Title: Affine Quantum Gravity: Review and Recent Results

Date: May 16, 2012 04:00 PM

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Abstract: Recent progress in the quantization of nonrenormalizable scalar fields has found that a suitable non-classical modification of the ground state wave function leads to a result that eliminates term-by-term divergences that arise in a conventional perturbation analysis. After a brief review of both the scalar field story and the affine quantum gravity program, examination of the procedures used in the latter surprisingly shows an analogous formulation which already implies that affine quantum gravity is not plagued by divergences that arise in a standard perturbation study. Additionally, guided by the projection operator method to deal with quantum constraints, trial reproducing kernels are introduced that satisfy the diffeomorphism constraints. Furthermore, it is argued that the trial reproducing kernels for the diffeomorphism constraints also satisfy the Hamiltonian constraint as well.
 This talk is based on arXiv:1203.0691 .

Basic Background

Classical actions : $A_g^{\pm} = \int \{\tfrac{1}{2}[\dot{x}^2 - x^2] - g x^{\pm 4}\}\ dt$

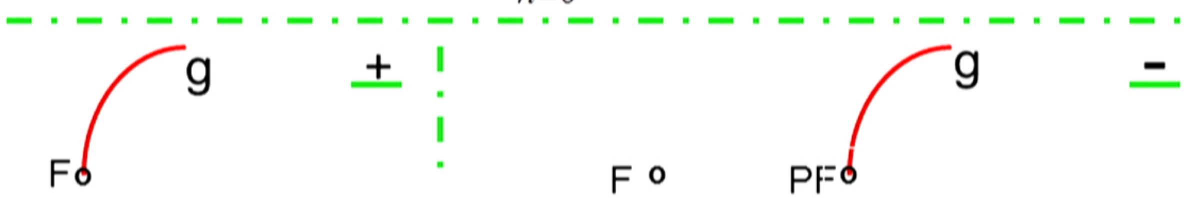
$$\lim_{g\to 0} A_g^{+} = A_o = \int \tfrac{1}{2}[\dot{x}^2 - x^2]\ dt \quad ; \quad \lim_{g\to 0} A_g^{-} = A'_0 \neq A_o$$

Solutions : $x_+(t) = B\cos(t+\beta) \quad ; \quad x_-(t) = \pm |\, B\cos(t+\beta)\,|$

Quantum propagators :

$$\langle x'', T | x', 0 \rangle_+ = \sum_{n=0}^{\infty} h_n(x'') h_n(x')\ e^{-i(n+1/2)T}$$

$$\langle x'', T | x', 0 \rangle_- = \theta(x''x') \sum_{n=0}^{\infty} h_n(x'') h_n(x')\ [1-(-1)^n]\ e^{-i(n+1/2)T}$$



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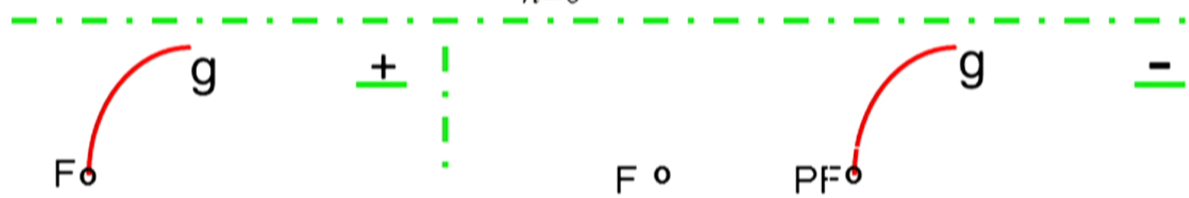

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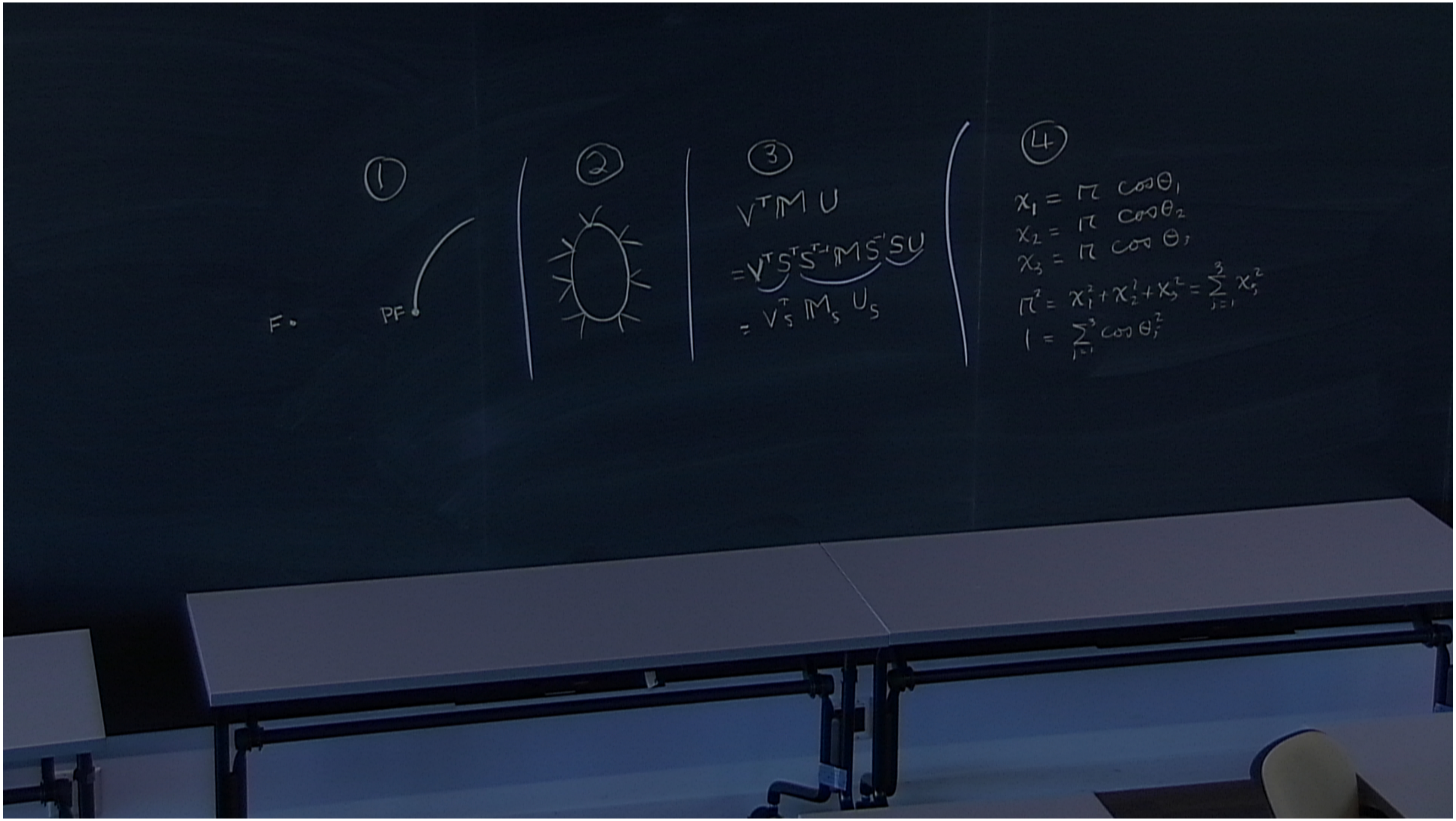

