

Title: On-shell Recursion Relation for String Tree-level Amplitude

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Abstract: It is well known that on-shell recursion relation can be applied to tree-level amplitude in string theory. One technical issue of the application is the sum of infinite middle on-shell states. We discuss how we can do the sum exactly to reproduce the known result.

* Reference:

Boels, Marmiroli, Obers 1002.5029

Boels, 1008.3101

Cheung, O'Connell, Wecht, 1002.4674

Fotopoulos, Prezas, 1009.3903

Fotopoulos, 1010.6265

On-shell Recursion Relation
for string tree-level amplitude.

— Based on work with

Yi Yang, Jenchi Lee, Logan

Zhihao Fu, Yihong Wang

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On-shell Recursion Relation
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on-shell. $(p_i, p_j) \rightarrow \begin{cases} p_i + z\epsilon \\ p_j - z\epsilon \end{cases}$

$$g^2 = g \cdot p_i = g \cdot p_j = 0$$

$A(z)$

on-shell $(p_i, p_j) \rightarrow \begin{cases} p_i + z\epsilon \\ p_j - z\epsilon \end{cases}$

$$g^2 = g \cdot p_i = g \cdot p_j = 0$$

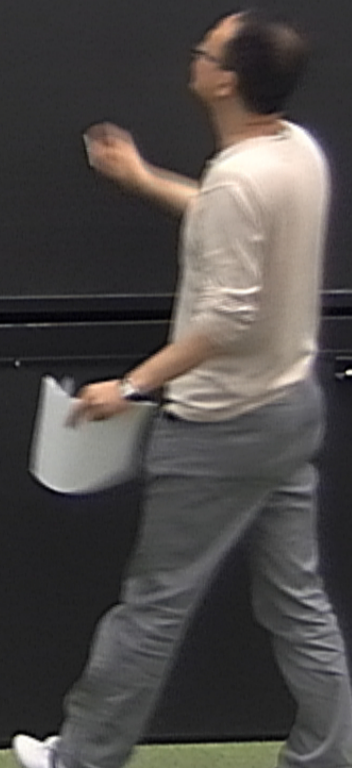
$$\oint \frac{dz}{z} A(z)$$

$$A = A_L^h(z_\alpha) \frac{1}{z - p_\alpha} A_R^h(z_\alpha)$$

On-shell $(p_i, p_j) \rightarrow \begin{cases} p_i + z\epsilon \\ p_j - z\epsilon \end{cases} \quad \left. \vphantom{\begin{cases} p_i + z\epsilon \\ p_j - z\epsilon \end{cases}} \right\} (a)$

$g^2 = g p_i = g p_j = 0$

$\oint \frac{dz}{z} A(z) \Rightarrow A = \sum_{\alpha, h} \bar{z} A_L^{\alpha, h}(z_\alpha) \frac{1}{p_\alpha^2} A_R^{\alpha, h}(z_\alpha)$

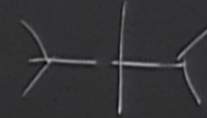


On-shell $(p_i, p_j) \rightarrow \begin{cases} p_i + z\epsilon \\ p_j - z\epsilon \end{cases}$

$$g^2 = g p_i = g p_j = 0$$

$$\oint \frac{dz}{z} A(z) \Rightarrow A = \sum_{\alpha, h} \frac{1}{p_\alpha^2} A_{\alpha, h}(z_\alpha)$$

- (a) pole-like $\frac{1}{p^2 + i\epsilon}$
- (b) factorization property

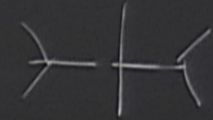


On-shell $(p_i, p_j) \rightarrow \begin{cases} p_i + z\epsilon \\ p_j - z\epsilon \end{cases}$

$$g^2 = 8 p_i = 8 - p_j = 0$$

$$\oint \frac{dz}{z} A(z) \Rightarrow A = \sum_{\alpha, h} z^{\alpha} A_{\alpha}^{(h)}(z) = A_n^{(-h)}(z_n)$$

- (a) pole-like $\frac{1}{p^2 + i\epsilon}$
- (b) factorization property
- (c) $z \rightarrow \infty, A(z) \rightarrow 0$



get rid boundary contribution

technical difficulties.

(1) Infinity number of middle
particle.

(2) physical.

technical difficulties.

(1) Infinity number of middle
particle.

(2) physical states are hard
to describe

technical difficulties.

(1) Infinity number of middle
particle.

(2) physical states are hard
to describe

gluon, $Im. Em.$

technical difficulties.

(1) Infinity number of middle
particle.

2) physical states are hard
to describe

gluon, $Im. \epsilon_m.$

Transverse
{
time
longitudinal

(2) physical states are hard to describe

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for string tree-level amplitude

— Based on work

Yi Yang, Jen-Chieh Logan

Zhihao Fu

(5). Hint: $A = A_L \left(\frac{I}{P^?} \right) A_R$

Scalar line

(5) Hint: $A = A_L \left(\frac{I}{P^?} \right) A_R$

$\frac{P_{my}}{P^?}$

Scalar line

$A_L = E_x(P)$

$A_R = E_y^*(P)$

(5) Hint: $A = A_L \left(\frac{I}{p^2} \right) A_R$

Scalar line

$$A_L = A_L^u \in_r^h(p)$$

$$A_R = A_R^r \in_r^{+h}(p)$$

wavy line

$$\frac{I_{uv}}{p^2}$$

$$A = A$$

(5) Hint. $A = A_L \left(\frac{1}{p^2} \right) A_R$

Scalar line

$\frac{g_{\mu\nu}}{p^2}$

$A = \bar{A}_L^{\mu} \frac{\epsilon_{\mu\nu}^k \epsilon_{\nu}^l(p)}{p^2} A_R^{\nu}(p)$

