Title: Background Independent Holographic Description : From Matrix Field Theory to Quantum Gravity

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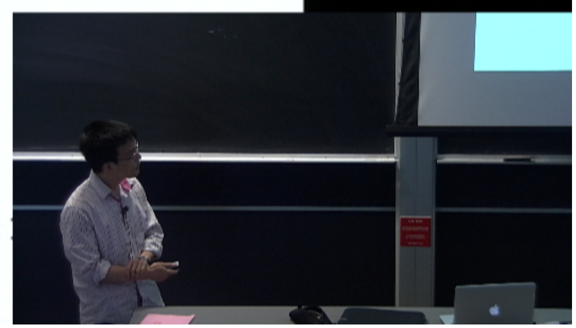
URL: http://pirsa.org/12050033

Abstract: A local renormalization group procedure is proposed where length scale is changed in spacetime dependent manner. Combining this scheme with an earlier observation that high energy modes in renormalization group play the role of dynamical sources for low energy modes at each scale, we provide a prescription to derive background independent holographic duals for field theories. From a first principle construction, it is shown that the holographic theory dual to a D-dimensional matrix field theory is a (D+1)-dimensional quantum theory of gravity coupled with matter fields of various spins.The (D+1)-dimensional diffeomorphism invariance is a consequence of the freedom to choose different local RG schemes.

Background independent holographic description : From QFT to Quantum Gravity

Sung-Sik Lee
McMaster University
Perimeter Institute

arXiv :



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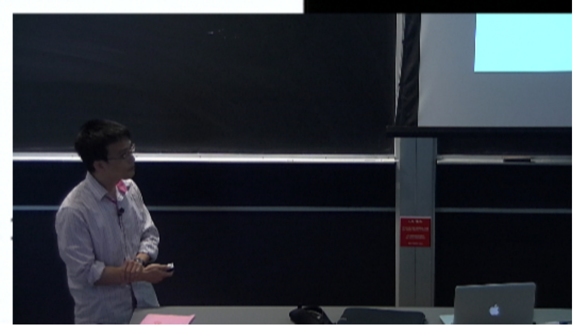

Maldacena's AdS/CFT = Wilson's RG + Sakharov's Induced gravity

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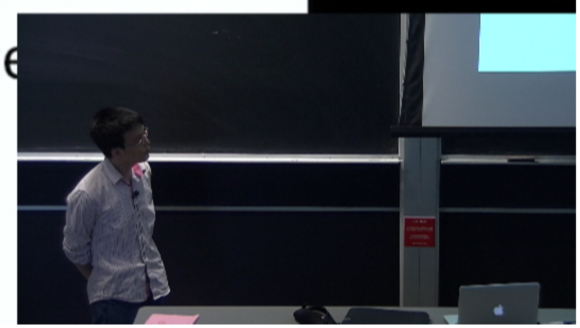
Motivation

- First principle construction of holographic duals for general QFT's

- Emergence of (D+1)-dim gravity from D-dim QFT

- Origin of the diffeomorphism invariance



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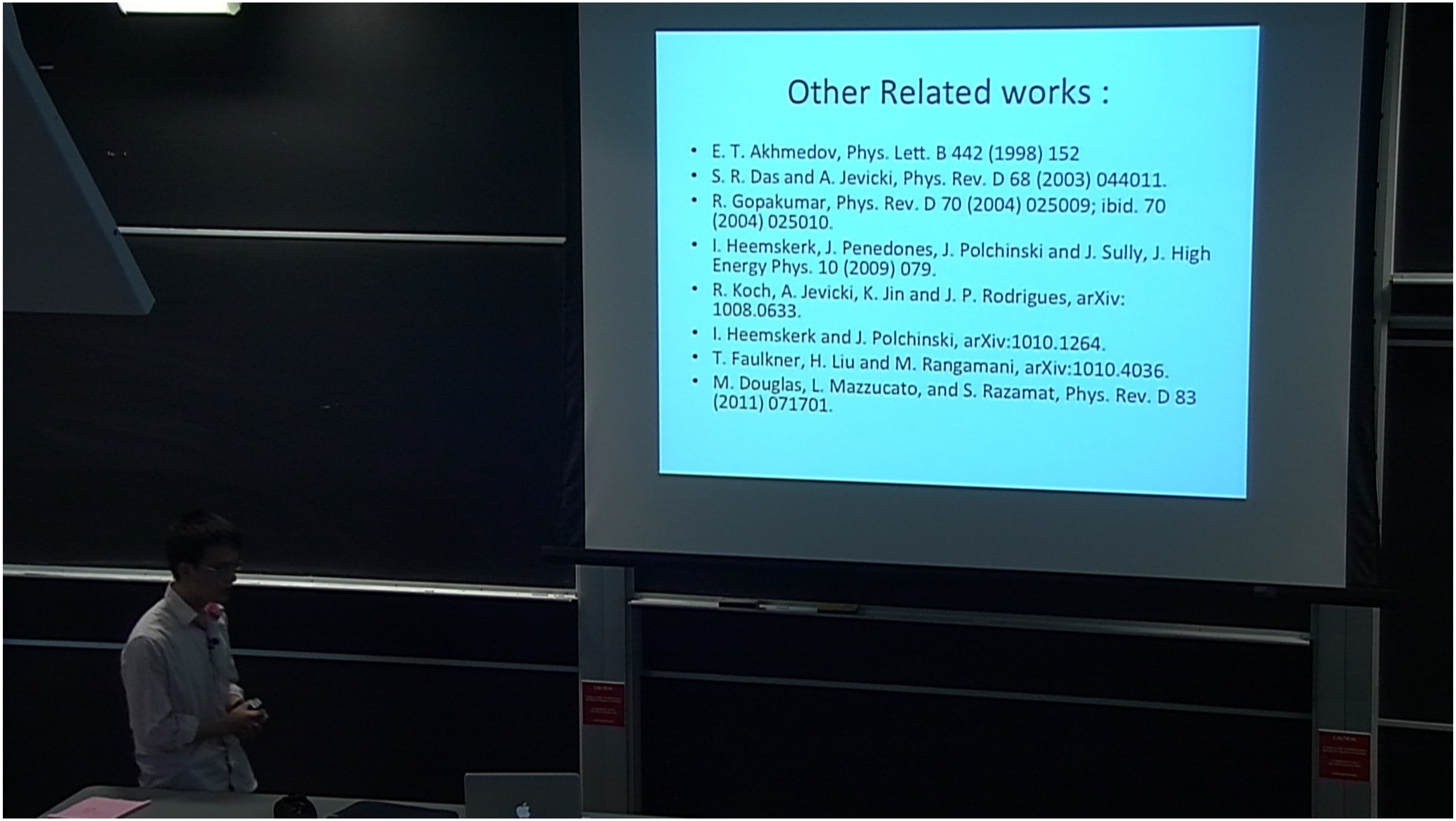
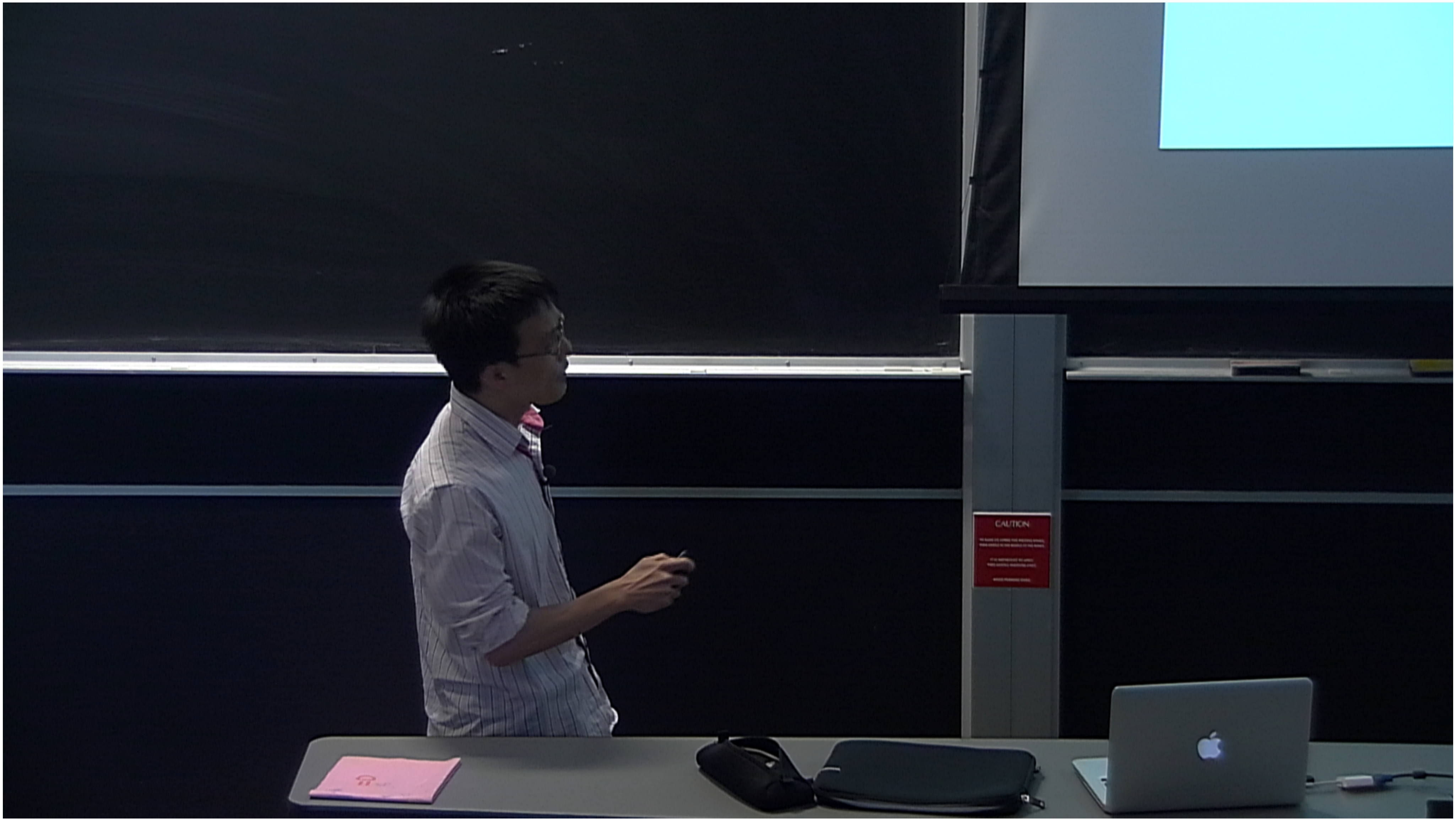
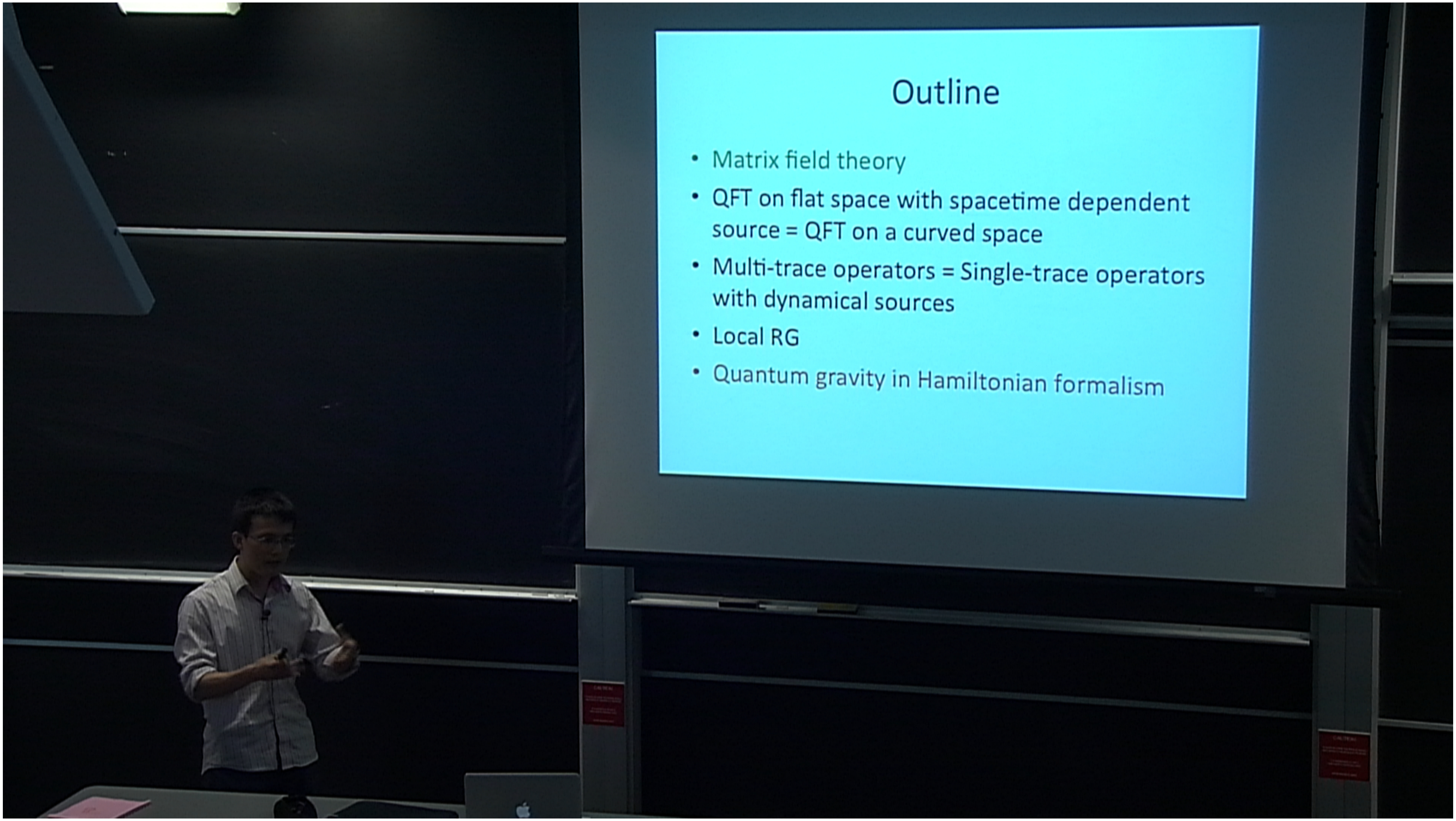
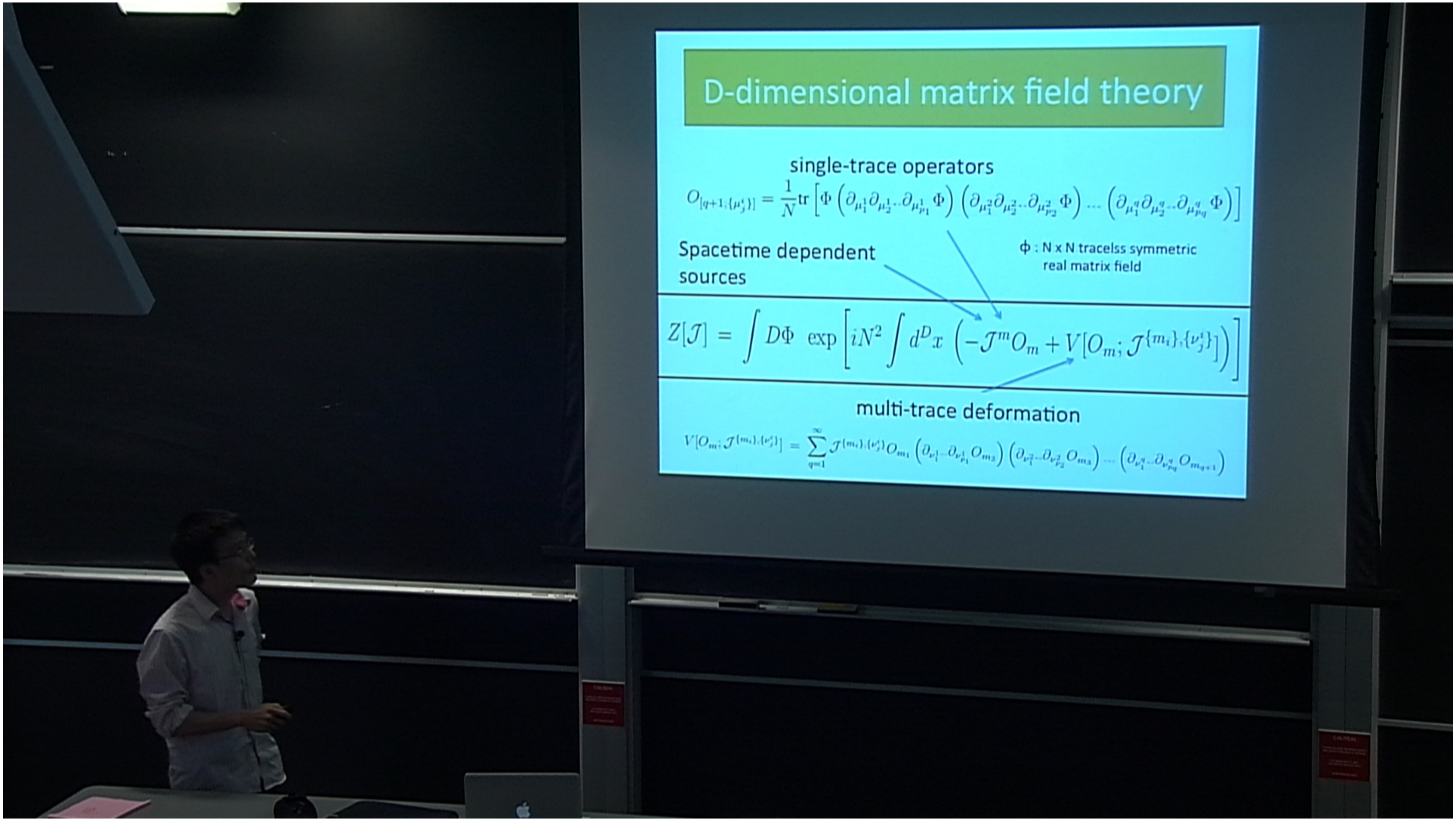

Other Related works :
E. T. Akhmedov, Phys. Lett. B 442 (1998) 152
S. R. Das and A. Jevicki, Phys. Rev. D 68 (2003) 044011.
R. Gopakumar, Phys. Rev. D 70 (2004) 025009; ibid. 70 (2004) 025010.
I. Heemskerk, J. Penedones, J. Polchinski and J. Sully, J. High Energy Phys. 10 (2009) 079.
R. Koch, A. Jevicki, K. Jin and J. P. Rodrigues, arXiv: 1008.0633.
I. Heemskerk and J. Polchinski, arXiv:1010.1264.
T. Faulkner, H. Liu and M. Rangamani, arXiv:1010.4036.
M. Douglas, L. Mazzucato, and S. Razamat, Phys. Rev. D 83 (2011) 071701.


CAUTION


Outline
• Matrix field theory
• QFT on flat space with spacetime dependent source = QFT on a curved space
• Multi-trace operators = Single-trace operators with dynamical sources
• Local RG
• Quantum gravity in Hamiltonian formalism


D-dimensional matrix field theory
single-trace operators
$O_{[q+1;\{\mu^i_j\}]} = \frac{1}{N}\mathrm{tr}\left[\Phi\left(\partial_{\mu^1_1}\partial_{\mu^1_2}..\partial_{\mu^1_{p_1}}\Phi\right)\left(\partial_{\mu^2_1}\partial_{\mu^2_2}..\partial_{\mu^2_{p_2}}\Phi\right)...\left(\partial_{\mu^q_1}\partial_{\mu^q_2}..\partial_{\mu^q_{p_q}}\Phi\right)\right]$
Spacetime dependent sources
Φ : N x N tracelss symmetric real matrix field
$Z[\mathcal{J}] = \int D\Phi \ \exp\left[iN^2\int d^Dx\left(-\mathcal{J}^m O_m + V[O_m;\mathcal{J}^{\{m_i\},\{\nu^i_j\}}]\right)\right]$
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$V[O_m;\mathcal{J}^{\{m_i\},\{\nu^i_j\}}] = \sum_{q=1}^{\infty}\mathcal{J}^{\{m_i\},\{\nu^i_j\}}O_{m_1}\left(\partial_{\nu^1_1}..\partial_{\nu^1_{p_1}}O_{m_2}\right)\left(\partial_{\nu^2_1}..\partial_{\nu^2_{p_2}}O_{m_3}\right)...\left(\partial_{\nu^q_1}..\partial_{\nu^q_{p_q}}O_{m_{q+1}}\right)$

