Title: Testing Gravity with Cosmology- a new Golden Age

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Abstract: With the emergence of the dark sector in cosmology, a variety of modified theories of gravity have come to the fore. I will discuss a framework which can be used to test gravity on large scales and the observational programmes that might lead to the tightest constraints.

Testing Gravity with Cosmology

(a new Golden Age)

Pedro Ferreira
Oxford

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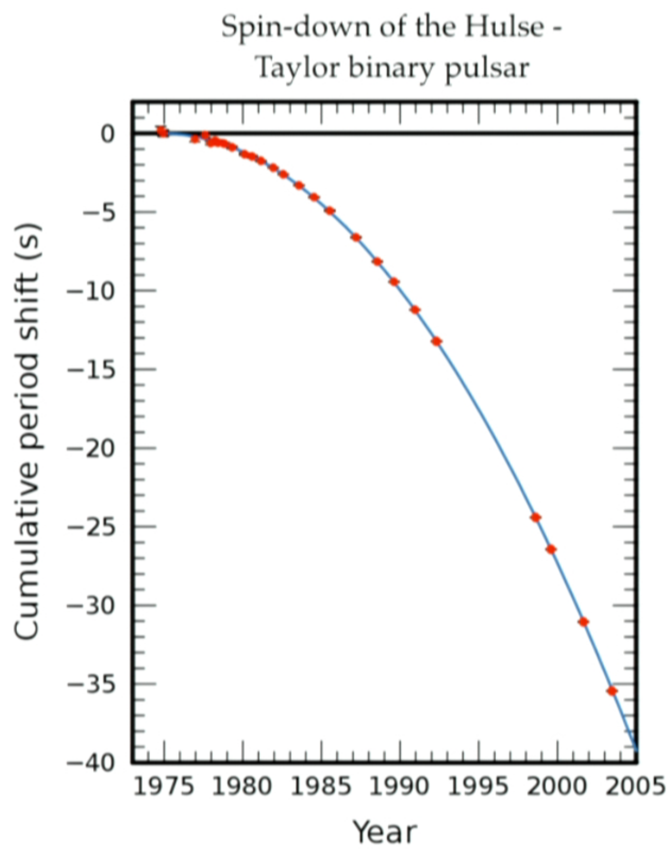
Spin-down of the Hulse -
Taylor binary pulsar
Cumulative period shift (s)
0
−5
−10
−15
−20
−25
−30
−35
−40
1975
1980
1985
1990
1995
2000
2005
Year

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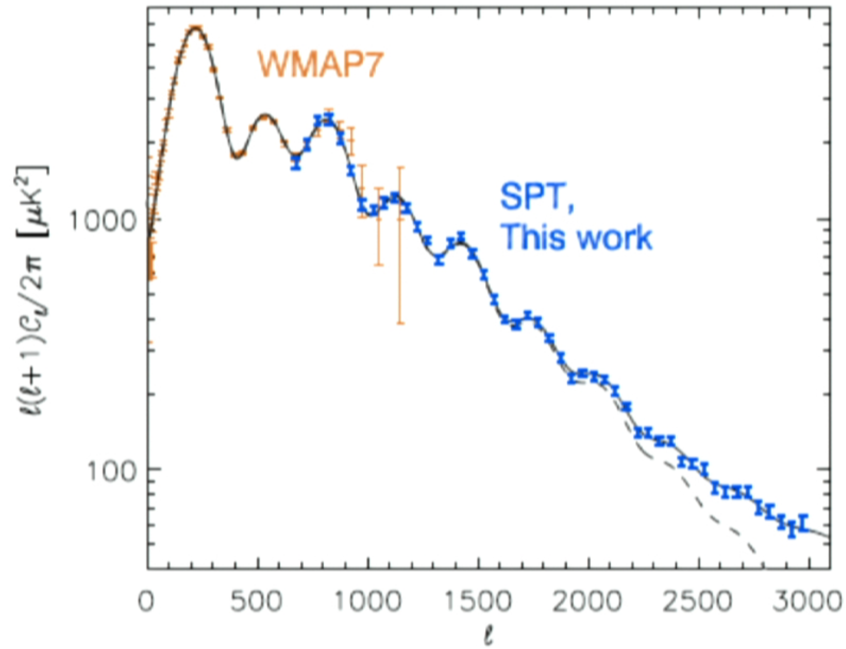
The Angular Power Spectrum of the CMB



Kriesler et al 2011

$$\ell \simeq \frac{180^o}{\theta}$$

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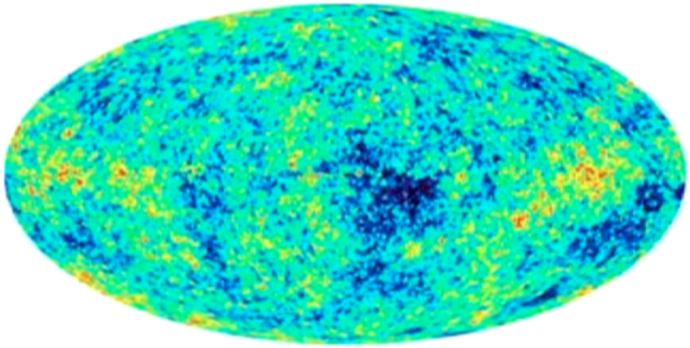
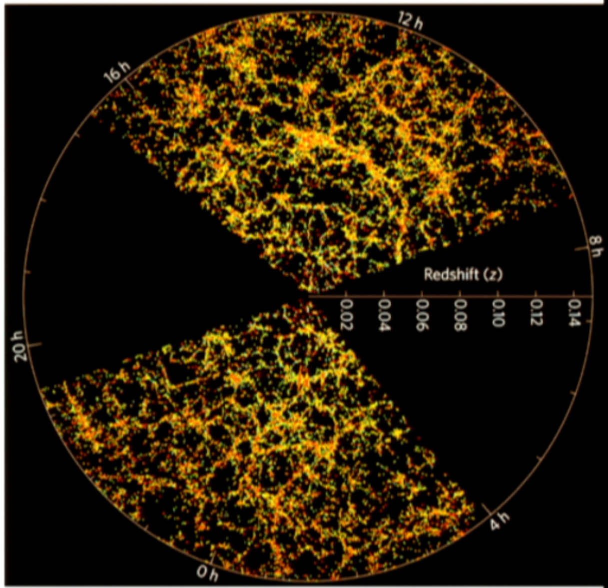
The Large Scale Structure of the Universe

WMAP



SDSS



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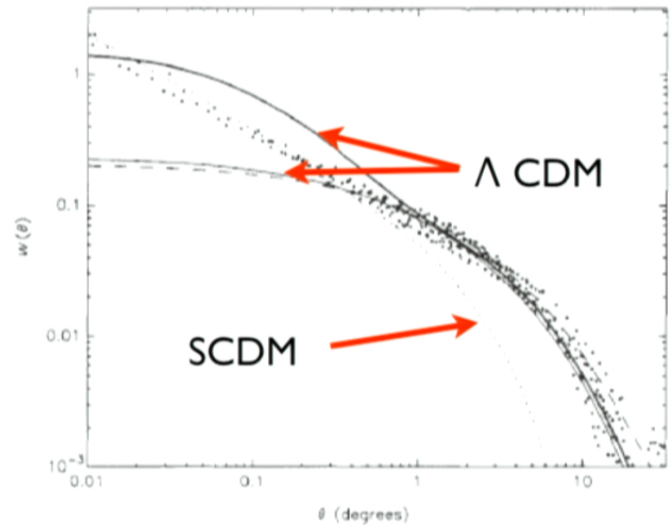
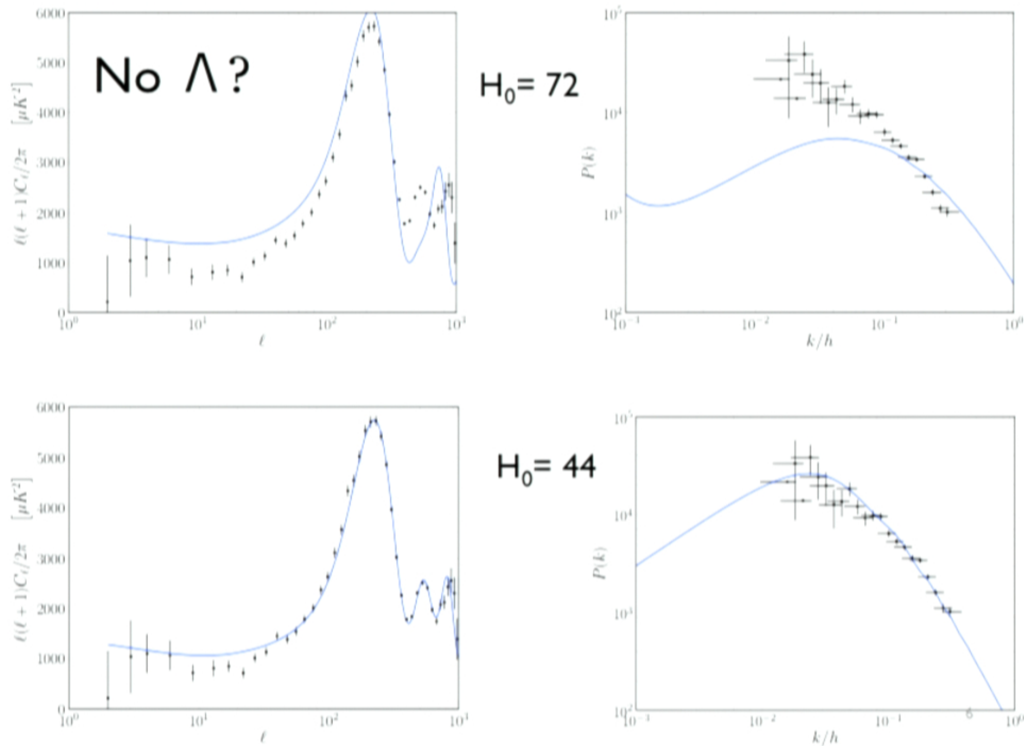
1990

Λ from large scale structure



Efstathiou, Sutherland and Maddox



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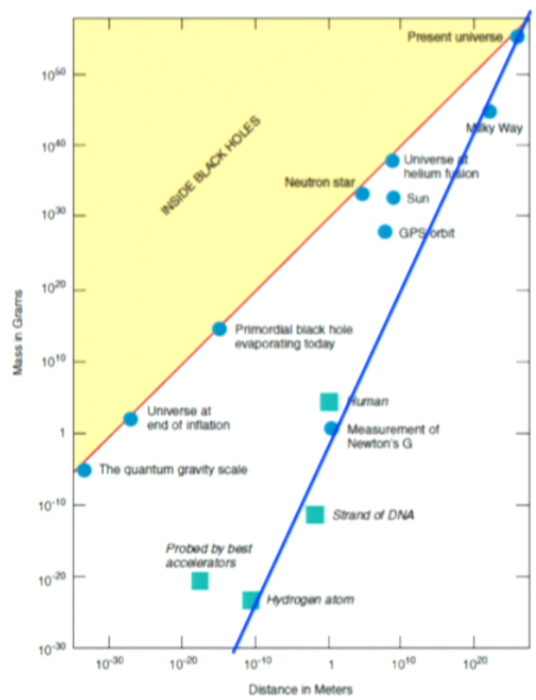
"The elegant logic of general relativity theory, and its precision tests, recommend GR as the first choice for a working model for cosmology. But the Hubble length is fifteen orders of magnitude larger than the length scale of the precision tests, at the astronomical unit and smaller, a spectacular extrapolation."