1
F
PF
2
3
$V^T M U$
$= V^T S^T S^{T-1} M S^{-1} S U$
$= V_S^T M_S U_S$
4
$x_1 = r \cos\theta_1$
$x_2 = r \cos\theta_2$
$x_3 = r \cos\theta_3$
$r^2 = x_1^2 + x_2^2 + x_3^2 = \sum_{j=1}^{3} x_j^2$
$1 = \sum_{j=1}^{3} \cos\theta_j^2$

Outline

- Scalar Fields & Measure Mashing **NR FINITE**
- Review of Affine Quantum Gravity **N-Can**
- Measure Mashing and AQG **FINITE**
- Reproducing Kernel Hilbert Spaces **HS REP**
- Diffeomorphism Constraints & Toys **EXAMPLES**
- Functional Integral Formulation **CON TIME REG**
- Enforcing All Constraints & Toys **EXAMPLES**
- Summary **QUESTION TIME**

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Scalar Fields & Measure Mashing

Classical action : $A = \int \{ \frac{1}{2} [\dot{\varphi}^2 - m_0^2 \varphi^2] - g_0 \varphi^4 \} \ dt \ d^3 x$ **PF**

Euclidean action : $I = \int \{ \frac{1}{2} [\dot{\varphi}^2 + m_0^2 \varphi^2] + g_0 \varphi^4 \} \ d^4 x$

Lattice spacetime : $\varphi(x) \to \varphi_k \quad ; \quad k = (\underline{k_0}; \underline{k_1, k_2, k_3}) \quad k^* = (k_0 + 1; k_1, k_2, k_3)$

Lattice action : $I = \Sigma_k \{ \frac{1}{2} [(\varphi_{k^*} - \varphi_k)^2 a^{-2} + m_0^2 \varphi_k^2] + g_0 \varphi_k^4 \} \ a^4$

$$S(h) = \Sigma_{p=0}^{\infty} (M / p!) \int e^{\Sigma_k \{ h_k \varphi_k a^4 - \frac{1}{2} [(\varphi_{k^*} - \varphi_k)^2 a^2 + m_0^2 \varphi_k^2 a^4 \}} [-\Sigma_k g_0 \varphi_k^4 a^4]^p \Pi_k d\varphi_k$$

$$| \langle [\Sigma_k V(\varphi_k) a^4]^p \rangle | \le a^p \Sigma_{k_{01}} \Sigma_{k_{02}} \cdots \Sigma_{k_{0p}} | \langle [\Sigma'_{k_1} V(\varphi_{k_1}) a^3] \cdots [\Sigma'_{k_p} V(\varphi_{k_p}) a^3] \rangle |$$

$$\le a^p \Sigma_{k_{01}} \Sigma_{k_{02}} \cdots \Sigma_{k_{0p}} | \langle [\Sigma'_{k_1} V(\varphi_{k_1}) a^3]^p \rangle^{1/p} \cdots \langle [\Sigma'_{k_p} V(\varphi_{k_p}) a^3]^p \rangle^{1/p} |$$

$$\langle [\Sigma'_k V(\varphi_k) a^3]^p \rangle = \int [\Sigma'_k V(\varphi_k) a^3]^p \, \Psi_0^2(\varphi) \, \Pi'_k \, d\varphi_k$$

S=>ST

Scalar Fields & Measure Mashing

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$$|\langle[\Sigma_k V(\varphi_k)a^4]^p\rangle| \le a^p \Sigma_{k_{01}}\Sigma_{k_{02}}\cdots\Sigma_{k_{0p}} |\langle[\Sigma'_{k_1} V(\varphi_{k_1})a^3]\cdots[\Sigma'_{k_p} V(\varphi_{k_p})a^3]\rangle|$$

$$\le a^p \Sigma_{k_{01}}\Sigma_{k_{02}}\cdots\Sigma_{k_{0p}} |\langle[\Sigma'_{k_1} V(\varphi_{k_1})a^3]^p\rangle^{1/p}\cdots\langle[\Sigma'_{k_p} V(\varphi_{k_p})a^3]^p\rangle^{1/p}|$$

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S=>ST

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Scalar Fields & Measure Mashing

Classical action : $A = \int \{\tfrac{1}{2}[\dot{\varphi}^2 - m_0^2\varphi^2] - g_0\varphi^4\} dt\ d^3x$ **NR**

Lattice Hamiltonian : $\mathrm{H} = \Sigma'_k \{\tfrac{1}{2}[\pi_k^2 + m_0^2\varphi_k^2] + g_0\varphi_k^4\}\ a^3$

