Title: Spin-liquid Phase in Spin-1/2 square J_1-J_2 Heisenberg Model: A Tensor Product State Approach

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Abstract: The ground state phase of spin-1/2 J1-J2 antiferromagnetic Heisenberg model on square lattice in the maximally frustrated regime (J2 ~ 0.5J1) has been debated for decades. Here we study this model by using a recently proposed novel numerical method - the cluster update algorithm for tensor product states (TPSs). The ground state energies at finite sizes and in the thermodynamic limit (with finite size scaling) are in good agreement with the state of art exact diagonalization study, and
the energy differences between these two studies are of the order of 0.001 J1 per site. At the largest bond dimension available D (D = 9), we find a paramagnetic ground state without any valence bond solid order in the thermodynamic limit in the range of 0.5 <= J2/J1 <= 0.6, which implies the emergence of a spin-liquid phase. Furthermore, we investigate the topologically ordered nature of such a spin-liquid phase by measuring a nonzero topological entanglement entropy.

The spin liquid state of the J1-J2 Antiferromagnetic Heisenberg model on square lattice -- a TPS study

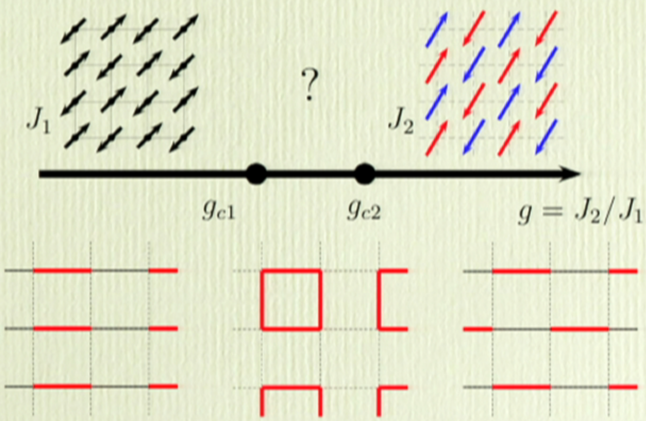
Ling Wang
University of Vienna
at PI, 2012. 04. 25

Zheng-cheng Gu (KITP)
Frank Verstraete (Univ. of Vienna)
Xiao-gang Wen (MIT)

The model

The spin 1/2 J1-J2 Antiferromagnetic Heisenberg model

$$H = J_1 \sum_{(i,j)} \mathbf{S}_i \cdot \mathbf{S}_j + J_2 \sum_{<i,j>} \mathbf{S}_i \cdot \mathbf{S}_j \quad (J_1 > 0, J_2 > 0)$$



Z_2 spin liquid state and the sRVB state

- Toric code — Kitaev, Ann. Phys. 303, 2 (2003)
- Rokhsar-Kivelson (RK) point of the quantum dimer model (QDM) on triangular and Kagome lattice — Moessner and Sondhi, PRL 86, 1881 (2000); Misguich et al, PRL 89, 137202, (2002); Yao and Kivelson, arXiv:1112.1702
- short range RVB states on Kagome lattice — Poilblanc et al, arXiv:1202.0947
- Proof of equivalence between: (a) the toric code, (b) the RK point of the QDM on Kagome lattice, and (c) the sRVB on Kagome lattice, by an isometry transformation — Schuch et al, arXiv:1203.4816

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RVB state on square lattice

--the stability of Z2 spin liquid state

- RK point of QDM on square lattice is a gapless spin liquid state with power law dimer correlation

  Rokhsar and Kivelson, PRL 61, 2376 (1988)

- equal weight superposition of NN RVB on square lattice is a U(1) spin liquid state confirmed by MC studies

  Henrey et al, PRB 84, 174427 (2011)

  Albuquerque and Alet, PRB 82, 180408 (2010)

- the above NN RVB on square lattice has parent Hamiltonian with short range interactions

  Cano and Fendley, PRL 105, 067205 (2010)

- the equal weight NN and NNN RVB on square and honeycomb lattices are gaped Z2 spin liquid states

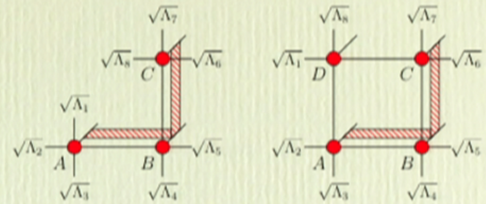
  H. Yao and S. A. Kivelson, arXiv:1112.1702

  The collapse of the extensive topological sectors for square lattice NN RVB states into 4 is accomplished by frustration

outline

- method
- results (spin liquid states at intermedia J2)
  - finite size energies
  - AF order parameter
  - Spin-Spin and dimer-dimer correlations
  - VBS order parameters
  - topological entanglement entropy
  - SVD spectral of the double tensor

The Method



1. Use A, B, C, D 4 sub-lattices to accommodate all possible valence bond orders and study their competing effect.

2. Optimize the ground state tensor for an infinite system with the cluster imaginary time evolution method. The ground state tensors with a bond dimension from D=2 to 9 are obtained.

Vidal, PRL 98, 070201 (2007)   Jiang et al., PRL 101, 090603 (2008)   Wang and Verstraete, arXiv:1110.4362

3. Set the same tensors on a finite size L by L torus (PBC).

4. Measure the expectation values of finite size PBC systems (L=4,6,8,12,16,32) with the variational Monte Carlo sampling method.   Wang et al., PRB **83**, 134421 (2011)

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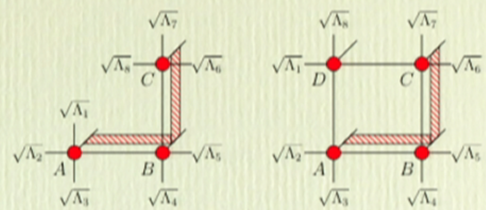

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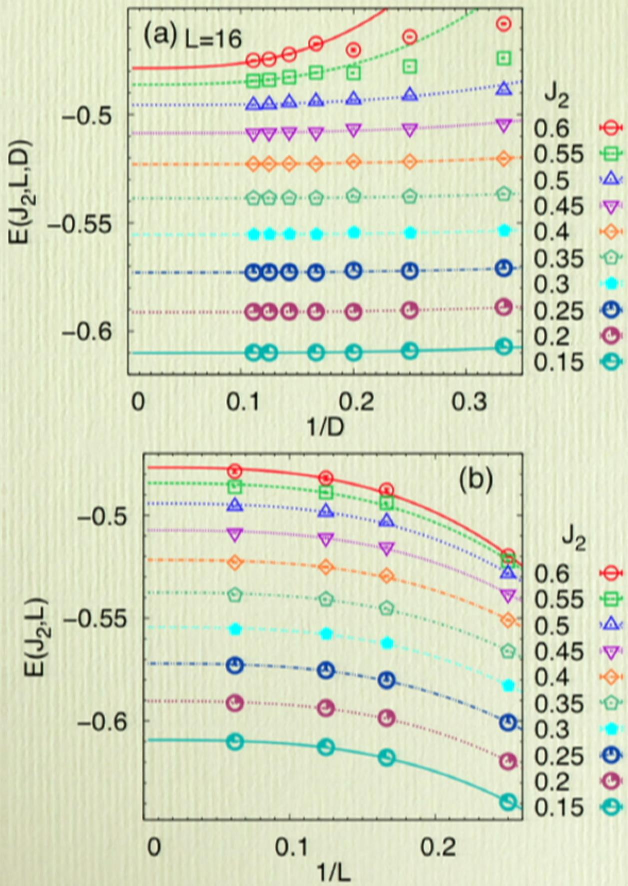
Finite size energies



Finite D scaling

$$E(J_2, L, D) = E(J_2, L) + c_{J_2}/D^3,$$

Finite size scaling of energy

$$E(J_2, L) = E(J_2) + d_{J_2}/L^3,$$

Sandvik, Phys. Rev. B 56, 11678 (1997)

Richter et al., Eur. Phys. J. B 73, 117 (2010)

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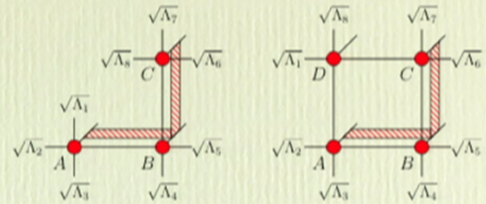

