM

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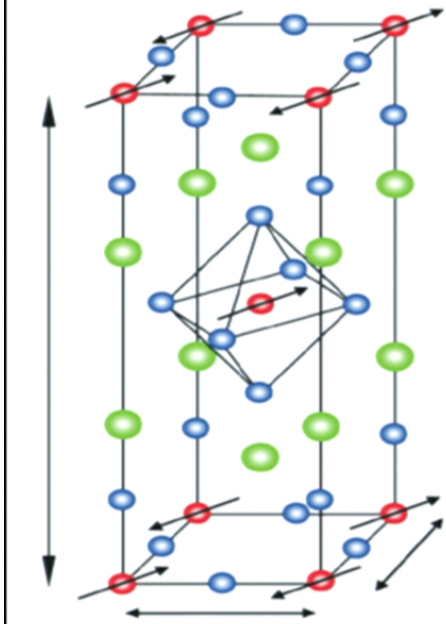
Abstract: The quantum spin liquid state is a prime example of an emergent phenomenon. Theory predicts that new particles such as spinons and gauge fields may emerge at low temperatures. However, for many years there have not been any examples in nature. The situation has changed in recent years in that a number of candidate materials have been discovered which may exhibit these exotic phenomena.

# strongly-correlated electron system example: Hi Tc cuprate.

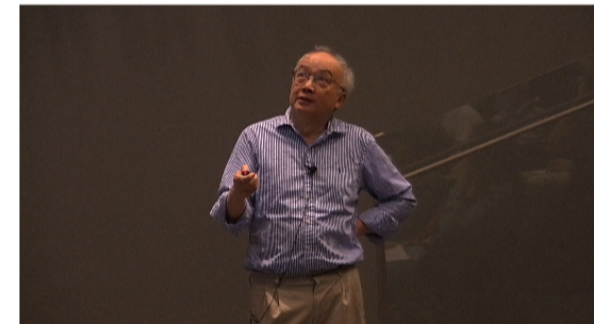
One hole per site: should be a metal according to band theory.  
Mott insulator.



*Undoped CuO<sub>2</sub> plane:  
Mott Insulator due to  
e<sup>-</sup> - e<sup>-</sup> interaction  
Virtual hopping induces  
AF exchange  $J=4t^2/U$*



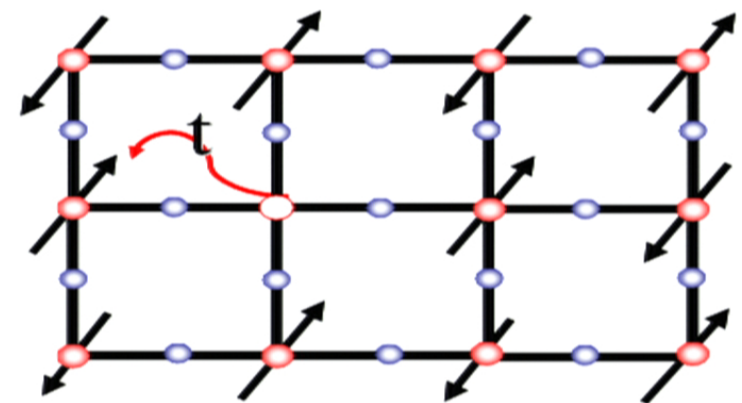
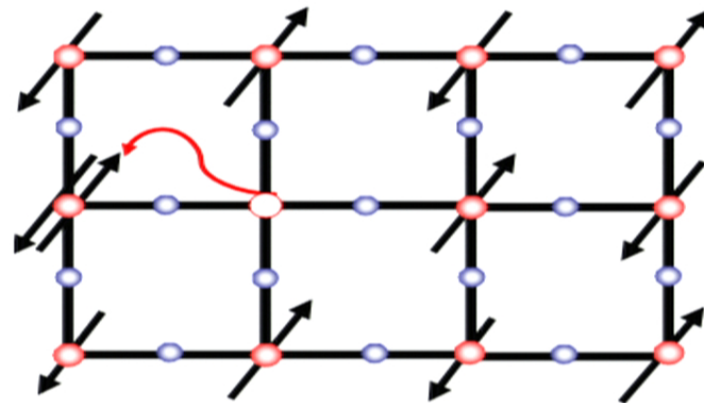
Cu<sup>2+</sup>     ●  
O<sup>2-</sup>     ●  
La<sup>3+</sup>    ●





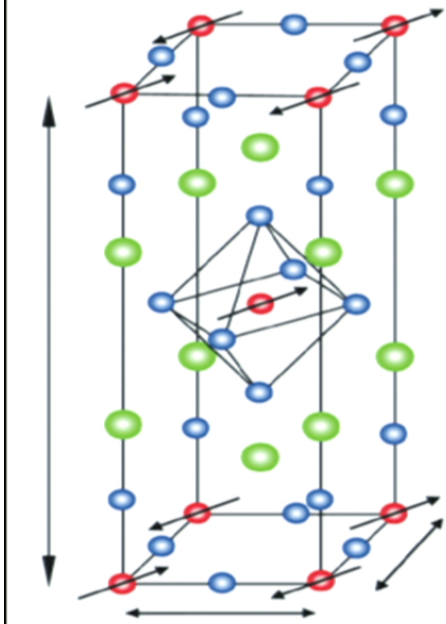
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*$\text{CuO}_2$  plane with doped holes:  
 $\text{La}^{3+} \rightarrow \text{Sr}^{2+}: \text{La}_{2-x}\text{Sr}_x\text{CuO}_4$*



$\text{Cu}^{2+}$  ●  
 $\text{O}^{2-}$  ●  
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## **Mott against Slater debate:**

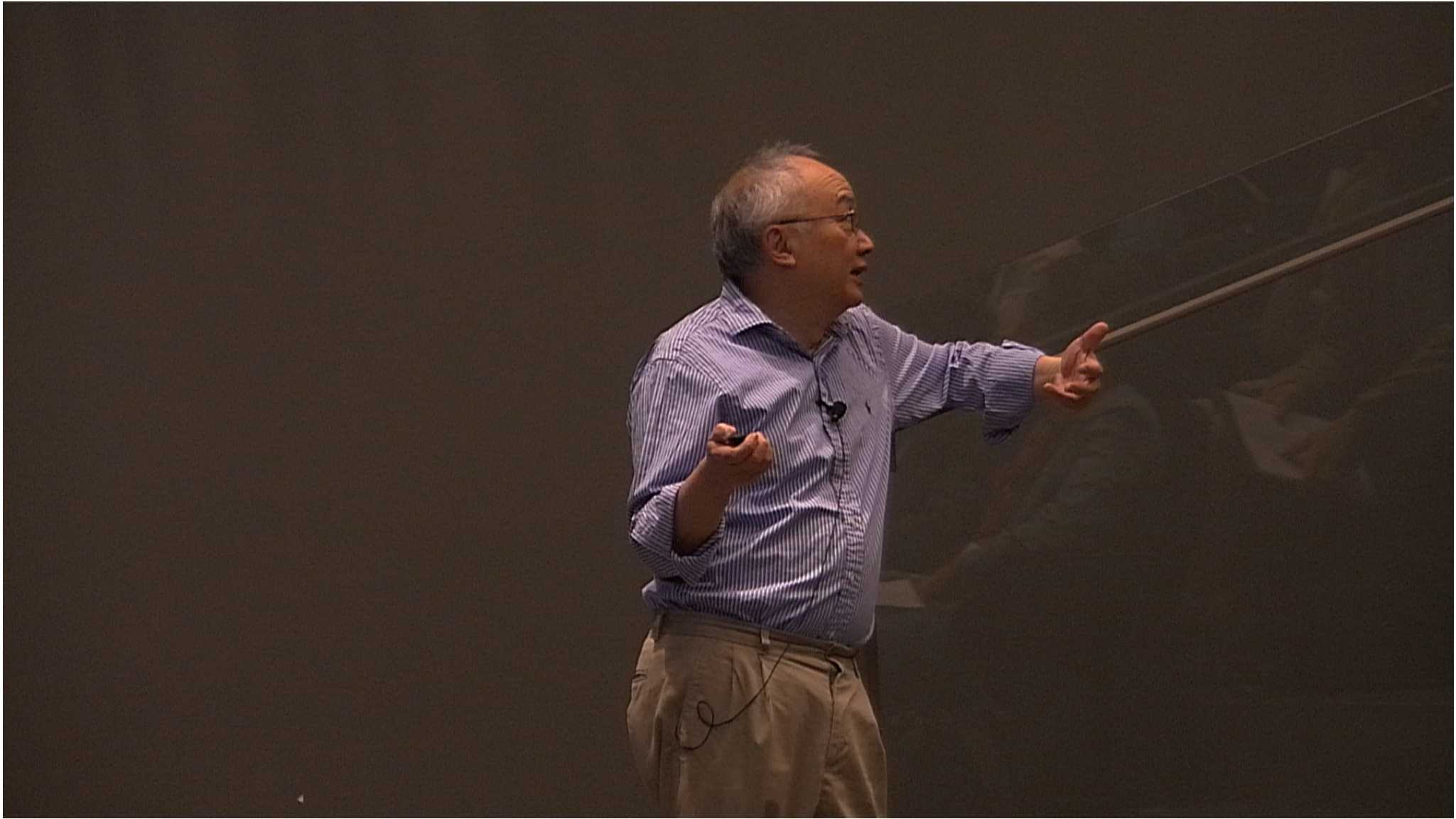
Slater:

Anti-ferromagnetic ground state.  
Unit cell is doubled. Then we have  
2 electrons per unit cell and the  
system can be an insulator,  
consistent with band theory.

