Title: The Electron's Link to the Kerr-Newman Metric

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Abstract: TBA

The link between the Kerr-Newman metric and the electron.

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THE MAGIC FIELD

Away from sources the
static electromagnetic field obeys

$$div\mathbf{E} = 0 \text{ and } curl\mathbf{E} = 0$$

further

$$curl\mathbf{B} = 0 \text{ and } div\mathbf{B} = 0$$

so $\mathbf{E} + i\mathbf{B} = -\nabla\Psi$

Where $\nabla^2\Psi = 0.$

What is the simplest complex potential ?

In electrostatics a charge, q at $\mathbf{b}$ has potential

$$\Phi = q/\sqrt{(\mathbf{r}-\mathbf{b})^2}$$

If we make q complex, we introduce a magnetic monopole which destroys parity.

But we could make $\mathbf{b}$ complex.

The real part merely shifts the origin, so we put

$$\mathbf{b} = \mathbf{ia}$$

and get the field of a charge at an imaginary point.

We orient the z axis along $\mathbf{a}$, so $\mathbf{a} = (0, 0, a)$.

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We write $\mathbf{r} = (x, y, z)$; $R^2 = x^2 + y^2$; $\mu = \cos\theta$

$$\Phi = q/\sqrt{r^2 - 2i\mathbf{r}.\mathbf{a} - a^2} => (q/r)\sum(ia/r)^n P_n(\mu)$$

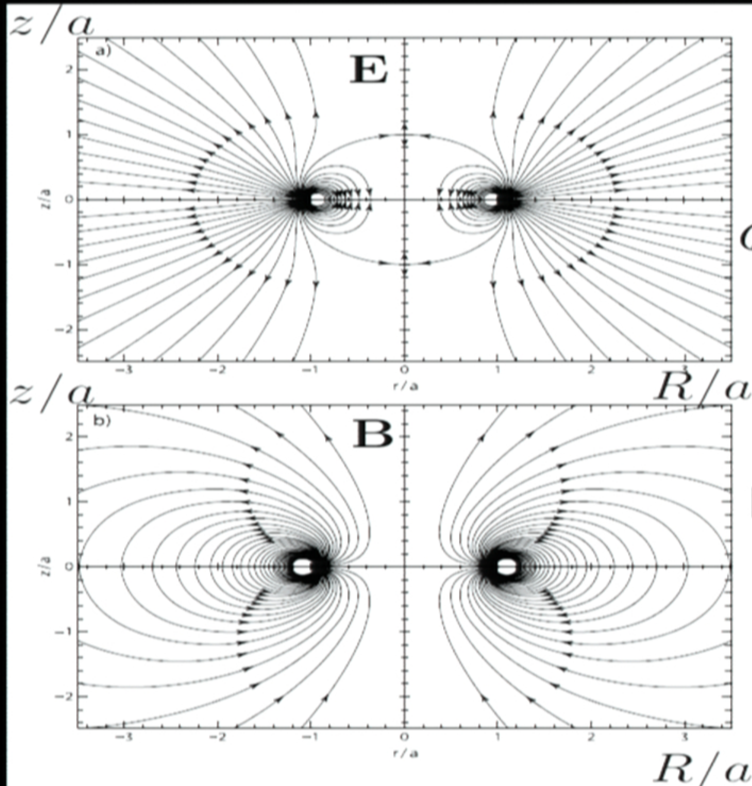
$$\mathbf{E} + i\mathbf{B} = -\nabla\Phi = q\frac{\mathbf{r}-i\mathbf{a}}{[\sqrt{(\mathbf{r}-i\mathbf{a})^2}]^3}, \text{ magnetic dipole } M = qa$$

Then on the plane $z = 0$ the argument of the surd becomes $R^2 - a^2$ which is real.
The surd itself is then pure imaginary inside the singular ring $R = a$ and real outside it.
Thus Inside the electric potential is zero, while $\mathbf{B}$ is along the plane. Outside $\mathbf{E}$ is along the plane while the magnetic field is downward.