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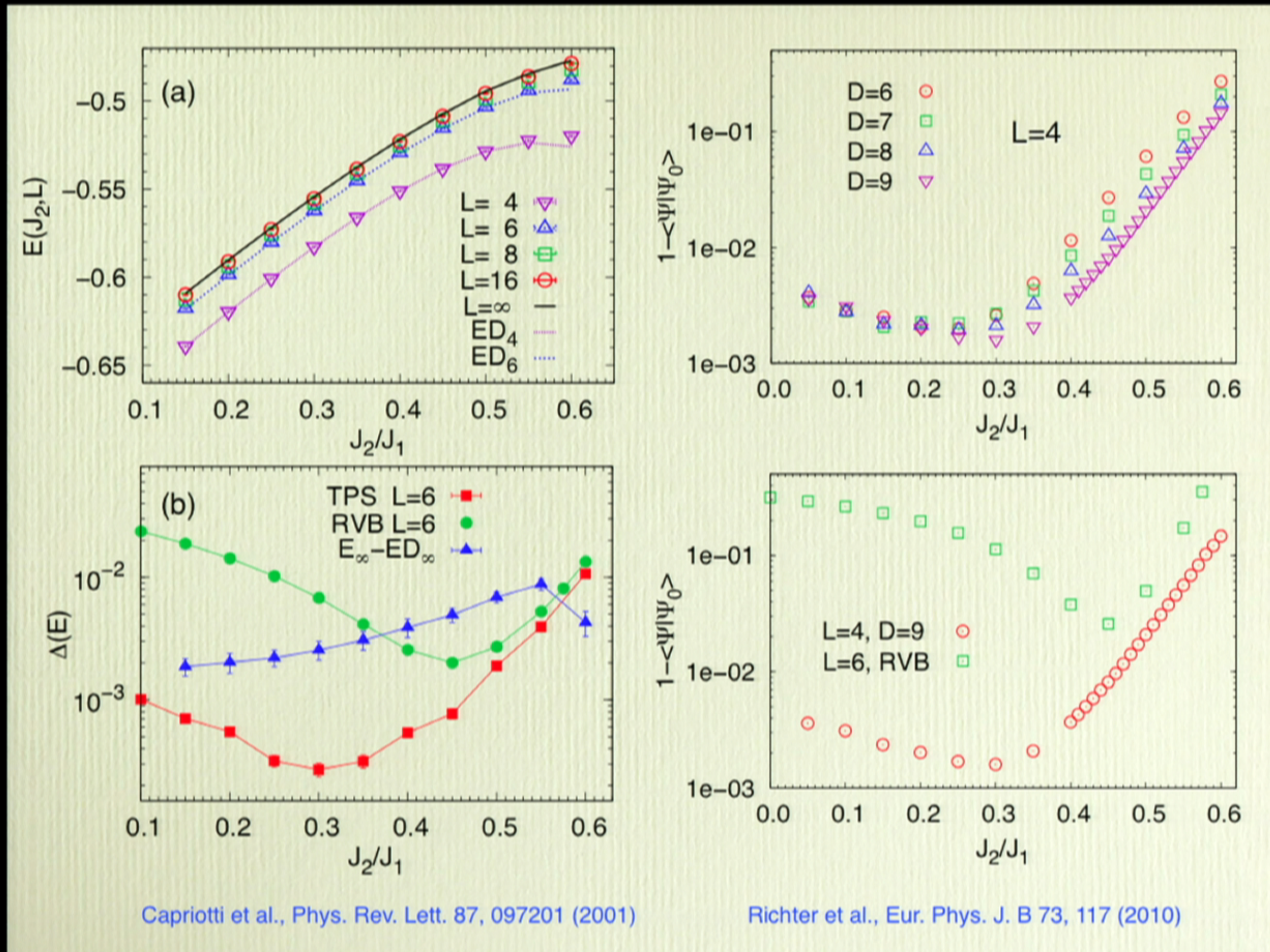
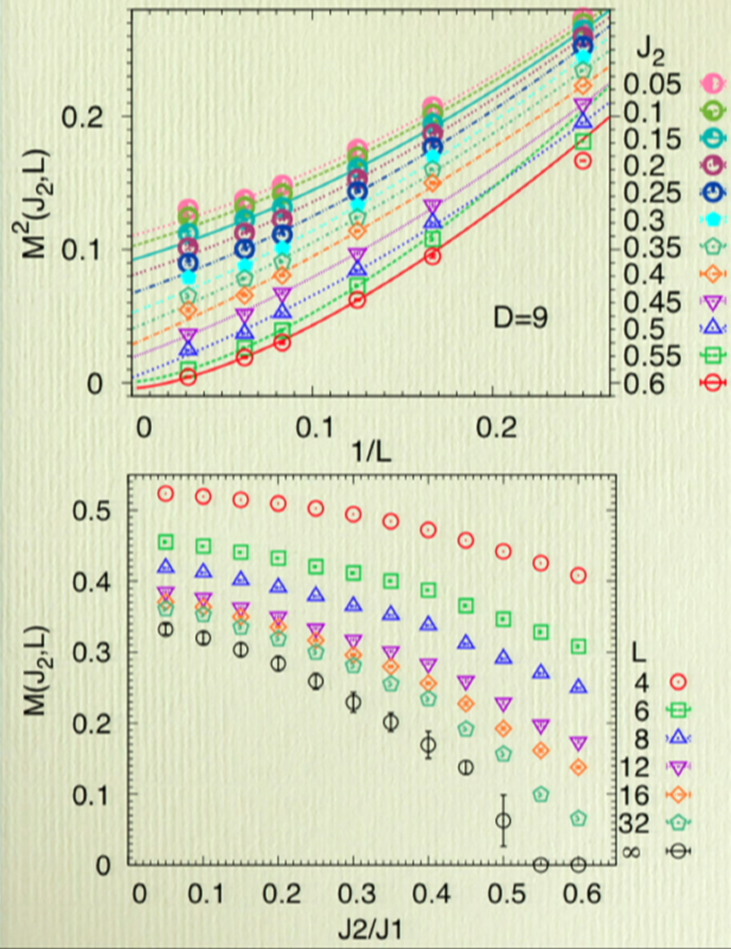

(a)
E(J2,L)
L= 4
L= 6
L= 8
L=16
L=∞
ED4
ED6
J2/J1
D=6
D=7
D=8
D=9
L=4
1−<Ψ|Ψ0>
(b)
TPS L=6
RVB L=6
E∞−ED∞
Δ(E)
L=4, D=9
L=6, RVB
Capriotti et al., Phys. Rev. Lett. 87, 097201 (2001)
Richter et al., Eur. Phys. J. B 73, 117 (2010)



AF order parameter

VMC sampling of the magnetization

$$M^2 = \frac{1}{4L^2} \sum_{x_i, y_i = 1}^{2} \sum_{x_j, y_j = 1}^{L} (-1)^{\phi} \mathbf{S}_{(x_i, y_i)} \cdot \mathbf{S}_{(x_j, y_j)}$$

