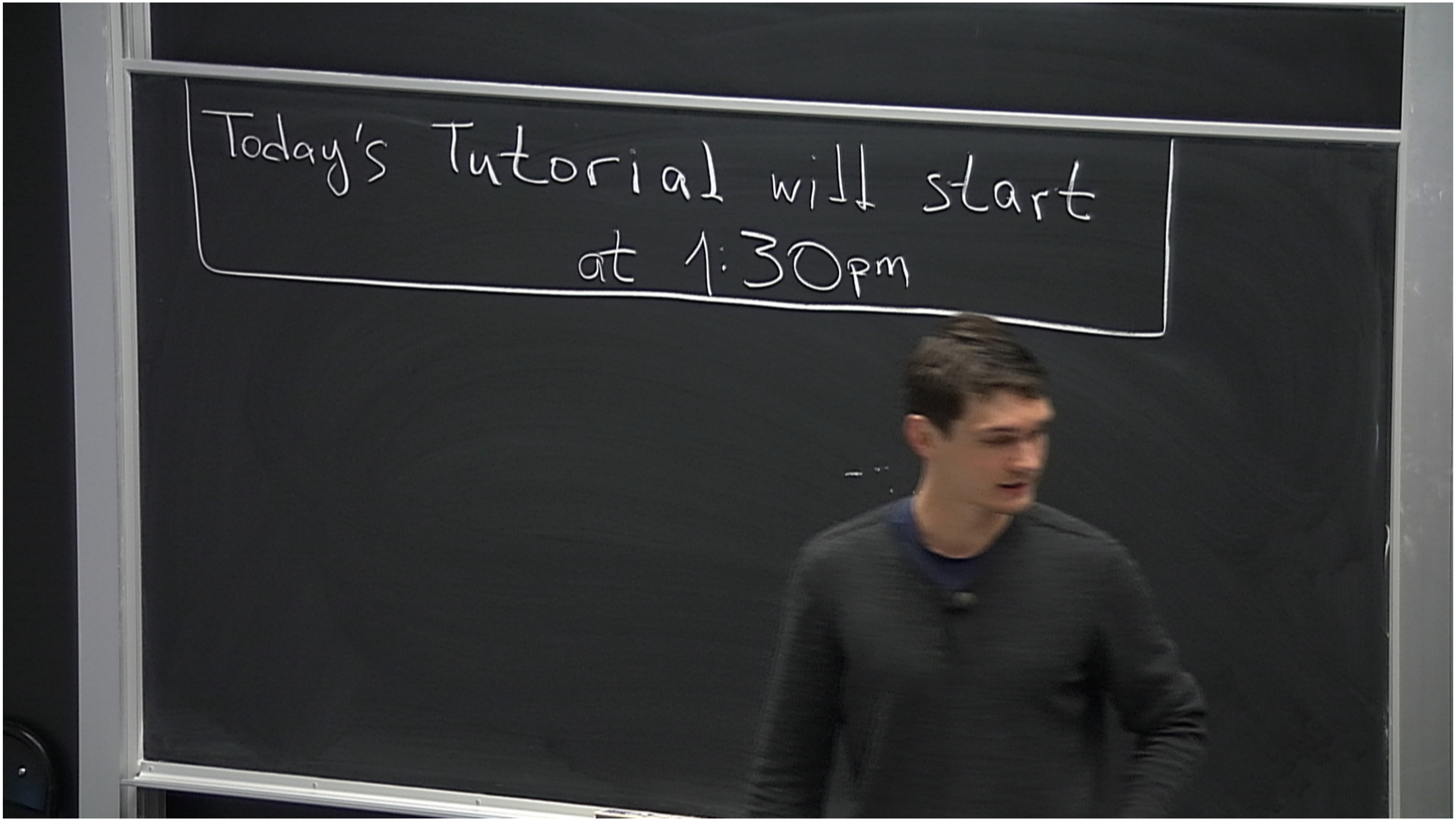
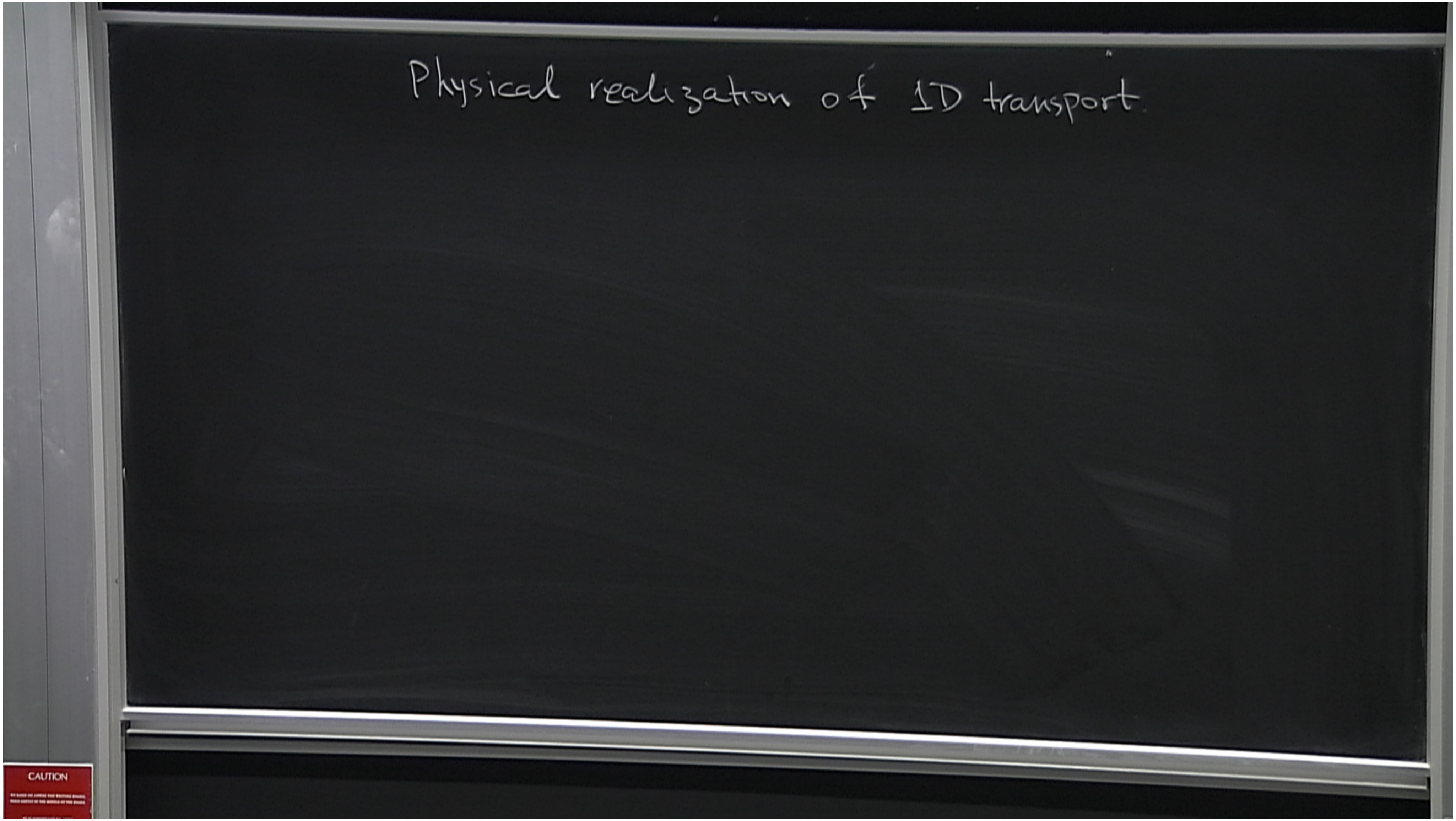
Title: Explorations in Condensed Matter - Lecture 4

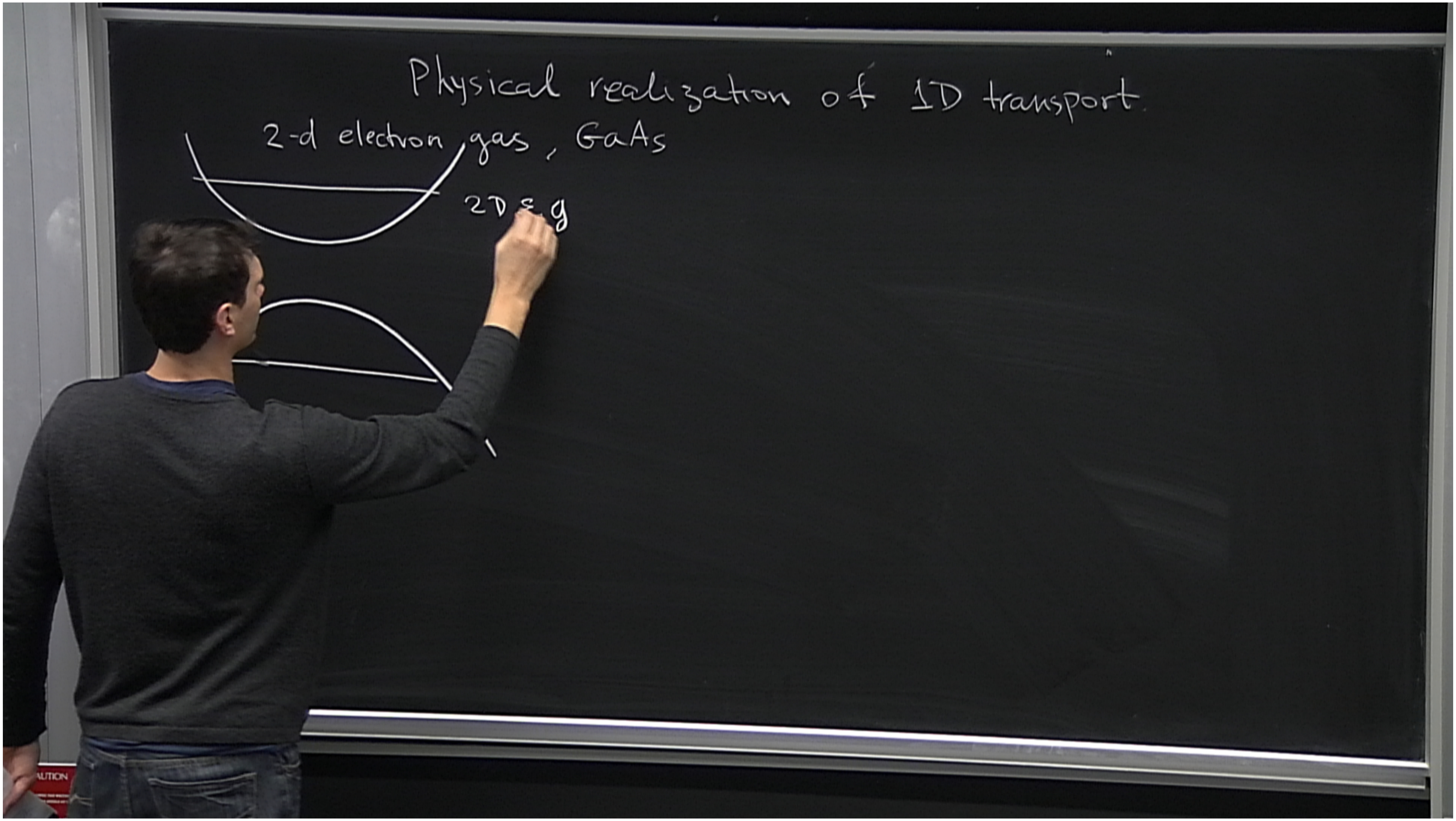
Date: Apr 05, 2012 10:15 AM

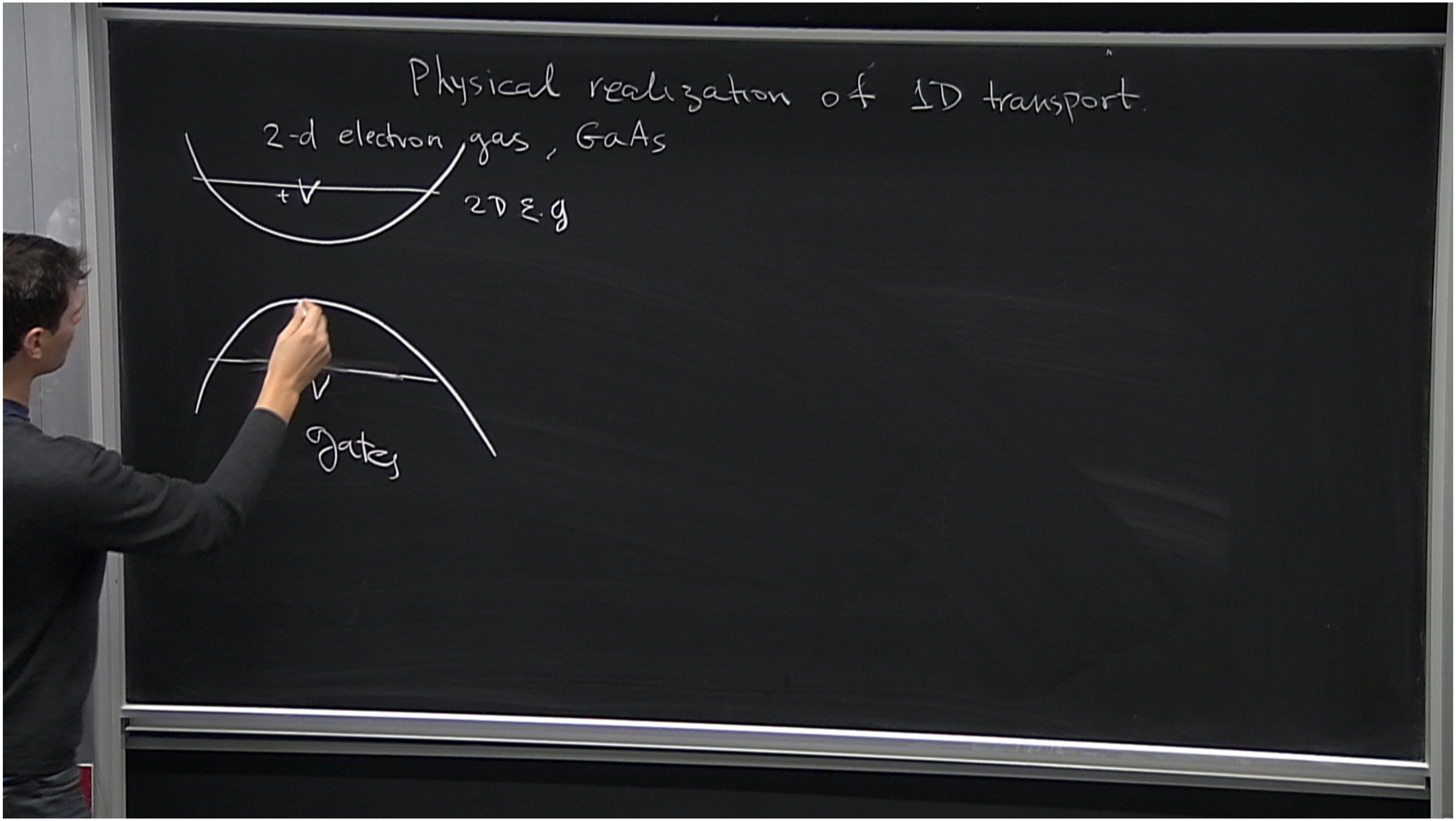
URL: http://pirsa.org/12040084

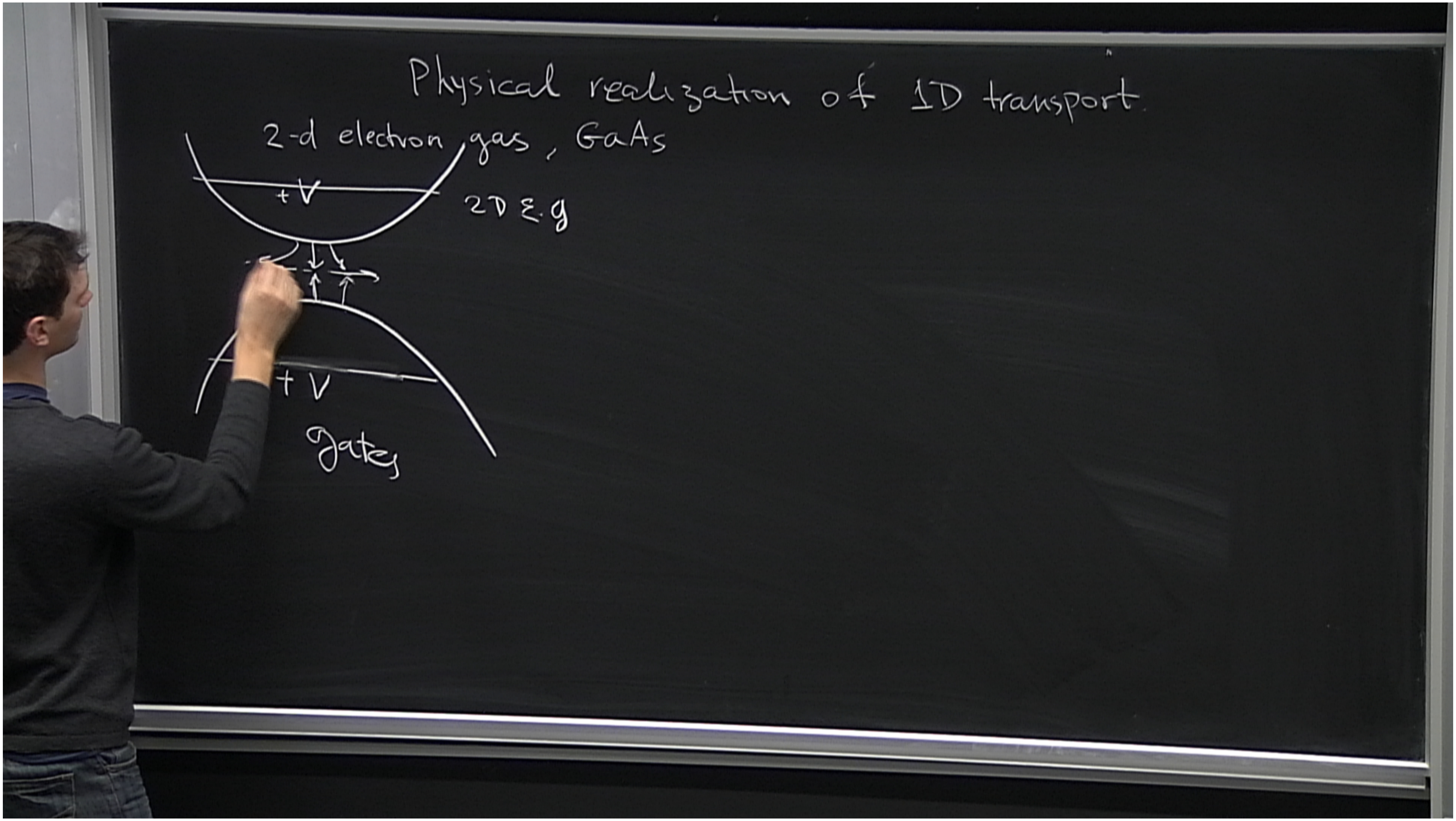
Abstract:


Today's Tutorial will start
at 1:30pm

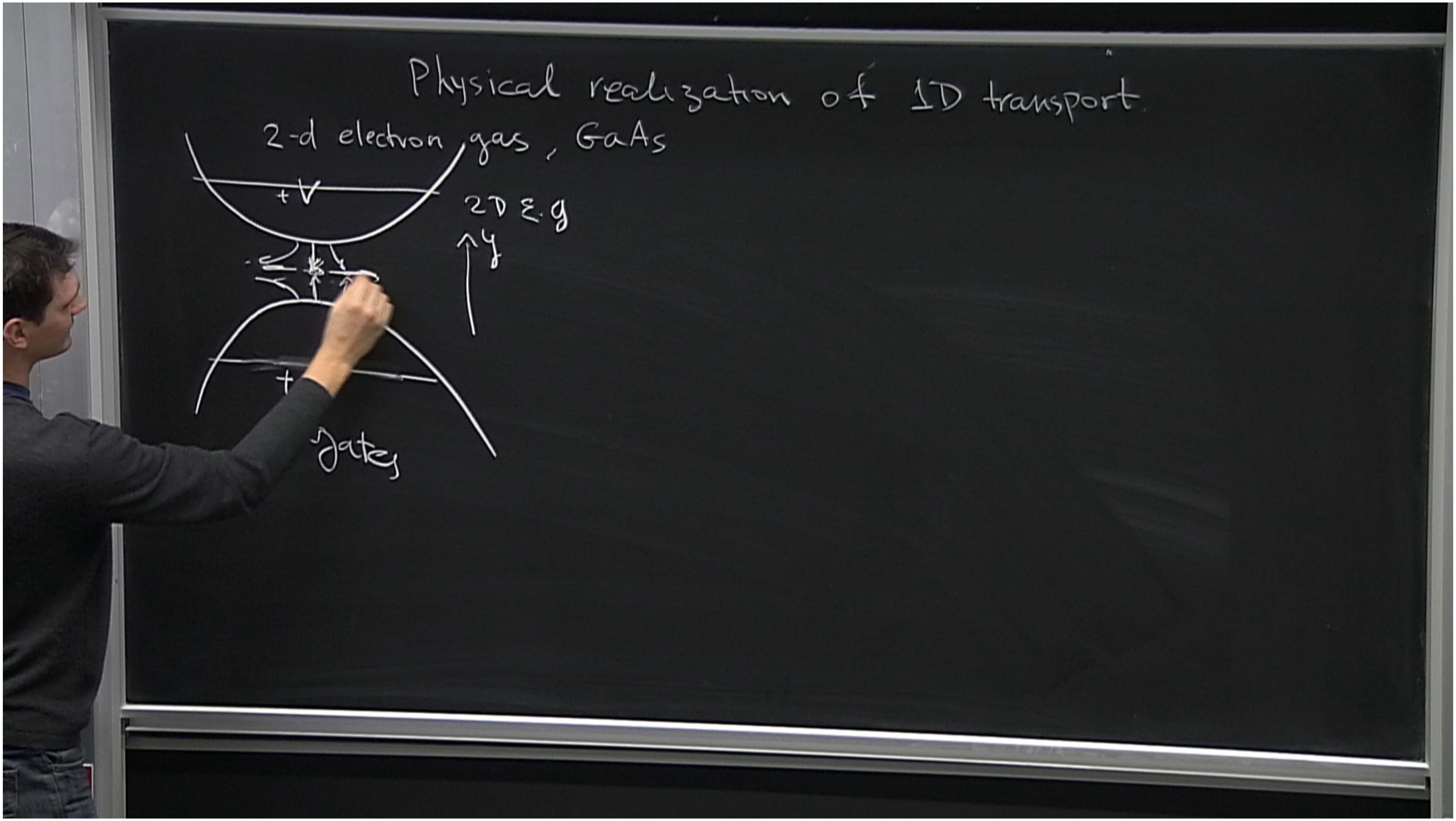

Physical realization of 1D transport
CAUTION

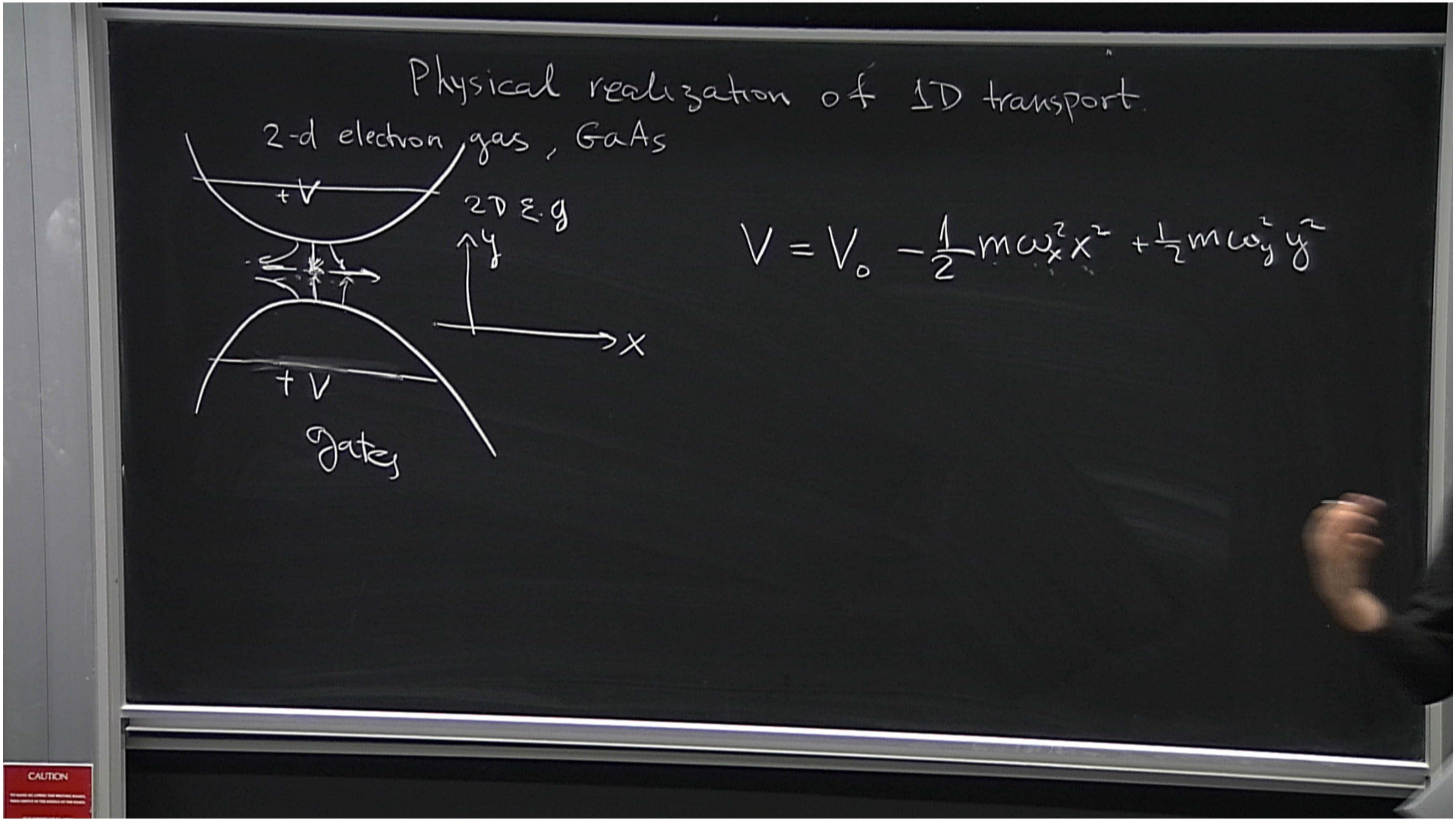

Physical realization of 1D transport
2-d electron gas, GaAs
2DEg

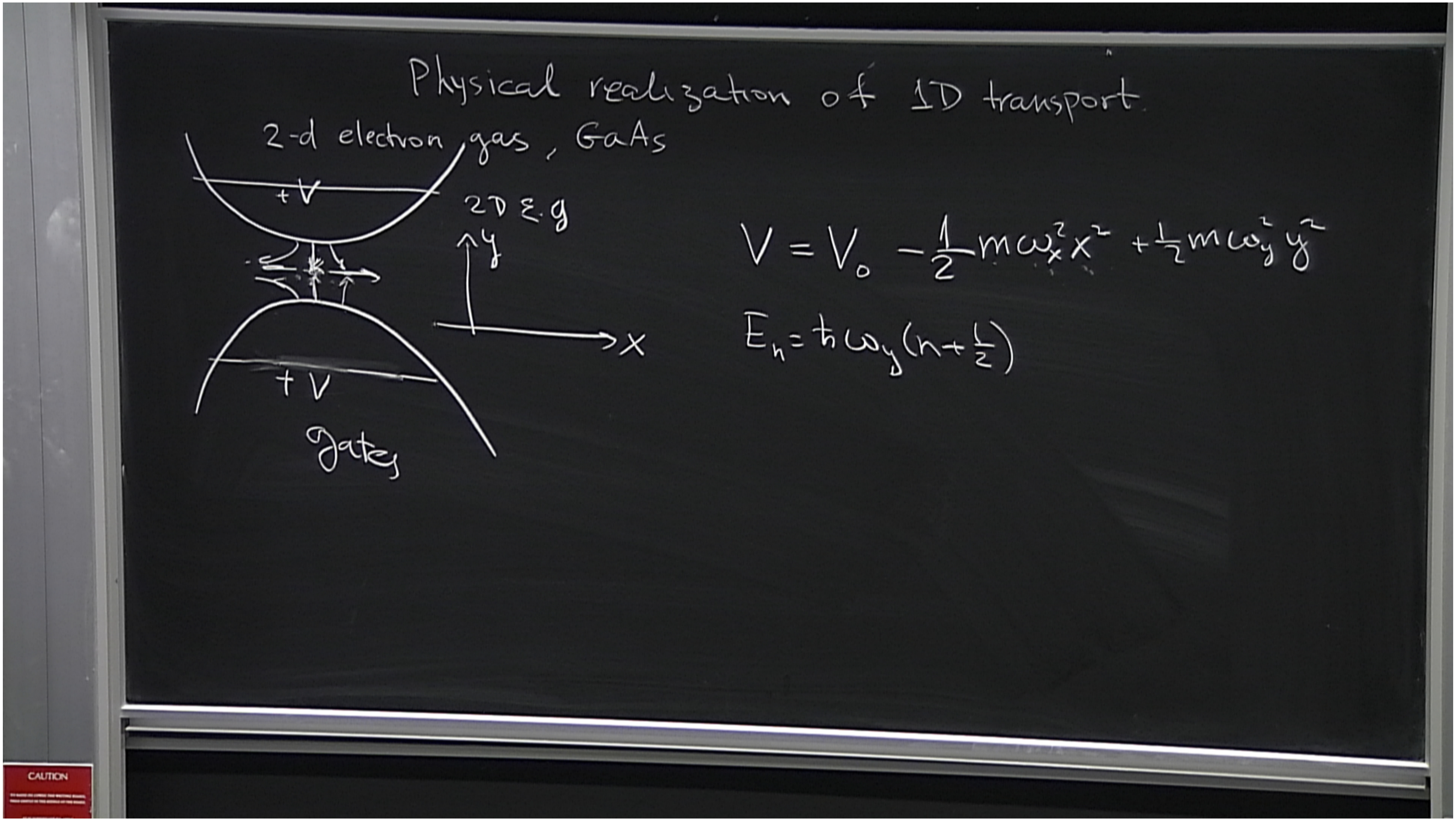

Physical realization of 1D transport.
2-d electron gas, GaAs
+V
2D E.g
V
gates

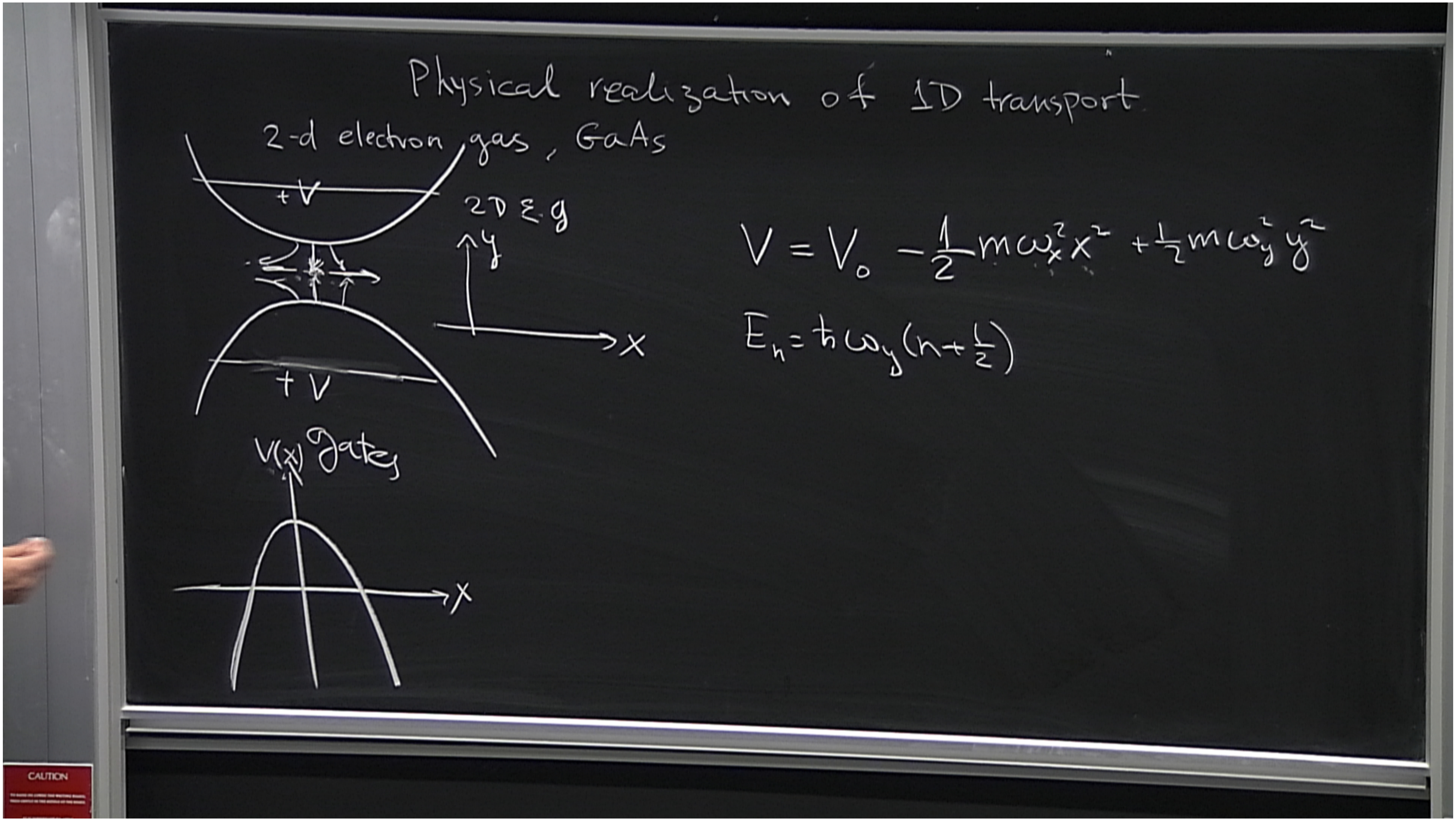


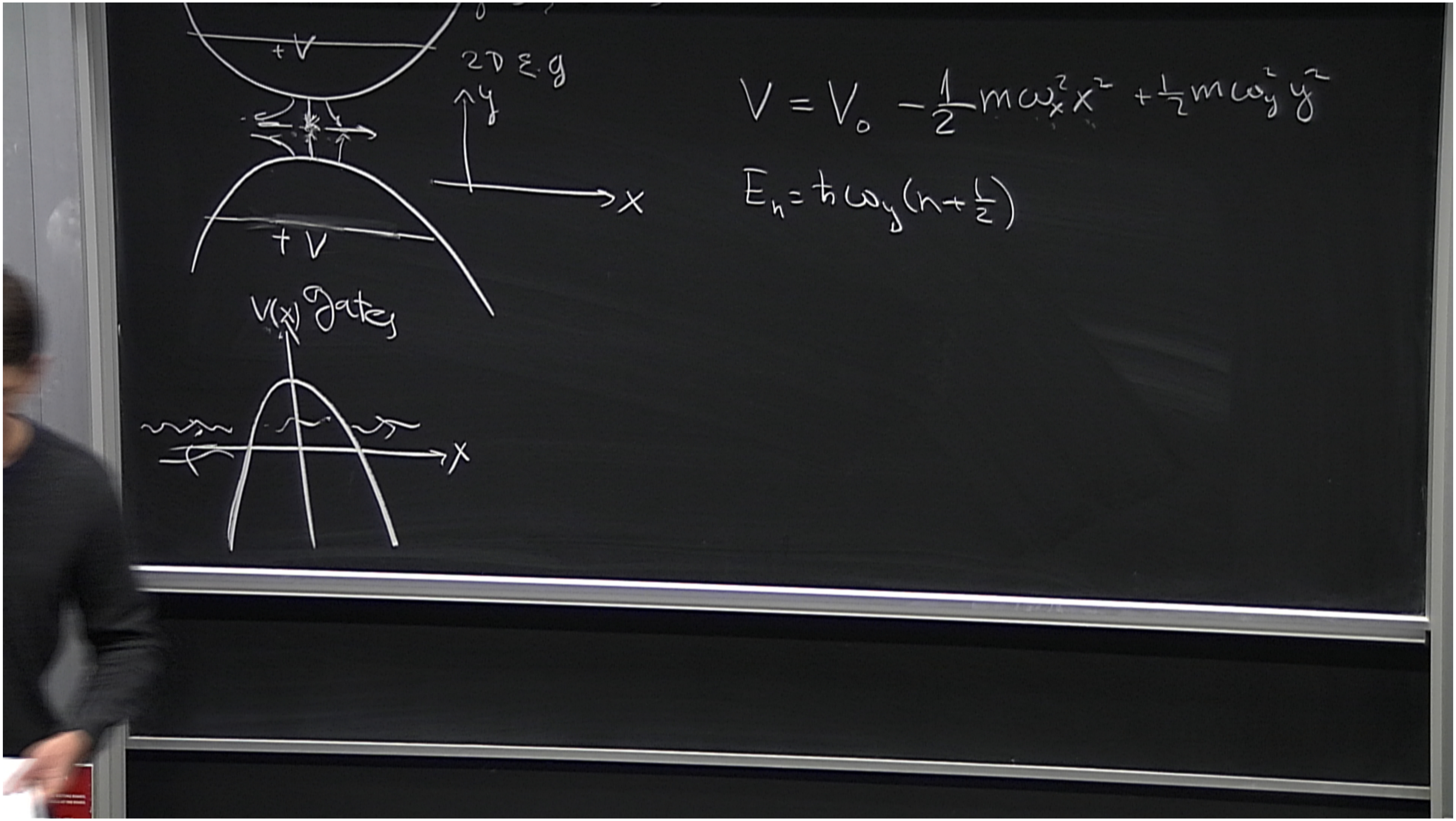
Physical realization of 1D transport
2-d electron gas, GaAs
+V
2D E.g
+V
Gates

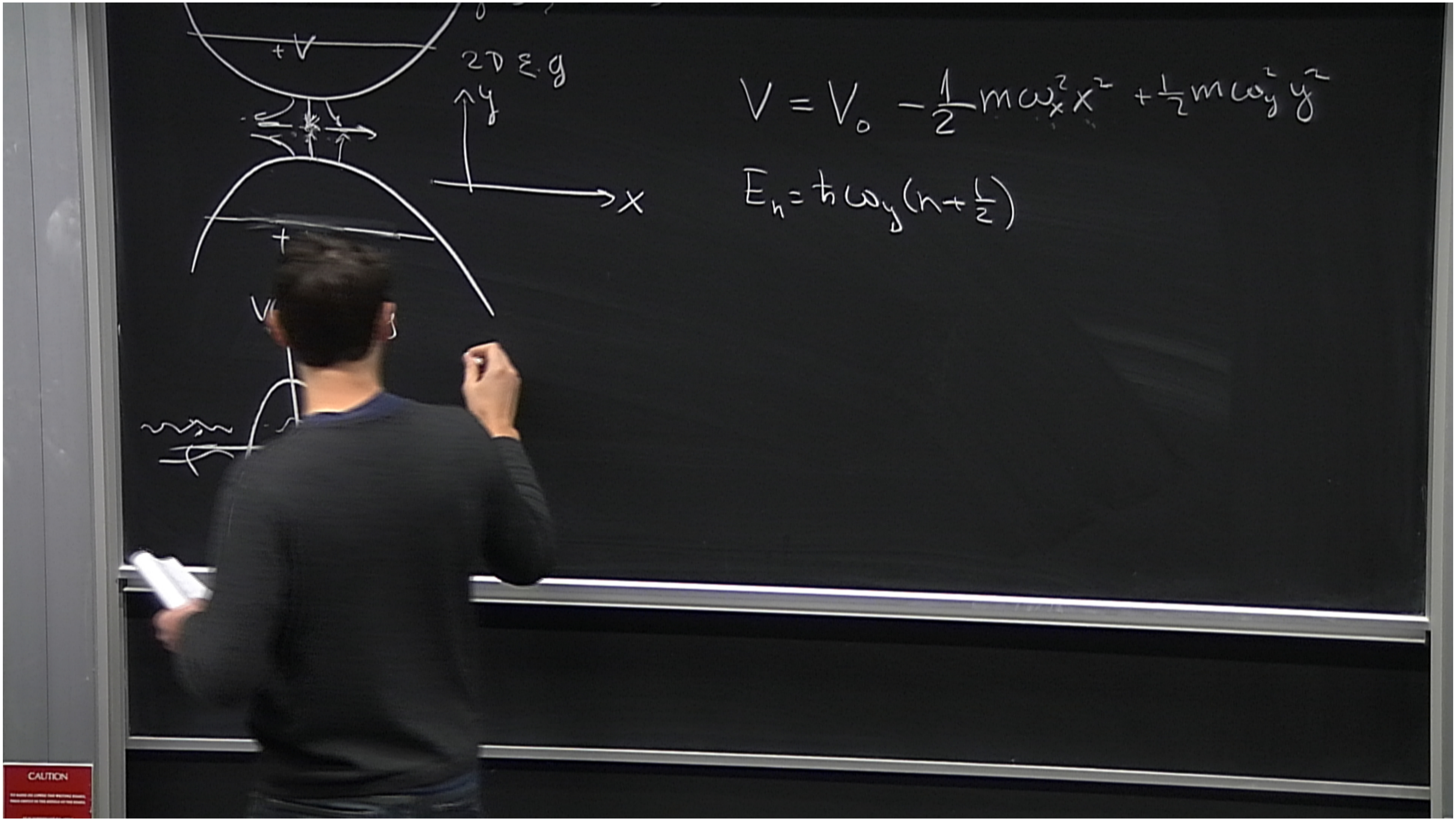

Physical realization of 1D transport
2-d electron gas, GaAs
+V
2D E.g
y
+
gates


Physical realization of 1D transport
2-d electron gas, GaAs
+V
2D E.g
y
x
+V
gates
$V = V_0 - \frac{1}{2} m \omega_x^2 x^2 + \frac{1}{2} m \omega_y^2 y^2$
CAUTION

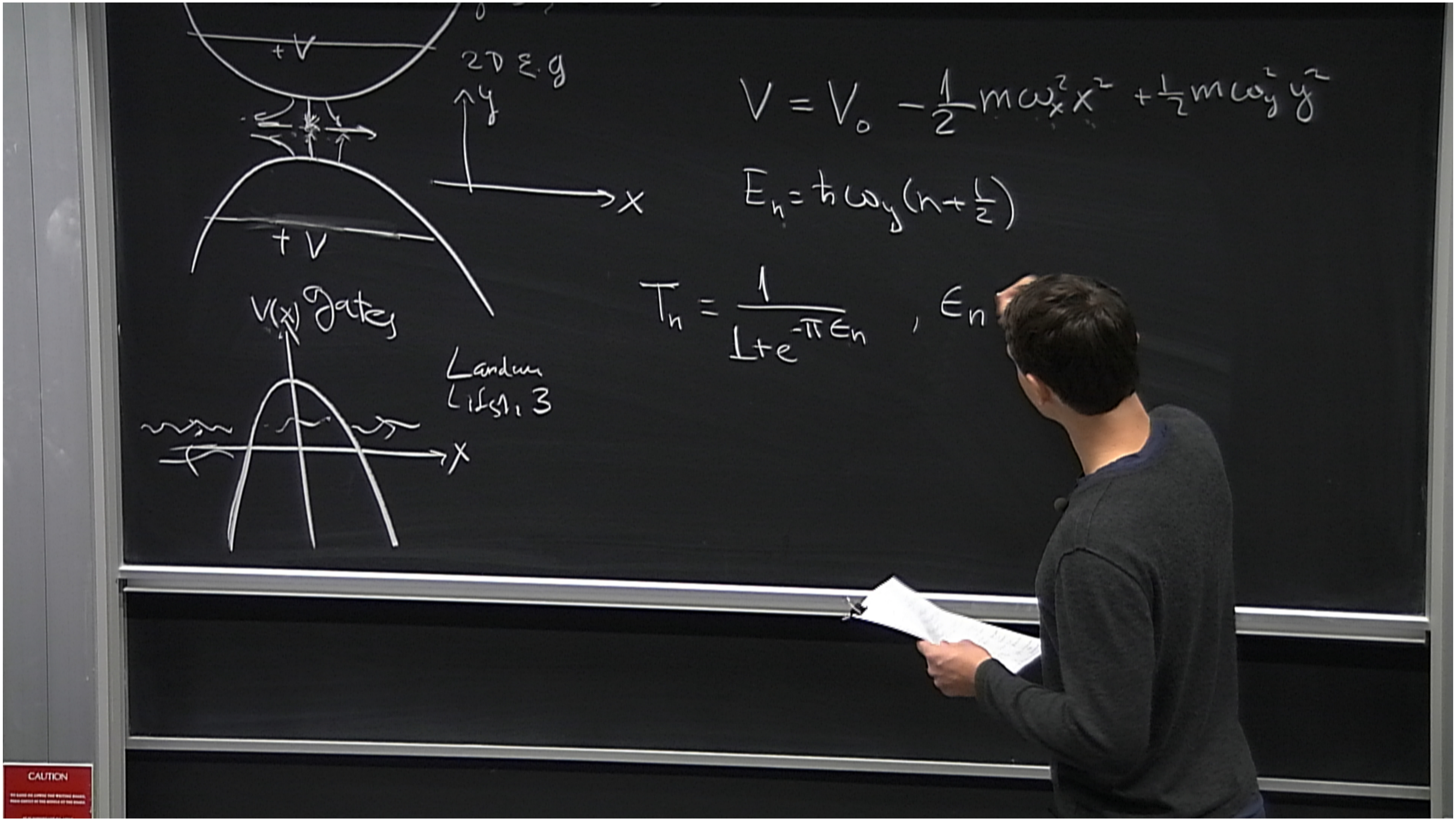

Physical realization of 1D transport
2-d electron gas, GaAs
+V
2D E.g
y
x
+V
gates
$V = V_0 - \frac{1}{2} m\omega_x^2 x^2 + \frac{1}{2} m\omega_y^2 y^2$
$E_n = \hbar\omega_y (n + \frac{1}{2})$
CAUTION