Any square root carries a sign ambiguity

We put the sign change on the disk inside R=a.
The fields change sign there but
no B across the disk => no magnetic monopoles

Outside the disk the magnetic field is
everywhere downwards.
How does it get back up ?



On upper side of the disk

$$\mathbf{E} = -q\mathbf{a}/(a^2 - R^2)^{3/2}$$

so the total charge within R is

$$Q(< R) = -q[(1 - R^2/a^2)^{-1/2} - 1]$$

negative! provided $R < a.$

But for $R > a, \quad Q(< R) = +q.$

The current is that formed by this charge rotating uniformly and reaching c at the edge. All the magnetic field lines go back up through the disk's edge.

How can this strange charge distribution be made ?

When I was at school !

If electron's mass is electrical .

$$e^2/(2r) = m_e c^2$$

this gives a radius of $e^2/(2m_e c^2) = 1.4089701447(29) \times 10^{-13} cm$

But quantum theory takes over at the much larger

Compton radius

$$\hbar/(m_e c) = 3.861592646(5) \times 10^{-11} cm$$

only a small fraction of the electron's mass

is in the field outside that.

Ratio of radii is half the fine structure constant .

Quantum electrodynamics has both h and c in it.

Perhaps its greatest failure is the lack of a theory why

charge comes in units of $e = \sqrt{\hbar c/137.03599967(9)}$.

The electron has a spin angular momentum $\hbar/2$
its magnetic dipole is very nearly one Bohr magneton $e\hbar/(2m_e c)$.
more exactly 1.001 159 652 181 11(74) times that.
One of the triumphs of quantum electrodynamics is the explanation of this correction up to 652!

The ratio of the magnetic dipole of a body to its angular momentum is called the gyromagnetic* ratio.
This is often described by a dimensionless g-factor.

$$\gamma = \frac{magnetic\ moment}{angular\ momentum} = g \times \frac{charge}{2\ c\ mass}.$$

For a classical charge in orbit $g = 1$

Can a g of 2, as for electron spin, occur classically ?

*Occasionally, and more accurately called magnetogyric

In 1963 Kerr's spinning black hole.

In 1965 Newman spinning charged black hole, has both angular momentum and a magnetic moment.

In 1968 Brandon Carter: all these metrics $g = 2$ the same as Dirac's electron.

How can this come out of Classical physics ? For electrons their own gravity is negligible.

What happens if we set G=0 in the Kerr-Newman metric ?

1. The space-time must become flat!
Actually Boyer- Lindquist coordinates become oblate spheroidal coordinates in Minkowski space.

2. The charge and its electric field remains.

3. The electric current and its magnetism remains.

THE FIELD BECOMES THE MAGIC FIELD!

More Magic

Carter showed that both the relativistic Hamilton-Jacobi equation and the Schrodinger equation separate in the Kerr-Newman metric.

Teukolski showed that the zero mass spin 1/2 Dirac equation separates.

Chandrasekhar and Page showed the massive Dirac equation does too.

These properties still hold when G=0.

In the frame that moves with velocity given by $\mathbf{V}/\sqrt{1+V^2/c^2} = \mathbf{E} \times \mathbf{B}/(E^2+B^2)$ the transformed fields $\mathbf{E}', \mathbf{B}'$ are parallel. Gair showed that these directions were normal to spheroids confocal to the disk and the velocities were uniform rotation on each spheroid.

BUT

the total electric field energy, $[1/(8\pi)] \int \mathbf{E^2} dV$,

the total magnetic field energy, $[1/(8\pi)] \int \mathbf{B^2} dV$,

and the angular momentum $[1/(4\pi c)] \int \mathbf{r} \times (\mathbf{E} \times \mathbf{B}) dV$,

all DIVERGE because of the singularity !

Is $g = 2$ a property of general relativity ?

It holds for black holes of any rotation

and it holds for highly relativistic bodies.