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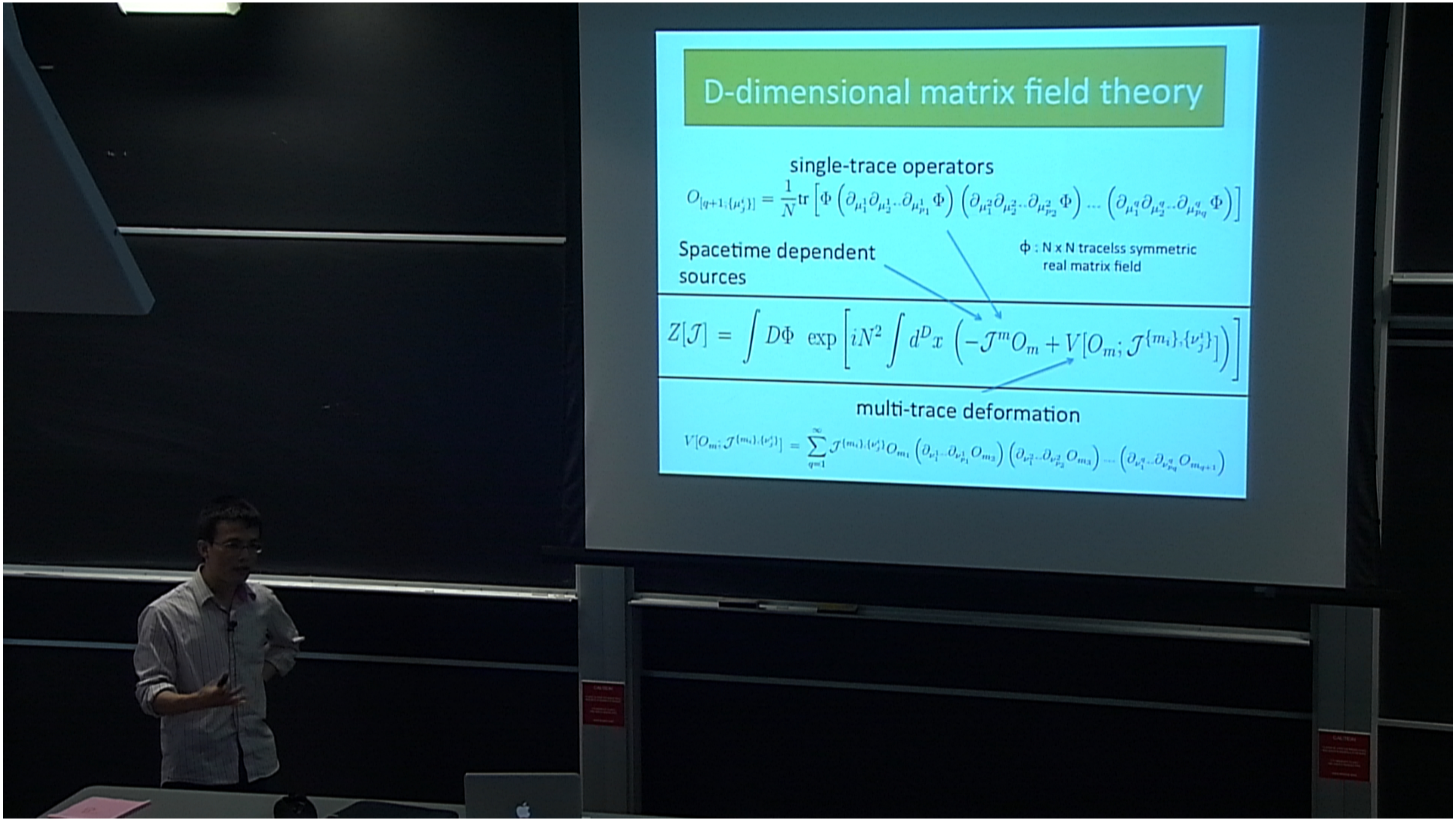
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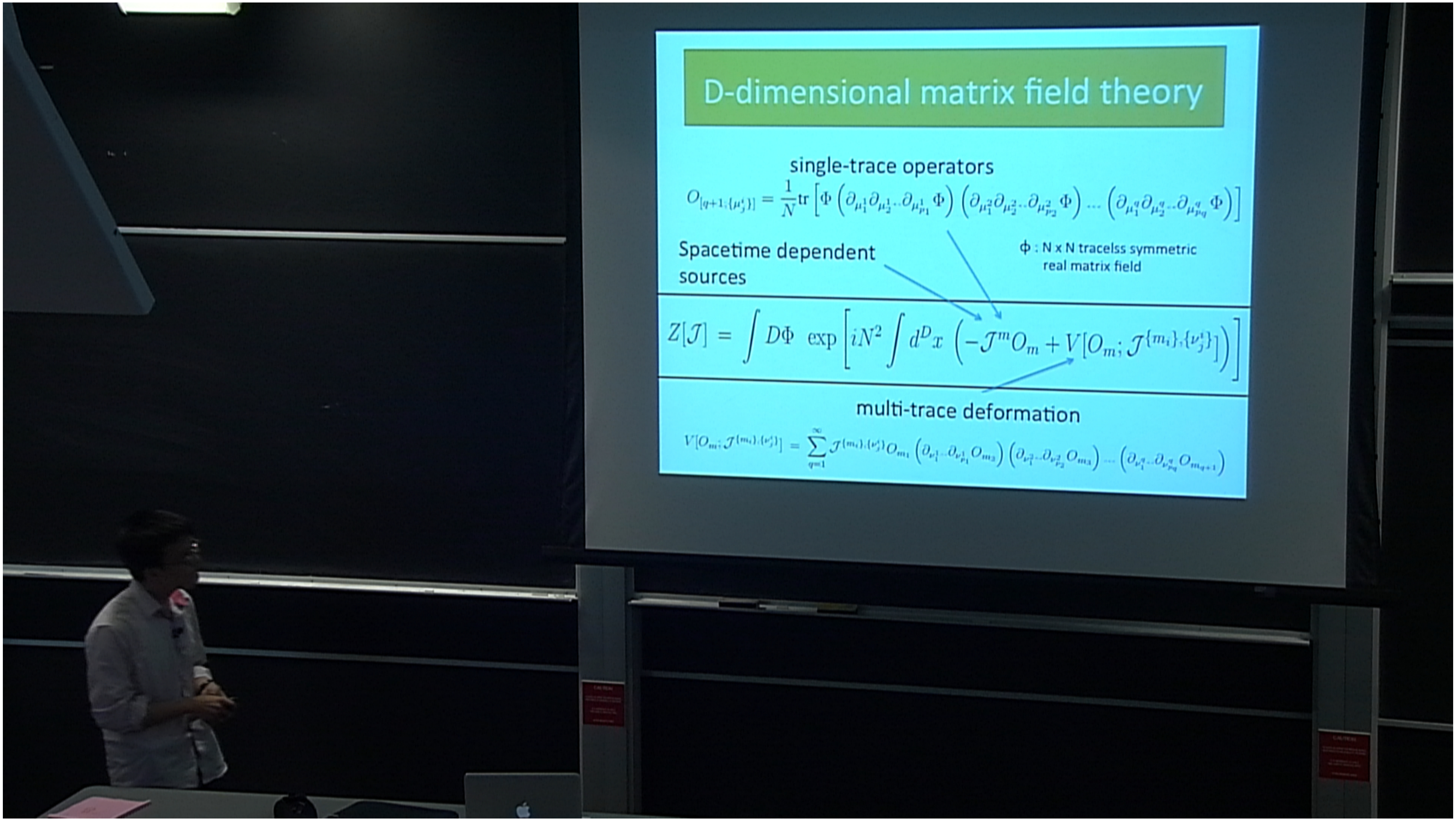
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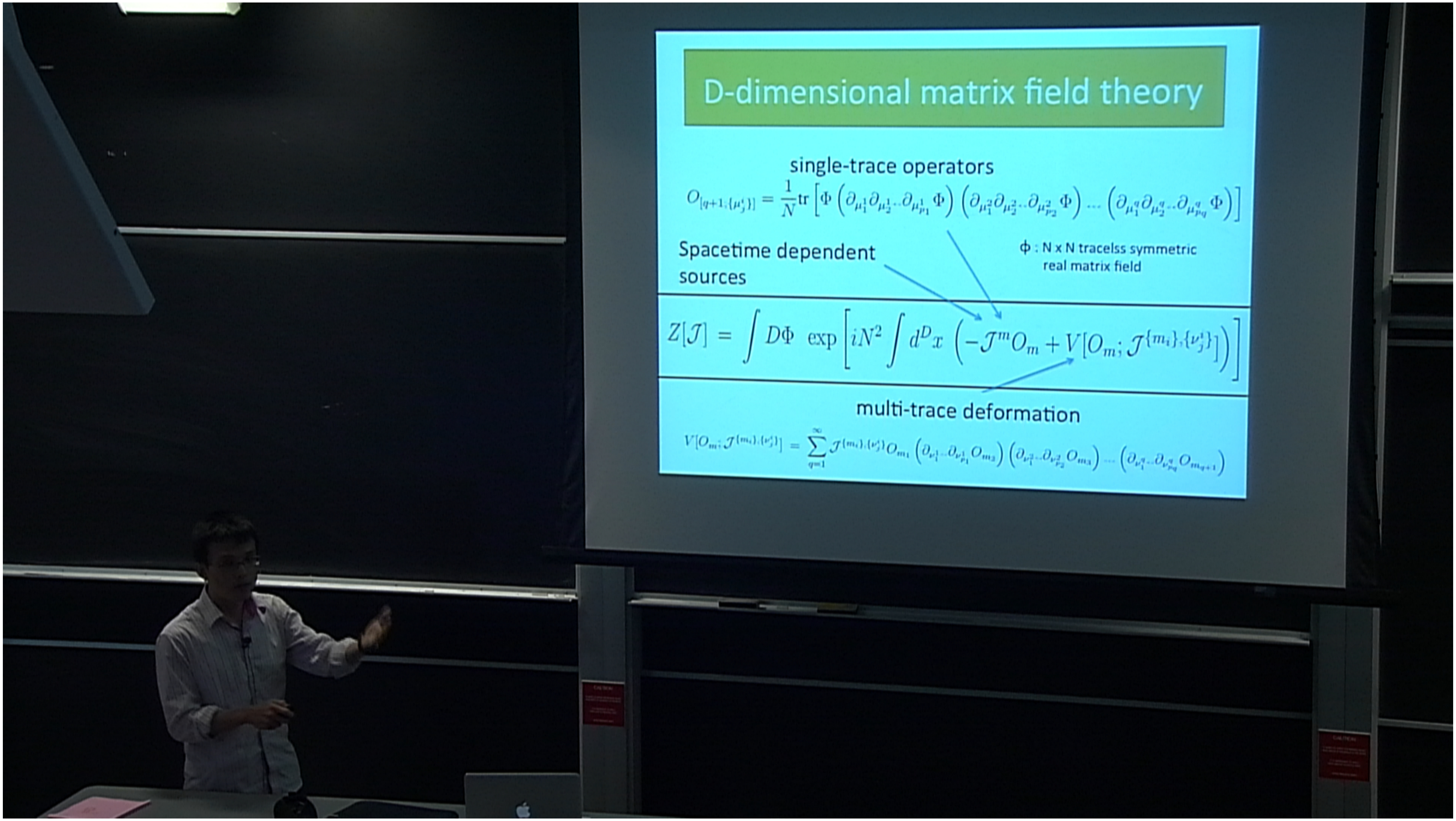
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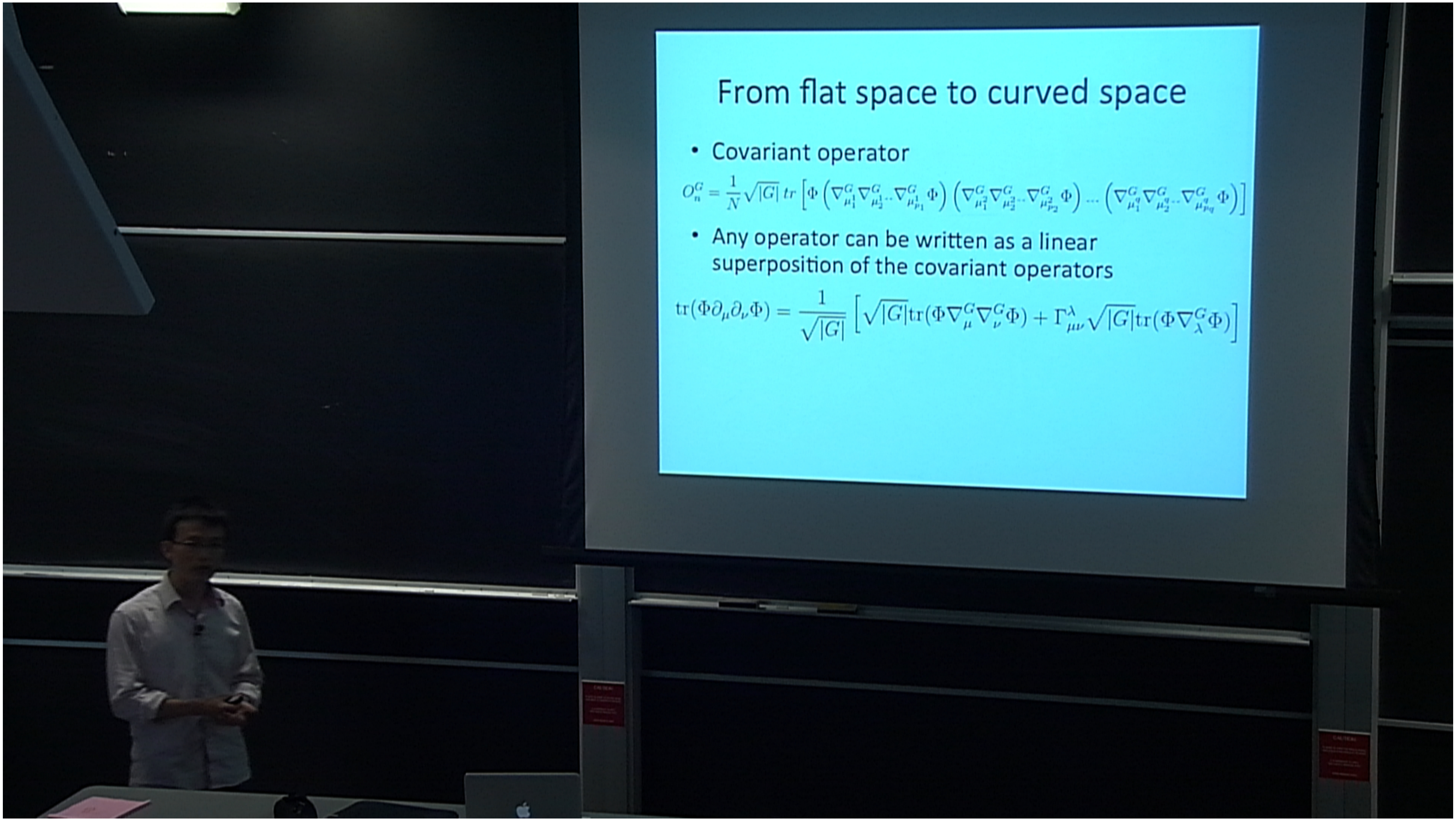
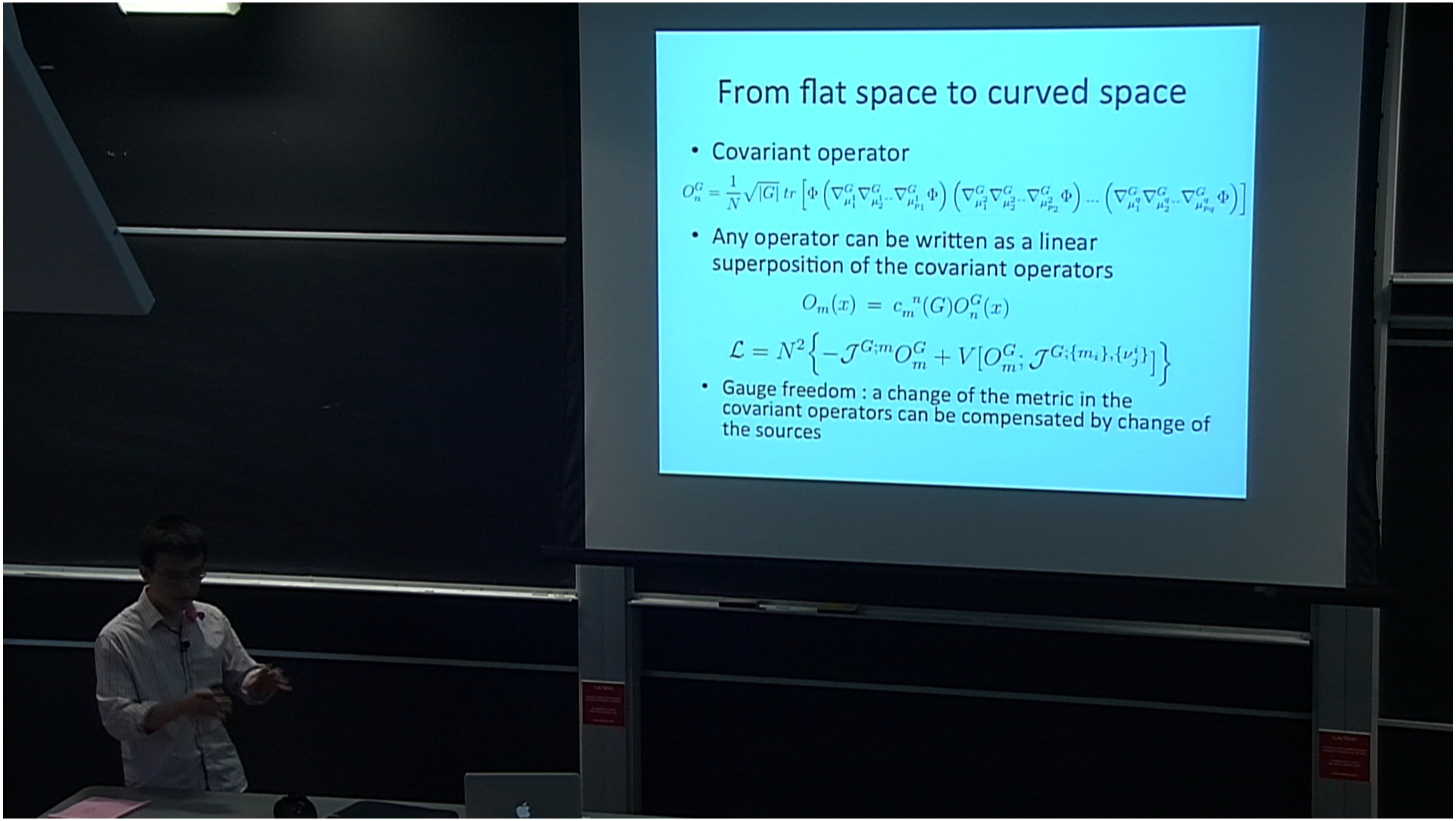

From flat space to curved space
• Covariant operator
$O_n^G = \frac{1}{N}\sqrt{|G|}\, tr\left[\Phi\left(\nabla^G_{\mu_1^1}\nabla^G_{\mu_2^1}..\nabla^G_{\mu^1_{p_1}}\Phi\right)\left(\nabla^G_{\mu_1^2}\nabla^G_{\mu_2^2}..\nabla^G_{\mu^2_{p_2}}\Phi\right)...\left(\nabla^G_{\mu_1^q}\nabla^G_{\mu_2^q}..\nabla^G_{\mu^q_{p_q}}\Phi\right)\right]$
• Any operator can be written as a linear superposition of the covariant operators
$\mathrm{tr}(\Phi\partial_\mu\partial_\nu\Phi) = \frac{1}{\sqrt{|G|}}\left[\sqrt{|G|}\mathrm{tr}(\Phi\nabla^G_\mu\nabla^G_\nu\Phi) + \Gamma^\lambda_{\mu\nu}\sqrt{|G|}\mathrm{tr}(\Phi\nabla^G_\lambda\Phi)\right]$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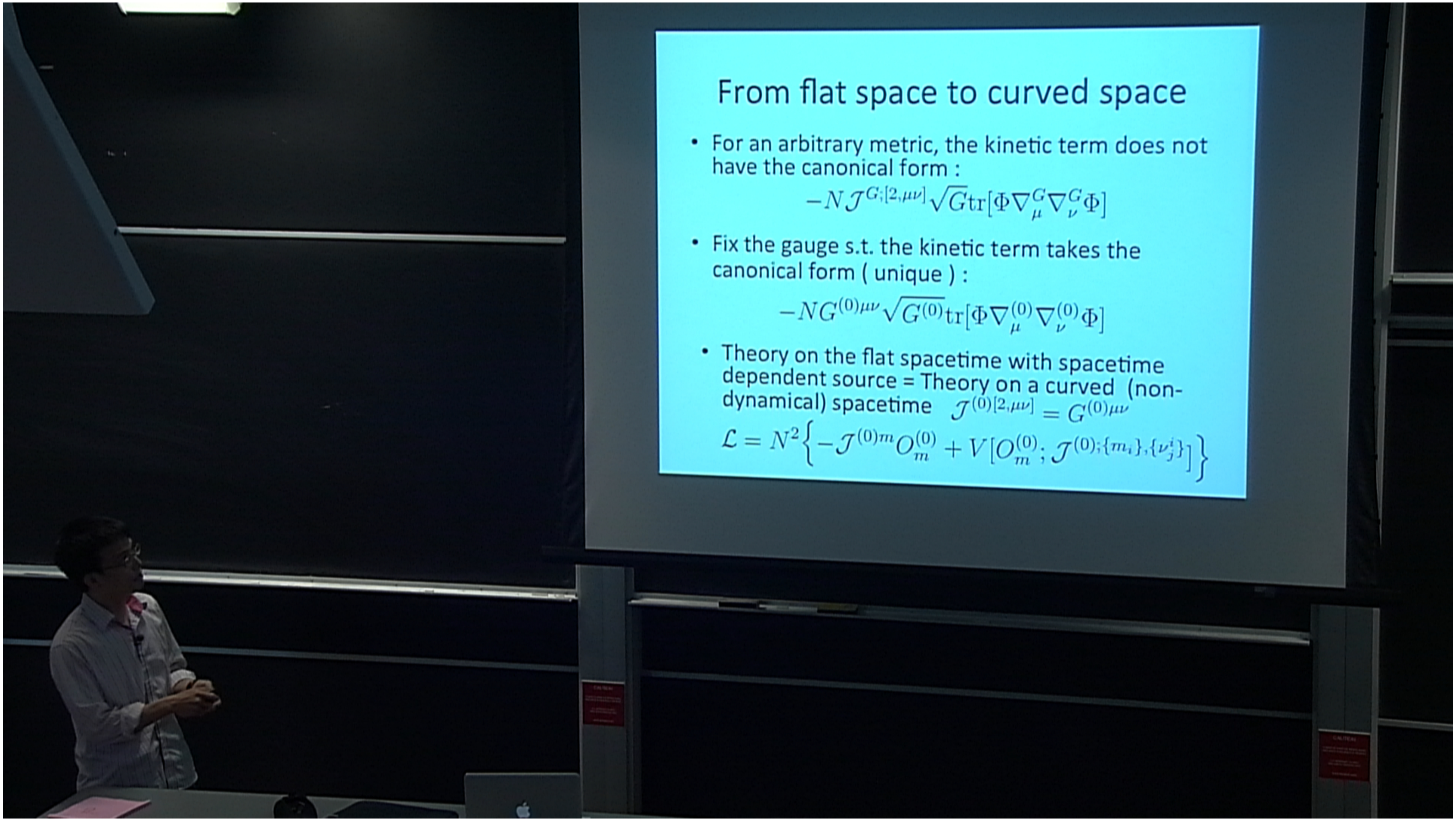
$O_m(x) = c_m{}^n(G) O_n^G(x)$
$\mathcal{L} = N^2\left\{-\mathcal{J}^{G;m} O_m^G + V[O_m^G; \mathcal{J}^{G;\{m_i\},\{\nu_j^i\}}]\right\}$
• Gauge freedom : a change of the metric in the covariant operators can be compensated by change of the sources


