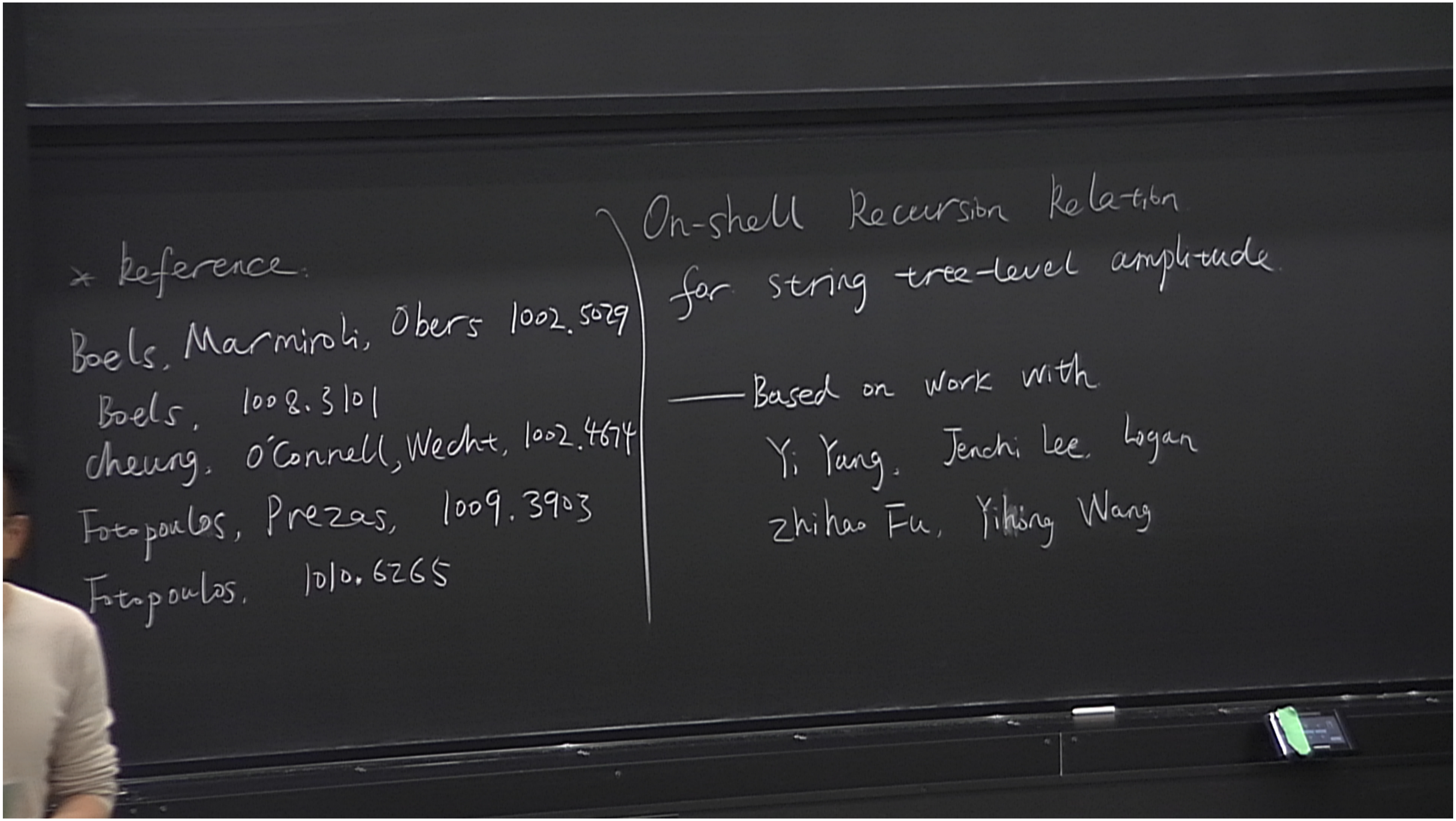
Title: On-shell Recursion Relation for String Tree-level Amplitude

Date: May 08, 2012 02:00 PM

URL: http://pirsa.org/12050003

Abstract: It is well known that on-shell recursion relation can be applied to tree-level amplitude in string theory. One technical issue of the application is the sum of infinite middle on-shell states. We discuss how we can do the sum exactly to reproduce the known result.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Fotopoulos, Prezas, 1009.3903
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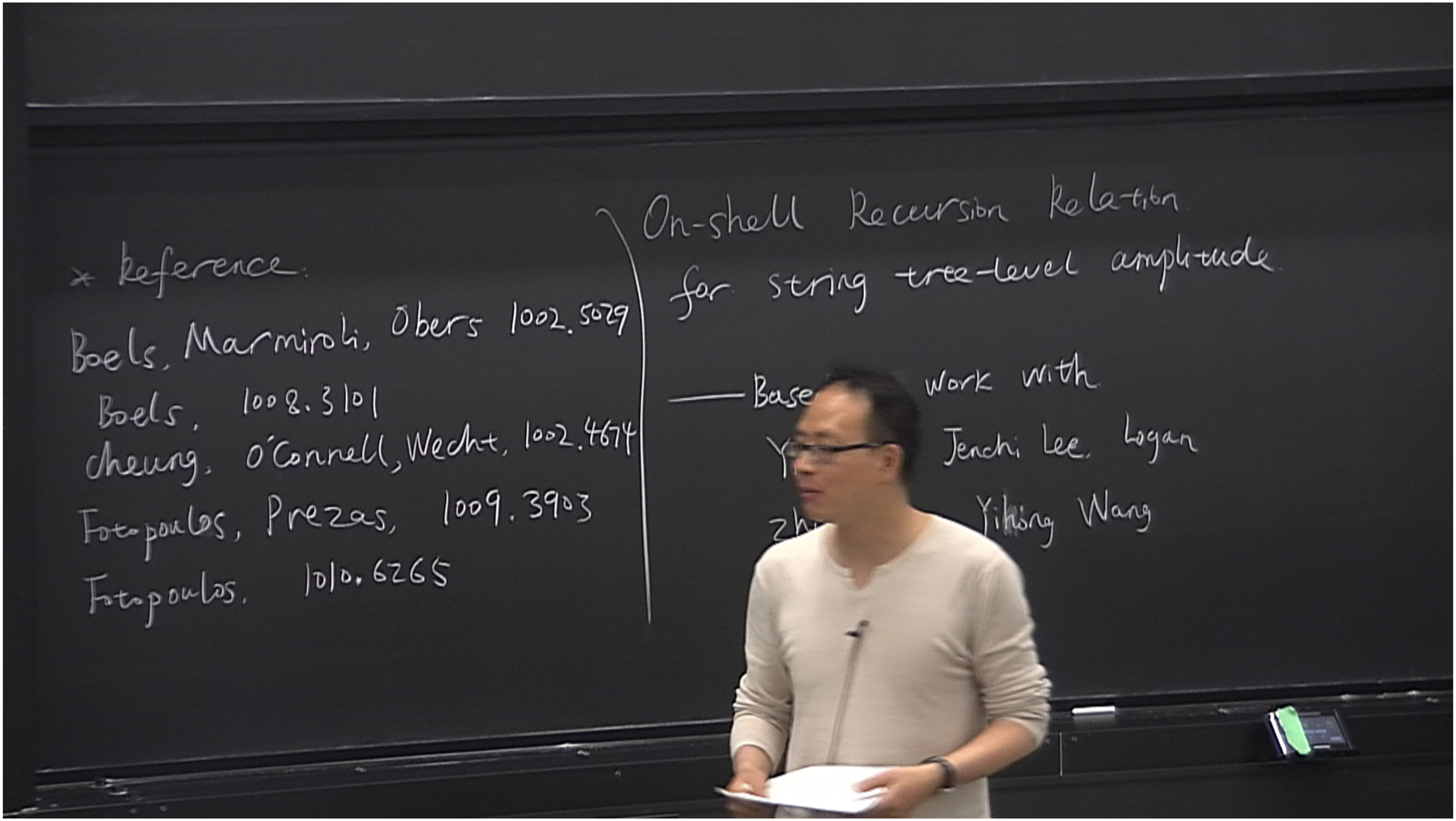
On-shell Recursion Relation
for string tree-level amplitude
— Based on work with
Yi Yang, Jenchi Lee, Logan
Zhihao Fu, Yihong Wang



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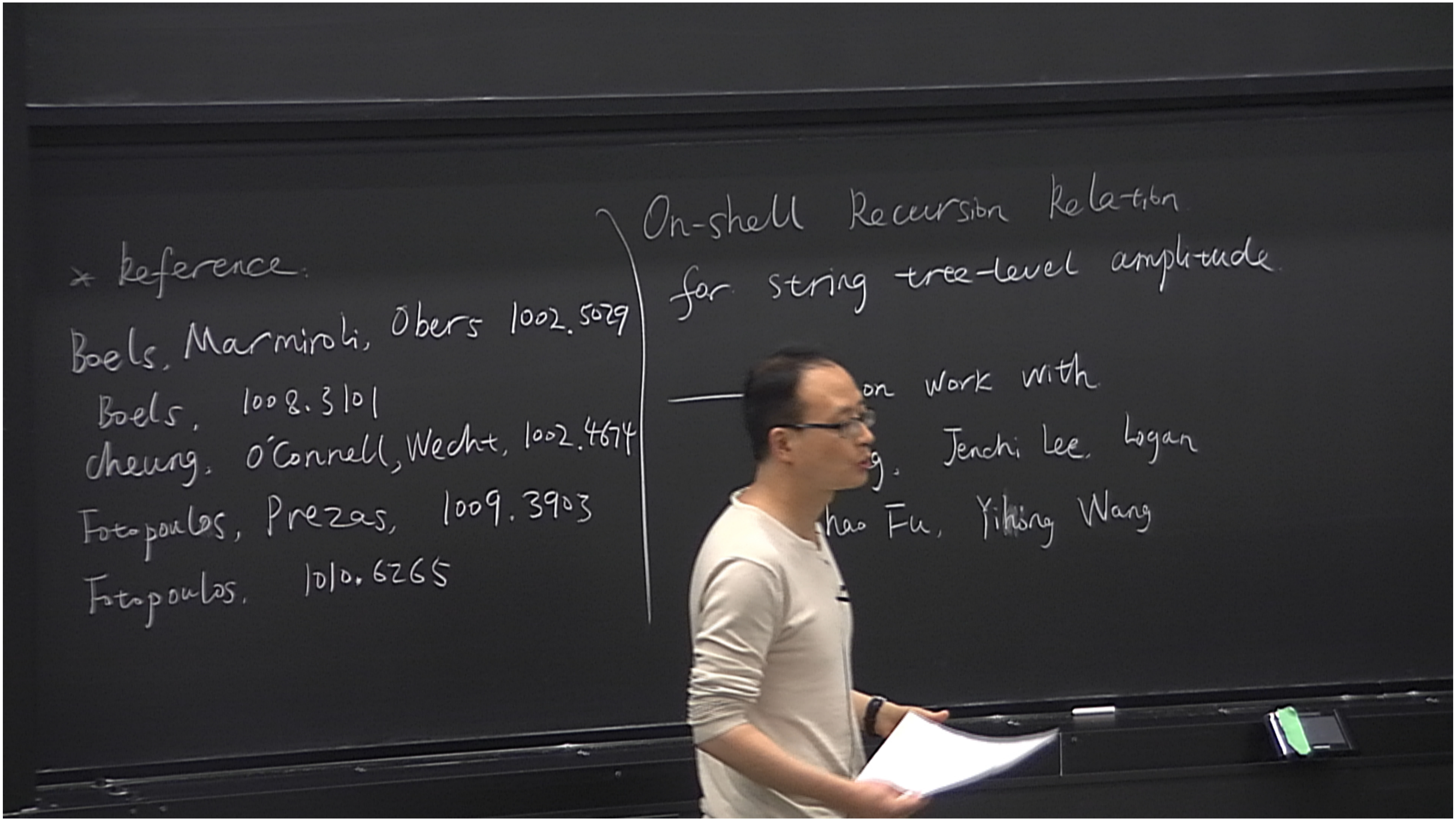

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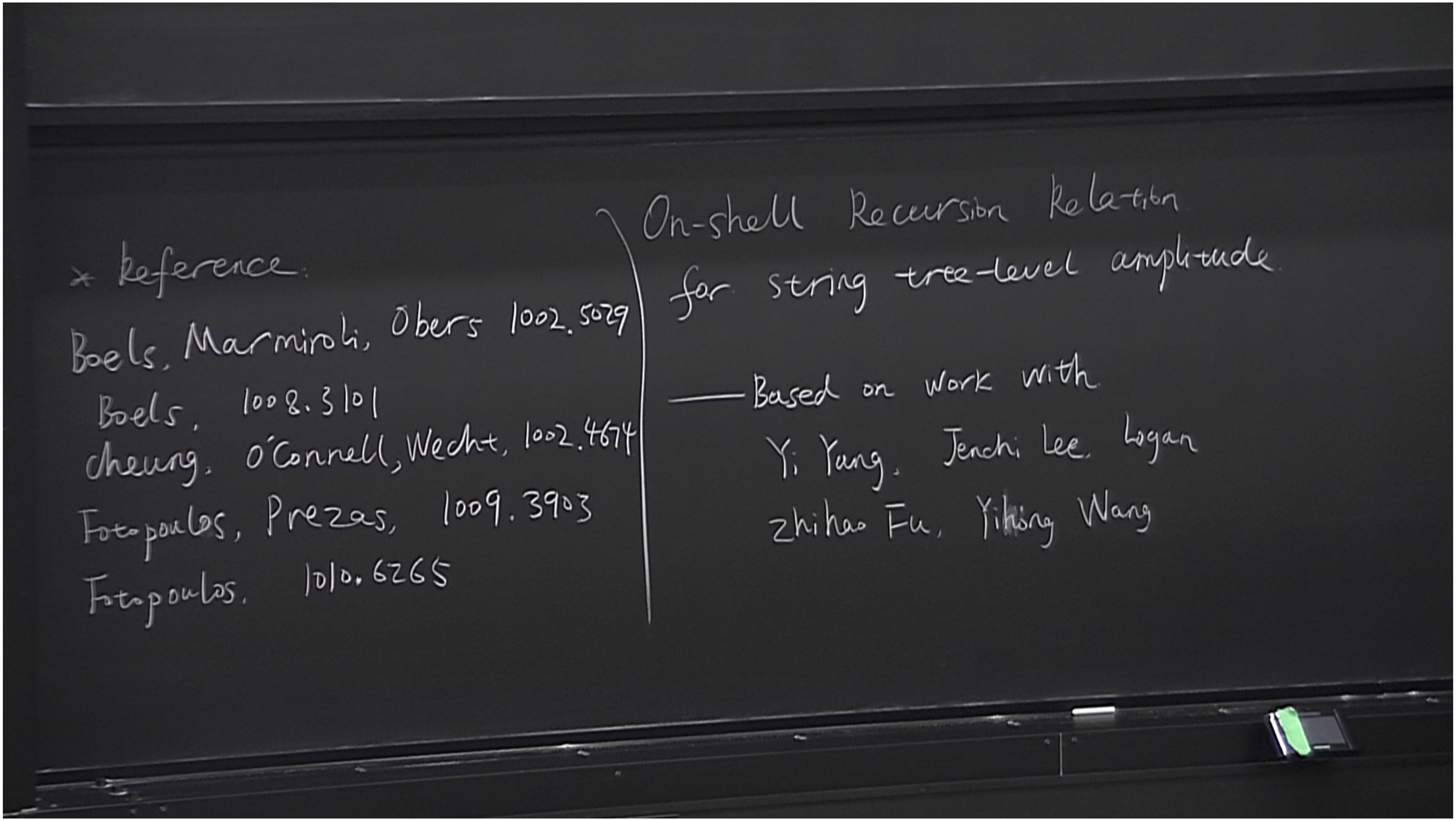
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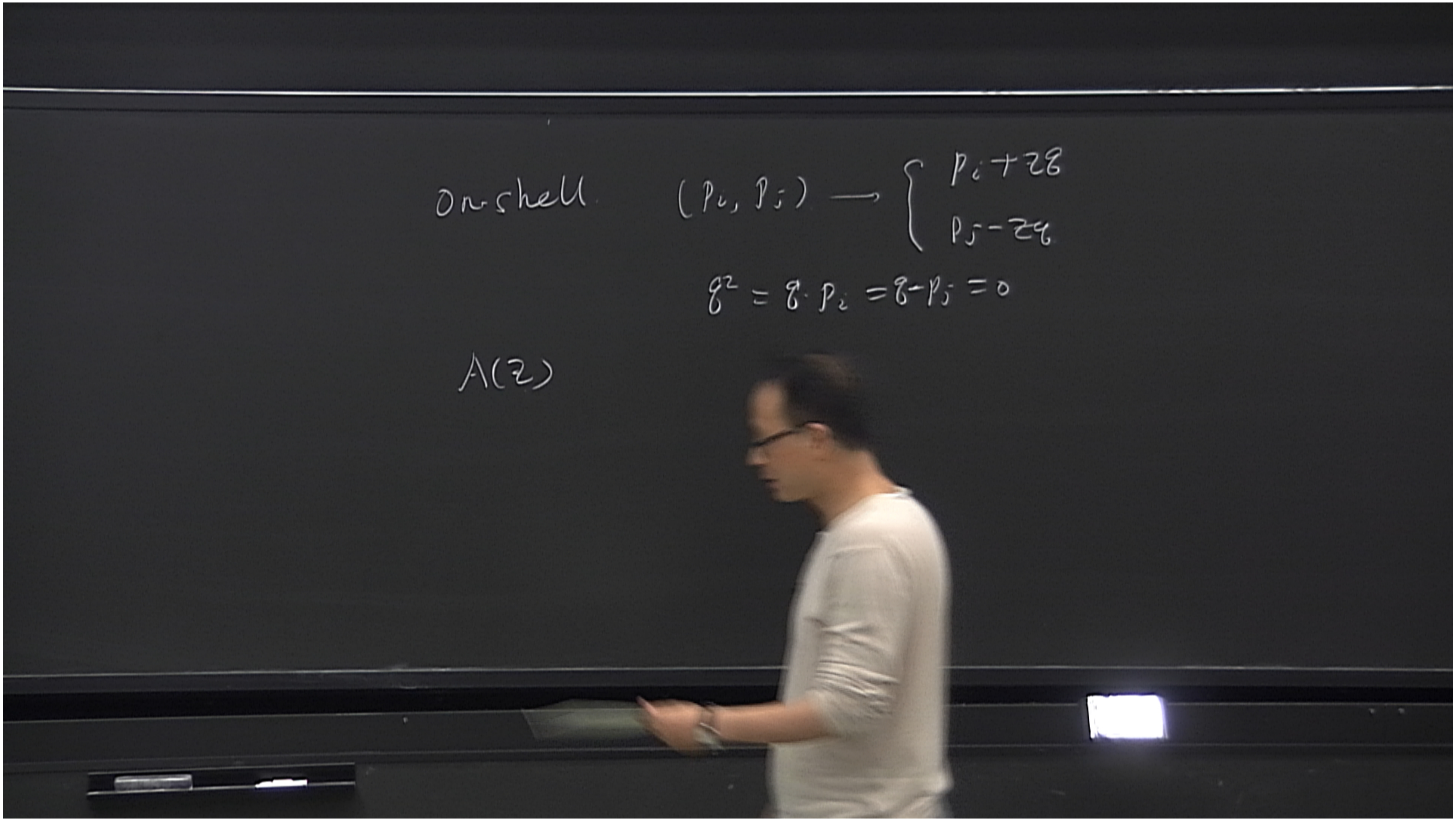
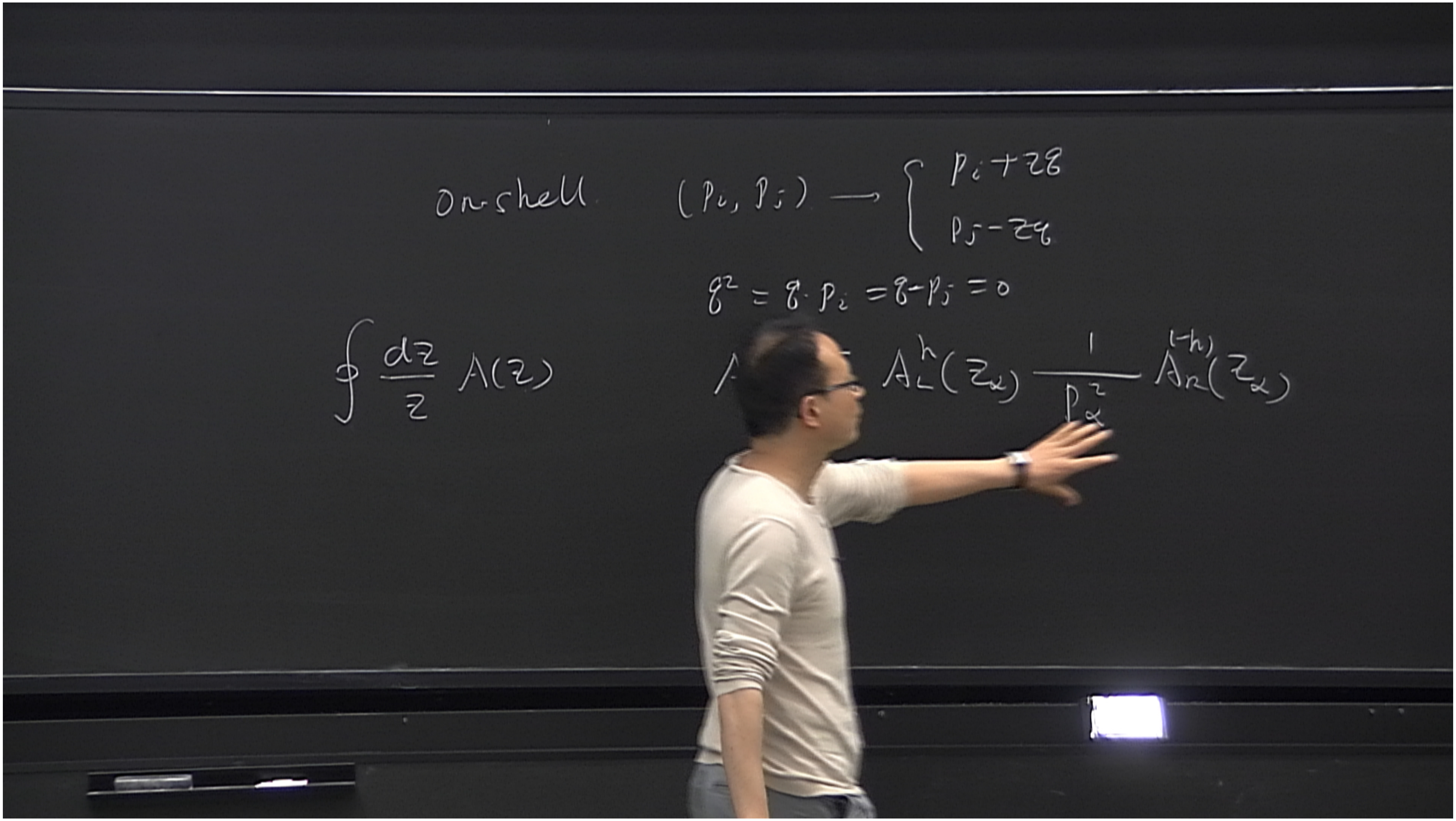

On-shell
$(p_i, p_j) \longrightarrow \begin{cases} p_i + zq \\ p_j - zq \end{cases}$
$q^2 = q \cdot p_i = q \cdot p_j = 0$
$A(z)$



