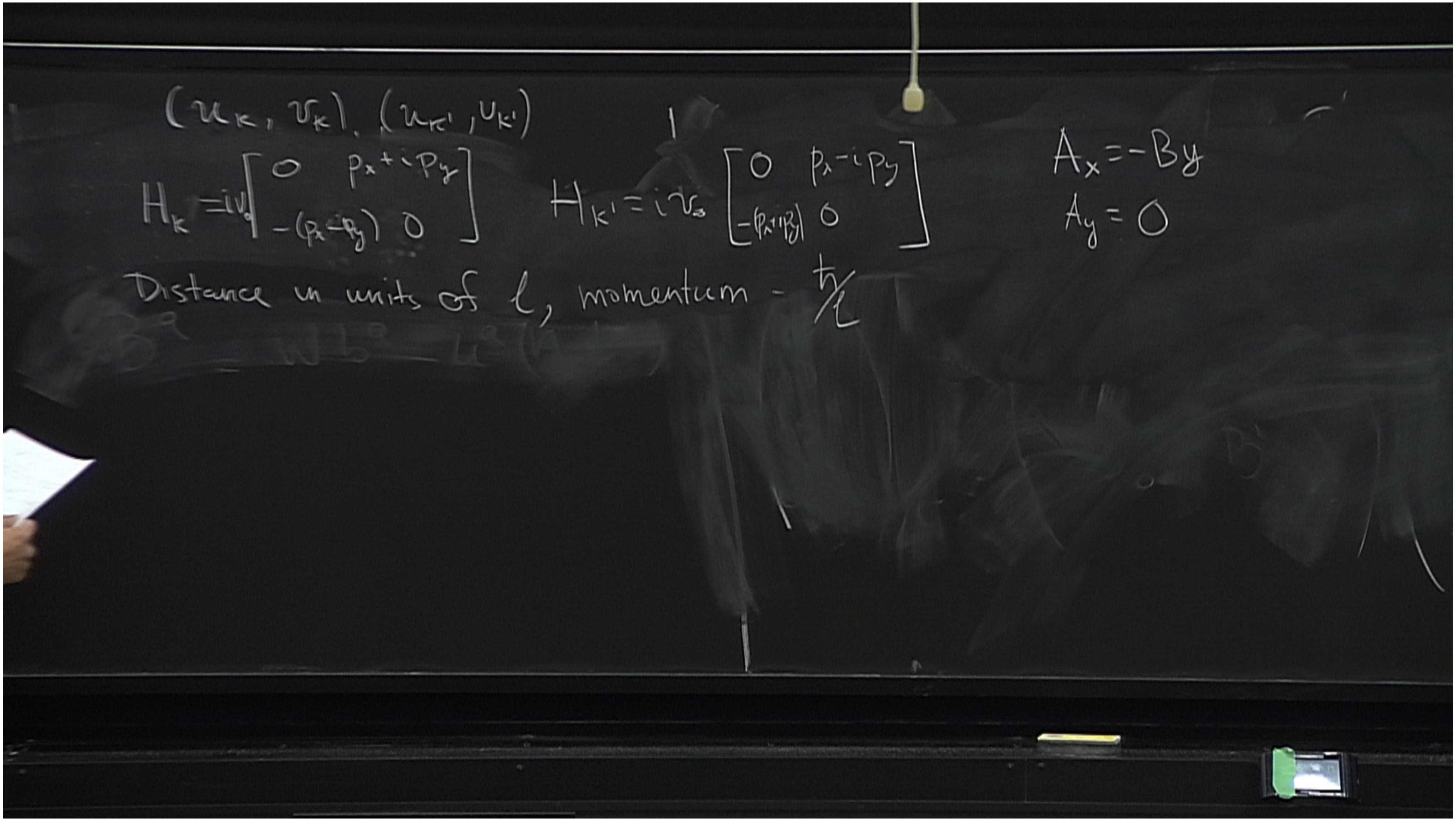
Title: Explorations in Condensed Matter - Lecture 8 B

Date: Apr 12, 2012 02:20 PM

URL: http://www.pirsa.org/12040118

Abstract:


$(u_K, v_K), (u_{K'}, u_{K'})$
$H_K = iv_0 \begin{bmatrix} 0 & p_x + ip_y \\ -(p_x - ip_y) & 0 \end{bmatrix}$
$H_{K'} = iv_0 \begin{bmatrix} 0 & p_x - ip_y \\ -(p_x + ip_y) & 0 \end{bmatrix}$
$A_x = -By$
$A_y = 0$
Distance in units of ℓ, momentum - $\hbar/\ell$

$(u_K, v_K),\ (u_{K'}, v_{K'})$

$$H_K = i v_0 \begin{bmatrix} 0 & p_x + i p_y \\ -(p_x - i p_y) & 0 \end{bmatrix} \qquad H_{K'} = i v_0 \begin{bmatrix} 0 & p_x - i p_y \\ -(p_x + i p_y) & 0 \end{bmatrix}$$

$A_x = -By$

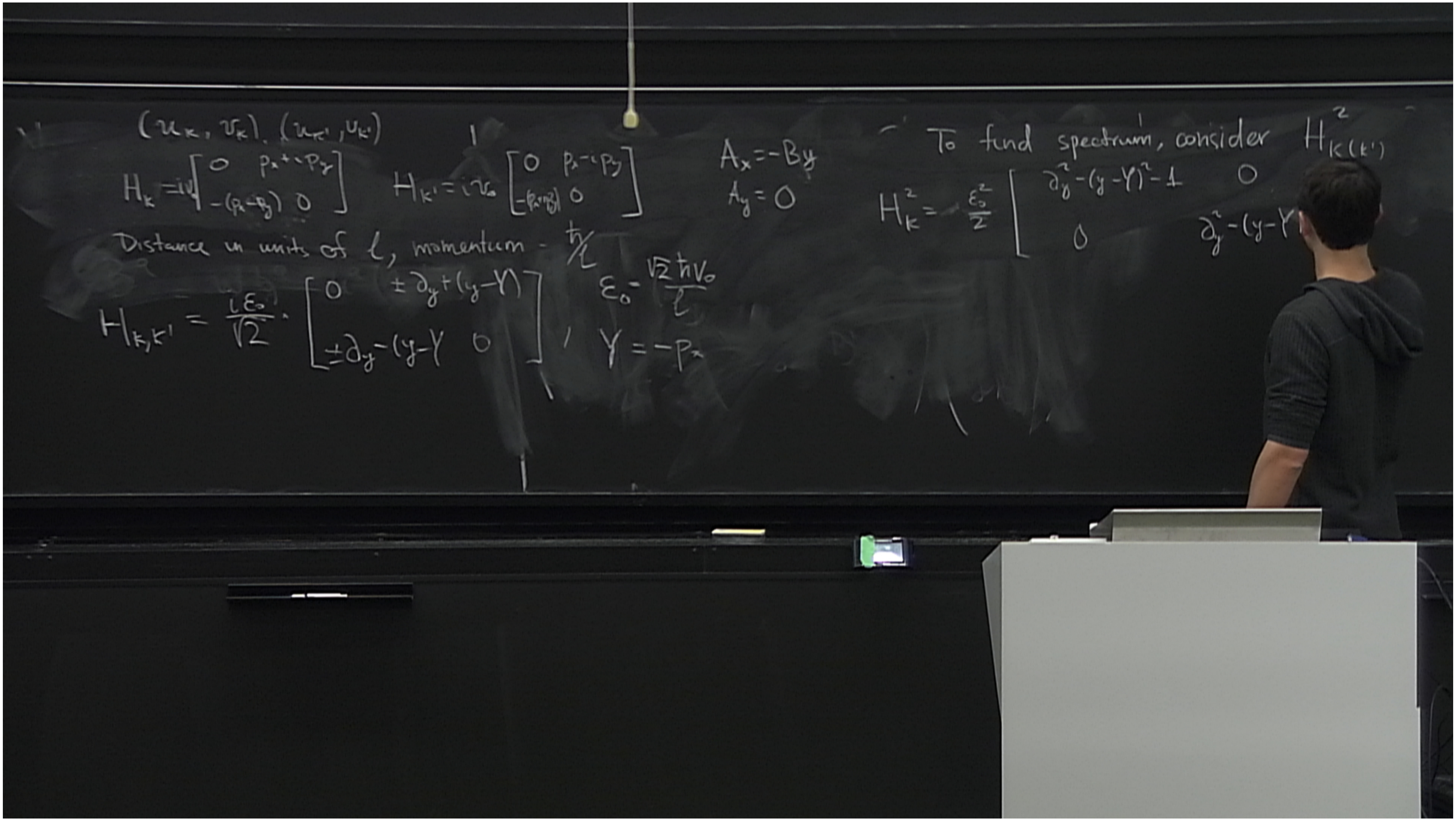
$A_y = 0$

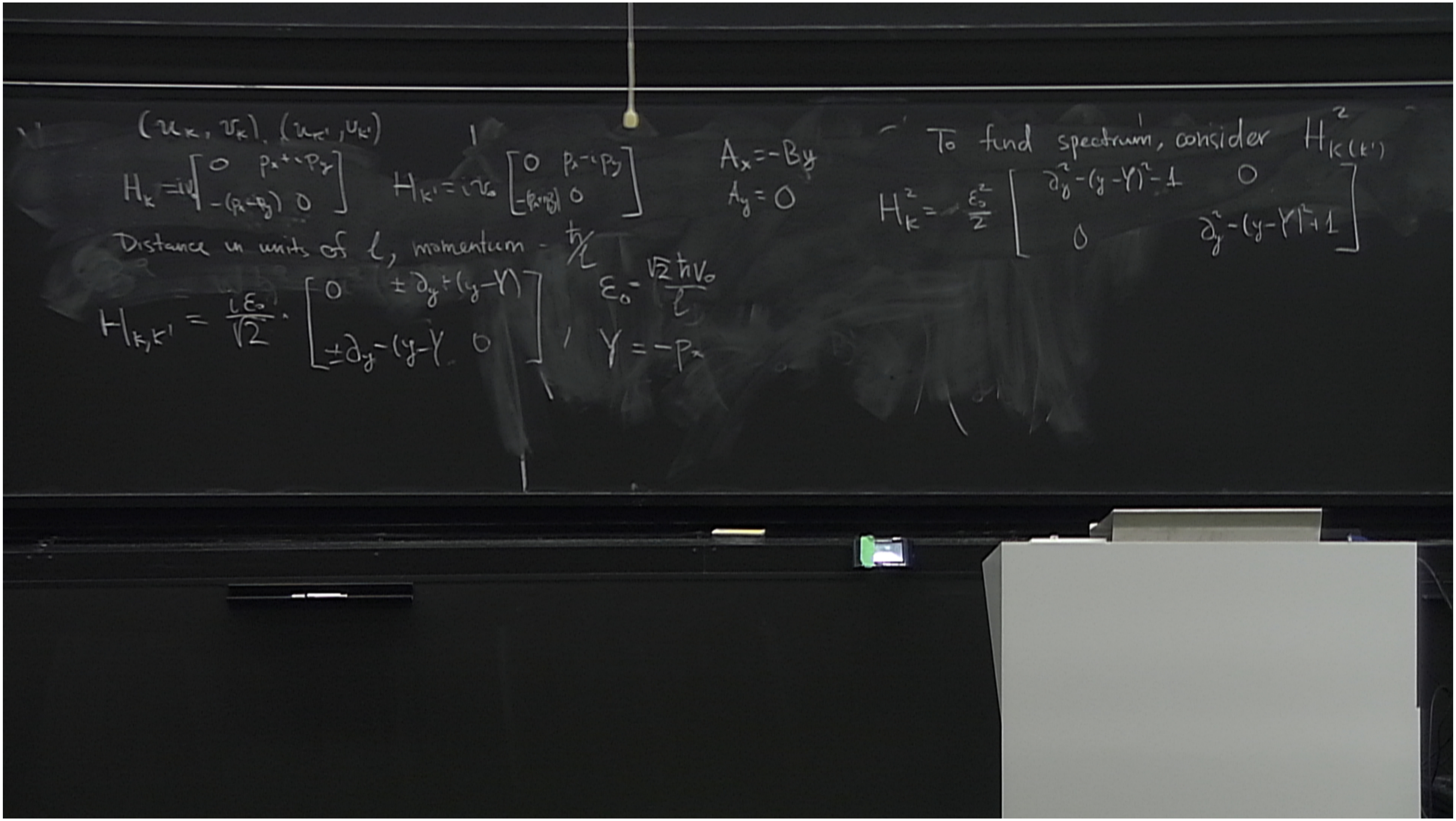
Distance in units of ℓ, momentum - $\hbar/\ell$

$$H_{K,K'} = \frac{i\varepsilon_0}{\sqrt{2}} \begin{bmatrix} 0 & \pm\partial_y + (y - Y) \\ \pm\partial_y - (y - Y) & 0 \end{bmatrix}$$

$\varepsilon_0 = \frac{\sqrt{2}\hbar v_0}{\ell}$

$Y = -p_x$


$(u_K, v_K), (u_{K'}, v_{K'})$
$H_K = iv\begin{bmatrix} 0 & p_x + ip_y \\ -(p_x - ip_y) & 0 \end{bmatrix}$
$H_{K'} = iv_0\begin{bmatrix} 0 & p_x - ip_y \\ -(p_x + ip_y) & 0 \end{bmatrix}$
$A_x = -By$
$A_y = 0$
Distance in units of ℓ, momentum - $\hbar/\ell$
$H_{K,K'} = \frac{i\varepsilon_0}{\sqrt{2}} \cdot \begin{bmatrix} 0 & \pm\partial_y + (y-Y) \\ \pm\partial_y - (y-Y) & 0 \end{bmatrix}$
$\varepsilon_0 = \frac{\sqrt{2}\hbar v_0}{\ell}$
$Y = -p_x$
To find spectrum, consider $H^2_{K(K')}$
$H^2_K = \frac{\varepsilon_0^2}{2}\begin{bmatrix} \partial_y^2 - (y-Y)^2 - 1 & 0 \\ 0 & \partial_y^2 - (y-Y \end{bmatrix}$


(u_K, v_K), (u_K', v_K')
H_K = iv [0 p_x + ip_y; -(p_x - ip_y) 0]
H_K' = iv_0 [0 p_x - ip_y; -(p_x + ip_y) 0]
A_x = -By
A_y = 0
To find spectrum, consider H^2_K(K')
H^2_K = ε_0^2/2 [∂_y^2 - (y - Y)^2 - 1 0; 0 ∂_y^2 - (y - Y)^2 + 1]
Distance in units of l, momentum - ħ/l
H_K,K' = iε_0/√2 [0 ±∂_y + (y - Y); ±∂_y - (y - Y) 0]
ε_0 = √2 ħv_0 / l
Y = -p_x

(u_K, v_K), $(u_{K'}, v_{K'})$

$$H_K = iv \begin{bmatrix} 0 & p_x + i p_y \\ -(p_x - i p_y) & 0 \end{bmatrix} \quad H_{K'} = i v_0 \begin{bmatrix} 0 & p_x - i p_y \\ -(p_x + i p_y) & 0 \end{bmatrix}$$

$A_x = -By$

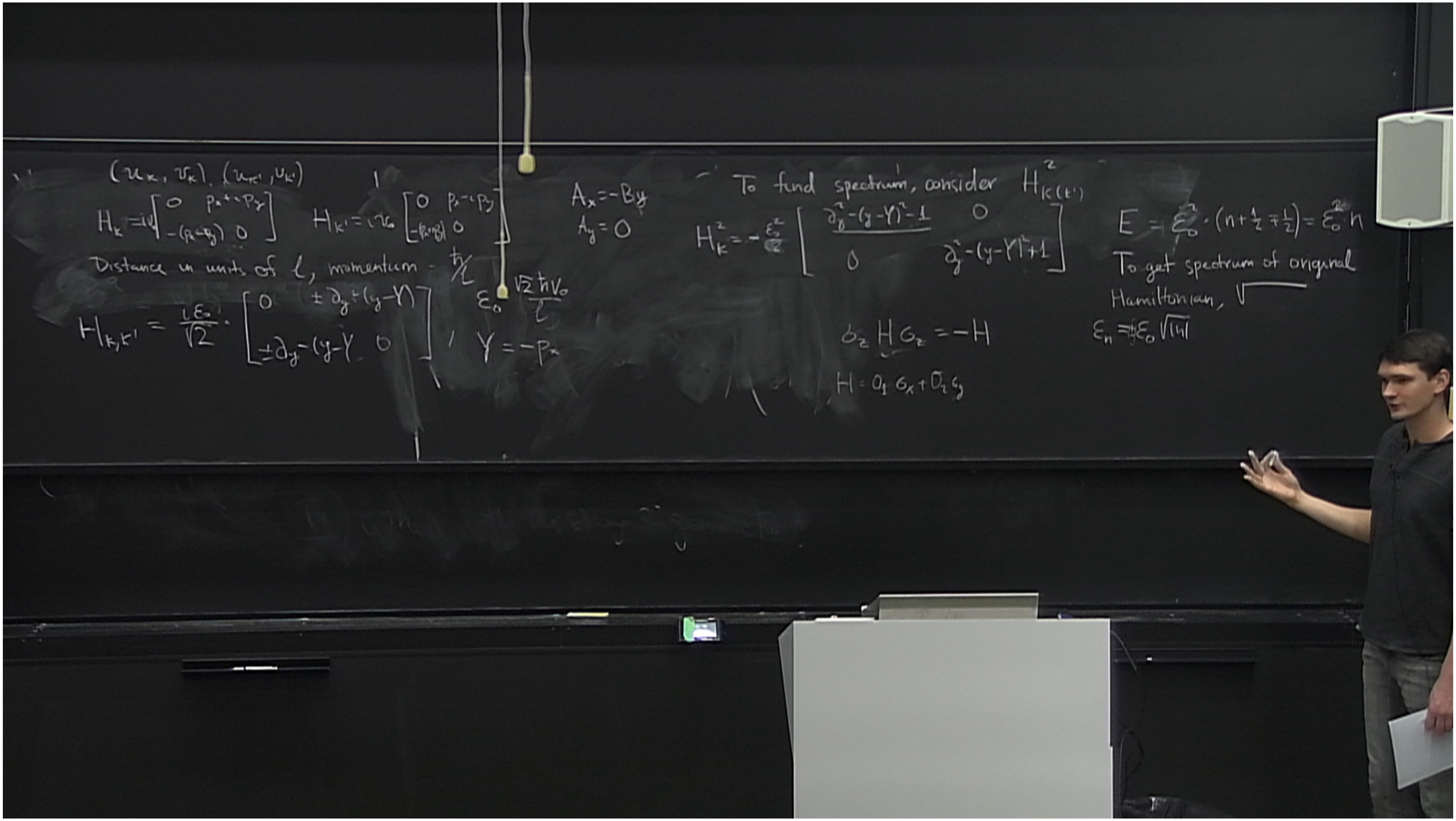
$A_y = 0$

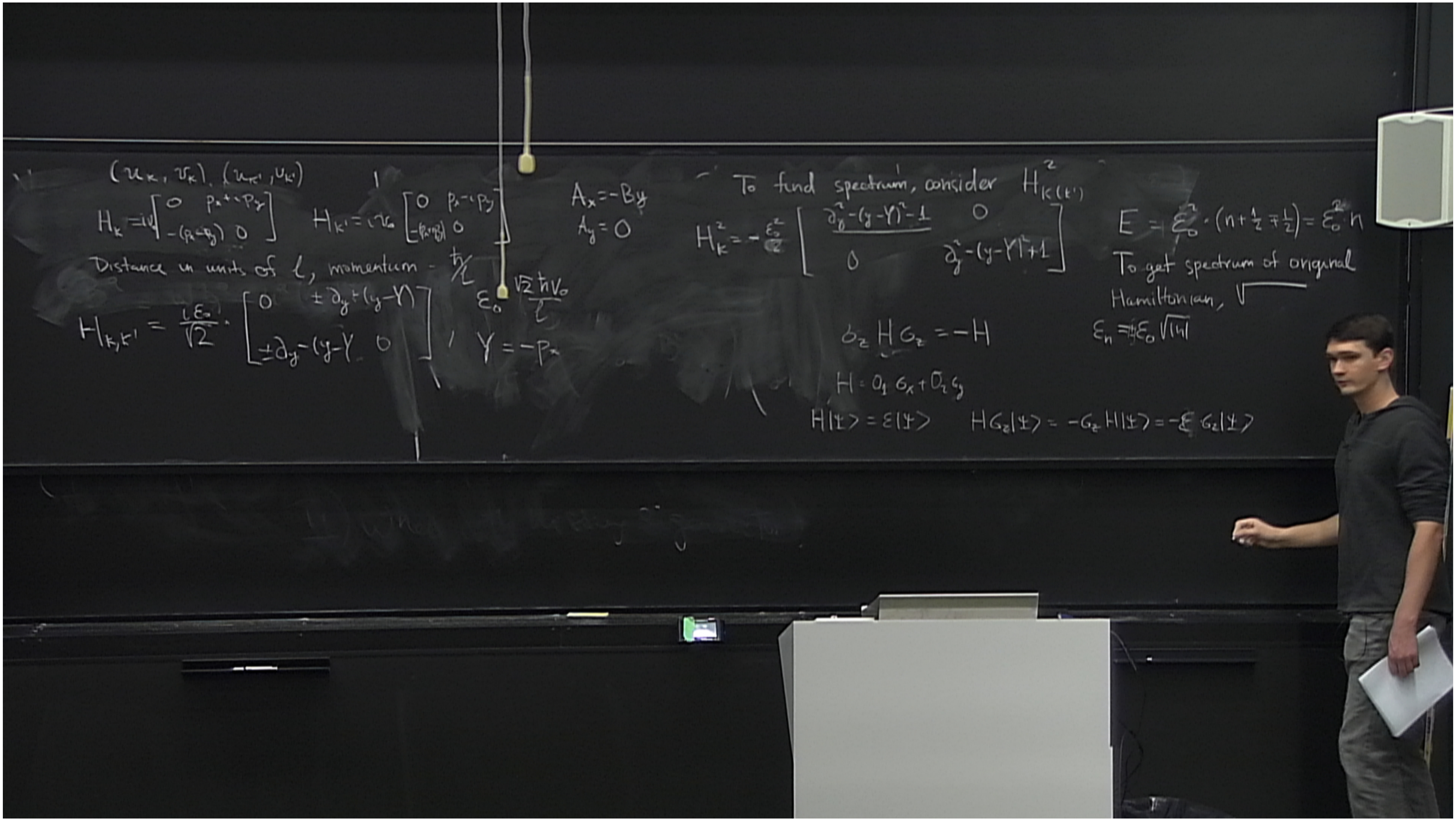
Distance in units of ℓ, momentum - $\hbar/\ell$