From flat space to curved space
For an arbitrary metric, the kinetic term does not have the canonical form :
$-N\mathcal{J}^{G;[2,\mu\nu]}\sqrt{G}\mathrm{tr}[\Phi\nabla^G_\mu\nabla^G_\nu\Phi]$
Fix the gauge s.t. the kinetic term takes the canonical form (unique) :
$-NG^{(0)\mu\nu}\sqrt{G^{(0)}}\mathrm{tr}[\Phi\nabla^{(0)}_\mu\nabla^{(0)}_\nu\Phi]$
Theory on the flat spacetime with spacetime dependent source = Theory on a curved (non-dynamical) spacetime $\mathcal{J}^{(0)[2,\mu\nu]} = G^{(0)\mu\nu}$
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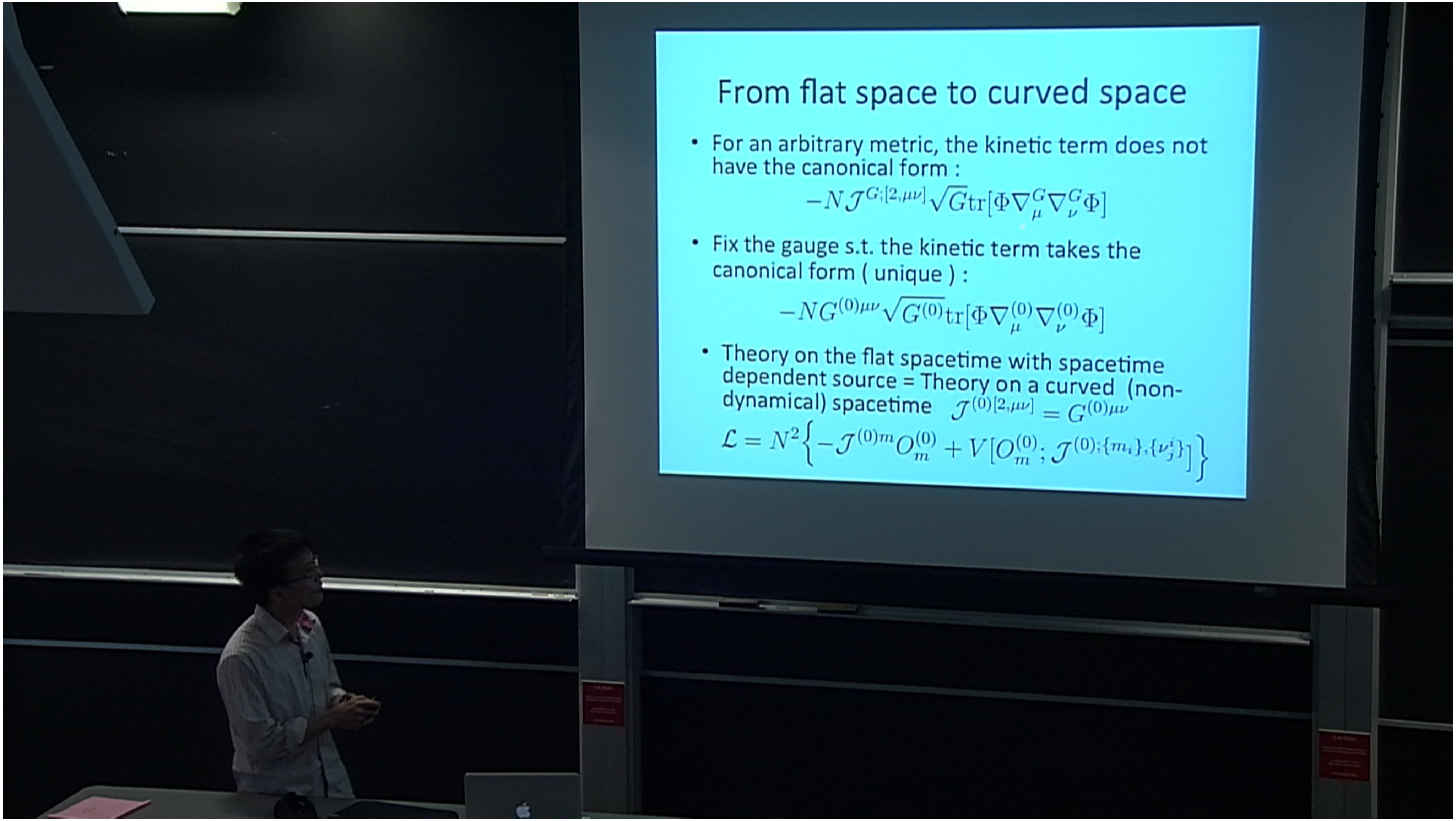
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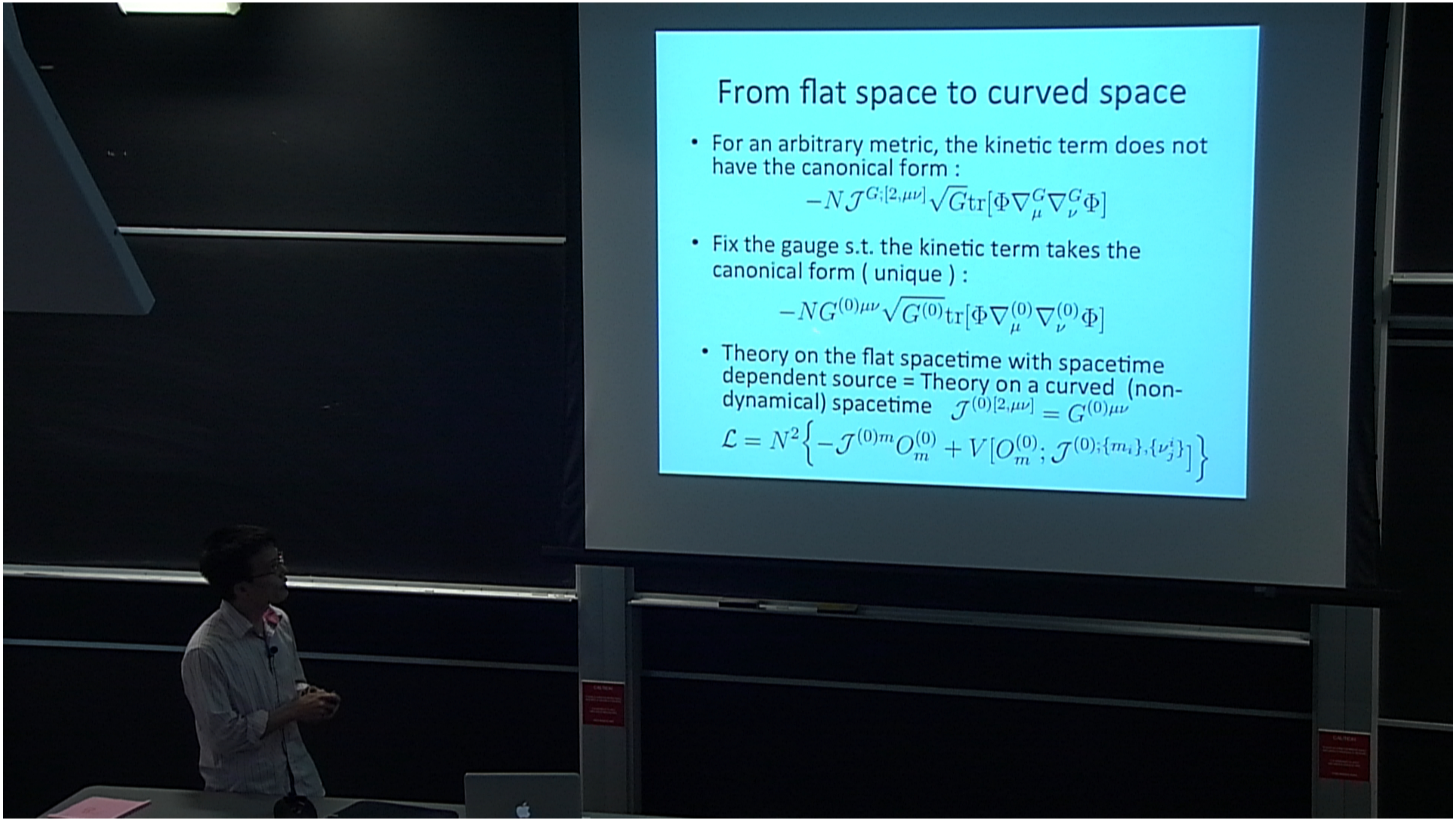
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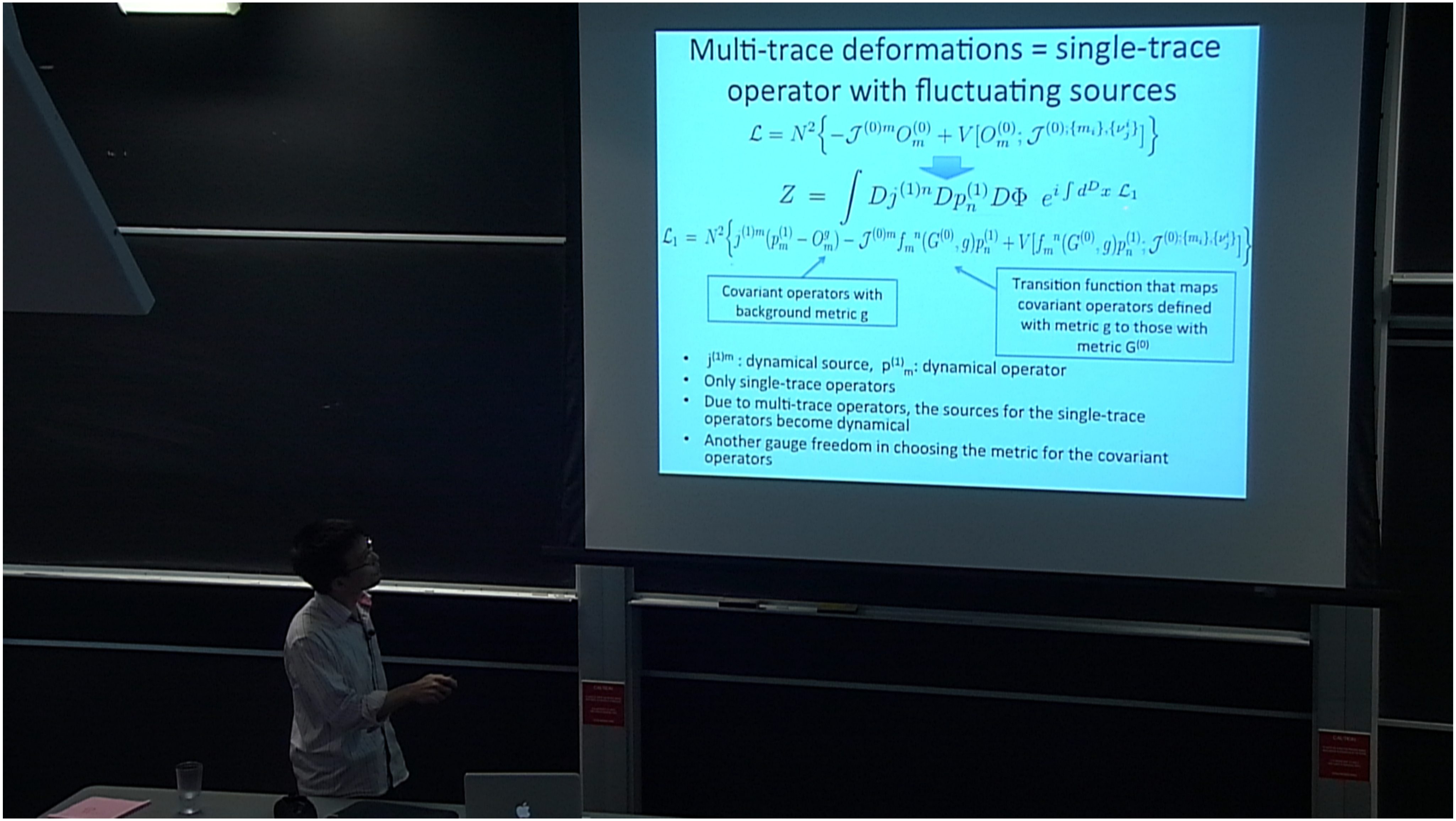
Multi-trace deformations = single-trace operator with fluctuating sources
$\mathcal{L} = N^2\left\{-\mathcal{J}^{(0)m}O_m^{(0)} + V[O_m^{(0)}; \mathcal{J}^{(0);\{m_i\},\{\nu_j^i\}}]\right\}$
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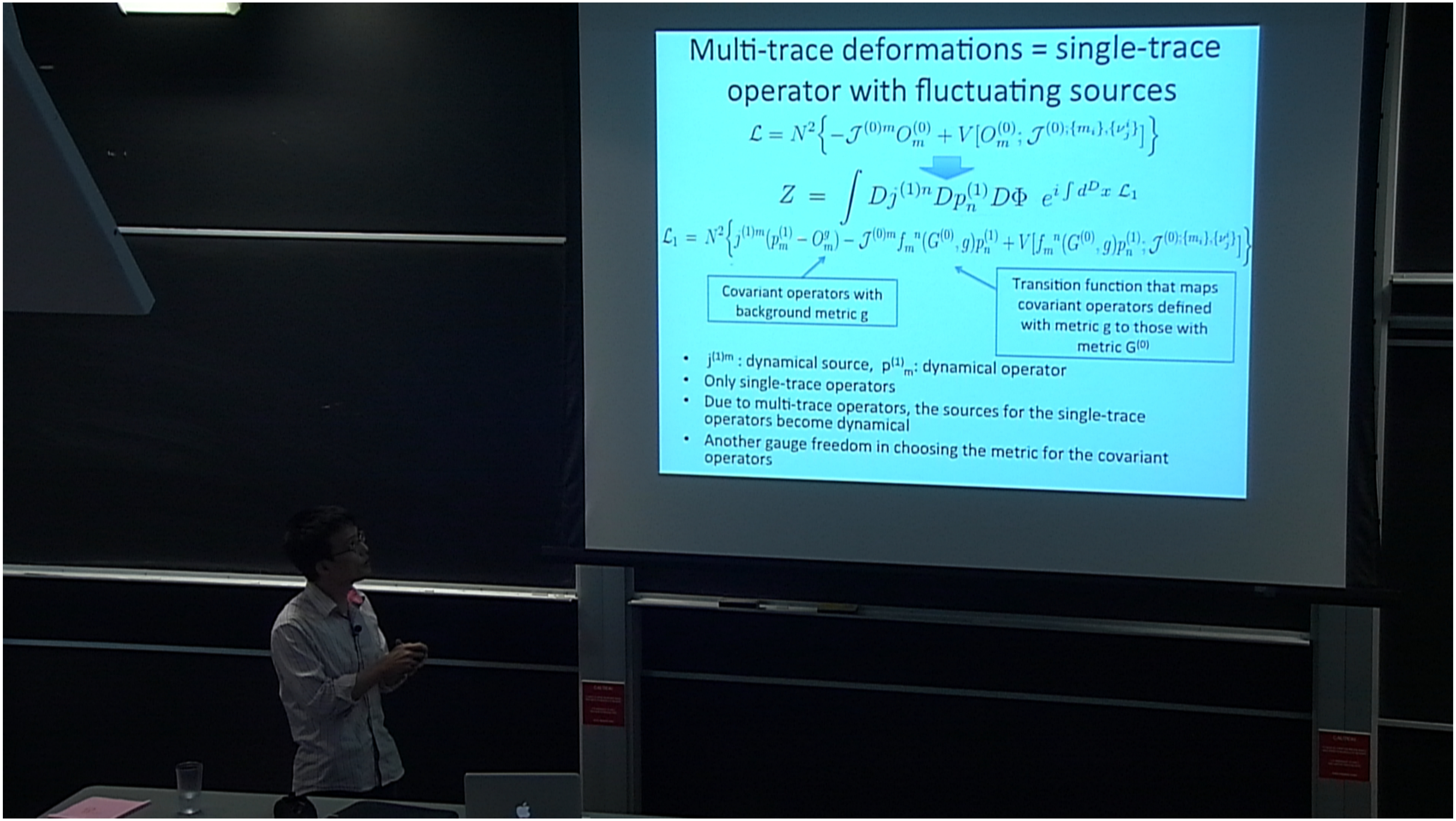
Covariant operators with background metric g
Transition function that maps covariant operators defined with metric g to those with metric $G^{(0)}$
• $j^{(1)m}$: dynamical source, $p^{(1)}{}_m$: dynamical operator
• Only single-trace operators
• Due to multi-trace operators, the sources for the single-trace operators become dynamical
• Another gauge freedom in choosing the metric for the covariant operators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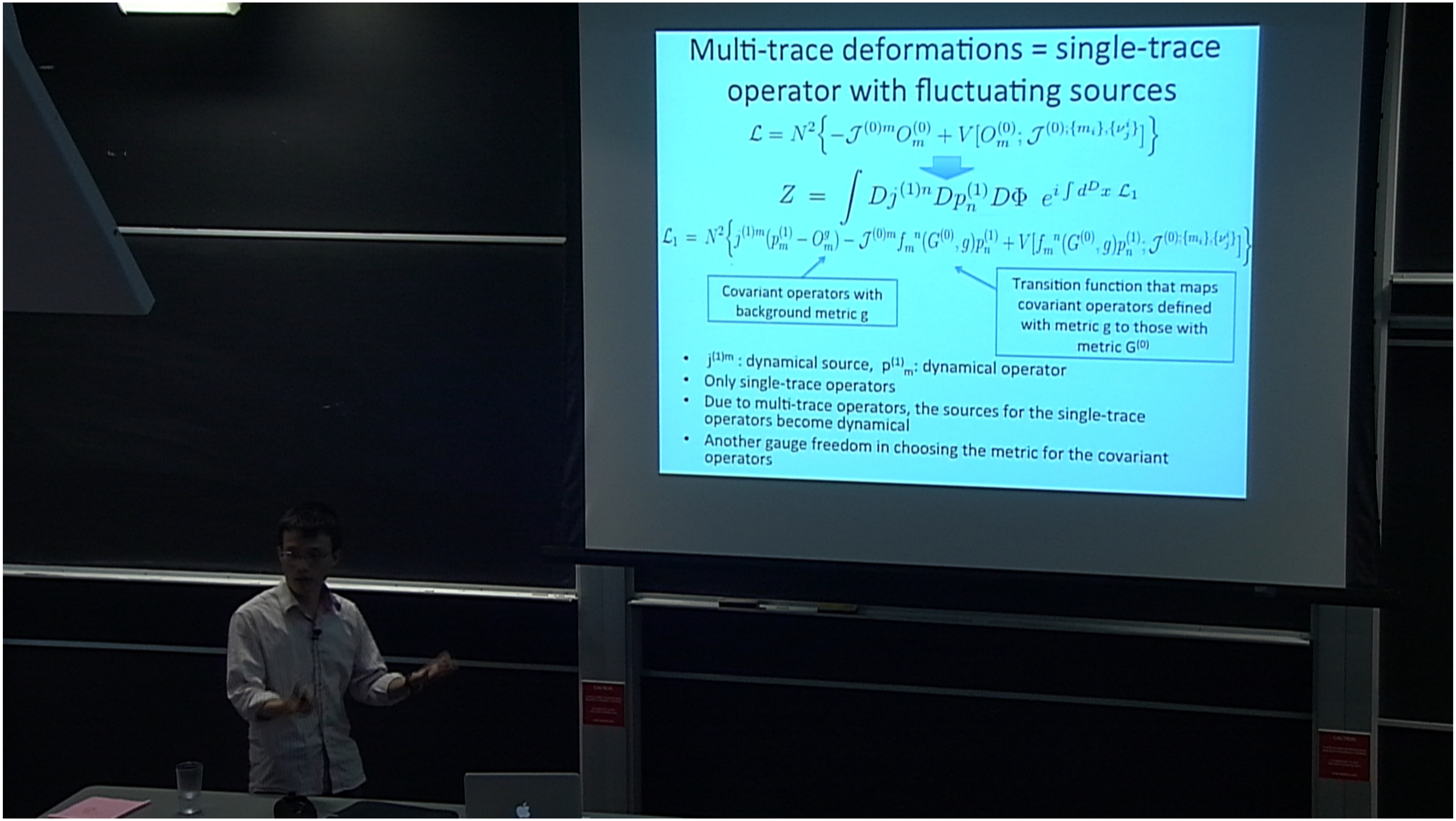

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Transition function that maps covariant operators defined with metric g to those with metric $G^{(0)}$
$j^{(1)m}$: dynamical source, $p^{(1)}{}_m$: dynamical operator
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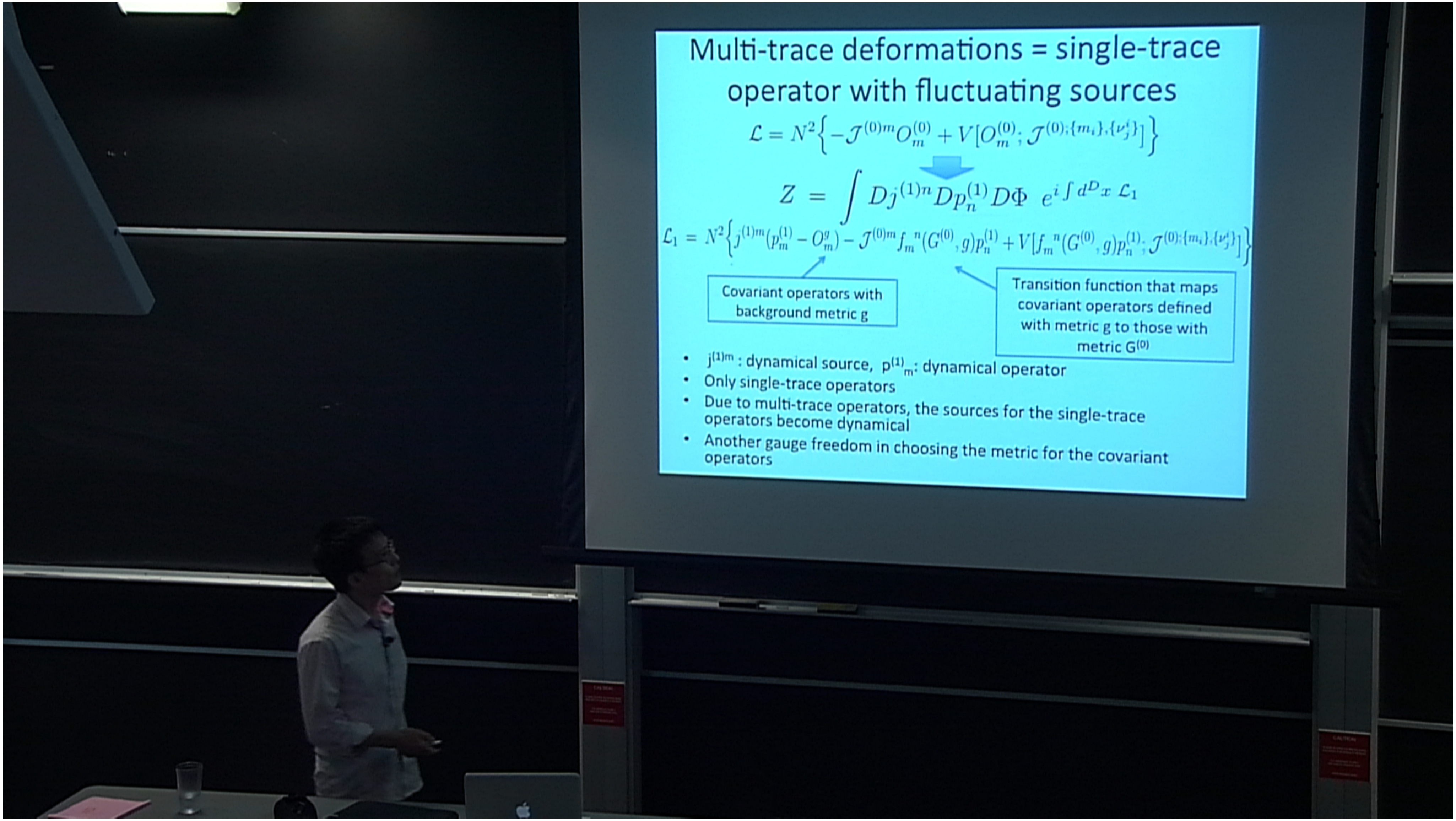
Due to multi-trace operators, the sources for the single-trace operators become dynamical
Another gauge freedom in choosing the metric for the covariant operators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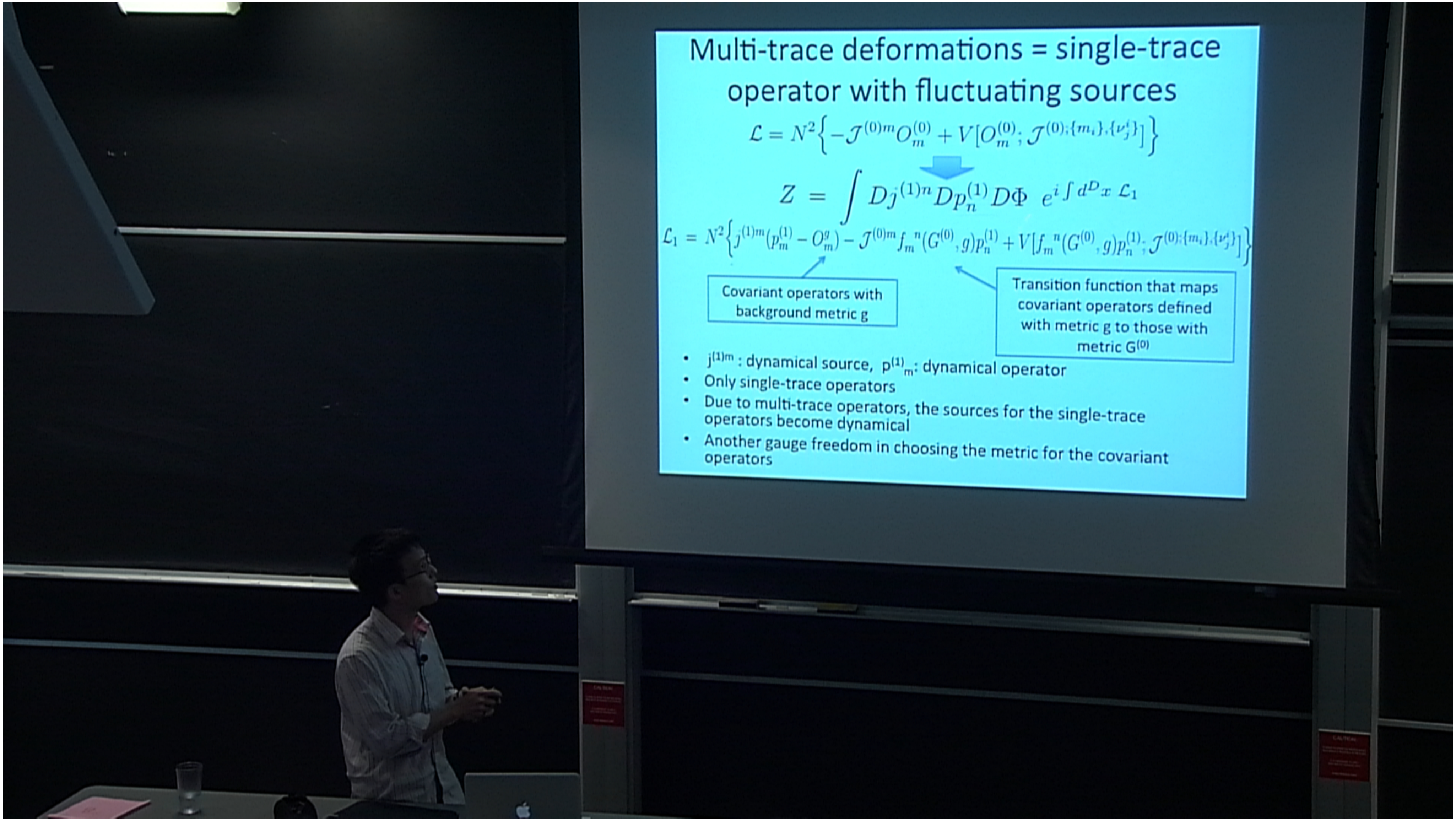

