Title: Large Mixing Angles From Many Right-Handed Neutrinos

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Abstract: A beautiful understanding of the smallness of the neutrino masses may be obtained via the seesaw mechanism, whereby one takes advantage of the key qualitative distinction between the neutrinos and the other fermions: right-handed neutrinos are gauge singlets, and may therefore have large Majorana masses. The standard seesaw mechanism, however, does not address the apparent lack of hierarchy in the neutrino masses compared to the quarks and charged leptons, nor the large leptonic mixing angles compared to the small angles of the CKM matrix. In this talk, I will show that the singlet nature of the right-handed neutrinos may be taken advantage of in one further way in order to solve these remaining problems: Unlike particles with gauge interactions, whose numbers are constrained by anomaly cancellation, the number of gauge singlet particles is essentially undetermined. If large numbers of gauge singlet fermions are present at high energies - as is suggested, for example, by various string constructions - then the effective low energy neutrino mass matrix may be determined as a sum over many distinct Yukawa couplings, with the largest ones being the most important. This can reduce hierarchy, and lead to large mixing angles. Assuming a statistical distribution of fundamental parameters, we will show that this scenario leads to a good fit to low energy phenomenology, with only a few qualitative assumptions guided by the known quark and lepton masses. The scenario leads to predictions of a normal hierarchy for the neutrino masses, and a value for the |m_ee| mass matrix element of about 1-6 meV.


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The Fundamental Mysteries of Flavor

- Quark and charged lepton masses have large (~6 orders of magnitude) hierarchies.

- With up and down quarks ordered by mass, the CKM matrix is approximately equal to the identity.

- Neutrino masses ~10 orders of magnitude smaller than other fermion masses, and seemingly less hierarchical.

The lepton and quark sector mixing angles seem fundamentally different, with the leptons having two near maximal mixing angles.

The See-Saw Mechanism

→ A simple explanation for the small neutrino masses.

The key point: Neutrinos are unique because ν_R's are standard model gauge singlets.

- Majorana masses for the ν_R's are allowed, and may be very large, perhaps close to the GUT scale, $M \sim 10^{15}$GeV.

→ Produce low energy Majorana ν masses ~ $\lambda^2 v^2/M$

→We obtain an extremely elegant understanding of the ique smallness of the neutrino masses.

(More generally, an operator $(LH)^2/M$ is allowed, and there are a few ways of generating it. We will consider only the type-I seesaw, but some of our results may extend to other types.)

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- For the other mysteries, we have many possibilities..

  → Flavor symmetries of many types..

  → Extra-dimensional wavefunctions

But perhaps nothing as singularly compelling as the see-saw mechanism..

→ How about an agnostic approach?

-Can we try to treat the Yukawas as just a bunch of “random” numbers?

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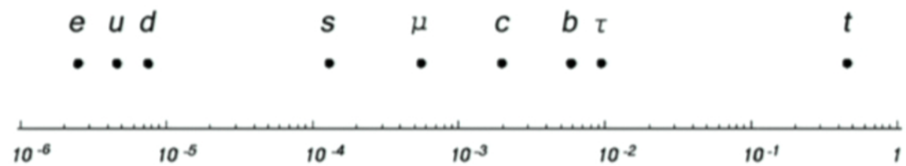
..Random in what way?

→ Quarks and charged leptons seem to prefer hierarchy.

- Well fit by a roughly scale invariant, hierarchical distribution.

-Donoghue, Dutta, Ross



→ Prediction of $V_{ub} < V_{cb} < V_{us}$

- Observed values statistically reasonable.

A disadvantage: “Generations” are an accident.. (1/6)

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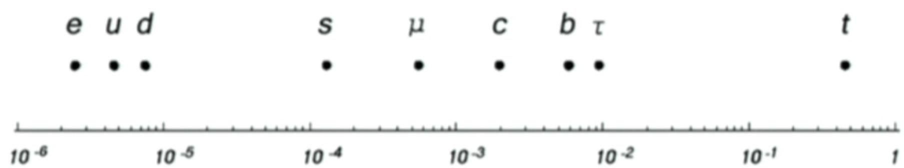

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-Hall, Murayama, Weiner
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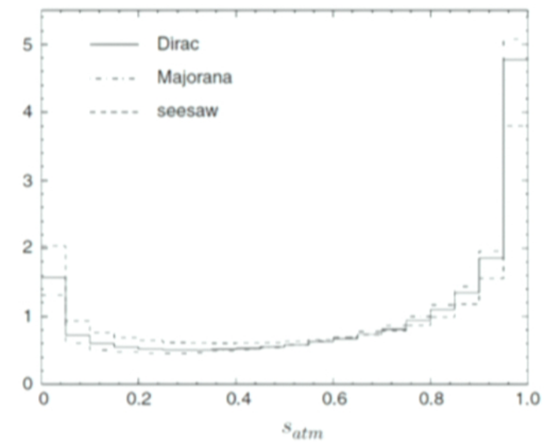

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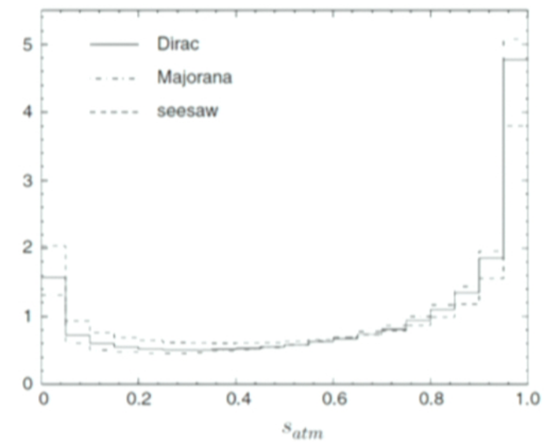

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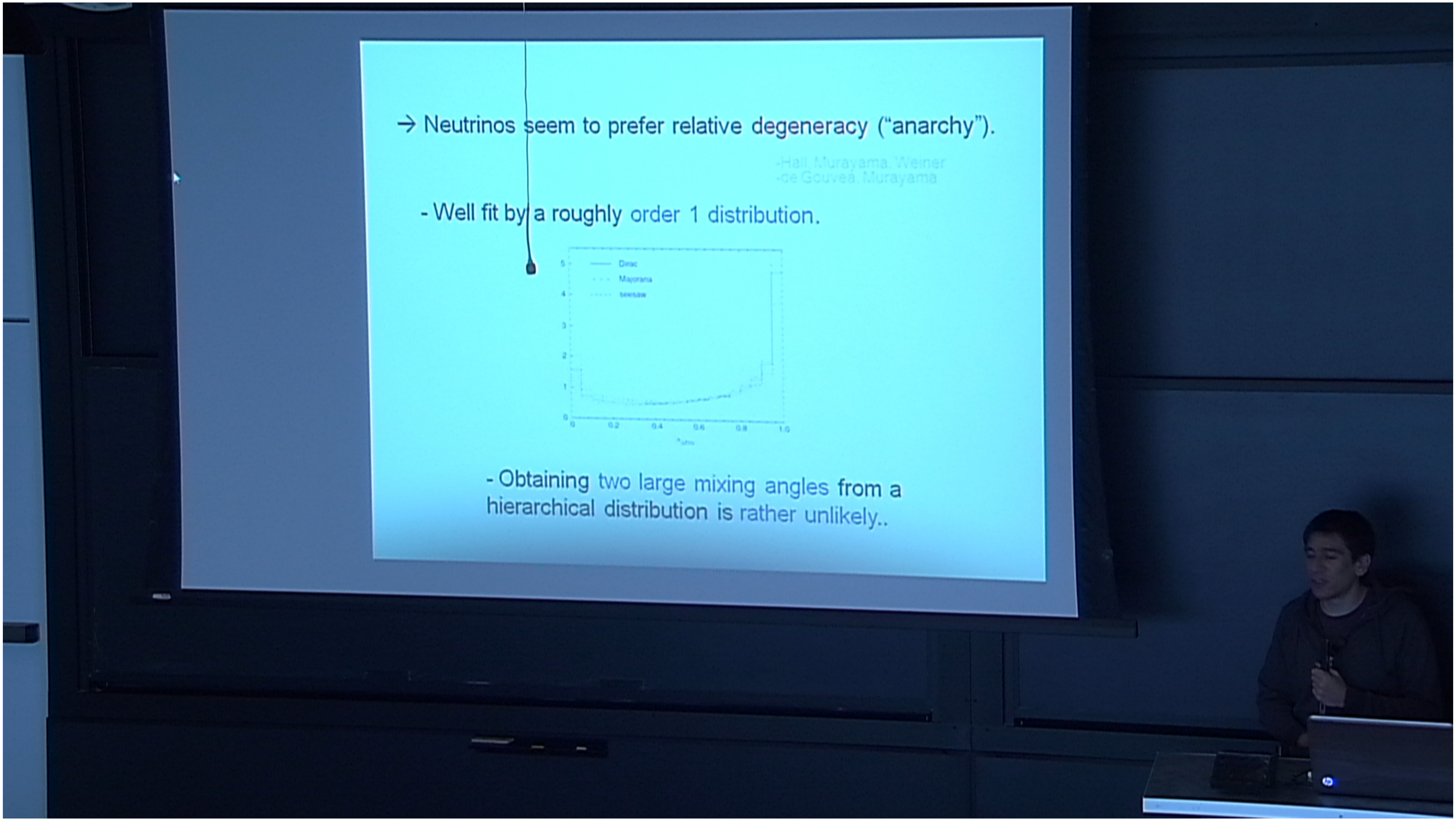
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Dirac
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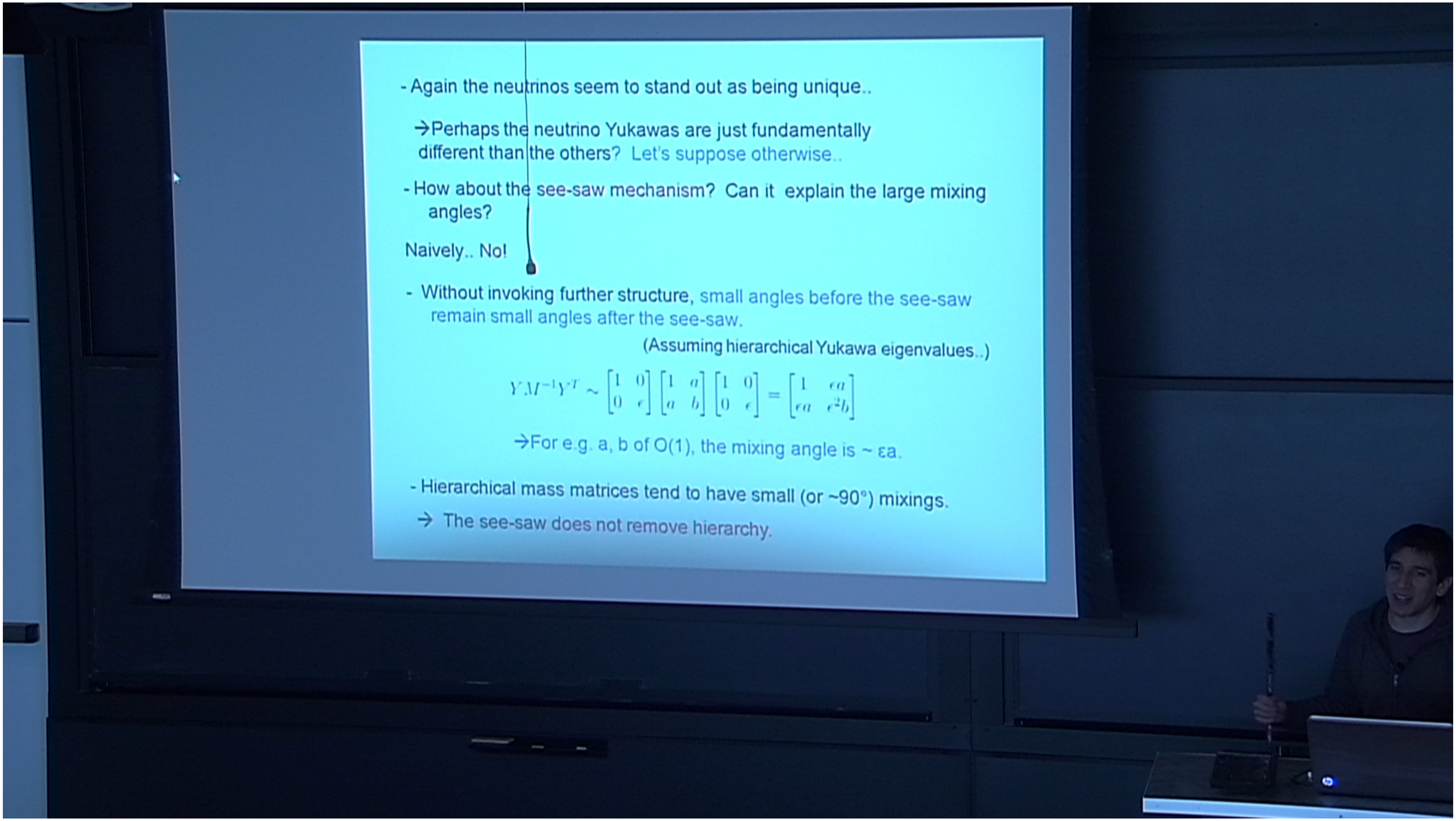
- Again the neutrinos seem to stand out as being unique..
→Perhaps the neutrino Yukawas are just fundamentally different than the others? Let's suppose otherwise..
- How about the see-saw mechanism? Can it explain the large mixing angles?
Naively.. No!
- Without invoking further structure, small angles before the see-saw remain small angles after the see-saw.
(Assuming hierarchical Yukawa eigenvalues..)
$YM^{-1}Y^T \sim \begin{bmatrix}1&0\\0&\epsilon\end{bmatrix}\begin{bmatrix}1&a\\a&b\end{bmatrix}\begin{bmatrix}1&0\\0&\epsilon\end{bmatrix} = \begin{bmatrix}1&\epsilon a\\ \epsilon a&\epsilon^2 b\end{bmatrix}$
→For e.g. a, b of O(1), the mixing angle is ~ εa.
- Hierarchical mass matrices tend to have small (or ~90°) mixings.
→ The see-saw does not remove hierarchy.