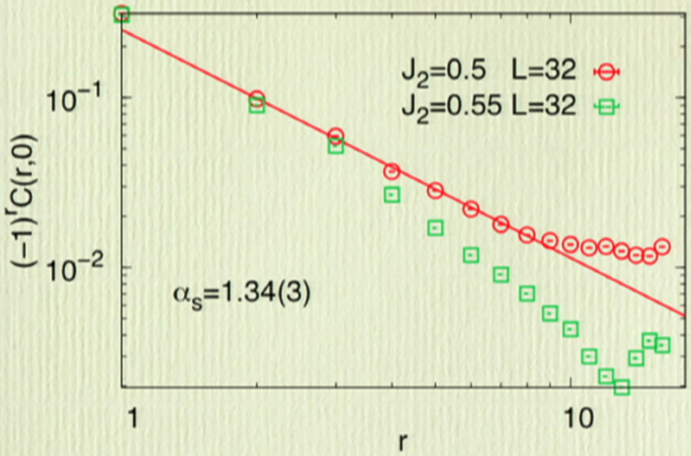
$$\phi = x_i - x_j + y_i - y_j$$

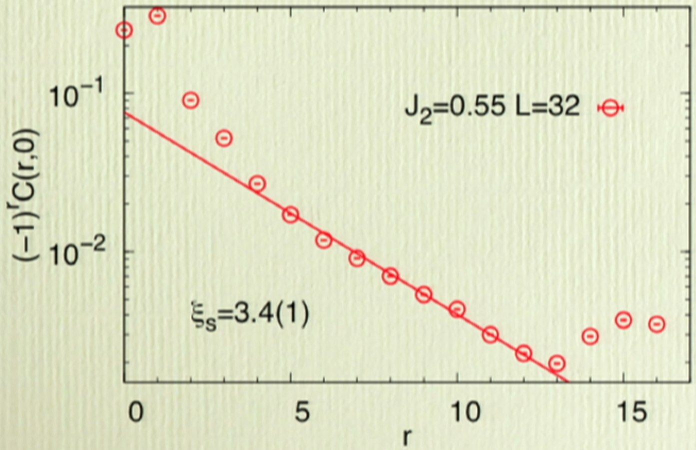
Finite size scaling of the magnetization

$$M^2(J_2, L) = M^2(J_2) + e^1_{J_2}/L + e^2_{J_2}/L^2$$

A continuous phase transition at J2 ~ 0.5 further confirmed by a power law spin-spin correlation function

spin-spin correlation





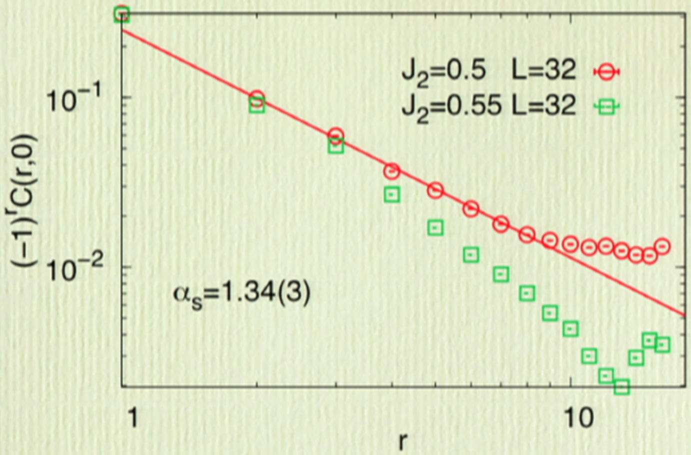
The spin-spin correlation at distance r

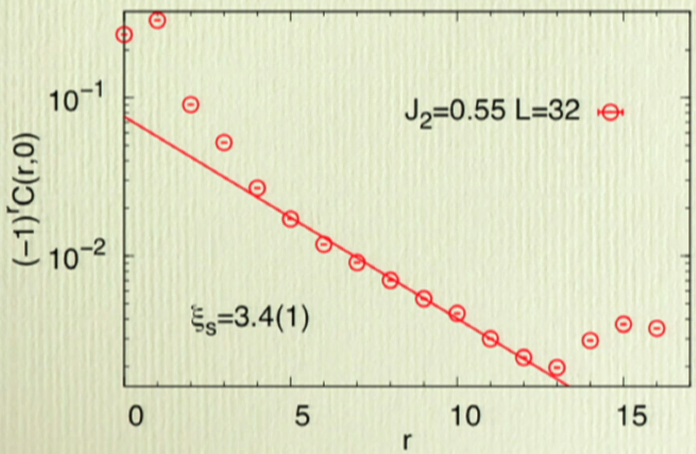
$$C(r,0) = \mathbf{S}_{(0,0)} \cdot \mathbf{S}_{(r,0)}$$

finite dcut set up a lingering of AF order

critical $(-1)^r C(r,0) \sim r^{-\alpha}$

spin liquid $(-1)^r C(r,0) \sim e^{-r/\xi_s}$





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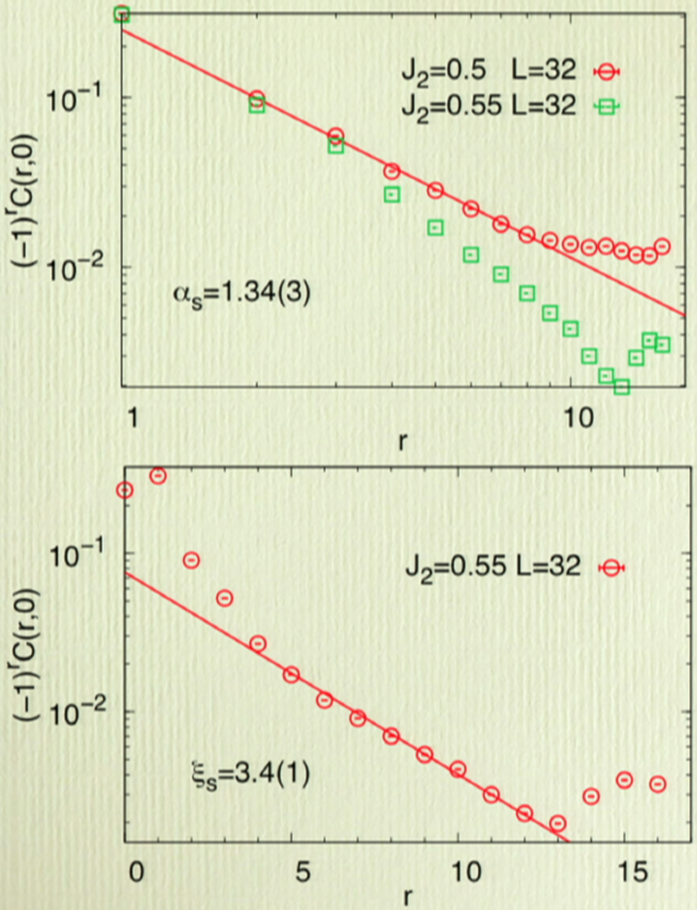

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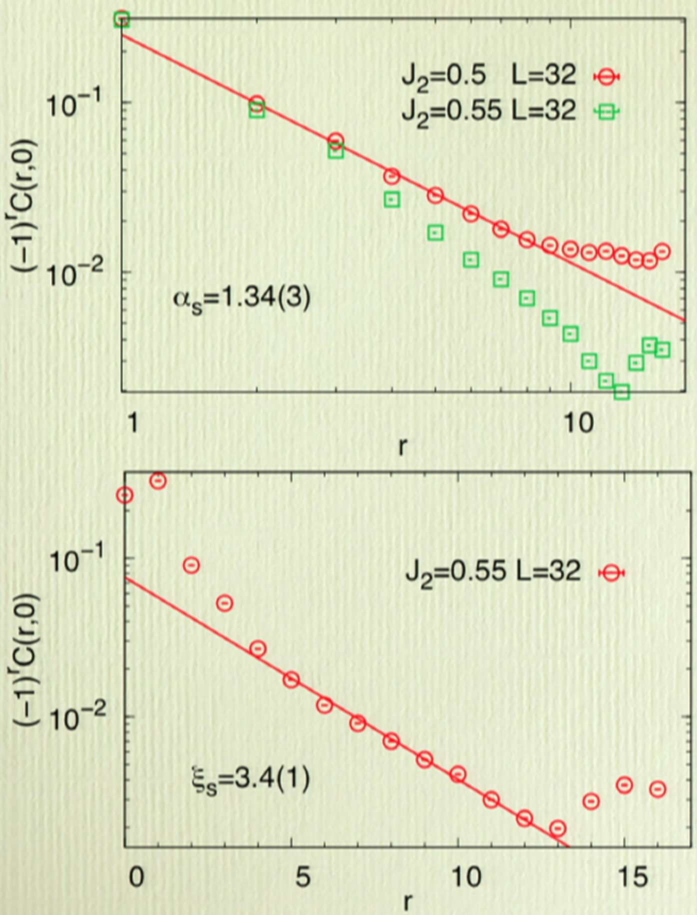

