Title: The Ubit Model in Real-Vector-Space Quantum Theory

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Abstract: It is certainly possible to express ordinary quantum mechanics in the framework of a real vector space: by adopting a suitable restriction on all operators--Stueckelbergâ€™s rule--one can make the real-vector-space theory exactly equivalent to the standard complex theory. But can we achieve a similar effect without invoking such a restriction? In this talk I explore a model within real-vector-space quantum theory in which the role of the complex phase is played by a separate physical system called the ubit (for â€œuniversal rebitâ€•). The ubit is a single binary real-vector-space quantum object that is allowed to interact with everything in the world. It also rotates in its two-dimensional state space. In the limit of infinitely fast rotation, one recovers standard quantum theory. When the rotation rate is large but not infinite, one finds small deviations from the standard theory. Here I describe a few such deviations that we have seen numerically and explained analytically.

The Ubit Model in Real-Vector-Space Quantum Theory

Antoniya Aleksandrova, Victoria Borish, and William Wootters

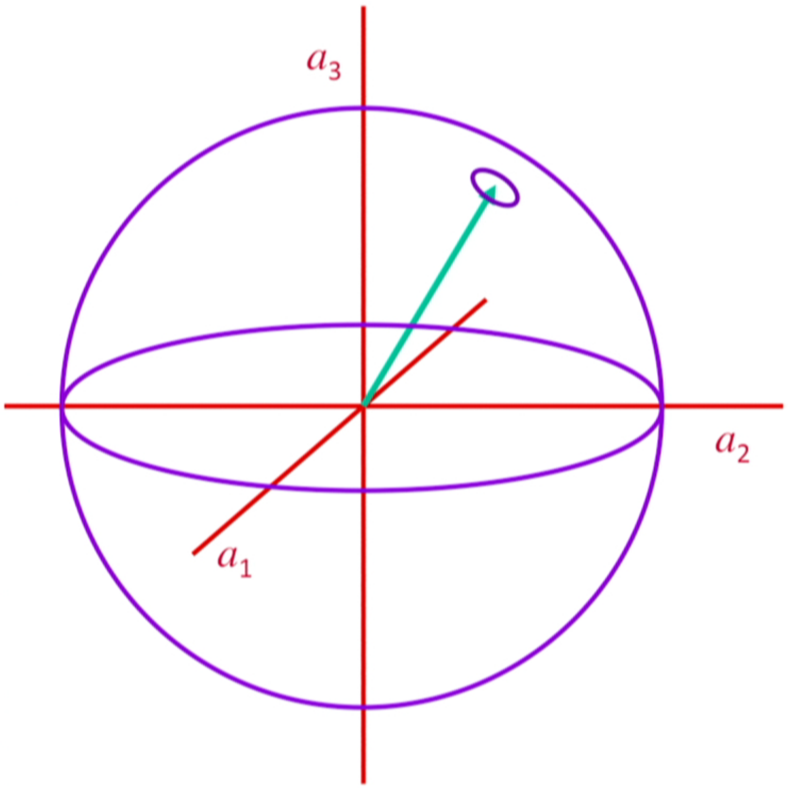
Williams College

Real-Vector-Space Quantum Theory

- A pure state is represented by a vector in a real vector space.
- A complete orthogonal measurement is a represented by an orthonormal basis. Probabilities are squared components.
- A reversible transformation is represented by an orthogonal matrix, generated by an antisymmetric real matrix S, the “Stueckelbergian”.

$$\frac{d|\psi\rangle}{dt} = S|\psi\rangle \qquad \text{OR} \qquad \frac{d\rho}{dt} = [S, \rho]$$

Real-vector-space quantum theory has a certain elegance.



