Title: What Can Stellar Kinematics Tell Us About Dark Matter in Dwarf Galaxies?

Date: Apr 10, 2012 11:00 AM

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Abstract: Dwarf galaxies are the most know dark matter dominated luminous objects in Universe. Observing the line of sight velocity and position of stars in Milky way satellites, and assuming the dark matter potential and a specific configuration of stellar orbits, one can obtain the mass profile of dark matter in galaxies. In this talk I will show that by considering a generic case of phase-space density as a function of energy and angular momentum and by relaxing the specific choice of orbital configuration (which is assumed in literature), we can find the dark matter potential and mass profile in this general case and restudy the challenges of Cold Dark Matter(CDM) paradigm like core-cusp or missing satellite problem.

Milky Way Satellites

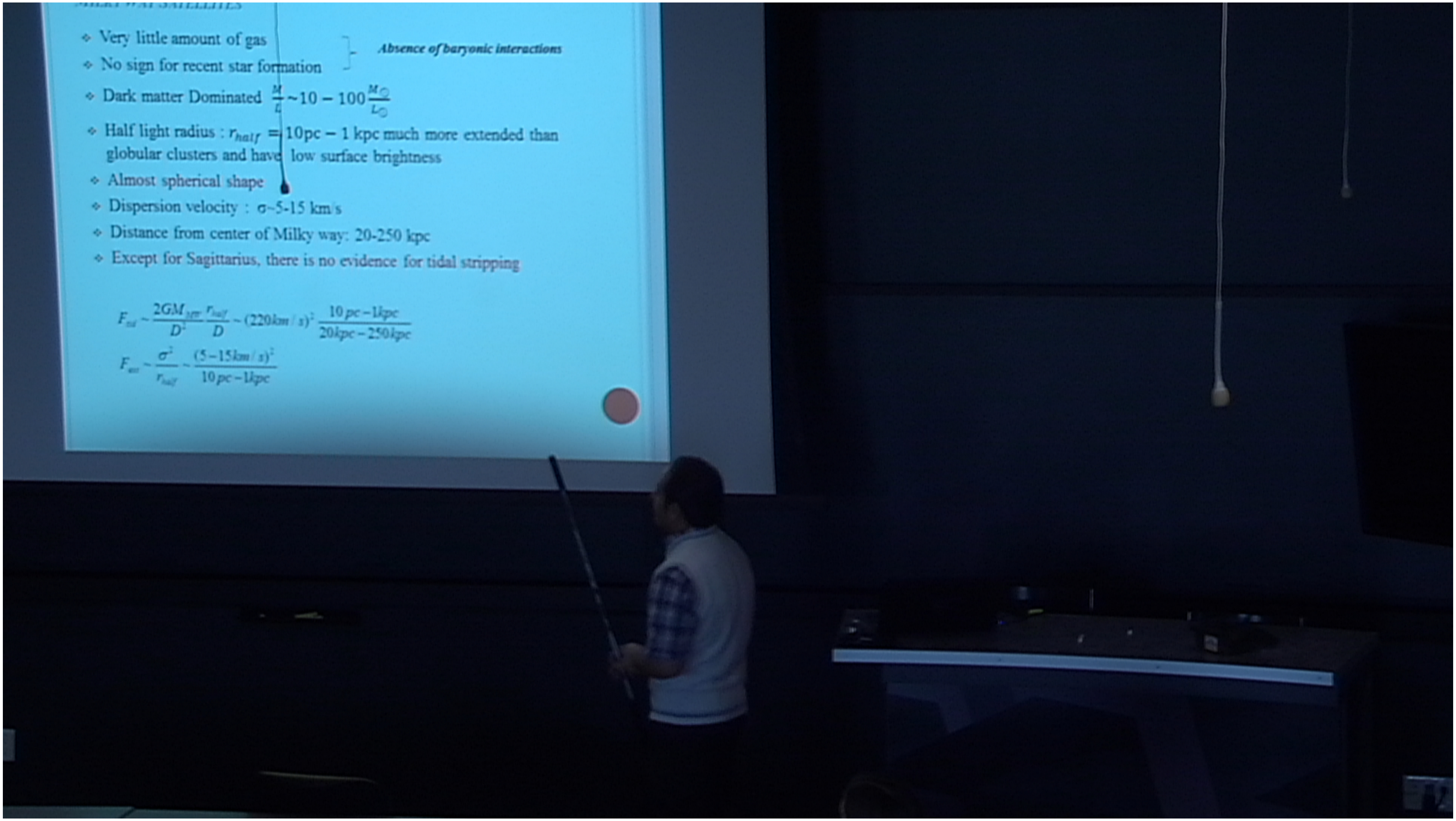
- Very little amount of gas
- No sign for recent star formation

} ***Absence of baryonic interactions***

- Dark matter Dominated $\frac{M}{L} \sim 10 - 100 \frac{M_\odot}{L_\odot}$
- Half light radius : $r_{half} = 10\text{pc} - 1\text{ kpc}$ much more extended than globular clusters and have low surface brightness
- Almost spherical shape
- Dispersion velocity : σ~5-15 km/s
- Distance from center of Milky way: 20-250 kpc
- Except for Sagittarius, there is no evidence for tidal stripping

$$F_{tid} \sim \frac{2GM_{MW}}{D^2}\frac{r_{half}}{D} \sim (220km/s)^2 \frac{10\,pc - 1kpc}{20kpc - 250kpc}$$

$$F_{ext} \sim \frac{\sigma^2}{r_{half}} \sim \frac{(5-15km/s)^2}{10\,pc - 1kpc}$$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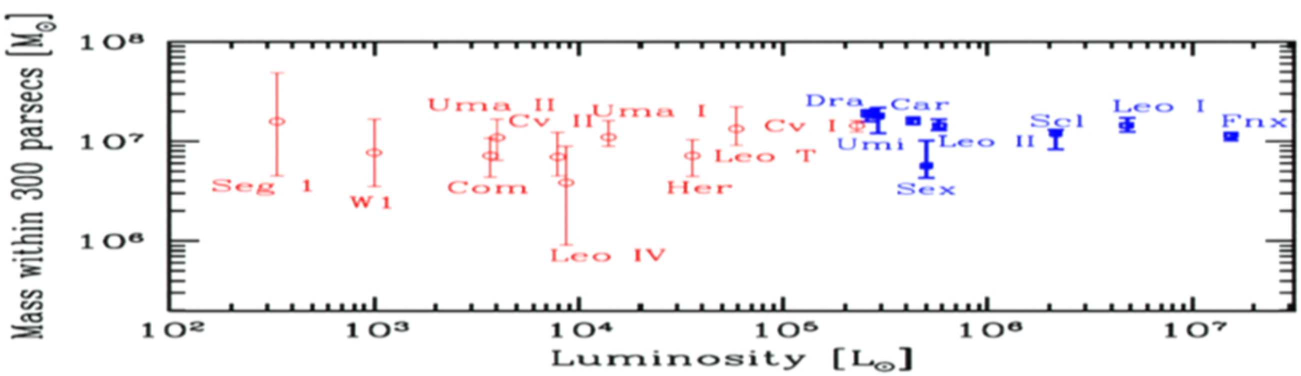

Strigari et al. , Nature 454:1096-1097, 2008

Outline :

- Dwarf Spheroidal Galaxies (Milky Way Satellites)

Why are we interested in studying them?

- Kinematics of stars and questions in Astrophysics and cosmology
- Non parametric Phase-Space density of stars
  and new mass profile for Milky way satellites
- Challenges of Cold Dark Matter Paradigm
- Conclusion and Future Prospects

THE MAJOR GOAL OF KINEMATICAL MODELING?

Major goal of kinematical modeling of astrophysical self gravitating systems:

1)Total Mass distribution (Visible + Dark Matter)

2)3D velocity mean + dispersions (Orbital properties of tracer)

OR

What is the Phase Space density of matter $f(x,v)$?

Binney and Tremaine(2008)- *Galactic Dynamics*

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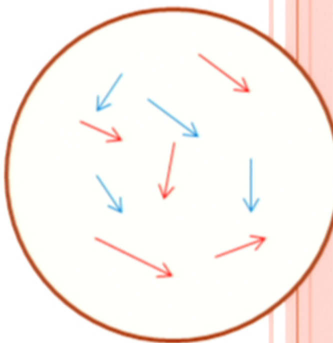
Observables:

1) Surface density (Surface Brightness)

2)Line of sight velocity distribution



Kinematical Modeling

- 1)Parametric functions for distribution of matter and anisotropy function
- 2) Non-parametric anisotropy function/ General form of phase-space density

In order to study the dynamic of phase-space density

We use the Collision less Boltzman Equation (CBE), (Vlasov/ Liouville eq.)