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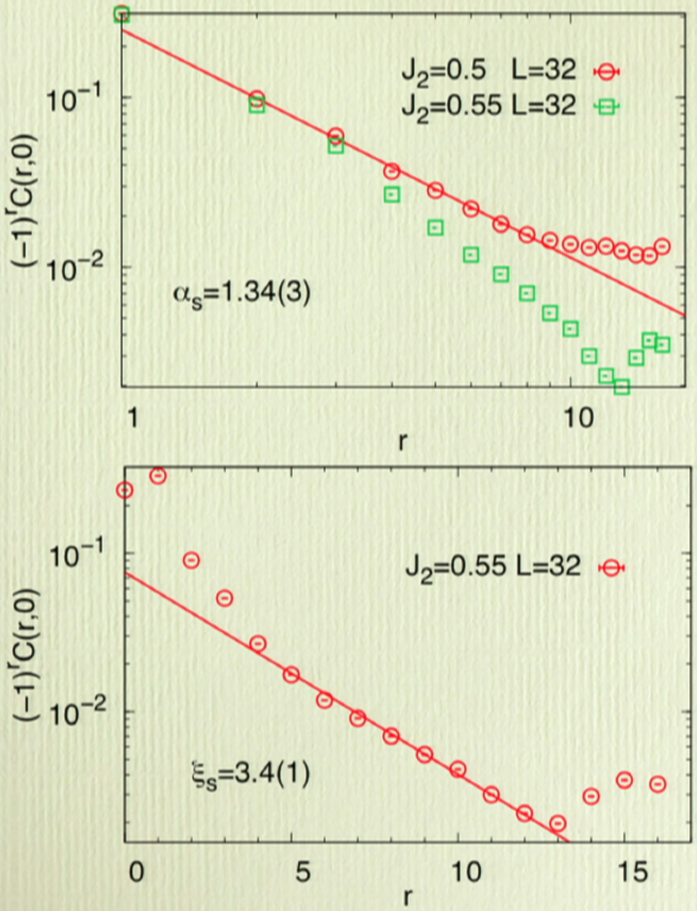

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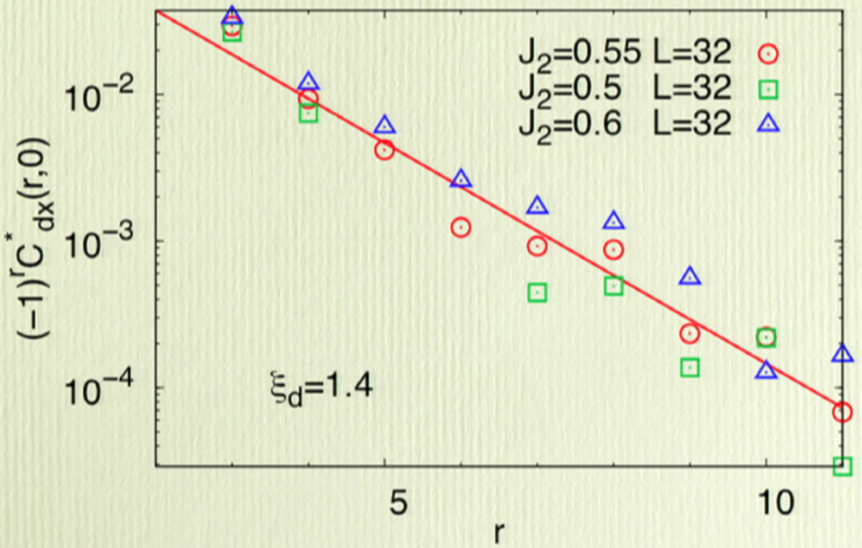
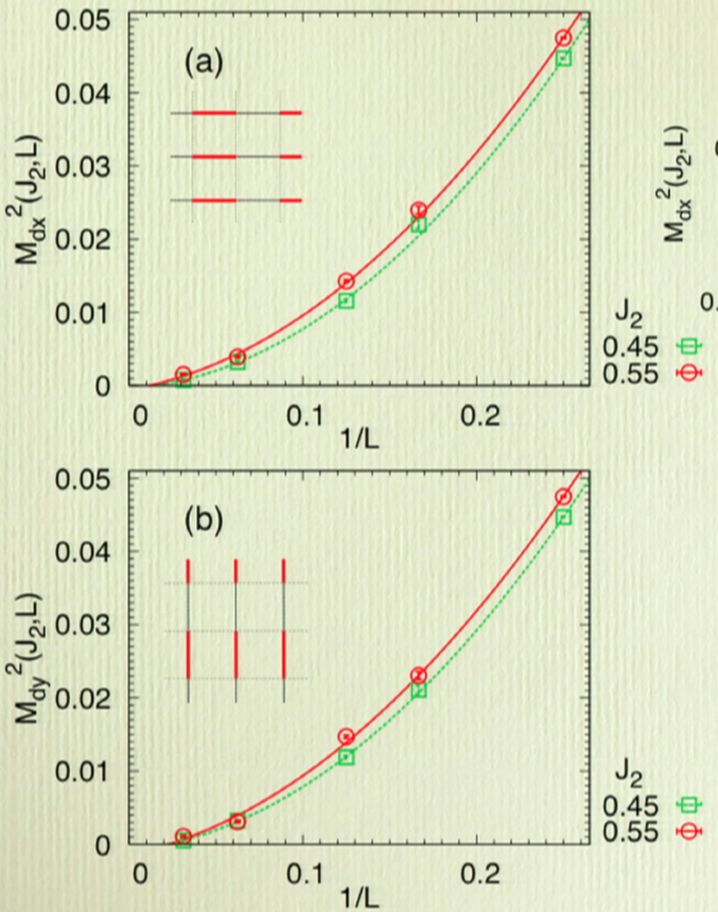
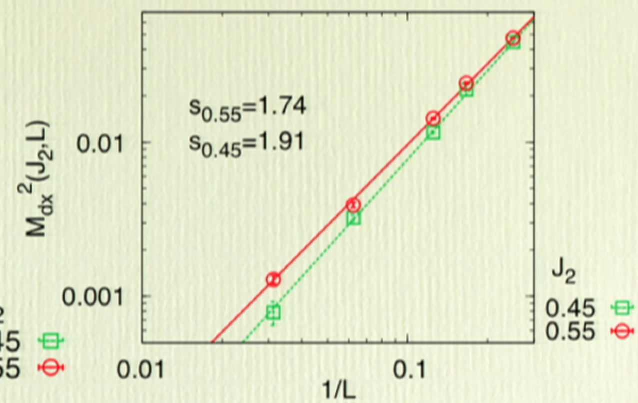
dimer-dimer correlation

$$C^*_{dx}(r,0) = \langle D^x_{(0,0)} D^x_{(r,0)} \rangle - \langle D^x_{(0,0)} D^x_{(r-1,0)} \rangle,$$

$$(-1)^r C^*_{dx}(r,0) \sim e^{-r/\xi_d}$$







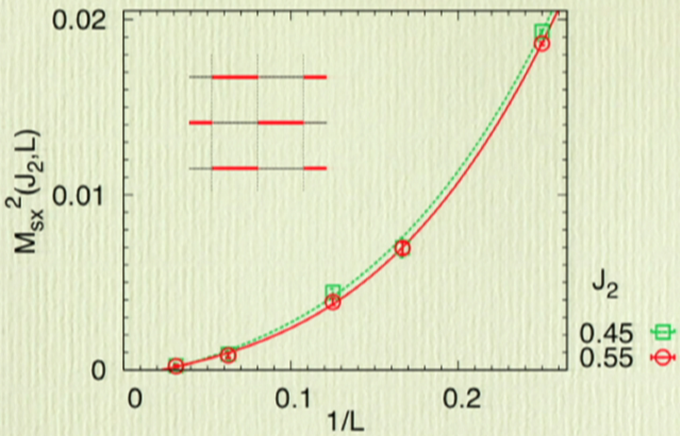
VBS order parameters

$$M_{dx}^2 = \frac{1}{N^2}\sum_{i,j=1}^{N}(-1)^{x_i-x_j}\langle D_i^x D_j^x\rangle,$$

$$M_{dy}^2 = \frac{1}{N^2}\sum_{i,j=1}^{N}(-1)^{y_i-y_j}\langle D_i^y D_j^y\rangle;$$

$$D_i^\alpha = \mathbf{S}_{(x_i,y_i)}\cdot\mathbf{S}_{(x_i,y_i)+\vec{\alpha}}$$

$$\vec{\alpha} = \vec{x},\vec{y}$$



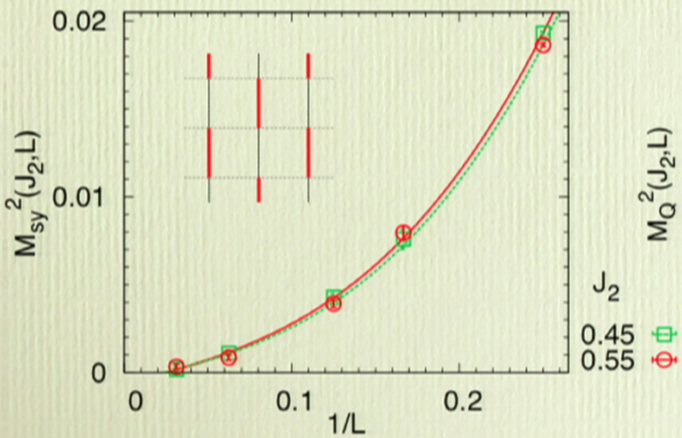
$$M_{sx}^2 = \frac{1}{N^2}\sum_{i,j=1}^{N}(-1)^{\phi_{ij}}\langle D_i^x D_j^x\rangle,$$