Jim Peebles, IAU 2000

Outline

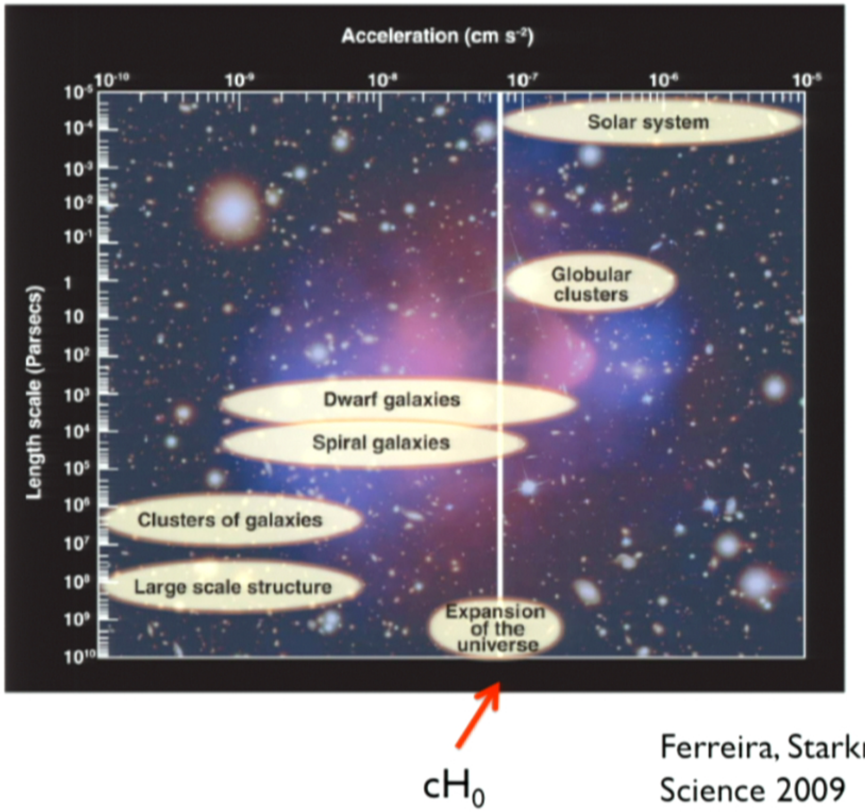
- The panorama of gravitation
- Cosmological linear perturbations
- How to parametrize the space of theories
- How to measure the parameters
- The future



The Rise of the Dark Sector

$$\frac{GM}{R^2} \simeq a_0$$

$$a_0 \simeq cH_0 \simeq 10^{-10} m/s^2$$



Ferreira, Starkman
Science 2009

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Einstein Gravity

Curvature

$$\frac{1}{16\pi G}\int d^4x\sqrt{-g}R(g) + \int d^4x\sqrt{-g}\mathcal{L}(g, \mathrm{matter})$$

Metric of space time

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Extending Gravity

- Lovelock's theorem (1971) :
  "The only second-order, local gravitational field equations derivable from an action containing solely the 4D metric tensor (plus related tensors) are the Einstein field equations with a cosmological constant."

- Ways of dodging Lovelock's theorem:
  - Extra degrees of freedom.
  - More than 4 dimensions.
  - Field equations > 2nd order in derivatives.
  - Non-locality.
  - (Sacrifice action principle).

Modified Gravity and Cosmology[1]

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Abstract

In this review we present a thoroughly comprehensive survey of recent work on modified theories of gravity and their cosmological consequences. Amongst other things, we cover General Relativity, Scalar-Tensor, Einstein-Aether, and Bimetric theories, as well as TeVeS, $f(R)$, general higher-order theories, Hořava-Lifschitz gravity, Galileons, Ghost Condensates, and models of extra dimensions including Kaluza-Klein, Randall-Sundrum, DGP, and higher co-dimension braneworlds. We also review attempts to construct a Parameterised Post-Friedmannian formalism, that can be used to constrain deviations from General Relativity in cosmology, and that is suitable for comparison with data on the largest scales. These subjects have been intensively studied over the past decade, largely motivated by rapid progress in the field of observational cosmology that now allows, for the first time, precision tests of fundamental physics on the scale of the observable Universe. The purpose of this review is to provide a reference tool for researchers and students in cosmology and gravitational physics, as well as a self-contained, comprehensive and up-to-date introduction to the subject as a whole.

Keywords: General Relativity, Gravitational Physics, Cosmology, Modified Gravity

Physics Reports (**arXiv:1106.2476)**

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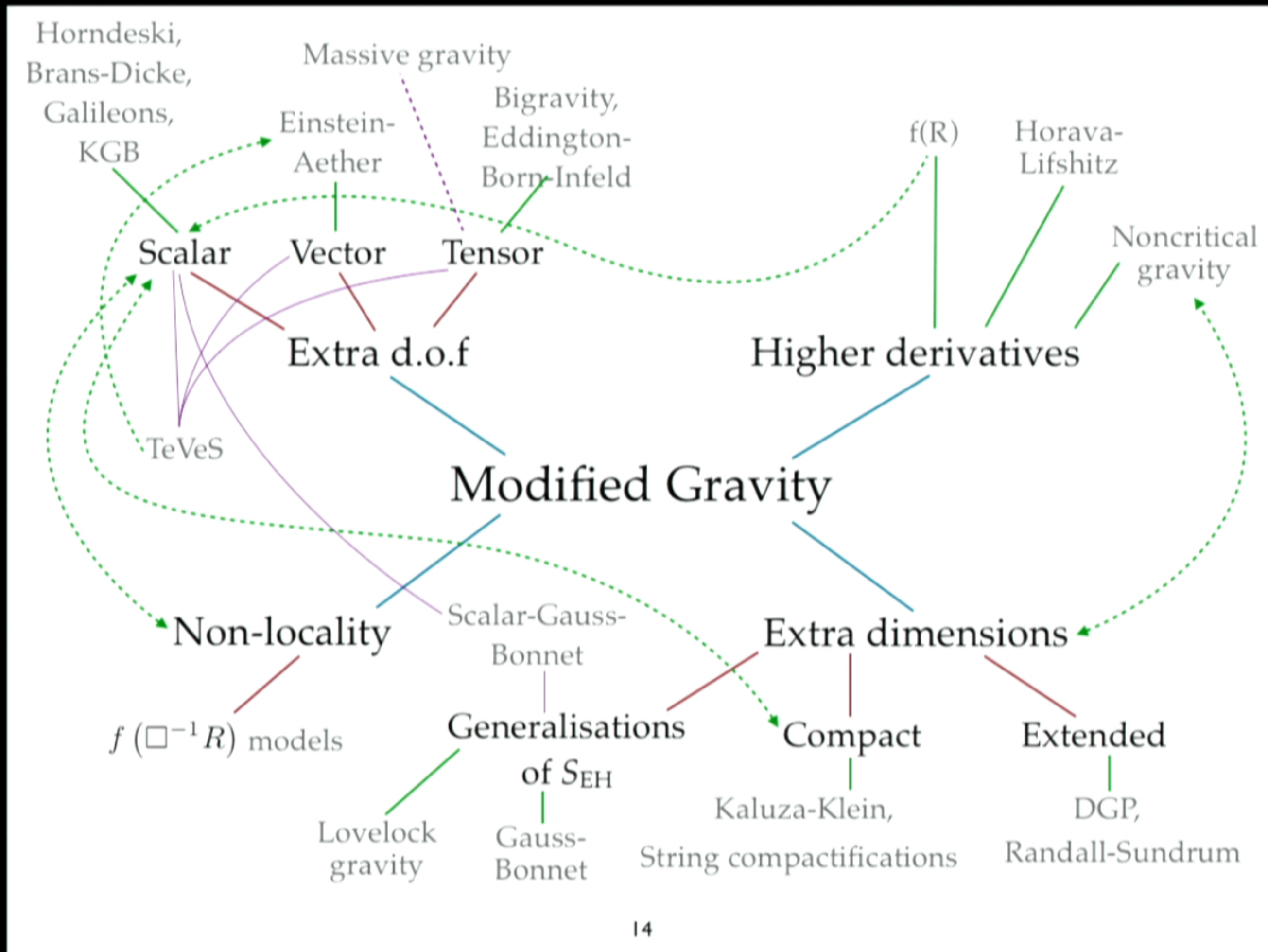
Horndeski,
Brans-Dicke,
Galileons,
KGB
Massive gravity
Einstein-
Aether
Bigravity,
Eddington-
Born-Infeld
f(R)
Horava-
Lifshitz
Noncritical
gravity
Scalar
Vector
Tensor
Extra d.o.f
Higher derivatives
TeVeS
Modified Gravity
Non-locality
Scalar-Gauss-
Bonnet
Extra dimensions
$f\left(\Box^{-1}R\right)$ models
Generalisations
of S_{EH}
Compact
Extended
Lovelock
gravity
Gauss-
Bonnet
Kaluza-Klein,
String compactifications
DGP,
Randall-Sundrum
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THE ASTROPHYSICAL JOURNAL, **163**:595–610, 1971 February 1

THEORETICAL FRAMEWORKS FOR TESTING RELATIVISTIC GRAVITY. I. FOUNDATIONS*

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Received 1970 August 24

ABSTRACT

This is the first in a series of theoretical papers which will discuss the experimental foundations of general relativity This paper reviews, modifies, and compares two very different theoretical frameworks, within which one devises and analyzes tests of gravity. The *Dicke framework* assumes almost nothing about the nature of gravity; and it uses a variety of experiments to delineate the gross features of the gravitational interaction. Two of its tentative conclusions (the presence of a metric, and the "gravitational response equation," $\nabla \cdot \boldsymbol{T} = 0$, for stressed matter) become the postulates of the *Parametrized Post-Newtonian framework*. The PPN framework encompasses most, if not all, of the theories of gravity that are currently compatible with experiment. Future papers in this series will develop the PPN framework in detail, and will use it to analyze a variety of relativistic gravitational effects that should be detectable in the solar system during the coming decade.

Lessons from Parametrized Post Newtonian Approach

- Post-Newtonian limit ⇒ weak-field, v << c.

- (Nearly?) all gravity theories share a metric structure:

$$
\begin{aligned}
g_{00} =& -1 + 2U - 2\beta U^2 - 2\xi\Phi_W + (2\gamma + 2 + \alpha_3 + \zeta_1 - 2\xi)\Phi_1 \\
&+ 2(3\gamma - 2\beta + 1 + \zeta_2 + \xi)\Phi_2 + 2(1 + \zeta_3)\Phi_3 \\
&+ 2(3\gamma + 3\zeta_4 - 2\xi)\Phi_4 - (\zeta_1 - 2\xi)\mathcal{A} - (\alpha_1 - \alpha_2 - \alpha_3)w^2 U \\
&- \alpha_2 w^i w^j U_{ij} + (2\alpha_3 - \alpha_1)w^i V_i \\
g_{0i} =& -\frac{1}{2}\left(4\gamma + 3 + \alpha_1 - \alpha_2 + \zeta_1 - 2\xi\right)V_i - \frac{1}{2}\left(1 + \alpha_2 - \zeta_1 + 2\xi\right)W_i \\
&- \frac{1}{2}\left(\alpha_1 - 2\alpha_2\right)w^i U - \alpha_2 w^j U_{ij} \\
g_{ij} =& \left(1 + 2\gamma U\right)\delta_{ij}
\end{aligned}
$$

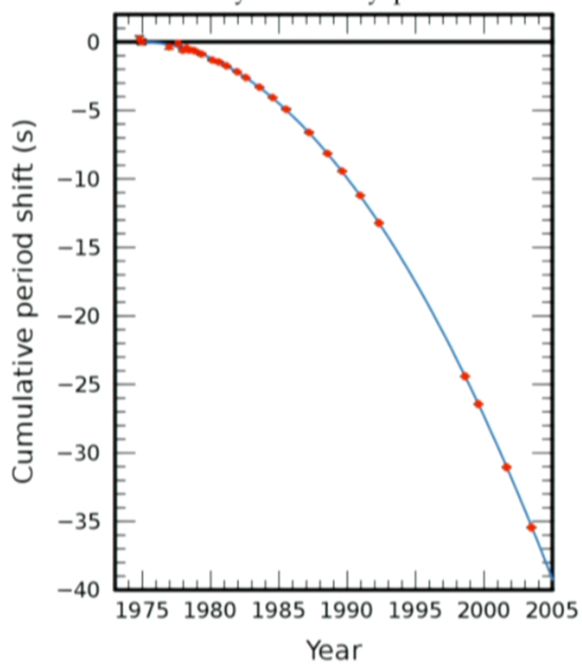
- Ten parameters quantifying non-GR phenomenology.

