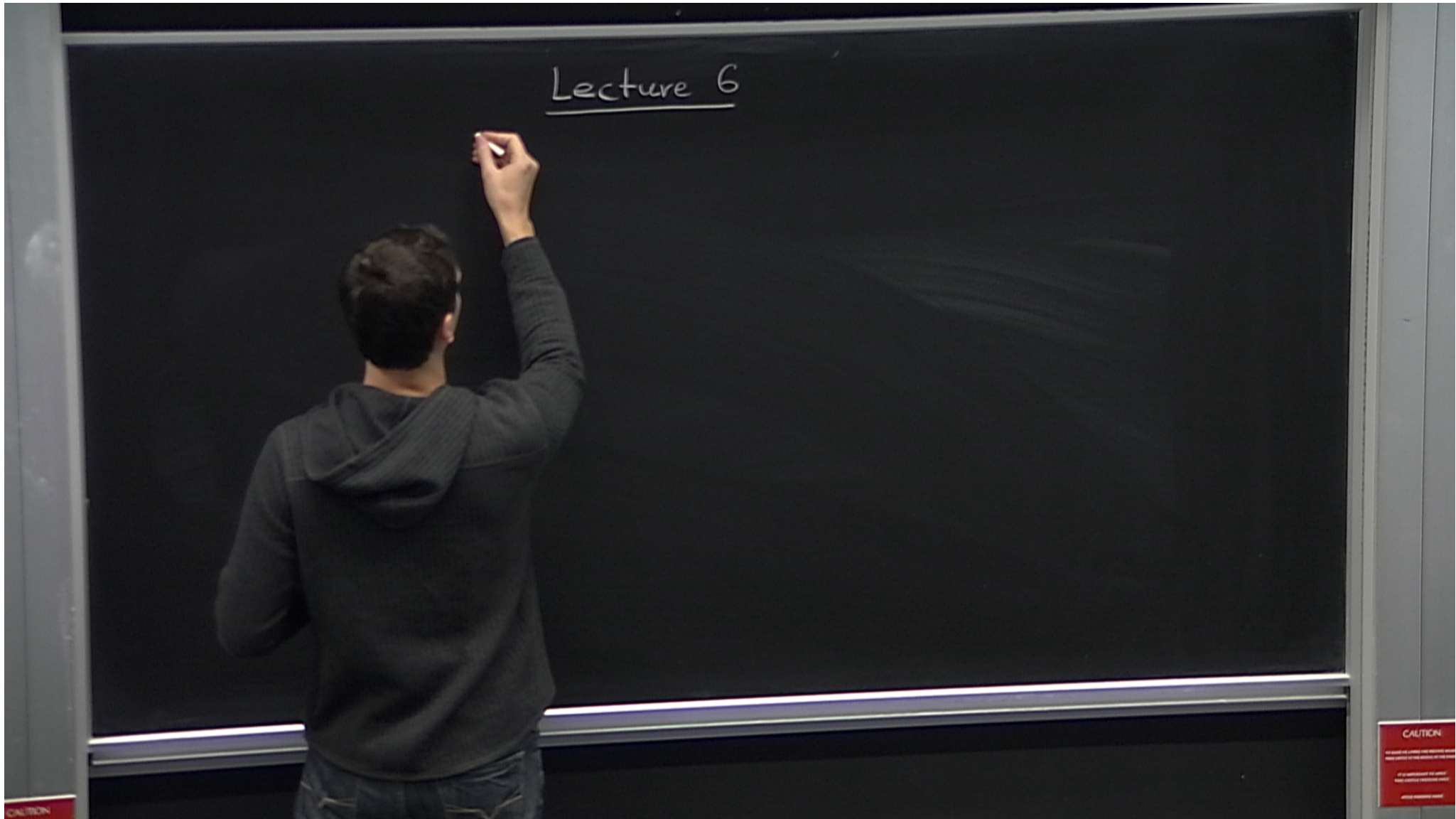


Title: Explorations in Condensed Matter - Lecture 7

Date: Apr 11, 2012 10:15 AM

URL: <http://www.pirsa.org/12040090>

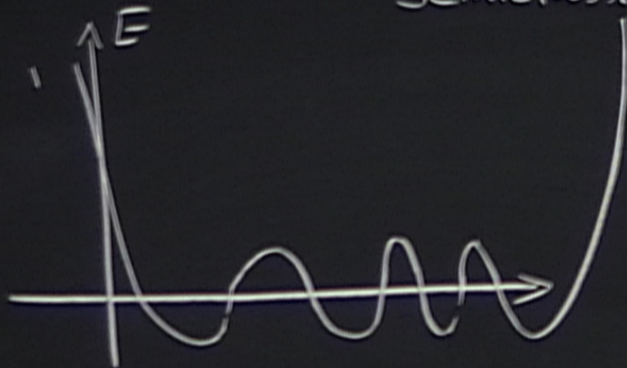
Abstract:





Lecture 6

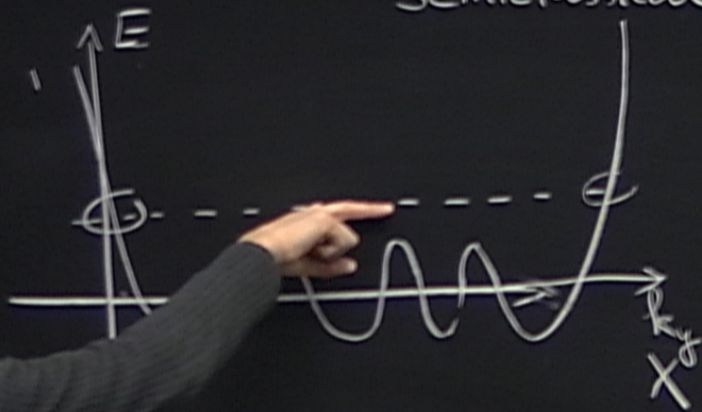
Semiclassical percolation picture of QH transitions





# Lecture 6

## Semiclassical percolation picture of QH transitions



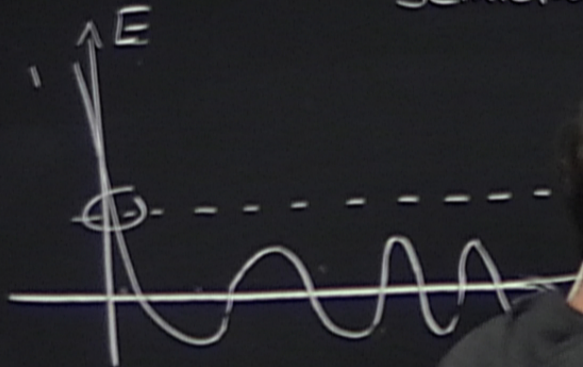


## Lecture 6

Semiclassical percolation picture of QH transitions

\* Are bulk states localized?

\*  $I_s$





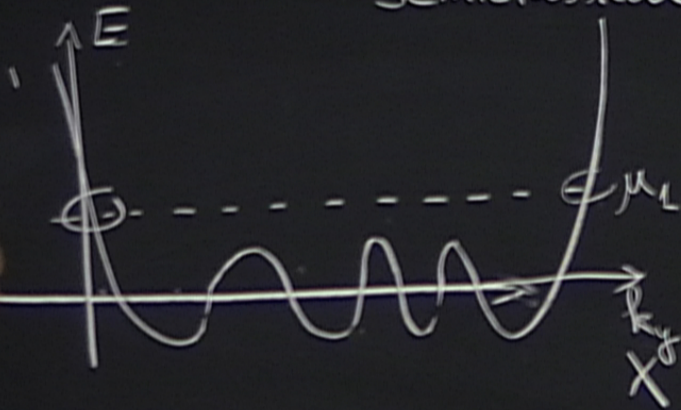
## Lecture 6

Semiclassical

percolation picture of QH transitions

\* Are bulk states localized?

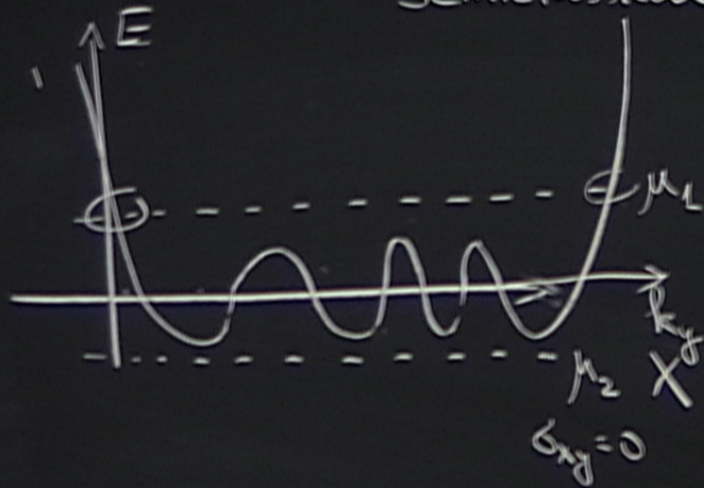
\* Is there extended state?





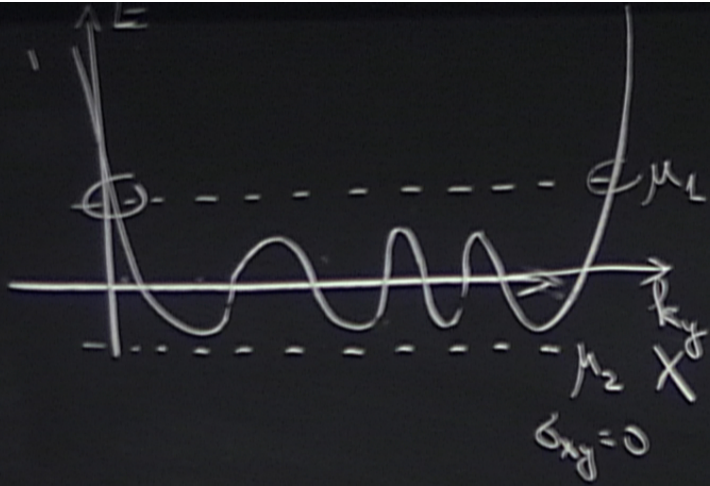
# Lecture 6

Semiclassical percolation picture of QH transitions



- \* Are bulk states localized?
- \* Is there external state?
- \* Transition  $\sigma_{xy} = \frac{e^2}{h}$

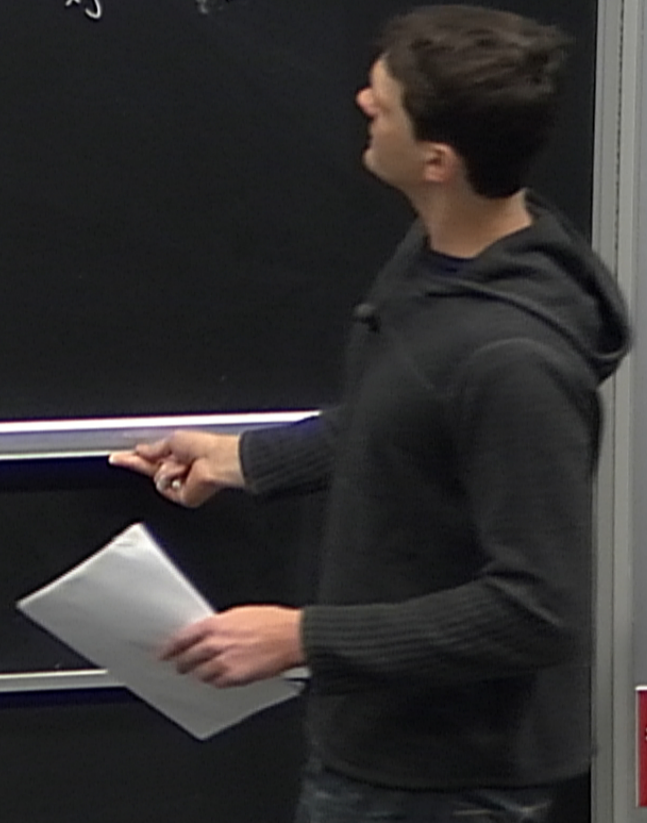




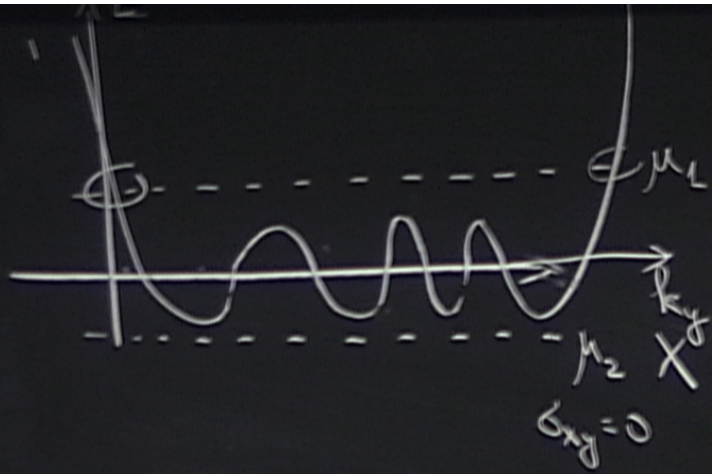
\* Are bulk states localized?

