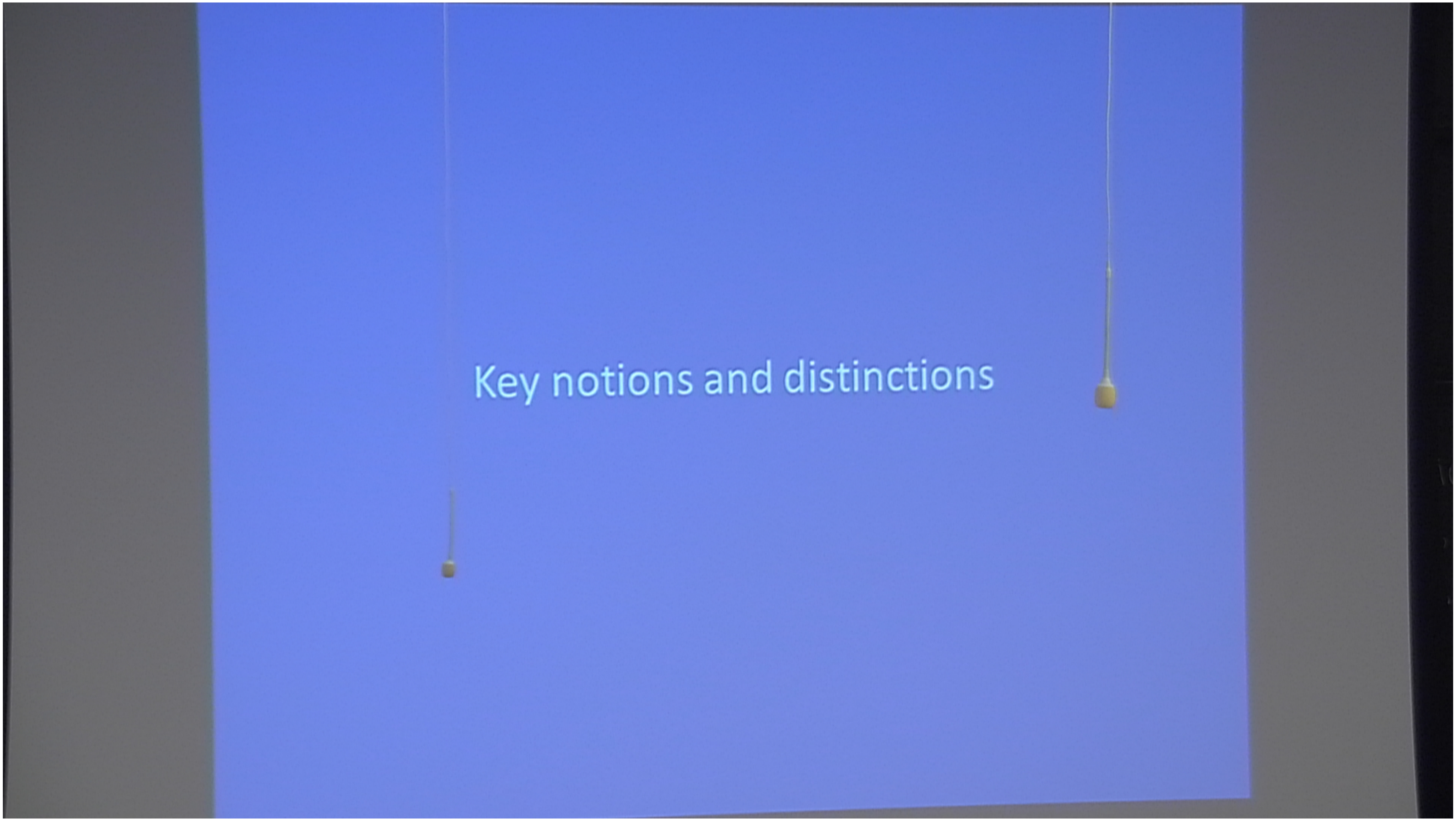
Title: Why I Am Not a Psi-ontologist

Date: May 08, 2012 03:30 PM

URL: http://pirsa.org/12050021

Abstract: The distinction between a realist interpretation of quantum theory that is psi-ontic and one that is psi-epistemic is whether or not a difference in the quantum state necessarily implies a difference in the underlying ontic state. Psi-ontologists believe that it does, psi-epistemicists that it does not. This talk will address the question of whether the PBR theorem should be interpreted as lending evidence against the psi-epistemic research program. I will review the evidence in favour of the psi-epistemic approach and describe the pre-existing reasons for thinking that if a quantum state represents knowledge about reality then it is not reality as we know it, i.e., it is not the kind of reality that is posited in the standard hidden variable framework. I will argue that the PBR theorem provides additional clues for "what has to give" in the hidden variable framework rather than providing a reason to retreat from the psi-epistemic position. The first assumption of the theorem - that holistic properties may exist for composite systems, but do not arise for unentangled quantum states - is only appealing if one is already predisposed to a psi-ontic view. The more natural assumption of separability (no holistic properties) coupled with the other assumptions of the theorem rules out both psi-ontic and psi-epistemic models and so does not decide between them. The connection between the PBR theorem and other no-go results will be discussed. In particular, I will point out how the second assumption of the theorem is an instance of preparation noncontextuality, a property that is known not to be achievable in any ontological model of quantum theory, regardless of the status of separability (though not in the form posited by PBR). I will also consider the connection of PBR to the failure of local causality by considering an experimental scenario which is in a sense a time-inversion of the PBR scenario.


Key notions and distinctions

Operational quantum theory

Preparation P
ρ

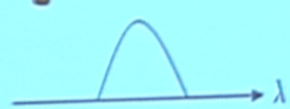
Measurement M
$\{E_x\}$

$$P(x|\mathsf{P},\mathsf{M}) = \mathrm{Tr}(\rho E_x)$$

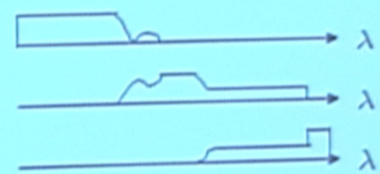
An ontological model of quantum theory

$\lambda \in \Lambda$ Ontic state space

$\mathsf{P} \leftrightarrow P(\lambda|\mathsf{P})$



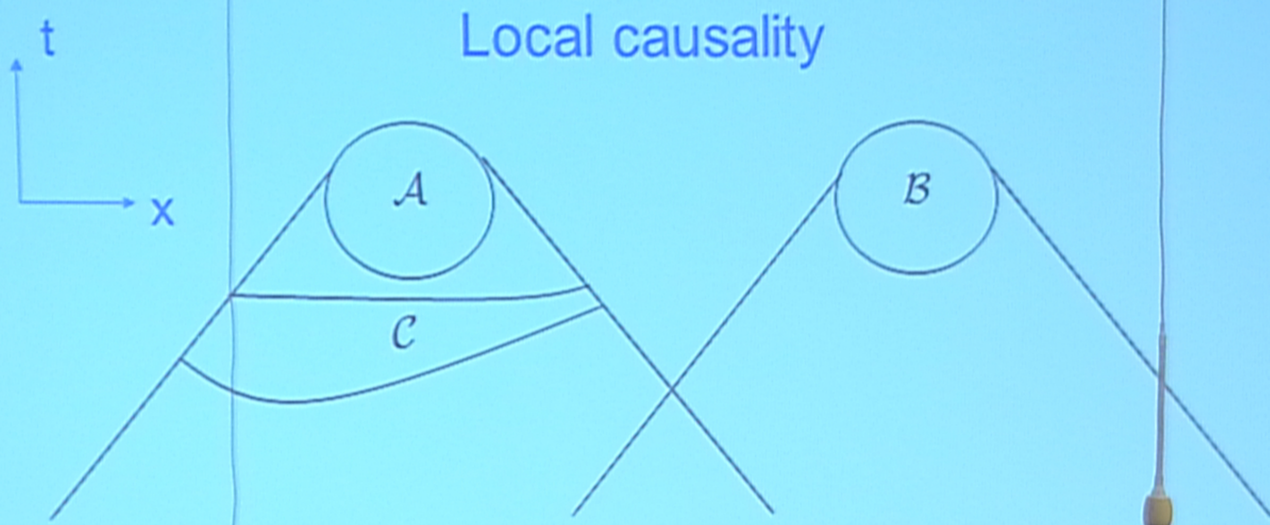
$\mathsf{M} \leftrightarrow P(X|\mathsf{M},\lambda)$



λ screens off P from M (λ-sufficiency)

$$P(X|\mathsf{P},\mathsf{M}) = \int P(X|\mathsf{M},\lambda)\, P(\lambda|\mathsf{P})\, \mathrm{d}\lambda$$
$$= \mathrm{Tr}(\rho E_x)$$

Local causality



$$p(X_{\mathcal{A}}|X_{\mathcal{B}}, \lambda_{\mathcal{C}}) = p(X_{\mathcal{A}}|\lambda_{\mathcal{C}})$$

Generalized noncontextuality

RWS, Phys. Rev. A 71, 052108 (2005)

Preparation noncontextuality

$$\forall \mathsf{M} : p(X|\mathsf{P},\mathsf{M}) = p(X|\mathsf{P}',\mathsf{M}) \longrightarrow p(\lambda|\mathsf{P}) = p(\lambda|\mathsf{P}')$$

In quantum theory $p(\lambda|\mathsf{P}) = p(\lambda|\rho)$

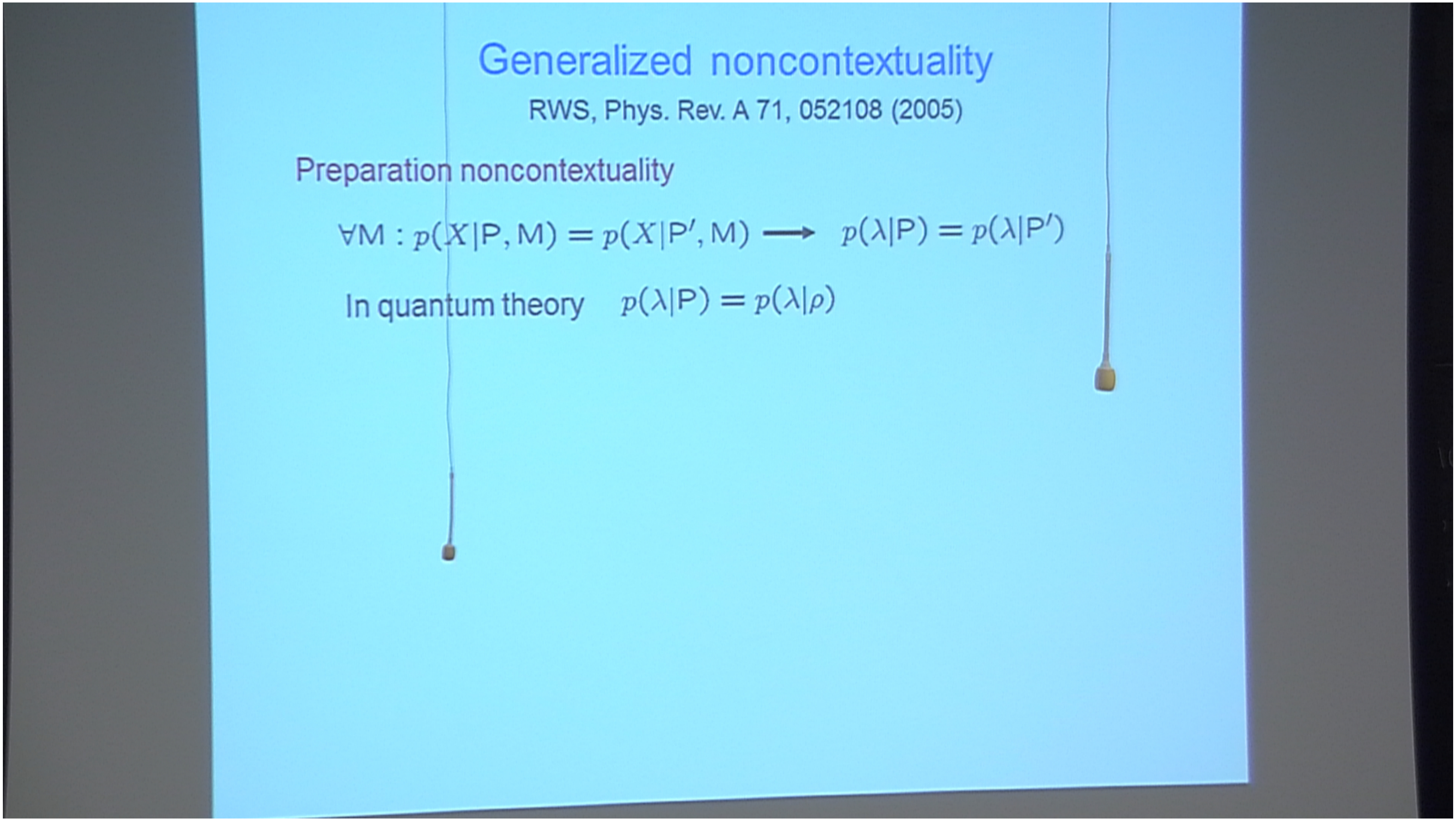
Generalized noncontextuality

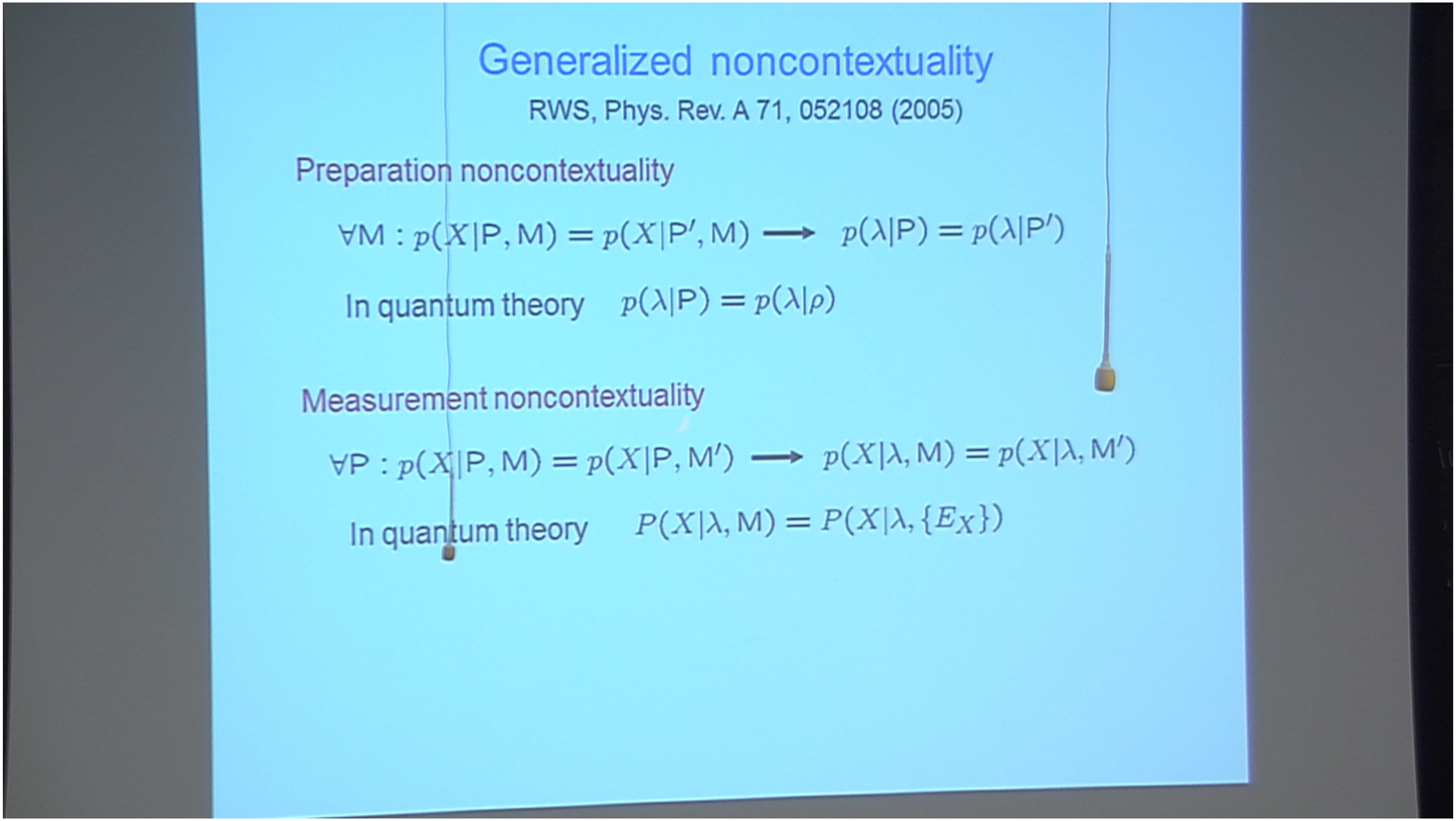
RWS, Phys. Rev. A 71, 052108 (2005)

Preparation noncontextuality

$$\forall \mathsf{M} : p(X|\mathsf{P},\mathsf{M}) = p(X|\mathsf{P}',\mathsf{M}) \longrightarrow p(\lambda|\mathsf{P}) = p(\lambda|\mathsf{P}')$$

In quantum theory $\quad p(\lambda|\mathsf{P}) = p(\lambda|\rho)$


Generalized noncontextuality
RWS, Phys. Rev. A 71, 052108 (2005)
Preparation noncontextuality
$\forall \mathsf{M} : p(X|\mathsf{P},\mathsf{M}) = p(X|\mathsf{P}',\mathsf{M}) \longrightarrow p(\lambda|\mathsf{P}) = p(\lambda|\mathsf{P}')$
In quantum theory $p(\lambda|\mathsf{P}) = p(\lambda|\rho)$