Lessons from PPN

		γ	β	ξ	α_1	α_2	α_3	ζ_1	ζ_2	ζ_3	ζ_4
	Einstein (1916) GR	1	1	0	0	0	0	0	0	0	0
Scalar-Tensor	Bergmann (1968), Wagoner (1970)		β	0	0	0	0	0	0	0	0
	Nordtvedt (1970), Bekenstein (1977)		β	0	0	0	0	0	0	0	0
	Brans-Dicke (1961)		1	0	0	0	0	0	0	0	0
Vector-Tensor	Hellings-Nordtvedt (1973)	γ	β	0	$\alpha1$	α_2	0	0	0	0	0
	Will-Nordtvedt (1972)	1	1	0	0	α_2	0	0	0	0	0
Bimetric	Rosen (1975)	1	1	0	0	$c_0/c_1 - 1$	0	0	0	0	0
	Rastall (1979)	1	1	0	0	$\alpha2$	0	0	0	0	0
	Lightman-Lee (1973)	γ	β	0	α_1	$\alpha2$	0	0	0	0	0
Stratified	Lee-Lightman-Ni (1974)	ac_0 / c_1	β	ξ	α_1	$\alpha2$	0	0	0	0	0
	Ni (1973)	ac_0 / c_1	bc_0	0	α_1	$\alpha2$	0	0	0	0	0
Scalar Field Theories	Einstein (1912) {Not GR}	0	0	0	-4	0	-2	0	-1	0	0
	Whitrow-Morduch (1965)	0	-1	0	-4	0	0	0	-3	0	0
	Rosen (1971)	λ		0	$-4-4\lambda$	0	-4	0	-1	0	0
	Papetrou (1954a, 1954b)	1	1	0	-8	-4	0	0	2	0	0
	Ni (1972) (stratified)	1	1	0	-8	0	0	0	2	0	0
	Yilmaz (1958, 1962)	1	1	0	-8	0	-4	0	-2	0	-1
	Page-Tupper (1968)	γ	β	0	$-4-4\gamma$	0	$-2-2\gamma$	0	ζ_2	0	ζ_4
	Nordström (1912)	-1	β	0	0	0	0	0	0	0	0
	Nordström (1913), Einstein-Fokker (1914)	-1	1	0	0	0	0	0	0	0	0
	Ni (1972) (flat)	-1	$1-q$	0	0	0	0	0	ζ_2	0	0
	Whitrow-Morduch (1960)	-1	$1-q$	0	0	0	0	0	q	0	0
	Littlewood (1953), Bergman(1956)	-1	β	0	0	0	0	0	-1	0	0

Lessons from PPN

Spin-down of the Hulse - Taylor binary pulsar



Parameter	Bound	Effects	Experiment
$\gamma - 1$	2.3×10^{-5}	Time delay, light deflection	Cassini tracking
$\beta - 1$	2.3×10^{-4}	Nordtvedt effect, Perihelion shift	Nordtvedt effect
ξ	0.001	Earth tides	Gravimeter data
α_1	10^{-4}	Orbit polarization	Lunar laser ranging
α_2	4×10^{-7}	Spin precession	Solar alignment with ecliptic
α_3	4×10^{-20}	Self-acceleration	Pulsar spin-down statistics
ζ_1	0.02	-	Combined PPN bounds
ζ_2	4×10^{-5}	Binary pulsar acceleration	PSR 1913+16
ζ_3	10^{-8}	Newton's 3rd law	Lunar acceleration
ζ_4	0.006	-	Usually not independent

Linear Perturbation Theory

$$G_{\mu\nu} = 8\pi G T_{\mu\nu} \xrightarrow{a,\rho,\cdots} H^2 = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3}$$

Tensor modes

$$\Box^2 h^T_{ij} = \pi_{ij}$$

Scalar modes

$$ds^2 = -a^2(1-2\Xi)dt^2 - 2a^2(\nabla_i \epsilon)dt dx^i + a^2\left[\left(1+\frac{1}{3}\beta\right)q_{ij} + D_{ij}\nu\right]dx^i dx^j$$

Gauge Invariant "Newtonian Potentials"

$$\hat{\Phi} = -\frac{1}{6}(\beta - \nabla^2\nu) + \frac{1}{2}\mathcal{H}V$$

$$\hat{\Psi} = -\Xi - \frac{1}{2}\dot{V} - \frac{1}{2}\mathcal{H}V$$

$$V = \dot{\nu} + 2\epsilon$$

Useful combination:

$$\hat{\Gamma} = \frac{1}{k}\left(\dot{\hat{\Phi}} + \mathcal{H}\hat{\Psi}\right)$$

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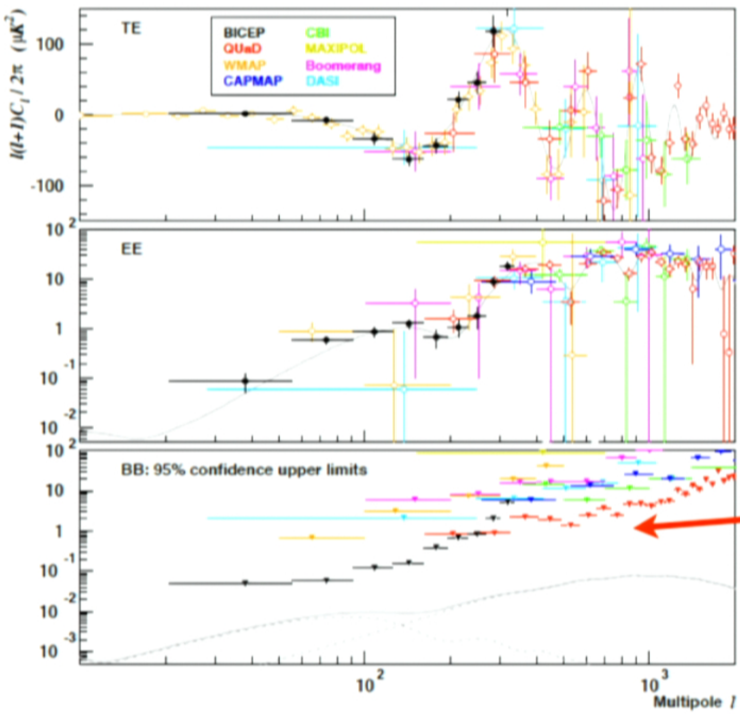
Linear Perturbation Theory



No Gravity Waves yet...

Chiang et al 2009

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Linear Perturbation Theory

$$\delta G_{\alpha\beta} = 8\pi G \delta T_{\alpha\beta}$$

+ E.M. Conservation

$$
\begin{aligned}
E_\Delta &= 2(\vec{\nabla}^2 + 3K)\hat{\Phi} - 6\mathcal{H}k\hat{\Gamma} - \frac{3}{2}\mathcal{H}EV &&= 8\pi G a^2 \sum_i \rho_i \delta_i \\
E_\Theta &= 2\,k\hat{\Gamma} + \frac{1}{2}EV &&= 8\pi G a^2 \sum_i (\rho_i + P_i)\theta_i \\
E_P &= 6k\frac{d\hat{\Gamma}}{d\tau} + 12\mathcal{H}k\hat{\Gamma} - 2(\vec{\nabla}^2 + 3K)(\hat{\Phi} - \hat{\Psi}) \\
&\quad -3E\hat{\Psi} + \frac{3}{2}\left(\dot{E}_R - 2\mathcal{H}E_R\right)V &&= 24\pi G a^2 \sum_i \rho_i \Pi_i \\
E_\Sigma &= \hat{\Phi} - \hat{\Psi} &&= 8\pi G a^2 \sum_i (\rho_i + P_i)\Sigma_i
\end{aligned}
$$

In fact- construct an algebraic equation: $(\nabla^2 + K)\hat{\Phi} = 4\pi G a^2 \sum_i \rho_i \Delta_i$

Linear Perturbation Theory

$$\delta G_{\alpha\beta} = 8\pi G \delta T_{\alpha\beta}$$

+ E.M. Conservation

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Linear Perturbation Theory

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E_\Theta &= 2\,k\hat{\Gamma} + \frac{1}{2}EV &&= 8\pi G a^2 \sum_i (\rho_i + P_i)\theta_i \\
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&\quad -3E\dot{\hat{\Psi}} + \frac{3}{2}\left(\dot{E}_R - 2\mathcal{H}E_R\right)V &&= 24\pi G a^2 \sum_i \rho_i \Pi_i \\
E_\Sigma &= \hat{\Phi} - \hat{\Psi} &&= 8\pi G a^2 \sum_i (\rho_i + P_i)\Sigma_i
\end{aligned}$$

In fact- construct an algebraic equation: $(\nabla^2 + K)\hat{\Phi} = 4\pi G a^2 \sum_i \rho_i \Delta_i$

From PPN to PPF

"Parameterised Post Friedmann"

PPN	PPF
Parameterise the metric elements	Parameterise the field equations
Up to 10 free parameters (but most zero)	Up to 20 free **functions** (some zero, some simply related)
Restricted to weak-field regime	Restricted to the linear regime

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Extending Einstein's equations

$$G_{\mu\nu} = 8\pi G_N T^M_{\mu\nu} + U_{\mu\nu}$$

$$\delta G_{\mu\nu} = 8\pi G_N \delta T^M_{\mu\nu} + \delta U_{\mu\nu}$$

- Scalar-tensor theory:

$$U^{\rm ST}_{\mu\nu} = G_{\mu\nu}(1-\phi) + \frac{\omega(\phi)}{\phi}\left(\nabla_\mu\phi\nabla_\nu\phi - \frac{1}{2}g_{\mu\nu}(\nabla\phi)^2\right) + \nabla_\mu\nabla_\nu\phi - g_{\mu\nu}\Box\phi$$

- f (R) gravity:

$$U^{f(R)}_{\mu\nu} = R_{\mu\nu}(1-f_R) - \frac{1}{2}g_{\mu\nu}(R - f(R)) + \nabla_u\nabla_v f_R - g_{\mu\nu}\Box f_R$$

- DGP:

$$U^{\rm DGP}_{\mu\nu} = -\Lambda_4 g_{\mu\nu} + \kappa_5^4\Pi_{\mu\nu} - E_{\mu\nu} - \kappa_4^2 T_{\mu\nu}$$

Extending Einstein's equations

$$G_{\mu\nu} = 8\pi G_N T^M_{\mu\nu} + U_{\mu\nu}$$

$$\delta G_{\mu\nu} = 8\pi G_N \delta T^M_{\mu\nu} + \boxed{\delta U_{\mu\nu}} \text{ — ?}$$

Considerations:

i) Can contain metric perturbations and/or new d.o.f.
ii) Need to retain gauge form-invariance.