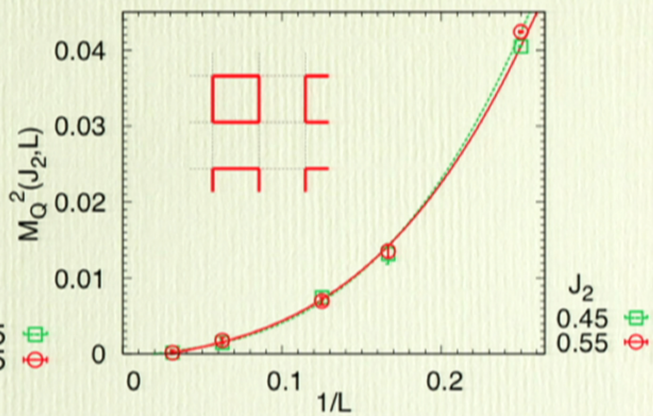
$$M_{sy}^2 = \frac{1}{N^2}\sum_{i,j=1}^{N}(-1)^{\phi_{ij}}\langle D_i^y D_j^y\rangle;$$

$$M_Q^2 = \frac{1}{N^2}\sum_{i,j=1}^{N}(-1)^{\phi_{ij}}\left(\langle Q_i Q_j\rangle - \langle Q_i\rangle\langle Q_j\rangle\right),$$

$$Q_i \equiv \frac{1}{2}(P_{\square_i} + P_{\square_i}^{-1})$$

$$\phi_{ij} = x_i - x_j + y_i - y_j$$





Topological entanglement entropy

Renyi entropy $S_n(\rho) = \frac{1}{1-n} \ln[\mathrm{Tr}(\rho^n)]$

The area law of entropy $S(\rho) = \alpha L - \gamma$

Topological entanglement entropy $S_{topo} = -\gamma$

Levin and Wen, PRL 96, 110405 (2006)

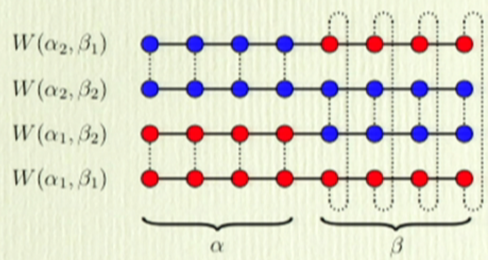
Kitaev and Preskill, PRL 96, 110404 (2006)

MC sampling of the swap operator

Hasting et al. PRL104, 157201 (2010)

$$\langle Swap \rangle = \frac{\sum_{\sigma_1,\sigma_2} W^2(\sigma_1) W^2(\sigma_2) Swap(\sigma_1,\sigma_2)}{\sum_{\sigma_1,\sigma_2} W^2(\sigma_1) W^2(\sigma_2)}$$

$$Swap(\sigma_1,\sigma_2) = \frac{W(\alpha_1,\beta_2) W(\alpha_2,\beta_1)}{W(\alpha_1,\beta_1) W(\alpha_2,\beta_2)},$$



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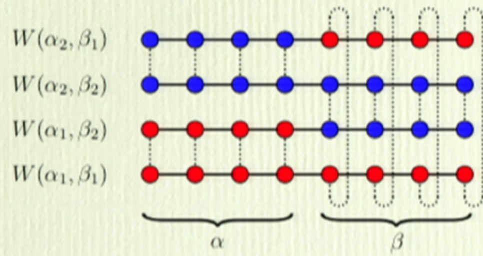
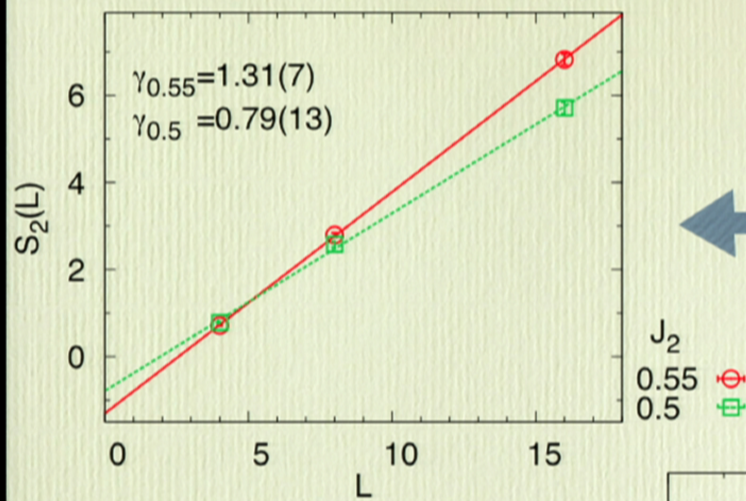
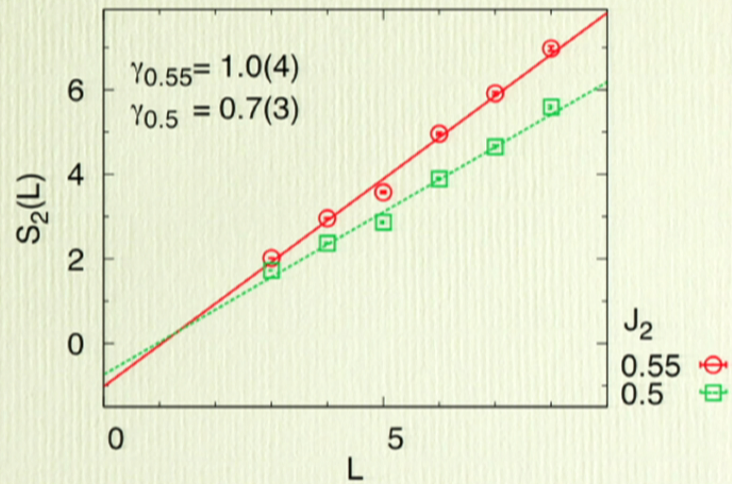