Generalized noncontextuality
RWS, Phys. Rev. A 71, 052108 (2005)
Preparation noncontextuality
$\forall \mathsf{M} : p(X|\mathsf{P},\mathsf{M}) = p(X|\mathsf{P}',\mathsf{M}) \longrightarrow p(\lambda|\mathsf{P}) = p(\lambda|\mathsf{P}')$
In quantum theory $p(\lambda|\mathsf{P}) = p(\lambda|\rho)$
Measurement noncontextuality
$\forall \mathsf{P} : p(X|\mathsf{P},\mathsf{M}) = p(X|\mathsf{P},\mathsf{M}') \longrightarrow p(X|\lambda,\mathsf{M}) = p(X|\lambda,\mathsf{M}')$
In quantum theory $P(X|\lambda,\mathsf{M}) = P(X|\lambda,\{E_X\})$

Generalized noncontextuality

RWS, Phys. Rev. A 71, 052108 (2005)

Preparation noncontextuality

$$\forall \mathsf{M} : p(X|\mathsf{P},\mathsf{M}) = p(X|\mathsf{P}',\mathsf{M}) \longrightarrow p(\lambda|\mathsf{P}) = p(\lambda|\mathsf{P}')$$

In quantum theory $p(\lambda|\mathsf{P}) = p(\lambda|\rho)$

Measurement noncontextuality

$$\forall \mathsf{P} : p(X|\mathsf{P},\mathsf{M}) = p(X|\mathsf{P},\mathsf{M}') \longrightarrow p(X|\lambda,\mathsf{M}) = p(X|\lambda,\mathsf{M}')$$

In quantum theory $P(X|\lambda,\mathsf{M}) = P(X|\lambda,\{E_X\})$

Generalized noncontextuality

RWS, Phys. Rev. A 71, 052108 (2005)

Preparation noncontextuality

$$\forall \mathsf{M} : p(X|\mathsf{P},\mathsf{M}) = p(X|\mathsf{P}',\mathsf{M}) \longrightarrow p(\lambda|\mathsf{P}) = p(\lambda|\mathsf{P}')$$

In quantum theory $p(\lambda|\mathsf{P}) = p(\lambda|\rho)$

Measurement noncontextuality

$$\forall \mathsf{P} : p(X|\mathsf{P},\mathsf{M}) = p(X|\mathsf{P},\mathsf{M}') \longrightarrow p(X|\lambda,\mathsf{M}) = p(X|\lambda,\mathsf{M}')$$

In quantum theory $P(X|\lambda,\mathsf{M}) = P(X|\lambda,\{E_X\})$

A universally noncontextual model does not exist (modulo loopholes)

ψ-ontic vs. ψ-epistemic ontological models

ψ-ontic model:

For all preparation procedures

$P_{|\psi_1\rangle}$, $P_{|\psi_2\rangle}$ with $|\psi_1\rangle \neq |\psi_2\rangle$

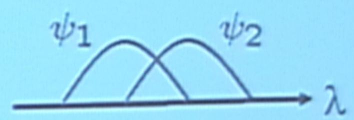
$P(\lambda|P_{|\psi_1\rangle})P(\lambda|P_{|\psi_2\rangle}) = 0$ for all λ

ψ-epistemic model:

Not ψ-ontic

$\exists |\psi_1\rangle \neq |\psi_2\rangle$

$P(\lambda|P_{|\psi_1\rangle})P(\lambda|P_{|\psi_2\rangle}) \neq 0$ for some λ

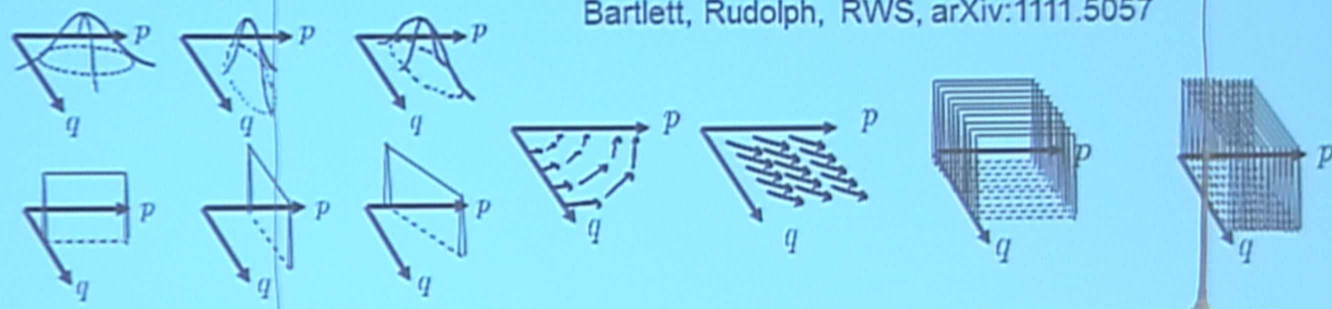


See Harrigan and RWS, Found. Phys. 40, 125 (2010)

Subtheories of QT with compelling ψ-epistemic models

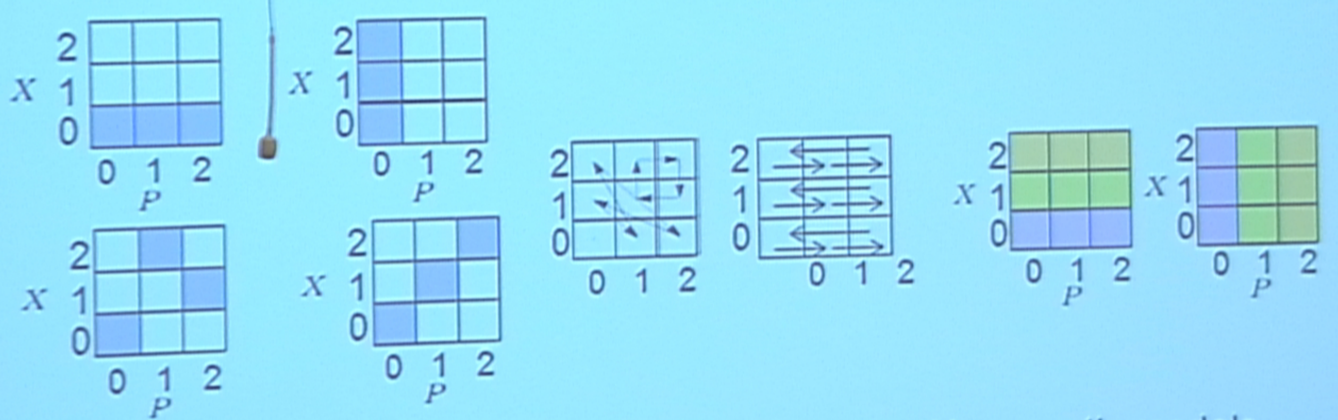
Gaussian quantum mechanics / linear quantum optics

Bartlett, Rudolph, RWS, arXiv:1111.5057



Stabilizer theory of qutrits

Schreiber, RWS, http://pirsa.org/09080009/.



These uphold principles of classical physics violated by ψ-ontic models

Question: Can we find ψ-epistemic ontological models of the *full* quantum theory? (First asked by Lucien Hardy)

Question: Can we find ψ-epistemic ontological models of the *full* quantum theory? (First asked by Lucien Hardy)

Answer: Yes!

Barrett, Hardy, RWS, unpublished 2006

Lewis, Jennings, Barrett, Rudolph, arXiv:1201.6554

These models are… *unappealing*

Are there interesting assumptions (criteria of appealingness) under which ψ-epistemic models are ruled out?

Are there any such assumptions that don't *also* rule out the ψ-ontic models?

Some interpretive options for the devoted realist

ψ-ontic
realist interpretations

deBroglie-Bohm
Everett
Collapse theories

ψ-epistemic
realist interpretations

Adhering to the standard ontological model framework w/ contextuality and nonlocality *hardwired* into the theory

e.g. Lewis, Jennings, Barrett & Rudolph, arXiv:1201.6554

Rejecting some implicit assumption in the standard ontological model framework and salvaging the *spirit* of noncontextuality and locality

Some interpretive options for the devoted realist

ψ-ontic
realist interpretations

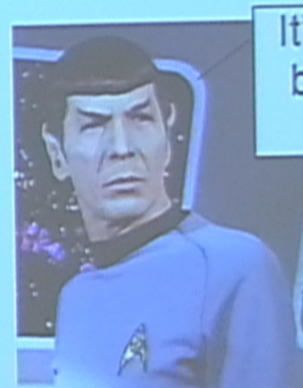
deBroglie-Bohm
Everett
Collapse theories

ψ-epistemic
realist interpretations

Adhering to the standard ontological model framework w/ contextuality and nonlocality *hardwired* into the theory

e.g. Lewis, Jennings, Barrett & Rudolph, arXiv:1201.6554

Rejecting some implicit assumption in the standard ontological model framework and salvaging the *spirit* of noncontextuality and locality



It's reality, Jim, but not as we know it

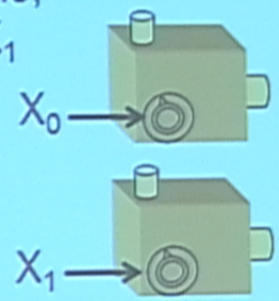
Pusey Barrett Rudolph theorem

Pusey, Barrett, Rudolph, arXiv:1111.3328

Consider only the basic version (for mutually unbiased states)

Four preparations,
labelled by X_0, X_1

$|0^A\rangle|0^B\rangle$
$|0^A\rangle|+^B\rangle$
$|+^A\rangle|0^B\rangle$
$|+^A\rangle|+^B\rangle$



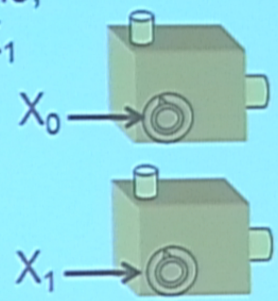
Pusey Barrett Rudolph theorem

Pusey, Barrett, Rudolph, arXiv:1111.3328

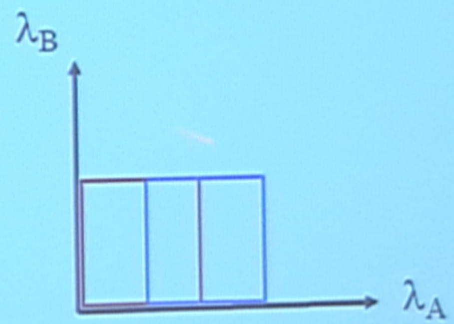
Consider only the basic version (for mutually unbiased states)

Four preparations,
labelled by X_0, X_1

$|0^A\rangle|0^B\rangle$
$|0^A\rangle|+^B\rangle$
$|+^A\rangle|0^B\rangle$
$|+^A\rangle|+^B\rangle$

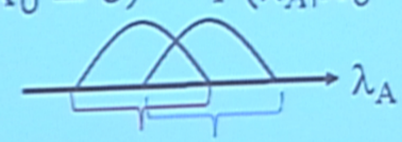


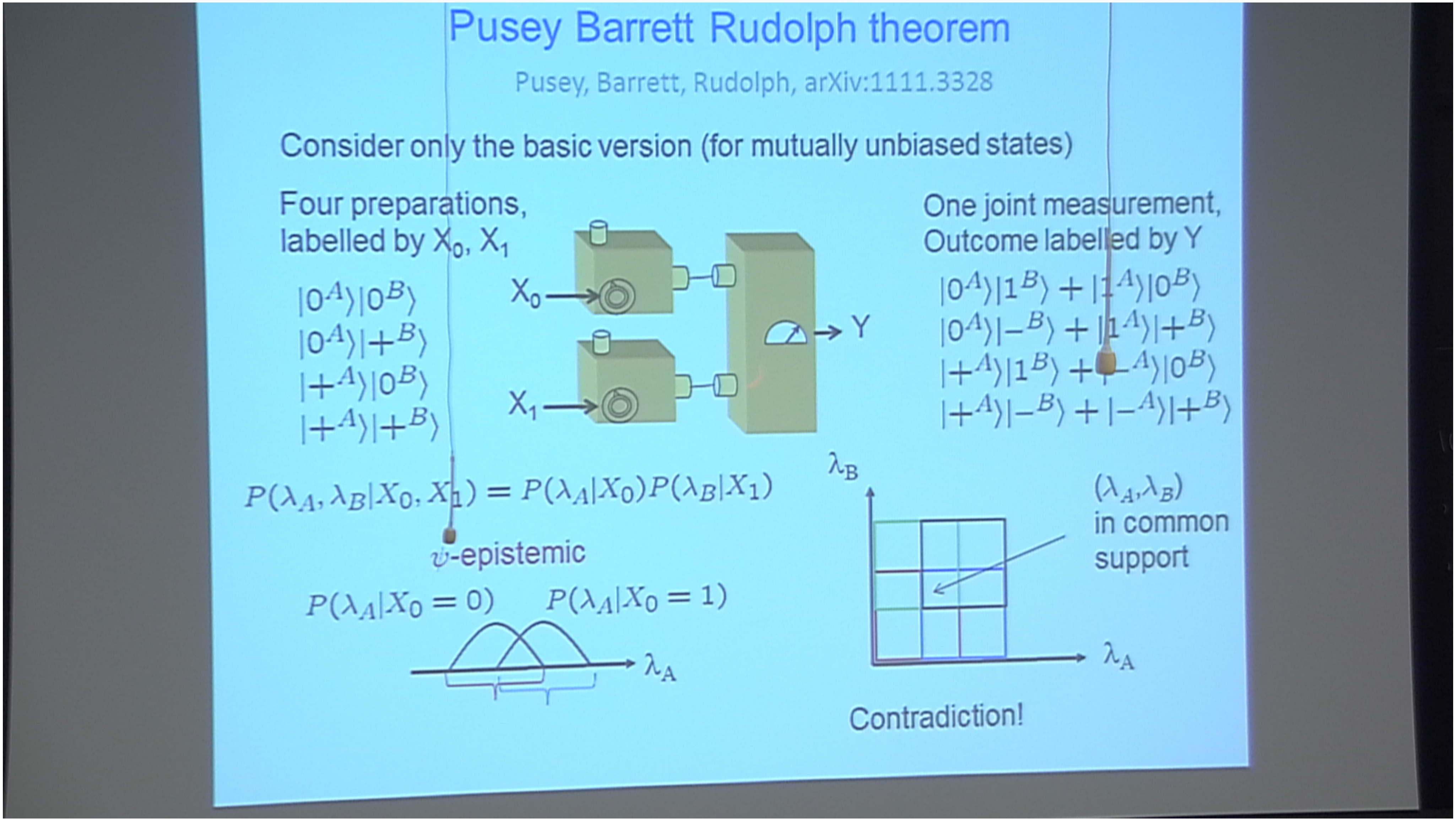
$$P(\lambda_A, \lambda_B|X_0, X_1) = P(\lambda_A|X_0)P(\lambda_B|X_1)$$

