Title: Accelerated Expansion and AdS/CFT

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Abstract: We review the notion of a quantum state of the universe and its role in fundamental cosmology. Then we discuss recent work which points towards a profound connection, at the level of the quantum state, between (asymptotic) Euclidean AdS spaces and Lorentzian de Sitter spaces. This gives a new framework in which (a mild generalization of) AdS/CFT can be applied to inflationary cosmology.   For the specific case of the Hartle-Hawking no-boundary quantum state the ADS/ de Sitter connection yields a natural proposal for a more precise `dual' formulation of the wave function, in terms of field theories on the future de Sitter boundary that are certain relevant deformations of the CFTs that occur in AdS/CFT.

Accelerated Expansion and AdS/CFT

Perimeter Institute

May 2012

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w/JB Hartle, SW Hawking

arXiv:1205......
arXiv:1111.6090
arXiv:0803.1663

- Can AdS/CFT be used for cosmology?

  [Horowitz, TH; Craps, Turok et al; Das et al; Hawking, Reall et al; Hubeny, Maloney, Myers, Shenker et al, ...]

- Analytic continuation from (Euclidean) AdS to de Sitter

  [Maldacena; Harlow, Stanford; Anninos et al.]

  → CMB correlators from dual CFT

  [Maldacena et al; McFadden et al]

- This only applies in certain limit

This talk: *a general and universal connection between Euclidean AdS and de Sitter, realized at the level of wave function of the universe.*

Wave function of the universe:

- Euclidean AdS/CFT
  [Horowitz & Maldacena '04]

$$\exp(-I^R_{ADS}[h,\chi]/\hbar) = Z_{QFT}[h,\chi]$$

- No-boundary State
  [Hartle & Hawking '83]

$$\Psi[h,\chi] = \exp(-I_{dS}[h,\phi]/\hbar)$$

$\rightarrow$ *profound connections between both ideas*

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No-Boundary State

$$\Psi[^3g, \chi] = \int_C \delta g \delta\phi \exp(-I_{dS}[g, \phi])$$

"The amplitude of configurations ($^3g, \chi$) on a three-surface Σ is given by the integral over all regular metrics g and matter fields ϕ that match ($^3g, \chi$) on their only boundary." [Hartle & Hawking '83]

Saddle Point Limit

$$\Psi[^3g, \chi] \approx \exp\{[-I(^3g, \chi)]/\hbar\}$$

Saddle points in general complex:

$$I(^3g, \chi) = -I_R(^3g, \chi) + iS(^3g, \chi)$$

WKB Interpretation

$$\Psi[^3g, \chi] \approx \exp\{[-I_R(^3g, \chi) + iS(^3g, \chi)]/\hbar\}$$

The semiclassical wave function predicts Lorentzian, classical evolution of space-time in regions of superspace where

$$|\nabla_A I_R| \ll |\nabla_A S|$$

The classical histories are the integral curves of S_L:

$$p_A = \nabla_A S$$

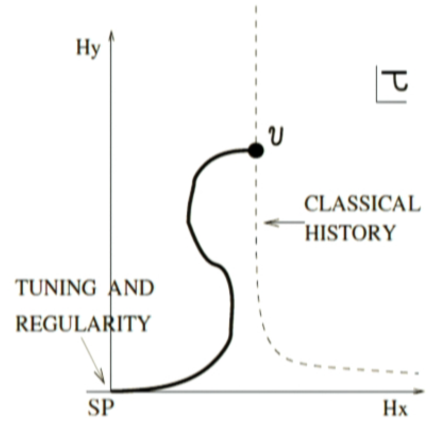
and have conserved probabilities

$$P_{history} \propto \exp[-2I_R/\hbar]$$

$\rightarrow$ no-boundary measure: prior on multiverse.