$= A_L^{\mu} \epsilon_{\mu}^k(p)$
 $= A_R^{\nu} \epsilon_{\nu}^{+k}(p)$

(5) Hint. $A = A_L \left(\frac{1}{p^2} \right) A_R$

Scalar line

$$A_L = A_L^{\mu} e_{\mu}^k(p)$$

$$A_R = A_R^{\nu} e_{\nu}^{+h}(p)$$

$\underbrace{\hspace{2cm}}_{\frac{g_{\mu\nu}}{p^2}}$

Iden

$$A = \sum_h \bar{A}_L^{\mu} \frac{\epsilon_{\mu\nu}^k \epsilon_{\nu}^{+h}(p)}{p^2} A_R^{\nu}(p)$$

$$= \sum_{k=\pm, T, L}$$

(5) Hint. $A = A_L \left(\frac{1}{p^2} \right) A_R$ ~~~~~

Scalar line

$$\frac{g_{uv}}{p^2}$$

$A_L = A_L^\mu e_\mu^h(p)$ Ward Idem

$A_R = A_R^\nu e_\nu^{+h}(p)$

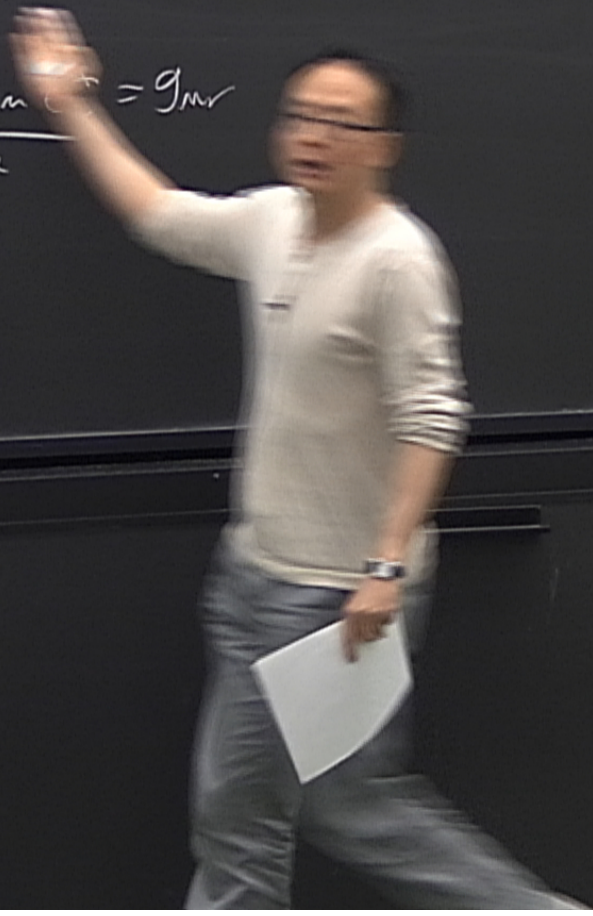
$$A = \frac{1}{h} \bar{A}_L^\mu \frac{\epsilon_{\mu\nu}^h \epsilon_\nu^{+h}(p)}{p^2} A_R^\nu(p)$$

$$= \bar{A}_L^\mu A_R^\nu \frac{\sum_{h=-1,1} \epsilon_{\mu\nu}^h \epsilon_\nu^{+h}}{p^2} = 0$$

$$\frac{\epsilon_{m\nu}^k \epsilon_{\nu}^{\mu l} \omega}{p^2} A_R^\nu(p)$$

on-shell recursion

$$A_R^\nu \frac{\sum_{k=-\infty}^{\infty} \epsilon_{m\nu}^k}{p^2} = g_{m\nu}$$



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for string

— Based
Yi Yan
zhikang

(5) Hline

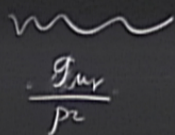
$$A = A_L \left(\frac{1}{p^2} \right) A_R$$

Scalar line

$$A_L = A_L^{\mu} \epsilon_{\mu}^k(p)$$

$$A_R = A_R^{\nu} \epsilon_{\nu}^{\dagger}(p)$$

Ward Idem



$$A = \frac{1}{h} \bar{A}_L^{\mu} \frac{\epsilon_{\mu}^k \epsilon_{\nu}^{\dagger}(p)}{p^2} A_R^{\nu}(p)$$

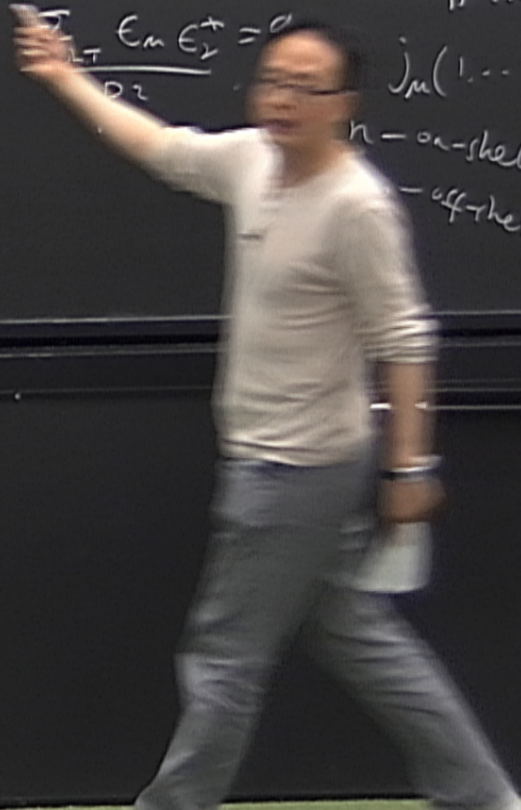
$$= \bar{A}_L^{\mu} A_R^{\nu} \frac{\epsilon_{\mu} \epsilon_{\nu}^{\dagger}}{p^2} = 0$$

on-shell recursion
off-shell property

$j_n(1, \dots, n)$

n - on-shell

- off-shell



(5) Hline

$$A = A_L \left(\frac{1}{p^2} \right) A_R$$

Scalar line

$\frac{g_{\mu\nu}}{p^2}$

Ward Idem

$$A_L = A_L^\mu \epsilon_\mu^k(p)$$

$$A_R = A_R^\nu \epsilon_\nu^{\pm}(p)$$


$$A = \frac{1}{h} \bar{A}_L^\mu \frac{\epsilon_\mu^k \epsilon_\nu^{\pm}(p)}{p^2} A_R^\nu$$

$$= \bar{A}_L^\mu A_R^\nu \frac{\sum_{k=\pm} \epsilon_\mu^k \epsilon_\nu^{\pm}(p)}{p^2} = g_{\mu\nu}$$

on-shell recursion
off-shell property

$j_n(1, \dots, -n)$ (1109 B21)

n - on-shell
(n+1) - off-shell

(5) Hline $A = A_L \left(\frac{1}{p^2} \right) A_R$ 

$A_L = A_L^{\mu} \epsilon_{\mu}^k(p)$ Ward Idem
 $A_R = A_R^{\nu} \epsilon_{\nu}^{+h}(p)$

Scalar line

$\frac{g_{\mu\nu}}{p^2}$

$A = \frac{1}{h} \bar{A}_L^{\mu} \frac{\epsilon_{\mu}^k \epsilon_{\nu}^{+h}(p)}{p^2} A_R^{\nu}(p)$

$= \bar{A}_L^{\mu} A_R^{\nu} \frac{\sum_{k=-h}^h \epsilon_{\mu}^k \epsilon_{\nu}^{+h} = g_{\mu\nu}}{p^2}$

on-shell recursion
off-shell property

$J_n(1, \dots, n)$ (1109 BBT)

n - on-shell
 $(n+1)$ - off-shell



x. 4 tachyon in Bosonic open.