6D – incompressibility is phase-space of (x,v)

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla f - \nabla\Phi \cdot \frac{\partial f}{\partial \mathbf{v}} = 0$$

3D velocity

Gravitational potential

Jeans Equation: for a system in local Equilibrium

Space density of tracer

Anisotropic dynamical pressure

$$\frac{\partial \overline{\mathbf{v}}}{\partial t} + (\overline{\mathbf{v}} \cdot \nabla)\overline{\mathbf{v}} = -\nabla\Phi - \frac{1}{\rho}\nabla \cdot \left(\rho\, \boldsymbol{\sigma}^{\mathbf{2}}\right)$$

$\overline{v_i v_j} - \overline{v_i}\,\overline{v_j}$

Dispersion tensor

Kinematical modeling and Dwarfs

- Absence of streaming motion
- Dynamical equilibrium
- Spherical symmetry

Spherical stationary non-streaming Jeans Equation:

Total Mass Profile

$$r\frac{d(\rho_\star \sigma_r^2)}{dr} = -\rho_\star(r)\frac{GM(r)}{r} - 2\beta(r)\rho_\star\sigma_r^2.$$

Radial velocity

Tracer velocity (Anisotropy Parameter):

$$\beta(r) = 1 - \frac{\sigma_\theta^2 + \sigma_\phi^2}{2\,\sigma_r^2} = 1 - \frac{\sigma_\theta^2}{\sigma_r^2}$$

$\beta = 1$ Radial orbits

$\beta = 0$ Isotropic orbits

$\beta \to -\infty$ Circular orbits

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Total Mass Profile ($M(r)$)

Radial velocity (σ_r)

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KINEMATICAL MODELING

- In solving the Jeans equation
  Now we have the MASS-ANISOTROPY degeneracy.

Radial velocity

Projected line of sight velocity

$$\sigma_t^2(R) = \frac{2}{I_\star(R)} \int_R^\infty \left(1 - \beta \frac{R^2}{r^2}\right) \frac{\rho_\star \sigma_r^2 r}{\sqrt{r^2 - R^2}} dr.$$

$$\beta(r) = (\beta_\infty - \beta_0) \frac{r^2}{r_\beta^2 + r^2} + \beta_0.$$

Stellar distribution

- Stellar distribution(king model- Plummer model…)

$$I_{\rm pl}(R) = \frac{4}{3} \frac{\rho_0 r_{\rm pl}}{[1 + (R/r_{\rm pl})^2]^2}. \qquad \rho_{\rm pl}(r) = \frac{\rho_0}{[1 + (r/r_{\rm pl})^2]^{5/2}}.$$

Smooth Parameterization for anisotropy parameter:
It can not capture the possible features of phase-space distribution

PHASE- SPACE DENSITY , ANISOTROPY RELATIONS

The anisotropy parameter define the dependence of phase-spaced density to angular momentum:

- *(a) Ergodic DF: isotropic orbits*

$$\beta = 0 \rightarrow f = f(E)$$

Eddington(1916) formula

$$f(\epsilon) = \frac{1}{\sqrt{8}\pi^2} \int_\epsilon^0 \frac{d^2\rho_\star}{d\Psi^2} \frac{d\Psi}{\sqrt{\Psi - \epsilon}},$$

- *(b)Models with constant anisotropy,*

$$\beta = cte \rightarrow f(E, L) = L^{-2\beta} f_1(E)$$

- *(c) Osipkov(1979)-Merritt (1985)models:*

Radial dependence from

"Radially biased"

$$\beta(r) = \frac{1}{1 + \frac{r_a^2}{r^2}} \rightarrow f = f(E - \frac{L^2}{2r_a^2})$$

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DEGENERACY OF ISOTROPY-MASS

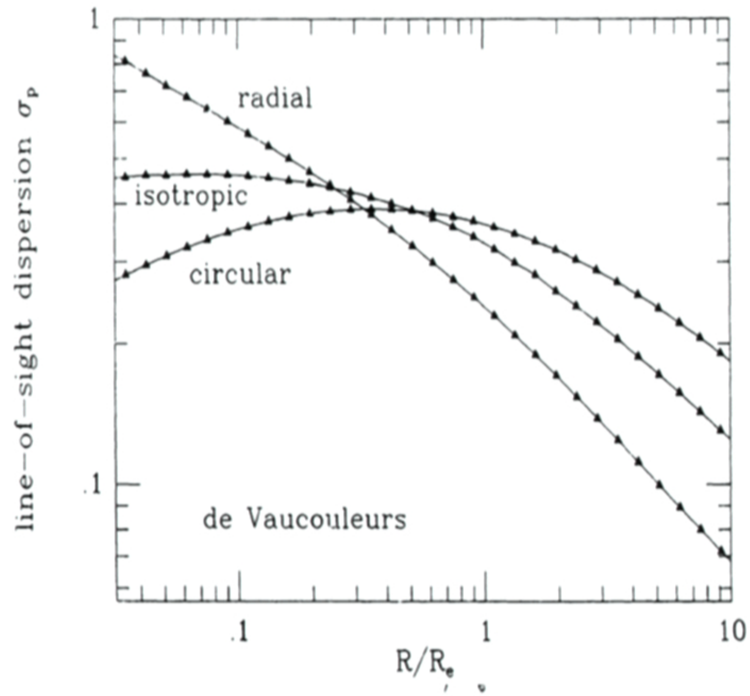

FIG. 1.—The root mean square line-of-sight dispersion σ_p as a function of projected radius R for a galaxy with constant mass-to-light ratio which obeys de Vaucouleurs's surface brightness law (eq. [11]). The dispersion is measured in units of $(GM/R_e)^{1/2}$. The curves are for galaxies with (*a*) isotropic orbits; (*b*) circular orbits; (*c*) nearly radial orbits (cf. eq. [45]).

Richstone and Tremaine :Astrophysical J. 1984

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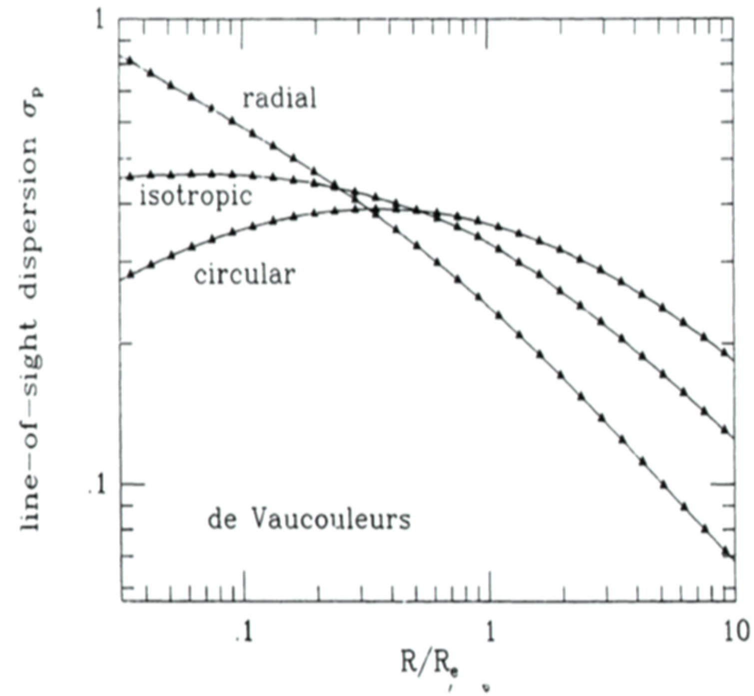