\* Is there extended state?

$\sigma_{xy} = \frac{e^2}{h}$  \* Transition b/w  $\sigma_{xy} = \frac{e^2}{h}$  and  $\sigma_{xy} = 0$  !?







$V(x,y)$

\* Are bulk states localized?

\* Is there extended state?

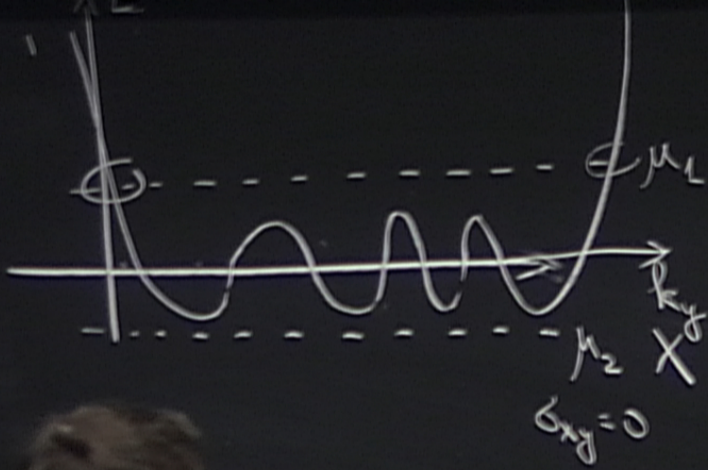
$\sigma_{xy} = \frac{e^2}{h}$

\* Transition b/w  $\sigma_{xy} = \frac{e^2}{h}$  and  $\sigma_{xy} = 0$ ?

$\sigma_{xy} = 0$

CAUTION

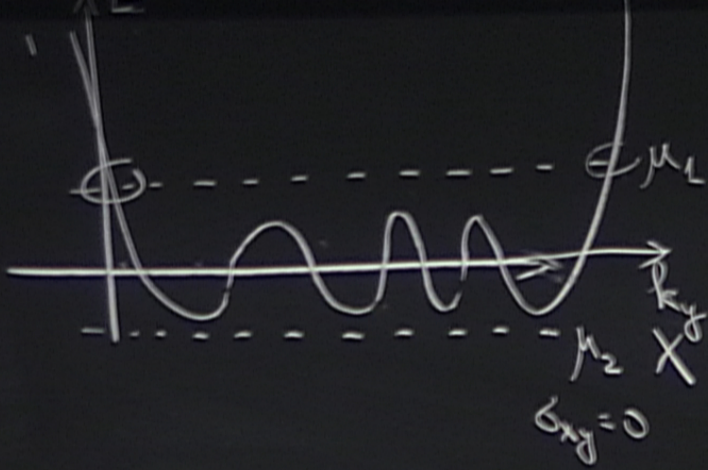




\* Are bulk states localized?  
 \* Is there extended state?  
 $\sigma_{xy} = \frac{e^2}{h}$  \* Transition b/w  $\sigma_{xy} = \frac{e^2}{h}$   
 and  $\sigma_{xy} = 0$  !?

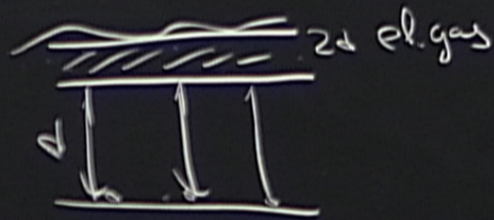
$(x,y)$   $\left| \frac{dV}{dx} \right| \ll \frac{\hbar \omega_c}{\ell}$



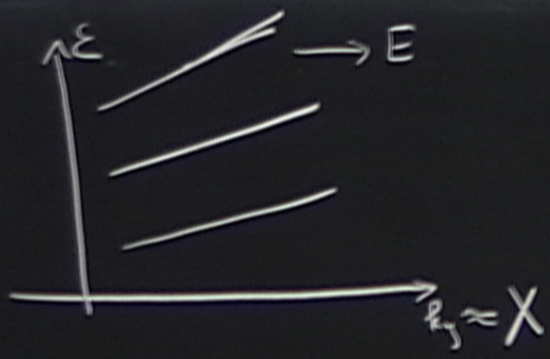


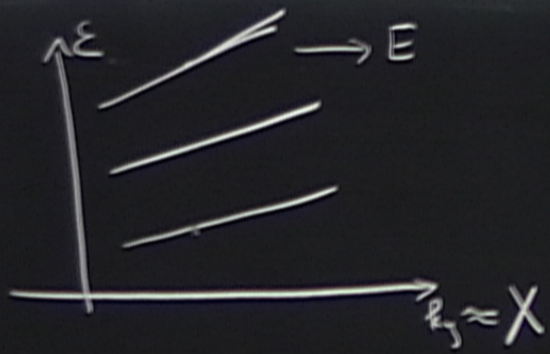
$$V(x,y) \quad \left| \frac{dV}{dx} \right| \ll \frac{\hbar \omega_c}{\ell}$$

\* Are bulk states localized?  
 \* Is there extended state?  
 $\sigma_{xy} = \frac{e^2}{h}$  \* Transition b/w  $\sigma_{xy} = \frac{e^2}{h}$   
 and  $\sigma_{xy} = 0$  !?





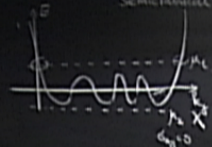




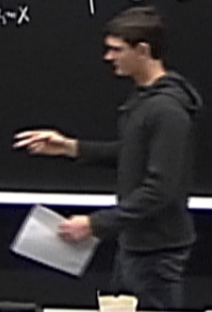
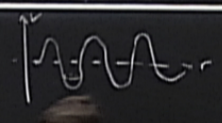
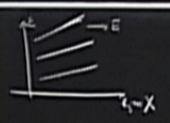
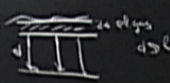


Tuesday 11/5/11  
Lecture 3:30-4:30pm (Bob room)

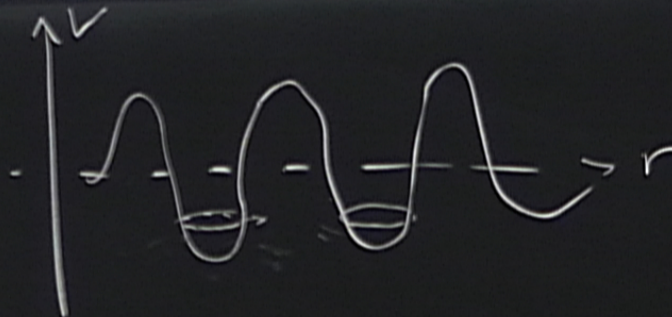
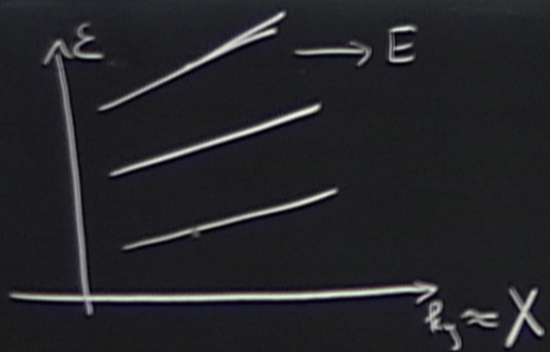
Semiclassical perturbation picture of QM transitions  
+ Are both states localized?  
+ Is there extended state?  
 $\omega_j \neq \omega_k \rightarrow$  Transitions b/w  $\omega_j \neq \omega_k$   
and  $\omega_j = \omega_k$ ?



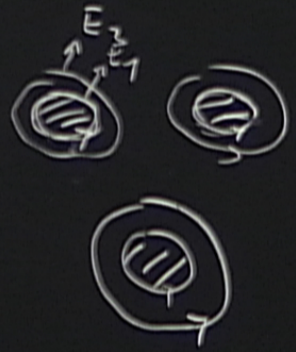
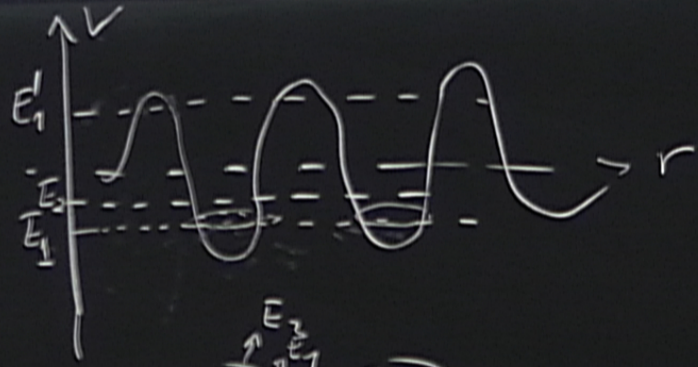
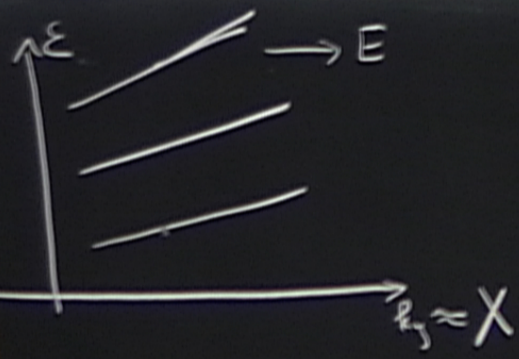
$V(x) \propto \frac{dV}{dx} \ll \frac{\hbar \omega}{c}$





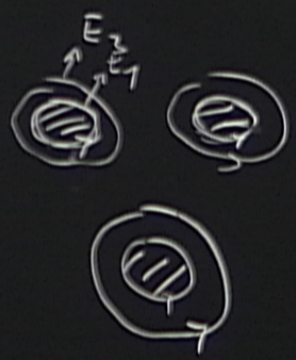
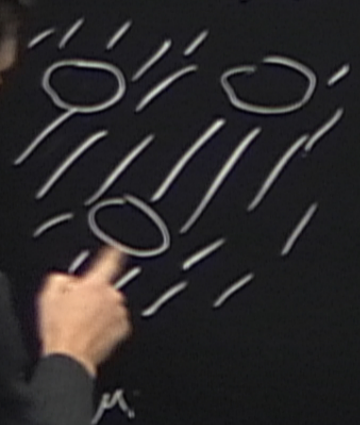
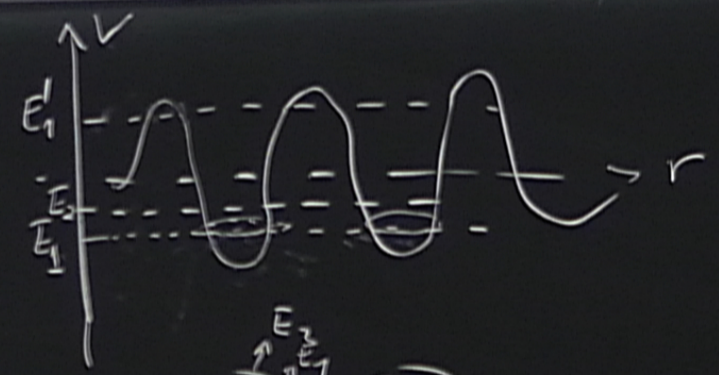
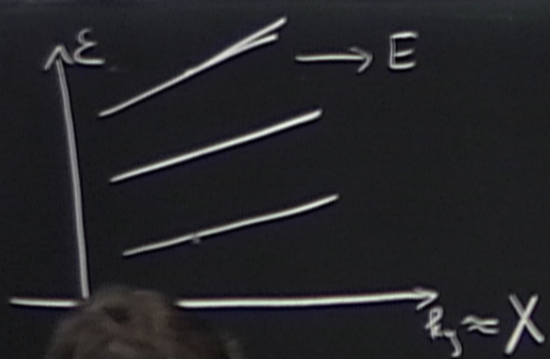