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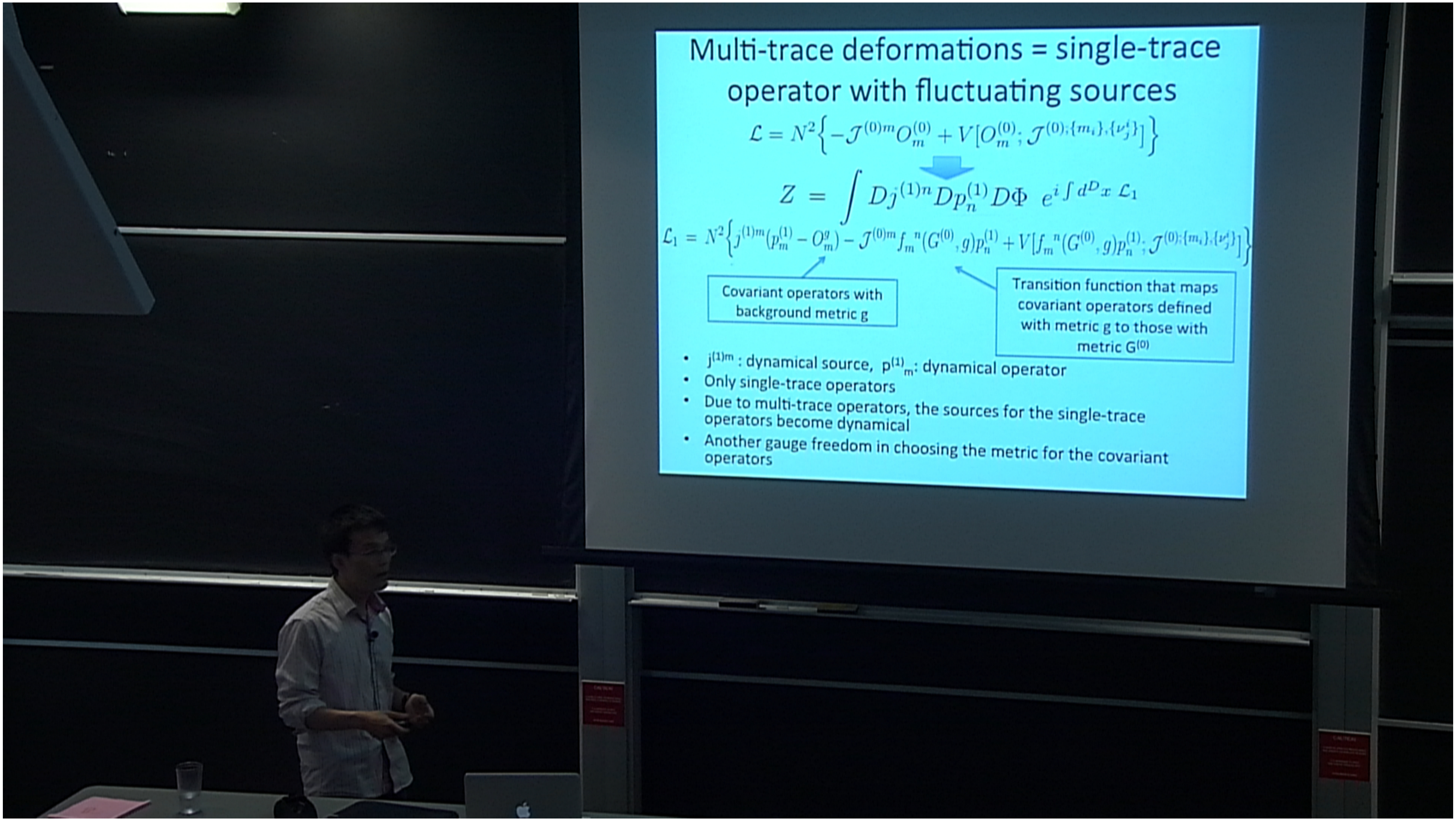

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$$Z = \int DJ^{n(1)} DP_n^{(1)} D\Phi \ \Delta(J^{(1)}) e^{i\int d^Dx \ \mathcal{L}_1'[J^{(1)},P^{(1)},J^{(1)[2,\mu\nu]},\Phi]}$$

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$$J^{(1)[2,\mu\nu]} = G^{(1)\mu\nu}$$

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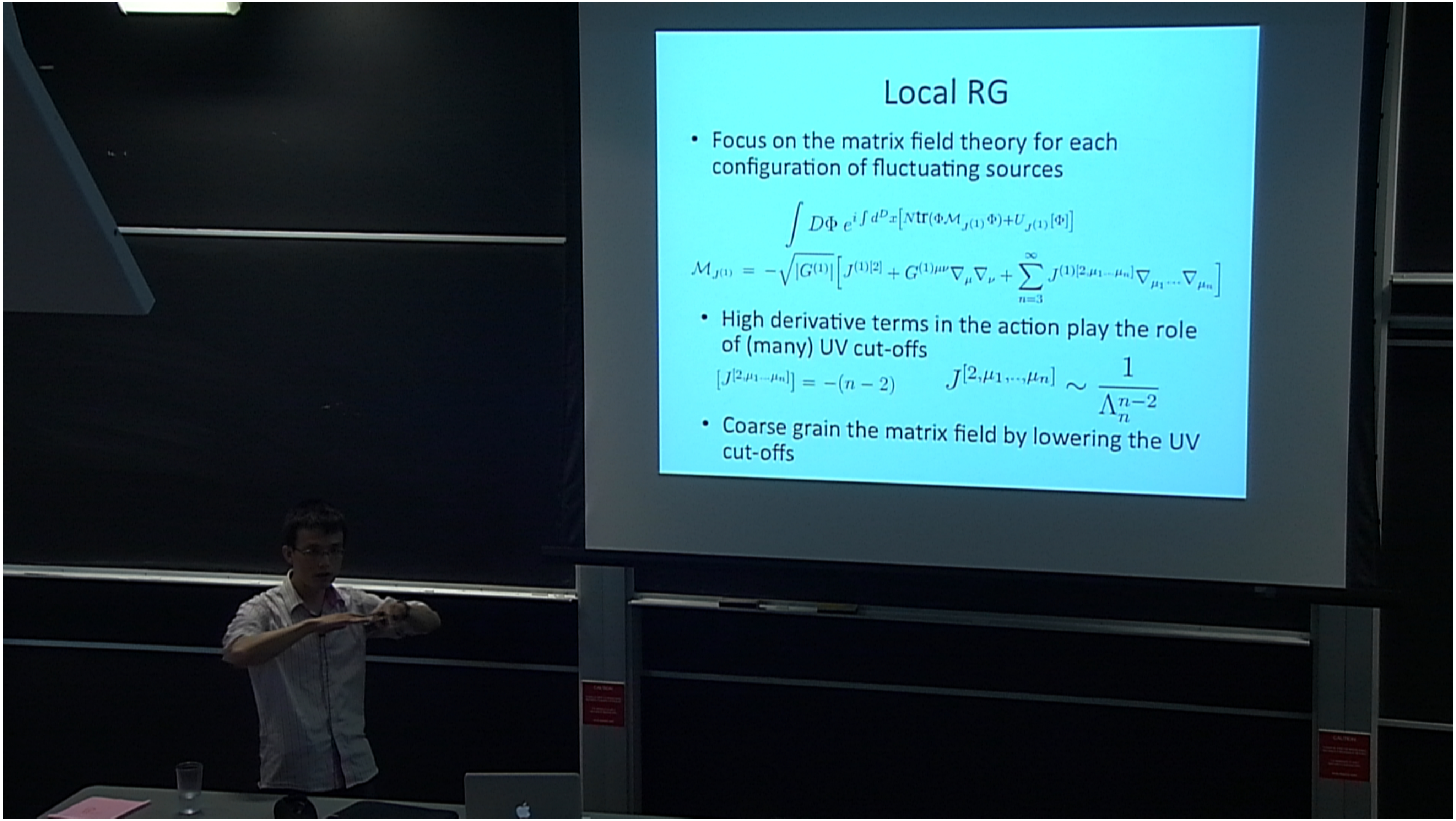
Local RG
• Focus on the matrix field theory for each configuration of fluctuating sources
$\int D\Phi\, e^{i\int d^Dx[N\mathrm{tr}(\Phi\mathcal{M}_{J^{(1)}}\Phi)+U_{J^{(1)}}[\Phi]]}$
$\mathcal{M}_{J^{(1)}} = -\sqrt{|G^{(1)}|}\left[J^{(1)[2]} + G^{(1)\mu\nu}\nabla_\mu\nabla_\nu + \sum_{n=3}^{\infty} J^{(1)[2,\mu_1\cdots\mu_n]}\nabla_{\mu_1}\cdots\nabla_{\mu_n}\right]$
• High derivative terms in the action play the role of (many) UV cut-offs
$[J^{[2,\mu_1\cdots\mu_n]}] = -(n-2)$
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• Coarse grain the matrix field by lowering the UV cut-offs