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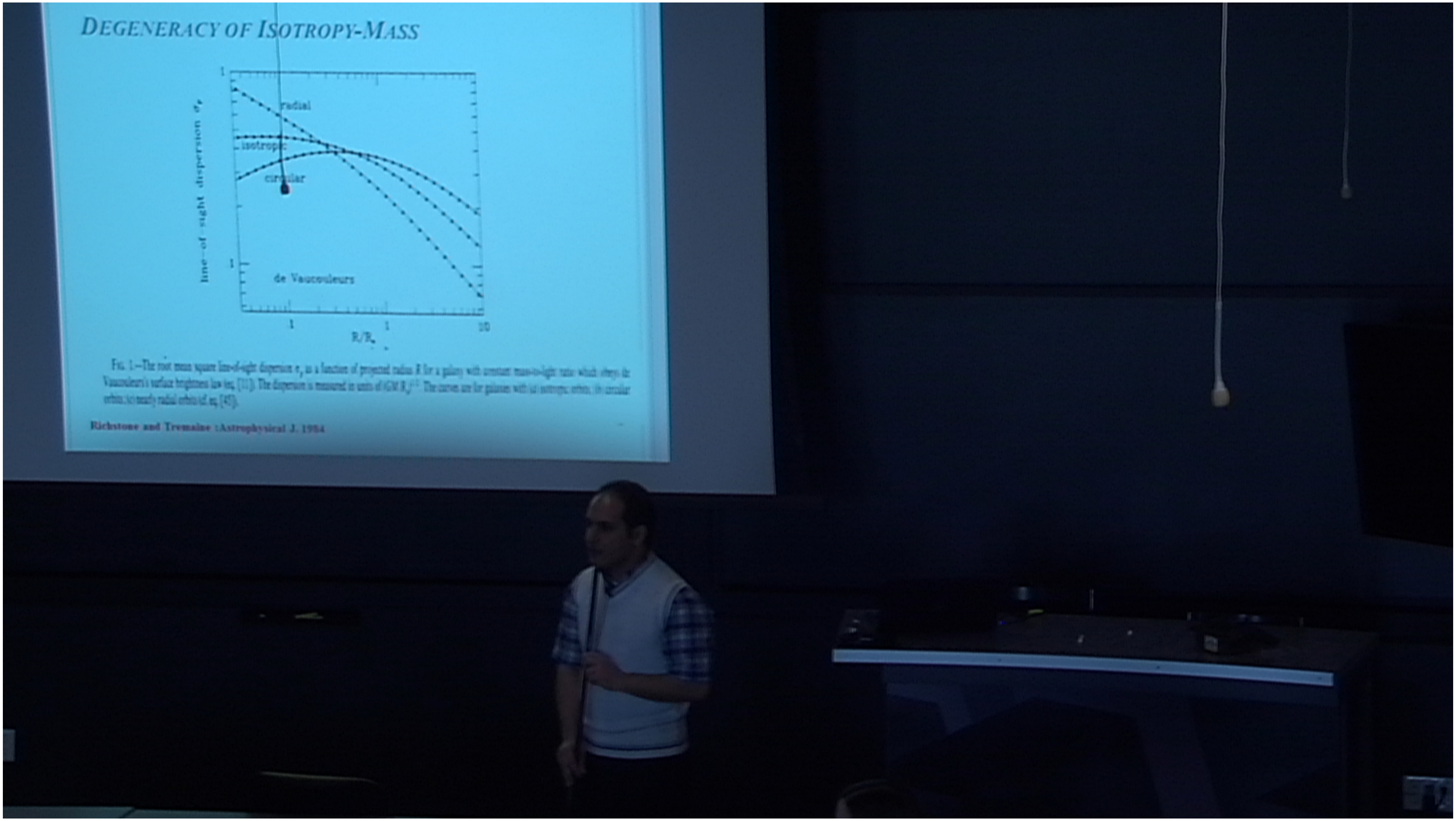
DEGENERACY OF ISOTROPY-MASS
radial
isotropic
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R/R
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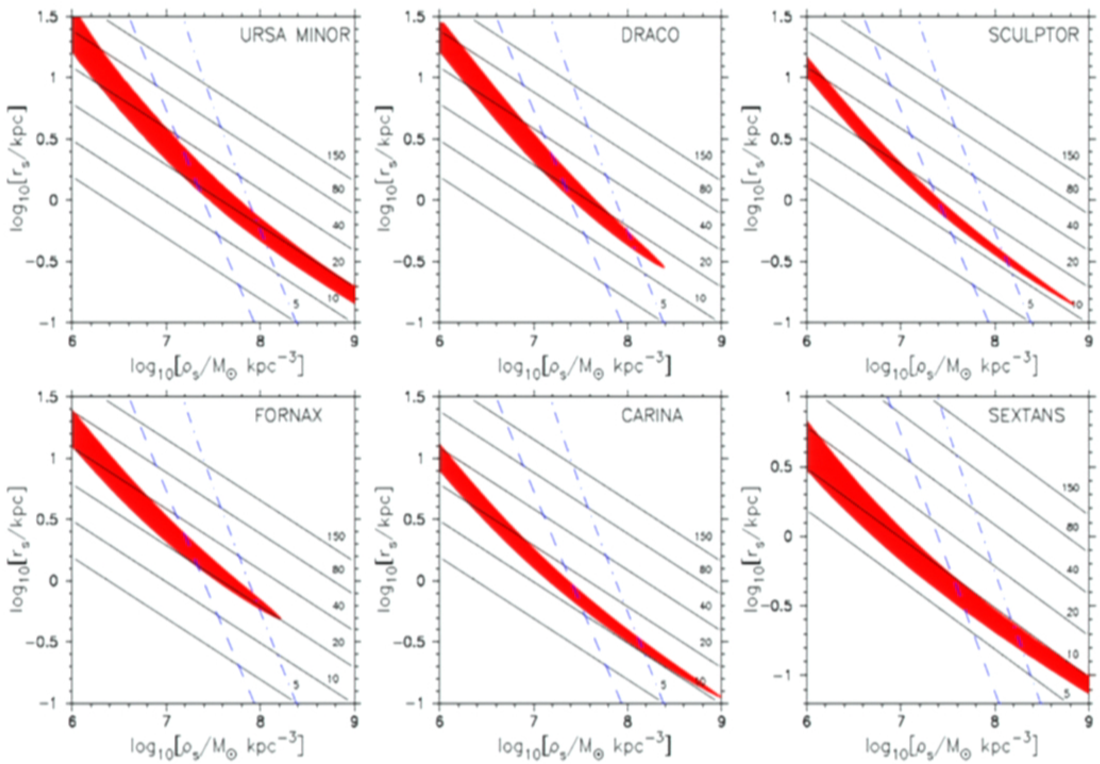
PRECISE MEASUREMENTS OF DWARF GALAXIES: CONSEQUENCES

- Constraints on profile of Dark matter → Mass profile

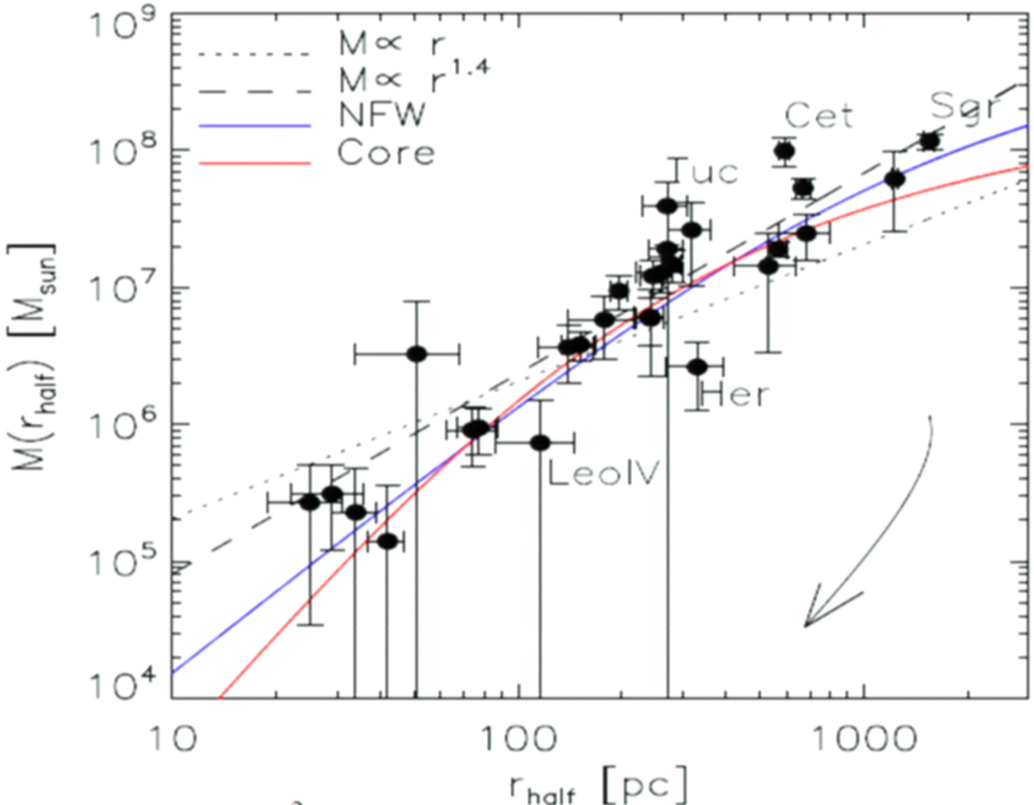


NFW-density profile

$$\rho = \frac{\rho_s}{\frac{r}{r_s}(1+\frac{r}{r_s})^2}$$

L.E. Strigari et al., Phys. Rev. D 76, 083526, (2007)