Physical realization of 1D transport
2-d electron gas, GaAs
+V
2D E.g
y
V = V0 − 1/2 mωx² x² + 1/2 mωy² y²
x
En = ħωy(n + 1/2)
+V
V(x) gates
x
CAUTION

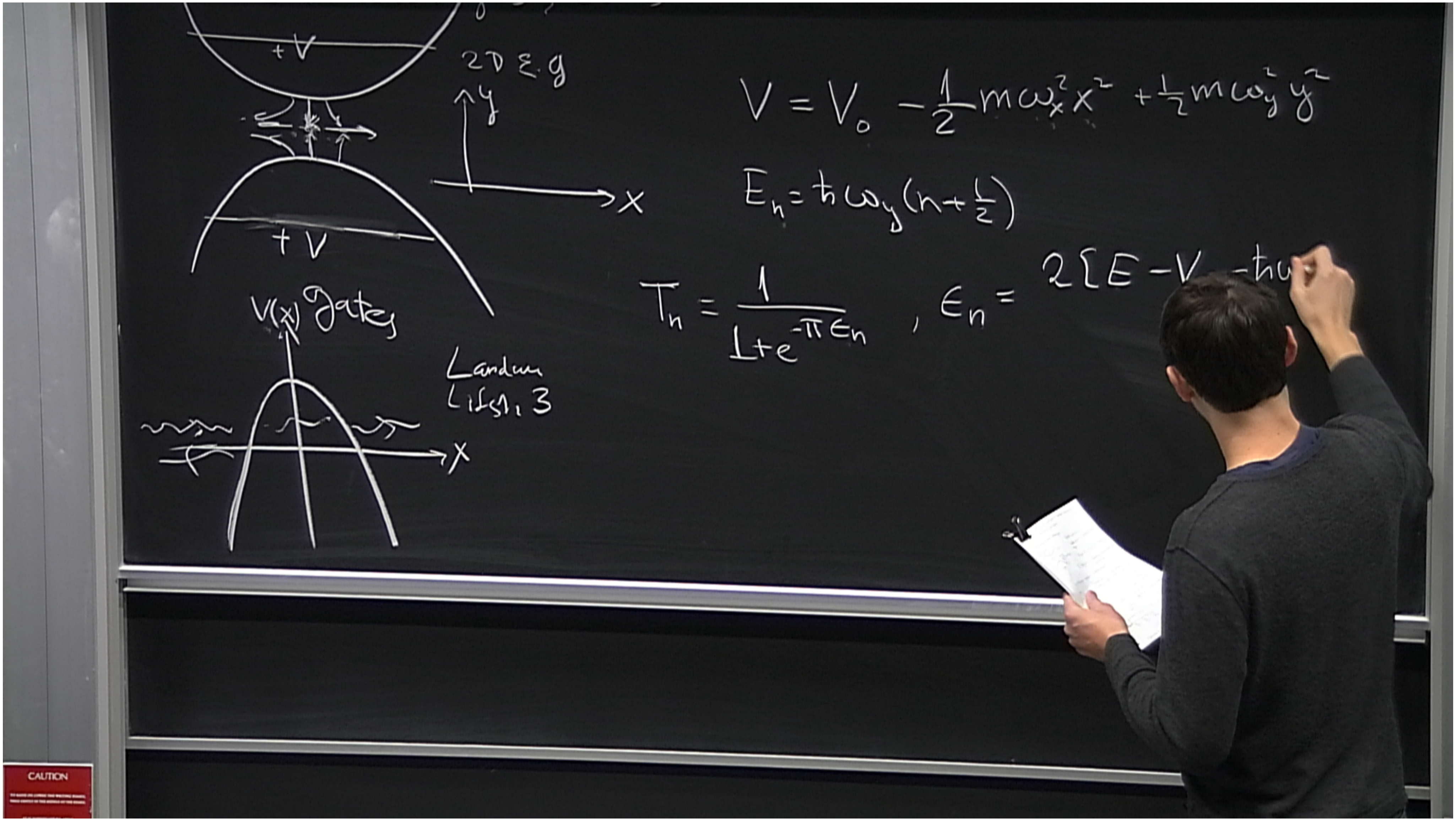

+V
2D E.g
y
x
+V
V(x) gates
x
V = V₀ − ½ m ω_x² x² + ½ m ω_y² y²
E_n = ħ ω_y (n + ½)



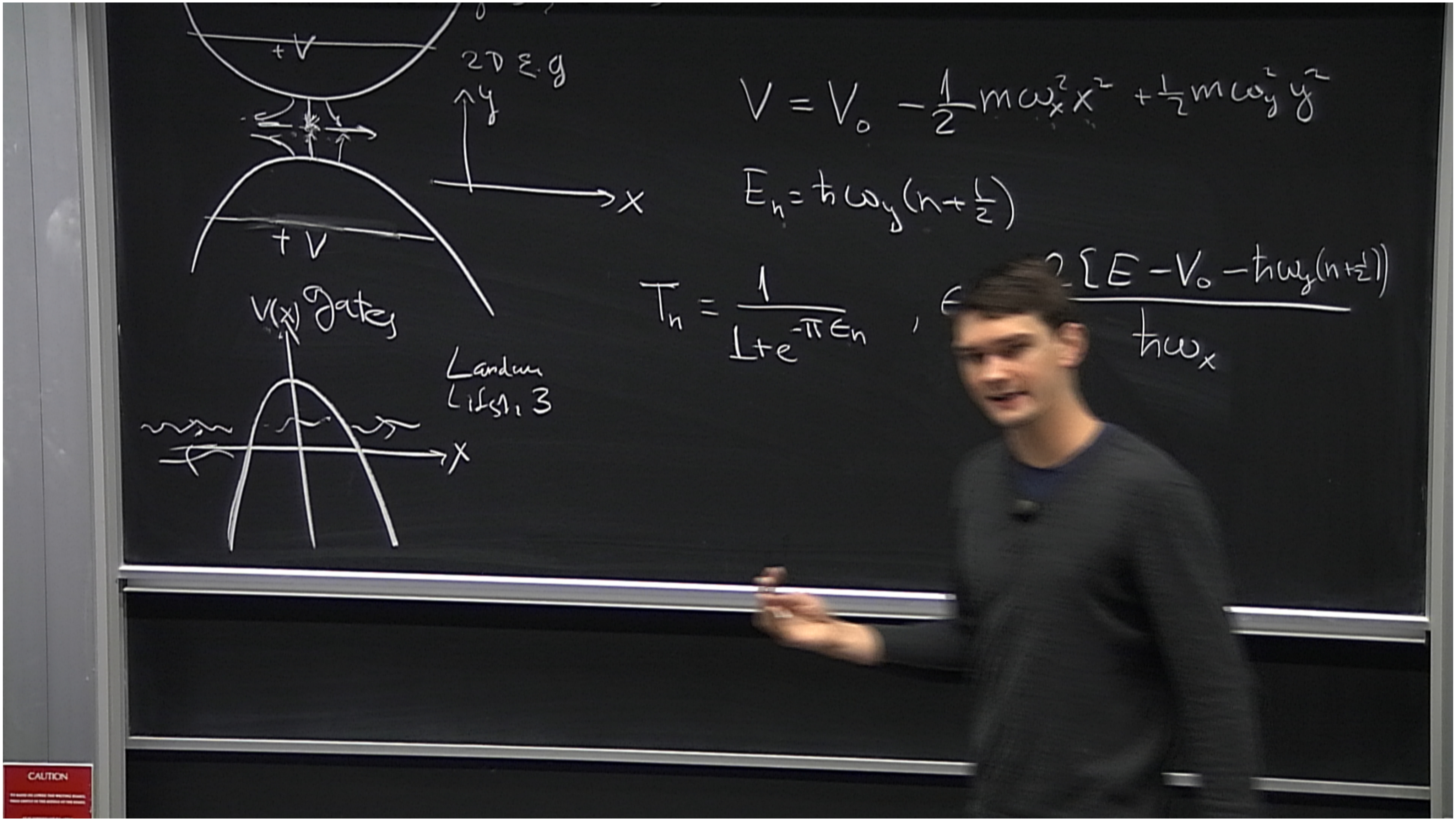
+V
2D E.g
y
x
$V = V_0 - \frac{1}{2} m \omega_x^2 x^2 + \frac{1}{2} m \omega_y^2 y^2$
$E_n = \hbar \omega_y (n + \frac{1}{2})$
CAUTION



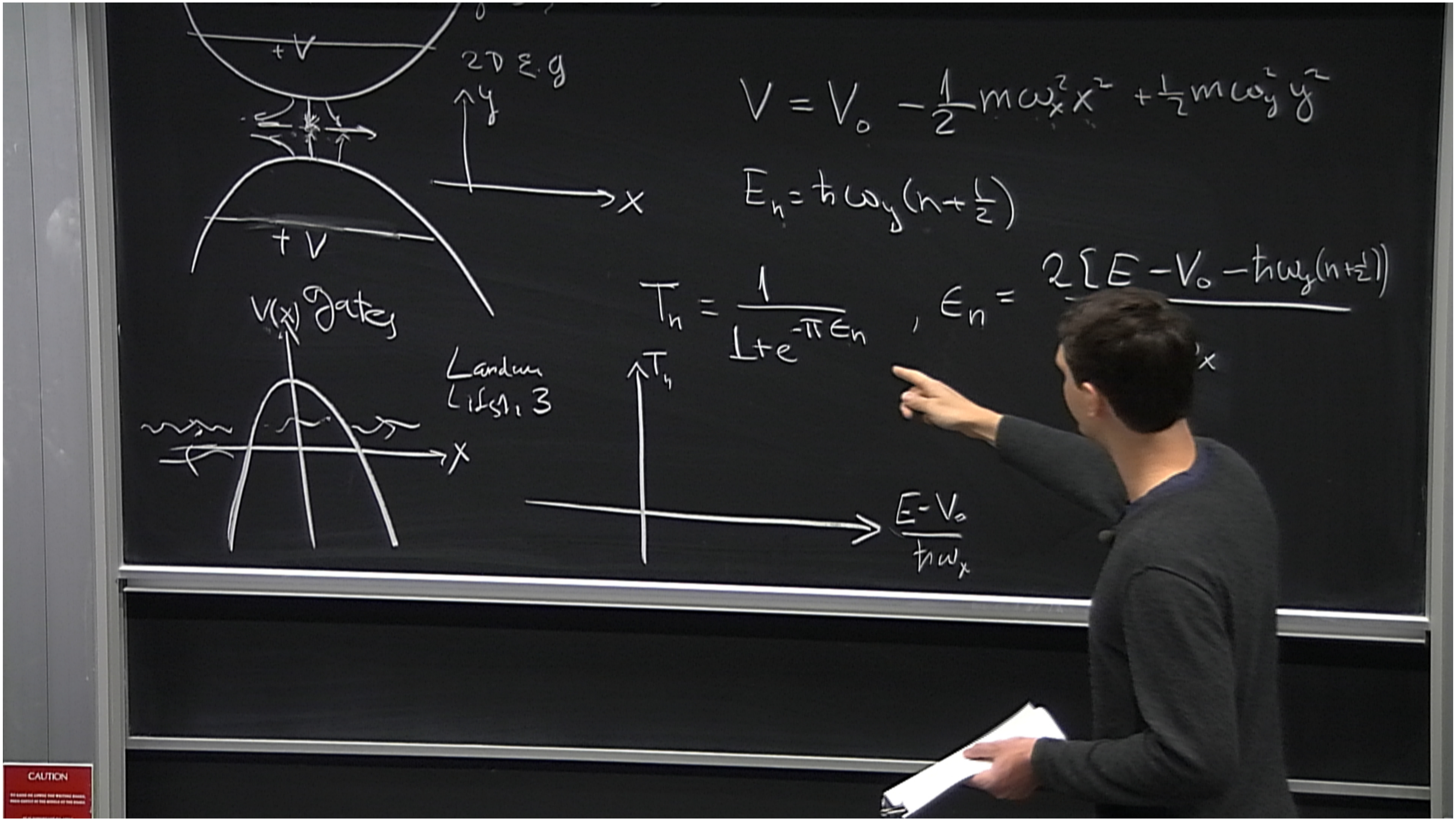
+V
2D e.g
y
x
+V
V(x) gates
Landau
Lifshitz 3
x
$V = V_0 - \frac{1}{2} m \omega_x^2 x^2 + \frac{1}{2} m \omega_y^2 y^2$
$E_n = \hbar \omega_y (n + \frac{1}{2})$
$T_n = \frac{1}{1 + e^{-\pi \epsilon_n}}$, ϵ_n

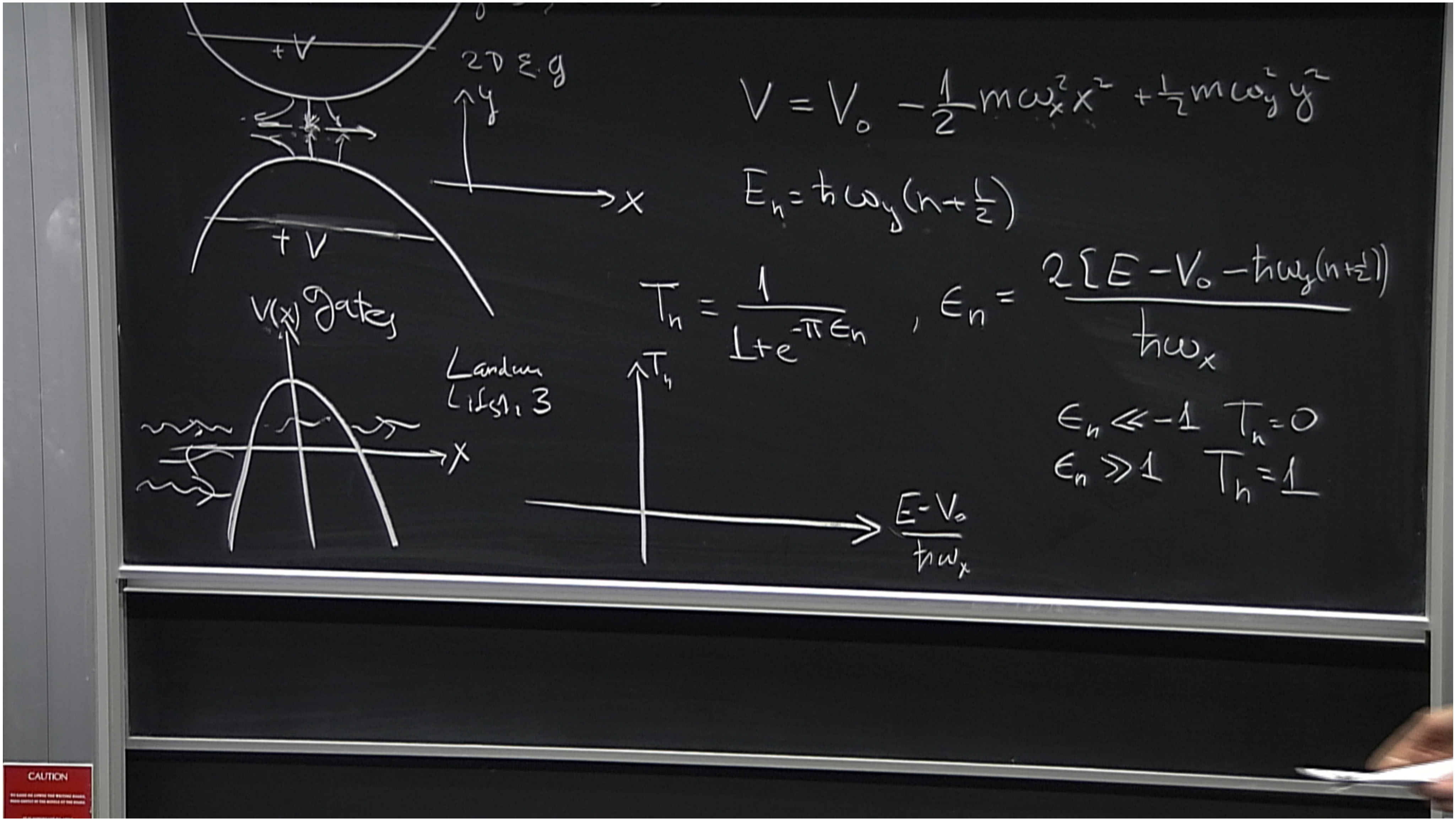


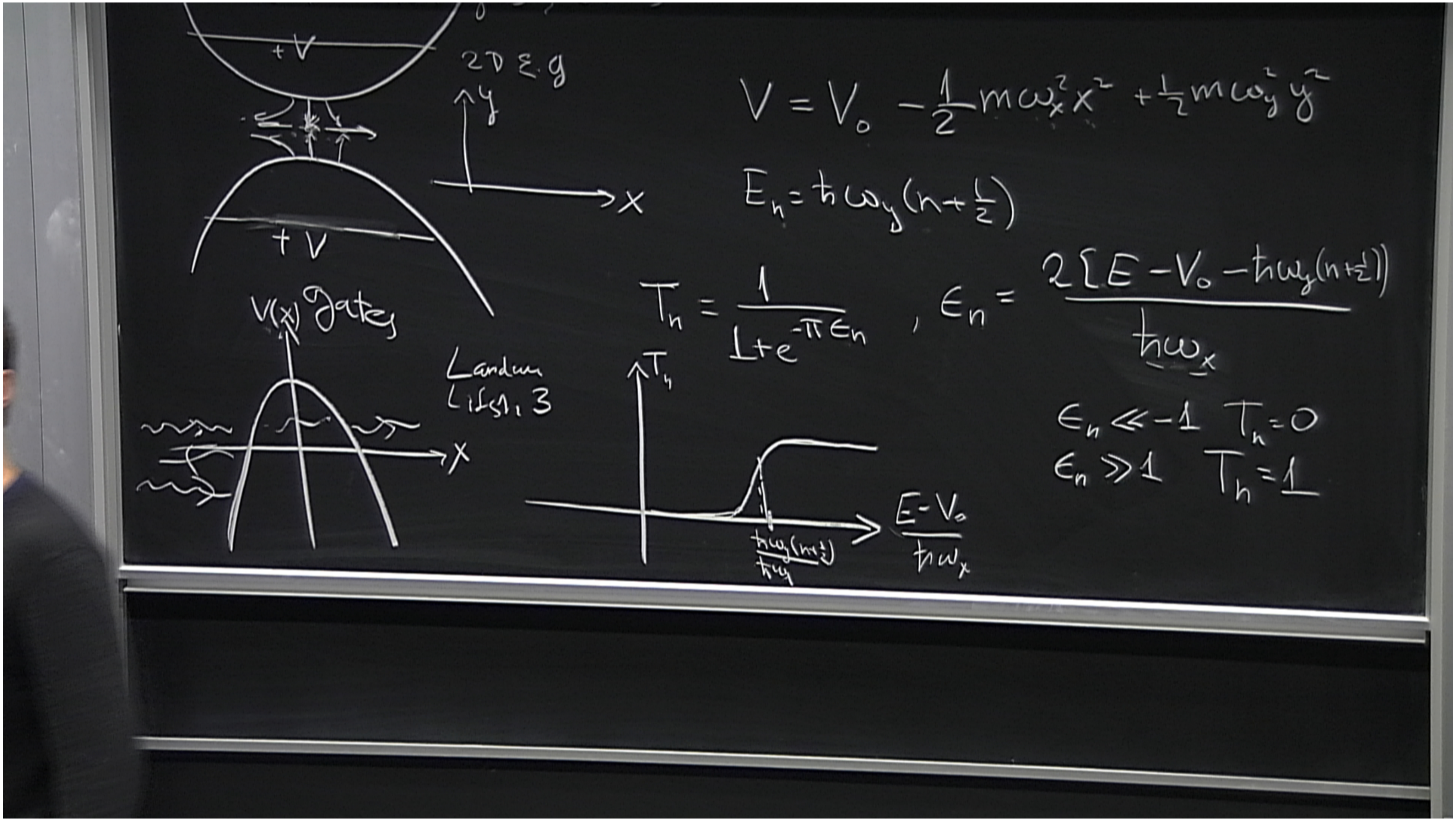
2D E.g
$V = V_0 - \frac{1}{2} m\omega_x^2 x^2 + \frac{1}{2} m\omega_y^2 y^2$
$E_n = \hbar\omega_y (n + \frac{1}{2})$
$T_n = \frac{1}{1+e^{-\pi \epsilon_n}}$, $\epsilon_n =$
V(x) gates
Landau
Lifshitz 3
CAUTION

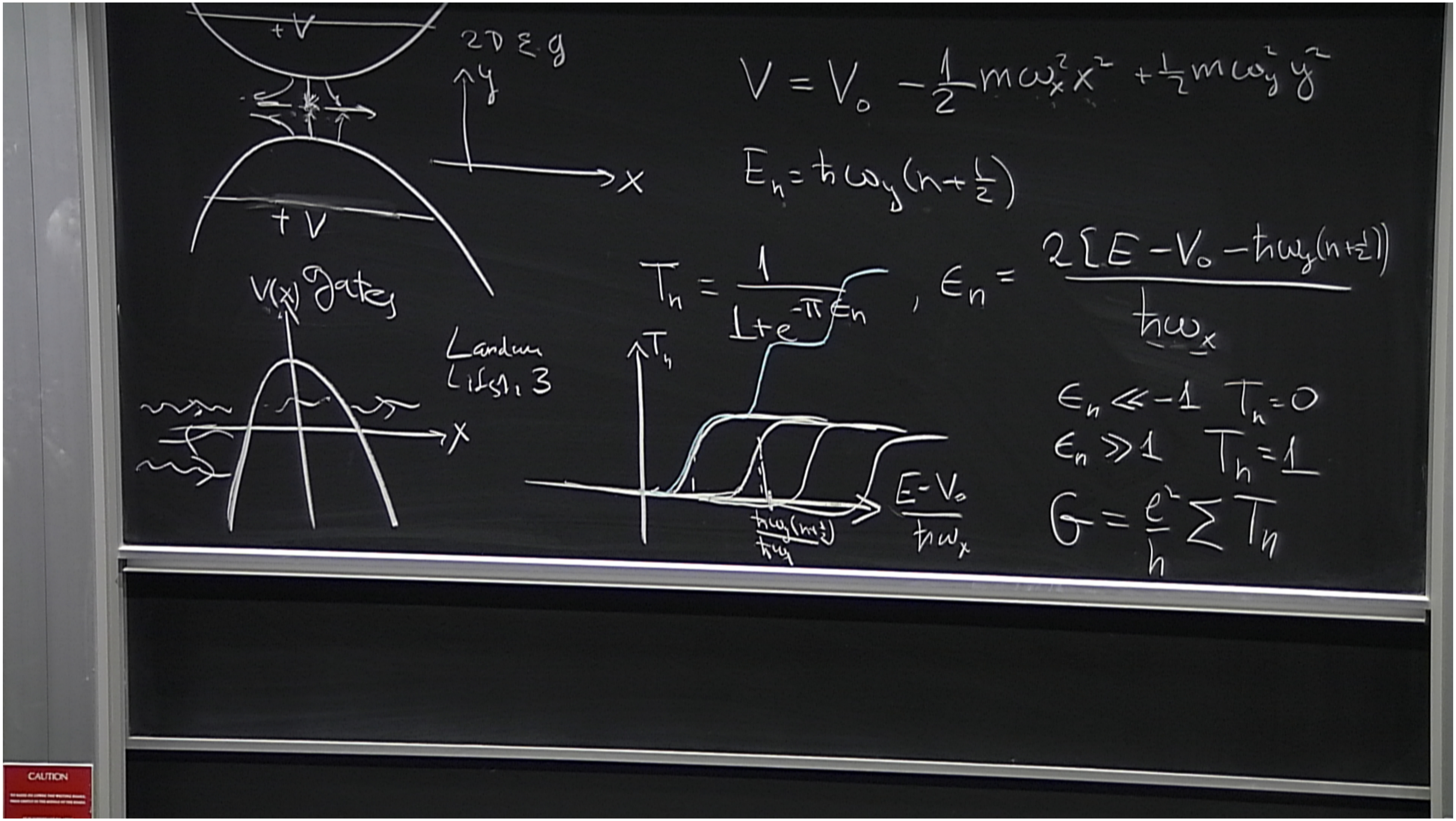


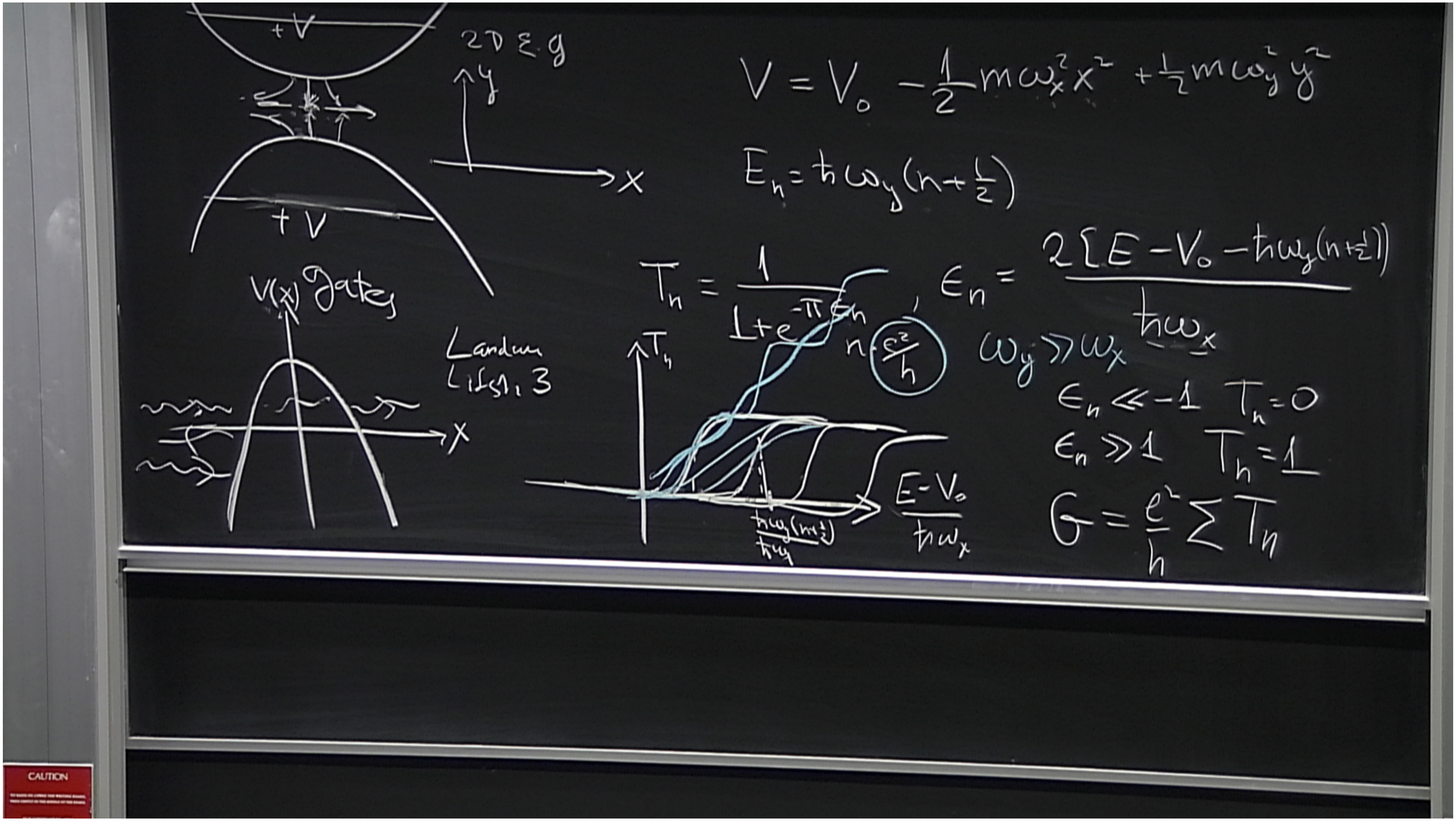
+V
2D E.g
y
x
$V = V_0 - \frac{1}{2} m\omega_x^2 x^2 + \frac{1}{2} m\omega_y^2 y^2$
$E_n = \hbar\omega_y (n + \frac{1}{2})$
$T_n = \frac{1}{1+e^{-\pi\epsilon_n}}$
$\hbar\omega_x$
V(x) gates
Landau
Lifsh, 3
CAUTION

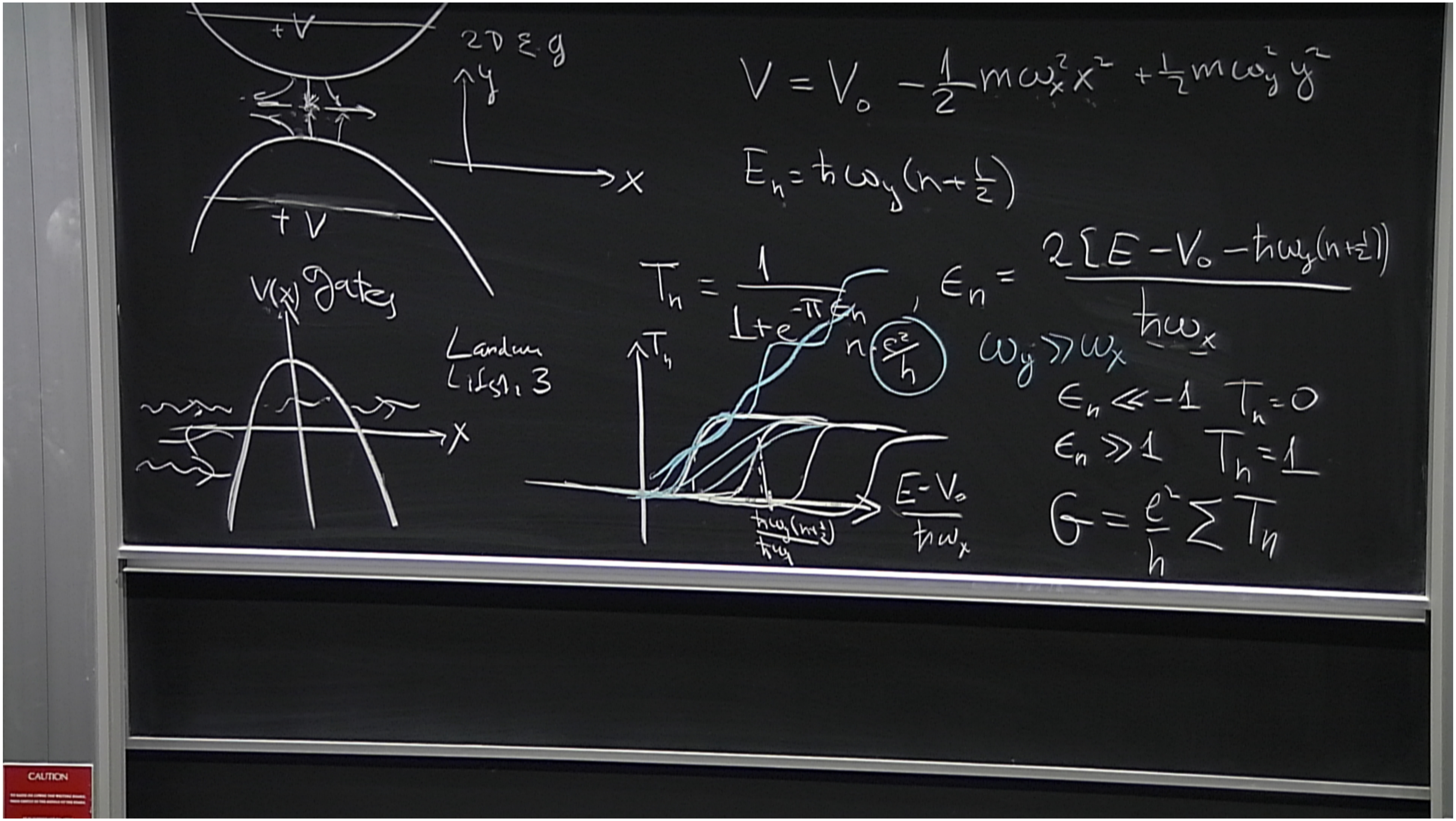

2D E.g
+V
+V
V(x) gates
Landau Lifsh. 3
V = V₀ − ½ mω_x² x² + ½ mω_y² y²
E_n = ħω_y(n+½)
T_n = 1 / (1+e^{−πε_n}) , ε_n = 2[E − V₀ − ħω_y(n+½)]
T_n
(E−V₀) / ħω_x
CAUTION

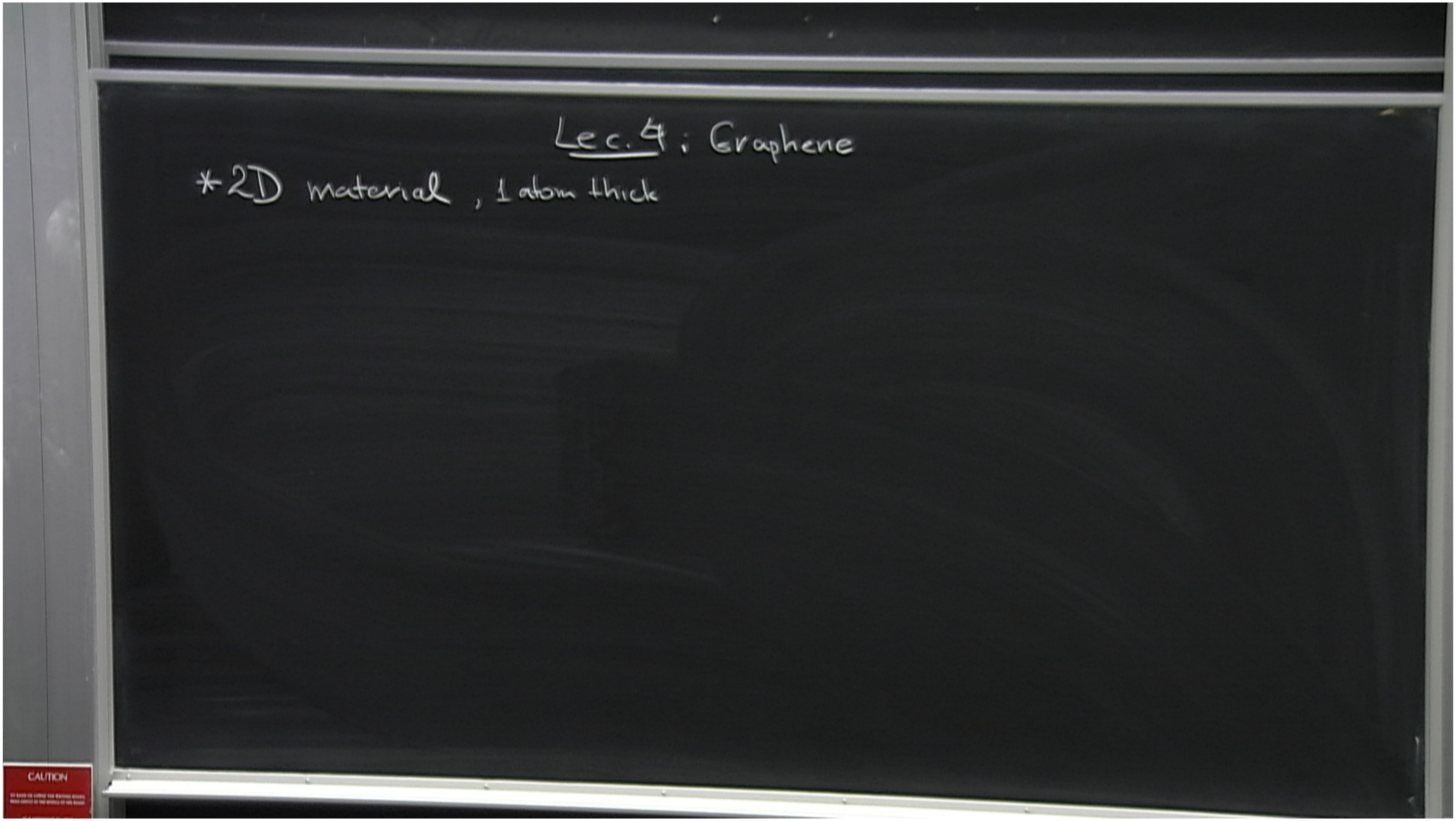

+V
2D E.g
y
x
+V
V(x) gates
Landau
Lifshitz 3
x
$V = V_0 - \frac{1}{2}m\omega_x^2 x^2 + \frac{1}{2}m\omega_y^2 y^2$
$E_n = \hbar\omega_y(n+\frac{1}{2})$
$T_n = \frac{1}{1+e^{-\pi\epsilon_n}}$, $\epsilon_n = \frac{2[E - V_0 - \hbar\omega_y(n+\frac{1}{2})]}{\hbar\omega_x}$
T_n
$\frac{E-V_0}{\hbar\omega_x}$
$\epsilon_n \ll -1$ $T_n = 0$
$\epsilon_n \gg 1$ $T_n = 1$
CAUTION

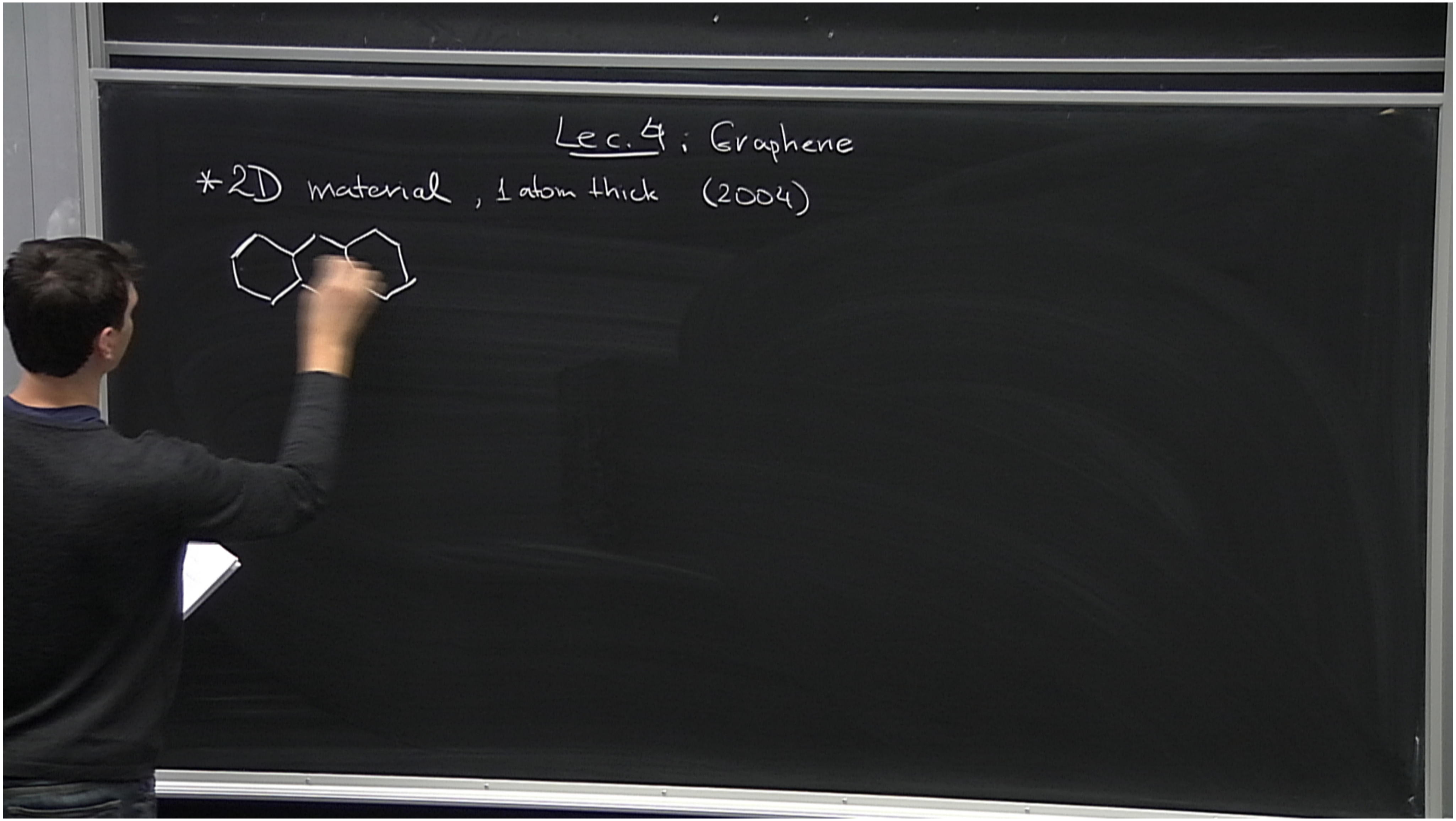

+V
2D e.g
y
x
+V
V(x) gates
Landau
Lifshitz 3
x
$V = V_0 - \frac{1}{2} m\omega_x^2 x^2 + \frac{1}{2} m\omega_y^2 y^2$
$E_n = \hbar\omega_y(n+\frac{1}{2})$
$T_n = \frac{1}{1+e^{-\pi\epsilon_n}}$, $\epsilon_n = \frac{2\{E - V_0 - \hbar\omega_y(n+\frac{1}{2})\}}{\hbar\omega_x}$
T_n
$\frac{\hbar\omega_y(n+\frac{1}{2})}{\hbar\omega_x}$
$\frac{E-V_0}{\hbar\omega_x}$
$\epsilon_n \ll -1 \quad T_n = 0$
$\epsilon_n \gg 1 \quad T_n = 1$


+V
2D Eg
y
x
+V
V(x) gates
Landau
Lifsh. 3
x
$V = V_0 - \frac{1}{2} m\omega_x^2 x^2 + \frac{1}{2} m\omega_y^2 y^2$
$E_n = \hbar\omega_y(n+\frac{1}{2})$
$T_n = \frac{1}{1+e^{-\pi\epsilon_n}}$, $\epsilon_n = \frac{2[E - V_0 - \hbar\omega_y(n+\frac{1}{2})]}{\hbar\omega_x}$
T_n
$\frac{\hbar\omega_y(n+\frac{1}{2})}{\hbar\omega_y}$
$\frac{E-V_0}{\hbar\omega_x}$
$\epsilon_n \ll -1 \quad T_n = 0$
$\epsilon_n \gg 1 \quad T_n = 1$
$G = \frac{e^2}{h}\sum T_n$
CAUTION


+V
2D E.g
y
x
+V
V(x) gates
Landau
$V = V_0 - \frac{1}{2} m\omega_x^2 x^2 + \frac{1}{2} m\omega_y^2 y^2$
$E_n = \hbar\omega_y(n+\frac{1}{2})$
$T_n = \frac{1}{1+e^{-\pi\epsilon_n}}$, $\epsilon_n = \frac{2[E - V_0 - \hbar\omega_y(n+\frac{1}{2})]}{\hbar\omega_x}$
T_n
$n \cdot \frac{e^2}{h}$
$\omega_y \gg \omega_x$
$\epsilon_n \ll -1 \quad T_n = 0$
$\epsilon_n \gg 1 \quad T_n = 1$
$G = \frac{e^2}{h} \sum T_n$
$\frac{E - V_0}{\hbar\omega_x}$
$\frac{\hbar\omega_y(n+\frac{1}{2})}{\hbar\omega_x}$
CAUTION

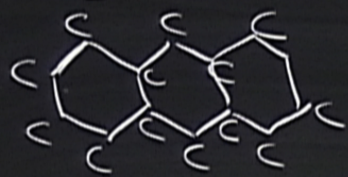

+V
2D E.g
+V
V(x) gates
Landau
Lifsh. 3
V = V_0 - 1/2 m ω_x^2 x^2 + 1/2 m ω_y^2 y^2
E_n = ħω_y(n + 1/2)
T_n = 1 / (1 + e^{-π ε_n}) , ε_n = 2[E - V_0 - ħω_y(n + 1/2)] / ħω_x
n·e^2/h
ω_y >> ω_x
ε_n << -1 T_n = 0
ε_n >> 1 T_n = 1
G = e^2/h Σ T_n
(E - V_0)/ħω_x
CAUTION