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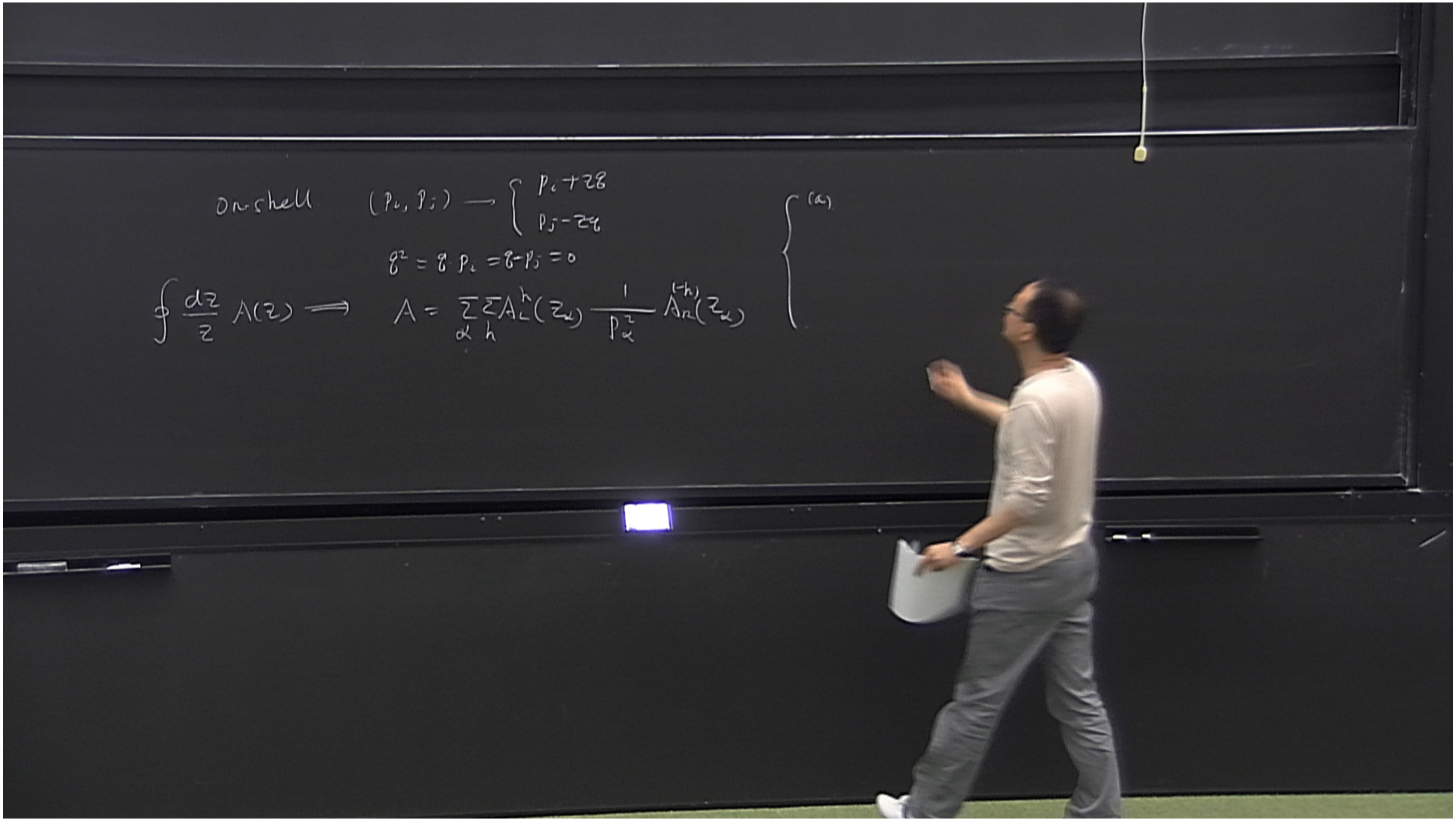
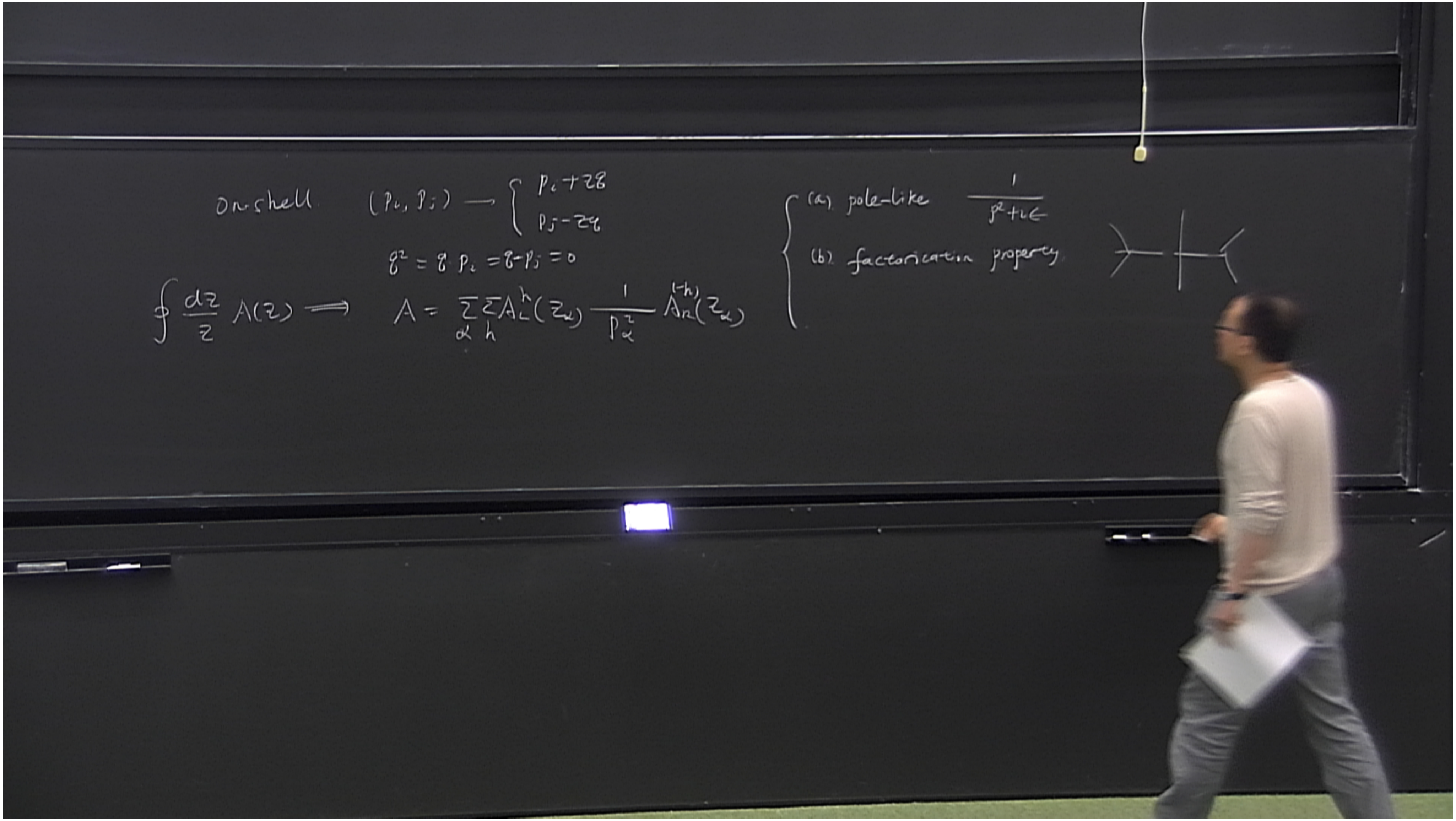
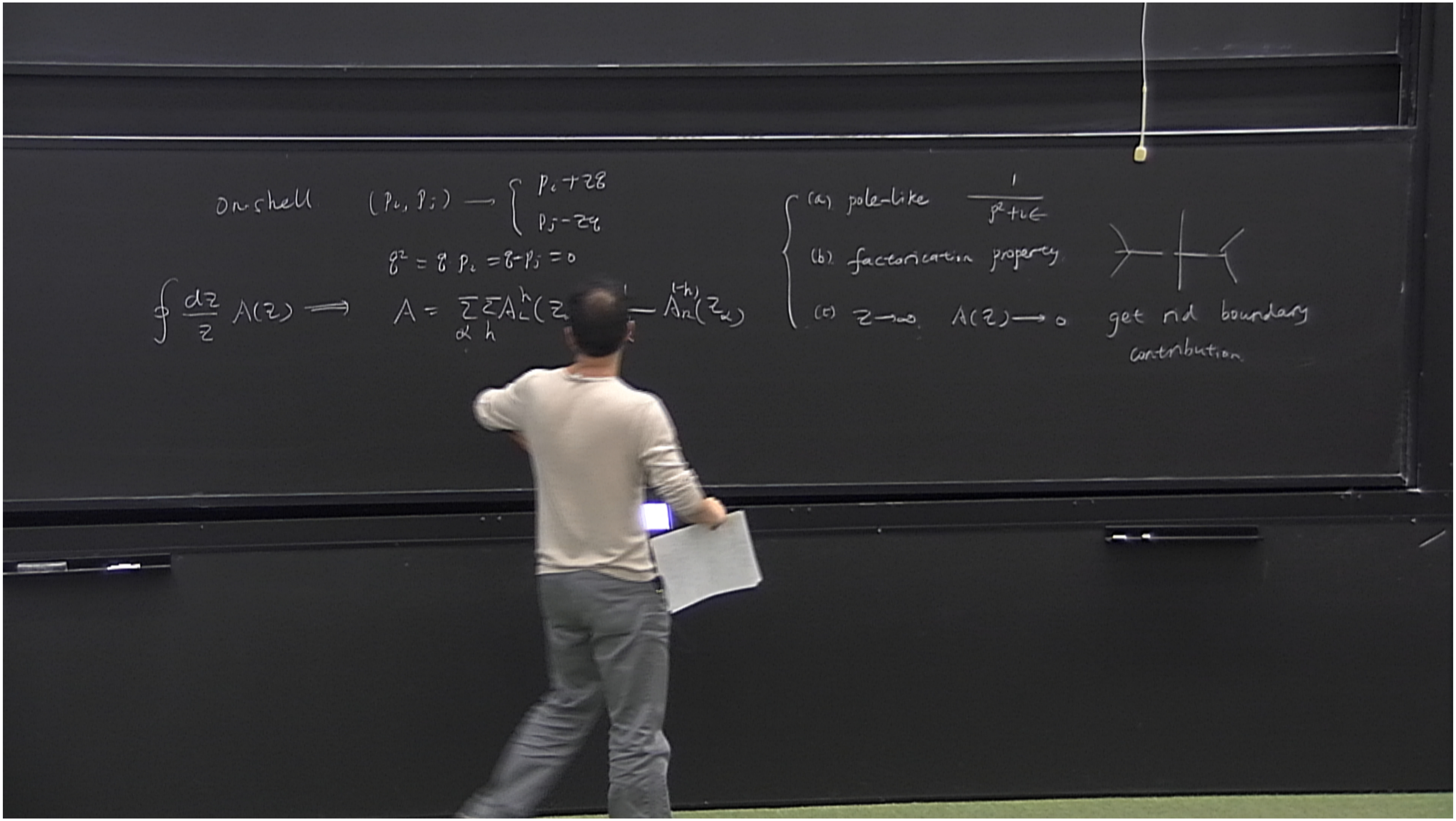
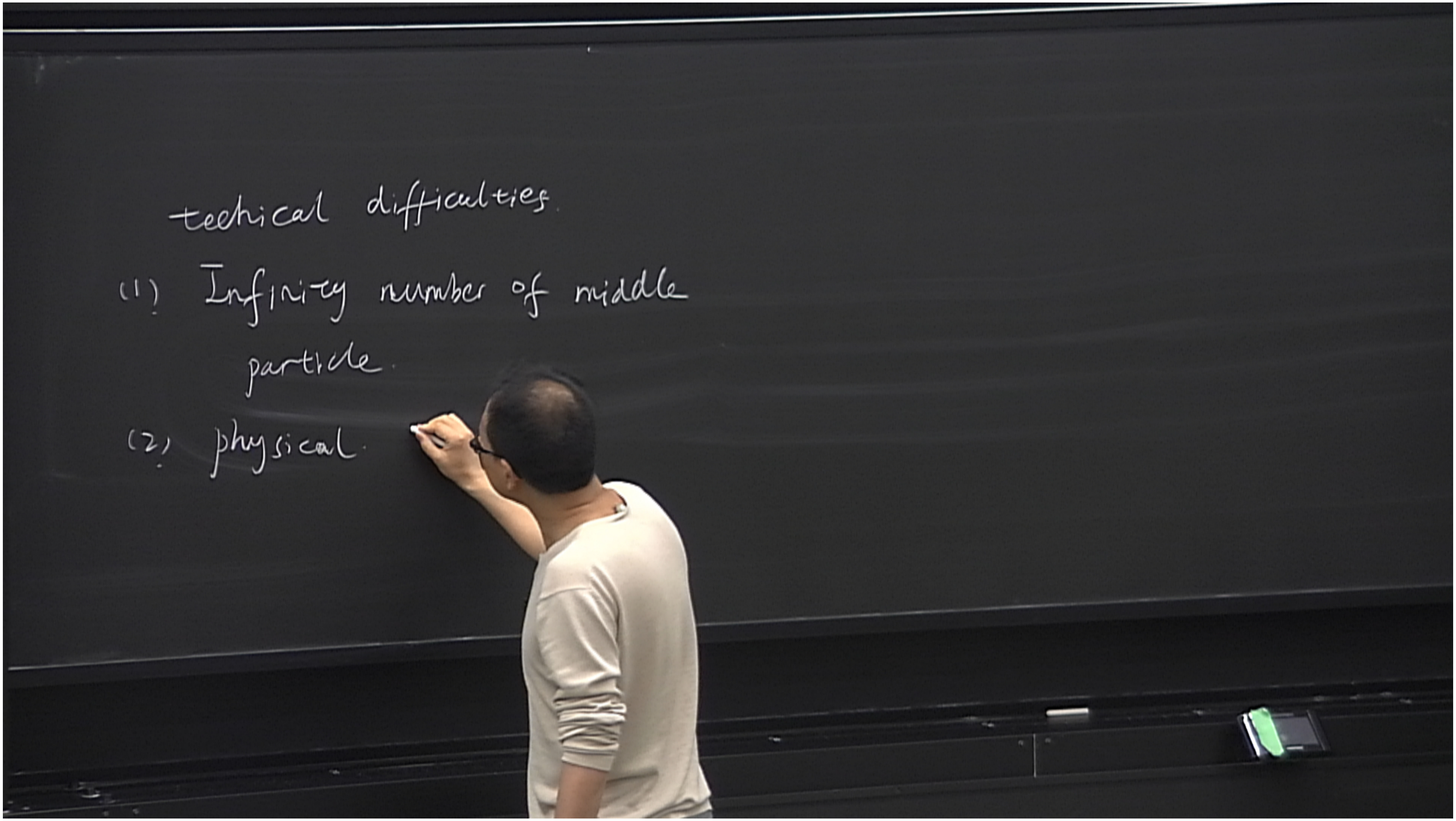
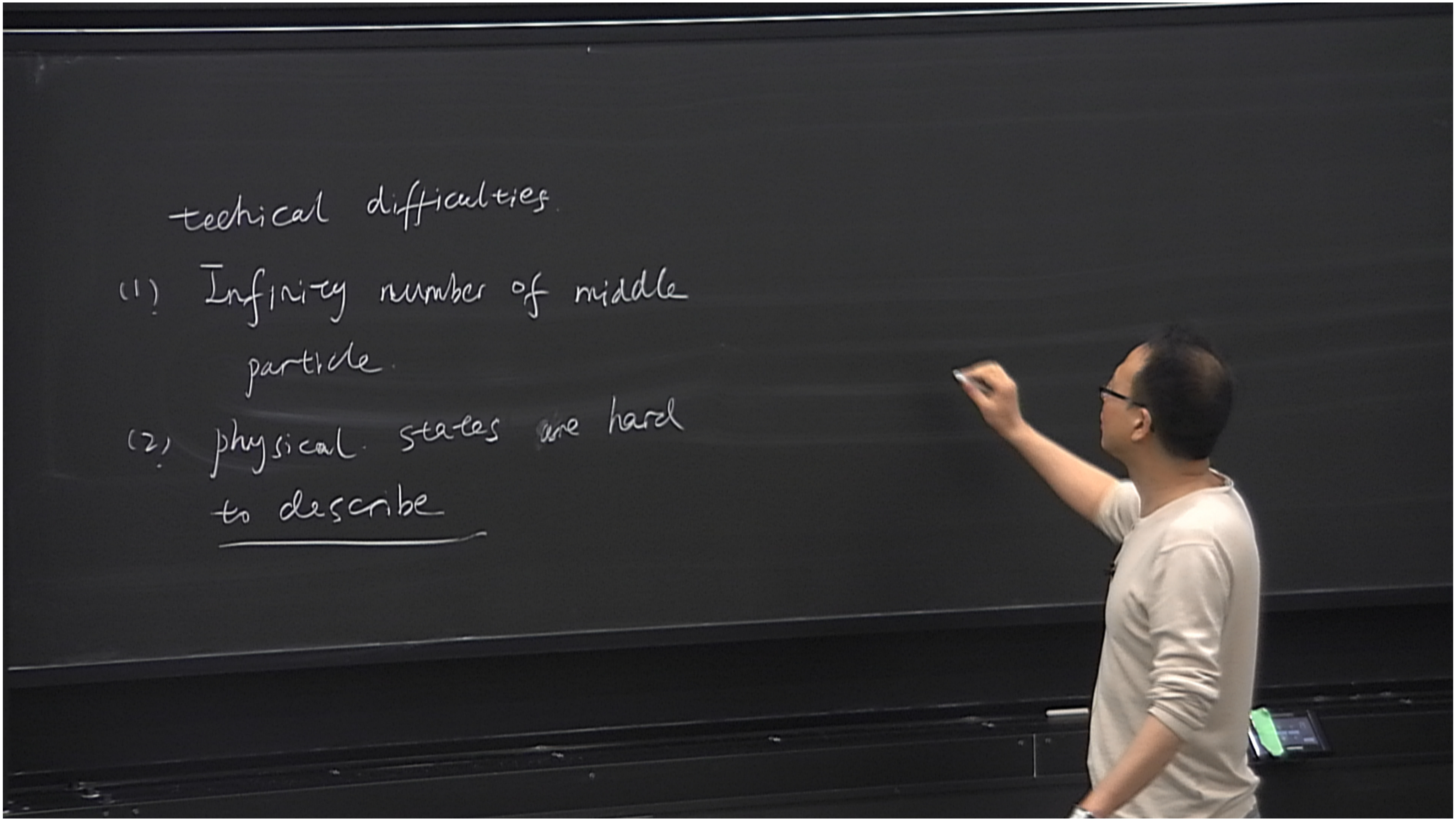
$q^2 = q \cdot p_i = q \cdot p_j = 0$
$\oint \frac{dz}{z} A(z)$
$A_L^h(z_\alpha) \frac{1}{P_\alpha^2} A_R^{(-h)}(z_\alpha)$


On-shell
$(p_i, p_j) \longrightarrow \begin{cases} p_i + zq \\ p_j - zq \end{cases}$
$q^2 = q \cdot p_i = q \cdot p_j = 0$
$\oint \frac{dz}{z} A(z) \Longrightarrow A = \sum_{\alpha} \sum_{h} A_L^{h}(z_\alpha) \frac{1}{P_\alpha^2} A_R^{(-h)}(z_\alpha)$
(a)


On-shell (p_i, p_j) → p_i + zq
p_j − zq
q^2 = q·p_i = q·p_j = 0
∮ dz/z A(z) ⟹ A = Σ_α Σ_h A_L^h(z_α) 1/P_α^2 A_R^(−h)(z_α)
(a) pole-like 1/(p^2+iε)
(b) factorization property


On-shell
(p_i, p_j) ⟶ { p_i + zq, p_j − zq
q² = q·p_i = q·p_j = 0
∮ dz/z A(z) ⟹ A = Σ_α Σ_h A_L^h(z
A_R^(−h)(z_α)
(a) pole-like 1/(p² + iε)
(b) factorization property
(c) z→∞ A(z) ⟶ 0
get rid boundary contribution