PRECISE MEASUREMENTS OF DWARF GALAXIES: CONSEQUENCES



$$M(r_{half}) \propto r_{half}\sigma_r^2$$

M.G. Walker e al., Astrophys.J.704:1274-1287,2009

$$\rho(r) = \rho_0\left(\frac{r}{r_0}\right)^{-\gamma}\left[1+\left(\frac{r}{r_0}\right)^{\alpha}\right]^{\frac{\gamma-3}{\alpha}},$$

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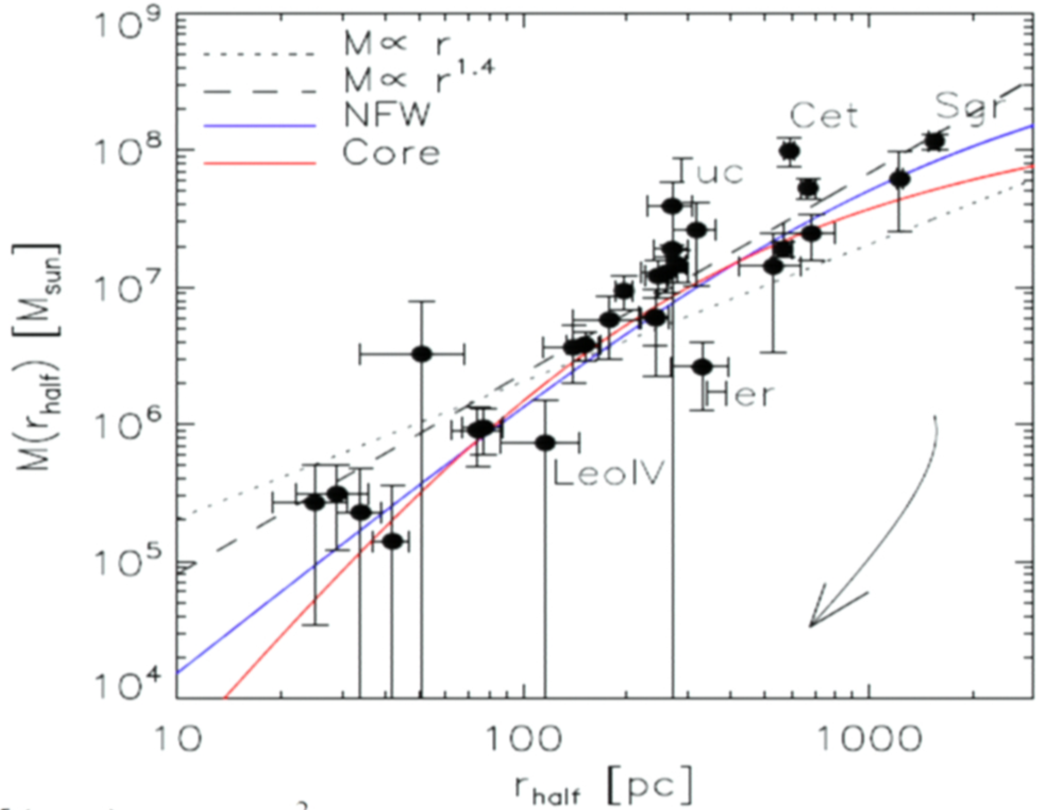

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APPROACHES TO KINEMATICAL MODELING

- 1)Parametric functions for distribution of matter and anisotropy function
- 2) Non-parametric anisotropy function/ General form of phase-space density

Previous Attempts:

a)Deriving the mass distribution of M87 from globular clusters

Xiaoan Wu and Scott Tremaine

Astrophys.J.643:210-221,2006

b)Axisymmetric two/three integral model of the dSphs

- **X. -A. Wu, astro-ph/0702233 [ASTRO-PH].**

c) Schwarzschild Method

Orbit based metod- Orbit Library

J. Jardel and K. Gebhardt, Astrophys. J. 746, 89 (2012) [arXiv:1112.0319 [astro-ph.CO]

OUR APPROACH

- Spherical Symmetric case, Equilibrium

JEANS THEOREM: Phase space density is a function Energy and angular Momentum.

$$f = f(E,L)$$

- We consider a NFW-profile for DM:

$$\Phi_{DM}(r) = -\frac{4\pi G \rho_s r_s^3 \ln(1 + r/r_s)}{r},$$

- For stars we assume the Plummer profile with the potential:

$$\Phi_*(r) = -\frac{GM_*}{\sqrt{r^2 + r_{pl}^2}}$$

$$M_* = \frac{4\pi}{3}\rho_{*0} r_{pl}^3$$

$$\frac{M_*}{L_*} = 1$$

- ***OBSERVABLE: The probability of finding stars in*** (R, v_z)

$$n(R, v_z; X) = \int dz dv_R dv_\phi f(E,L)$$

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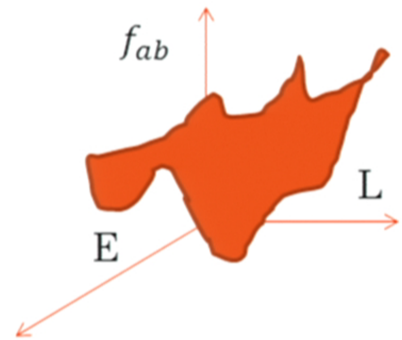
GENERAL CASE OF PHASE SPACE DENSITY



*Binning in the Energy and Angular momentum
*Non- smooth phase-space density

$$f(E,L) = \sum_{ab} f_{ab}\delta(E - E_a)\delta(L - L_b).$$

Number of binning = number of free parameters

- Now we can rewrite the observable density of stars, by plugging the phase space density function definition as :

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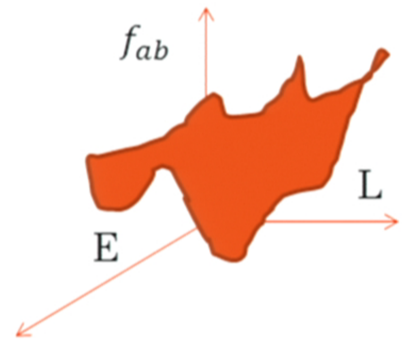

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FINDING THE BEST PARAMETERS OF MODEL

- As the distribution of stars are discrete and independent, so we have to minimize the Poisson χ^2:

$$\chi^2 = -2\sum_s \ln n(R_s, v_{z,s}) + 2\int d^2R dv_z n(R, v_z).$$

- The second term is the total number of stars, which can obtained from integration of phase-space density:

$$\int d^2R dv_z n(R, v_z) = \int d^3x d^3v f(E, L) = \sum_{ab} f_{ab} N_{ab},$$

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WHAT ABOUT STARS?

- Adding the light profile of stars, assuming the M/L~1:
- We can find the surface density of stars:

$$\Sigma(R) = \int dv_z n(R, v_z)$$

- Also we can write the light profile terms as:

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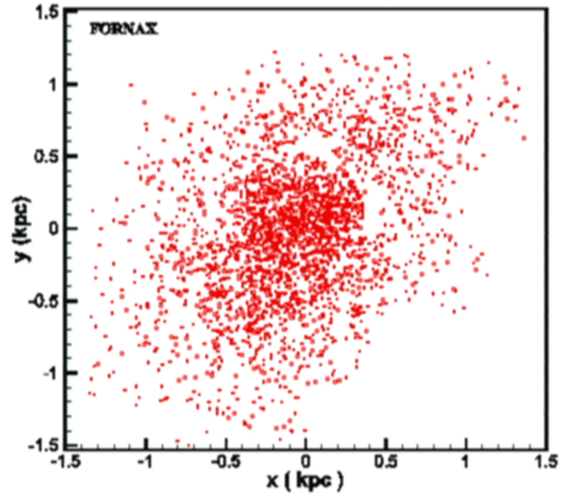
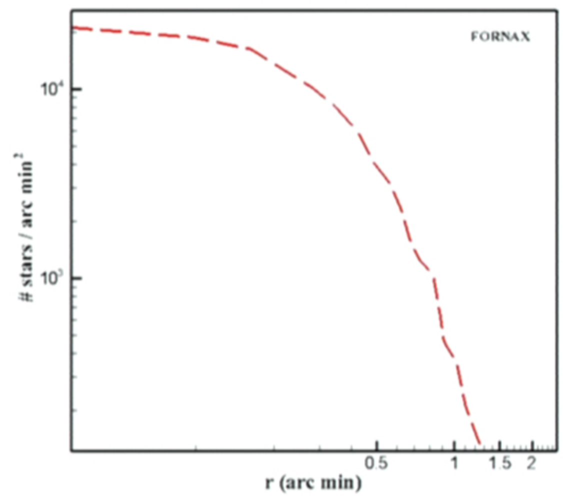
Spectroscopic Stars-Fornax