Mott:

Charge gap is due to correlation.  
Antiferromagnetism is secondary.  
Mott insulator violate band theory.

**Can there be a Mott insulator which does not have AF order?**





P. W. Anderson introduced the RVB idea in 1973.

**Key idea:** spin singlet can give a better energy than anti-ferromagnetic order.

What is special about  $S=1/2$ ?

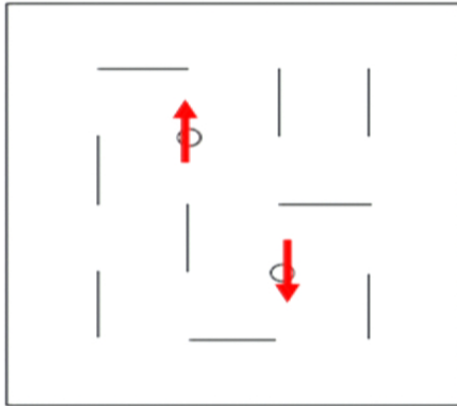
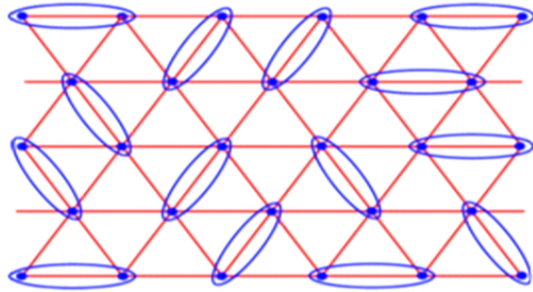
1 dimensional chain:

Energy per bond of singlet trial wavefunction is

$(1/2)S(S+1)J = (3/8)J$  vs.  $(1/4)J$  for AF.



$$\uparrow\downarrow = \frac{1}{\sqrt{2}} (\uparrow\downarrow - \downarrow\uparrow)$$



Spin liquid: destruction of Neel order due to quantum fluctuations.

In 1973 Anderson proposed a spin liquid ground state (RVB) for the triangular lattice Heisenberg model.. It is a linear superposition of singlet pairs. (not restricted to nearest neighbor.)

**New property of spin liquid:**

Excitations are spin  $\frac{1}{2}$  particles (called spinons), as opposed to spin 1 magnons in AF. These spinons may even form a Fermi sea.

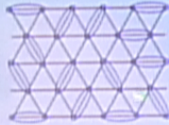
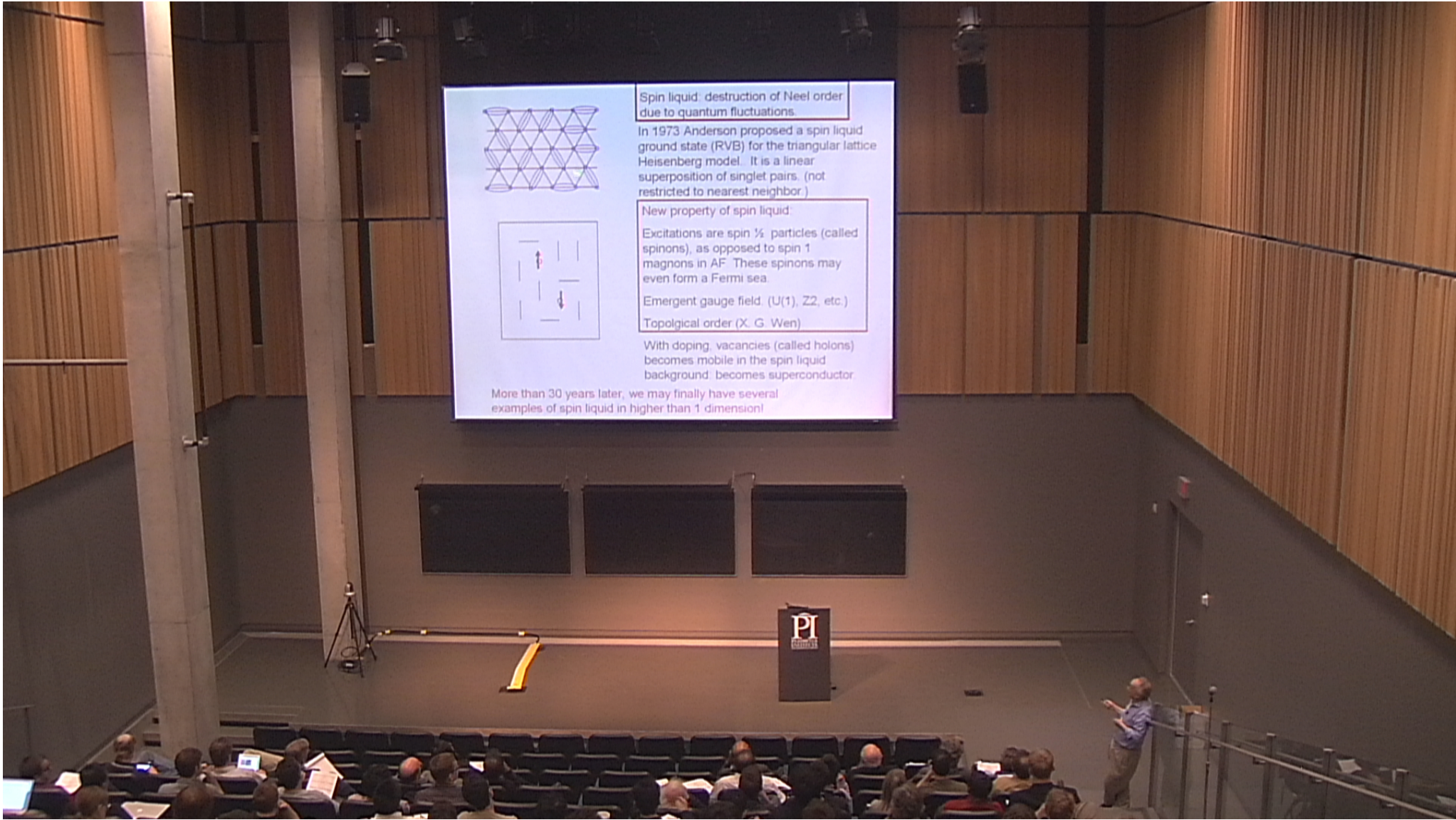
Emergent gauge field. (U(1), Z2, etc.)

Topological order (X. G. Wen)

With doping, vacancies (called holons) becomes mobile in the spin liquid background: becomes superconductor.

More than 30 years later, we may finally have several examples of spin liquid in higher than 1 dimension!