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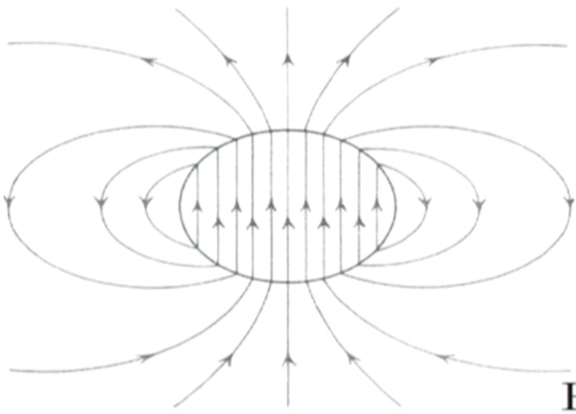
Can we devise a non-singular magic field ?

I looked at charged conducting spheres and disks rotating at tip speeds up to c.

Tiomno kept the magic field externally, cut it on a spheroid and devised a consistent internal field.

For a static conducting sphere the surface charge is uniform. In steady rotation electrical resistance ensures that the charge moves with the sphere. Jackson rotates a spherical shell of uniform surface charge and gets, this



<= magnetic field.

Field Energies $(8\pi)^{-1}\int E^2 dV = q^2/(2a)$

$(8\pi)^{-1}\int B^2 dV = (1/9)q^2 a^{-1} v^2/c^2$

Angular momentum in the field

$\mathbf{L} = (4\pi c)^{-1}\int \mathbf{E}\times\mathbf{B}dV = (2/9)(q^2/c)v/c$

"Fine Structure" $q^2/(2Lc) = 9/4$ *

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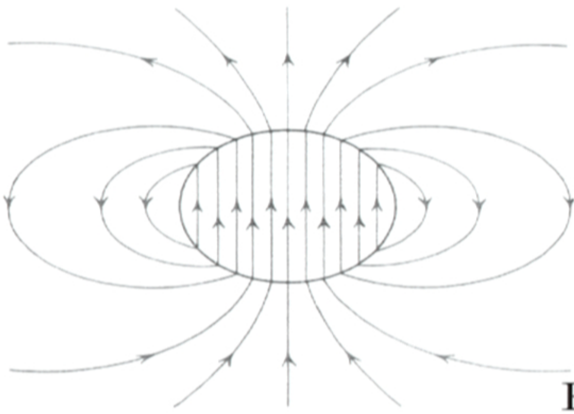

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But is that the solution for a conducting shell ?

No the magnetic force is unbalanced

Rotating a charged conducting spherical shell

Both outside and inside $\mathbf{E} = -\nabla\Phi; \quad \mathbf{B} = -\nabla\chi$ and $\nabla^2\Phi = 0 = \nabla^2\chi$

i)E_θ is continuous across the shell

ii)$(E_r)_{out} - (E_r)_{in} = 4\pi\sigma$

iii)B_r is continuous

iv)$(B_\theta)_{out} - (B_\theta)_{in} = 4\pi J_\phi$

v)$J_\phi = \sin\theta\ \omega\sigma; \quad \omega = a\Omega/c$

vi)$E_\theta + \omega\sin\theta B_r = 0$

v) says the current is due to the moving charge;
vi) says there is no Lorentz force in the conductor.

Some Calculations

$$\Phi_{in} = q/a \sum \phi_{2n}(r/a)^{2n} P_{2n}(\mu); \quad \mu = \cos\theta$$

$$\Phi_{out} = q/r \sum \phi_{2n}(a/r)^{2n} P_{2n}(\mu); \; P_n = \text{Legendre poly.}$$

These ensure that Φ is harmonic, and continuous across the shell. The jump in $\partial\Phi/\partial r$ gives $4\pi \times \sigma$ the surface density of charge whose total is q.

$$4\pi\sigma a^2 = q \sum (4n+1)\phi_{2n} P_{2n}(\mu) \; ; \qquad \phi_0 = 1$$

$$\chi_{in} = -q/a \sum \chi_{2n+1}/(2n+1)(r/a)^{2n+1} P_{2n+1}(\mu)$$

$$\chi_{out} = q/r \sum \chi_{2n+1}/(2n+2)(a/r)^{2n+1} P_{2n+1}(\mu)$$

These ensure that χ is harmonic, and that $\partial\chi/\partial r$ is continuous. **This gives the continuity of the radial magnetic field. The discontinuity in** $B_\theta = -r^{-1}\partial\chi/\partial\theta$ **is**

$$q a^{-2} \sin\theta \sum \frac{4n+3}{(2n+2)(2n+1)} \chi_{2n+1} P'_{2n+1}(\mu) = 4\pi J_\phi$$

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We aim to discover the fields so we solve for

ϕ_{2n} and $\chi_{2n+1} = (4n+3)\eta_{2n+1}$

which turns out to be simpler.