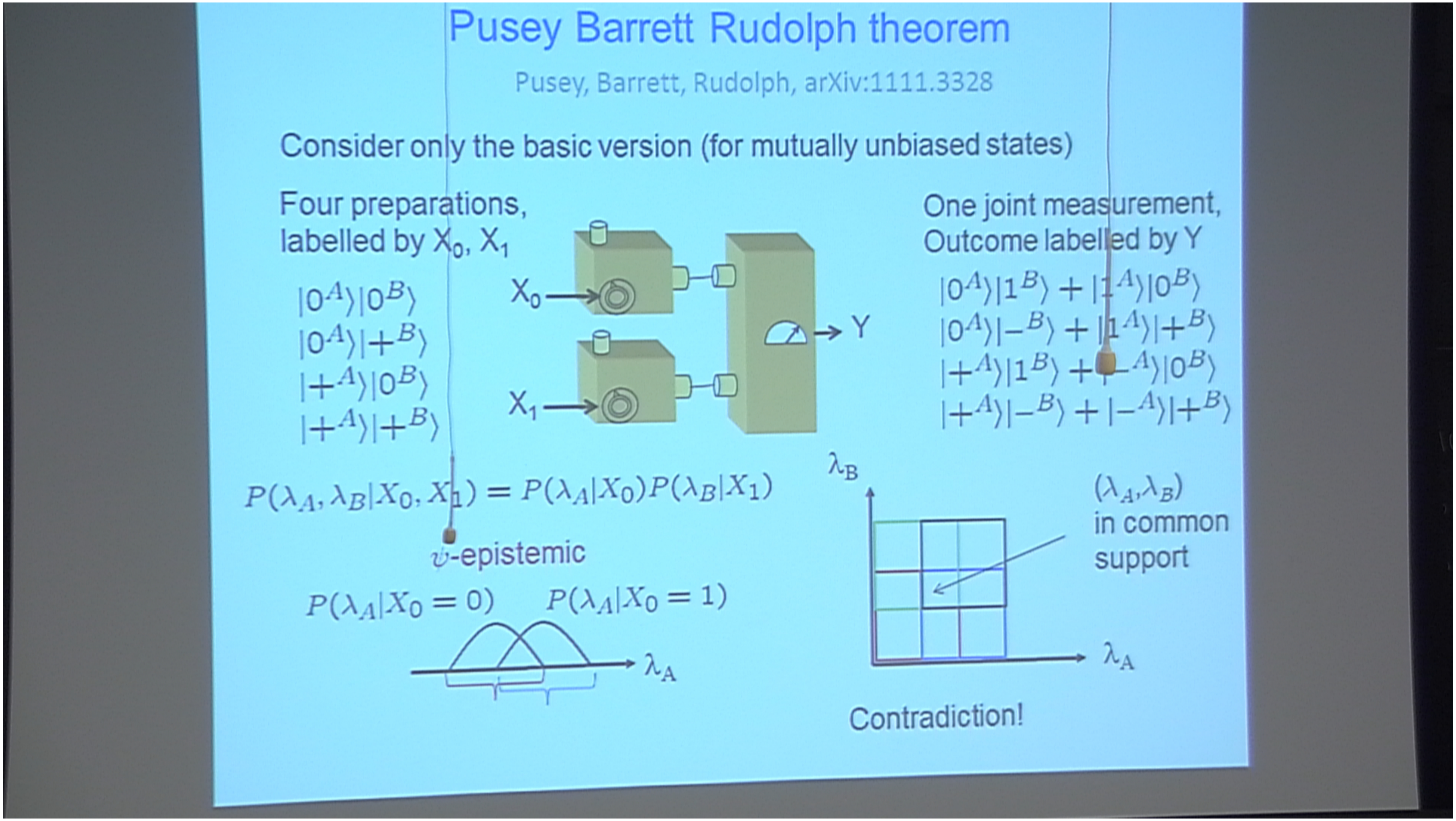


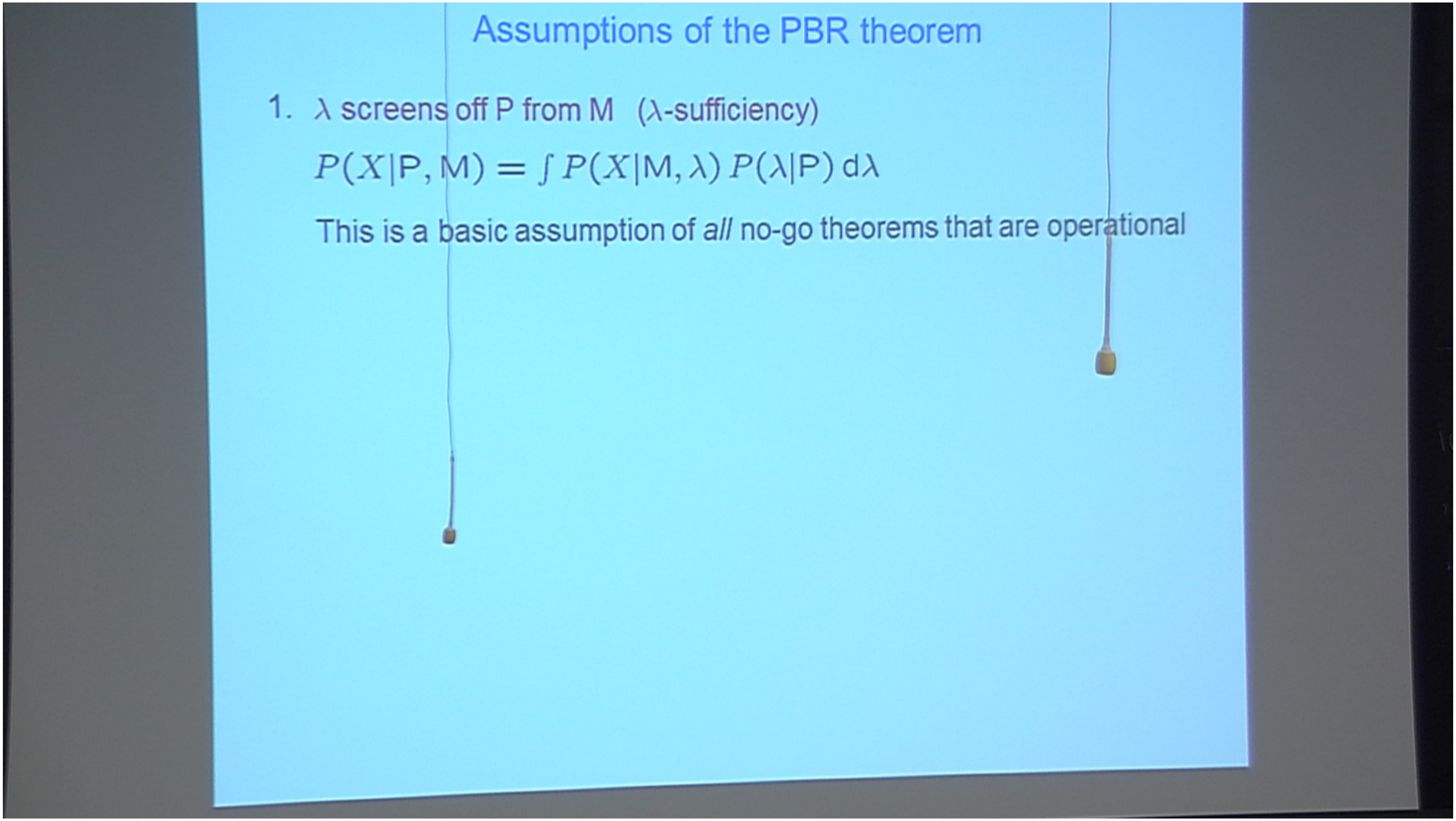
ψ-epistemic

$P(\lambda_A|X_0 = 0)$ $P(\lambda_A|X_0 = 1)$

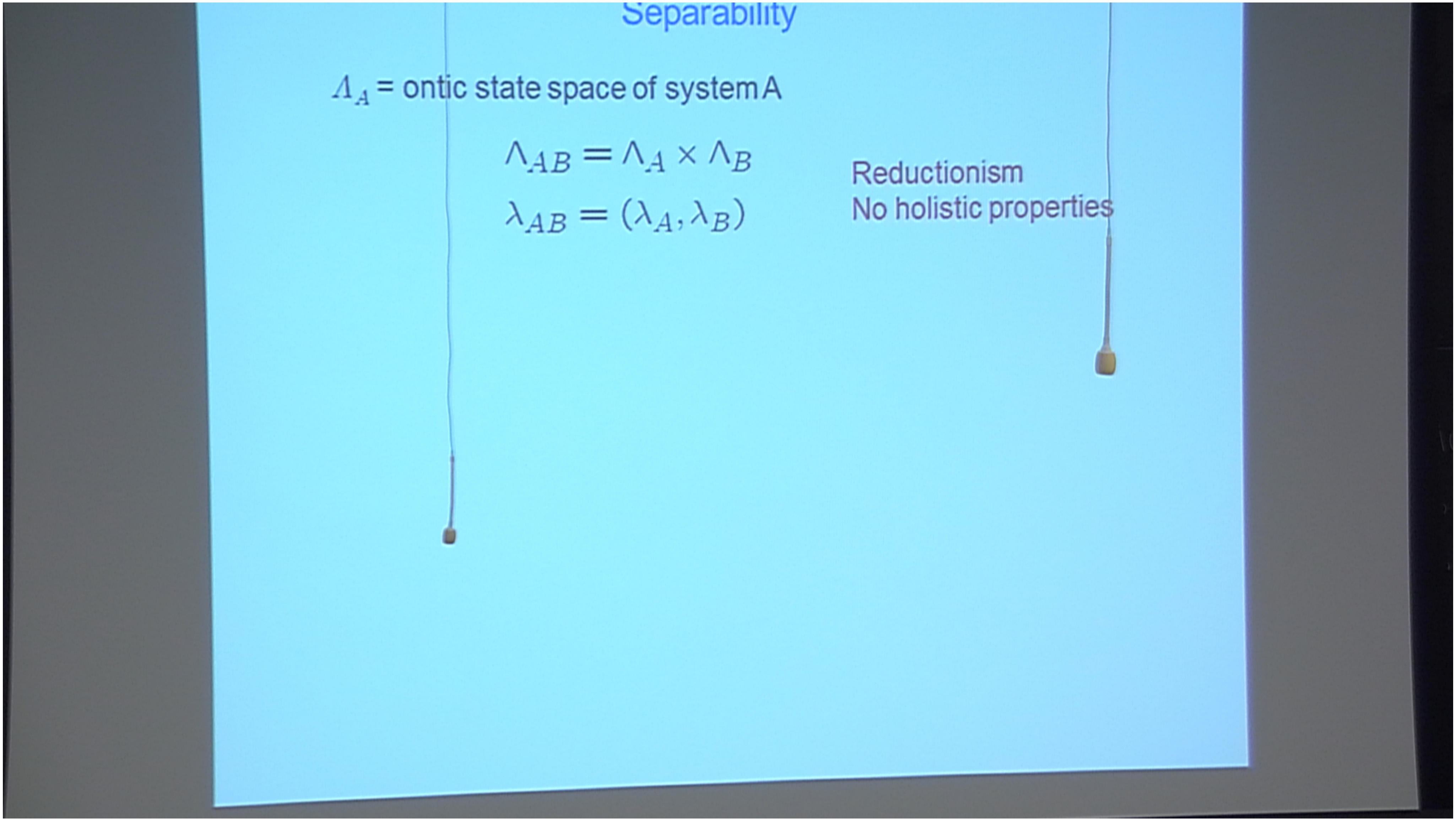



Pusey Barrett Rudolph theorem
Pusey, Barrett, Rudolph, arXiv:1111.3328
Consider only the basic version (for mutually unbiased states)
Four preparations, labelled by X_0, X_1
$|0^A\rangle|0^B\rangle$
$|0^A\rangle|+^B\rangle$
$|+^A\rangle|0^B\rangle$
$|+^A\rangle|+^B\rangle$
X_0
X_1
Y
One joint measurement, Outcome labelled by Y
$|0^A\rangle|1^B\rangle + |1^A\rangle|0^B\rangle$
$|0^A\rangle|-^B\rangle + |1^A\rangle|+^B\rangle$
$|+^A\rangle|1^B\rangle + |-^A\rangle|0^B\rangle$
$|+^A\rangle|-^B\rangle + |-^A\rangle|+^B\rangle$
$P(\lambda_A, \lambda_B|X_0, X_1) = P(\lambda_A|X_0)P(\lambda_B|X_1)$
ψ-epistemic
$P(\lambda_A|X_0 = 0)$
$P(\lambda_A|X_0 = 1)$
λ_A
λ_B
(λ_A, λ_B) in common support
λ_A
Contradiction!


Pusey Barrett Rudolph theorem
Pusey, Barrett, Rudolph, arXiv:1111.3328
Consider only the basic version (for mutually unbiased states)
Four preparations, labelled by X_0, X_1
$|0^A\rangle|0^B\rangle$
$|0^A\rangle|+^B\rangle$
$|+^A\rangle|0^B\rangle$
$|+^A\rangle|+^B\rangle$
X_0
X_1
Y
One joint measurement, Outcome labelled by Y
$|0^A\rangle|1^B\rangle + |1^A\rangle|0^B\rangle$
$|0^A\rangle|-^B\rangle + |1^A\rangle|+^B\rangle$
$|+^A\rangle|1^B\rangle + |-^A\rangle|0^B\rangle$
$|+^A\rangle|-^B\rangle + |-^A\rangle|+^B\rangle$
$P(\lambda_A, \lambda_B|X_0, X_1) = P(\lambda_A|X_0)P(\lambda_B|X_1)$
ψ-epistemic
$P(\lambda_A|X_0 = 0)$
$P(\lambda_A|X_0 = 1)$
λ_A
λ_B
(λ_A, λ_B) in common support
λ_A
Contradiction!



Assumptions of the PBR theorem
1. λ screens off P from M (λ-sufficiency)
$P(X|\mathrm{P},\mathrm{M}) = \int P(X|\mathrm{M},\lambda)\, P(\lambda|\mathrm{P})\, \mathrm{d}\lambda$
This is a basic assumption of *all* no-go theorems that are operational


Separability
Λ_A = ontic state space of system A
$\Lambda_{AB} = \Lambda_A \times \Lambda_B$
$\lambda_{AB} = (\lambda_A, \lambda_B)$
Reductionism
No holistic properties

Separability

Λ_A = ontic state space of system A

$$\Lambda_{AB} = \Lambda_A \times \Lambda_B$$

$$\lambda_{AB} = (\lambda_A, \lambda_B)$$

Reductionism
No holistic properties

Separability

Λ_A = ontic state space of system A

$$\Lambda_{AB} = \Lambda_A \times \Lambda_B$$

$$\lambda_{AB} = (\lambda_A, \lambda_B)$$

Reductionism
No holistic properties

$\mathcal{L}(\Lambda_A)$ = space in which the epistemic states live

$$\mathcal{L}(\Lambda) \equiv \{f : \Lambda \rightarrow R^+\}$$

$$\mathcal{L}(\Lambda_{AB}) = \mathcal{L}(\Lambda_A) \otimes \mathcal{L}(\Lambda_B)$$

$$P(\lambda_A, \lambda_B) = \sum_i w_i P_i(\lambda_A) P_i(\lambda_b)$$

Separability

Λ_A = ontic state space of system A

$$\Lambda_{AB} = \Lambda_A \times \Lambda_B$$

$$\lambda_{AB} = (\lambda_A, \lambda_B)$$

Reductionism
No holistic properties

$\mathcal{L}(\Lambda_A)$ = space in which the epistemic states live

$$\mathcal{L}(\Lambda) \equiv \{f : \Lambda \rightarrow R^+\}$$

$$\mathcal{L}(\Lambda_{AB}) = \mathcal{L}(\Lambda_A) \otimes \mathcal{L}(\Lambda_B)$$

$$P(\lambda_A, \lambda_B) = \sum_i w_i P_i(\lambda_A) P_i(\lambda_b)$$

Assumptions of the PBR theorem

1. λ screens off P from M (λ-sufficiency)

   $P(X|\mathrm{P},\mathrm{M}) = \int P(X|\mathrm{M},\lambda)\, P(\lambda|\mathrm{P})\, \mathrm{d}\lambda$

   This is a basic assumption of *all* no-go theorems (to my knowledge)

2. Separability in ontic support of product states (SeparabilityPS)

   $|\phi^A_{X_0}\rangle|\phi^B_{X_1}\rangle \leftrightarrow P(\lambda_{AB}|X_0, X_1)$

   $\forall\lambda_{AB} : P(\lambda_{AB}|X_0, X_1) > 0 \quad \lambda_{AB} = (\lambda_A, \lambda_B)$

3. Product quantum states represented by product dist'ns (FactorizationPS)

   $P(\lambda_A, \lambda_B|X_0, X_1) = P(\lambda_A|X_0)P(\lambda_B|X_1)$

λ-sufficiency $\wedge$ SeparabilityPS $\wedge$ FactorizationPS $\wedge$ ψ-epistemic
→ contradiction

How a small difference in the assumptions can make a big difference in the conclusions

Replace SeparabilityPS by Separability

<u>ψ-ontic models</u>:

$$\lambda_{AB} = (\psi_{AB}, \omega_{AB})$$

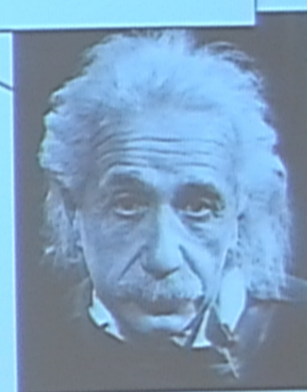
$$\mathcal{H}_{AB} = \mathcal{H}_A \otimes \mathcal{H}_B$$

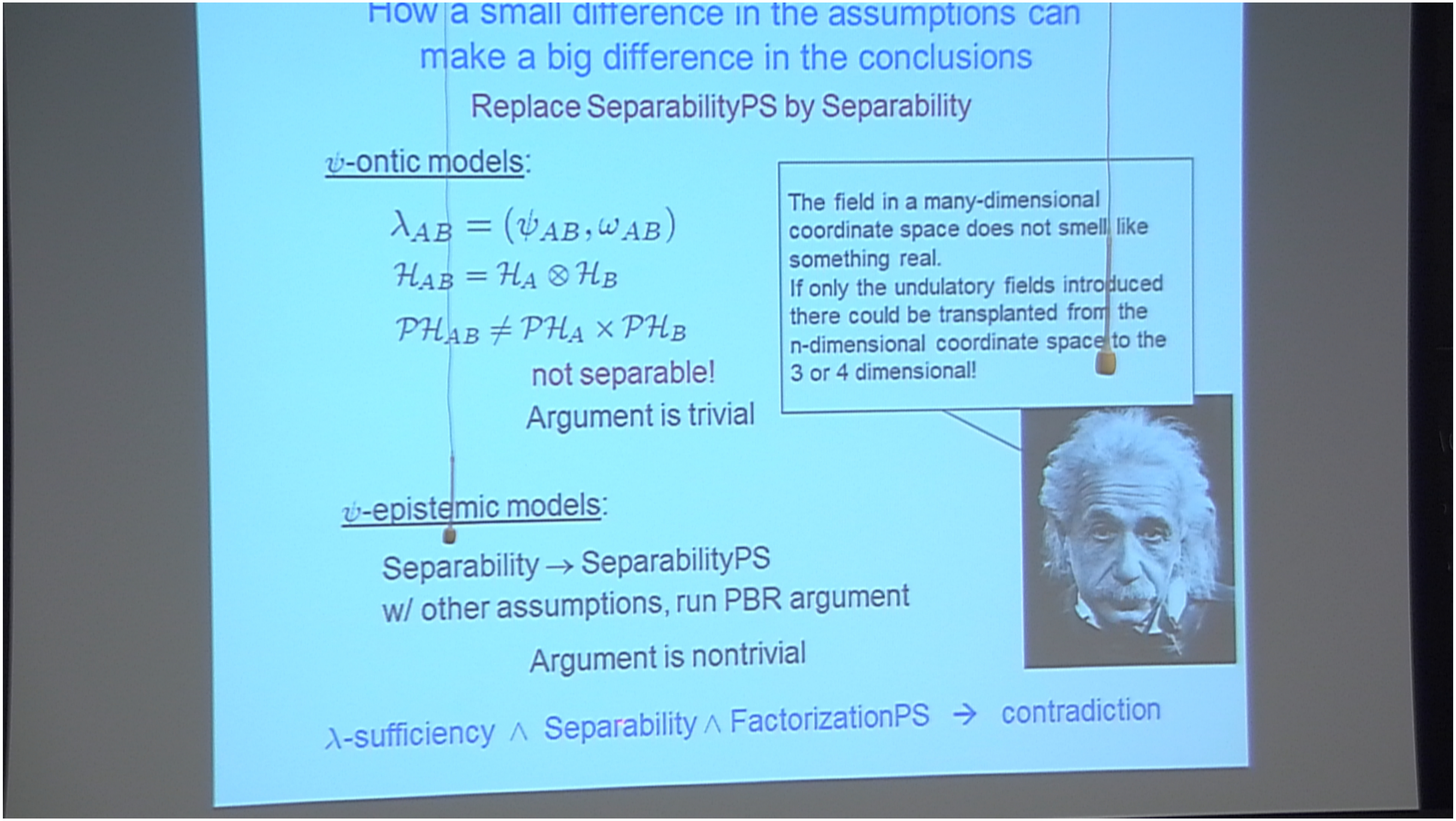
$$\mathcal{P}\mathcal{H}_{AB} \neq \mathcal{P}\mathcal{H}_A \times \mathcal{P}\mathcal{H}_B$$

not separable!