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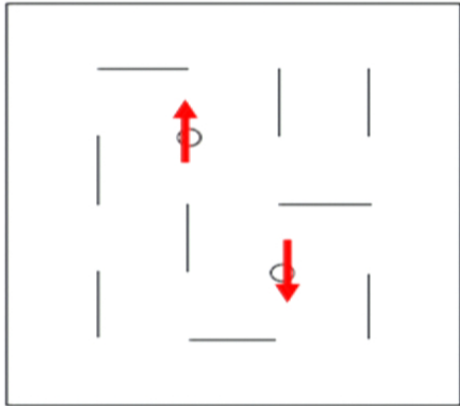
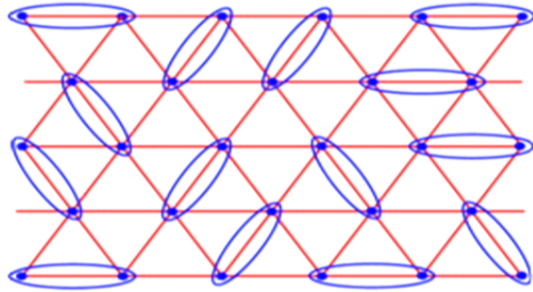
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$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j,$$

Introduce fermions which carry spin index  $\vec{S}_i = \frac{1}{2} f_{i\alpha}^\dagger \vec{\sigma}_{\alpha\beta} f_{i\beta}$

$$\mathbf{S}_i \cdot \mathbf{S}_j = -\frac{1}{4} f_{i\alpha}^\dagger f_{j\alpha} f_{j\beta}^\dagger f_{i\beta} - \frac{1}{4} (f_{i\uparrow}^\dagger f_{j\downarrow}^\dagger - f_{i\downarrow}^\dagger f_{j\uparrow}^\dagger) (f_{j\downarrow} f_{i\uparrow} - f_{j\uparrow} f_{i\downarrow}) + \frac{1}{4} (f_{i\alpha}^\dagger f_{i\alpha})$$

Constraint of single occupation,  
no charge fluctuation allowed.

$$f_{\uparrow}^\dagger f_{\uparrow} + f_{\downarrow}^\dagger f_{\downarrow} = 1$$

Two ways to proceed:

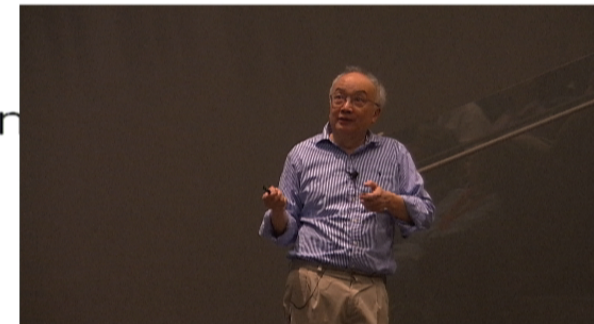
1. Numerical: Projected trial wavefunction.

$$|\Psi_{proj}(\chi_{ij})\rangle = P_D |\Psi_{mean}(\chi_{ij})\rangle$$

$$P_D = \prod_i (1 - n_{i\uparrow} n_{i\downarrow})$$

Extended Hilbert space: many to one representation

2. Analytic: gauge theory.





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## General problem of compact gauge field coupled to fermions. **Emergence!**

Mean field (saddle point) solutions:

1. For  $\chi_{ij}$  real and constant: fermi sea.
2. For  $\chi_{ij}$  complex: flux phases and Dirac sea.

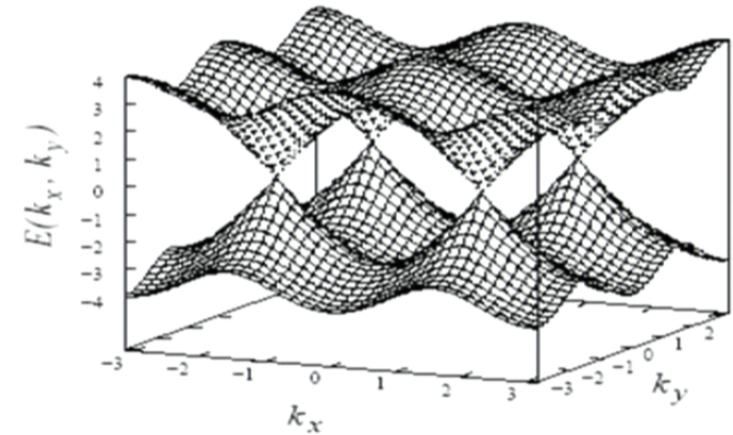
**Enemy of spin liquid is confinement:**

( $\pi$  flux state and SU(2) gauge field leads to chiral symmetry breaking, ie AF order)

If we are in the **de-confined phase**, fermions and gauge fields emerge as new particles at low energy. (**Fractionalization**)

**The fictitious particles introduced formally takes on a life of its own!**

They are not free but interaction leads to a new critical state. This is the spin liquid.



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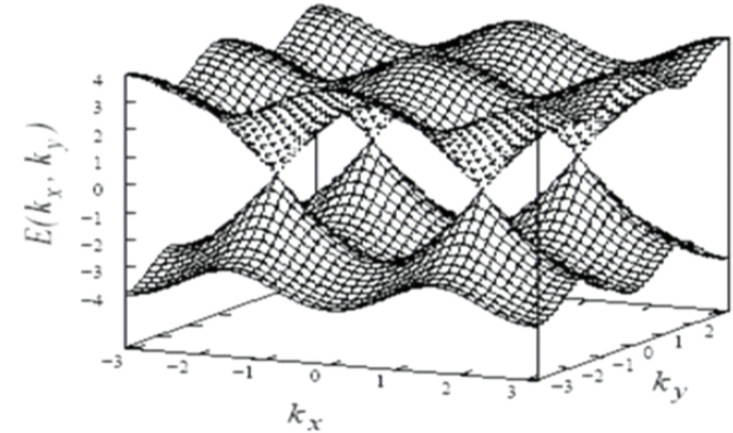
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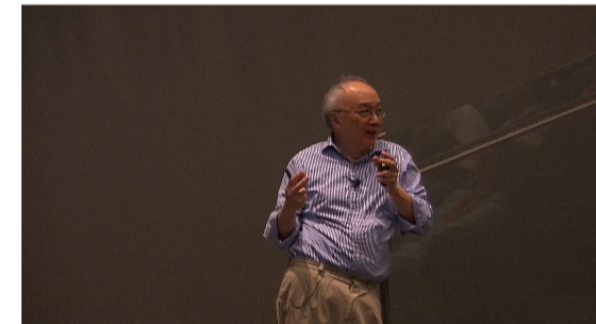
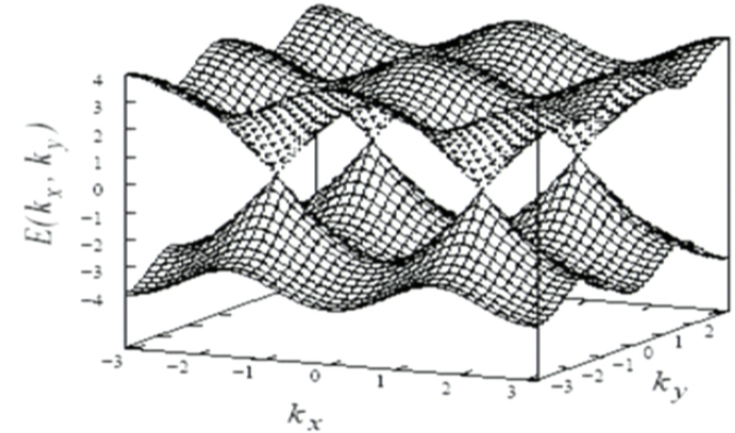
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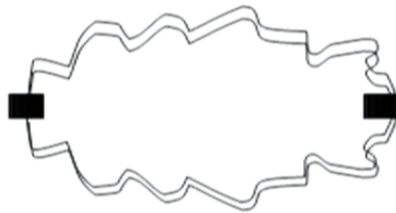
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# Physical Consequence

$$\text{Specific heat : } C \sim T^{2/3}$$



**Gauge fluctuations dominate entropy at low temperatures. (See also Motrunich, 2005)**

[Reizer (89); Nagaosa and Lee (90)]

Physical meaning of gauge field:

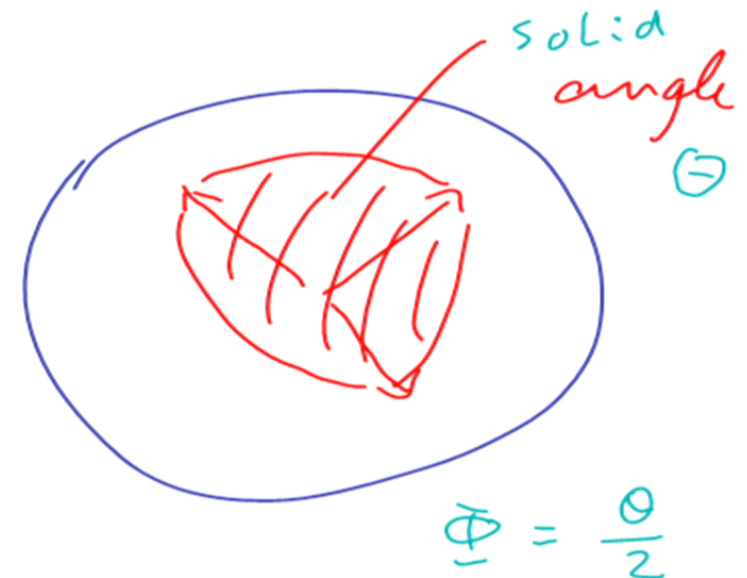
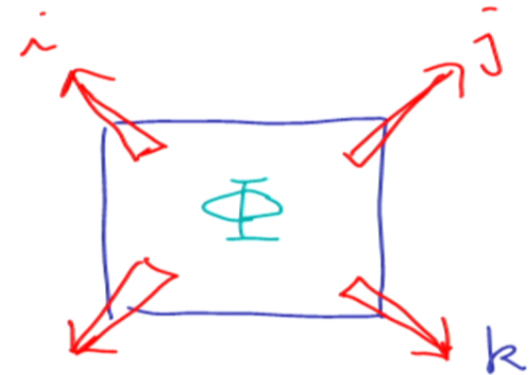
gauge flux is gauge invariant

$$\mathbf{b} = \nabla \times \mathbf{a}$$

Fermions hopping around a plaquette picks up a Berry's phase due to the meandering quantization axes. This is represented by a gauge flux through the plaquette.