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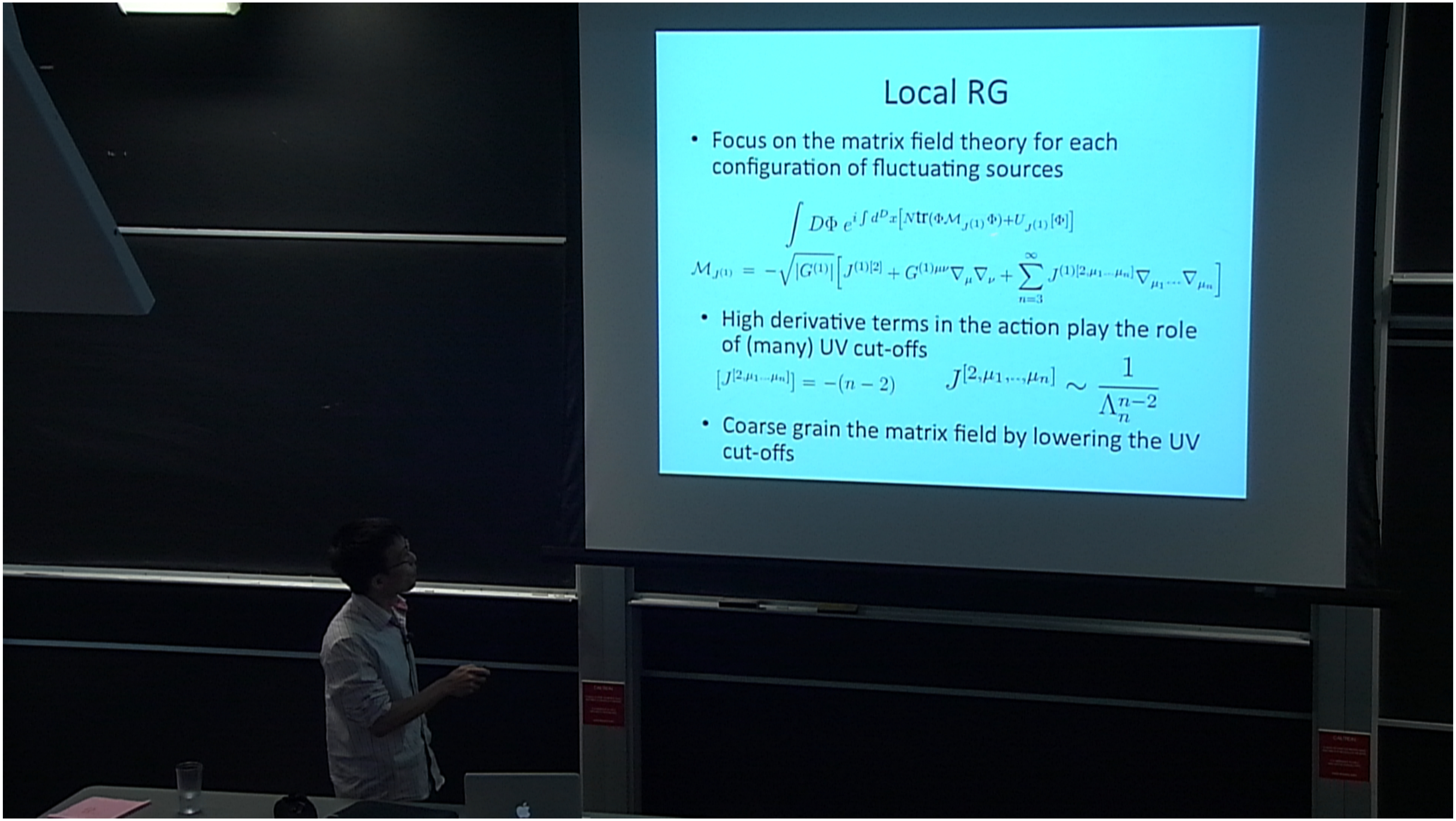
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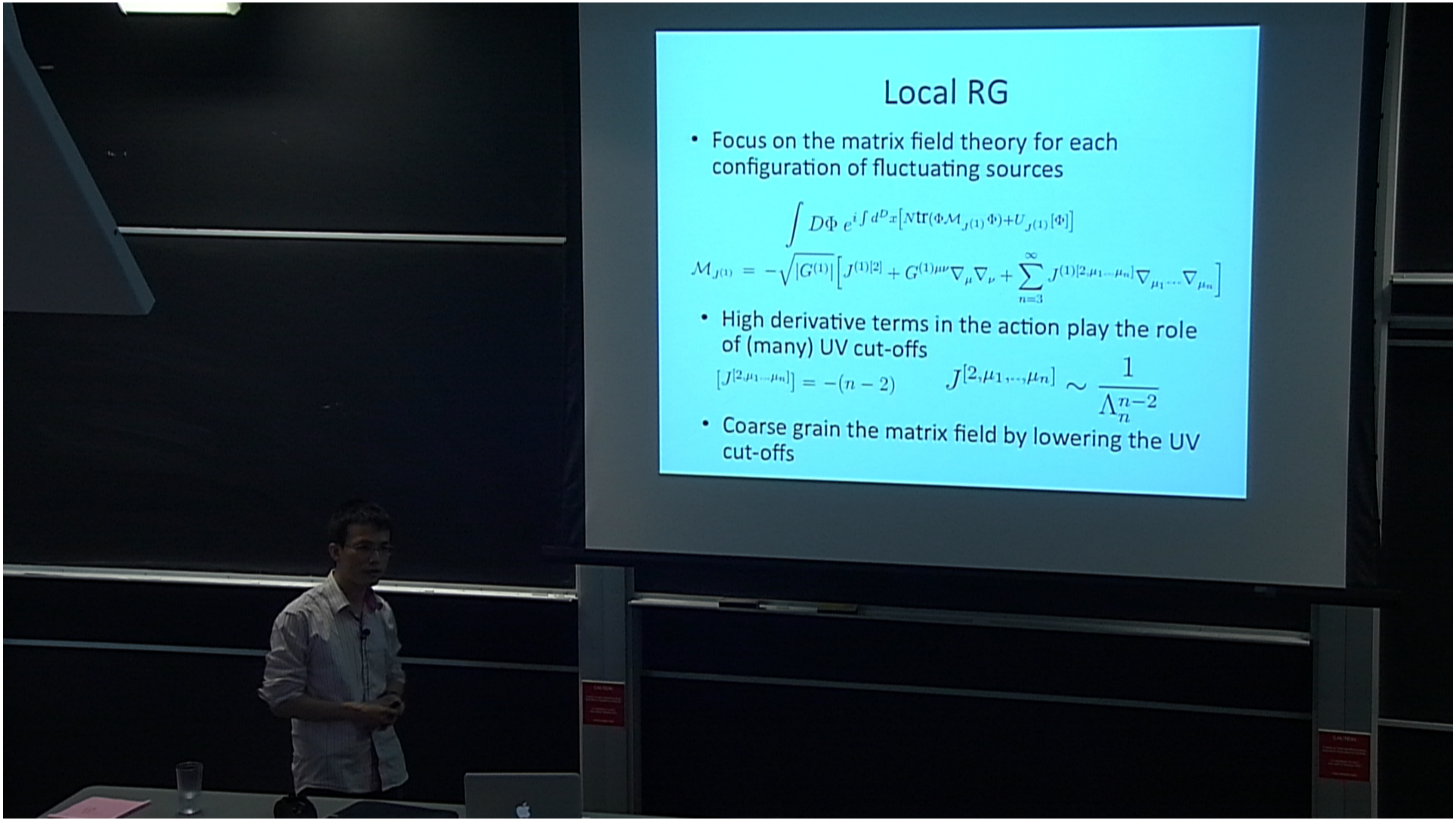
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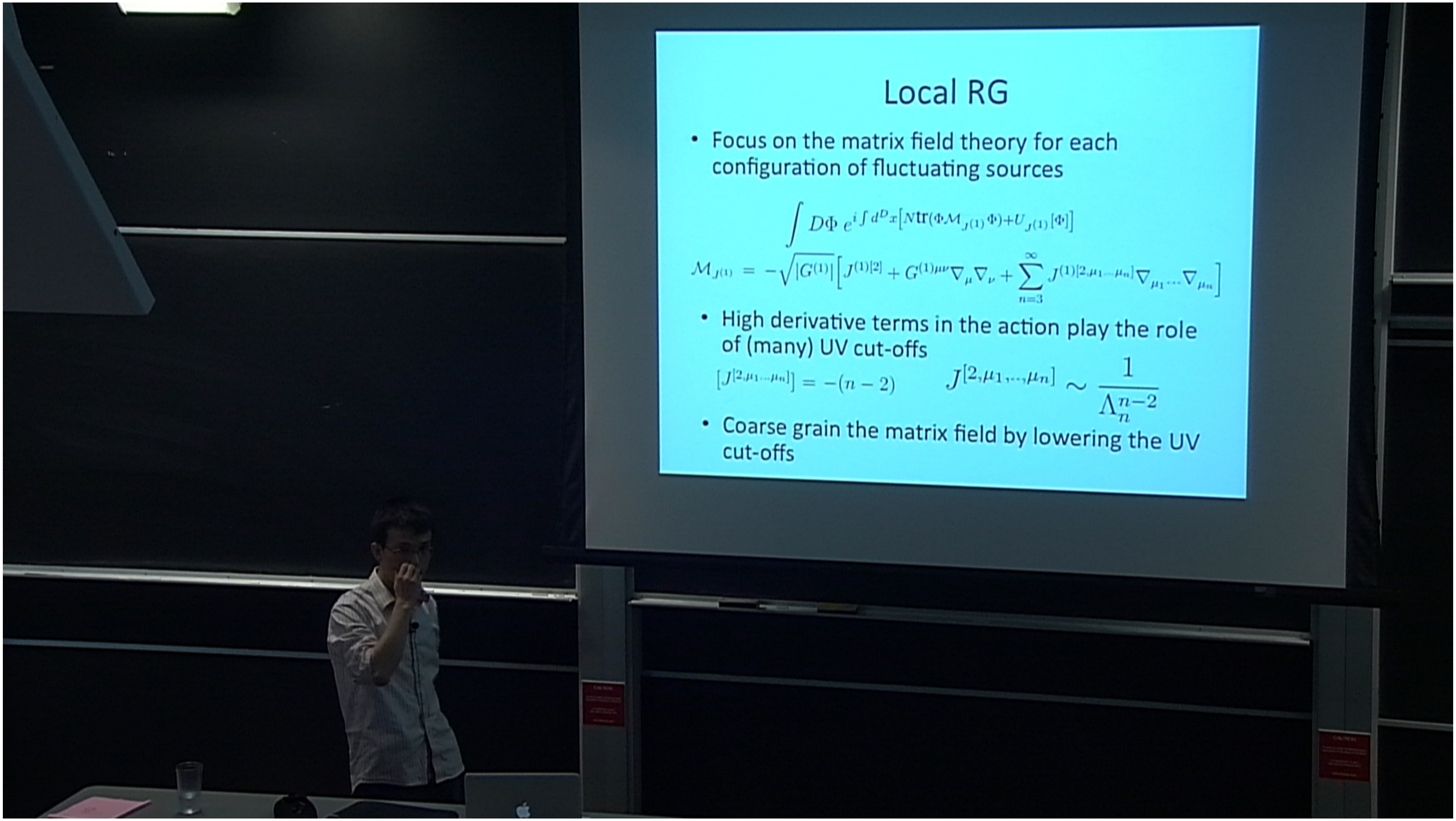
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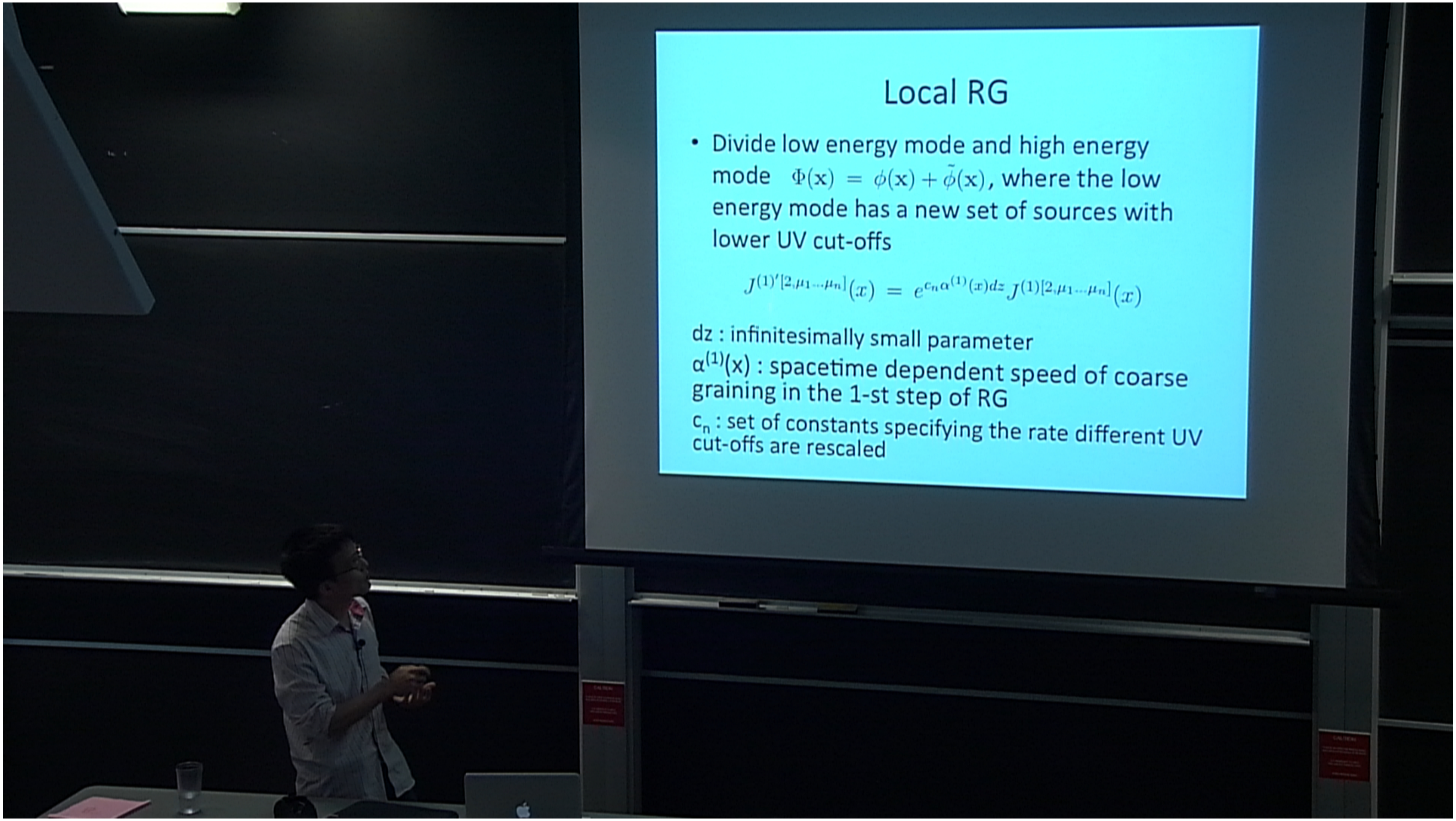
CAUTION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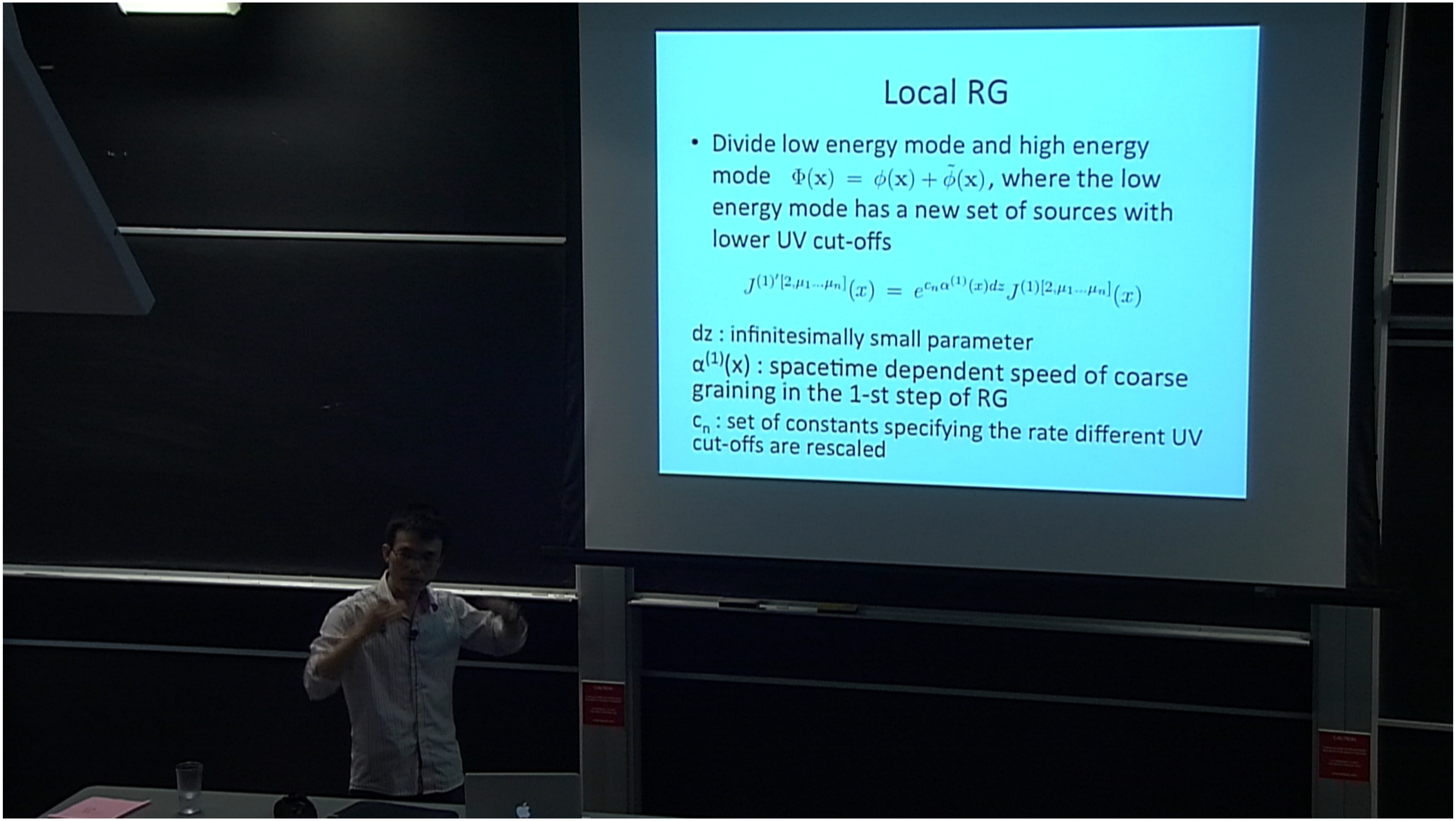
dz : infinitesimally small parameter
$\alpha^{(1)}(x)$: spacetime dependent speed of coarse graining in the 1-st step of RG
c_n : set of constants specifying the rate different UV cut-offs are rescaled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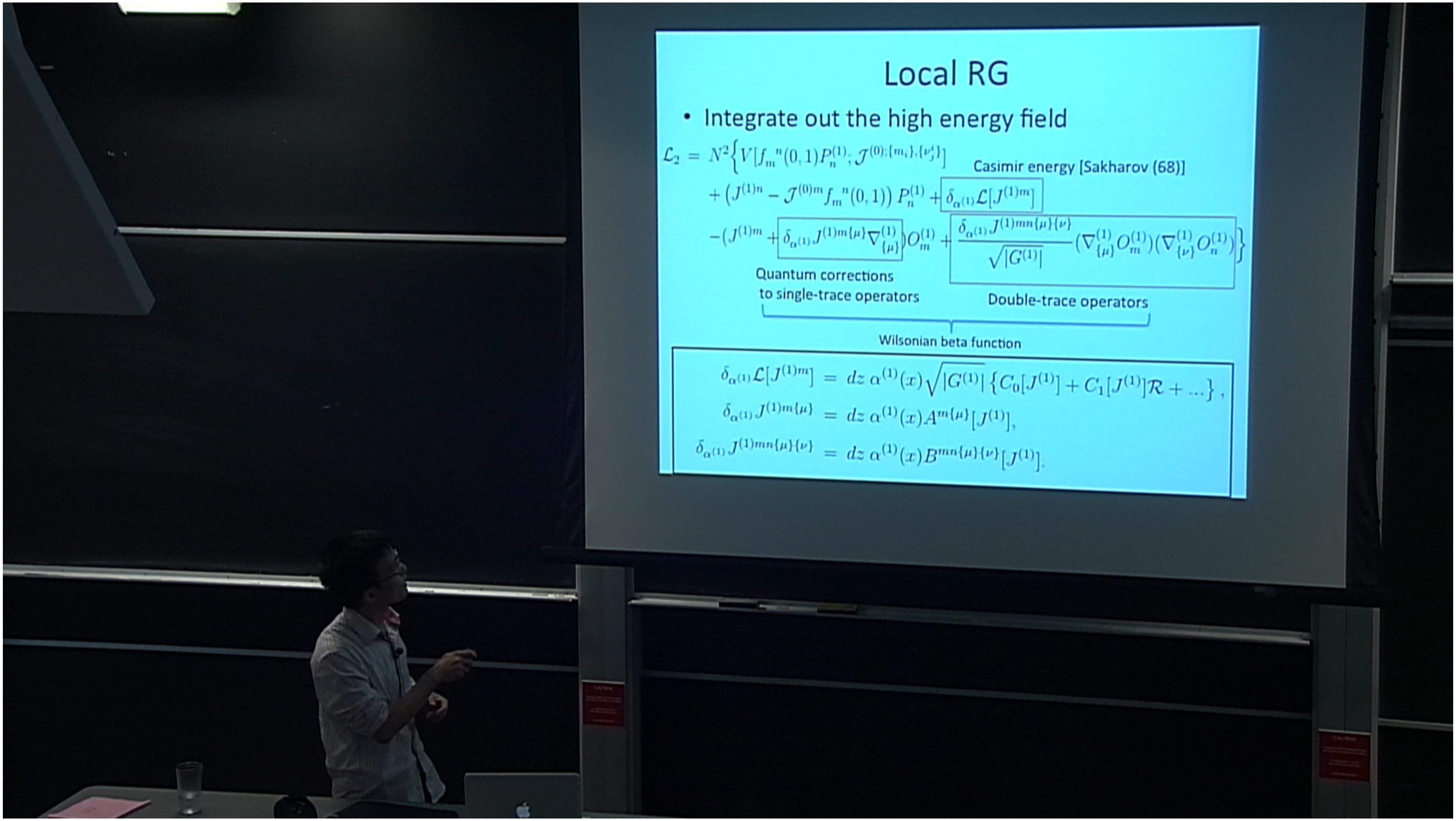
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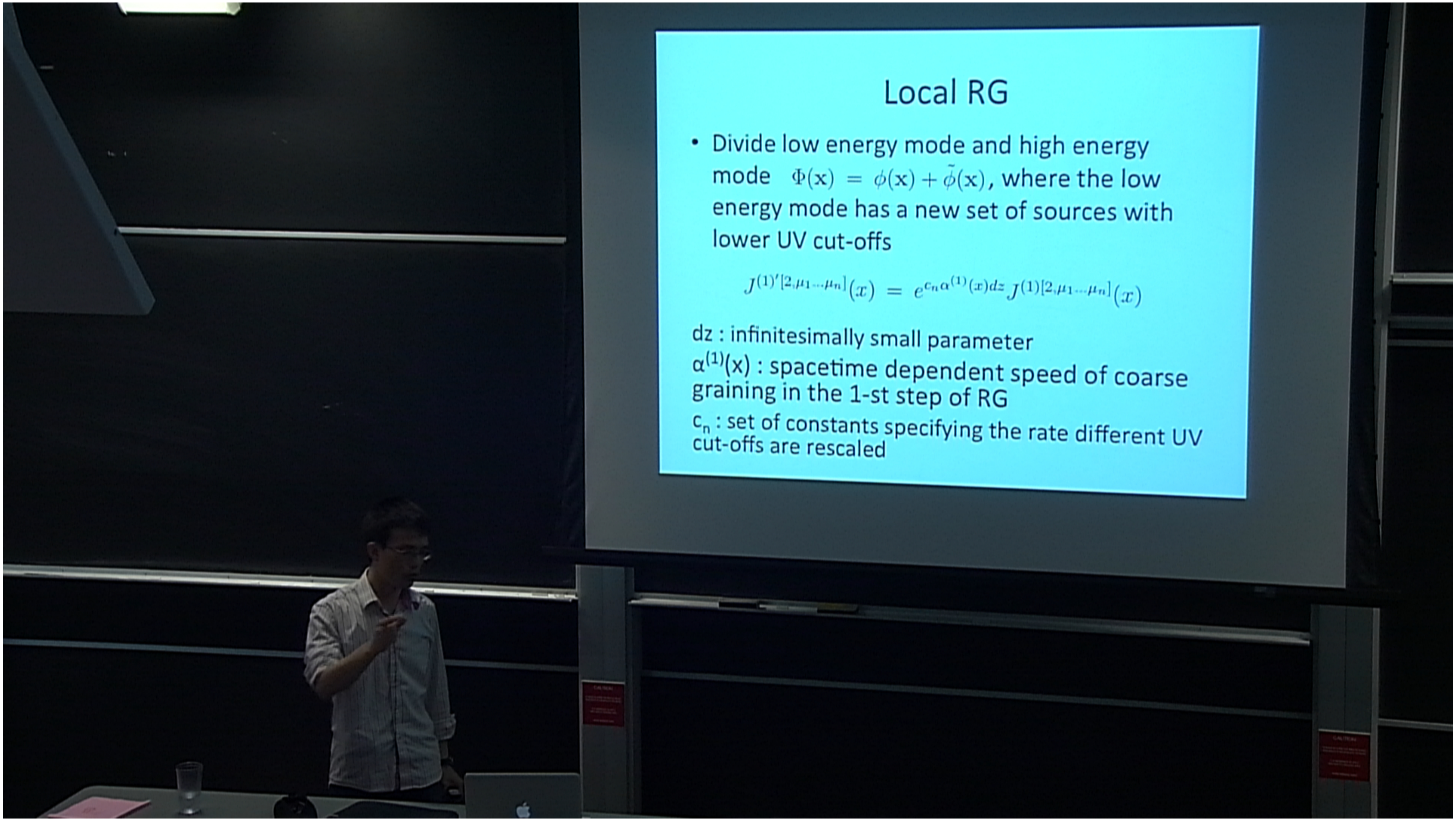
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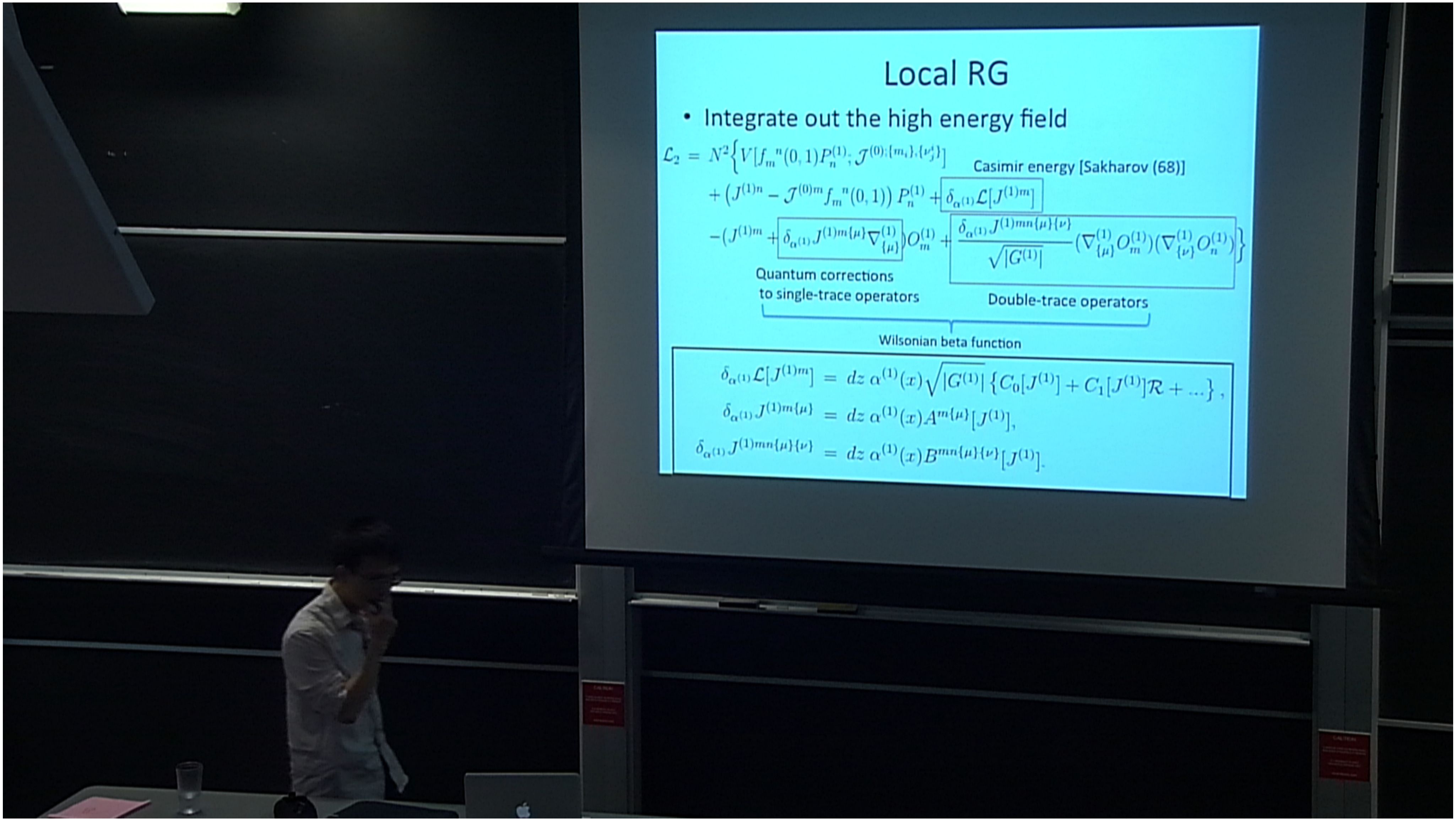
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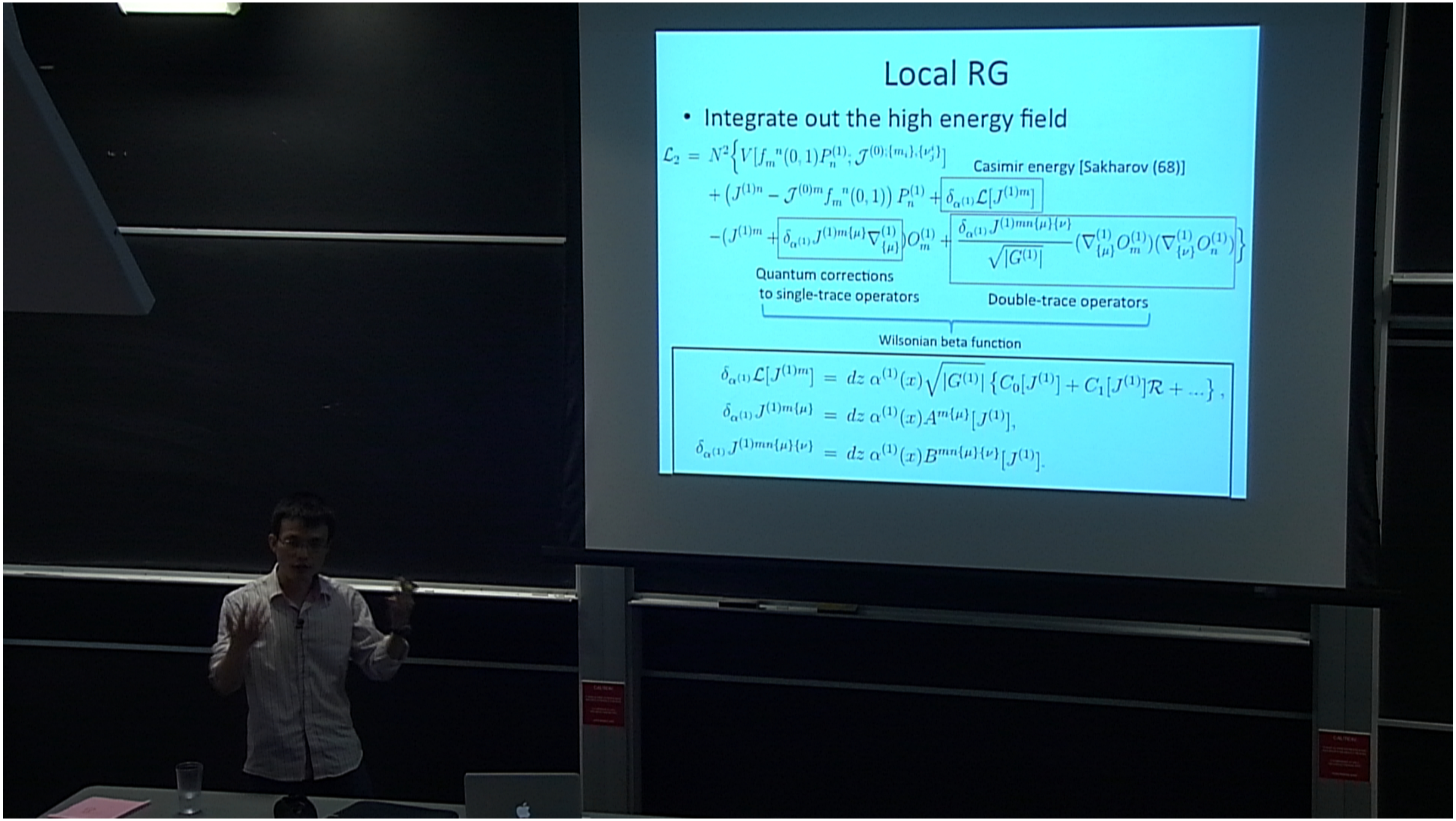
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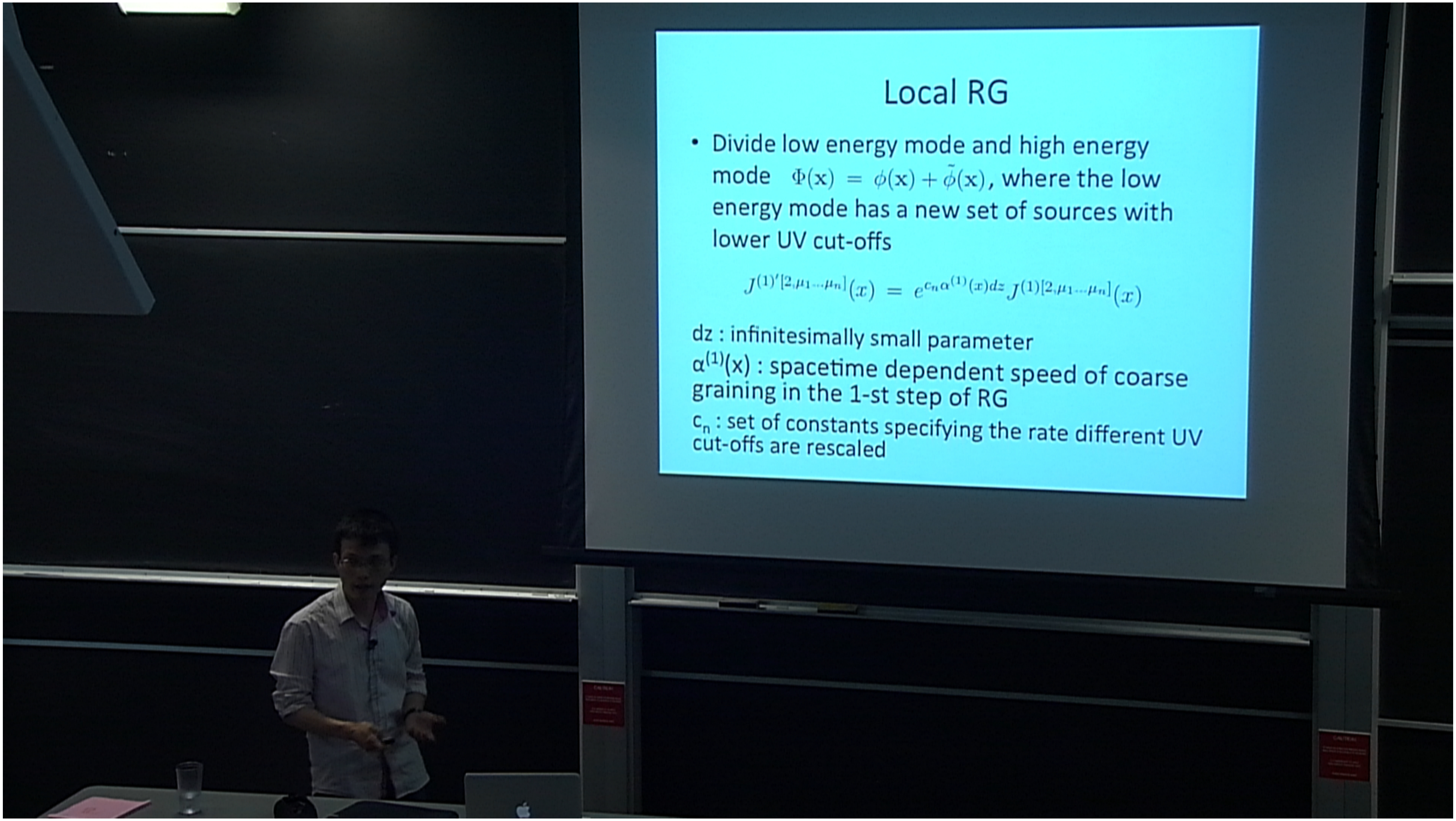
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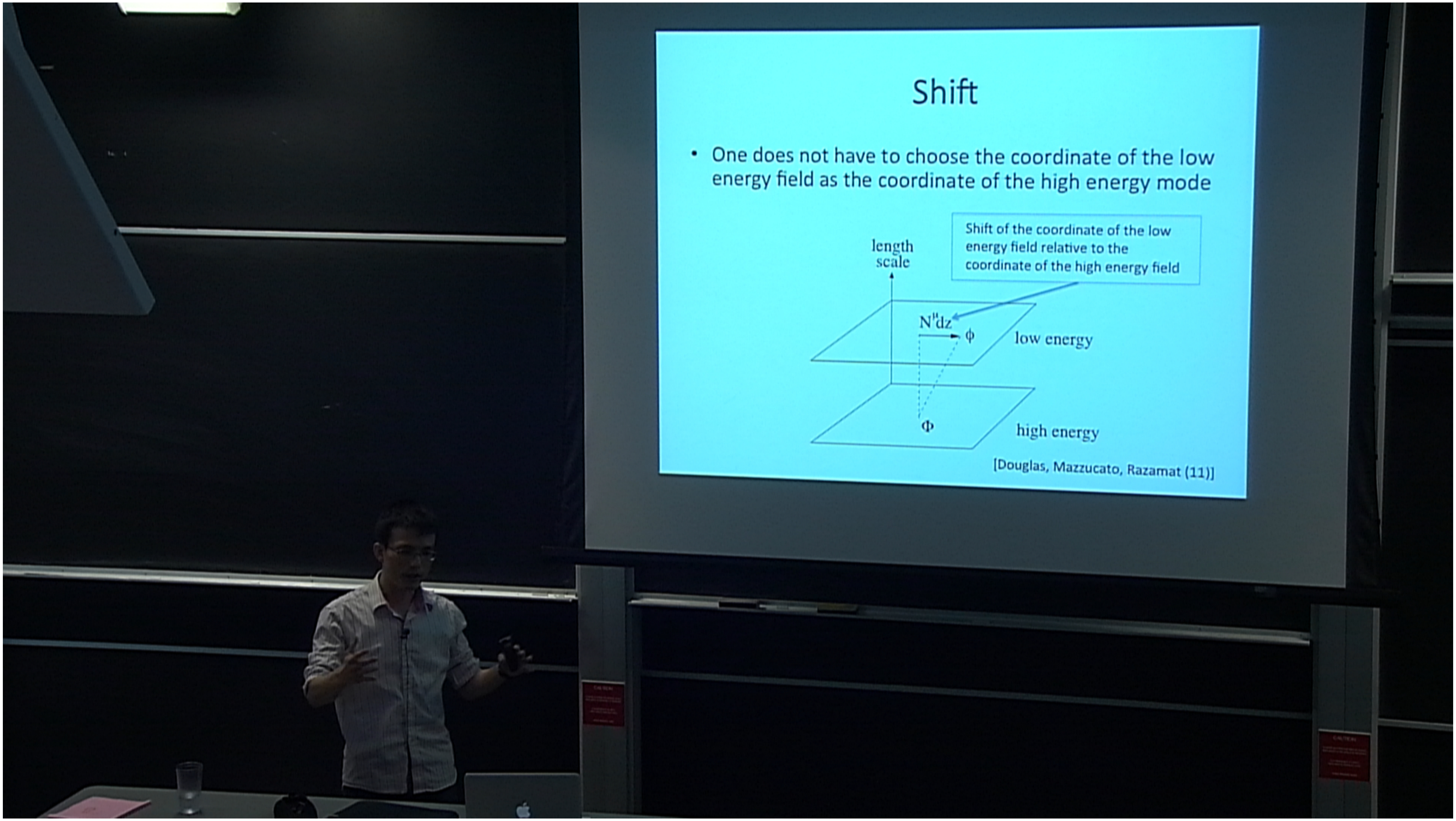
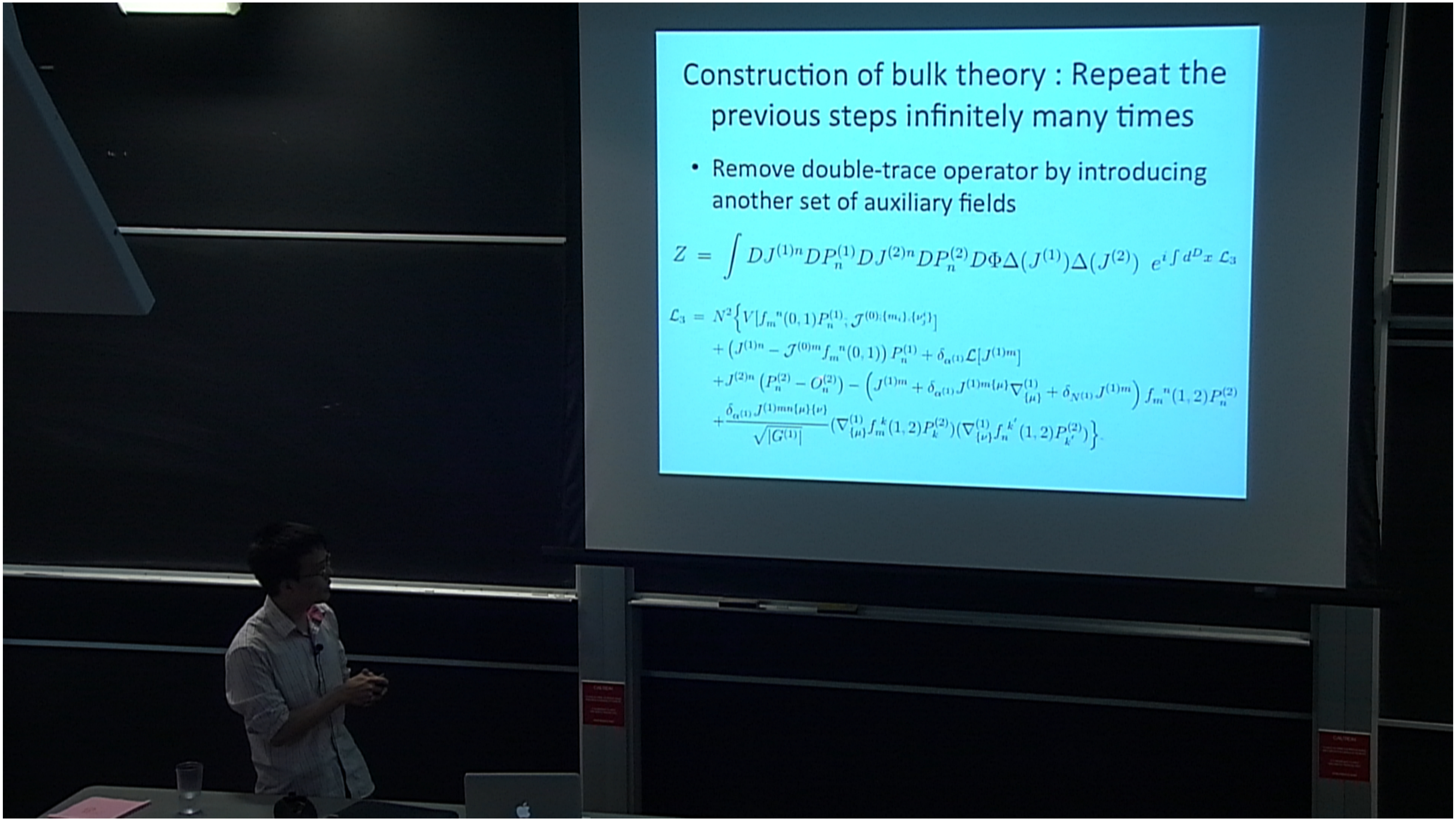

