Title: Cosmic Magnetic Fields

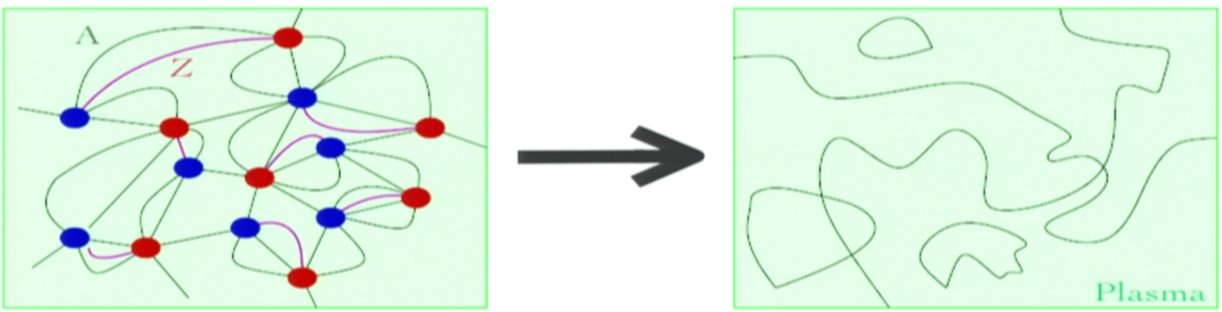
Date: Apr 30, 2012 02:00 PM

URL: http://www.pirsa.org/12040111

Abstract: I will describe the tight connection between cosmic baryon number and cosmic magnetic fields, and also some recent work on chiral magnetic effects in cosmology.

Magnetic fields from the electroweak phase transition

TV, 1991, 1994



Magnetic fields with large coherence scale are frozen-in.

Frozen-in magnetic fields

$$\vec{\nabla} \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \times \vec{B} = \frac{4\pi}{c}\vec{J} + \frac{1}{c}\frac{\partial \vec{E}}{\partial t}$$

$$\vec{J} = \sigma\left(\vec{E} + \frac{\vec{v}}{c} \times \vec{B}\right)$$

Therefore, large electrical conductivity

$$\frac{\partial \vec{B}}{\partial t} = \vec{\nabla} \times (\vec{v} \times \vec{B}) + \frac{c^2}{4\pi\sigma}\left[\nabla^2 \vec{B} - \frac{1}{c^2}\frac{\partial^2 \vec{B}}{\partial t^2}\right]$$

Implies flux conserved through comoving contour.

More accurately -- diffusion time scale is larger than Hubble time scale for fields on cosmological length scales.