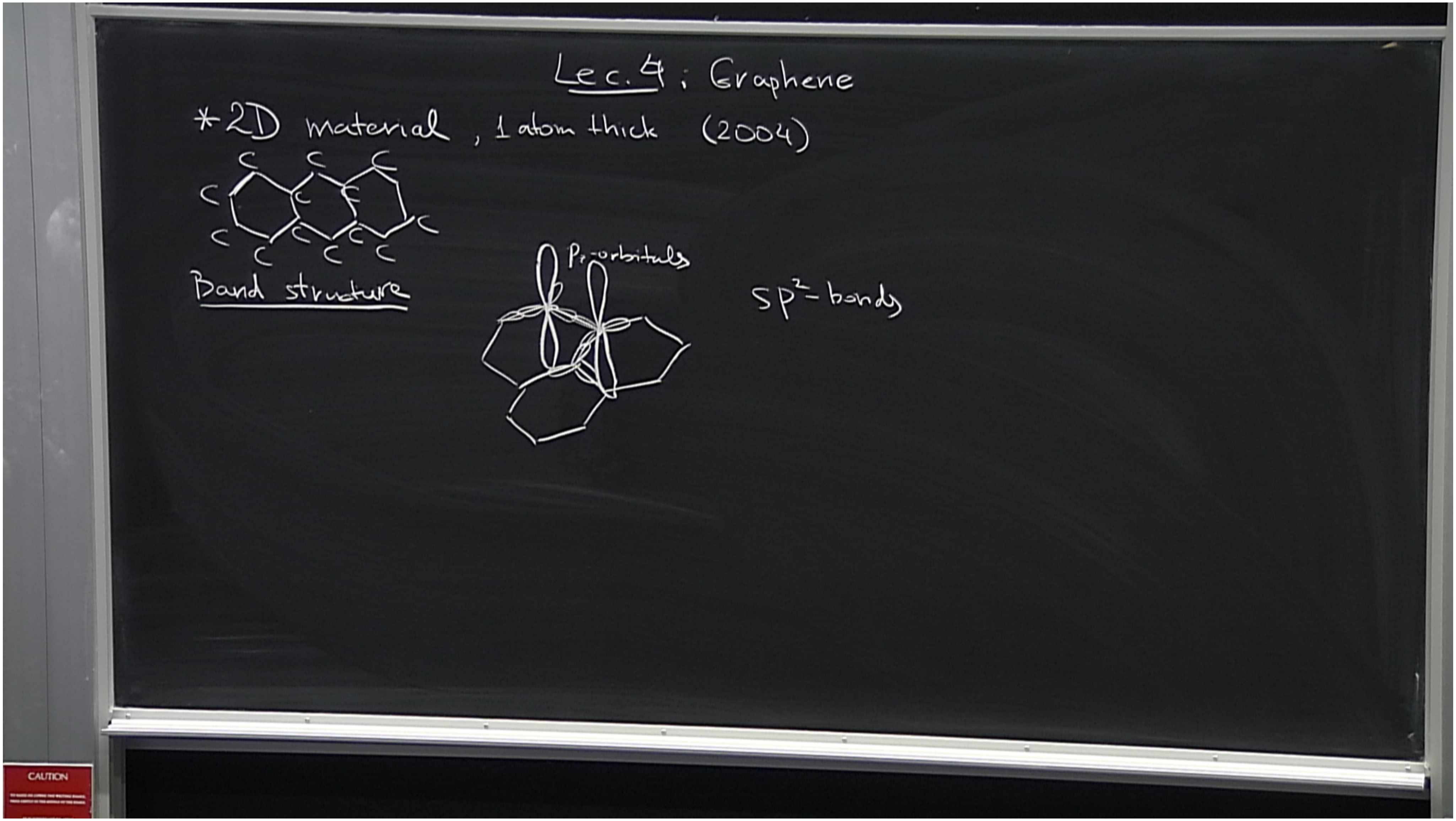

Lec. 4 : Graphene
* 2D material , 1 atom thick
CAUTION

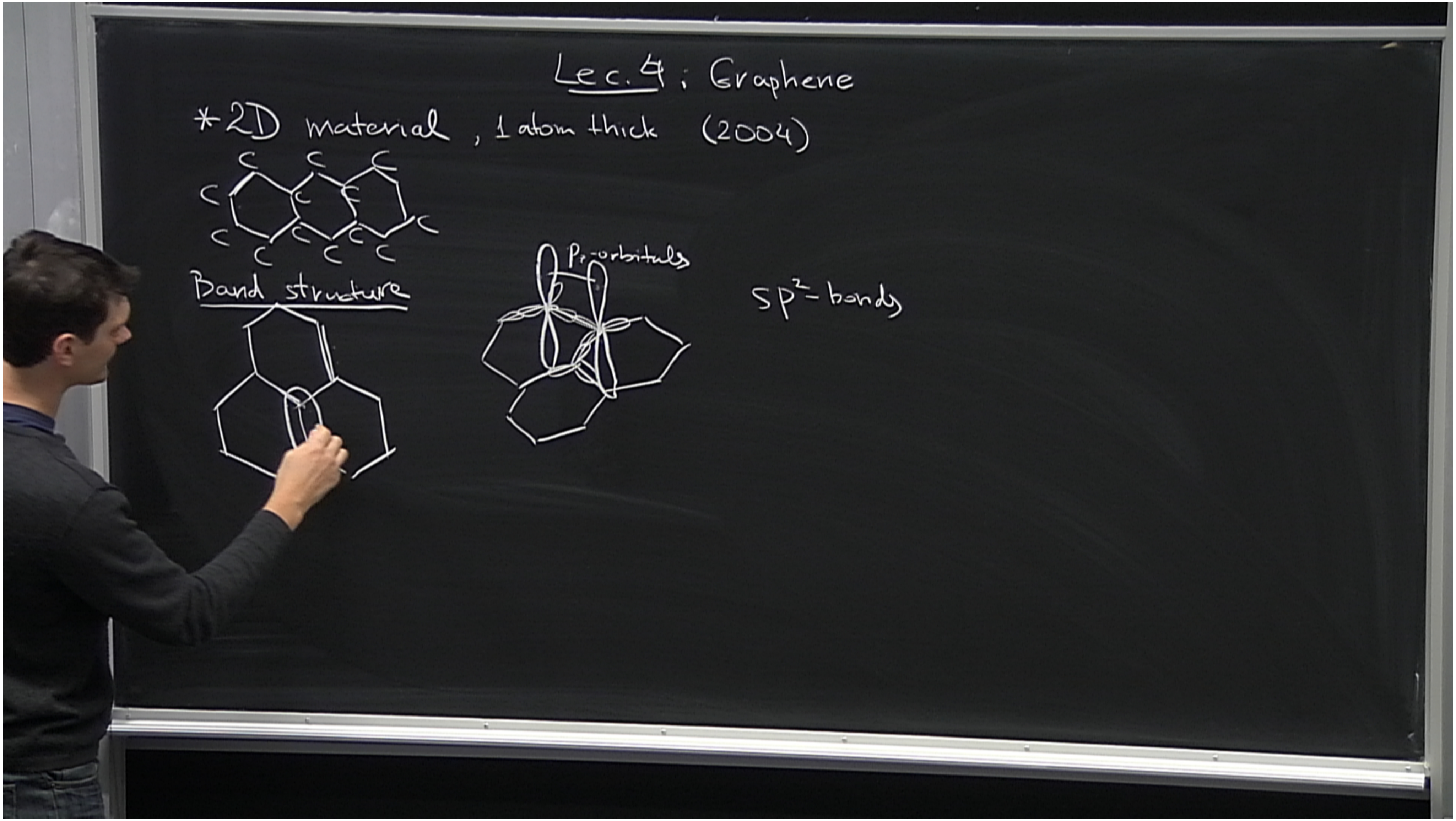

Lec. 4 : Graphene
* 2D material , 1 atom thick (2004)

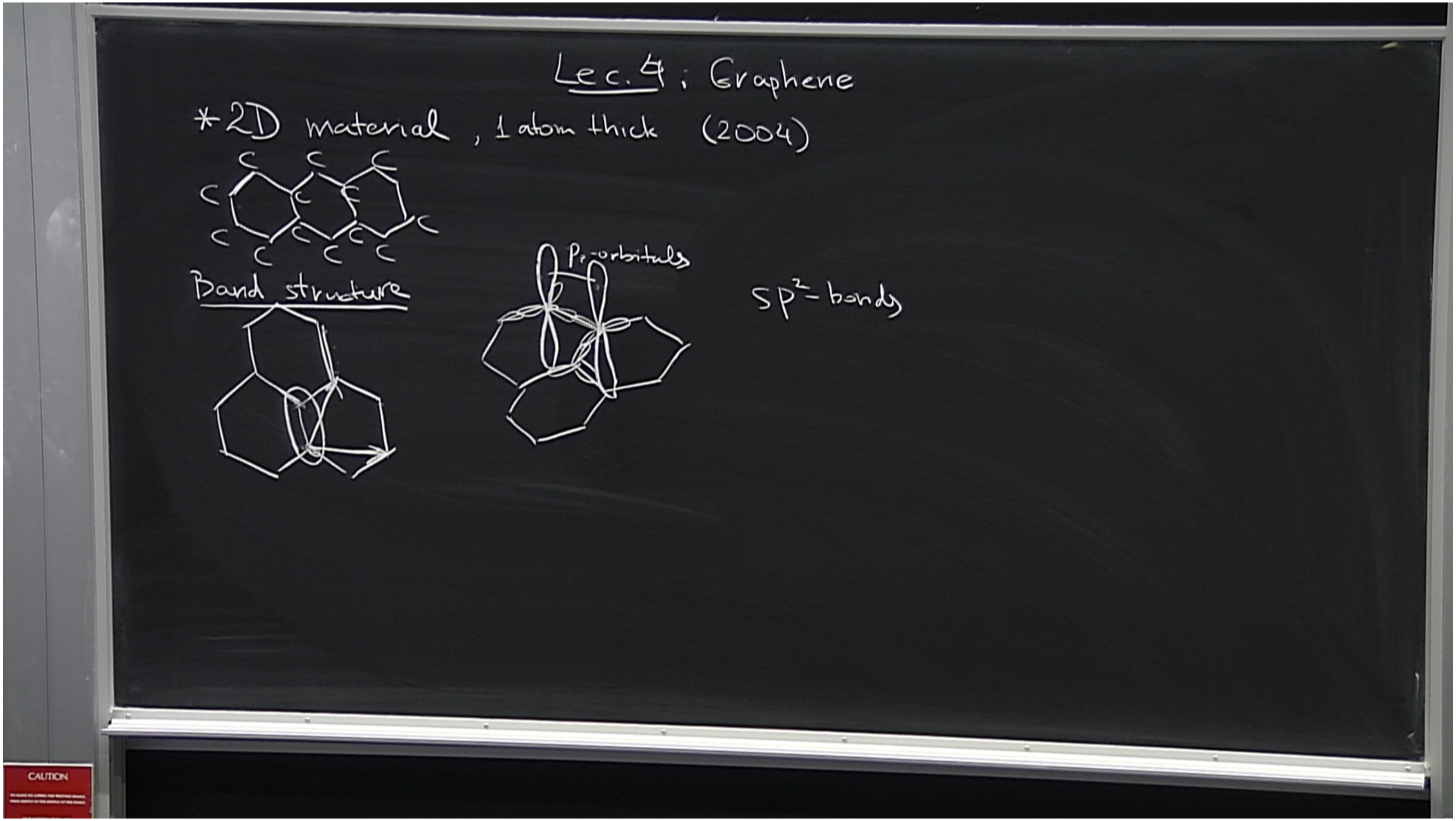
Lec. 4 : Graphene

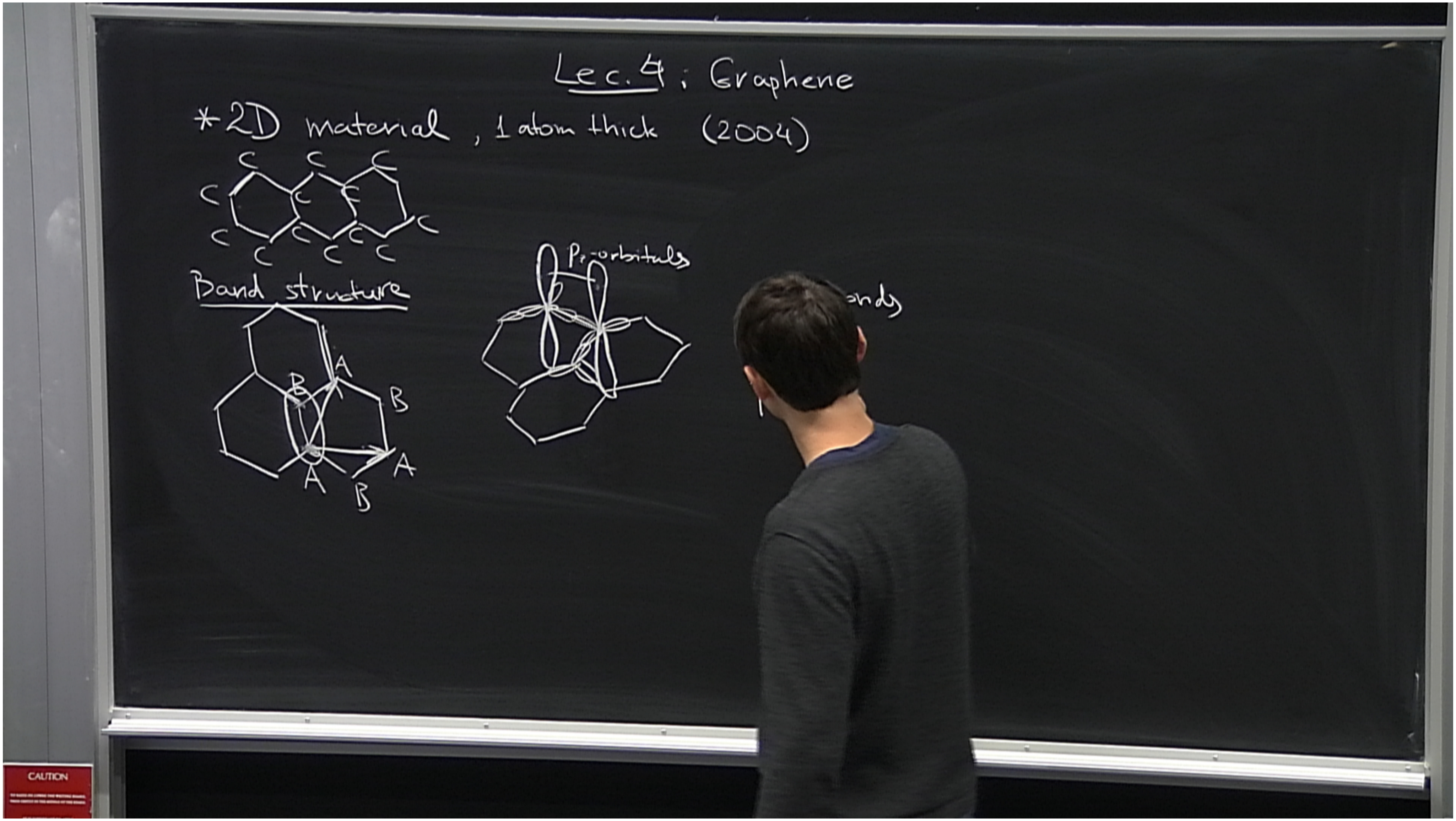
* 2D material, 1 atom thick (2004)

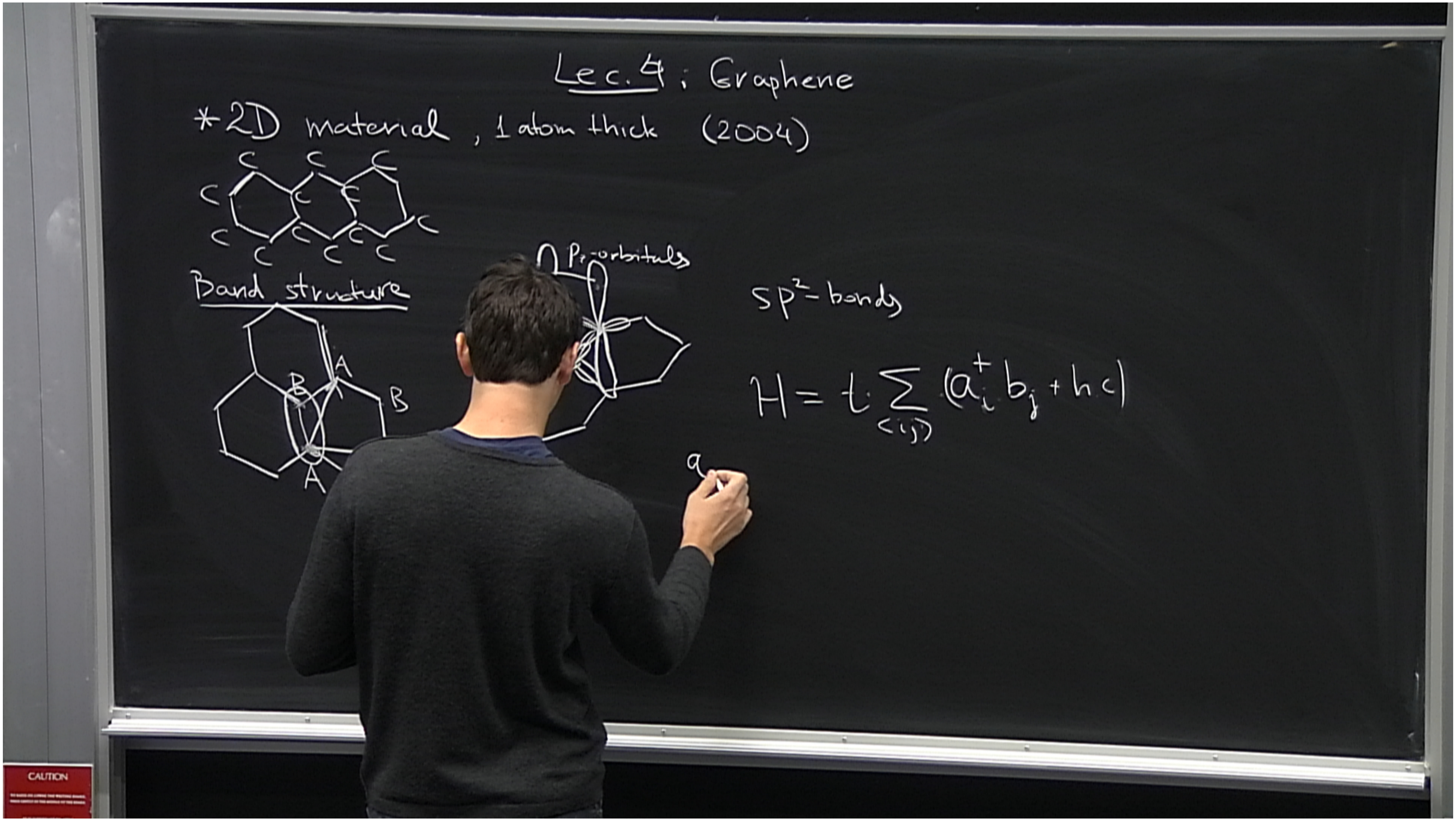



Lec. 4 : Graphene
* 2D material , 1 atom thick (2004)
Band structure
Pz-orbitals
sp²- bonds

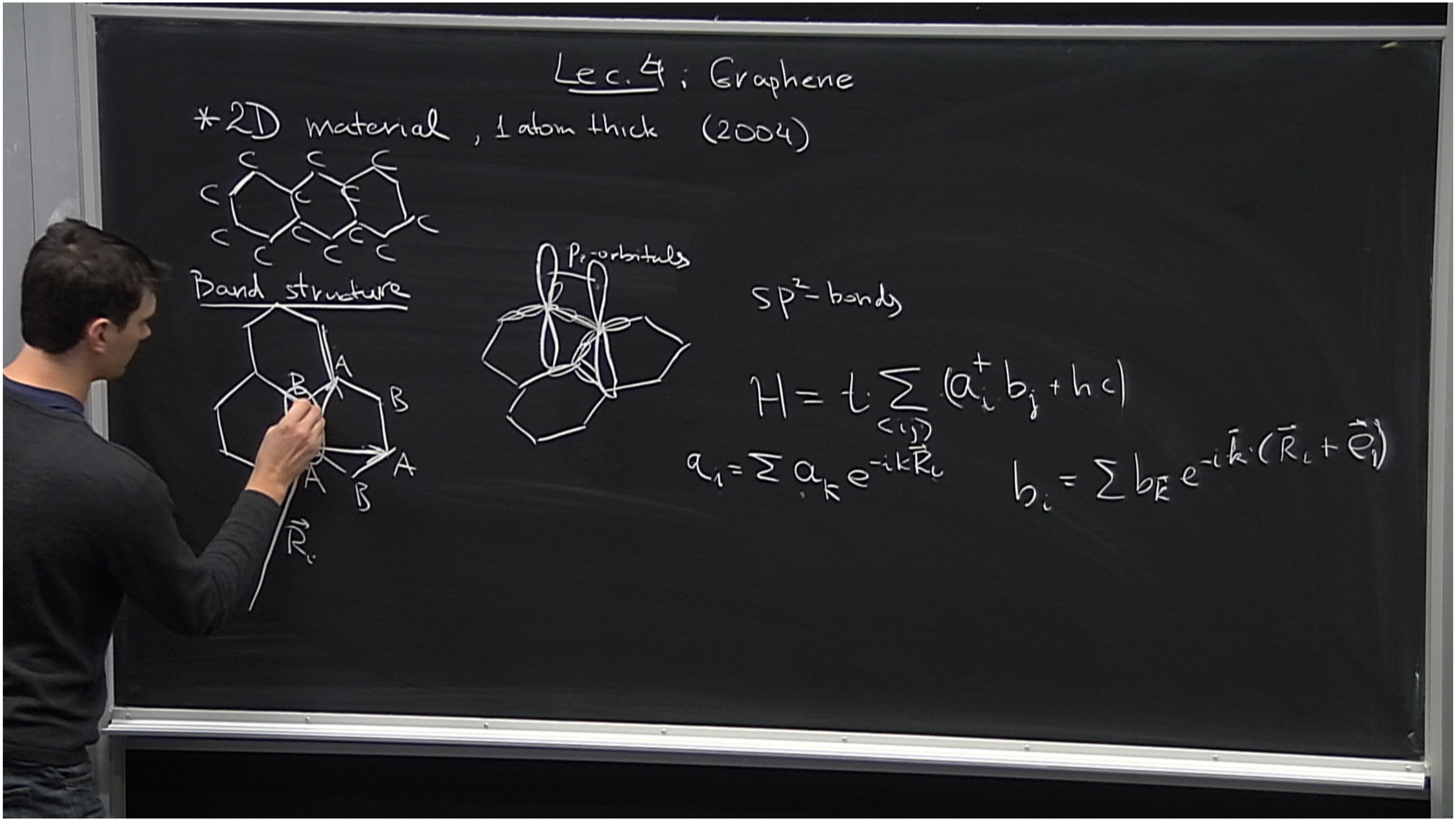

Lec. 4 : Graphene
* 2D material , 1 atom thick (2004)
C C C C C C C C C C C C C C
Band structure
Pz-orbitals
sp2 - bonds

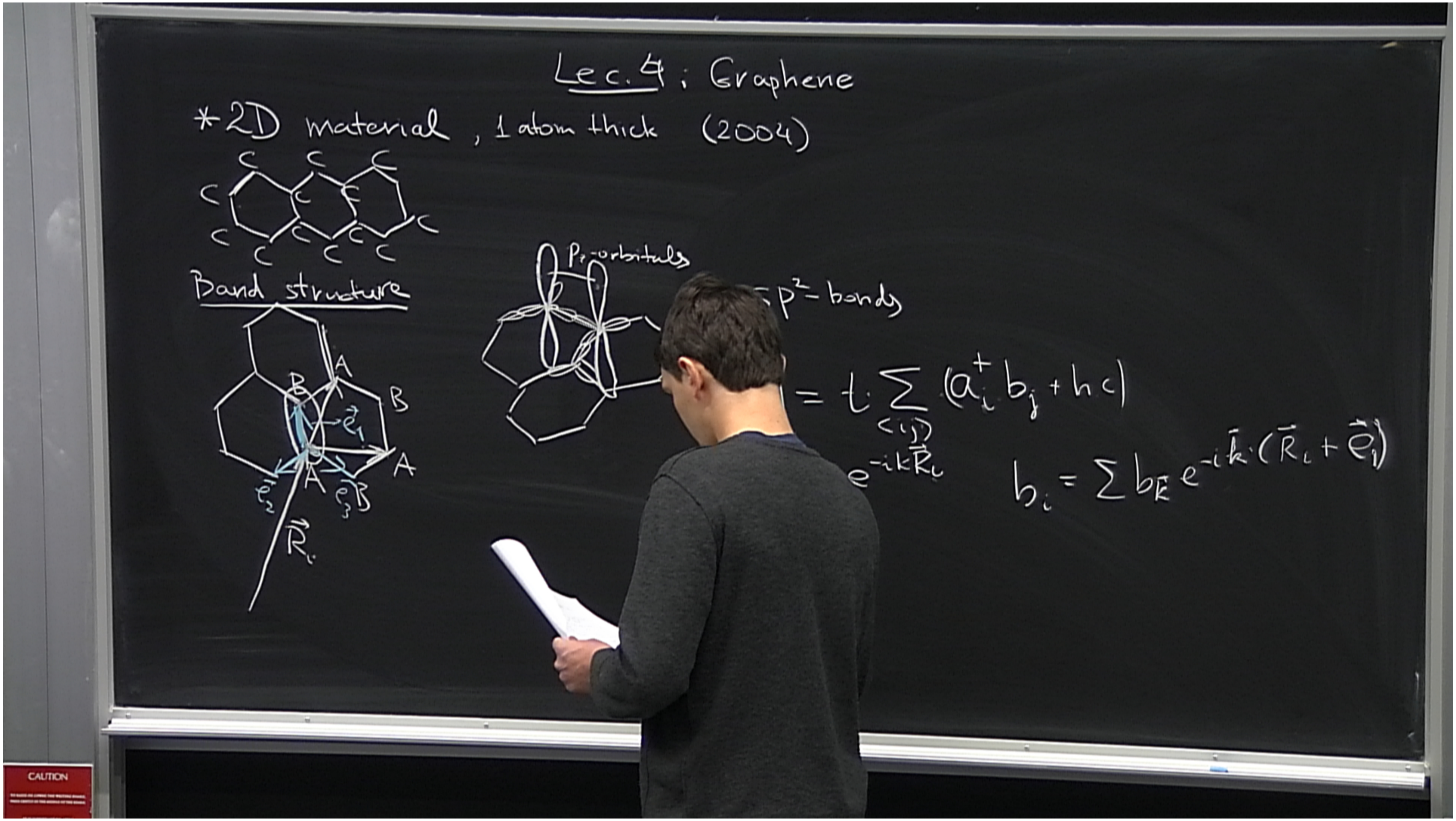

Lec. 4 : Graphene
* 2D material , 1 atom thick (2004)
Band structure
p_z-orbitals
sp^2-bonds
CAUTION

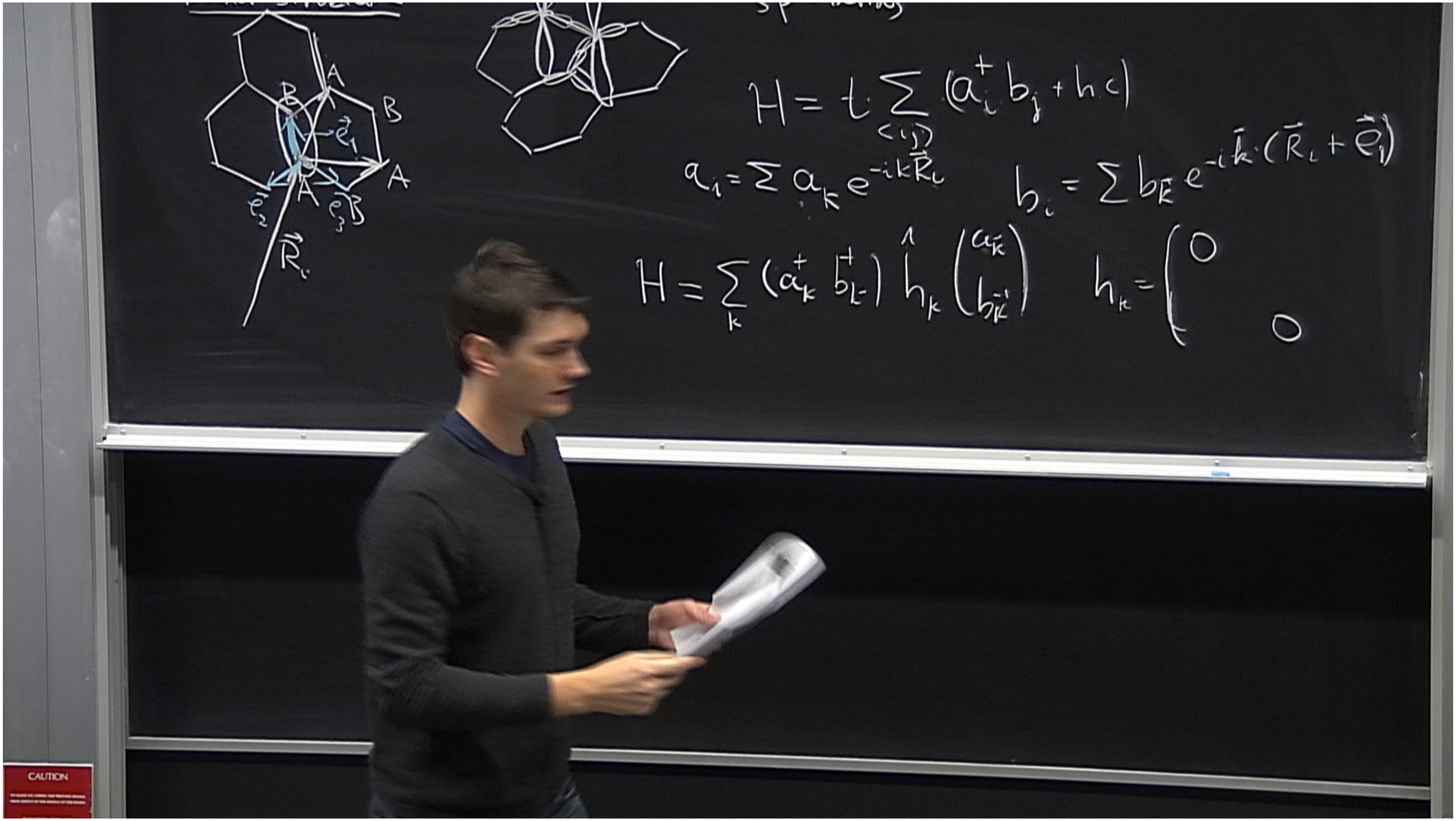

Lec. 4 : Graphene
* 2D material, 1 atom thick (2004)
Band structure
A
B
B
A
A
B
CAUTION

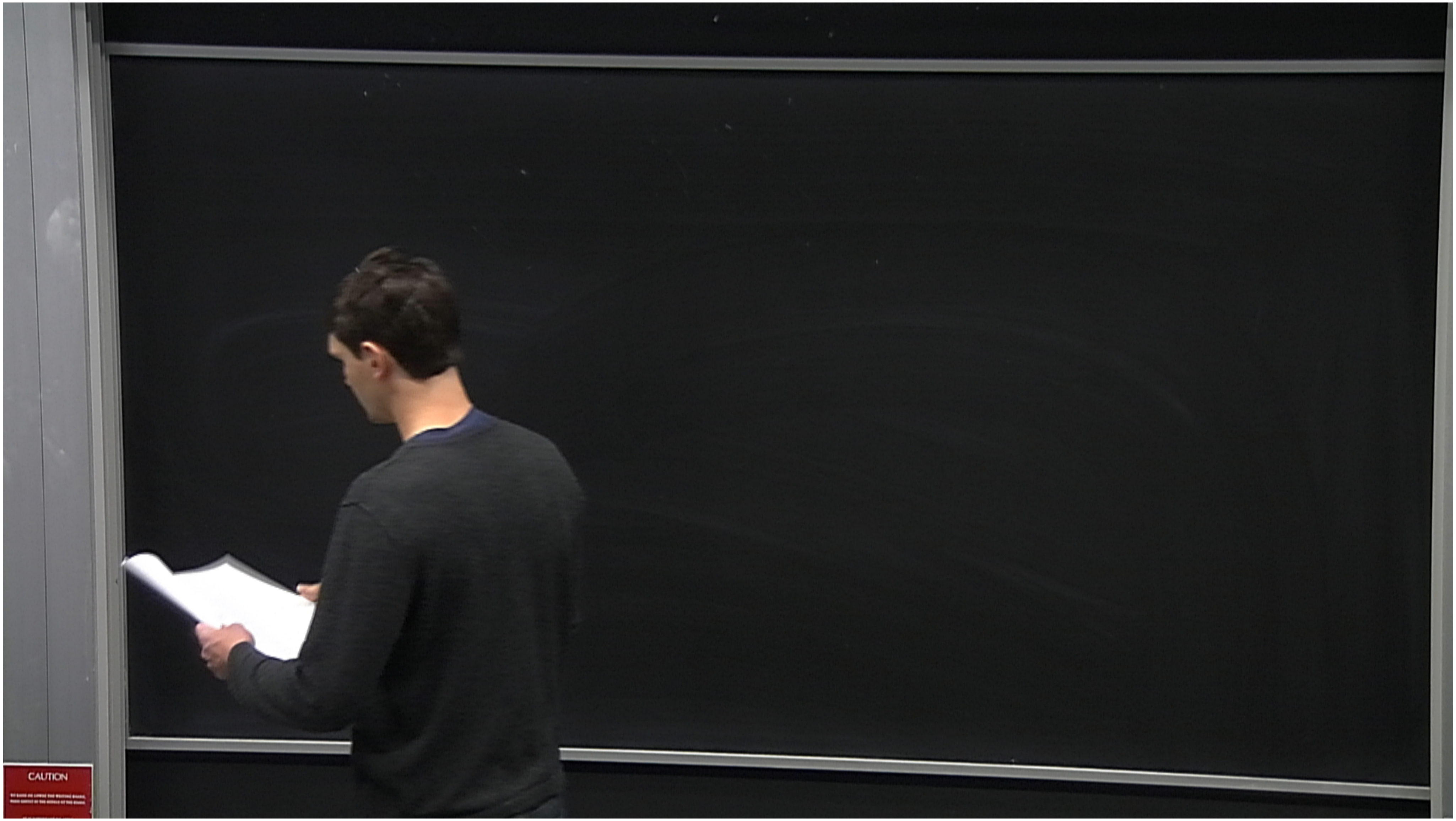


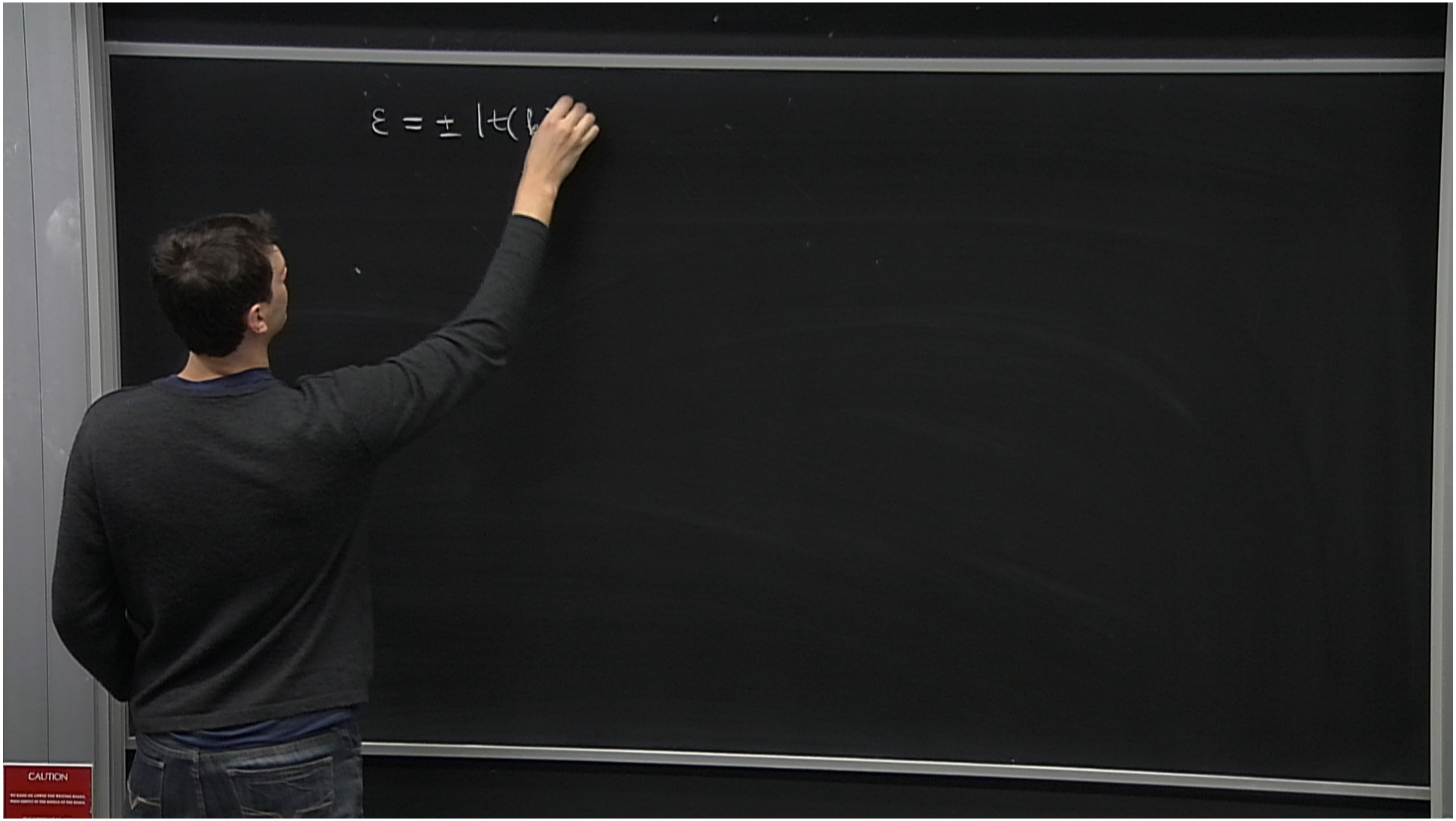
Lec. 4 : Graphene
* 2D material , 1 atom thick (2004)
Band structure
A
B
B
A
p_z-orbitals
sp^2-bonds
$H = t \sum_{\langle ij \rangle} (a_i^+ b_j + h.c)$
a
CAUTION

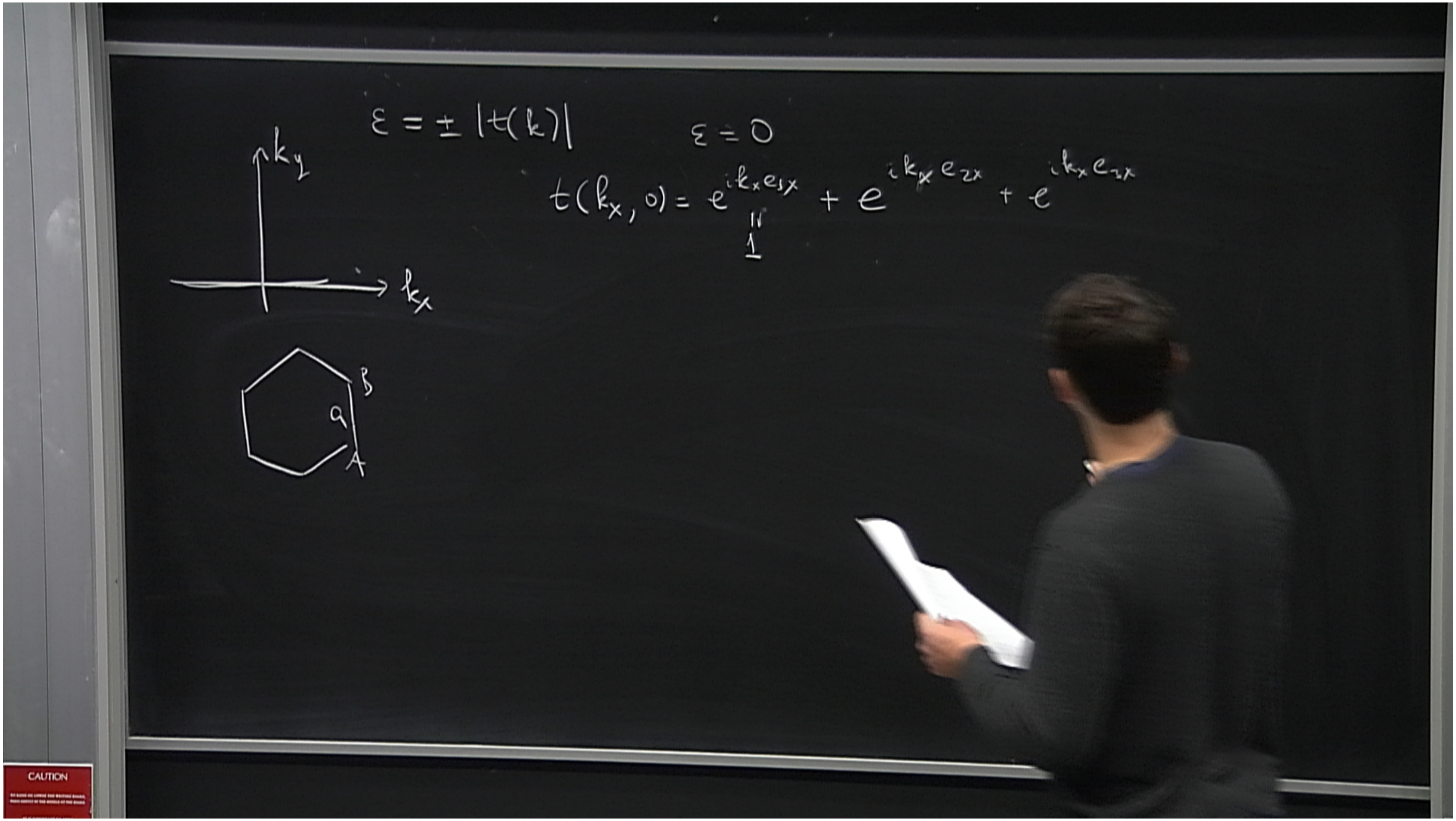

Lec. 4 : Graphene
* 2D material , 1 atom thick (2004)
Band structure
p_z-orbitals
sp²- bonds
H = t Σ (a⁺_i b_j + h.c)
a_1 = Σ a_k e^{-ik R_i}
b_i = Σ b_k e^{-ik·(R_i + e_1)}