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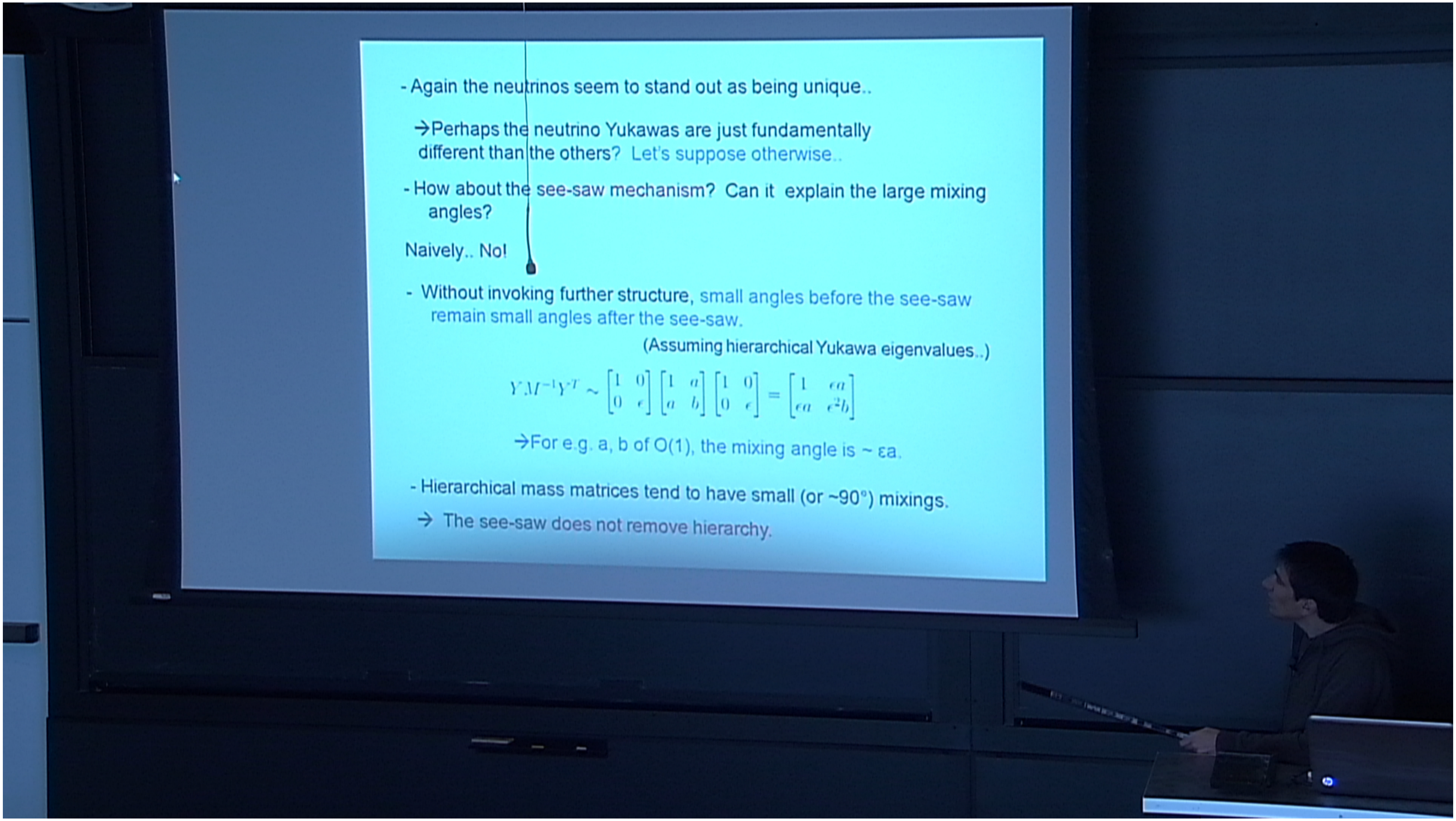
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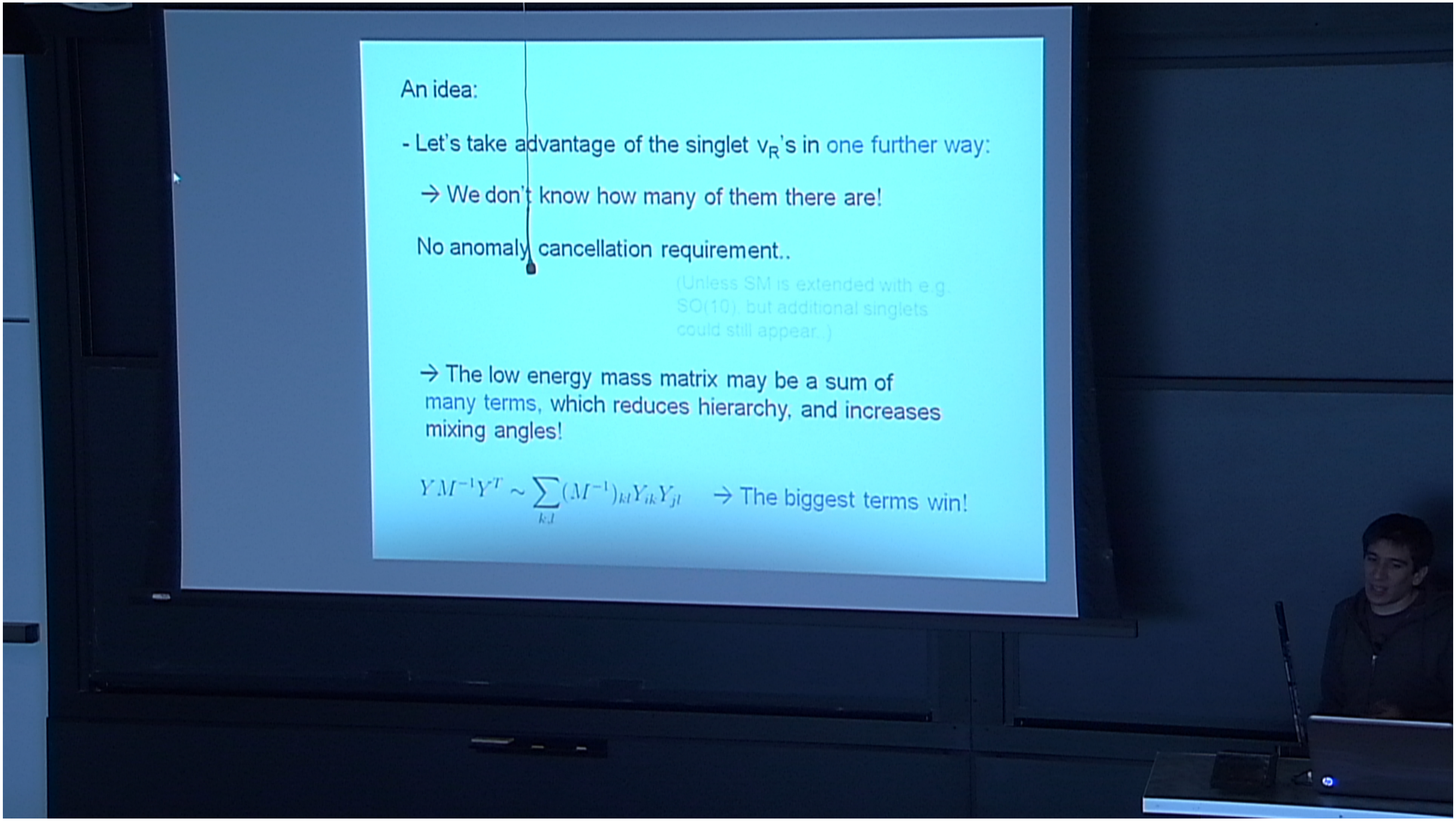
An idea:
- Let's take advantage of the singlet ν_R's in one further way:
→ We don't know how many of them there are!
No anomaly cancellation requirement..
(Unless SM is extended with e.g. SO(10), but additional singlets could still appear..)
→ The low energy mass matrix may be a sum of many terms, which reduces hierarchy, and increases mixing angles!
$YM^{-1}Y^T \sim \sum_{k,l}(M^{-1})_{kl}Y_{ik}Y_{jl}$
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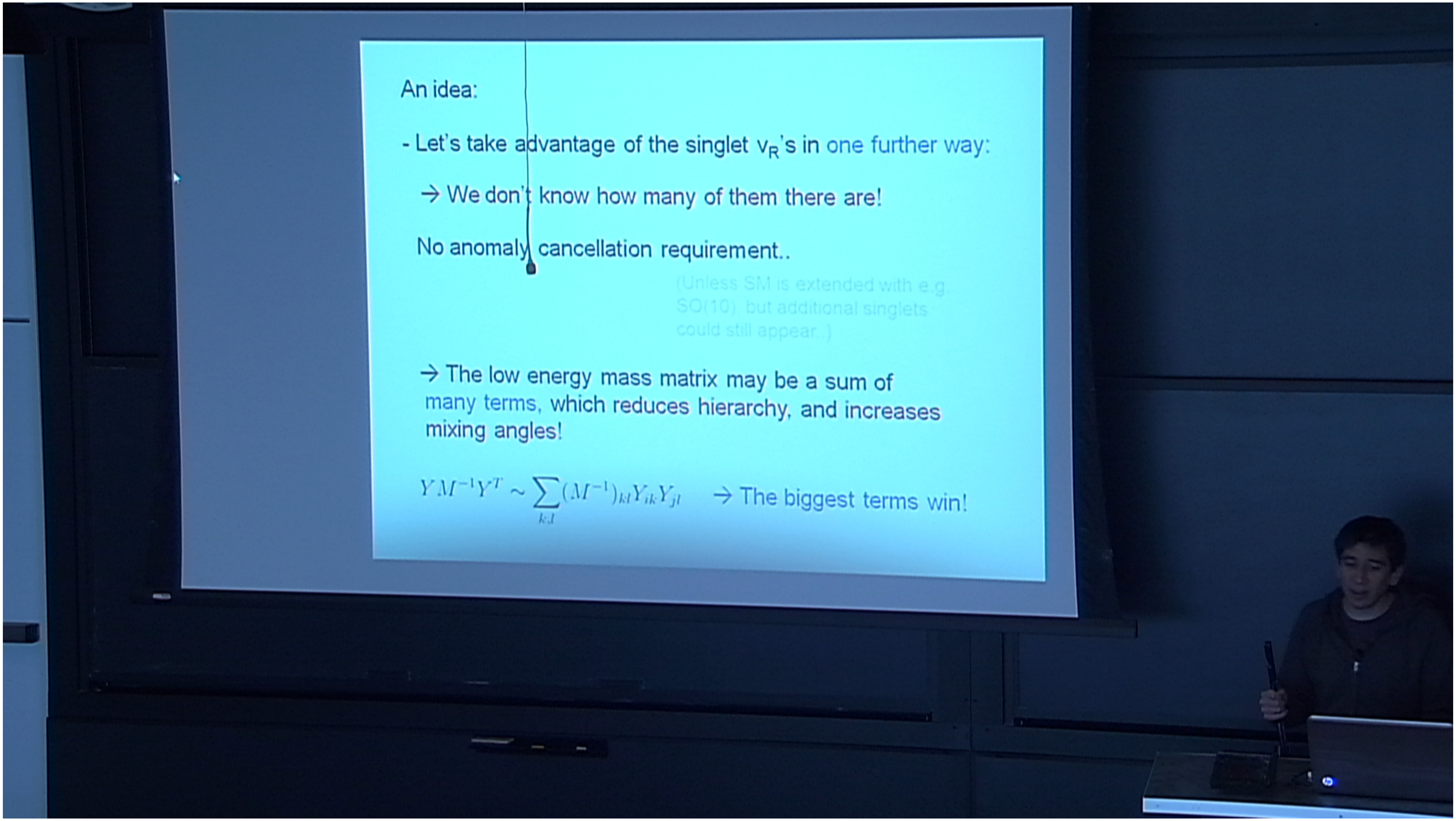
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- KK modes, or Moduli fields