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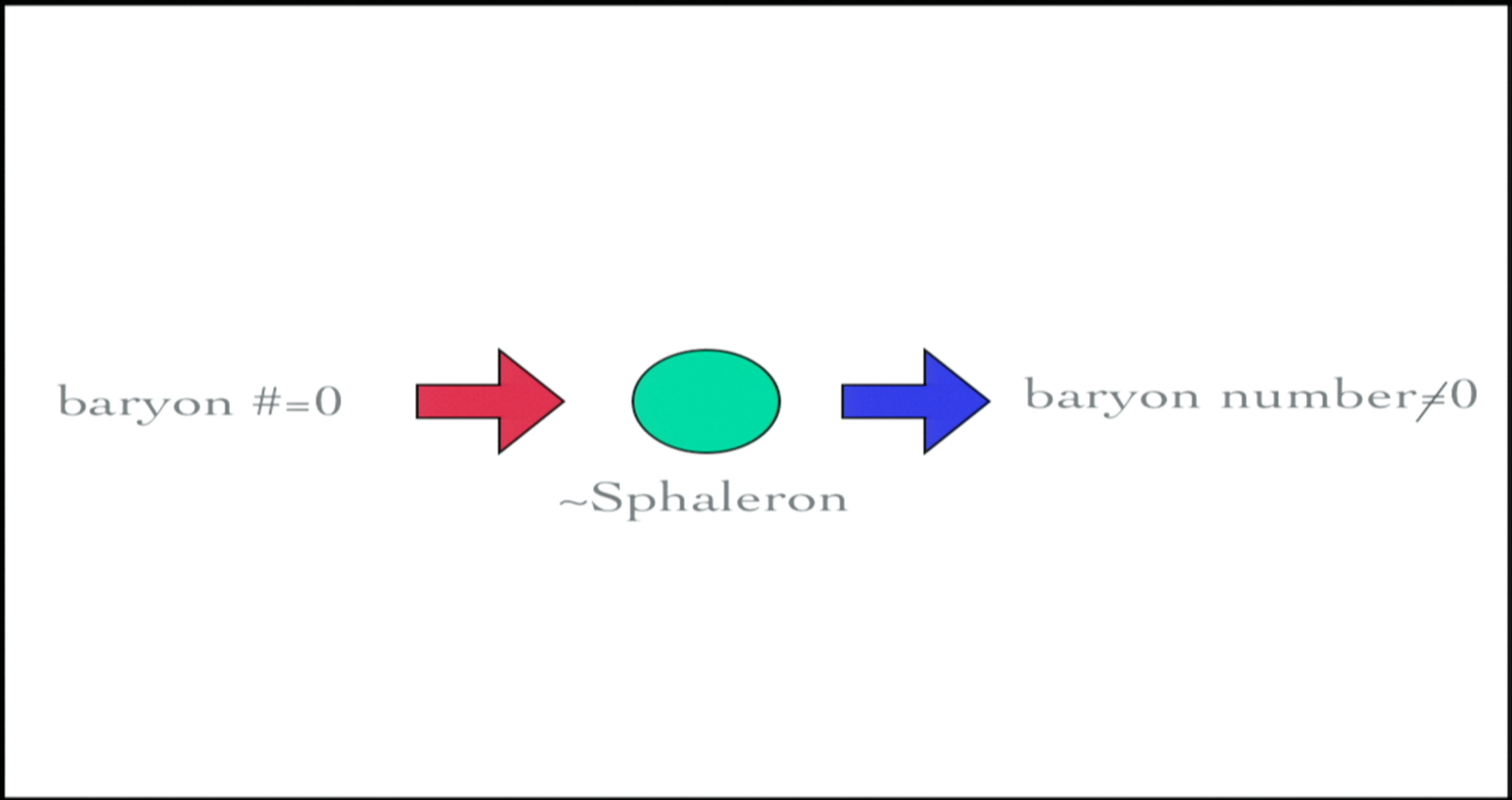
baryon #=0
~Sphaleron
baryon number≠0

Sphaleron decay

Copi, Ferrer, TV & Achucarro
Y-Z. Chu, J. Dent & TV

A decay path for the sphaleron is known.

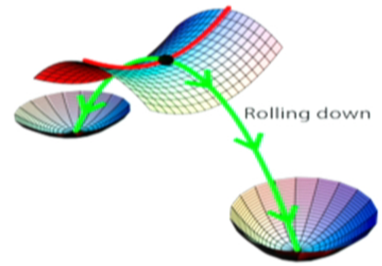

Image- http://spie.org/x31524.xml?ArticleID=x31524

Therefore currents can be calculated up to one function (flow velocity).

$$(\partial_t^2 - \nabla^2) A^\mu = J^\mu$$

Calculate magnetic helicity generation along this path:

$$\mathcal{H}(t) = \int d^3x \; \mathbf{A} \cdot \mathbf{B}$$

Asymptotic helicity is independent of flow velocity.

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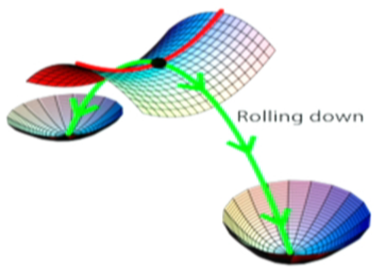

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Evolution of helicity in vacuum

Jackiw & Pi, 1999:

Consider Maxwell's equations with $\vec{E}(t=0)=0$.

$$\vec{A}(t,\vec{x}) = \int \frac{d^3x}{(2\pi)^3}\cos(kt)e^{-i\vec{k}\cdot\vec{x}}\vec{a}(\vec{k})$$

$$\mathcal{H} = -i\int \frac{d^3k}{(2\pi)^3}\cos^2(kt)\epsilon^{ijk}k^i a_j(\vec{k})a_k^*(\vec{k})$$

$$= -\frac{i}{2}\int \frac{d^3k}{(2\pi)^3}(1+\cos(2kt))\epsilon^{ijk}k^i a_j(\vec{k})a_k^*(\vec{k})$$

Therefore: $\mathcal{H}(t=\infty) = \frac{1}{2}\mathcal{H}(t=0)$

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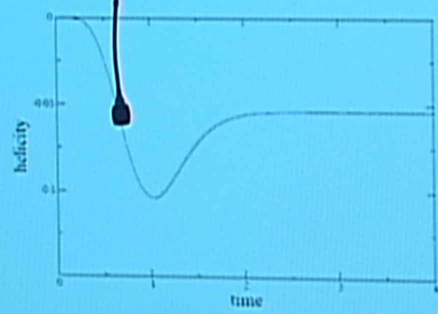
Magnetic helicity



Helicity is conserved at late times. $\mathcal{H}(\infty) \sim -\frac{\sin^2\theta_w}{g^2}$

Baryon production implies left-handed helicity.