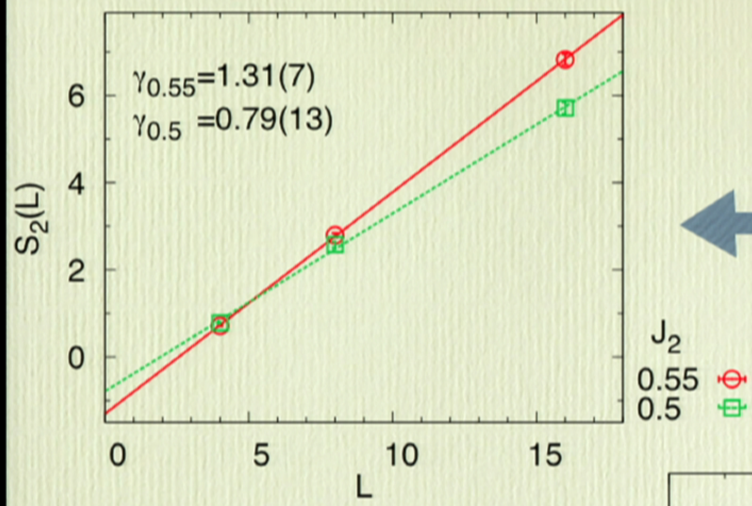


Cylinder $2L \times L$

Disk $L \times L$

from a 16×16 system





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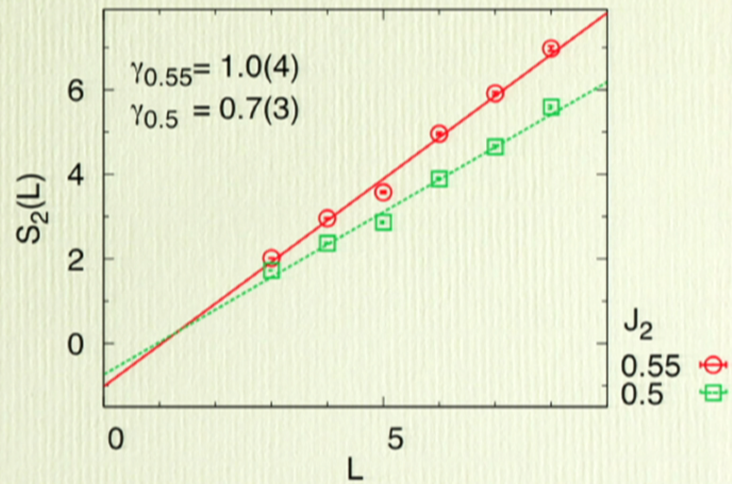
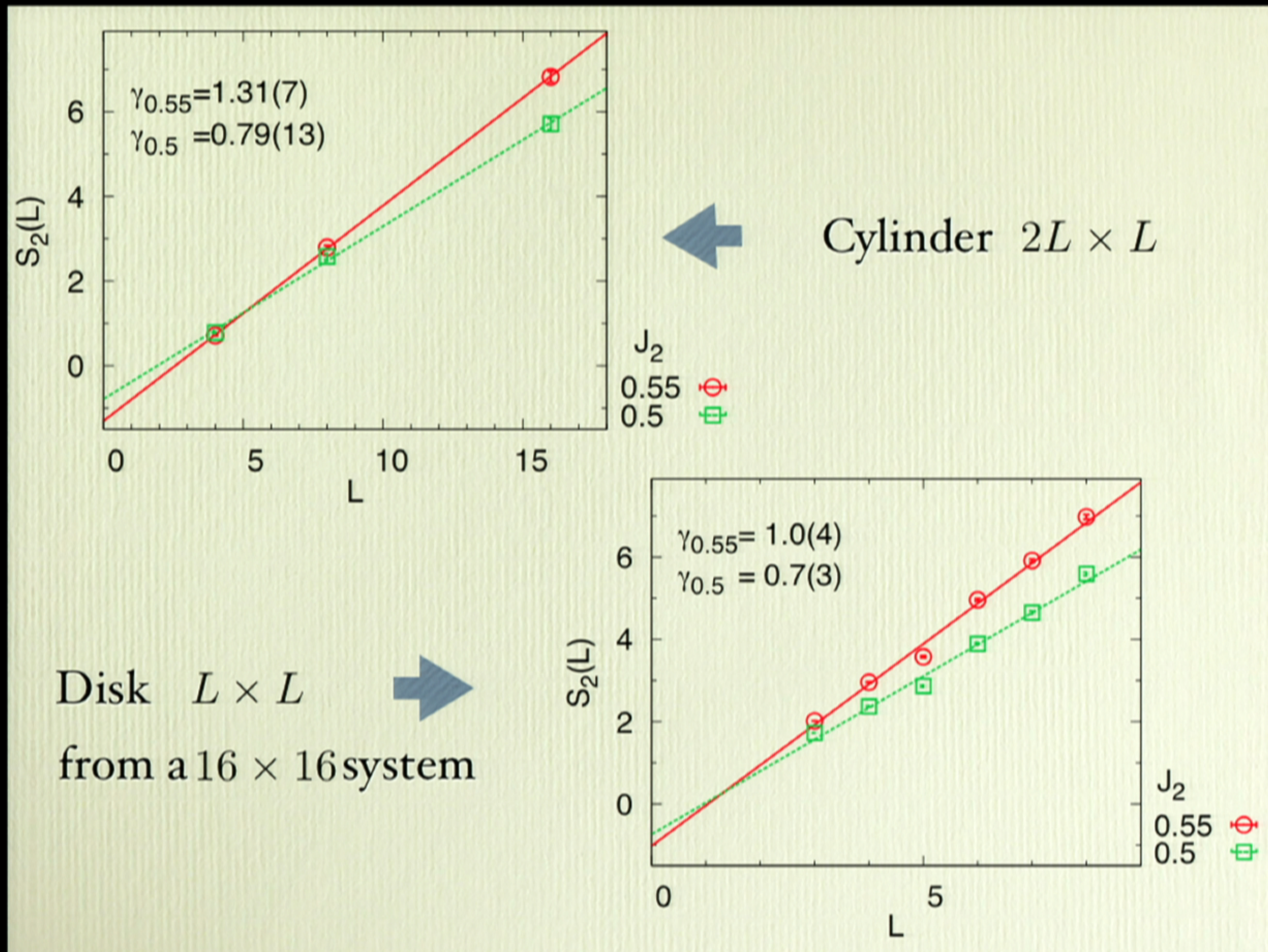
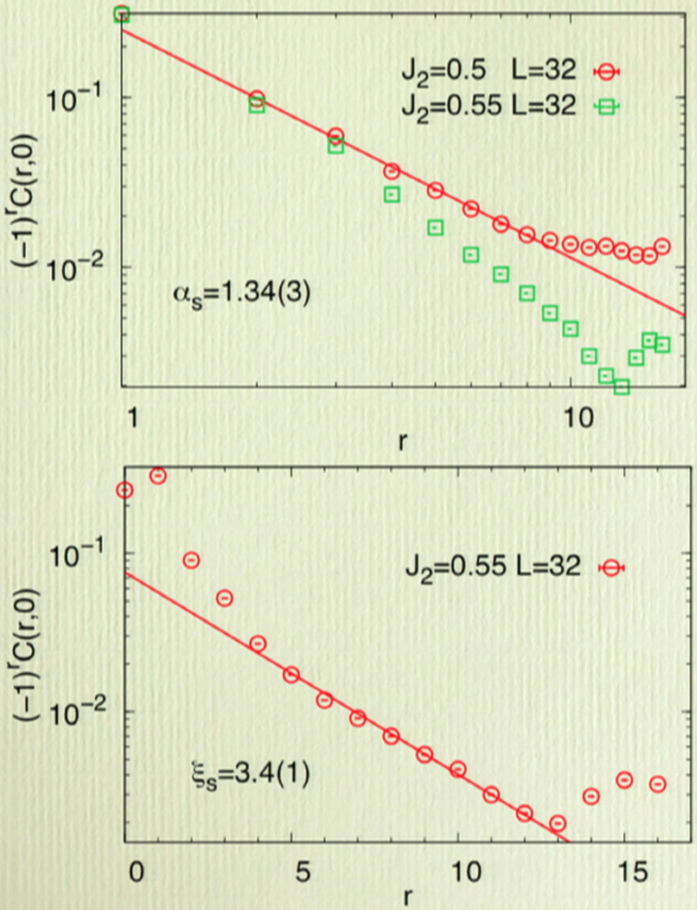



γ0.55=1.31(7)
γ0.5 =0.79(13)
S2(L)
L
J2
0.55
0.5
Cylinder 2L × L
γ0.55= 1.0(4)
γ0.5 = 0.7(3)
Disk L × L
from a 16 × 16 system



spin-spin correlation

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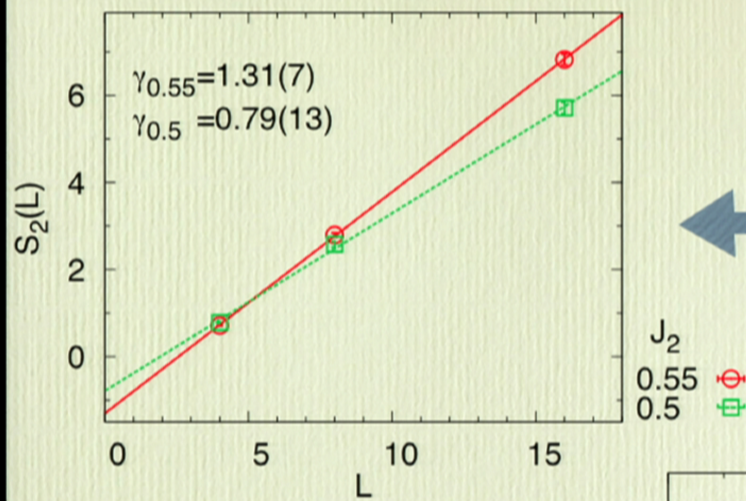