METRIC **NEW D.O.F.**

$$\delta U_{\mu\nu} = \delta U^{\hat{\Phi}}_{\mu\nu} + \delta U^{\widehat{\delta\phi}}_{\mu\nu} + \text{gauge-form invariance fixing term}$$

Built from Bardeen potentials × function of background

Fixed by background equations

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The Beyond-Einstein equations

iii) We have a perturbed Bianchi identity for $U_{\mu\nu}$:

$$\delta\left(\nabla^{\mu}G_{\mu\nu}\right) = 0 \Rightarrow \delta\left(\nabla^{\mu}U_{\mu\nu}\right) = 0$$

(Will assume $\nabla^{\mu}T^{M}_{\mu\nu} = 0$)

If unmodified or Λ-like background the Bianchi identity imposes additional constraints to ensure consistency with the field equations.

iv) We want second-order equations of motion, therefore:

Constraint equations ⇒ $\delta U_{00},\, \delta U_{0i}$ must be first order in time derivatives.

Propagation equations ⇒ $\delta U_{ii},\, \delta U_{ij}$ can be second order.

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Adding New Scalars

$$-a^2\delta U^0_0 = U_\Delta = A_0 k^2\hat{\Phi} + F_0 k^2\hat{\Gamma} + \alpha_0 k^2\hat{\chi} + \alpha_1 k\dot{\hat{\chi}} + M_\Delta k^3 V$$

$$U_\Theta = B_0 k\hat{\Phi} + I_0 k\hat{\Gamma} + \beta_0 k\hat{\chi} + \beta_1\dot{\hat{\chi}} + M_\Theta k^2 V$$

$$a^2\delta U^i_i = U_P = C_0 k^2\hat{\Phi} + C_1 k\dot{\hat{\Phi}} + J_0 k^2\hat{\Gamma} + J_1 k\dot{\hat{\Gamma}}$$
$$+ \gamma_0 k^2\hat{\chi} + \gamma_1 k\dot{\hat{\chi}} + \gamma_2\ddot{\hat{\chi}} + M_P k^3 V$$

$$U_\Sigma = D_0\hat{\Phi} + \frac{D_1}{k}\dot{\hat{\Phi}} + K_0\hat{\Gamma} + \frac{K_1}{k}\dot{\hat{\Gamma}}$$
$$+ \epsilon_0\hat{\chi} + \frac{\epsilon_1}{k}\dot{\hat{\chi}} + \frac{\epsilon_2}{k^2}\ddot{\hat{\chi}}$$

Functions of background (a, k, φ_i...)

Gauge form-fixing term, zero in CN gauge.

$$\nabla_i U_\Theta = -a^2\delta U^0_i, \quad \left(\nabla^i\nabla_j - \frac{1}{3}\delta^i_j\nabla^2\right) U_\Sigma = a^2\delta U^i_j$$

What is $\hat{\chi}$?

- $\hat{\chi}$ is a gauge-invariant scalar perturbation to a degree of freedom (d.o.f.) introduced by a gravitational theory.

Examples: a scalar field ⇒ $\delta\phi$

a fluid ⇒ $\rho\delta,\ \rho(1+\omega)\theta,\ \delta P,\ \rho(1+\omega)\pi$

a time-like vector field ⇒ $\nabla_i\alpha$

+ metric parts to ensure gauge-invariance.

- How to evolve $\hat{\chi}$?
  - eq. of motion from Bianchi identity?
  - In DGP and Eddington-Born-Infeld (EBI) gravity the new d.o.f. are effective fluids. Could we apply this generally?

$$\alpha_0 k^2\hat{\chi} + \alpha_1 k\dot{\hat{\chi}} + M_\Delta k^3 V \equiv \rho_{\rm eff}\delta_{\rm eff}$$
$$\gamma_0 k^2\hat{\chi} + \gamma_1 k\dot{\hat{\chi}} + \gamma_2\ddot{\hat{\chi}} + M_P k^3 V \equiv \delta P_{\rm eff}$$

Adding New Scalars

$$-a^2\delta U^0_0 = U_\Delta = A_0 k^2\hat{\Phi} + F_0 k^2\hat{\Gamma} + \alpha_0 k^2\hat{\chi} + \alpha_1 k\dot{\hat{\chi}} + M_\Delta k^3 V$$

$$U_\Theta = B_0 k\hat{\Phi} + I_0 k\hat{\Gamma} + \beta_0 k\hat{\chi} + \beta_1\dot{\hat{\chi}} + M_\Theta k^2 V$$

$$a^2\delta U^i_i = U_P = C_0 k^2\hat{\Phi} + C_1 k\dot{\hat{\Phi}} + J_0 k^2\hat{\Gamma} + J_1 k\dot{\hat{\Gamma}} + \gamma_0 k^2\hat{\chi} + \gamma_1 k\dot{\hat{\chi}} + \gamma_2\ddot{\hat{\chi}} + M_P k^3 V$$

$$U_\Sigma = D_0\hat{\Phi} + \frac{D_1}{k}\dot{\hat{\Phi}} + K_0\hat{\Gamma} + \frac{K_1}{k}\dot{\hat{\Gamma}} + \epsilon_0\hat{\chi} + \frac{\epsilon_1}{k}\dot{\hat{\chi}} + \frac{\epsilon_2}{k^2}\ddot{\hat{\chi}}$$

1) Scalar-Tensor Theory

$$S_{ST} = \frac{1}{16\pi}\int \mathrm{d}^4x\sqrt{-g}\left[\phi R - \frac{\omega(\phi)}{\phi}(\nabla\phi)^2\right] + S_m[\psi^{(i)}, g_{\mu\nu}]$$

$$A_0 = -2(1-\phi) \qquad F_0 = \frac{3}{k}\left[\dot{\phi} - 2\mathcal{H}(1-\phi)\right]$$

$$B_0 = 0 \qquad I_0 = 2(1-\phi)$$

$$C_0 = 2(1-\phi) \qquad C_1 = \frac{2}{\mathcal{H}}(1-\phi)\left(k^2 - 3\dot{\mathcal{H}} + 3\mathcal{H}^2\right)$$

$$D_0 = (1-\phi) \qquad D_1 = \frac{k}{\mathcal{H}}(1-\phi)$$

$$J_0 = \frac{2}{\mathcal{H}k}(1-\phi)(3\dot{\mathcal{H}} + 3\mathcal{H}^2 - k^2) - 6\frac{\dot{\phi}}{k} \qquad J_1 = 6(1-\phi)$$

$$K_0 = -\frac{k}{\mathcal{H}}(1-\phi) \qquad K_1 = 0$$

2) Horndeski Theory

$$S_{\rm HD} = \int d^4x\sqrt{-g}\left(\sum_{i=2}^{5}\mathcal{L}_i + \mathcal{L}_m\right)$$

where:

$$\begin{aligned}
\mathcal{L}_2 &= K(\phi, X) \\
\mathcal{L}_3 &= -G_3(\phi, X)\Box\phi \\
\mathcal{L}_4 &= G_4(\phi, X)\,R + G_{4,X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)\,(\nabla^\mu\nabla^\nu\phi)\right], \\
\mathcal{L}_5 &= G_5(\phi, X)\,G_{\mu\nu}\,(\nabla^\mu\nabla^\nu\phi) - \frac{1}{6}\,G_{5,X}\left[(\Box\phi)^3 - 3(\Box\phi)\,(\nabla_\mu\nabla_\nu\phi)\,(\nabla^\mu\nabla^\nu\phi)\right. \\
&\quad \left. +2(\nabla^\mu\nabla_\alpha\phi)\,(\nabla^\alpha\nabla_\beta\phi)\,(\nabla^\beta\nabla_\mu\phi)\right]
\end{aligned}$$

and $X = -\partial^\mu\phi\partial_\mu\phi/2$.

Encompasses scalar-tensor, *f (R)*, covariant Galileons, kinetic gravity braiding, the Fab Four, plus all scalar dark energy models (quintessence, k-essence...) .

Define useful combinations $\mathcal{G}_\mathcal{T}$, $\mathcal{F}_T$, Θ in terms of the G_i and their derivatives.

2) Horndeski Theory

$$S_{\rm HD} = \int d^4x\sqrt{-g}\left(\sum_{i=2}^{5}\mathcal{L}_i + \mathcal{L}_m\right)$$

where:

$$\begin{aligned}
\mathcal{L}_2 &= K(\phi, X) \\
\mathcal{L}_3 &= -G_3(\phi, X)\Box\phi \\
\mathcal{L}_4 &= G_4(\phi, X)\, R + G_{4,X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)\,(\nabla^\mu\nabla^\nu\phi)\right], \\
\mathcal{L}_5 &= G_5(\phi, X)\, G_{\mu\nu}\,(\nabla^\mu\nabla^\nu\phi) - \frac{1}{6}\, G_{5,X}\left[(\Box\phi)^3 - 3(\Box\phi)\,(\nabla_\mu\nabla_\nu\phi)\,(\nabla^\mu\nabla^\nu\phi)\right. \\
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Define useful combinations $\mathcal{G}_T$, $\mathcal{F}_T$, Θ in terms of the G_i and their derivatives.

2) Horndeski Theory

$$S_{\rm HD} = \int d^4x\sqrt{-g}\left(\sum_{i=2}^{5}\mathcal{L}_i + \mathcal{L}_m\right)$$

$$A_0 = -2\left(1-\frac{\mathcal{G}_T}{M_{Pl}^2}\right)$$

$$B_0 = 0$$

$$C_0 = 2\left(1-\frac{\mathcal{F}_T}{M_{Pl}^2}\right)$$

$$D_0 = 1-\frac{\mathcal{F}_T}{M_{Pl}^2}$$

$$J_0 = \frac{2}{k\mathcal{H}}\Big[\left(1-\frac{\mathcal{G}_T}{M_{Pl}^2}\right)(-k^2+3\dot{\mathcal{H}}+3\mathcal{H}^2) - 3\mathcal{H}\frac{\dot{\mathcal{G}}_T}{M_{Pl}^2}\Big]$$

$$K_0 = -\frac{k}{\mathcal{H}}\left(1-\frac{\mathcal{G}_T}{M_{Pl}^2}\right)$$

$$F_0 = \frac{6}{k}\left(\frac{\Theta}{M_{Pl}^2}-\mathcal{H}\right)$$

$$I_0 = 2\left(1-\frac{\mathcal{G}_T}{M_{Pl}^2}\right)$$

$$C_1 = \frac{2}{k\mathcal{H}}\left(1-\frac{\mathcal{G}_T}{M_{Pl}^2}\right)\left[k^2-3(\dot{\mathcal{H}}-\mathcal{H}^2)\right]$$

$$D_1 = \frac{k}{\mathcal{H}}\left(1-\frac{\mathcal{G}_T}{M_{Pl}^2}\right)$$

$$J_1 = 6\left(1-\frac{\mathcal{G}_T}{M_{Pl}^2}\right)$$

$$K_1 = 0$$

3) DGP

$$S_{DGP} = \frac{1}{2\kappa_5^2}\int d^5\tilde{x}\sqrt{-{}^{(5)}g}\left[{}^{(5)}R - 2\Lambda_5\right] + \int d^4x\,\sqrt{-{}^{(4)}g}\left[\frac{1}{2\kappa_4^2}{}^{(4)}R + \mathcal{L}_B\right]$$

$$A_0 = -\frac{3}{r_c^2 X} \qquad F_0 = -\frac{9\mathcal{H}_k}{r_c^2 X}$$

$$B_0 = 0 \qquad I_0 = \frac{3}{r_c^2 X}$$

$$C_0 = \frac{3}{r_c^2 X}(2-\omega_E) \qquad C_1 = \frac{3}{r_c^2 X\mathcal{H}k}(k^2 + 3\mathcal{H}^2 - 3\dot{\mathcal{H}})$$