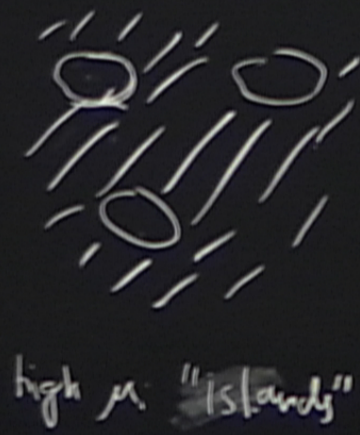
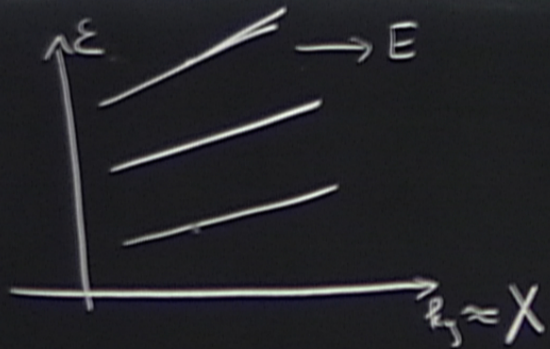
Low chem. potential. "lakes"



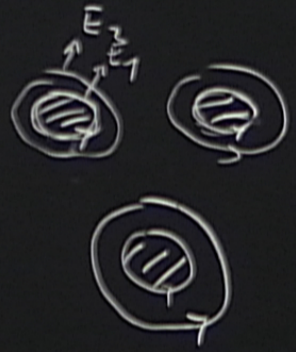
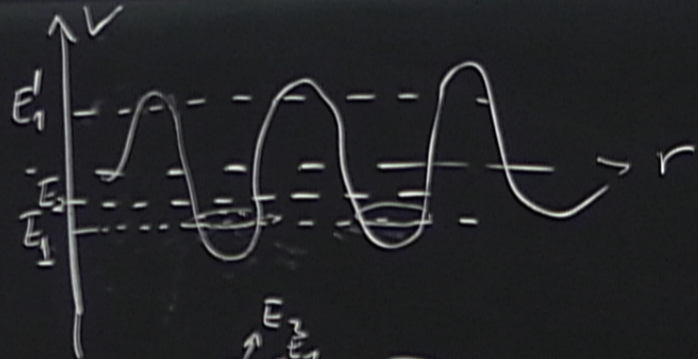


Low chem. potential. "lakes"



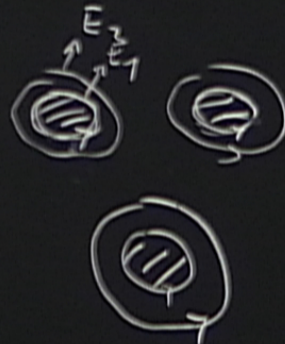
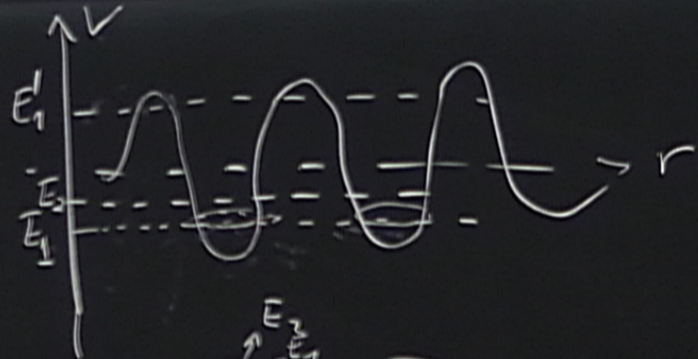
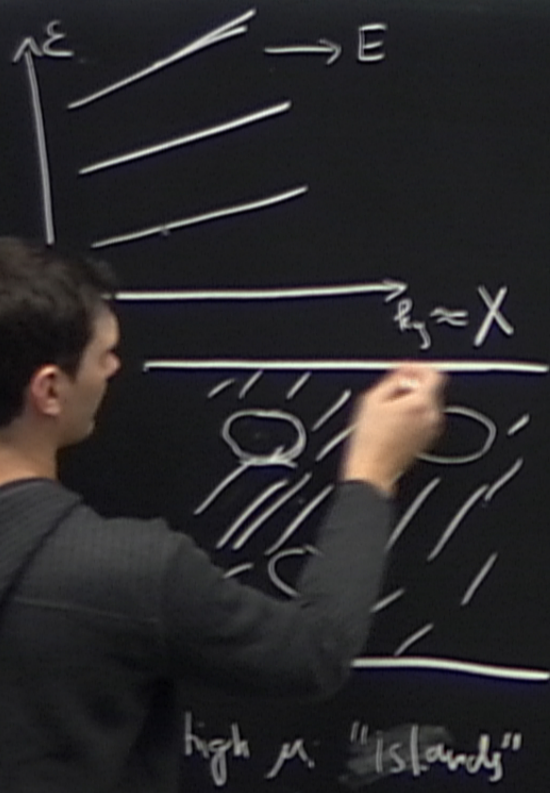


high  $\mu$ : "islands"



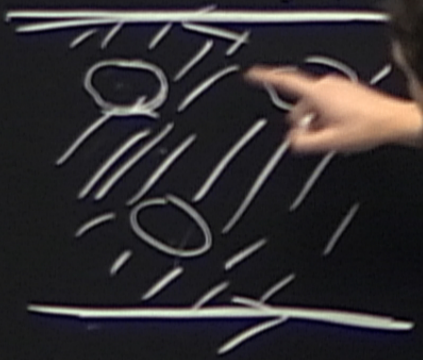
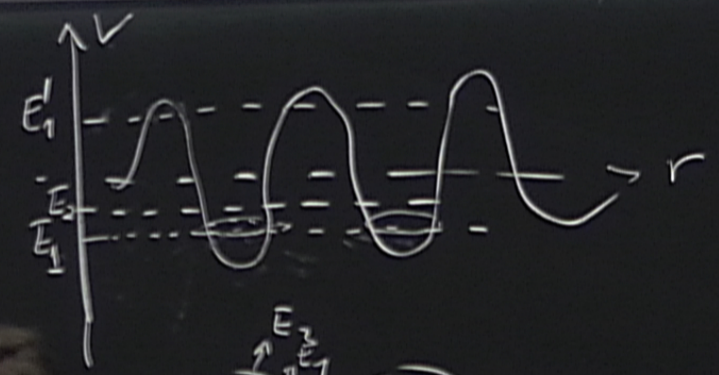
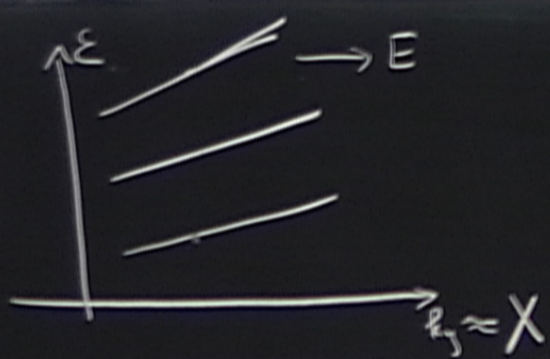
Low chem. potential. "lakes"



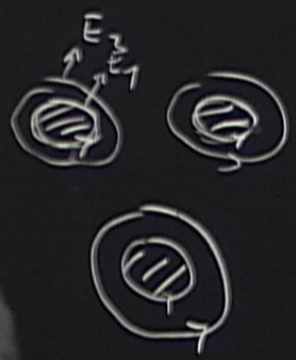


Low chem. potential. "lakes"



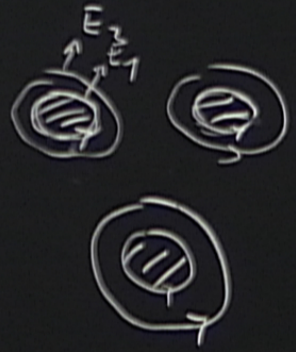
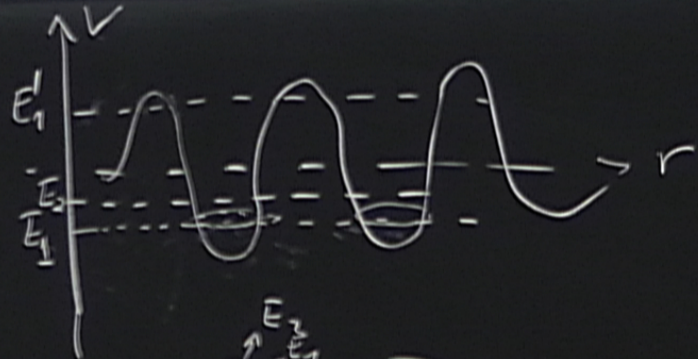
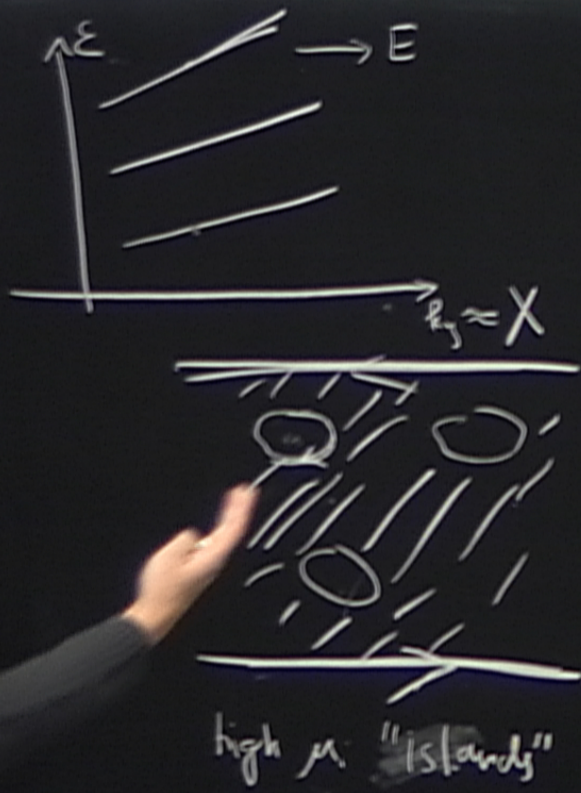


high  $\mu$  "islands"



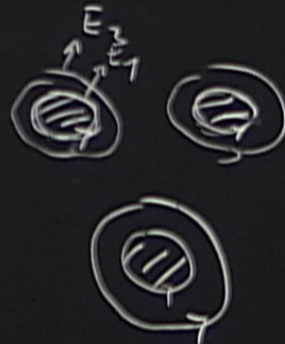
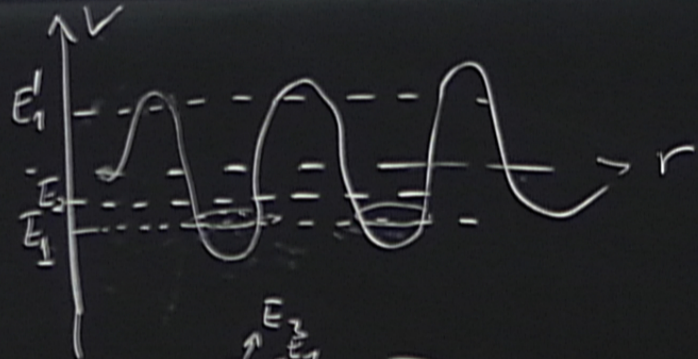
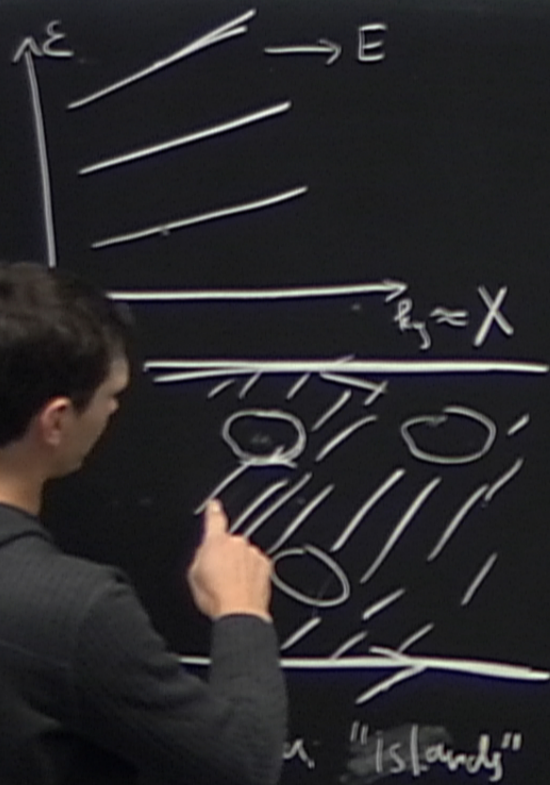
Low chem. potential. "lakes"