Argument is trivial

The field in a many-dimensional coordinate space does not smell like something real.
If only the undulatory fields introduced there could be transplanted from the n-dimensional coordinate space to the 3 or 4 dimensional!




How a small difference in the assumptions can make a big difference in the conclusions
Replace SeparabilityPS by Separability
ψ-ontic models:
$\lambda_{AB} = (\psi_{AB}, \omega_{AB})$
$\mathcal{H}_{AB} = \mathcal{H}_A \otimes \mathcal{H}_B$
$\mathcal{PH}_{AB} \neq \mathcal{PH}_A \times \mathcal{PH}_B$
not separable!
Argument is trivial
The field in a many-dimensional coordinate space does not smell like something real.
If only the undulatory fields introduced there could be transplanted from the n-dimensional coordinate space to the 3 or 4 dimensional!
ψ-epistemic models:
Separability → SeparabilityPS
w/ other assumptions, run PBR argument
Argument is nontrivial
λ-sufficiency ∧ Separability ∧ FactorizationPS → contradiction

Separability versus SeparabilityPS

For a ψ-onticist, assuming that entangled states are associated with holistic properties is very natural

But for a ψ-epistemicist, the entangled states are not themselves part of the ontology – they are *merely* an epistemic notion, indicating a kind of mutual information

Separability versus SeparabilityPS

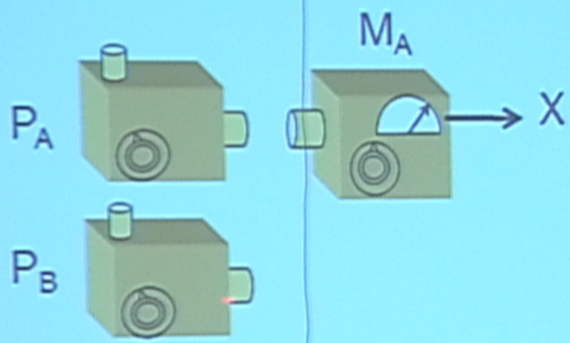
For a ψ-onticist, assuming that entangled states are associated with holistic properties is very natural

But for a ψ-epistemicist, the entangled states are not themselves part of the ontology – they are *merely* an epistemic notion, indicating a kind of mutual information

For the PBR theorem to count as evidence in favour of a ψ-ontic approach, an argument must be provided for why separabilityPS is a natural assumption when one can't have separability itself (without an appeal to ψ-ontic intuitions)

FactorizationPS

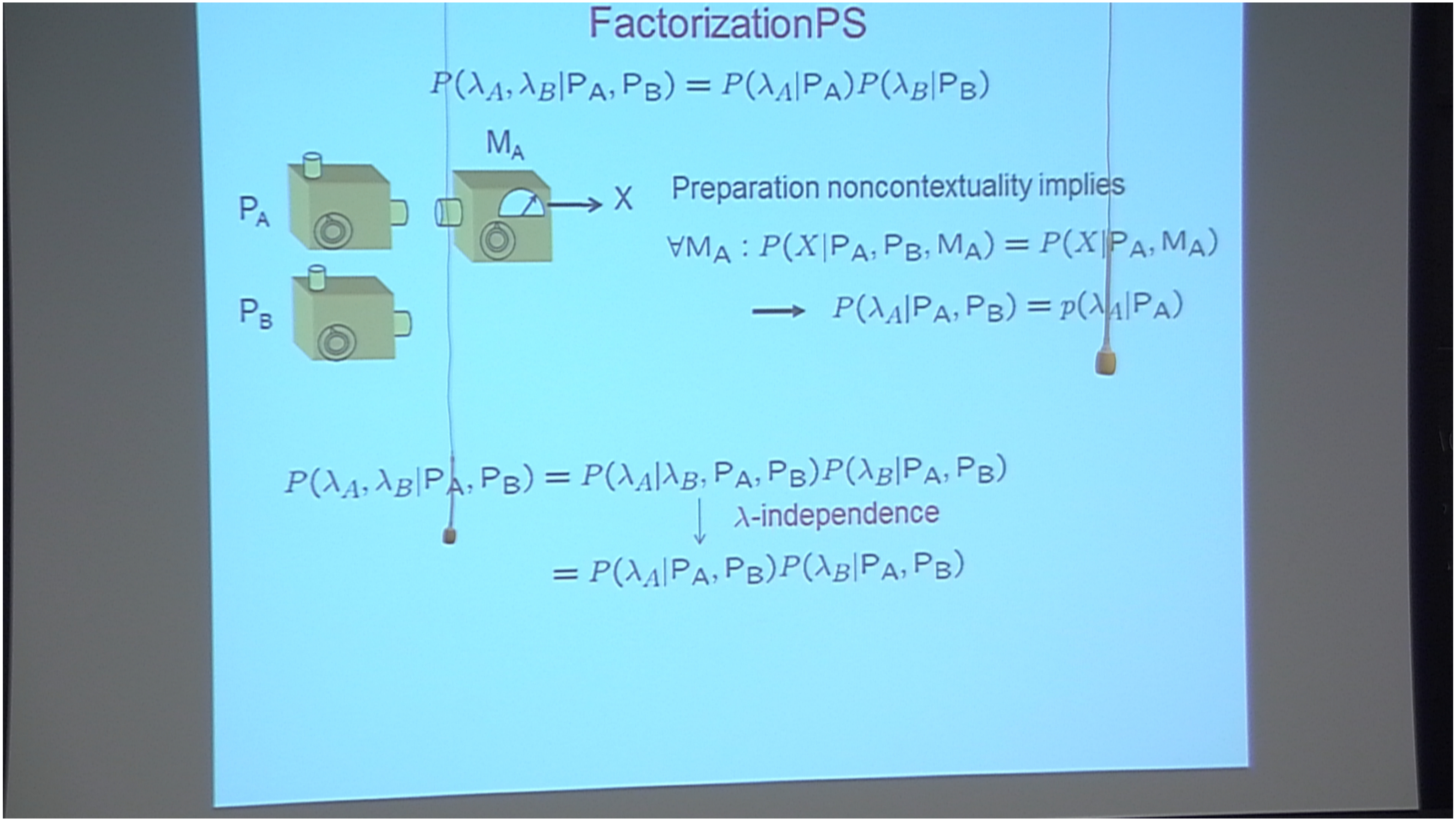
$$P(\lambda_A, \lambda_B | \mathsf{P_A}, \mathsf{P_B}) = P(\lambda_A | \mathsf{P_A}) P(\lambda_B | \mathsf{P_B})$$



Preparation noncontextuality implies

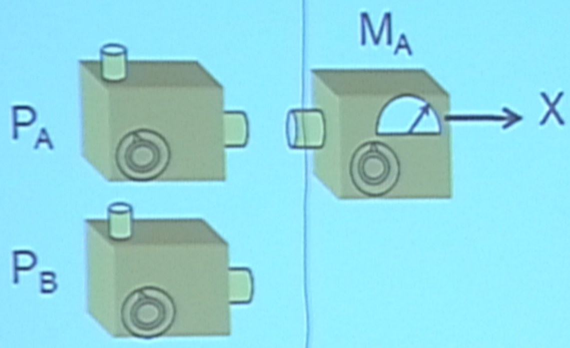
$$\forall \mathsf{M_A} : P(X | \mathsf{P_A}, \mathsf{P_B}, \mathsf{M_A}) = P(X | \mathsf{P_A}, \mathsf{M_A})$$

$$\longrightarrow \quad P(\lambda_A | \mathsf{P_A}, \mathsf{P_B}) = p(\lambda_A | \mathsf{P_A})$$


FactorizationPS
$P(\lambda_A, \lambda_B | \mathsf{P_A}, \mathsf{P_B}) = P(\lambda_A | \mathsf{P_A}) P(\lambda_B | \mathsf{P_B})$
$\mathsf{M_A}$
$\mathsf{P_A}$
X
$\mathsf{P_B}$
Preparation noncontextuality implies
$\forall \mathsf{M_A} : P(X | \mathsf{P_A}, \mathsf{P_B}, \mathsf{M_A}) = P(X | \mathsf{P_A}, \mathsf{M_A})$
$\longrightarrow P(\lambda_A | \mathsf{P_A}, \mathsf{P_B}) = p(\lambda_A | \mathsf{P_A})$
$P(\lambda_A, \lambda_B | \mathsf{P_A}, \mathsf{P_B}) = P(\lambda_A | \lambda_B, \mathsf{P_A}, \mathsf{P_B}) P(\lambda_B | \mathsf{P_A}, \mathsf{P_B})$
$\downarrow$ λ-independence
$= P(\lambda_A | \mathsf{P_A}, \mathsf{P_B}) P(\lambda_B | \mathsf{P_A}, \mathsf{P_B})$

FactorizationPS

$$P(\lambda_A, \lambda_B | \mathsf{P_A}, \mathsf{P_B}) = P(\lambda_A | \mathsf{P_A}) P(\lambda_B | \mathsf{P_B})$$



Preparation noncontextuality implies

$$\forall \mathsf{M_A} : P(X | \mathsf{P_A}, \mathsf{P_B}, \mathsf{M_A}) = P(X | \mathsf{P_A}, \mathsf{M_A})$$

$$\longrightarrow \quad P(\lambda_A | \mathsf{P_A}, \mathsf{P_B}) = p(\lambda_A | \mathsf{P_A})$$

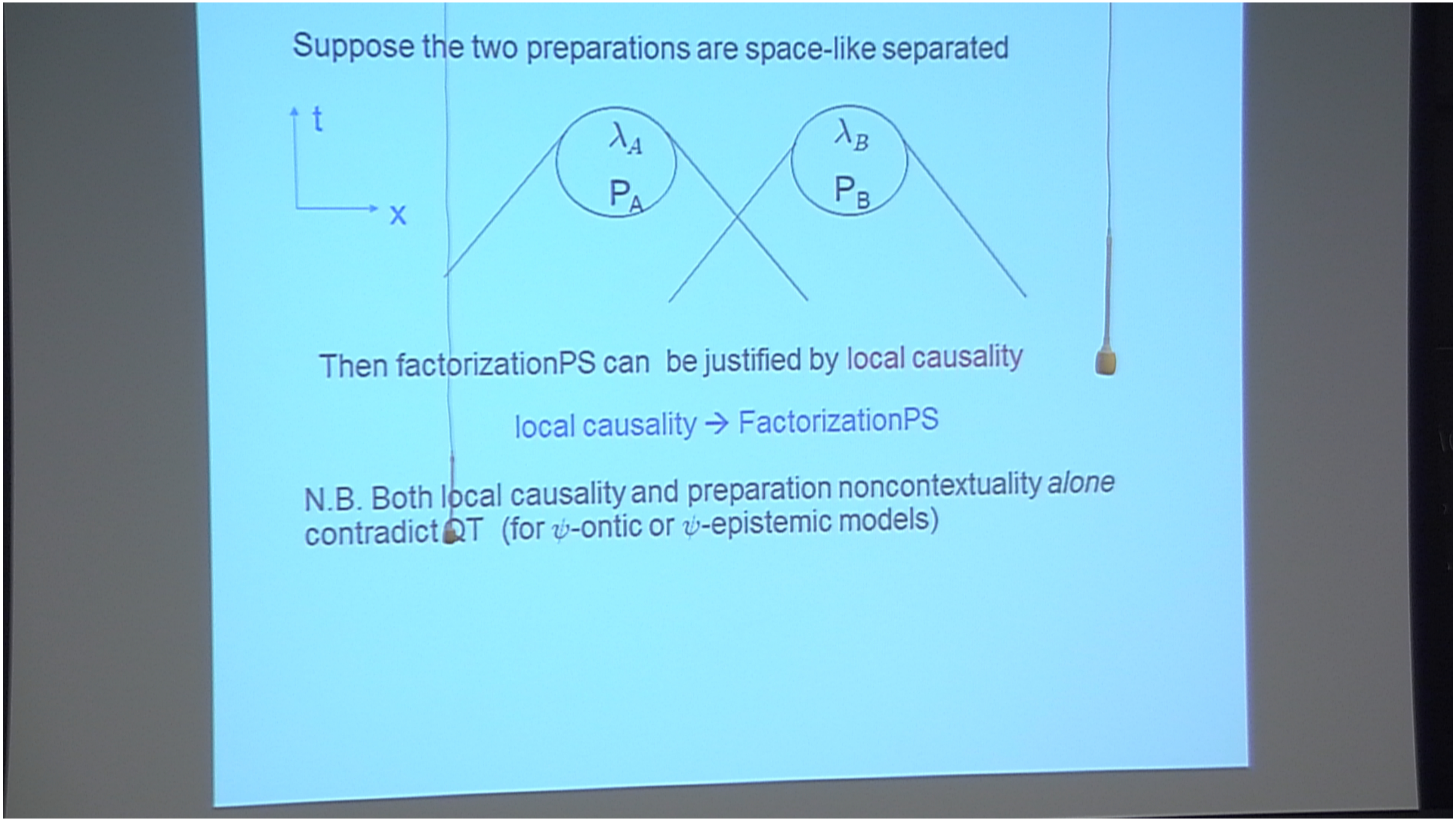
$$P(\lambda_A, \lambda_B | \mathsf{P_A}, \mathsf{P_B}) = P(\lambda_A | \lambda_B, \mathsf{P_A}, \mathsf{P_B}) P(\lambda_B | \mathsf{P_A}, \mathsf{P_B})$$

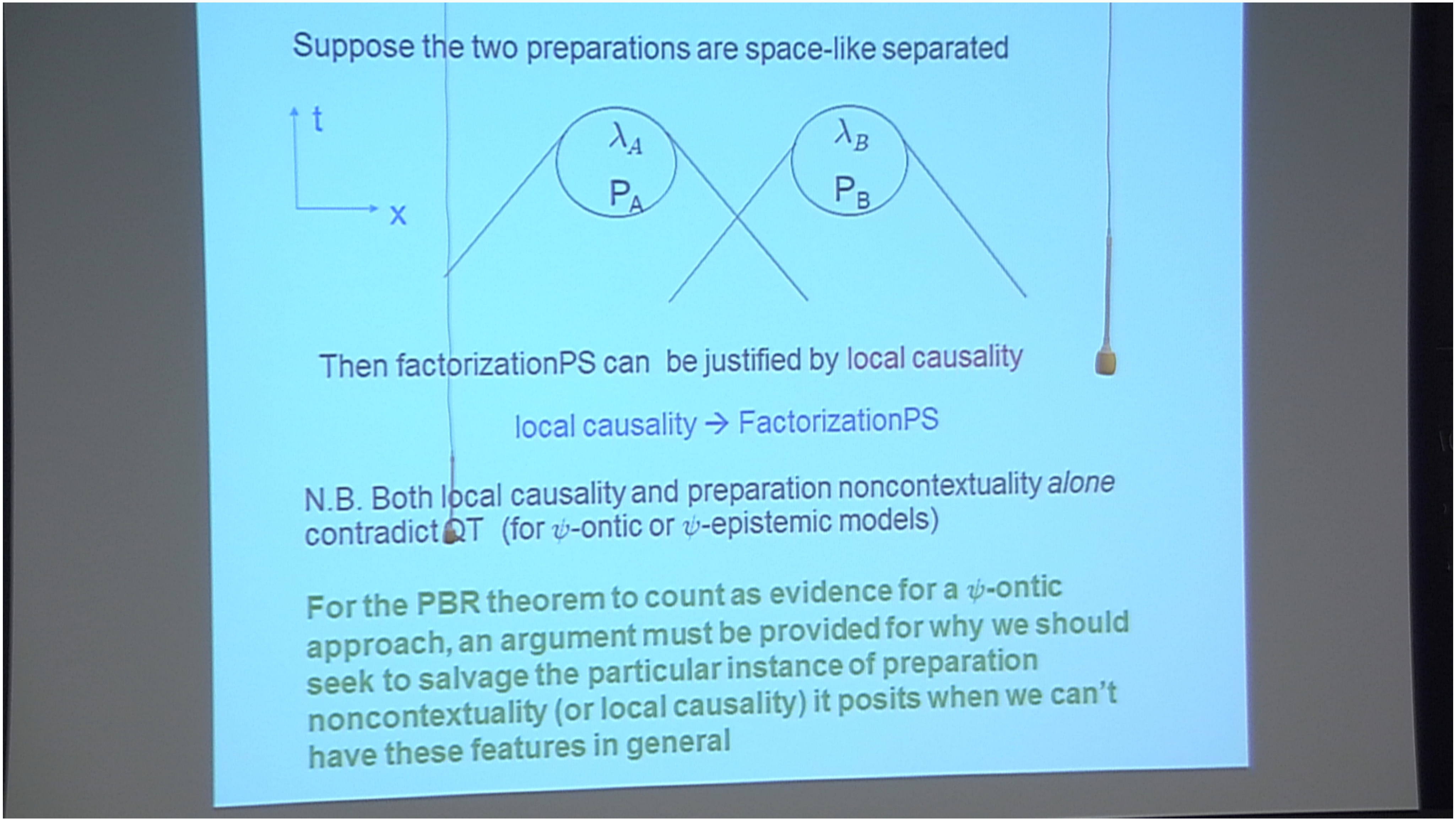
$\downarrow$ λ-independence

$$= P(\lambda_A | \mathsf{P_A}, \mathsf{P_B}) P(\lambda_B | \mathsf{P_A}, \mathsf{P_B})$$