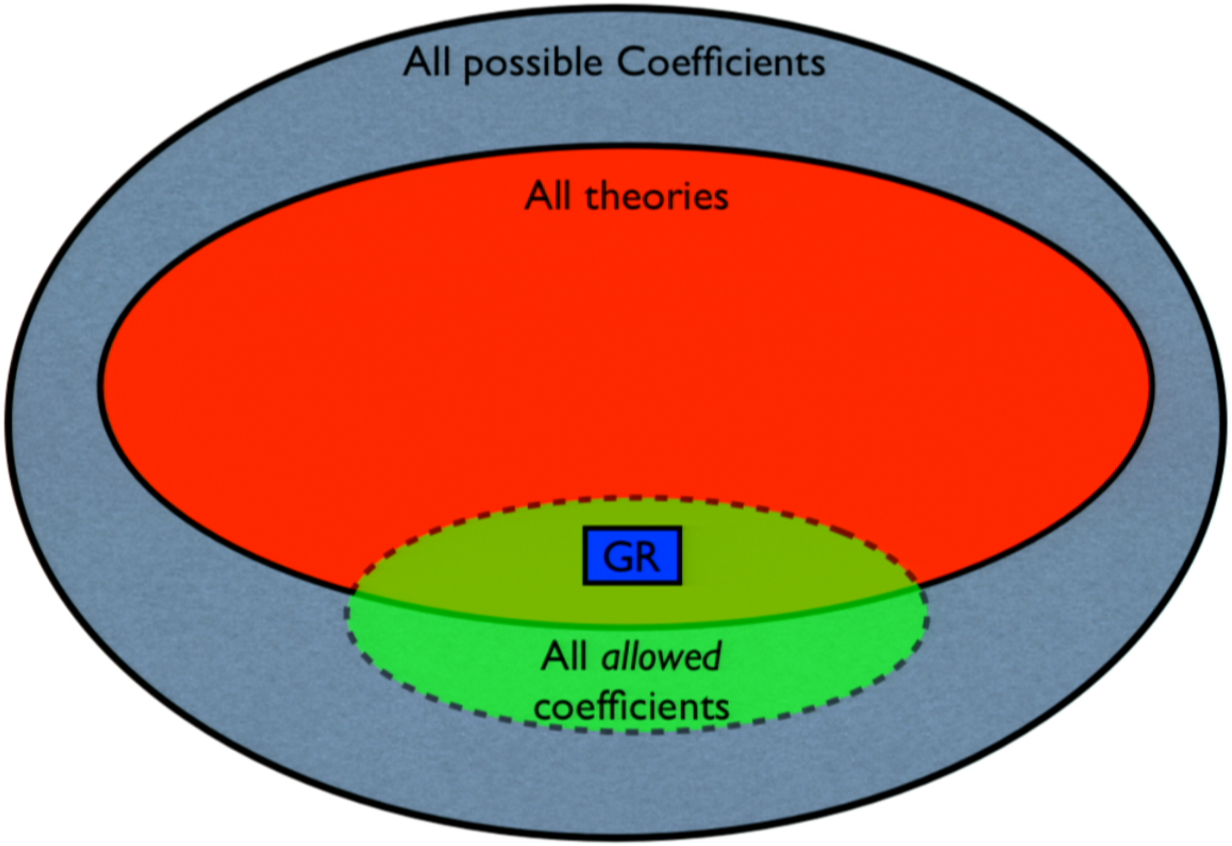
$$D_0 = -\frac{3}{r_c^2(X+3Y)} \qquad D_1 = -\frac{3k}{\mathcal{H}r_c^2(X+3Y)}$$

$$J_0 = -\frac{3}{r_c^2 X k\mathcal{H}}\left(k^2 - 3\mathcal{H}^2(2-\omega_E) - 3\dot{\mathcal{H}}\right) \qquad J_1 = \frac{9}{r_c^2 X}$$

crossover scale (r_c)

$$K_0 = \frac{3k}{\mathcal{H}r_c^2(X+3Y)} \qquad K_1 = 0$$

where $X = \frac{3\epsilon}{r_c}H$ $\qquad Y = -\epsilon\frac{\frac{dH}{dt} + 3H^2}{r_c H}$


All possible Coefficients
All theories
GR
All *allowed*
coefficients

34

Alternative Approach

Zhang, Liguori, Bean and Dodelson
Caldwell, Cooray and Melchiorri
Amendola, Kunz and Sapone
Bertschinger and Zukin
Amin, Blandford and Wagoner
Pogosian, Silvestri, Koyama and Zhao
Bean and Tangmatitham

- One only works with 2 field equations to close the system:

Poisson $-2k^2\Phi = 8\pi G_N a^2 \rho \Delta$

Slip $\Phi - \Psi = 0$

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- One only works with 2 field equations to close the system:

  So test for violations of these equations.

  Pick F_1 and F_2 to be related to LHS by dimensionless functions.

  Rearrange.

Poisson $$-2k^2\Psi = 8\pi G_N a^2 \mu(a,k,\phi_i...)\rho\Delta$$

Slip $$\Phi = \gamma(a,k,\phi_i...)\Psi$$

where $\gamma(a,k,\phi_i...) = \frac{1}{1-f_2(a,k,\phi_i...)}$ $\quad \mu(a,k,\phi_i...) = \frac{1-f_2(a,k,\phi_i...)}{1+\frac{1}{2}f_1(a,k,\phi_i...)}$

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- There are a few variants, e.g.

$$-k^2(\Phi+\Psi) = 8\pi G_N a^2 \Sigma(a,k,\phi_i...)\rho\Delta$$

Alternative Approach

pick any two

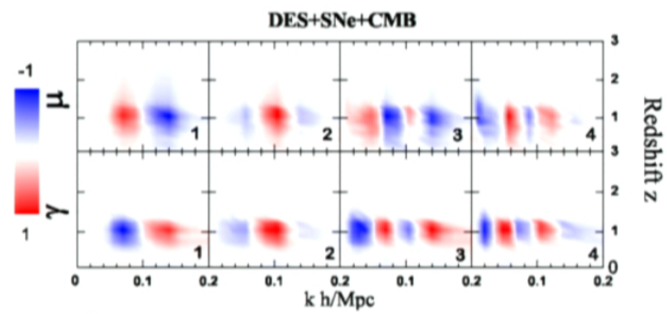
$$\begin{cases} \Phi = \gamma(a,k)\Psi \\ -2k^2\Psi = 8\pi G_N a^2 \mu(a,k)\rho\Delta \\ -k^2(\Phi+\Psi) = 8\pi G_N a^2 \Sigma(a,k)\rho\Delta \end{cases}$$

- This is fine if you only want to raise a flag for non-GR behaviour.
- BUT the Poisson equation and slip relation don't generally look like this.
- γ, μ and Σ will be complicated functions of initial conditions and environmental dependence.

Hojjati *et al.* 2011

⇒ Each function must be

constrained on a grid of (a,k) using Principal Component Analysis .



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Light vs. Matter

- For a perturbed line element of the form:

$$ds^2 = a(\tau)^2 \left[-(1+2\Psi)d\tau^2 + (1-2\Phi)\gamma_{ij}dx^i dx^j \right]$$

the equations of motion are:

$$\frac{1}{a}\frac{d(a\mathbf{v})}{d\tau} = -\nabla\Psi \qquad \text{(non-relativistic particles)}$$

$$\frac{d\mathbf{v}}{d\tau} = -\nabla_\perp \left(\Phi + \Psi\right) \qquad \text{(relativistic particles)}$$

- These expressions are independent of the underlying gravity theory.
- Hence we need **both** CDM/baryons and photons to test for the equality of the potentials.

Growth of structure,
RSDs and PVs.

ISW
Lensing (CMB, weak)

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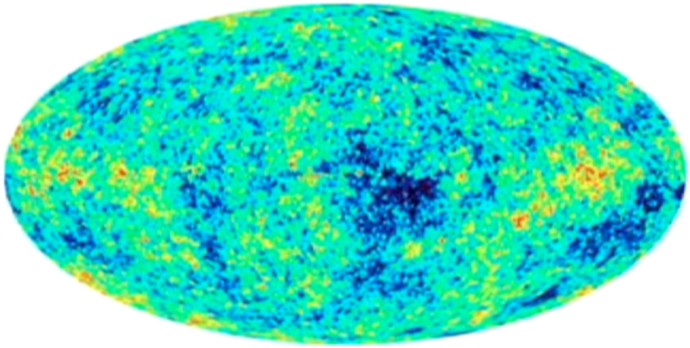
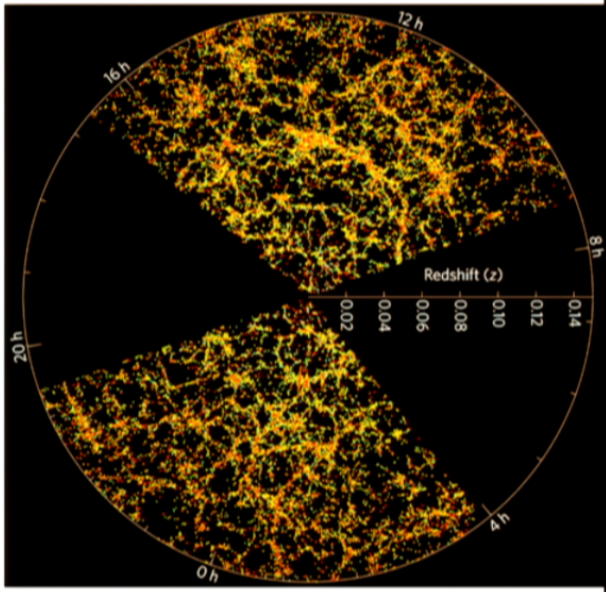
The Large Scale Structure of the Universe

WMAP



SDSS



39

Growth of Structure

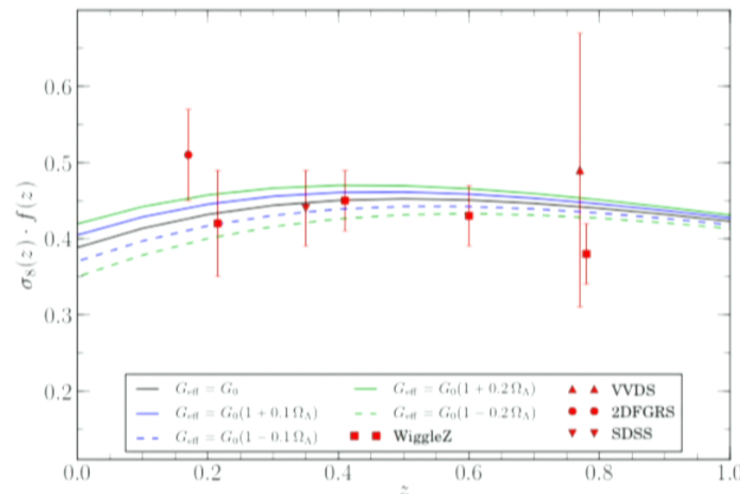
- Evolution of CDM energy density perturbations:

$$\ddot{\delta}_M + \mathcal{H}\dot{\delta}_M - 3\ddot{\Phi} - 3\mathcal{H}\dot{\Phi} + k^2\Psi = 0$$

- The growth rate of structure is quantified via f:

$$f(k,a) = \frac{d\ln\delta_M(k,a)}{d\ln a}$$

- In GR $\delta_M \propto a$ during matter domination, so $f = 1$ (independent of k for linear scales).



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Relation to δ_M has changed. Relation to Φ has changed.