* 4-tachyon in Bosonic open.

$$A(1234) = \int_0^1 dz_2 (1-z_2)^{F_2 K_2} z_2^{F_2 K_1}$$

$$z_1 = 0, \quad z_3 = 1, \quad z_4 = +\infty$$

* 4 tachyon in Bosonic open.

$$(1) \quad A(1234) = \int_0^1 dz_2 (1-z_2)^{k_2 \cdot k_3} z_2^{k_2 \cdot k_4}$$

$$z_1 = 0, \quad z_3 = 1, \quad z_4 = +\infty$$

$$(2) \quad (x-y)^w = \sum_{a=0}^w \binom{w}{a} x^{w-a} y^a$$

$$\binom{w}{a} = \frac{w(w-1)\dots(w-a+1)}{1 \cdot 2 \dots a}$$

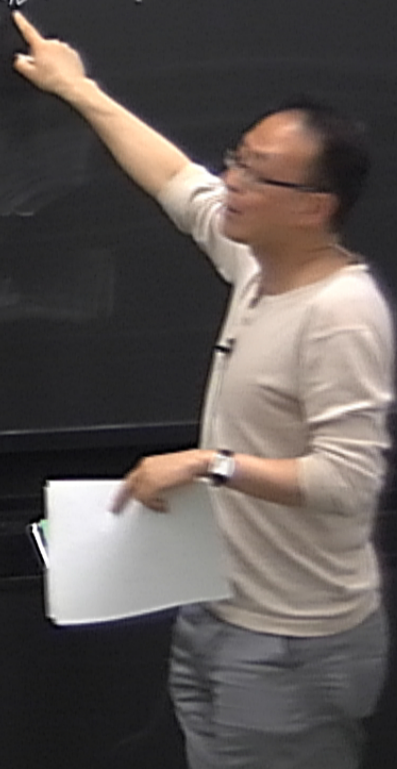
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$$z_1 = 0, \quad z_3 = 1, \quad z_4 = +\infty$$

$$(2) (x-y)^w = \sum_{a=0}^w \binom{w}{a} x^{w-a} y^a$$

$$A(1234) = \sum_{a=0}^w \binom{F_3 k_2}{a} \frac{z^{(-)^a}}{k_2^2 + 2(a-1)}$$



* 4 tachyon in Bosonic open

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(3) deformation $(k_1 + 2\alpha \quad k_4 - 2\alpha$

$$\text{pole at } z_a = \frac{k_n^2 + 2(a-1)}{-2\alpha k_n}$$

* 4 tachyon in Bosonic open

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(3) deformation $(k_1 + 2\alpha \quad k_4 - 2\alpha$

$$\text{pole at } z_a = \frac{k_n^2 + 2(a-1)}{-2\alpha k_n}$$

$\sum A_L$
physical
state

* 4 tachyon in Bosonic open

$$(1) A(1234) = \int_0^1 dz_2 (1-z_2)^{F_1 F_2} z_2^{F_3 F_4}$$

$$z_1=0 \quad z_3=1 \quad z_4=\infty$$

$$(2) (x-y)^w = \sum_{a=0}^w \binom{w}{a} x^{w-a} y^a$$

$$A(1234) = \sum_{a=0}^w \binom{F_1 F_2}{a} \frac{z^{(-)^a}}{k_{12}^2 + 2(a-1)}$$

(3) deformation
pole at $z_a = \frac{k_1 + 2\alpha}{k_{12}^2 + 2(a-1)}$

$$A_L(1,2,p^k) A_R(-p^k,3,4) = (-)^a \binom{F_1 F_2}{a}$$

$$(1) \quad h = \pm 1 \rightarrow h = \pm, T, L$$

no-ghost theorem

$$\begin{array}{ccc} \overline{\Sigma} & \longrightarrow & \overline{\Sigma} \\ \text{physical} & & \text{Fock space} \\ & & \text{at mass level} \\ & & a \end{array}$$

$$(1) \quad h = \pm 1 \rightarrow h = \pm, T, L$$

no-ghost theorem

$\bar{\Sigma}$
physical \longrightarrow $\bar{\Sigma}$
Fock space
at mass level
a

$$A^\mu \in E_\mu^{T,L} = 0$$

$$A^\mu \in E_{\text{unphysical}} = 0$$

$$(1) h = \pm 1 \rightarrow h = \pm, T, L$$

Σ physical \rightarrow $\bar{\Sigma}$
 Fock space
 at mass level
 a

no-ghost theorem

$$A^\mu \in T, L = 0$$

$$A^\mu \in \text{unphysical} = 0$$

oscillation basis

$$| \{ N_{\mu, \nu} \}, k \rangle = \prod_{\mu=0}^{d-1} \prod_{\nu=1}^{\infty} \frac{(\alpha_{-\nu})}{\dots}$$



$$\sum_{\text{middle}} |\{N_S, p_a\}\rangle \langle \{N_S, p_a\}|$$

oscillation basis

$$|\{N_{\mu, n}, k\}\rangle = \prod_{m=0}^{\mu-1} \prod_{k=1}^n \frac{(\alpha_{\mu, n})^{N_{\mu, n}}}{\sqrt{N_{\mu, n}!}} |0, k\rangle$$

$$\sum_{\text{middle}} |\{N_S, Pa\}\rangle \geq T < \{N_S, -Pa\}$$

level $a=0$ $T=1$
 $a=1$

oscillation basis

$$|\{N_{\mu, n}, k\}\rangle = \prod_{m=0}^{a-1} \prod_{k=1}^{N_{\mu, m}} \frac{(\alpha_{\mu, m})^{N_{\mu, m}}}{\sqrt{N_{\mu, m}!}} |0, k\rangle$$