techical difficulties.
(1) Infinity number of middle
particle.
(2) physical.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(1) Infinity number of middle particle
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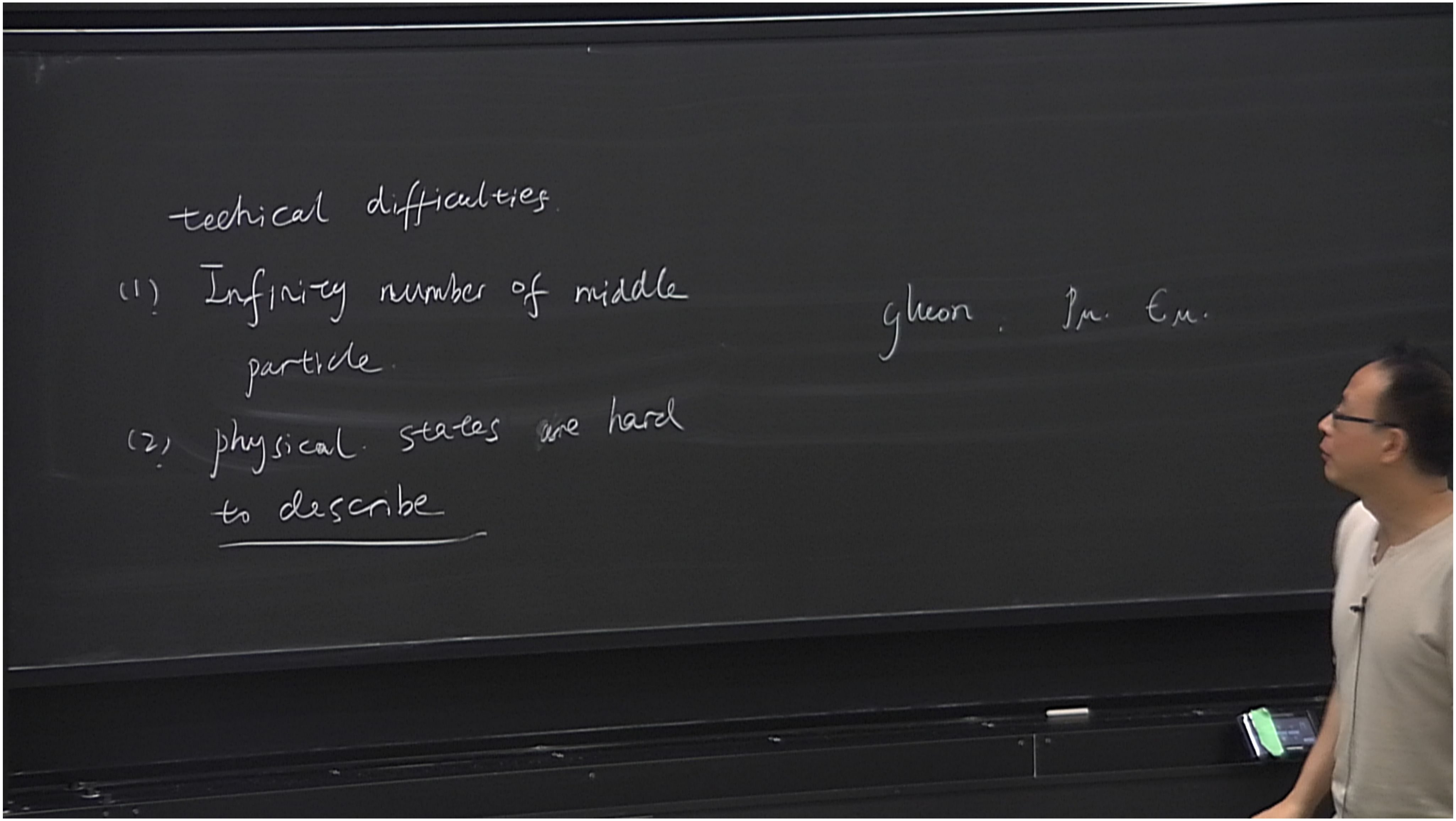
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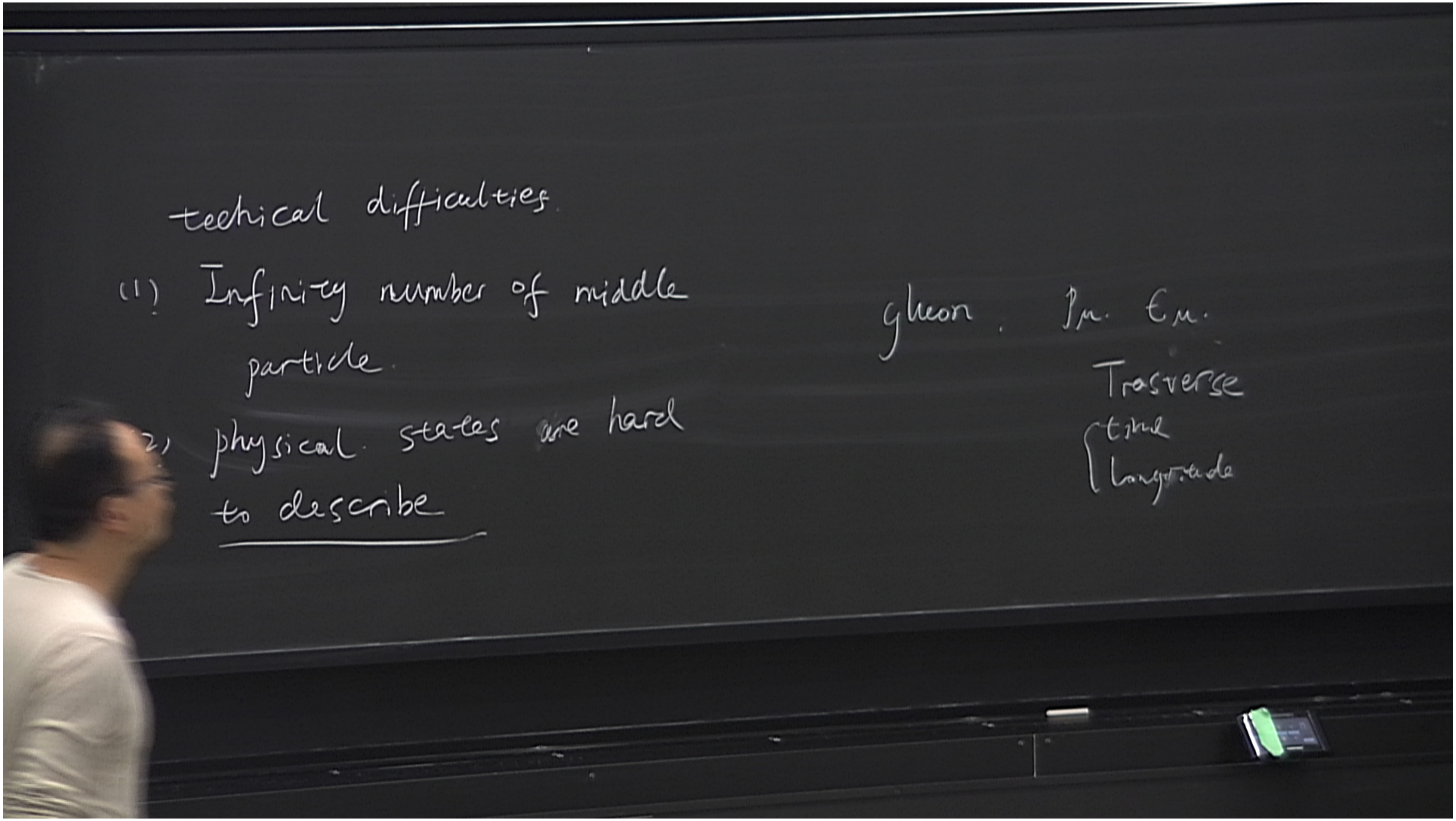
gluon. p_μ. ϵ_μ.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(1) Infinity number of middle particle
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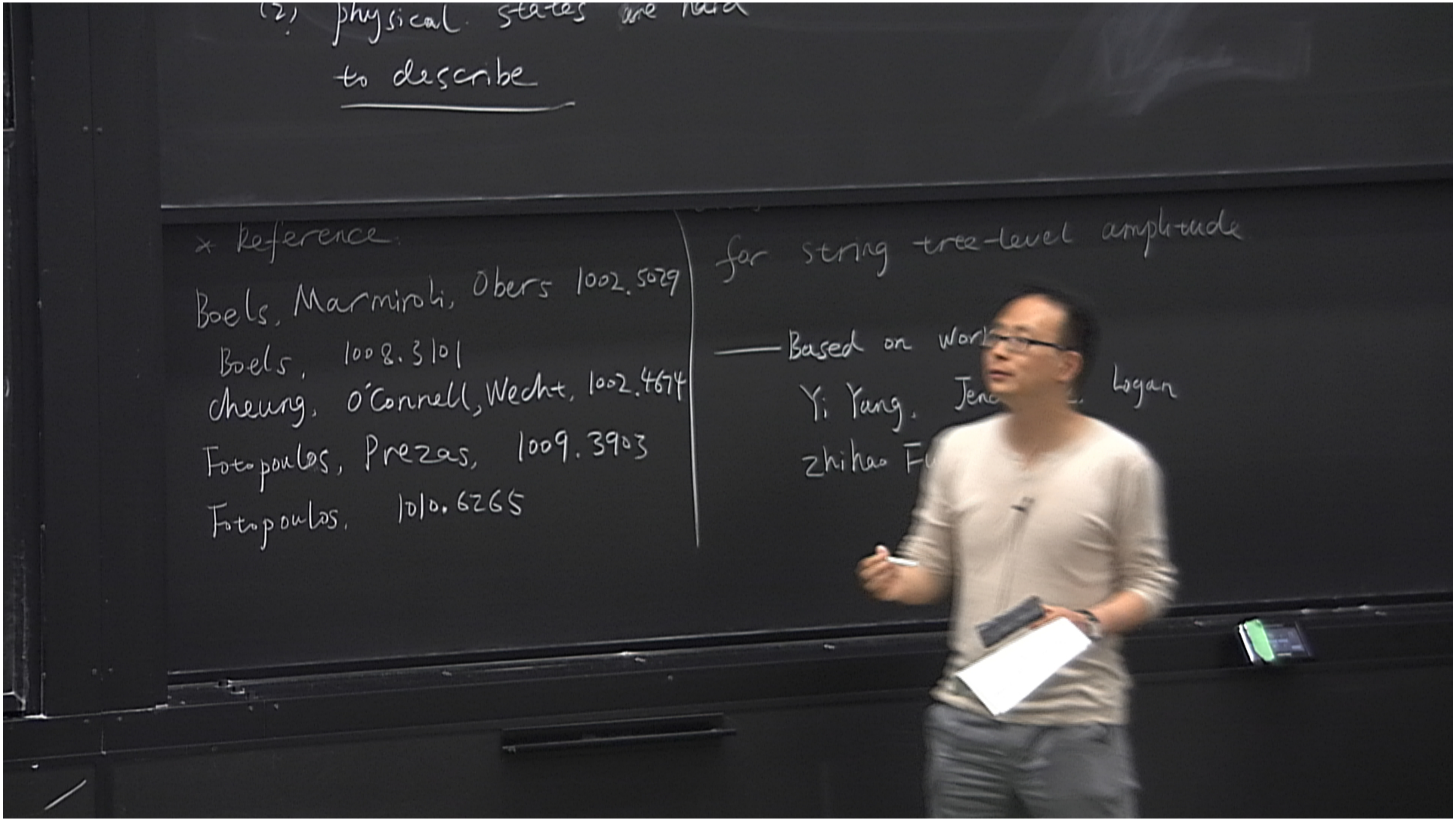
gluon . Pμ. εμ.
Trasverse
time
Longitude



physical states
to describe
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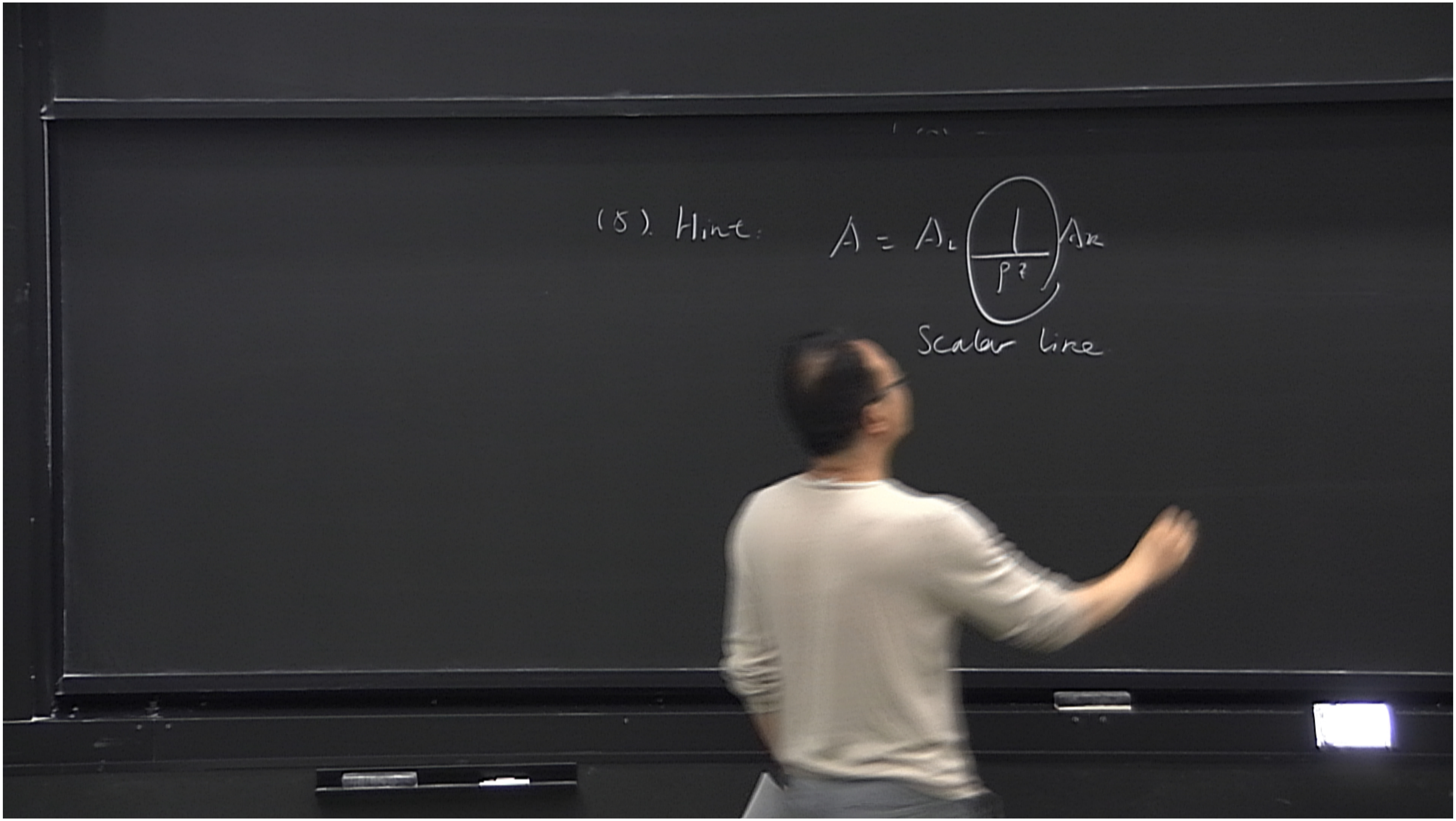
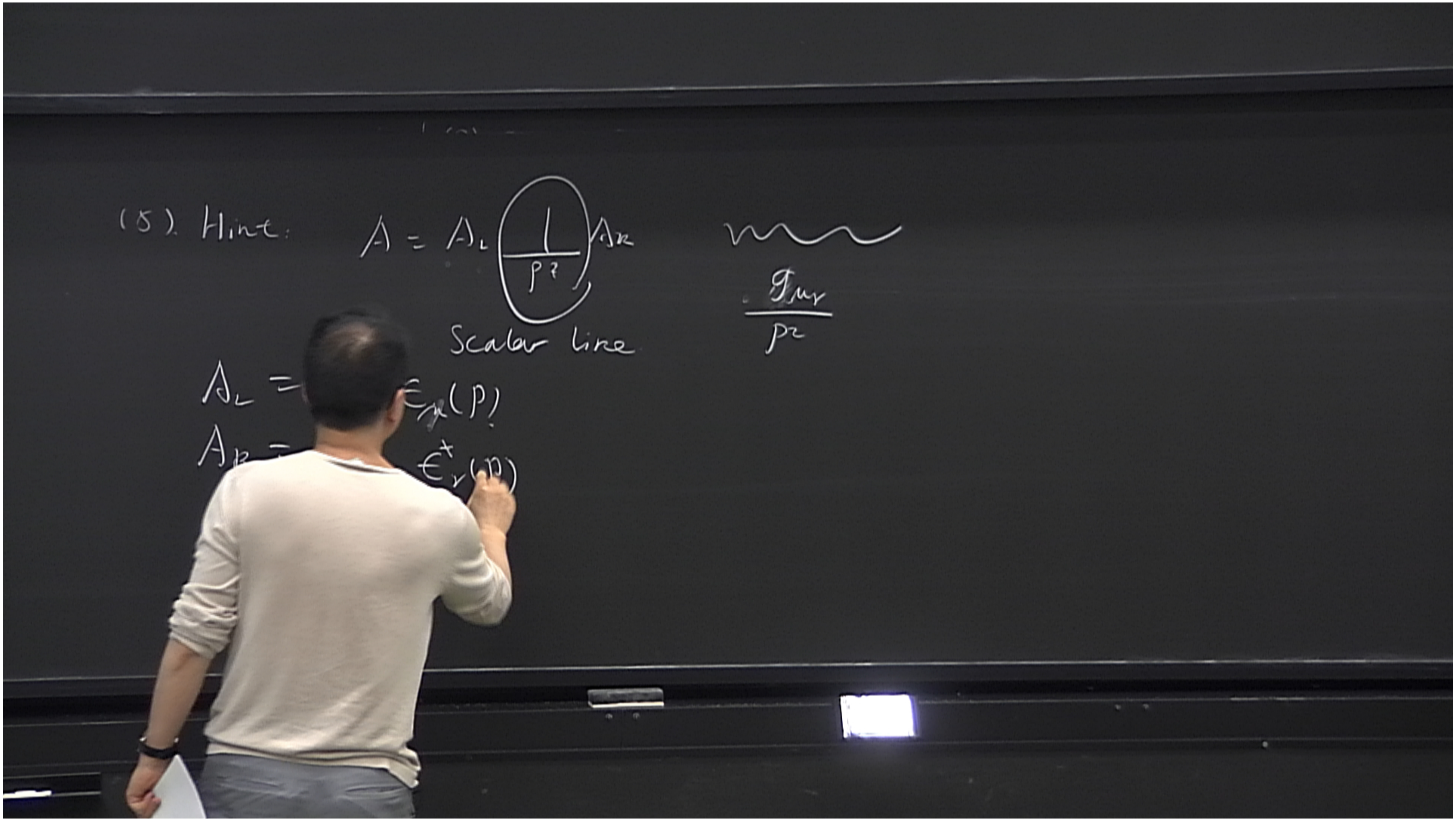