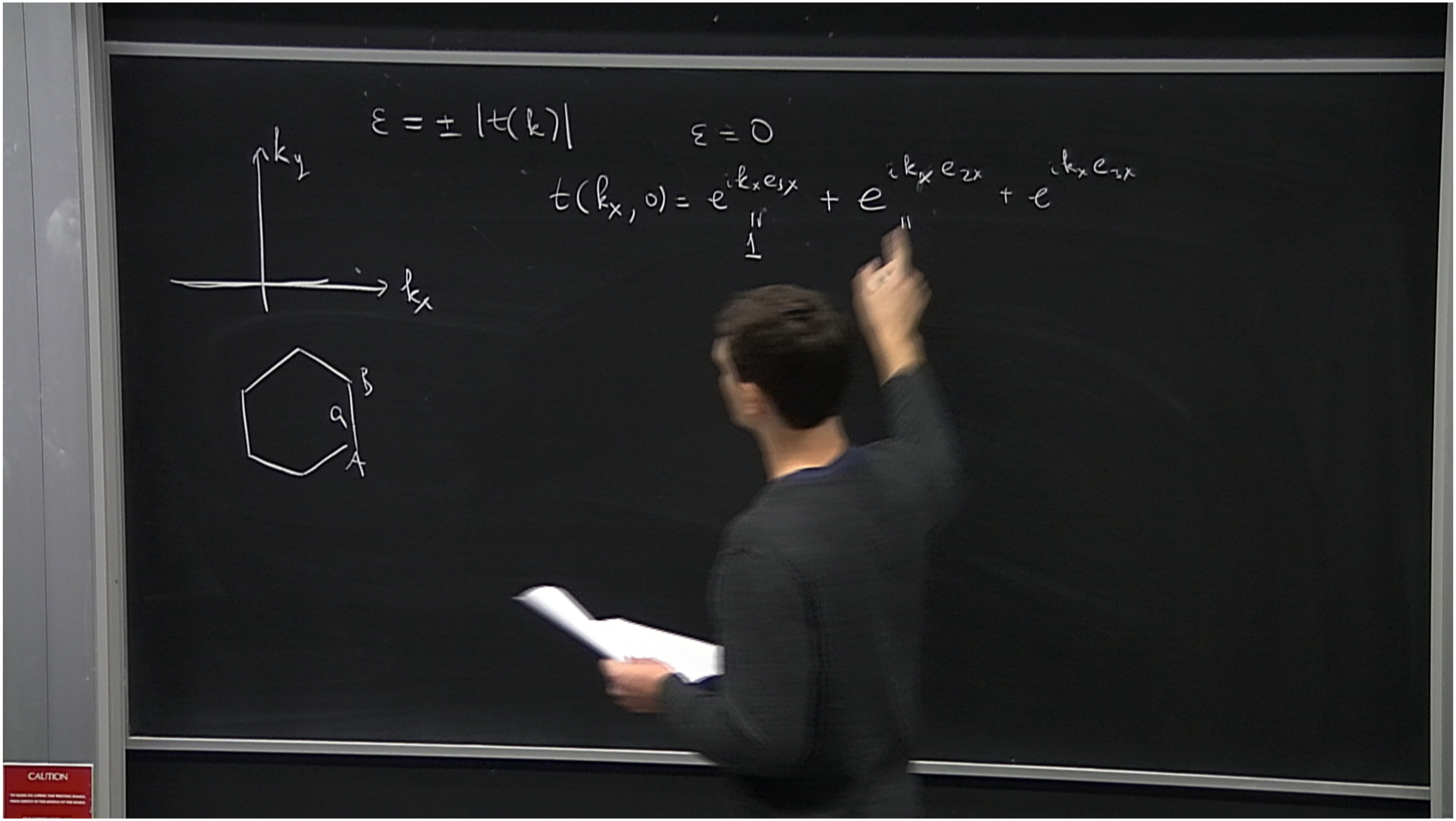

Lec. 4 : Graphene
* 2D material, 1 atom thick (2004)
Band structure
p_z-orbitals
sp²-bonds
= t ∑ (a_i⁺ b_j + h.c)
b_i = ∑ b_k e^{-ik·(R_i + e_1)}
CAUTION

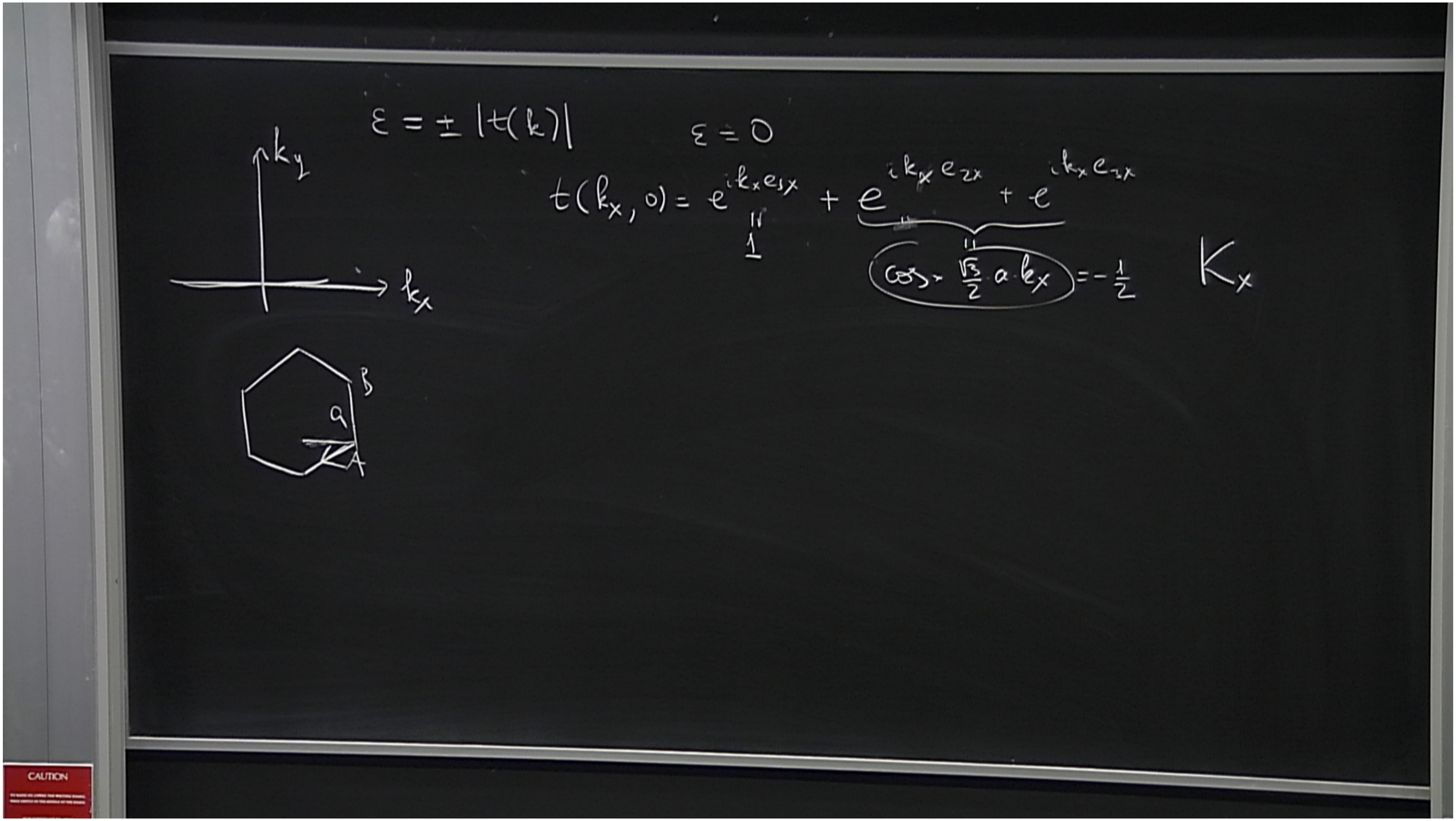

A
B
B
A
A
B
$\vec{e}_1$
$\vec{e}_2$
$\vec{e}_3$
$\vec{R}_i$
$H = t \sum_{<ij>} (a_i^+ b_j + h.c)$
$a_i = \sum a_k e^{-ik\vec{R}_i}$
$b_i = \sum b_{\vec{k}} e^{-i\vec{k}\cdot(\vec{R}_i + \vec{e}_1)}$
$H = \sum_k (a_k^+ \; b_k^+) \, \hat{h}_k \begin{pmatrix} a_k \\ b_k \end{pmatrix}$
$h_k = \begin{pmatrix} 0 & \\ & 0 \end{pmatrix}$
CAUTION

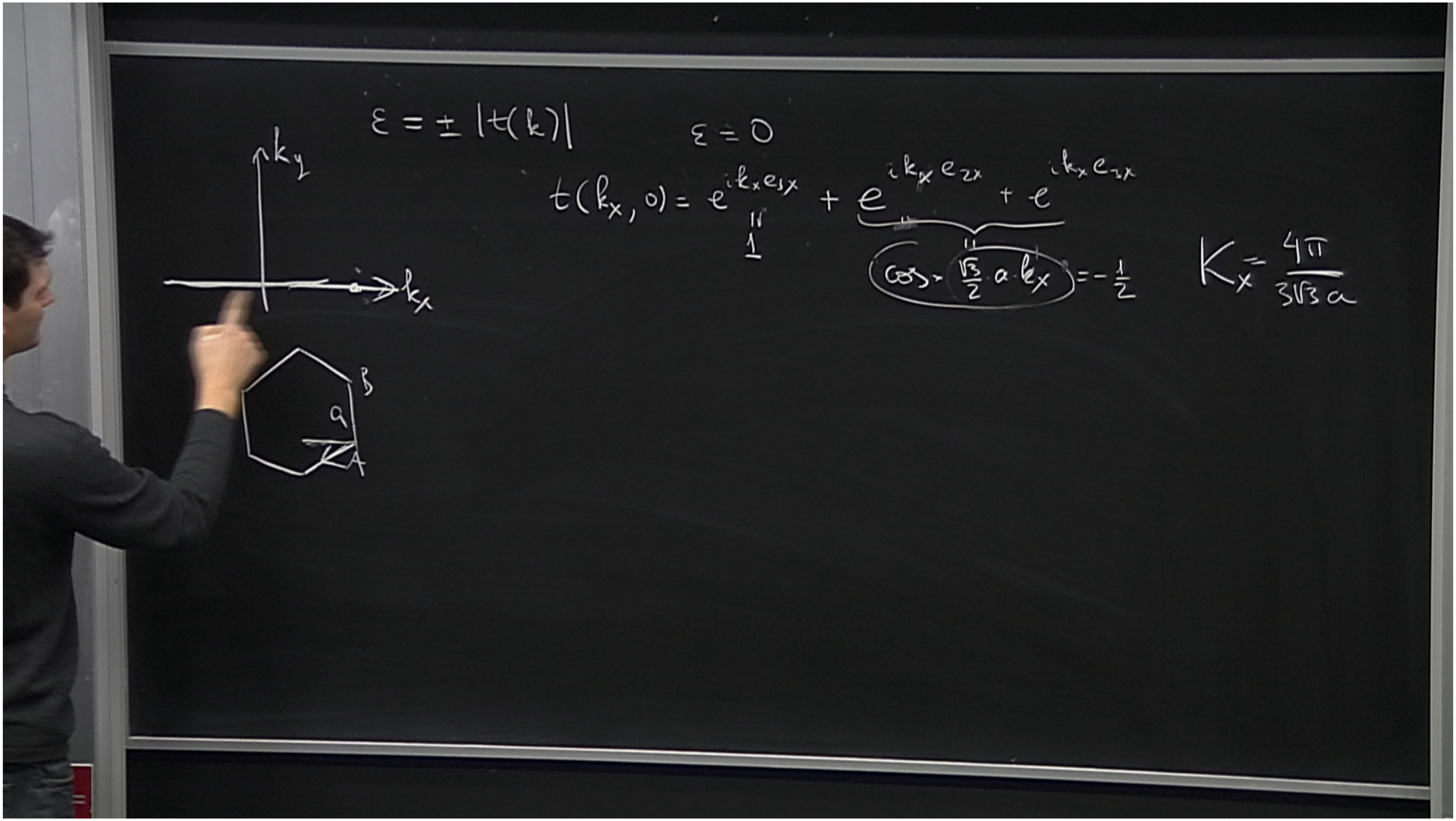


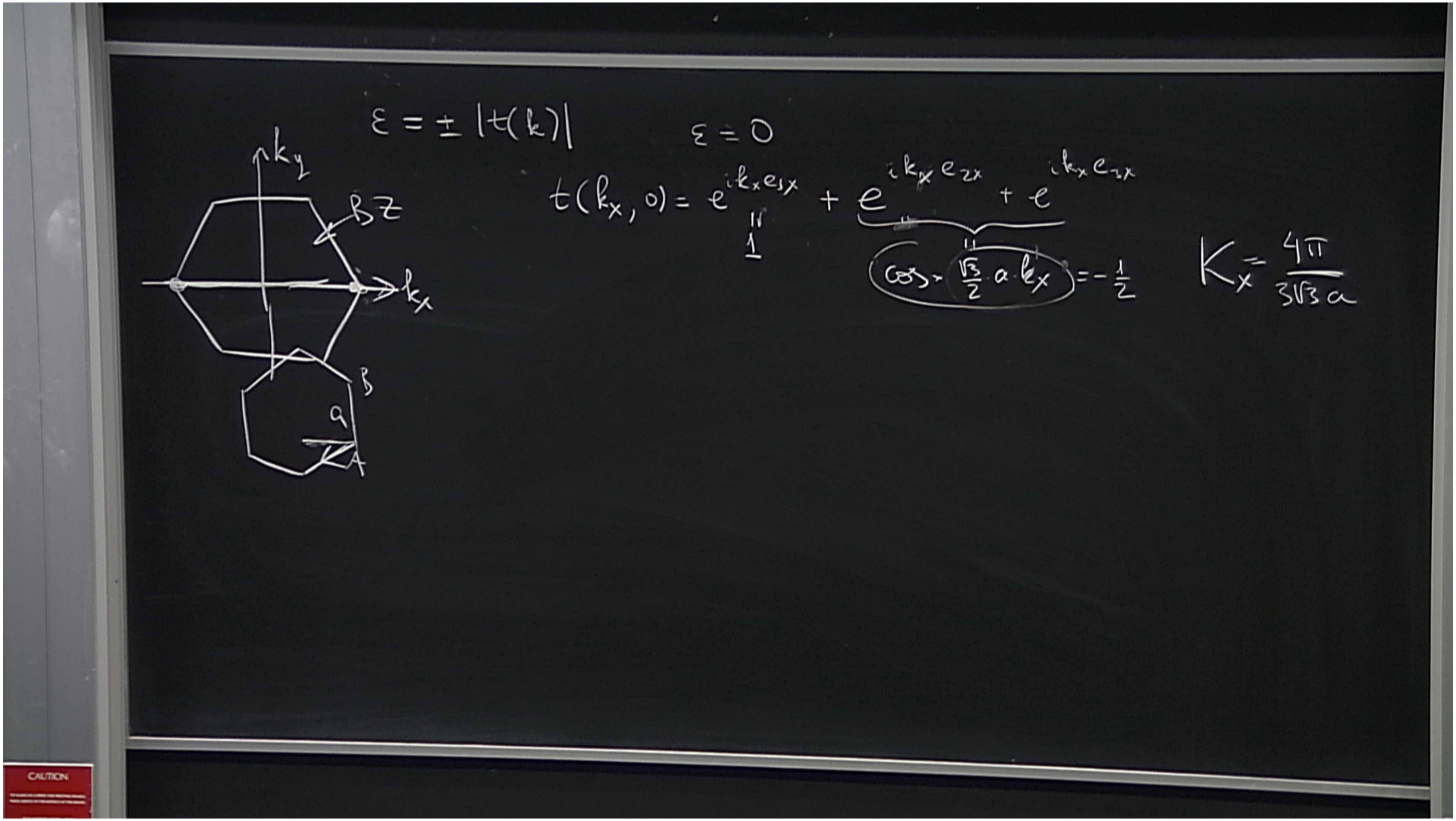

$\varepsilon = \pm |t(k$
CAUTION

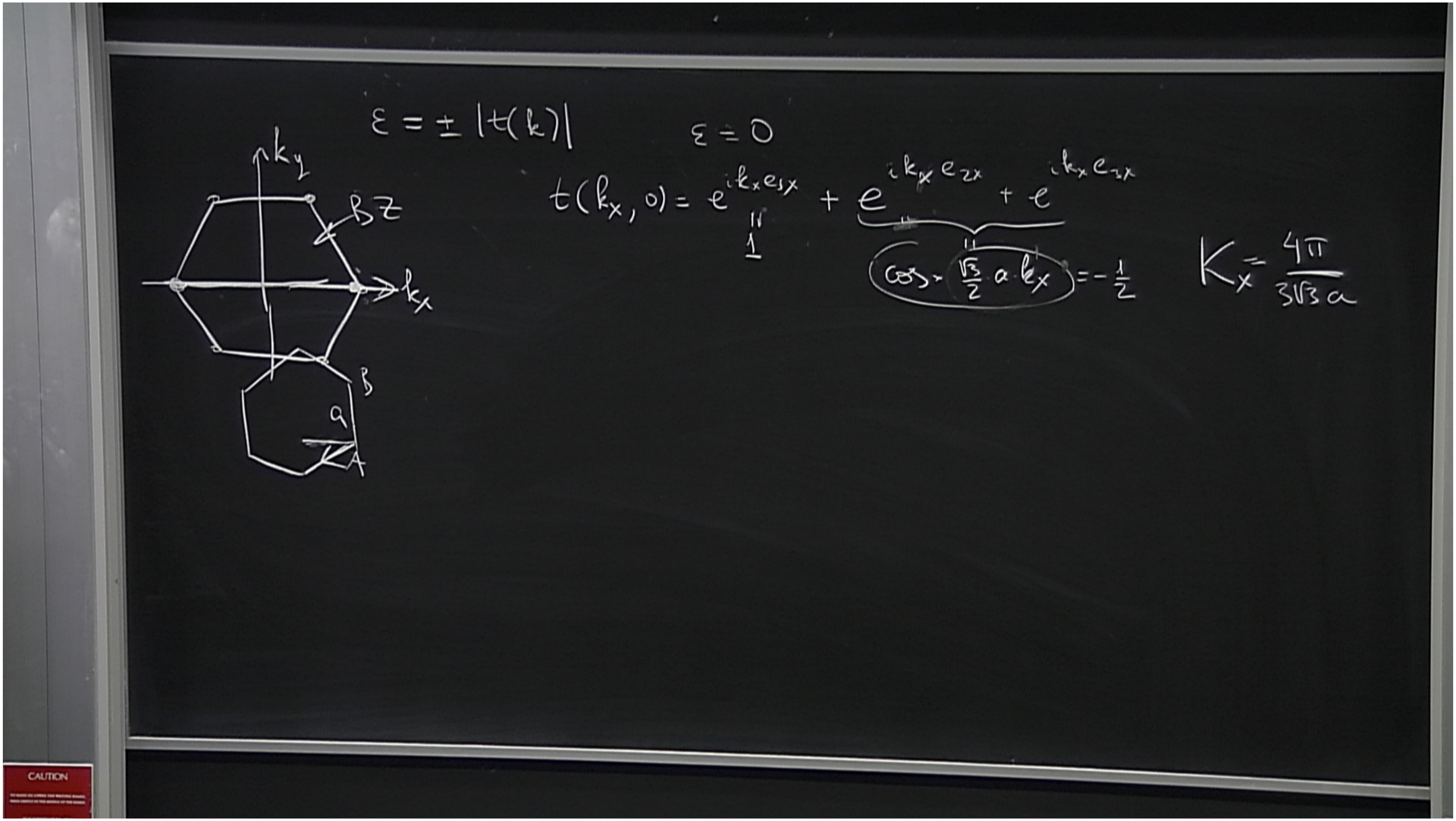

ε = ± |t(k)|
ε = 0
t(kx, 0) = e^{i kx e1x} + e^{i kx e2x} + e^{i kx e3x}
1
ky
kx
B
a
A
CAUTION

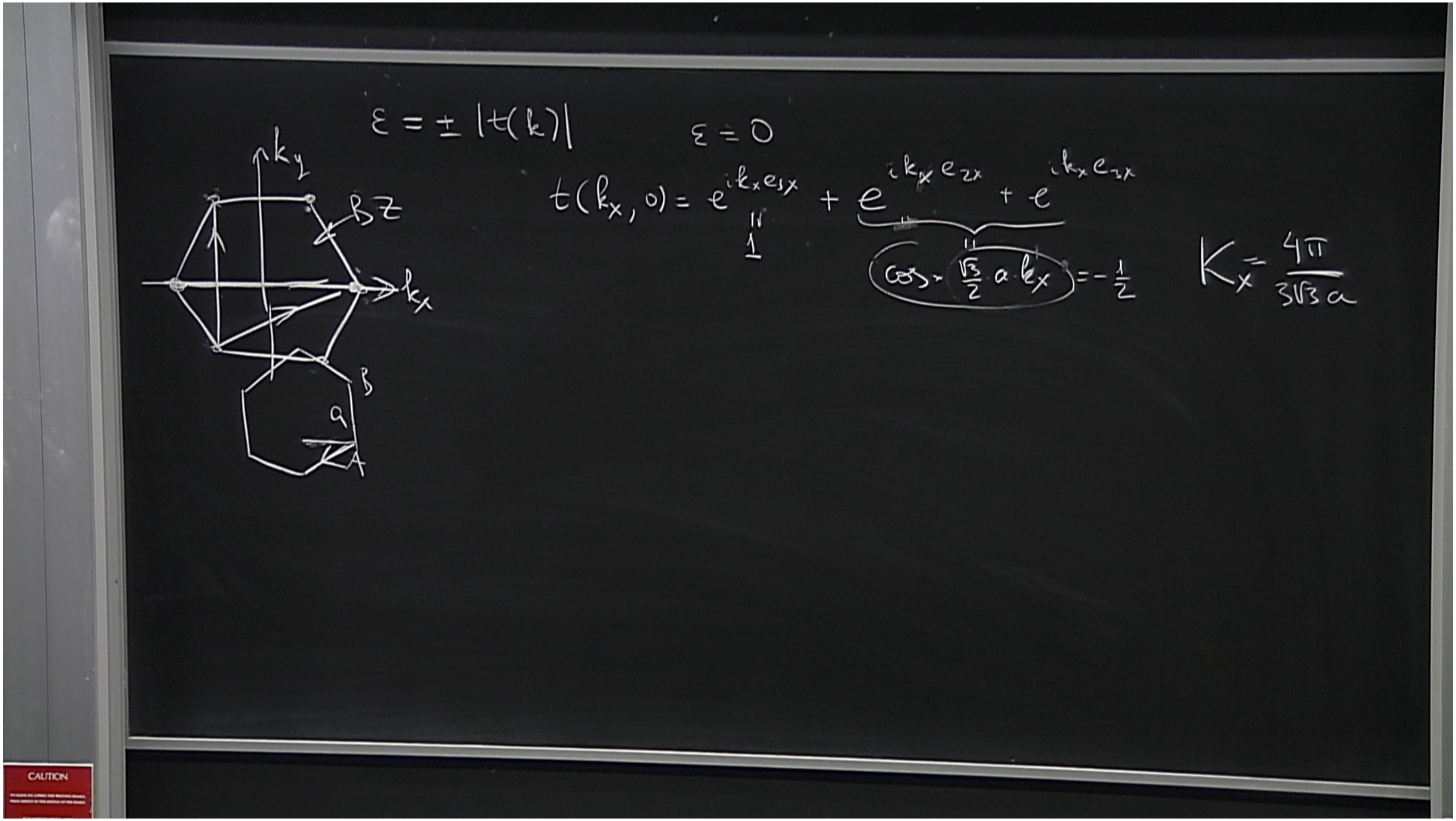

ε = ± |t(k)|
ε = 0
t(k_x, 0) = e^{ik_x e_1x} + e^{ik_x e_2x} + e^{ik_x e_3x}
1
k_y
k_x
B
a
A
CAUTION

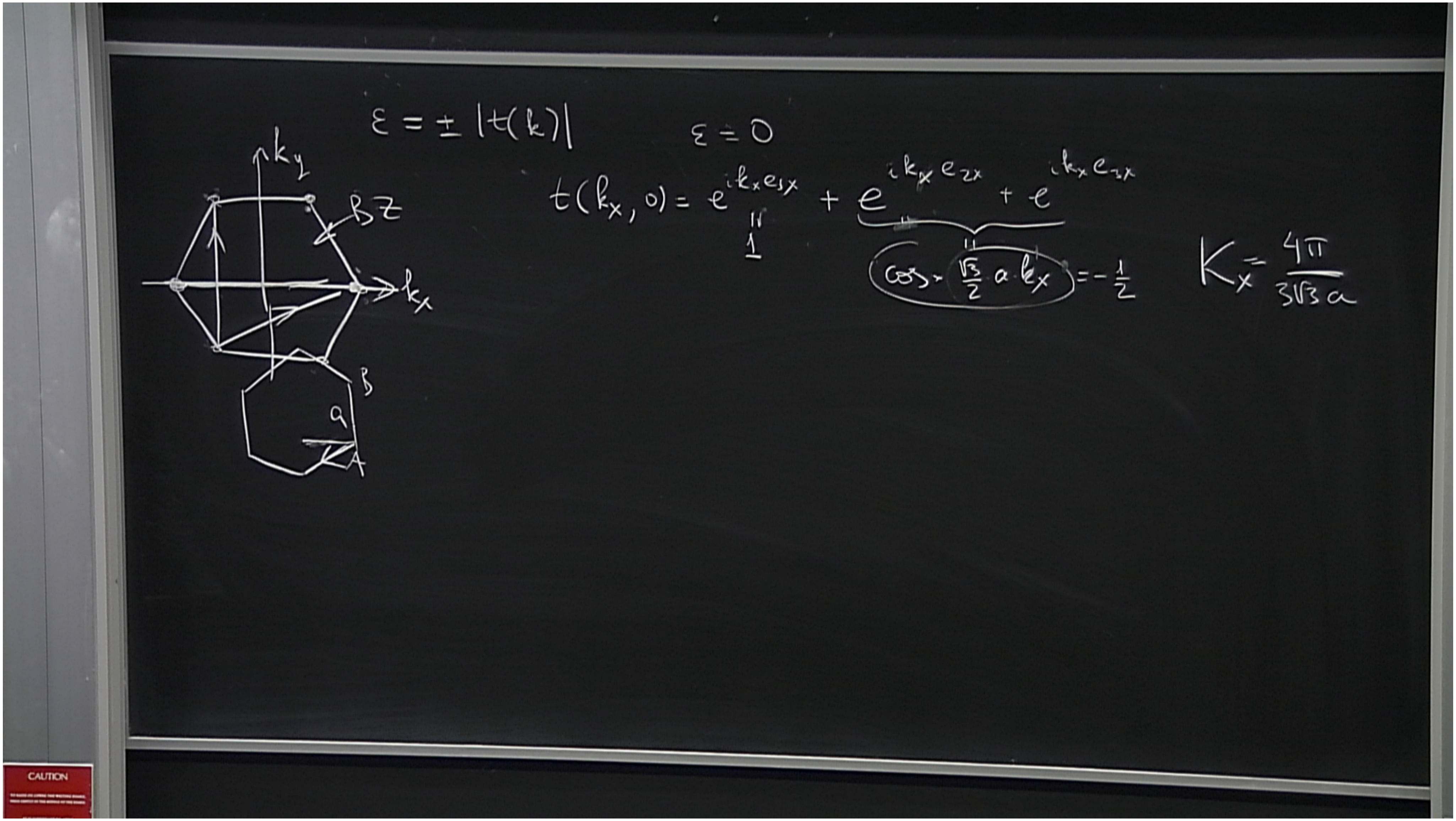

$\varepsilon = \pm |t(k)|$
$\varepsilon = 0$
$t(k_x, 0) = e^{i k_x e_{1x}} + e^{i k_x e_{2x}} + e^{i k_x e_{3x}}$
1
$\cos \frac{\sqrt{3}}{2} a \cdot k_x = -\frac{1}{2}$
K_x
k_y
k_x
B
a
A
CAUTION

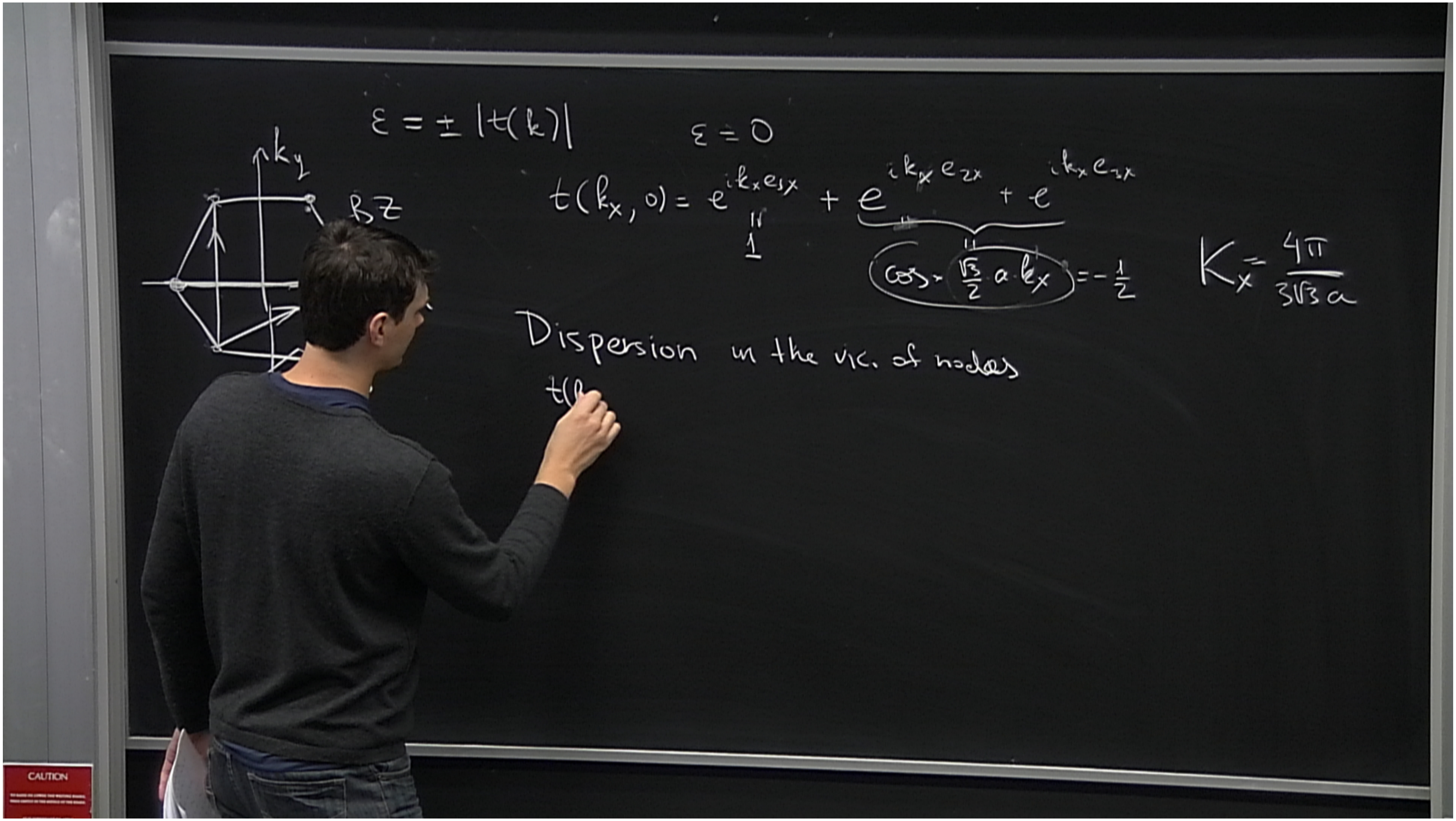

ε = ± |t(k)|
ε = 0
t(k_x, 0) = e^{i k_x e_{1x}} + e^{i k_x e_{2x}} + e^{i k_x e_{3x}}
1
cos (√3/2 a k_x) = −1/2
K_x = 4π / (3√3 a)
k_y
k_x
B
a
A

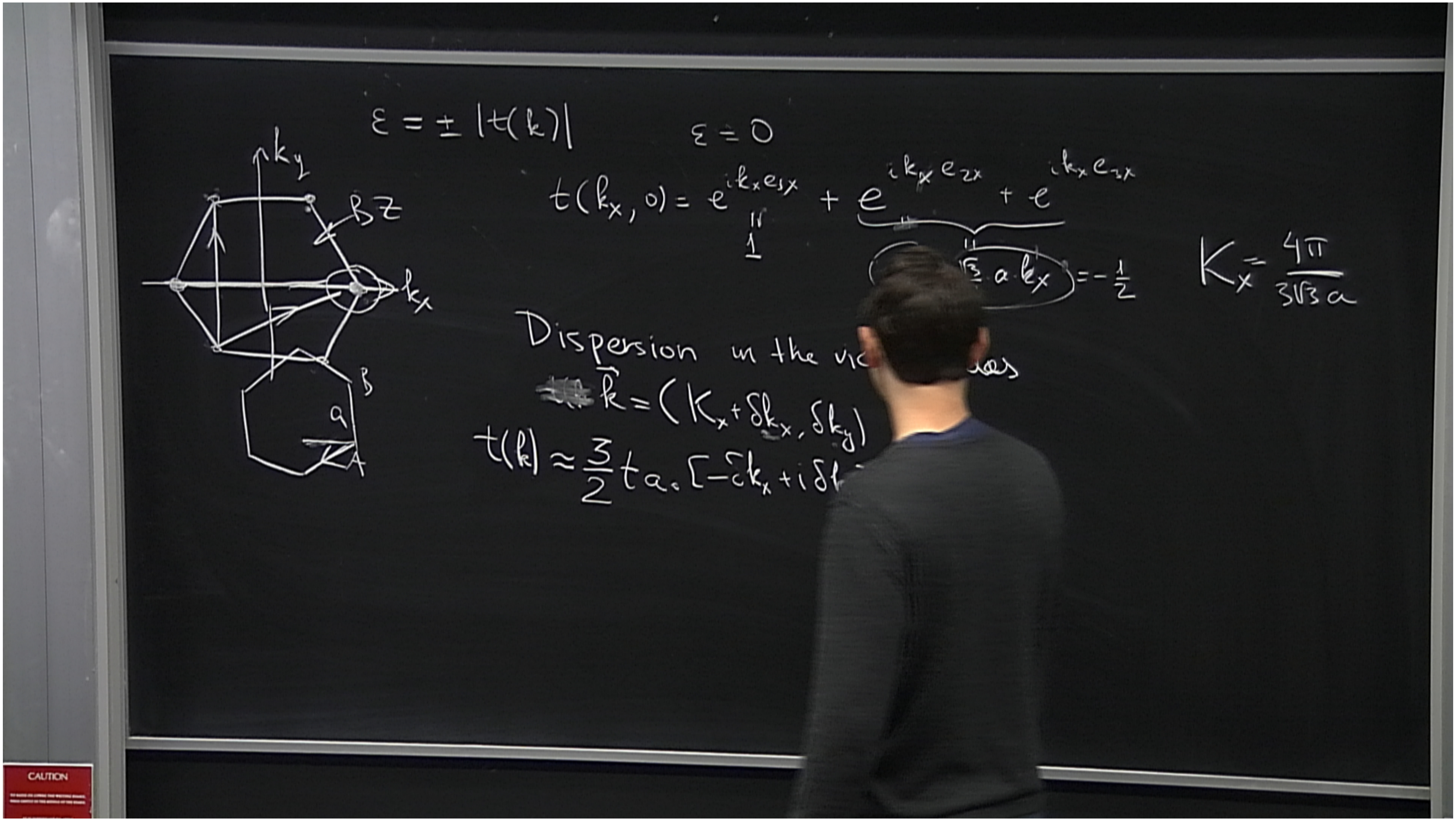

ε = ± |t(k)|
ε = 0
k_y
BZ
k_x
B
a
A
t(k_x, 0) = e^{i k_x e_{1x}} + e^{i k_x e_{2x}} + e^{i k_x e_{3x}}
1
cos (√3/2 a k_x) = −1/2
K_x = 4π / (3√3 a)

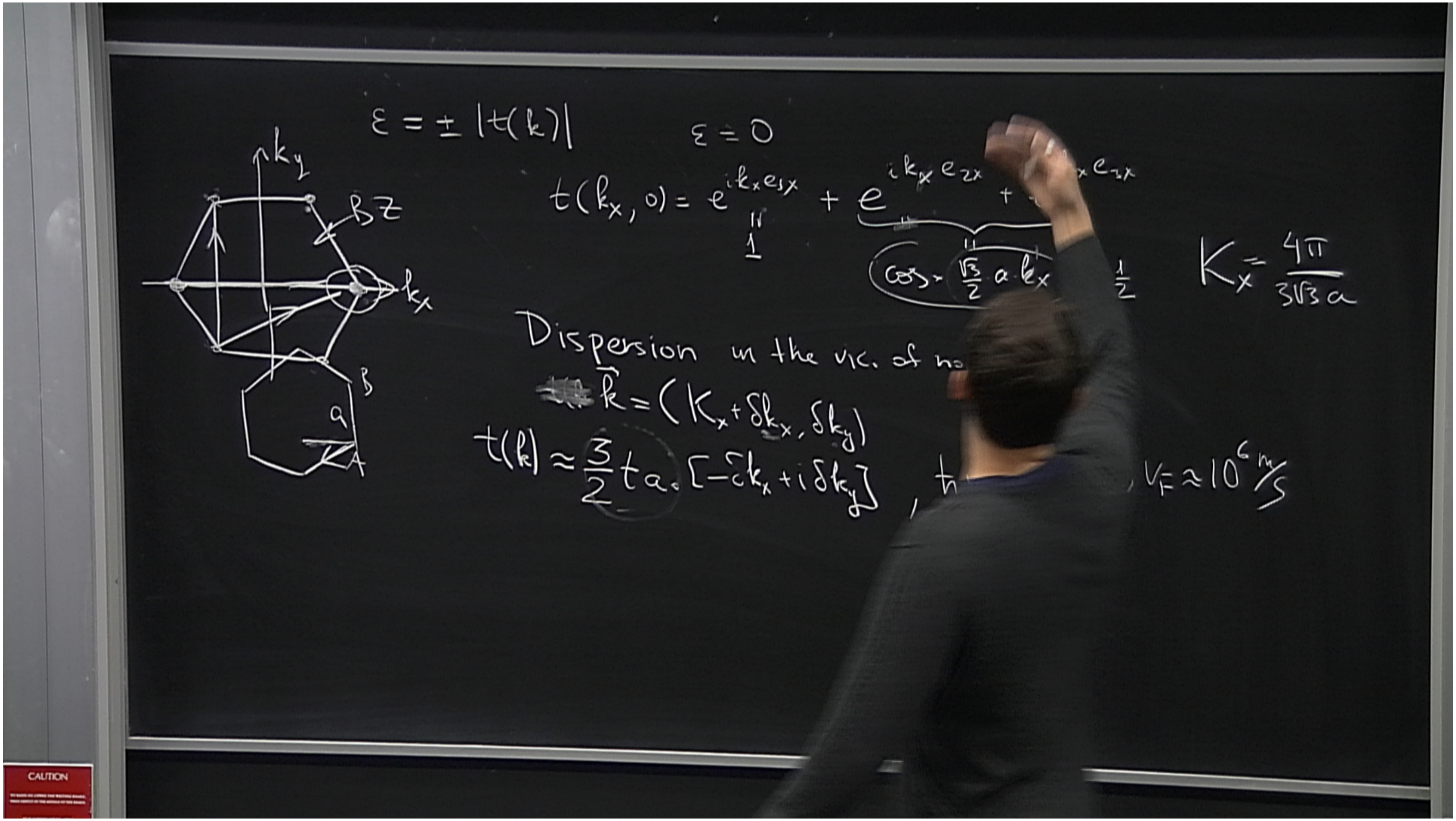

ε = ± |t(k)|
ε = 0
t(k_x, 0) = e^{i k_x e_1x} + e^{i k_x e_2x} + e^{i k_x e_3x}
1
cos √3/2 a·k_x = -1/2
K_x = 4π / 3√3 a
k_y
k_x
BZ
B
a
A
CAUTION

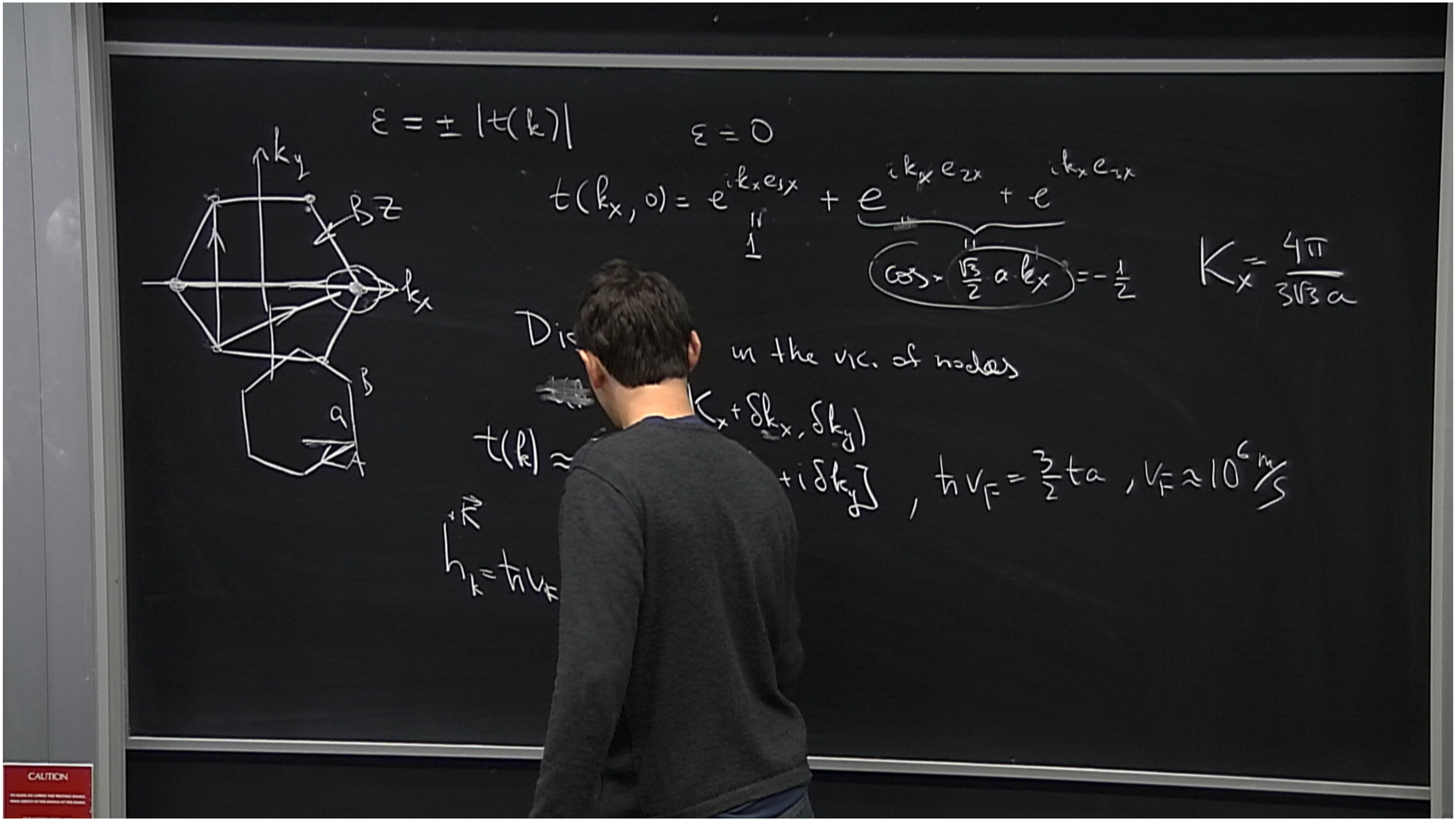

ε = ± |t(k)|
ε = 0
t(kx, 0) = e^{ikx e1x} + e^{ikx e2x} + e^{ikx e3x}
1
cos √3/2 a kx = -1/2
Kx = 4π / 3√3 a
ky
kx
BZ
B
a
A
CAUTION


$\varepsilon = \pm |t(k)|$
$\varepsilon = 0$
$t(k_x, 0) = e^{i k_x e_{1x}} + e^{i k_x e_{2x}} + e^{i k_x e_{3x}}$
1
$\cos \frac{\sqrt{3}}{2} a \cdot k_x = -\frac{1}{2}$
$K_x = \frac{4\pi}{3\sqrt{3} a}$
k_y
k_x
BZ
B
a
A
CAUTION