$\downarrow \quad \downarrow$ PS preparation noncontextuality

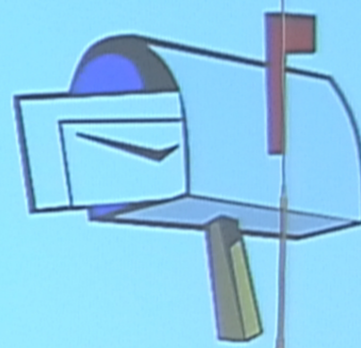
$$= P(\lambda_A | \mathsf{P_A}) P(\lambda_B | \mathsf{P_B})$$


Suppose the two preparations are space-like separated
t
x
λ_A
P_A
λ_B
P_B
Then factorizationPS can be justified by local causality
local causality → FactorizationPS
N.B. Both local causality and preparation noncontextuality alone contradict QT (for ψ-ontic or ψ-epistemic models)

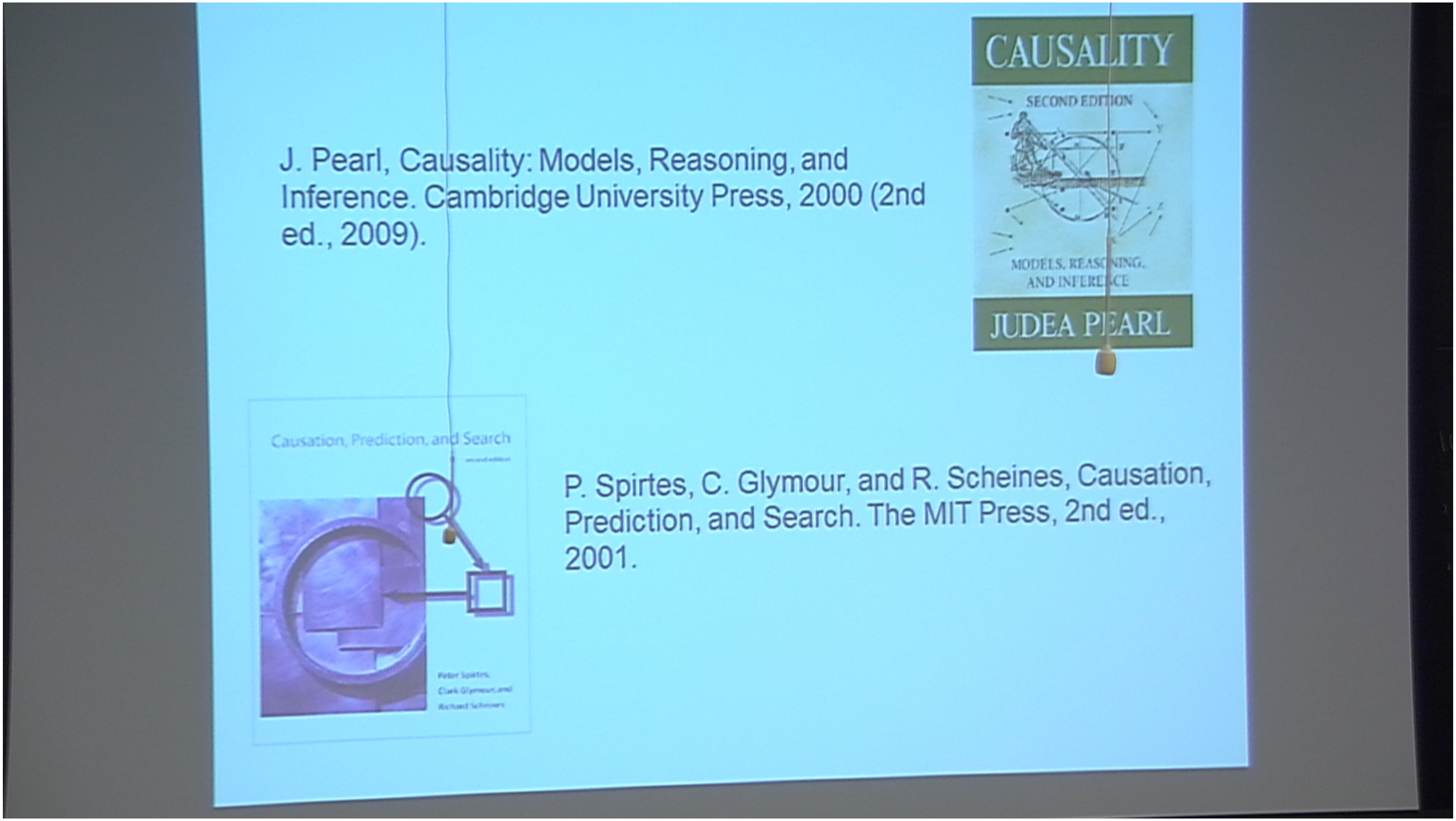

Suppose the two preparations are space-like separated
t
x
λA
PA
λB
PB
Then factorizationPS can be justified by local causality
local causality → FactorizationPS
N.B. Both local causality and preparation noncontextuality alone contradict QT (for ψ-ontic or ψ-epistemic models)
For the PBR theorem to count as evidence for a ψ-ontic approach, an argument must be provided for why we should seek to salvage the particular instance of preparation noncontextuality (or local causality) it posits when we can't have these features in general

Separability reconsidered

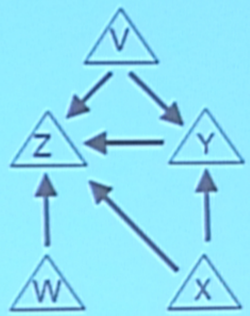
First glance – separability clearly fails for ψ-ontic models



More careful look – it depends on the assumed association of ψ to regions of space-time


J. Pearl, Causality: Models, Reasoning, and Inference. Cambridge University Press, 2000 (2nd ed., 2009).
CAUSALITY
SECOND EDITION
MODELS, REASONING, AND INFERENCE
JUDEA PEARL
Causation, Prediction, and Search
P. Spirtes, C. Glymour, and R. Scheines, Causation, Prediction, and Search. The MIT Press, 2nd ed., 2001.

Causal structure



Causal-statistical parameters

$P(V)$

$P(W)$

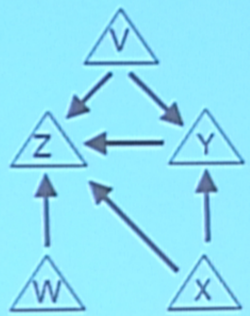
$P(X)$

$Y = f(V, X)$

$Z = g(VWXY)$

Causal model

Causal structure



Causal-statistical parameters

$P(V)$

$P(W)$

$P(X)$

$P(Y|V,X)$

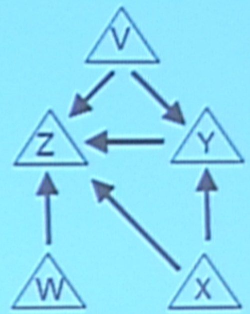
$P(Z|VWXY)$

Causal model

Assumptions:
- Conditionals arise from *autonomous* causal connections
- *Parentless* nodes are uncorrelated

i.e. all correlations must be explained causally

Classical causal models



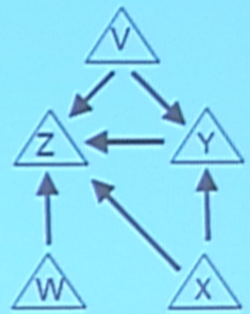
$P(V)$
$P(W)$
$P(X)$
$P(Y|V,X)$
$P(Z|VWXY)$

Def'n: X and Y are conditionally independent given Z

$$P(X|YZ) = P(X|Z)$$
$$P(Y|XZ) = P(Y|Z)$$
$$P(XY|Z) = P(X|Z)P(Y|Z)$$

Denote this
$(X \perp Y|Z)$

Classical causal models



$P(V)$
$P(W)$
$P(X)$
$P(Y|V,X)$
$P(Z|VWXY)$

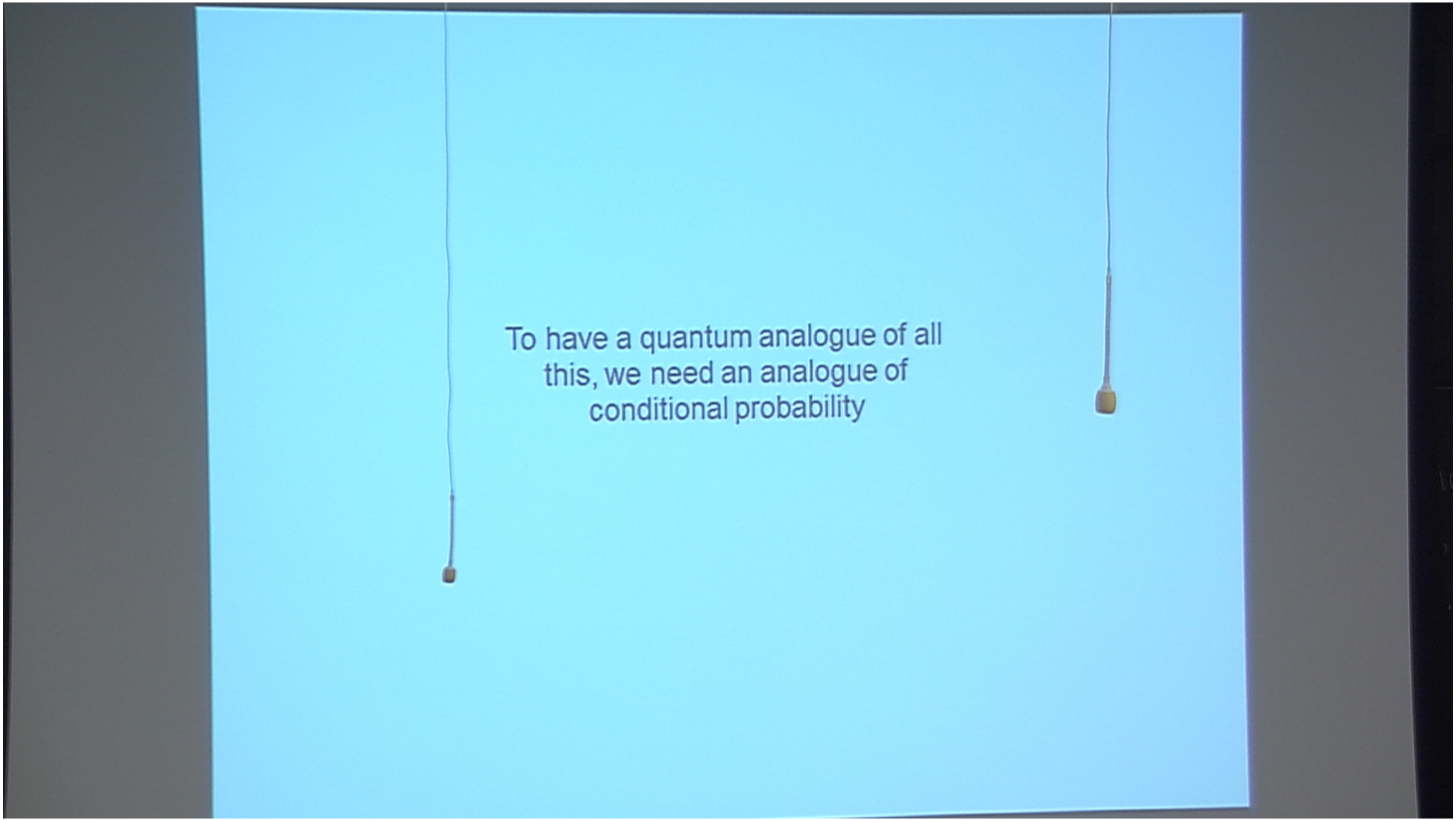
Def'n: X and Y are conditionally independent given Z

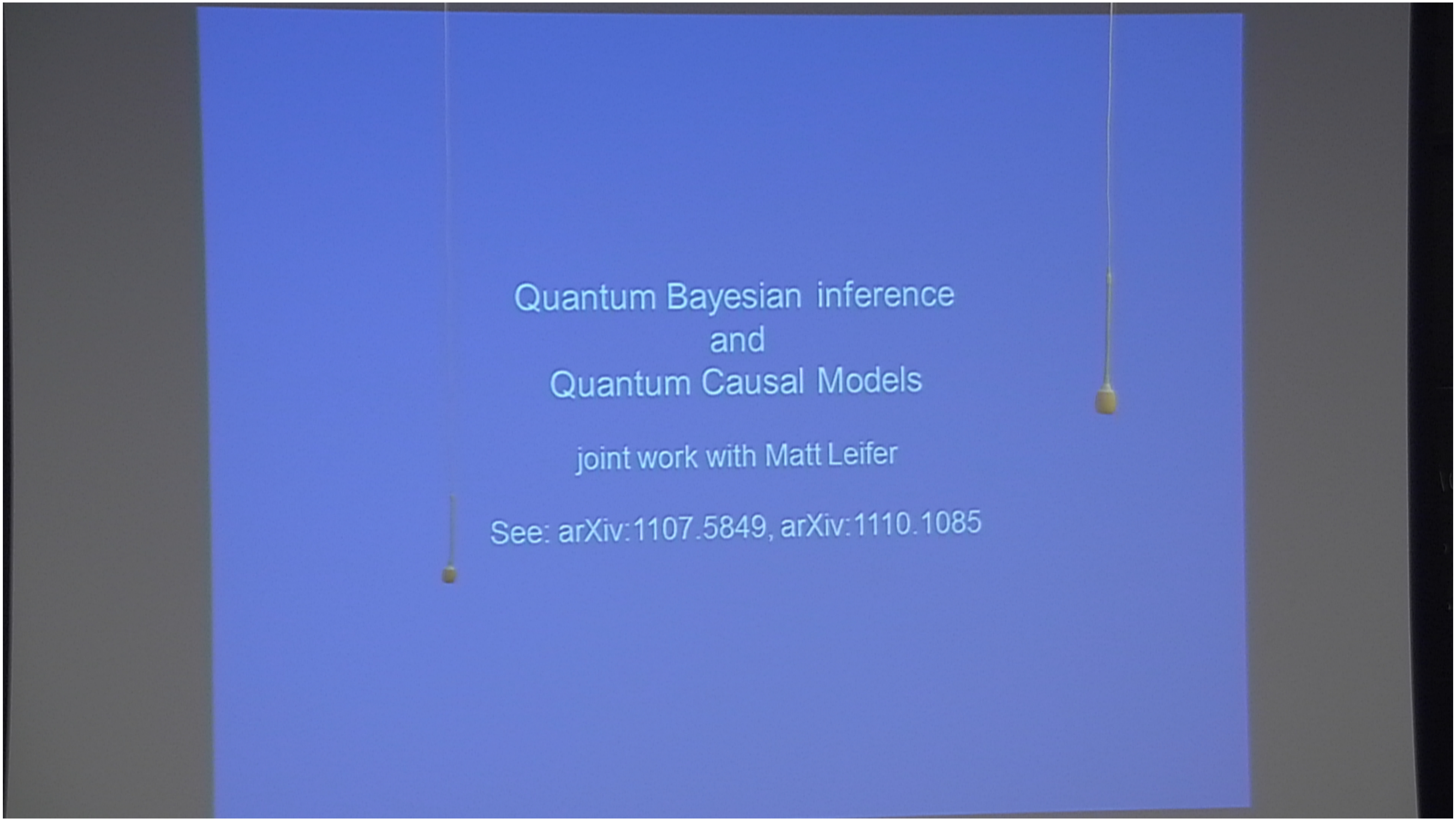
$$P(X|YZ) = P(X|Z)$$
$$P(Y|XZ) = P(Y|Z)$$
$$P(XY|Z) = P(X|Z)P(Y|Z)$$