Cosmological magnetic helicity

J. Cornwall
TV

$$\text{Every } \Delta B \implies \Delta\mathcal{H}$$

$$\implies \boxed{h \approx -\# \frac{n_b}{\alpha}}$$

Independent of details of electroweak baryogenesis scenario.

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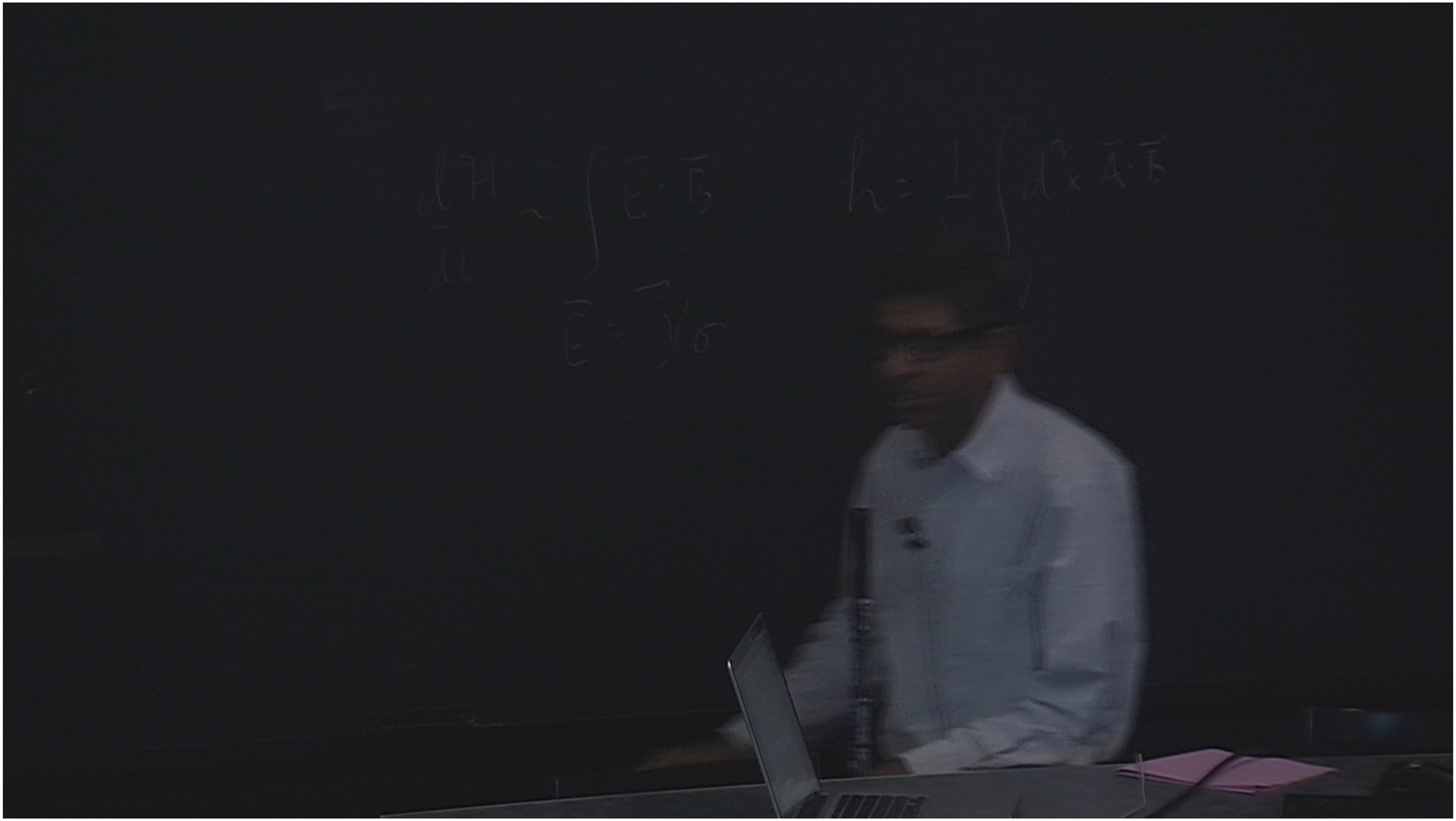
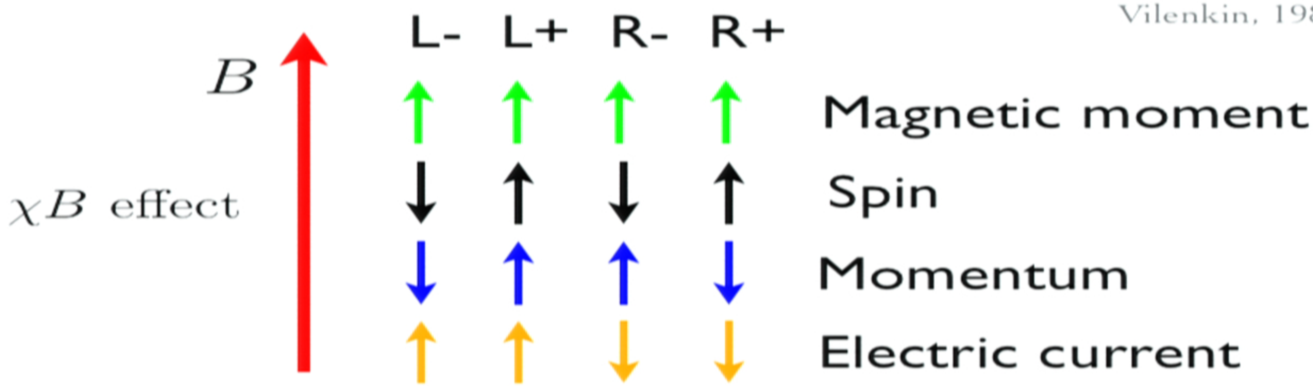