- As many real parameters as independent probabilities (for a complete orthogonal measurement).
- Mutual information between the (pure) preparation and the measurement outcomes (for a large number of identically prepared systems) is maximal.

$$|\psi\rangle = a_1|1\rangle + a_2|2\rangle + a_3|3\rangle$$

Stueckelberg's Rule

To make the real theory equivalent to the standard theory:

1. Add one rebit—the ubit (that is, double the dimension).
2. Insist that every operator commute with $J_U \otimes I_A$.

$$J = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

This forces every operator to be of the form

$$I_U \otimes (\text{something}) + J_U \otimes (\text{something})$$

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Translating ordinary quantum theory into real terms:

1. Density matrix:

$$\rho \rightarrow \frac{1}{2}\left[I_U \otimes \mathrm{Re}(\rho) + J_U \otimes \mathrm{Im}(\rho)\right]$$

2. Hamiltonian to Stueckelbergian:

$$S = I_U \otimes \mathrm{Re}\left(-iH/\hbar\right) + J_U \otimes \mathrm{Im}\left(-iH/\hbar\right)$$

3. Projection operator for a measurement:

$$P \rightarrow I_U \otimes \mathrm{Re}(P) + J_U \otimes \mathrm{Im}(P)$$

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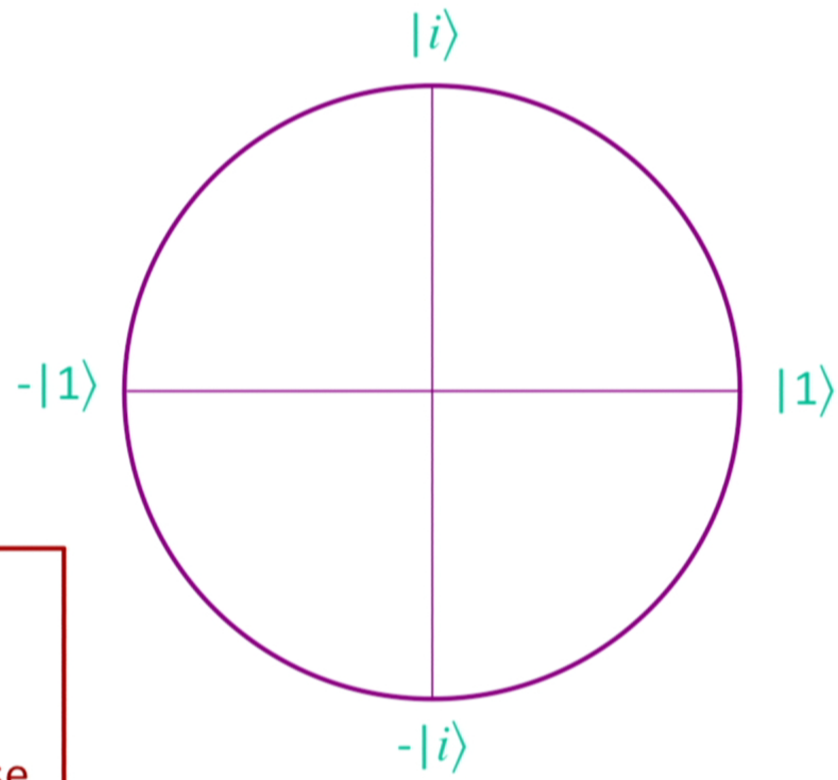
The Ubit Plays the Role of the Phase Factor

The ubit's state space:

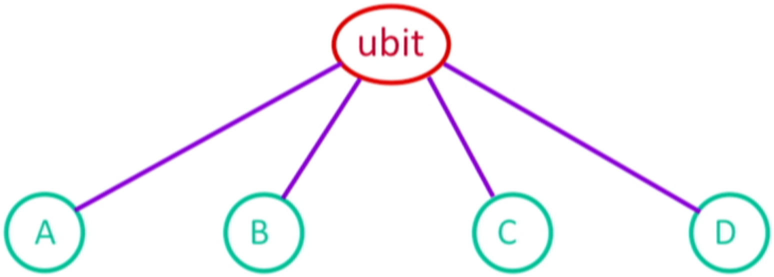


The Stueckelbergian

$$S = -\omega J$$

rotates the ubit clockwise.