Low chem. potential. "lakes"





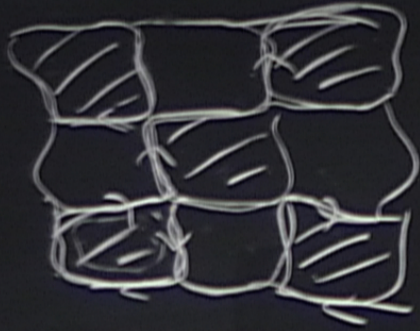
Low chem. potential. "lakes"





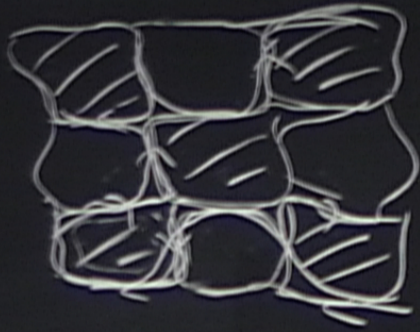
$$\mu = \frac{h\omega_c}{2} + \langle V \rangle$$





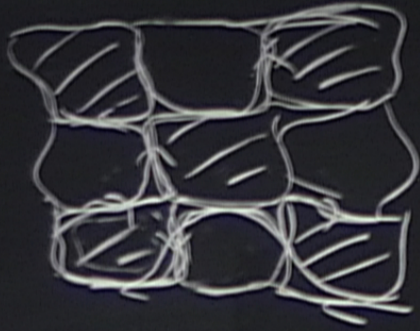
$$\underline{\underline{\mu = \frac{hw_c}{2} + \langle V \rangle}}$$





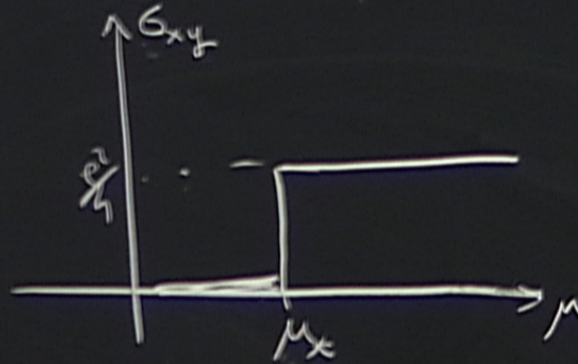
$$\underline{\underline{\mu = \frac{hw_c}{2} + \langle V \rangle}}$$



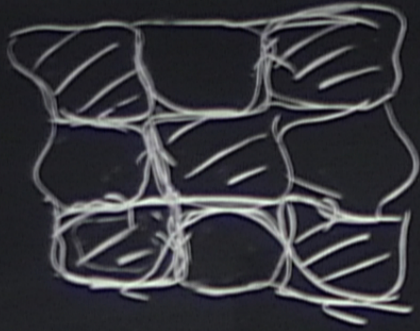


$$\mu_* = \frac{hw_c}{2} + \langle \nabla \rangle$$

↓  
Critical point

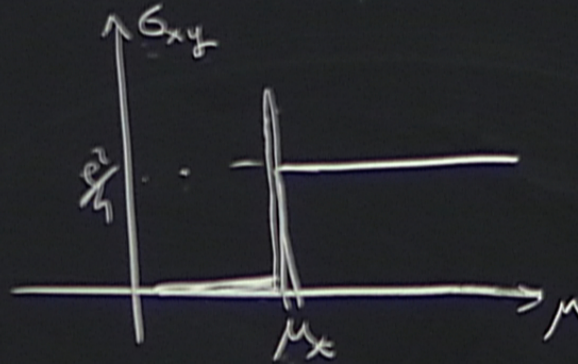




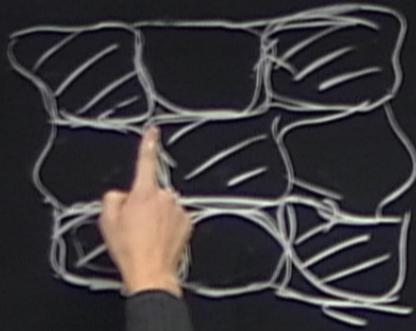


$$\mu_* = \frac{hw_c}{2} + \langle \nabla \rangle$$

↓  
Critical point

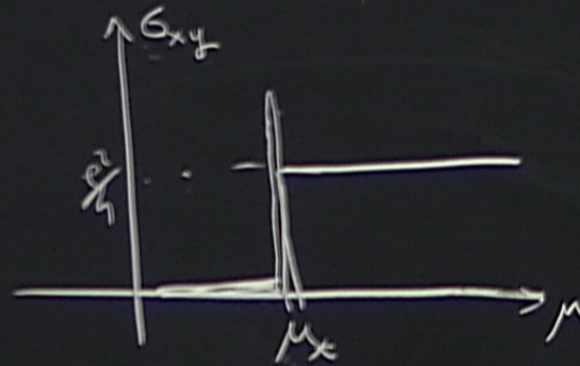




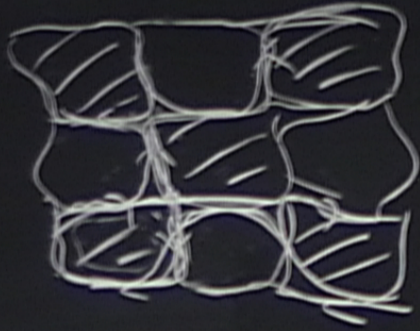


$$\mu_* = \frac{hw_c}{2} + \langle \nabla \rangle$$

↓  
Critical point

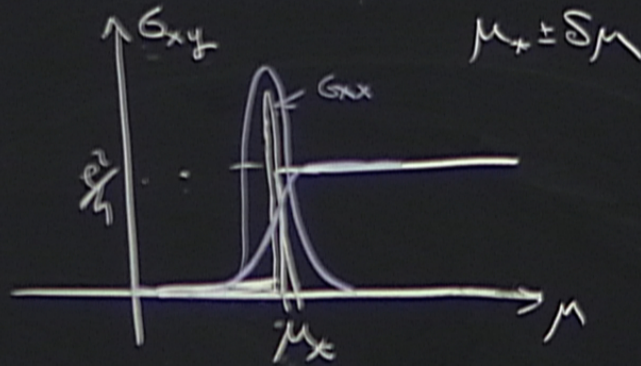




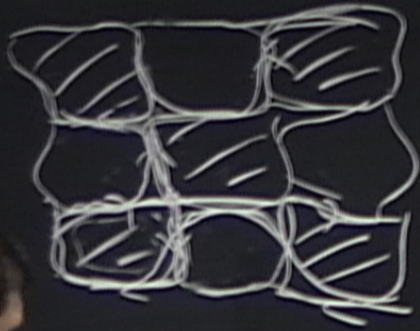


$$\mu_* = \frac{hw_c}{2} + \langle V \rangle$$

↓  
Critical point

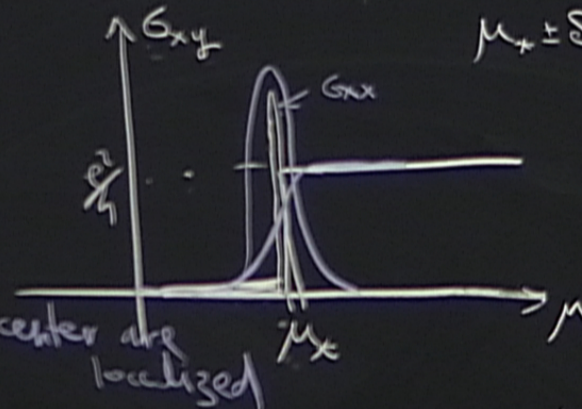






$$\mu_* = \frac{t w_c}{2} + \langle V \rangle$$

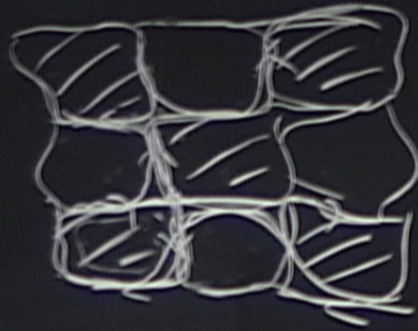
↓  
Critical point



States away from  $\mu_*$  center are localized

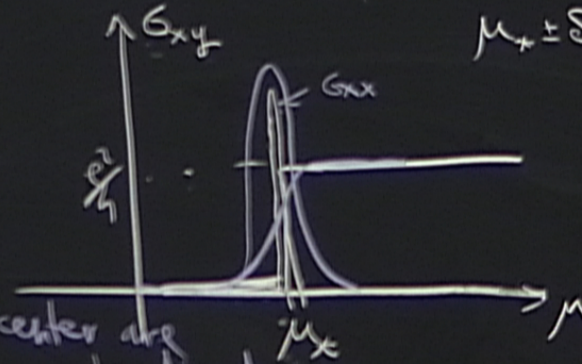
At center of  $\mu_*$ , extended state must exist





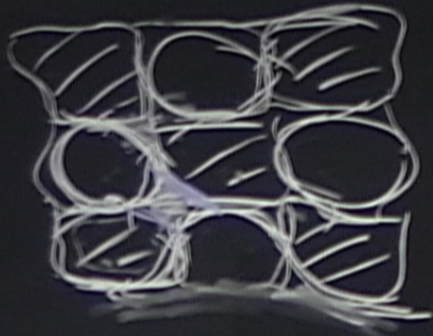
$$\mu_* = \frac{\hbar\omega_c}{2} + \langle V \rangle$$

↓  
Critical point



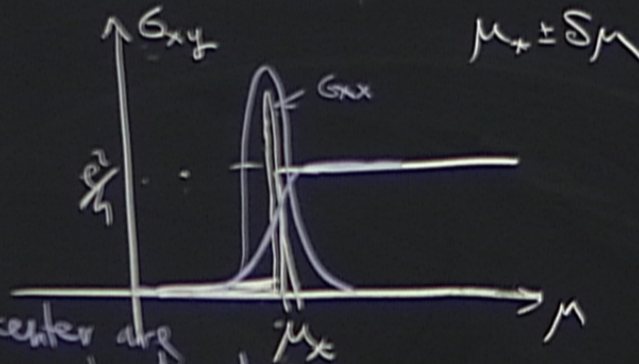
- \* States away from hh center are localized at energy  $E_0$
- \* At the center of hh, extended state must exist
- \*  $\mu_* = E_0$  - phase transition





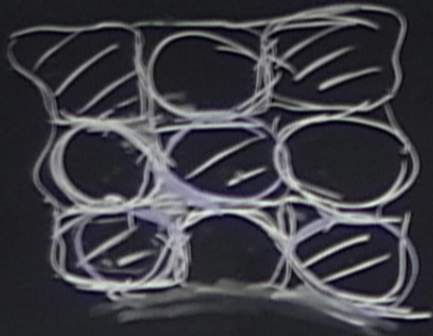
$$\mu_* = \frac{\hbar\omega_c}{2} + \langle V \rangle$$

↓  
Critical point



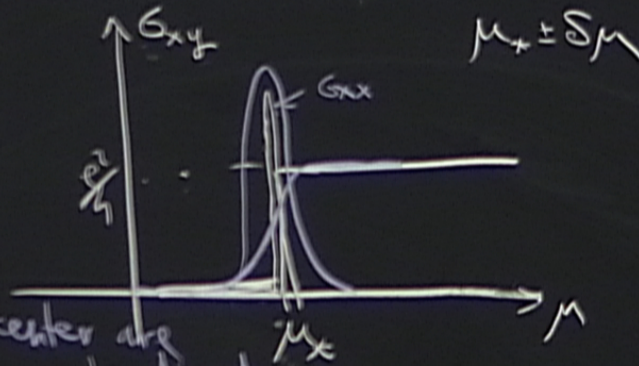
- \* States away from  $\mu$  center are localized at energy  $E_0$
- \* At the center of  $\mu$ , extended state must exist
- \*  $\mu_* = E_0$  - phase transition