Complex Saddle points

$$ds^2 = d\tau^2 + g_{ij}(\tau, x)dx^i dx^j, \quad \phi(\tau, x)$$



Regularity at SP: $g_{ij}(0) \to 0, \quad \dot{\phi}(0) \to 0$

At final boundary: $g_{ij}(\upsilon) = {}^3g, \quad \phi(\upsilon) = \chi$

Tuning at SP: $\phi(0) = \phi_0 e^{i\gamma}, \ldots$

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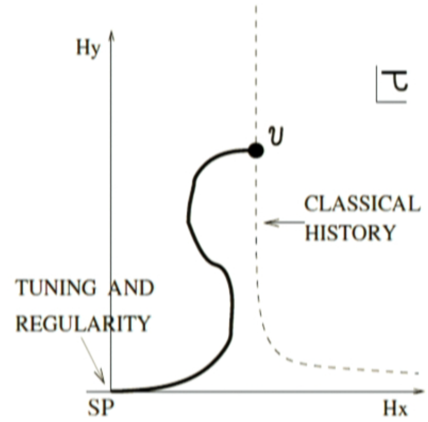

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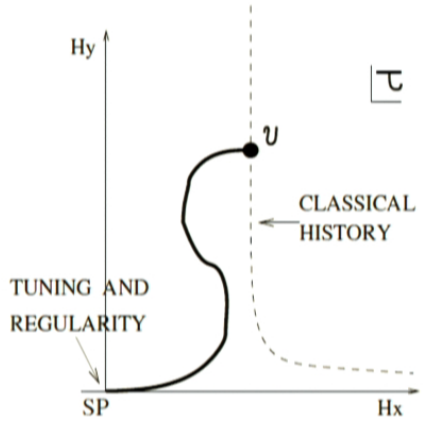

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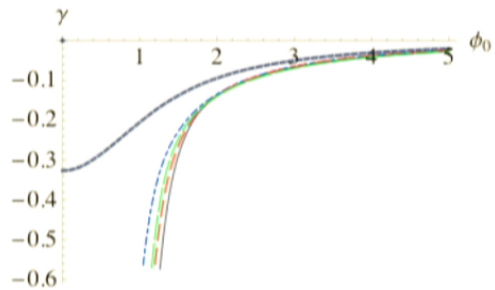
Example

Homogeneous/isotropic ensemble:

$$ds^2 = d\tau^2 + a^2(\tau)d\Omega_3, \quad \phi(\tau), \quad \Psi[b,\chi]$$

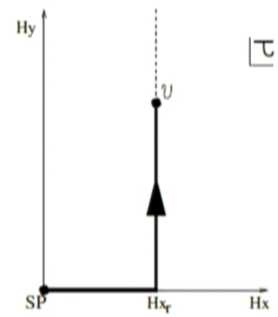
$$V(\phi) = \Lambda + \frac{1}{2}m^2\phi^2$$

Classical evolution requires tuning:



→ 1-parameter set of FLRW backgrounds

Saddle point Action



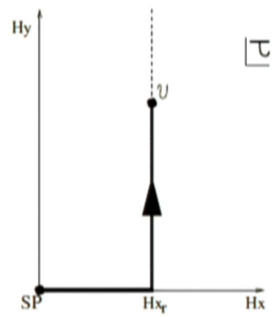
$$I(\upsilon) = \frac{3\pi}{2}\int_{C(0,\upsilon)} d\tau a[a^2(H^2 + 2V(\phi)) - 1]$$

I_R tends to a constant along vertical part

→ probability measure

$$I_R(\upsilon) \approx -\frac{\pi}{4V(\phi_0)}$$

Saddle point Action



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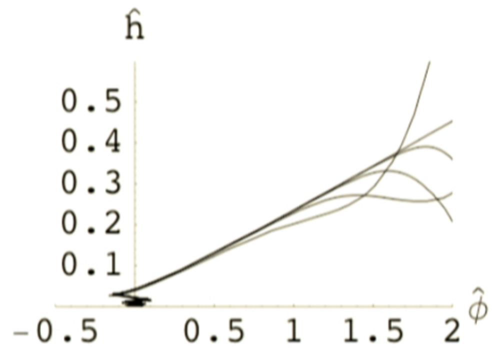
Inflation

Lorentzian background histories:

$$p_A = \nabla_A S$$



$\rightarrow$ no-boundary state predicts inflation

Was there a Beginning?