After some work we find

$\phi_{2n} = \omega(\eta_{2n+1} - \eta_{2n-1})$, and

$\frac{(n+3/4)^2}{(n+1)(n+1/2)}\eta_{2n+1} = -\frac{\omega}{4}(\phi_{2n+2} - \phi_{2n})$

which lead to the recurrence relation

$$\eta_{2n+3} + 2[2\omega^{-2}(1 + \frac{1}{16(n+1)(n+1/2)}) - 1]\eta_{2n+1} + \eta_{2n-1} = 0$$

Now $\frac{1}{16(n+1)(n+1/2)} << 1$ for $n > 0$

$= 1/8\ ; \quad n = 0$

$= 1/48; \quad n = 1$

$= 1/120; \quad n = 2$

$= 1/224; \quad n = 3$

so neglect n dependence for n>1

$$\eta_{2n+3} + 2(2\omega^{-2} - 1)\eta_{2n+1} + \eta_{2n-1} = 0$$

the convergent solution to this is

$$\eta_{2n+1} = C\left(-\frac{\omega^2}{1+\sqrt{1-\omega^2}}\right)^n = C(-u)^n$$

We use the exact recurrence relations to correct the monopole and quadrupole terms .

Apart from those correction terms the field outside the sphere is as though it came from a Magic disc of the smaller radius $\sqrt{u}\ a$!

in the limit $\omega \to 1, \quad u \to 1$

and the disk touches the sphere.

At an equatorial speed of 0.93c
the charge density at the poles
becomes zero.
At greater speeds the polar caps
have negative charge
and this spreads to lower latitudes.
The total charge in the equatorial
belt diverges as c is approached!
Gyromagnetic ratio of dipole on string.

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Rotating Disk

Field Energy: $\omega = v/c$

$$W = (8\pi)^{-1} \int (E^2 + B^2) dV = q^2/a \frac{\pi\omega^2}{(1-\omega^2)[\ln(\frac{1+\omega}{1-\omega})]^2}.$$

Now imagine speeding up the disk via a couple (torque) C

$$dL = C dt; \quad dW = C\Omega dt = C(c/a)\omega dt$$

$$dL/dt = (a/c)\omega^{-1} dW/dt;$$

hence $Lc/a = \int_0 \omega^{-1} \frac{dW}{d\omega} d\omega = W/\omega + \int W/\omega^2 d\omega.$

Now $d\ln(\frac{1+\omega}{1-\omega})/d\omega = 2/(1-\omega^2)$;So

Inverse "fine structure" $2Lc/q^2 = \pi \frac{2\omega - (1-\omega^2)\ln\frac{1+\omega}{1-\omega}}{(1-\omega^2)[\ln\frac{1+\omega}{1-\omega}]^2}$

To give 137.036 assuming only this L we need

$$\omega = .999706454$$

$\omega = .999706454$ gives a field energy

$$= L(c/a) \frac{\omega^2}{(\omega - \frac{1}{2}(1-\omega^2)\ln\frac{1+\omega}{1-\omega}}$$

with $L = \hbar/2$ and a given by the electron's dipole moment we get

$$W = 1.001142804 \ m_e c^2$$

Thus all the mass may be in field energy!