It is related to spin chirality (Wen, Wilczek and Zee, PRB 1989)

$$\vec{S}_i \cdot (\vec{S}_j \times \vec{S}_k)$$



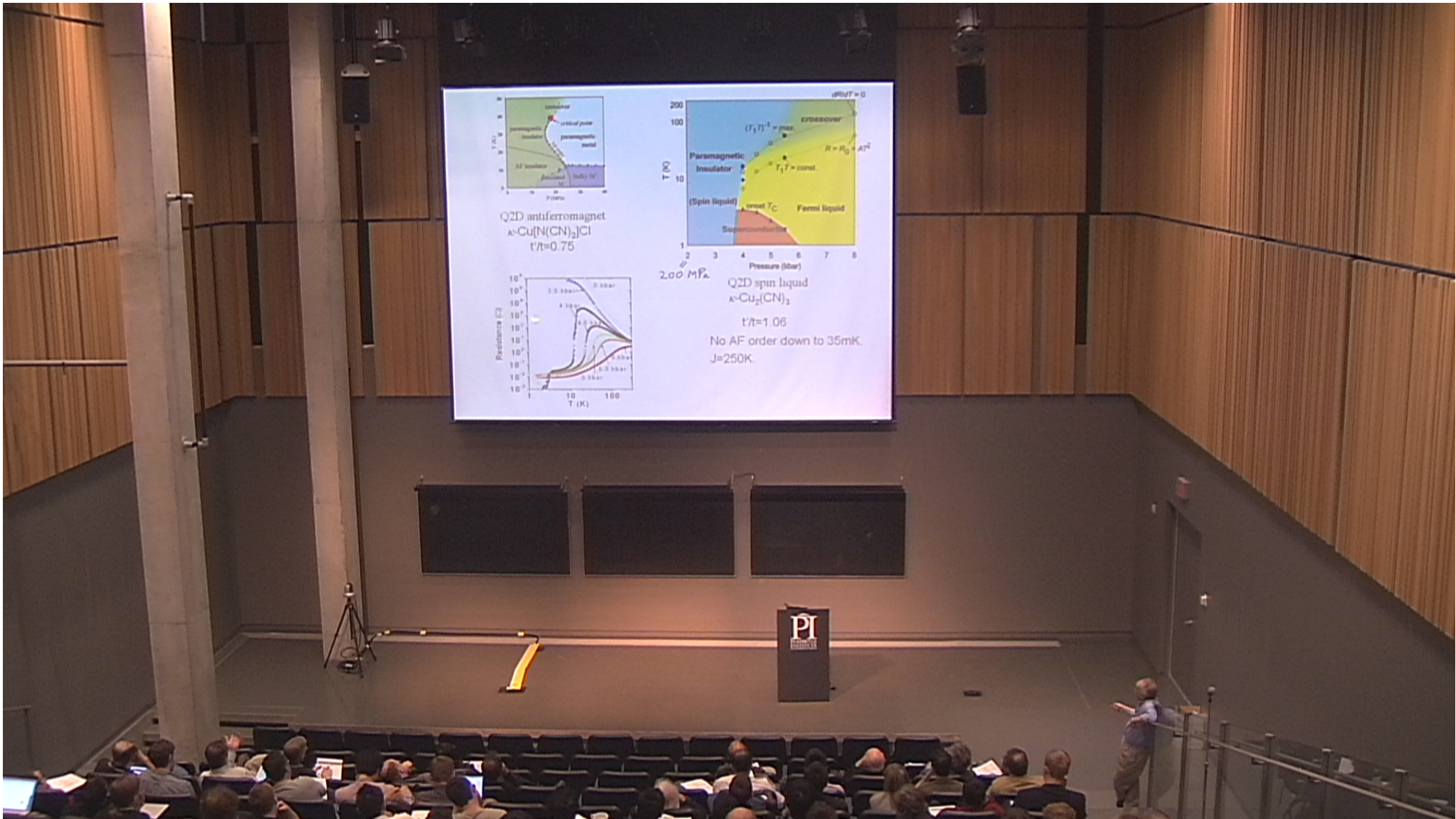
Four examples:

1. Organic triangular lattice near the Mott transition.
2. Kagome lattice, more frustrated than triangle.
3. Hyper-Kagome, 3D.
4.  $S=1/2$  triangular lattice:  $\text{Ba}_3\text{CuSb}_2\text{O}_9$  and  $S=1$  version.

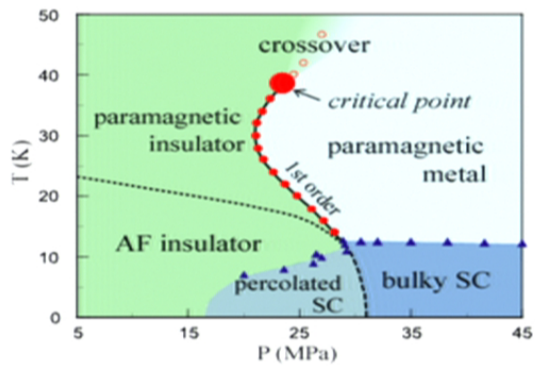
Other disordered ground states:

Pyrochlore, such as  $\text{Yb}_2\text{Ti}_2\text{O}_7$ ; competition between single ion anisotropy and exchange. Next talk.

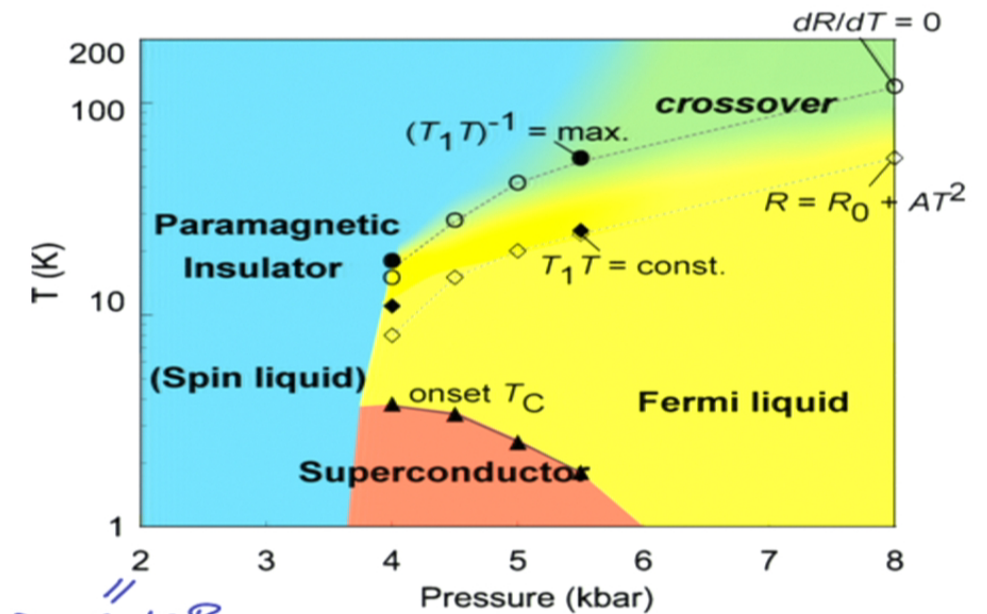
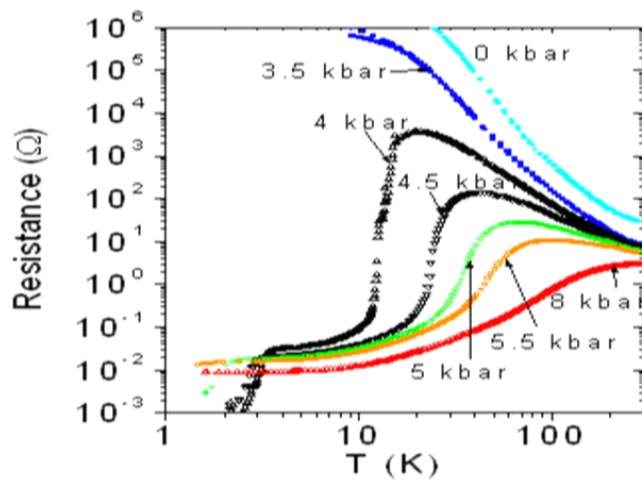
We are not talking about spin glass, spin ice etc.