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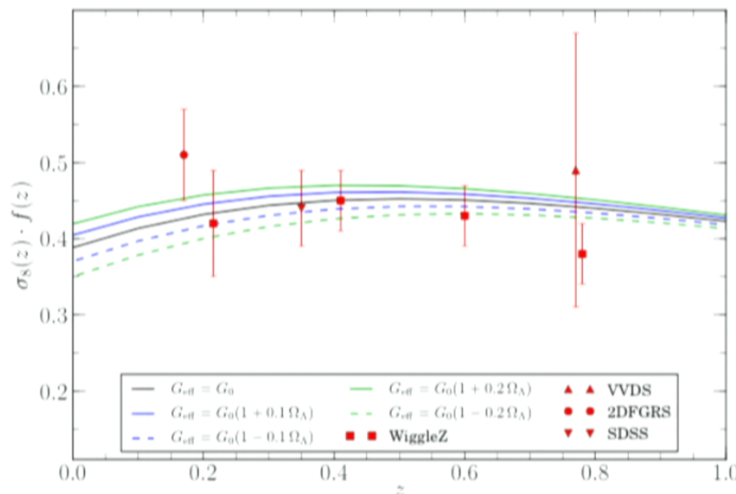

40

Growth of Structure

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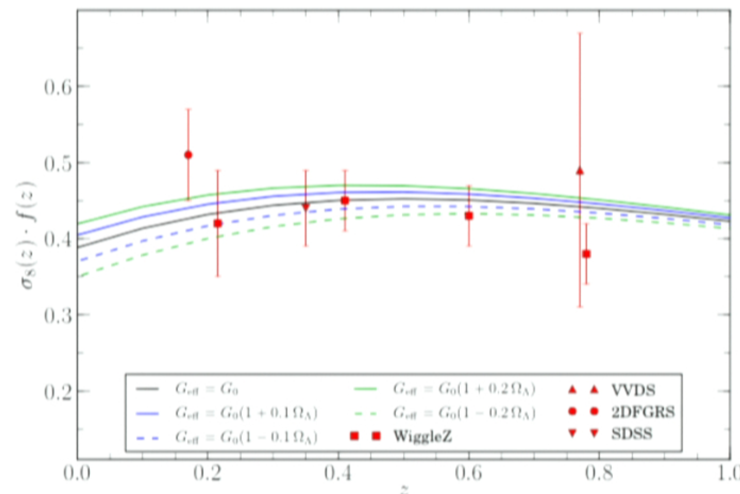
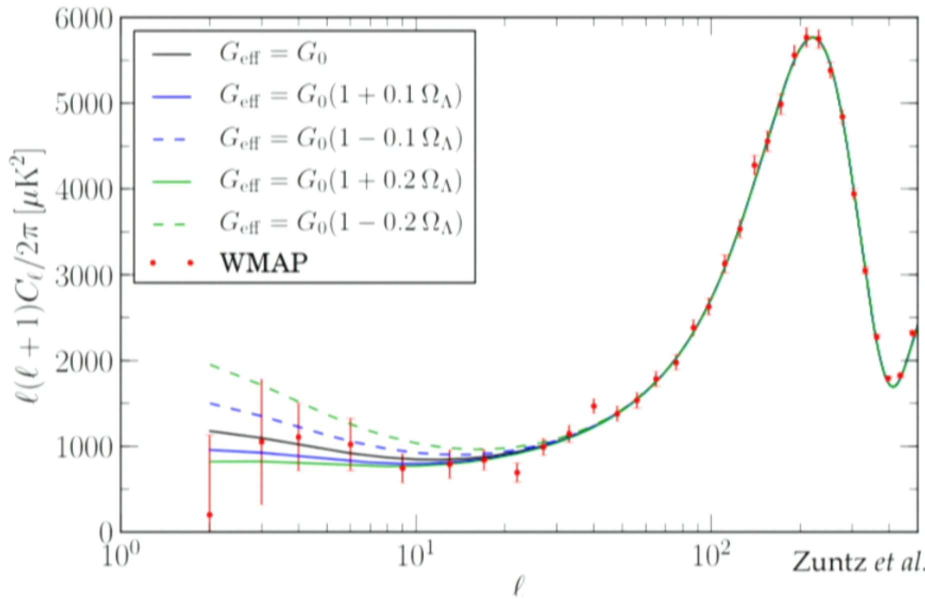
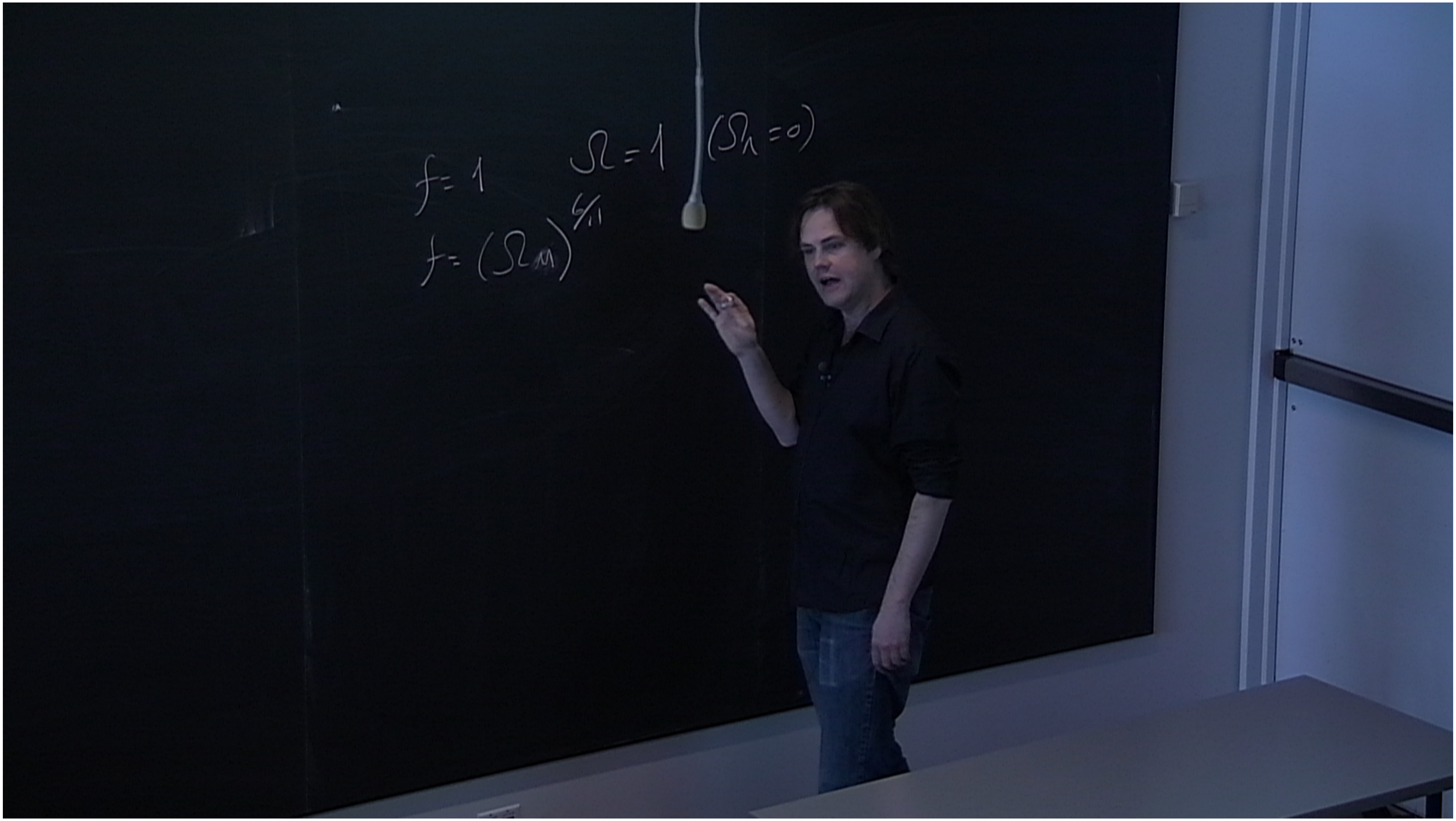


The ISW Effect

Late-time modifications affect the ISW plateau via $\int \left(\dot{\Phi} + \dot{\Psi}\right) d\tau$

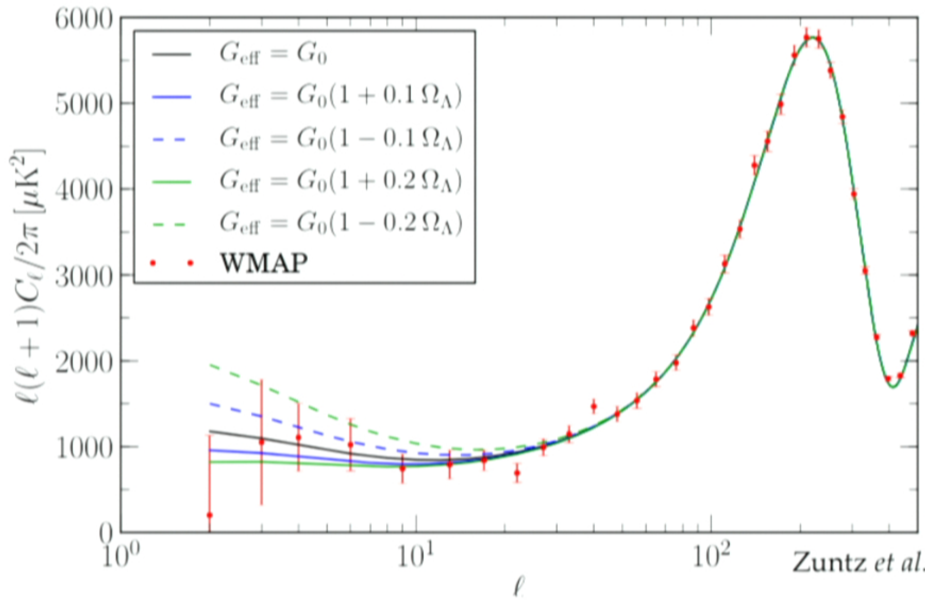


Zuntz *et al.* 2011


f = 1
Ω = 1 (Ω_Λ = 0)
f = (Ω_M)^{6/11}

The ISW Effect

Late-time modifications affect the ISW plateau via $\int \left(\dot{\Phi} + \dot{\Psi}\right) d\tau$



Zuntz *et al.* 2011

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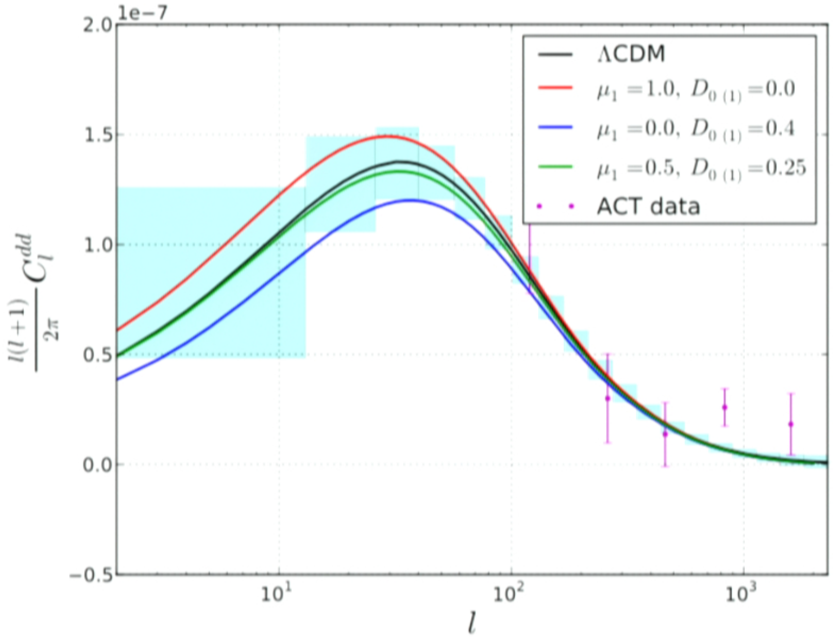
CMB Lensing