Chiral Magnetic Effect

Vilenkin, 1980



$$J_{\chi B} \propto [n(e_L^-) - n(e_R^+)] - [n(e_R^-) - n(e_L^+)]$$

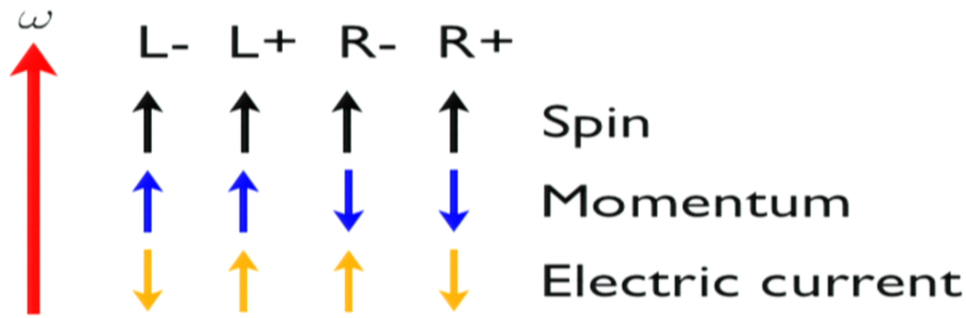
$$\mathbf{J}_{\chi B} = \frac{e^2}{2\pi^2} \Delta\mu \, \mathbf{B}$$

For massive particles, flipping between chiral states can damp the current.

Chiral-vorticity Effect

Vilenkin, 1979

$\chi\omega$ effect



$$J_{\chi\omega} \propto [n(e_L^-) + n(e_R^+)] - [n(e_R^-) + n(e_L^+)]$$

$$\mathbf{J}_{\chi\omega} = \frac{e}{4\pi^2}\Delta\mu^2\ \omega$$

$\chi - \mathbf{B}$ Effect at Electroweak

Boyarsky, Frohlich & Ruchayskiy

Also see: Joyce & Shaposhnikov

Chiral anomaly:

$$\frac{d(n_L - n_R)}{dt} = -\frac{\alpha}{\pi}\frac{dh}{dt} = \frac{2\alpha}{\pi V}\int_V d^3x\ \mathbf{E}\cdot\mathbf{B}$$

Maxwell & Ohm & \chi-B:

$$\nabla \times \mathbf{B} = \sigma\mathbf{E} + \frac{\alpha}{\pi}\Delta\mu(t)\mathbf{B}$$

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Evolution of modes

$$\frac{\partial \mathcal{H}_k}{\partial \eta} = -\frac{2k^2}{\sigma_c}\mathcal{H}_k + \frac{\alpha}{\pi}\frac{k\Delta\mu}{\sigma_c}\mathcal{H}_k$$ Maxwell eq.