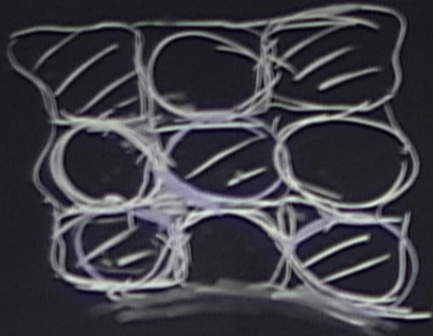
$$\mu_* = \frac{\hbar\omega_c}{2} + \langle V \rangle$$

↓  
Critical point



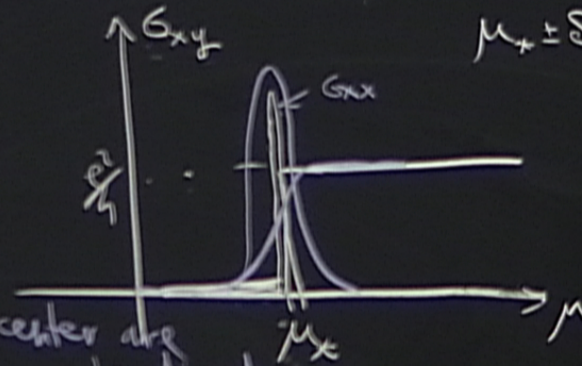
- \* States away from the center are localized at energy  $E_0$
- \* At the center of the band, extended state must exist
- \*  $\mu_* = E_0$  - phase transition





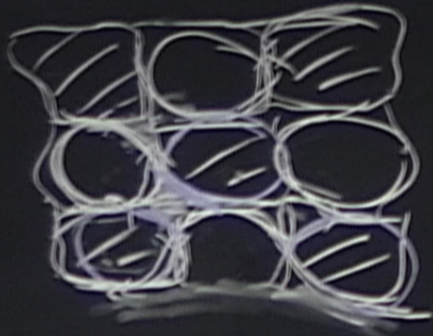
$$\mu_* = \frac{\hbar \omega_c}{2} + \langle V \rangle$$

↓  
Critical point



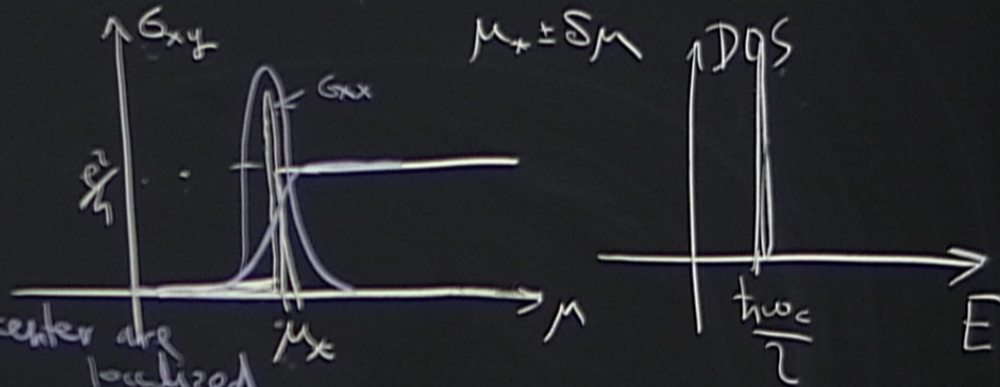
- \* States away from hh center are localized at energy  $E_0$
- \* At the center of hh, extended state must exist
- \*  $\mu_* = E_0$  - phase transition





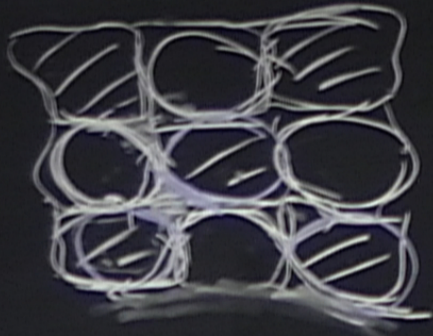
$$\mu_* = \frac{\hbar\omega_c}{2} + \langle V \rangle$$

↓  
Critical point



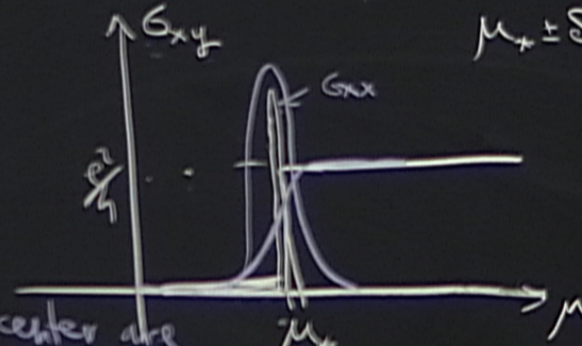
- \* States away from  $\mu$  center are localized at energy  $E_0$
- \* At the center of  $\mu$ , extended state must exist
- \*  $\mu_x = E_0$  - phase transition



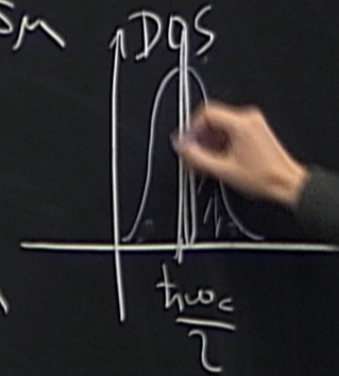


$$\mu_* = \frac{\hbar\omega_c}{2} + \langle V \rangle$$

↓  
Critical point

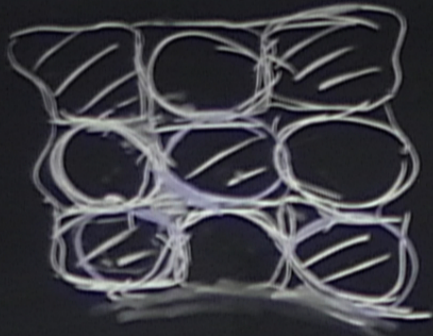


$$\mu_* \pm \delta\mu$$



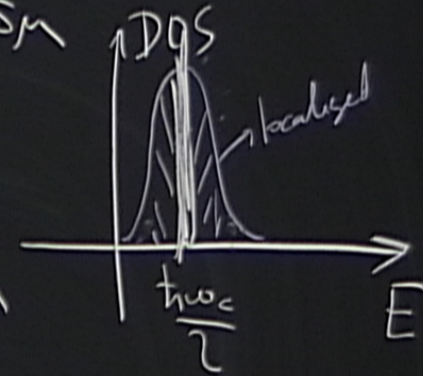
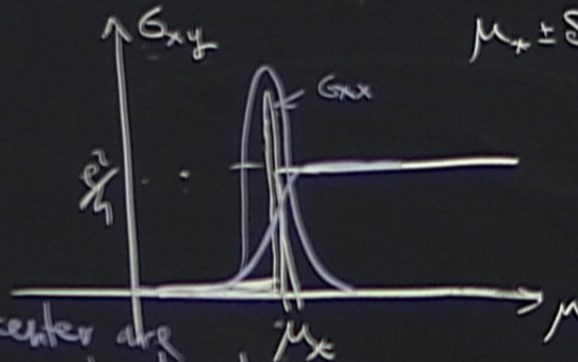
- \* States away from hh center are localized at energy  $E_0$
- \* At the center of hh, extended state must exist
- \*  $\mu_* = E_0$  - phase transition





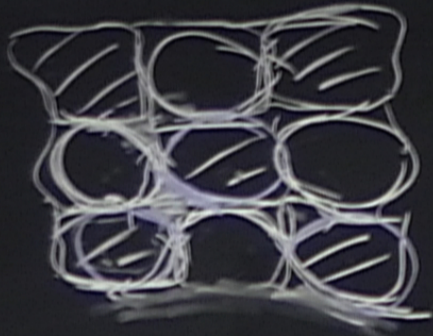
$$\mu_* = \frac{\hbar\omega_c}{2} + \langle V \rangle$$

↓  
Critical point



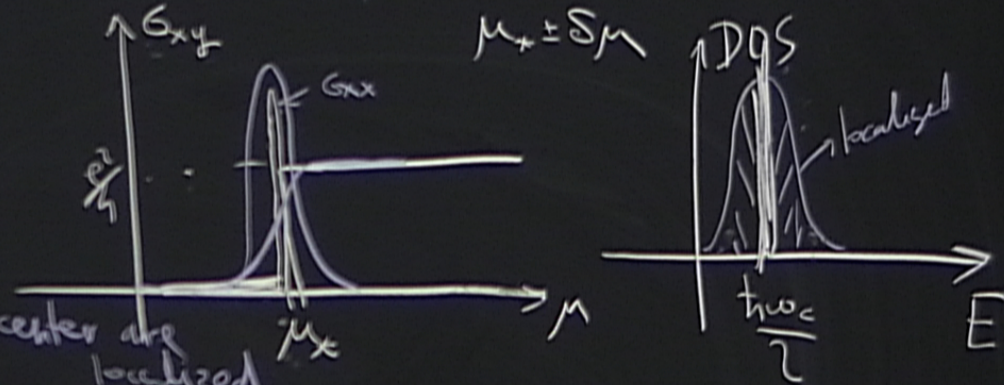
- \* States away from hh center are localized at energy  $E_0$
- \* At the center of hh, extended state must exist
- \*  $\mu_* = E_0$  - phase transition





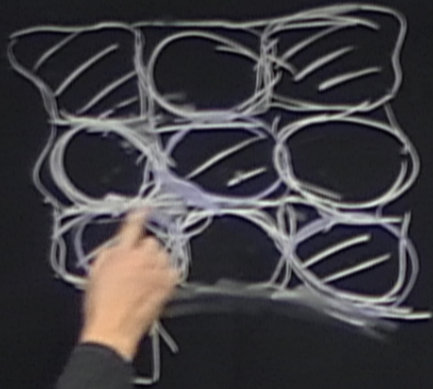
$$\mu_* = \frac{\hbar\omega_c}{2} + \langle V \rangle$$

↓  
Critical point



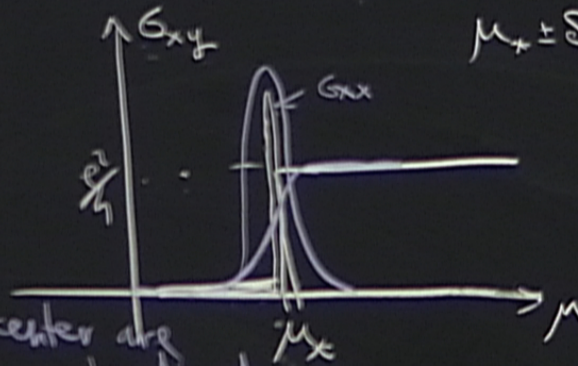
- \* States away from hh center are localized at energy  $E_0$
- \* At the center of hh, extended state must exist
- \*  $\mu_* = E_0$  - phase transition



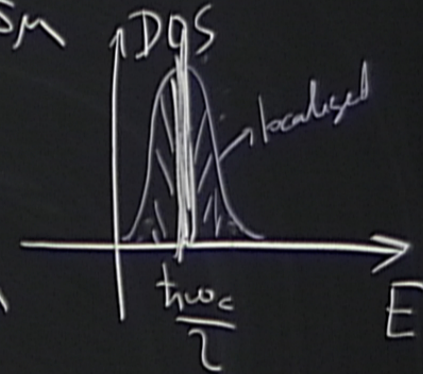


$$\mu_* = \frac{\hbar\omega_c}{2} + \langle \nabla \rangle$$

↓  
Critical point



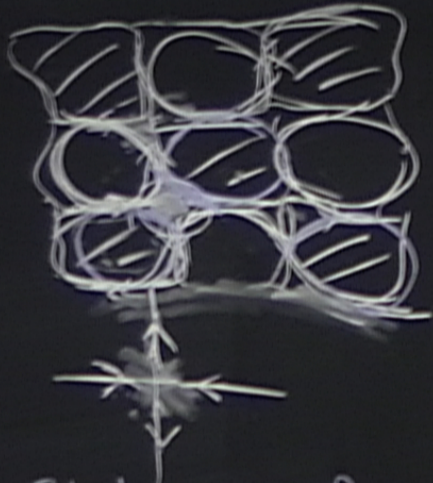
$$\mu_* \pm \delta\mu$$



states away from hh center are localized at energy  $E_0$

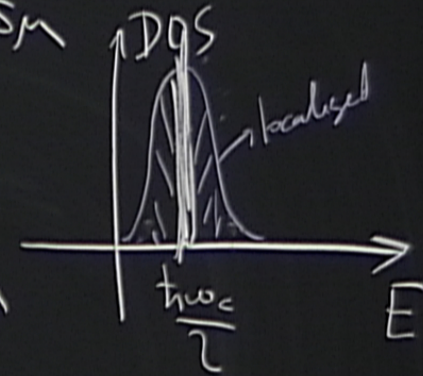
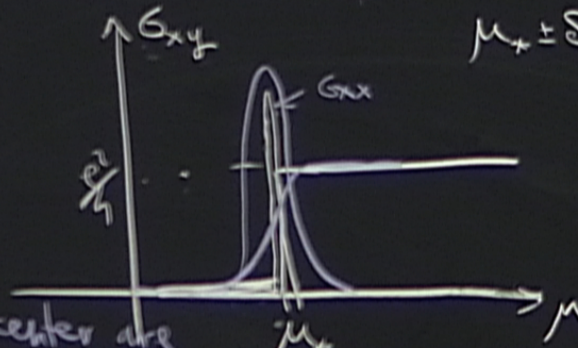
- \* At the center of hh, extended state must exist
- \*  $\mu_* = E_0$  - phase transition