(5). Hint:
A = A_L 1/p² A_R
Scalar line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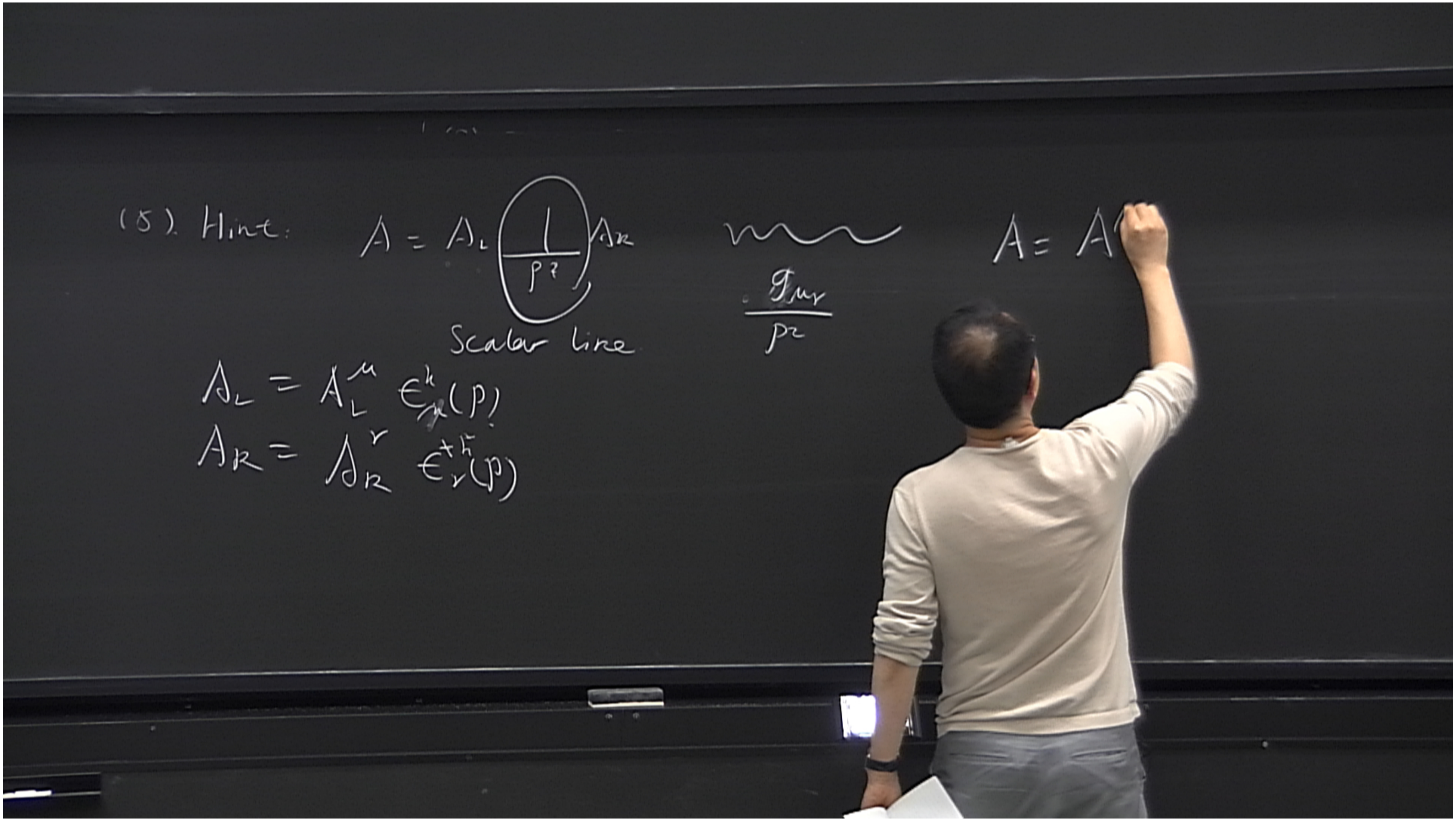
A = A_L 1/p² A_R
Scalar line
g_μν / p²
A_L =
A_R =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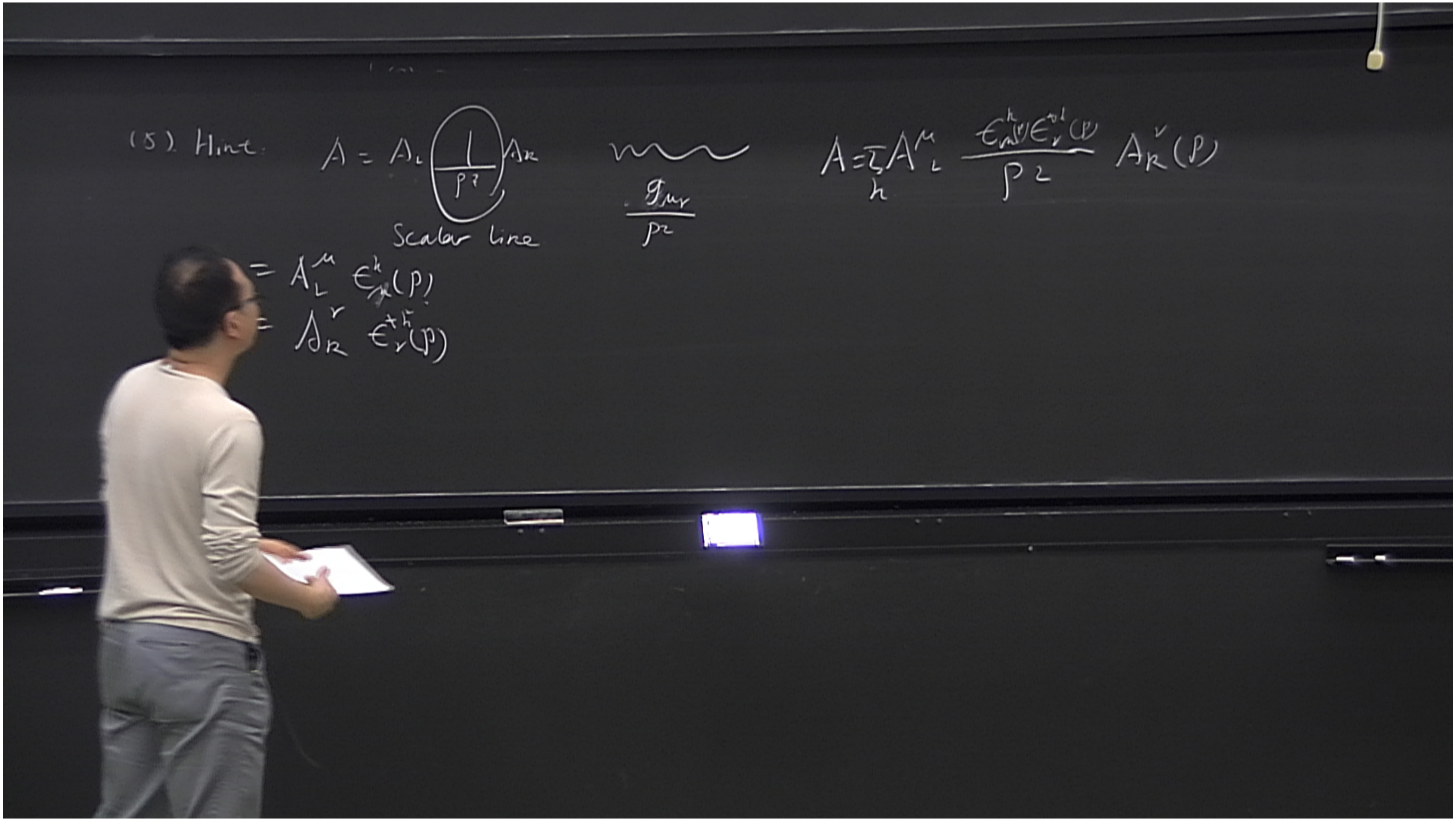
$A = A_L \frac{1}{p^2} A_R$
Scalar line
$\frac{g_{\mu\nu}}{p^2}$
$A = A$
$A_L = A_L^\mu \epsilon^h_\mu(p)$
$A_R = A_R^\nu \epsilon^{+h}_\nu(p)$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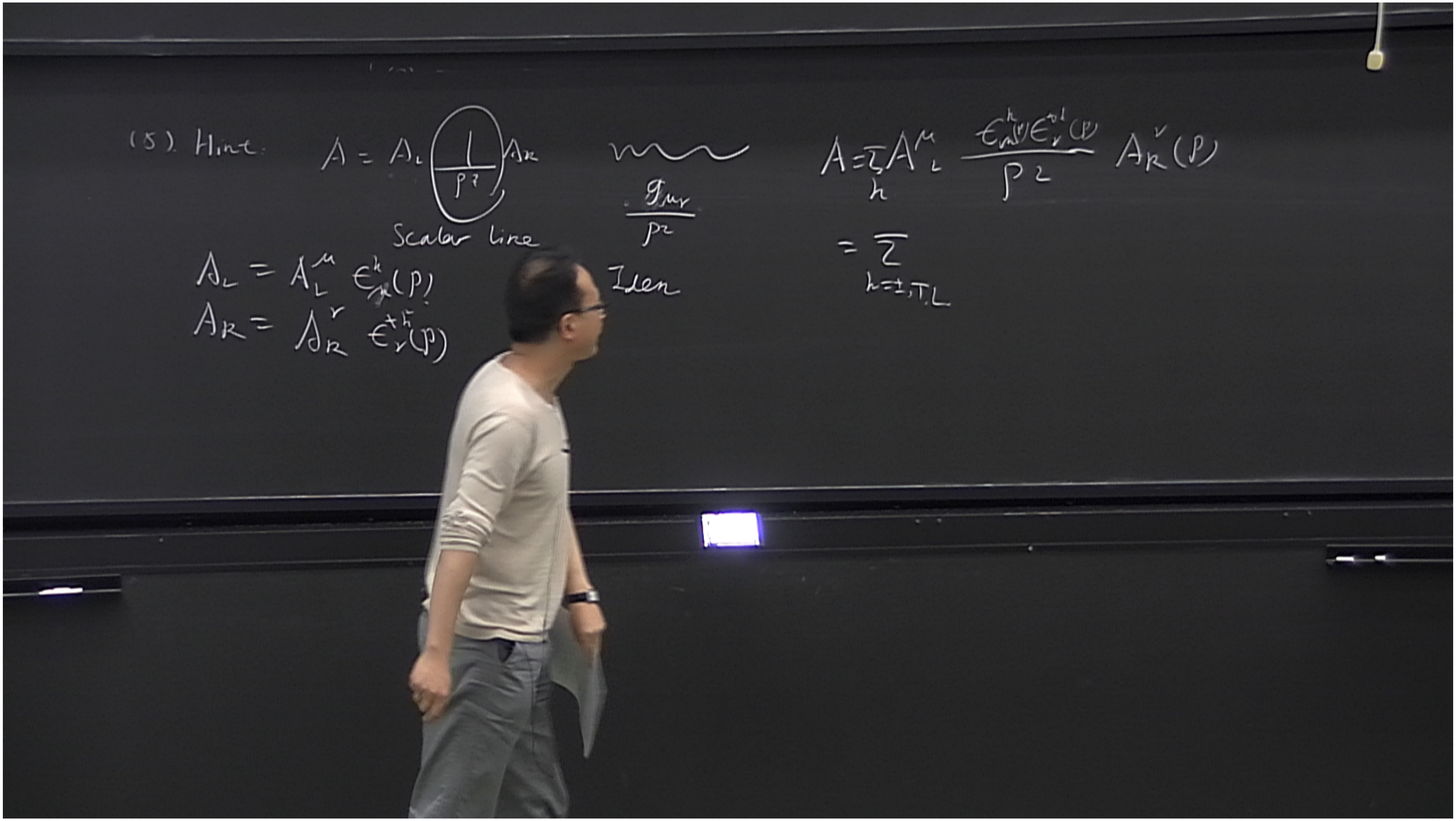
$A = A_L \frac{1}{p^2} A_R$
Scalar line
$\frac{g_{\mu\nu}}{p^2}$
$A = \sum_h A^\mu_L \frac{\epsilon^h_\mu(p)\epsilon^{*h}_\nu(p)}{p^2} A^\nu_R(p)$
$= A^\mu_L \epsilon^h_\mu(p)$
$= A^\nu_R \epsilon^{*h}_\nu(p)$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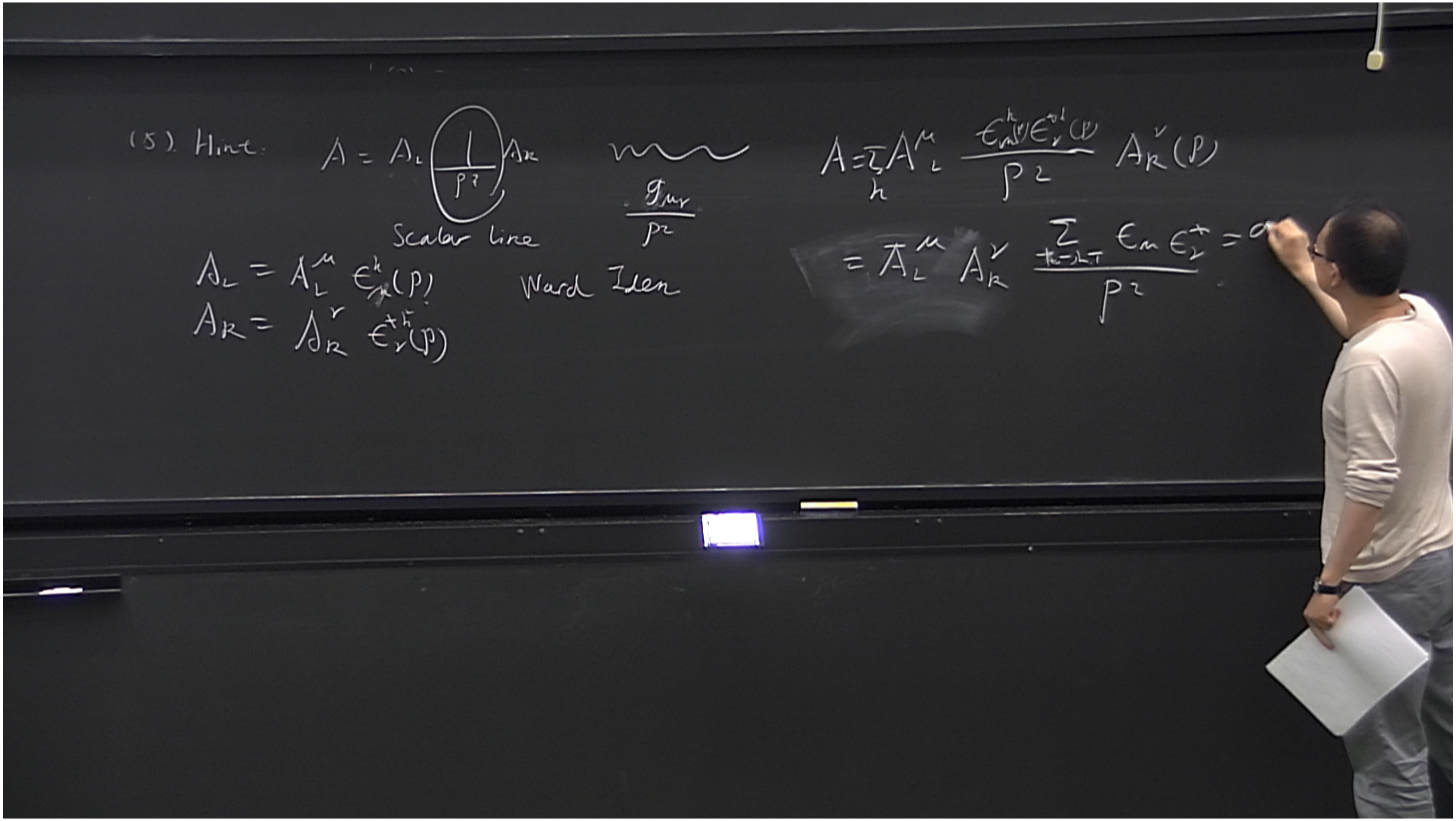
Iden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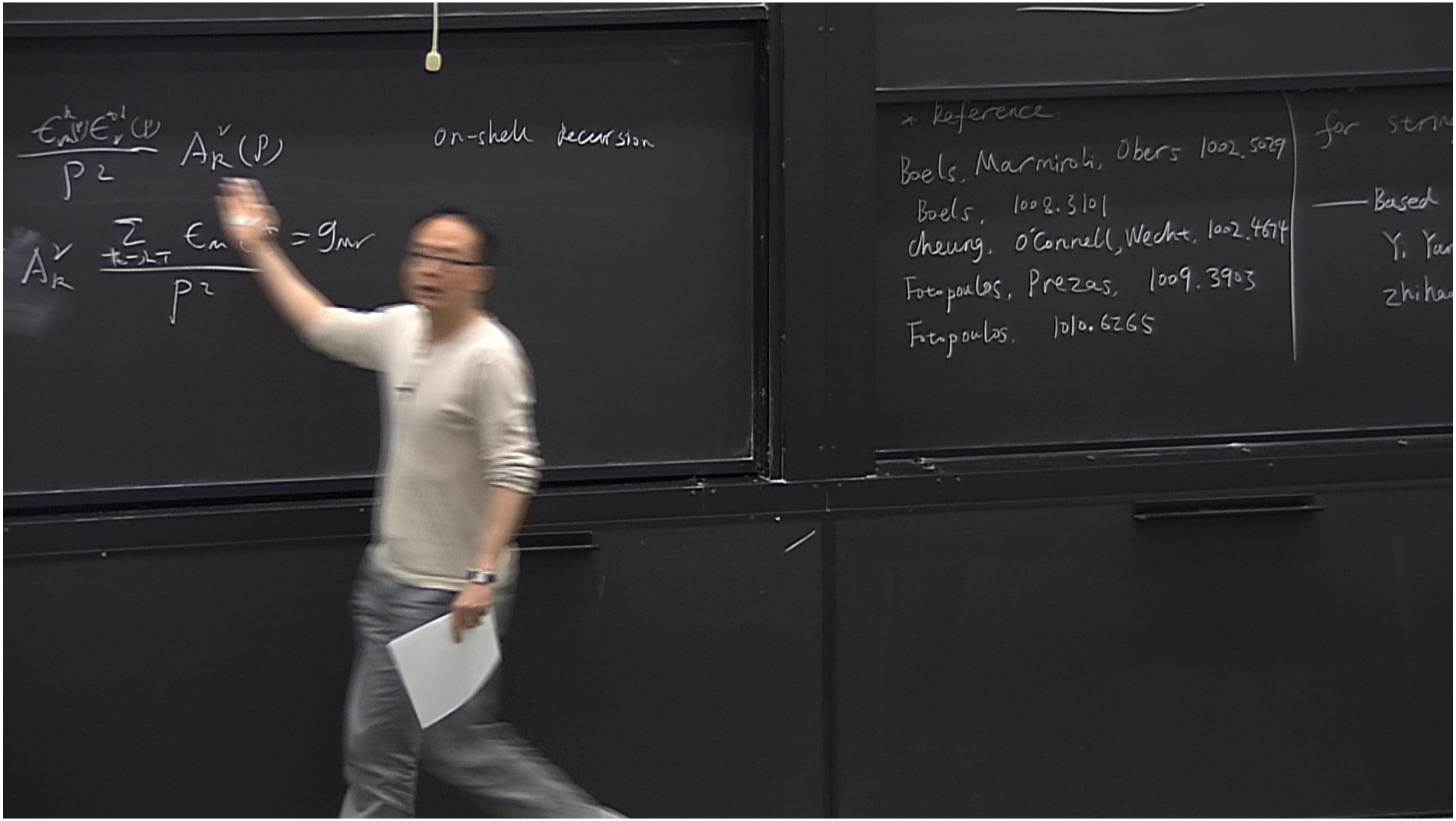
Scalar line
Ward Iden



on-shell recursion
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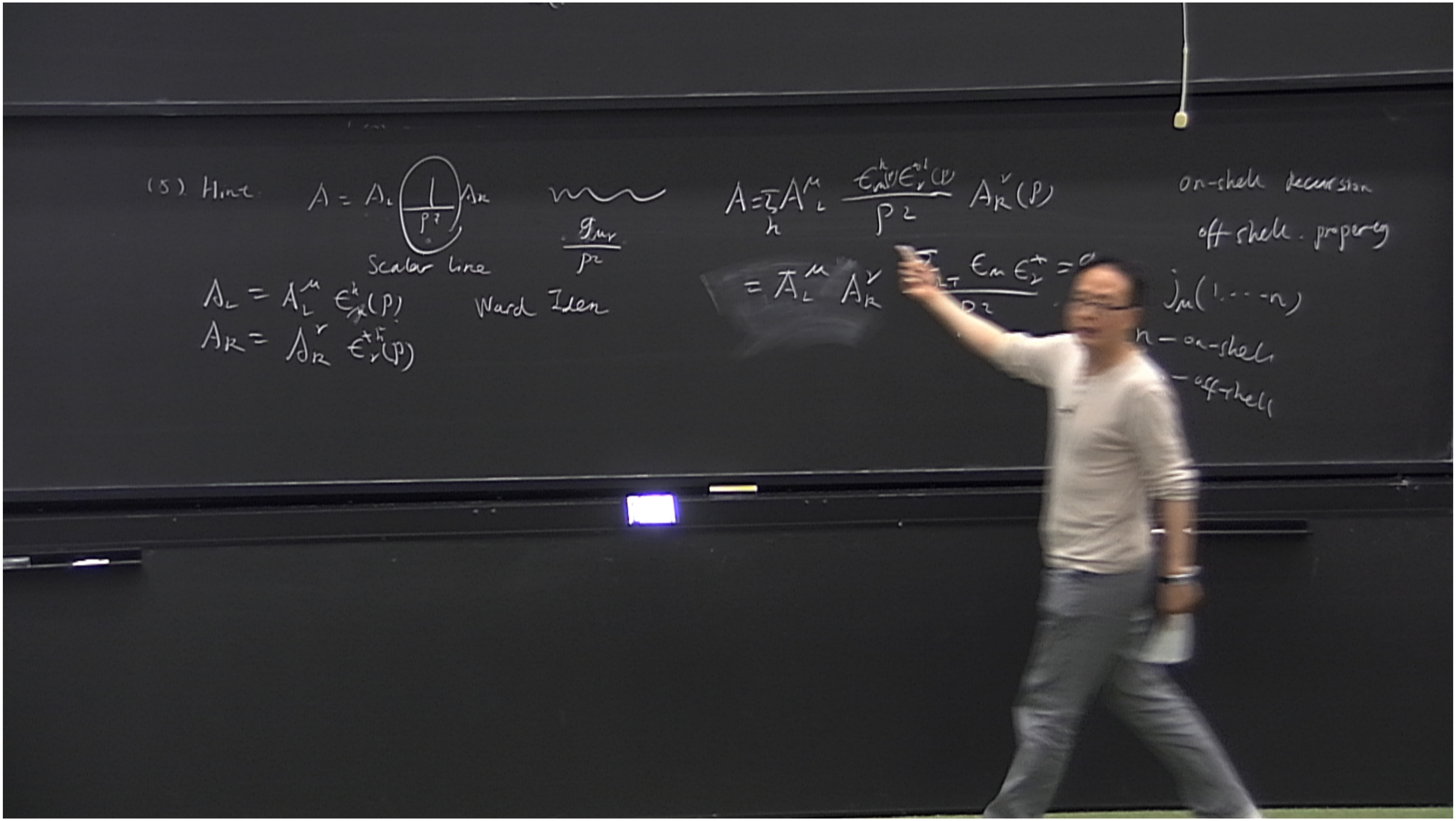
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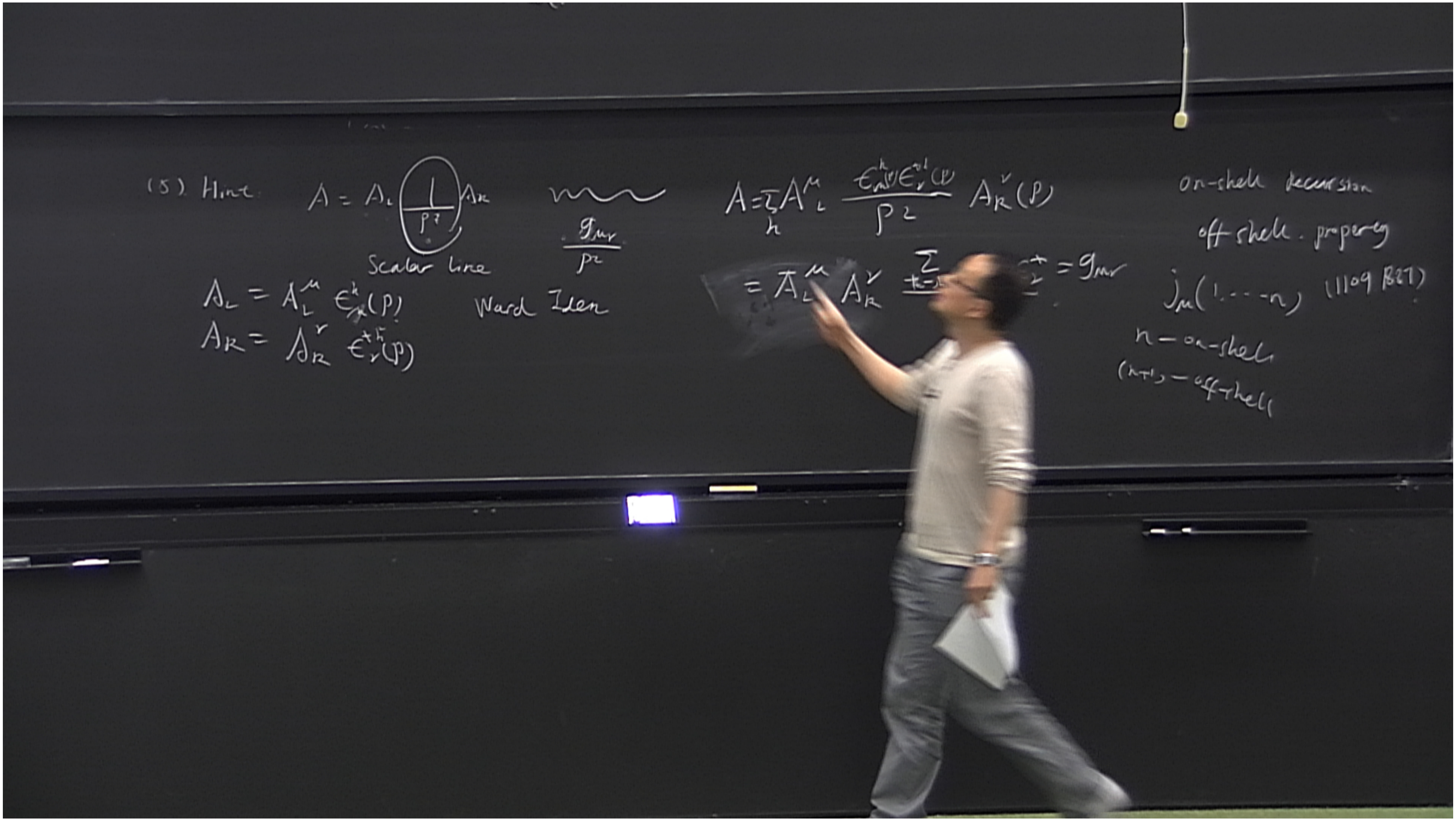
Scalar line
g_μν / p²
A_L = A_L^μ ε^h_μ(p)
Ward Iden
A_R = A_R^ν ε^{+h}_ν(p)
A = Σ_h A_L^μ ε^h_μ(p) ε^{-h}_ν(p) / p² A_R^ν(p)
= A_L^μ A_R^ν
on-shell recursion
off shell. property
j_μ(1, ..., n)
n — on-shell
— off-shell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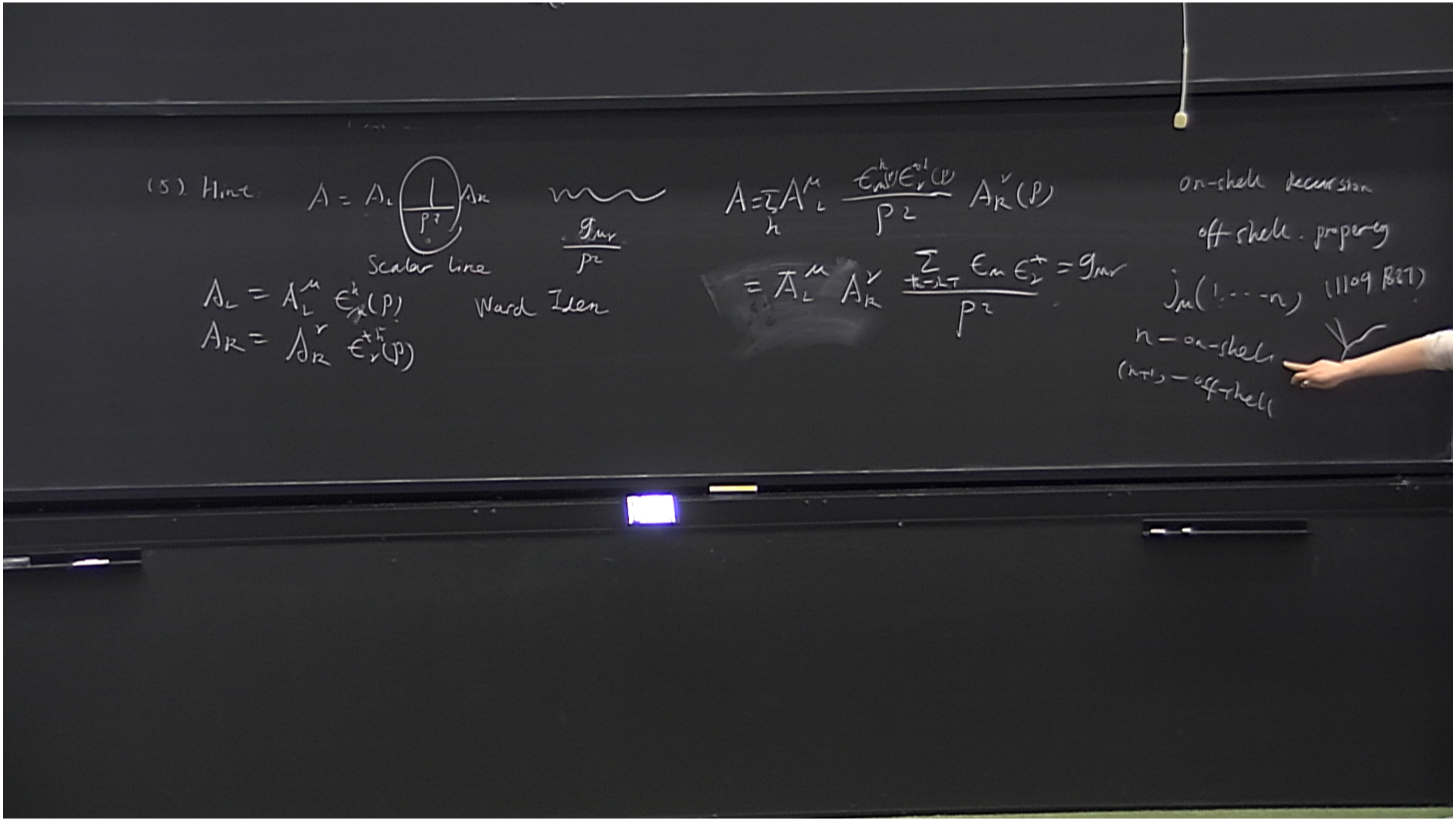
Scalar line
Ward Iden
on-shell recursion
off shell . property
n — on-shell
(n+1) — off-shell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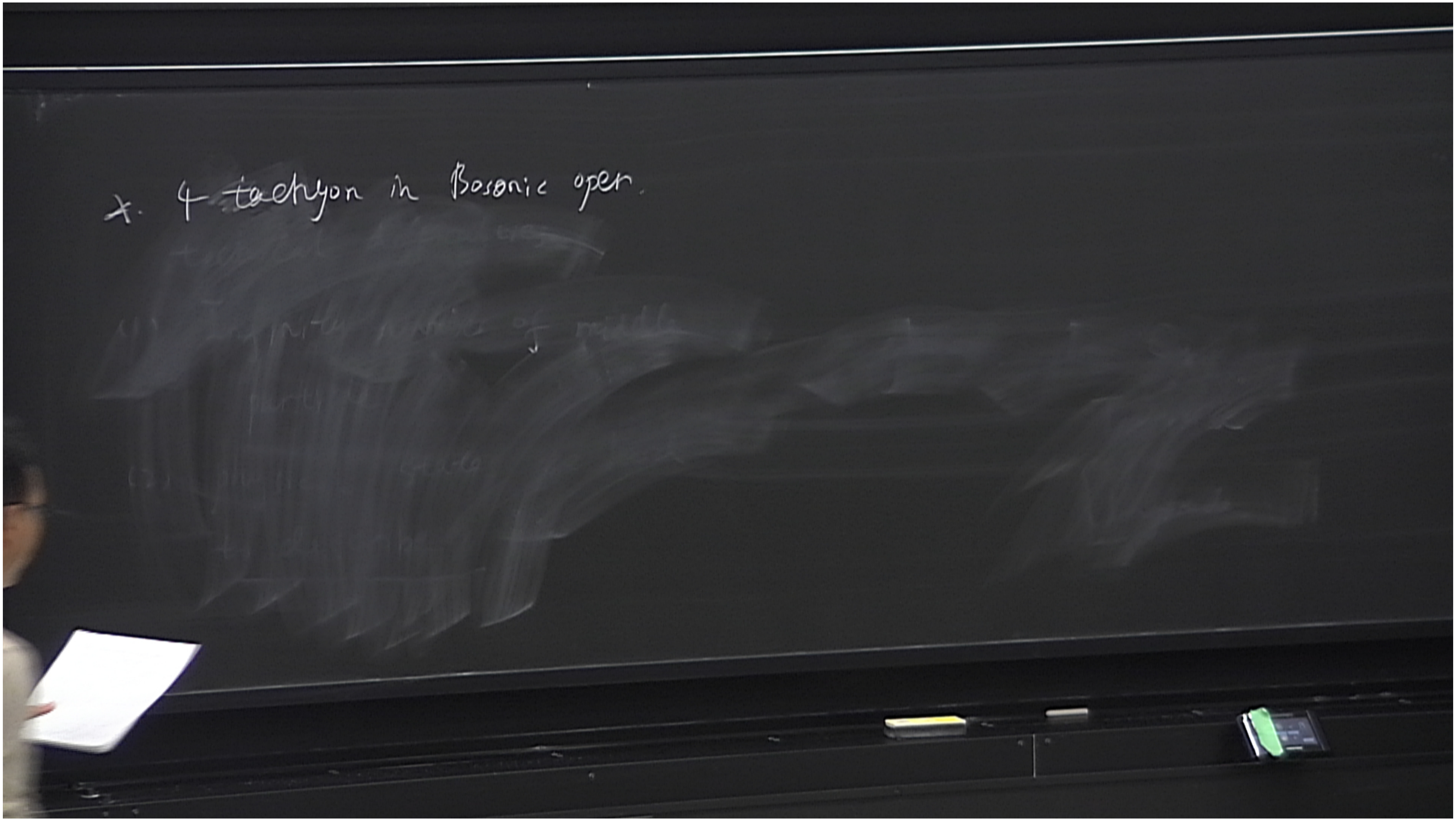
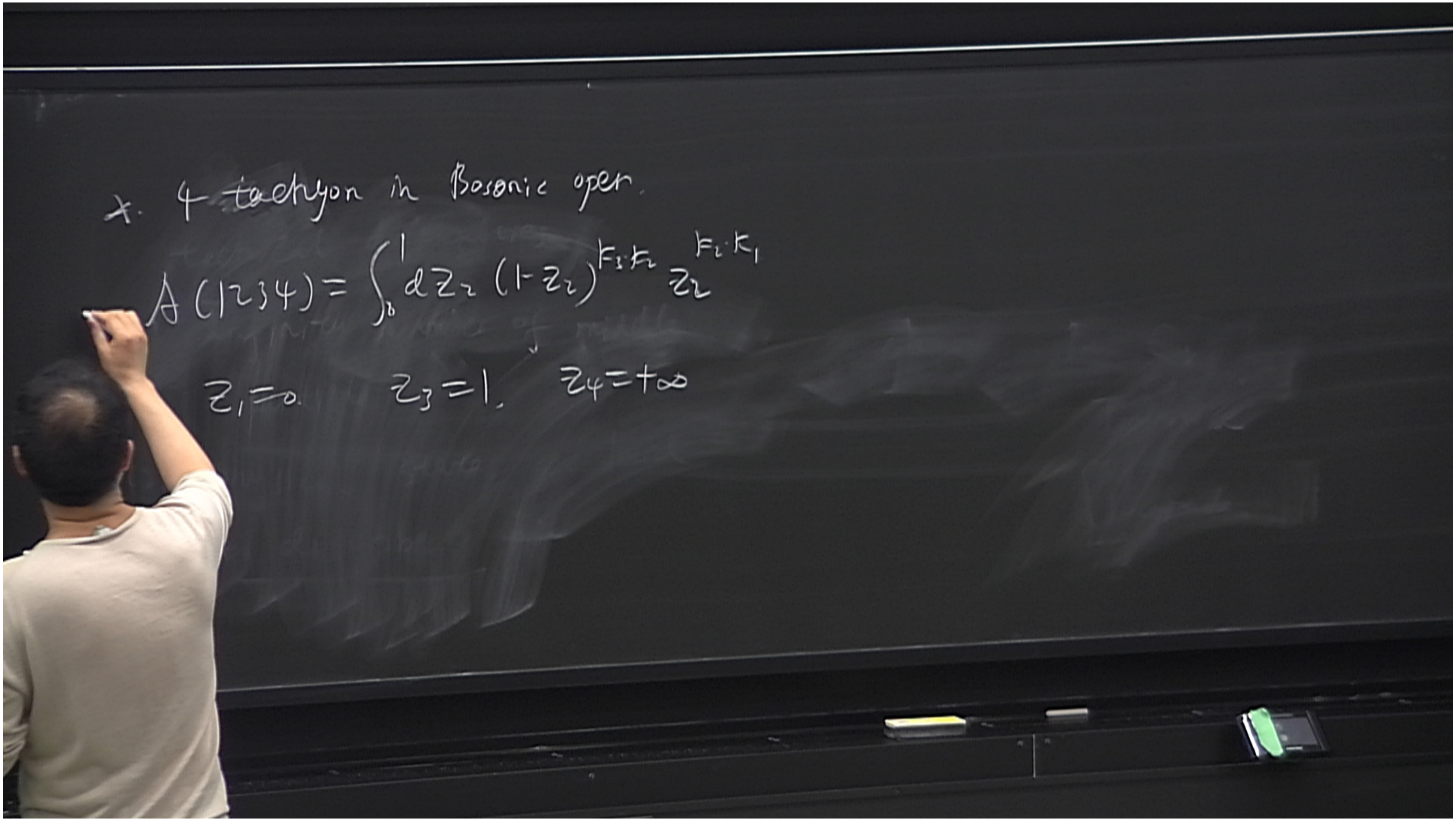