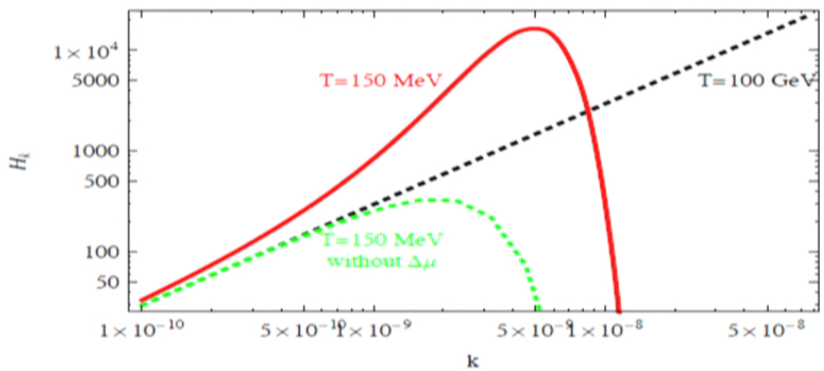
$$\frac{d(\Delta\mu)}{d\eta} = -(c_\Delta \alpha)\int dk \frac{\partial \mathcal{H}_k}{\partial \eta} - \Gamma_f \Delta\mu$$ chiral anomaly eq.

$\sigma_c \equiv \sigma/T$, $\Delta\mu \equiv \mu_L - \mu_R$, $c_\Delta \sim 1$, $\Gamma_f \equiv$ chirality flipping rate

Solve with B injected due to baryogenesis.

$\chi - \mathbf{B}$ Amplification of Magnetic Helicity

Boyarsky, Frohlich & Ruchayskiy



What is the role of fluid velocity?

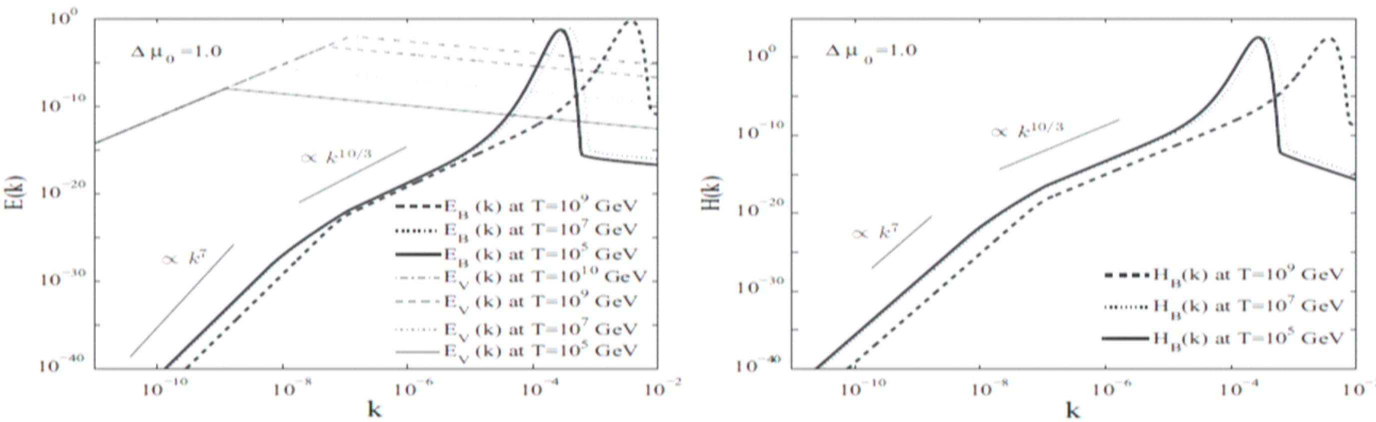
Chiral Effects on B

Hiroyuki Tashiro, TV & Alex Vilenkin (in progress)

$$\partial_t \mathbf{B} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \gamma_D \nabla^2 \mathbf{B} + \gamma_\omega \nabla \times \omega + \gamma_B \nabla \times \mathbf{B}$$