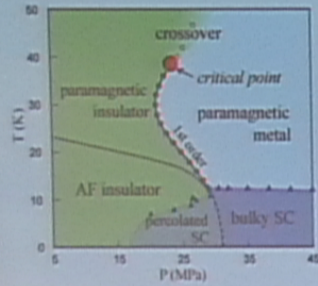
Q2D antiferromagnet  
 $\kappa$ -Cu[N(CN)<sub>2</sub>]Cl  
 $t'/t=0.75$



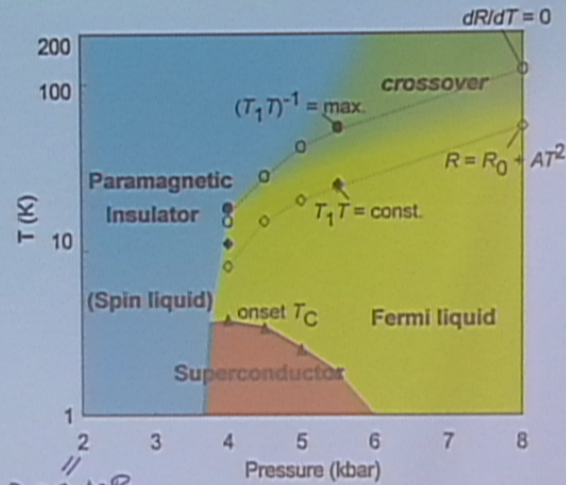
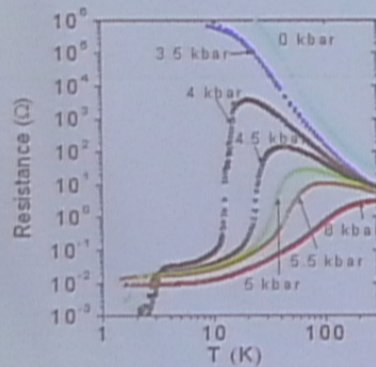
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No AF order down to 35mK.  
 $J=250\text{K}$ .



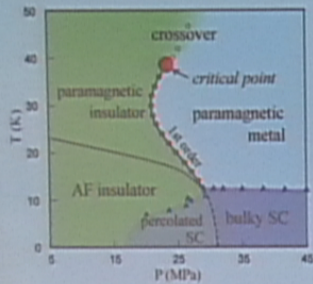
Q2D antiferromagnet  
 $\kappa\text{-Cu}[\text{N}(\text{CN})_2]\text{Cl}$   
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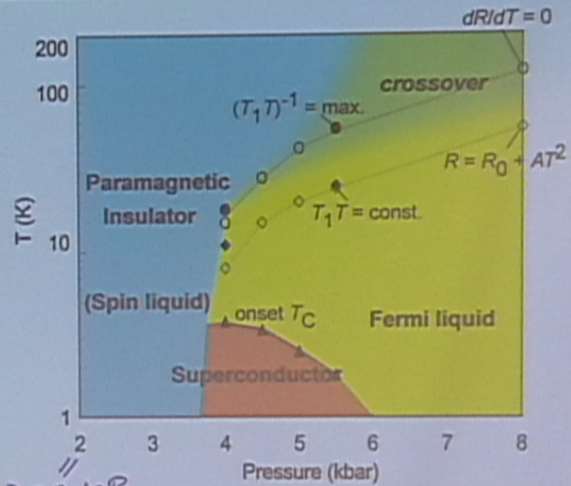
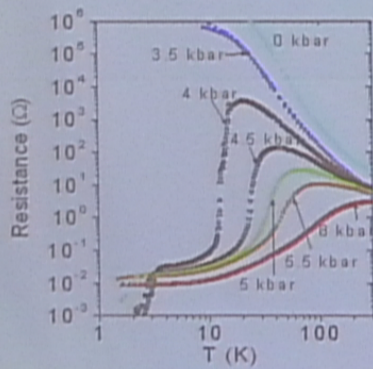
$200 \text{ MPa}$   
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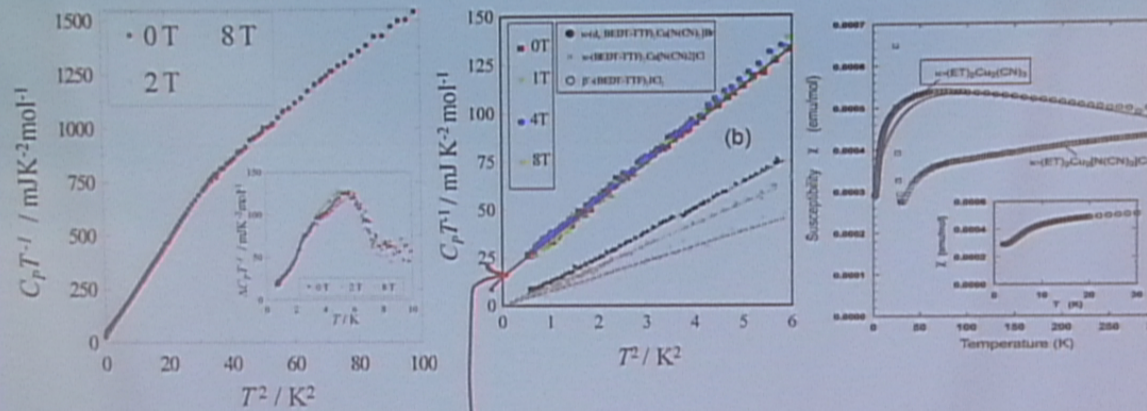


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From Y. Nakazawa and K. Kanoda, Nature Physics 2008.



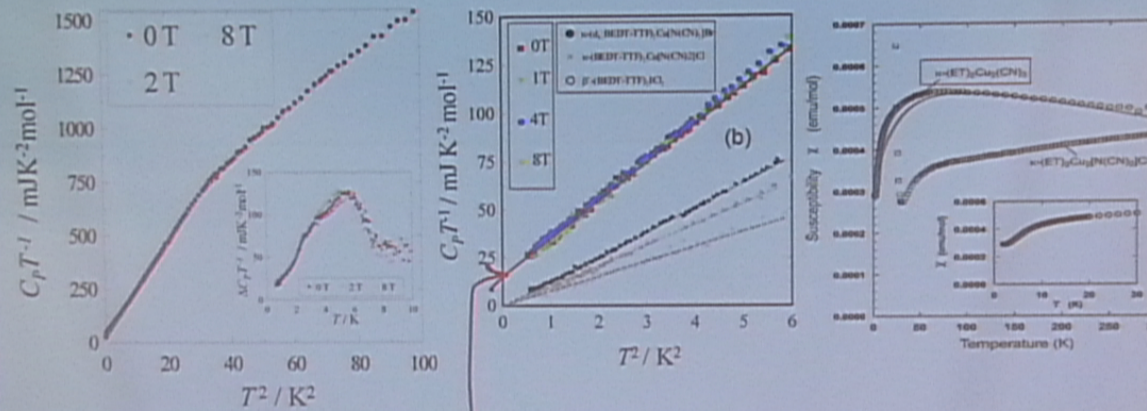
$\gamma$  is about  $15 \text{ mJ/K}^2 \text{mole}$

Something happens around 6K.  
 Partial gapping of spinon Fermi surface due to spinon pairing?

Wilson ratio is approx. one at  $T=0$ .