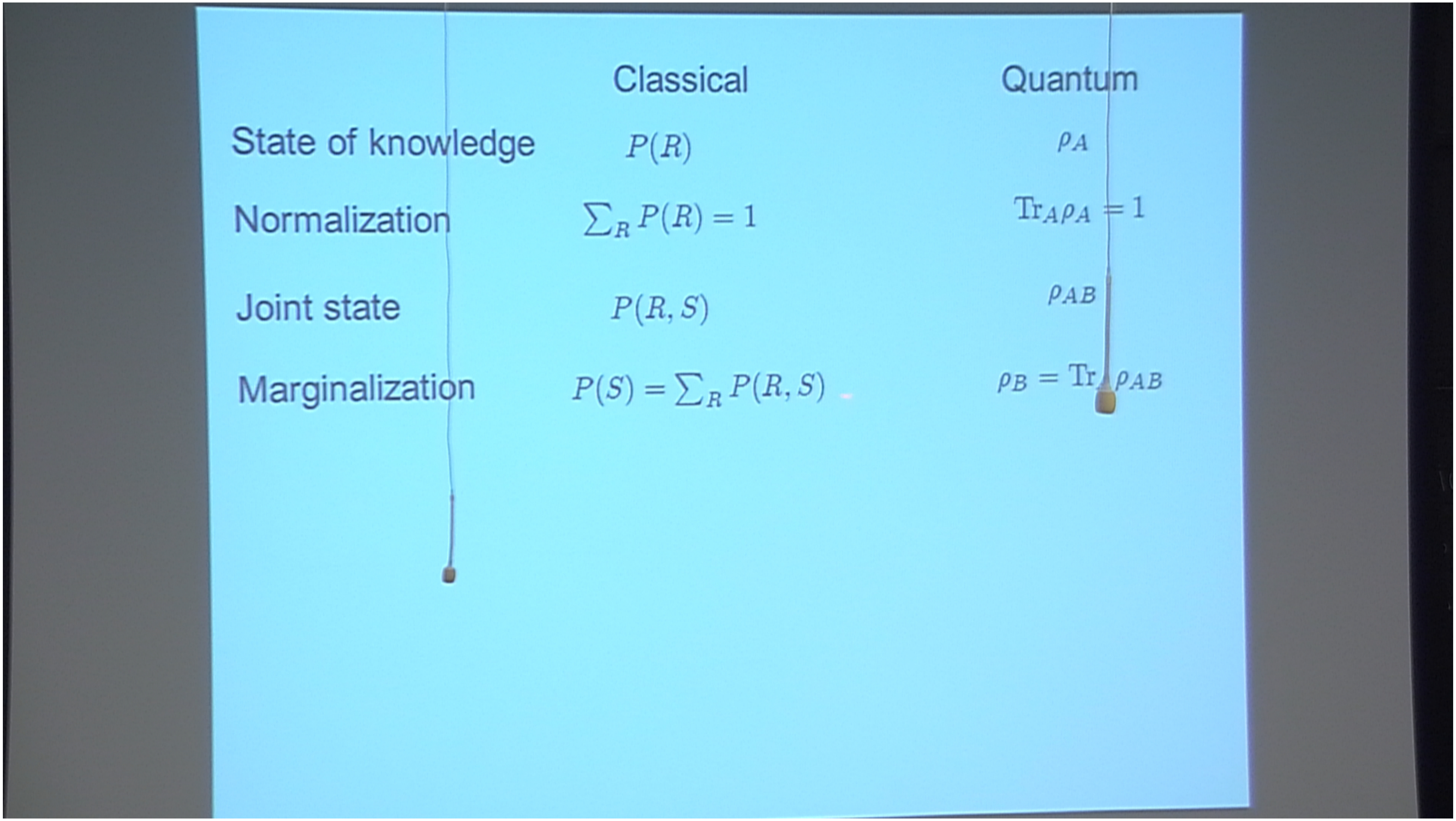
Denote this
$(X \perp Y|Z)$

Markov condition: The joint distribution induced by a causal model is such that every variable X is conditionally independent of its nondescendants given its parents,

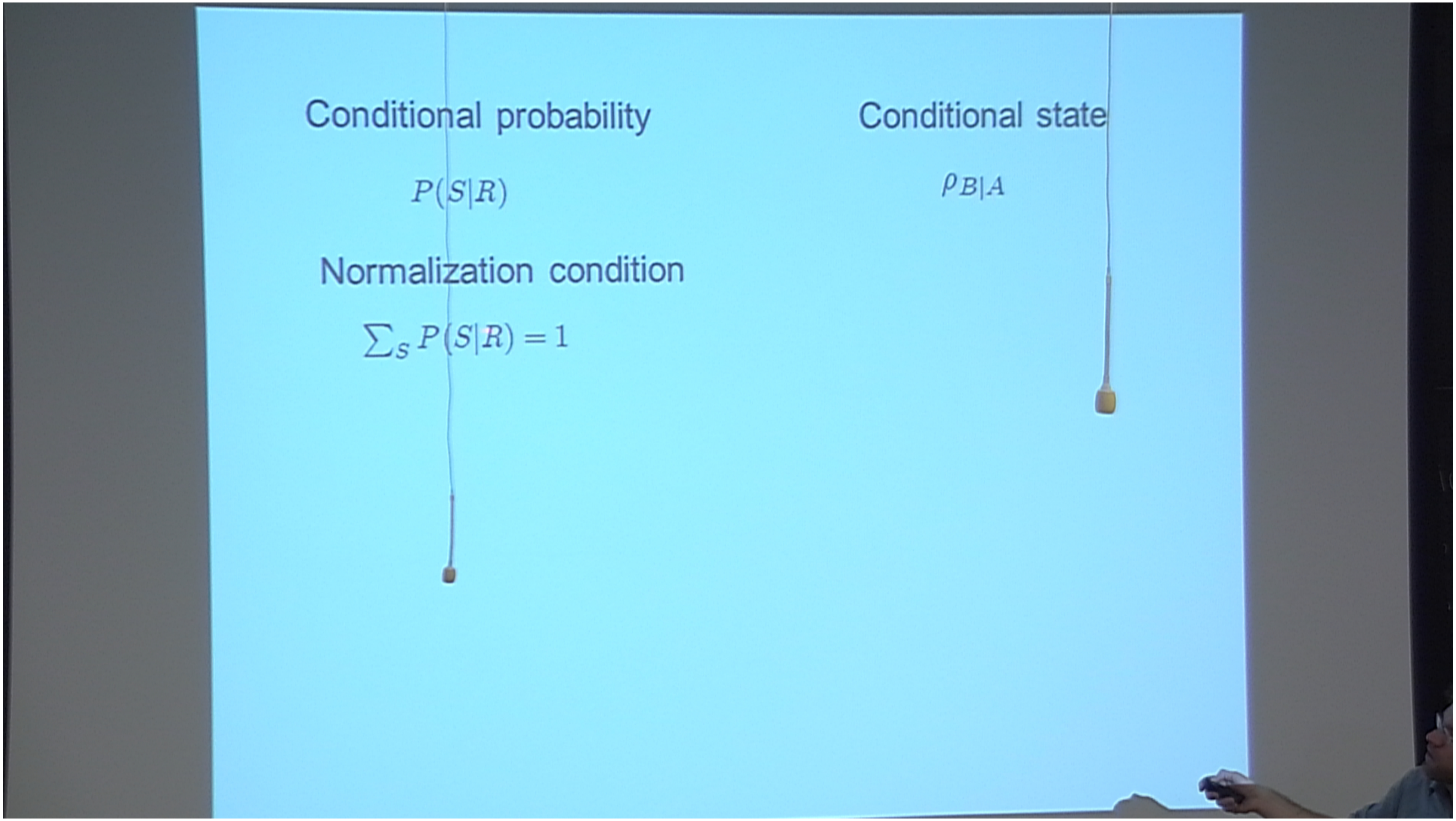
$$(X \perp \text{Nondescendents}(X)|\text{Parents}(X))$$

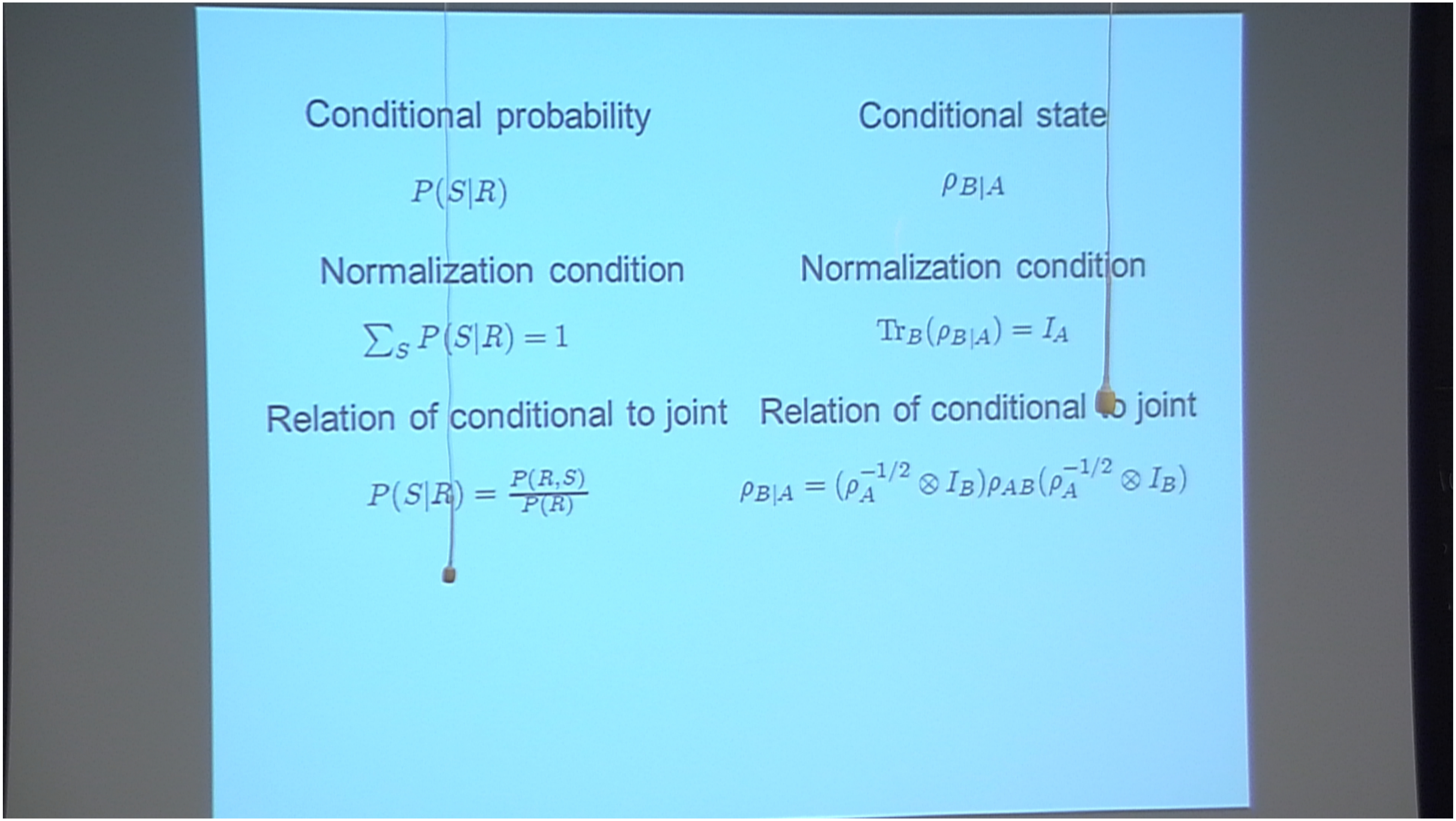

To have a quantum analogue of all
this, we need an analogue of
conditional probability

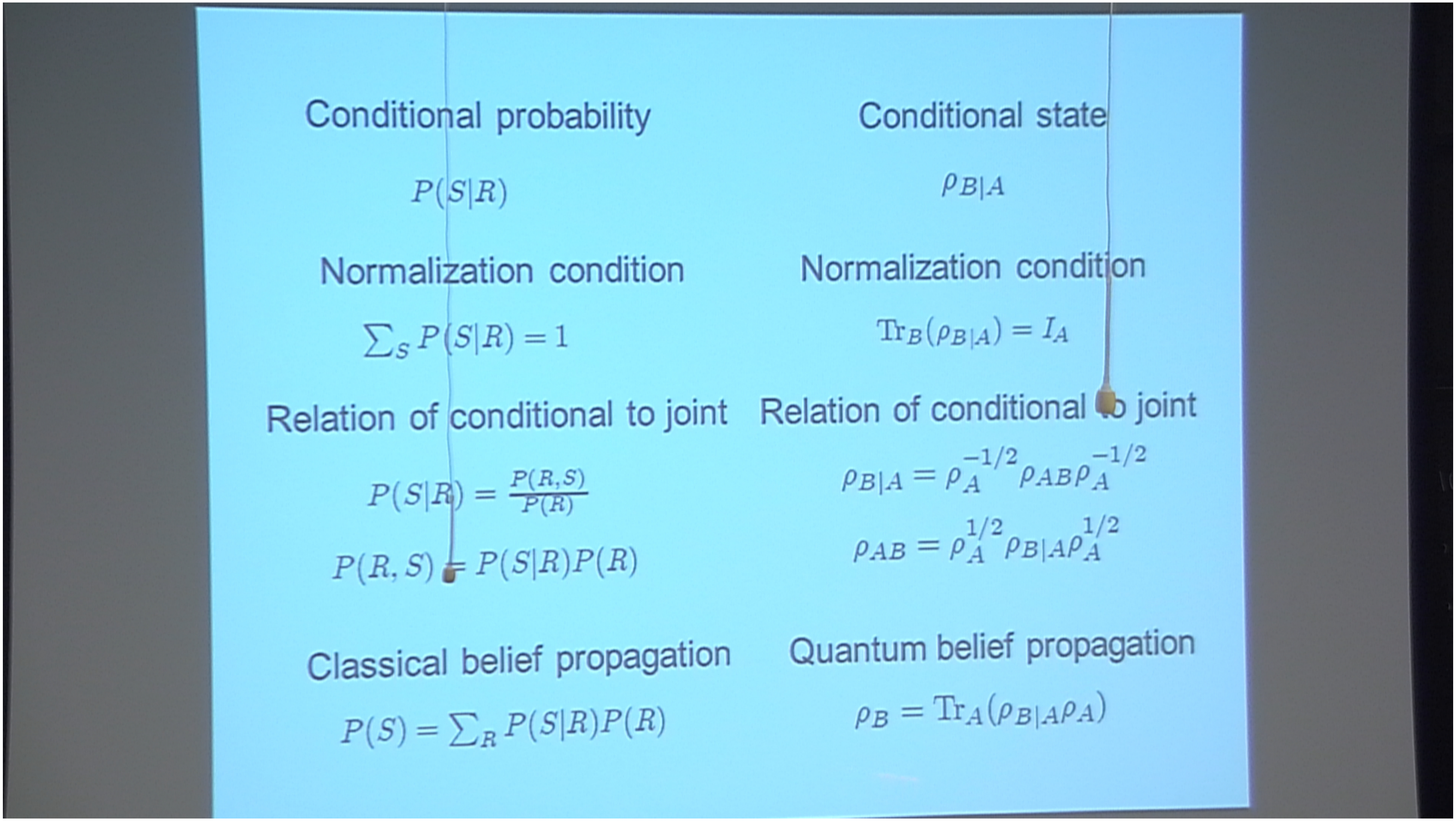

Quantum Bayesian inference
and
Quantum Causal Models
joint work with Matt Leifer
See: arXiv:1107.5849, arXiv:1110.1085



Classical
Quantum
State of knowledge
$P(R)$
ρ_A
Normalization
$\sum_R P(R) = 1$
$\mathrm{Tr}_A \rho_A = 1$
Joint state
$P(R, S)$
ρ_{AB}
Marginalization
$P(S) = \sum_R P(R, S)$
$\rho_B = \mathrm{Tr}_A \rho_{AB}$


Conditional probability
Conditional state
$P(S|R)$
$\rho_{B|A}$
Normalization condition
$\sum_S P(S|R) = 1$


Conditional probability
$P(S|R)$
Normalization condition
$\sum_S P(S|R) = 1$
Relation of conditional to joint
$P(S|R) = \frac{P(R,S)}{P(R)}$
Conditional state
$\rho_{B|A}$
Normalization condition
$\mathrm{Tr}_B(\rho_{B|A}) = I_A$
Relation of conditional to joint
$\rho_{B|A} = (\rho_A^{-1/2} \otimes I_B)\rho_{AB}(\rho_A^{-1/2} \otimes I_B)$


Conditional probability
$P(S|R)$
Normalization condition
$\sum_S P(S|R) = 1$
Relation of conditional to joint
$P(S|R) = \frac{P(R,S)}{P(R)}$
$P(R,S) = P(S|R)P(R)$
Classical belief propagation
$P(S) = \sum_R P(S|R)P(R)$
Conditional state
$\rho_{B|A}$
Normalization condition
$\mathrm{Tr}_B(\rho_{B|A}) = I_A$
Relation of conditional to joint
$\rho_{B|A} = \rho_A^{-1/2}\rho_{AB}\rho_A^{-1/2}$
$\rho_{AB} = \rho_A^{1/2}\rho_{B|A}\rho_A^{1/2}$
Quantum belief propagation
$\rho_B = \mathrm{Tr}_A(\rho_{B|A}\rho_A)$

States for classical systems

$$\rho_X = \sum_x P(X=x)|x\rangle\langle x|_X$$

Classical-given-quantum conditional is associated with a POVM

$$\rho_{Y|A} = \sum_y |y\rangle\langle y|_Y \otimes E_y^A$$

The Born rule: $\rho_Y = \mathrm{Tr}_A(\rho_{Y|A}\rho_A)$

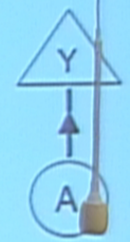
$$\forall y : P(Y=y) = \mathrm{Tr}_A(E_y^A \rho_A)$$