$$\sum_{\text{middle}} |\{N_S, Pa\}\rangle \geq T < \{N_S, -Pa\}$$

level $a=0$ $T=1$
 $a=1$ $=1$ other zero $T=g_{mr}$
 $a=2$

oscillation basis

$$|\{N_{\mu, n}\}, k\rangle = \prod_{\mu=0}^{\infty} \prod_{n=1}^{\infty} \frac{(\alpha_{\mu, n})^{N_{\mu, n}}}{\sqrt{N_{\mu, n}!}} |0, k\rangle$$

$$\sum_{\text{middle}} |\{NS, Pa\} \rangle \geq T < \{NS, -Pa\}$$

level $a=0$. $T=1$

$$N_{r,1} = 1 \quad \text{other zero} \quad T = g_{m,r}$$

$$(\alpha_{-c}^{\wedge}) |k\rangle \longrightarrow T = g_{m,r}$$

$$\neq r \quad (\alpha_{-1}^{\wedge}) (\alpha_{-1}^{\wedge}) \longrightarrow T = g_{m,r_1} g_{m,r_2}$$

oscillation basis

$$|\{N_{m,n}, k\}\rangle = \prod_{m=0}^{\infty} \prod_{n=1}^{\infty} \frac{(\alpha_{-n})^{N_{m,n}}}{\sqrt{N_{m,n}!}} |0, k\rangle$$

$$\sum_{\text{middle}} |\{NS, Pa\} \rangle \geq T < \{NS, -Pa\}$$

level $a=0$ $T=1$

$a=1$ $N_{n,1}=1$ other zero $T=g_{nr}$

$a=2$ $(\alpha_{-1}^n) |k\rangle \longrightarrow T=g_{nr}$

$n+r$ $(\alpha_{-1}^n)(\alpha_{-1}^r) \longrightarrow T=g_{nr} g_{nr}$
 $[(\alpha_{-1}^n)]^2 \longrightarrow T=(g_{nr})^2$

oscillation basis

$$|\{N_{n,r}, k\}\rangle = \prod_{n=0}^{n-1} \prod_{r=1}^r \frac{(\alpha_{-n})^{N_{n,r}}}{\sqrt{N_{n,r}!}} |0, k\rangle$$

$$\sum_{\text{middle}} |\{N_S, Pa\}\rangle \geq T < \{N_S, -Pa\}$$

level $a=0$ $T=1$

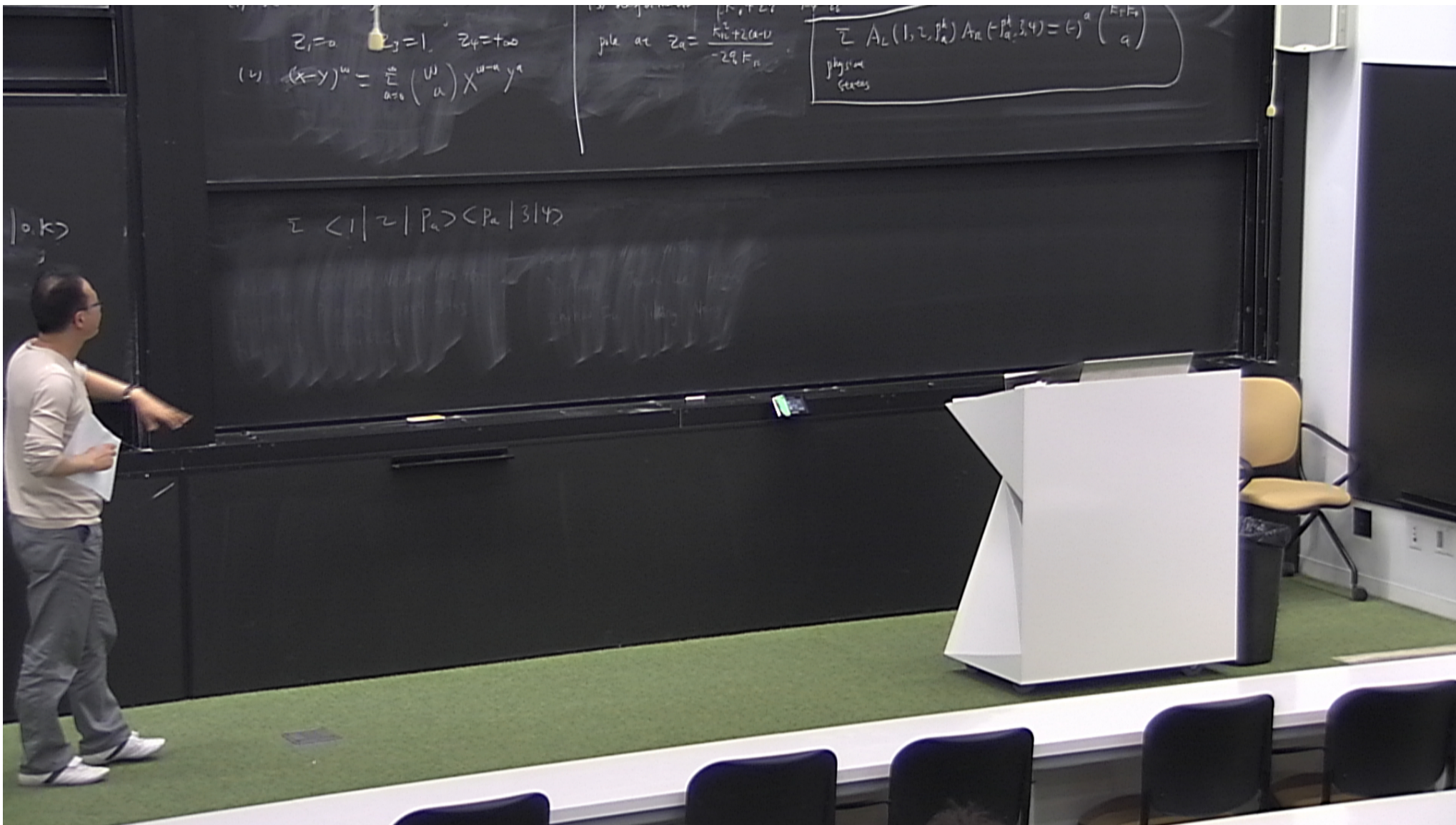
$a=1$ $N_{N,1}=1$ other zero $T=g_{m,r}$