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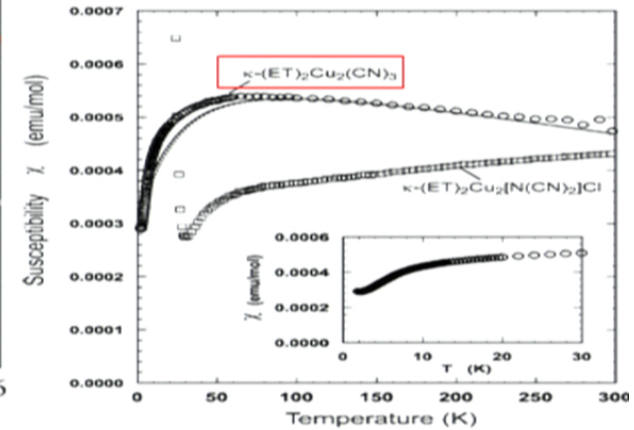
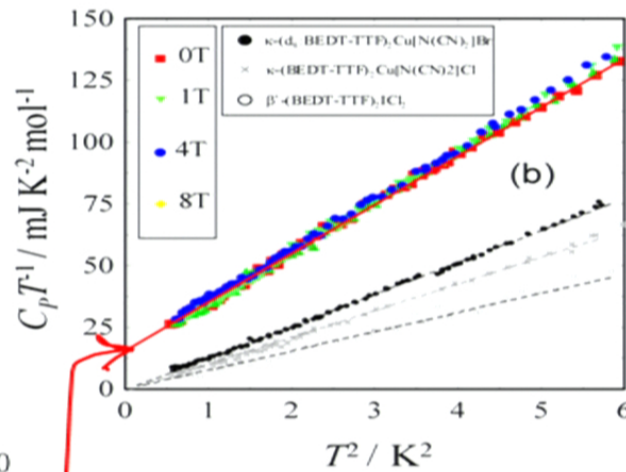
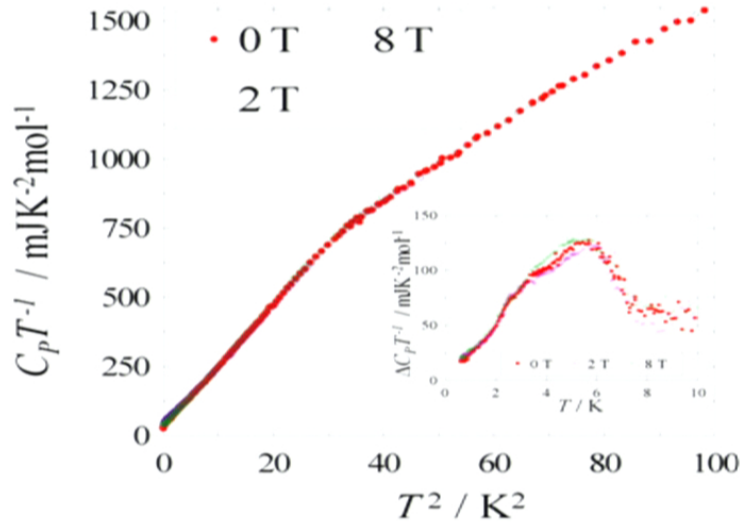


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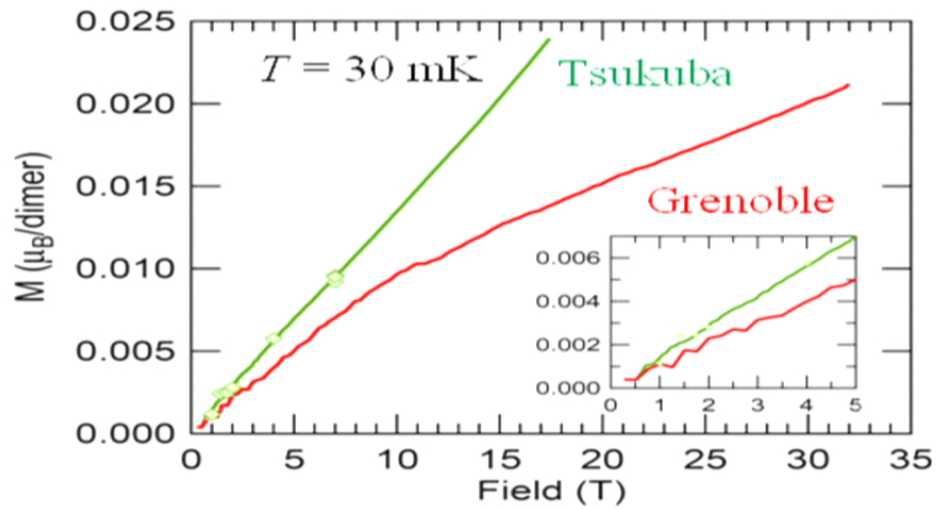
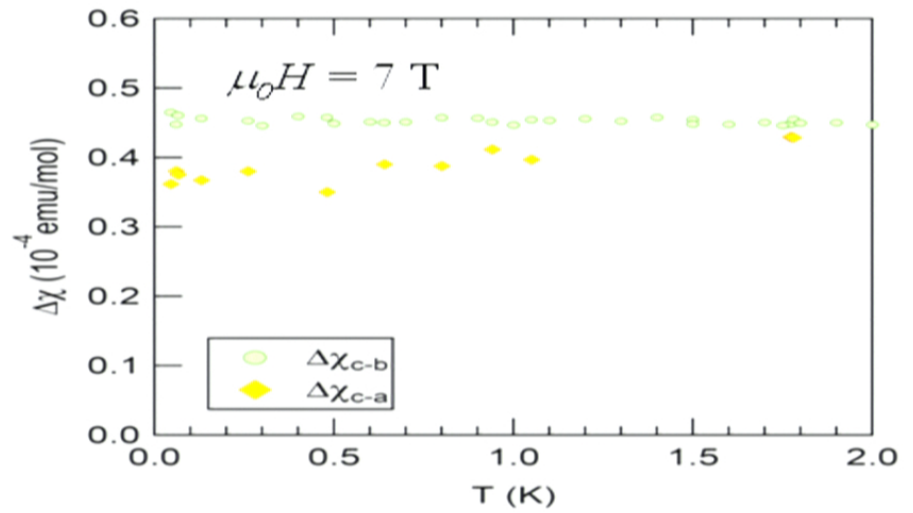
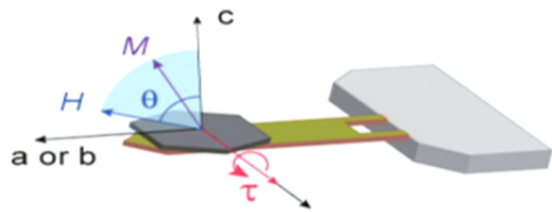
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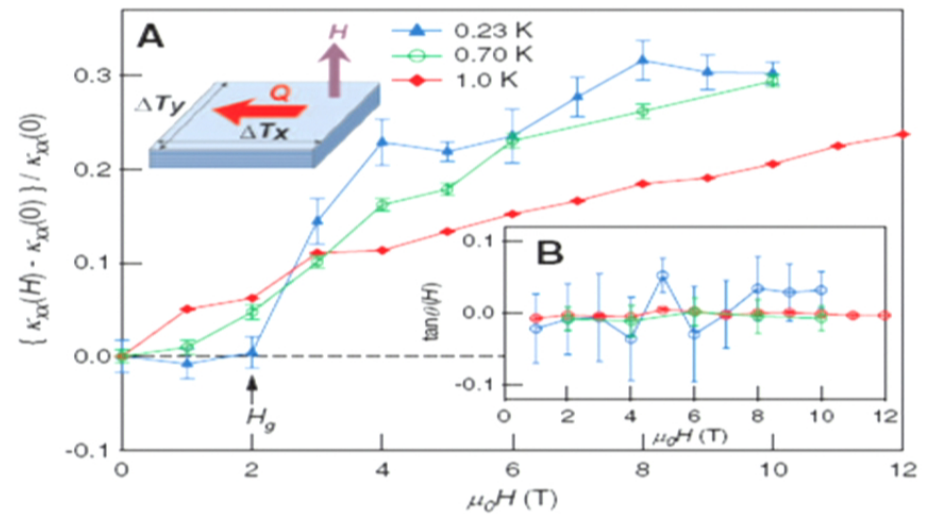
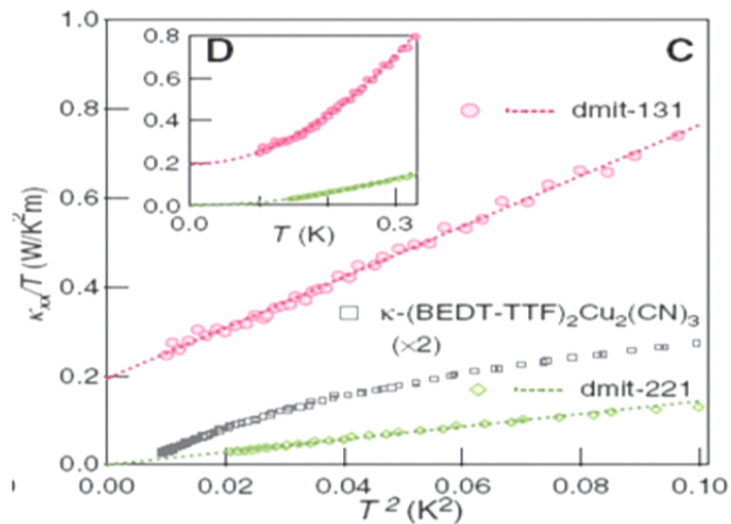
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Watanabe .... Matsuda, APS march 2012