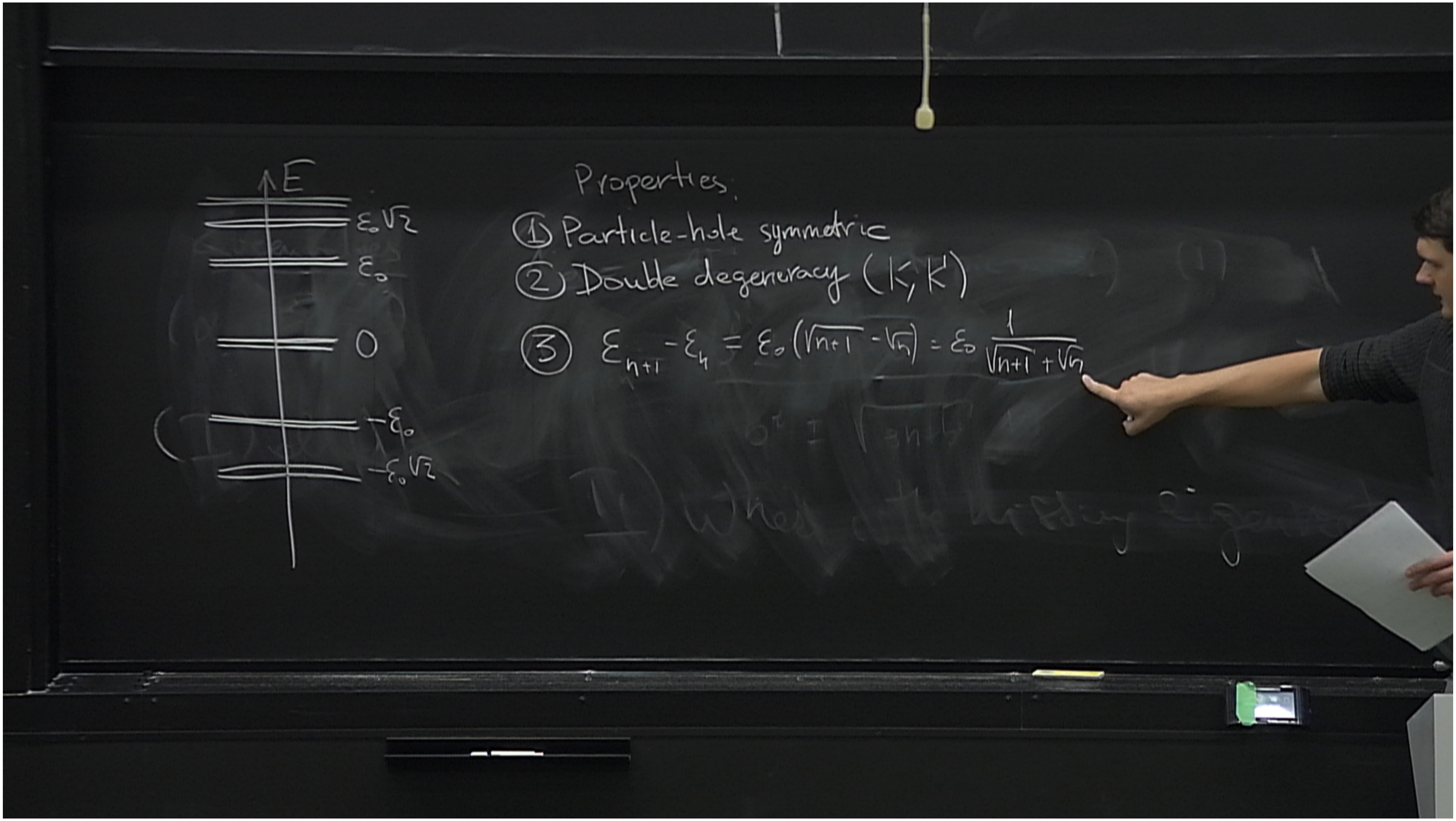
$$H_{K,K'} = \frac{i\varepsilon_0}{\sqrt{2}} \begin{bmatrix} 0 & \pm\partial_y + (y-Y) \\ \pm\partial_y - (y-Y) & 0 \end{bmatrix}, \quad \varepsilon_0 = \frac{\sqrt{2}\hbar v_0}{\ell}, \quad Y = -p_x$$

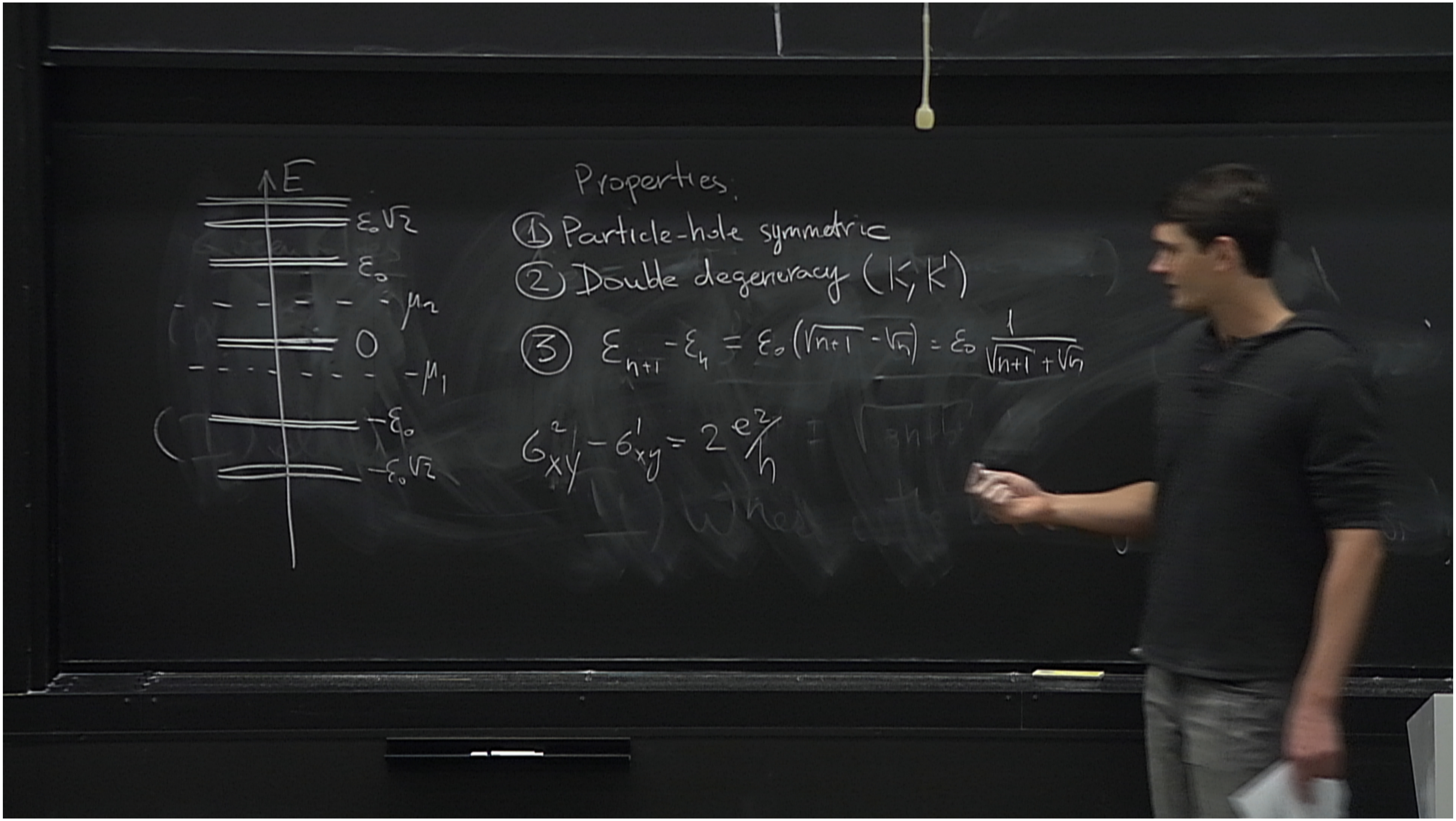
To find spectrum, consider $H^2_{K(K')}$

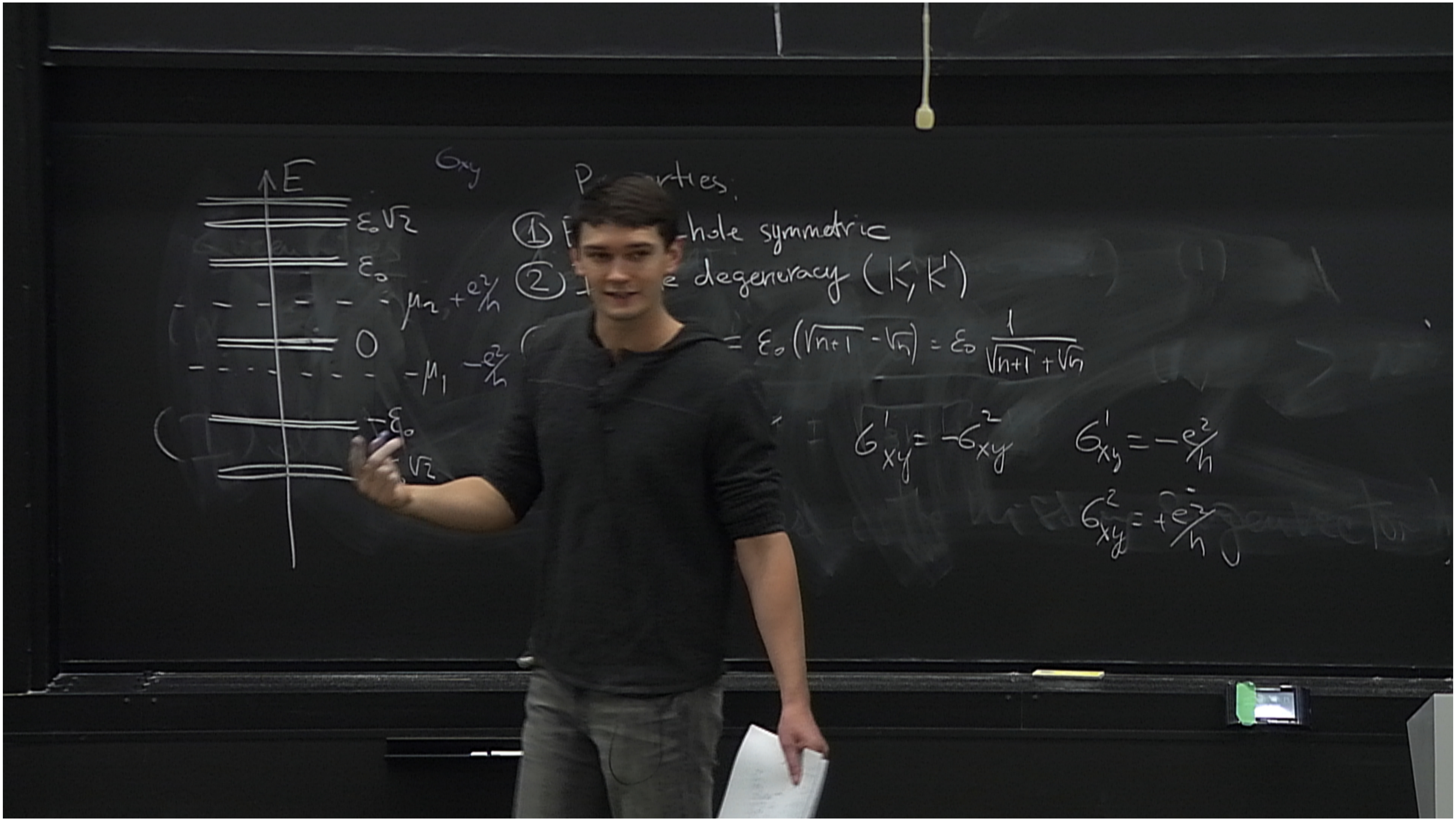
$$H_K^2 = -\frac{\varepsilon_0^2}{2} \begin{bmatrix} \partial_y^2 - (y-Y)^2 - 1 & 0 \\ 0 & \partial_y^2 - (y-Y)^2 + 1 \end{bmatrix}$$


To find spectrum, consider
Distance in units of ℓ, momentum
To get spectrum of original
Hamiltonian,
A_x = -By
A_y = 0

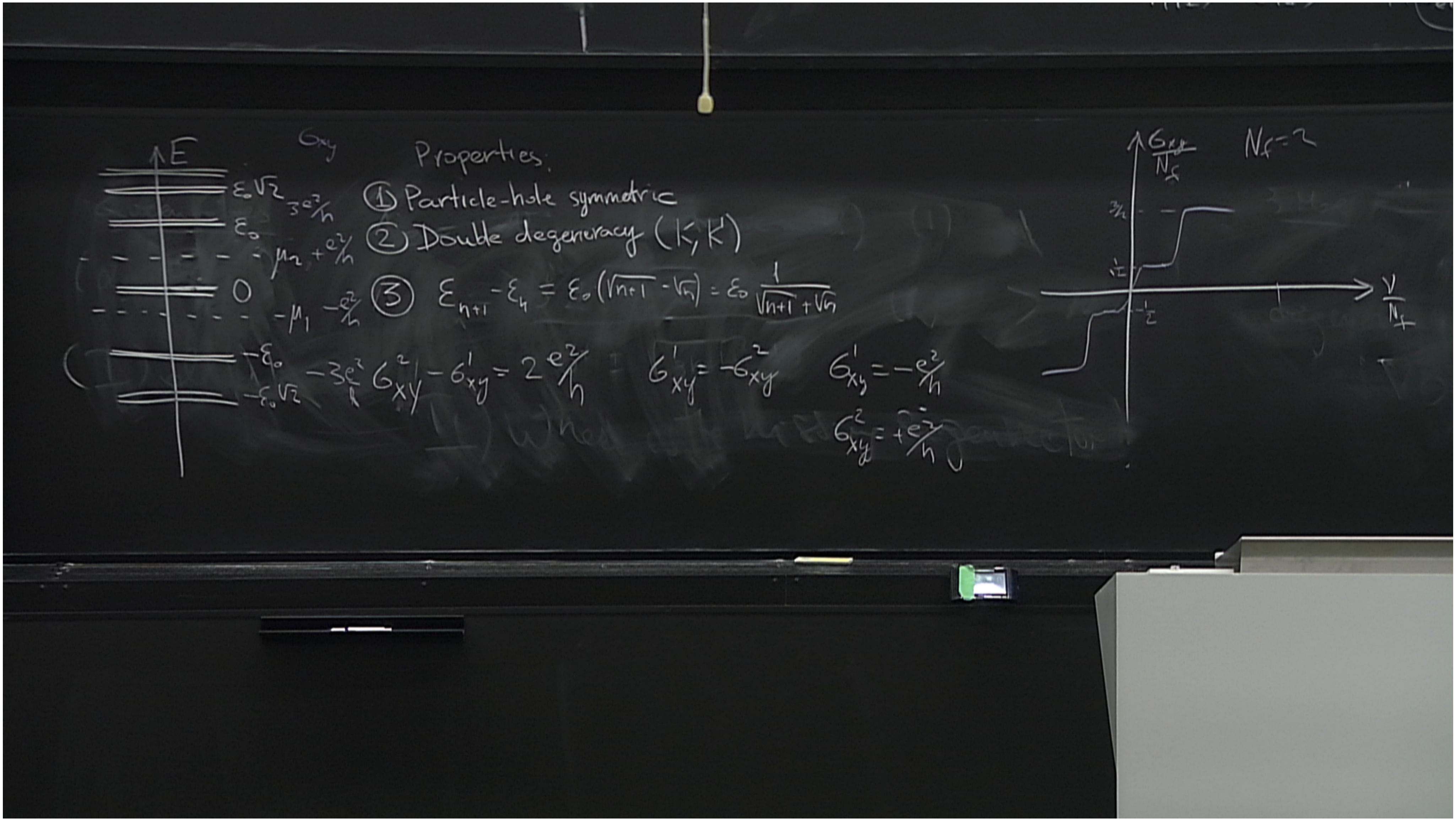

To find spectrum, consider
Distance in units of ℓ, momentum
To get spectrum of original
Hamiltonian,

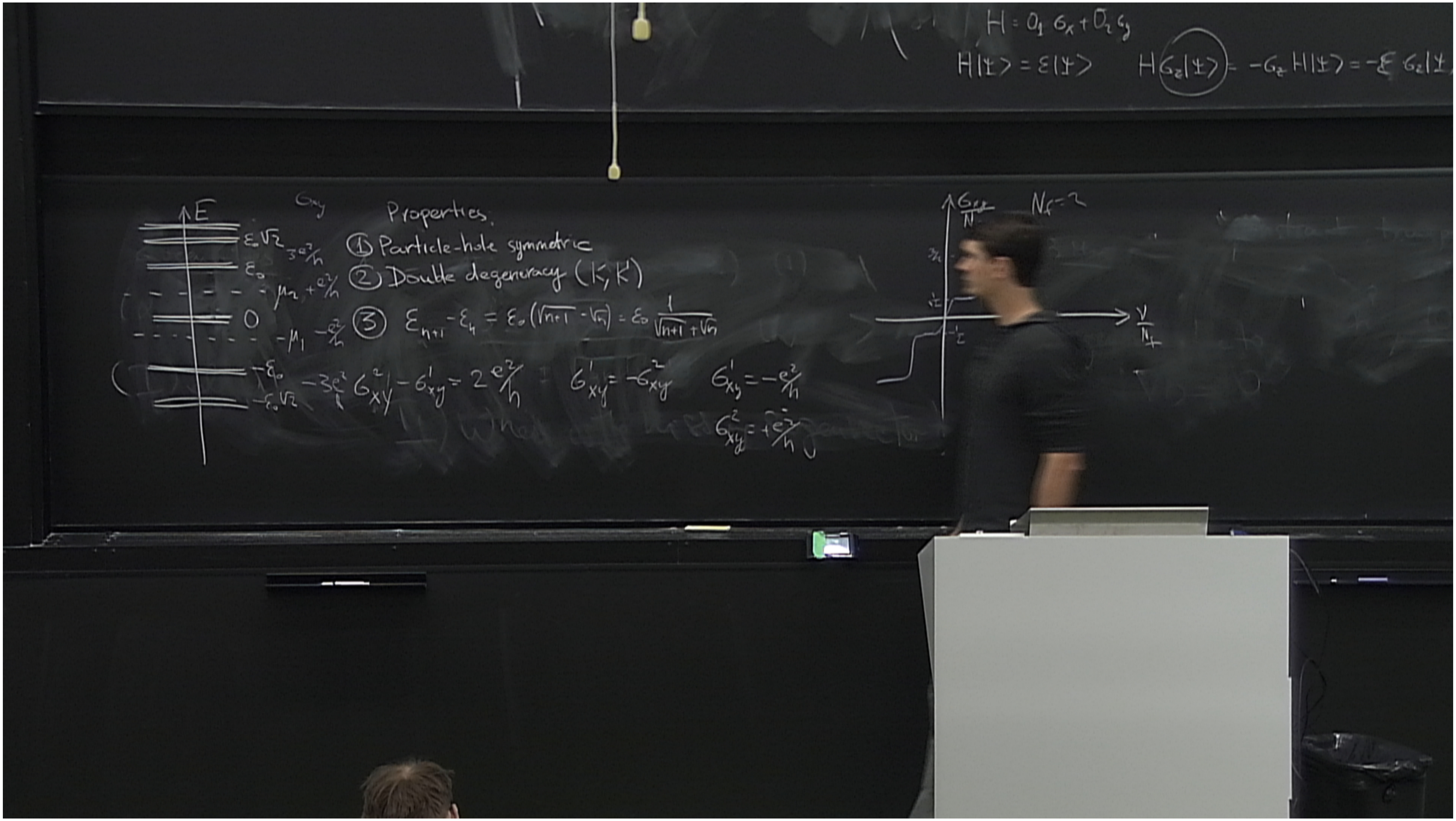

E
$\varepsilon_0\sqrt{2}$
ε_0
0
$-\varepsilon_0$
$-\varepsilon_0\sqrt{2}$
Properties:
(1) Particle-hole symmetric
(2) Double degeneracy (K, K')
(3) $\varepsilon_{n+1}-\varepsilon_n = \varepsilon_0(\sqrt{n+1}-\sqrt{n}) = \varepsilon_0 \frac{1}{\sqrt{n+1}+\sqrt{n}}$


E
$\varepsilon_0\sqrt{2}$
ε_0
μ_2
0
$-\mu_1$
$-\varepsilon_0$
$-\varepsilon_0\sqrt{2}$
Properties:
① Particle-hole symmetric
② Double degeneracy (K, K')
③ $\varepsilon_{n+1} - \varepsilon_n = \varepsilon_0(\sqrt{n+1} - \sqrt{n}) = \varepsilon_0 \frac{1}{\sqrt{n+1}+\sqrt{n}}$
$\sigma^2_{xy} - \sigma^1_{xy} = 2\, e^2/h$