FT of FGSD : $C_0(f) = M_0 \int e^{\Sigma'_k[if_k\varphi_k a^3 - m_0\varphi_k^2 a^3]} \Pi_k d\varphi_k \rightarrow e^{-1/4m_0 \int f(x)^2 d^3x}$

FT of GSD : $C(f) = M \int e^{\Sigma'_k[if_k\varphi_k a^3 - W(\varphi_k, a)]} \Pi_k d\varphi_k \rightarrow e^{-1/4\mathrm{m} \int f(x)^2 d^3x}$ **G CLT**

Divergences : $I_p = M_0 \int [\Sigma'_k \varphi_k^2 a^3]^p e^{-\Sigma'_k \phi_k^2 a^3} \Pi'_k\ d\varphi_k = O(N'^p)$

New coordinates : $\varphi_k = \kappa\eta_k\ ,\ \kappa^2 = \Sigma'_k \varphi_k^2\ ,\ 1 = \Sigma'_k \eta_k^2$

Divergences : $I_p = M_0 \int [\kappa^2 a^3]^p e^{-\kappa^2 a^3} \kappa^{(N'-1)} d\kappa\ \ d\mu(\eta) = O(N'^p)$

Divergences : Cause & Cure : $\kappa^{(N'-1)} \rightarrow \kappa^{(R-1)}\ ,\ R = 2ba^3N' < \infty$

Scalar Fields & Measure Mashing

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Scalar Fields & Measure Mashing

Free to pseudofree: $M\,\Pi'_k\{e^{-m_0\varphi_k^2a^3}\} \to M'\,\Pi'_k\{e^{-m_0\varphi_k^2a^3}\,|\varphi_k|^{-(1-2ba^3)}\}$

FT of PFGSD: $C'_0(f) = M'\int e^{\Sigma'_k i f_k\varphi_k a^3 - m_0\Sigma'_k\varphi_k^2a^3}\,\Pi'_k|\varphi_k|^{-(1-2ba^3)}\,d\varphi_k$

$$[m_0 = ba^3m\;, \quad \lambda = \varphi a^3]$$

$$\to \exp\{-b\int d^3x\int[1-\cos(f(x)\lambda)]e^{-mb\lambda^2}\,d\lambda/|\lambda|\}$$

P

Divergences ? : $M_0\int[\Sigma'_k\varphi_k^2a^3]^p\,e^{-\Sigma'_k\phi_k^2a^3}\,\Pi'_k|\varphi_k|^{-(1-2ba^3)}\,d\varphi_k$

Divergences ? : $M_0\int[\kappa^2a^3]^p\,e^{-\kappa^2a^3}\,\kappa^{-(N'-R)}\,\Pi'_k|\eta_k|^{-(1-2ba^3)}$ **MM**

$$\times\,\kappa^{(N'-1)}d\kappa\;\;2\delta(1-\Sigma'_k\eta_k^2)\,\Pi'_k\,d\eta_k = O(1)\;,$$

NO!

EXAMPLE of *AFFINE QUANTIZATION* :

OVERCOMES TRIVIALITY and NONRENORMALIZABILITY

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Scalar Fields & Measure Mashing

Free to pseudofree: $M\,\Pi'_k\,\{e^{-m_0\varphi_k^2a^3}\} \rightarrow M'\Pi'_k\,\{e^{-m_0\varphi_k^2a^3}\;|\varphi_k|^{-(1-2ba^3)}\}$

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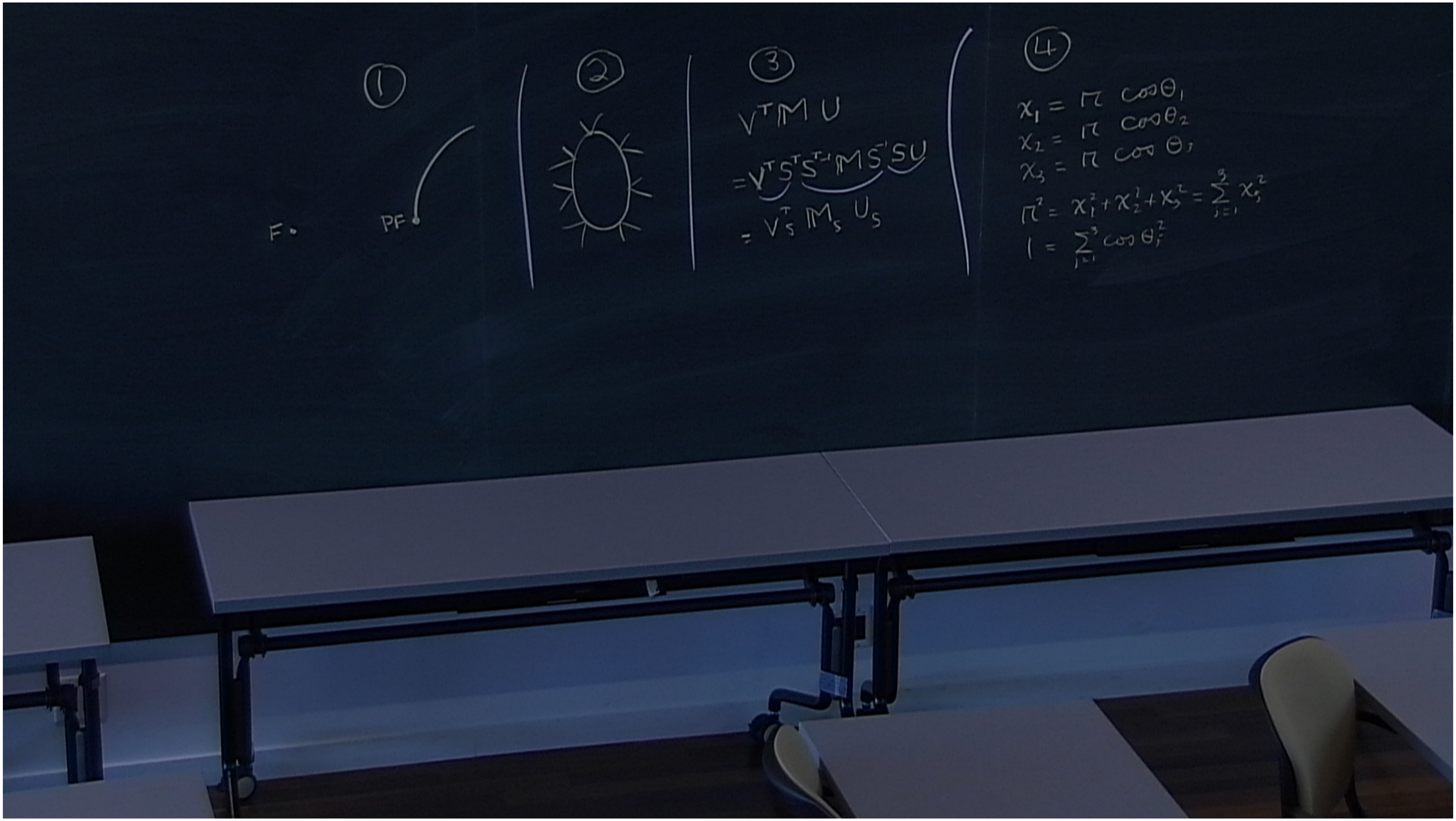
OVERCOMES TRIVIALITY and NONRENORMALIZABILITY 8