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$$M_{KK} \sim M_{pl} \left(\frac{M_{KK}}{M_s} \right)^{\frac{n}{2}+1}$$

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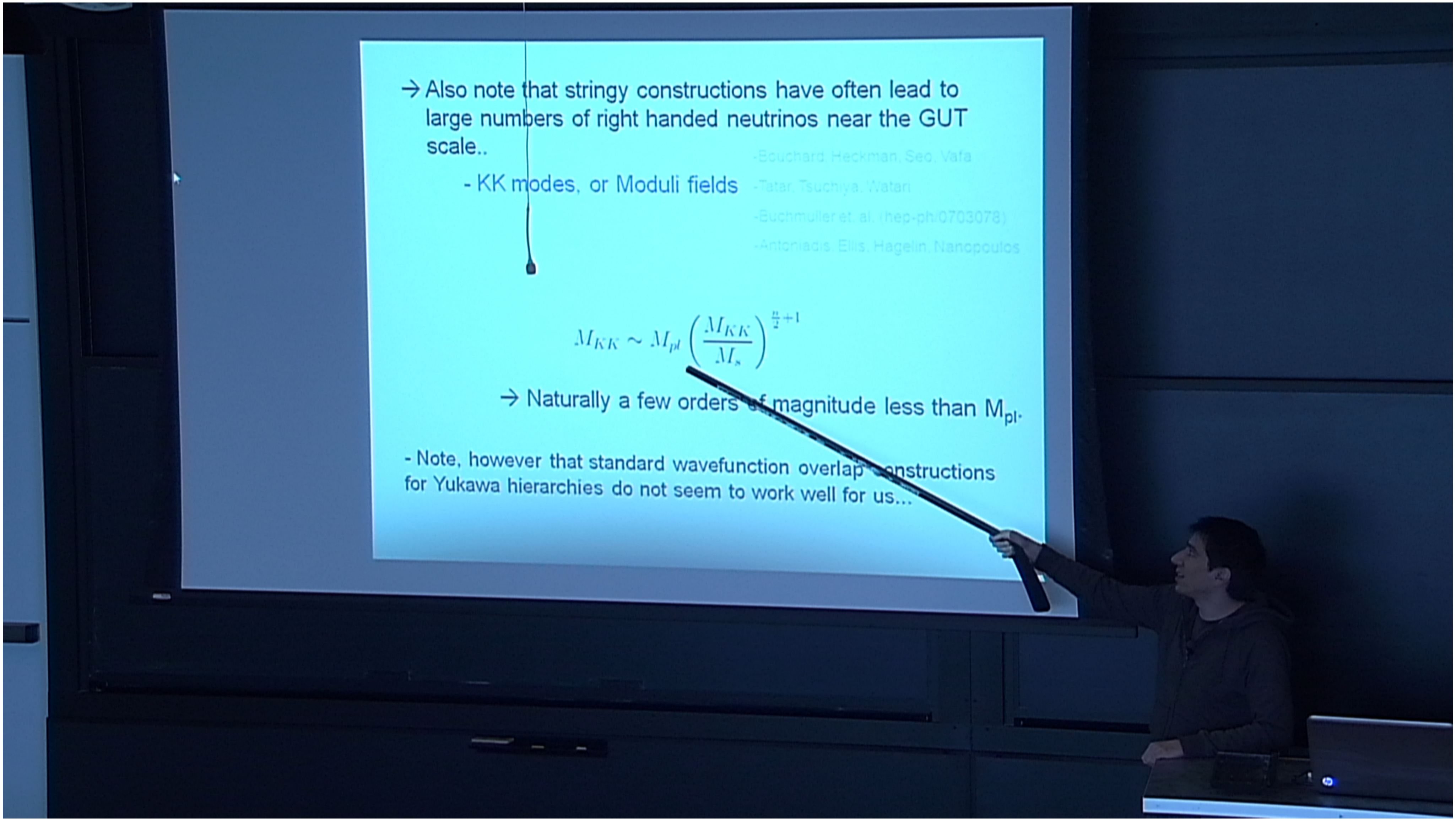
- Note, however that standard wavefunction overlap constructions for Yukawa hierarchies do not seem to work well for us...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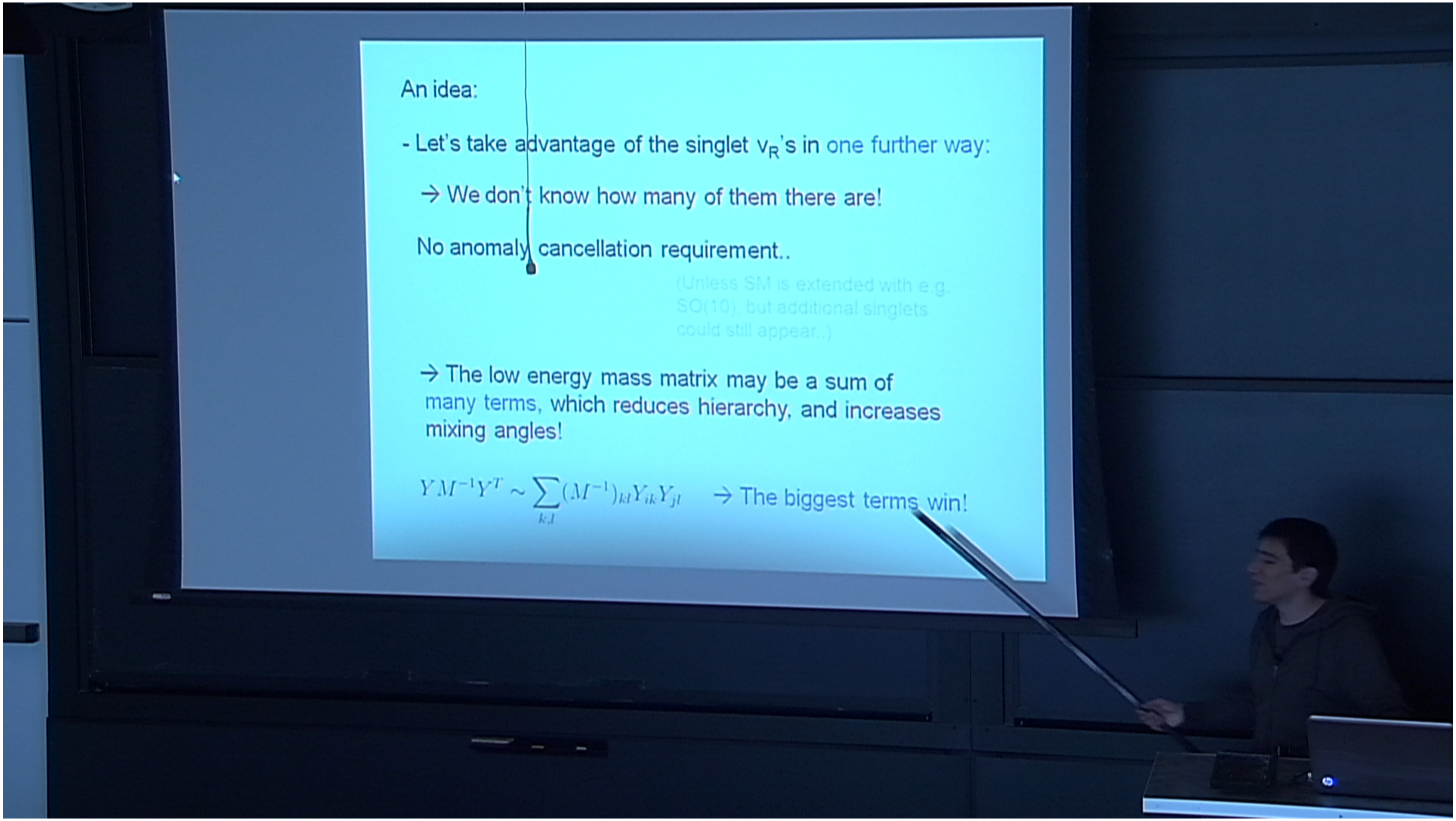
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- A simple qualitative idea.. How to quantify it?