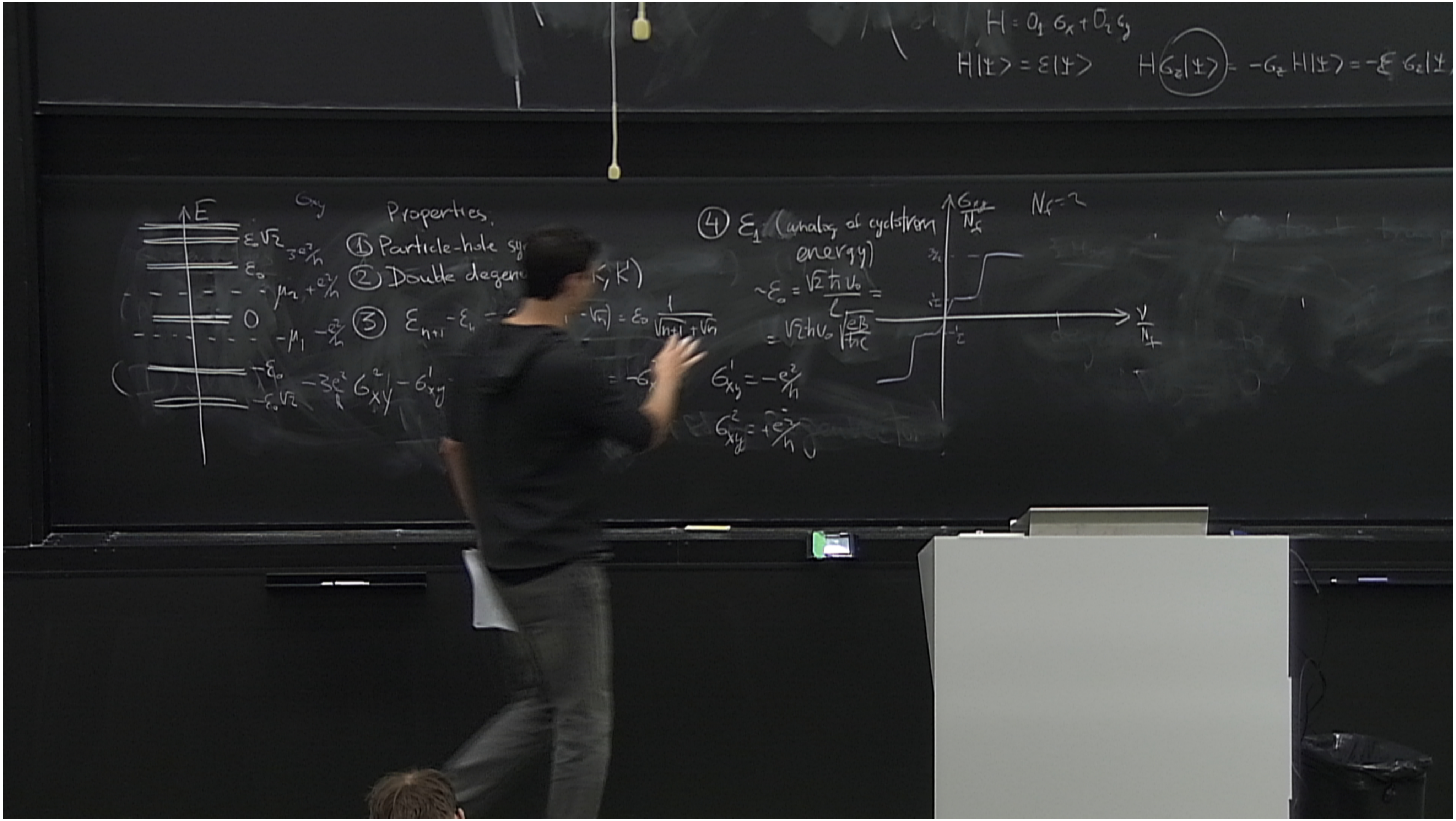


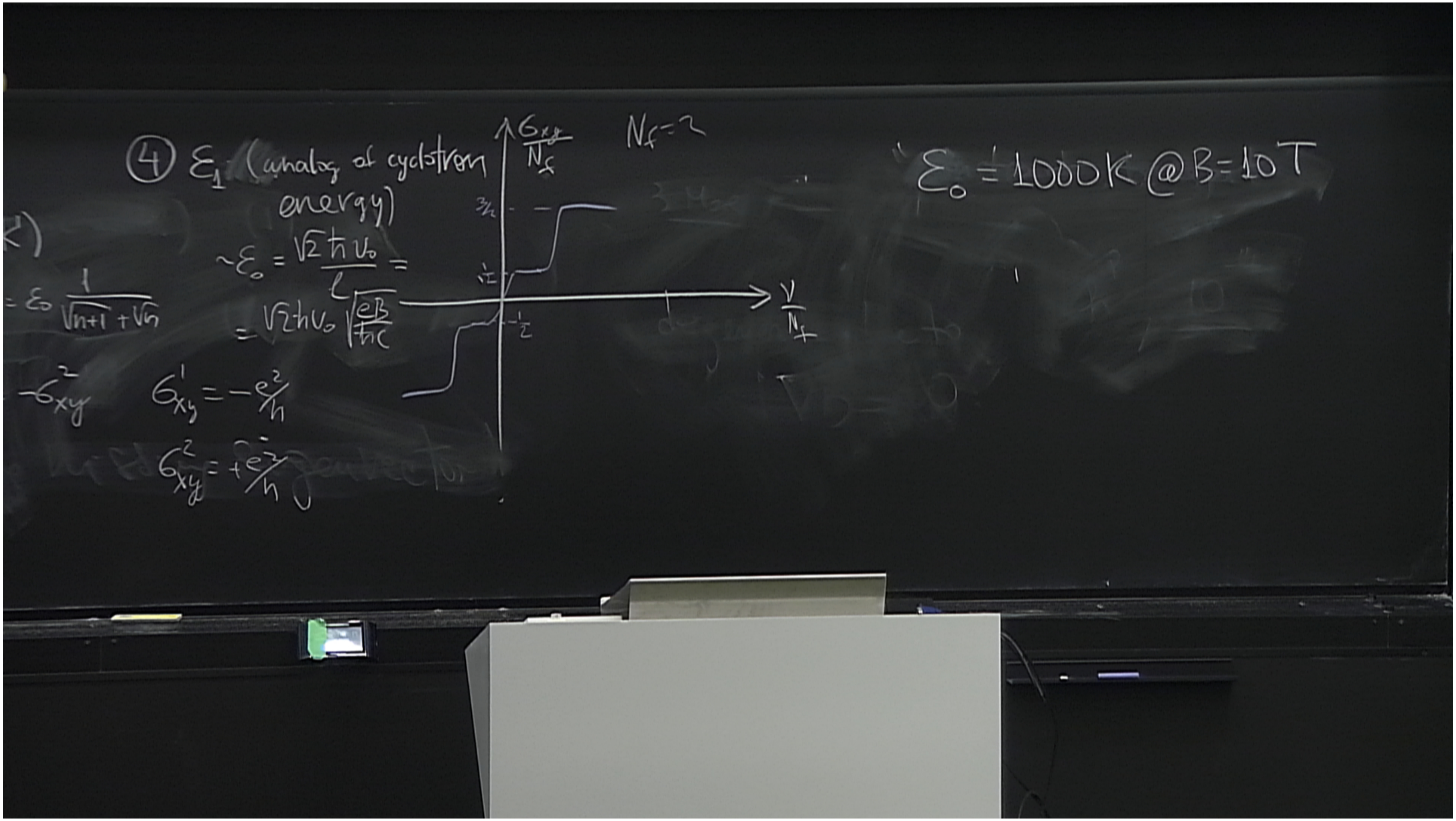
E
$\varepsilon_0\sqrt{2}$
ε_0
$\mu_2, +e^2/h$
0
$-\mu_1 \; -e^2/h$
$-\varepsilon_0$
σ_{xy}
Properties:
① -hole symmetric
② degeneracy (K, K')
$= \varepsilon_0(\sqrt{n+1}-\sqrt{n}) = \varepsilon_0 \frac{1}{\sqrt{n+1}+\sqrt{n}}$
$\sigma^1_{xy} = -\sigma^2_{xy}$
$\sigma^1_{xy} = -e^2/h$
$\sigma^2_{xy} = +e^2/h$

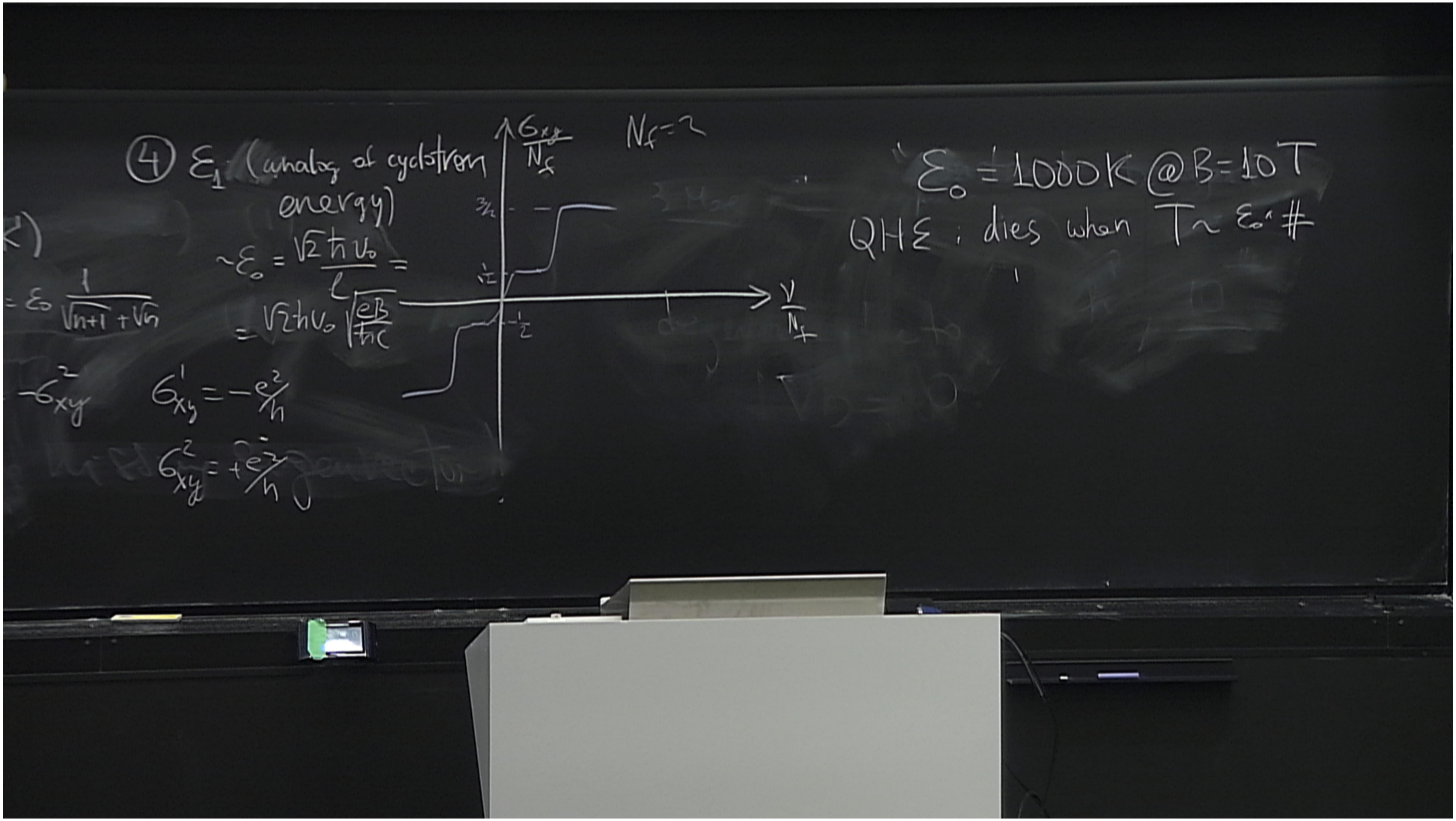

Properties:
① Particle-hole symmetric
② Double degeneracy (K, K')
③ ε_{n+1} − ε_n = ε_0 (√(n+1) − √n) = ε_0 1/(√(n+1) + √n)
N_f = 2

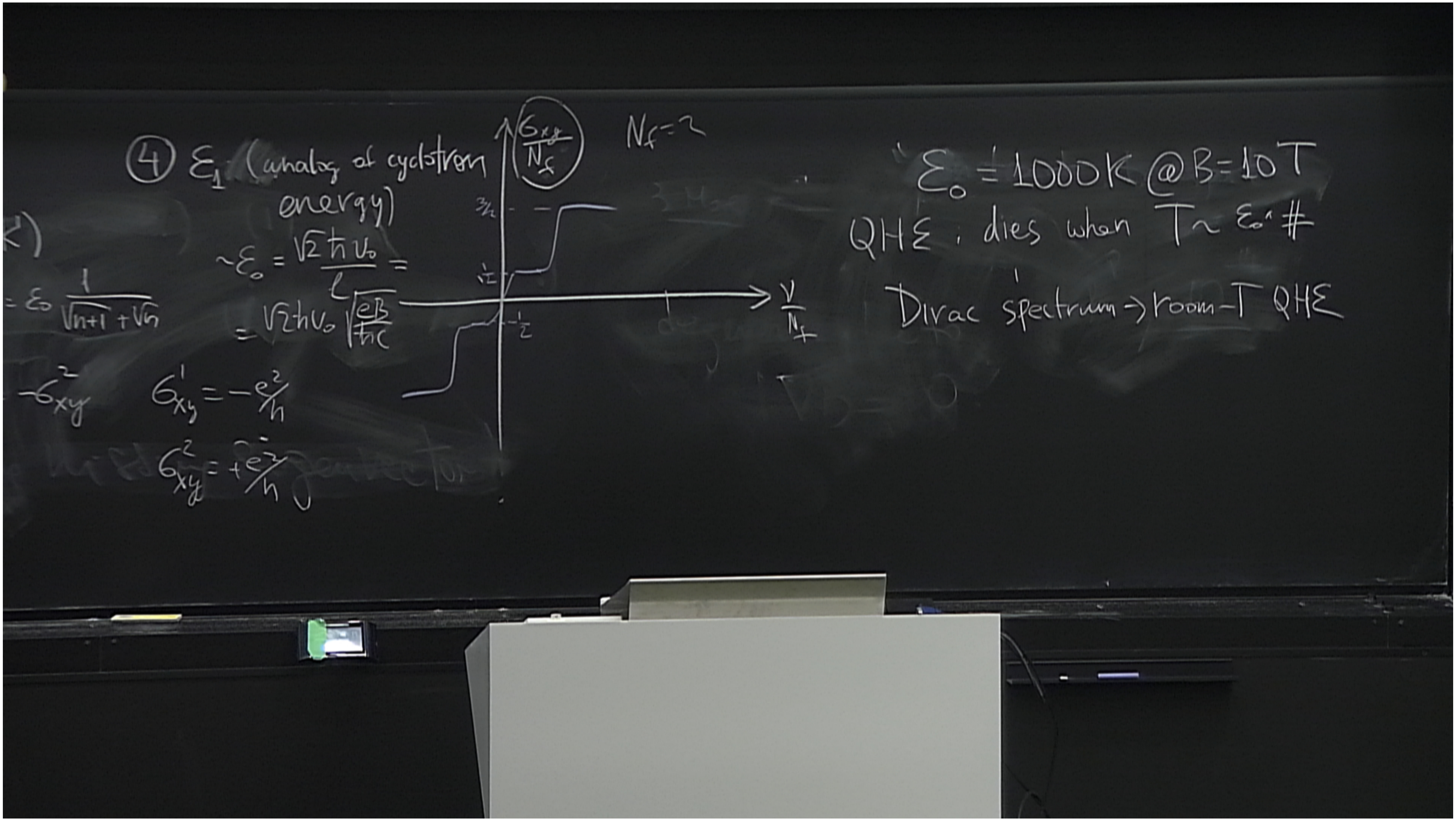


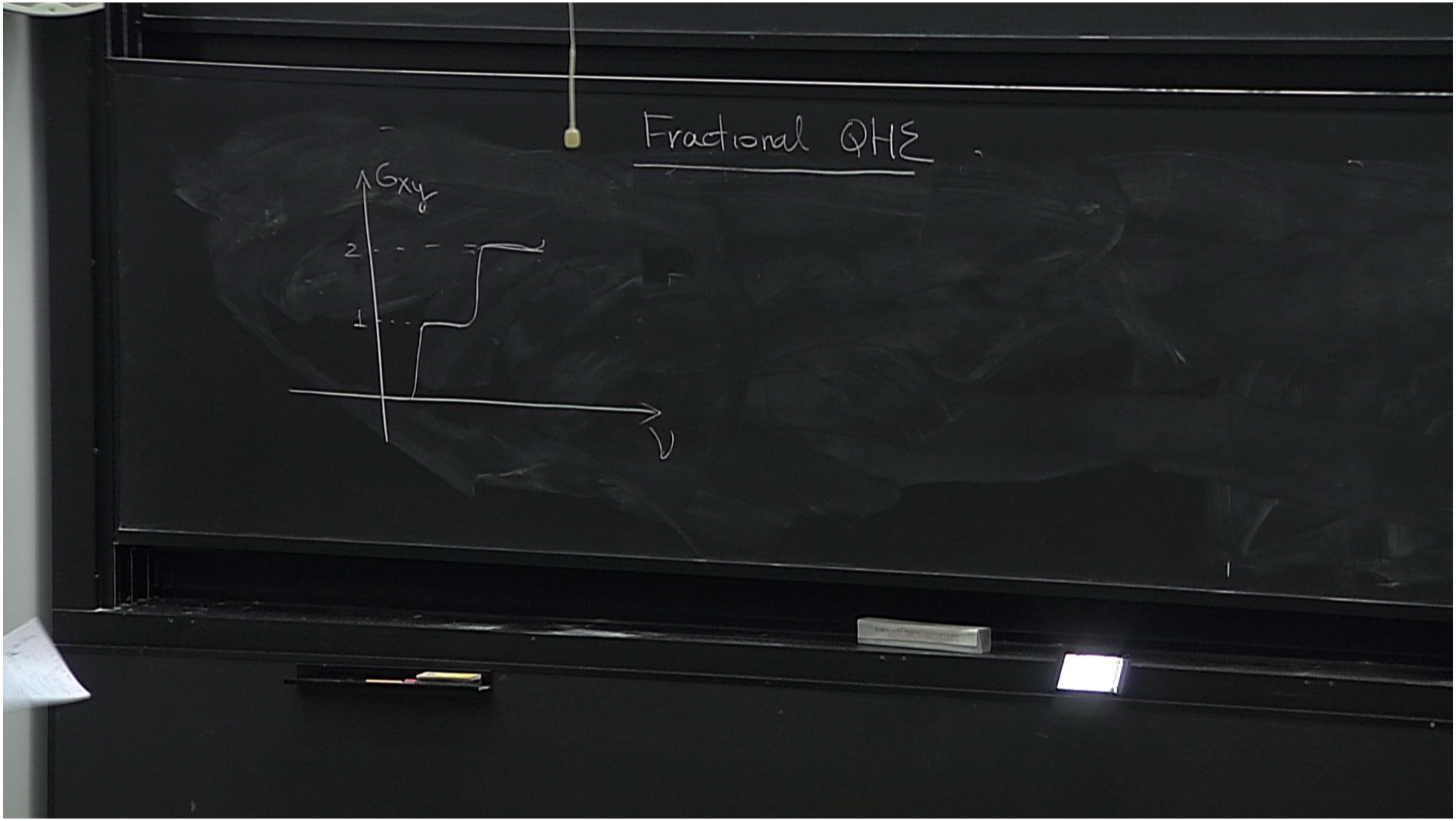
$H = \mathcal{O}_1 \sigma_x + \mathcal{O}_2 \sigma_y$
$H|\Psi\rangle = \varepsilon|\Psi\rangle$
Properties.
① Particle-hole symmetric
② Double degeneracy (K, K')
③ $\varepsilon_{n+1} - \varepsilon_n = \varepsilon_0(\sqrt{n+1} - \sqrt{n}) = \varepsilon_0 \frac{1}{\sqrt{n+1}+\sqrt{n}}$
$\sigma^2_{xy} - \sigma^1_{xy} = 2\, e^2/h$
$\sigma^1_{xy} = -\sigma^2_{xy}$
$\sigma^1_{xy} = -e^2/h$
$\sigma^2_{xy} = +e^2/h$

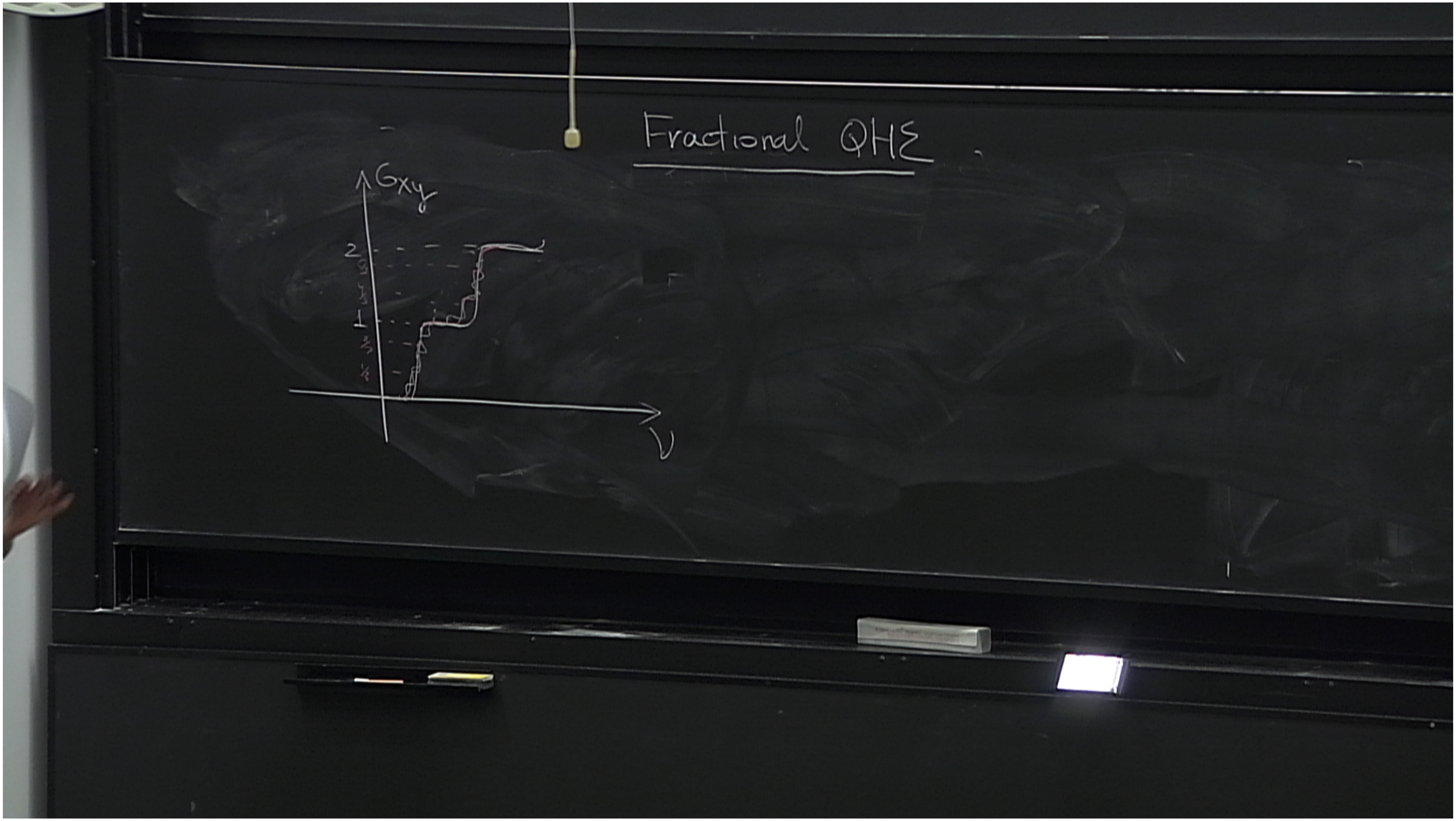

Properties.
energy)
Nf=2

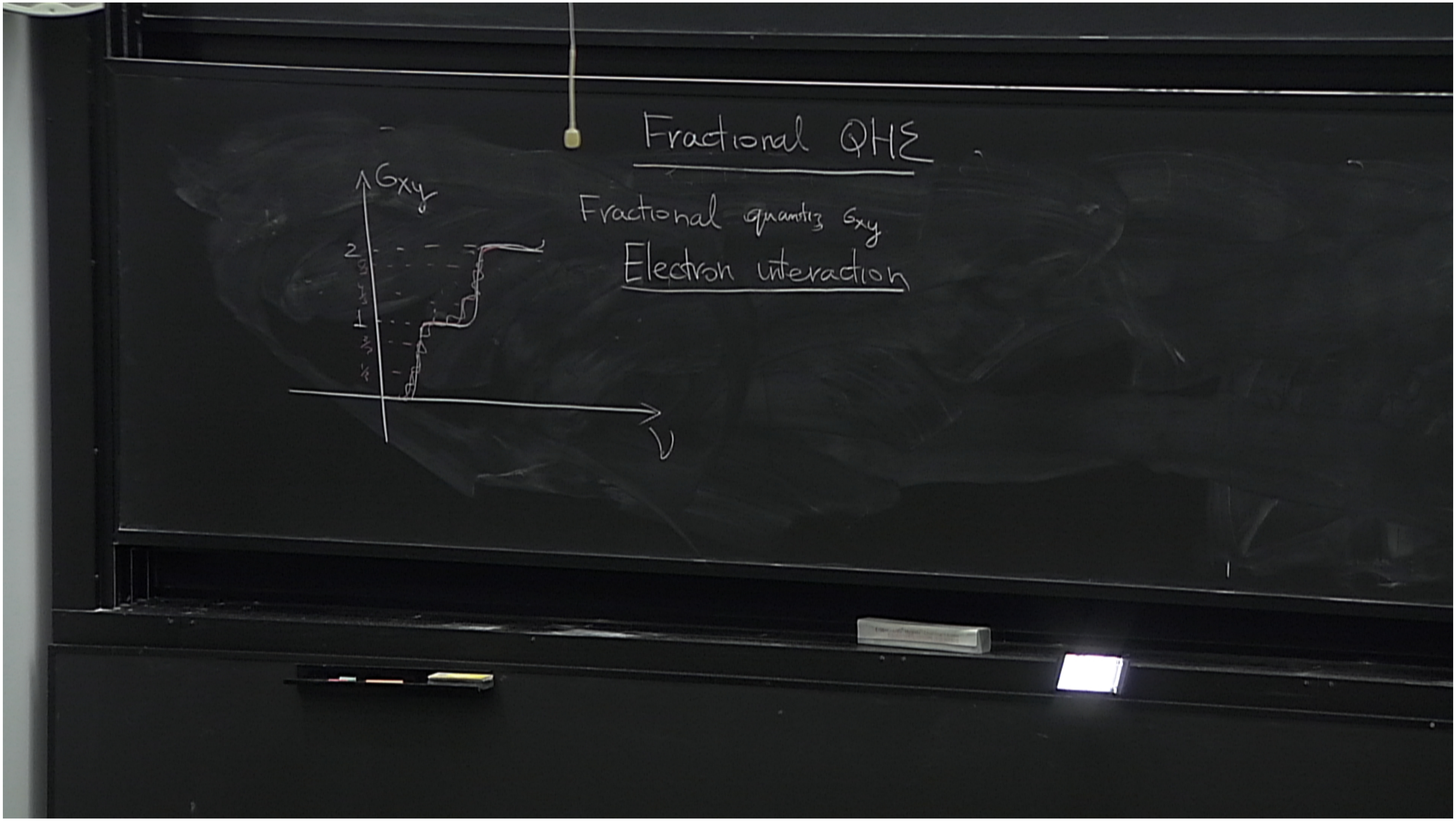

④ E1 (analog of cyclotron energy)
Nf = 2
E0 ≐ 1000K @ B = 10T
~E0 = √2 ħ v0 / l =
= √2 ħ v0 √(eB/ħc)
= E0 1/(√(n+1) + √n)
σ¹xy = −e²/h
σ²xy = +e²/h
σxy/Nf
ν/Nf