## Thermal conductivity of dmit salts.



$$\kappa_{xx}^{\text{spin}} = C_s v_s \ell_s / 3$$

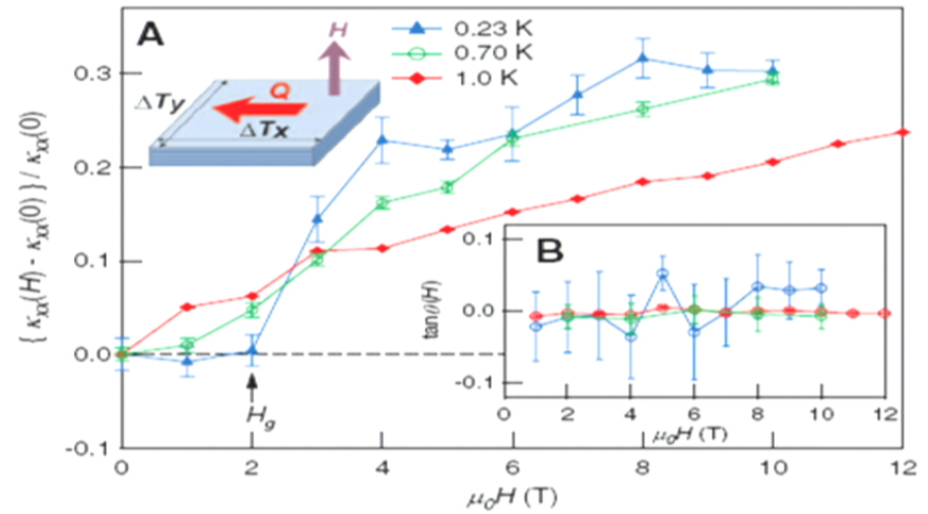
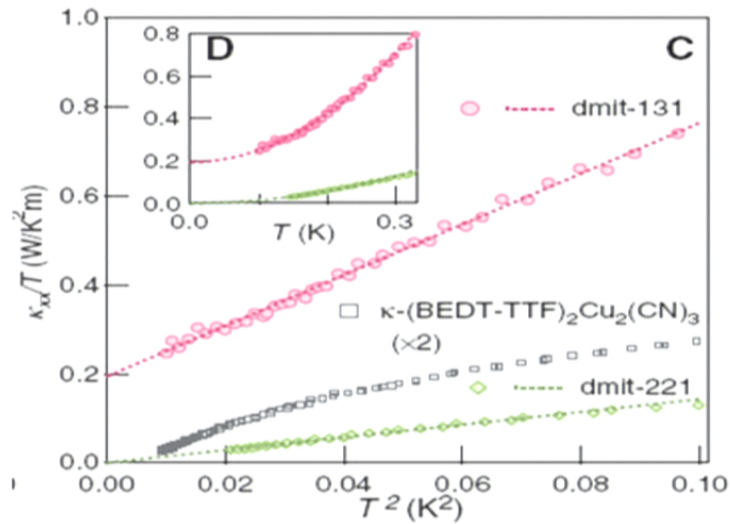
mean free path  $\ell_s$  reaches 500 inter-spin spacing.

M. Yamashita et al, Science 328, 1246 (2010)

However, ET salt seems to develop a small gap below 0.2 K.



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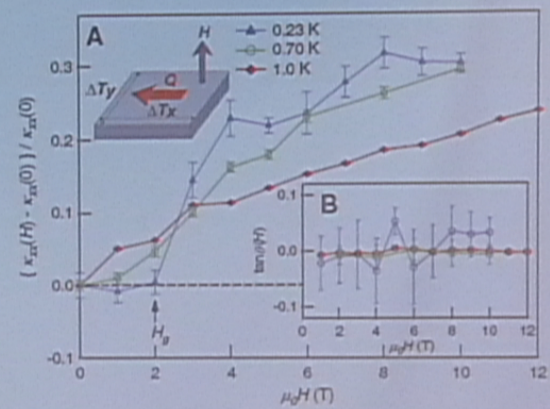
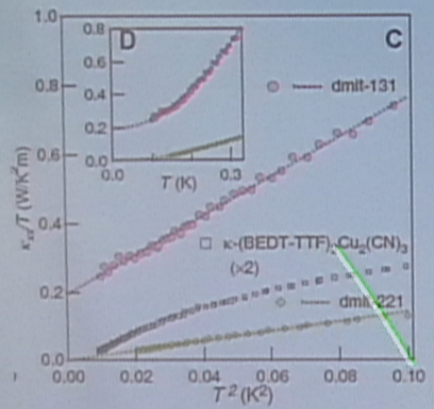
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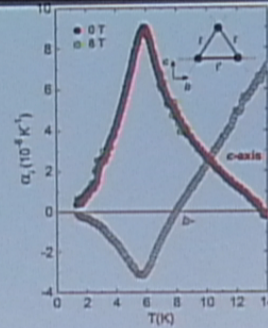
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What about experiment?

Linear T specific heat, not  $T^{2/3}$ .

Evidence for phase transition at 6K.



*Thermal expansion coefficient*

Manna et al., *PRL* 104 (2010) 016403

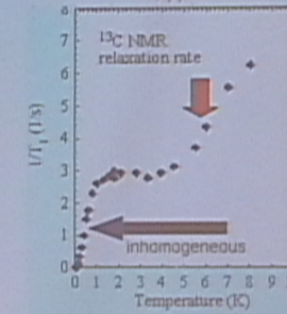
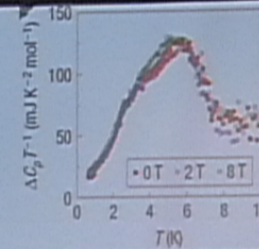
### Spinon pairing?

U(1) breaks down to Z2 spin liquid. The gauge field is gapped.

What kind of pairing?

One candidate is d wave pairing. With disorder the node is smeared and gives finite density of states.  $\kappa/T$  is universal constant (independent on impurity conc.) However, singlet pairing seems ruled out by smooth behavior of spin susceptibility up to 30T.

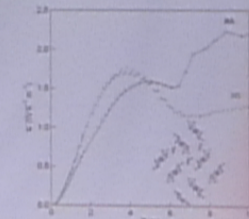
More exotic pairing? Amperian pairing, SS Lee, PL, Senthil.