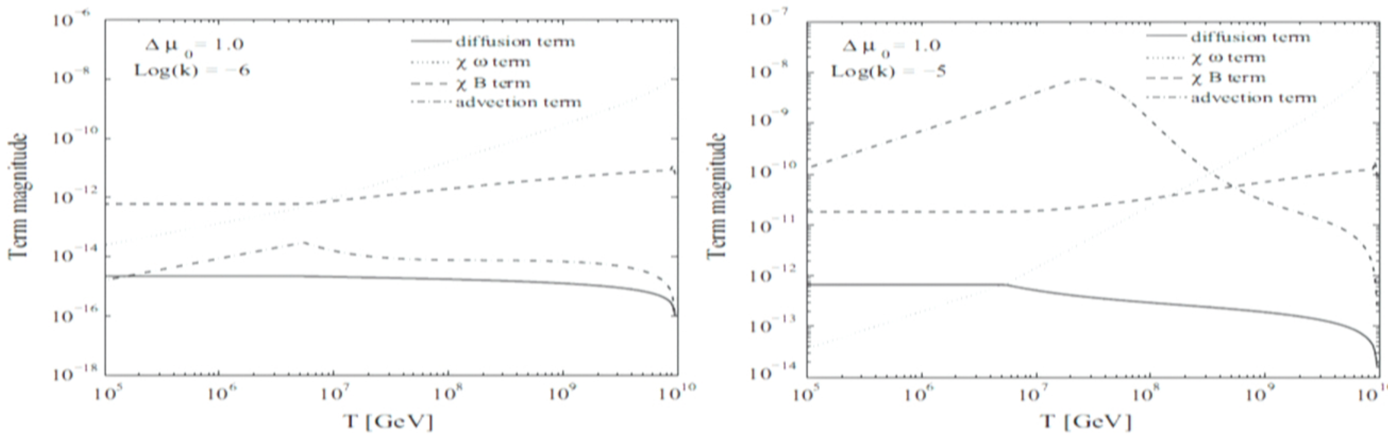
$$\gamma_D = \frac{1}{4\pi\sigma} \ , \quad \gamma_\omega = \frac{e\Delta\mu^2}{4\pi^2\sigma} \ , \quad \gamma_B = \frac{e^2\Delta\mu}{2\pi^2\sigma}$$

Evolution without advection



Peak shifts to larger length scales with time.

Self-consistency check



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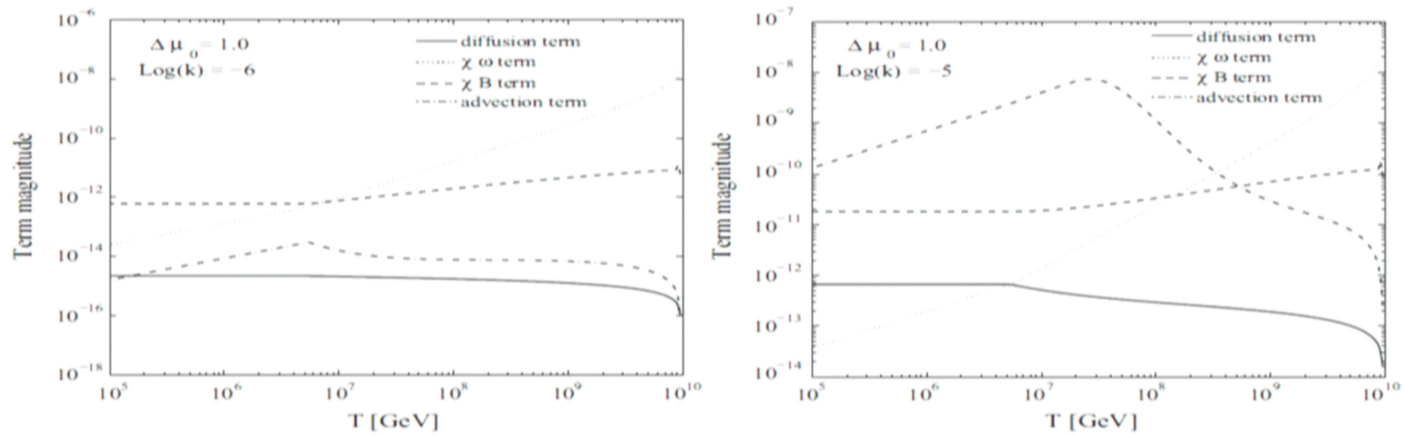

Inverse Cascade

Magnetic helicity can cause an "inverse cascade" i.e. transfer power from small to large length scales.