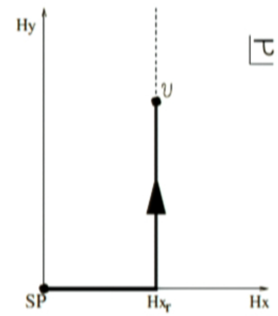
large ϕ_0 small ϕ_0

Main Points

- A semiclassical quantum state describes an ensemble of space-time histories.
- Classical histories are interesting for cosmology
- Their relative probabilities are given by complex solutions of a given Euclidean action.

No-Boundary State:
ADS form

Complex Saddle points



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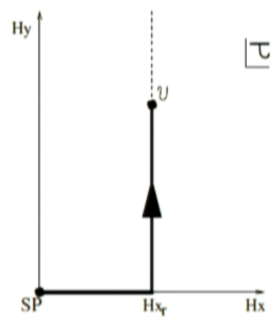

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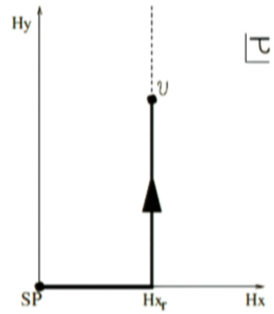
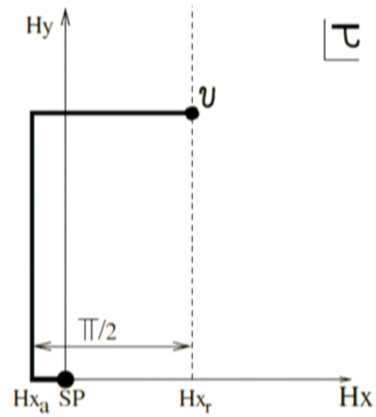


e.g. no matter: $a(\tau) = \frac{1}{H}\sin(H\tau)$

horizontal part: $ds^2 = d\tau^2 + \frac{1}{H^2}\sin^2(H\tau)d\Omega_3^2$

vertical part: $ds^2 = -dy^2 + \frac{1}{H^2}\cosh^2(Hy)d\Omega_3^2$

Representations



vertical part: Euclidean ADS

$$ds^2 = -dy^2 - \frac{1}{H^2}\sinh^2(Hy)d\Omega_3^2$$

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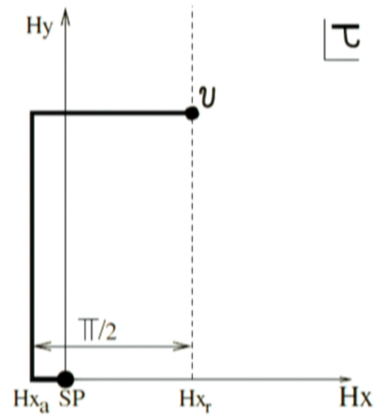

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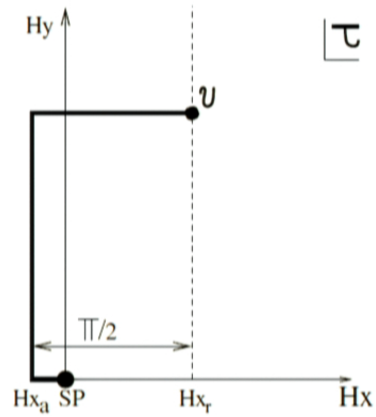


vertical part: Euclidean ADS

$$ds^2 = -dy^2 - \frac{1}{H^2}\sinh^2(Hy)d\Omega_3^2$$

- V and Λ can act as $-V$ and $-\Lambda$ because the signature of complex saddle point metrics varies in τ-plane

$$I[g, V, \Lambda] = -I[-g, -V, -\Lambda]$$

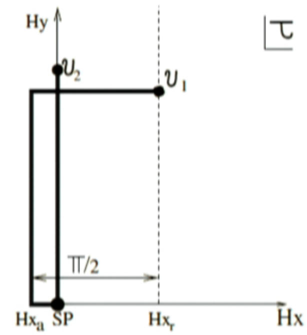
- Somewhat reminiscent of Domain Wall/Cosmology correspondence in SUGRA, realized here at the level of the universe's quantum state [Cvetic; Skenderis,Townsend, Van Proeyen]

$$V_{AdS}^{eff} = -\Lambda - V$$

- Yet in general this is different from a continuation between theories.