States for classical systems

$$\rho_X = \sum_x P(X=x)|x\rangle\langle x|_X$$

Classical-given-quantum conditional is associated with a POVM

$$\rho_{Y|A} = \sum_y |y\rangle\langle y|_Y \otimes E_y^A$$



The Born rule: $\rho_Y = \mathrm{Tr}_A(\rho_{Y|A}\rho_A)$

$$\forall y : P(Y=y) = \mathrm{Tr}_A(E_y^A \rho_A)$$

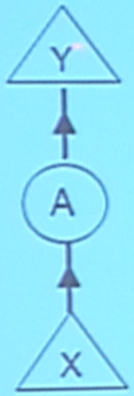
Quantum-given-classical conditional is associated with a set of states

$$\rho_{A|X} = \sum_x |x\rangle\langle x|_X \otimes \rho_x^A$$

Ensemble averaging: $\rho_A = \mathrm{Tr}_X(\rho_{A|X}\rho_X)$

$$\rho_A = \sum_x P(X=x)\rho_x^A$$

Prepare and Measure experiment



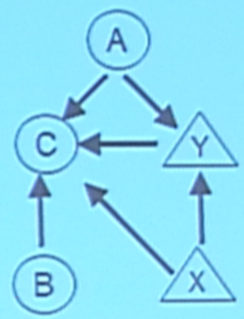
$\rho_{Y|A}$

$\rho_{A|X}$

ρ_X

$$\rho_Y = \mathrm{Tr}_{AX}(\rho_{Y|A}\rho_{A|X}\rho_X)$$

Quantum causal models



ρ_A

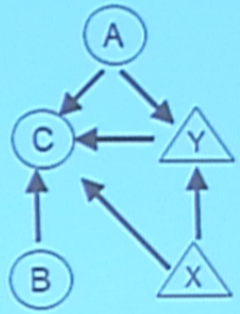
ρ_B

ρ_X

$\rho_{Y|AX}$

$\rho_{C|ABXY}$

Quantum causal models



ρ_A

ρ_B

ρ_X

$\rho_{Y|AX}$

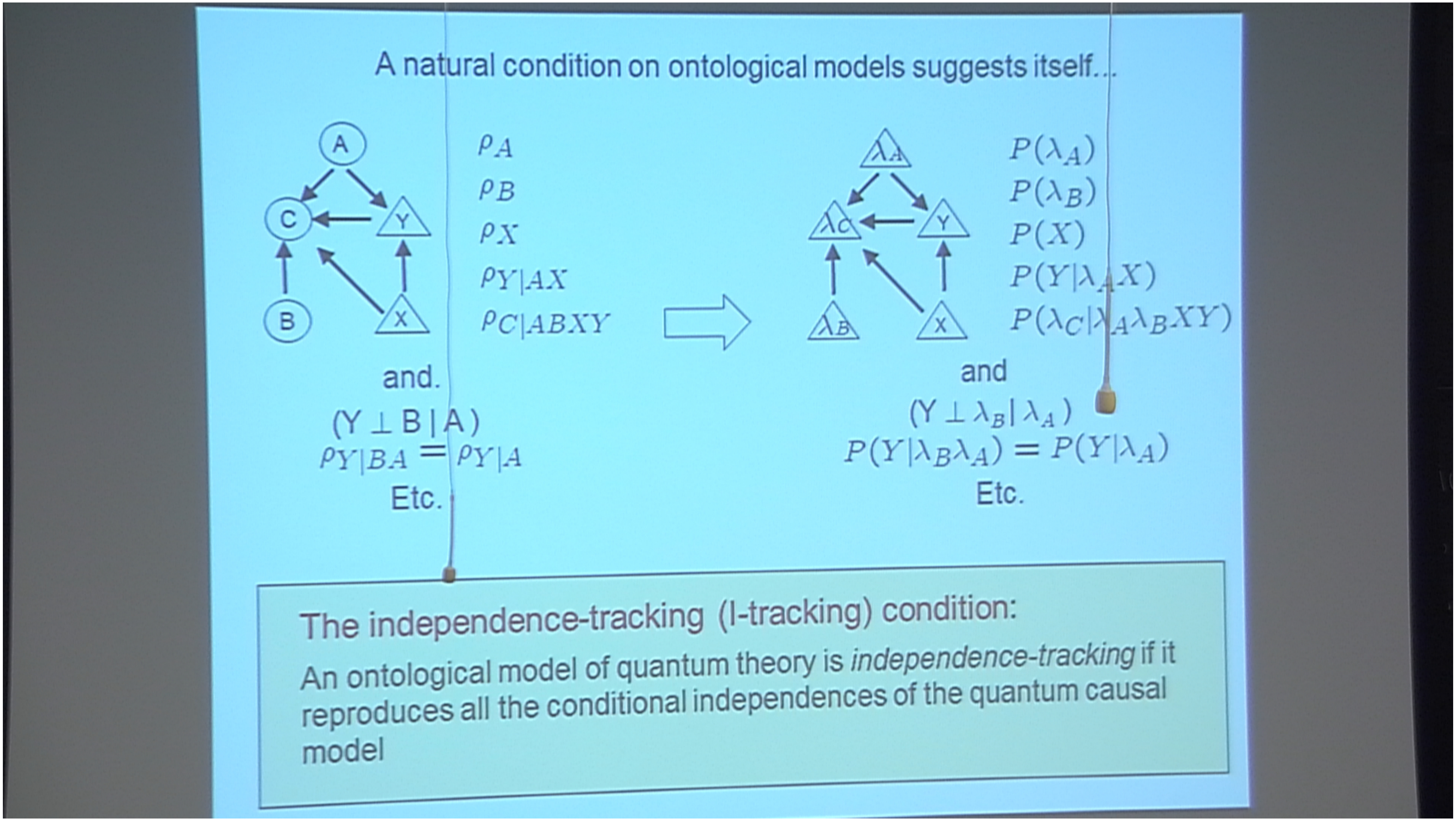
$\rho_{C|ABXY}$

Def'n: A and B are conditionally independent given C

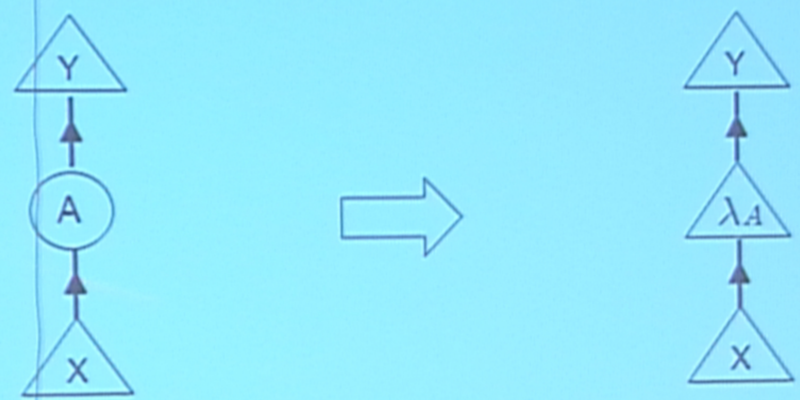
$$\rho_{A|BC} = \rho_{A|C}$$
$$\rho_{B|AC} = \rho_{B|C}$$
$$\rho_{AB|C} = \rho_{A|C}\rho_{B|C}$$

Denote this $(A \perp B|C)$

Actually, it is only this simple if two of the variables are classical but we only consider this case


A natural condition on ontological models suggests itself…
A
B
C
Y
X
ρ_A
ρ_B
ρ_X
$\rho_{Y|AX}$
$\rho_{C|ABXY}$
and
(Y ⊥ B | A)
$\rho_{Y|BA} = \rho_{Y|A}$
Etc.
λ_A
λ_B
λ_C
Y
X
$P(\lambda_A)$
$P(\lambda_B)$
$P(X)$
$P(Y|\lambda_A X)$
$P(\lambda_C|\lambda_A\lambda_B XY)$
and
$(Y \perp \lambda_B | \lambda_A)$
$P(Y|\lambda_B\lambda_A) = P(Y|\lambda_A)$
Etc.
The independence-tracking (I-tracking) condition:
An ontological model of quantum theory is *independence-tracking* if it reproduces all the conditional independences of the quantum causal model

Example #1: λ-sufficiency



$$(Y \perp X | A)$$

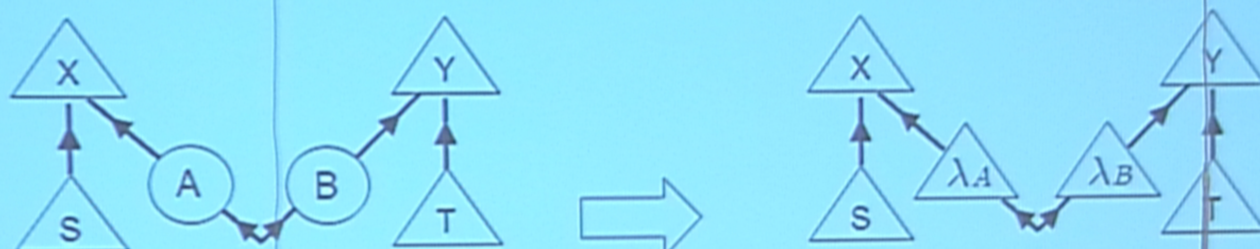
$$\rho_{Y|AX} = \rho_{Y|A}$$

$$(Y \perp X | \lambda_A)$$

$$P(Y|\lambda_A X) = P(Y|\lambda_A)$$

This is λ-sufficiency
A common assumption
of no-go theorems

Example #4: Local causality assuming separability



$$(X \perp YTB|AS)$$
$$(Y \perp XSA|BT)$$

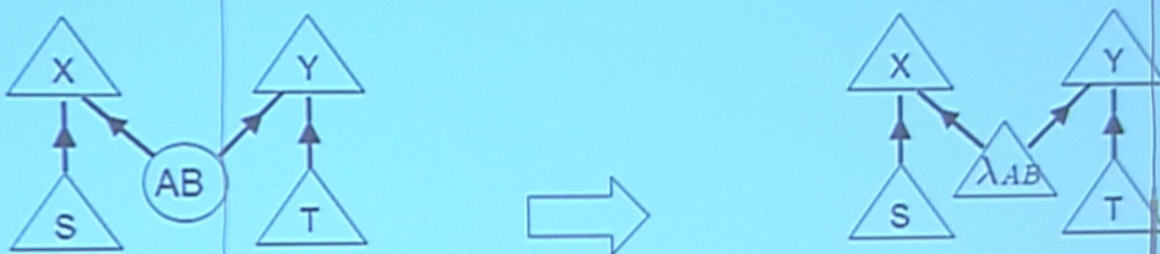
$$\rho_{XY|STAB} = \rho_{X|SA}\rho_{Y|TB}$$
$$E_{xy}^{(st)AB} = (E_x^{(s)A} \otimes I_B)(I_A \otimes E_y^{(t)B})$$

$$(X \perp YT\lambda_B|\lambda_A S)$$
$$(Y \perp XS\lambda_A|\lambda_B T)$$

$$P(XY|ST\lambda_A\lambda_B) = P(X|S\lambda_A)P(Y|T\lambda_B)$$

This is local causality for the case where one assumes separability

Example #5: Local causality *without* assuming separability



$(X \perp YT|(AB)S)$
$(Y \perp XS|(AB)T)$

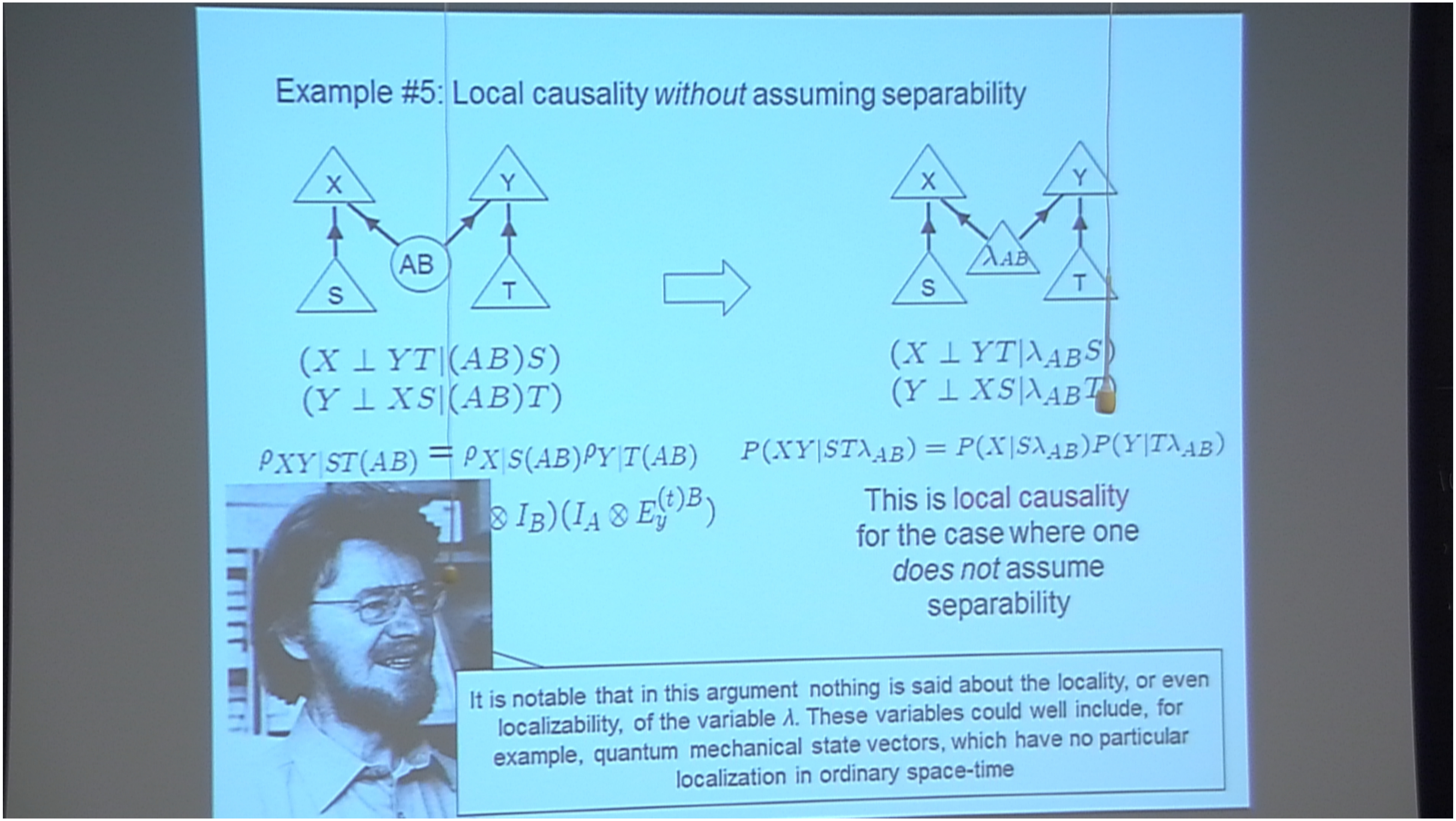
$(X \perp YT|\lambda_{AB}S)$
$(Y \perp XS|\lambda_{AB}T)$

$$\rho_{XY|ST(AB)} = \rho_{X|S(AB)}\rho_{Y|T(AB)}$$

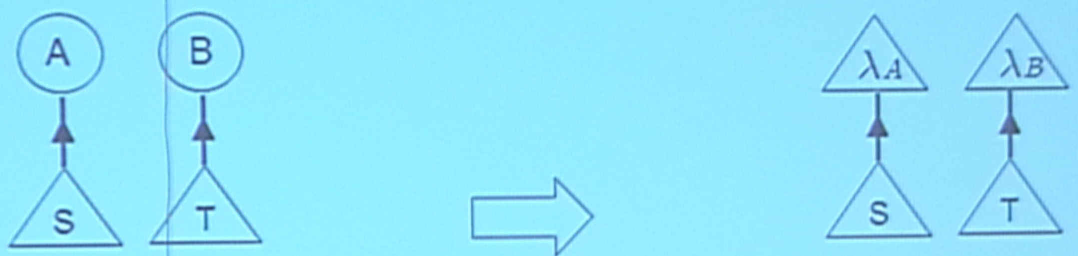
$$E^{(st)AB}_{xy} = (E^{(s)A}_x \otimes I_B)(I_A \otimes E^{(t)B}_y)$$

$$P(XY|ST\lambda_{AB}) = P(X|S\lambda_{AB})P(Y|T\lambda_{AB})$$

This is local causality for the case where one *does not* assume separability


Example #5: Local causality *without* assuming separability
X
Y
AB
S
T
X
Y
λ_{AB}
S
T
$(X \perp YT|(AB)S)$
$(Y \perp XS|(AB)T)$
$(X \perp YT|\lambda_{AB}S)$
$(Y \perp XS|\lambda_{AB}T)$
$\rho_{XY|ST(AB)} = \rho_{X|S(AB)}\rho_{Y|T(AB)}$
$P(XY|ST\lambda_{AB}) = P(X|S\lambda_{AB})P(Y|T\lambda_{AB})$
$\otimes I_B)(I_A \otimes E_y^{(t)B})$
This is local causality for the case where one *does not* assume separability
It is notable that in this argument nothing is said about the locality, or even localizability, of the variable λ. These variables could well include, for example, quantum mechanical state vectors, which have no particular localization in ordinary space-time