Shift
One does not have to choose the coordinate of the low energy field as the coordinate of the high energy mode
Shift of the coordinate of the low energy field relative to the coordinate of the high energy field
length scale
$N^\mu dz$
ϕ
low energy
Φ
high energy
[Douglas, Mazzucato, Razamat (11)]


Construction of bulk theory : Repeat the previous steps infinitely many times
• Remove double-trace operator by introducing another set of auxiliary fields
$Z = \int DJ^{(1)n} DP_n^{(1)} DJ^{(2)n} DP_n^{(2)} D\Phi \Delta(J^{(1)}) \Delta(J^{(2)}) \; e^{i \int d^D x \, \mathcal{L}_3}$
$\mathcal{L}_3 = N^2 \Big\{ V[f_m{}^n(0,1) P_n^{(1)}; \mathcal{J}^{(0):\{m_i\},\{\nu_j^i\}}]$
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$+ \frac{\delta_{\alpha^{(1)}} J^{(1)mn\{\mu\}\{\nu\}}}{\sqrt{|G^{(1)}|}} (\nabla_{\{\mu\}}^{(1)} f_m{}^k(1,2) P_k^{(2)}) (\nabla_{\{\nu\}}^{(1)} f_n{}^{k'}(1,2) P_{k'}^{(2)}) \Big\}.$

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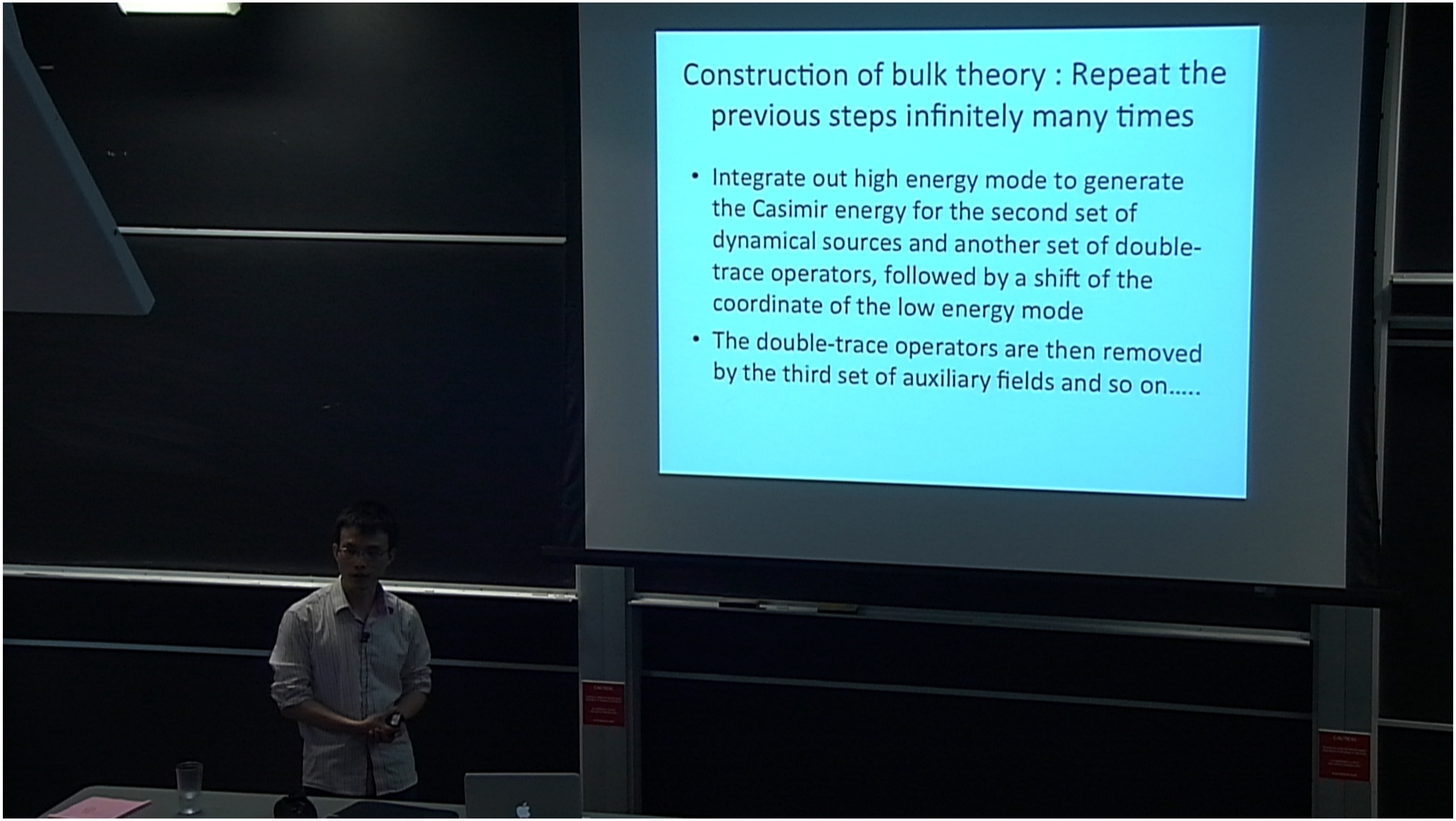
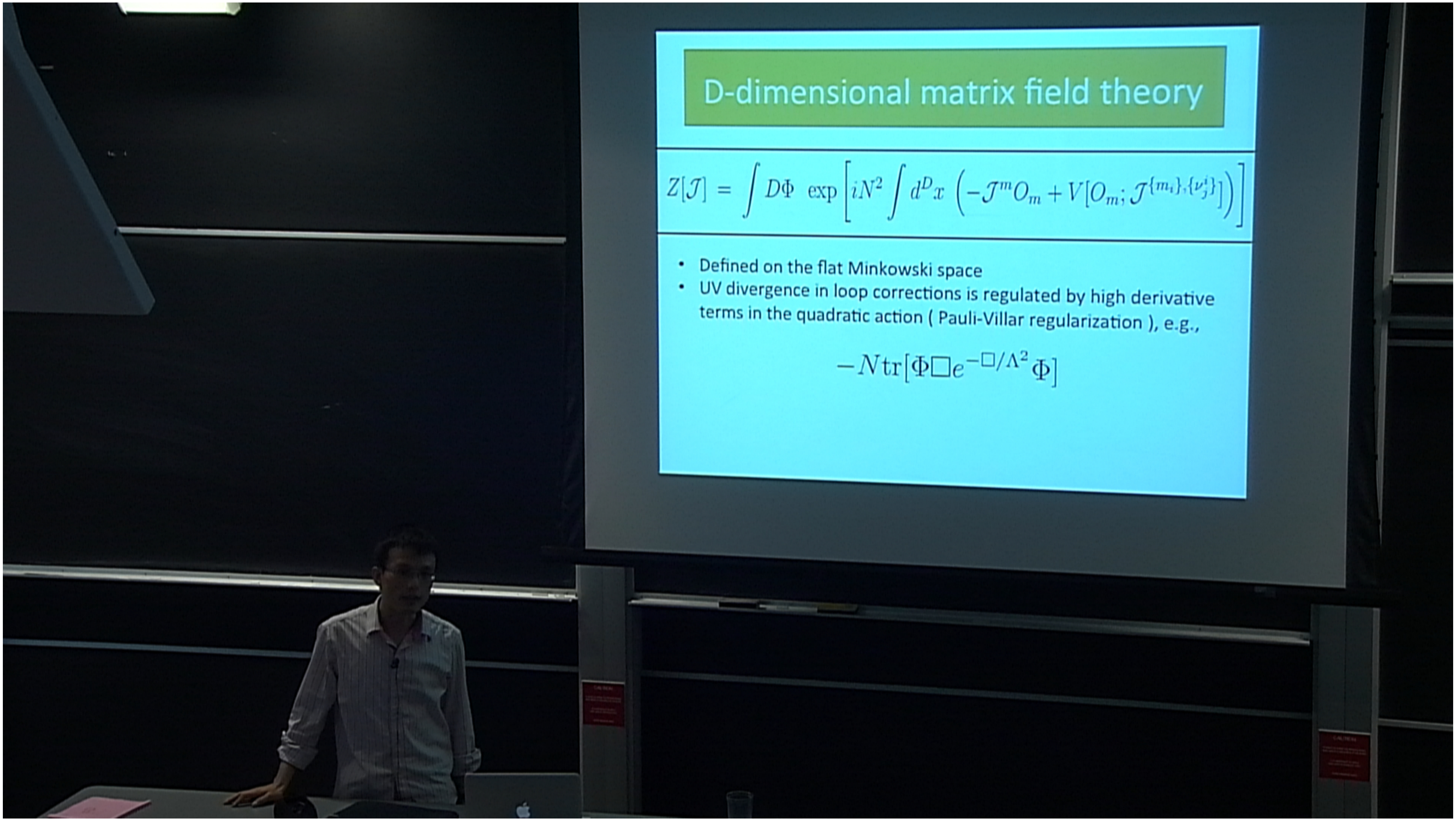
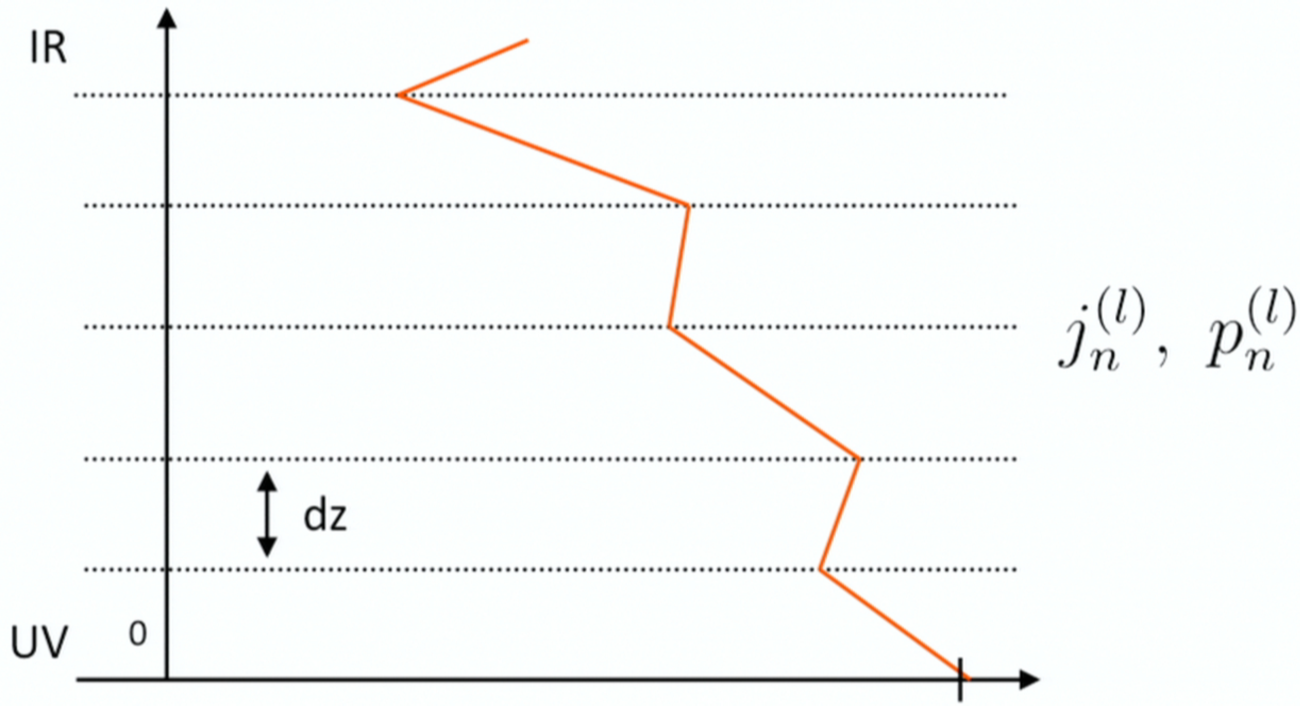

Construction of bulk theory : Repeat the previous steps infinitely many times
• Integrate out high energy mode to generate the Casimir energy for the second set of dynamical sources and another set of double-trace operators, followed by a shift of the coordinate of the low energy mode
• The double-trace operators are then removed by the third set of auxiliary fields and so on.....


D-dimensional matrix field theory
$Z[\mathcal{J}] = \int D\Phi \; \exp\left[iN^2 \int d^D x \left(-\mathcal{J}^m O_m + V[O_m; \mathcal{J}^{\{m_i\},\{\nu_j^i\}}] \right) \right]$
• Defined on the flat Minkowski space
• UV divergence in loop corrections is regulated by high derivative terms in the quadratic action (Pauli-Villar regularization), e.g.,
$-N\mathrm{tr}[\Phi \Box e^{-\Box/\Lambda^2} \Phi]$

Extra dimension as a length scale



Boundary condition $j_n^{(0)}(x) = J_n(x)$

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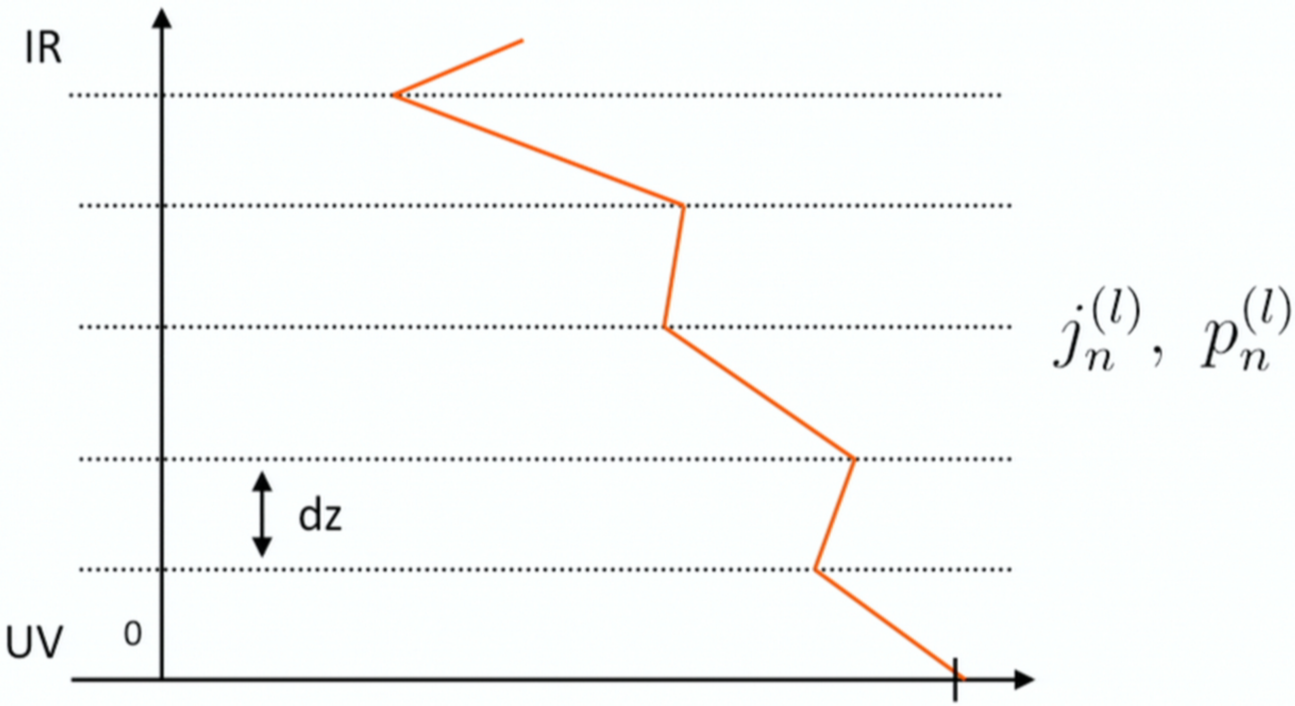