Basic starting point: Assume a universal, (almost) scale invariant distribution for Yukawas, with upper and lower cutoffs, random phases.

→ Following DDR, take a simple ansatz consistent with phenomenology:

$$\rho(\lambda) \propto 1/\lambda^{1.1} \qquad \lambda_{max} = 1 \qquad \lambda_{min} = 1.2 \times 10^{-6}$$

	Median			% below exp.		
δ	V_{us}	V_{ub}	V_{cb}	V_{us}	V_{ub}	V_{cb}
1	.162	.018	.060	60	19	42
1.1	.130	.012	.032	65	24	52
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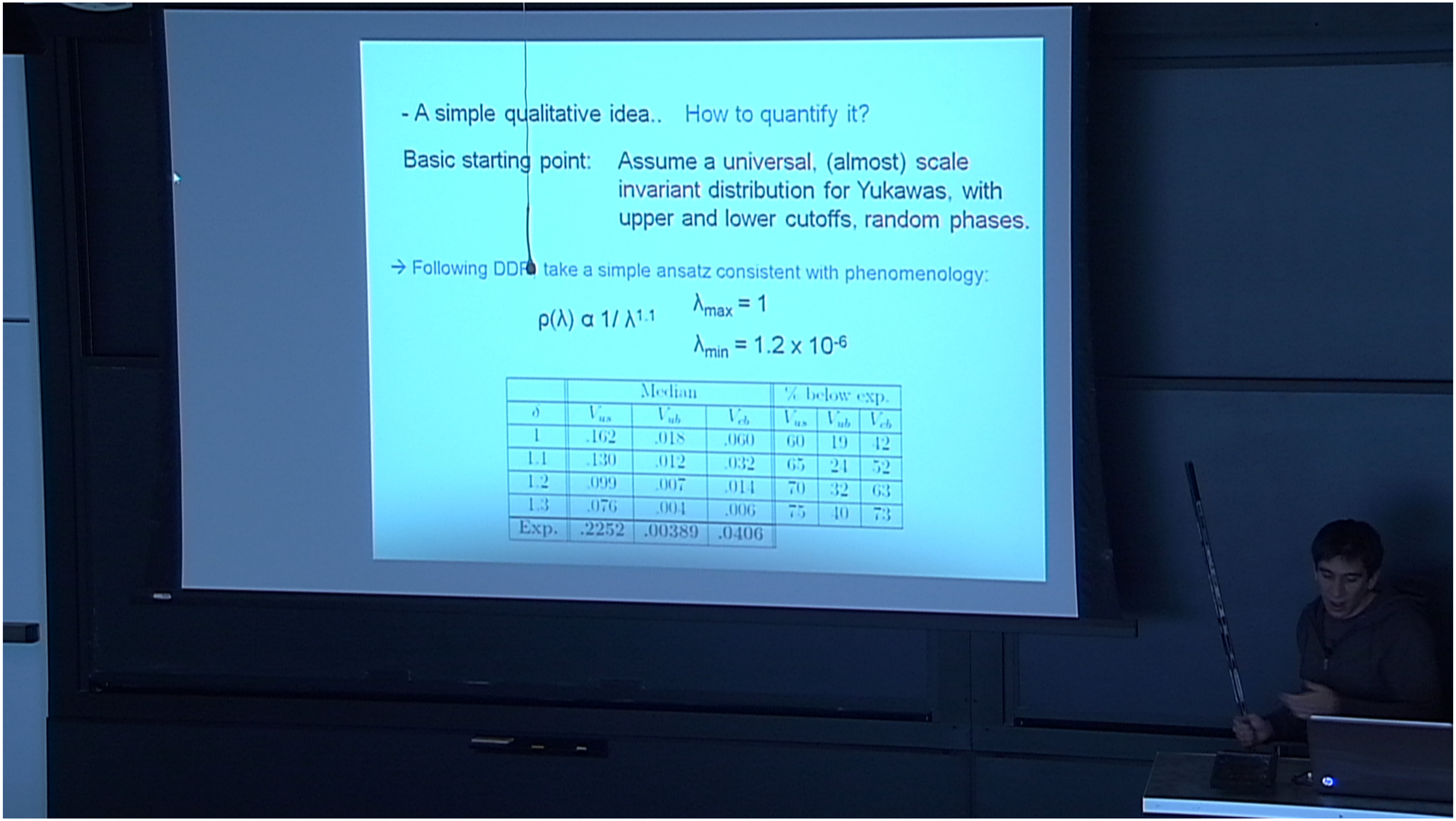

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$$m_{ij} = \sum_k (M^{-1})_{kk} Y_{ik} Y_{jk}$$

Now compare, e.g.:

$$\sum_k (M^{-1})_{kk} Y_{1k} Y_{1k} \qquad \text{vs} \qquad \sum_k (M^{-1})_{kk} Y_{1k} Y_{2k}$$

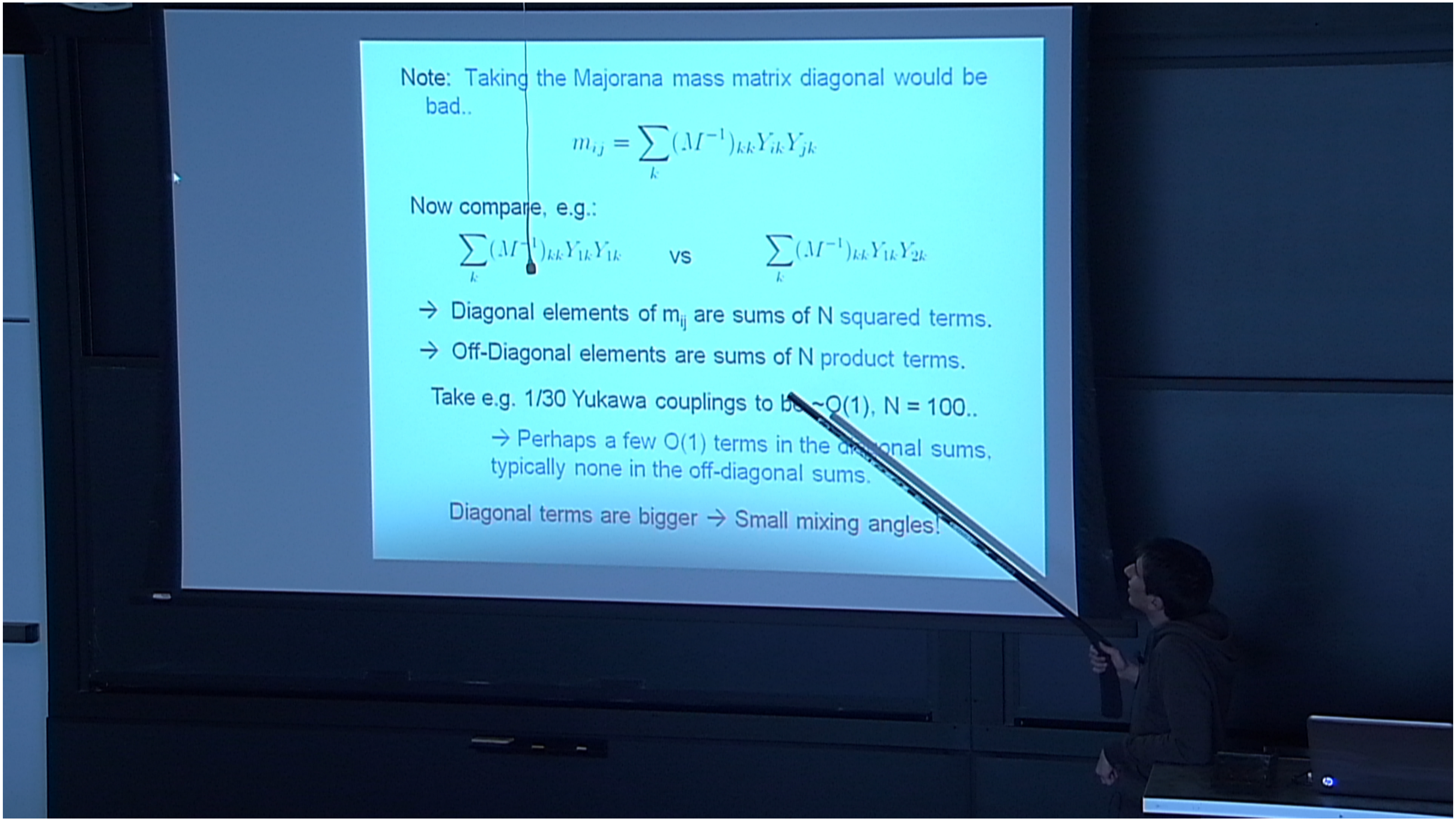
→ Diagonal elements of m_{ij} are sums of N squared terms.

→ Off-Diagonal elements are sums of N product terms.

Take e.g. 1/30 Yukawa couplings to be ~O(1), N = 100..

→ Perhaps a few O(1) terms in the diagonal sums, typically none in the off-diagonal sums.