Cylinder $2L \times L$

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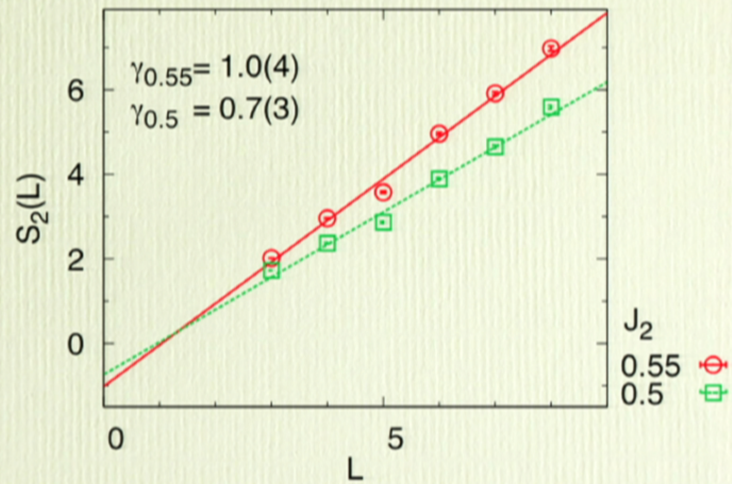
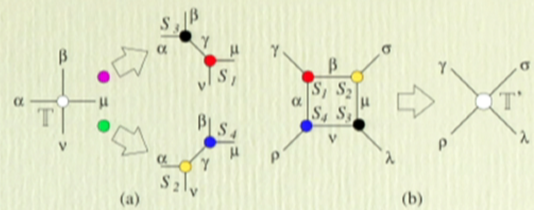
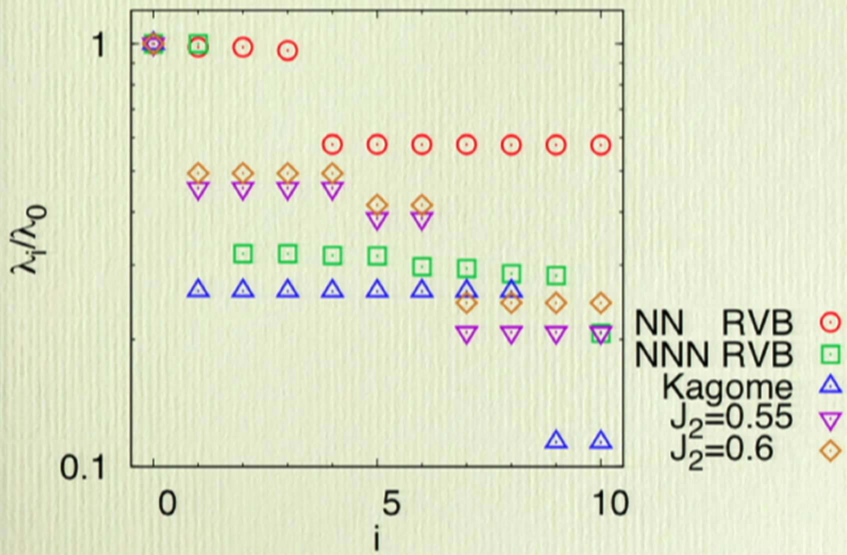
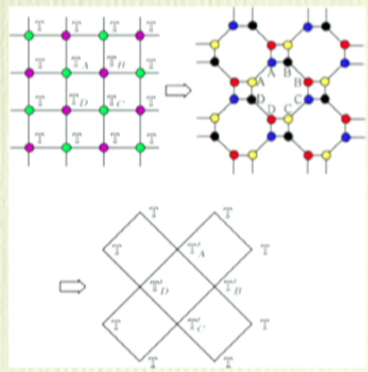


the SVD spectral



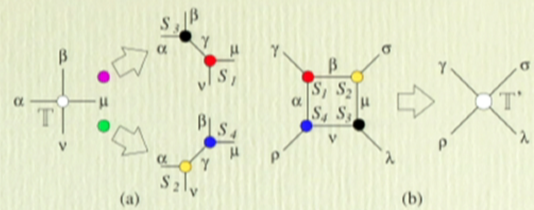


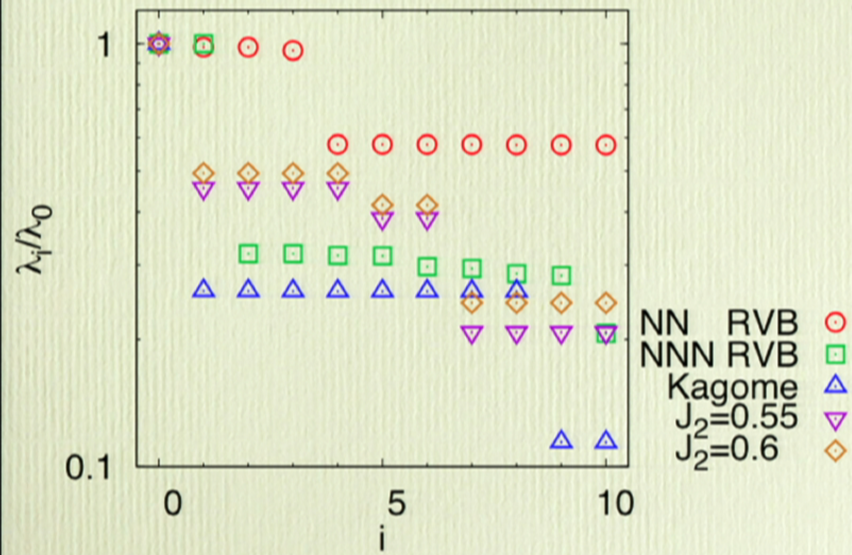


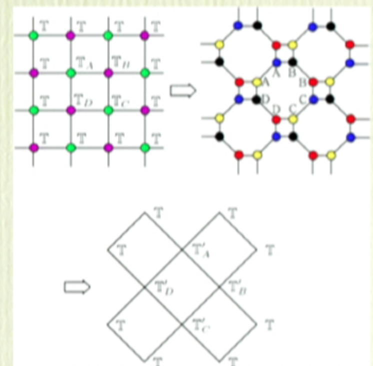
Gu et al. PRB 78, 205116 (2008)

Two fold degeneracy of the SVD spectral is a sufficient but not a necessary signature of a Z2 spin liquid state; a gap is confirmed in the singular value spectral of the spin liquid state in the J1-J2 model.

the SVD spectral







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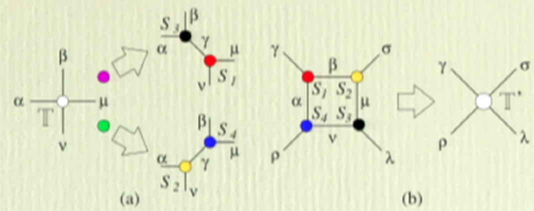
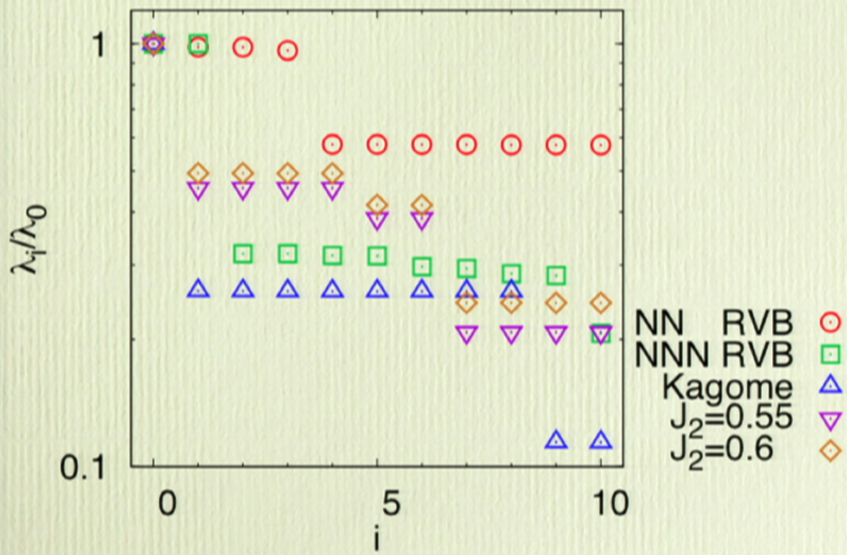
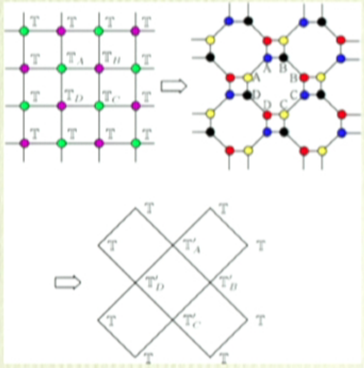
the SVD spectral