x. 4 tachyon in Bosonic open.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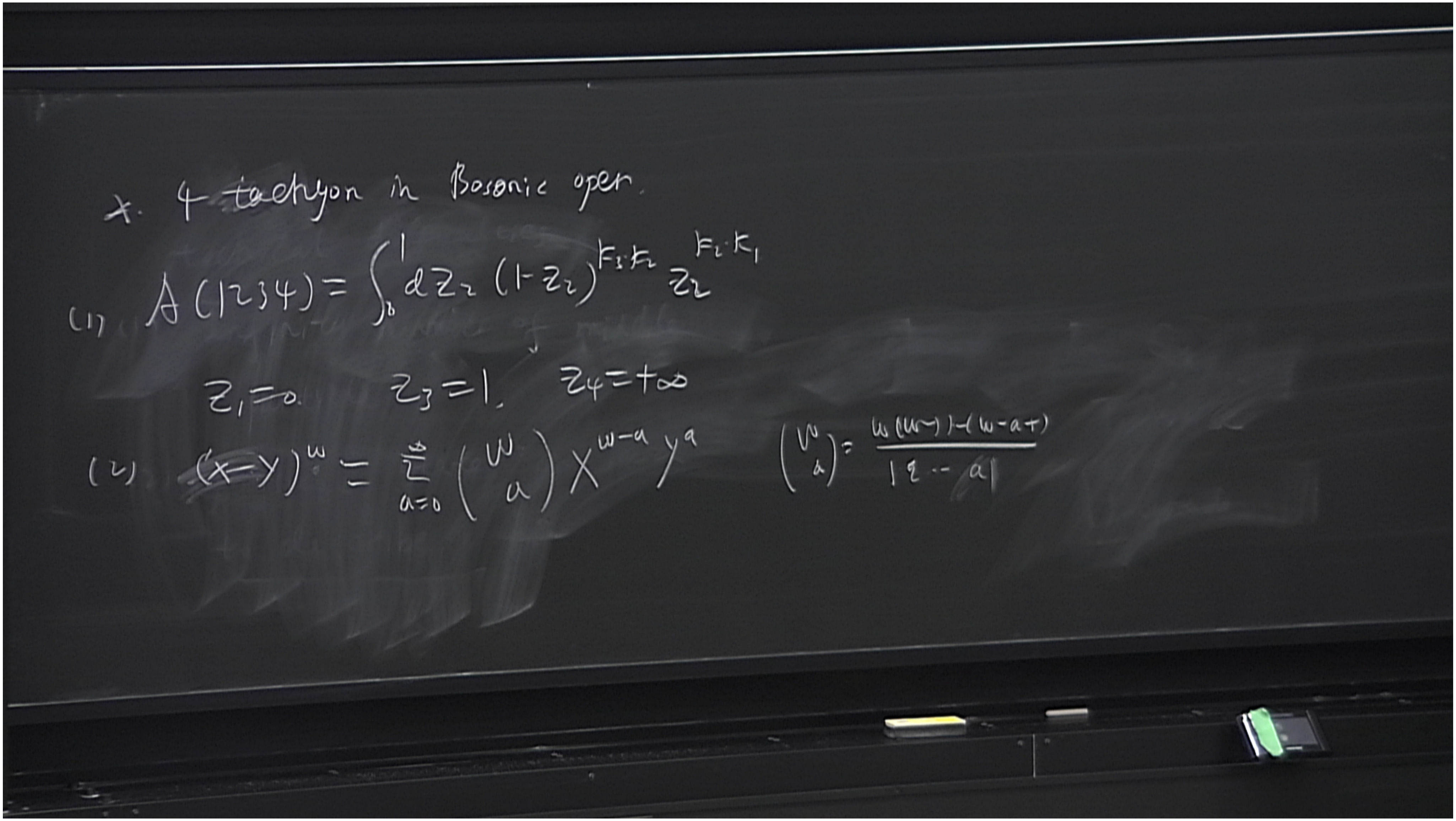
$A(1234) = \int_0^1 dz_2 \, (1-z_2)^{k_3 \cdot k_2} \, z_2^{k_2 \cdot k_1}$
$z_1 = 0.$ $z_3 = 1.$ $z_4 = +\infty$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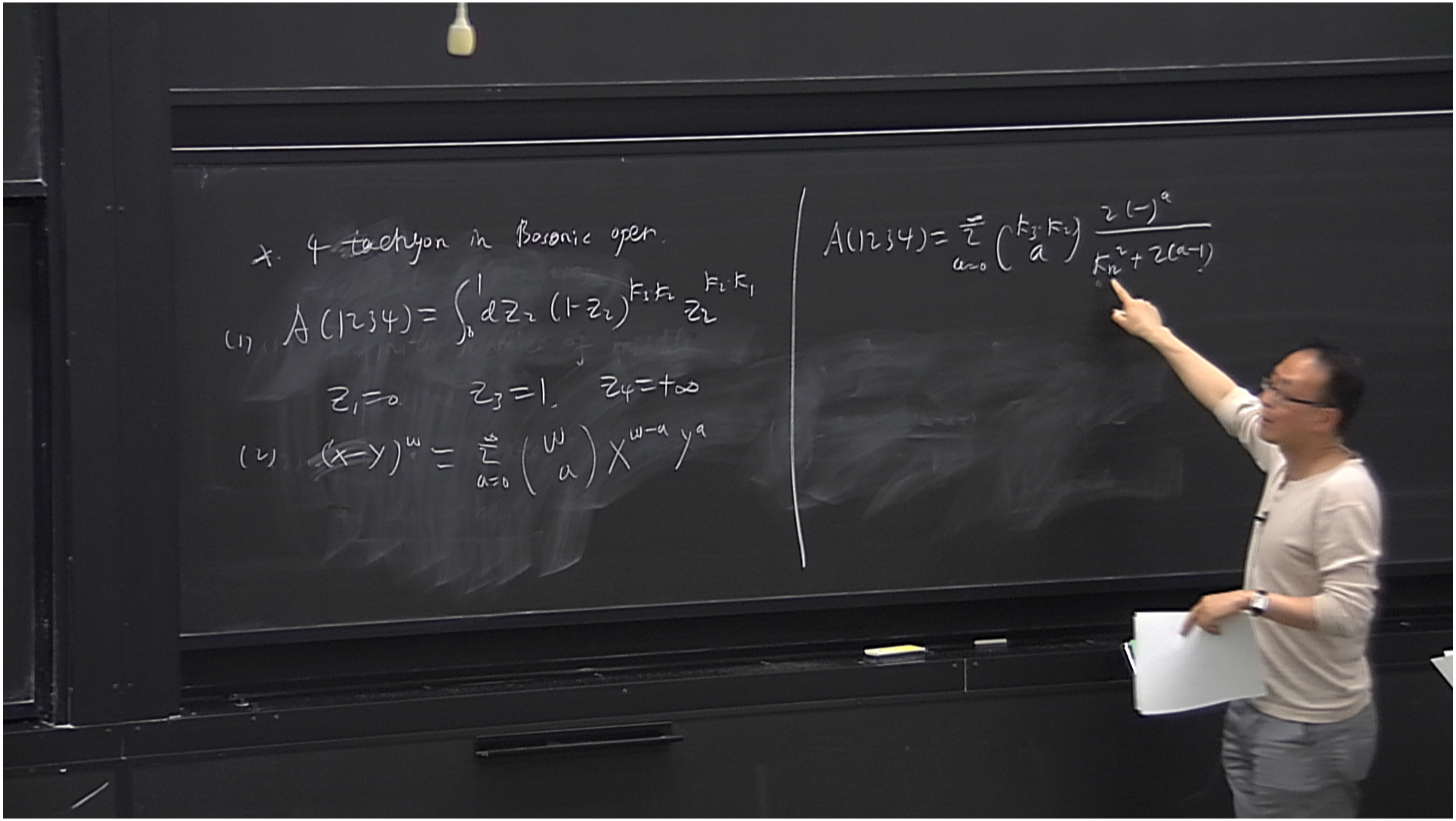
(2) $(x-y)^w = \sum_{a=0}^{\infty} \binom{w}{a} x^{w-a} y^a$
$\binom{w}{a} = \frac{w(w-1)\cdots(w-a+1)}{1 \, 2 \cdots a}$



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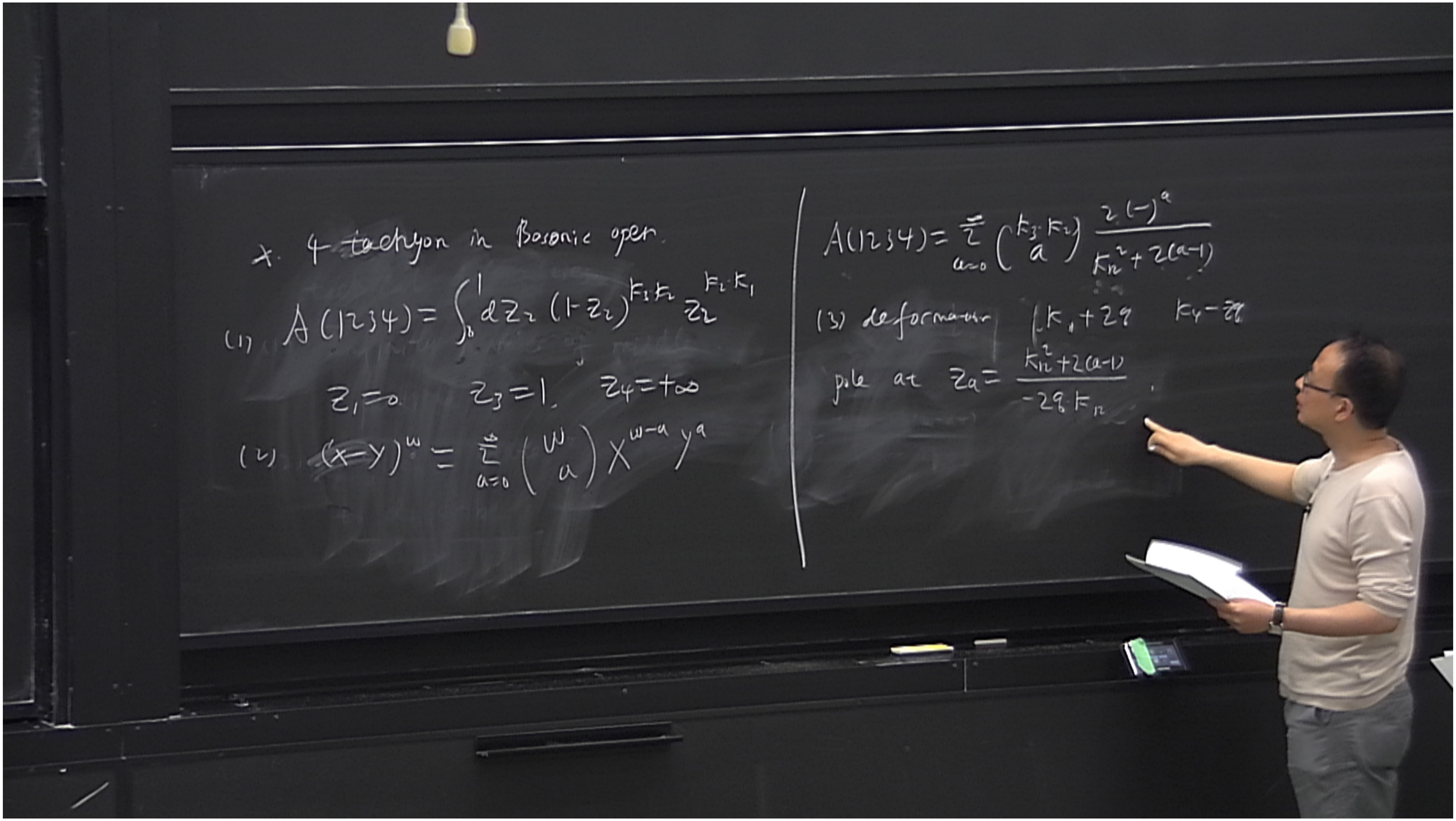
(1) A(1234) = ∫_0^1 dz_2 (1-z_2)^{k_3·k_2} z_2^{k_2·k_1}
z_1=0   z_3=1   z_4=+∞
(2) (x-y)^w = Σ_{a=0}^∞ (w a) x^{w-a} y^a
A(1234) = Σ_{a=0}^∞ (k_3·k_2 a) 2(-)^a / (k_12^2 + 2(a-1))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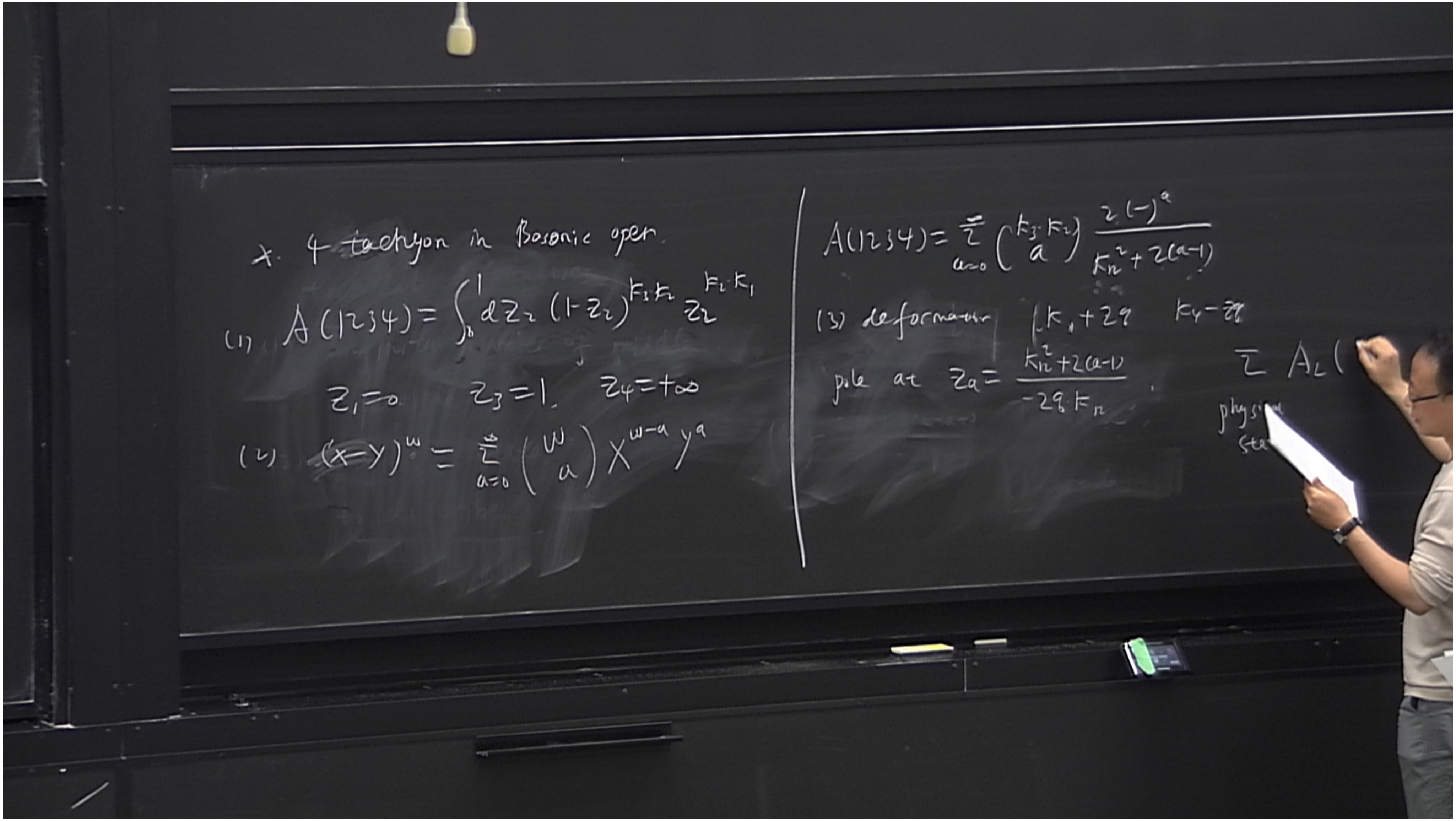
(3) deformation $k_1 + 2q \quad k_4 - 2q$
pole at $z_a = \frac{k_{12}^2 + 2(a-1)}{-2q \cdot k_{12}}$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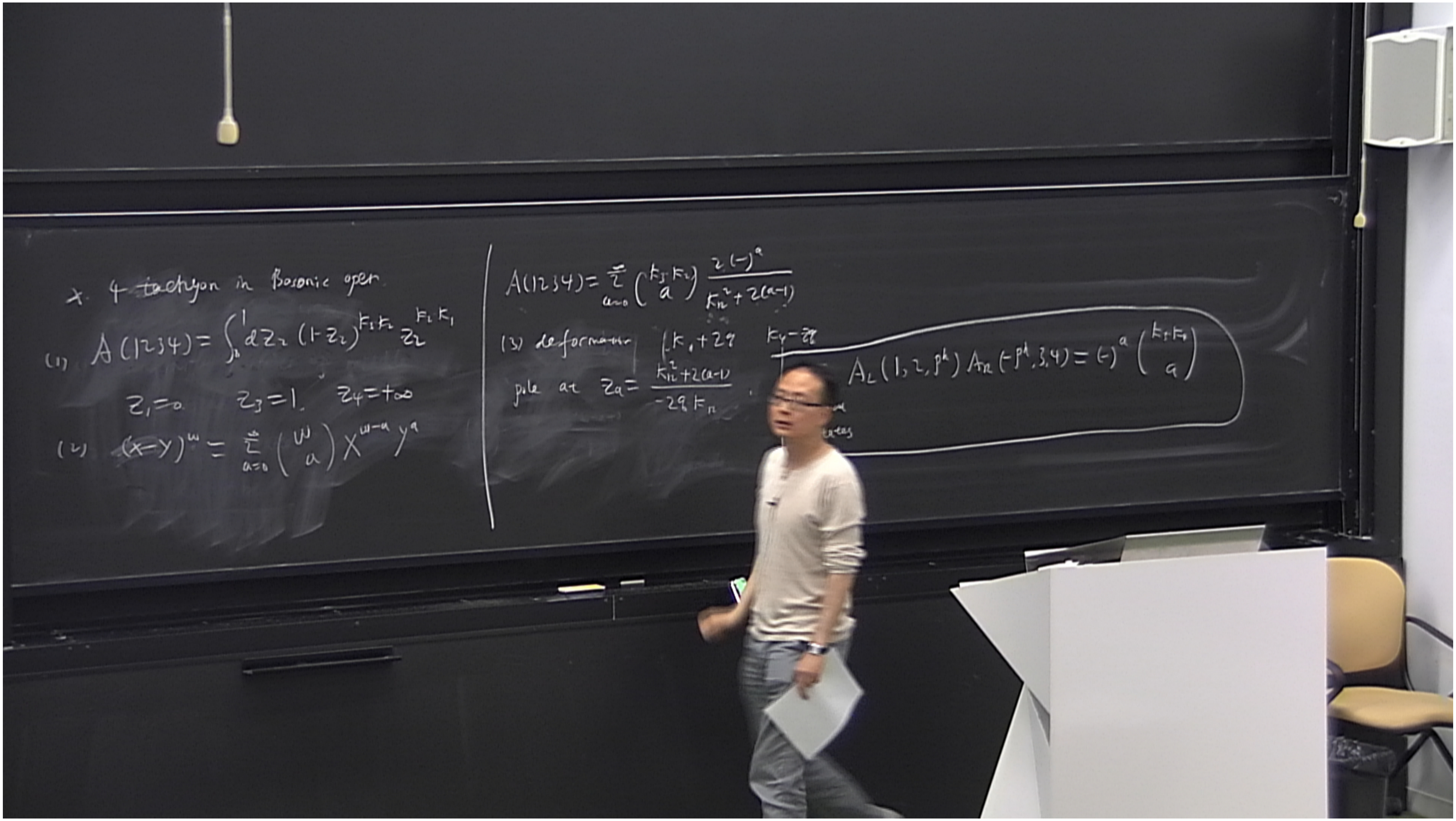
$\sum A_L($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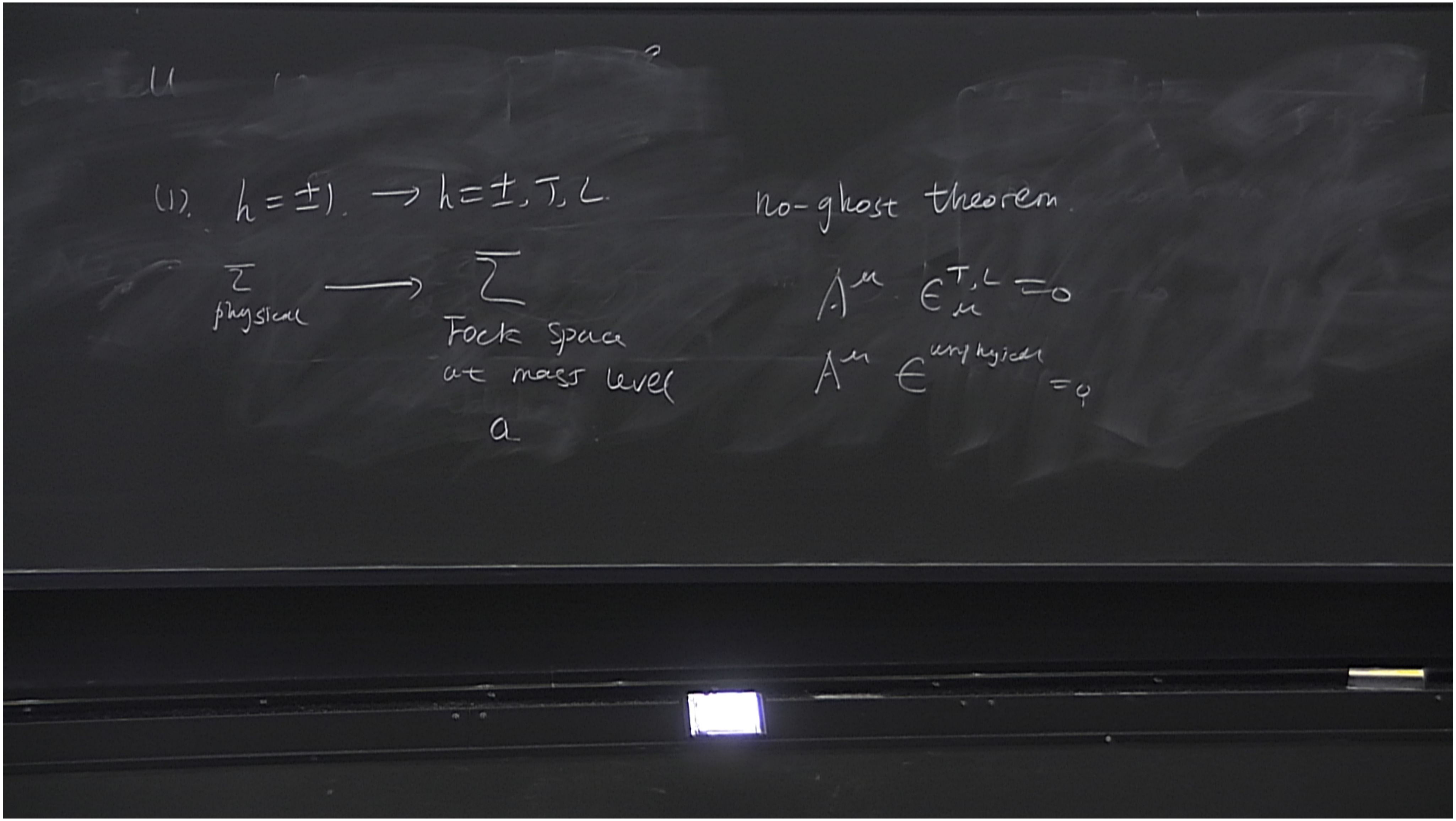
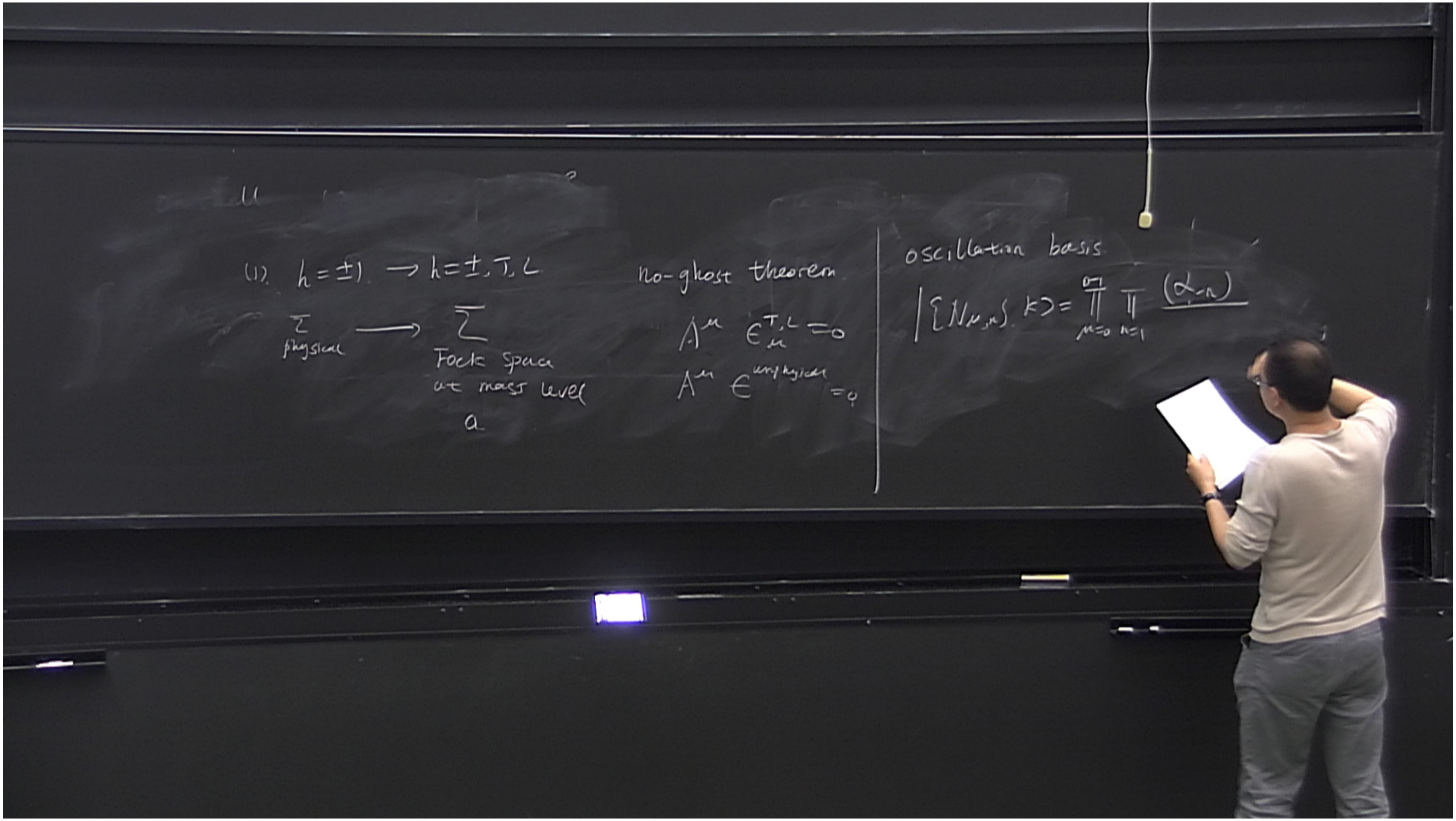
(2) (x-y)^w = Σ_{a=0} (w a) x^{w-a} y^a
A(1234) = Σ_{a=0} (k_3 k_2 a) 2(-)^a / (k_12^2 + 2(a-1))
(3) deformation
pole at z_a = (k_12^2 + 2(a-1)) / (-2q k_12)
A_L(1,2,p^A) A_R(-p^A,3,4) = (-)^a (k_3 k_4 a)