Our Questions



- Can we obtain standard quantum theory ***without*** invoking Stueckelberg's rule?
  It seems we can, in a certain limit (ubit rotating infinitely fast).

- What theory do we get when we don't take that limit?

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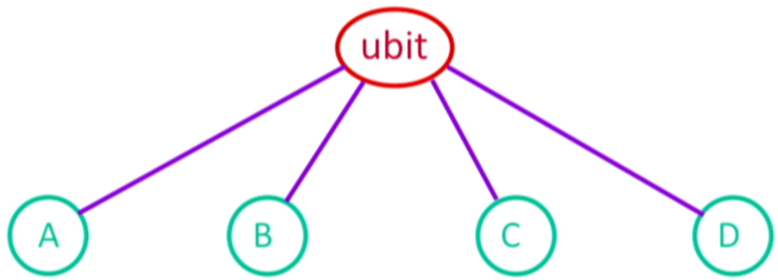

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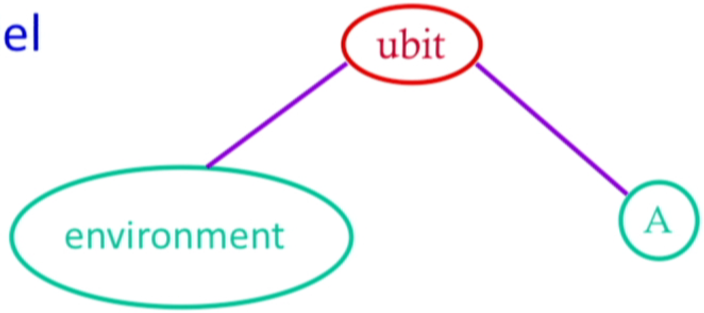
Our Model



$$S = -\omega I_E \otimes J_U \otimes I_A + B_{EU} \otimes I_A + I_E \otimes S_{UA}$$

- B_{EU} is chosen randomly. The size of a typical eigenvalue is s.
- s and ω are both much larger than the size of S_{UA}.
- The ratio s/ω is an adjustable parameter in our model. We'll get standard quantum mechanics when s/ω goes to zero.

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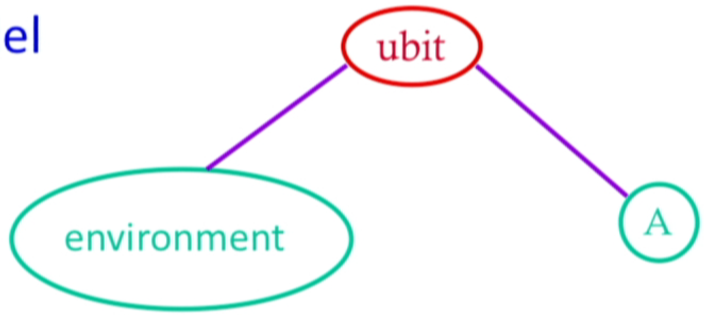

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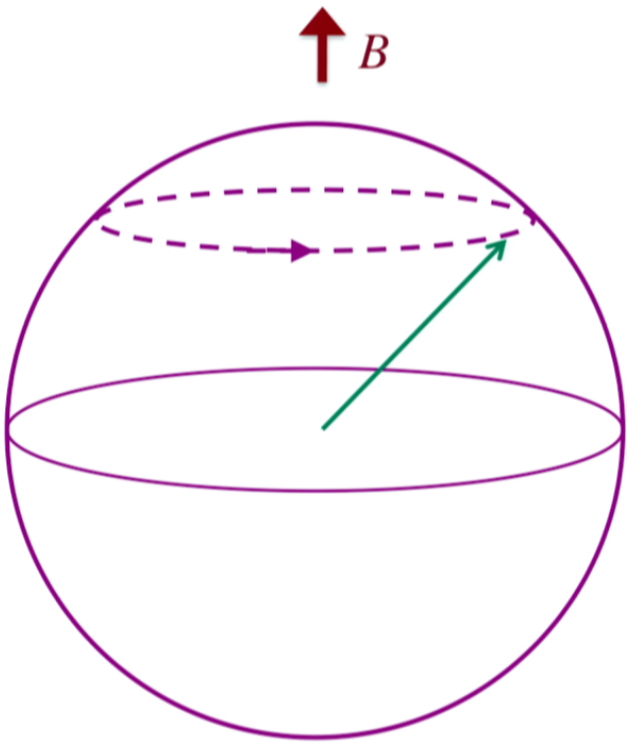
What we've mostly studied: a precessing spin-1/2 particle