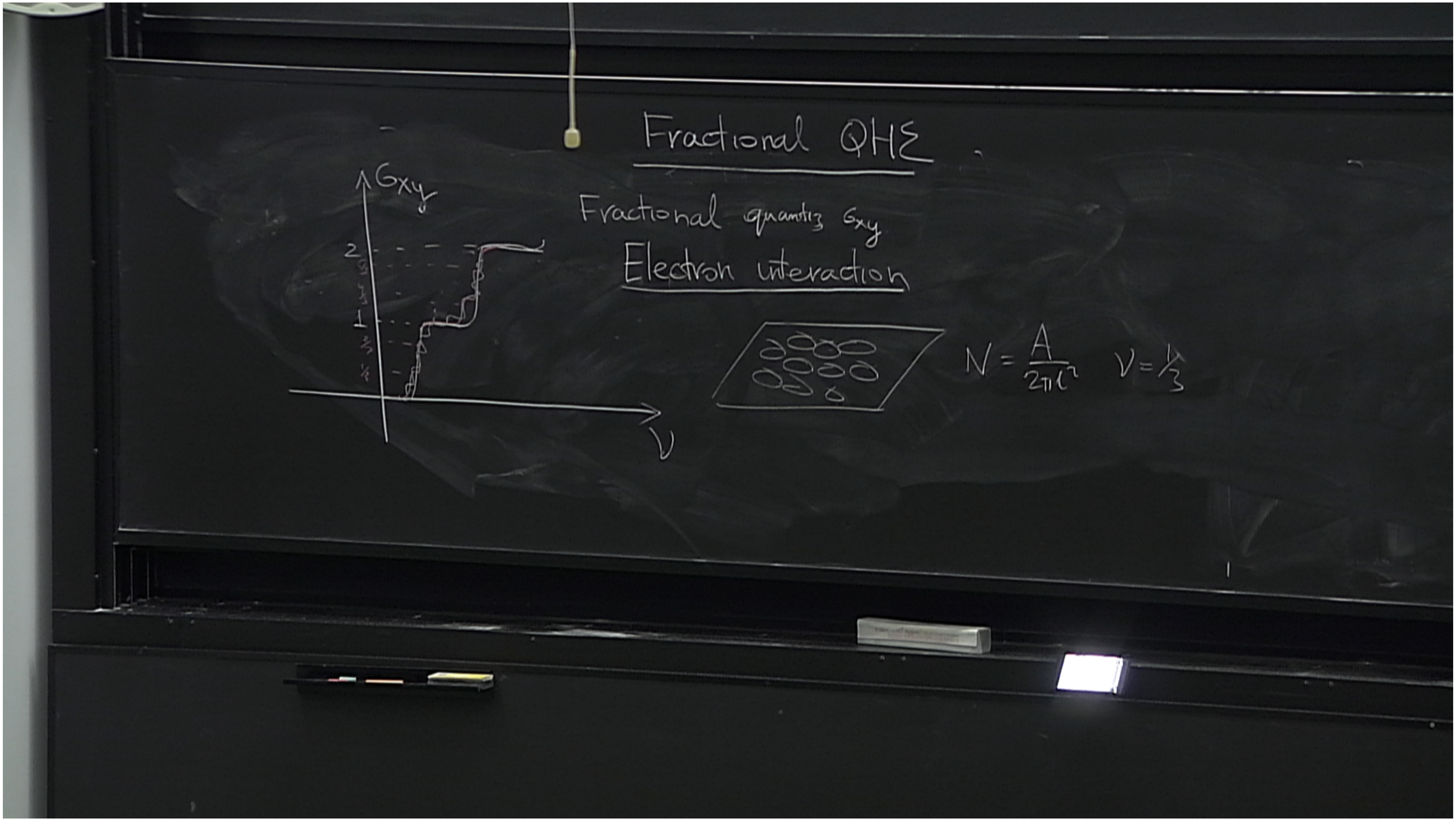

④ ε₁ (analog of cyclotron energy)
~ε₀ = √2 ħ v₀ / ℓ = √2 ħ v₀ √(eB/ħc)
σ¹xy = −e²/h
σ²xy = +e²/h
σxy / Nf
Nf = 2
ν / Nf
ε₀ ≐ 1000K @ B = 10T
QHE : dies when T ~ ε₀ · #

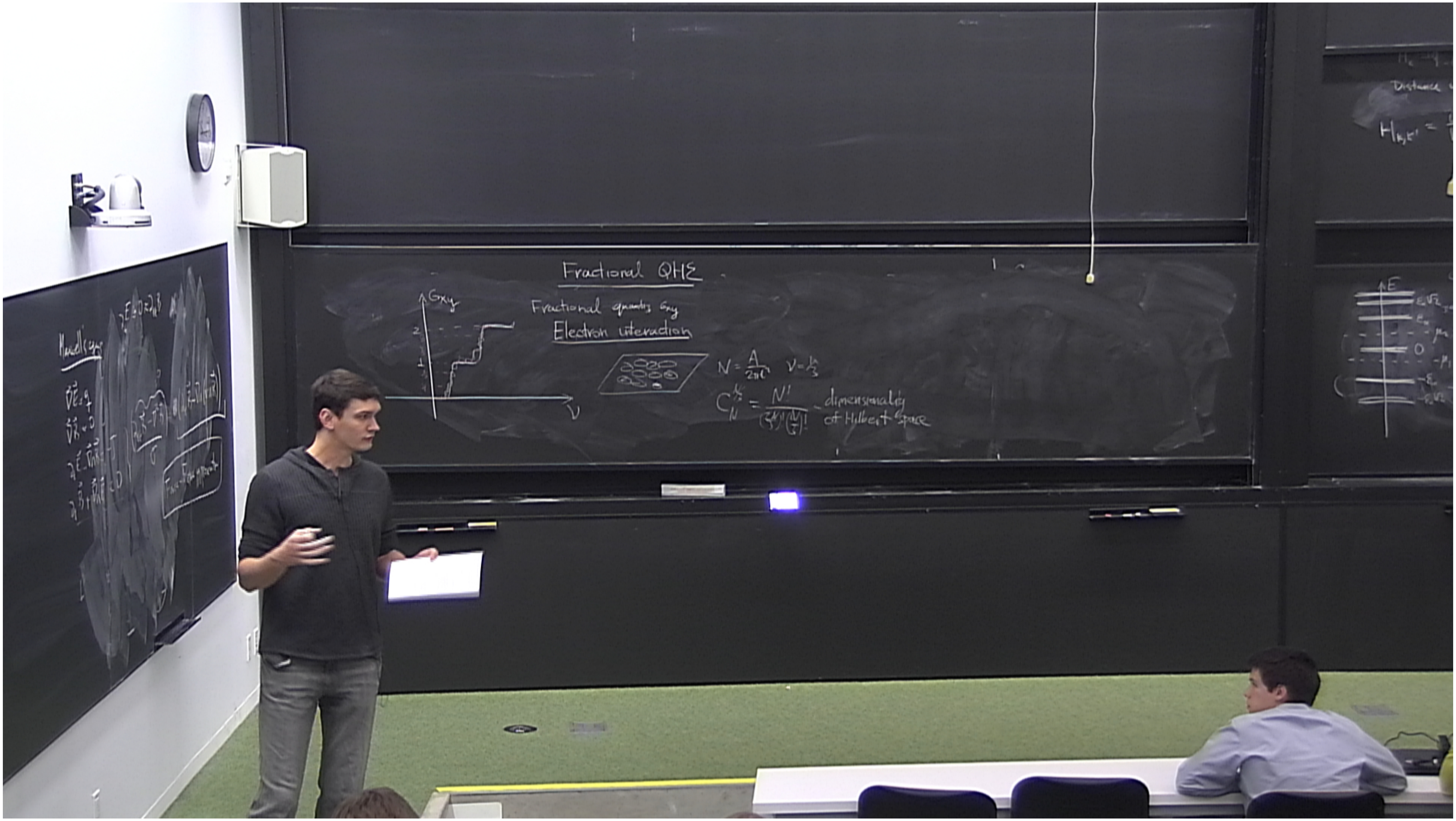

④ ε₁ (analog of cyclotron energy)
~ε₀ = √2 ħ v₀ / ℓ =
= √2 ħ v₀ √(eB/ħc)
= ε₀ 1/(√(n+1) + √n)
σ¹xy = −e²/h
σ²xy = +e²/h
−σ²xy
σxy/Nf
Nf = 2
3/2
1/2
−1/2
ν/Nf
ε₀ ≐ 1000K @ B = 10 T
QHE : dies when T ~ ε₀ #
Dirac spectrum → room-T QHE

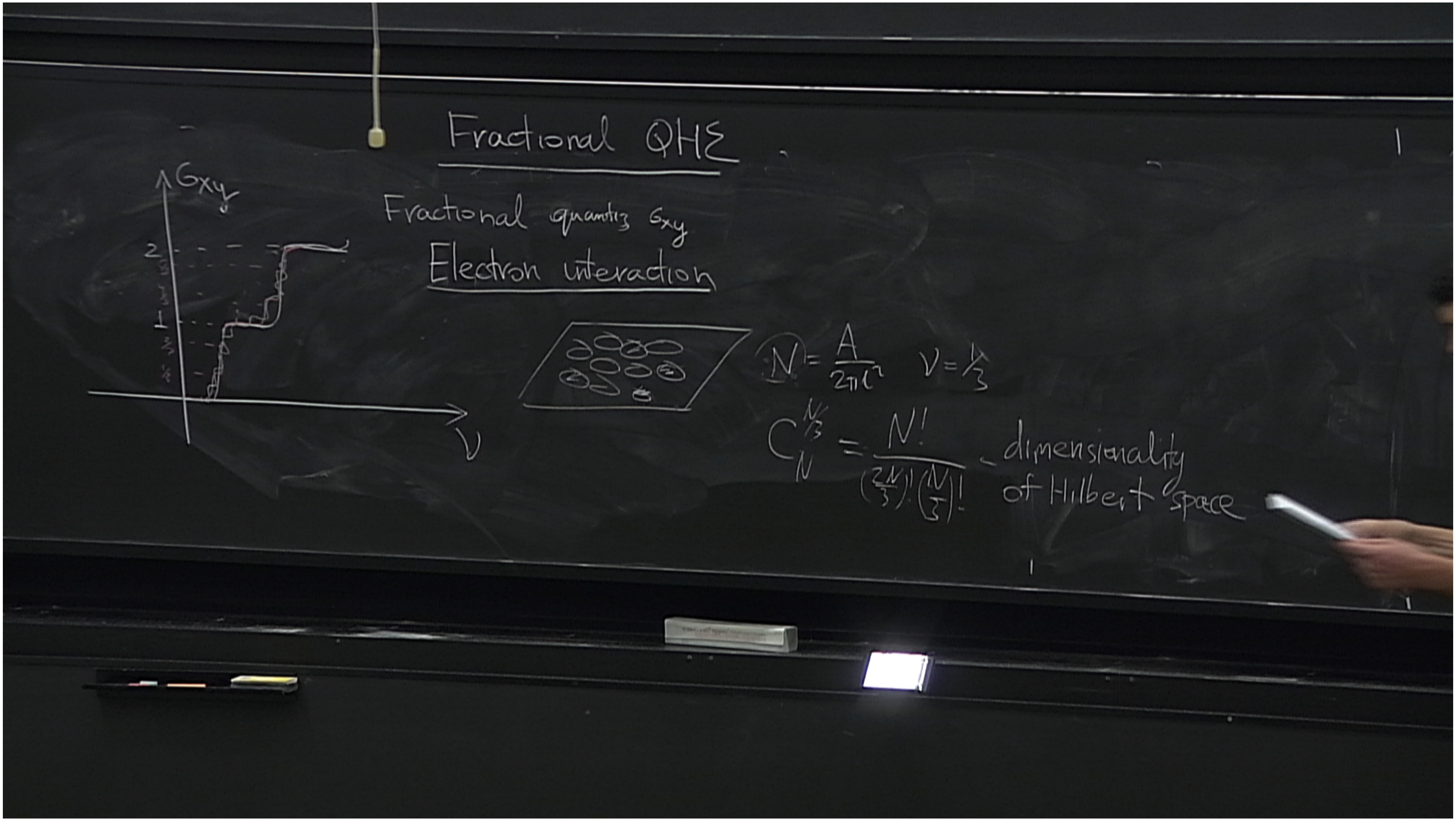

Fractional QHE
σxy
2
1
ν

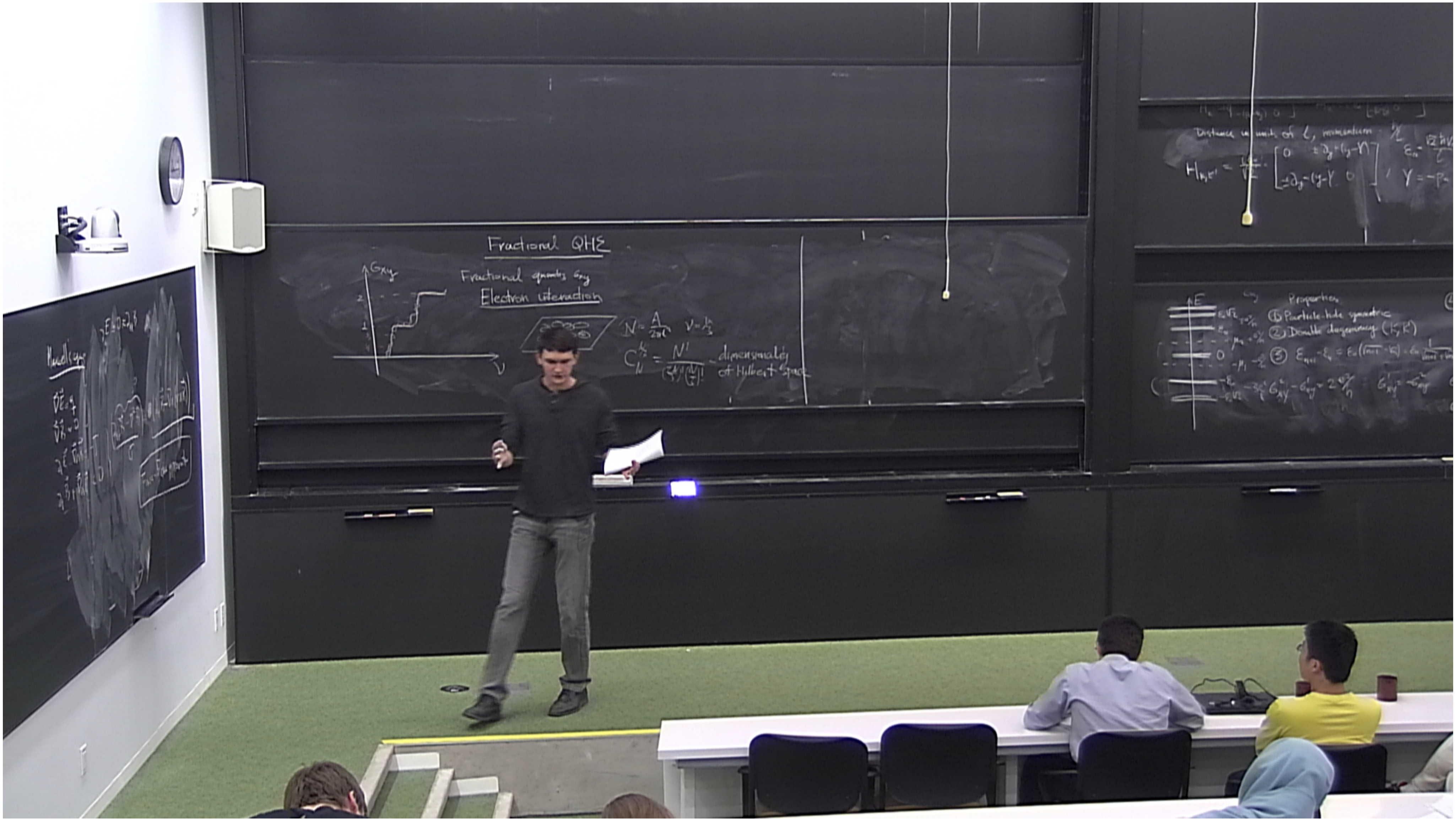

Fractional QHE
σxy
2
1
ν

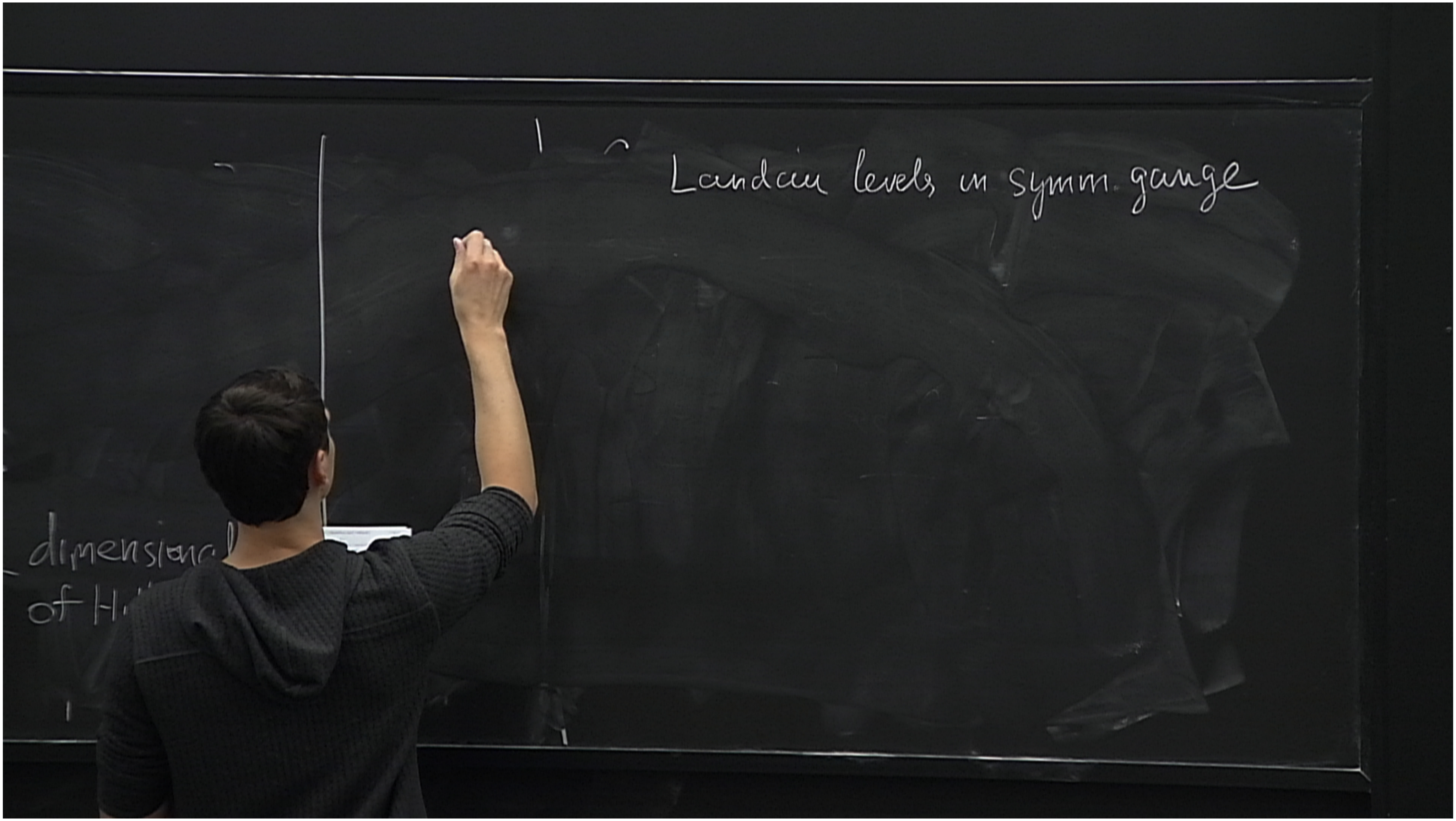

Fractional QHE
σxy
Fractional quantiz σxy
Electron interaction
ν

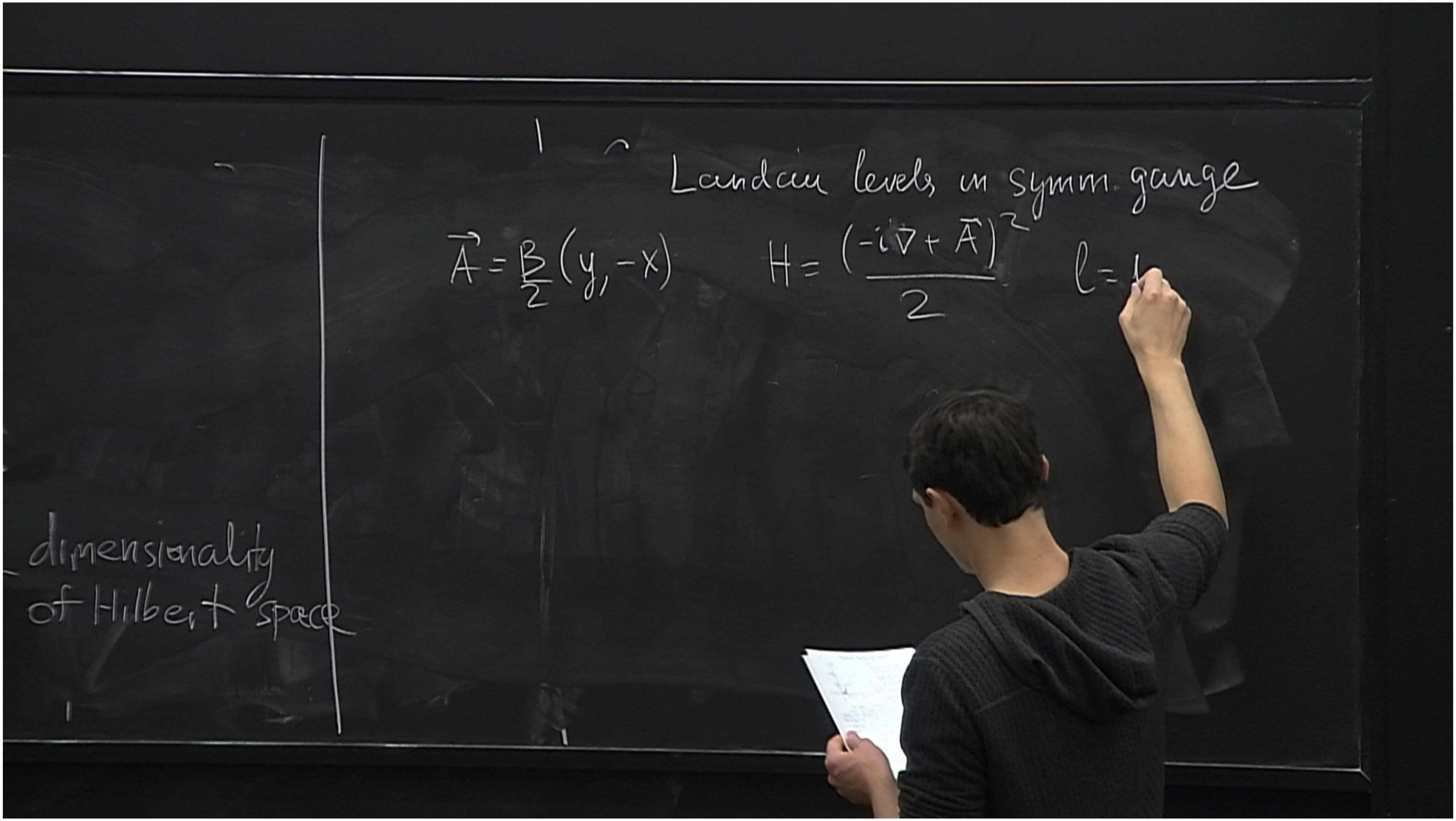

Fractional QHE
σ_{xy}
2
1
ν
Fractional quantiz σ_{xy}
Electron interaction
$N = \frac{A}{2\pi l^2}$
$\nu = 1/3$