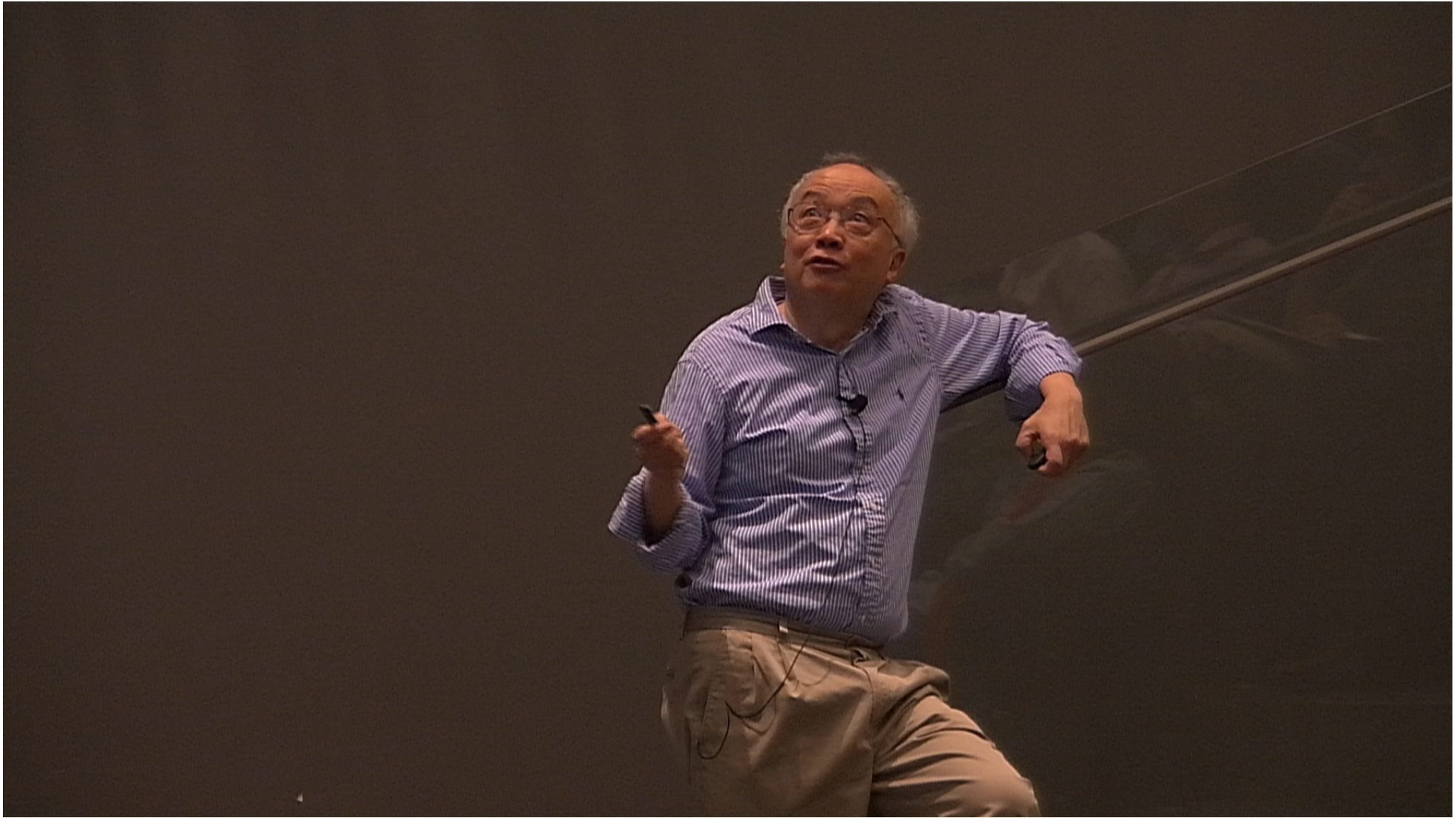
*NMR Relaxation rate*

Shimizu et al., *PRB* 70 (2006)

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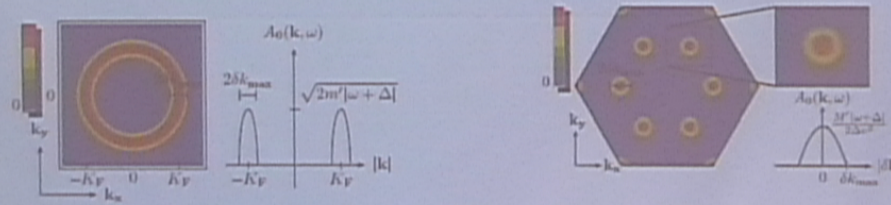
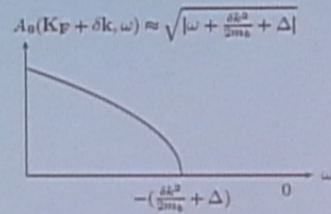


### Other experiments?

How to see spinon Fermi surface?

Electron spectrum = convolution of fermion with boson with gap  $\Delta$ .

$$c_{\sigma}^{\dagger} = f_{\sigma}^{\dagger} b$$



Location of the lowest threshold traces out the spinon Fermi surface.

Another idea:  $2k_F$  Friedel oscillations may be observable by STM.  
Mross and Senthil, PRB 2010.

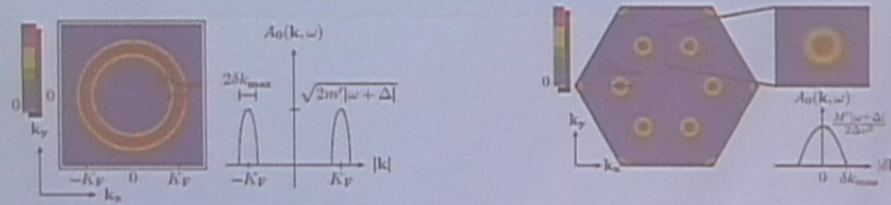
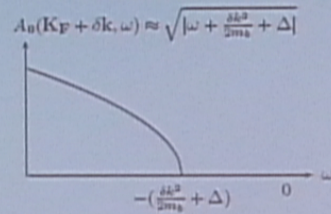


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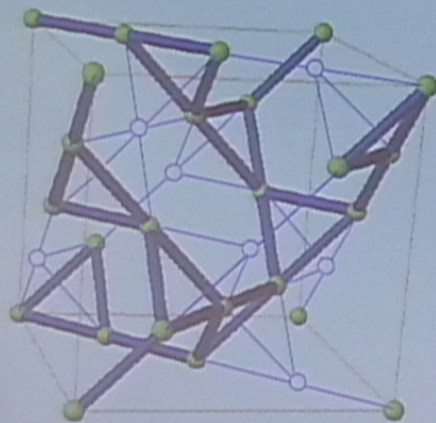
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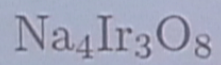
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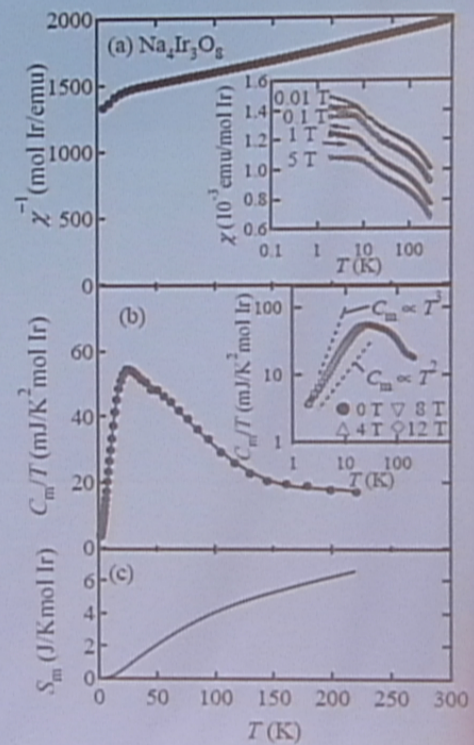


3 dim example?  
Hyper-Kagome.

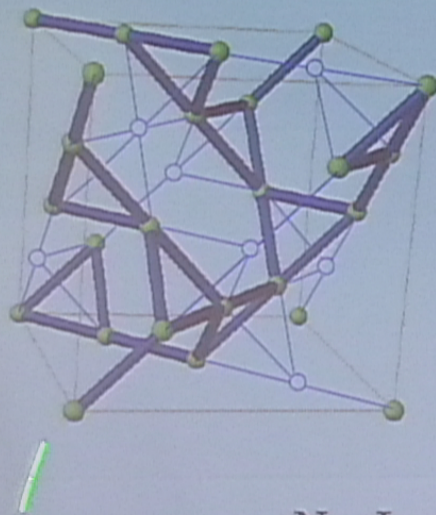


Okamoto .. Takagi  
PRL 07

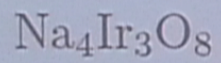
Near Mott transition: becomes metallic under pressure.





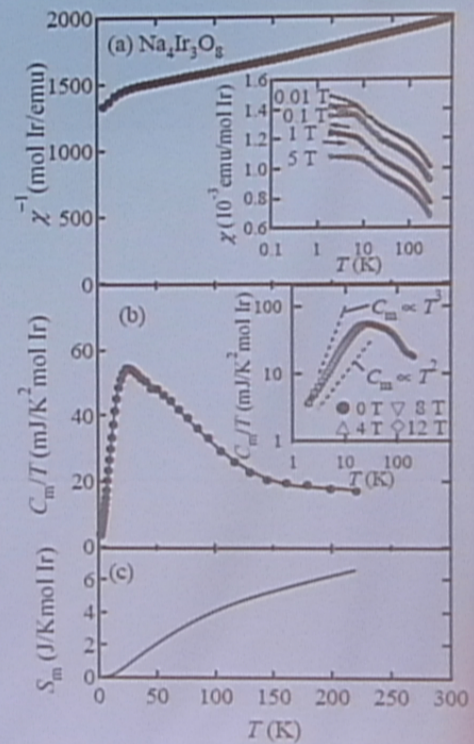


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Conclusion:

There is an excellent chance that the long sought after spin liquid state in 2 dimension has been discovered experimentally.

organic: spinon Fermi surface

Kagome and Hyper-Kagome.

S=1/2 triangular lattice: Ba<sub>3</sub>CuSb<sub>2</sub>O<sub>9</sub> ( 1d chain?) and S=1 version.

More experimental confirmation needed.

New phenomenon of emergent spinons and gauge field may now be studied.

If the same set of tools (slave boson theory, projected wavefunctions) are successful in describing the spin liquids, this should strengthen the case for a spin liquid description of the pseudogap and superconducting state in the cuprates.