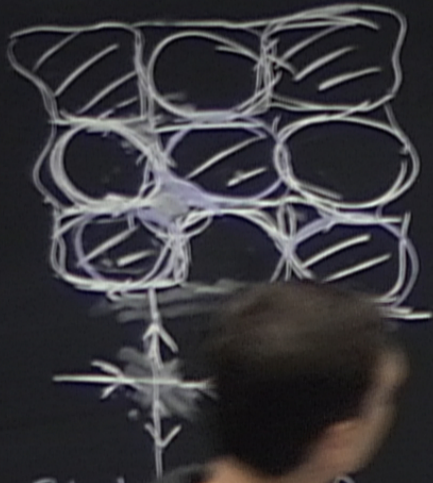
$$\mu_* = \frac{\hbar\omega_c}{2} + \langle V \rangle$$

↓  
Critical point



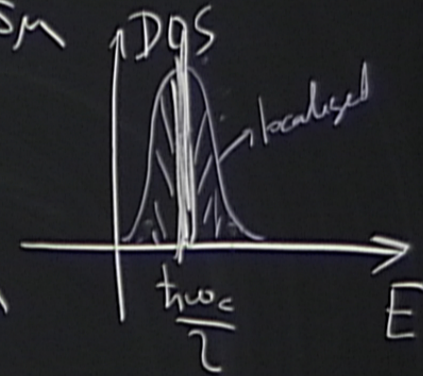
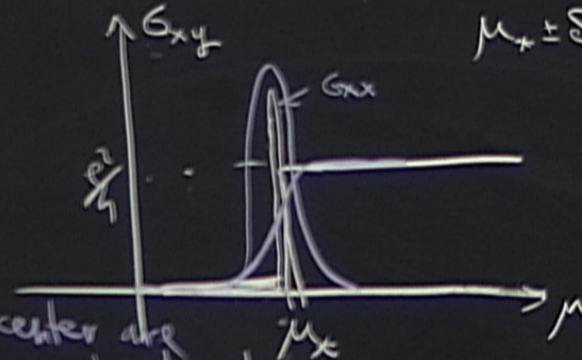
- \* States away from hh center are localized at energy  $E_0$
- \* At the center of hh, extended state must exist
- \*  $\mu_* = E_0$  - phase transition





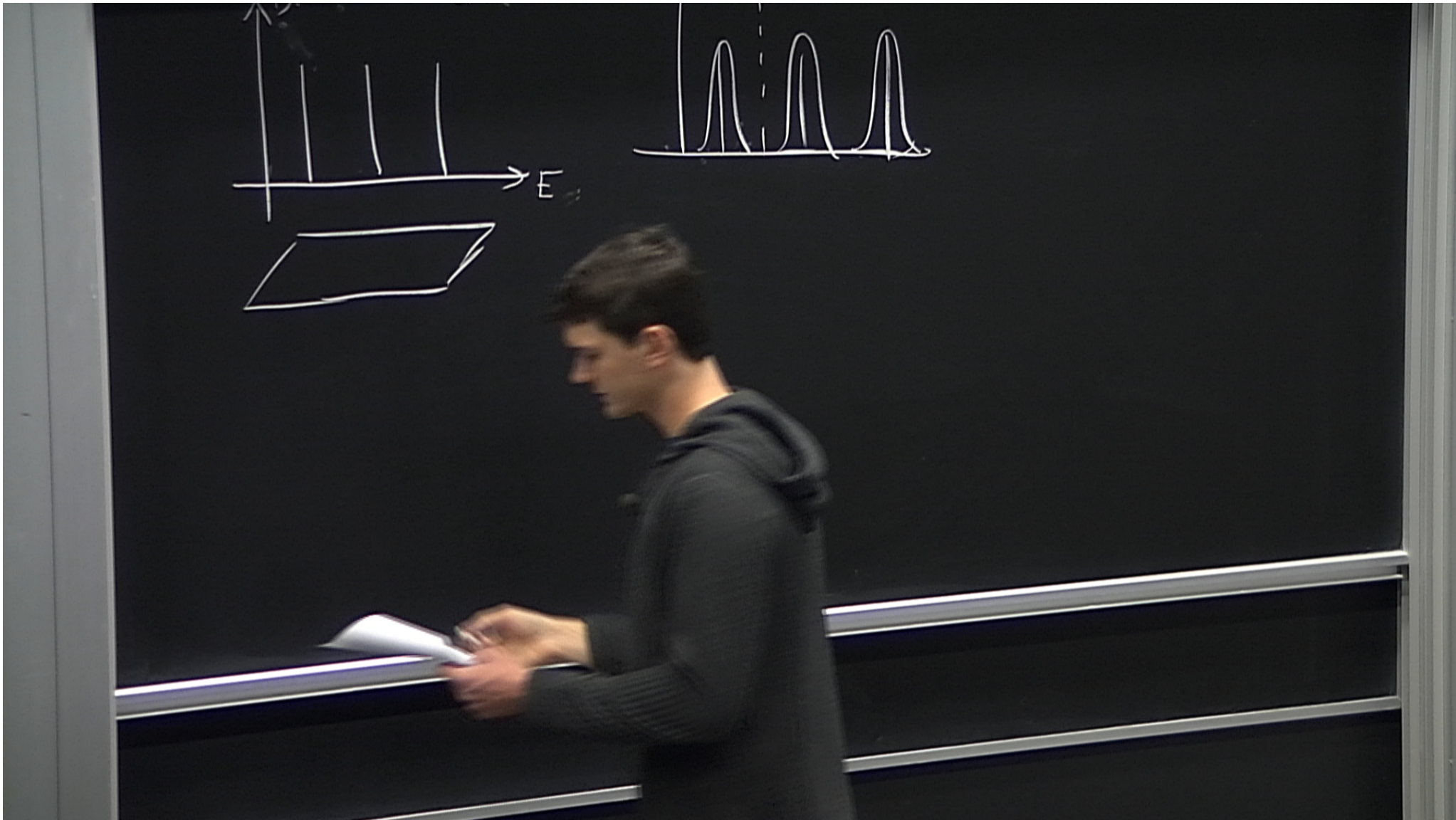
$$\mu_* = \frac{\hbar\omega_c}{2} + \langle V \rangle$$

↓  
Critical point

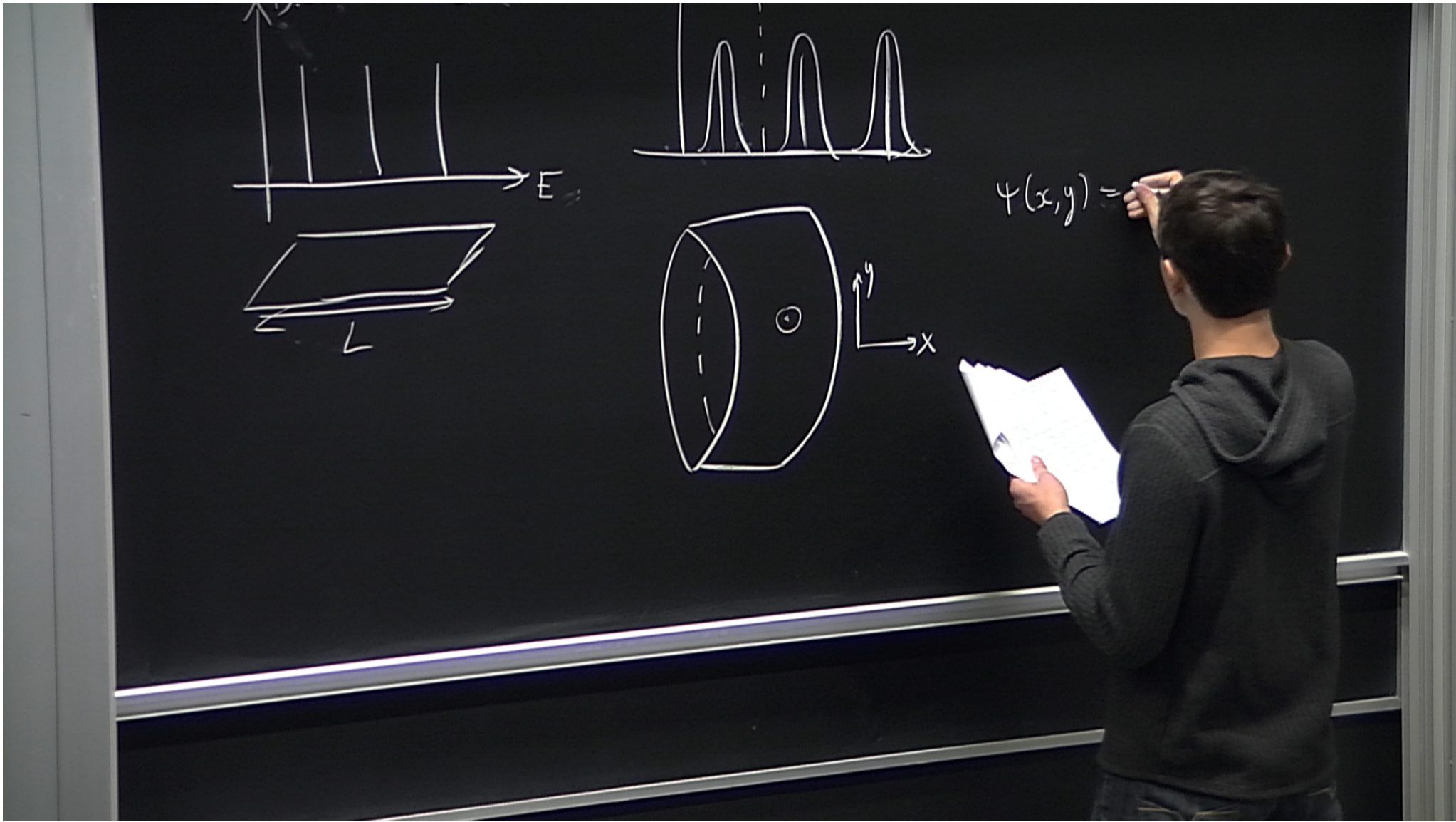


- \* Start from hh center are localized
- \* At at energy  $E_0$  hh, extended state must exist
- \*  $\mu_*$

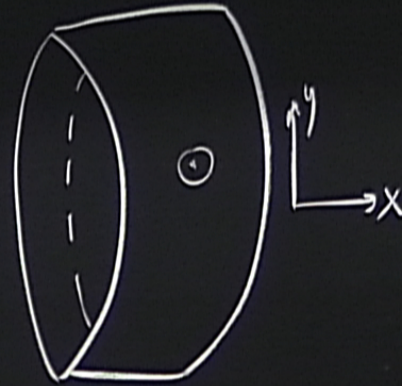
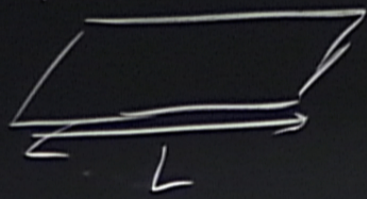