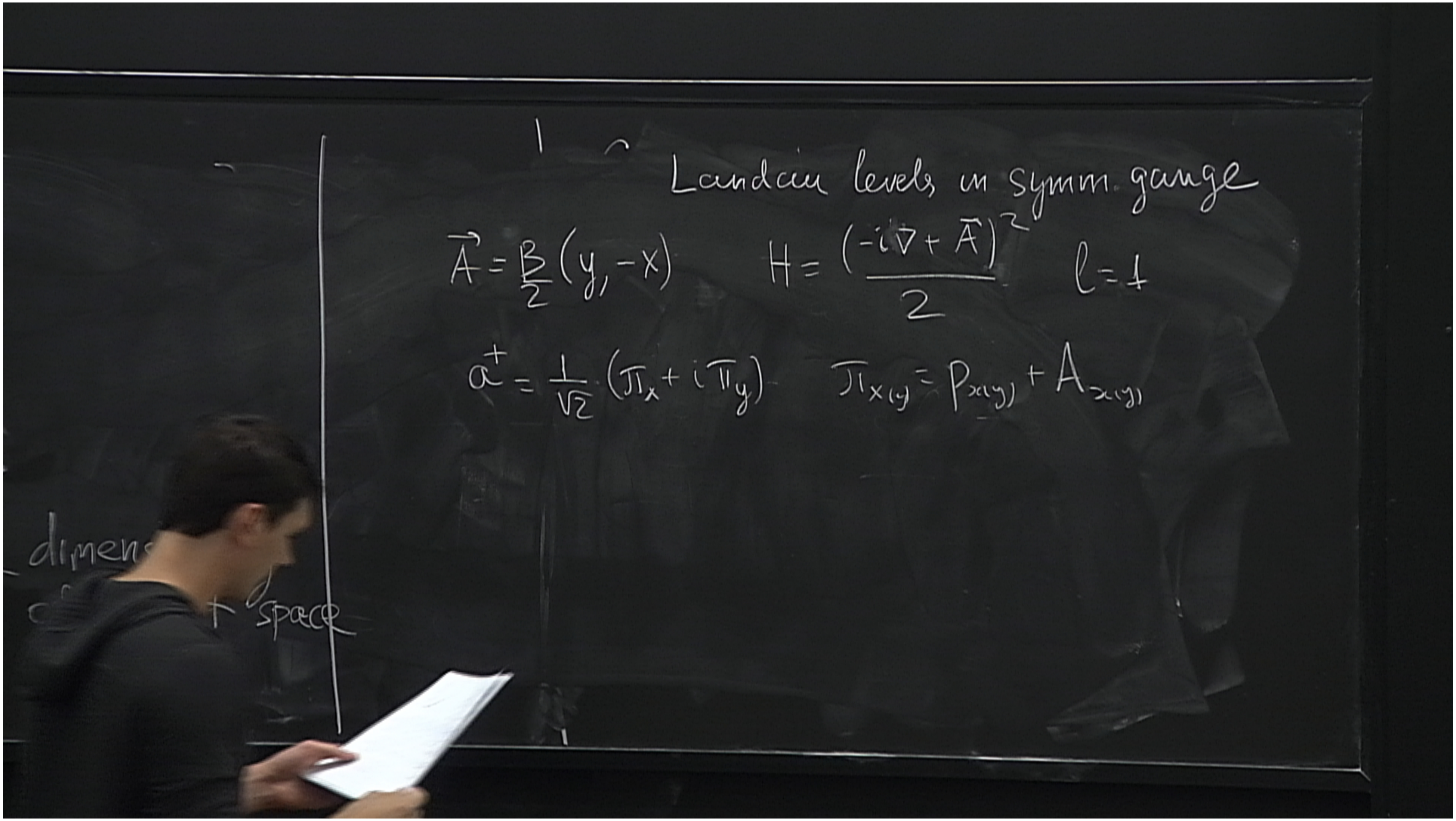

Fractional QHE
Fractional quantiz σxy
Electron interaction
σxy
dimensionality of Hilbert space

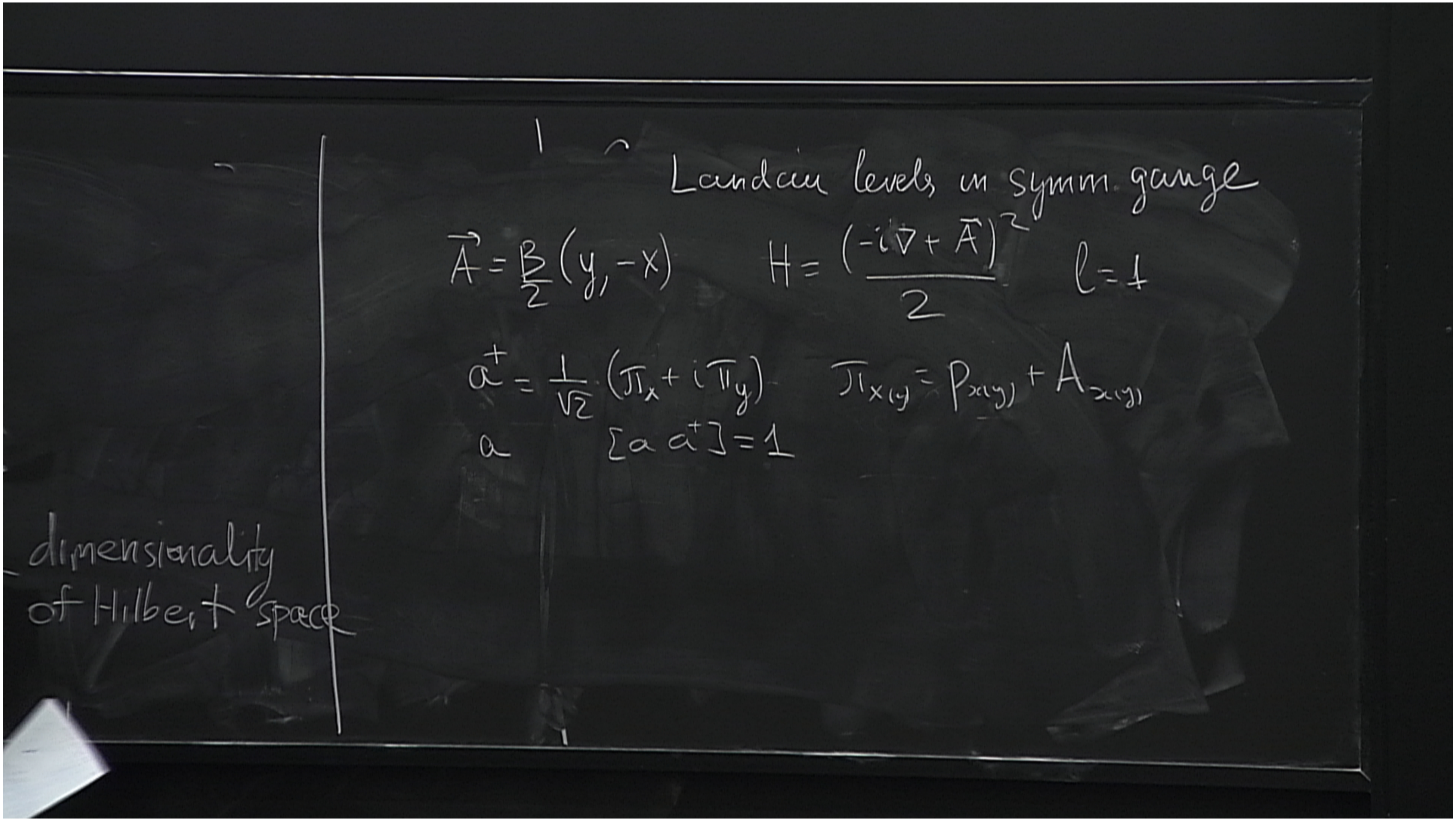

Fractional QHE
Fractional quantiz σxy
Electron interaction
σxy
ν
N = A/2πl² ν = 1/3
C_N^{N/3} = N!/((2N/3)!(N/3)!) - dimensionality of Hilbert space

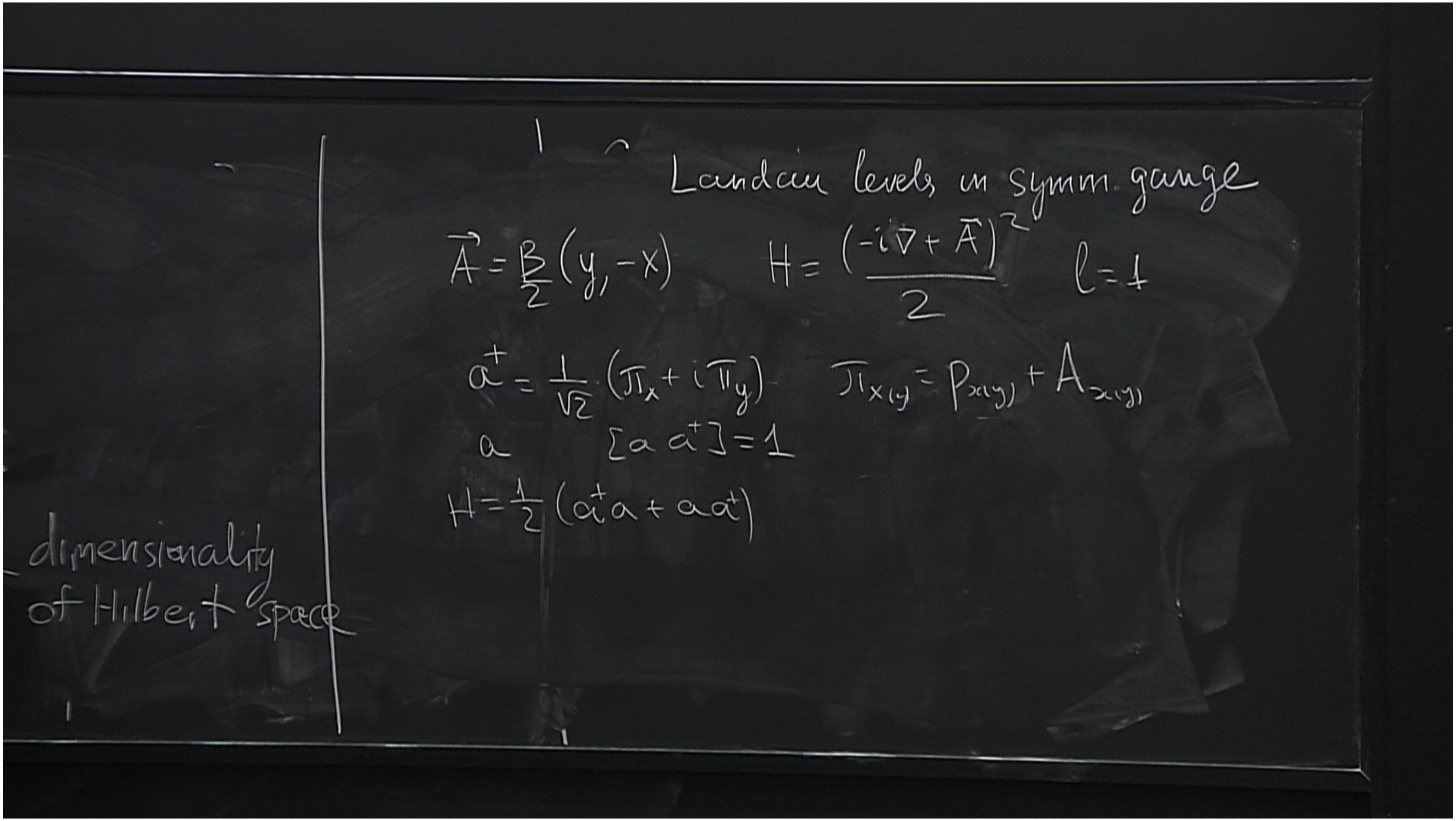

Fractional QHE
Electron interaction
dimensionality of Hilbert Space
Properties
Particle-hole symmetric
Double degeneracy


Landau levels in symm. gauge
dimensional
of H


Landau levels in symm. gauge
A = B/2 (y, -x)
H = (-i∇ + A)^2 / 2
l =
dimensionality
of Hilbert space


Landau levels in symm. gauge
$\vec{A} = \frac{B}{2}(y, -x)$
$H = \frac{(-i\nabla + \vec{A})^2}{2}$
$l = 1$
$a^+ = \frac{1}{\sqrt{2}}(\pi_x + i\pi_y)$
$\pi_{x(y)} = p_{x(y)} + A_{x(y)}$


Landau levels in symm. gauge
A = B/2 (y, -x)
H = (-i∇ + A)^2 / 2
l = 1
a^+ = 1/√2 (π_x + i π_y)
π_x(y) = p_x(y) + A_x(y)
a
[a a^+] = 1
dimensionality of Hilbert space


Landau levels in symm. gauge
$\vec{A} = \frac{B}{2}(y, -x)$
$H = \frac{(-i\nabla + \vec{A})^2}{2}$
$l = 1$
$a^+ = \frac{1}{\sqrt{2}}(\pi_x + i\pi_y)$
$\pi_{x(y)} = p_{x(y)} + A_{x(y)}$
a
$[a\, a^+] = 1$
$H = \frac{1}{2}(a^+a + aa^+)$
dimensionality of Hilbert space

dimensionality of Hilbert space

Landau levels in symm. gauge

$\vec{A} = \frac{B}{2}(y, -x)$ $\qquad H = \frac{(-i\nabla + \vec{A})^2}{2}$ $\qquad l = 1$

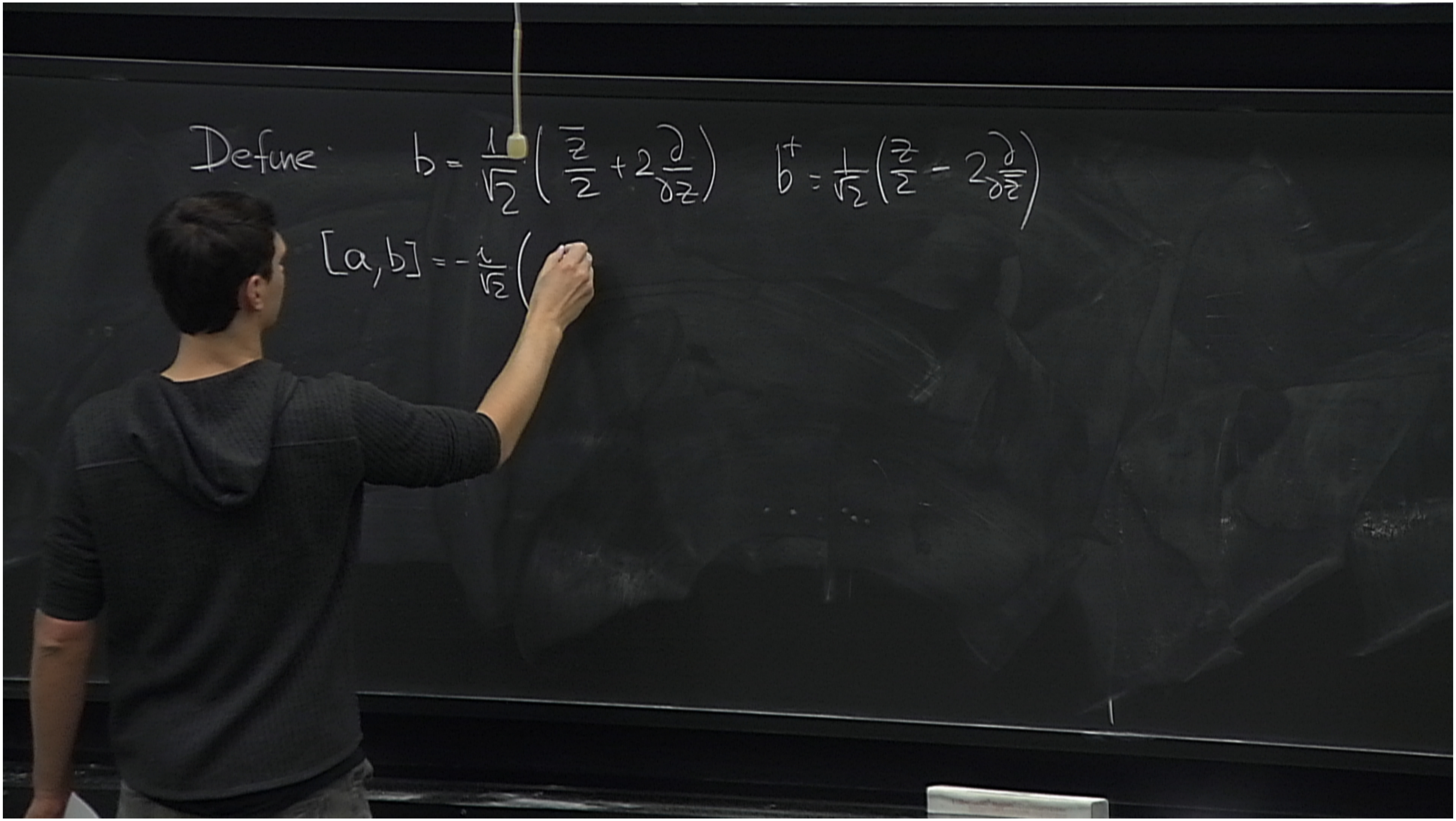
$a^+ = \frac{1}{\sqrt{2}}(\pi_x + i\pi_y)$ $\qquad \pi_{x(y)} = p_{x(y)} + A_{x(y)}$

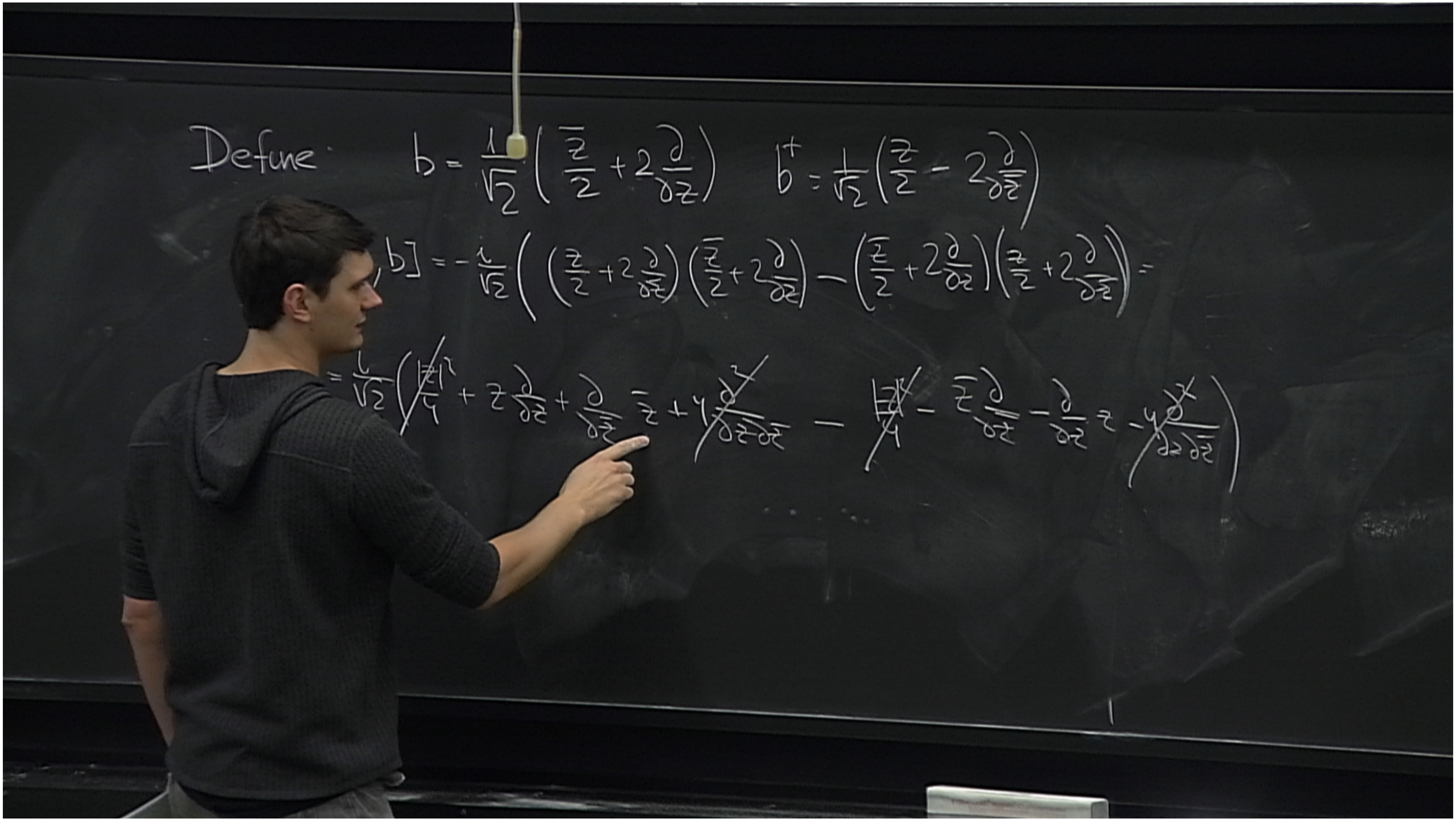
a $\qquad [a\, a^+] = 1$

$H = \frac{1}{2}(a^+ a + a a^+)$ $\qquad \langle a^+ a \rangle$ - Landau level number

$z = x + iy$;

Via z: $a = -\frac{i}{\sqrt{2}}\left(\frac{z}{2} + 2\frac{\partial}{\partial \bar{z}}\right)$, $a^+ = \frac{i}{\sqrt{2}}\left(\frac{\bar{z}}{2} - 2\frac{\partial}{\partial z}\right)$


Define:
$b = \frac{1}{\sqrt{2}}\left(\frac{\bar{z}}{2} + 2\frac{\partial}{\partial z}\right)$
$b^{\dagger} = \frac{1}{\sqrt{2}}\left(\frac{z}{2} - 2\frac{\partial}{\partial \bar{z}}\right)$
$[a, b] = -\frac{1}{\sqrt{2}}($