Planck forecast errors

Power spectrum of the CMB lensing potential

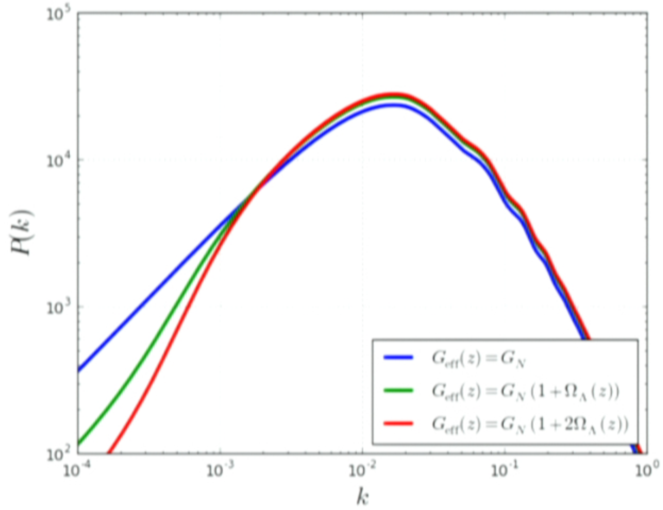
Lensing probes both potentials via:

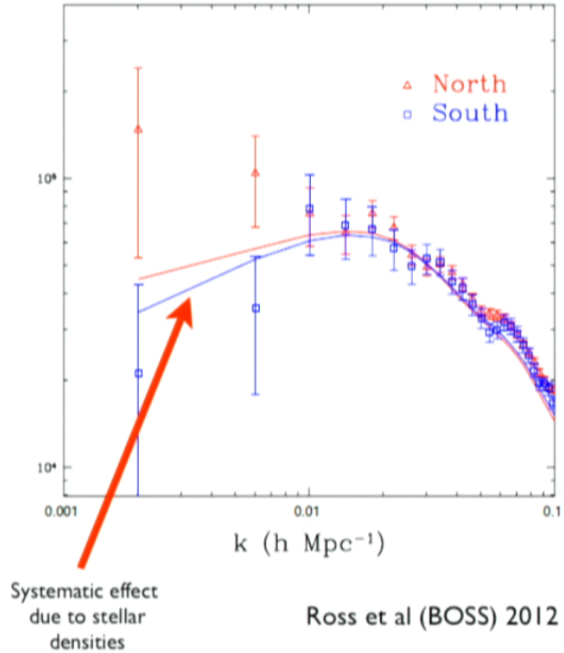
$$\int (\Phi + \Psi)\, W(\chi) d\chi$$

↑ geometry

$\Rightarrow$ Essentially no constraints at the moment.

Ultra-large scale surveys





Ross et al (BOSS) 2012

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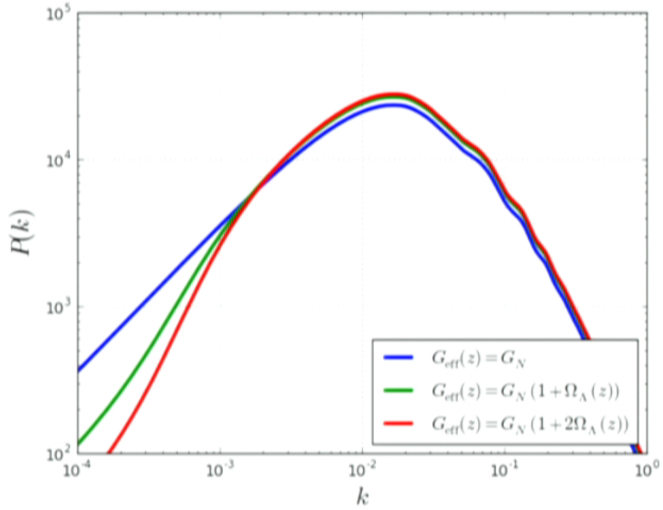
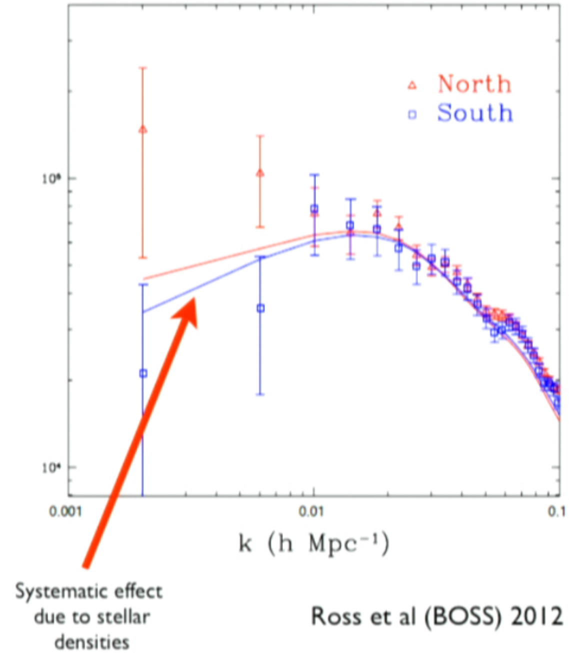
Ultra-large scale surveys





Ross et al (BOSS) 2012

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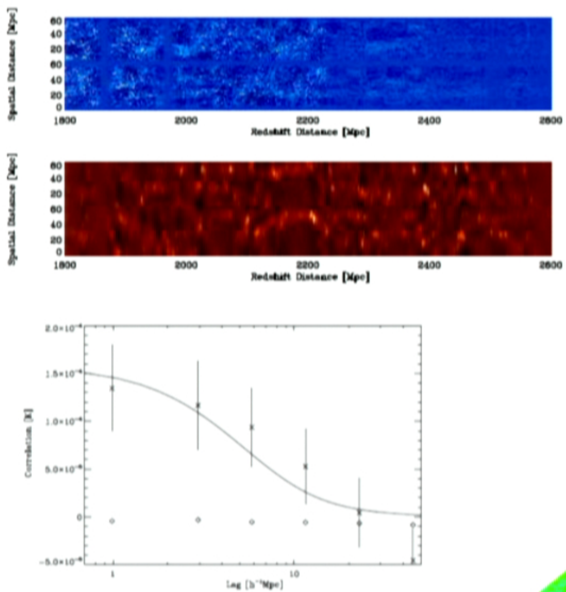
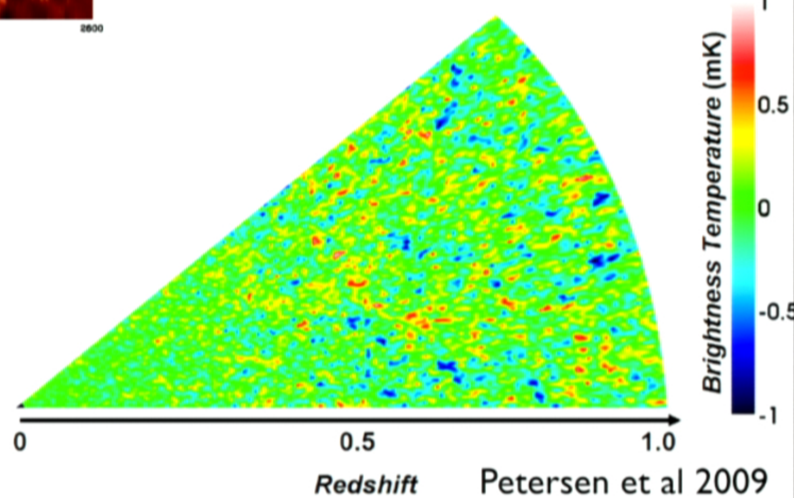
Radio surveys and total intensity mapping



Chang et al 2010



Petersen et al 2009

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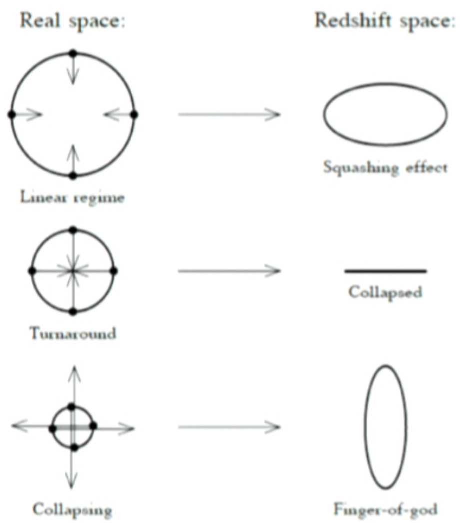
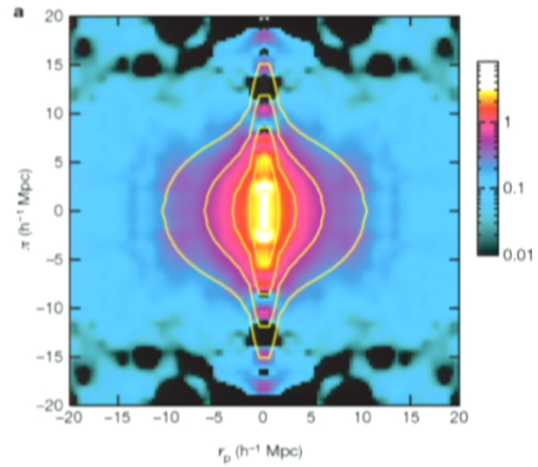
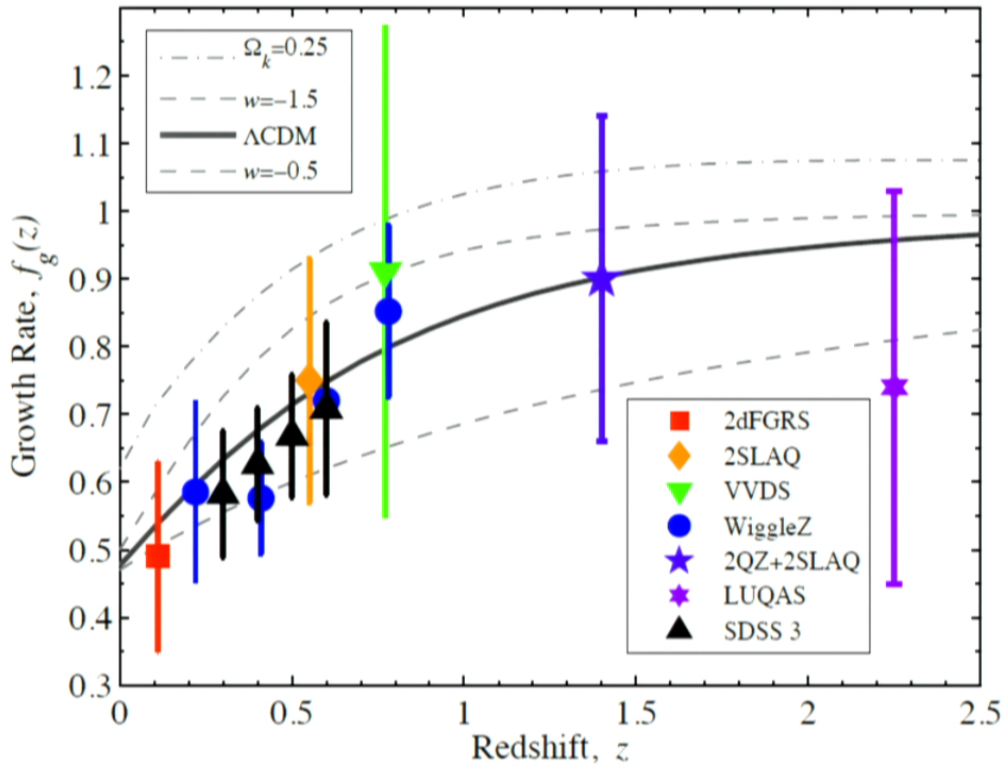
Redshift Space Distortions