①
②
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1 = Σ_{i=1}^{3} cos θ_i^2

Review of Affine Quantum Gravity

Canonical kinematical variables: $g_{ab}(x)$, $\pi^{ac}(x)$

Metric positivity: $u^a g_{ab}(x) u^b > 0$ if $u^a \neq 0$

Canonical transformation: $\pi^{\%} \equiv \int u_{cd}(y)\pi^{cd}(y)\ d^3y$

$$e^{\{\bullet,\pi^{\%}\}} g_{ab}(x) \equiv g_{ab}(x) + \{g_{ab}(x),\pi^{\%}\} + \tfrac{1}{2}\{\{g_{ab}(x),\pi^{\%}\},\pi^{\%}\} + \cdots$$

$$= g_{ab}(x) + u_{ab}(x)$$ [May *fail* positivity]

BAD NEWS

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Scalar Fields & Measure Mashing

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MM

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EXAMPLE of *AFFINE QUANTIZATION* :

OVERCOMES TRIVIALITY and NONRENORMALIZABILITY

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Review of Affine Quantum Gravity

Kinematical variables: $g_{ab}(x)$, $\pi^{ac}(x)g_{bc}(x) = \pi_b^a(x)$

Metric positivity: $u^a g_{ab}(x) u^b > 0$ if $u^a \neq 0$

Canonical transformation: $\pi^* \equiv \int \gamma_b^a(y)\pi_a^b(y)\; d^3y$

$$e^{\{\bullet,\pi^*\}} g_{ab}(x) \equiv g_{ab}(x) + \{g_{ab}(x), \pi^*\} + \tfrac{1}{2}\{\{g_{ab}(x),\pi^*\},\pi^*\} + \cdots$$

$$= M_a^c(x) g_{cd}(x) M_b^d(x) \quad , \quad [M(x) = e^{\gamma(x)/2}]$$

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$$= g_{ab}(x) + u_{ab}(x)$$

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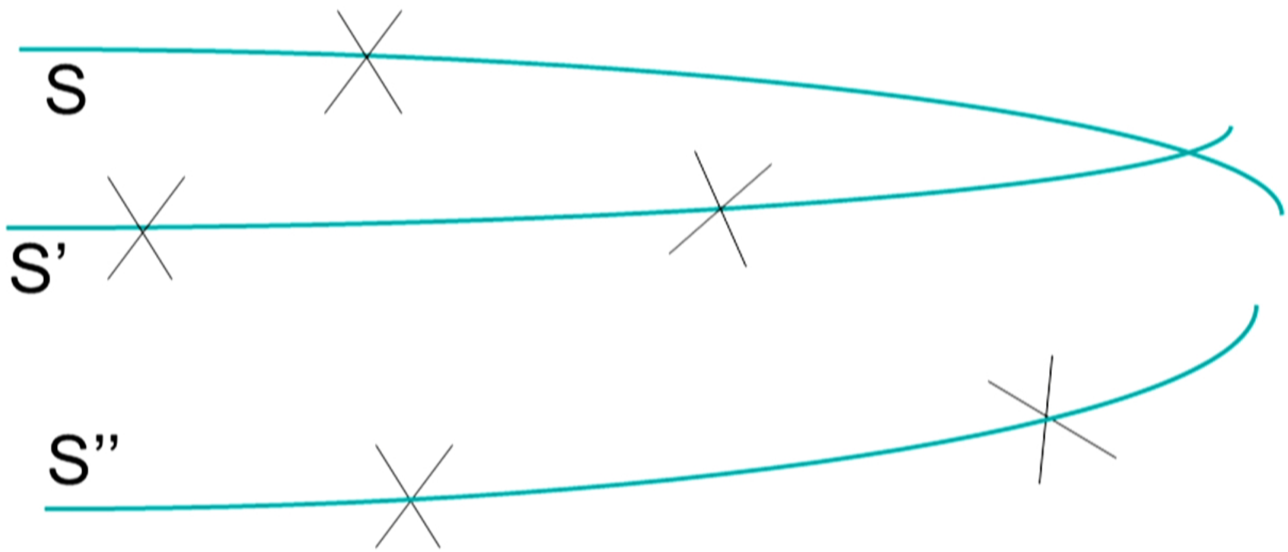
Review of Affine Quantum Gravity

Space-like surfaces: S, S', S'' , etc.