Diagonal terms are bigger → Small mixing angles!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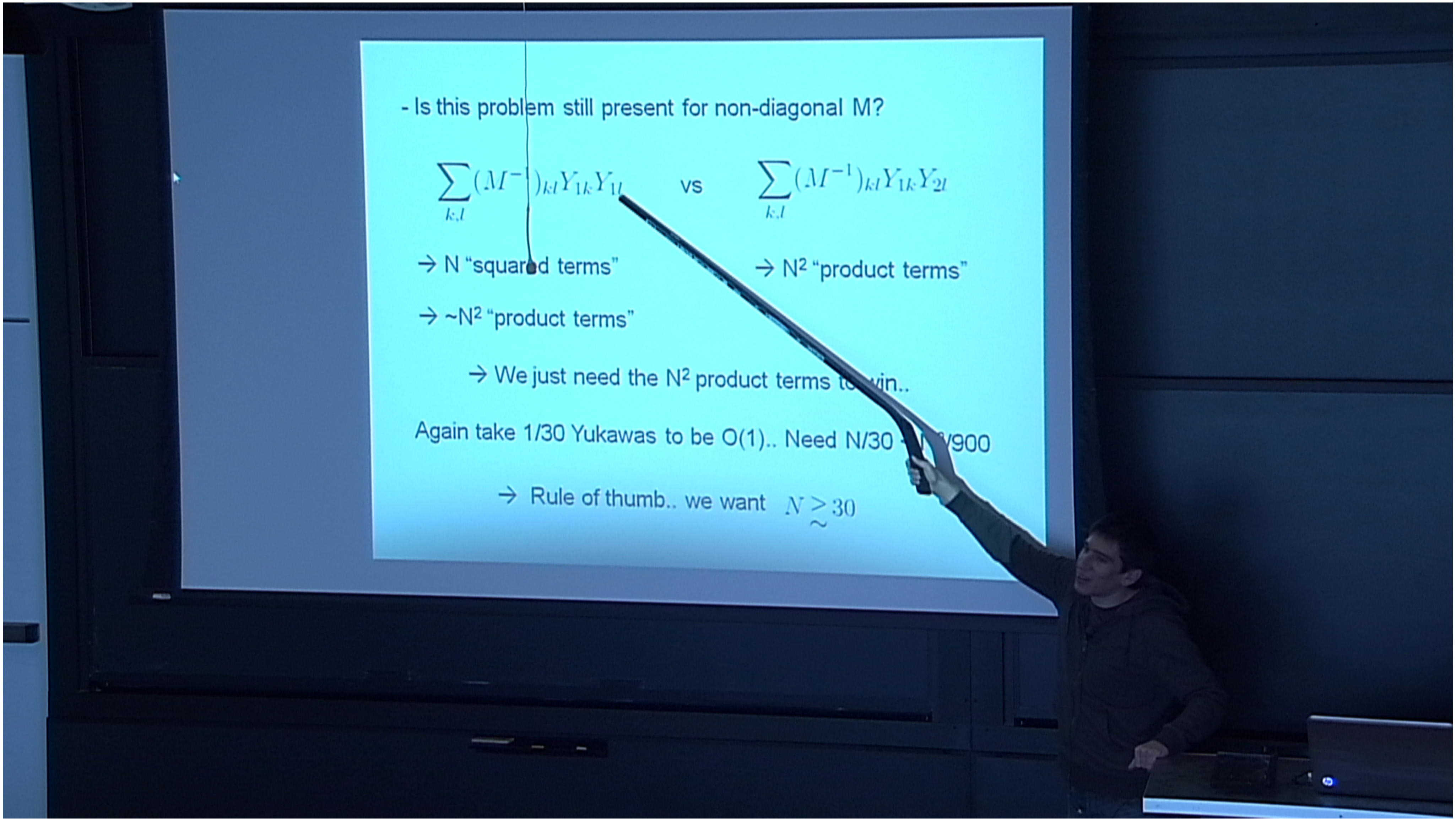
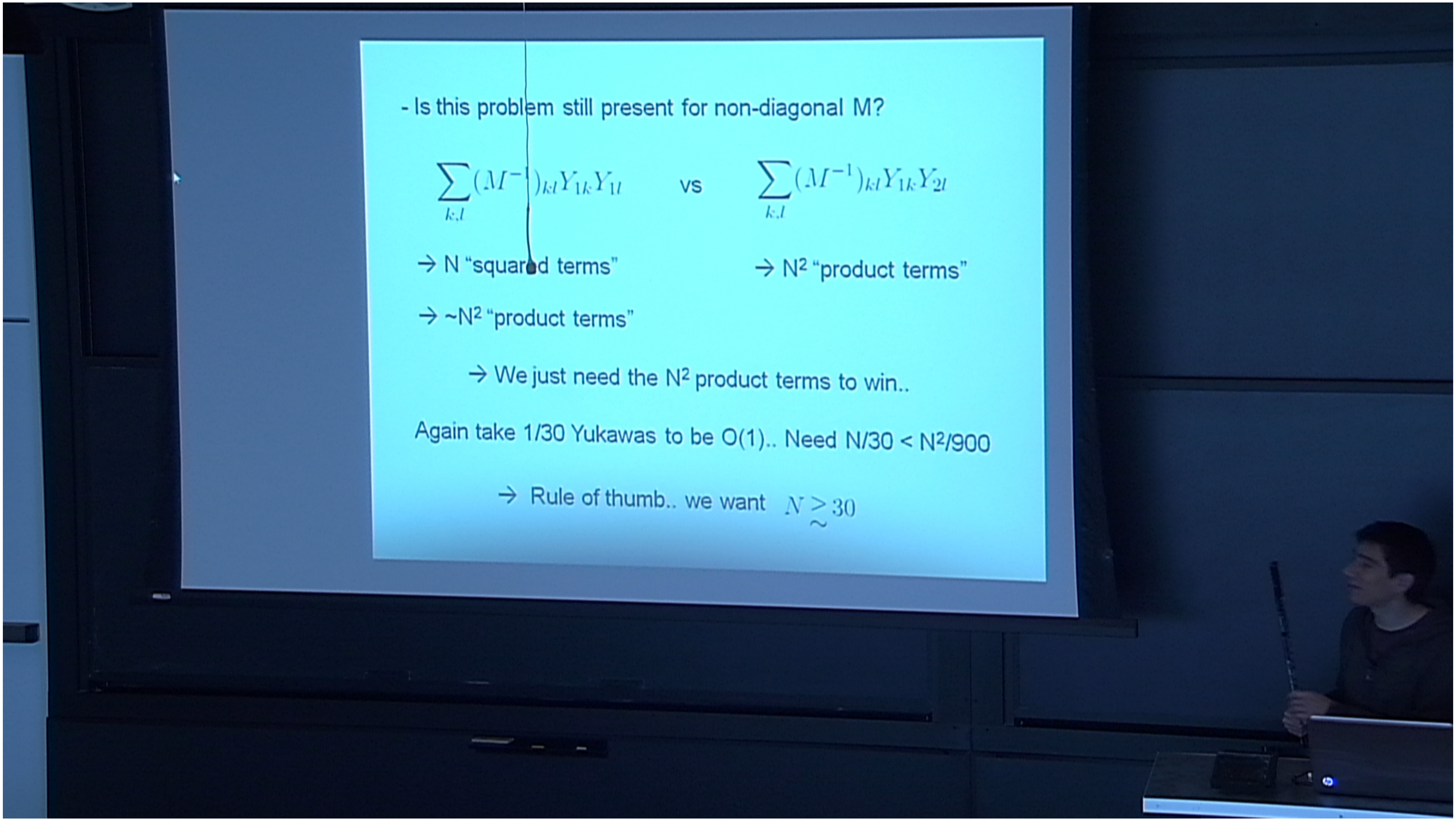

- Is this problem still present for non-diagonal M?
$\sum_{k,l} (M^{-1})_{kl} Y_{1k} Y_{1l}$ vs $\sum_{k,l} (M^{-1})_{kl} Y_{1k} Y_{2l}$
→ N "squared terms"
→ N^2 "product terms"
→ ~N^2 "product terms"
→ We just need the N^2 product terms
Again take 1/30 Yukawas to be O(1).. Need N/30
→ Rule of thumb.. we want $N \gtrsim 30$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Again take 1/30 Yukawas to be O(1).. Need $N/30 < N^2/900$
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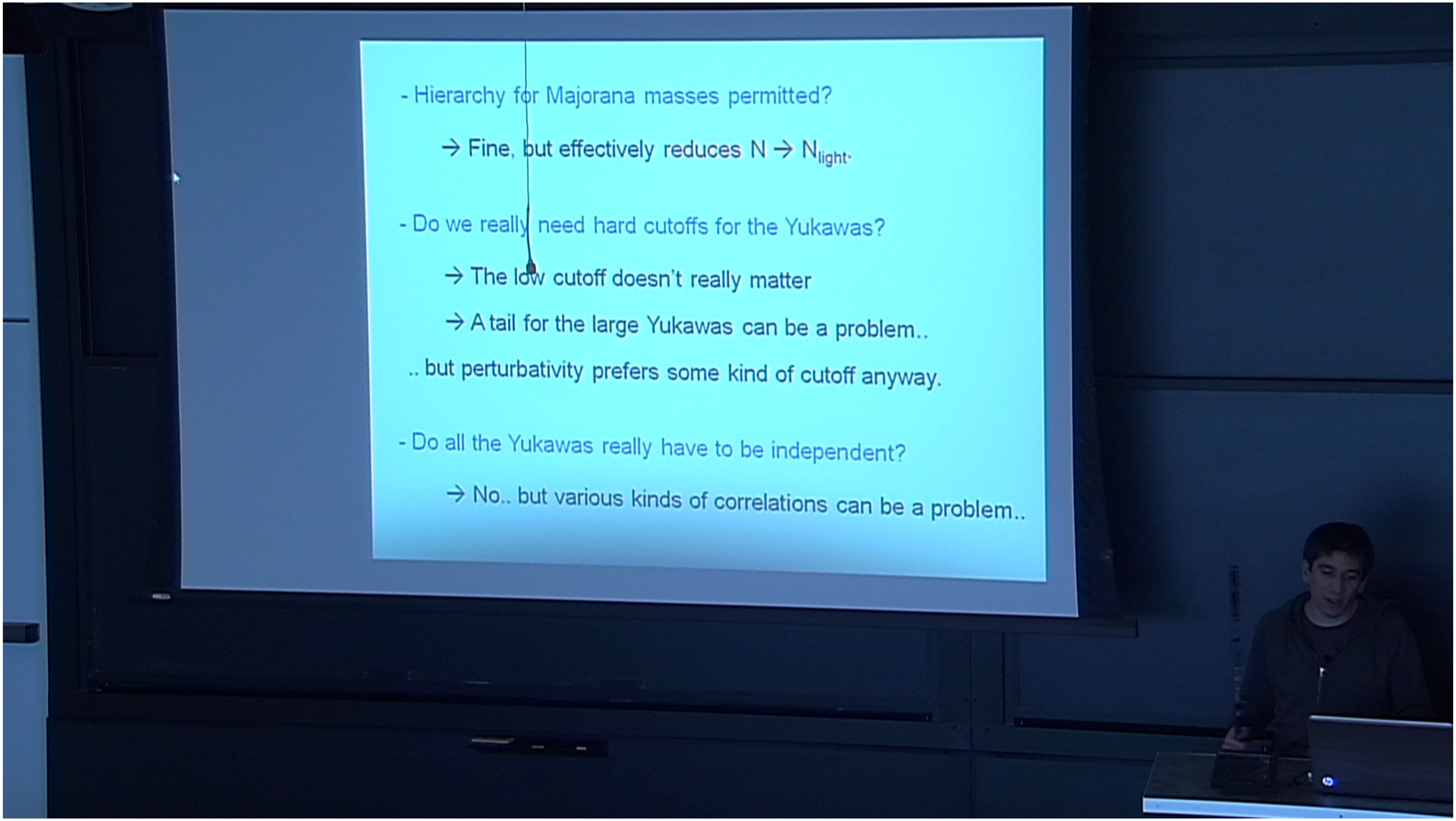
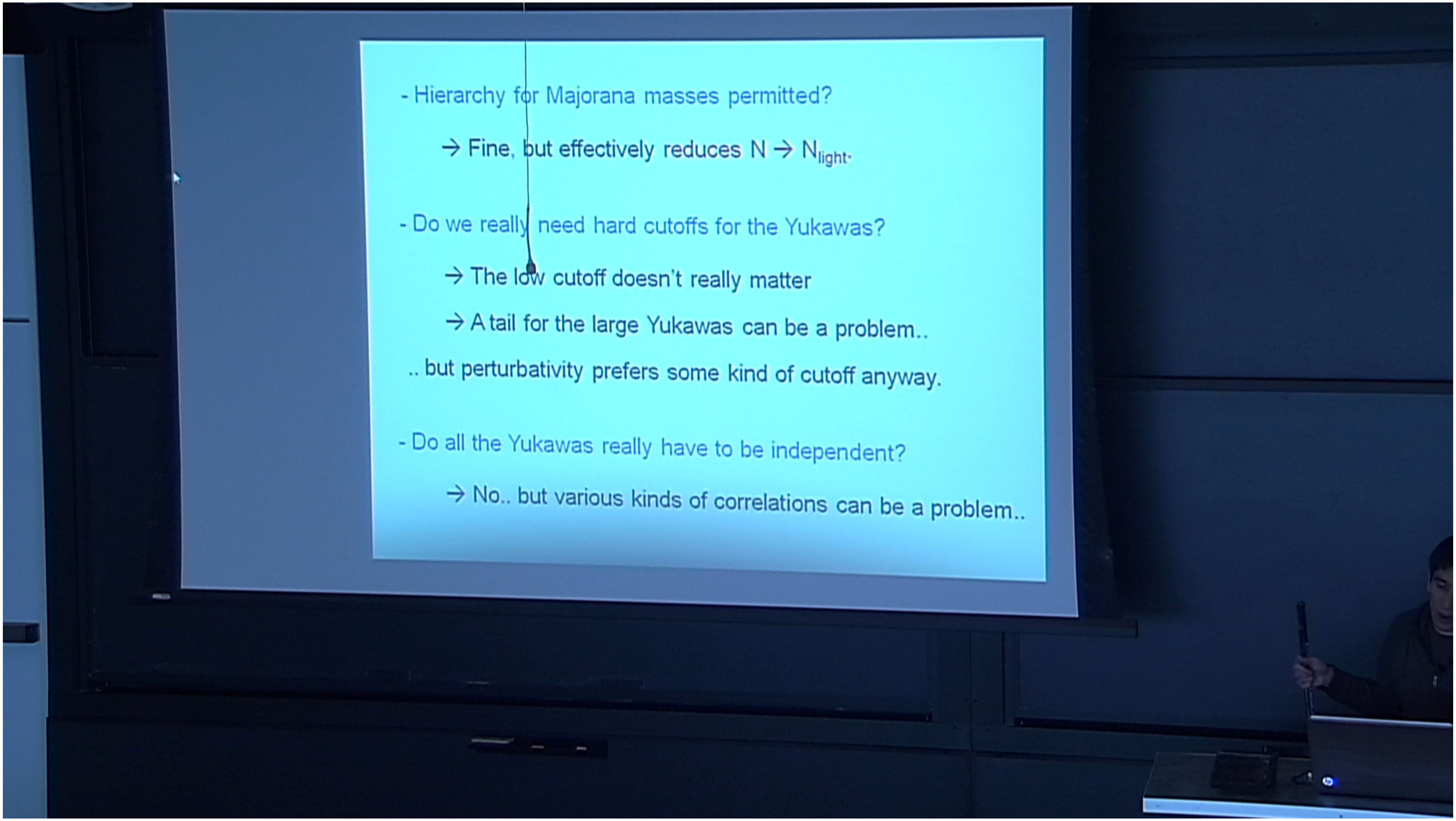