MHD simulations & models in flat spacetime:

Numerical: $\xi \propto t^{1/2}$ — Christensson, Hindmarsh & Brandenburg, 2005

Analytical: $\xi \propto t^{2/3}$ — D. Biskamp, 1993; P. Olesen, 1997; D.T. Son, 1999; Field & Carroll, 2000

Translate these exponents to expanding universe by interpreting t as the conformal time.

$$\xi \propto \tau^{\alpha} \propto t^{\alpha/2} \quad \text{radiation}$$

$$\xi \propto t^{\alpha/3} \quad \text{matter}$$

Also see: Kahniashvili, Brandenburg, Tevzadze & Ratra, 2010

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Coherence Scale

Length scale grows by Hubble expansion and inverse cascade, in radiation- and matter-dominated epochs.

$$\begin{aligned} \xi_{\rm eq} &= \xi_{\rm inj} \left(\frac{a_{\rm eq}}{a_{\rm inj}}\right)^{1+\alpha} \left(\frac{a_0}{a_{\rm eq}}\right)^{1+\alpha/2} \\ &\lesssim (1\ {\rm cm}) \left(\frac{T_{\rm ew}}{1{\rm eV}}\right)^{1+2/3} 10^{4+1} \\ &\sim 10^{20}\ {\rm cm} \\ &\sim 0.1\ {\rm kpc} \end{aligned}$$

Expect - power law fall off on length scales larger than 0.1 kpc.

Vanishing correlation outside the null cone gives $k^{5/2}$ fall off. Durrer & Caprini

Random superposition of magnetic dipoles gives $k^{3/2}$ fall off. Jedamzik & Sigl

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Field Strength: helicity alone (no chiral effects, no antibaryons)

$$|h| \sim n_b \implies \xi B^2 \sim n_b$$

$$B \sim \sqrt{\frac{n_b}{\xi}}$$

$$B(t_0) \sim 10^{-21}\ \mathrm{G}$$

Better to think of this as:

$$h \sim (10^{-21}\ \mathrm{G})^2 - \mathrm{kpc}$$

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Field Strength Re-visited

Sphaleron transitions produce baryons and anti-baryons.
CP violation implies a slight excess of baryons.

The baryons and anti-baryons annihilate but magnetic fields are spread out and cannot annihilate completely.

$$B(t_0) \approx 10^{-21}\mathrm{G} \left(\frac{N_b + N_{\bar{b}}}{N_b - N_{\bar{b}}} \right)^{\gamma}$$

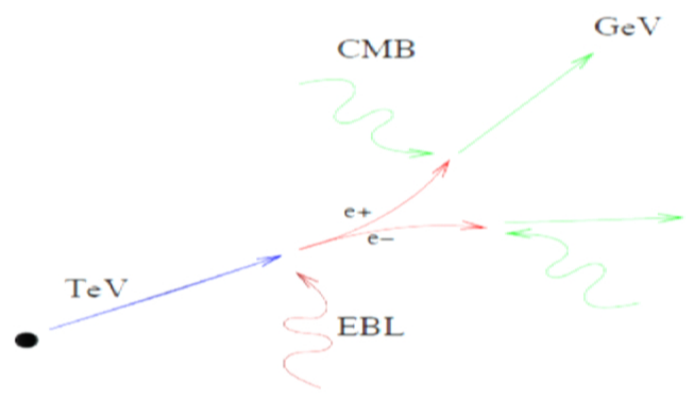
In standard model, CP violation gives -- $\frac{N_b + N_{\bar{b}}}{N_b - N_{\bar{b}}} \approx 10^{20}$

If $\gamma = 1/2 :$ $B(t_0) \approx 10^{-11}$ G

(In a baryogenesis model that actually works, CP violation would be larger. Then the CP enhancement would be smaller but gamma could compensate.)

Observations

TeV gamma ray sources should have GeV halos in the absence of magnetic fields. Absence of GeV halos indicates a *lower* bound on intergalactic magnetic fields.



Claim:
FERMI data gives *lower* bound of ~10^{-16} Gauss magnetic field.

Neronov & Vovk, 2010

Claim:
Measured magnetic field at ~3x10^{-16} Gauss.

Ando & Kusenko, 2010

Counterpoint: PSF of telescope.

Neronov, Semikoz, Tinyakov & Tkachev, 2010

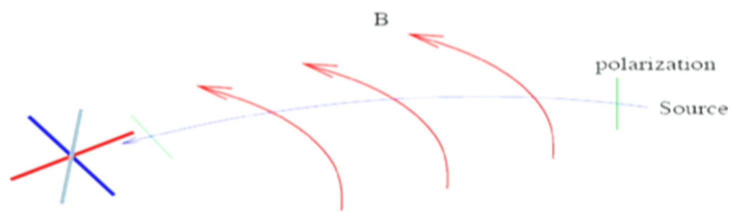
Claim:
MHD instabilities can give rise to absence of GeV gamma rays.