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Review of Affine Quantum Gravity

Affine relations : $\pi^a_b(x) \to \hat{\pi}^a_b(x)$, $g_{ab}(x) \to \hat{g}_{ab}(x)$

$$[\hat{\pi}^a_b(x), \hat{\pi}^c_d(y)] = i\tfrac{1}{2}\{\delta^c_b \hat{\pi}^a_d(x) - \delta^a_d \hat{\pi}^c_b(x)\}\delta(x,y)$$

L.a.

$$[\hat{g}_{ab}(x), \hat{\pi}^c_d(y)] = i\tfrac{1}{2}\{\delta^c_a \hat{g}_{db}(x) + \delta^c_b \hat{g}_{ad}(x)\}\delta(x,y)$$

c.c.r.

$$[\hat{g}_{ab}(x), \hat{g}_{cd}(y)] = 0$$

Unitary transformation : $\hat{\pi}^* \equiv \int \gamma^a_b(y)\, \hat{\pi}^b_a(y)\ d^3y$

$$e^{i\hat{\pi}^*}\hat{g}_{ab}(x)e^{-i\hat{\pi}^*} = \hat{g}_{ab}(x) - i[\hat{g}_{ab}(x), \hat{\pi}^*] + \tfrac{1}{2}i^2[[\hat{g}_{ab}(x), \hat{\pi}^*], \hat{\pi}^*] + \cdots$$

$$= M^c_a(x)\hat{g}_{cd}(x)M^d_b(x) \quad , \quad [M(x) = e^{\gamma(x)/2}]$$

Review of Affine Quantum Gravity

Affine coherent states : $|\pi, g\rangle \equiv e^{i\hat{g}^*} e^{-i\hat{\pi}^*} |\eta\rangle$

$$\hat{g}^* \equiv \int \pi^{ab}(y)\hat{g}_{ab}(y)\ d^3y \quad , \quad \hat{\pi}^* \equiv \int \gamma_b^a(y)\hat{\pi}_a^b(y)\ d^3y$$

Fiducial vector : $\langle\eta|\hat{g}_{ab}(x)|\eta\rangle = \tilde{g}_{ab}(x) \quad , \quad \langle\eta|\hat{\pi}_b^a(x)|\eta\rangle = 0$

Choose $|\eta\rangle$ as an extremal weight vector : $[M(x) = e^{\gamma(x)/2}]$

Coherent state overlap function : $g_{ab}(x) \equiv M_a^c(x)\tilde{g}_{cd}(x)M_b^d(x)$

$$\langle\pi'', g''|\pi', g'\rangle = \exp\Big[-2\int b(x)\ d^3x$$

$$\times \ln\left(\frac{\det\{\frac{1}{2}[g''^{ab}(x) + g'^{ab}(x)] + \frac{1}{2}ib(x)^{-1}[\pi''^{ab}(x) - \pi'^{ab}(x)]\}}{[\det\{g''^{ab}(x)\}\ \det\{g'^{ab}(x)\}]^{1/2}}\right)\Big]$$

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Review of Affine Quantum Gravity

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Measure Mashing and AQG

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$$\times \ln\Big(\frac{\det\{\frac{1}{2}[\bar{g}''^{ab}(x) + \bar{g}'^{ab}(x)] + \frac{1}{2} i b(x)^{-1}[\bar{\pi}''^{ab}(x) - \bar{\pi}'^{ab}(x)]\}}{[\det\{g''^{ab}(x)\}\;\det\{g'^{ab}(x)\}]^{1/2}}\Big)\Big]$$

ICT

$$\hat{g}_{ab}(x) \to k_{n\;ab}\;,\quad k_n = \{k_{n\;ab}\} > 0\;,\quad dk_n = \Pi_{a\le b}\, dk_{n\;ab}$$

$$= \lim_{\Delta\to 0} \int_+ \Pi_n C_n e^{-i\Delta Tr[(\pi_n'' - \pi_n')k_n] - b_n \Delta Tr[(g_n^{-1}{}'' + g_n^{-1}{}')k_n]} \det(k_n)^{2(b_n\Delta - 1)} dk_n$$

$$\kappa^{(6|N|-1)} \Pi_n \det(k_n)^{2(b_n\Delta-1)} = \Pi_n \det(\eta_n)^{2(b_n\Delta-1)} \kappa^{(6b_n\Delta-1)}$$ **MM**

$$k_{n\;ab} = \kappa\eta_{n\;ab}\;,\quad \kappa^2 = \Sigma_n \Sigma_{a\le b} k_{n\;ab}^2\;,\quad \underline{R = 6\Sigma_n b_n \Delta < \infty}$$ **DF**

18

Measure Mashing and AQG

Coherent state overlap function : $g_{ab}(x) \equiv M_a^c(x)\widetilde{g}_{cd}(x)M_b^d(x)$

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$$\times \ln\left(\frac{\det\{\frac{1}{2}[\overline{g''^{ab}}(x) + \overline{g'^{ab}}(x)] + \frac{1}{2} i b(x)^{-1} [\overline{\pi''^{ab}}(x) - \overline{\pi'^{ab}}(x)]\}}{[\det\{g''^{ab}(x)\}\ \det\{g'^{ab}(x)\}]^{1/2}} \right) \Big]$$

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$$\hat{g}_{ab}(x) \rightarrow k_{n\ ab}\ , \quad k_n = \{k_{n\ ab}\} > 0\ , \quad dk_n = \Pi_{a\le b}\, dk_{n\ ab}$$

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DF 18

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DF

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Reproducing Kernel Hilbert Spaces

$L-$dim. topological space : $\mathfrak{L}$, $\ell \in \mathfrak{L}$, $\ell' \to \ell = (l_1, \ldots, l_L)$

Cont. func. of pos. type : $\mathcal{K}(l';l)$, $\Sigma_{i,j=1}^{J,J} \overline{\alpha}_i \, \alpha_j \mathcal{K}(l_i;l_j) \geq 0$

GNS Theorem : $\exists\, \mathbf{H}\ \&\ |l\rangle \in \mathbf{H}$: $\mathcal{K}(l';l) \equiv \langle l'|l\rangle$