- Hierarchy for Majorana masses permitted?
→ Fine, but effectively reduces N → N_light.
- Do we really need hard cutoffs for the Yukawas?
→ The low cutoff doesn't really matter
→ A tail for the large Yukawas can be a problem..
.. but perturbativity prefers some kind of cutoff anyway.
- Do all the Yukawas really have to be independent?
→ No.. but various kinds of correlations can be a problem..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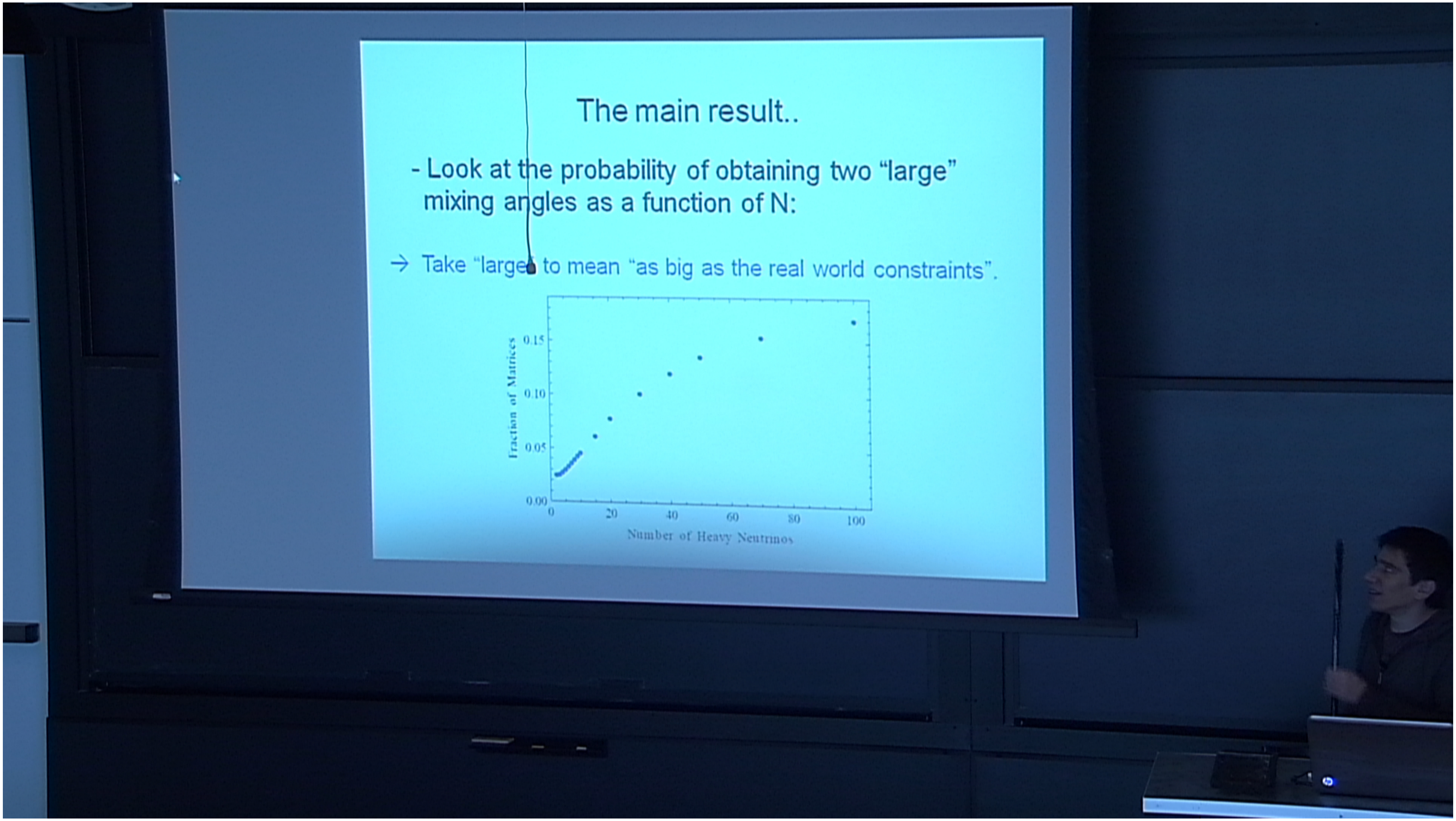
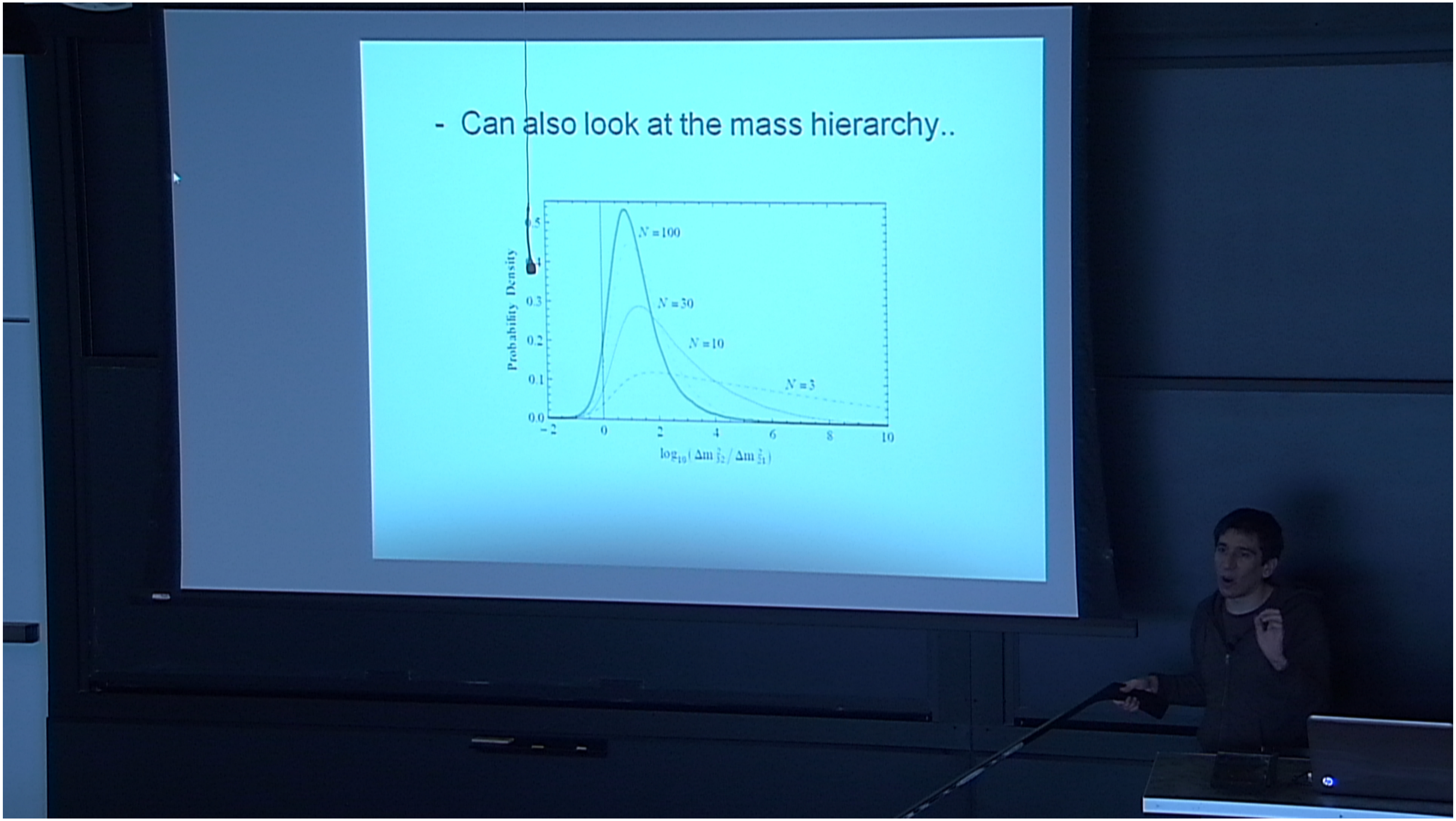
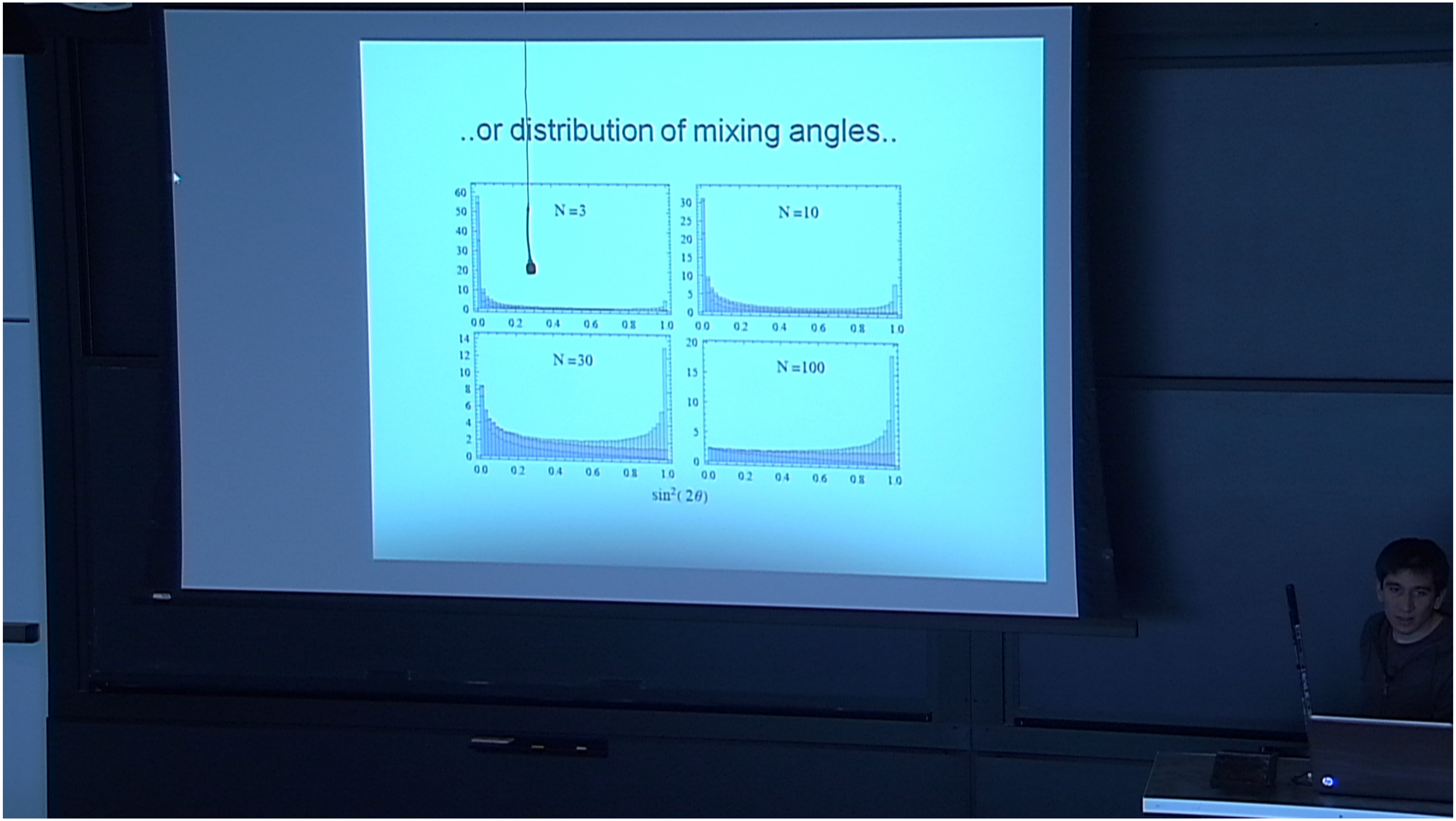
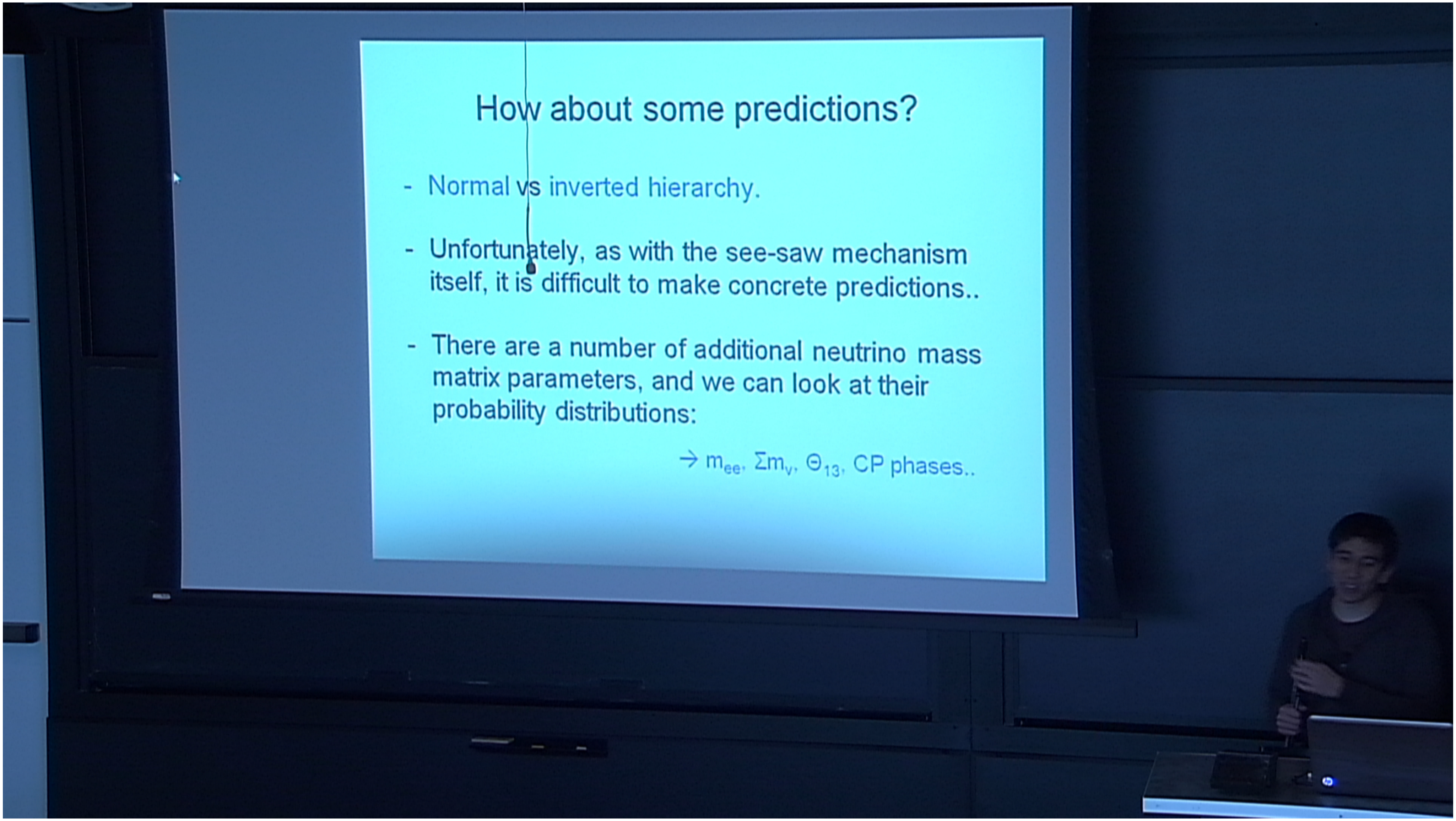