Broderick, Chang & Pfrommer, 2011

B in CMB

(Earlier literature mostly focused on very coherent B.)

Pogosian, Yadav, Ng & TV



$$\Delta\theta = \lambda^2 \ \mathrm{RM} = \frac{3\lambda^2}{2\pi e} \int \dot{\tau}(\mathbf{x})\mathbf{B} \cdot d\mathbf{l}$$

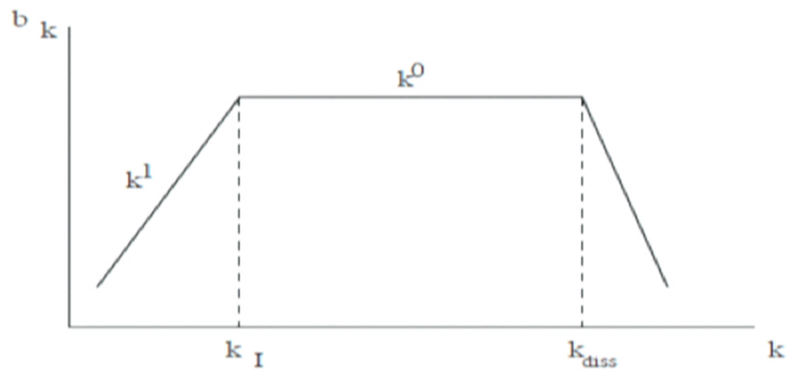
Faraday rotation of CMB polarization converts E-modes to B-modes.

Frequency dependent.

Correlated with temperature fluctuations.

Grows at small angular scales.

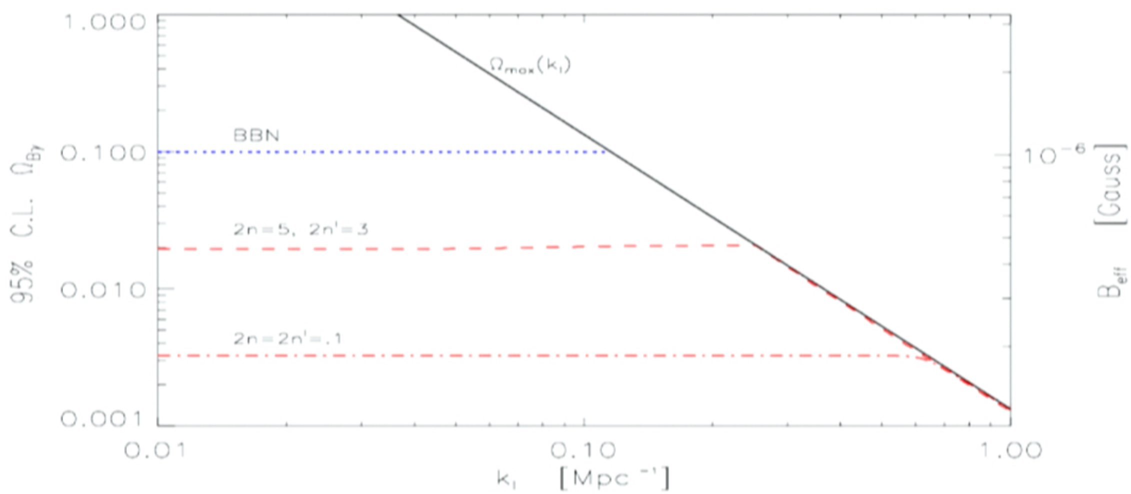
Spectrum of B



Jedamzik & Sigl
Durrer & Caprini

Current Constraints

Pogosian, Yadav, Ng & TV



Omega_{max} motivated by Alfven wave dissipation.

Jedamzik, Katalinic & Olinto
Kahniashvili, Tevzadze, Sethi, Pandey & Ratra

Coming Opportunities

- Tight connection between baryogenesis and magnetogenesis.
- Fields today can have "interesting" amplitude on astrophysical (though not Mpc) scales.
- Properties of cosmic magnetic fields can be a window to the electroweak phase transition.
- TeV gamma rays, cosmic rays, and CMB can probe primordial magnetic fields.

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Open Questions

How do magnetic fields evolve in the early universe?

How well will future experiments at low frequency and high angular resolution be able to constrain B?
(ACT, SPT, BICEP, QUAD....)

How can one directly detect magnetic helicity?
(Indirect detection possible using spectral exponents.)

Cosmic rays: Kahniashvili & TV

What are the implications of strong, small-scale fields for the first generation of structure?