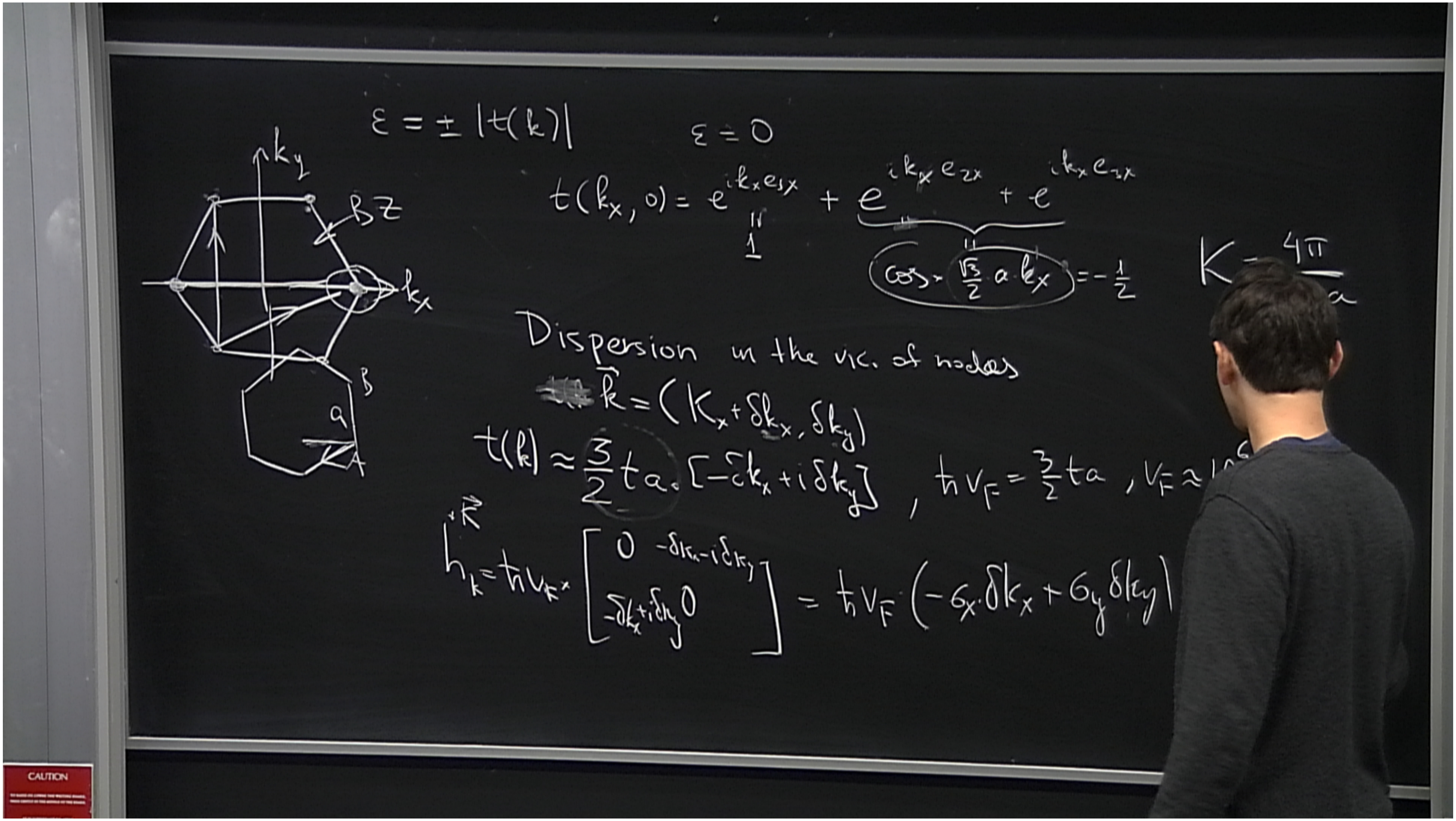

ε = ± |t(k)|
ε = 0
k_y
BZ
t(k_x, 0) = e^{ik_x e_1x} + e^{ik_x e_2x} + e^{ik_x e_3x}
1
cos(√3/2 a·k_x) = −1/2
K_x = 4π / 3√3 a
Dispersion in the vic. of nodes
t(k

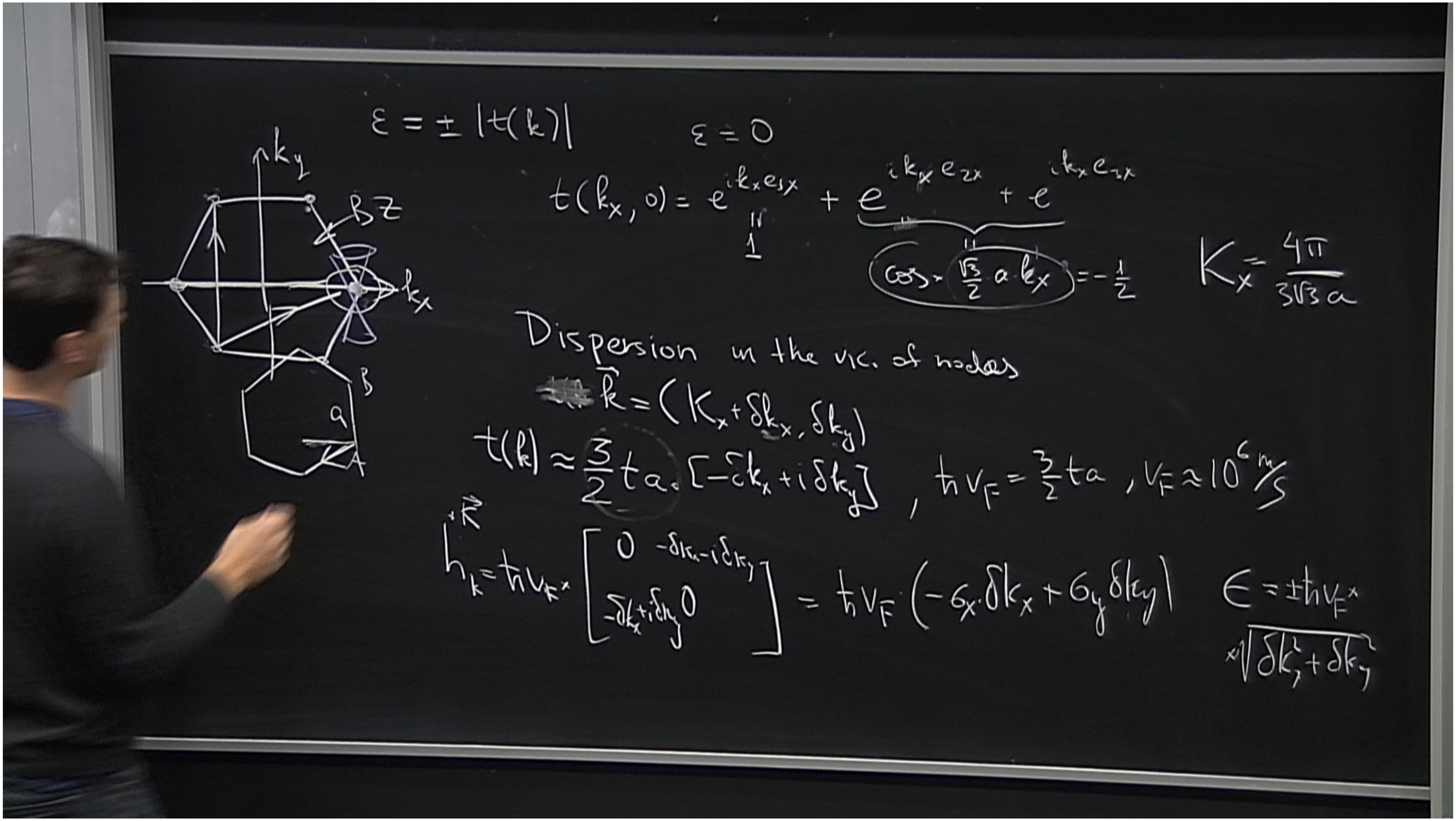

ε = ± |t(k)|
ε = 0
k_y
BZ
k_x
t(k_x, 0) = e^{ik_x e_1x} + e^{ik_x e_2x} + e^{ik_x e_3x}
K_x = 4π / 3√3 a
Dispersion in the
k = (K_x + δk_x, δk_y)

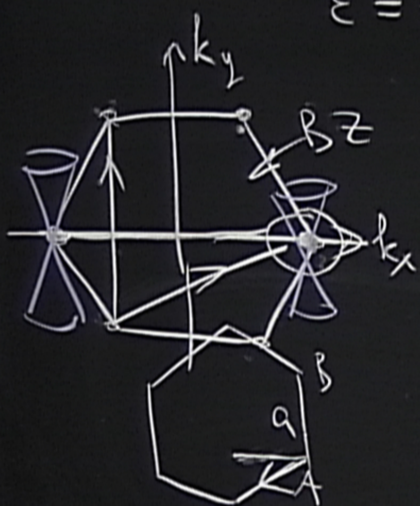

ε = ± |t(k)|
ε = 0
k_y
BZ
k_x
t(k_x, 0) = e^{i k_x e_{1x}} + e^{i k_x e_{2x}} +
1
cos
K_x = 4π / 3√3 a
Dispersion in the vic. of
k = (K_x + δk_x, δk_y)
t(k) ≈ 3/2 t a [−i δk_x + i δk_y]
v_F ≈ 10^6 m/s



ε = ± |t(k)|
ε = 0
k_y
BZ
k_x
B
a
A
t(k_x, 0) = e^{ik_x e_{1x}} + e^{ik_x e_{2x}} + e^{ik_x e_{3x}}
1
cos √3/2 a k_x = -1/2
K_x = 4π / 3√3 a
Di in the vic. of nodes
K_x + δk_x, δk_y)
t(k) ≈
+ iδk_y] , ħv_F = 3/2 ta , v_F ≈ 10^6 m/s
h_k = ħv_F


ε = ± |t(k)|
ε = 0
t(k_x, 0) = e^{i k_x e_{1x}} + e^{i k_x e_{2x}} + e^{i k_x e_{3x}}
cos √3/2 a k_x = −1/2
BZ
k_y
k_x
Dispersion in the vic. of nodes
k = (K_x + δk_x, δk_y)
t(k) ≈ 3/2 ta [−δk_x + iδk_y] , ħv_F = 3/2 ta
h_k = ħv_F [0 −δk_x − iδk_y; −δk_x + iδk_y 0] = ħv_F (−σ_x δk_x + σ_y δk_y)
CAUTION


$\varepsilon = \pm |t(k)|$
$\varepsilon = 0$
k_y
BZ
k_x
a
$t(k_x, 0) = e^{ik_x e_{1x}} + e^{ik_x e_{2x}} + e^{ik_x e_{3x}}$
$\cos \frac{\sqrt{3}}{2} a \cdot k_x = -\frac{1}{2}$
$K_x = \frac{4\pi}{3\sqrt{3}a}$
Dispersion in the vic. of nodes
$\vec{k} = (K_x + \delta k_x, \delta k_y)$
$t(k) \approx \frac{3}{2} t a \cdot [-\delta k_x + i \delta k_y]$, $\hbar v_F = \frac{3}{2} t a$, $v_F \approx 10^6\, m/s$
$h_k^{K} = \hbar v_F \begin{bmatrix} 0 & -\delta k_x - i\delta k_y \\ -\delta k_x + i\delta k_y & 0 \end{bmatrix} = \hbar v_F (-\sigma_x \delta k_x + \sigma_y \delta k_y)$
$\epsilon = \pm \hbar v_F \sqrt{\delta k_x^2 + \delta k_y^2}$



$$\varepsilon = \pm |t(k)| \qquad \varepsilon = 0$$

$$t(k_x, 0) = \underbrace{e^{i k_x e_{1x}}}_{=1} + \underbrace{e^{i k_x e_{2x}} + e^{i k_x e_{3x}}}_{\cos\left(\frac{\sqrt{3}}{2} a k_x\right) = -\frac{1}{2}} \qquad K_x = \frac{4\pi}{3\sqrt{3}a}$$

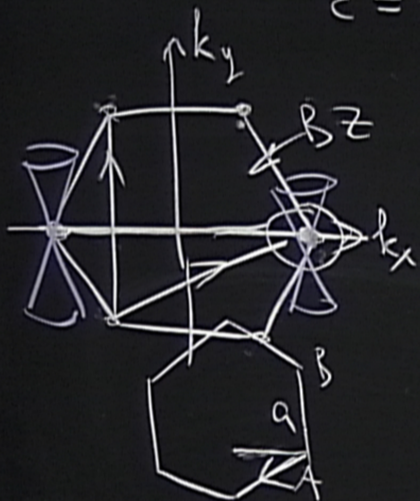
Dispersion in the vic. of nodes

$$\vec{k} = (K_x + \delta k_x, \delta k_y)$$

$$t(k) \approx \frac{3}{2} t a \left[-\delta k_x + i \delta k_y\right], \quad \hbar v_F = \frac{3}{2} t a, \quad v_F \approx 10^6 \, m/s$$

$$h_{\vec{k}} = \hbar v_F \begin{bmatrix} 0 & -\delta k_x - i\delta k_y \\ -\delta k_x + i\delta k_y & 0 \end{bmatrix} = \hbar v_F \left(-\sigma_x \delta k_x + \sigma_y \delta k_y\right) \qquad \epsilon = \pm \hbar v_F \sqrt{\delta k_x^2 + \delta k_y^2}$$

$$\varepsilon = \pm |t(k)| \qquad \varepsilon = 0$$



$$t(k_x, 0) = e^{ik_x e_{1x}} + e^{ik_x e_{2x}} + e^{ik_x e_{3x}}$$

$e^{ik_x e_{1x}} = 1$; $\cos\left(\frac{\sqrt{3}}{2} a \cdot k_x\right) = -\frac{1}{2}$

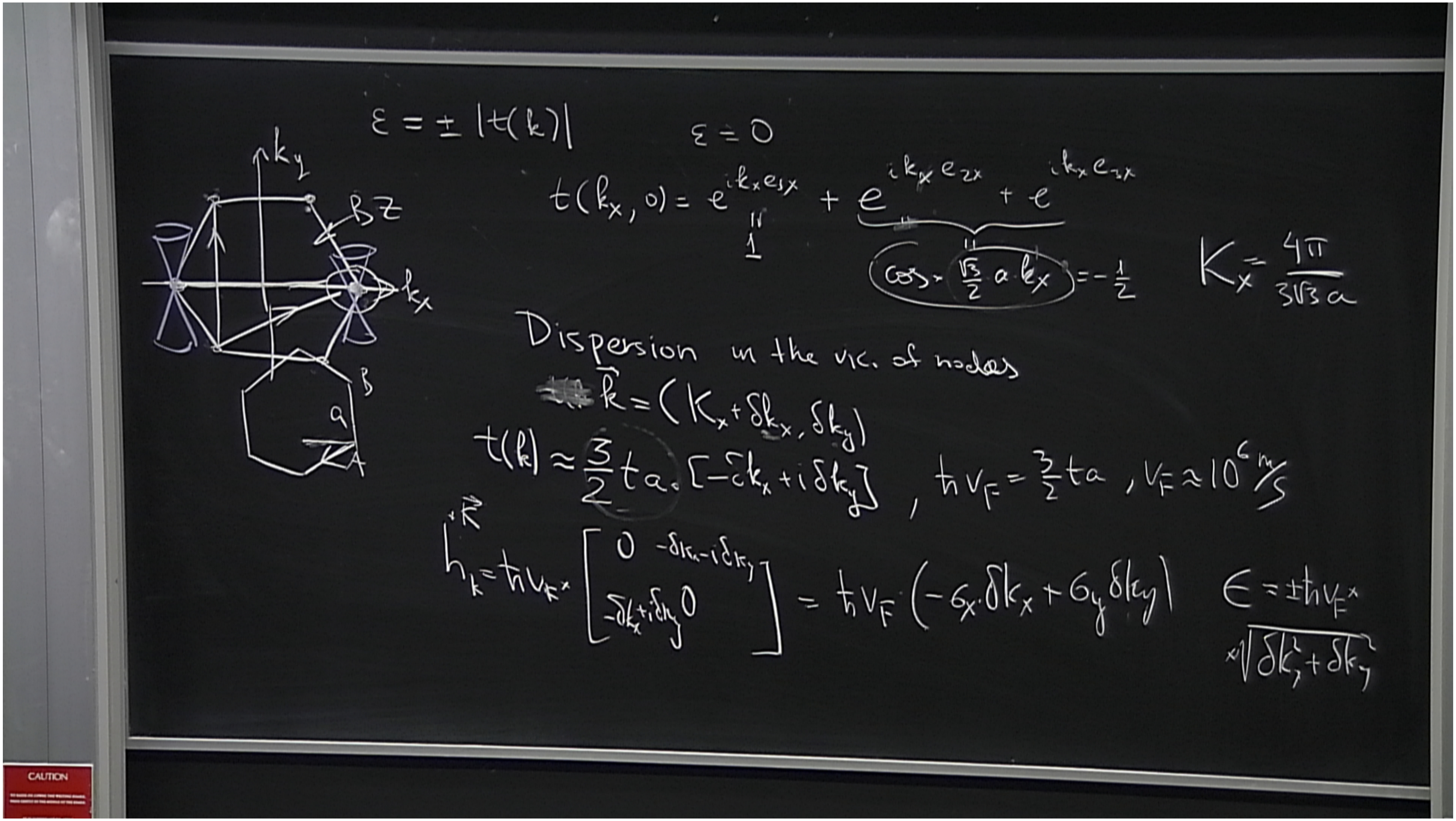
$$K_x = \frac{4\pi}{3\sqrt{3}a}$$

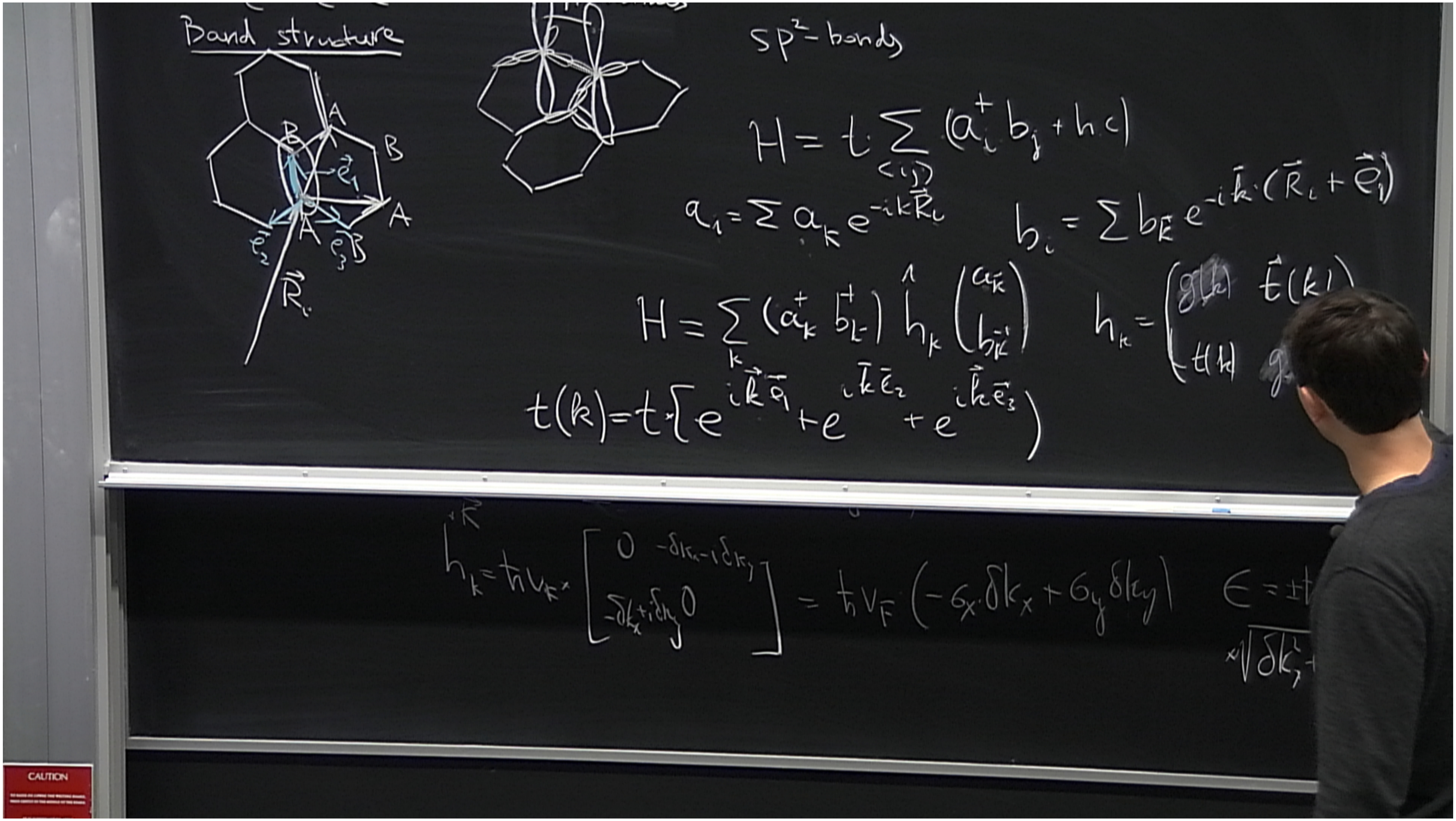
Dispersion in the vic. of nodes

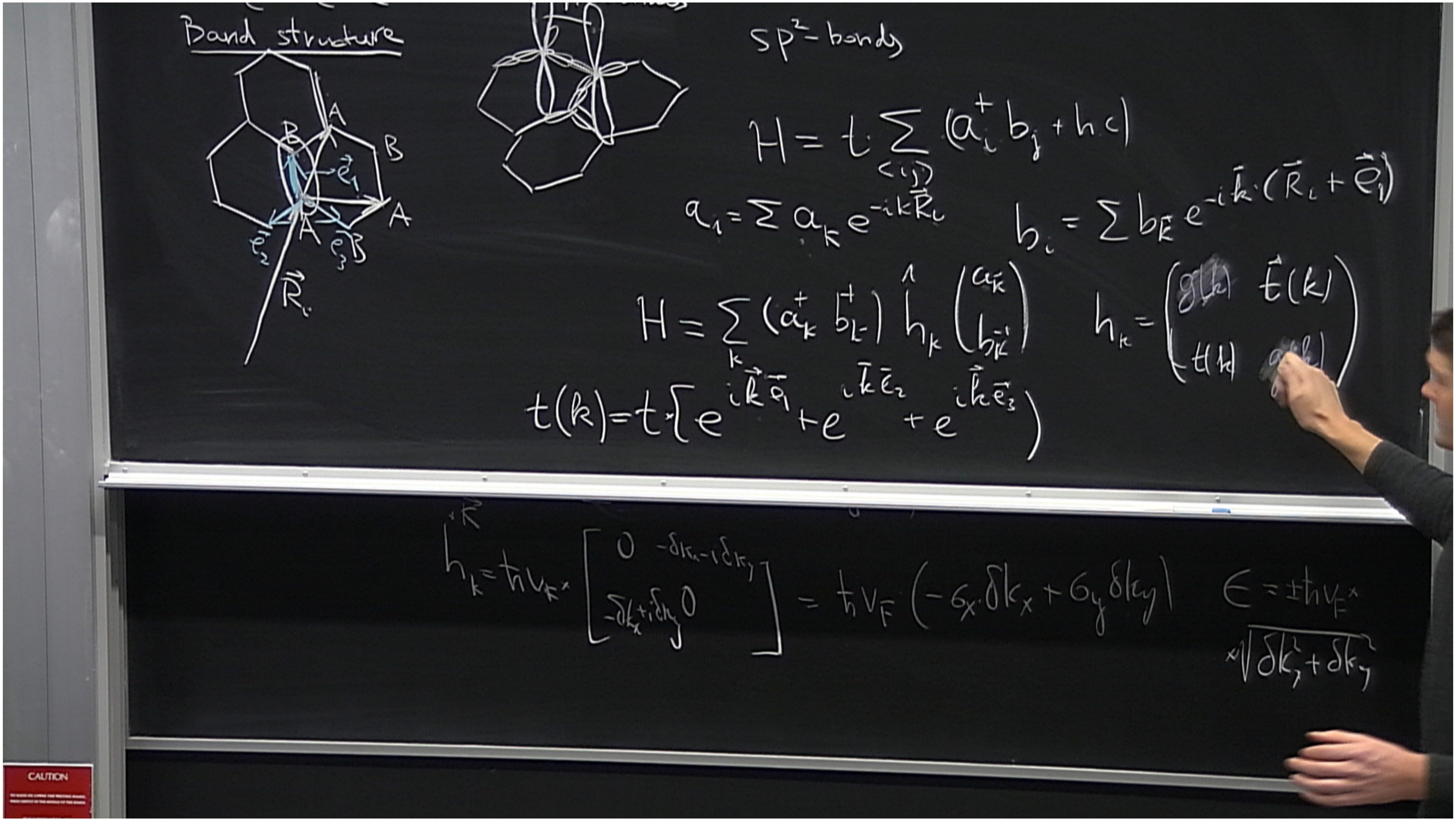
$$\vec{k} = (K_x + \delta k_x, \delta k_y)$$

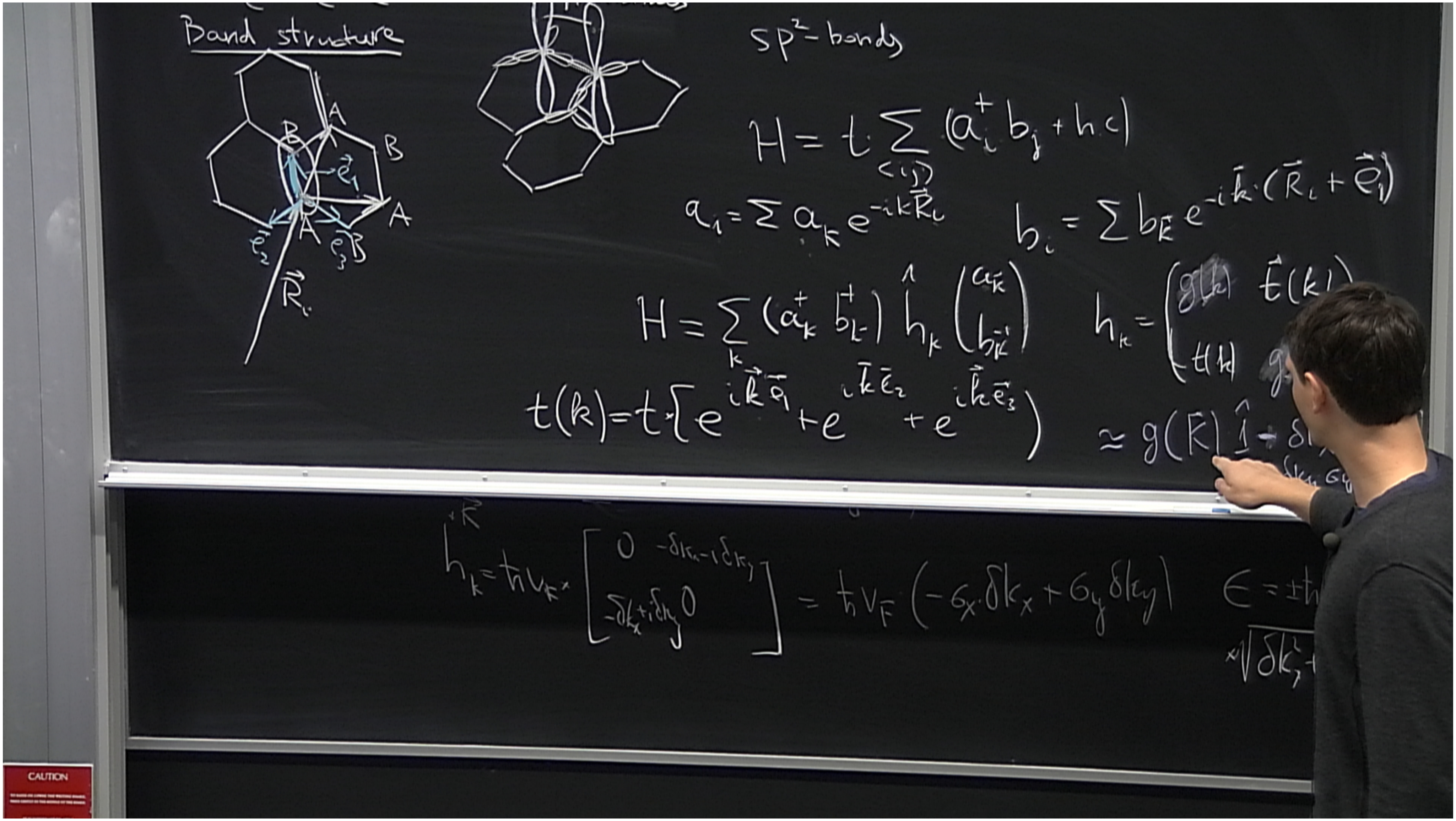
$$t(k) \approx \frac{3}{2} t a \cdot [-\delta k_x + i\delta k_y], \quad \hbar v_F = \frac{3}{2} t a, \quad v_F \approx 10^6 \, m/s$$

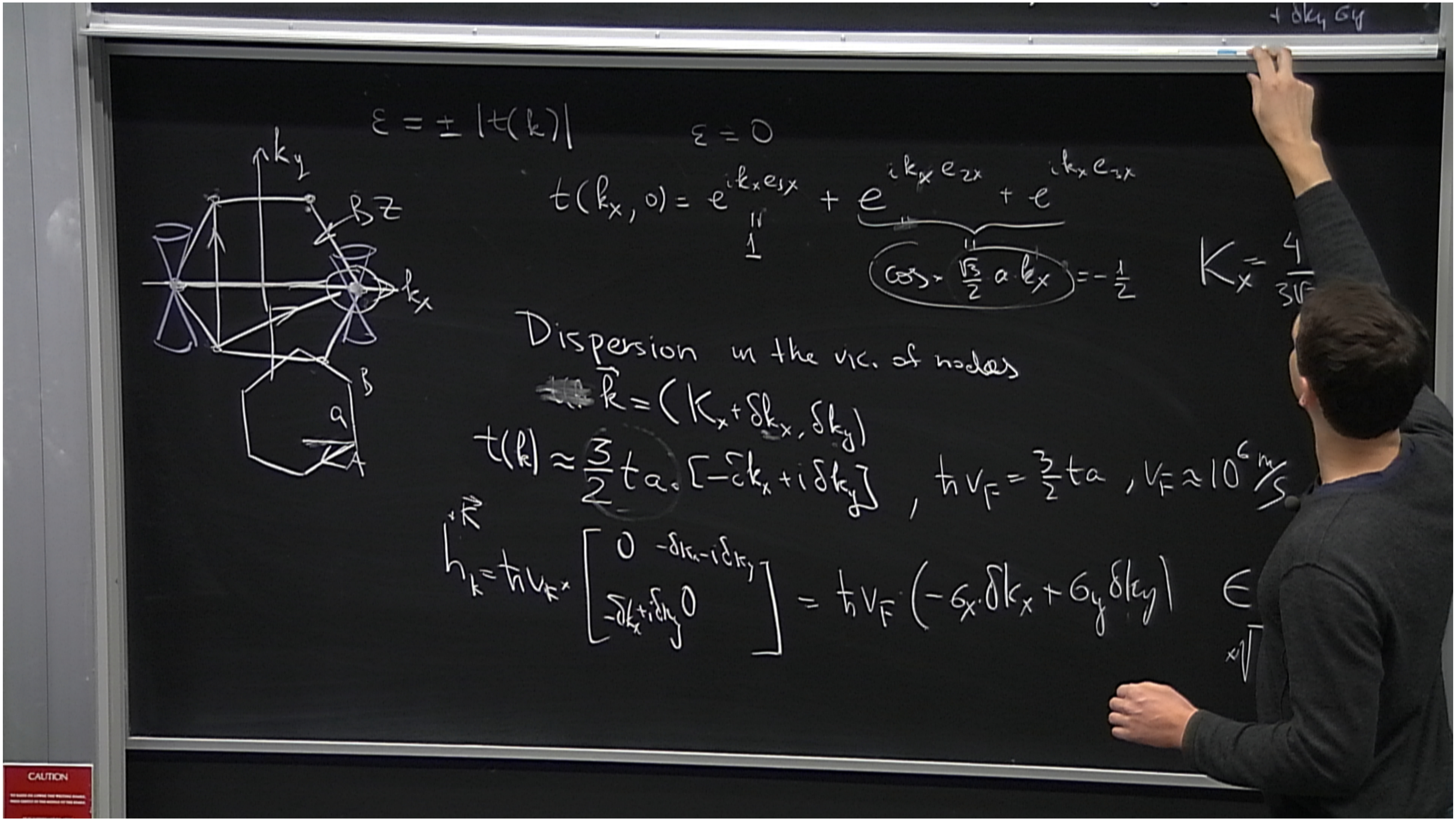
$$h_k = \hbar v_F \cdot \begin{bmatrix} 0 & -\delta k_x - i\delta k_y \\ -\delta k_x + i\delta k_y & 0 \end{bmatrix} = \hbar v_F \cdot (-\sigma_x \cdot \delta k_x + \sigma_y \delta k_y) \qquad \epsilon = \pm \hbar v_F \cdot \sqrt{\delta k_x^2 + \delta k_y^2}$$

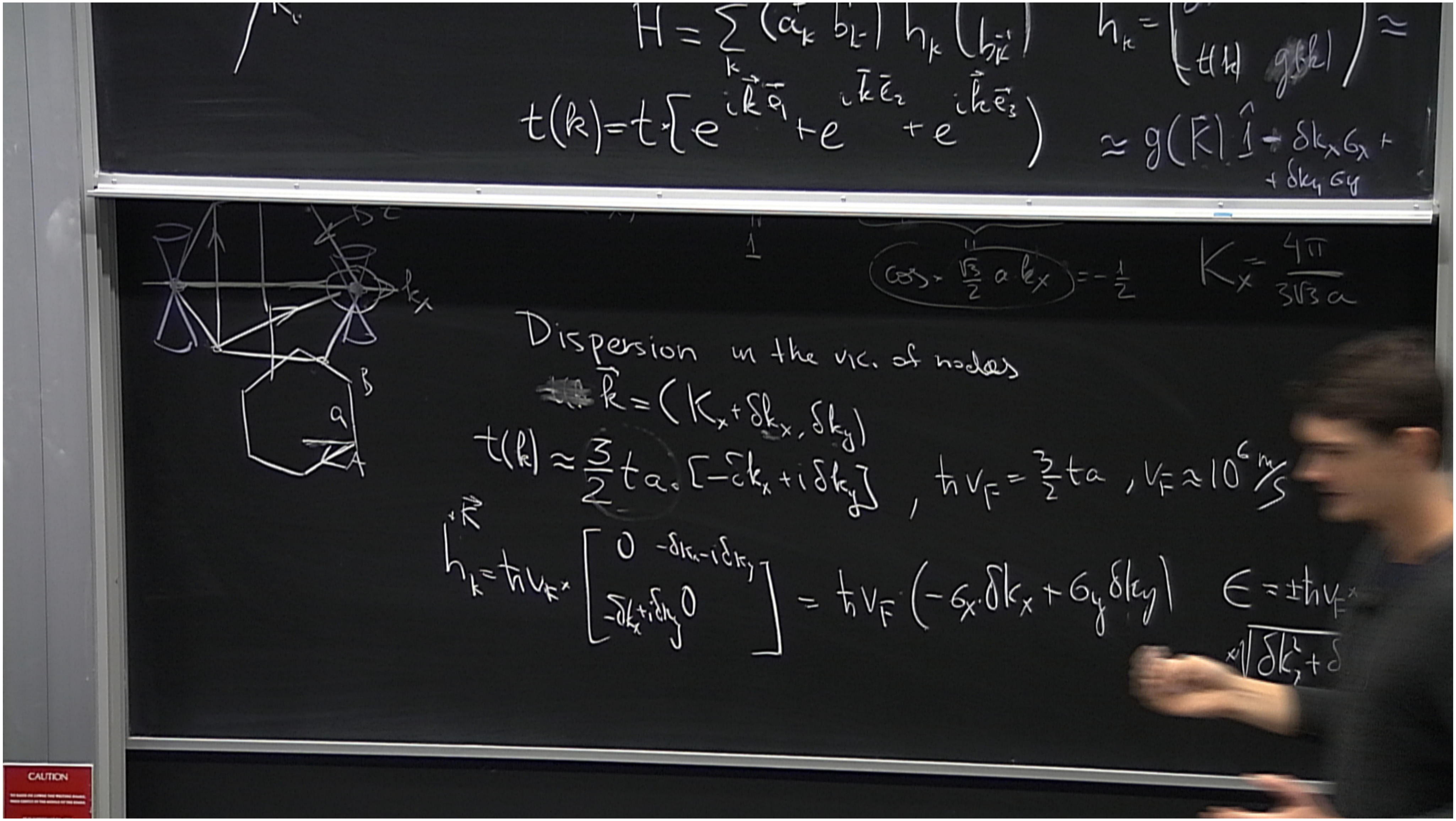

ε = ± |t(k)|
ε = 0
t(kx, 0) = e^{i kx e1x} + e^{i kx e2x} + e^{i kx e3x}
cos √3/2 a kx = -1/2
Kx = 4π / 3√3 a
BZ
ky
kx
a
Dispersion in the vic. of nodes
k = (Kx + δkx, δky)
t(k) ≈ 3/2 ta [-δkx + i δky] , ħvF = 3/2 ta , vF ≈ 10^6 m/s
h_k = ħvF [0 -δkx-iδky ; -δkx+iδky 0] = ħvF (-σx δkx + σy δky)
ε = ±ħvF √(δkx² + δky²)
CAUTION

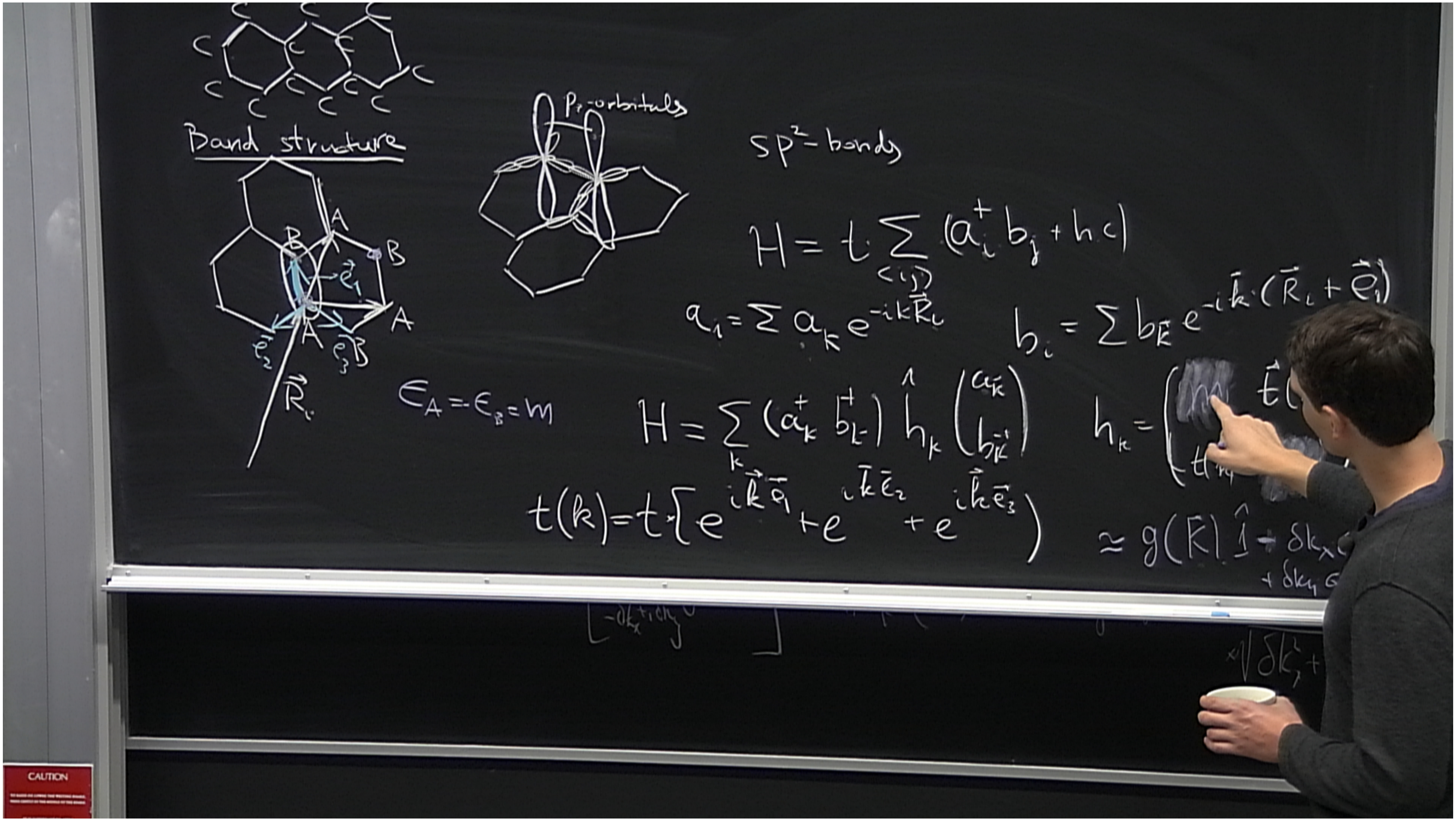

Band structure
sp²-bonds
H = t Σ_⟨i,j⟩ (a_i^+ b_j + h.c)
a_i = Σ a_k e^{-ikR_i}
b_i = Σ b_k e^{-ik·(R_i + e_1)}
H = Σ_k (a_k^+ b_k^+) h_k (a_k b_k)
h_k = (g(k) t(k); t(k) g)
t(k) = t·[e^{ik·e_1} + e^{ik·e_2} + e^{ik·e_3})
h_k = ħv_F [0, -δk_x - iδk_y; -δk_x + iδk_y, 0] = ħv_F (-σ_x δk_x + σ_y δk_y)

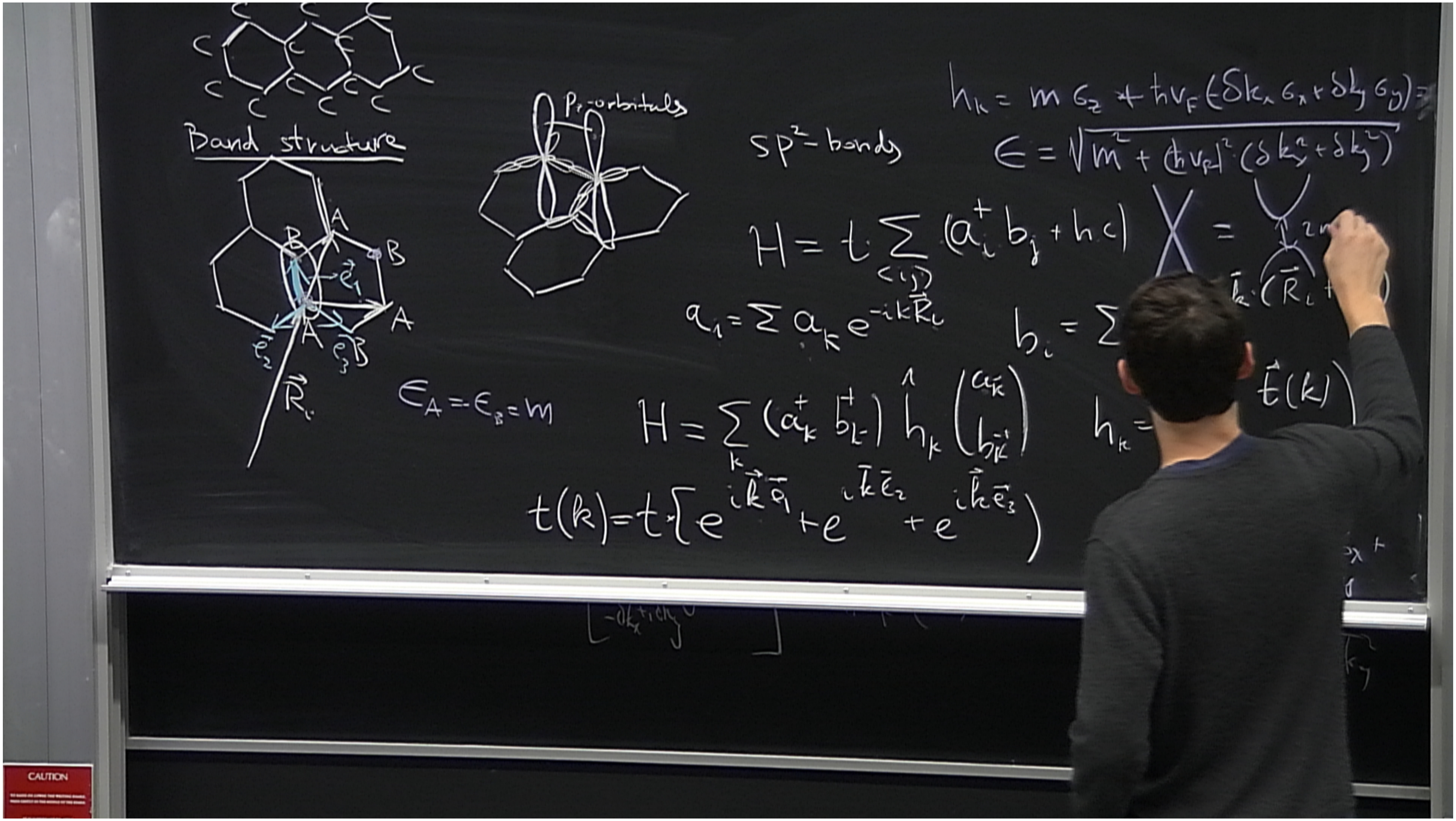

Band structure
A
B
B
A
A
B
sp²-bonds
H = t Σ_⟨ij⟩ (a_i^+ b_j + h.c)
a_i = Σ a_k e^{-ikR_i}
b_i = Σ b_k e^{-ik·(R_i + e_1)}
H = Σ_k (a_k^+ b_k^+) ĥ_k (a_k b_k)
t(k) = t·[e^{ik·e_1} + e^{ik·e_2} + e^{ik·e_3}]
ϵ = ±ħv_F × √(δk_x² + δk_y²)