Functional representation of $\mathbf{H}$: $[I, J < \infty]$

$$|\psi\rangle = \Sigma_{i=1}^{I} \alpha_i \, |l_{(i)}\rangle \quad , \quad |\varphi\rangle = \Sigma_{j=1}^{J} \beta_j \, |l_{[j]}\rangle$$

$$\psi(l) \equiv \Sigma_{i=1}^{I} \alpha_i \, \langle l | l_{(i)} \rangle = \langle l | \psi \rangle, \quad \varphi(l) \equiv \Sigma_{j=1}^{J} \beta_j \, \langle l | l_{[j]} \rangle = \langle l | \varphi \rangle$$

$$(\psi, \varphi) \equiv \Sigma_{i,j=1}^{I,J} \overline{\alpha}_i \, \beta_j \, \langle l_{(i)} | l_{[j]} \rangle = \langle \psi | \varphi \rangle$$

Complete space by including limits of Cauchy sequences

Everything from the RK : $\mathcal{K}(l';l)$; $\langle \pi', g' | \pi, g \rangle$

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Reproducing Kernel Hilbert Spaces

Reproducing kernel : $\langle \pi'', g'' | \pi', g' \rangle = N'' N' \langle g'' + i\pi''/b | g' - i\pi'/b \rangle$

Rescaling of RK : $\langle g'' + i\pi''/b | g' - i\pi'/b \rangle$

Reduction of RK : $\langle g'' | g' \rangle \equiv \langle g'' + i\pi''/b | g' - i\pi'/b \rangle |_{\pi''=\pi'=0}$

spans the same abstract Hilbert space $\mathbf{H}$

$$\langle \pi, g | \hat{g}_{ab}(x) | \pi, g \rangle = g_{ab}(x)$$

$$\langle \pi, g | \hat{\pi}^a_c(x) | \pi, g \rangle = \pi^{ab}(x) g_{bc}(x) \equiv \pi^a_c(x)$$

Projection operator : $\mathsf{E} = \mathsf{E}^{+} = \mathsf{E}^2$

Constraints lead to subspace : $\mathbf{H}_{phys} = \mathsf{E}\,\mathbf{H}$

Subspace RK : $\langle g'' + i\pi''/b | \mathsf{E} | g' - i\pi'/b \rangle$

Reduced subspace RK : $\langle g'' | \mathsf{E} | g' \rangle$

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Constraint Example

Original RK : $\langle \vec{z}'' | \vec{z}' \rangle = \exp[(\vec{z}'' \cdot \vec{z}')]$, $\vec{z} \in V(N, \mathbf{R})$, $N \geq 2$

Rotation : $\vec{z}'' \to O\vec{z}''$ & $\vec{z}' \to O\vec{z}'$, $O \in \mathrm{O}(N, \mathbf{R})$

Invariance : $\langle \vec{z}'' | \vec{z}' \rangle = \exp[(O\vec{z}'' \cdot O\vec{z}')] = \langle O\vec{z}'' | O\vec{z}' \rangle$

Constraints : $\mathbf{J} = 0 \Rightarrow$ separate invariance for *each* vector

Projection operator : $\mathsf{E} = O\mathsf{E} = \mathsf{E}O = O\mathsf{E}O$

$$\langle \vec{z}'' | \mathsf{E} | \vec{z}' \rangle = \langle O\vec{z}'' | \mathsf{E} | \vec{z}' \rangle = \langle \vec{z}'' | \mathsf{E} | O\vec{z} \rangle = \langle O\vec{z}'' | \mathsf{E} | O\vec{z}' \rangle$$

$$\langle \vec{z}'' | \mathsf{E} | \vec{z}' \rangle = \int \langle \vec{z}'' | O(g) | \vec{z}' \rangle dg = M \int \exp[|\vec{z}''||\vec{z}'| \cos(\theta)] \sin(\theta)^{N-2} d\theta$$

$\langle \vec{z}'' | \mathsf{E} | \vec{z}' \rangle \approx \underline{\exp[A\ (\vec{z}'' \cdot \vec{z}'')(\vec{z}' \cdot \vec{z}')]}$: $\langle \vec{z}'' | \mathsf{E}_T | \vec{z}' \rangle \approx$

$\underline{\Sigma_{l=1}^{L} r_l [(\vec{z}'' \cdot \vec{z}'')(\vec{z}' \cdot \vec{z}')]^l}$; $\underline{\exp[B\ (\vec{z}'' \cdot \vec{z}'')^7 (\vec{z}' \cdot \vec{z}')^7]}$; $\cdots$