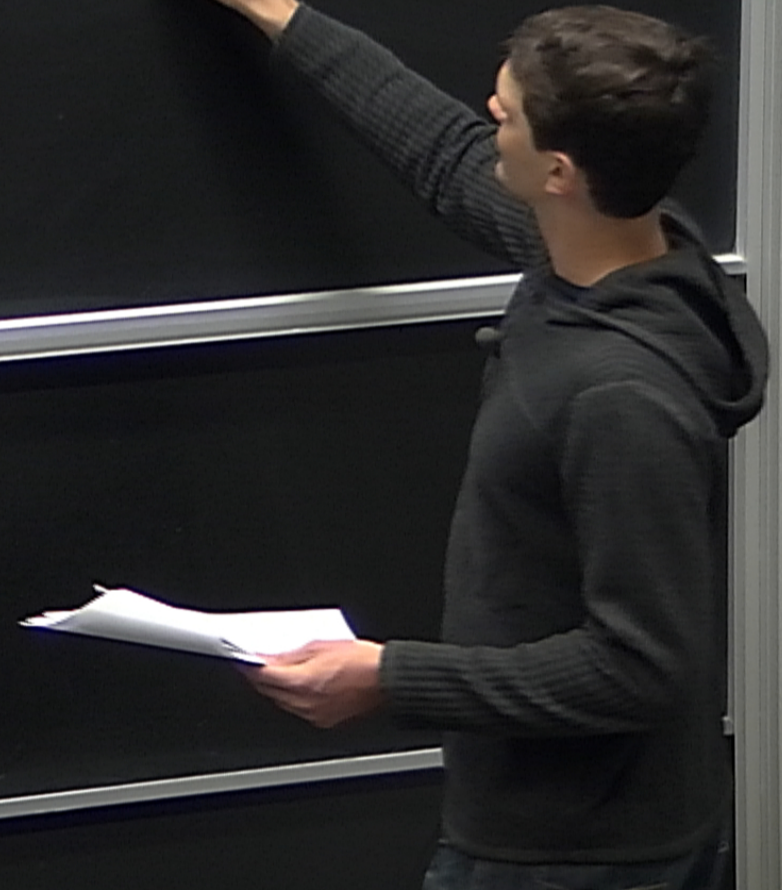




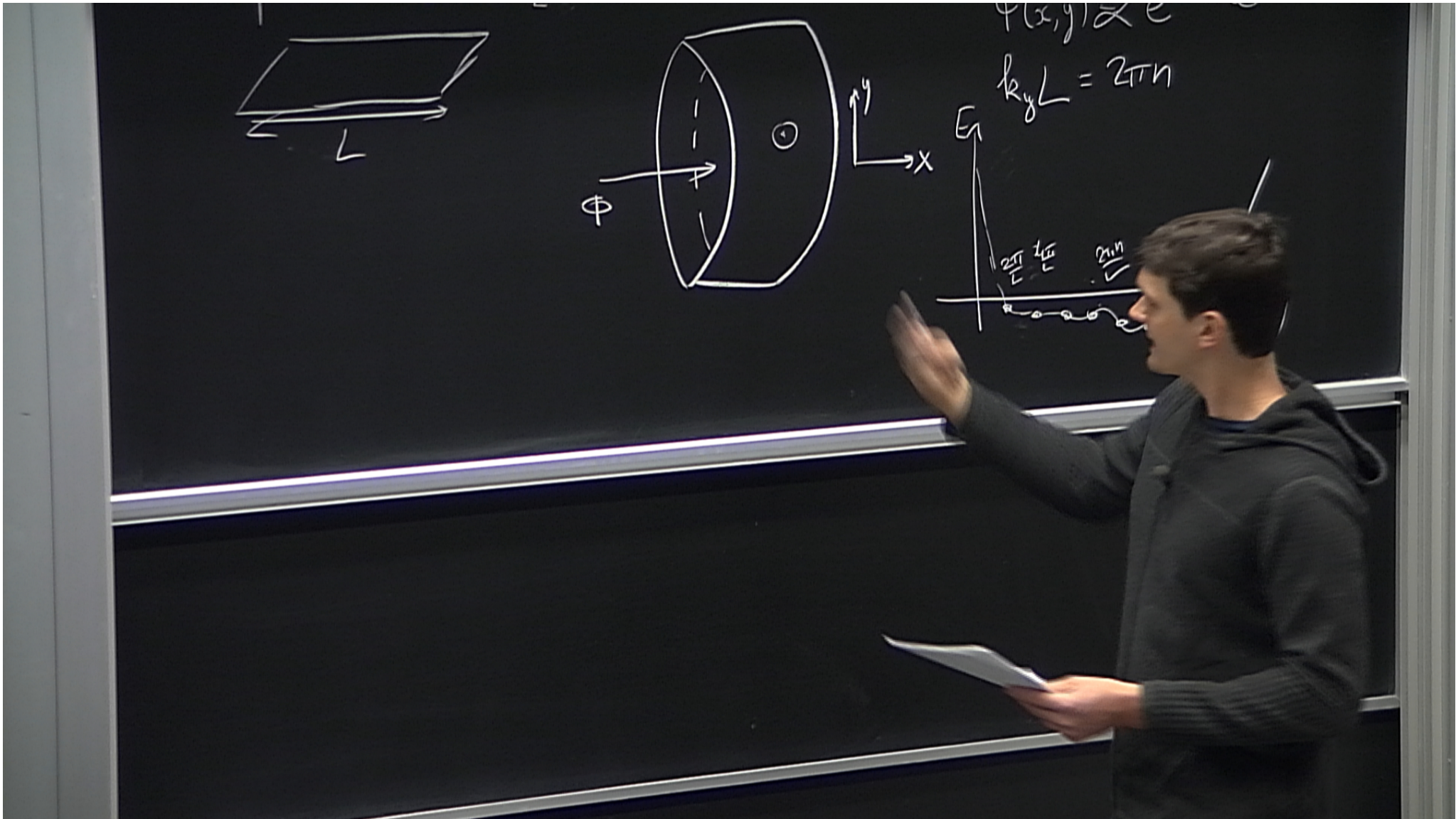




$$k_y L = 2\pi n$$









$\delta\Phi \rightarrow$  vector-potential

$$\delta A_y = \frac{\delta\Phi}{L}$$

$$\delta\phi = \frac{1}{\hbar} \int \frac{e}{c} A_y dy = \frac{e}{\hbar c} \delta\Phi =$$



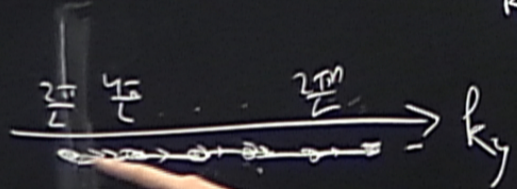
$\delta\Phi$   $\rightarrow$  vector-potential

$$\delta A_y = \frac{\delta\Phi}{L}$$

$$\delta\Phi = \frac{1}{\pi} \int \frac{e}{c} A_y dy = \frac{e}{\hbar c} \delta\Phi =$$

$$\Phi_0 = \frac{\hbar c}{e} \quad \delta\Phi = 2\pi \frac{\delta\Phi}{\Phi_0}$$

$$k_y L = 2\pi$$





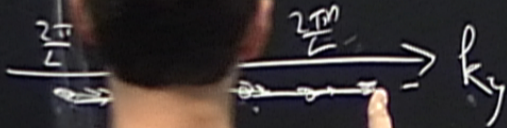
$\delta\Phi \rightarrow$  vector-potential

$$\delta A_y = \frac{\delta\Phi}{L}$$

$$\delta\Phi = \frac{1}{\hbar} \int \frac{e}{c} A_y dy = \frac{e}{\hbar c} \delta\Phi =$$

$$\Phi_0 = \frac{hc}{e} \quad \delta\Phi = 2\pi \frac{\delta\Phi}{\Phi_0}$$

$$k_y L = 2\pi n + \delta\Phi$$



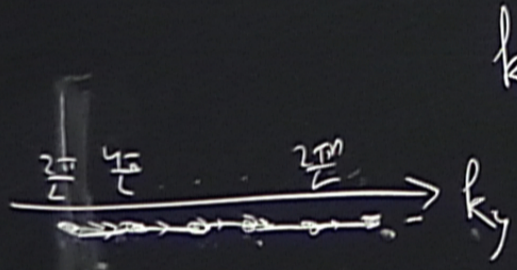


$\delta\phi$   $\rightarrow$  vector-potential

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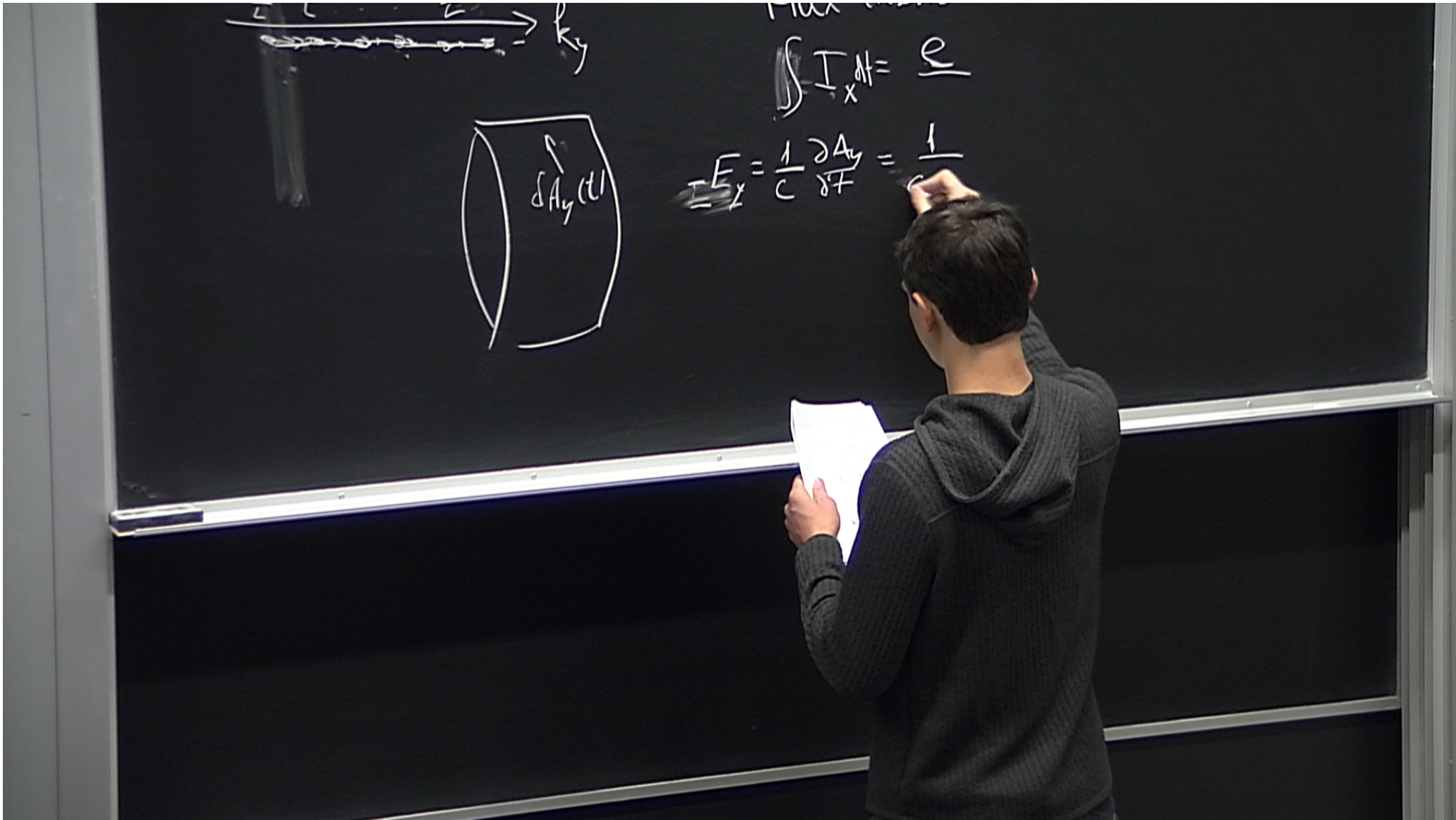


$$k_y L = 2\pi n + \delta\phi$$

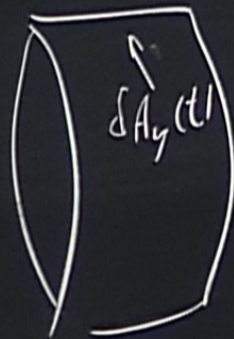
Flux insertion. Charge transfer  $e$ .