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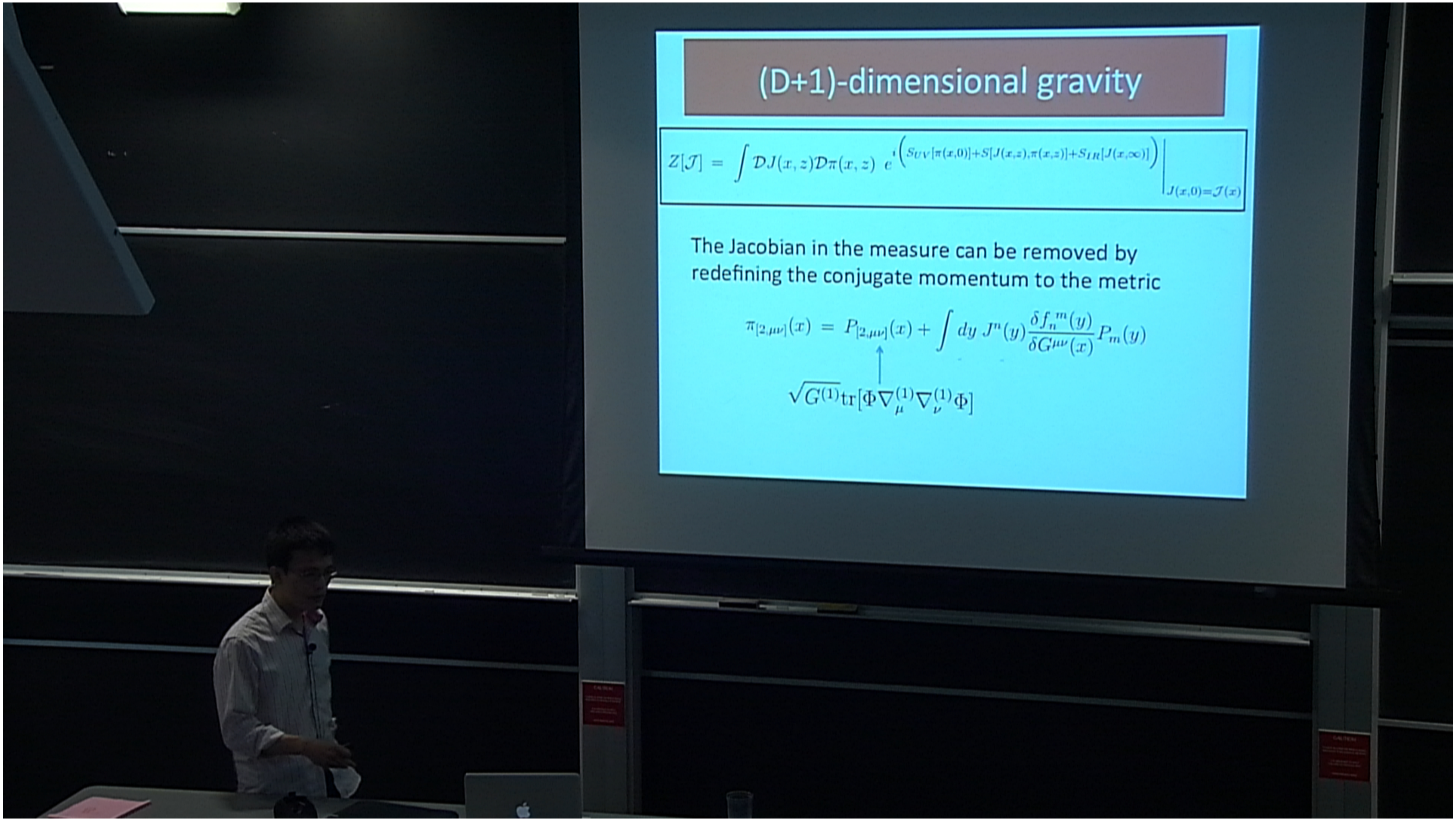
(D+1)-dimensional gravity
$Z[\mathcal{J}] = \int \mathcal{D}J(x,z)\mathcal{D}\pi(x,z)\ e^{i\left(S_{UV}[\pi(x,0)]+S[J(x,z),\pi(x,z)]+S_{IR}[J(x,\infty)]\right)}\Big|_{J(x,0)=\mathcal{J}(x)}$
The Jacobian in the measure can be removed by redefining the conjugate momentum to the metric
$\pi_{[2,\mu\nu]}(x) = P_{[2,\mu\nu]}(x) + \int dy\ J^n(y)\frac{\delta f_n^{\ m}(y)}{\delta G^{\mu\nu}(x)}P_m(y)$
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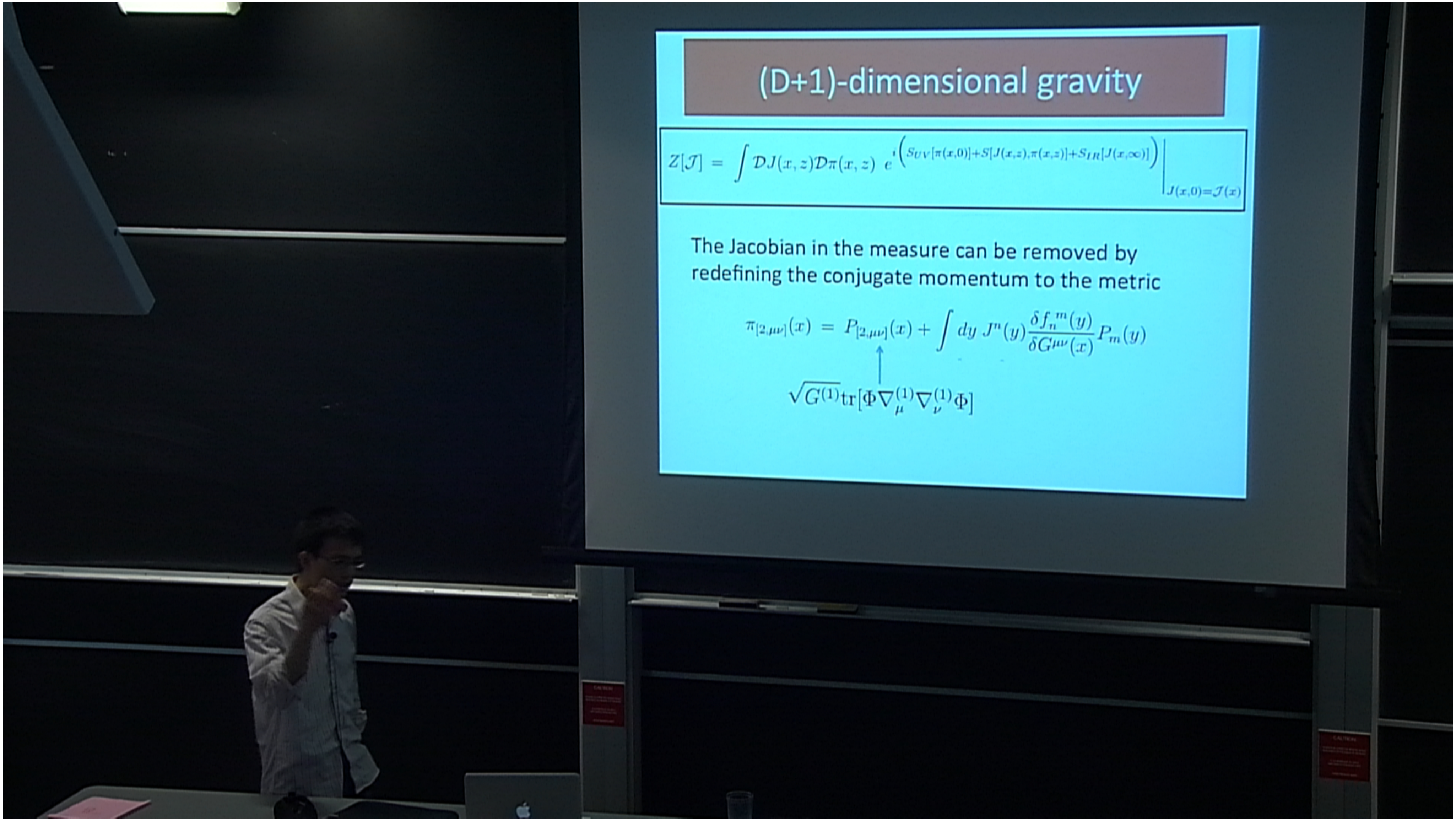
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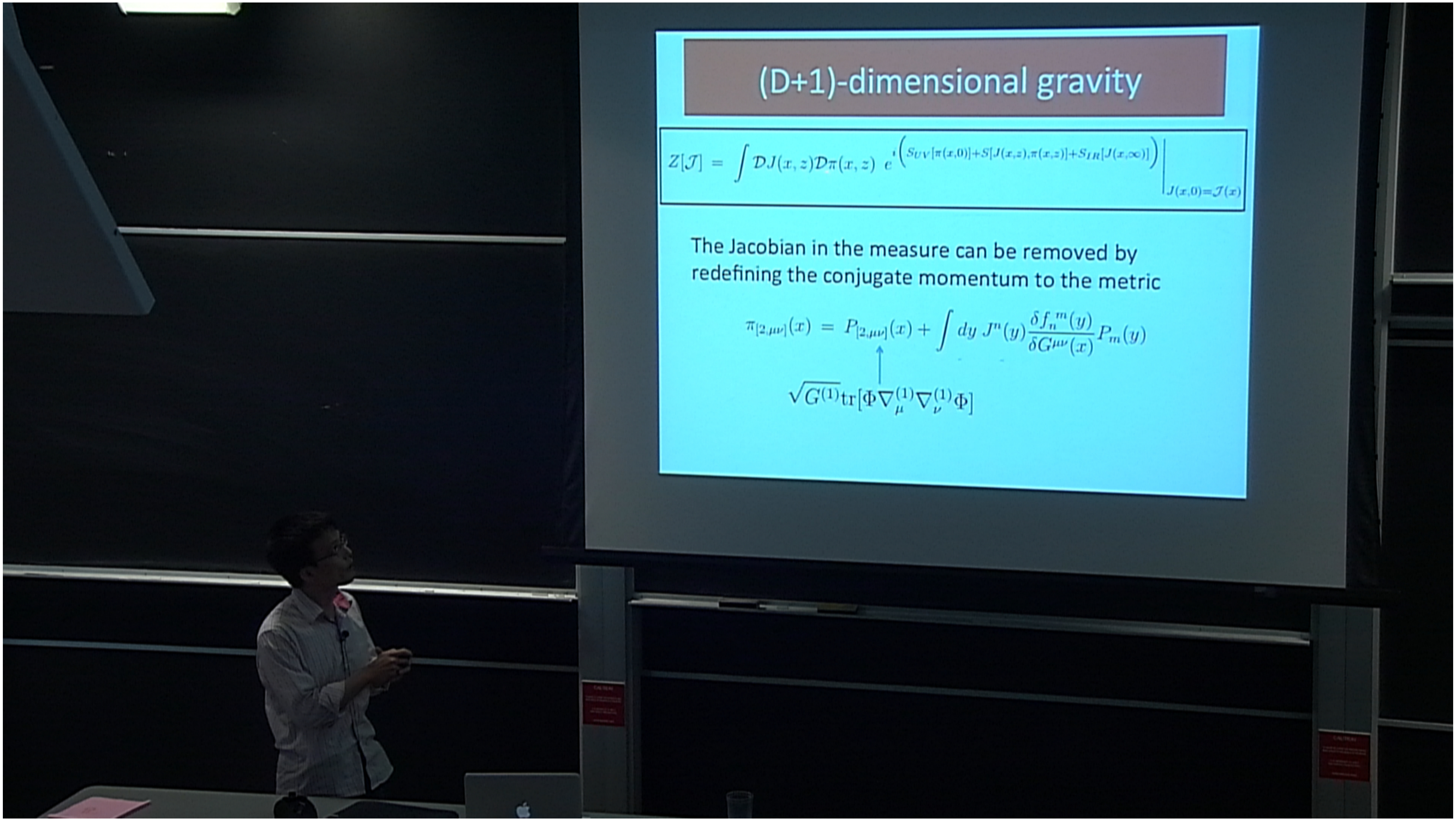
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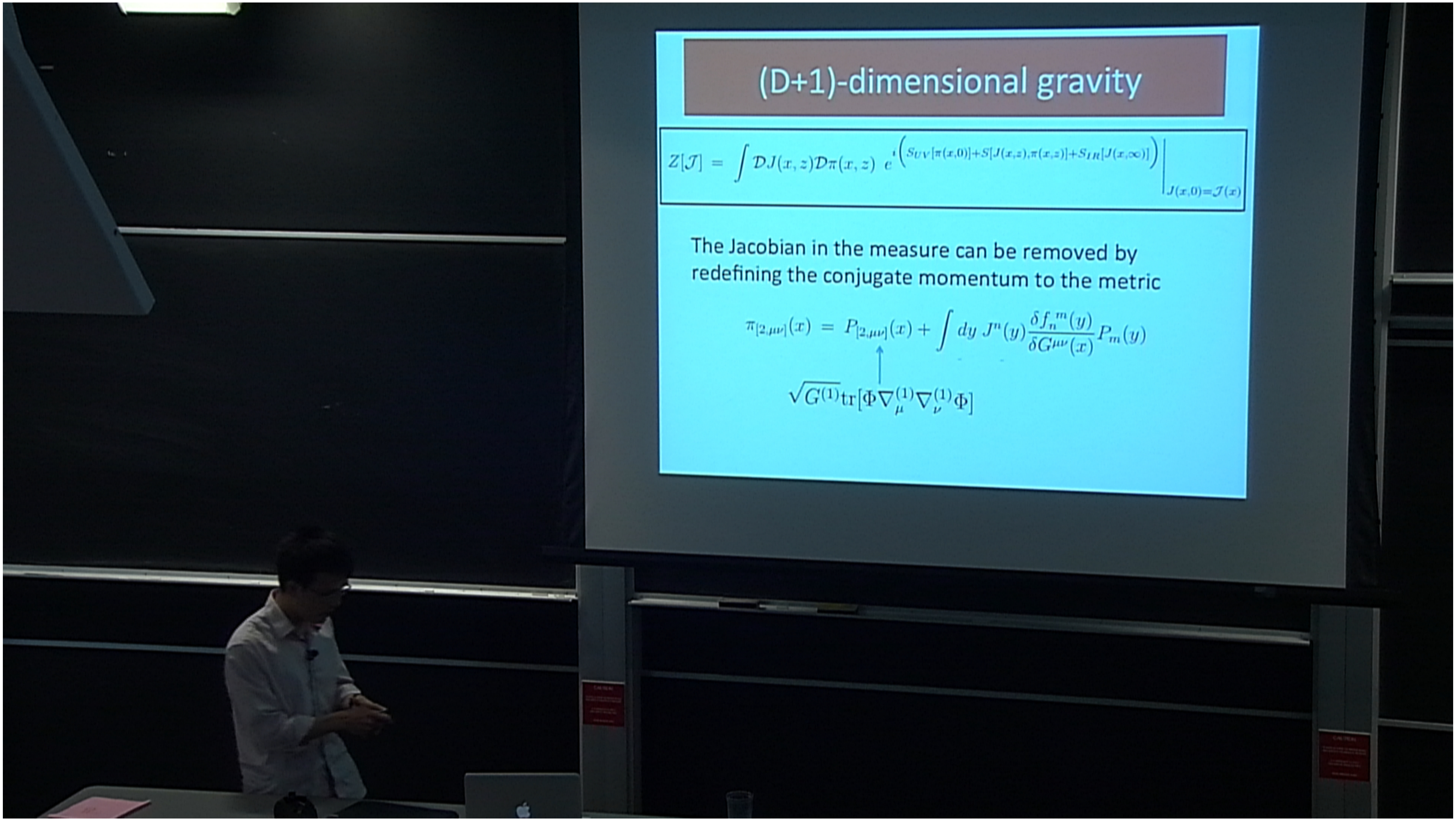
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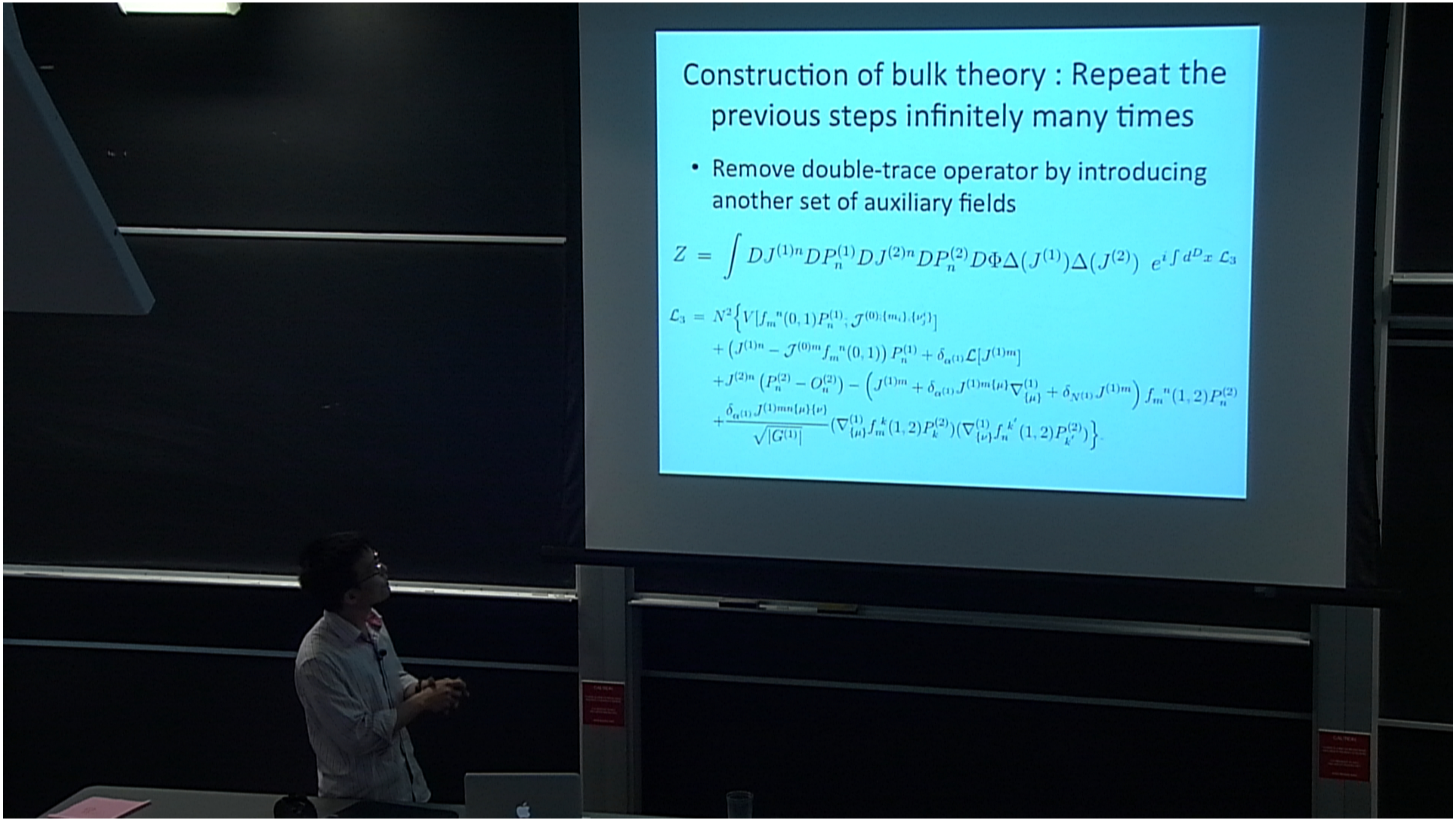
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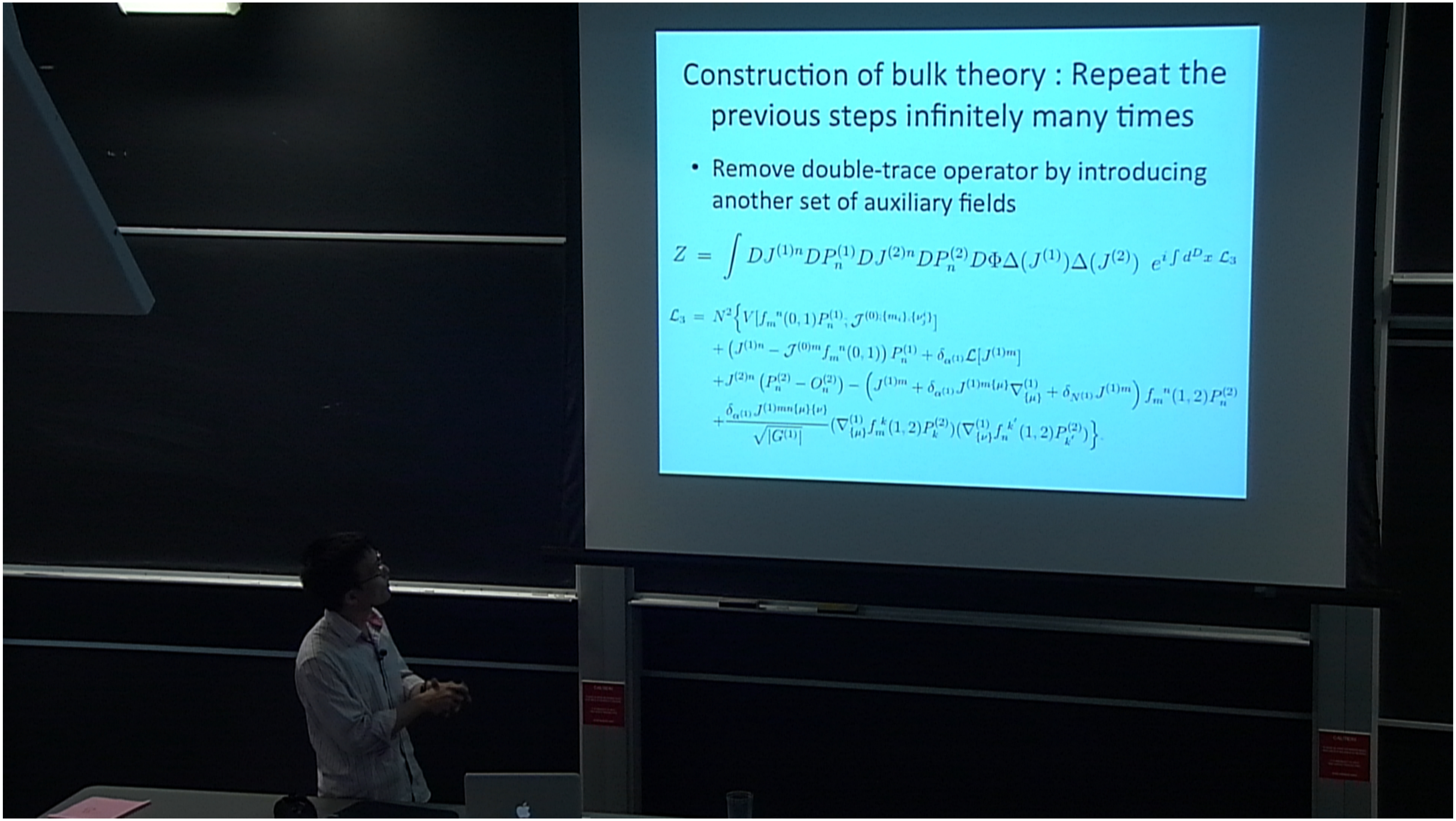
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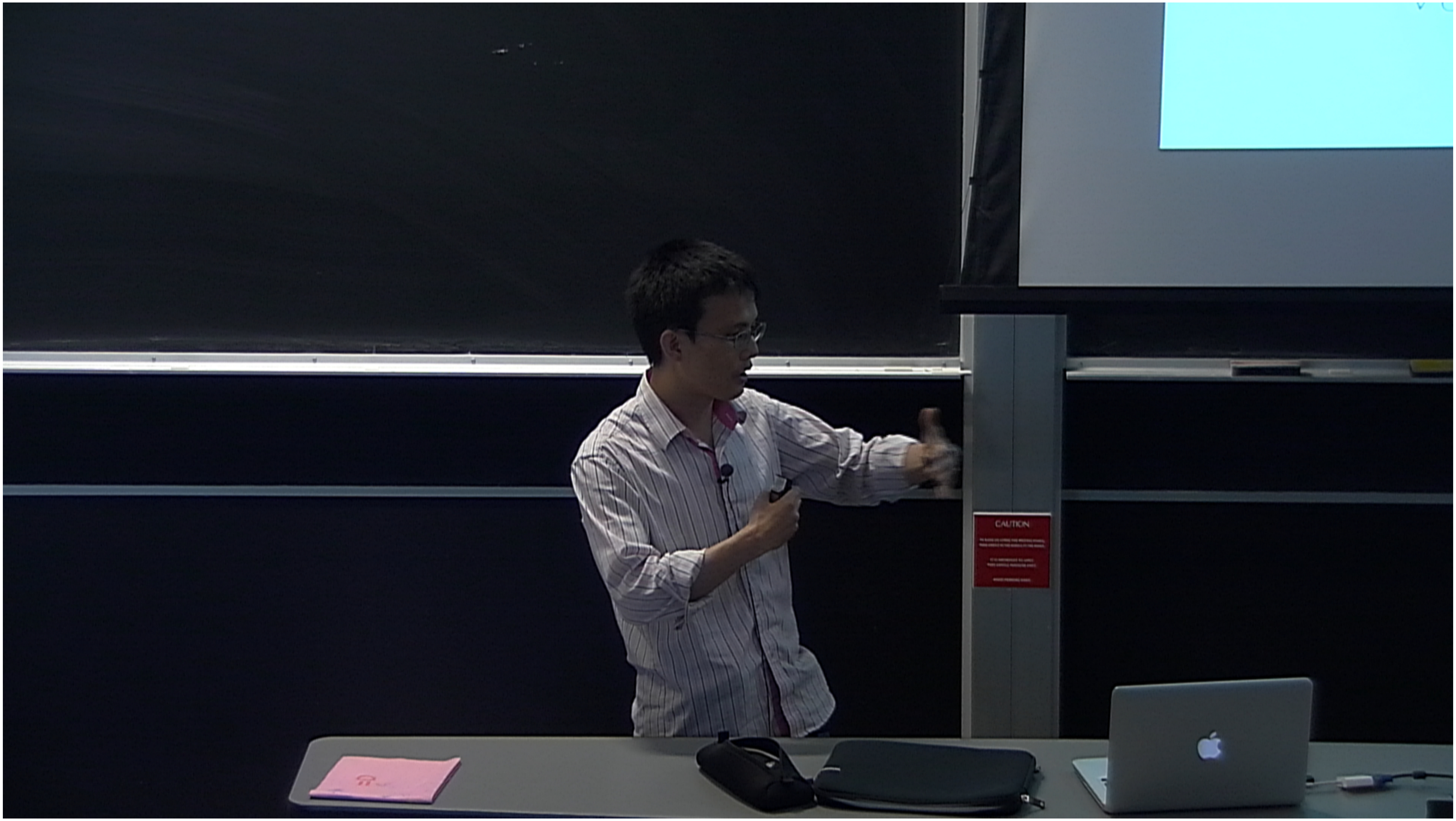