the correction is surprisingly close to

$$(g-2)/2 = \ 1.001159652$$

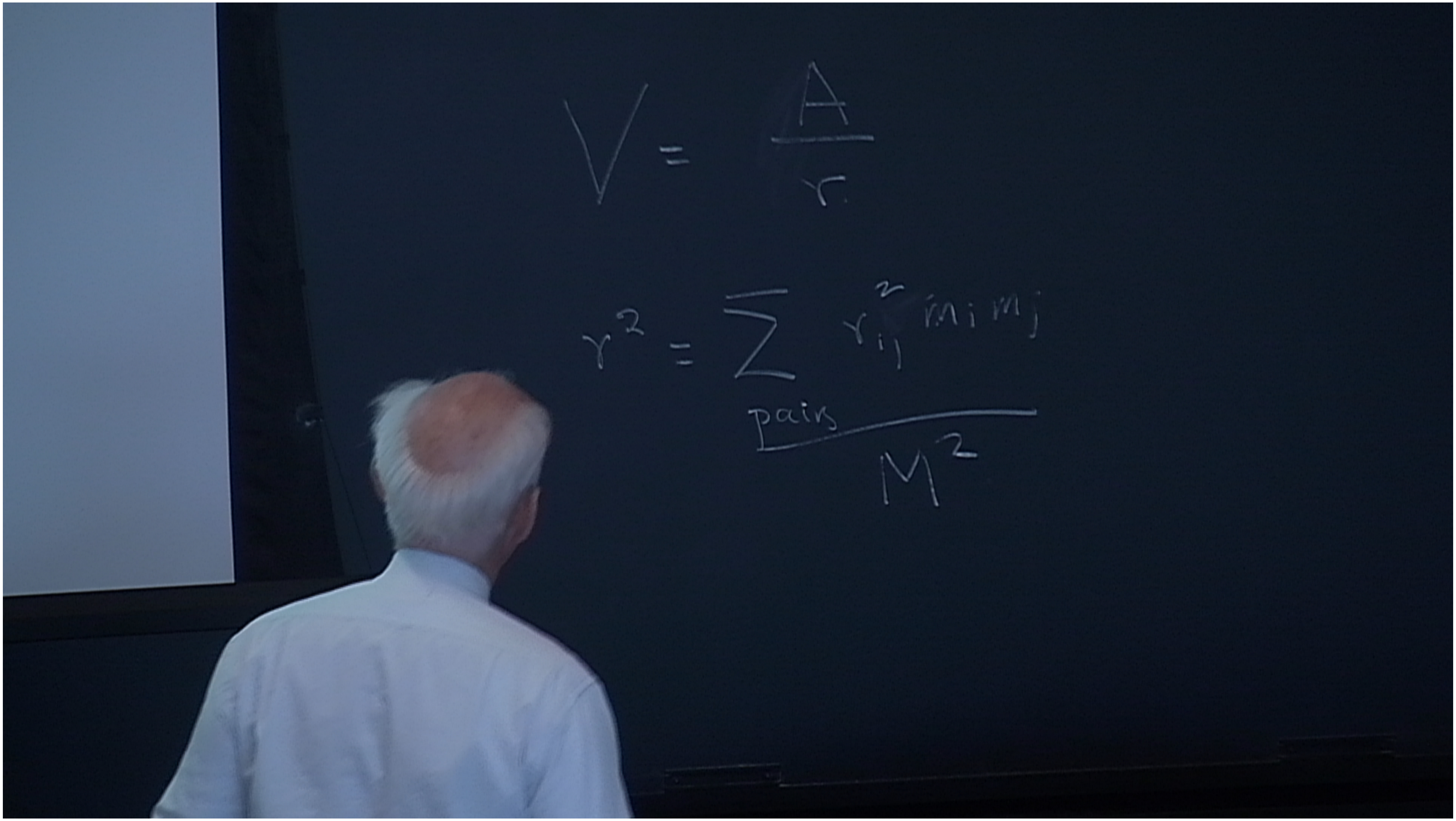
ALL Charge is quantised in units of

$$e = \sqrt{\hbar c/137.035999679}$$

Think of a non-linear topological quantum reason not involving masses of elementary particles.

I suspect the answer is simple
Despite many attempts, no-one has it.

Think about it!


V = A / r
r² = Σ pairs r_ij² m_i m_j / M²