The main result..
- Look at the probability of obtaining two "large" mixing angles as a function of N:
→ Take "large" to mean "as big as the real world constraints".
Fraction of Matrices
0.00
0.05
0.10
0.15
0
20
40
60
80
100
Number of Heavy Neutrinos



- Can also look at the mass hierarchy..
Probability Density
N = 100
N = 30
N = 10
N = 3
$\log_{10}(\Delta m^2_{32}/\Delta m^2_{21})$


..or distribution of mixing angles..
N =3
N =10
N =30
N =100
sin²(2θ)


How about some predictions?
- Normal vs inverted hierarchy.
- Unfortunately, as with the see-saw mechanism itself, it is difficult to make concrete predictions..
- There are a number of additional neutrino mass matrix parameters, and we can look at their probability distributions:
→ m_{ee}, Σm_ν, Θ_{13}, CP phases..

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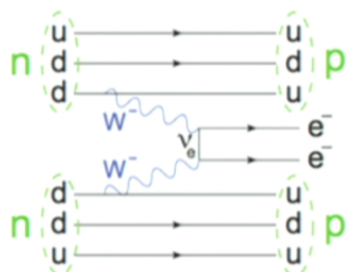
(The 1,1 element of the neutrino mass matrix after diagonalizing the charged leptons)

→ Determines the neutrinoless double beta decay rate.

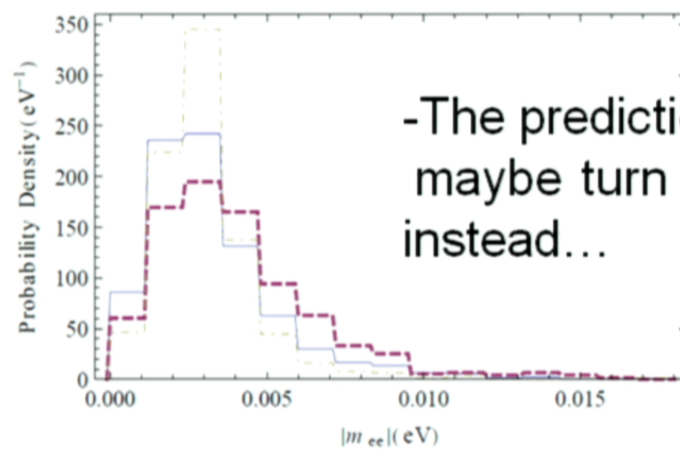


- Determines Majorana vs Dirac.

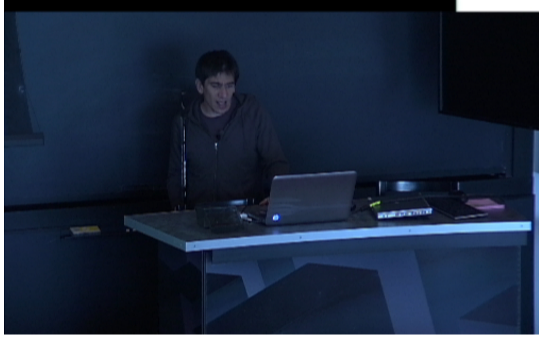
- Must be larger than a few x 10^{-2} eV in order that the effect may be seen in planned experiments.

Our distribution:

$m_{ee} \sim 4 \pm 2$ meV



-The prediction is small.. maybe turn to cosmology instead…



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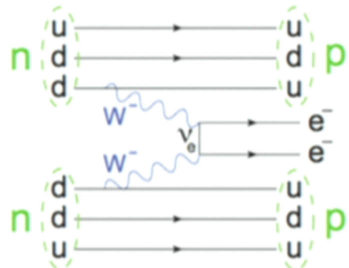
(The 1,1 element of the neutrino mass matrix after diagonalizing the charged leptons)

→ Determines the neutrinoless double beta decay rate.

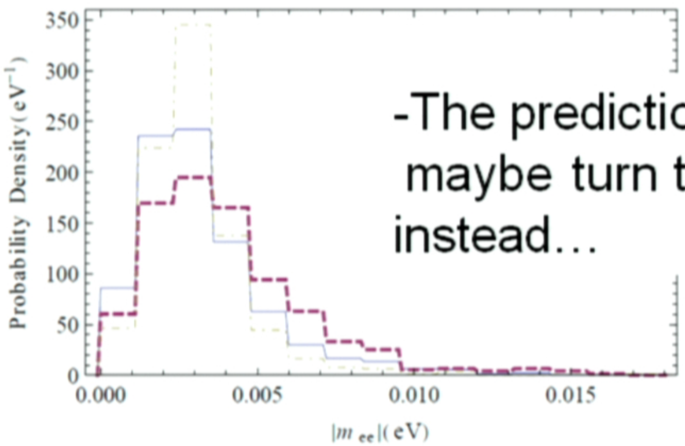


- Determines Majorana vs Dirac.