Example #6: FactorizationPS in PBR assuming separabilityPS



$$(A \perp BT|S)$$
$$(B \perp AS|T)$$
$$\rho_{AB|ST} = \rho_{A|S}\rho_{B|T}$$

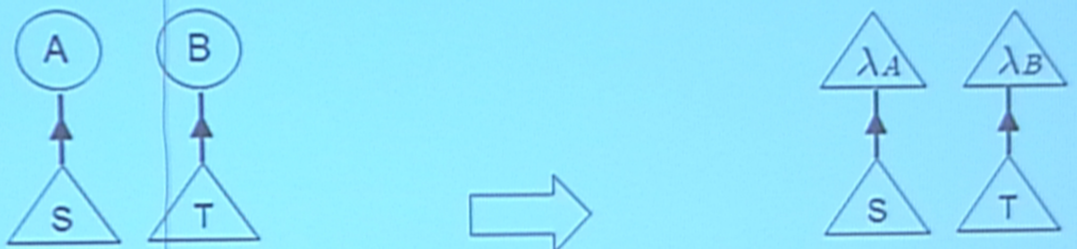
$$(\lambda_A \perp \lambda_B T|S)$$
$$(\lambda_B \perp \lambda_A S|T)$$
$$P(\lambda_A\lambda_B|ST) = P(\lambda_A|S)P(\lambda_B|T)$$

This is factorizationPS assuming separabilityPS

Note: $(S \perp T|AB)$

Proof: $\rho_{ST|AB} = \rho_{AB}^{-1/2}\rho_{AB|ST}\rho_{AB}^{-1/2}\rho_{ST}$

Example #6: FactorizationPS in PBR assuming separabilityPS



$(A \perp BT|S)$

$(B \perp AS|T)$

$\rho_{AB|ST} = \rho_{A|S}\rho_{B|T}$

$(\lambda_A \perp \lambda_B T|S)$

$(\lambda_B \perp \lambda_A S|T)$

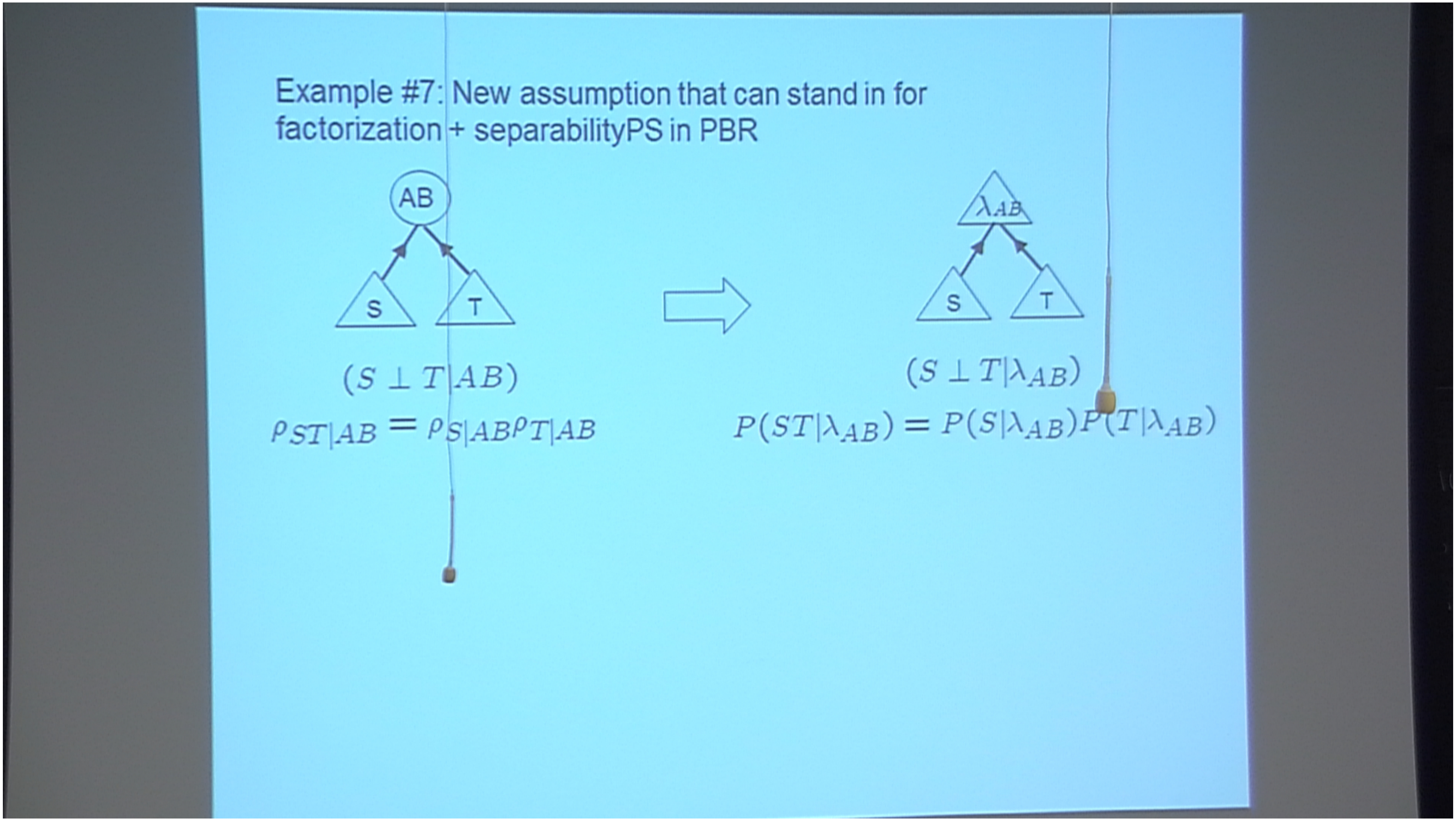
$P(\lambda_A \lambda_B|ST) = P(\lambda_A|S)P(\lambda_B|T)$

This is factorizationPS assuming separabilityPS

Note: $(S \perp T|AB)$

Proof: $\rho_{ST|AB} = \rho_{AB}^{-1/2}\rho_{AB|ST}\rho_{AB}^{-1/2}\rho_{ST}$

$= (\rho_A^{-1/2}\rho_B^{-1/2})\rho_{A|S}\rho_{B|T}(\rho_A^{-1/2}\rho_B^{-1/2})\rho_S\rho_T$

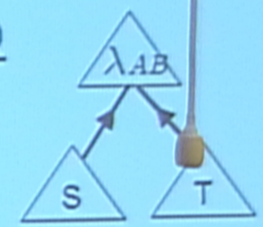

Example #7: New assumption that can stand in for factorization + separabilityPS in PBR
AB
S
T
λ_{AB}
S
T
$(S \perp T | AB)$
$\rho_{ST|AB} = \rho_{S|AB}\rho_{T|AB}$
$(S \perp T | \lambda_{AB})$
$P(ST|\lambda_{AB}) = P(S|\lambda_{AB})P(T|\lambda_{AB})$

SeparabilityPS
+
FactorizationPS

can be replaced by poolabilityPS

Variant of PBR theorem:

poolabilityPS $\rightarrow P(\lambda_{AB}|ST) \propto \frac{P(\lambda_{AB}|S)P(\lambda_{AB}|T)}{P(\lambda_{AB})}$



ψ-epistemic $\rightarrow \exists\, \lambda_{AB}$: $P(\lambda_{AB}|S) > 0\ \forall S$

$P(\lambda_{AB}|T) > 0\ \forall T$

$\rightarrow \exists\, \lambda_{AB}$: $P(\lambda_{AB}|ST) > 0\ \forall S, T$

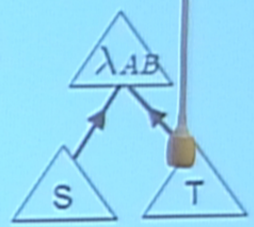
Then the proof runs as before

SeparabilityPS
+
FactorizationPS

can be replaced by poolabilityPS

Variant of PBR theorem:

poolabilityPS $\rightarrow P(\lambda_{AB}|ST) \propto \frac{P(\lambda_{AB}|S)P(\lambda_{AB}|T)}{P(\lambda_{AB})}$



ψ-epistemic $\rightarrow \exists\, \lambda_{AB}$: $P(\lambda_{AB}|S) > 0\ \forall S$

$P(\lambda_{AB}|T) > 0\ \forall T$

$\rightarrow \exists\, \lambda_{AB}$: $P(\lambda_{AB}|ST) > 0\ \forall S,T$

Then the proof runs as before

λ-sufficiency $\wedge$ PoolabilityPS $\wedge$ ψ-epistemic → contradiction

A similar modification is described in:
M. Hall, arXiv:1111.6304

The I-tracking condition implies

- λ-sufficiency
- Measurement noncontextuality
- Preparation noncontextuality
- Local causality
- Factorization for product states
- Poolability for product states

Claim: The I-tracking condition is the overarching principle that makes all of these assumptions seem plausible

Suppose one is commited to:
- the standard ontological model framework
- The principle of I-traction

Presumably then, one should seek to salvage instances of I-traction *as much as one can*. But how do we quantify this?

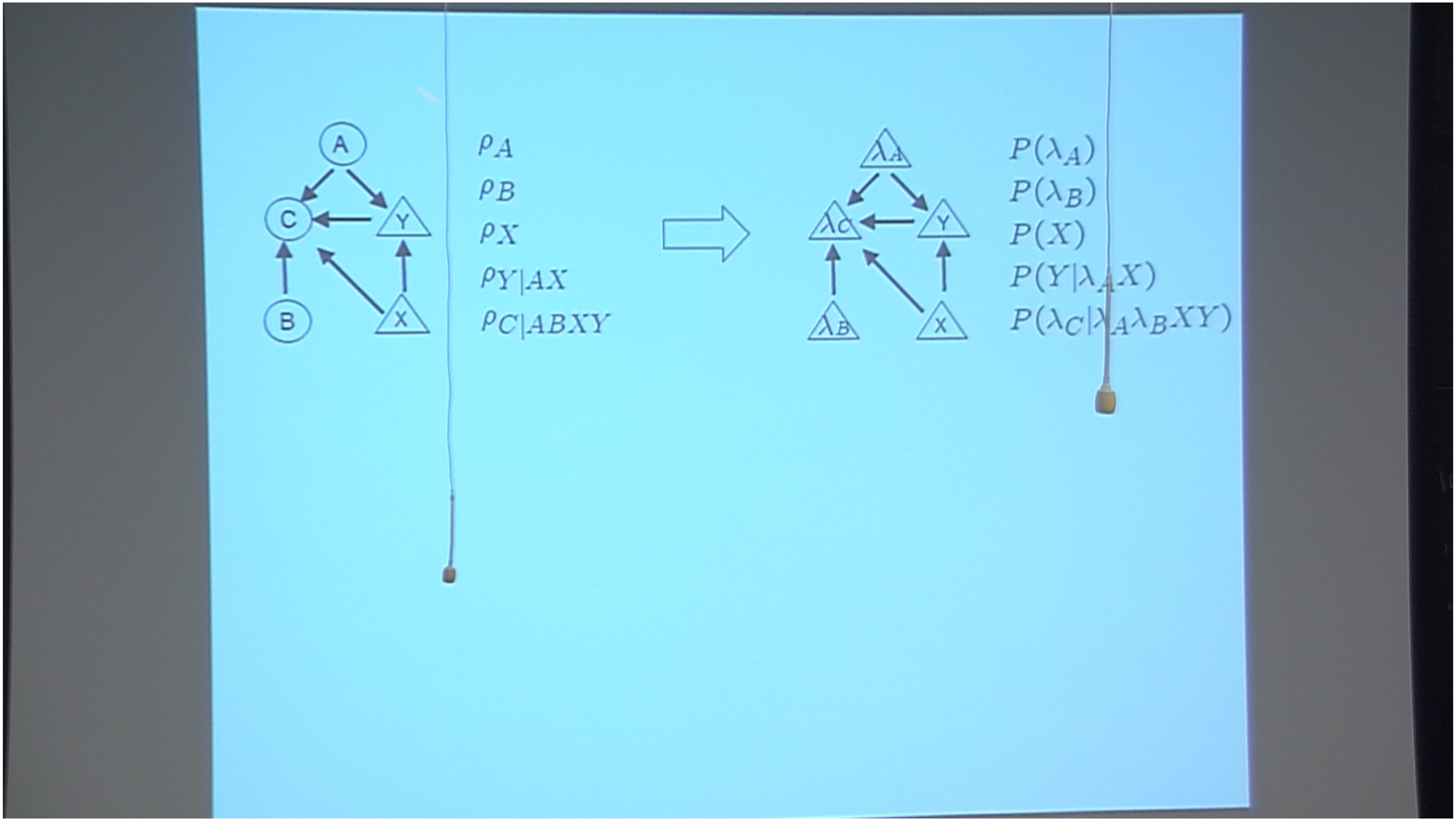
- ψ-ontic models can achieve FactorizationPS or PoolabilityPS in the full quantum theory, while ψ-epistemic models can only achieve them in certain subtheories

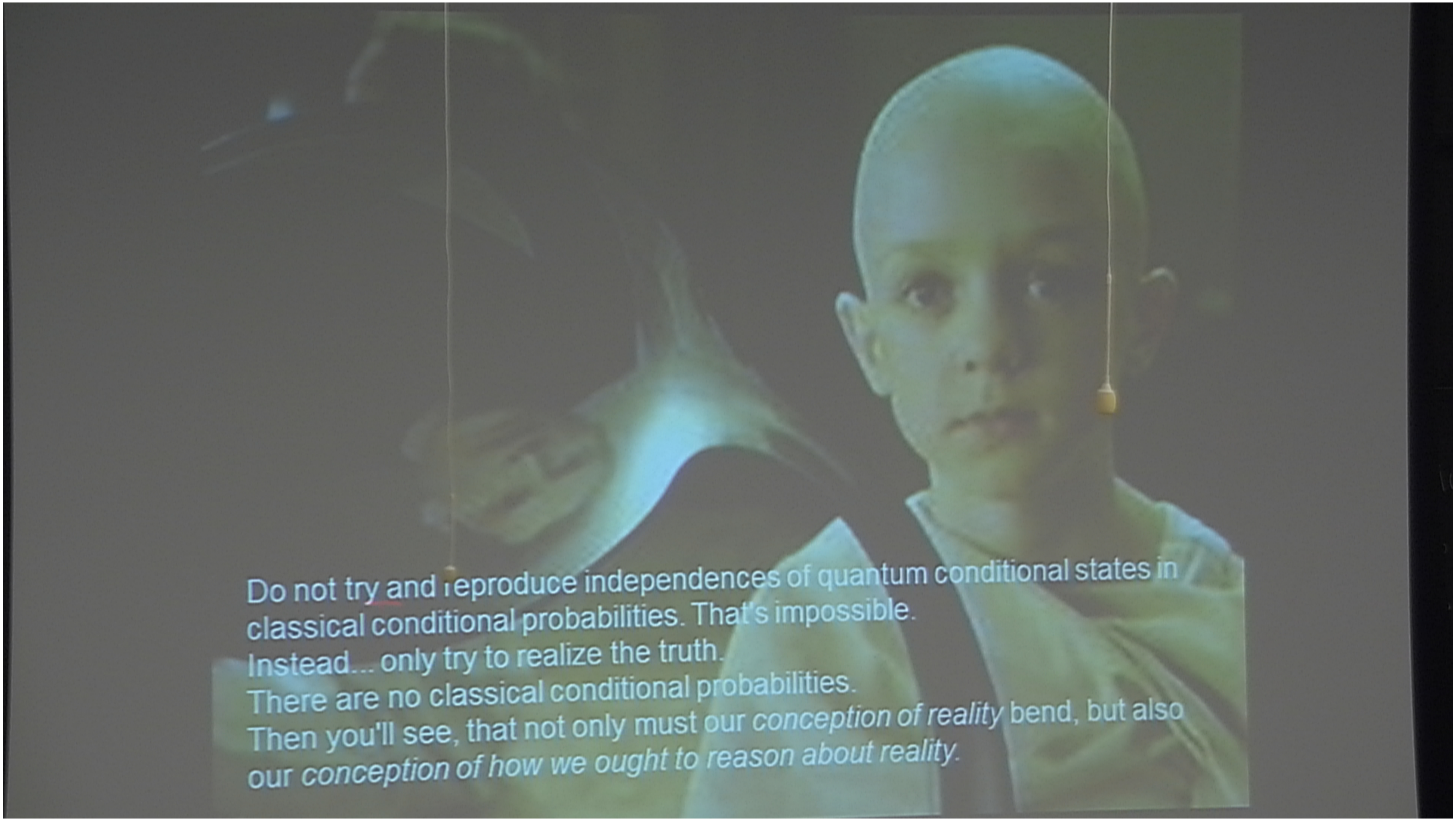
- ψ-epistemic models can achieve local causality and preparation noncontextuality in certain subtheories while ψ-ontic models cannot achieve them at all

All of something versus some of everything

One can't satisfy the I-tracking condition for the full quantum theory in the standard framework for ontological models (ψ-ontic or ψ-epistemic)

To me, this suggests that **"something has to give"** in this framework


A
C
Y
B
X
ρ_A
ρ_B
ρ_X
$\rho_{Y|AX}$
$\rho_{C|ABXY}$
λ_A
λ_C
Y
λ_B
X
$P(\lambda_A)$
$P(\lambda_B)$
$P(X)$
$P(Y|\lambda_A X)$
$P(\lambda_C|\lambda_A \lambda_B XY)$


Do not try and reproduce independences of quantum conditional states in classical conditional probabilities. That's impossible.
Instead... only try to realize the truth.
There are no classical conditional probabilities.
Then you'll see, that not only must our *conception of reality* bend, but also our *conception of how we ought to reason about reality*.