Photometric Stars



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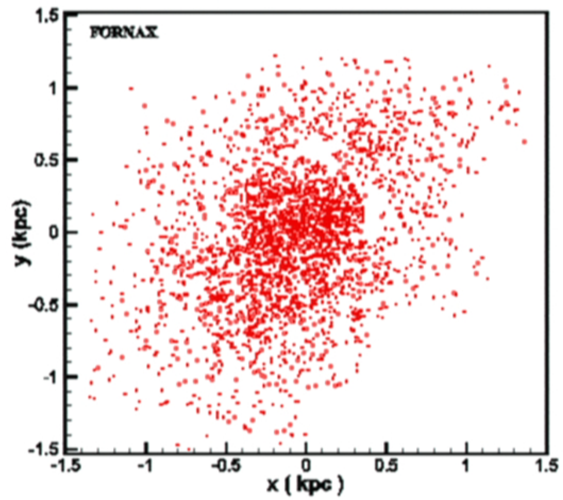

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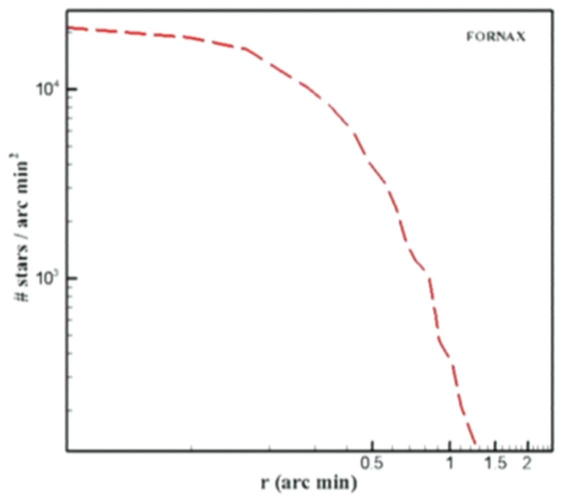

Finding the best parameters of model

- Now we have to minimize:

Number of photometric stars

$n(R, v_z)$ $\#stars$ $light - profile$

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observables

Normalization factor

Considering the errors of line of sight velocity measurements

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$$M_{ab}(R, v_z) = L_b \int_{\max[R, r_{\min,ab}]}^{r_{\max,ab}} \frac{r dr}{\sqrt{r^2 - R^2}} \sum_{\pm} |R v_{\phi\pm}^{z\pm} (v_{R\pm}^{z\pm} R \pm z v_z)|^{-1}.$$

$$P_{ab}(R) = 4\pi L_b \int_{r_{min}}^{r_{max}} \frac{dr}{\sqrt{R^2 - r^2 \lambda}}$$

$$\lambda \equiv \sqrt{2r^2(E - \Phi_T(r)) - L_b^2}.$$

$$N_{ab} = 4\pi \int d^3x \frac{L_b}{r^2 |v_r|} = (4\pi)^2 L_b \int_{r_{\min,ab}}^{r_{\max,ab}} \frac{r dr}{\sqrt{2r^2[E_a - \Phi_T(r)] - L_b^2}}$$

$$v_{R\pm} = R^{-1}\left\{-z v_z \pm \sqrt{2r^2[E_a - \Phi_T(r)] - L_b^2}\right\}$$

$$v_{\phi\pm} = r^{-1}\sqrt{L_b^2 - (z v_{R\pm} - R v_z)^2}.$$

$$r = \sqrt{R^2 + z^2}.$$

MW Satellites

- ***Draco, Fornax, Sculptor, Sextan***

Info./Galaxy	Draco	Sextan	Fornax	Sculptor	References
$\#stars(kinematic)$	588	423	2400	1352	[7]
$r_{half}(pc)$	196 ± 12	682 ± 117	668 ± 34	260 ± 39	[7]
$r_{pl}(pc)(Plummer)$	281	625	610	430	[6]
$\rho_*(M_\odot/kpc^3)(Plummer)$	2.79×10^6	4.3×10^5	1.6×10^7	6.4×10^6	[6]
$Luminosity[10^6 L_\odot]$	0.26	0.50	15.5	2.15	[6]

Catalogues:
1)Walker et al. 2006
2)Wilkinson et al. 2007
3)Unpublished catalogue :
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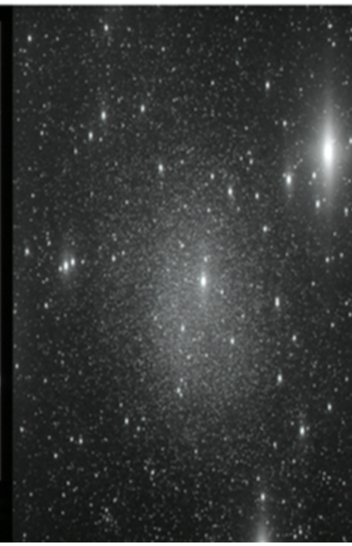
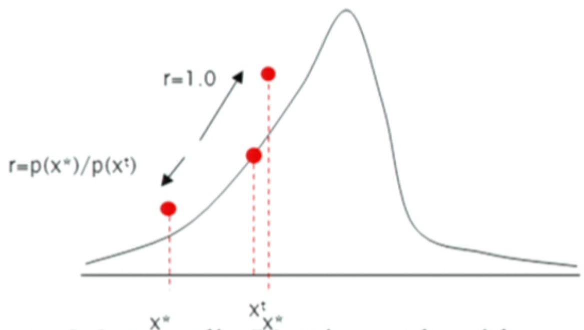
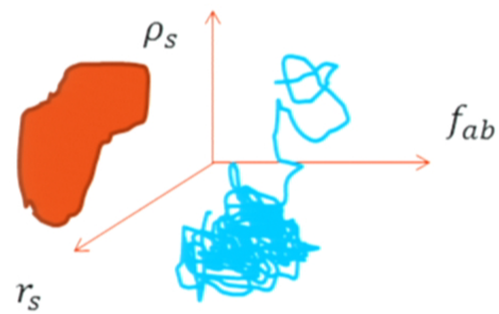


MCMC METHOD, FOR MARGINALIZING OVER PARAMETERS

- Free parameters are the number of $f_{ab} + \rho_s + r_s$
- Fixing the Plummer profile parameters .
- ***Instead of doing Maximum Likelihood Analysis , we marginalize over all***

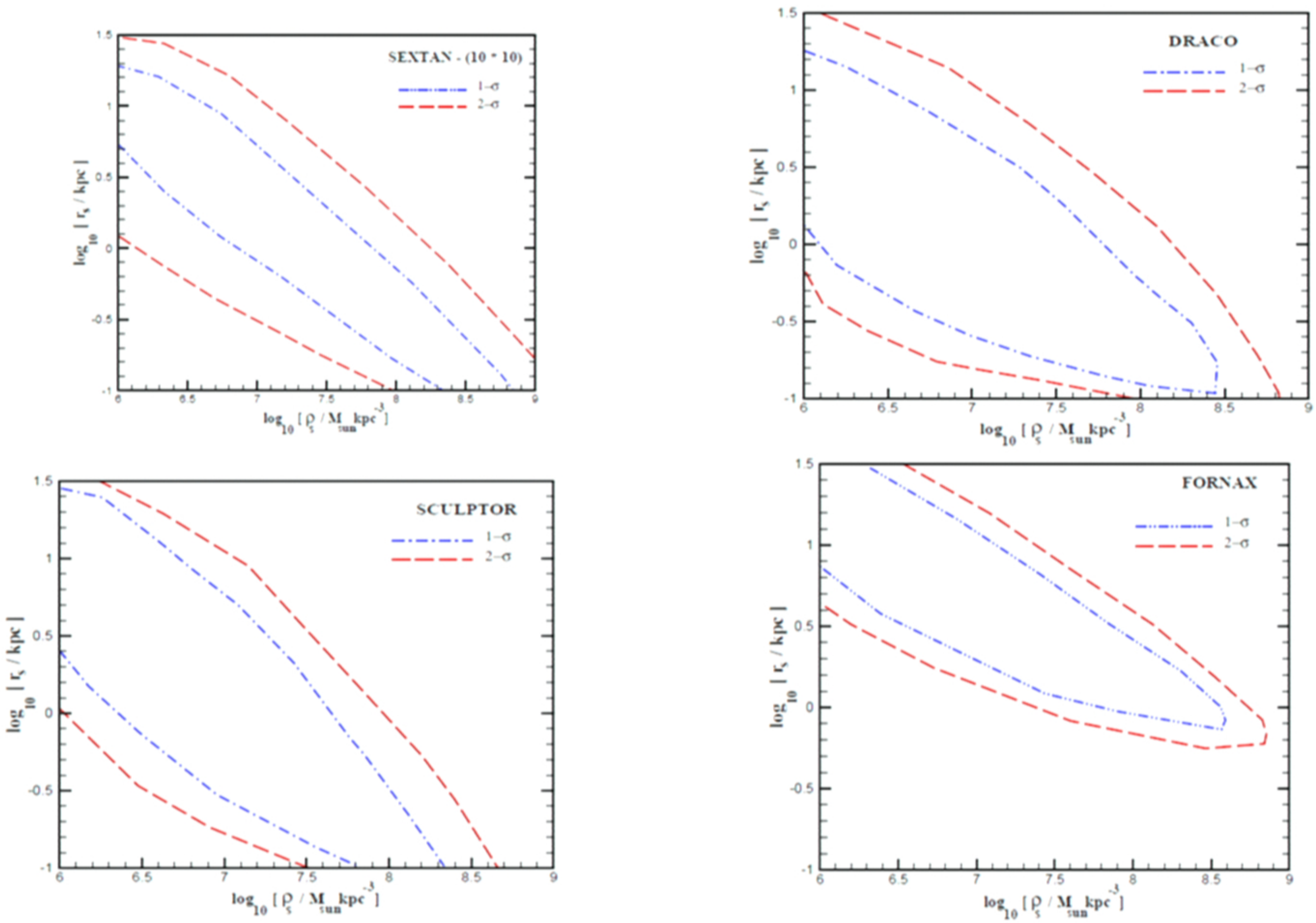
f_{ab} parameters . Relaxing the assumption of Gaussian distribution in phase space