Guzzo et al 2008



Ω_k=0.25
w=−1.5
ΛCDM
w=−0.5
Growth Rate, $f_g(z)$
1.2
1.1
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0
0.5
1
1.5
2
2.5
Redshift, z
2dFGRS
2SLAQ
VVDS
WiggleZ
2QZ+2SLAQ
LUQAS
SDSS 3

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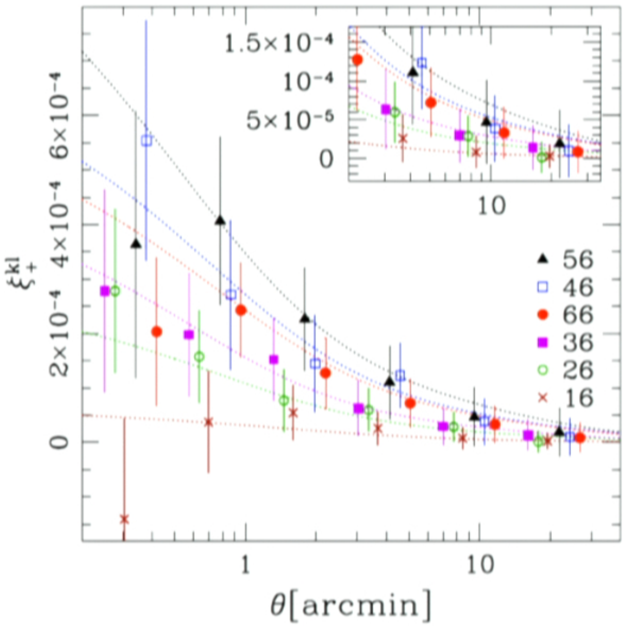
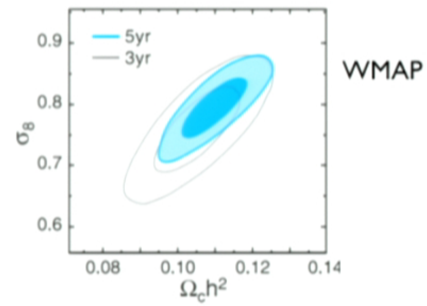
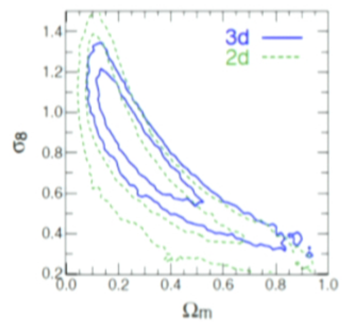
Weak Lensing: state of play

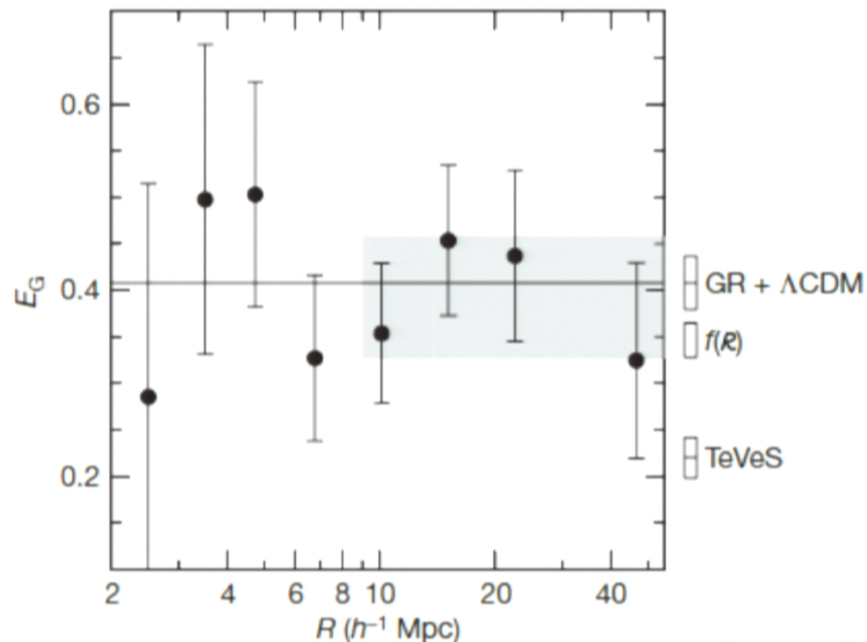






COSMOS: Schrabback et al 2010

Cross correlating data sets



Reyes et al 2010

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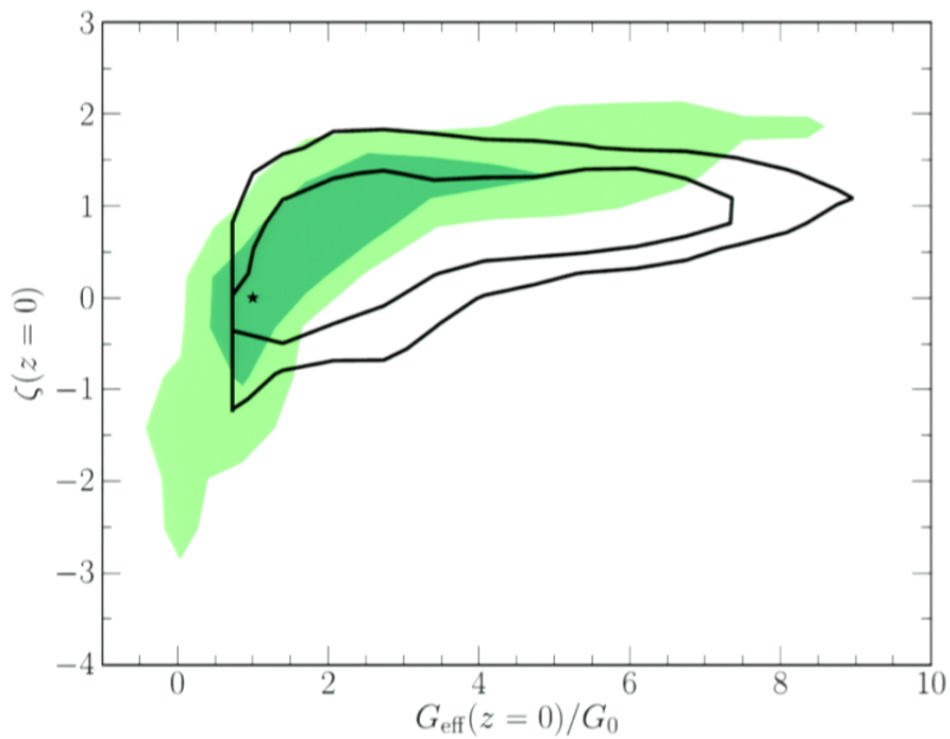
The State of Play



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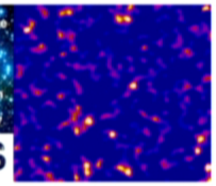
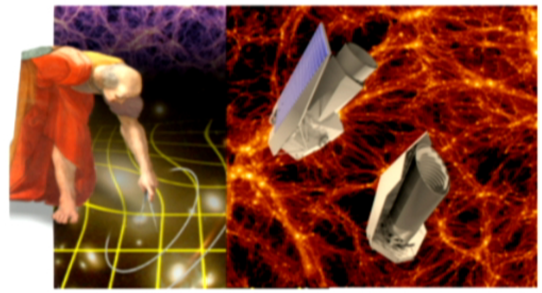
The Future

Soon: ground-based galaxy weak lensing.

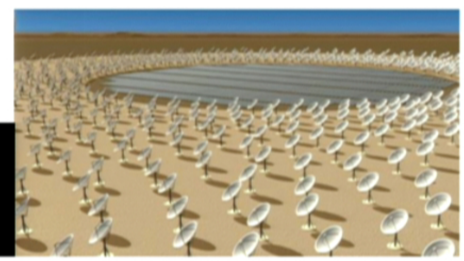


Soon-ish (2018): Euclid mission. Space-based weak lensing and redshift-space distortions.



The future? The Square Kilometer Array (2020 onwards). An almighty survey of radio galaxies to high z $\Rightarrow$ RSDs,

peculiar velocities, BAO; also continuum mapping.



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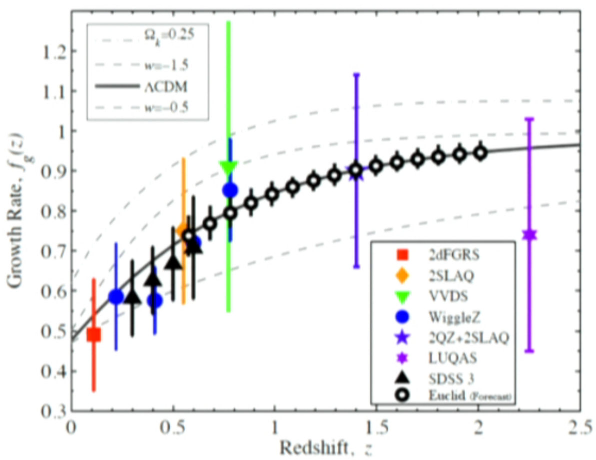
The Future



Growth Rate, $f_g(z)$
Redshift, z
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w=-1.5
ΛCDM
w=-0.5
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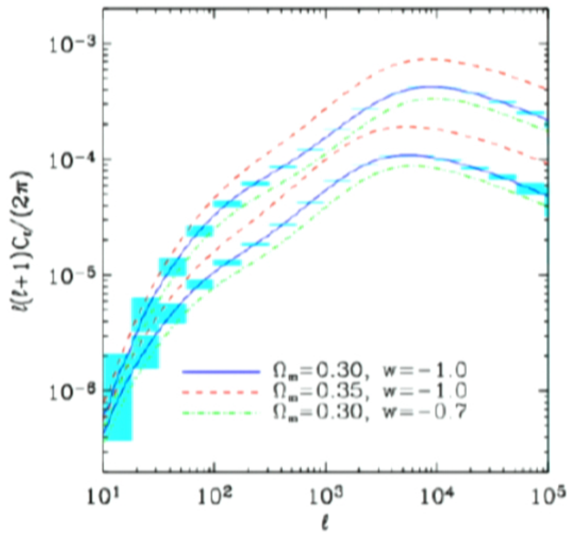
2SLAQ
VVDS
WiggleZ
2QZ+2SLAQ
LUQAS
SDSS 3
Euclid (Forecast)



$\ell(\ell+1)C_\ell/(2\pi)$
ℓ
Ω_m=0.30, w=−1.0
Ω_m=0.35, w=−1.0
Ω_m=0.30, w=−0.7

Summary

- The large scale structure of the Universe can be used to test gravity.
- There is a immense landscape of gravitational theories.
- We need a unified framework- “PPF”- for constraining such theories.
- We need to focus on linear scales (for now).
- Signatures on large scales and growth rate.
- There are a plethora of new experiments to look forward to.

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Collaborators

- Tessa Baker (Oxford)
- Constantinos Skordis (Nottingham)
- Mario Santos (IST/Lisbon)
- Joseph Zuntz (Oxford/UCL)