Define
$b = \frac{1}{\sqrt{2}}\left(\frac{\bar{z}}{2} + 2\frac{\partial}{\partial z}\right)$ $b^{+} = \frac{1}{\sqrt{2}}\left(\frac{z}{2} - 2\frac{\partial}{\partial \bar{z}}\right)$

Define $b = \frac{1}{\sqrt{2}}\left(\frac{\bar{z}}{2} + 2\frac{\partial}{\partial z}\right) \quad b^{\dagger} = \frac{1}{\sqrt{2}}\left(\frac{z}{2} - 2\frac{\partial}{\partial \bar{z}}\right)$

$$[a,b] = -\frac{i}{\sqrt{2}}\left(\left(\frac{z}{2}+2\frac{\partial}{\partial \bar{z}}\right)\left(\frac{\bar{z}}{2}+2\frac{\partial}{\partial z}\right) - \left(\frac{\bar{z}}{2}+2\frac{\partial}{\partial z}\right)\left(\frac{z}{2}+2\frac{\partial}{\partial \bar{z}}\right)\right) =$$

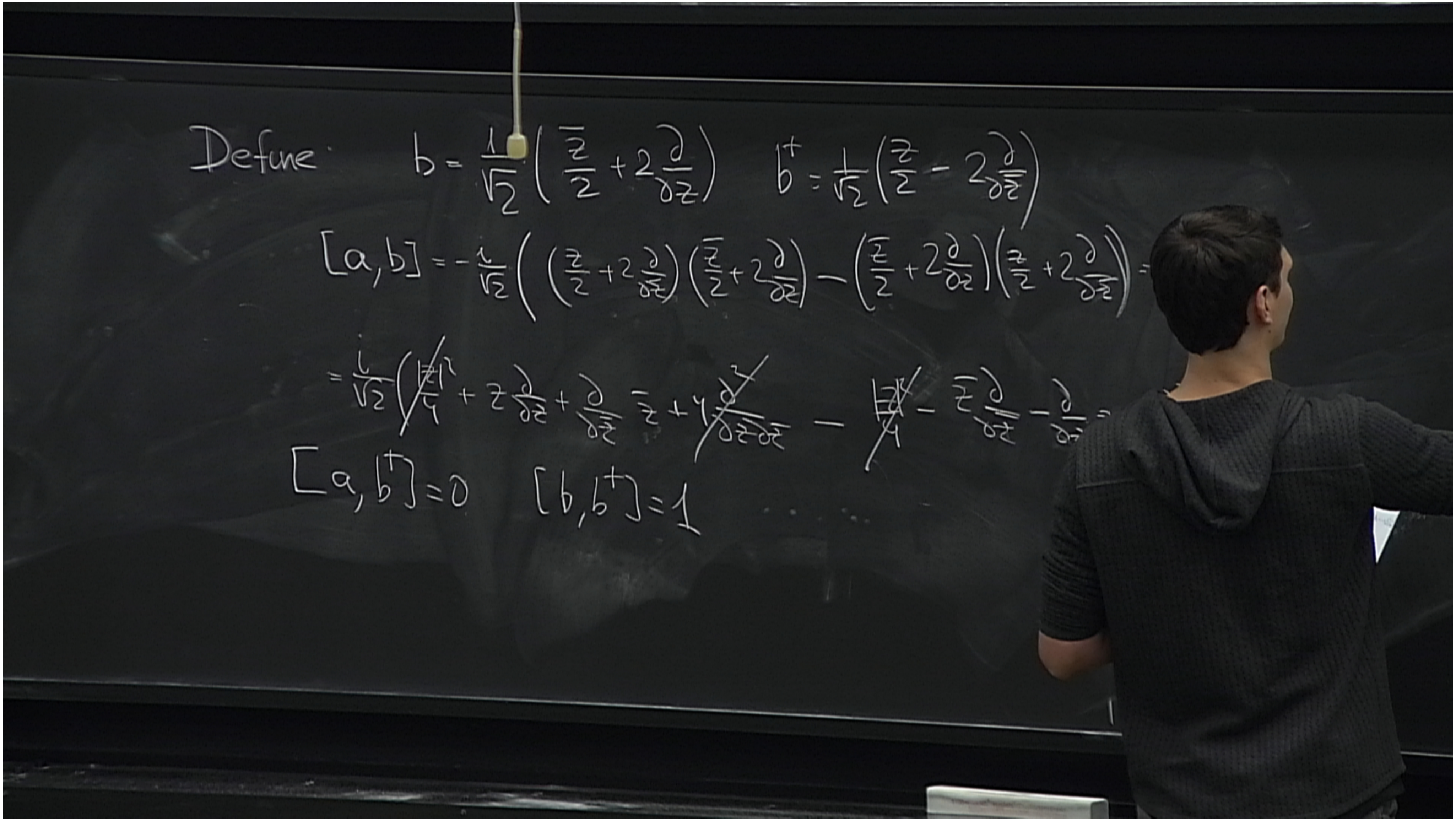
$$= \frac{i}{\sqrt{2}}\left(\frac{|z|^2}{4} + z\frac{\partial}{\partial z} + \frac{\partial}{\partial \bar{z}}\bar{z} + 4\frac{\partial^2}{\partial \bar{z}\partial z} - \frac{|z|^2}{4} - \bar{z}\frac{\partial}{\partial \bar{z}} - \frac{\partial}{\partial z}z - 4\frac{\partial^2}{\partial z\partial \bar{z}}\right) = 0$$

Define $b = \frac{1}{\sqrt{2}}\left(\frac{\bar z}{2} + 2\frac{\partial}{\partial z}\right)$ $\quad b^+ = \frac{1}{\sqrt{2}}\left(\frac{z}{2} - 2\frac{\partial}{\partial \bar z}\right)$

$$[a,b] = -\frac{i}{\sqrt{2}}\left(\left(\frac{z}{2}+2\frac{\partial}{\partial \bar z}\right)\left(\frac{\bar z}{2}+2\frac{\partial}{\partial z}\right) - \left(\frac{\bar z}{2}+2\frac{\partial}{\partial z}\right)\left(\frac{z}{2}+2\frac{\partial}{\partial \bar z}\right)\right) =$$

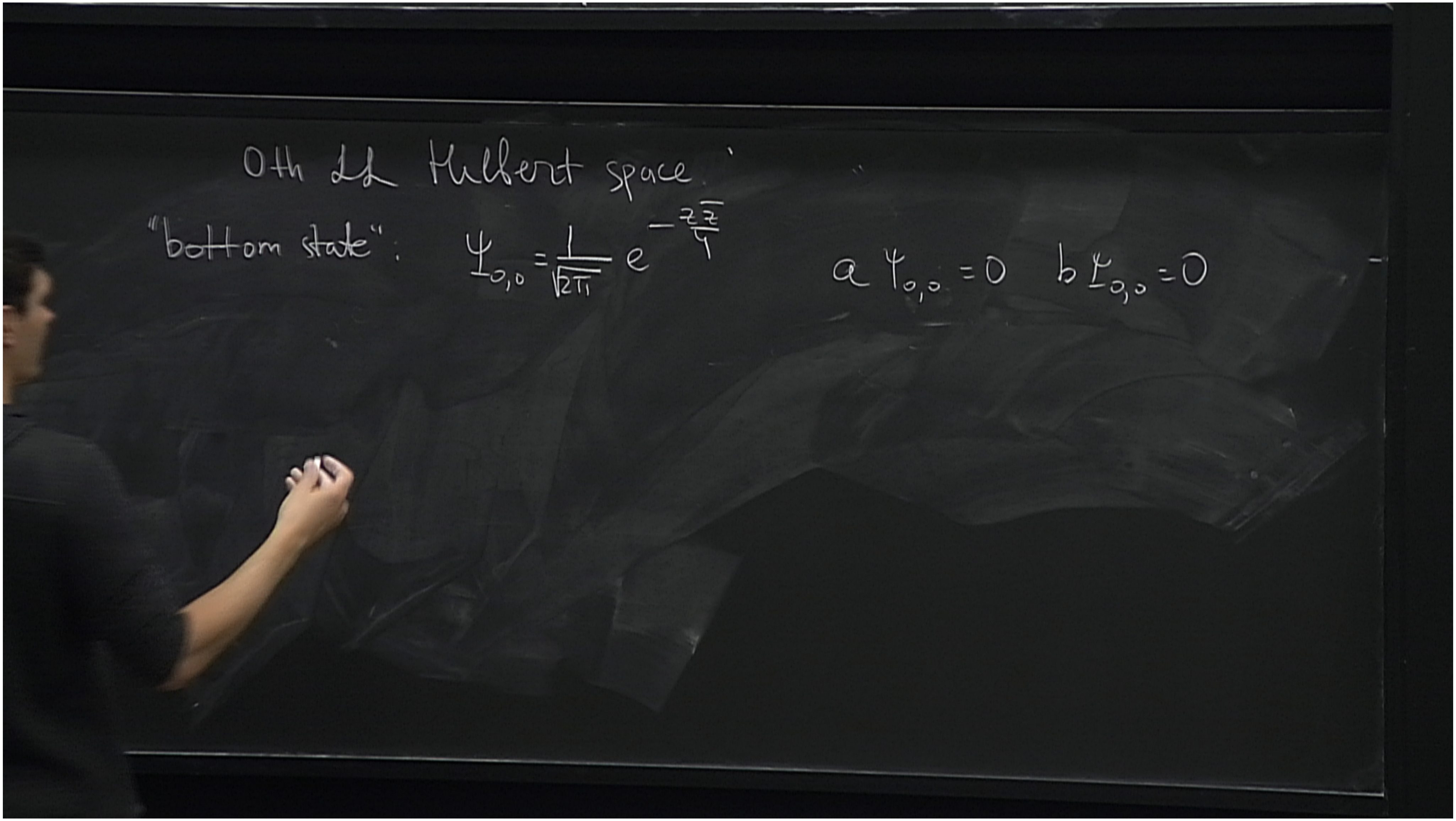
$$= \frac{i}{\sqrt{2}}\left(\frac{|z|^2}{4} + z\frac{\partial}{\partial z} + \frac{\partial}{\partial \bar z}\bar z + 4\frac{\partial^2}{\partial \bar z\,\partial z} - \frac{|z|^2}{4} - \bar z\frac{\partial}{\partial \bar z} - \frac{\partial}{\partial z} z - 4\frac{\partial^2}{\partial z\,\partial \bar z}\right) = 0$$

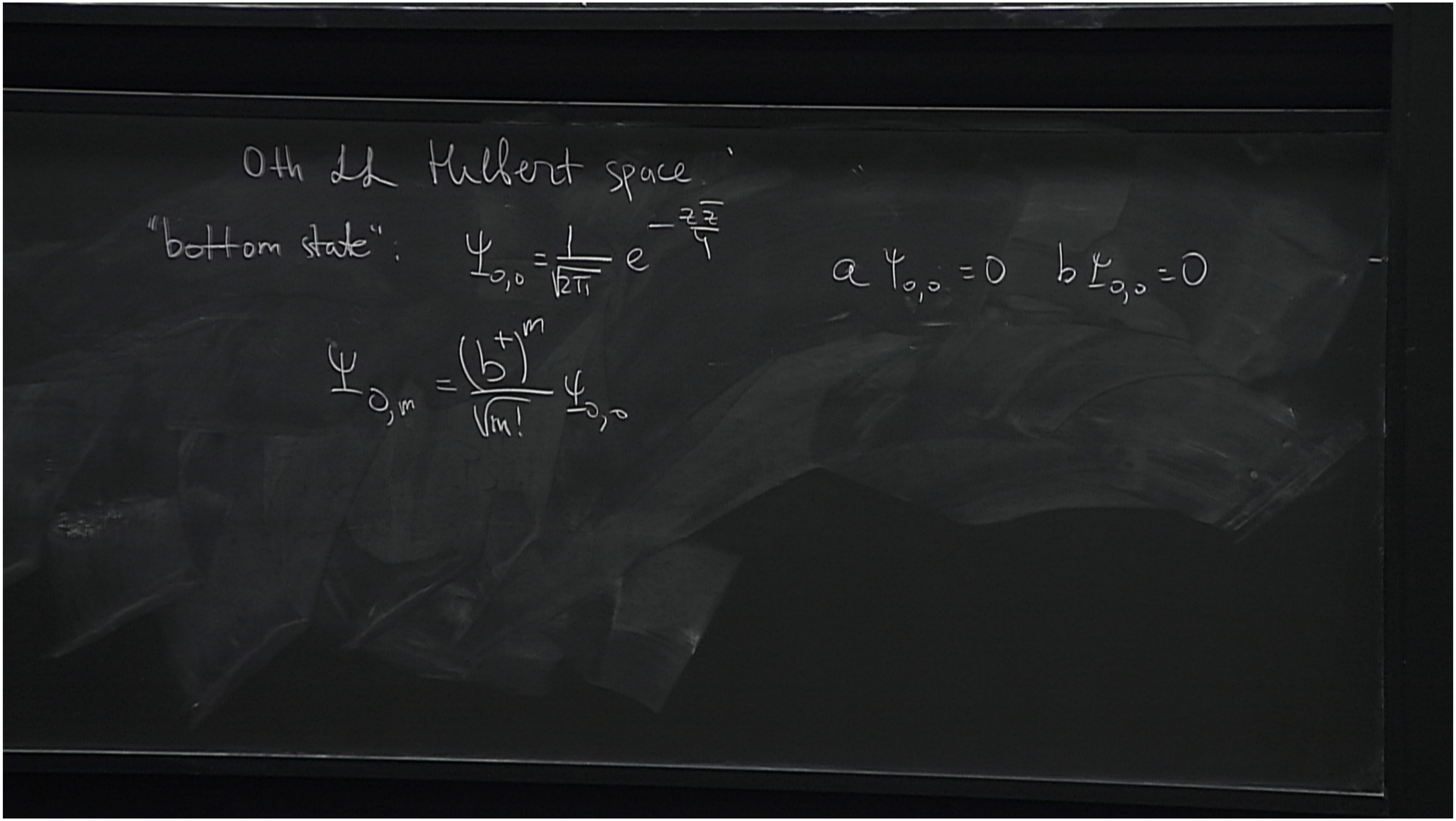
$[a, b^+] = 0 \qquad [b, b^+] = 1$


Define b = 1/√2 (z̄/2 + 2 ∂/∂z)   b† = 1/√2 (z/2 − 2 ∂/∂z̄)
[a, b] = −1/√2 ((z/2 + 2 ∂/∂z̄)(z̄/2 + 2 ∂/∂z) − (z̄/2 + 2 ∂/∂z)(z/2 + 2 ∂/∂z̄)) =
= 1/√2 (|z|²/4 + z ∂/∂z + ∂/∂z̄ z̄ + 4 ∂²/∂z̄∂z − |z|²/4 − z̄ ∂/∂z̄ − ∂/∂z
[a, b†] = 0   [b, b†] = 1

0th LL Hilbert space:

"bottom state": $\psi_{0,0} = \frac{1}{\sqrt{2\pi}} e^{-\frac{z\bar{z}}{4}}$


0th LL Hilbert space:
"bottom state": $\Psi_{0,0} = \frac{1}{\sqrt{2\pi}} e^{-\frac{z\bar{z}}{4}}$
$a \Psi_{0,0} = 0$ $b \Psi_{0,0} = 0$


0th LL Hilbert space :
"bottom state": $\Psi_{0,0} = \frac{1}{\sqrt{2\pi}} e^{-\frac{z\bar{z}}{4}}$
$a\,\Psi_{0,0} = 0 \quad b\,\Psi_{0,0} = 0$
$\Psi_{0,m} = \frac{(b^{+})^{m}}{\sqrt{m!}} \Psi_{0,0}$

0th LL Hilbert space:

"bottom state": $\Psi_{0,0} = \frac{1}{\sqrt{2\pi}} e^{-\frac{z\bar{z}}{4}}$ $\qquad a\,\Psi_{0,0} = 0 \quad b\,\Psi_{0,0} = 0$

$$\Psi_{0,m} = \frac{(b^{+})^{m}}{\sqrt{m!}}\Psi_{0,0} \qquad \Psi_{0,1} = \frac{1}{\sqrt{2}}\left(\frac{z}{2} - 2\frac{\partial}{\partial\bar{z}}\right)\cdot\frac{1}{\sqrt{2\pi}}\, e^{-\frac{z\bar{z}}{4}} =$$

$$= \frac{1}{\sqrt{2}}\, z \cdot \frac{1}{\sqrt{2\pi}}\, e^{-\frac{z\bar{z}}{4}}$$

0th LL Hilbert space:

"bottom state": $\Psi_{0,0} = \frac{1}{\sqrt{2\pi}} e^{-\frac{z\bar{z}}{4}}$

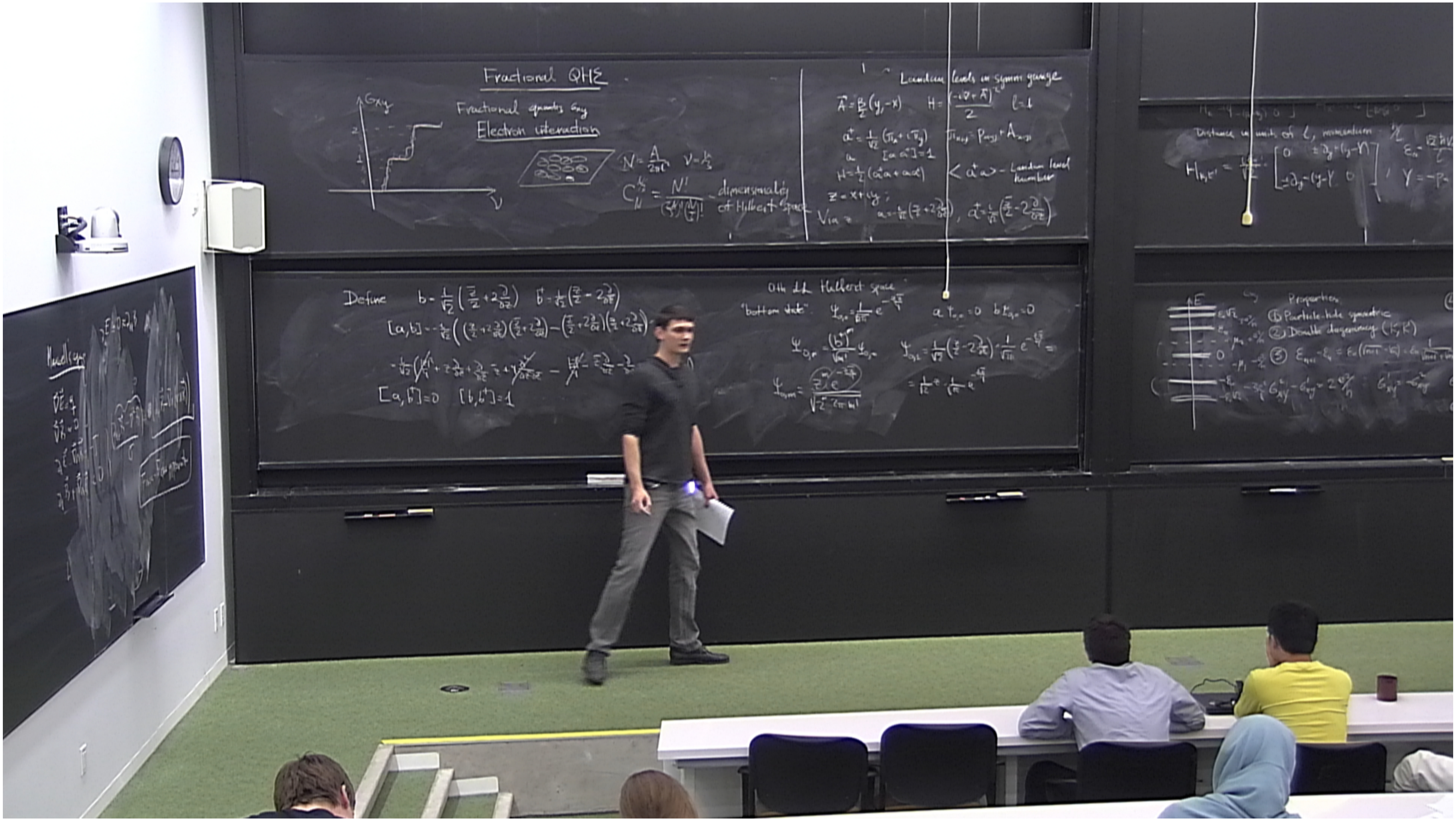
$a\,\Psi_{0,0} = 0 \qquad b\,\Psi_{0,0} = 0$

$$\Psi_{0,m} = \frac{(b^{+})^{m}}{\sqrt{m!}}\Psi_{0,0}$$

$$\Psi_{0,1} = \frac{1}{\sqrt{2}}\left(\frac{z}{2} - 2\frac{\partial}{\partial \bar{z}}\right)\cdot\frac{1}{\sqrt{2\pi}}\, e^{-\frac{z\bar{z}}{4}} =$$

$$= \frac{1}{\sqrt{2}}\, z \cdot \frac{1}{\sqrt{2\pi}}\, e^{-\frac{z\bar{z}}{4}}$$

$$\Psi_{0,m} = \frac{z^{m}\, e^{-\frac{z\bar{z}}{4}}}{\sqrt{2^{m}\, 2\pi\, m!}}$$



0th LL Hilbert space:

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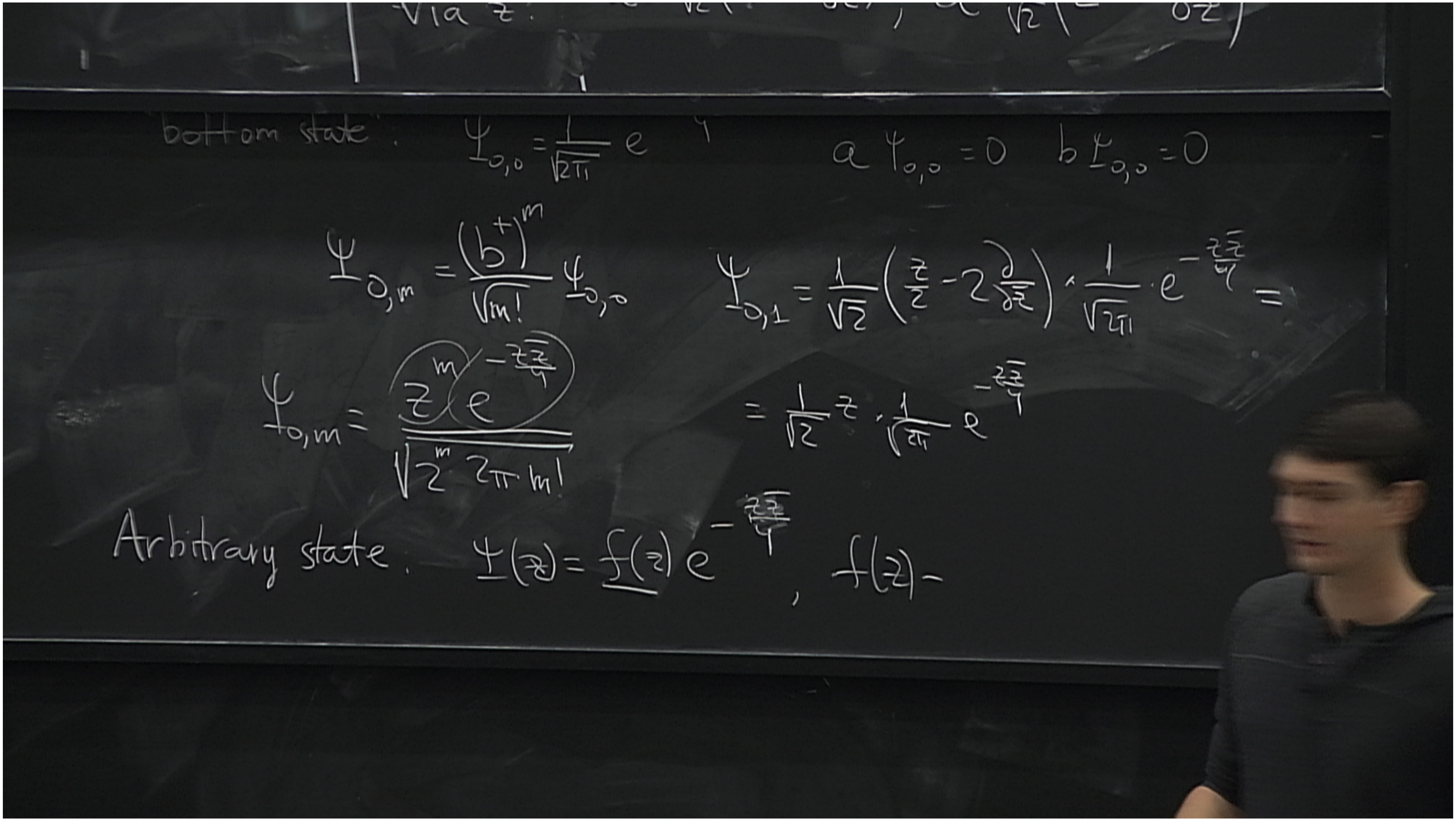
$$= \frac{1}{\sqrt{2}} z \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{z\bar{z}}{4}}$$

$$\Psi_{0,m} = \frac{z^{m} e^{-\frac{z\bar{z}}{4}}}{\sqrt{2^{m}\, 2\pi\, m!}}$$



bottom state: $\Psi_{0,0} = \frac{1}{\sqrt{2\pi}} e$
$a\Psi_{0,0} = 0 \quad b\Psi_{0,0} = 0$
$\Psi_{0,m} = \frac{(b^+)^m}{\sqrt{m!}} \Psi_{0,0}$
$\Psi_{0,1} = \frac{1}{\sqrt{2}}\left(\frac{z}{2} - 2\frac{\partial}{\partial \bar{z}}\right) \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{z\bar{z}}{4}} =$
$= \frac{1}{\sqrt{2}} z \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{z\bar{z}}{4}}$
$\Psi_{0,m} = \frac{z^m e^{-\frac{z\bar{z}}{4}}}{\sqrt{2^m 2\pi \, m!}}$
Arbitrary state: $\Psi(z) = f(z) e^{-\frac{z\bar{z}}{4}}$, $f(z)$ -

via z: $\quad \ldots \frac{1}{\sqrt{2}}\left(\ldots \partial_{\bar z} \right), \quad \ldots \frac{1}{\sqrt{2}}\left(\ldots \partial_{z} \right)$

bottom state: $\Psi_{0,0} = \frac{1}{\sqrt{2\pi}} e^{\ \ 4} \qquad a\Psi_{0,0} = 0 \quad b\Psi_{0,0} = 0$

$$\Psi_{0,m} = \frac{(b^+)^m}{\sqrt{m!}} \Psi_{0,0} \qquad \Psi_{0,1} = \frac{1}{\sqrt{2}}\left(\frac{z}{2} - 2\frac{\partial}{\partial \bar z}\right) \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{z\bar z}{4}} =$$

$$\Psi_{0,m} = \frac{z^m e^{-\frac{z\bar z}{4}}}{\sqrt{2^m\, 2\pi\, m!}} \qquad = \frac{1}{\sqrt{2}} z \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{z\bar z}{4}}$$

Arbitrary state: $\Psi(z) = f(z)\, e^{-\frac{z\bar z}{4}}$, $f(z)$ - any analytic function via z: ... $\sqrt{2}(\ldots \partial \bar z)$, ... $\sqrt{2}(\ldots \partial z)$

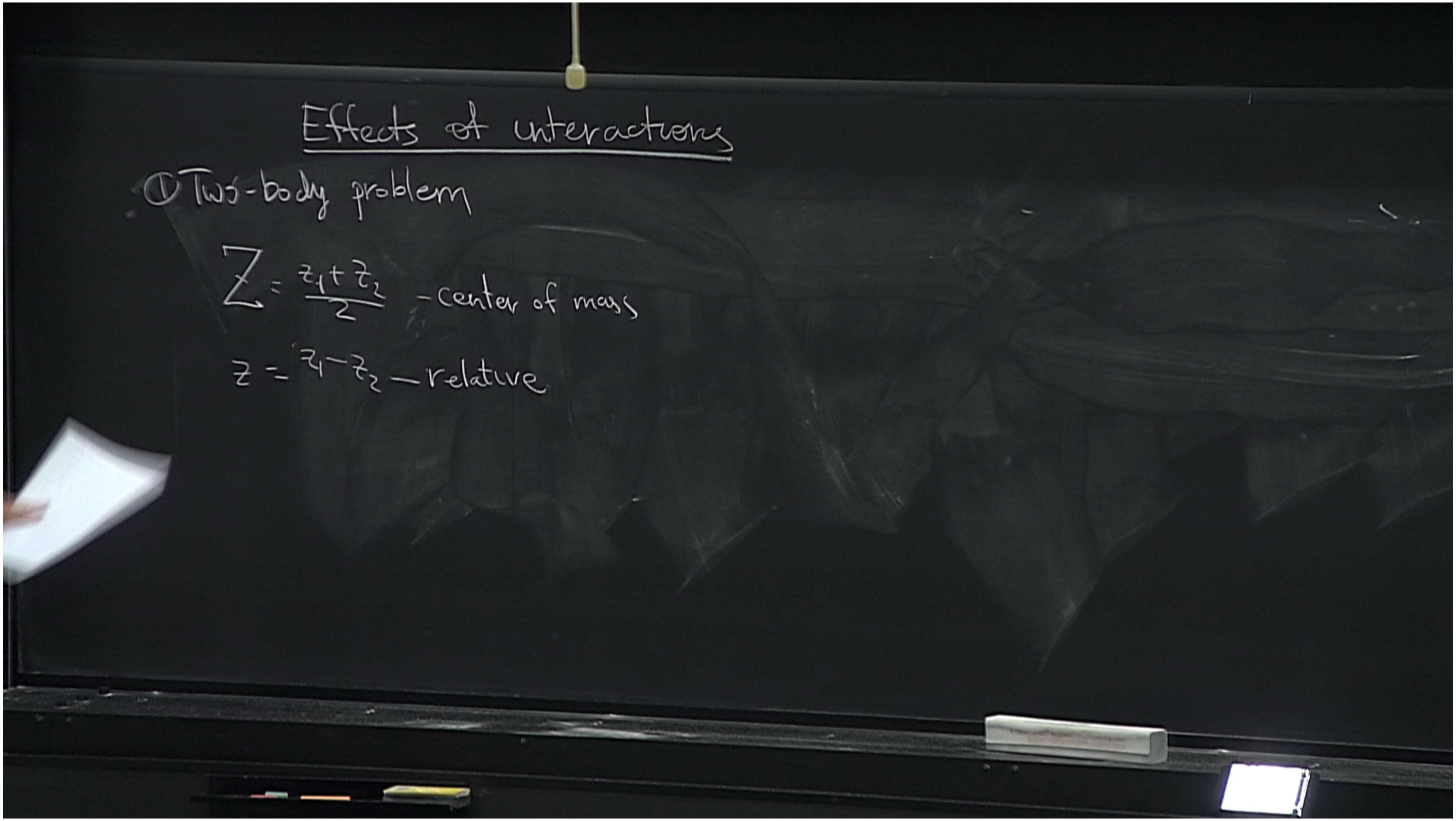
bottom state: $\Psi_{0,0} = \frac{1}{\sqrt{2\pi}} e^{-\frac{z\bar z}{4}}$ $\qquad a\,\Psi_{0,0} = 0 \quad b\,\Psi_{0,0} = 0$

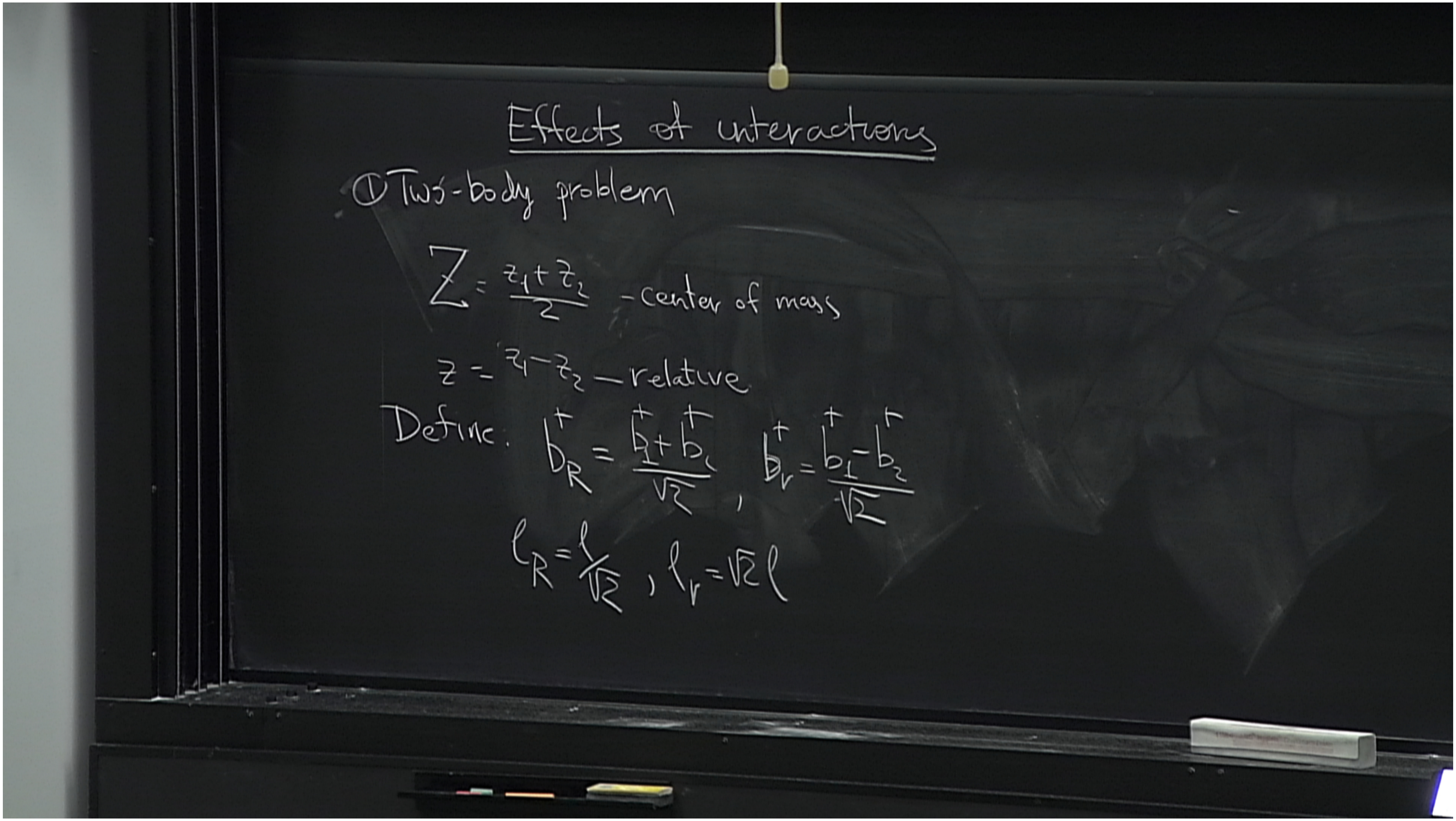
$$\Psi_{0,m} = \frac{(b^{+})^m}{\sqrt{m!}}\Psi_{0,0}$$

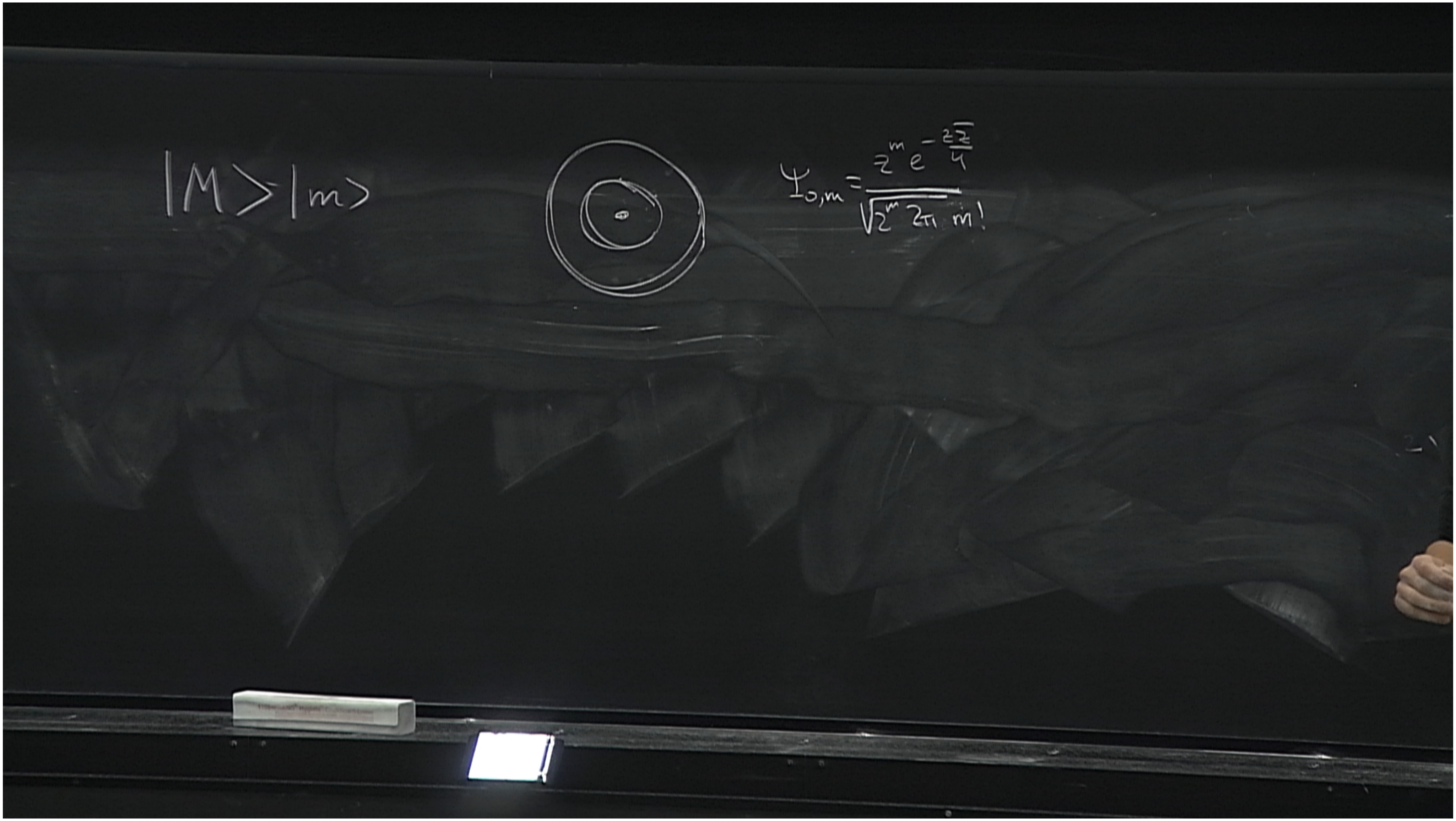
$$\Psi_{0,1} = \frac{1}{\sqrt{2}}\left(\frac{z}{2} - 2\frac{\partial}{\partial \bar z}\right)\cdot\frac{1}{\sqrt{2\pi}}\, e^{-\frac{z\bar z}{4}} = \frac{1}{\sqrt{2}}\, z \cdot \frac{1}{\sqrt{2\pi}}\, e^{-\frac{z\bar z}{4}}$$

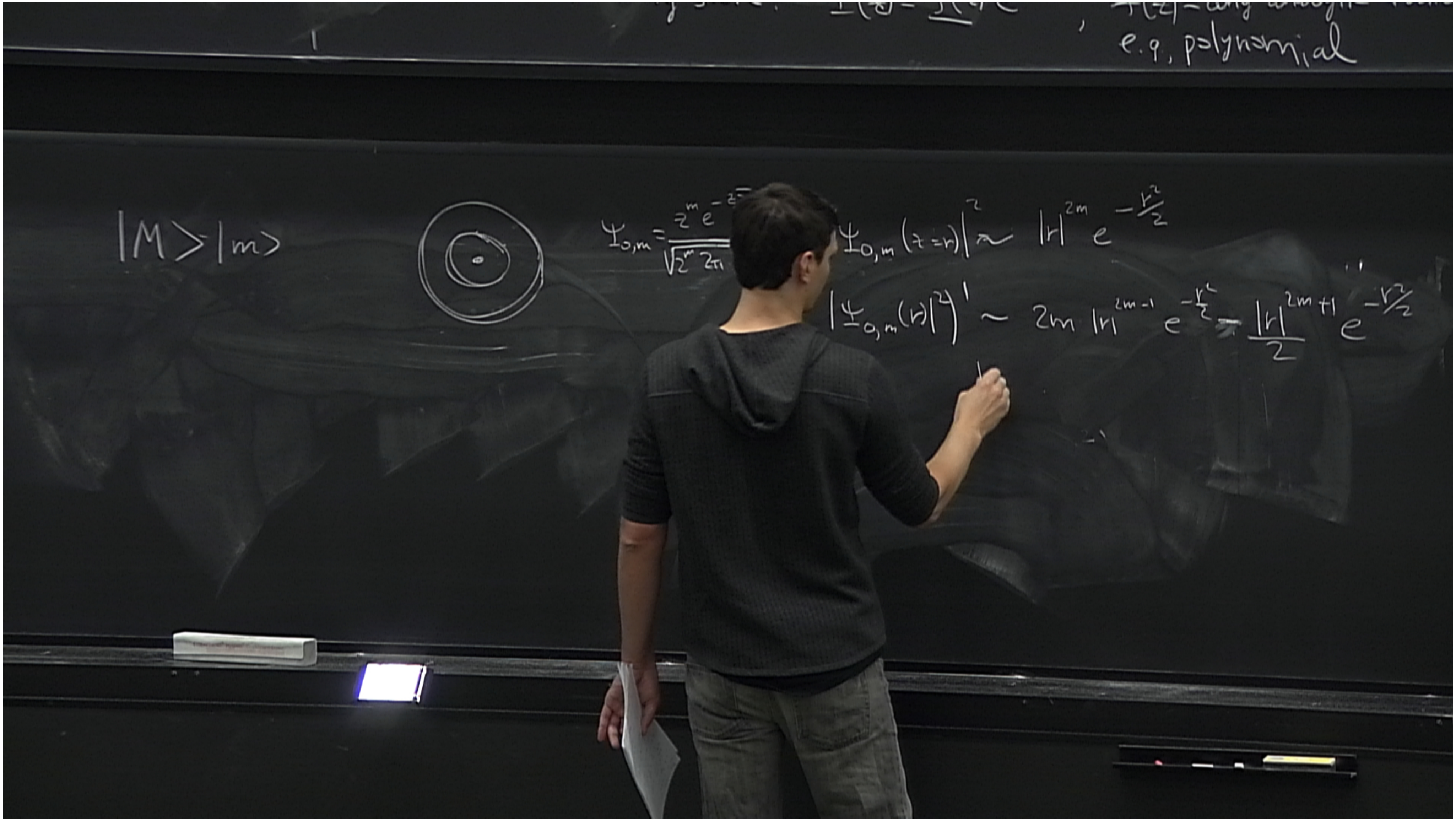
$$\Psi_{0,m} = \frac{z^m\, e^{-\frac{z\bar z}{4}}}{\sqrt{2^m\, 2\pi\, m!}}$$

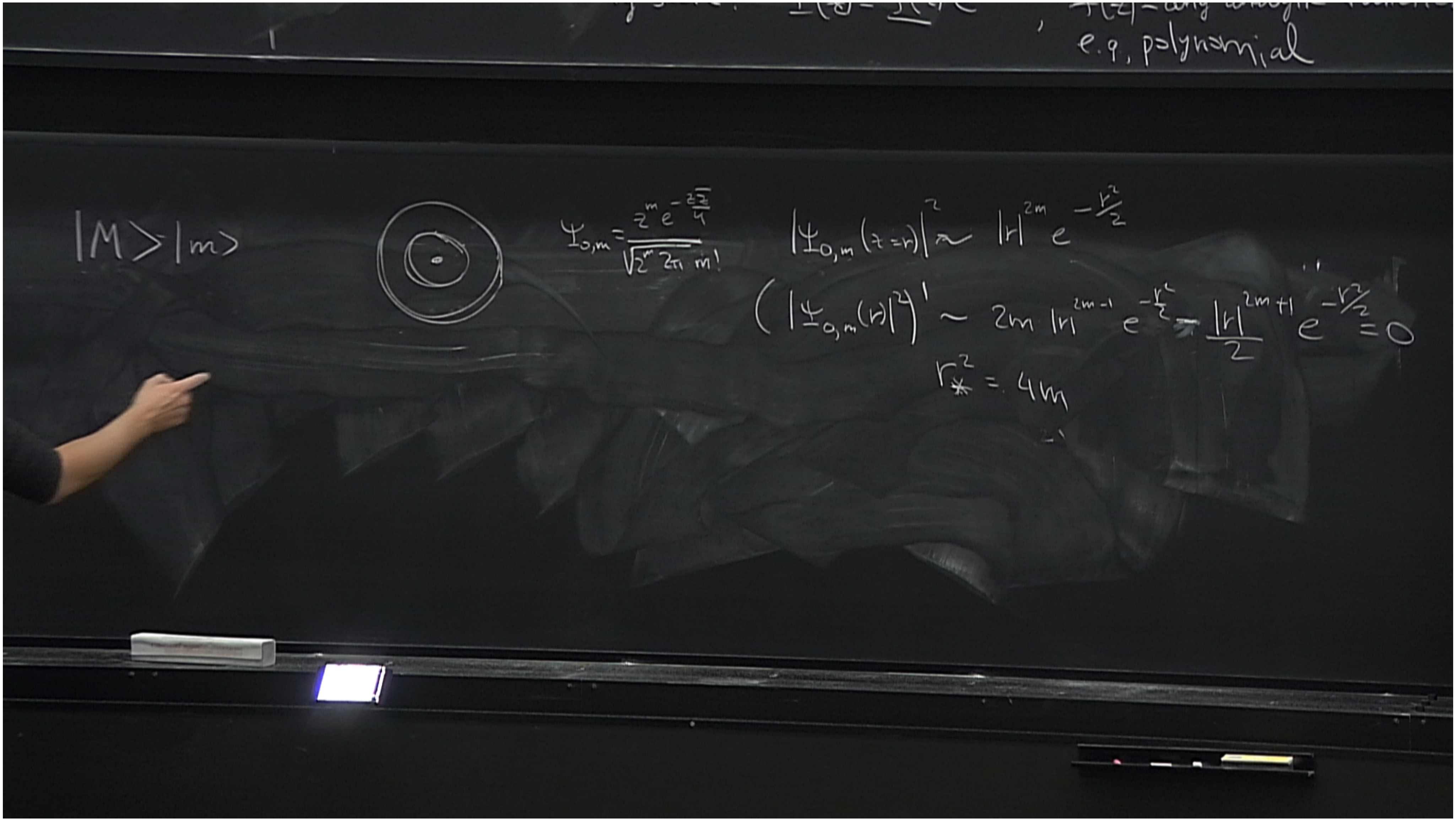
Arbitrary state: $\Psi(z) = f(z)\, e^{-\frac{z\bar z}{4}}$, $f(z)$ – any analytic function e.g., polynomial