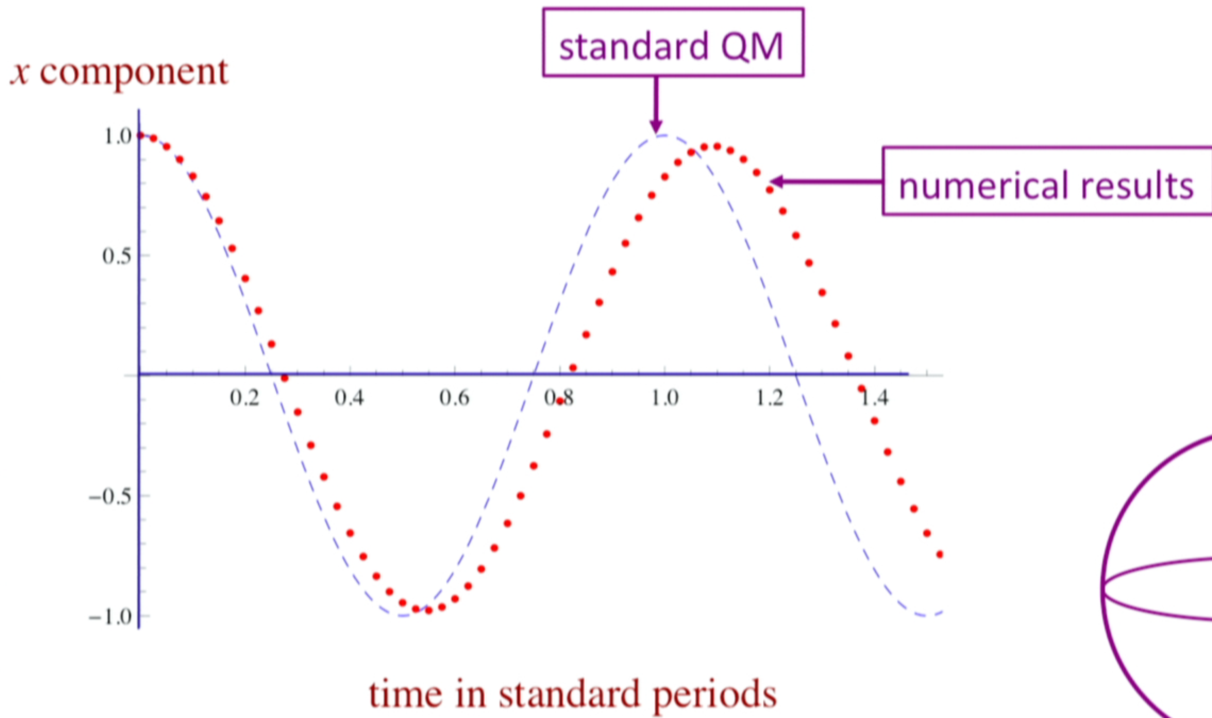
The spin is represented as a rebit A together with the ubit.