(1). $h = \pm 1 \rightarrow h = \pm, T, L$

no-ghost theorem

$\sum_{\text{physical}} \longrightarrow \sum$ Fock space at mass level a


(1). h = ±1 → h = ±, T, L
Σ physical → Σ Fock space at mass level a
no-ghost theorem
A^μ · ε_μ^{T, L} = 0
A^μ ε^{unphysical} = 0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Σ physical → Σ Fock space at mass level a
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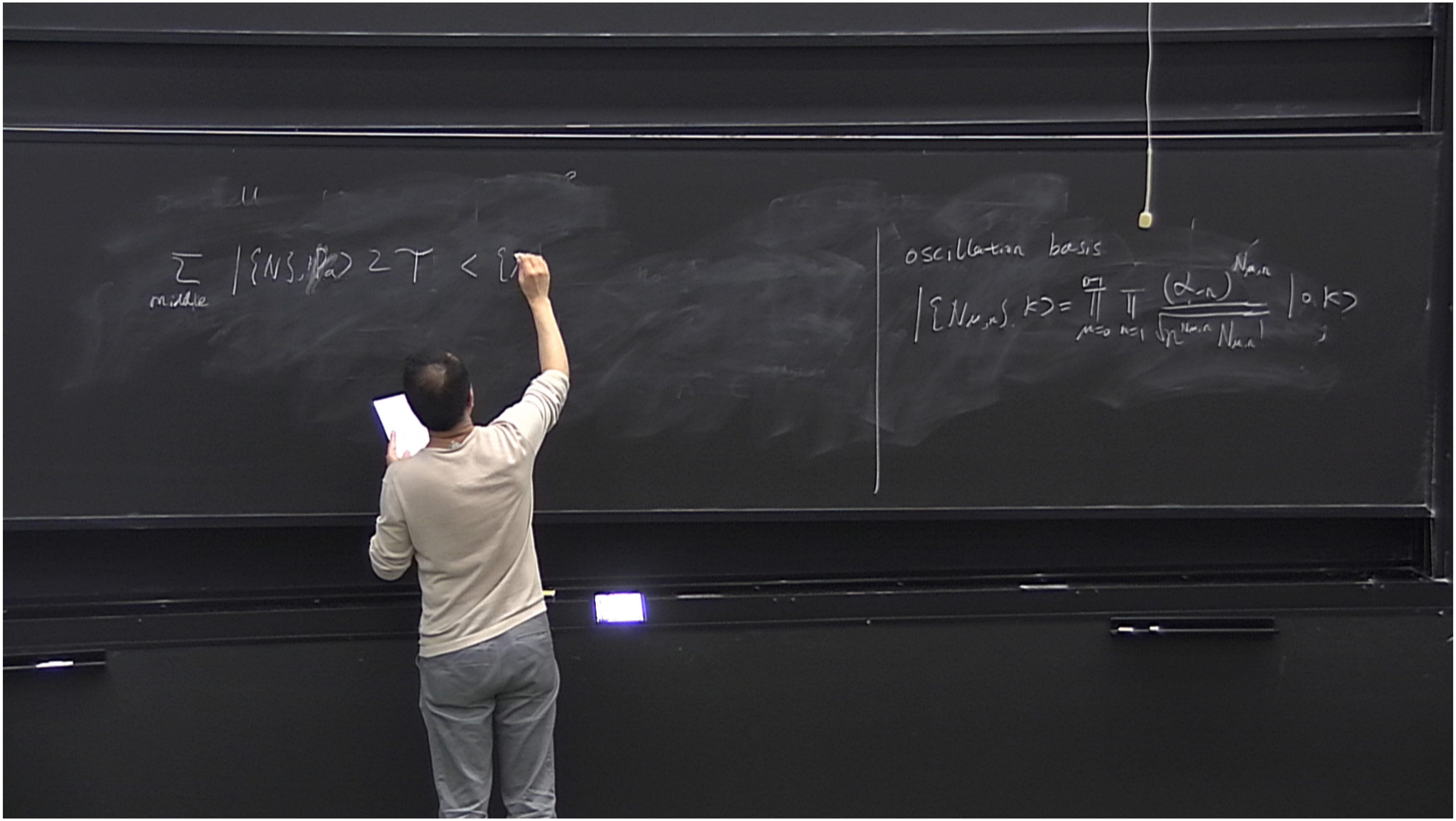
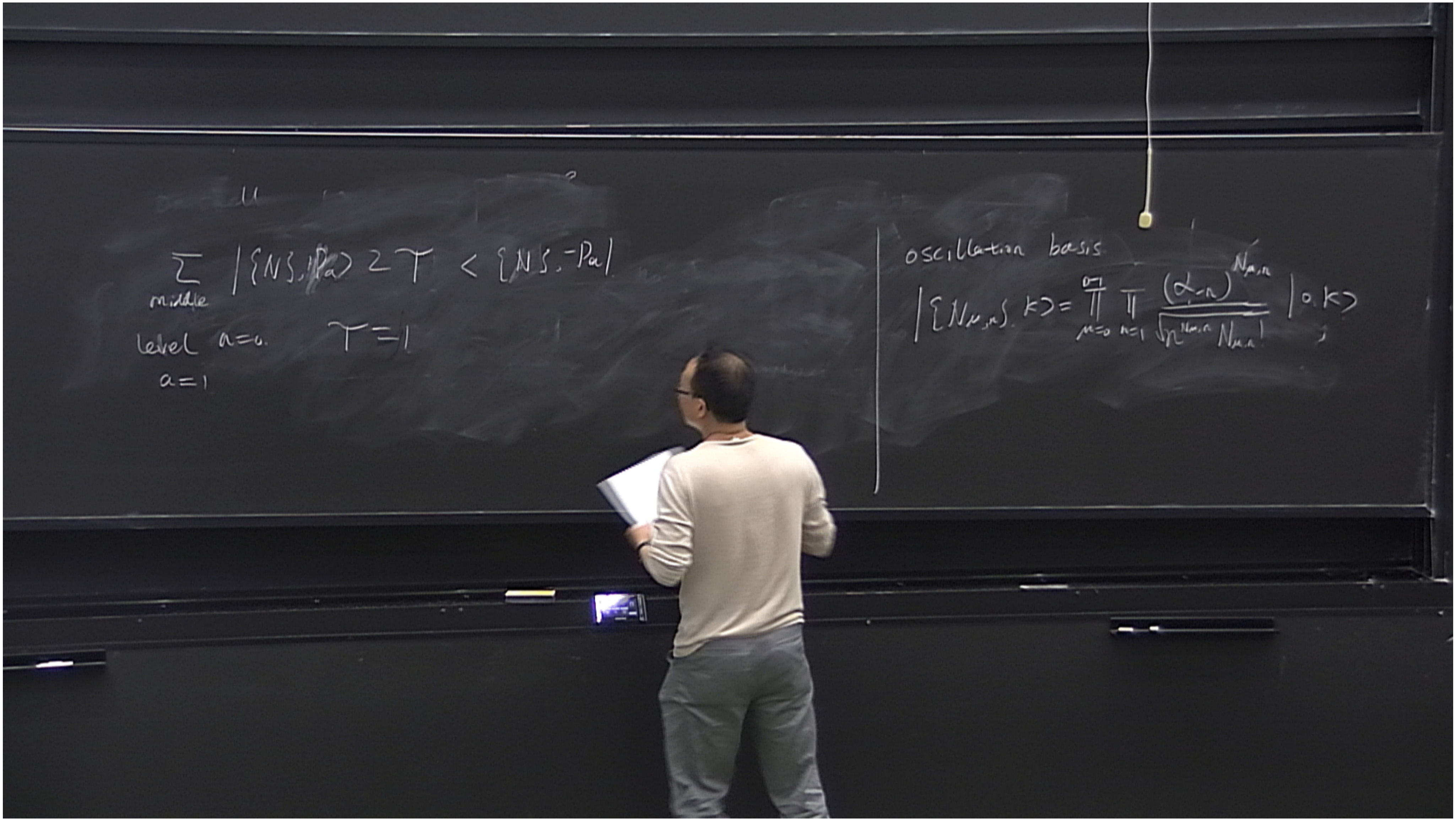
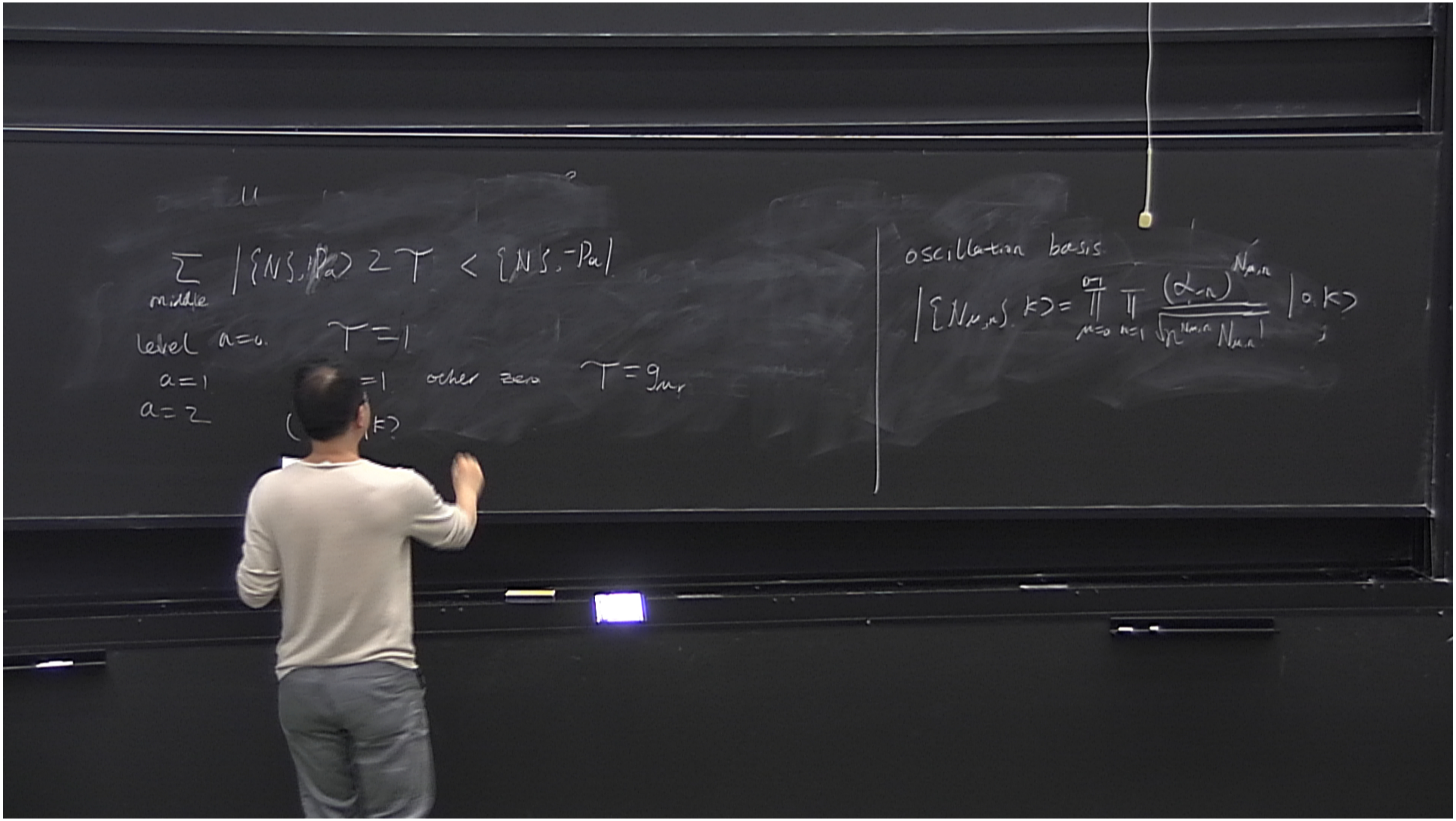
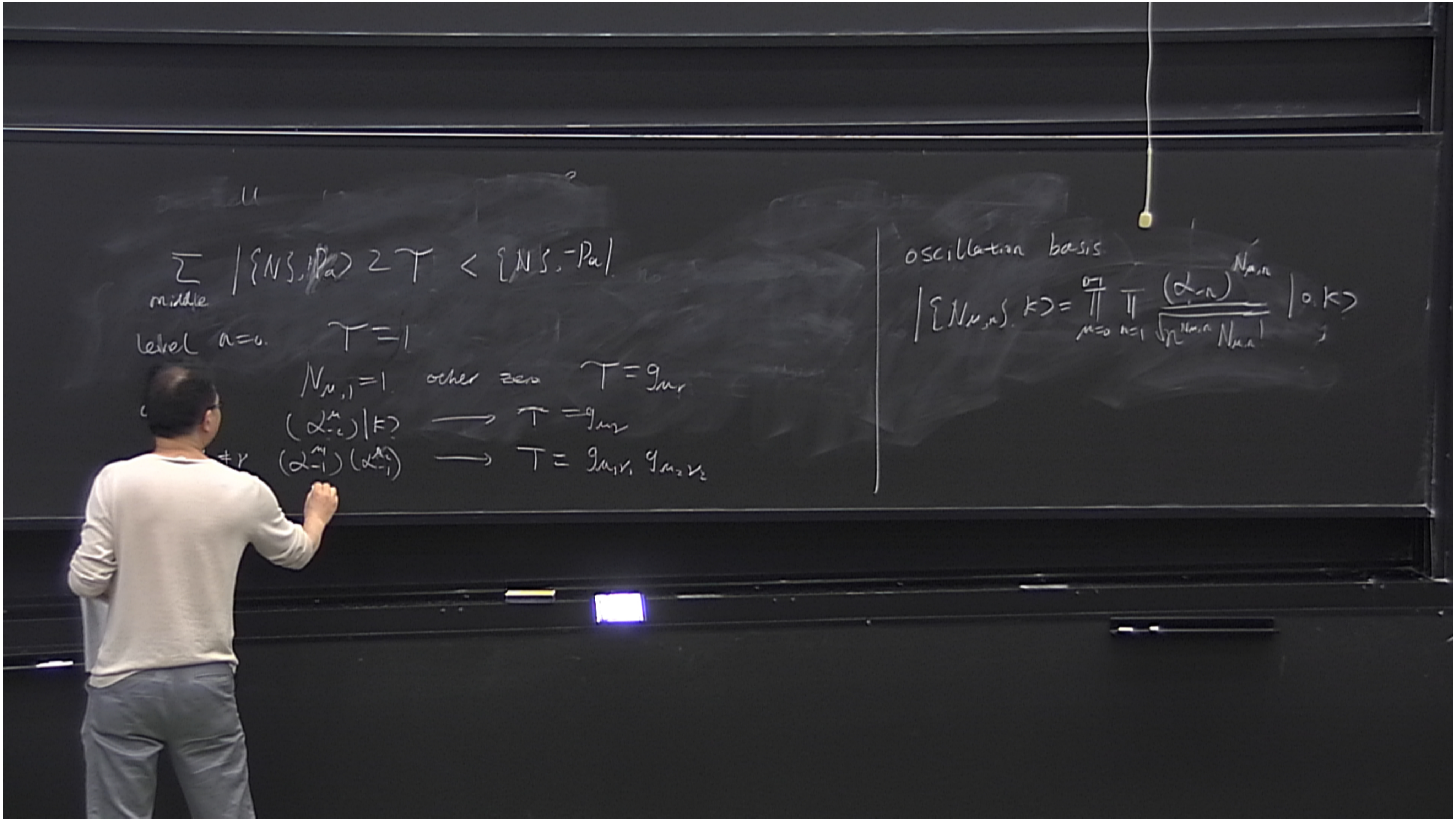
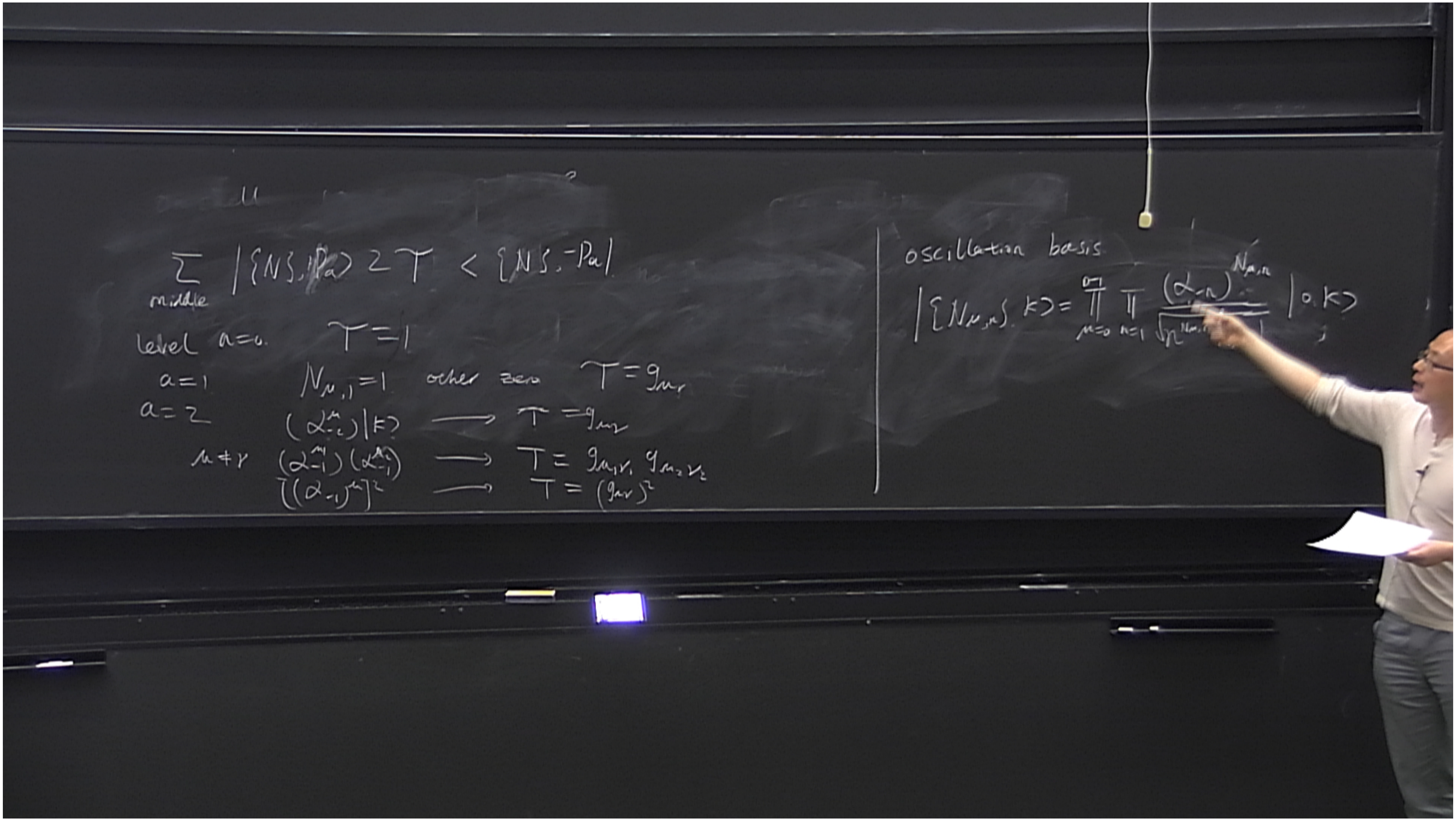
oscillation basis
|{N_{μ,n}}, k⟩ = Π Π (α_{-n})