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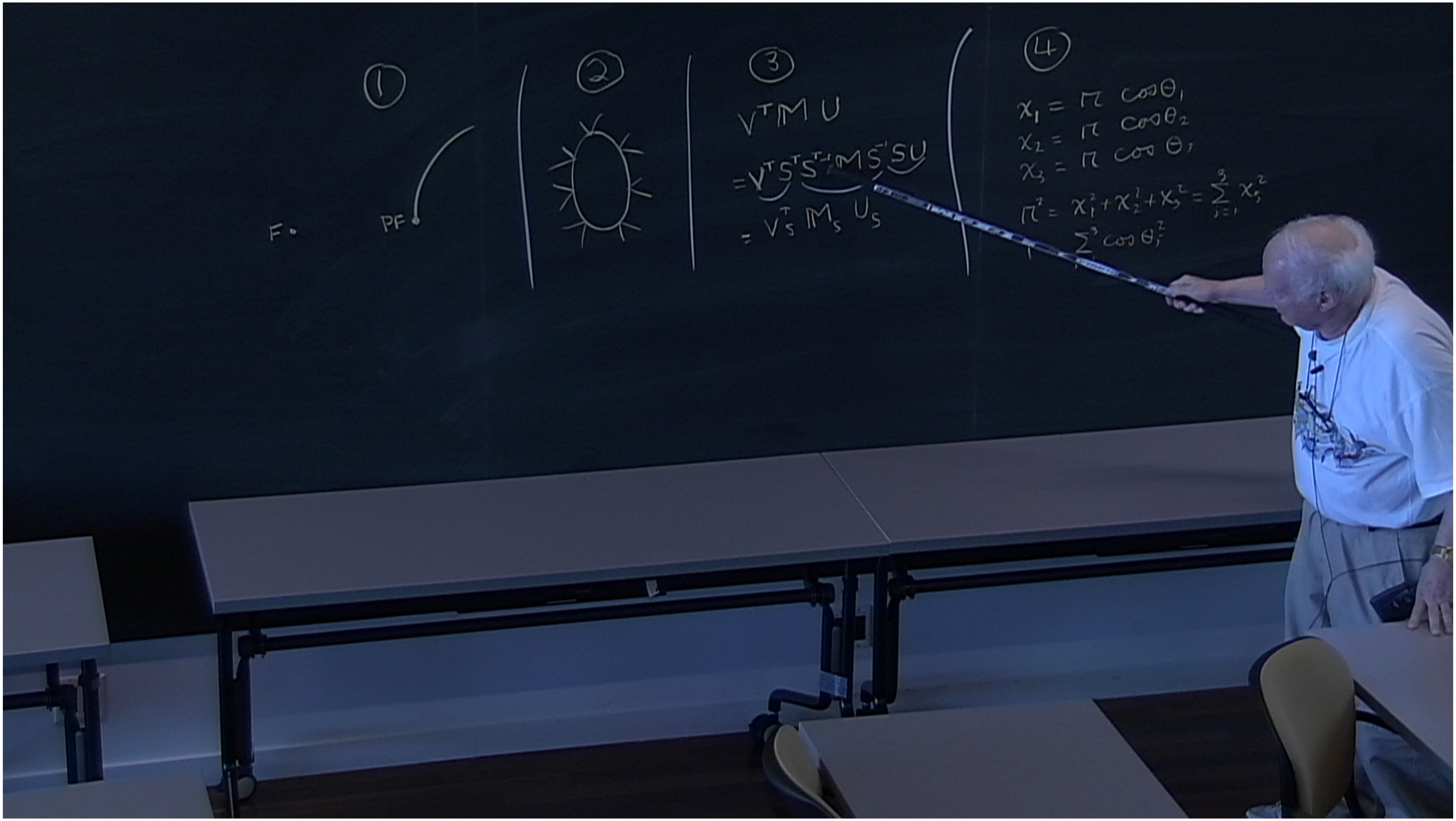
$\Sigma_{l=1}^{L} r_l [(\vec{z}'' \cdot \vec{z}'')(\vec{z}' \cdot \vec{z}')]^l$; $\exp[B\ (\vec{z}'' \cdot \vec{z}'')^7 (\vec{z}' \cdot \vec{z}')^7]$; $\cdots$


1
F
PF
2
3
V^T M U
= V^T S^T S^T-1 M S^-1 S U
= V_s^T M_s U_s
4
x_1 = r cos θ_1
x_2 = r cos θ_2
x_3 = r cos θ_3
r^2 = x_1^2 + x_2^2 + x_3^2 = Σ x_i^2
Σ cos θ_i^2

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Diffeomorphism Constraints & Toys

CSOF : $\tilde{x} = \tilde{x}(x)$; $\langle \pi', g' | \pi, g \rangle = \langle \tilde{\pi}', \tilde{g}' | \tilde{\pi}, \tilde{g} \rangle$

$$\tilde{\pi}^{ab}(\tilde{x}) / \tilde{b}(\tilde{x}) = \frac{\partial \tilde{x}^a}{\partial x^c} \frac{\partial \tilde{x}^b}{\partial x^d} \pi^{cd}(x) / b(x) \quad , \quad \tilde{g}_{ab}(\tilde{x}) = \frac{\partial x^c}{\partial \tilde{x}^a} \frac{\partial x^d}{\partial \tilde{x}^b} g_{cd}(x)$$

Diffeomorphism invariant RK :

$$\langle \pi', g' | \mathsf{E} | \pi, g \rangle = \langle \pi', g' | \mathsf{E} | \tilde{\pi}, \tilde{g} \rangle = \langle \tilde{\pi}', \tilde{g}' | \mathsf{E} | \pi, g \rangle = \langle \tilde{\pi}', \tilde{g}' | \mathsf{E} | \tilde{\pi}, \tilde{g} \rangle$$

$$\langle g' | \mathsf{E} | g \rangle = \langle g' | \mathsf{E} | \tilde{g} \rangle = \langle \tilde{g}' | \mathsf{E} | g \rangle = \langle \tilde{g}' | \mathsf{E} | \tilde{g} \rangle$$

Toy models : $\mathsf{E}_T \subset \mathsf{E}$ [plus $\int \sqrt{g(x)}\, d^3x$] [plus derivatives]

$$\langle g' | \mathsf{E}_T | g \rangle = \Sigma_{n=1}^{5} \{ \int R'(x') \sqrt{g'(x')}\; d^3x' \}^n \; \{ \int R(x) \sqrt{g(x)}\; d^3x \}^n$$

$$\langle g' | \mathsf{E}_T | g \rangle = \Sigma_{n=1}^{5} \int\int R'^n(x') R^n(x) \sqrt{g'(x') g(x)}\; d^3x' \; d^3x$$

$$\langle g' | \mathsf{E}_T | g \rangle = \int\int \{ \exp[R'^n(x') R^n(x)] - 1 \} \sqrt{g'(x') g(x)}\; d^3x' \; d^3x$$

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Functional Integral Formulation

Simple affine variables: $[Q, D] = iQ \;,\quad Q > 0$

Affine coherent states: $|p,q\rangle = e^{ipQ} e^{-i\ln(q)D} |\tilde{\beta}\rangle$

ACSOF: $\langle p'',q''|p',q'\rangle \;;\; [q''>0,\; q'>0] \;\; [\hbar = 1]$

$$= \exp\left\{-2\tilde{\beta}\ln\left[\frac{\frac{1}{2}(q''^{-1}+q'^{-1}) + i\frac{1}{2}\tilde{\beta}^{-1}(p''-p')}{(q''q')^{-1/2}}\right]\right\}$$

$$= \lim_{\upsilon\to\infty} \mathfrak{N}\int \exp[-i\int q\dot{p}\; dt]$$

$$\times \exp\{-(1/2\upsilon)\int[\tilde{\beta}^{-1}q^2\dot{p}^2 + \tilde{\beta}q^{-2}\dot{q}^2]\; dt\}\; \mathfrak{D}p\; \mathfrak{D}q$$