- Metropolis-Hastings Algorithm,
- Convergence (Autocorrelation) – different initial conditions in f_{ab}
- Number of steps $7.5 * 10^5$
- Unity prior for free parameters in log-scale
- Burn-in process ~20%
- Acceptance rate 7%
- Fornax-run : ~ 60 hours

CONTOUR PLOTS FOR PARAMETERS OF NFW-PROFILE



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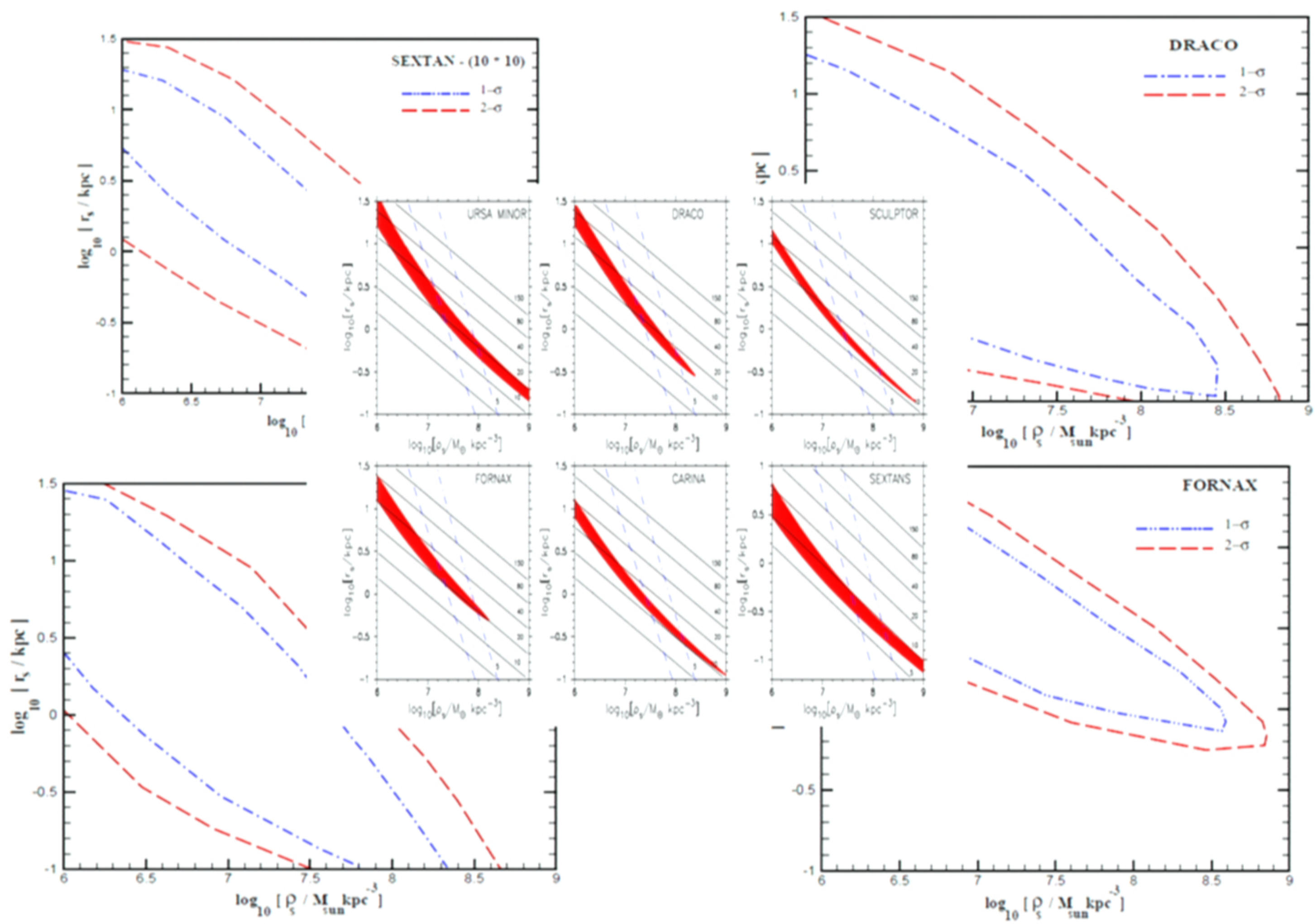


MASS OF DWARF GALAXIES:

Using the accepted points of MCMC; we can find the mass of each dwarf galaxy:

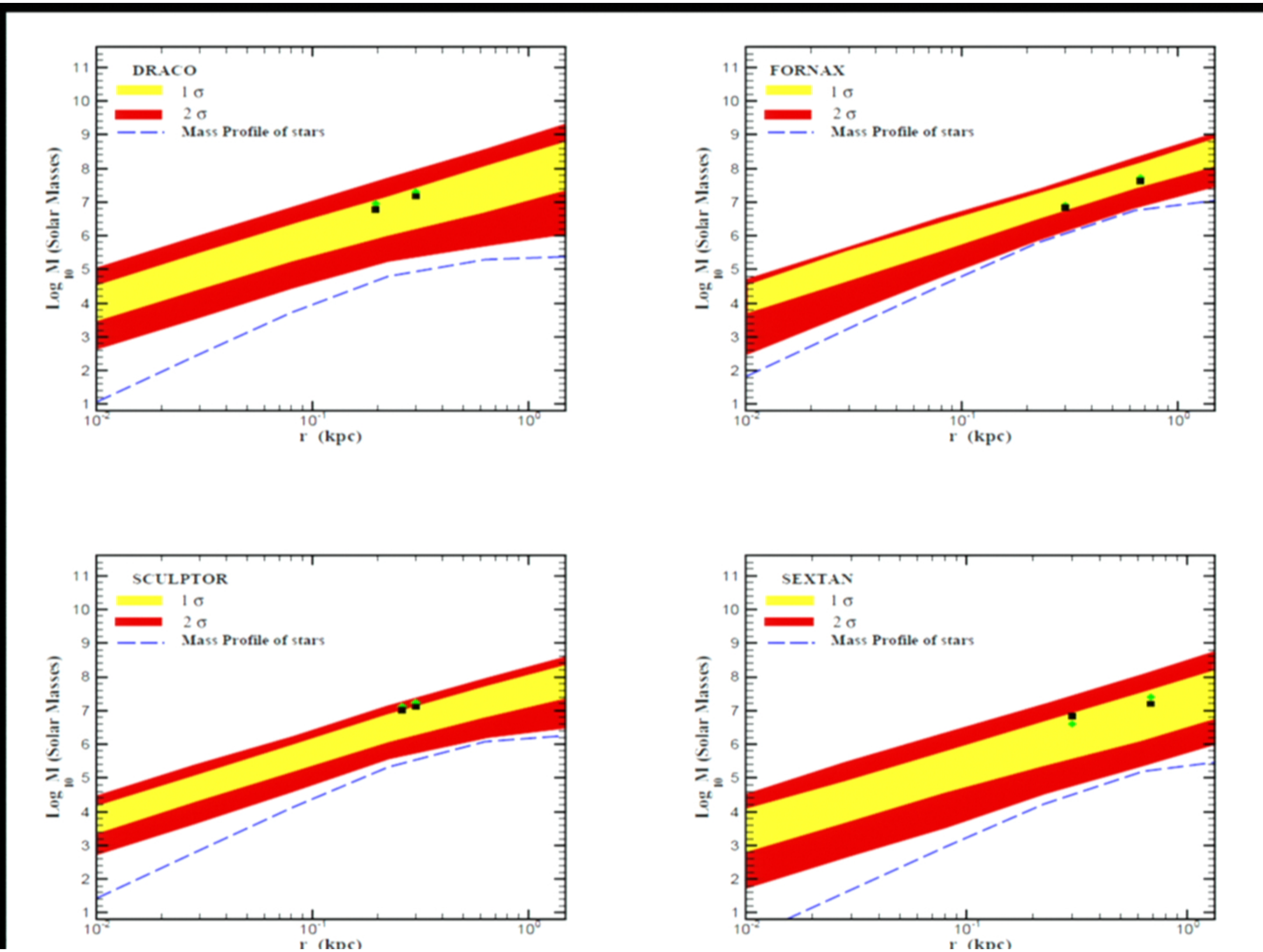
$$M_{DM}(r) = 4\pi\rho_s^{acc}(r_s^{acc})^3 \left[\ln(1 + r/r_s^{acc}) - \frac{r/r_s^{acc}}{1 + r/r_s^{acc}})\right],$$