oscillation basis
middle


oscillation basis
middle
level a=0.
a=1


middle
level a=0. T=1
a=1
other zero
a=2
oscillation basis


oscillation basis
middle
level
other zero


oscillation basis
middle
level a=0. T=1
a=1 other zero
a=2

$$\sum_{\text{middle}} |\{N\}, P_a\rangle \; 2T \; \langle \{N\}, -P_a|$$

level $a=0$. $\quad T=1$

$a=1 \qquad N_{\mu,1}=1$ other zero $\quad T = g_{\mu\nu}$

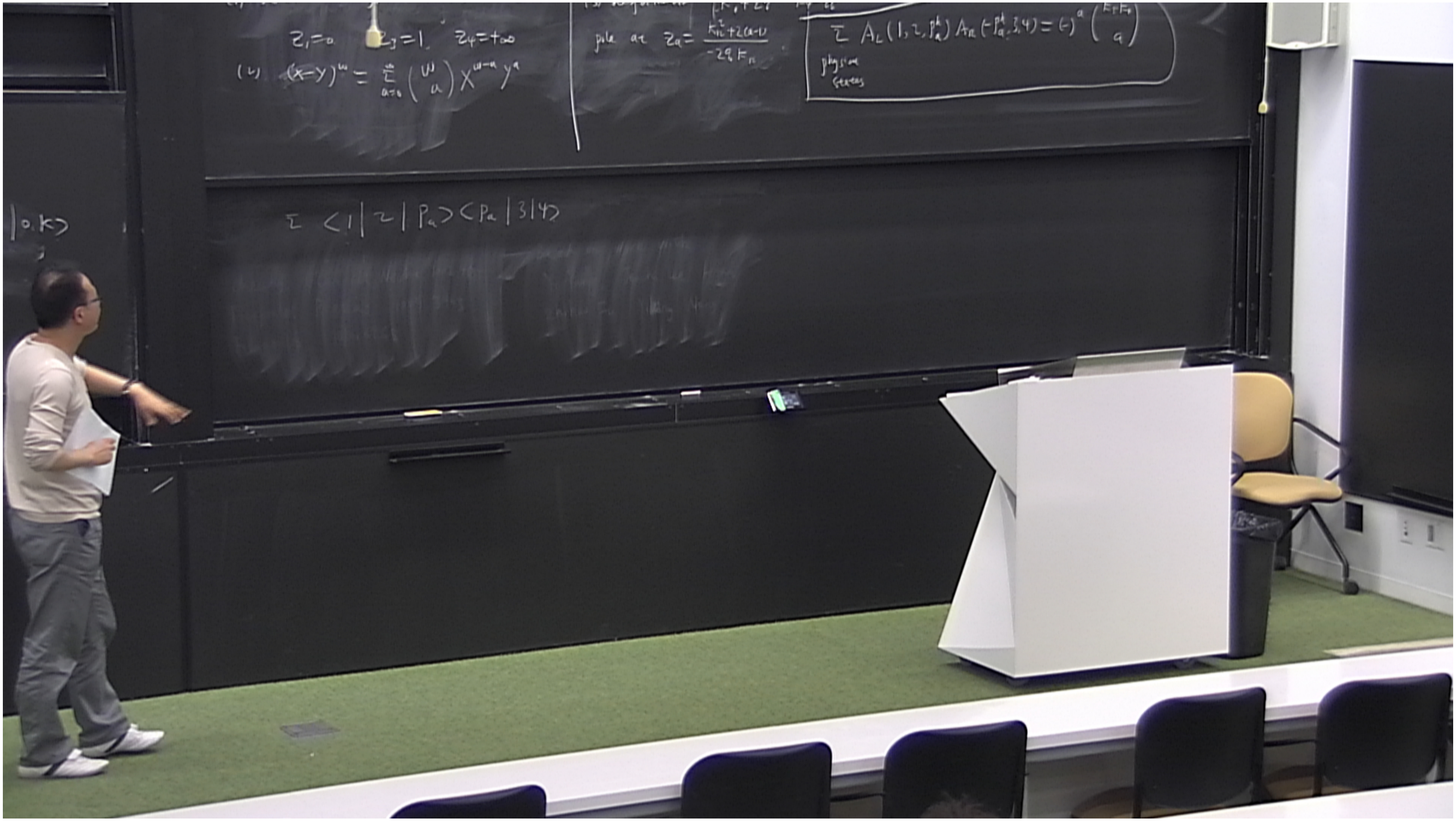
$a=2 \qquad (\alpha^{\mu}_{-2})|k\rangle \longrightarrow T = g_{\mu\nu}$

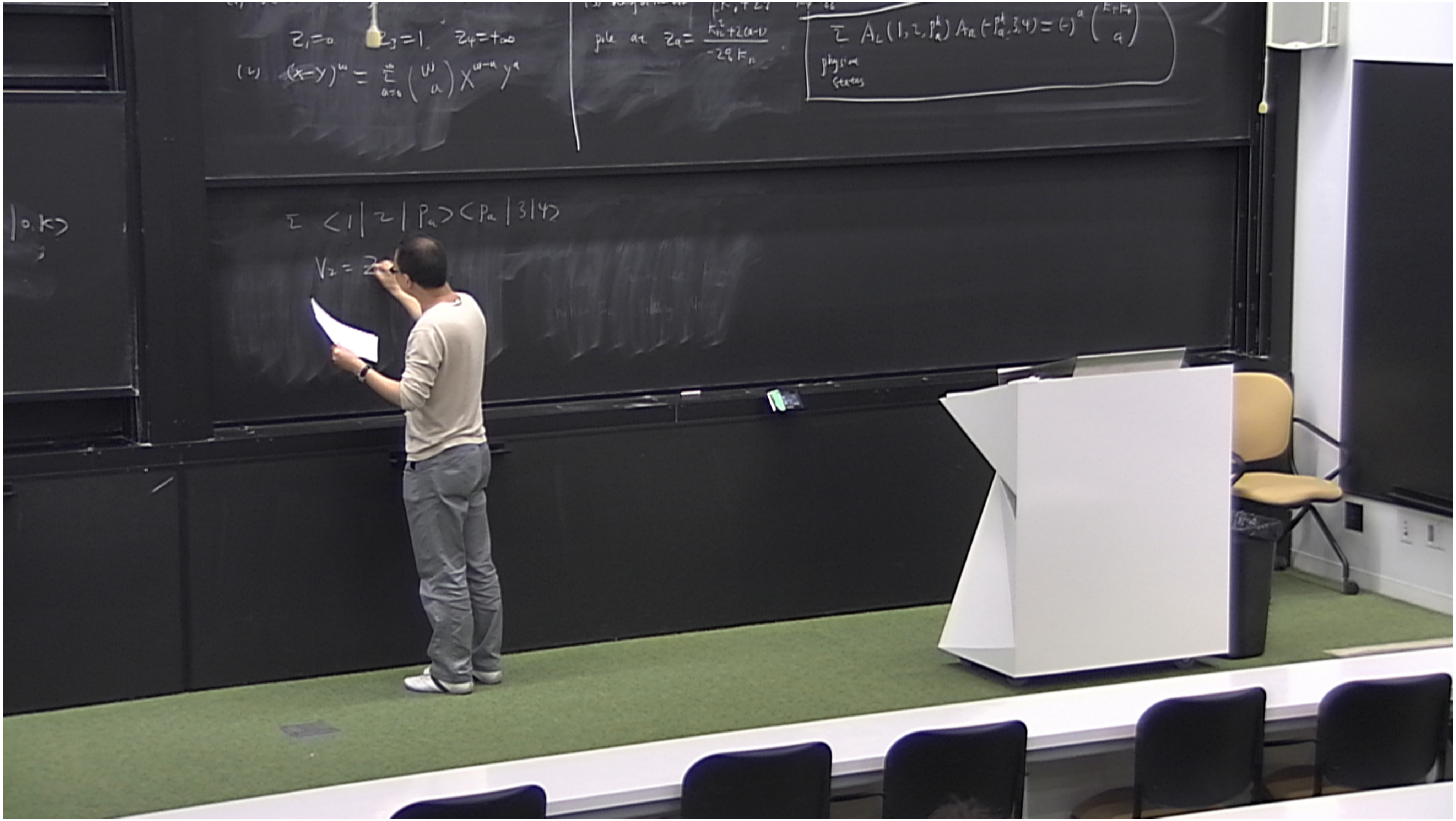
$\mu \neq \nu \quad (\alpha^{\mu_1}_{-1})(\alpha^{\mu_2}_{-1}) \longrightarrow T = g_{\mu_1\nu_1}\, g_{\mu_2\nu_2}$

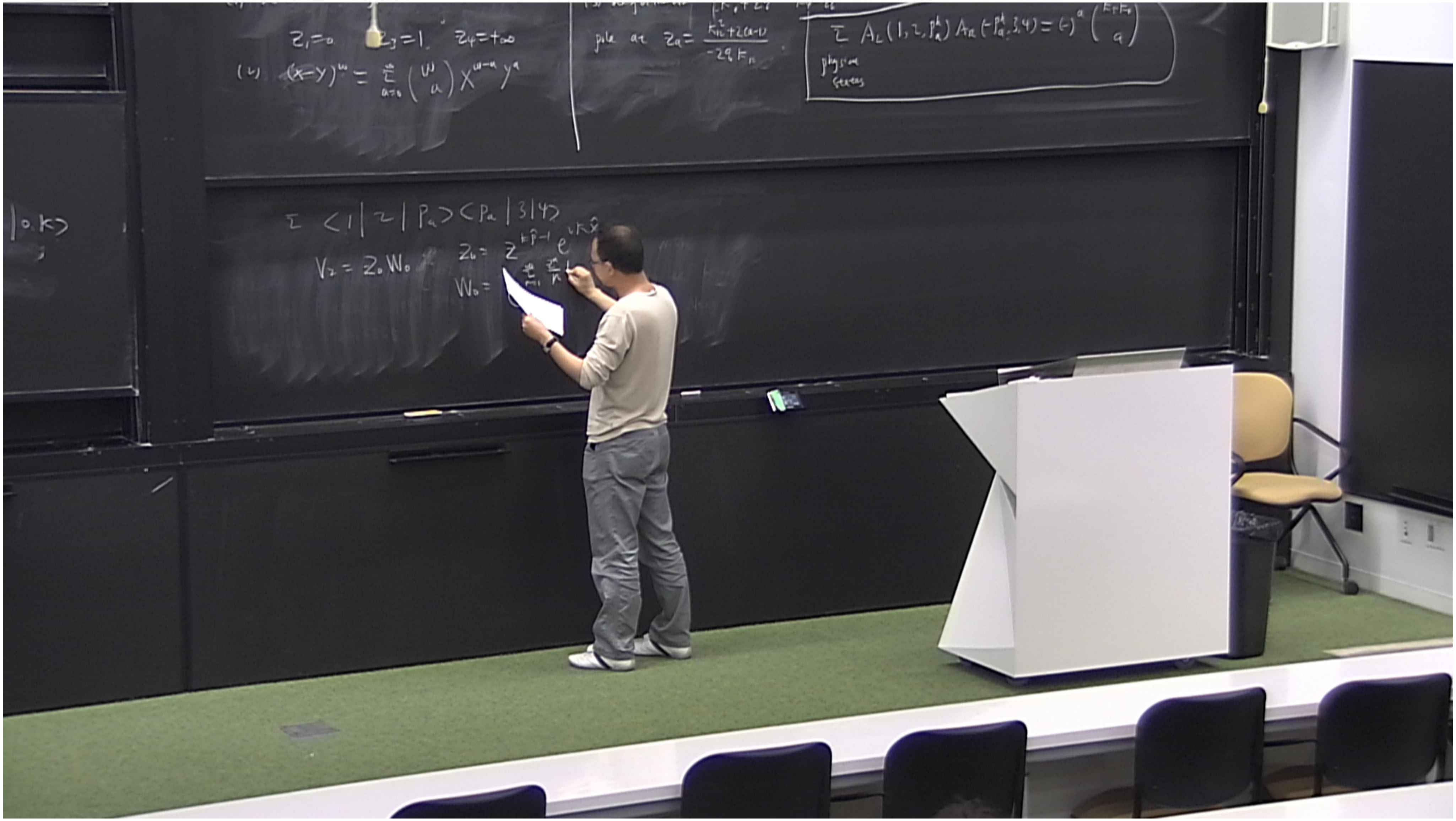
$[(\alpha_{-1})^{\mu}]^2 \longrightarrow T = (g_{\mu\nu})^2$

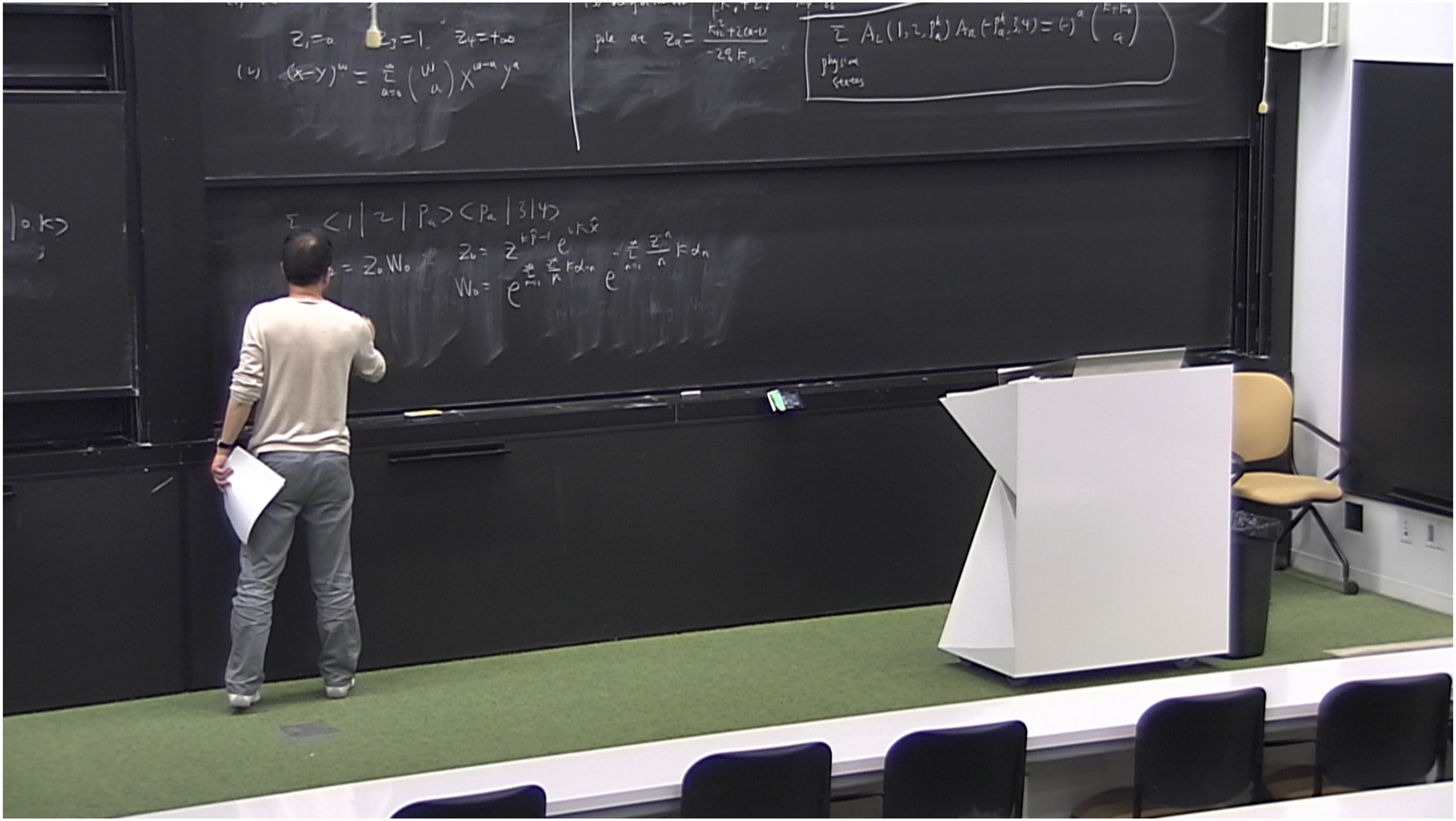
oscillation basis

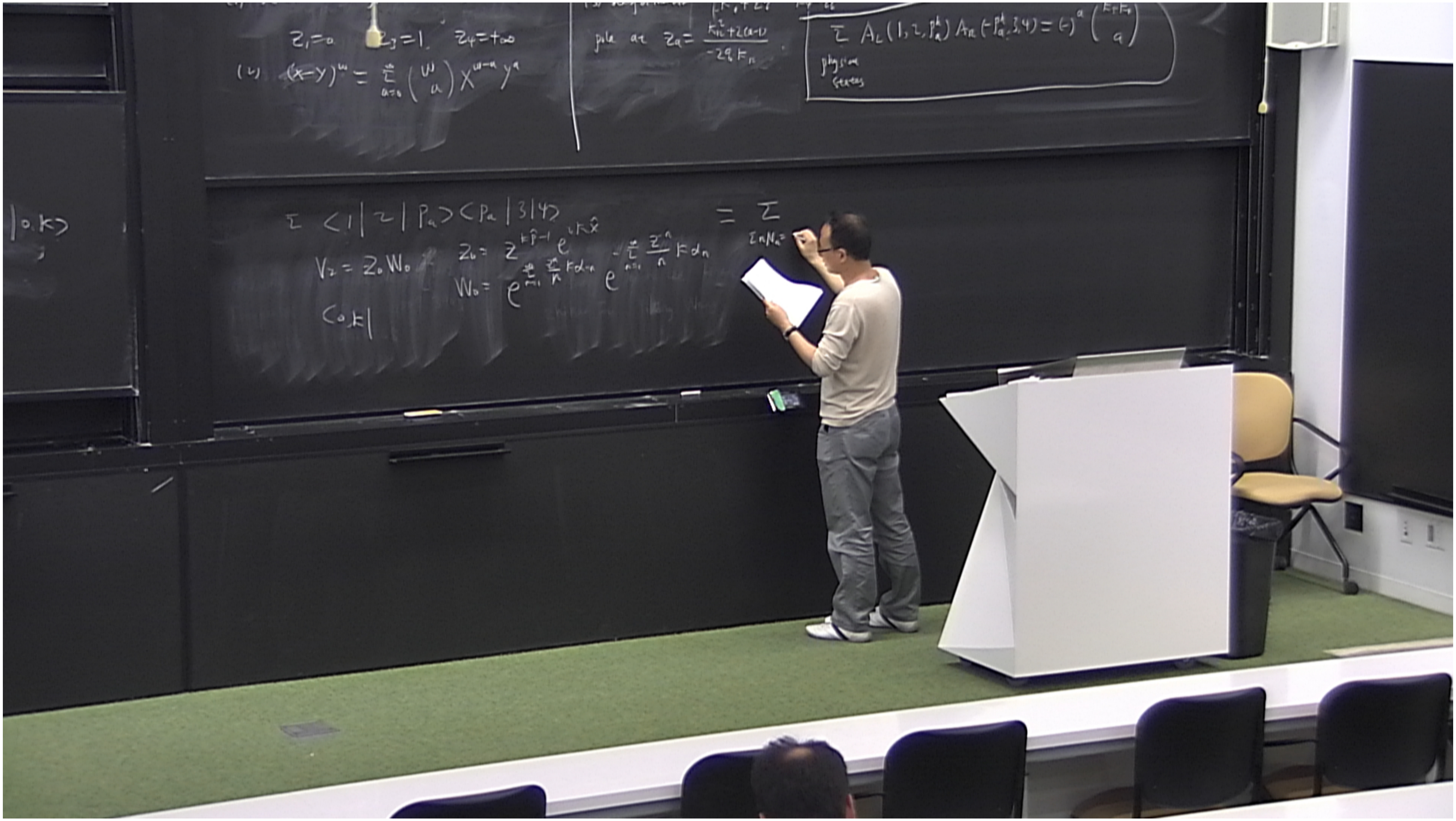
$$|\{N_{\mu,n}\}, k\rangle = \prod_{\mu=0}^{D-1} \prod_{n=1} \frac{(\alpha^{\mu}_{-n})^{N_{\mu,n}}}{\sqrt{n^{N_{\mu,n}}\, N_{\mu,n}!}} |0, k\rangle$$