AdS Saddle points

$$\phi(\upsilon_1) = \phi(\upsilon_2) = \chi, \qquad \tilde{g}_{ij}(\upsilon_1) = \tilde{g}_{ij}(\upsilon_2)$$



Asymptotic AdS saddle pt: REAL ϕ_0 at SP.

Asymptotic dS saddle pt: COMPLEX ϕ_0 at SP.

$$\phi \sim \frac{\alpha}{\rho^{\lambda_-}} + \frac{\beta}{\rho^{\lambda_+}}, \quad \alpha = e^{i\lambda_- \pi/2} \chi \rho^{\lambda_-}$$

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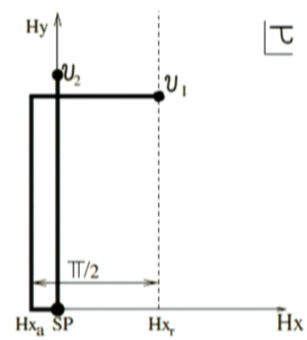

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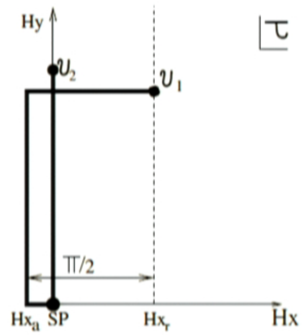

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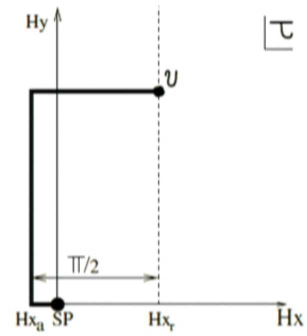

- AdS/de Sitter connection for general saddle points?
- Beyond saddle pt limit?
- Action along horizontal part of the contour?

Universal Semiclassical Limit

[Hartle, TH, to appear]

Consider gravity with non-zero Λ.

Any state Ψ must obey Wheeler-DeWitt eq.

$$\left(-\hbar^2 \frac{d^2}{db^2} + b^2 + \frac{b^4}{l_{ads}^2} + \cdots\right) \Psi(b,\chi) = 0$$

WDW eq predicts a universal asymptotic form,

$$\Psi(b,\chi) \equiv \exp[-I(b,\chi)/\hbar]$$

This implies asymptotic Einstein eqs.

Asymptotic Structure

Expanded in small $u \equiv e^{i\tau} = e^{-y+ix}$,

$$g_{ij}(u,\Omega) = \frac{-1}{4u^2}[h_{ij}(\Omega) + h_{ij}^{(2)}(\Omega)u^2 + h_{ij}^{(-)}(\Omega)u^{\lambda_-} + h_{ij}^{(3)}(\Omega)u^3 + \cdots]$$

$$\phi(u,\Omega) = u^{\lambda_-}(\alpha(\Omega) + \alpha_1(\Omega)u + \cdots) + u^{\lambda_+}(\beta(\Omega) + \beta_1(\Omega)u + \cdots)$$

with $\lambda_\pm \equiv \frac{3}{2}[1 \pm \sqrt{1-(2m/3)^2}]$
and arbitrary 'boundary values' (h_{ij}, α).

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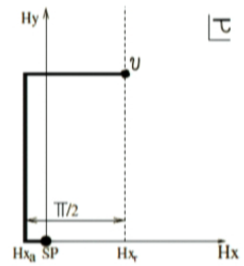
Saddle Point Action



- Action integral along vertical part:

$$I_v = \int_v I[g,\phi] = -I^R_{AdS}(^3\tilde{g},\tilde{\chi}) + S_{ct}(^3g,\tilde{\chi})$$

where I^R_{AdS} is finite when $a \to \infty$.