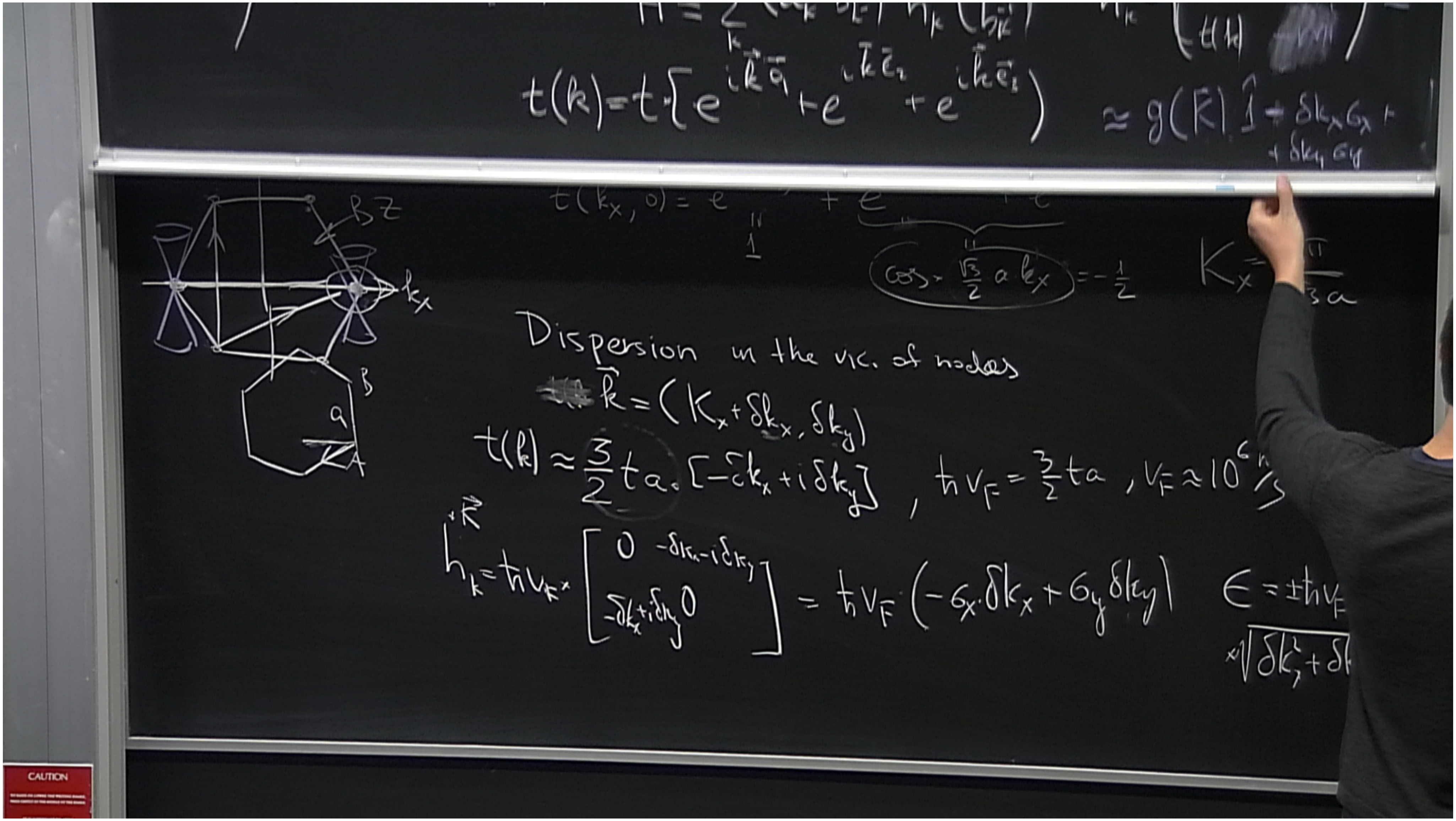

Band structure
sp² - bonds
H = t Σ (a†_i b_j + h.c)
a_i = Σ a_k e^{-ikR_i}
b_i = Σ b_k e^{-ik·(R_i + e_1)}
H = Σ_k (a†_k b†_k) ĥ_k (a_k b_k)
t(k) = t·(e^{ik·e_1} + e^{ik·e_2} + e^{ik·e_3})
ĥ_k = ħv_F [0, -δk_x - iδk_y; -δk_x + iδk_y, 0] = ħv_F (-σ_x δk_x + σ_y δk_y)
CAUTION

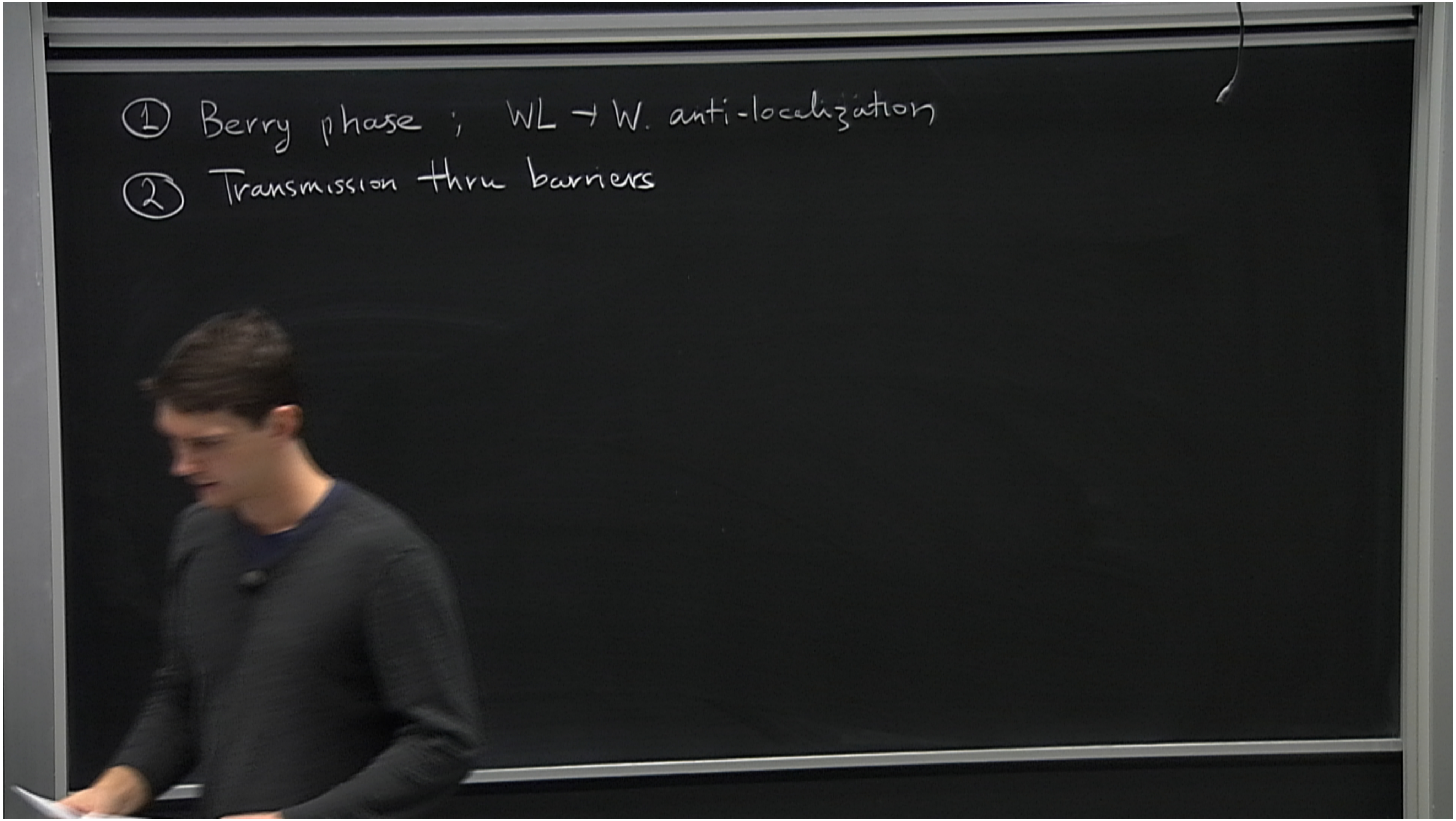

ε = ± |t(k)|
ε = 0
k_y
BZ
k_x
t(k_x, 0) = e^{ik_x e_{1x}} + e^{ik_x e_{2x}} + e^{ik_x e_{3x}}
1
cos √3/2 a·k_x = −1/2
K_x = 4/3√
Dispersion in the vic. of nodes
k = (K_x + δk_x, δk_y)
t(k) ≈ 3/2 ta [−δk_x + iδk_y] , ħv_F = 3/2 ta , v_F ≈ 10^6 m/s
h_k = ħv_F [0 −δk_x−iδk_y ; −δk_x+iδk_y 0] = ħv_F (−σ_x δk_x + σ_y δk_y)
CAUTION

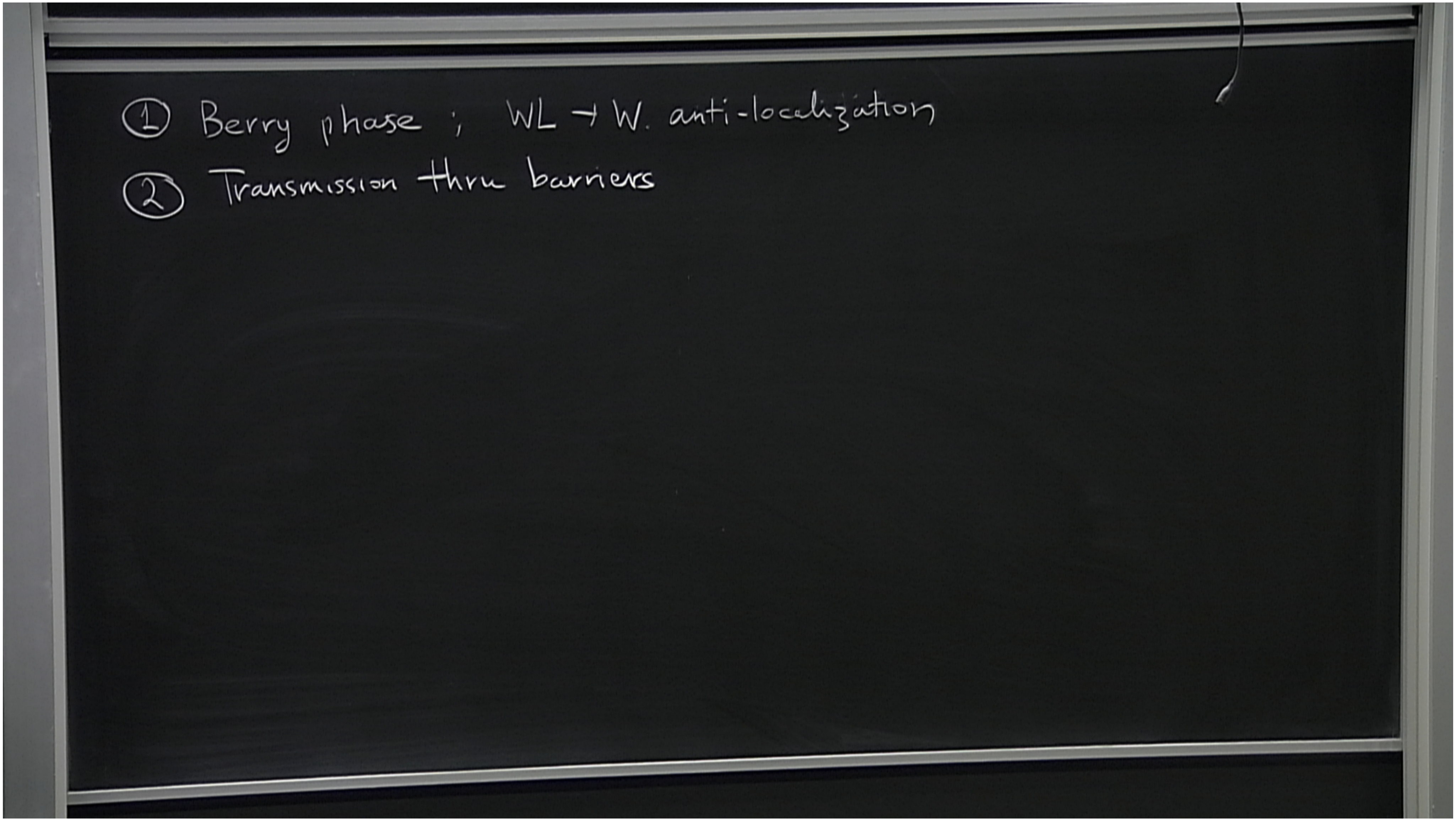

Dispersion in the vic. of nodes
$\vec{k} = (K_x + \delta k_x, \delta k_y)$
$t(k) \approx \frac{3}{2} t a \cdot [-i\delta k_x + i\delta k_y]$, $\hbar v_F = \frac{3}{2} t a$, $v_F \approx 10^6 m/s$
$K_x = \frac{4\pi}{3\sqrt{3} a}$
$t(k) = t \cdot [e^{i\vec{k}\vec{a}_1} + e^{i\vec{k}\vec{e}_2} + e^{i\vec{k}\vec{e}_3})$
$\approx g(\vec{k}) \hat{1} + \delta k_x \sigma_x + \delta k_y \sigma_y$
$= \hbar v_F (-\sigma_x \cdot \delta k_x + \sigma_y \delta k_y)$
k_x

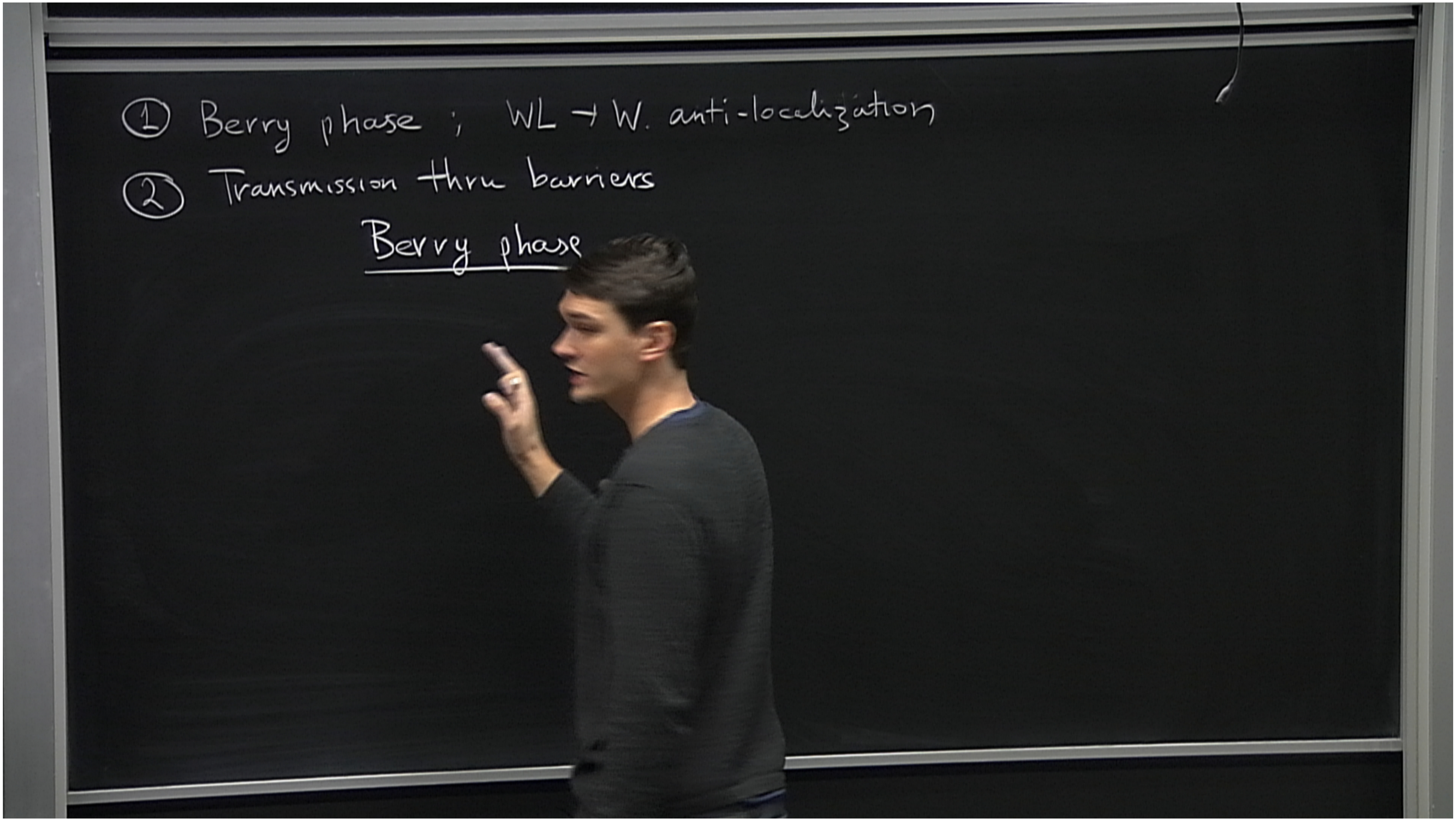

Band structure
sp²-bonds
CAUTION

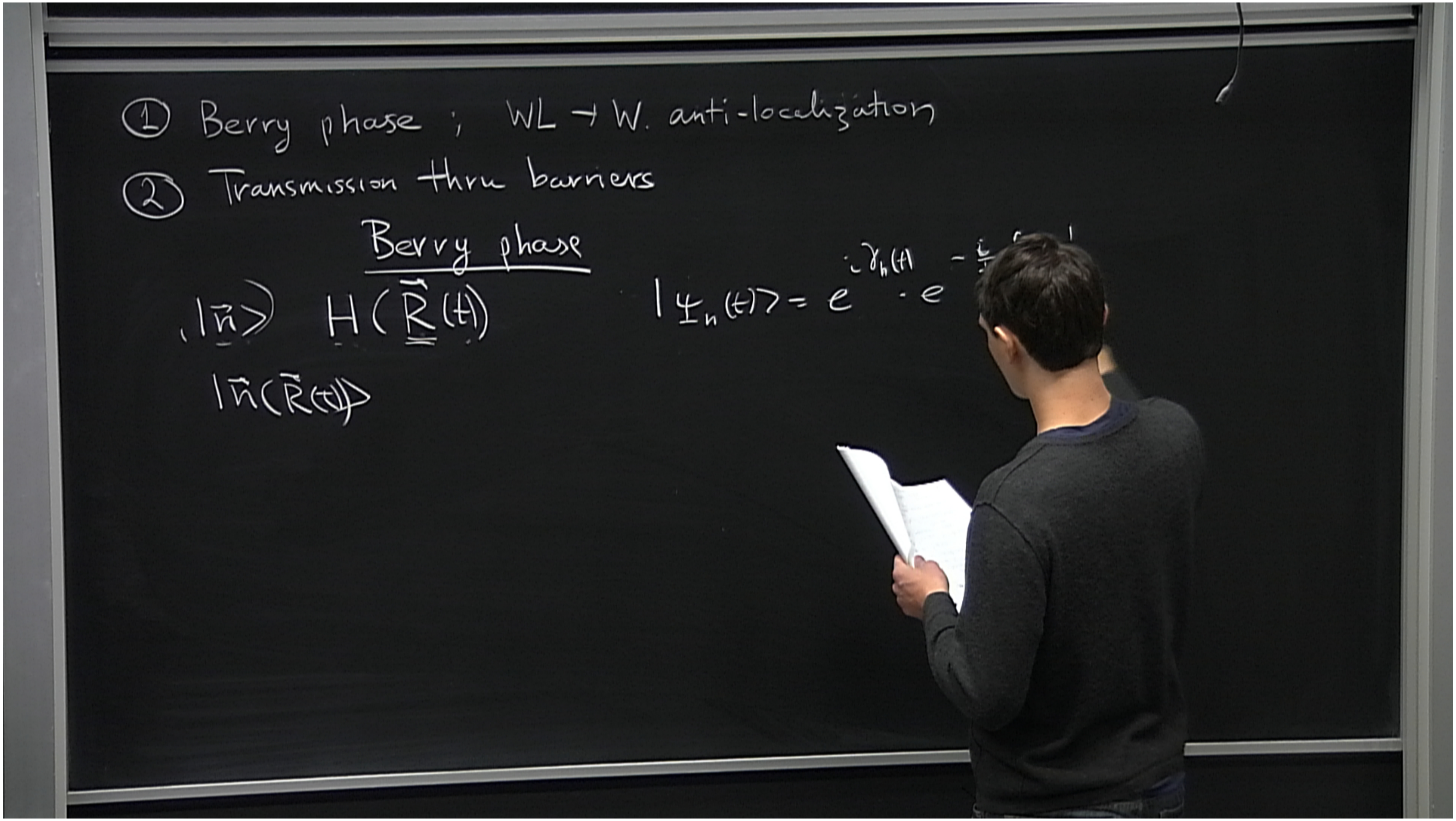

Band structure
sp²-bonds
CAUTION

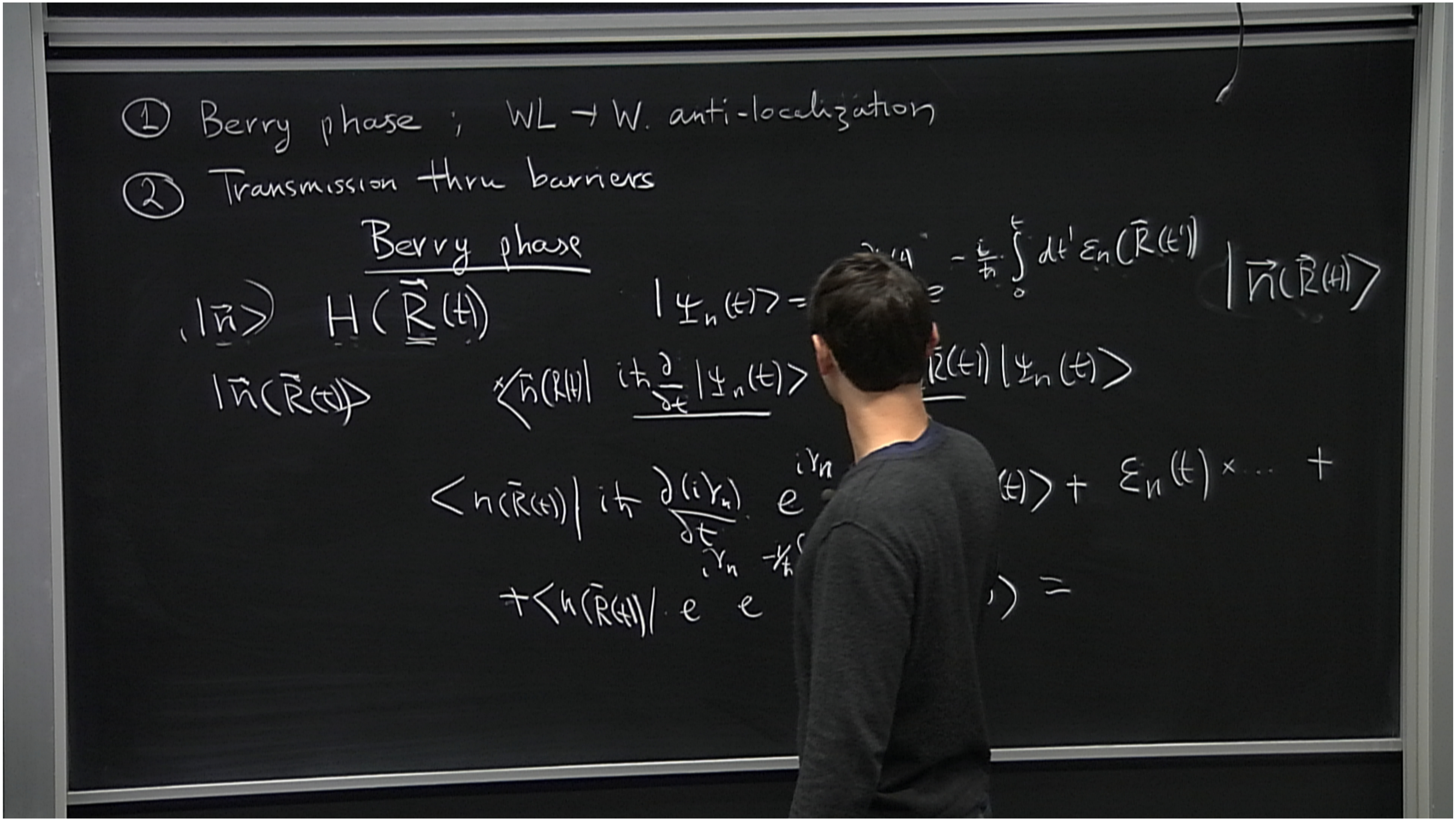

BZ
Dispersion in the vic. of nodes
CAUTION


① Berry phase ; WL → W. anti-localization
② Transmission thru barriers


① Berry phase ; WL → W. anti-localization
② Transmission thru barriers


① Berry phase ; WL → W. anti-localization
② Transmission thru barriers
Berry phase


① Berry phase ; WL → W. anti-localization
② Transmission thru barriers
Berry phase
|n⟩ H(R(t))
|n(R(t))⟩
|Ψn(t)⟩ = e^{iγn(t)} · e


① Berry phase ; WL → W. anti-localization
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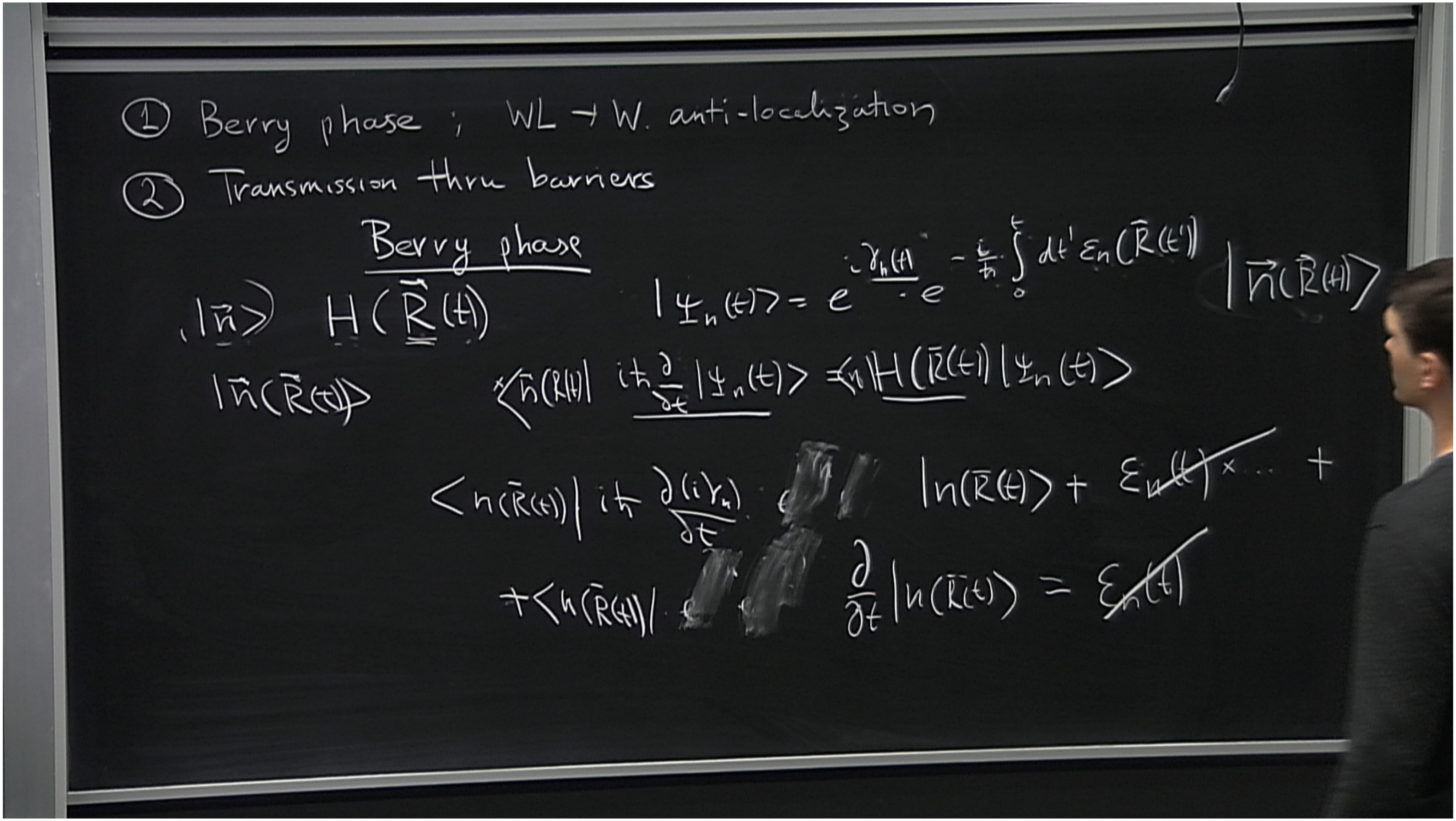
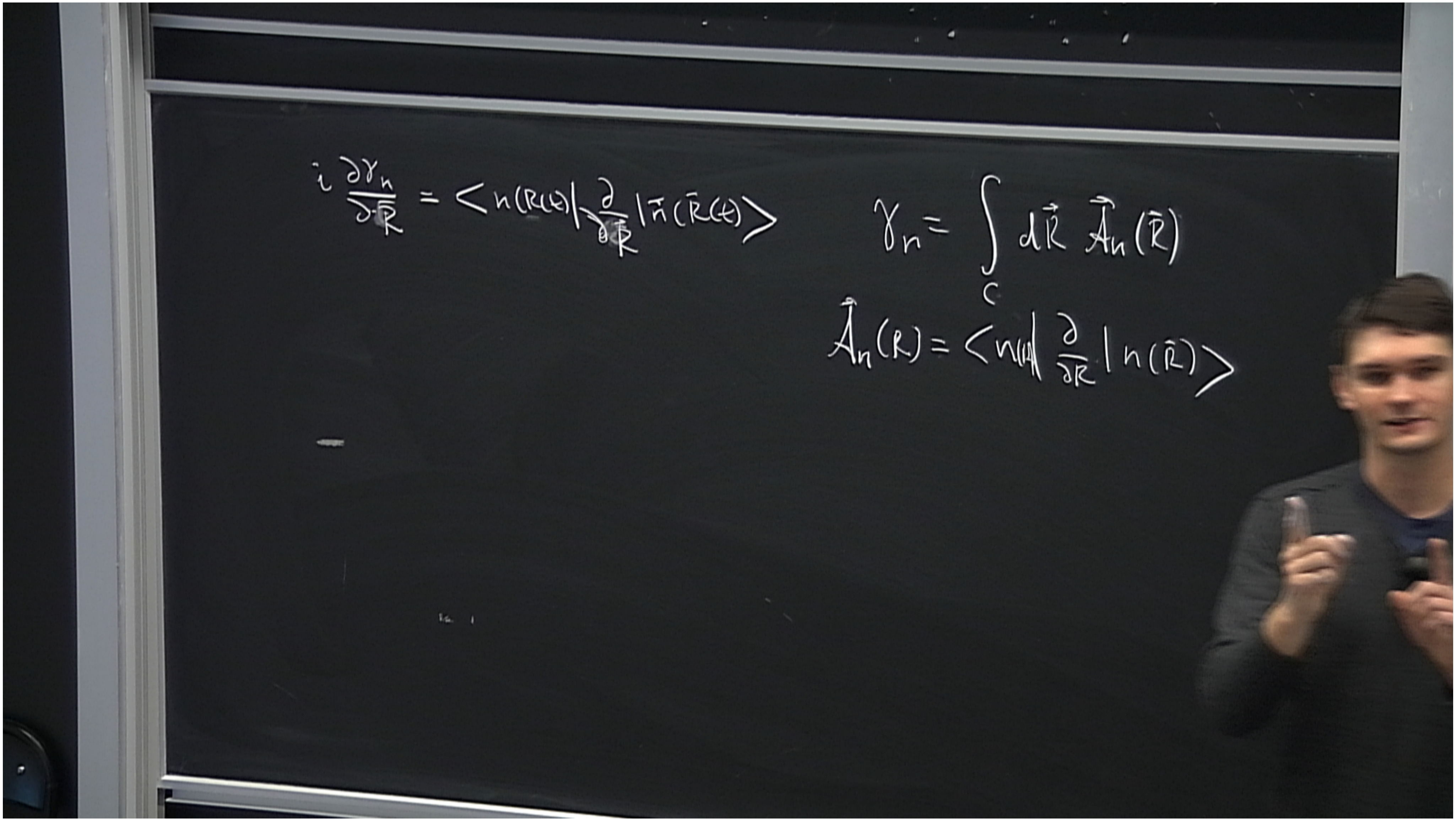
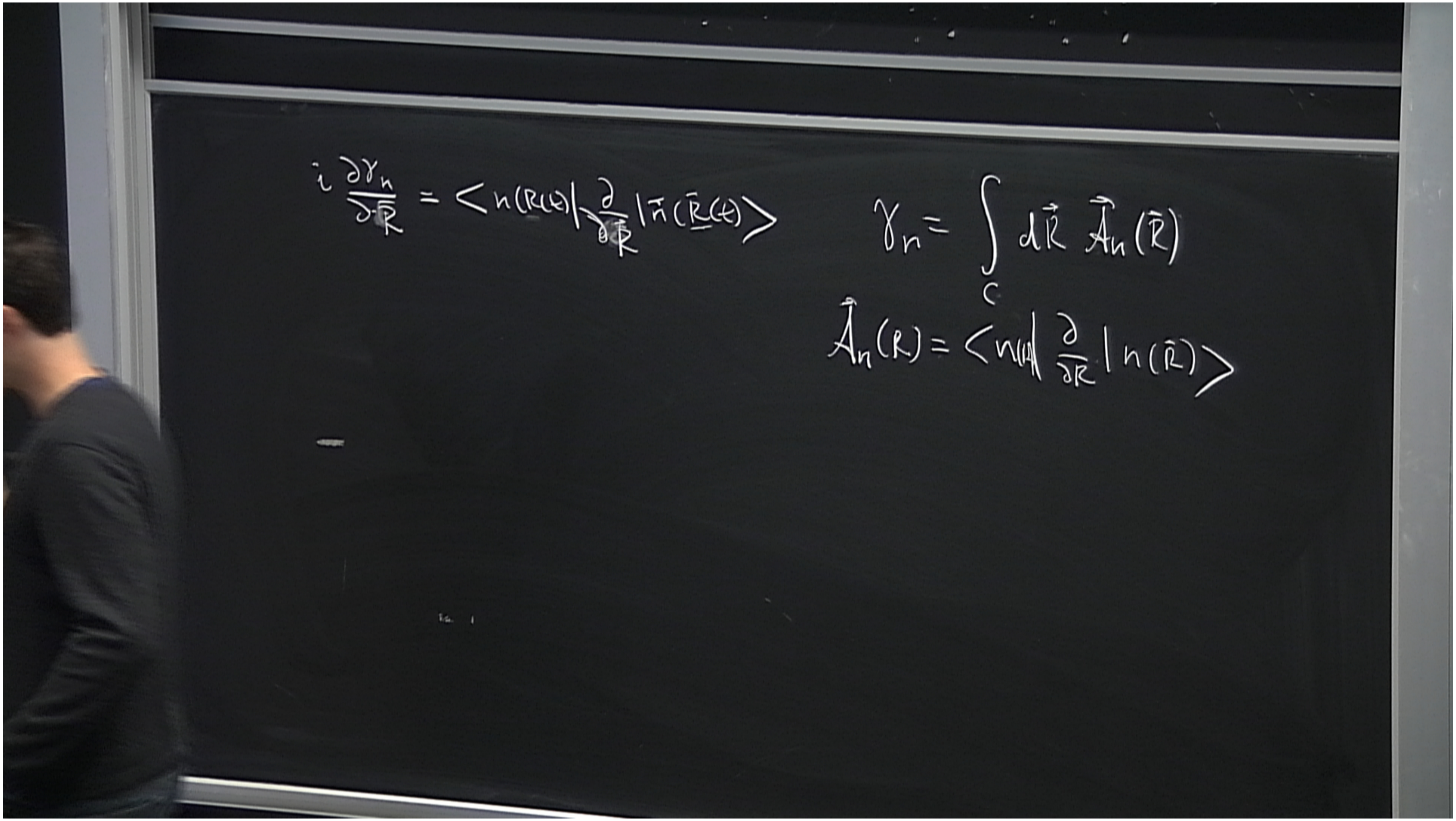
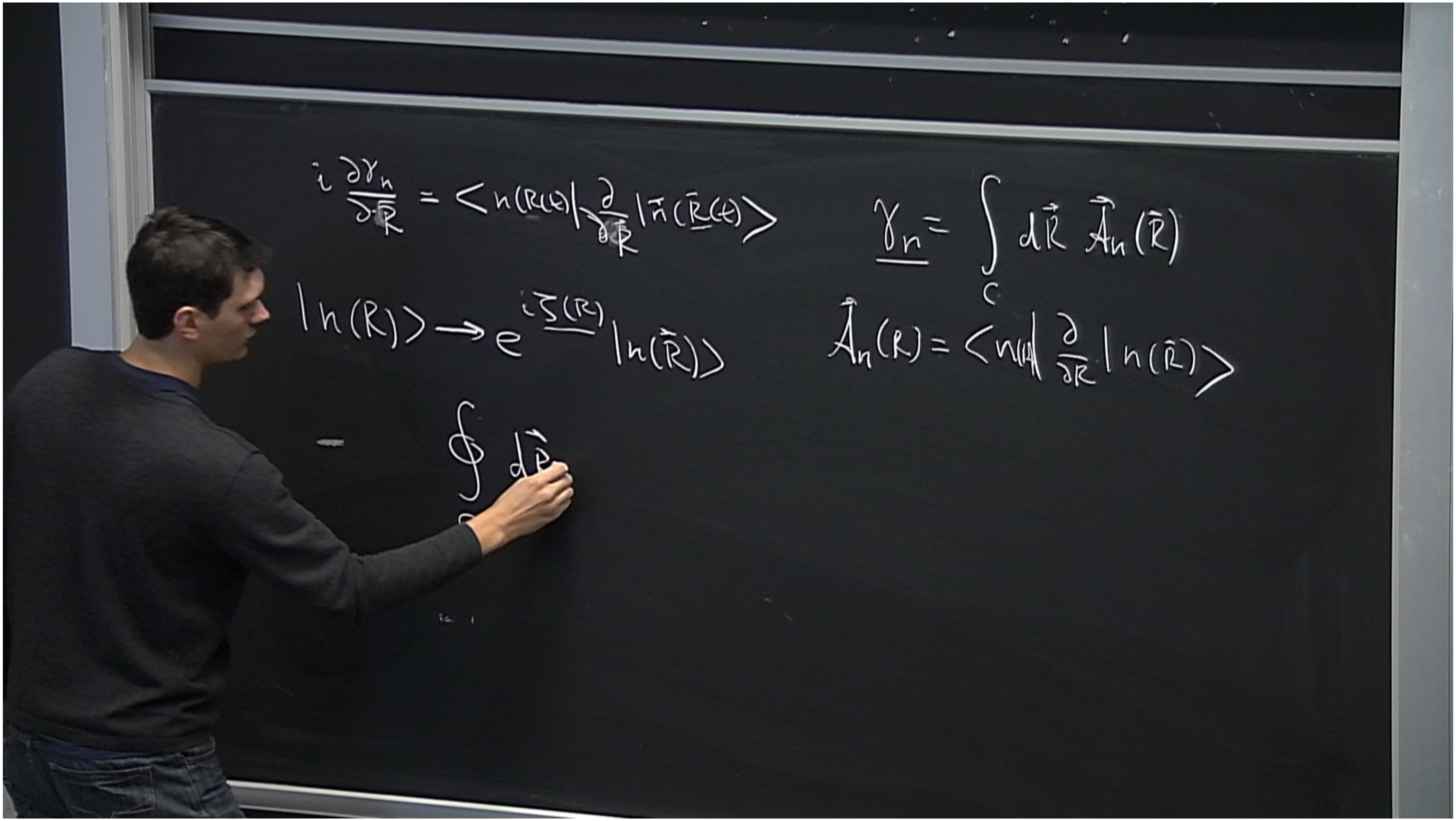
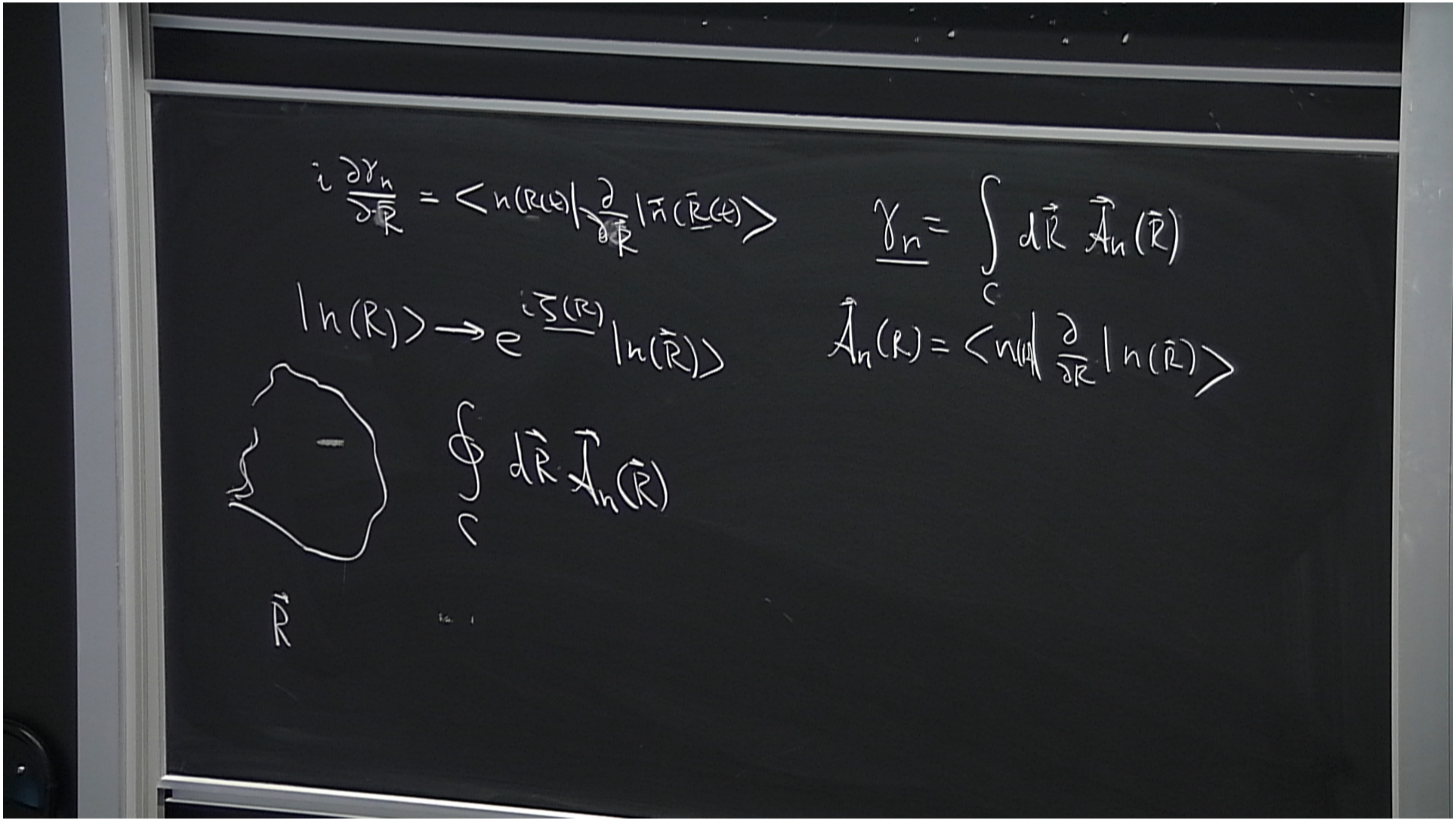
Berry phase
|n⟩ H(R(t))
|n(R(t))⟩
|Ψn(t)⟩ =
|n(R(t))⟩
⟨n(R(t))| iħ ∂/∂t |Ψn(t)⟩
|Ψn(t)⟩


① Berry phase ; WL → W. anti-localization
② Transmission thru barriers
Berry phase
|n⟩ H(R(t))
|n(R(t))⟩
|Ψn(t)⟩ = e^{iγn(t)} · e^{-i/ħ ∫0^t dt' εn(R(t'))} |n(R(t))⟩
⟨n(R(t))| iħ ∂/∂t |Ψn(t)⟩ = ⟨n| H(R(t)) |Ψn(t)⟩
⟨n(R(t))| iħ ∂(iγn)/∂t |n(R(t))⟩ + εn(t) × ... +
+⟨n(R(t))| ∂/∂t |n(R(t))⟩ = εn(t)