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Functional Integral Formulation

$$\langle \pi'', g'' | \pi', g' \rangle \;,\; \langle \pi'', g'' | \mathsf{E} | \pi', g' \rangle \;,\; \langle \pi'', g'' | \mathsf{E}^* | \pi', g' \rangle$$

$$= \lim_{\nu\to\infty} \mathscr{N} \int e^{-i\int g_{ab}\dot{\pi}^{ab} d^3x\, dt - i\int [N^a H_a + NH] d^3x\, dt}$$

$$\times\, e^{-(1/2\nu)\int [b(x)^{-1} g_{bc} g_{da} \dot{\pi}^{ab}\dot{\pi}^{cd} + b(x) g^{bc} g^{da} \dot{g}_{ab} \dot{g}_{cd}] d^3x\, dt}$$

$$\times [\Pi_{x,t;a\le b} d\pi^{ab}(x,t)\;\; dg_{ab}(x,t)]\;\; \mathscr{D}R(N^a, N)$$

Note phase space metric in the regularization 25

Functional Integral Formulation

$$\langle \pi'', g'' | \pi', g' \rangle \,,\ \ \langle \pi'', g'' | \mathsf{E} | \pi', g' \rangle \,,\ \ \langle \pi'', g'' | \mathsf{E}^{*} | \pi', g' \rangle$$

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Note phase space metric in the regularization 27

Enforcing All Constraints & Toys

Expected relations : $\mathsf{E}_T \subset \mathsf{E}$, $\mathsf{E}^* \subset \mathsf{E}$

A possible relation : $\mathsf{E}_T \subset \mathsf{E}^* \subset \mathsf{E}$

Diffeomorphism constraints hold on any space-like surface.

$$\underline{n_\perp^\alpha \sigma_\perp^\beta G_{\alpha\beta}(x) = 0} \Rightarrow n^\alpha \sigma^\beta [G_{\alpha\beta}(x) + \lambda(x) g_{\alpha\beta}(x)] = 0 \Rightarrow$$

$$\Rightarrow u^\alpha v^\beta [G_{\alpha\beta}(x) + \lambda(x) g_{\alpha\beta}(x)] = 0 \Rightarrow \quad G_{\alpha\beta}(x) = -\lambda(x) g_{\alpha\beta}(x) \Rightarrow$$

$$G^\alpha_{\beta;\alpha}(x) = 0 = -[\lambda(x)\delta^\alpha_\beta]_{;\alpha} \Rightarrow \underline{G_{\alpha\beta}(x) + \Lambda g_{\alpha\beta}(x) = 0}$$

K.K.

The Hamiltonian constraint with CC is valid on any space-like surface; if asymptotically flat, then $\Lambda = 0$.

Toy examples already satisfy all constraints with CC.

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Diffeomorphism constraints hold on any space - like surface.

$$\underline{n_\perp^\alpha \sigma_\perp^\beta G_{\alpha\beta}(x) = 0} \Rightarrow n^\alpha \sigma^\beta [G_{\alpha\beta}(x) + \lambda(x) g_{\alpha\beta}(x)] = 0 \Rightarrow$$

$$\Rightarrow u^\alpha v^\beta [G_{\alpha\beta}(x) + \lambda(x) g_{\alpha\beta}(x)] = 0 \Rightarrow \quad G_{\alpha\beta}(x) = -\lambda(x) g_{\alpha\beta}(x) \Rightarrow$$

$$G^\alpha_{\beta;\alpha}(x) = 0 = -[\lambda(x)\delta^\alpha_\beta]_{;\alpha} \Rightarrow \underline{G_{\alpha\beta}(x) + \Lambda g_{\alpha\beta}(x) = 0}$$

K.K.

The Hamiltonian constraint with CC is valid on any space - like surface; if asymptotically flat, then $\Lambda = 0$.

Toy examples already satisfy all constraints with CC .

Speculation

Compact space : $\langle g' | \mathrm{E_T} | g \rangle = \langle\langle g' | g \rangle\rangle$

$$= [\int\int \sqrt{g'(x')}\sqrt{g(x)}\ dx'\,dx]^{1/2}$$

$$\times \exp\{[\int\int \sqrt{g'(x')}\sqrt{g(x)}\ dx'\,dx]^{1/2}\}$$

$$\mathrm{A} \mid g\rangle\rangle = [\int \sqrt{g(x)}\ dx]^{1/2} \mid g\rangle\rangle$$

$$\frac{\langle\langle g \mid \mathrm{A}^{+}\mathrm{A} \mid g\rangle\rangle}{\langle\langle g \mid g\rangle\rangle} = \int \sqrt{g(x)}\ dx$$

$$\mathrm{Spectrum}(\mathrm{A}^{+}\mathrm{A}) = \cancel{0}, 1, 2, 3, \ldots$$

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Speculation

$$\text{Compact space}: \langle g'|\mathrm{E_T}|g\rangle = \langle\langle g'|\, g\rangle\rangle$$

$$= [\int\int \sqrt{g'(x')}\sqrt{g(x)}\ dx'\,dx]^{1/2}$$

$$\times \exp\{[\int\int \sqrt{g'(x')}\sqrt{g(x)}\ dx'\,dx]^{1/2}\}$$

$$\mathrm{A}\,|\,g\rangle\rangle = [\int \sqrt{g(x)}\ dx]^{1/2}\,|\,g\rangle\rangle$$

$$\frac{\langle\langle g\,|\,\mathrm{A}^+\mathrm{A}\,|\,g\rangle\rangle}{\langle\langle g\,|\,g\rangle\rangle} = \int \sqrt{g(x)}\ dx$$

$$\mathrm{Spectrum}(\mathrm{A}^+\mathrm{A}) = \not{0}, 1, 2, 3, \ldots$$

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Summary

- Affine variables preserve strict metric positivity!! *Do your variables do the same?*
- Affine coherent state overlap function is divergence free thanks to mashing!!
- Reproducing kernels are very useful; may have representations via path integrals!!
- Toy diffeomorphism invariant reproducing kernels satisfy all constraints!!

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