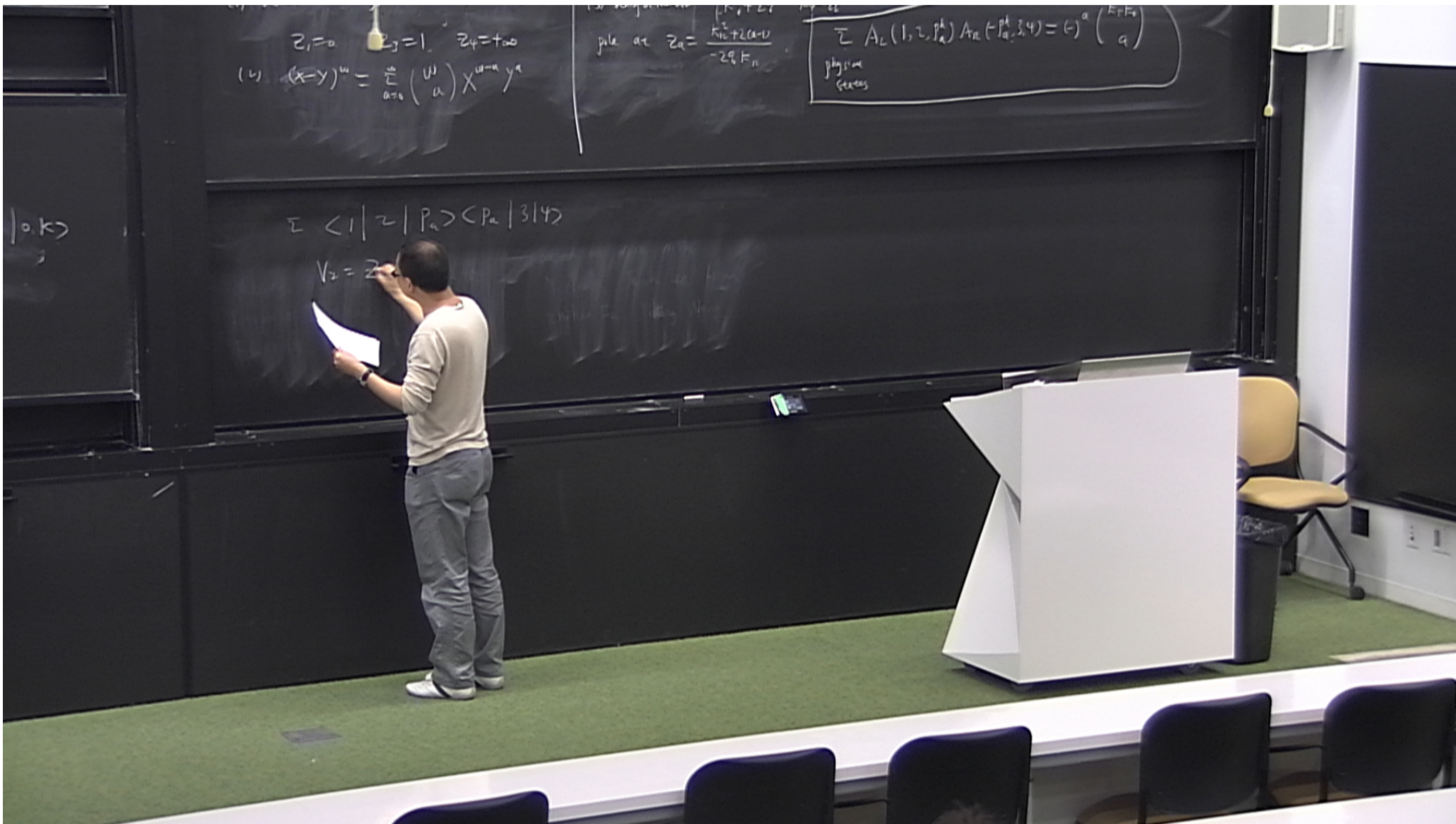
$a=2$ $(\alpha_{-1}^m) |k\rangle \longrightarrow T=g_{m,r}$

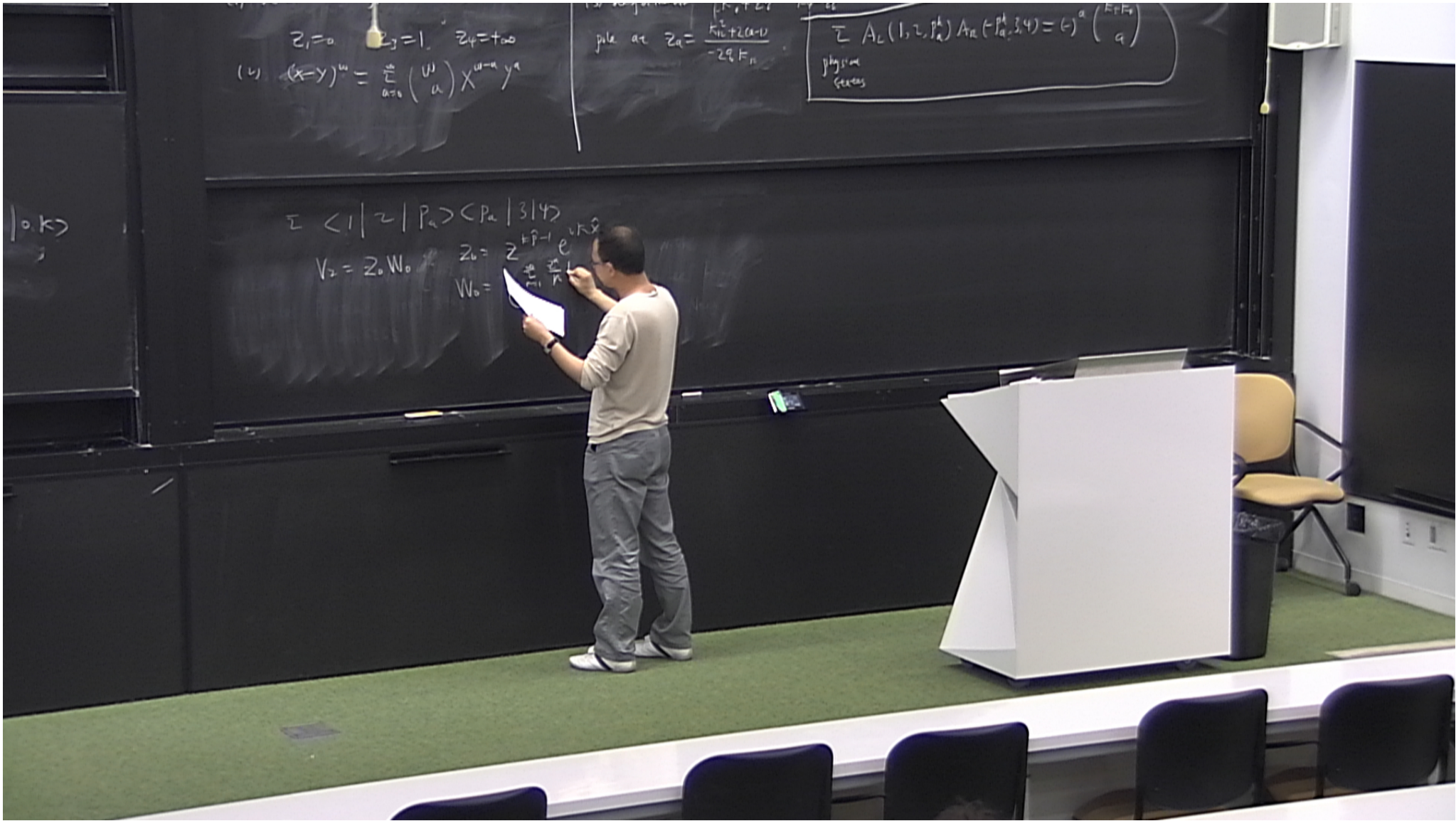
$n+r$ $(\alpha_{-1}^m)(\alpha_{-1}^n) \longrightarrow T=g_{m,r} g_{m_2,r_2}$
 $[(\alpha_{-1}^m)]^2 \longrightarrow T=(g_{m,r})^2$