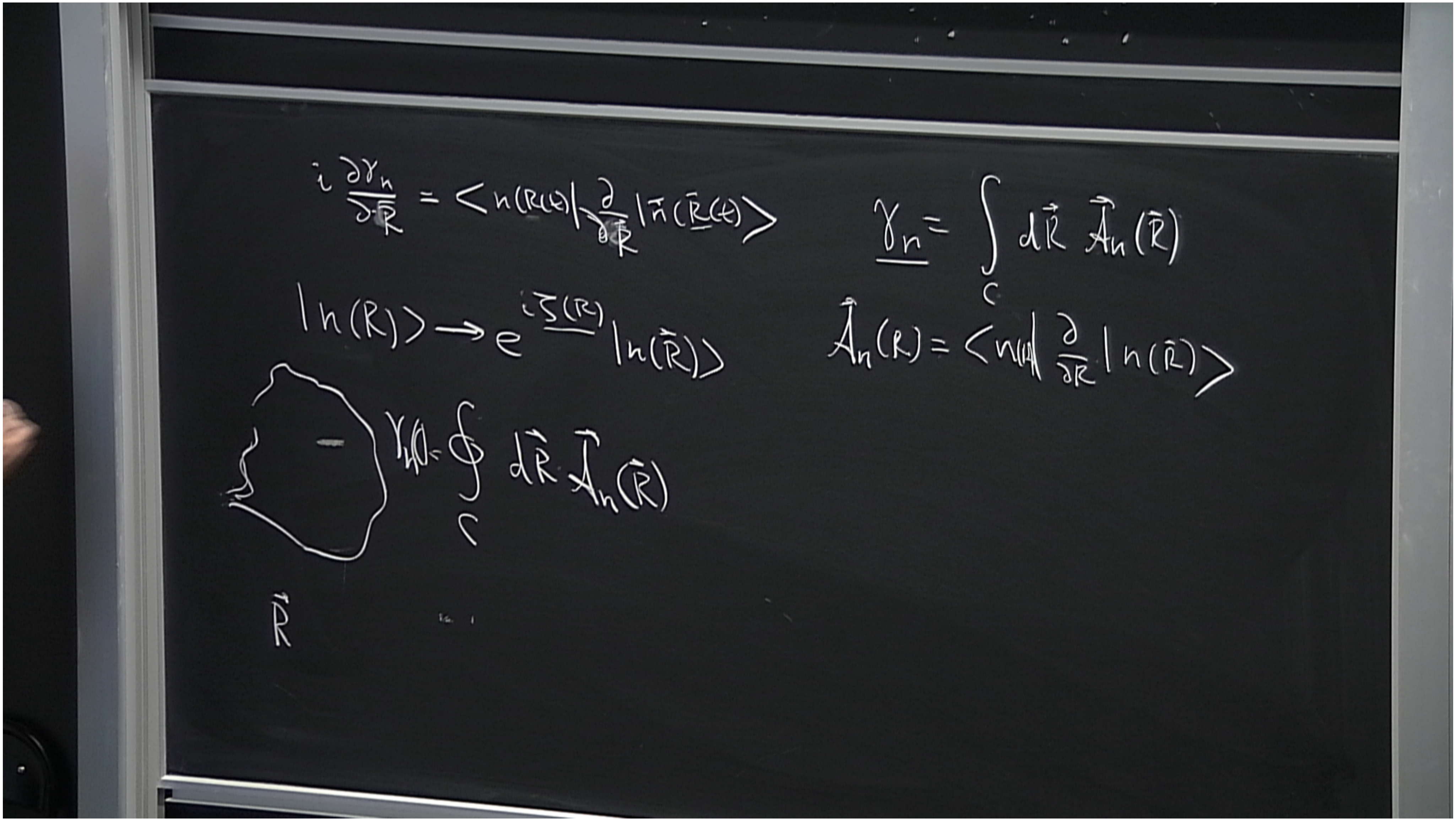

$i \frac{\partial \gamma_n}{\partial \vec{R}} = \langle n(R(t))| \frac{\partial}{\partial \vec{R}} |n(\vec{R}(t))\rangle$
$\gamma_n = \int_C d\vec{R}\, \vec{A}_n(\vec{R})$
$\vec{A}_n(R) = \langle n(R)| \frac{\partial}{\partial R} |n(\vec{R})\rangle$

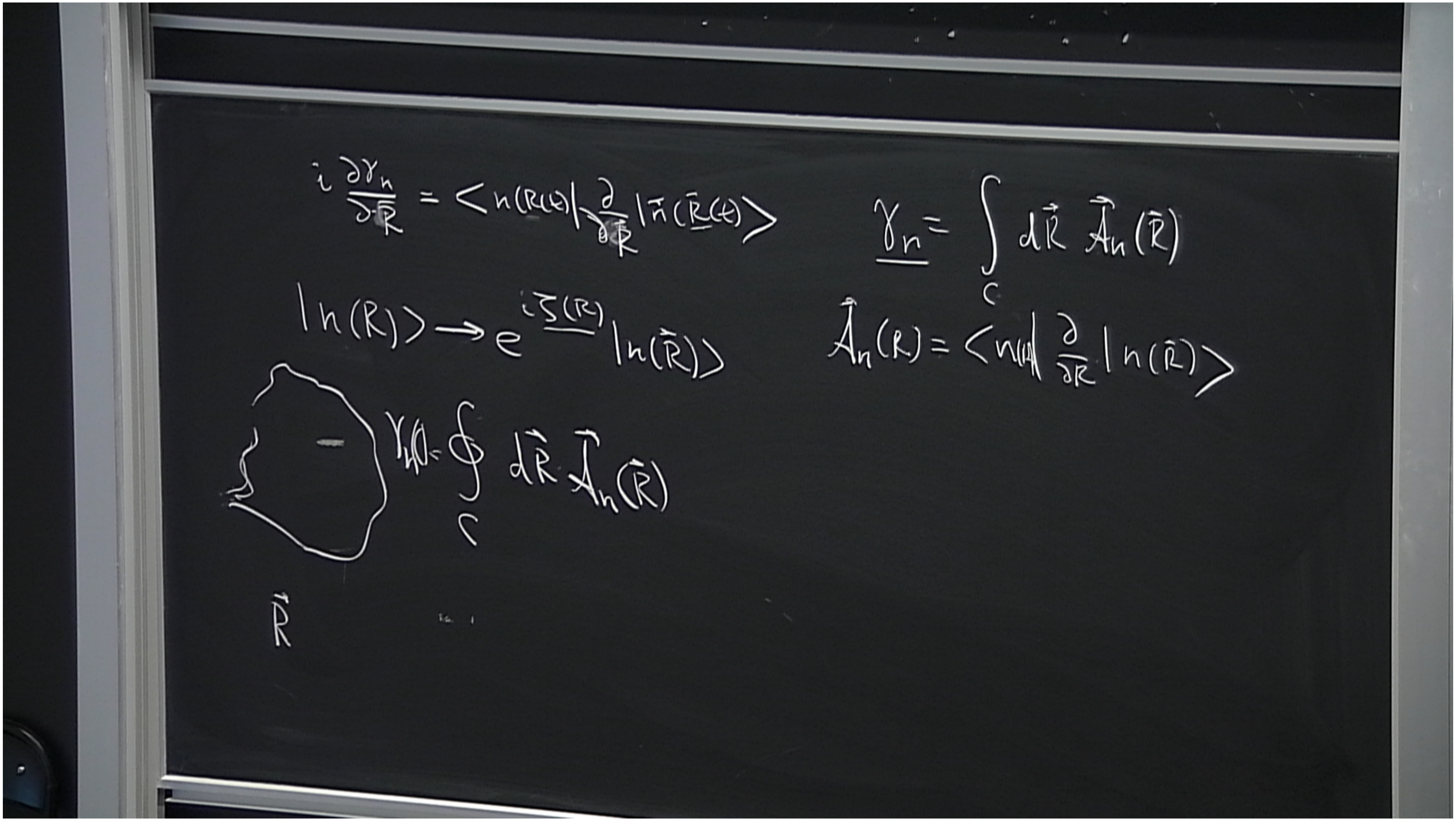

$i \frac{\partial \gamma_n}{\partial \vec{R}} = \langle n(R(t)) | \frac{\partial}{\partial \vec{R}} | n(\vec{R}(t)) \rangle$
$\gamma_n = \int_C d\vec{R}\, \vec{A}_n(\vec{R})$
$\vec{A}_n(R) = \langle n(R) | \frac{\partial}{\partial \vec{R}} | n(\vec{R}) \rangle$


$i \frac{\partial \gamma_n}{\partial \vec{R}} = \langle n(\vec{R}(t))| \frac{\partial}{\partial \vec{R}} |n(\vec{R}(t))\rangle$
$\gamma_n = \int_C d\vec{R}\, \vec{A}_n(\vec{R})$
$|n(R)\rangle \rightarrow e^{i\xi(R)} |n(\vec{R})\rangle$
$\vec{A}_n(R) = \langle n(R)| \frac{\partial}{\partial R} |n(\vec{R})\rangle$
$\oint d\vec{R}$

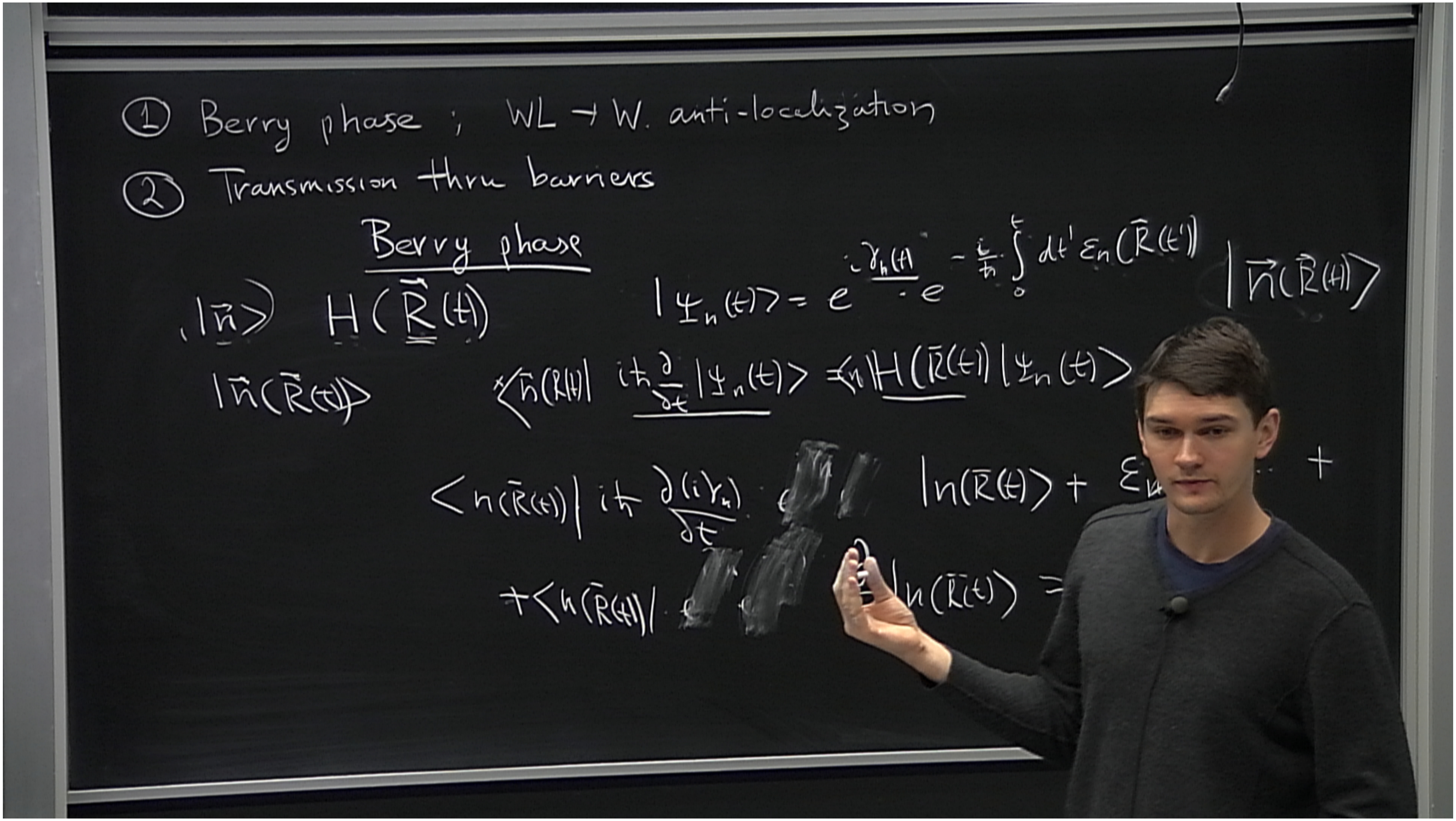


$i \frac{\partial \gamma_n}{\partial \vec{R}} = \langle n(R(t)) | \frac{\partial}{\partial \vec{R}} | n(\vec{R}(t)) \rangle$
$\gamma_n = \int_C d\vec{R}\, \vec{A}_n(\vec{R})$
$|n(R)\rangle \rightarrow e^{i\zeta(R)} |n(\vec{R})\rangle$
$\vec{A}_n(R) = \langle n(R) | \frac{\partial}{\partial R} | n(\vec{R}) \rangle$
$\oint_C d\vec{R}\, \vec{A}_n(\vec{R})$
$\vec{R}$


$i\frac{\partial \gamma_n}{\partial \vec{R}} = \langle n(R(t))| \frac{\partial}{\partial \vec{R}} |n(\vec{R}(t))\rangle$
$\gamma_n = \int_C d\vec{R}\, \vec{A}_n(\vec{R})$
$|n(R)\rangle \rightarrow e^{i\xi(R)} |n(\vec{R})\rangle$
$\vec{A}_n(R) = \langle n(R)| \frac{\partial}{\partial R} |n(\vec{R})\rangle$
$\gamma_n(C) = \oint_C d\vec{R}\, \vec{A}_n(\vec{R})$
$\vec{R}$



i ∂γn/∂R = <n(R(t))| ∂/∂R |n(R(t))>
γn = ∫C dR An(R)
|n(R)> → e^{iξ(R)} |n(R)>
An(R) = <n(R)| ∂/∂R |n(R)>
γn(C) = ∮C dR An(R)
R


① Berry phase ; WL → W. anti-localization
② Transmission thru barriers
Berry phase

$$i \frac{\partial \gamma_n}{\partial \vec{R}} = \langle n(R(t))| \frac{\partial}{\partial \vec{R}} |n(\vec{R}(t))\rangle$$

$$\gamma_n = \int_C d\vec{R} \, \vec{A}_n(\vec{R})$$

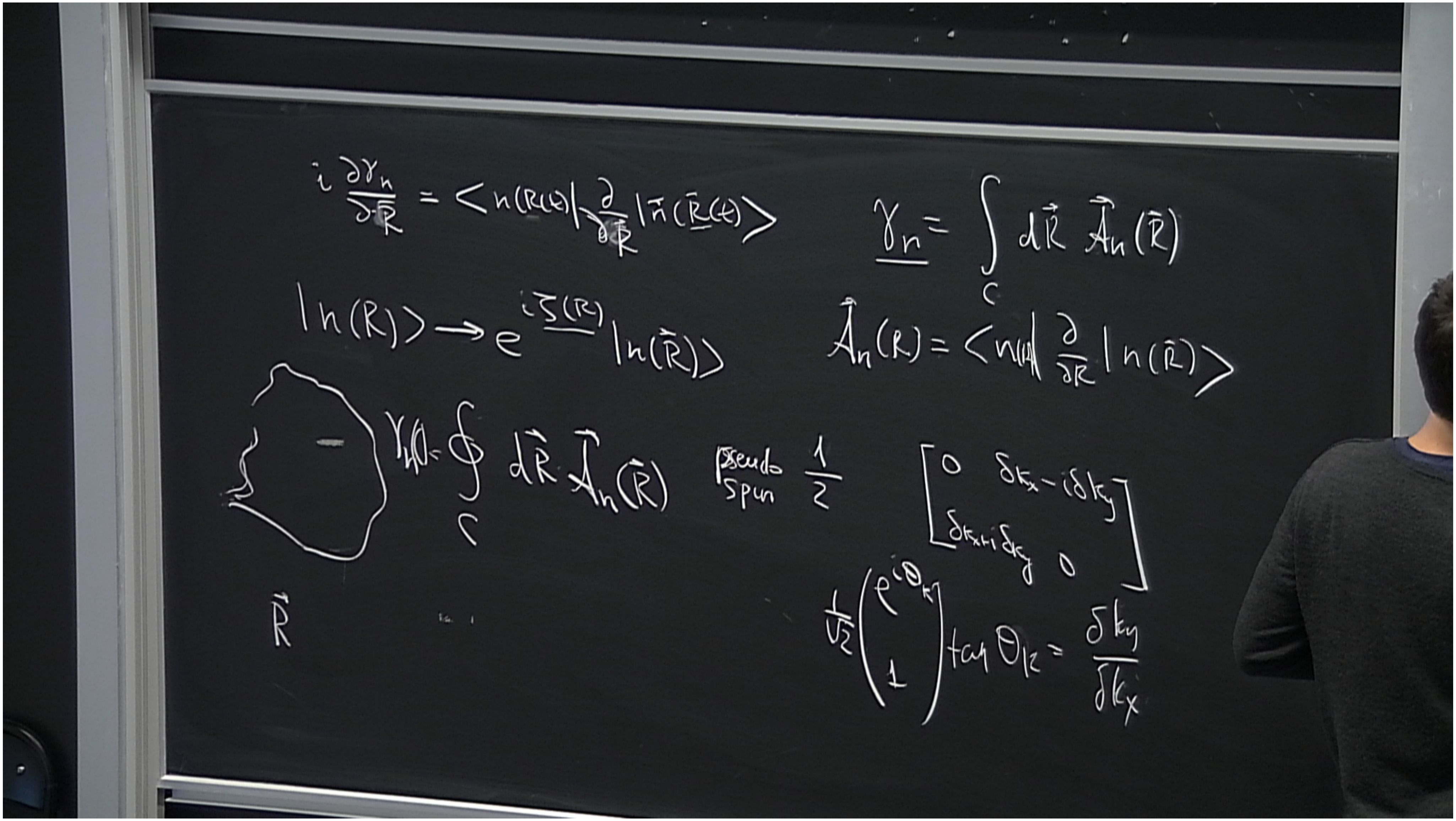
$$|n(R)\rangle \rightarrow e^{i\xi(R)} |n(\vec{R})\rangle$$

$$\vec{A}_n(R) = \langle n(R)| \frac{\partial}{\partial R} |n(\vec{R})\rangle$$

$$\gamma_n(C) = \oint_C d\vec{R} \, \vec{A}_n(\vec{R})$$

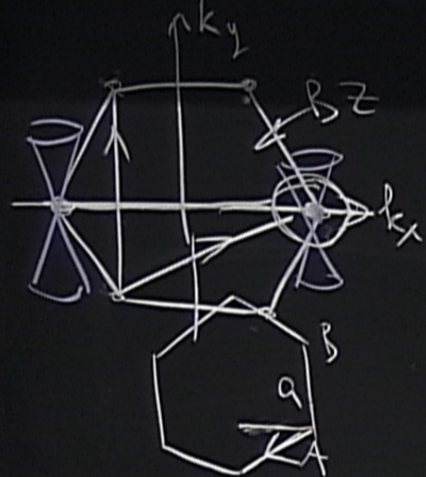
pseudo spin $\frac{1}{2}$

$\vec{R}$



$i \frac{\partial \gamma_n}{\partial \vec{R}} = \langle n(\vec{R}(t)) | \frac{\partial}{\partial \vec{R}} | n(\vec{R}(t)) \rangle$
$\gamma_n = \int_C d\vec{R}\, \vec{A}_n(\vec{R})$
$|n(\vec{R})\rangle \rightarrow e^{i\xi(\vec{R})} |n(\vec{R})\rangle$
$\vec{A}_n(\vec{R}) = \langle n(\vec{R}) | \frac{\partial}{\partial \vec{R}} | n(\vec{R}) \rangle$
$\gamma_n(C) = \oint_C d\vec{R}\, \vec{A}_n(\vec{R})$
pseudo spin $\frac{1}{2}$
$\begin{bmatrix} 0 & \delta k_x - i\delta k_y \\ \delta k_x + i\delta k_y & 0 \end{bmatrix}$
$\frac{1}{\sqrt{2}} \begin{pmatrix} e^{i\theta_k} \\ 1 \end{pmatrix}$ $\tan \theta_k = \frac{\delta k_y}{\delta k_x}$
$\vec{R}$

$$t(k) = t\cdot[e^{ik e_1} + e^{ik e_2} + e^{ik \vec{e}_3}) \approx g(\vec K)\,\hat{1} + \delta k_x \sigma_x + \delta k_y \sigma_y$$



$$t(k_x, 0) = \underbrace{e^{ik_x e_{1x}}}_{1} + \underbrace{e^{ik_x e_{2x}} + e^{ik_x e_{3x}}}_{\cos\frac{\sqrt{3}}{2} a\, k_x \,=\, -\frac{1}{2}}$$

$$K_x = \frac{4\pi}{3\sqrt{3}\,a}$$

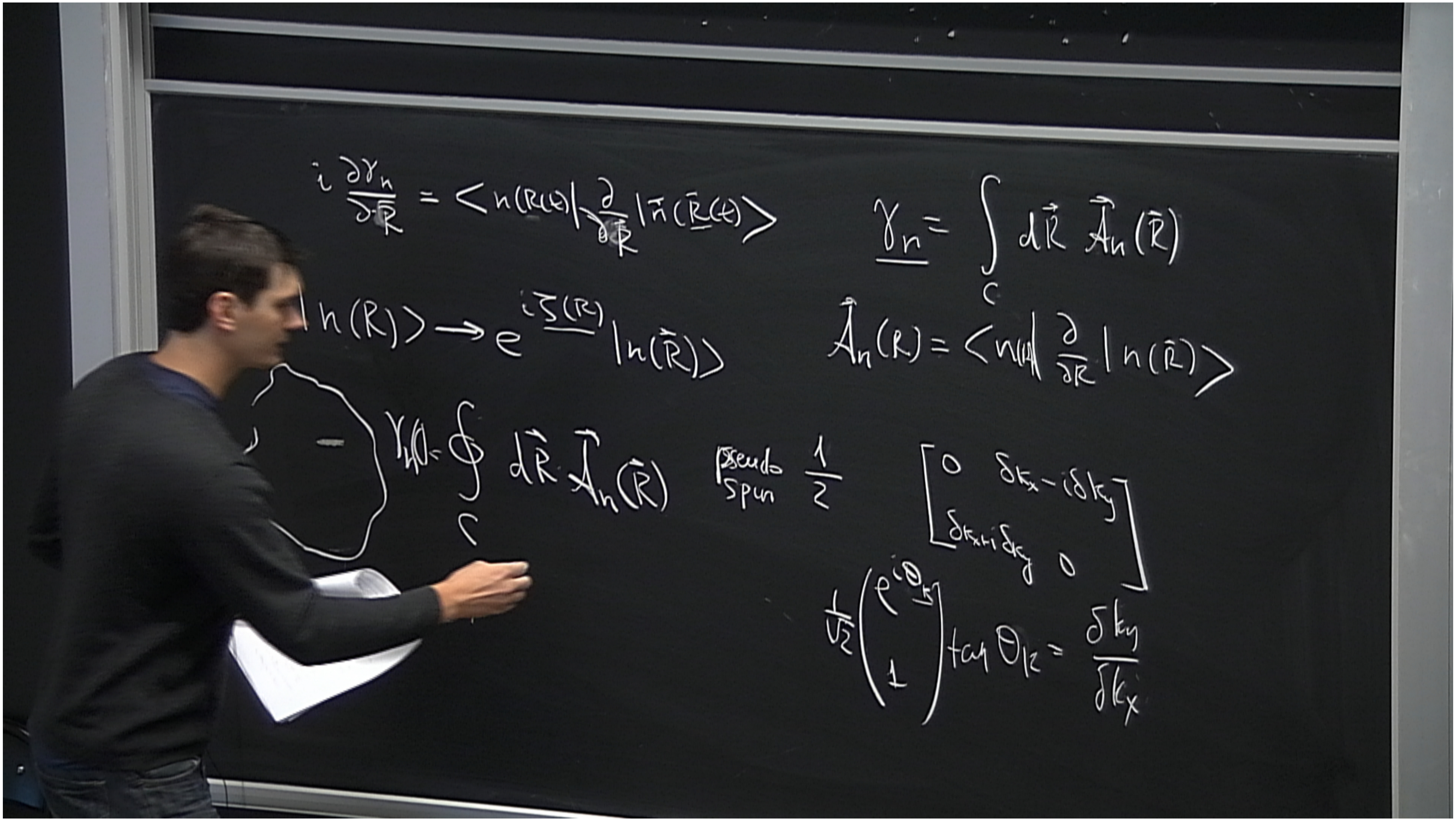
Dispersion in the vic. of nodes

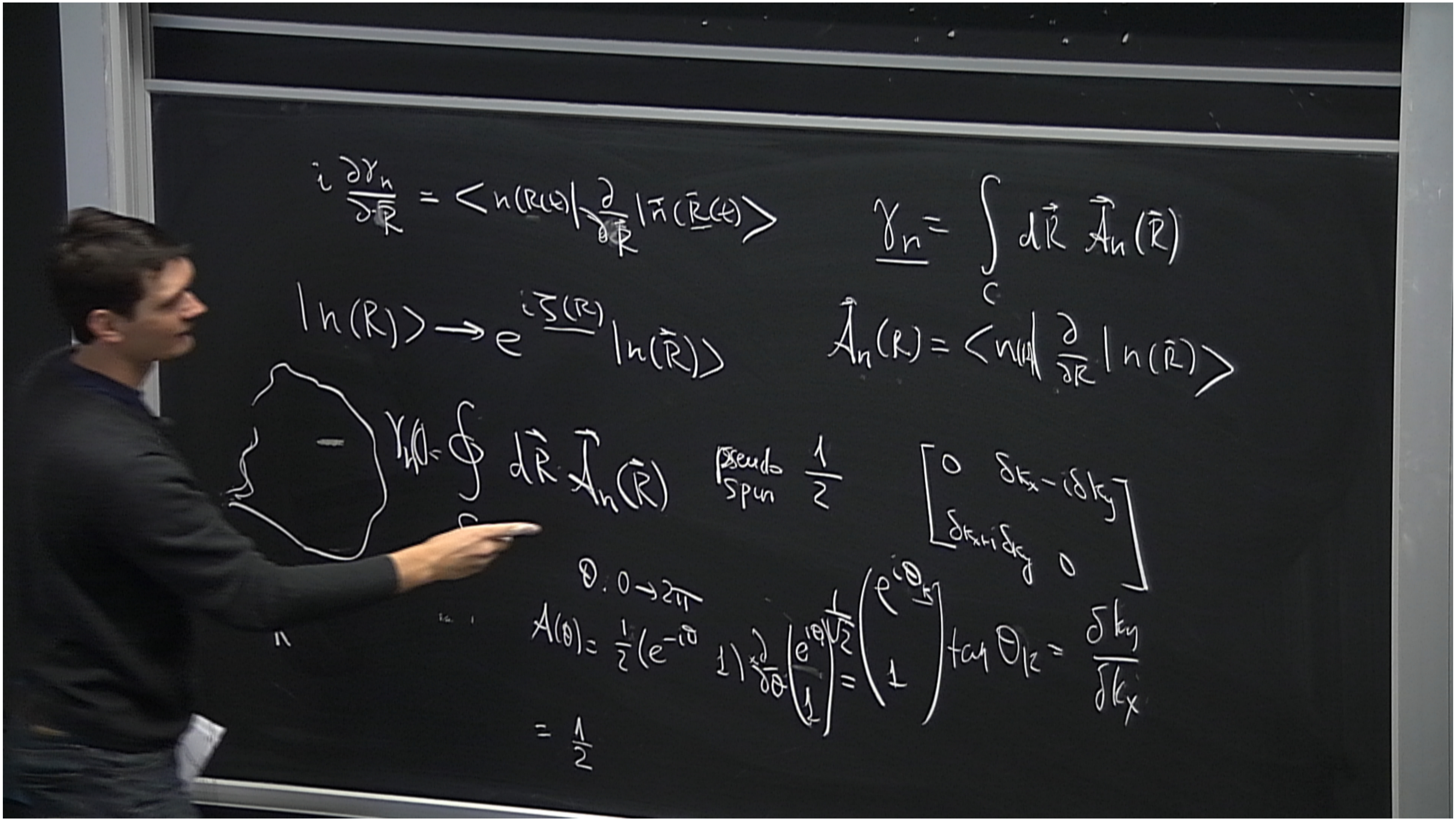
$$\vec k = (K_x + \delta k_x, \delta k_y)$$

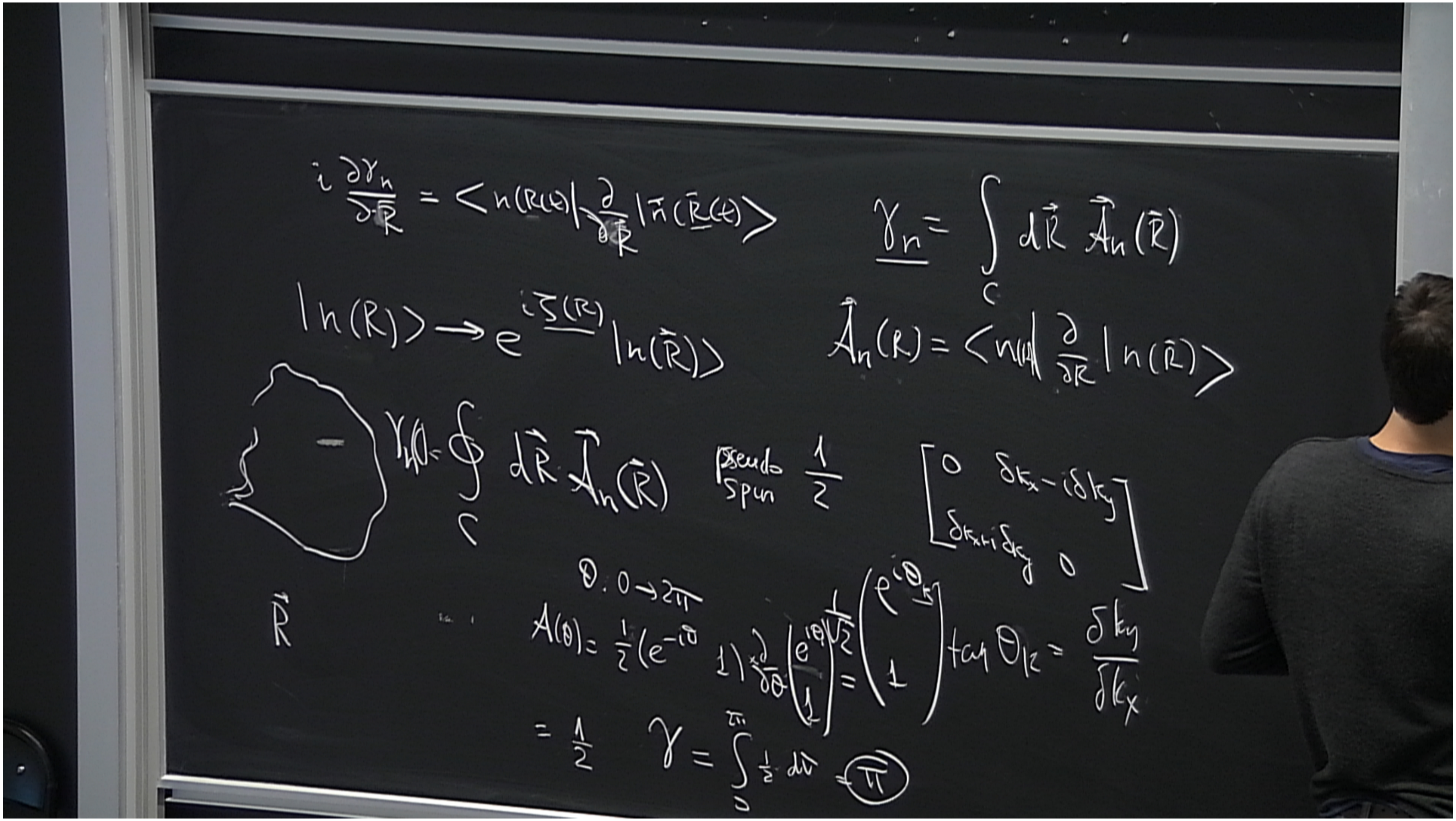
$$t(k) \approx \frac{3}{2} t a \,[-i\delta k_x + i\delta k_y], \quad \hbar v_F = \frac{3}{2} t a, \quad v_F \approx 10^6\ \text{m/s}$$

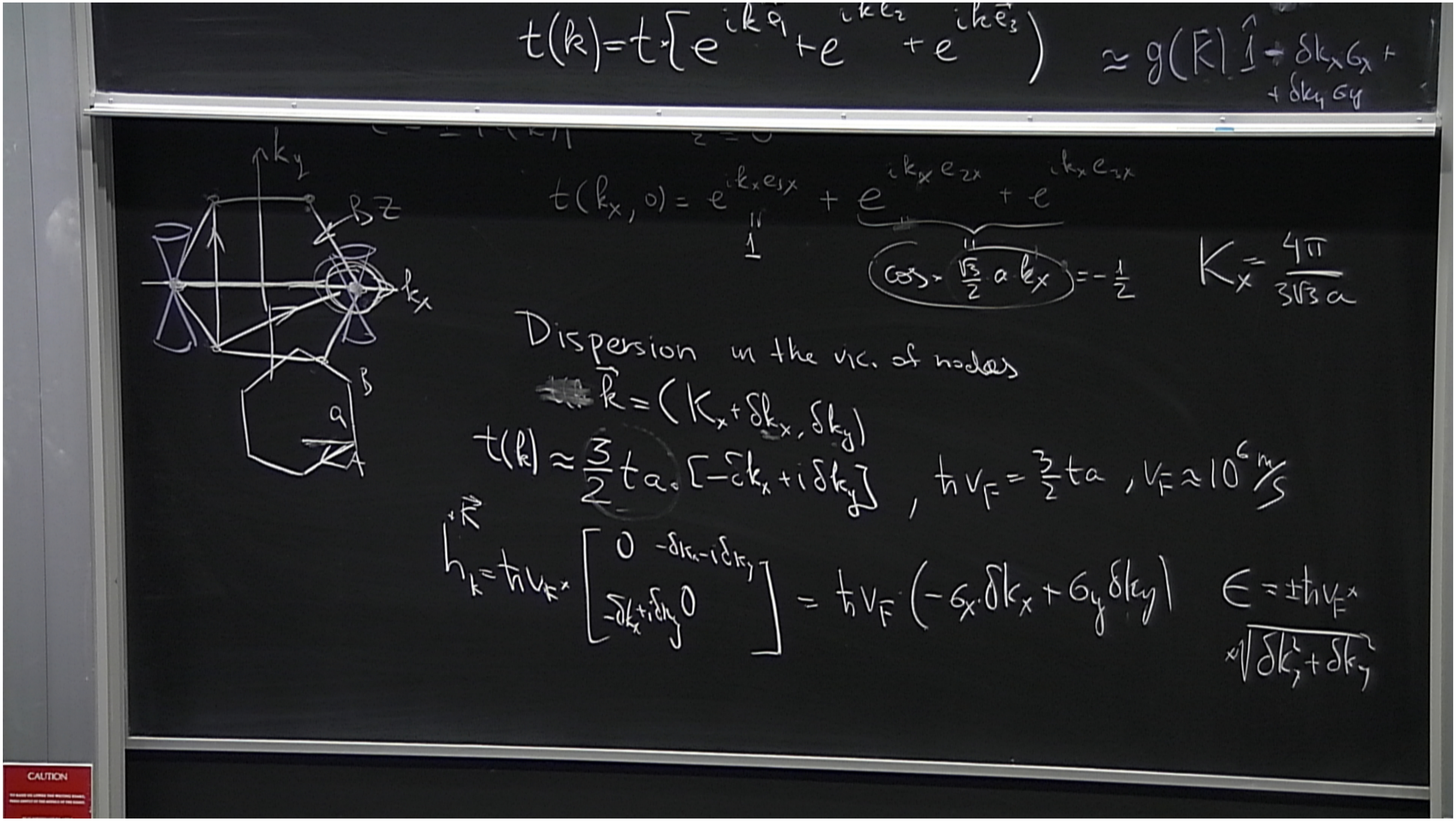
$$h_k^{\vec K} = \hbar v_F \begin{bmatrix} 0 & -\delta k_x - i\delta k_y \\ -\delta k_x + i\delta k_y & 0 \end{bmatrix} = \hbar v_F \left(-\sigma_x \delta k_x + \sigma_y \delta k_y\right)$$

$$\epsilon = \pm \hbar v_F \sqrt{\delta k_x^2 + \delta k_y^2}$$


i ∂γn/∂R = <n(R(t))| ∂/∂R |n(R(t))>
γn = ∫C dR An(R)
|n(R)> → e^{iξ(R)} |n(R)>
An(R) = <n(R)| ∂/∂R |n(R)>
γn(C) = ∮C dR · An(R)
pseudo spin 1/2
0 δkx − iδky
δkx + iδky 0
1/√2 (e^{iθk}, 1)
tan θk = δky/δkx


pseudo spin 1/2
θ: 0 → 2π
= 1/2


$i \frac{\partial \gamma_n}{\partial \vec{R}} = \langle n(R(t)) | \frac{\partial}{\partial \vec{R}} | n(\vec{R}(t)) \rangle$
$\gamma_n = \int_C d\vec{R}\, \vec{A}_n(\vec{R})$
$|n(R)\rangle \rightarrow e^{i\xi(R)} |n(\vec{R})\rangle$
$\vec{A}_n(R) = \langle n(R) | \frac{\partial}{\partial \vec{R}} | n(\vec{R}) \rangle$
$\gamma_n(C) = \oint_C d\vec{R}\, \vec{A}_n(\vec{R})$
pseudo spin $\frac{1}{2}$
$\begin{bmatrix} 0 & \delta k_x - i\delta k_y \\ \delta k_x + i\delta k_y & 0 \end{bmatrix}$
$\vec{R}$
$\theta: 0 \rightarrow 2\pi$
$A(\theta) = \frac{1}{2}(e^{-i\theta}\ 1) i\frac{\partial}{\partial\theta} \begin{pmatrix} e^{i\theta} \\ 1 \end{pmatrix}$
$\frac{1}{\sqrt{2}} \begin{pmatrix} e^{i\theta_k} \\ 1 \end{pmatrix}$
$\tan\theta_k = \frac{\delta k_y}{\delta k_x}$
$= \frac{1}{2}$
$\gamma = \int_0^{2\pi} \frac{1}{2}\, d\theta = \pi$



$t(k) = t\cdot[e^{ik e_1} + e^{ik e_2} + e^{ik e_3}) \simeq g(K)\hat{1} + \delta k_x \sigma_x + \delta k_y \sigma_y$
k_y
BZ
k_x
B
a
$t(k_x, 0) = e^{i k_x e_{1x}} + e^{i k_x e_{2x}} + e^{i k_x e_{3x}}$
1
$\cos \frac{\sqrt{3}}{2} a k_x = -\frac{1}{2}$
$K_x = \frac{4\pi}{3\sqrt{3} a}$
Dispersion in the vic. of nodes
$\vec{k} = (K_x + \delta k_x, \delta k_y)$
$t(k) \simeq \frac{3}{2} t a \cdot [-i\delta k_x + i\delta k_y]$, $\hbar v_F = \frac{3}{2} t a$, $v_F \approx 10^6$ m/s
$h_{\vec{k}} = \hbar v_F \times \begin{bmatrix} 0 & -\delta k_x - i\delta k_y \\ -\delta k_x + i\delta k_y & 0 \end{bmatrix} = \hbar v_F (-\sigma_x \delta k_x + \sigma_y \delta k_y)$
$\epsilon = \pm \hbar v_F \times \sqrt{\delta k_x^2 + \delta k_y^2}$
CAUTION

$$i\frac{\partial \gamma_n}{\partial \vec R} = \langle n(R(t))| \frac{\partial}{\partial \vec R} |n(\vec R(t))\rangle \qquad \underline{\gamma_n} = \int_C d\vec R\, \vec A_n(\vec R)$$

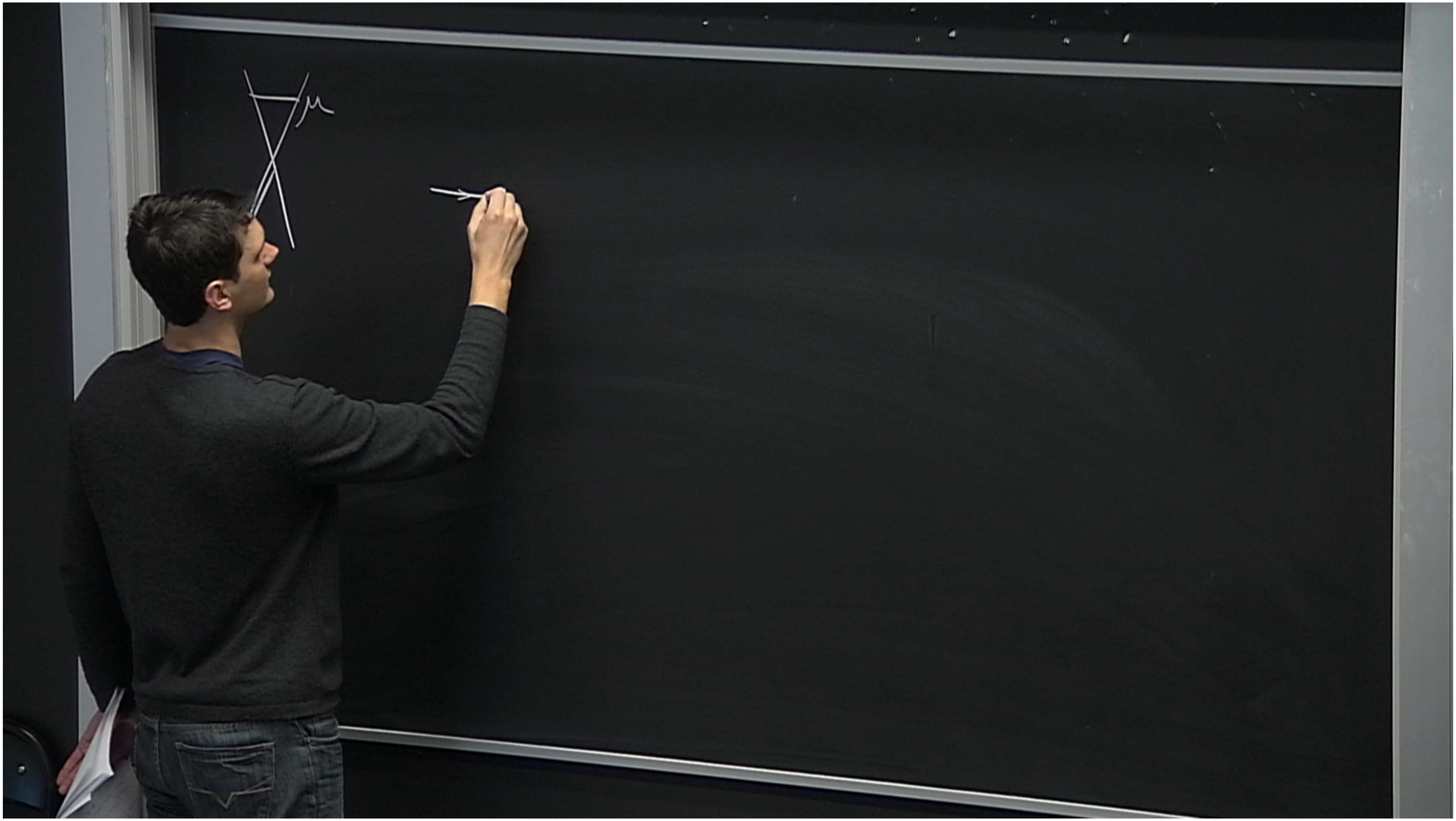
$$|n(R)\rangle \rightarrow e^{i\zeta(R)} |n(\vec R)\rangle \qquad \vec A_n(R) = \langle n(R)| \frac{\partial}{\partial R} |n(\vec R)\rangle$$