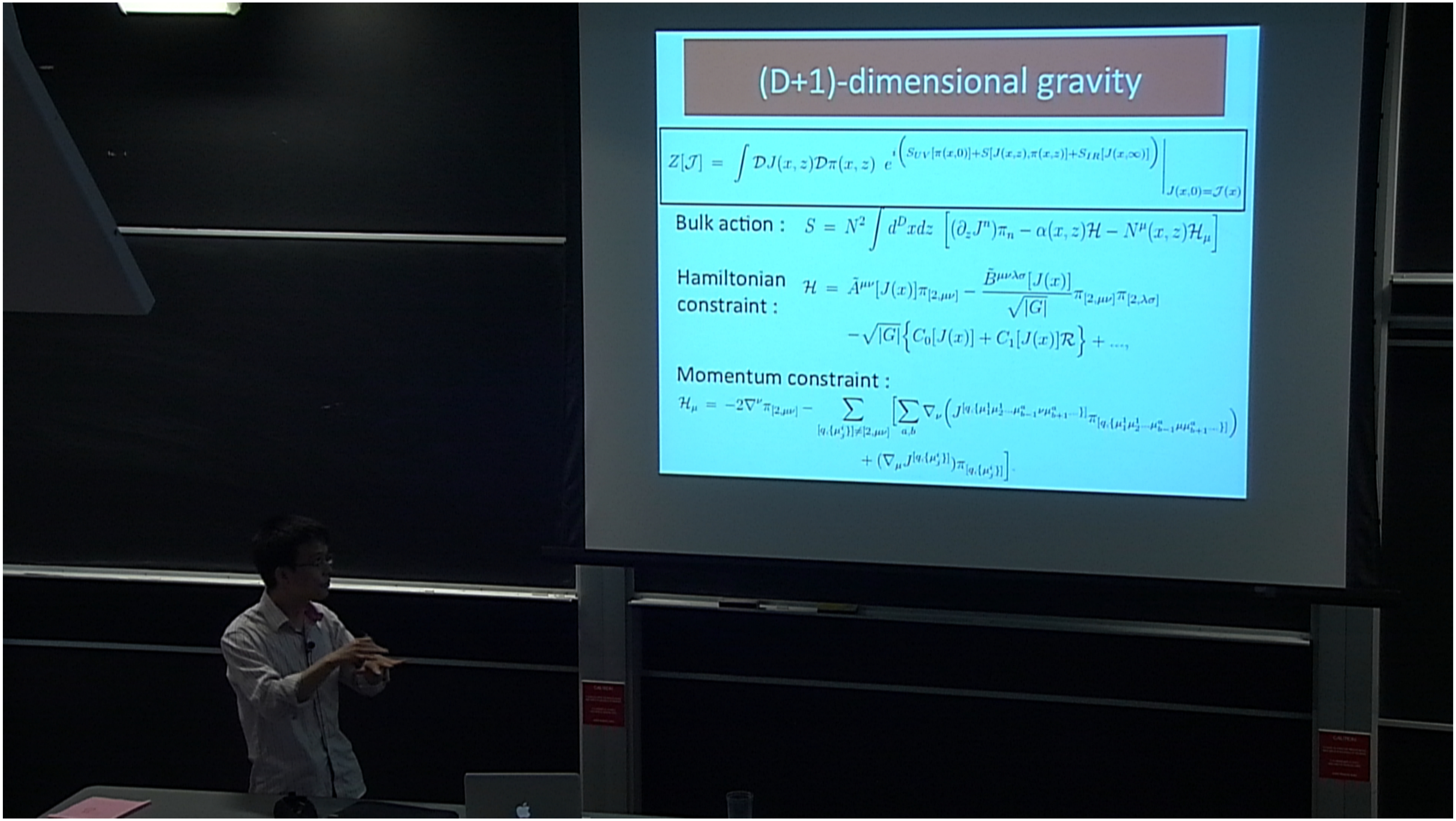
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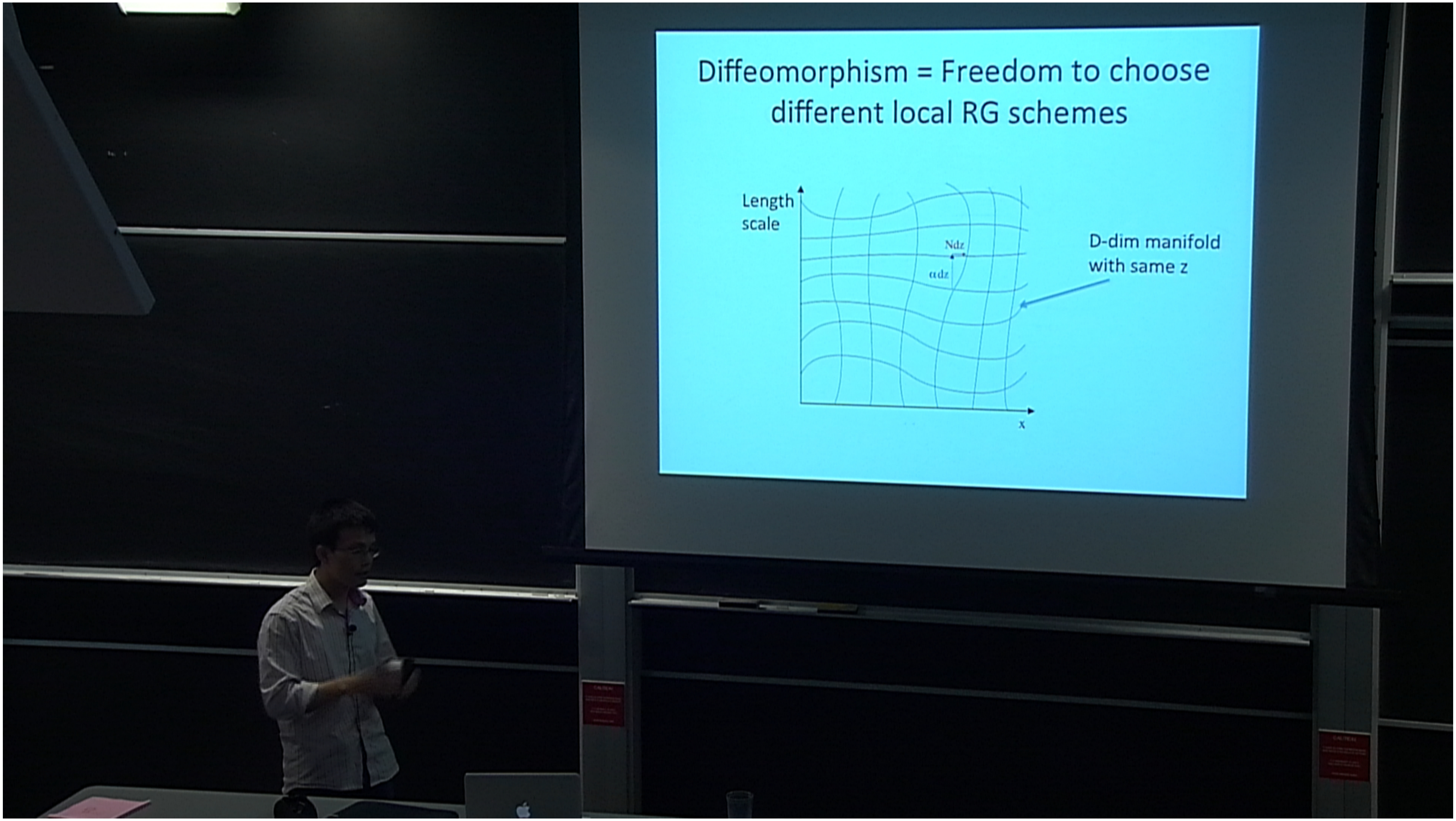
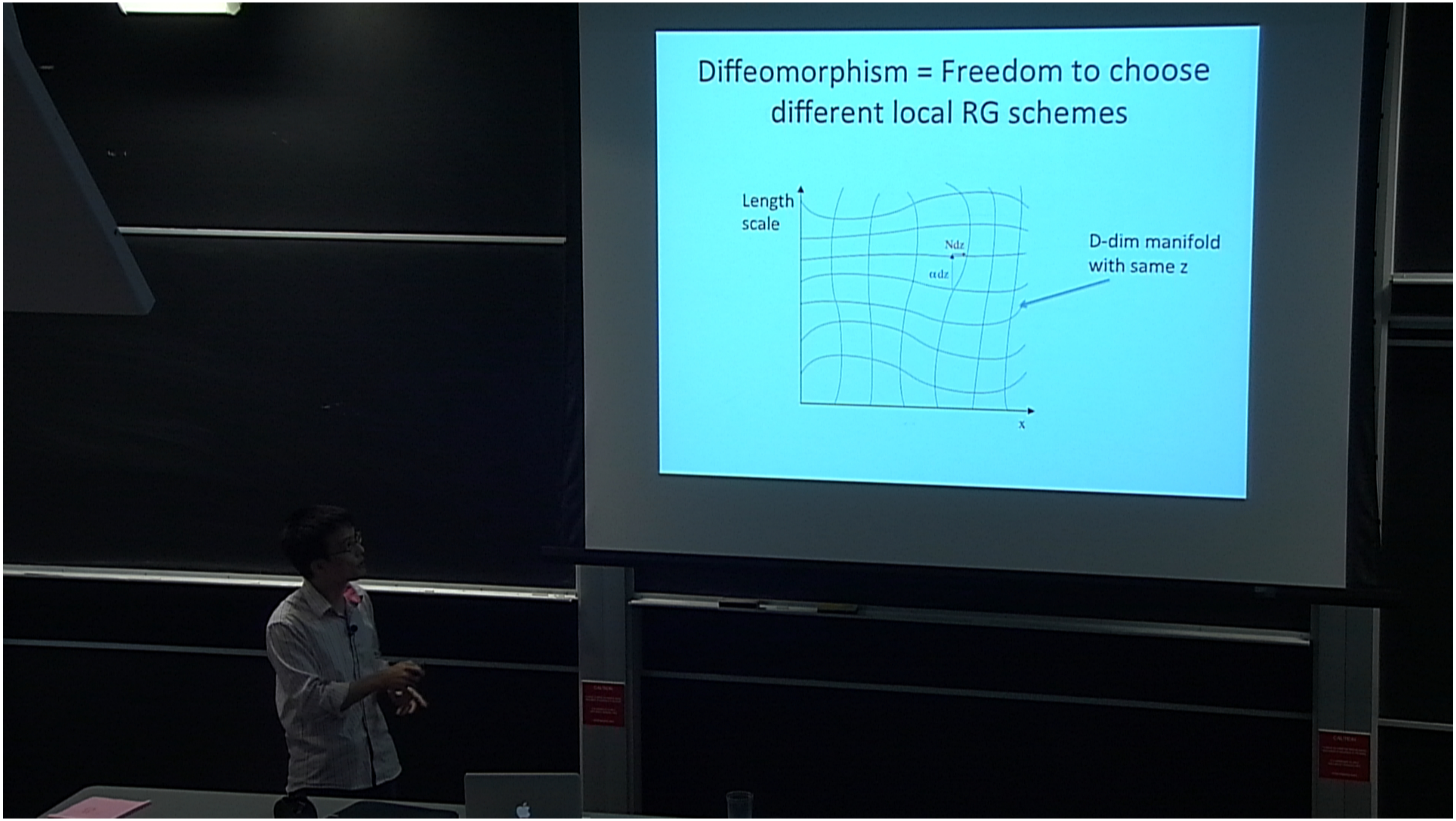
Bulk action : $S = N^2 \int d^D x dz \left[(\partial_z J^n)\pi_n - \alpha(x,z)\mathcal{H} - N^\mu(x,z)\mathcal{H}_\mu\right]$
Hamiltonian constraint : $\mathcal{H} = \tilde{A}^{\mu\nu}[J(x)]\pi_{[2,\mu\nu]} - \frac{\tilde{B}^{\mu\nu\lambda\sigma}[J(x)]}{\sqrt{|G|}}\pi_{[2,\mu\nu]}\pi_{[2,\lambda\sigma]}$
$-\sqrt{|G|}\left\{C_0[J(x)] + C_1[J(x)]\mathcal{R}\right\} + \dots,$
Momentum constraint :
$\mathcal{H}_\mu = -2\nabla^\nu \pi_{[2,\mu\nu]} - \sum_{[q,\{\mu_j^i\}]\neq[2,\mu\nu]}\left[\sum_{a,b}\nabla_\nu\left(J^{[q,\{\mu_1^1\mu_2^1\cdots\mu_{b-1}^a\nu\mu_{b+1}^a\cdots\}]}\pi_{[q,\{\mu_1^1\mu_2^1\cdots\mu_{b-1}^a\mu\mu_{b+1}^a\cdots\}]}\right)\right.$
$\left.+ (\nabla_\mu J^{[q,\{\mu_j^i\}]})\pi_{[q,\{\mu_j^i\}]}\right].$


Diffeomorphism = Freedom to choose different local RG schemes
Length scale
Ndz
αdz
D-dim manifold with same z
x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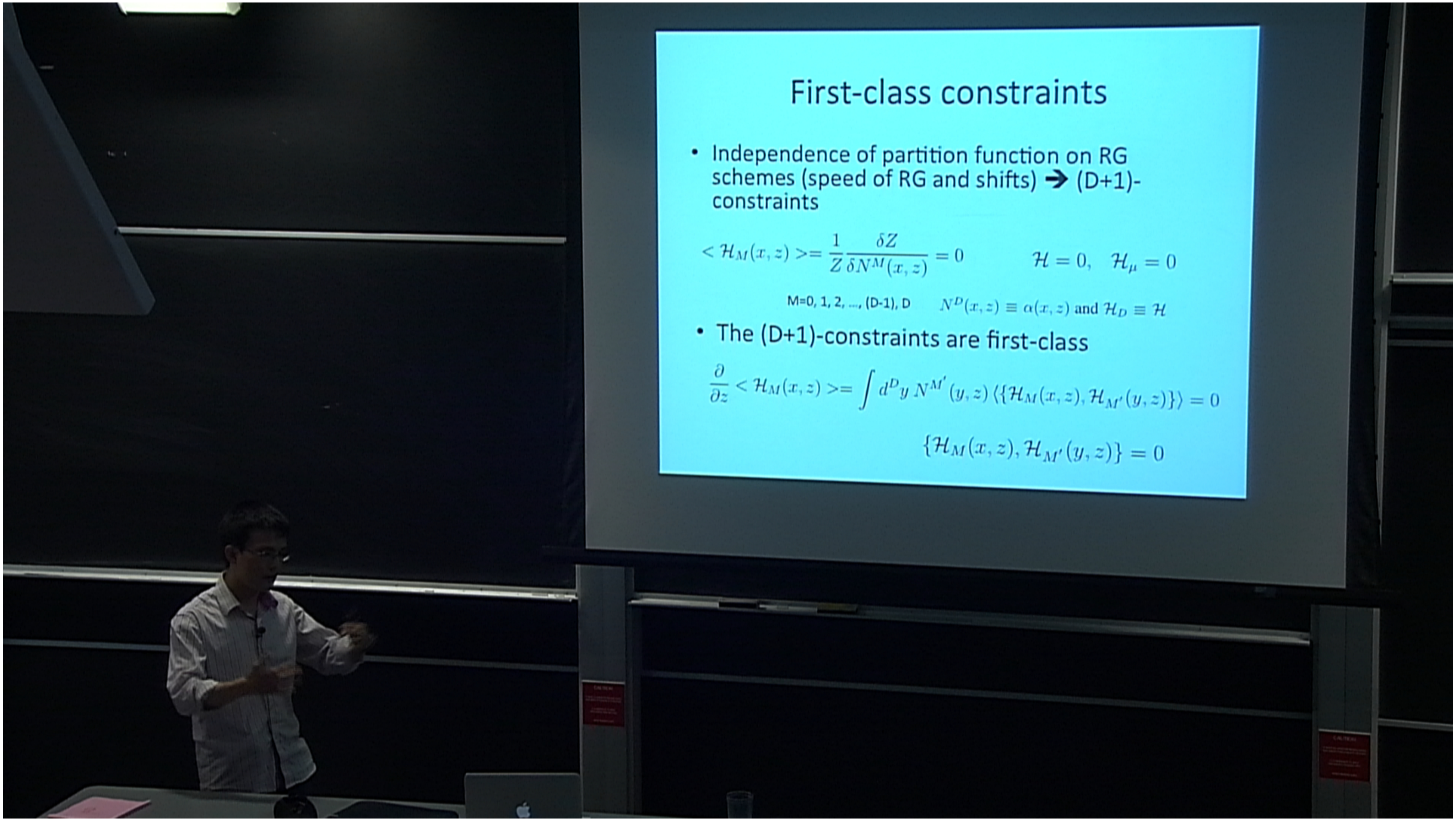
First-class constraints
• Independence of partition function on RG schemes (speed of RG and shifts) → (D+1)-constraints
$< \mathcal{H}_M(x,z) > = \frac{1}{Z}\frac{\delta Z}{\delta N^M(x,z)} = 0$
$\mathcal{H} = 0, \quad \mathcal{H}_\mu = 0$
M=0, 1, 2, …, (D-1), D
$N^D(x,z) \equiv \alpha(x,z)$ and $\mathcal{H}_D \equiv \mathcal{H}$
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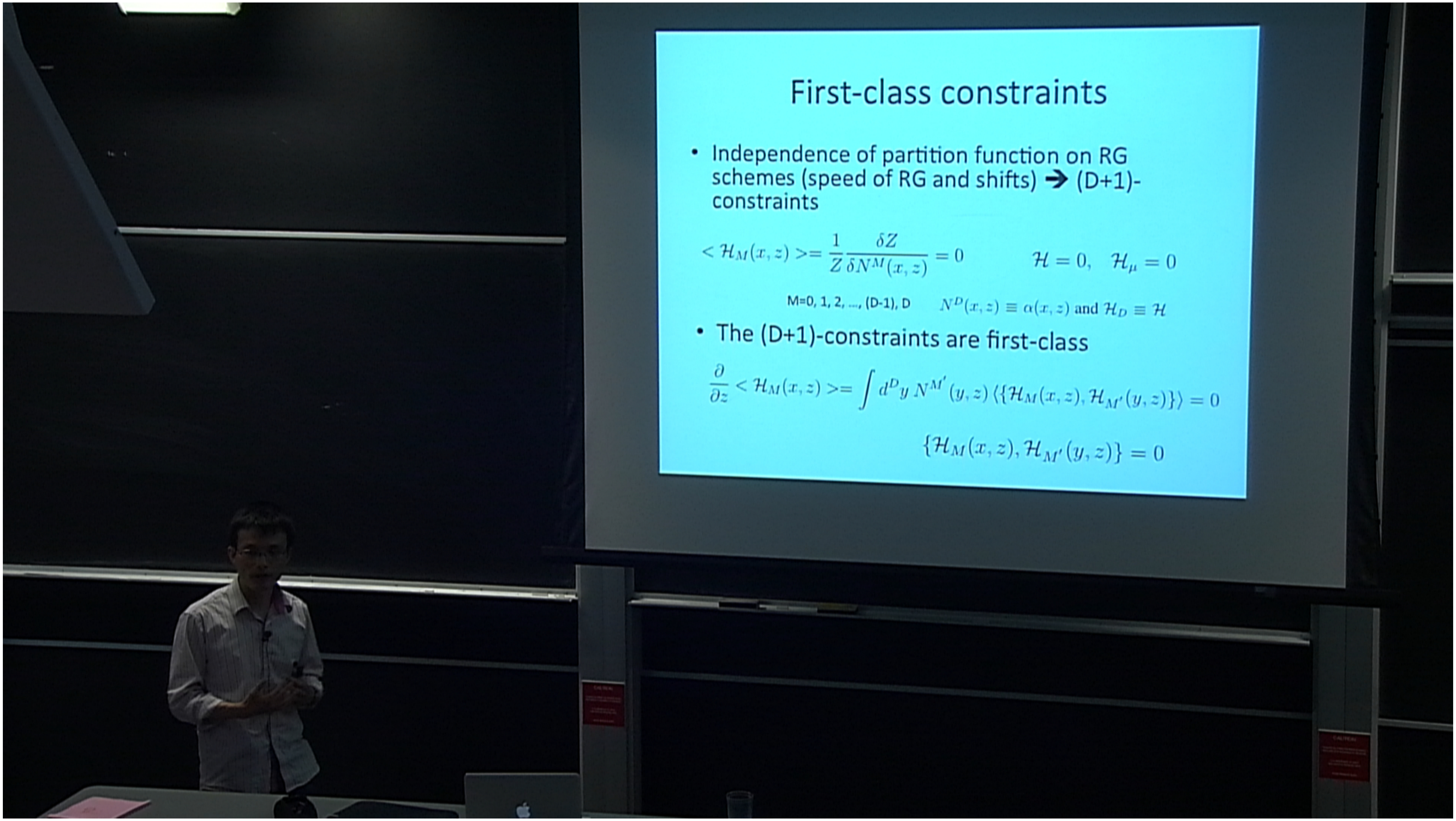
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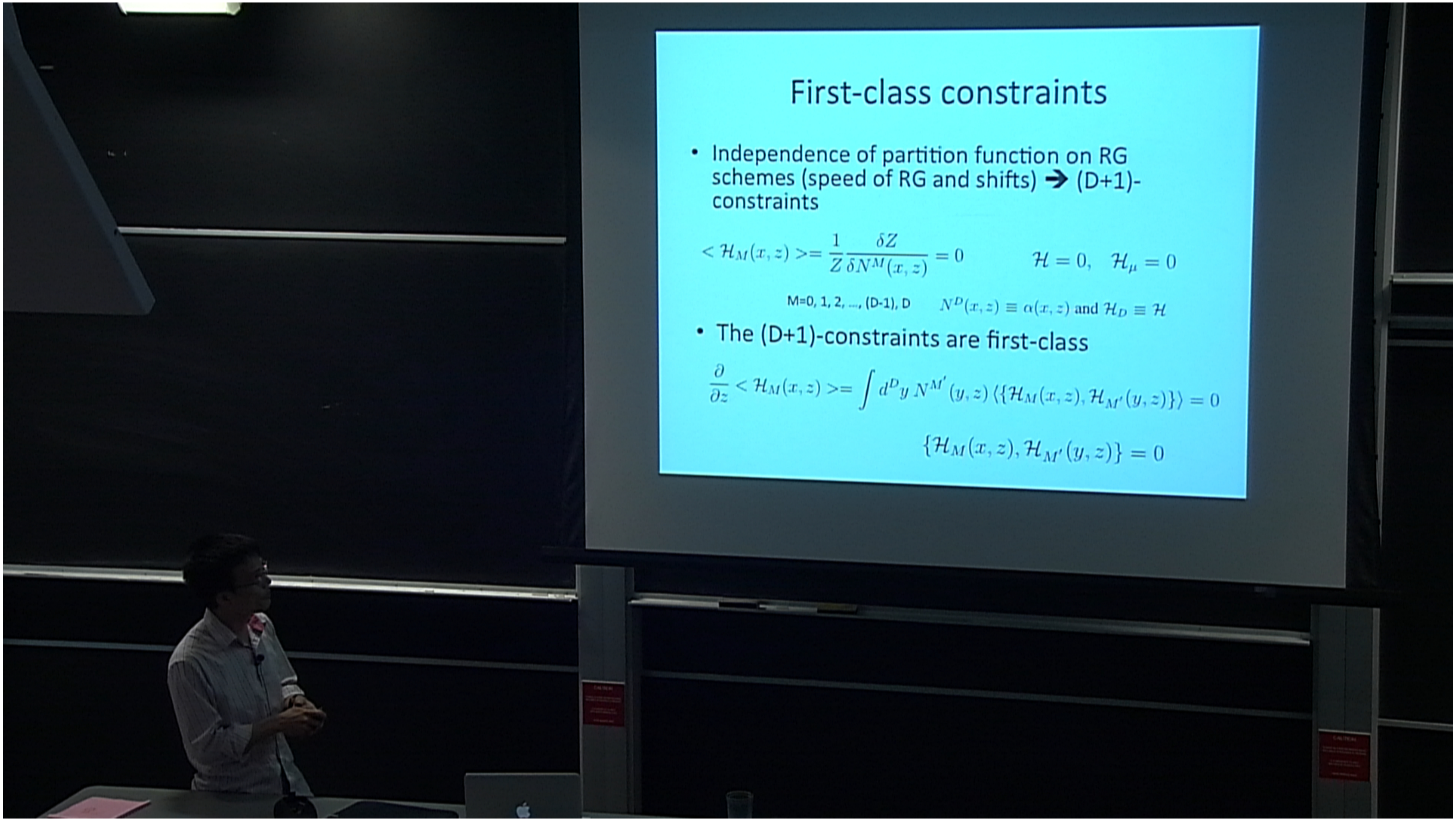
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