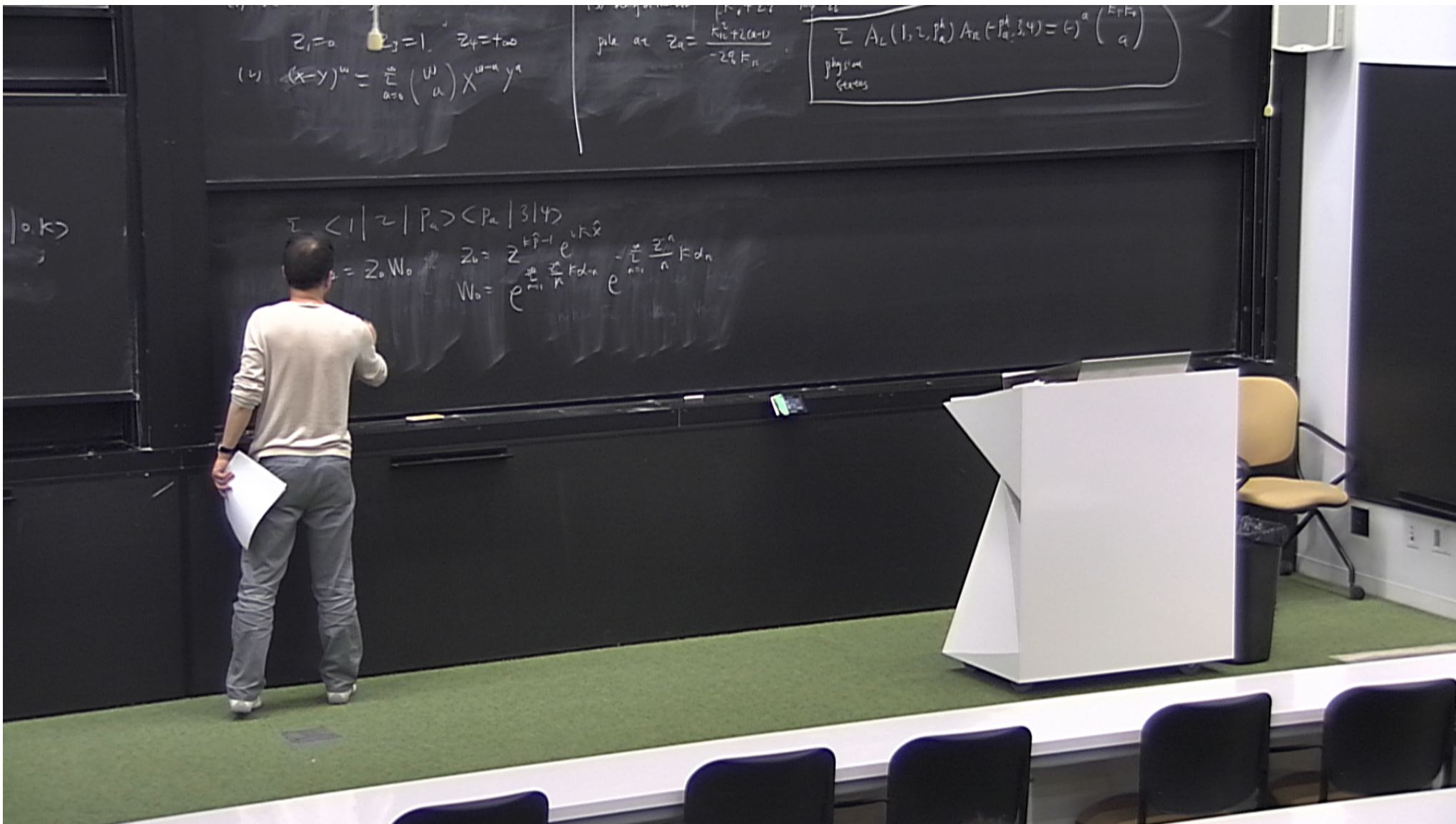
oscillation basis

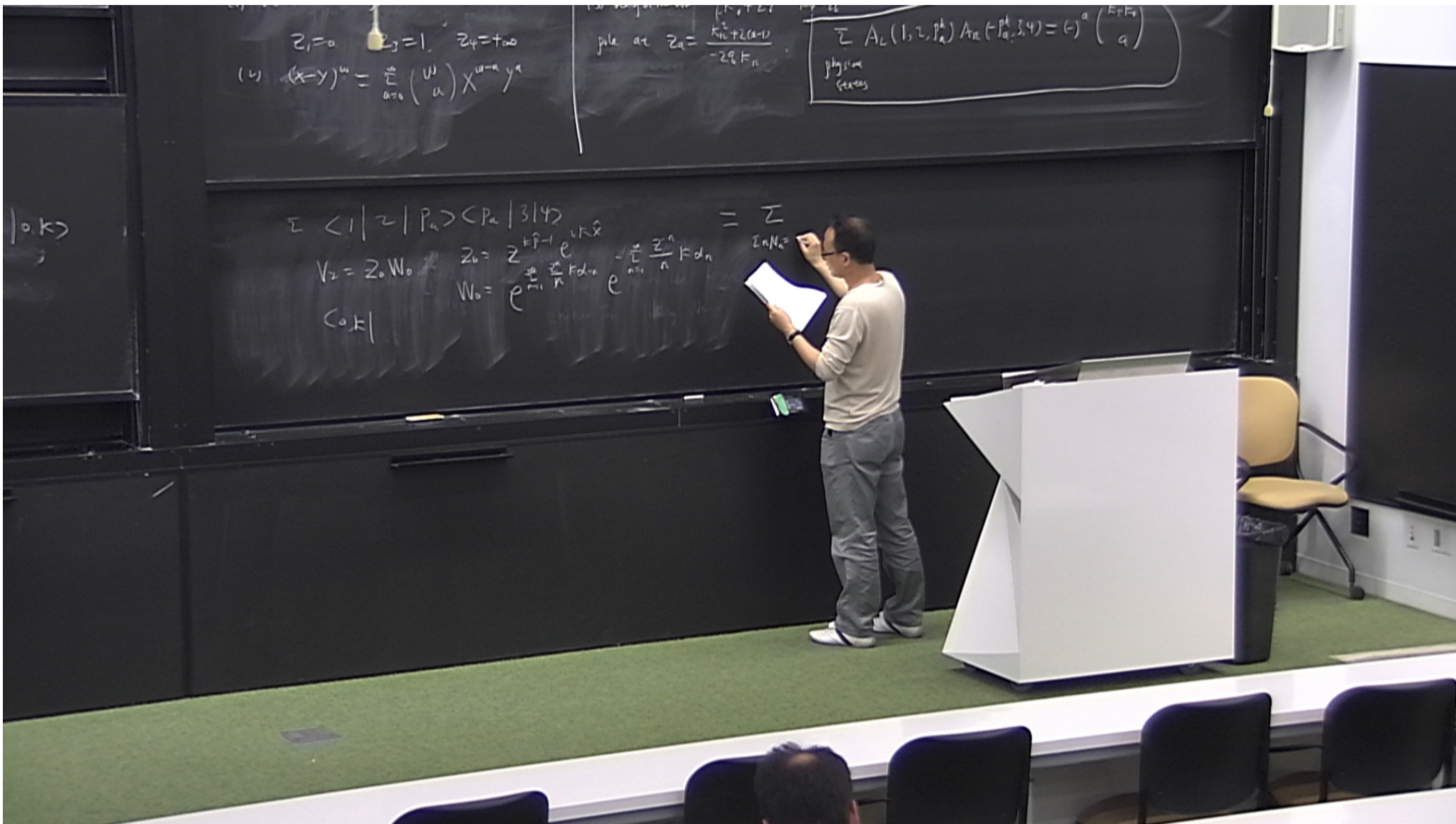
$$|\{N_{m,n}, k\}\rangle = \prod_{m=0}^{a-1} \prod_{n=1}^{\infty} \frac{(\alpha_{-n})^{N_{m,n}}}{\sqrt{N_{m,n}!}} |0, k\rangle$$