- Action integral along horizontal part:

$$I_h = \int_h I[g,\phi] = -S_{ct}(^3g,\tilde{\chi}) + iS_{ct}(^3g,\chi)$$

and no finite contribution.

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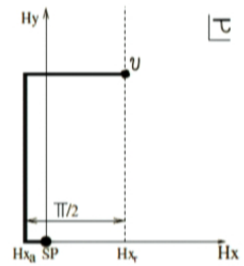

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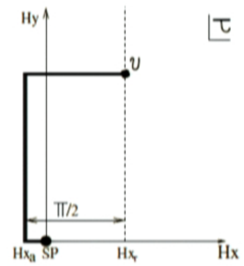

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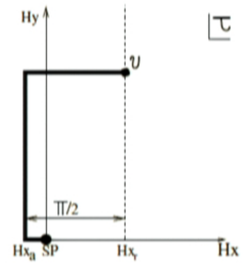


- Total action:

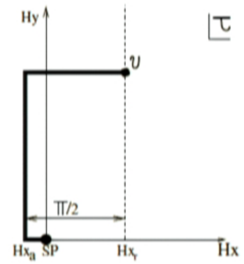
$$I_{dS}({}^3g,\chi) = I_v + I_h = -I^R_{AdS}({}^3\tilde{g},\tilde{\chi}) + iS_{ct}({}^3g,\chi)$$

with

$$S_{ct}(h,\phi) = a_0\int\sqrt{h} + a_1\int\sqrt{h}R^{(3)} + a_2\int\sqrt{h}\phi^2$$

$$\Psi[b,\tilde{h},\chi] = \exp\{[+I^R_{AdS}(\tilde{h},\tilde{\chi}) - iS_{ct}(b,\tilde{h},\chi)]/\hbar\}$$

Saddle Point Action



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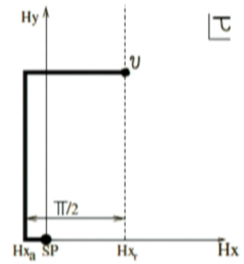

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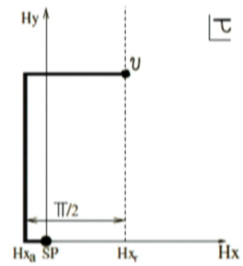