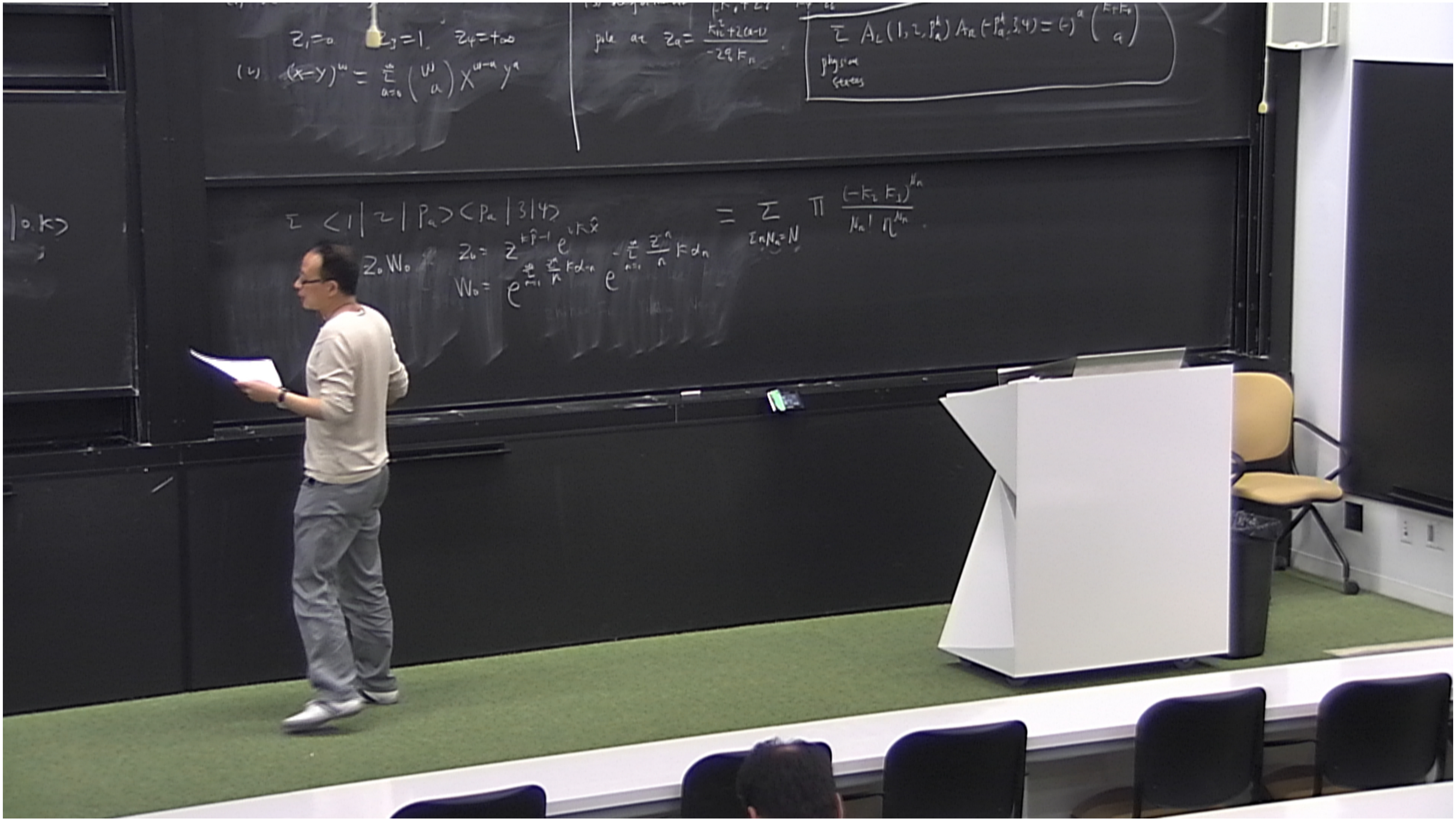


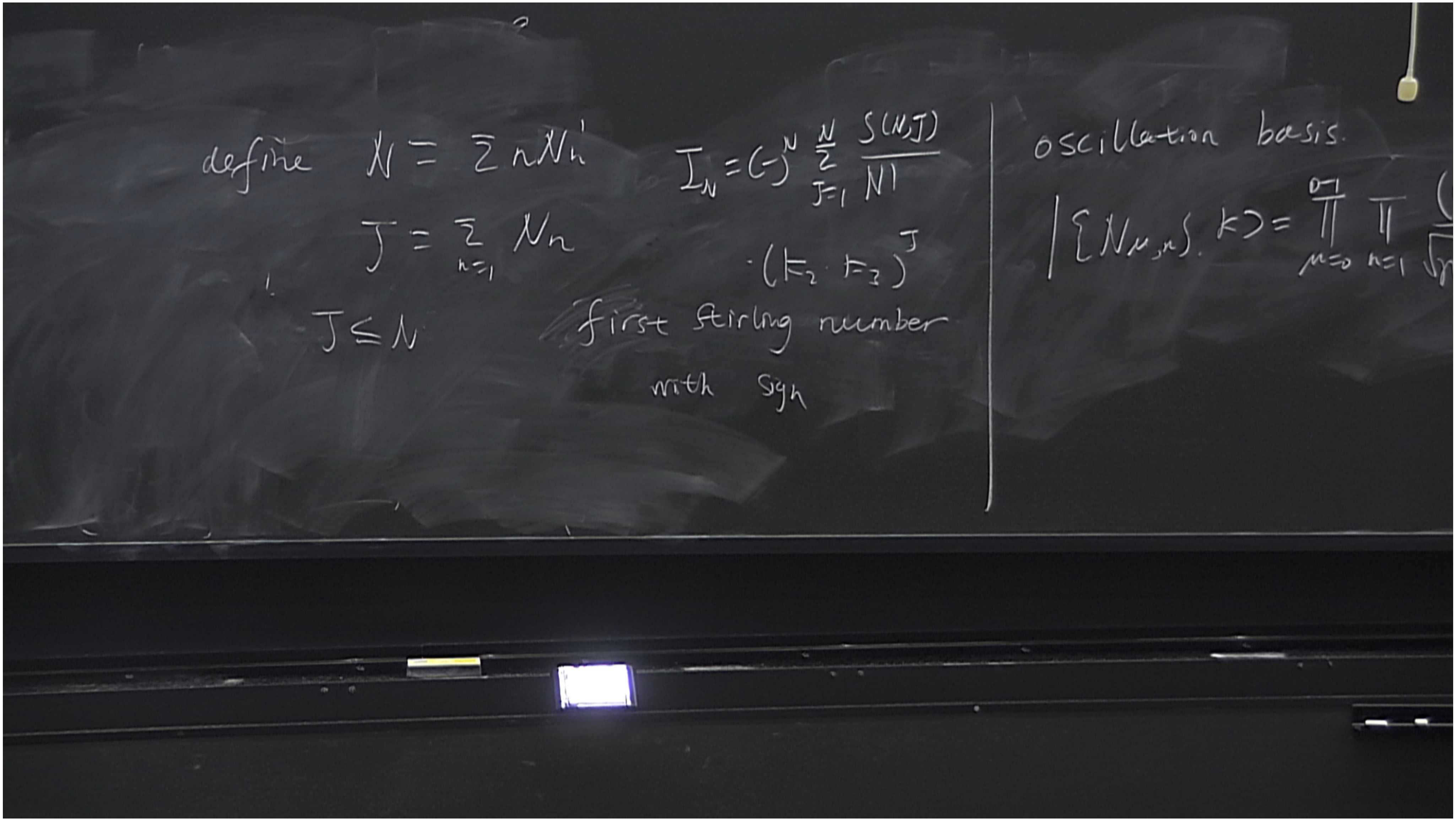


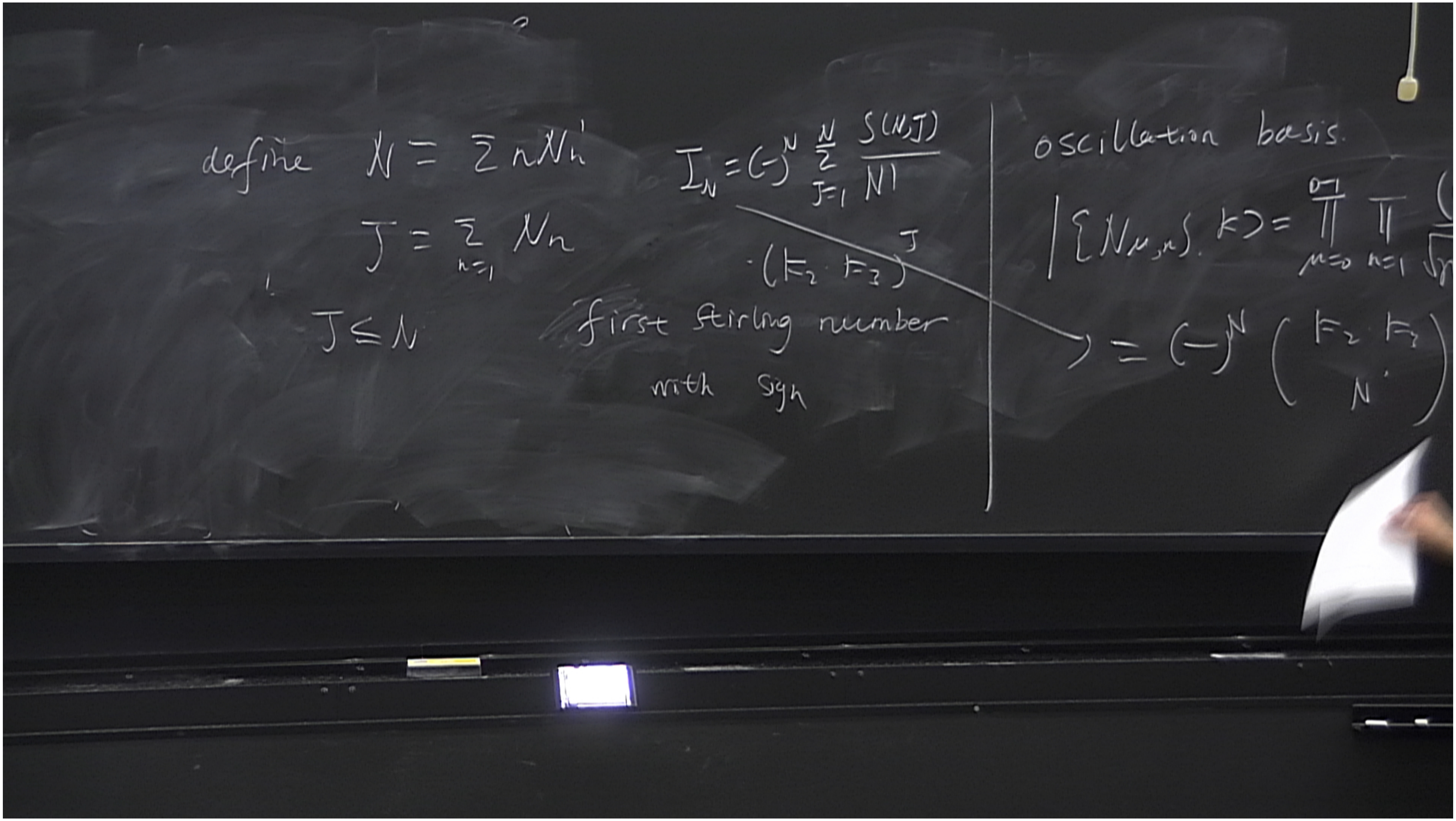


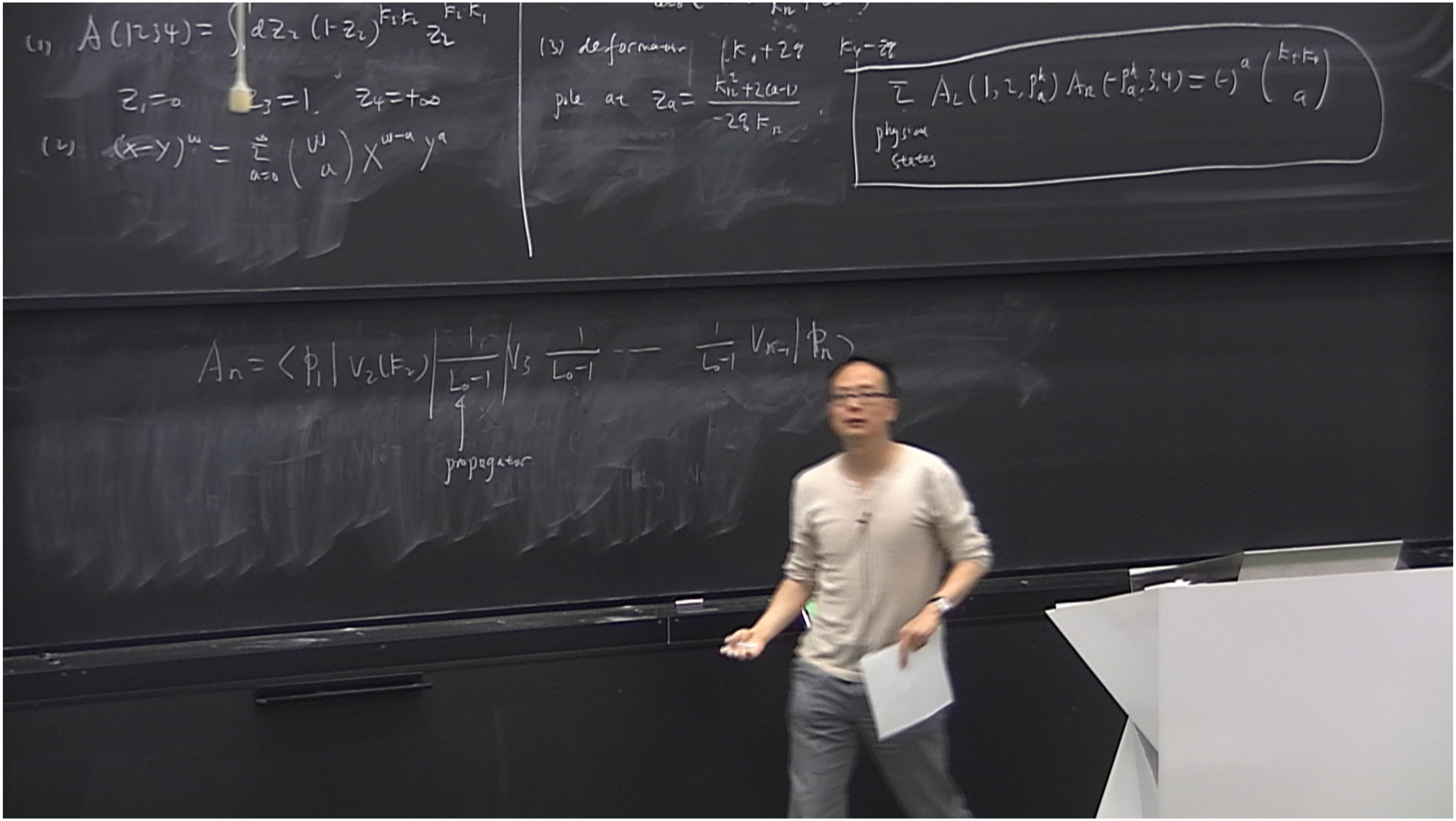


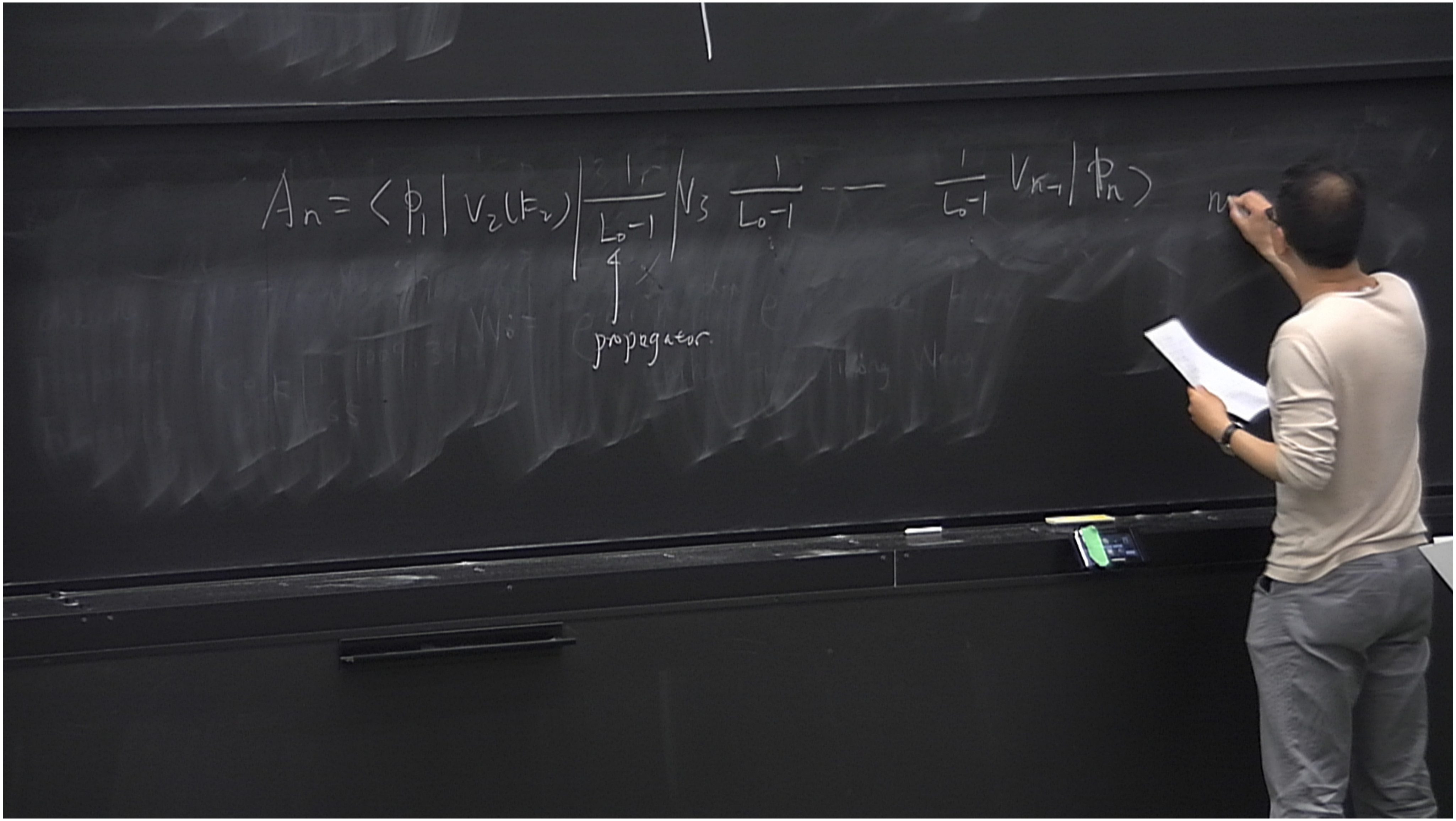


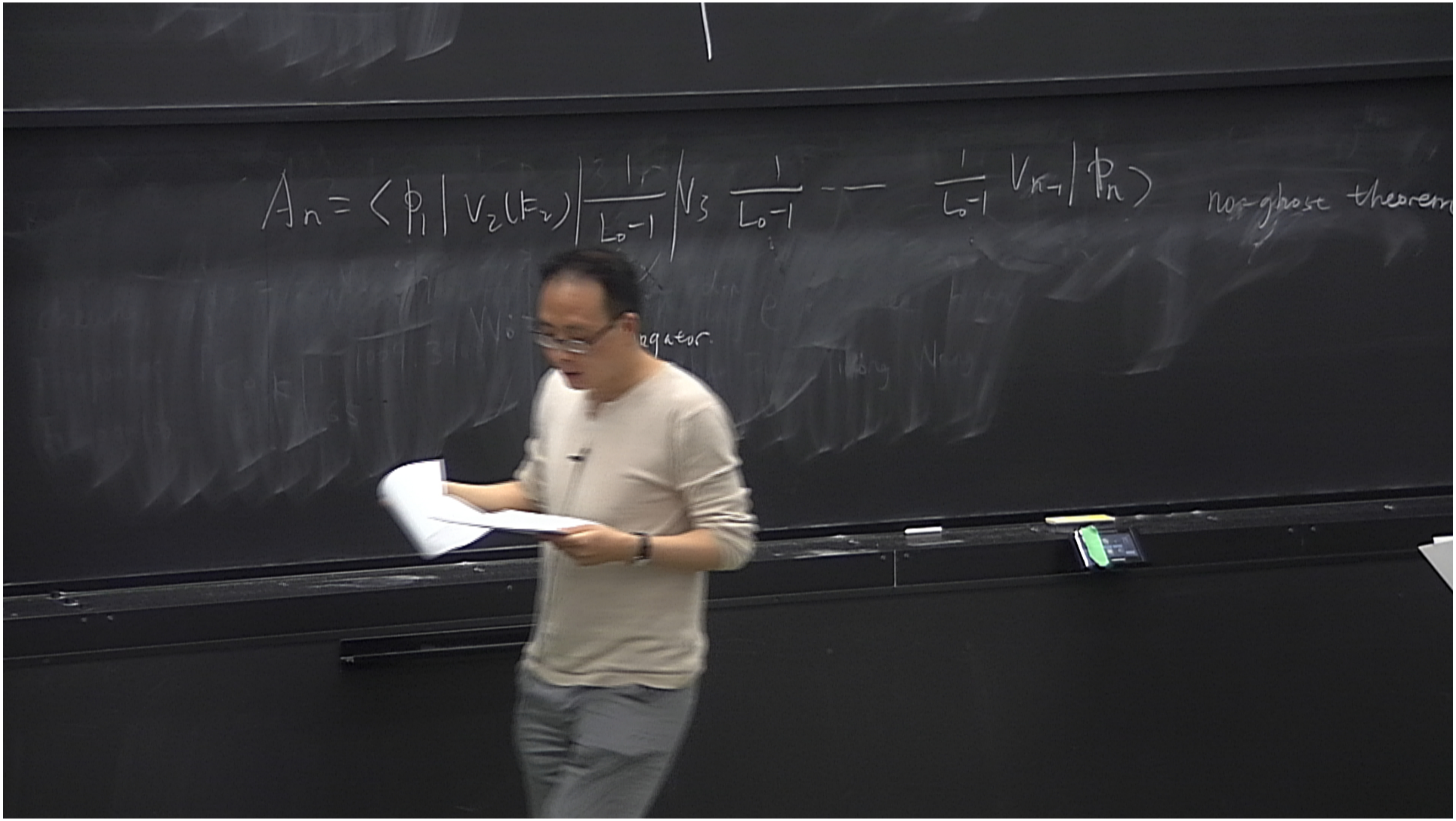



define N = Σ n N_n
J = Σ_{n=1} N_n
J ≤ N
I_N = (-)^N Σ_{J=1}^{N} S(N,J) / N! · (k_2 · k_3)^J
first stirling number
with sign
oscillation basis.
|{N_{μ,n}}, k> = Π_{μ=0}^{D-1} Π_{n=1}


define
J ≤ N
first stirling number
with sign
oscillation basis.


z1 = 0   z3 = 1.   z4 = +∞
(3) deformation
pole at
physical states
An = ⟨p1| V2(k2) 1/(L0−1) V3 1/(L0−1) --- 1/(L0−1) Vn−1 |pn⟩
propagator


A_n = < p_1 | V_2(k_2) | 1/(L_0 - 1) | V_3 1/(L_0 - 1) --- 1/(L_0 - 1) V_{n-1} | p_n >
propagator


no-ghost theorem