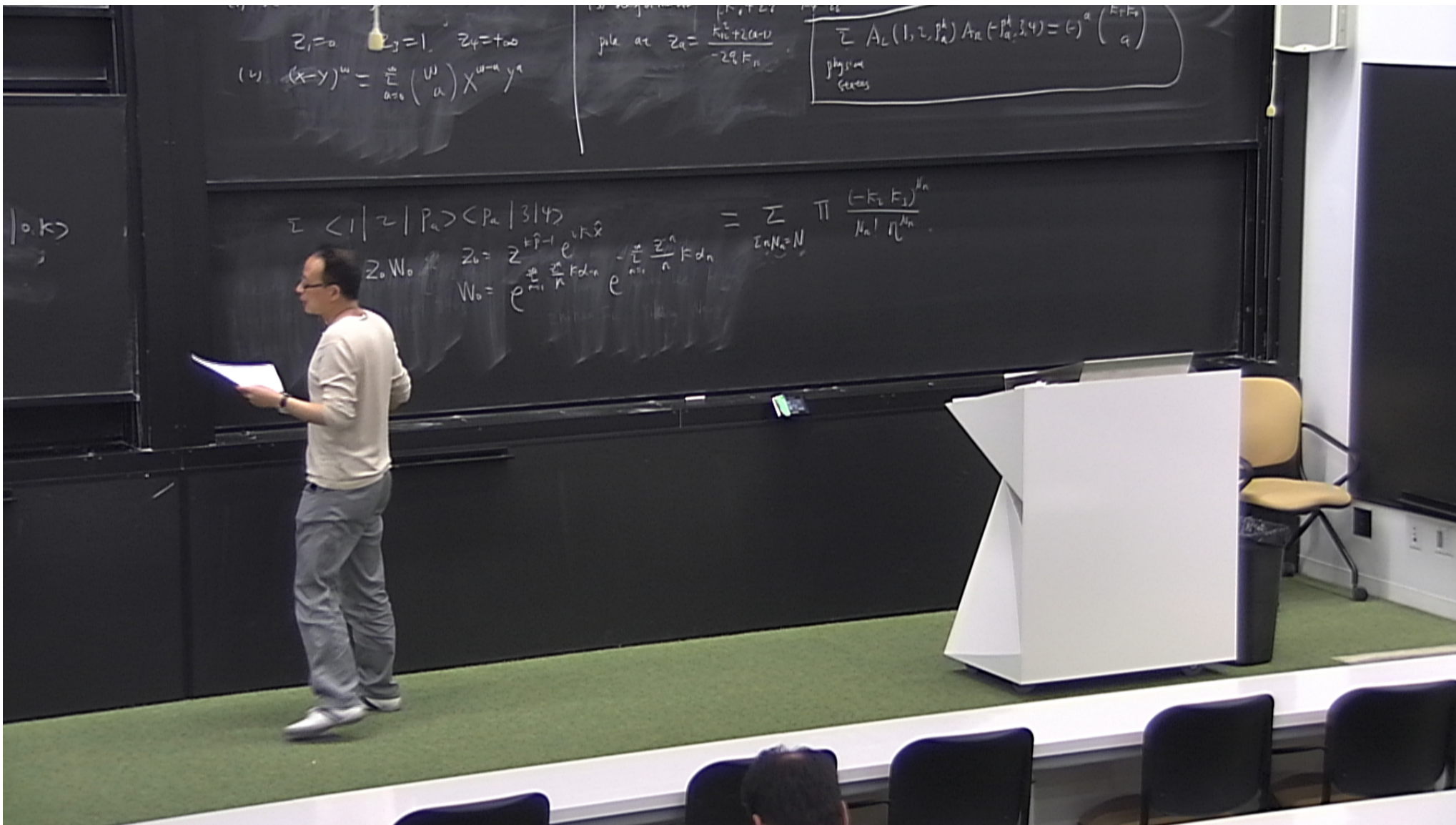












$$z_1=0 \quad z_2=1 \quad z_3=\infty$$

$$(x-y)^w = \sum_{a=0}^w \binom{w}{a} x^{w-a} y^a$$

pole at $z_0 = \frac{F_1 + 2(a-1)}{-2F_1}$

$$\sum A_L(1, z, p_a^L) A_R(-p_a^R, z, 4) = (-)^a \binom{F_1 + F_2}{a}$$

physical series

$|0, k\rangle$

$$\sum \langle 1 | z | p_n \rangle \langle p_n | 3 | 4 \rangle = \sum_{\sum n_i = N} \prod \frac{(-F_1 F_2)^{n_i}}{n_i! n_i^{d_i}}$$

$$z_0 W_0 = z^{\frac{F_1-1}{2}} e^{i k \hat{x}}$$

$$W_0 = e^{\sum_{n=1}^{\infty} \frac{F_1}{n} F_{d_n}} e^{-\sum_{n=1}^{\infty} \frac{z^n}{n} F_{d_n}}$$

define $N = \sum n N_n$

$$J = \sum_{n=1} N_n$$

$$J \leq N$$

$$I_N = (-)^N \sum_{J=1}^N \frac{S(N, J)}{N!}$$

$$= (-)^J \binom{N}{J}$$

first stirling number
with sign

oscillation basis.

$$| \{N, \mu, n\}, k \rangle = \prod_{m=0}^{J-1} \prod_{n=1}^{\infty} \frac{1}{\sqrt{m!}} \left(\frac{a_n}{\sqrt{n}} \right)^m$$

define $N = \sum n N_n$

$$J = \sum_{n=1} N_n$$

$$J \leq N$$

$$I_N = (-1)^N \sum_{J=1}^N \frac{S(N, J)}{N!}$$

$$= (-1)^J \binom{N}{J}$$

first stirling number
with sign

oscillation basis.