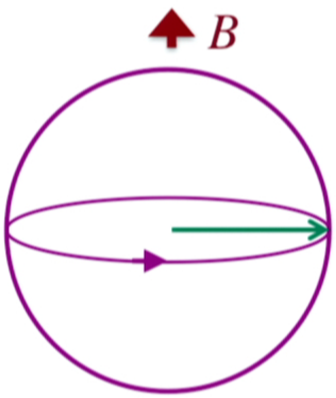
$$S_{UA} = -\frac{1}{2}\Omega J_U \otimes Z_A$$

Ω is the nominal angular frequency of precession.

Numerical Results—Deviations from Quantum Mechanics

1. Reduced frequency





To explain these results, we'll use perturbation theory, expanding in powers of s/ω.

We begin by turning off S_{UA} and keeping just the ubit rotation and the ubit-environment interaction.

We ask what happens to each of these components of the UA state:

$$X_U \otimes (\text{something})$$
$$Z_U \otimes (\text{something})$$
$$J_U \otimes (\text{something})$$

We assume s and ω are so large that these decays happen instantaneously and continually.

We assume this means that ρ is continually being projected onto the space of matrices that commute with S_{EU}.

This assumption forces ρ to have the form

$$\rho(t) = \sum_k |\Psi_k\rangle\langle\Psi_k| \otimes A_k(t)$$

where the $|\Psi_k\rangle$ are the eigenstates of S_{EU}.

The A_k's evolve according to

$$\frac{dA_k}{dt} = [\langle\Psi_k|I_E \otimes S_{UA}|\Psi_k\rangle, A_k]$$

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Explaining Our Numerical Results

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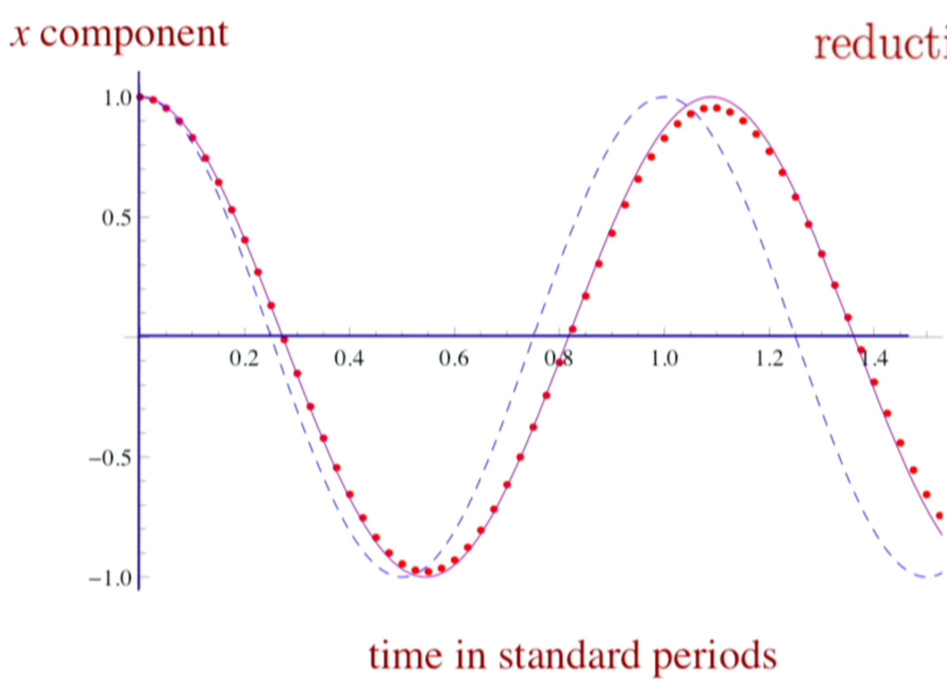

reduction factor: $1 - \frac{1}{2}\left(\frac{s}{\omega}\right)^2$

The Ghost Part

When the Bloch vector gets shorter, another component of the total density matrix grows.

At maximum, this "ghost part" is

$$-\frac{1}{4\omega}[B+(I\otimes J)B(I\otimes J)]_{EU}\otimes J_A$$

It has zero partial trace over the environment, but "Alice" (with the A system) can easily access it.