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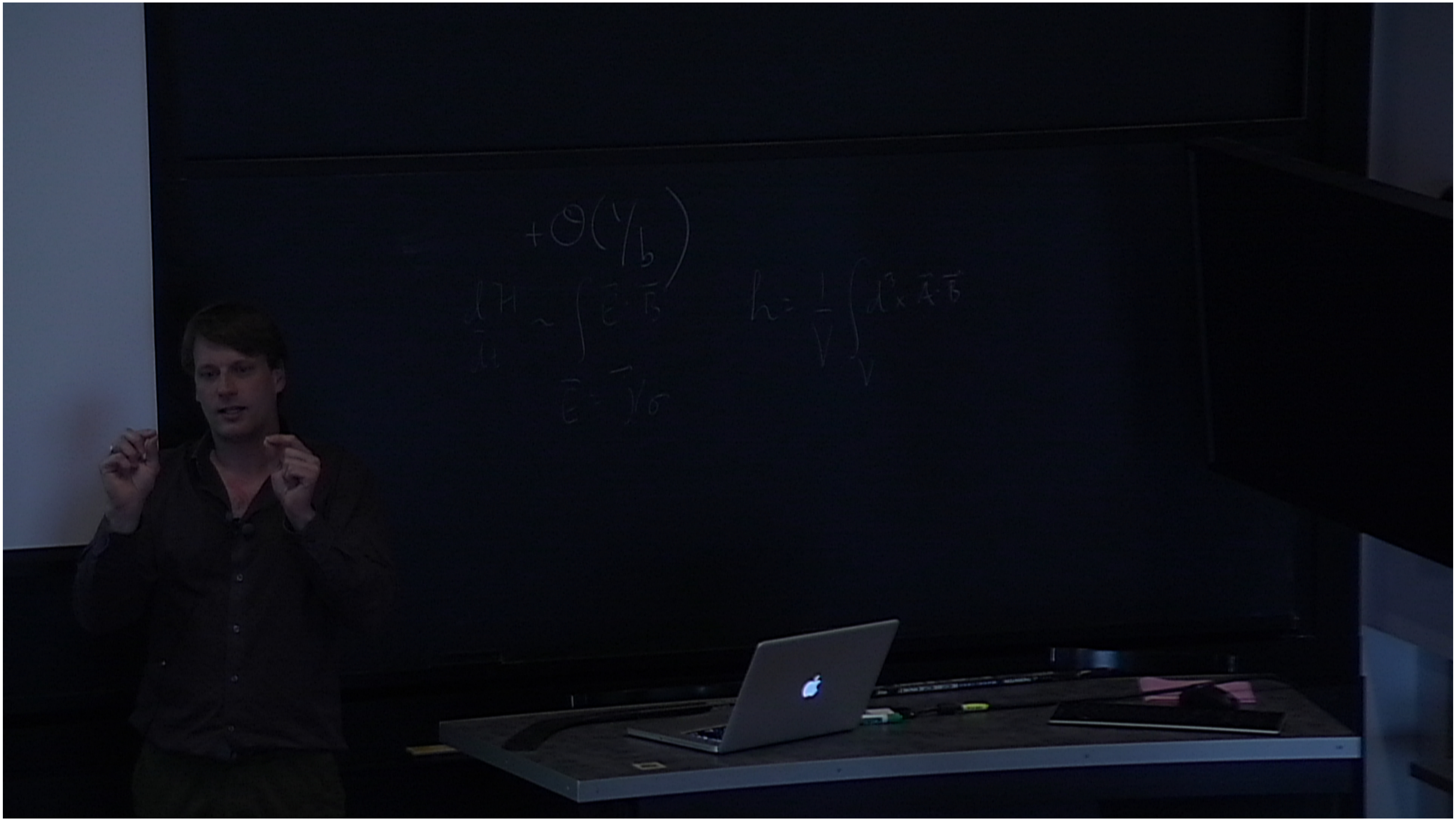

Holographic Cosmology

- No-boundary State

$$\Psi[b, \tilde{h}, \chi] = \exp\{[+I^R_{AdS}(\tilde{h}, \tilde{\chi}) iS_{ct}(b, \tilde{h}, \chi)]/\hbar\}$$

- Euclidean AdS/CFT

$$\exp(-I^R_{ADS}[\tilde{h}, \tilde{\chi}]/\hbar) = Z_{QFT}[\tilde{h}, \tilde{\chi}]$$

Combination:

$$\Psi[b, \tilde{h}, \chi] = \frac{1}{Z_{QFT}[\tilde{h}, \tilde{\chi}, \epsilon]} \exp\{[iS_{ct}(b, \tilde{h}, \chi)]/\hbar\}$$

with UV cutoff $\epsilon \sim 1/b$

Remarks

$$Z_{QFT}[\tilde{h}, \tilde{\chi}] = \langle \exp \int d^3x \sqrt{\tilde{h}} \tilde{\chi} \mathcal{O} \rangle$$

- The dependence of Z on the external sources provides a cosmological measure on the space of configurations $(b, \tilde{h}, \chi)$.
- AdS/CFT implements no-boundary condition of regularity in saddle point limit
- Scale factor evolution arises as inverse RG flow
- Wave function interpretation of AdS/CFT provides physical meaning of counterterms in AdS
- 1/Z factor resonates with higher-spin dS/CFT

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Conclusion

The universe's quantum state connects Euclidean asymptotic AdS spaces and Lorentzian inflationary cosmologies.

Implications:

- A wave function defined in terms of a gravitational theory with a negative cosmological constant Λ can predict expanding universes with an 'effective' positive cosmological constant $-\Lambda$.
- The Euclidean AdS/CFT correspondence provides a dual 'holographic' formulation of the semiclassical no-boundary state.
- This may be useful to get a better handle on eternal inflation.