$$\int -I_x dt = e$$









Max

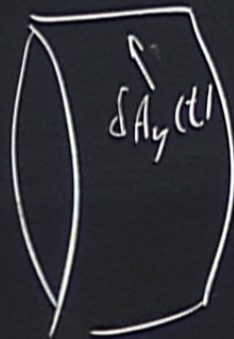
$$\int I_x dt = e$$

$$E_y = \frac{1}{c} \frac{\partial A_y}{\partial t} = \frac{1}{cL} \frac{\partial \phi}{\partial t}$$

$$j_x = \epsilon_{xy} E_y$$

Charge:  $\int j_x dt \times L = \int \frac{1}{cL} \frac{\partial \phi}{\partial t} dt = \frac{1}{c} \frac{\partial \phi}{\partial t} dt = \frac{1}{c} \Delta \phi$





$$\int I_x dt = \underline{e}$$

$$E_y = \frac{1}{c} \frac{\partial A_y}{\partial t} = \frac{1}{cL} \frac{\partial \Phi}{\partial t}$$

$$j_x = \epsilon_{xy} E_y$$

$$\text{Charge } Q = \int j_x dt \times L = \int \epsilon_{xy} \frac{1}{cL} \frac{\partial \Phi}{\partial t} dt \times L$$

$$= \frac{\epsilon_{xy}}{c} \Phi = \frac{\epsilon_{xy}}{c} \frac{hc}{e} = \dots$$



$$G_{xy} = \frac{e_r}{h}$$



$$G_{xy} = \frac{e^2}{h}$$

Implications:

- ① There are gapless edge states
- ②



$$\sigma_{xy} = \frac{e^2}{h}$$

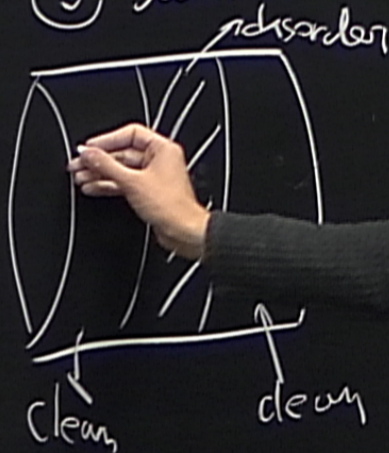
- Implications:
- ① There are gapless edge states
  - ② Extended state always exists in the bulk



$$\sigma_{xy} = \frac{e^2}{h}$$

Implications:

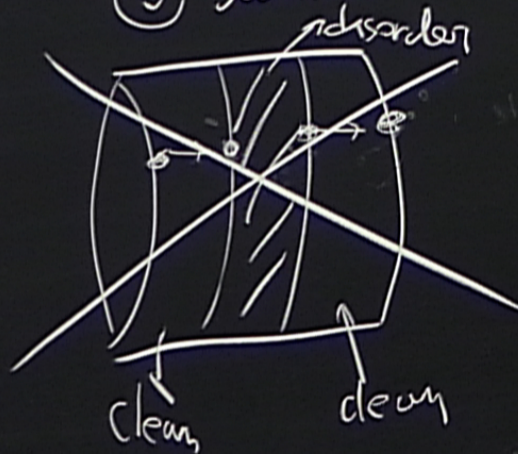
- ① There are gapless edge states
- ② Extended states exist in the bulk
- ③ Survival of disorder





$$\sigma_{xy} = \frac{e^2}{h}$$

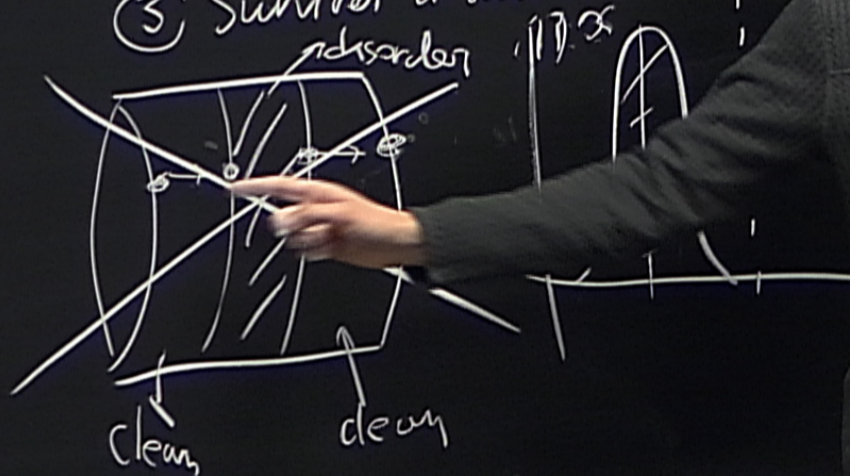
- Implications:
- ① There are gapless edge states
  - ② Extended state always exists in the bulk
  - ③ Survives if disorder added.





$$G_{xy} = \frac{e^2}{h}$$

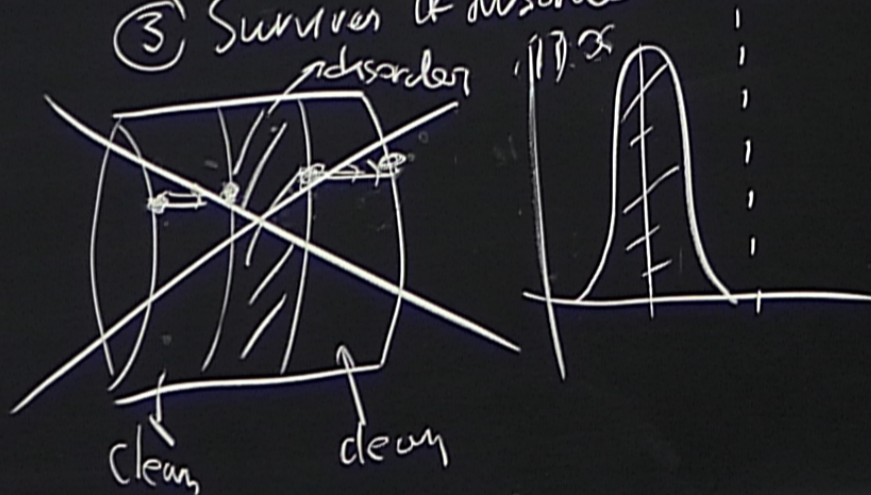
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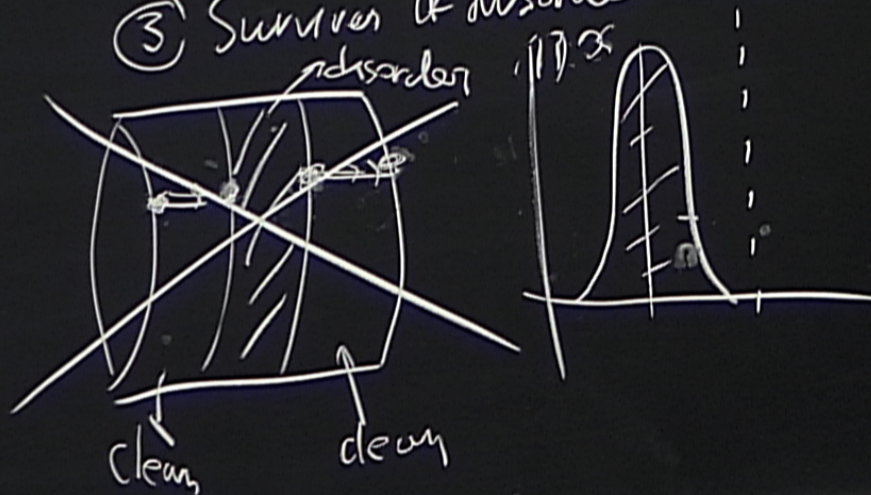
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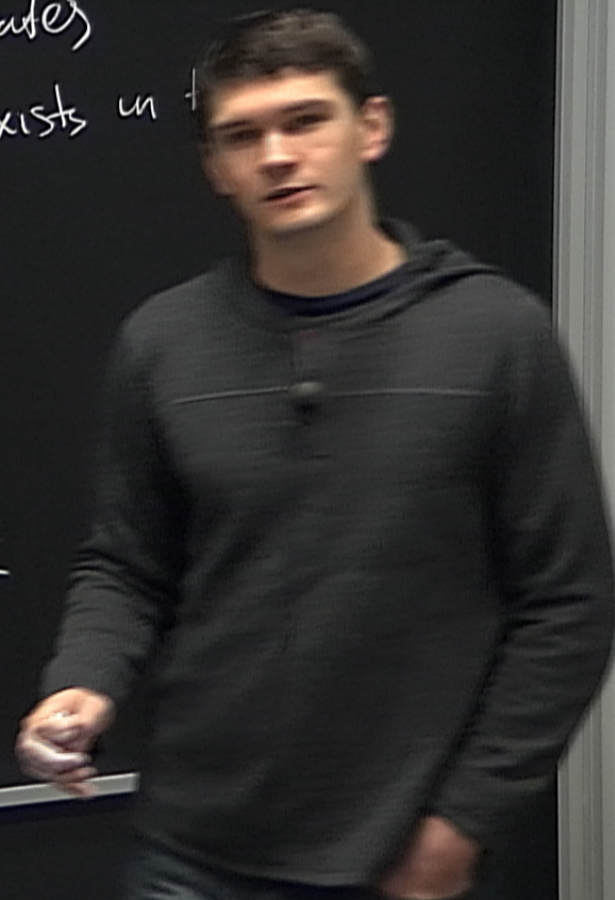
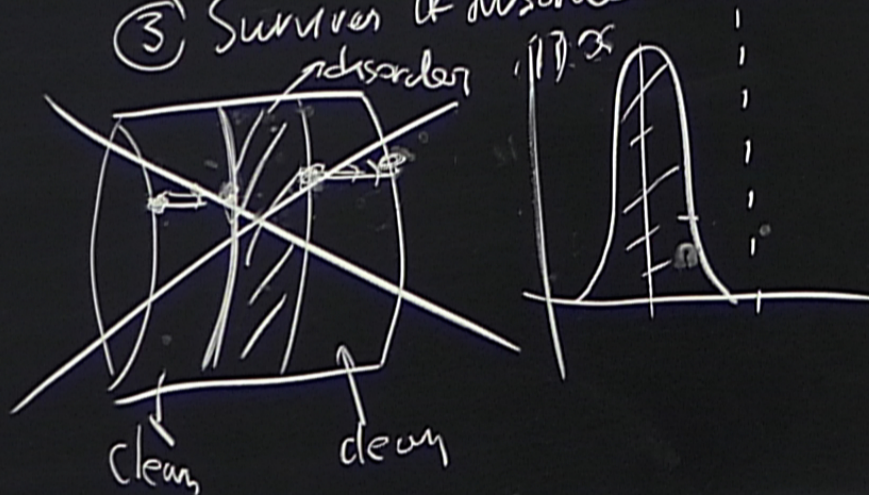




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Implications:

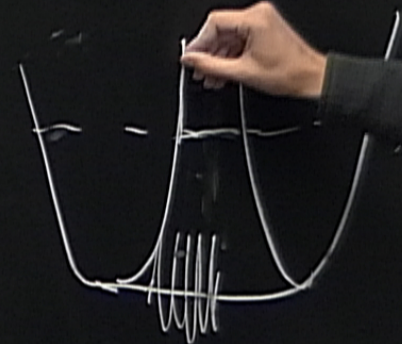
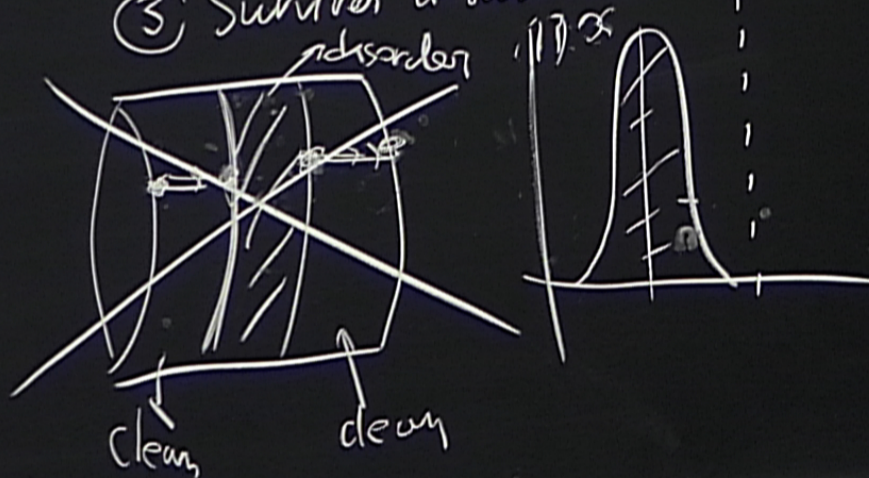
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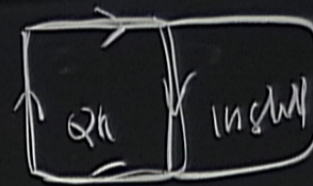
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