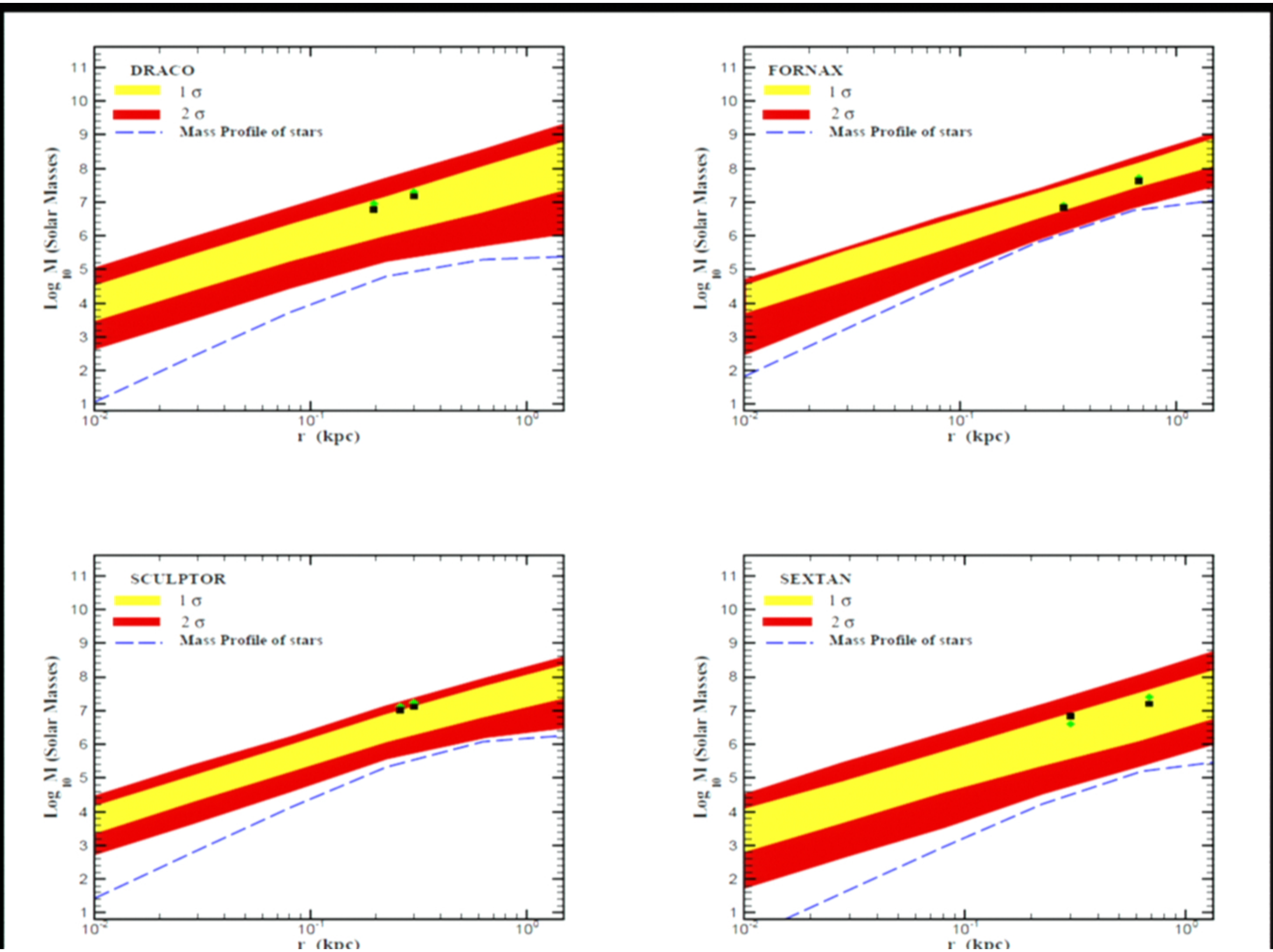
Add the stellar mass due to Plummer distribution:

$$M_*(r) = M_* \frac{r^3}{(r^2 + r_{pl}^2)^{3/2}},$$

The total mass $M_T = M_{DM} + M_*$

Finding the median + $1\sigma, 2\sigma$ level of confidance


DRACO
FORNAX
SCULPTOR
SEXTAN
1 σ
2 σ
Mass Profile of stars
Log$_{10}$ M (Solar Masses)
r (kpc)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CHALLENGES FOR DARK MATTER PARADIGM

Missing satellite problem:

* Sub-halo formed in higher redshifts

*Survive in the host halos like MW

*Observed Satellites1 order of magnitude less than Simulation's prediction.

SOLUTIONS:

1)Modification of CDM

WDM-SIDM-…

2)Astrophysical Reasoning:

Reionization, SN feedback,…

A. Klypin, A.V. Kravtsov, O. Valenzuela and F. Prada, ApJ, **522**, 82 (1999).

Springel et al. 2001

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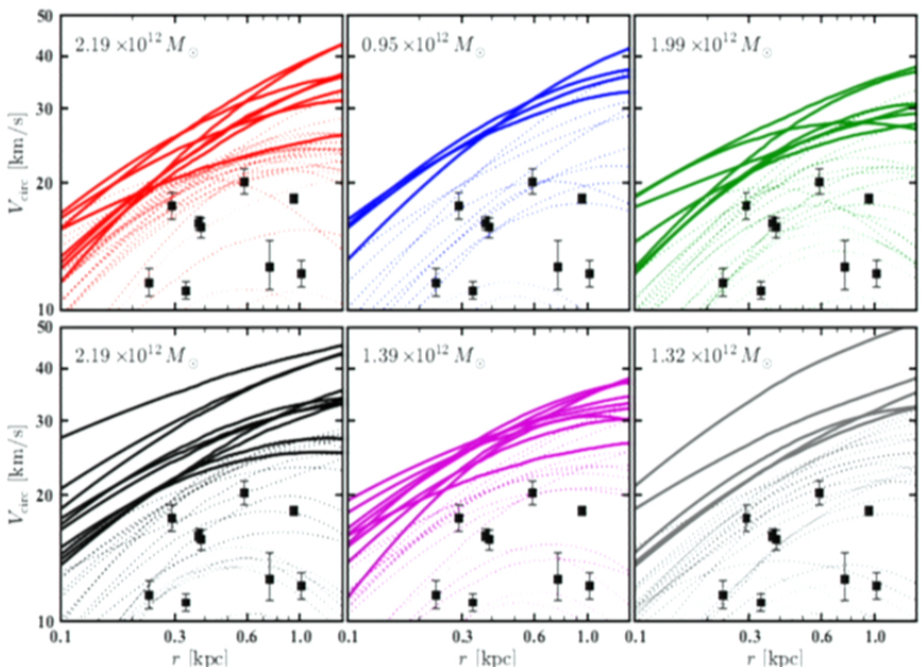
Too Big to Fail Problem?

- Subhalos in ΛCDM are dynamically can not host the most luminous dwarf satellites of Milky Way: (Too big to fail?)