Gu et al. PRB 78, 205116 (2008)

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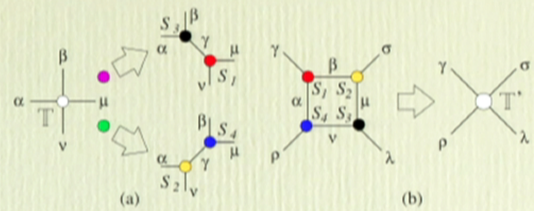

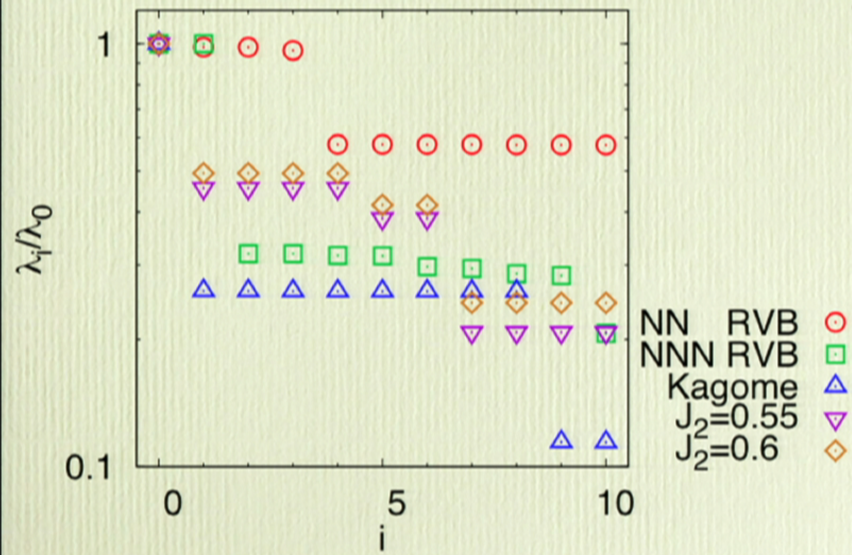

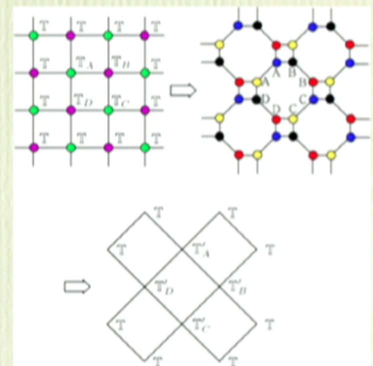

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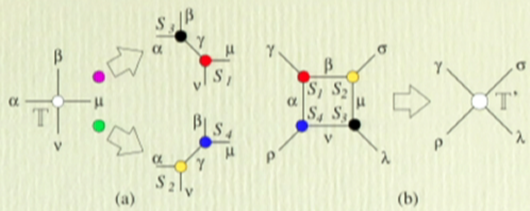

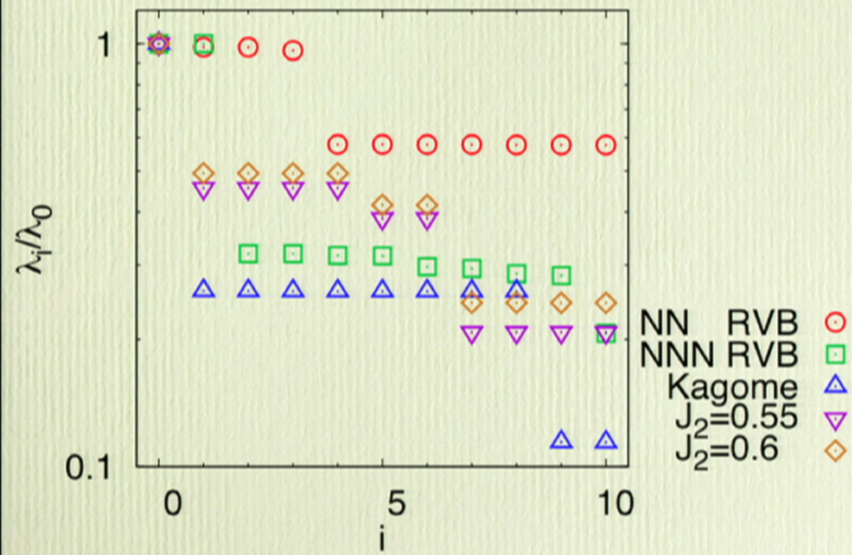

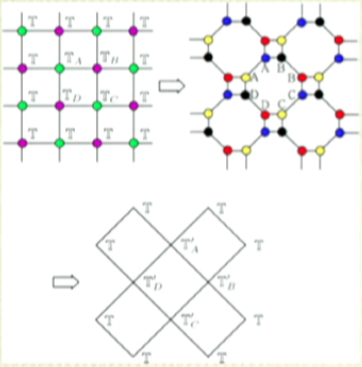

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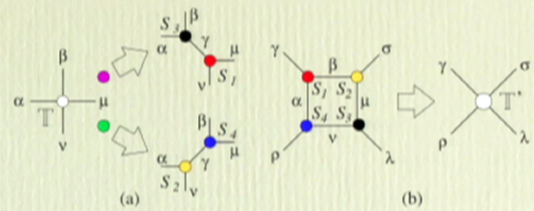

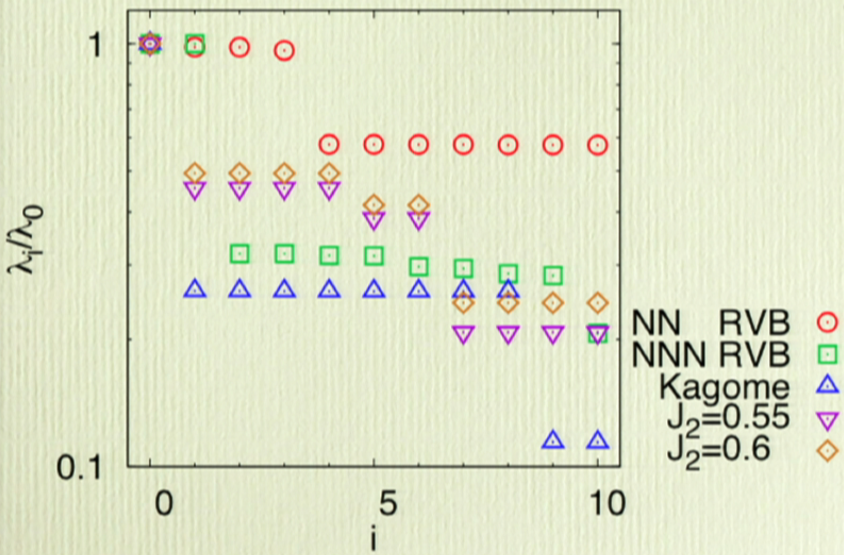

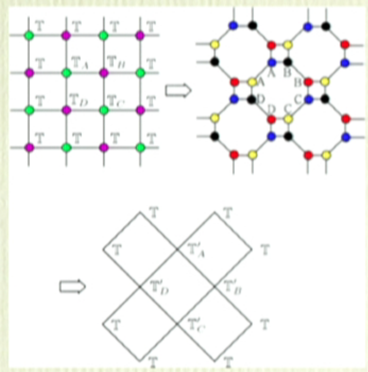

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