$$| \{N_{\mu, n}\}, k \rangle = \prod_{n=0}^{N-1} \prod_{\mu=1}^n \frac{1}{\sqrt{N_{\mu, n}}}$$

$$\rangle = (-1)^N \binom{N}{J}$$

$$(1) A(1234) = \int dZ_2 (1-Z_2)^{F_1 F_2} Z_2^{F_1 F_1}$$

$$Z_1=0 \quad Z_3=1 \quad Z_4=\infty$$

$$(2) (x-y)^w = \sum_{a=0}^{\infty} \binom{w}{a} x^{w-a} y^a$$

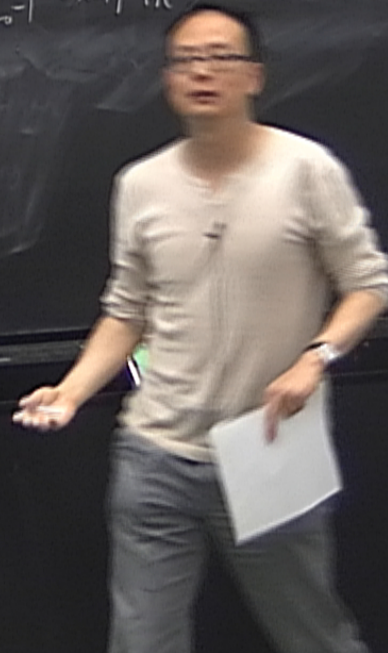
(3) deformation $|k_1+2g\rangle$ $|k_2-2g\rangle$
 pole at $z_a = \frac{k_{11}^2 + 2(a-1)}{-2g F_n}$

$$\sum A_L(1,2,p_a) A_R(-p_a,3,4) = (-)^a \binom{k_1+k_2}{a}$$

physical states

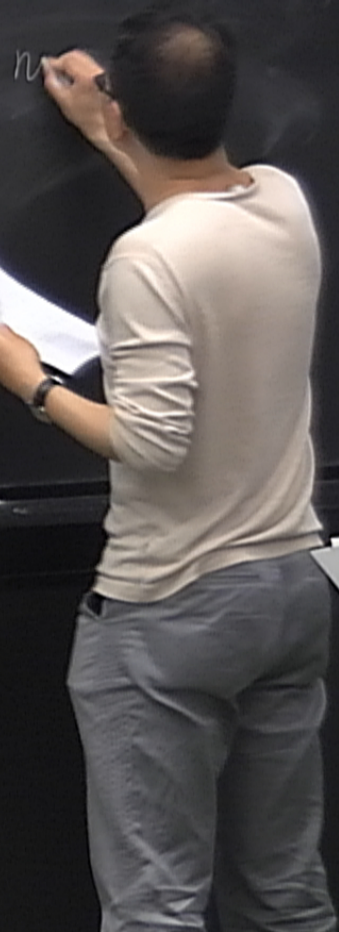
$$A_n = \langle p_1 | V_2(k_2) \left| \frac{1}{L_0-1} \right| V_3 \left| \frac{1}{L_0-1} \right\rangle - \left| \frac{1}{L_0-1} \right\rangle V_{k_1} | p_n \rangle$$

propagator



$$A_n = \langle \phi_1 | V_2(E_2) \left[\frac{1}{L_0 - 1} \right] V_3 \left[\frac{1}{L_0 - 1} \right] \dots \left[\frac{1}{L_0 - 1} \right] V_{n-1} | \phi_n \rangle$$

\uparrow
 propagator



$$A_n = \langle \phi_1 | V_2(K_2) \frac{1}{L_0-1} V_3 \frac{1}{L_0-1} \dots \frac{1}{L_0-1} V_{n-1} | \phi_n \rangle \quad \text{no-ghost theorem}$$