Effects of interactions
① Two-body problem
Z = (z1 + z2)/2 - center of mass
z = z1 - z2 - relative

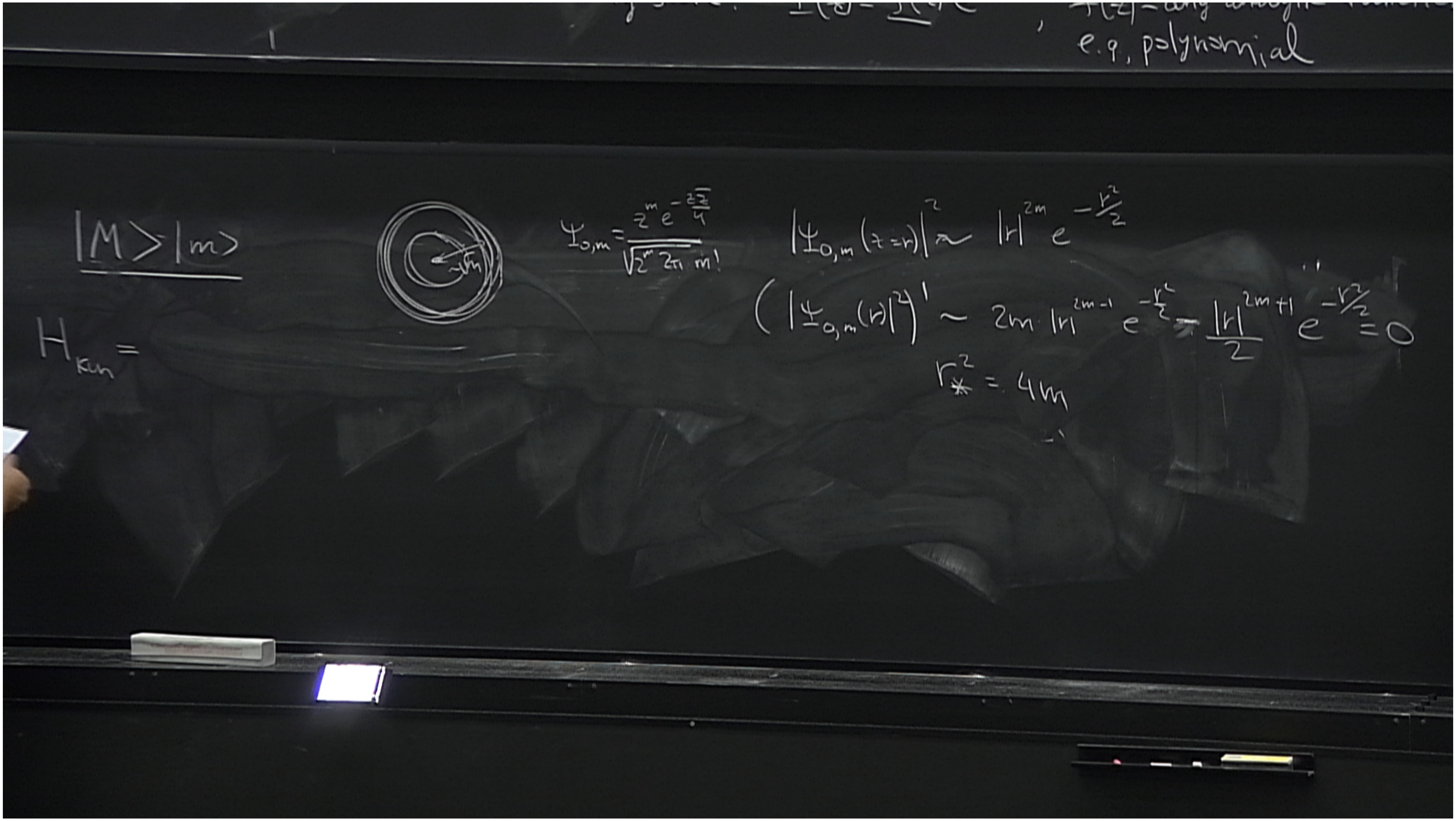

Effects of interactions
① Two-body problem
$Z = \frac{z_1 + z_2}{2}$ - center of mass
$z = z_1 - z_2$ — relative
Define: $b_R^+ = \frac{b_1^+ + b_2^+}{\sqrt{2}}$, $b_r^+ = \frac{b_1^+ - b_2^+}{\sqrt{2}}$
$\ell_R = \ell/\sqrt{2}$, $\ell_r = \sqrt{2}\,\ell$

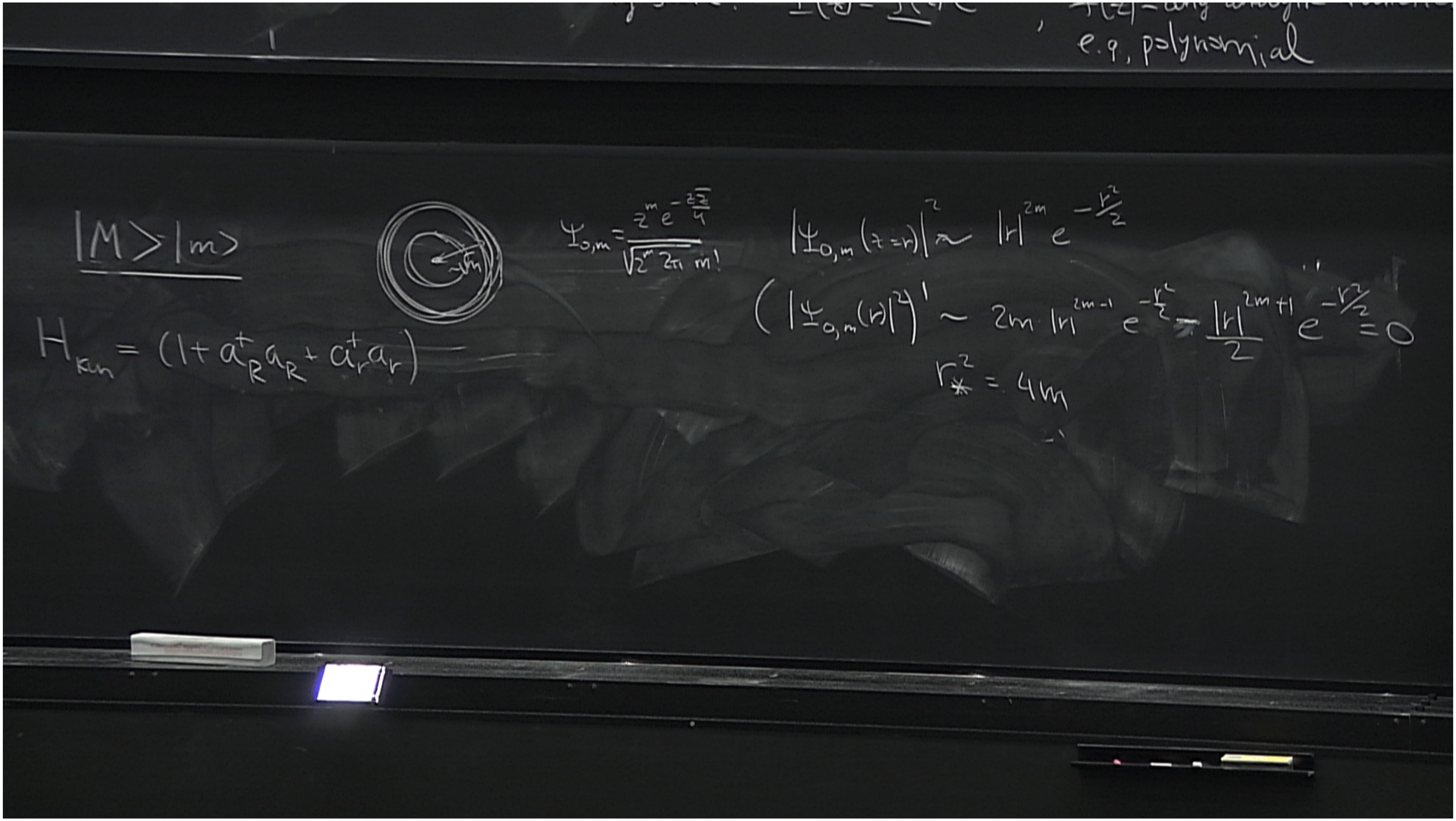

|M>|m>
$\Psi_{0,m} = \frac{z^m e^{-\frac{z\bar{z}}{4}}}{\sqrt{2^m 2\pi\, m!}}$

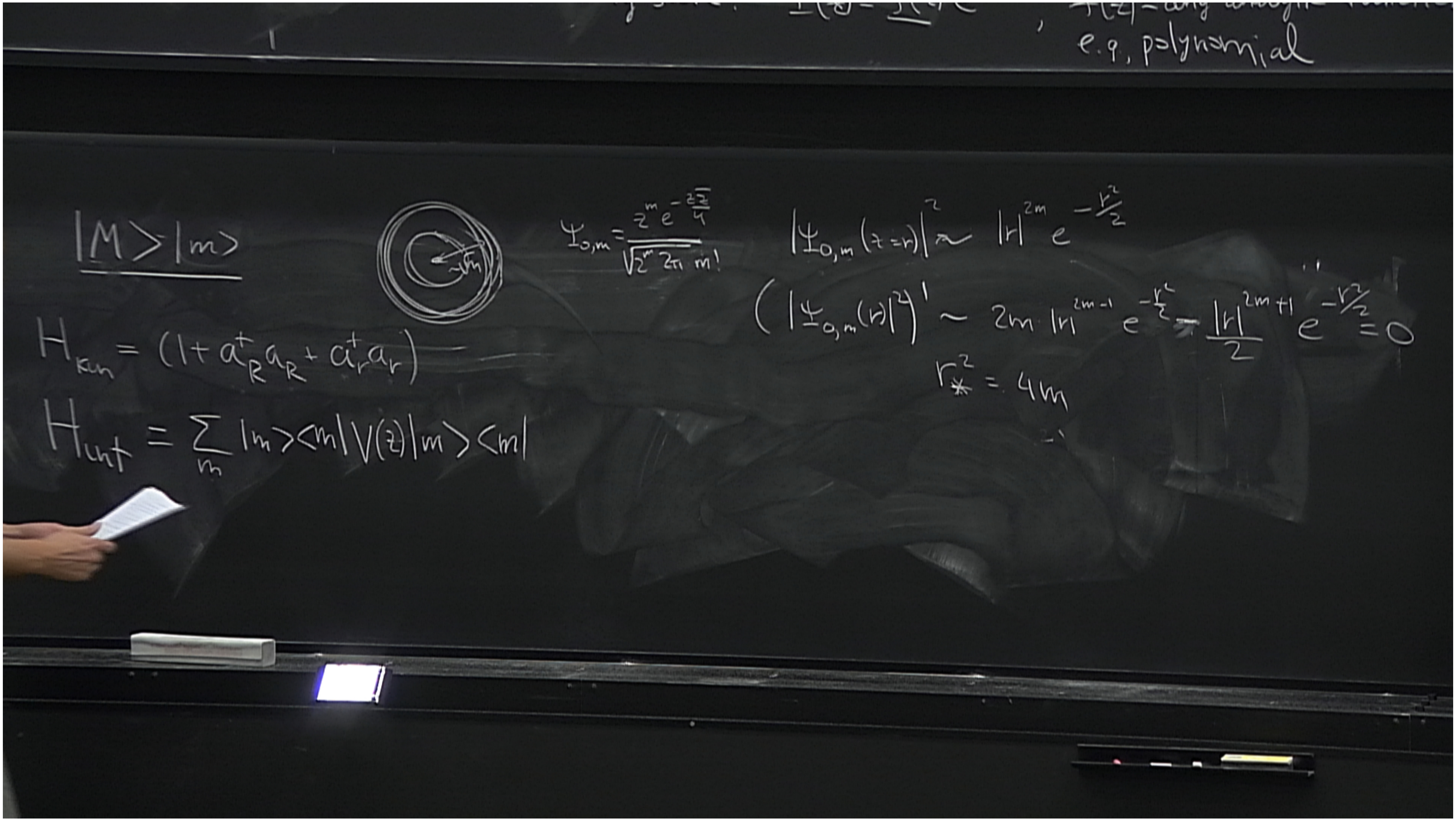

e.g., polynomial
|M> |m>
$\Psi_{0,m} = \frac{z^m e^{-z\bar{z}}}{\sqrt{2^m 2\pi}}$
$|\Psi_{0,m}(z=r)|^2 \sim |r|^{2m} e^{-r^2/2}$
$(|\Psi_{0,m}(r)|^2)' \sim 2m\, |r|^{2m-1} e^{-r^2/2} - \frac{|r|^{2m+1}}{2} e^{-r^2/2}$

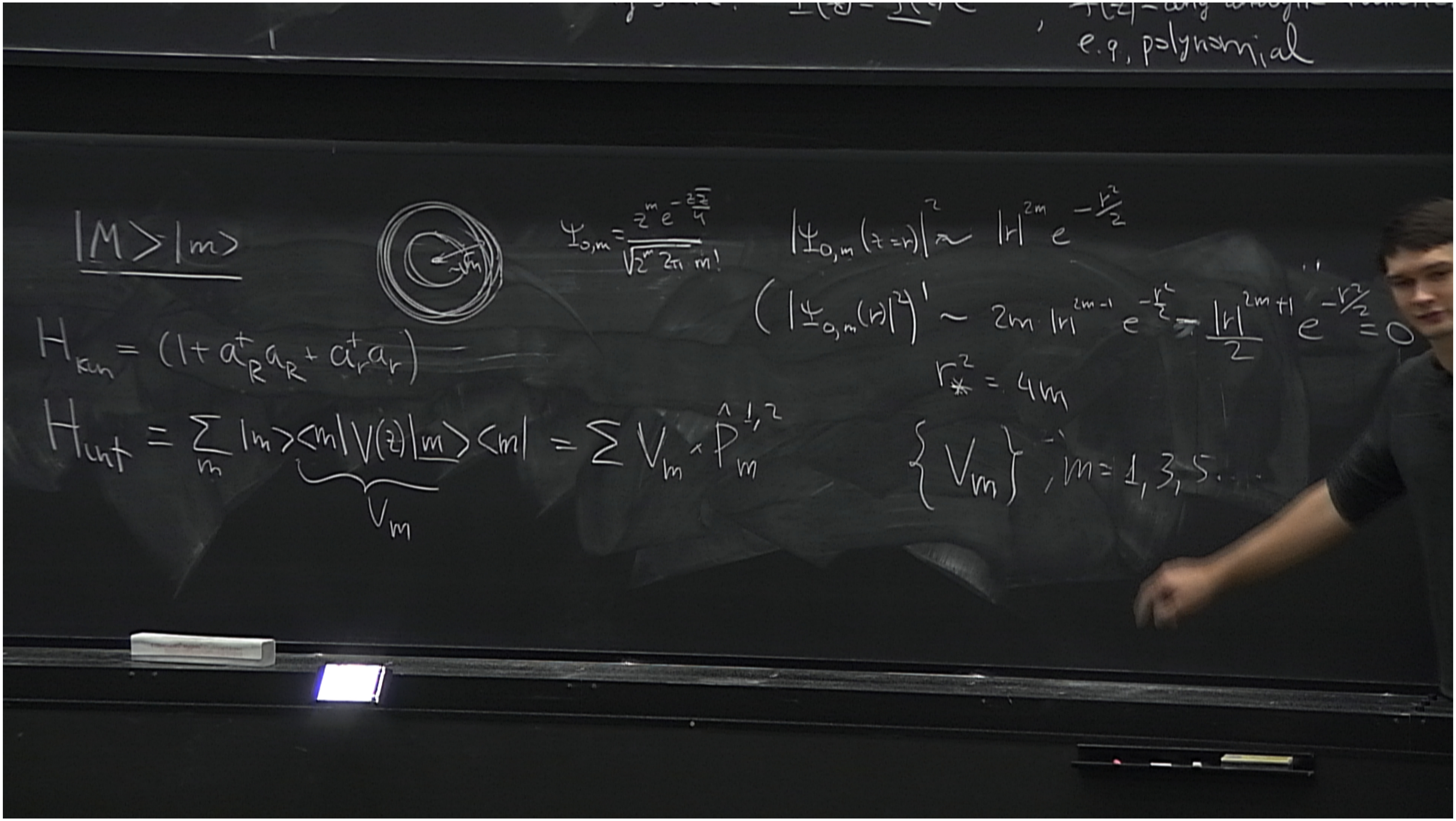


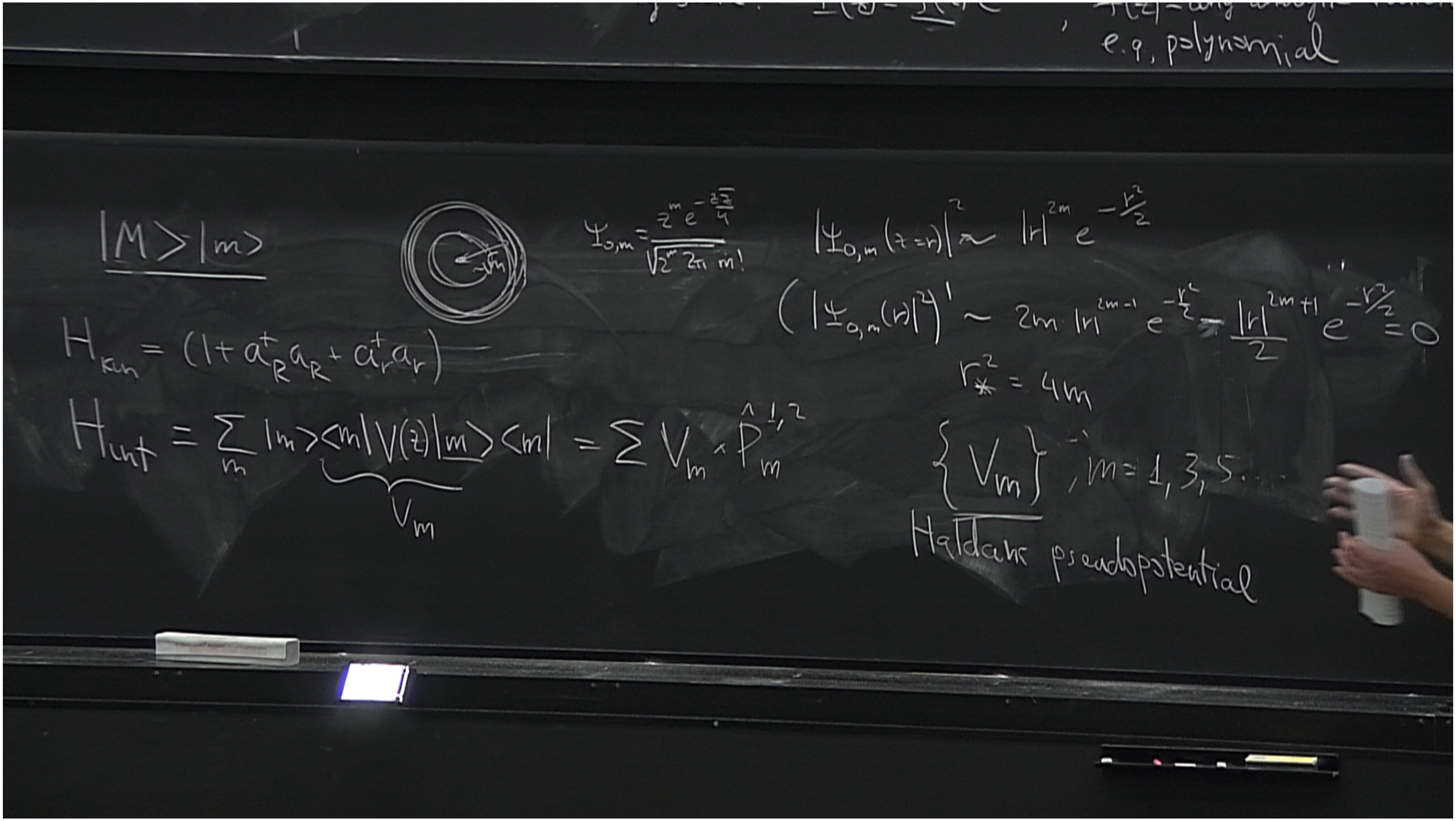
e.g. polynomial
$|M\rangle |m\rangle$
$\Psi_{0,m} = \frac{z^m e^{-\frac{z\bar{z}}{4}}}{\sqrt{2^m 2\pi\, m!}}$
$|\Psi_{0,m}(z=r)|^2 \sim |r|^{2m} e^{-r^2/2}$
$(|\Psi_{0,m}(r)|^2)' \sim 2m\,|r|^{2m-1} e^{-\frac{r^2}{2}} - \frac{|r|^{2m+1}}{2} e^{-r^2/2} = 0$
$r_*^2 = 4m$


e.g, polynomial
|M>|m>
$H_{kin} =$
$\Psi_{0,m} = \frac{z^m e^{-\frac{z\bar{z}}{4}}}{\sqrt{2^m 2\pi\, m!}}$
$|\Psi_{0,m}(z=r)|^2 \sim |r|^{2m} e^{-\frac{r^2}{2}}$
$(|\Psi_{0,m}(r)|^2)' \sim 2m\,|r|^{2m-1} e^{-\frac{r^2}{2}} - \frac{|r|^{2m+1}}{2} e^{-\frac{r^2}{2}} = 0$
$r_*^2 = 4m$


e.g. polynomial
|M>|m>
H_kin = (1 + a_R^† a_R + a_r^† a_r)
Ψ_0,m = z^m e^(-z z̄/4) / √(2^m 2π m!)
|Ψ_0,m (z=r)|^2 ~ |r|^2m e^(-r^2/2)
(|Ψ_0,m (r)|^2)' ~ 2m |r|^(2m-1) e^(-r^2/2) - |r|^(2m+1)/2 e^(-r^2/2) = 0
r_*^2 = 4m


e.g. polynomial
|M>|m>
Ψ_{0,m} = z^m e^{-z z̄/4} / √(2^m 2π m!)
|Ψ_{0,m}(z=r)|^2 ~ |r|^{2m} e^{-r^2/2}
(|Ψ_{0,m}(r)|^2)' ~ 2m |r|^{2m-1} e^{-r^2/2} - |r|^{2m+1}/2 e^{-r^2/2} = 0
r_*^2 = 4m
H_kin = (1 + a_R^† a_R + a_r^† a_r)
H_int = Σ_m |m><m|V(z)|m><m|


e.g. polynomial
|M>|m>
Hkin = (1 + a+R aR + a+r ar)
Hint = Σm |m><m|V(z)|m><m| = Σ Vm × P̂m
Vm
Ψ0,m = z^m e^(-zz̄/4) / √(2^m 2π m!)
|Ψ0,m(z=r)|² ~ |r|^2m e^(-r²/2)
r*² = 4m
{Vm}, m = 1, 3, 5...


e.g. polynomial
|M>|m>
Ψ0,m = z^m e^(-zz̄/4) / √(2^m 2π m!)
|Ψ0,m(z=r)|^2 ~ |r|^2m e^(-r^2/2)
(|Ψ0,m(r)|^2)' ~ 2m·|r|^(2m-1) e^(-r^2/2) - |r|^(2m+1)/2 e^(-r^2/2) = 0
r*^2 = 4m
Hkin = (1 + aR+ aR + ar+ ar)
Hint = Σm |m><m|V(z)|m><m| = Σ Vm × P̂m^1,2
Vm
{Vm}, m = 1, 3, 5 ...
Haldane pseudopotential