(Michael Boylan-Kolchin, James S. Bullock, Manoj Kaplinghat, arXiv :1103.0007)

Aquarius simulation: high resolution simulation of 6 Milky- way type halo



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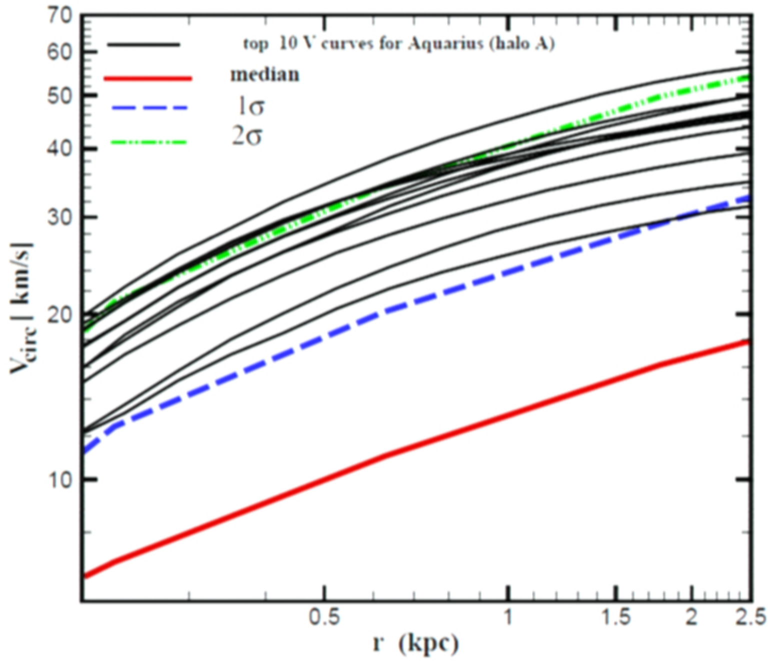

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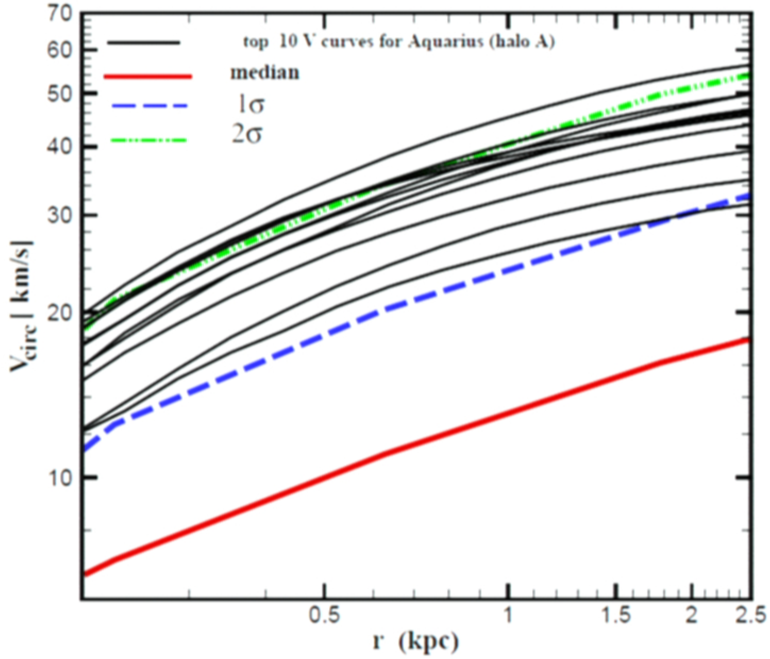

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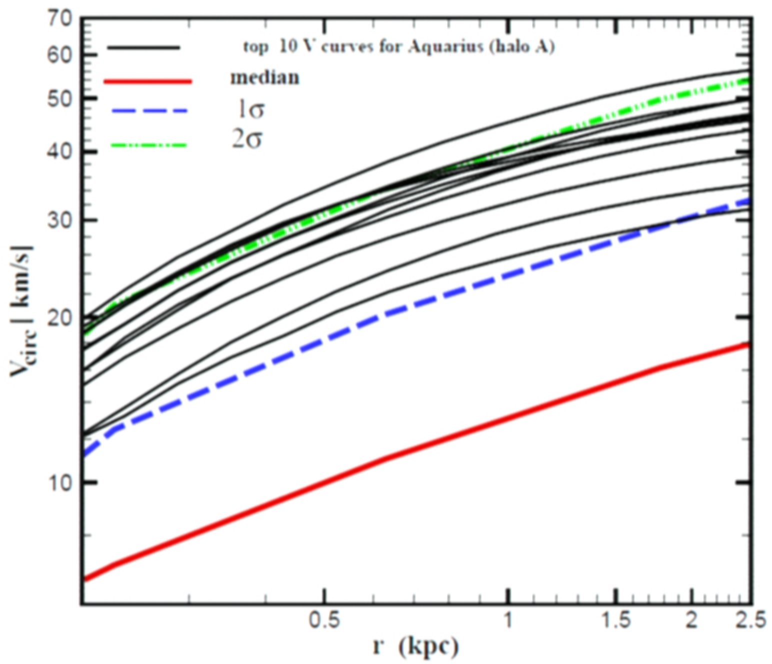


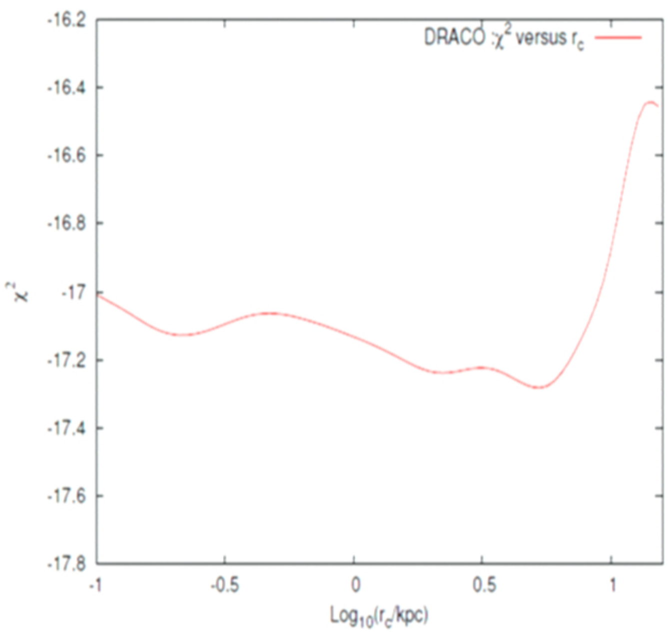
$$M_A = 2.19 \times 10^{12} M_{\odot}$$

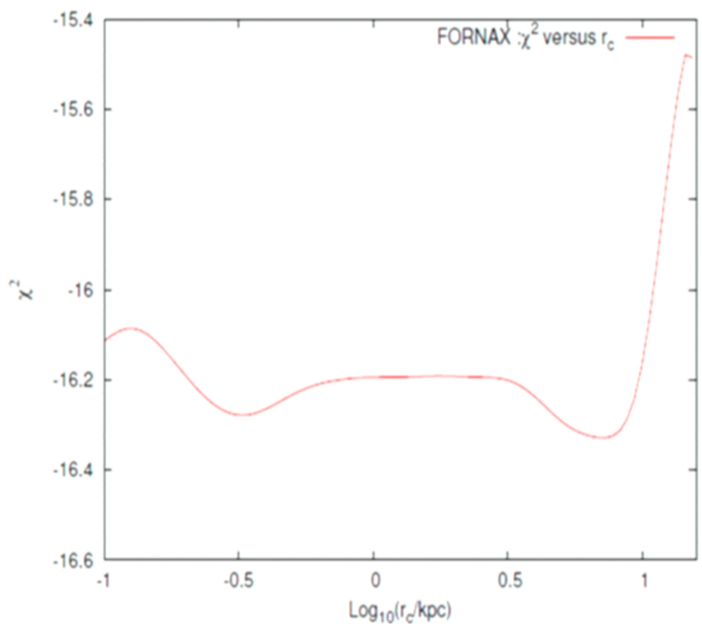
CORE- CUSP PROBLEM:

- In order to address, the core-cusp problem in our modeling, we introduce a new potential as:

$$\Phi(r) = -\frac{4\pi G\rho_s r_s^3 \ln(1+\tilde{r}/r_s)}{\tilde{r}}$$

$$\tilde{r} \equiv \sqrt{r^2 + r_c^2}$$





Conclusion and Future Prospects

Main Points:

- Jeans Theorem for a spherical symmetric case:

Non-parametric general phase-space density: $f = f(E, L)$

- Marginalizing over f_{ab} instead of Maximum Likelihood analyses
- A probable solution for the problem of Too Big to be fail?

Prospects:

- Priors on f_{ab} phase-space density

Niayesh Afshordi, R. Mohayaee, E. Bertschinger Phys.Rev.D79:083526,2009

- Number of free parameters, and more sophisticated numerical methods.
- Apply to other Astrophysical systems: Milky way, search for Black holes,…

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