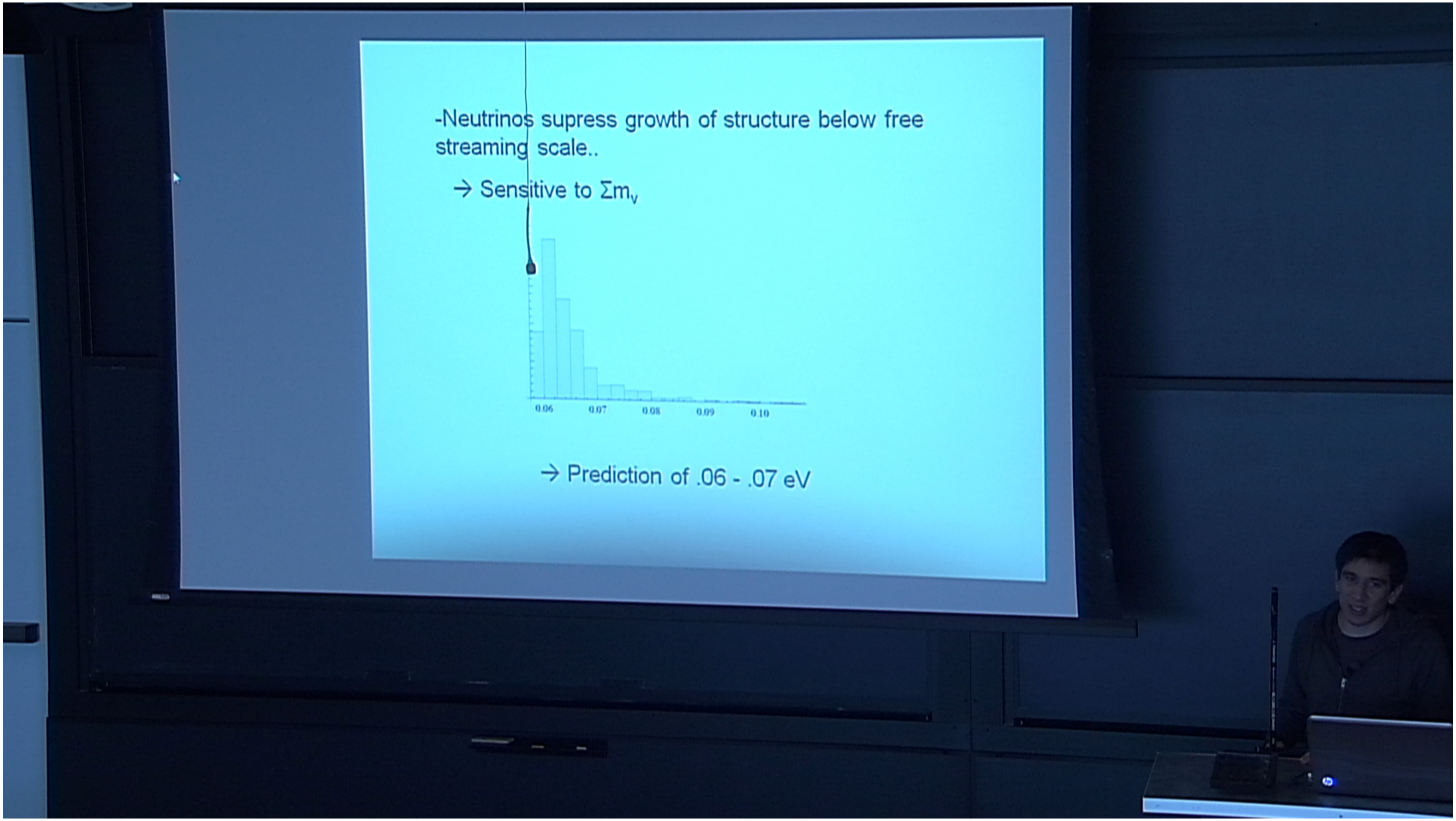
- Must be larger than a few x 10^{-2} eV in order that the effect may be seen in planned experiments.

Our distribution:

$m_{ee} \sim 4 \pm 2$ meV

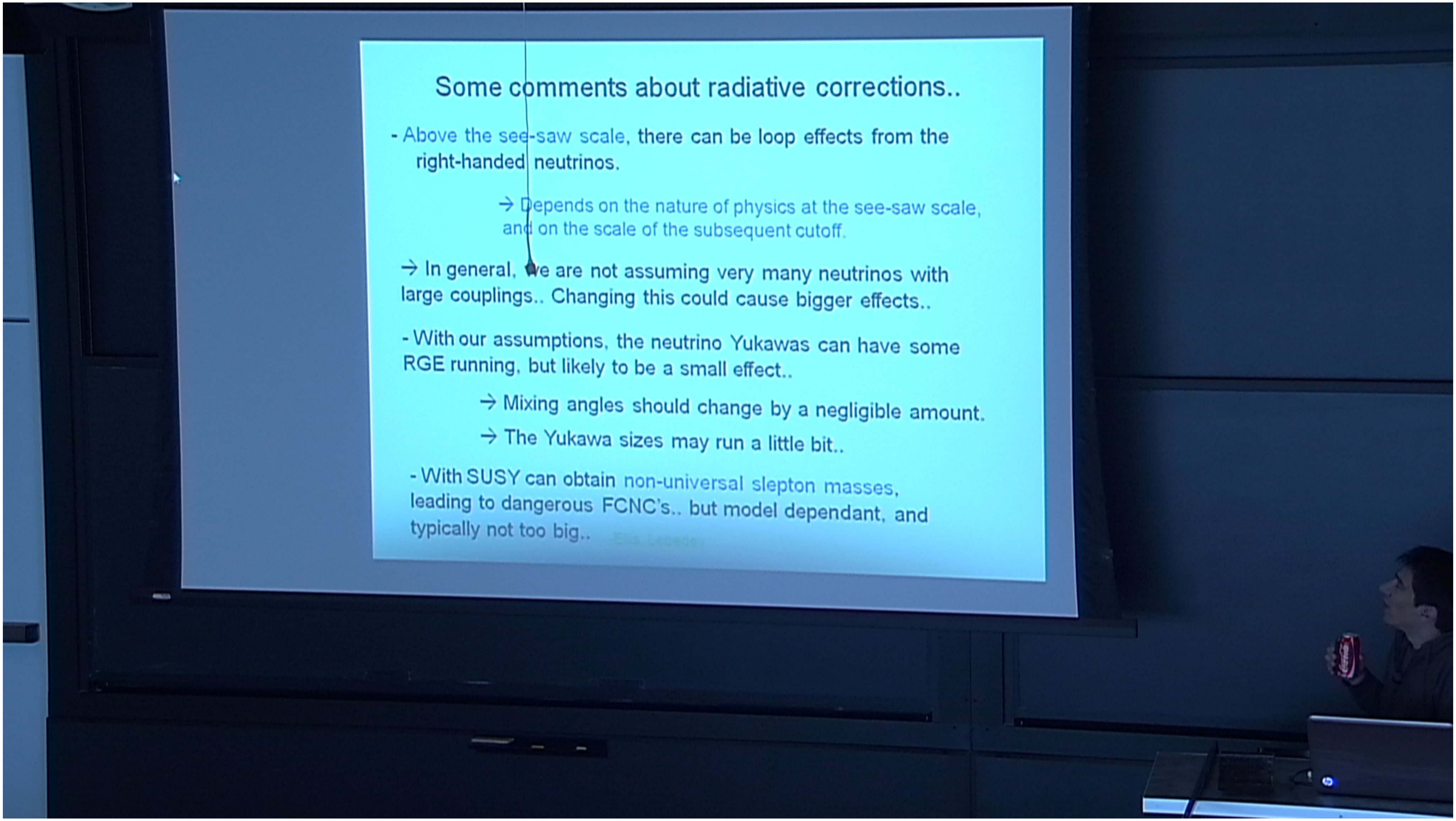


-The prediction is small.. maybe turn to cosmology instead…



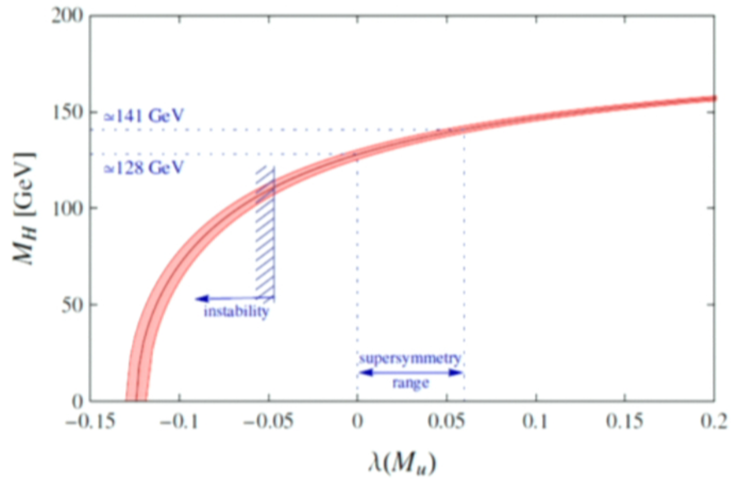
-Neutrinos supress growth of structure below free streaming scale..
→ Sensitive to Σm_ν
0.06
0.07
0.08
0.09
0.10
→ Prediction of .06 - .07 eV


- Θ_{13} distribution:
Cumulative Probability
0.14
0.12
0.10
0.08
0.06
0.04
0.02
0.00
0.00
0.05
0.15
0.20
$\sin(\theta_{13})$
- N = 30
- N = 100
→ Prefer large angles..


Some comments about radiative corrections..
- Above the see-saw scale, there can be loop effects from the right-handed neutrinos.
→ Depends on the nature of physics at the see-saw scale, and on the scale of the subsequent cutoff.
→ In general, we are not assuming very many neutrinos with large couplings.. Changing this could cause bigger effects..
- With our assumptions, the neutrino Yukawas can have some RGE running, but likely to be a small effect..
→ Mixing angles should change by a negligible amount.
→ The Yukawa sizes may run a little bit..
- With SUSY can obtain non-universal slepton masses, leading to dangerous FCNC's.. but model dependant, and typically not too big..

-Without high scale SUSY, can obtain Higgs quartic corrections..

→In the standard model, can cause problems.. the Higgs may be predicted to be too heavy given LHC constraints..



$$\Delta\lambda \sim \frac{1}{6\pi^2} N_{\rm big}^2 \lambda_{\rm big}^4 \log(\Lambda/M)\text{...perhaps} \sim (\frac{\rm N}{300})^2 \log(\Lambda/{\rm M})$$

→ Depends on the maximum Yukawa size, high scale physics, and possible cancellations..

Conclusions

- The neutrinos apparently have several unusual flavor properties; very small masses, and large mixing angles.
- Can be explained in the see-saw framework, if we assume that there are many (perhaps 10-100) gauge singlet right-handed neutrinos.
- The framework is flexible, with non-hierarchical masses generically coming from a sum of many hierarchical terms.
- We predict a normal hierarchy, $m_{ee} \sim 4 \pm 2$ meV, $\Sigma m_\nu \sim .06\text{-}.07$eV.
- Large values of Θ_{13} are preferred, in good agreement with recent experimental results.