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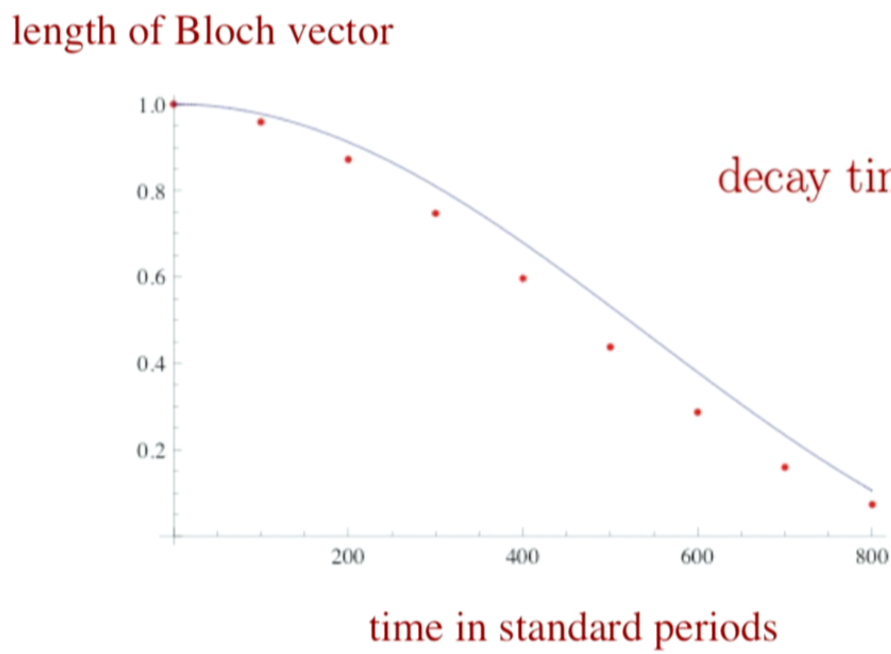
Explaining Our Numerical Results

3. Long-term decoherence (harder to explain in detail)



$$\text{decay time } \approx \frac{1}{\Omega}\left(\frac{\omega}{s}\right)^3$$

Stueckelberg's Rule without Stueckelberg's Rule

Recall the environment-ubit Stueckelbergian:

$$S_{EU} = -\omega I_E \otimes J_U + B_{EU}$$

The density matrix is made of eigenstates of S_{EU}.
Why does it commute with J_U when we trace over the environment?

$\omega = 0$ ➔ random state of EU. *E* is big ➔ ubit state proportional to I_U.
(This would cut out too much: not just X_U and Z_U but also J_U.)

ω huge ➔ J_U dominates. Commuting with S_{EU} ➔ commuting with J_U.

Intermediate case ➔ get identity and some J_U.

Can We Capture the Environment's Effects in a Local Equation?

Our best attempt so far (here ρ is for the UA system):

$$\frac{d\rho}{dt} = Q\left([S, Q(\rho)]\right)$$

Q removes X_U and Z_U from the ubit state (Stueckelberg) and takes J_U to γJ_U, where $\gamma < 1$.

This equation captures the frequency reduction (including the anisotropy) but not the long-term decoherence.

But it immediately generalizes to higher dimensions.
If H is real, this equation just slows things down—no other effect.

Conclusions

It seems we can recover ordinary quantum theory from the real theory with a ubit. We need:

(i) Large environment to effect Stueckelberg's rule.
(ii) Fast-rotating ubit to prevent the elimination of desired states.

Not taking the limit of infinitely fast rotation, we get a modified theory that includes:

(i) Spontaneous decoherence of an isolated system.
(ii) Part of the environment absorbed into the system (the ghost part).
(iii) Possibly another physical entity to determine how each operator is to be broken in an I_U part and a J_U part.

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