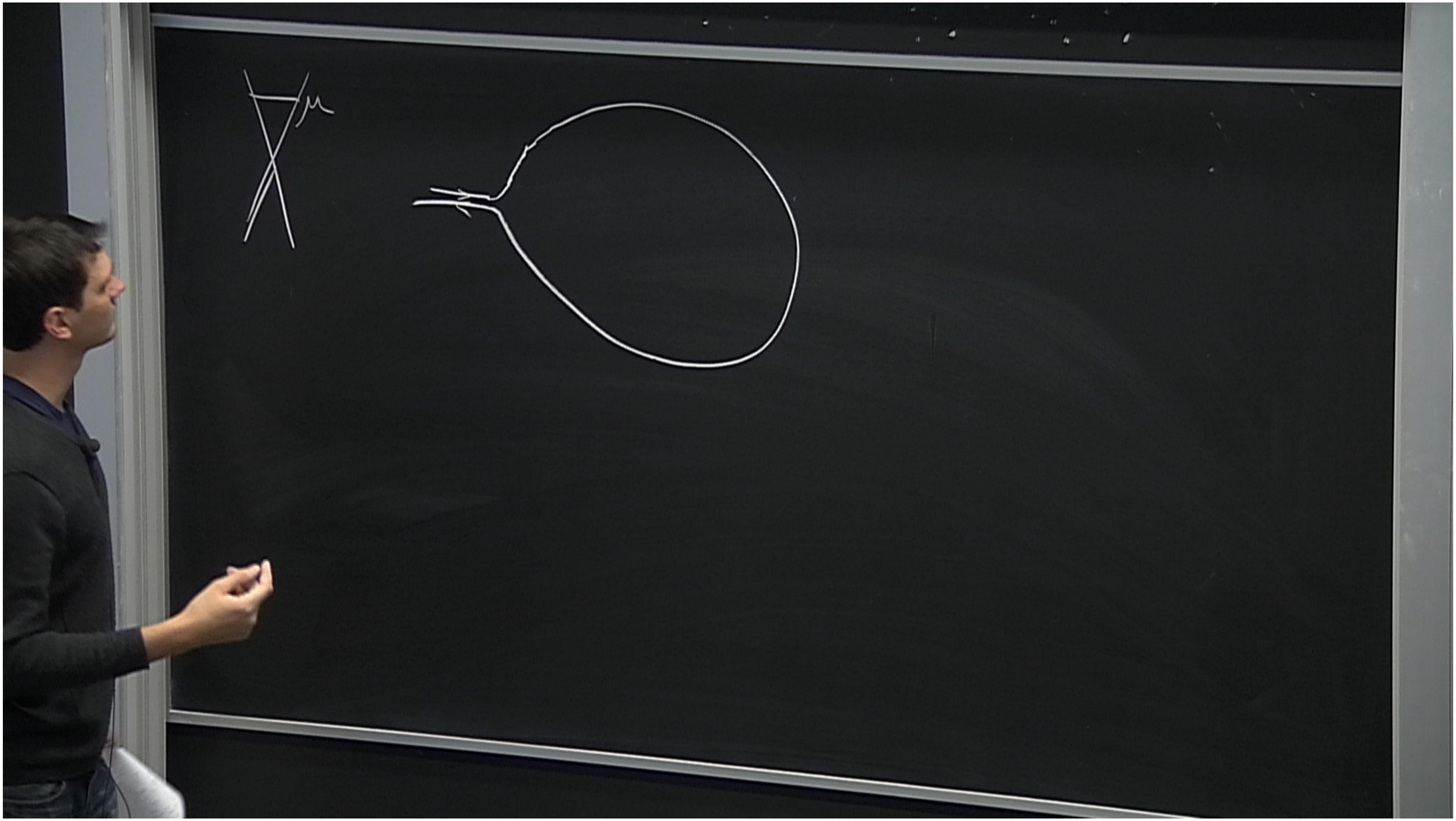
$$\gamma_n(C) = \oint_C d\vec R \cdot \vec A_n(\vec R) \qquad \text{pseudo spin } \frac{1}{2} \qquad \begin{bmatrix} 0 & \delta k_x - i\delta k_y \\ \delta k_x + i\delta k_y & 0 \end{bmatrix}$$

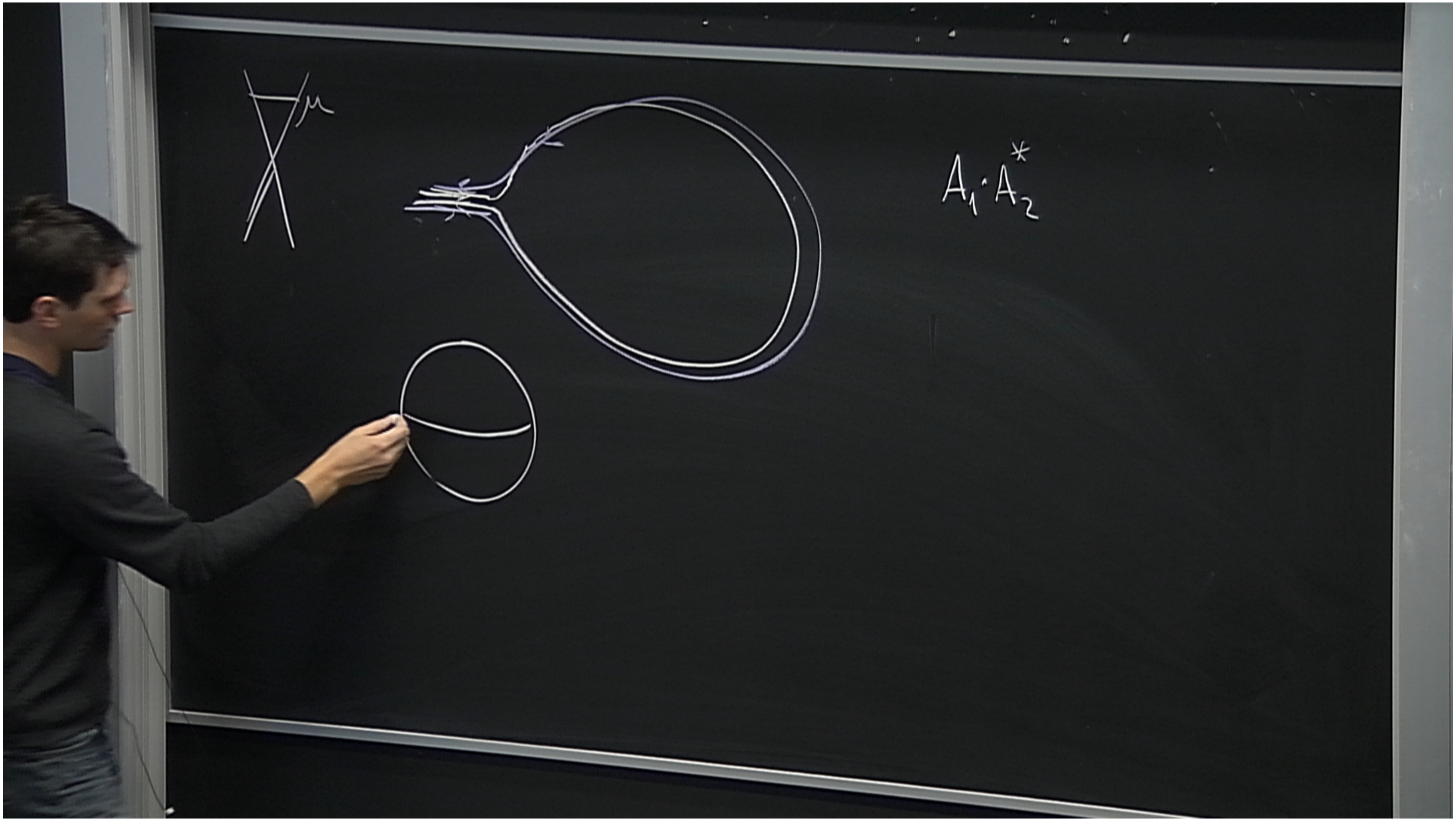
$$\vec R \qquad \theta : 0 \rightarrow 2\pi$$

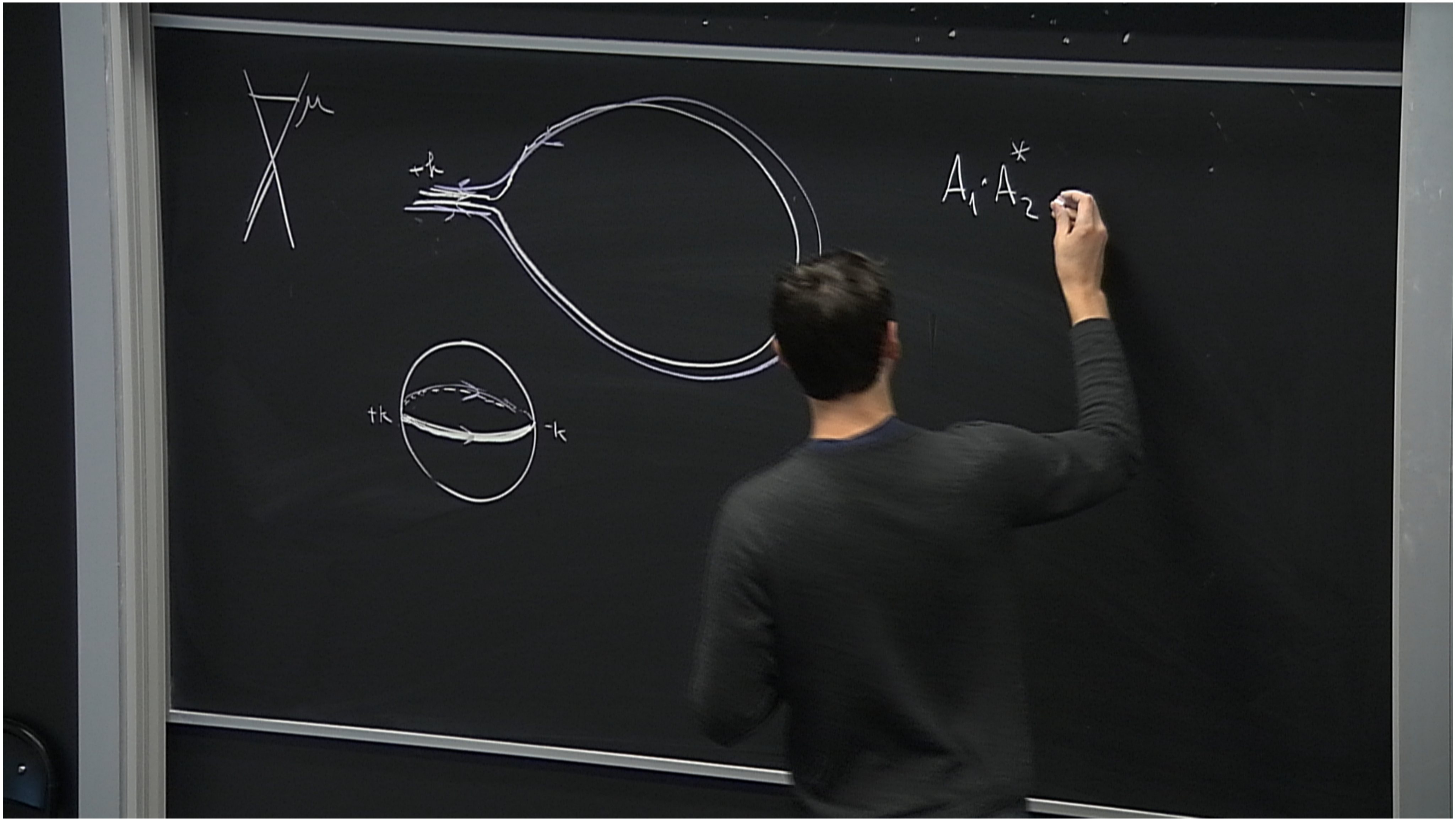
$$A(\theta) = \frac{1}{2}(e^{-i\theta} \;\; 1) \frac{\partial}{\partial\theta} \begin{pmatrix} e^{i\theta} \\ 1 \end{pmatrix} \qquad \frac{1}{\sqrt 2}\begin{pmatrix} e^{i\theta_k} \\ 1 \end{pmatrix} \qquad \tan\theta_k = \frac{\delta k_y}{\delta k_x}$$

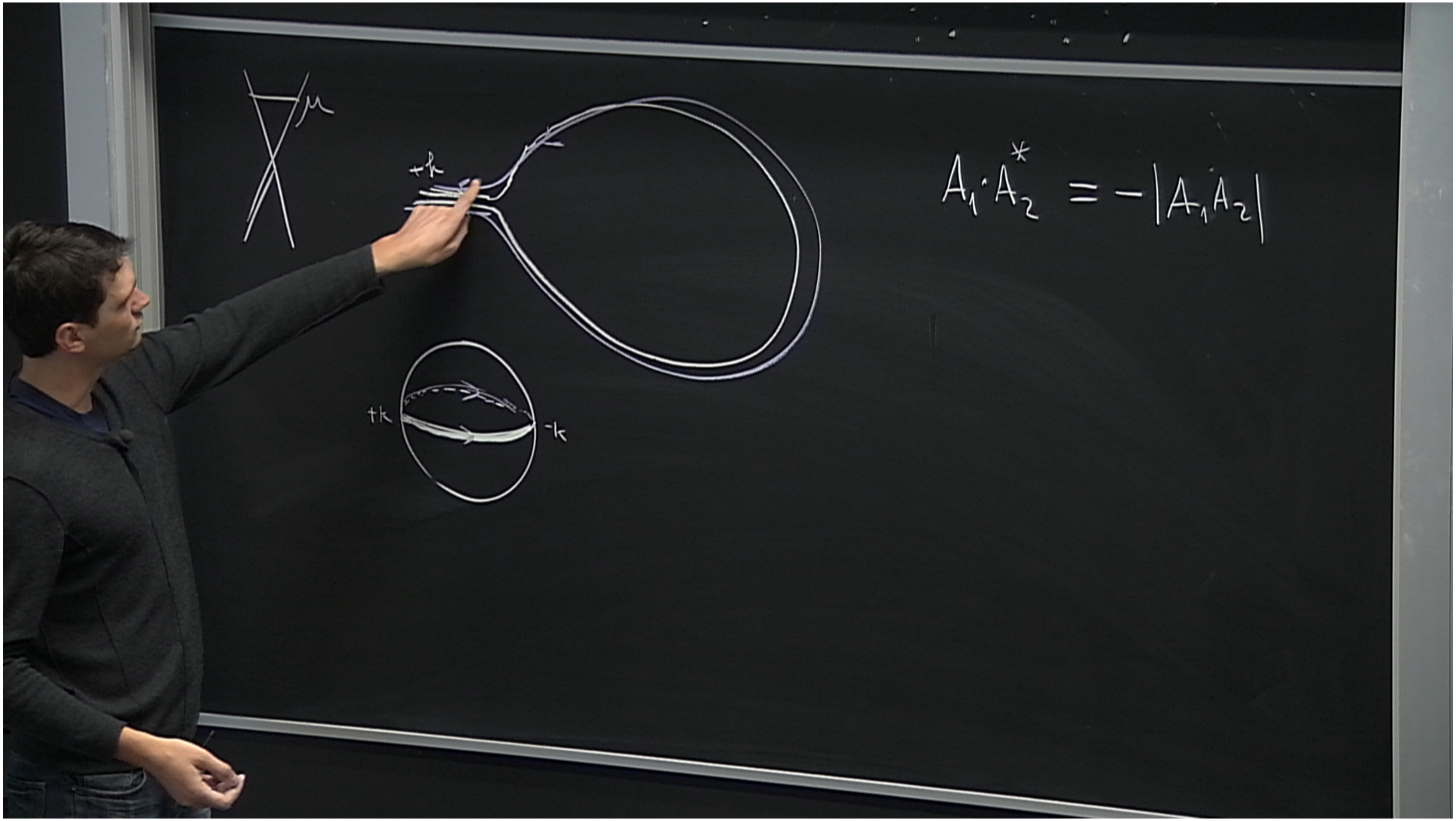
$$= \frac{1}{2} \qquad \gamma = \int_0^{2\pi} \frac{1}{2}\, d\theta = \pi$$



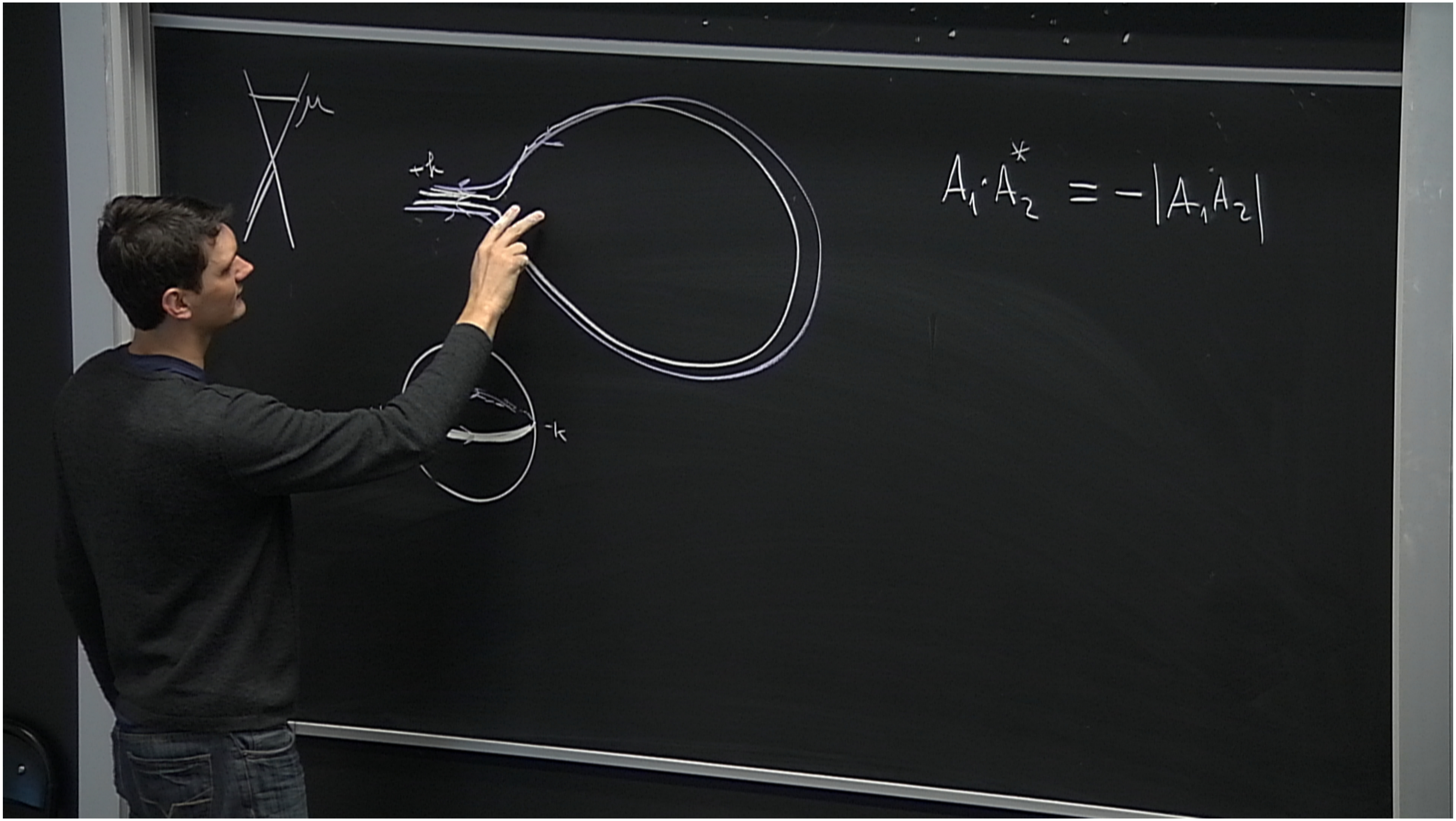


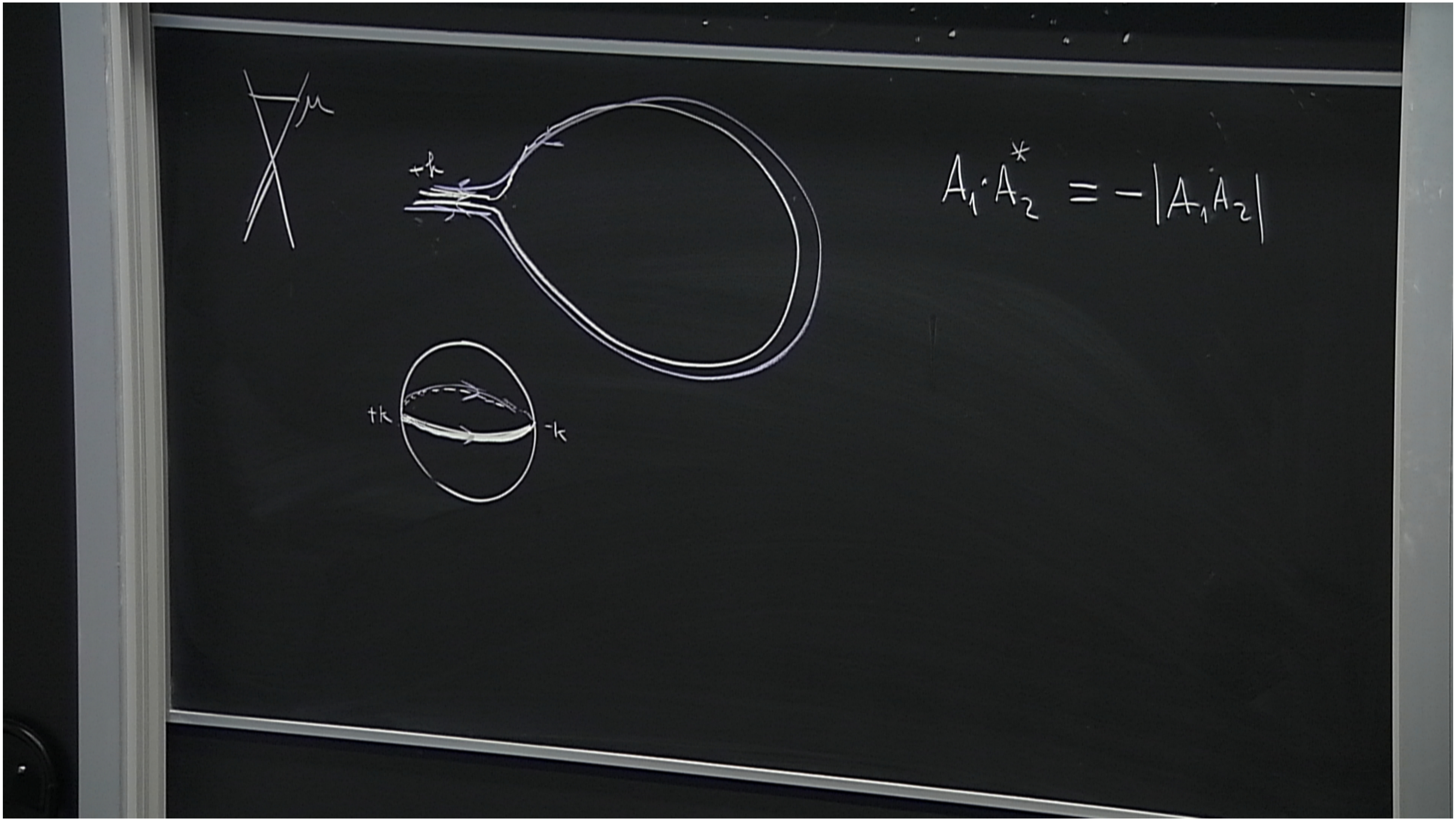

μ
A₁·A₂*

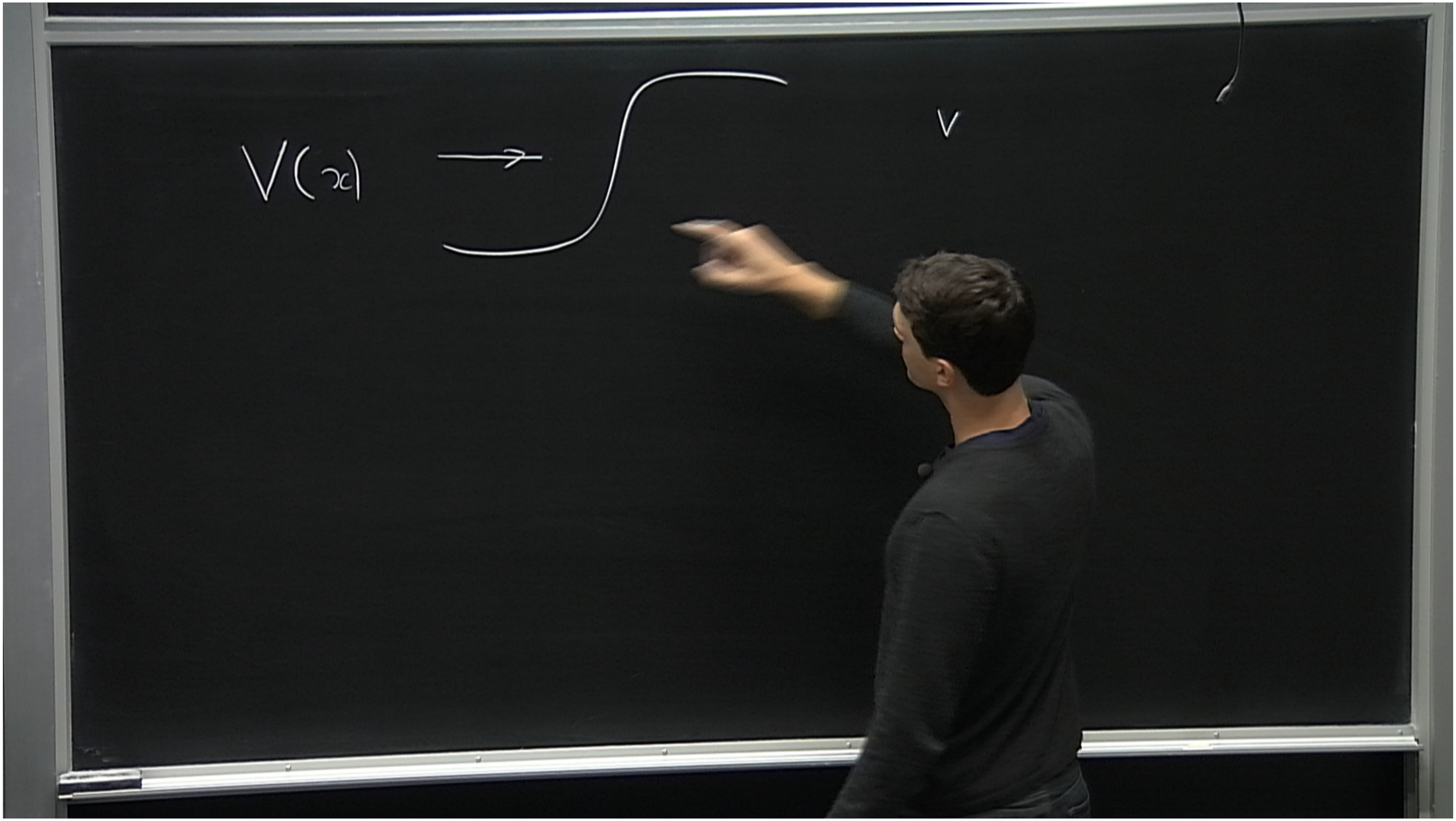

μ
+k
+k
-k
A1 · A2*

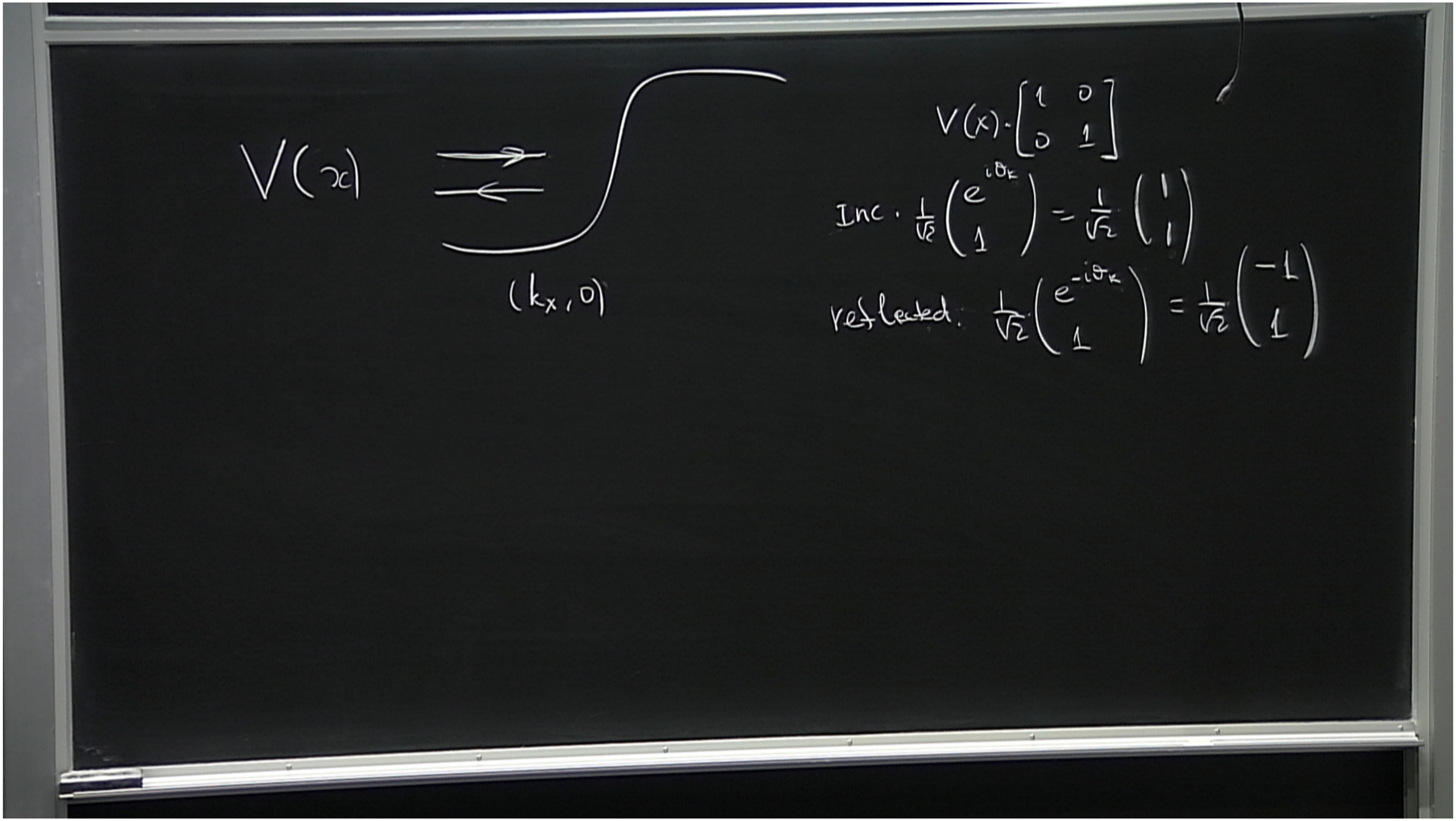


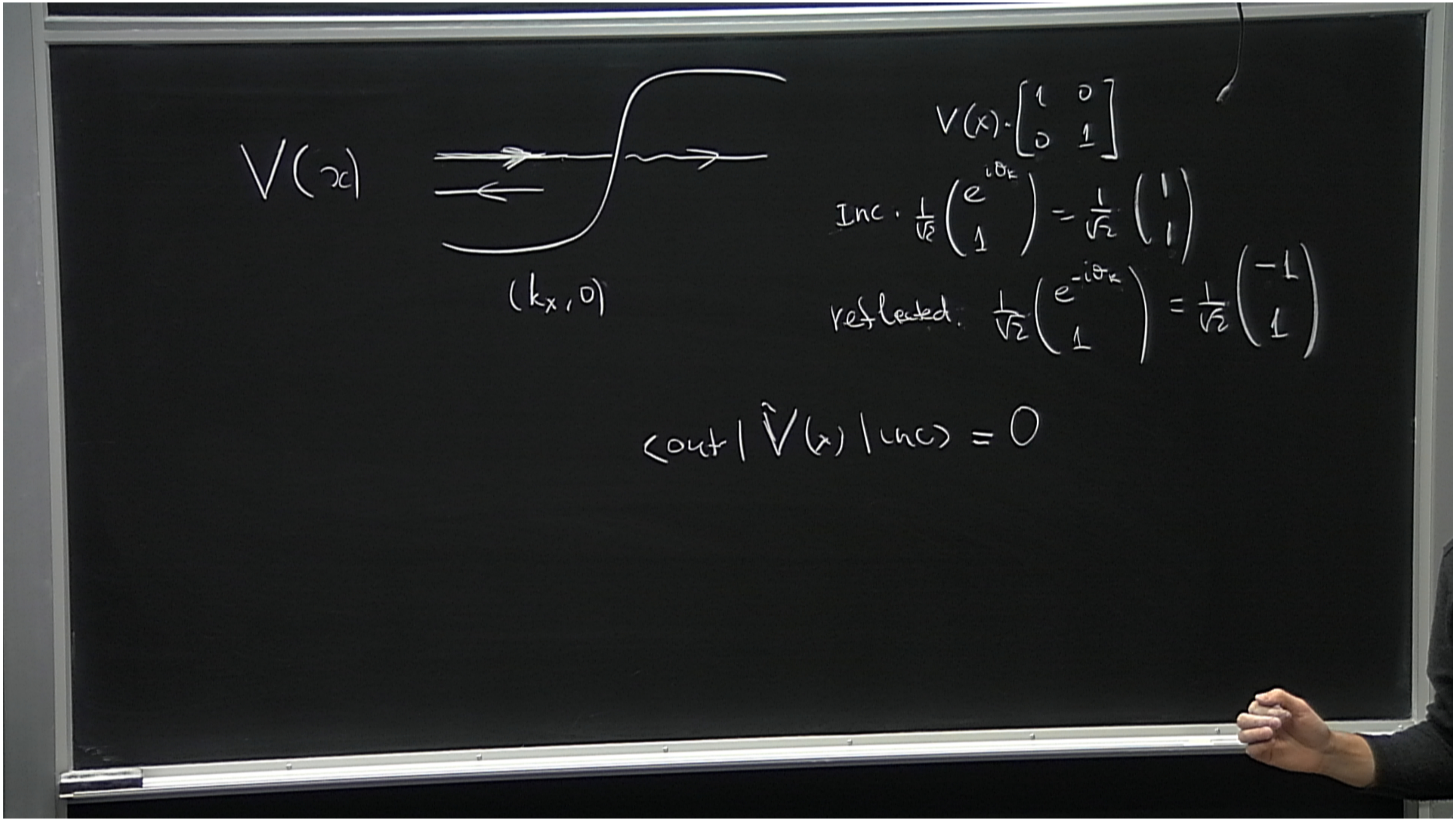
μ
+k
+k
-k
$A_1 \cdot A_2^* = -|A_1 A_2|$

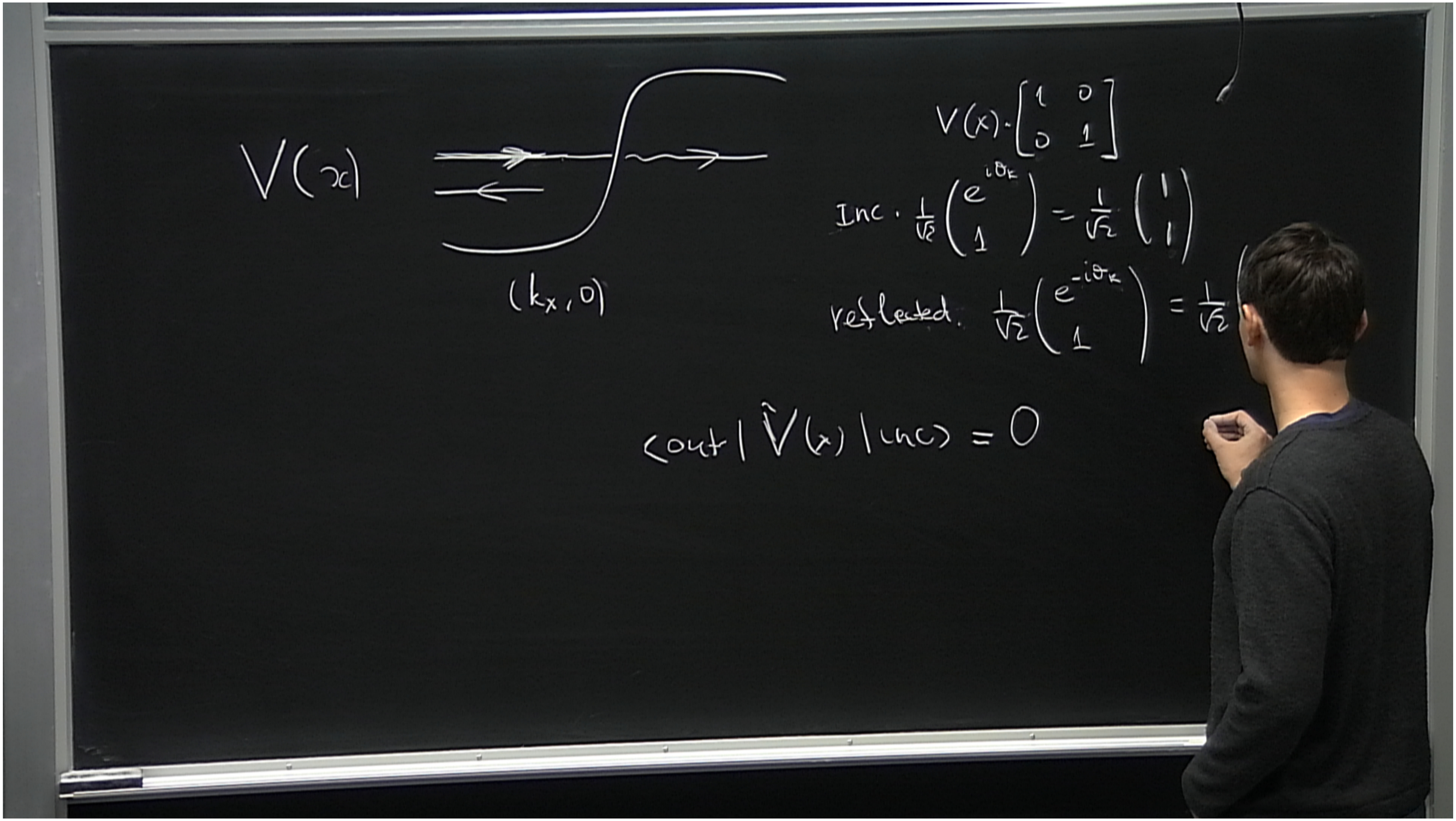

μ
+k
-k
A1·A2* = -|A1A2|

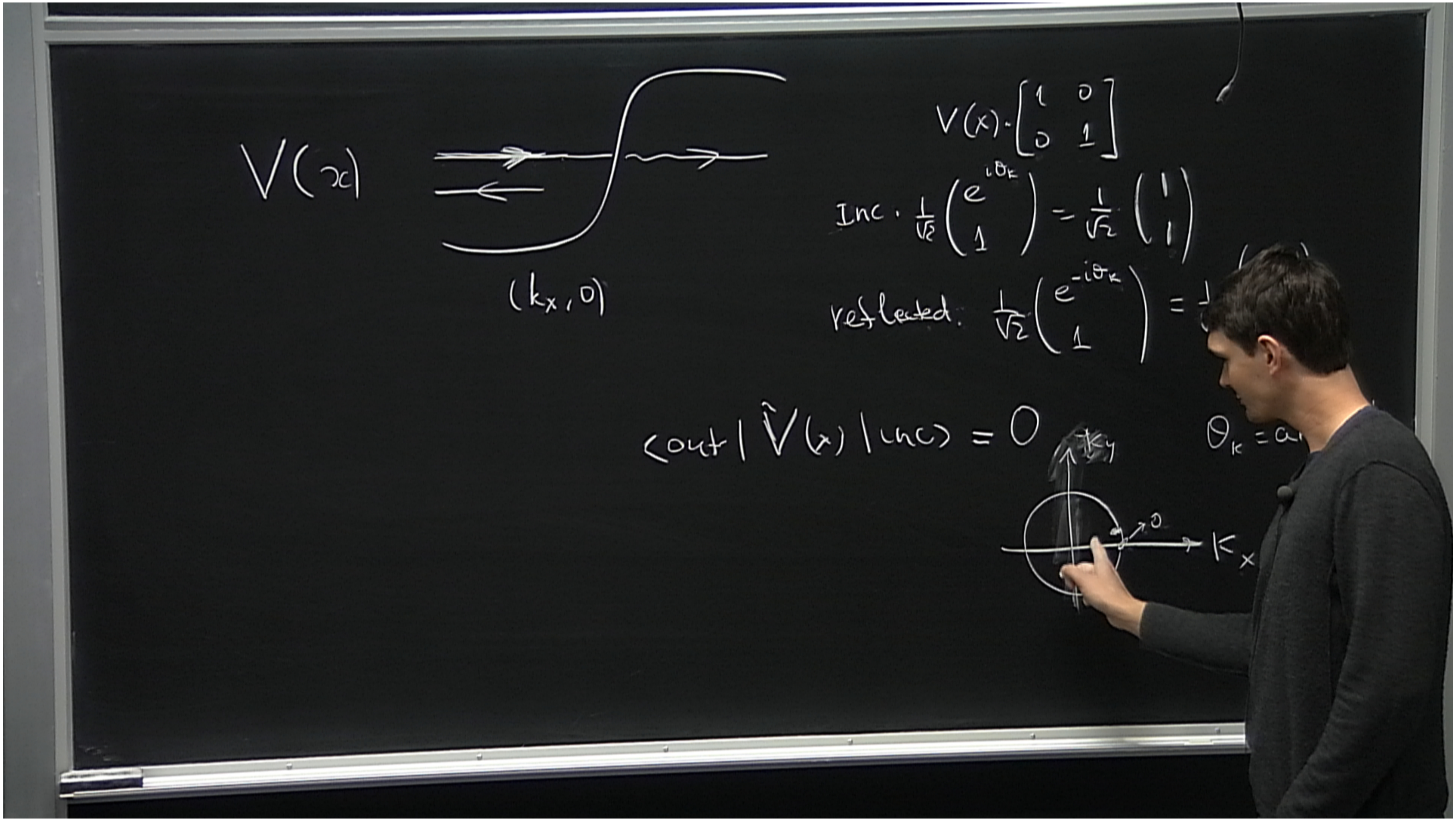

μ
+k
A1·A2* = −|A1A2|
+k
-k

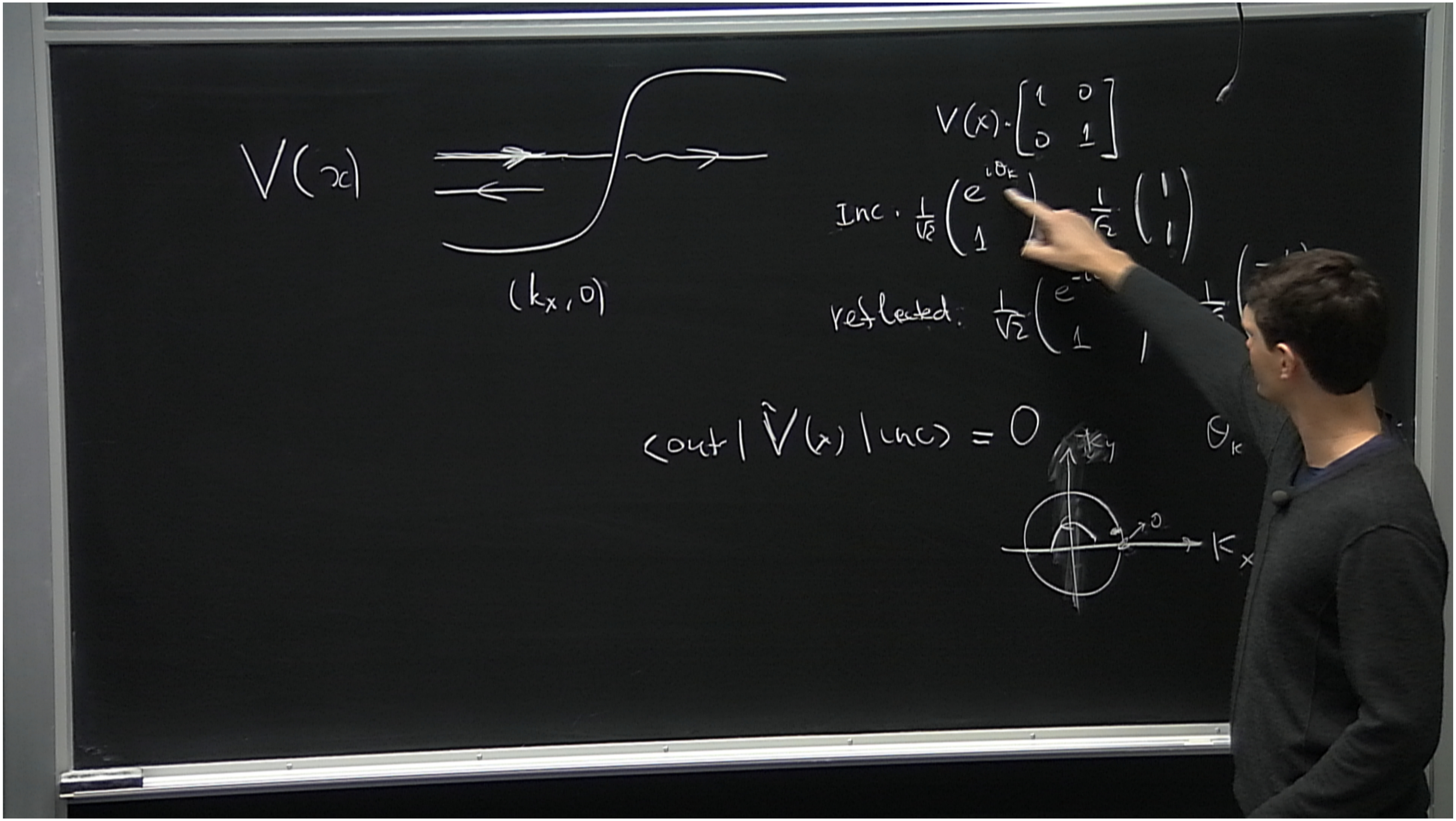

V(x)
V

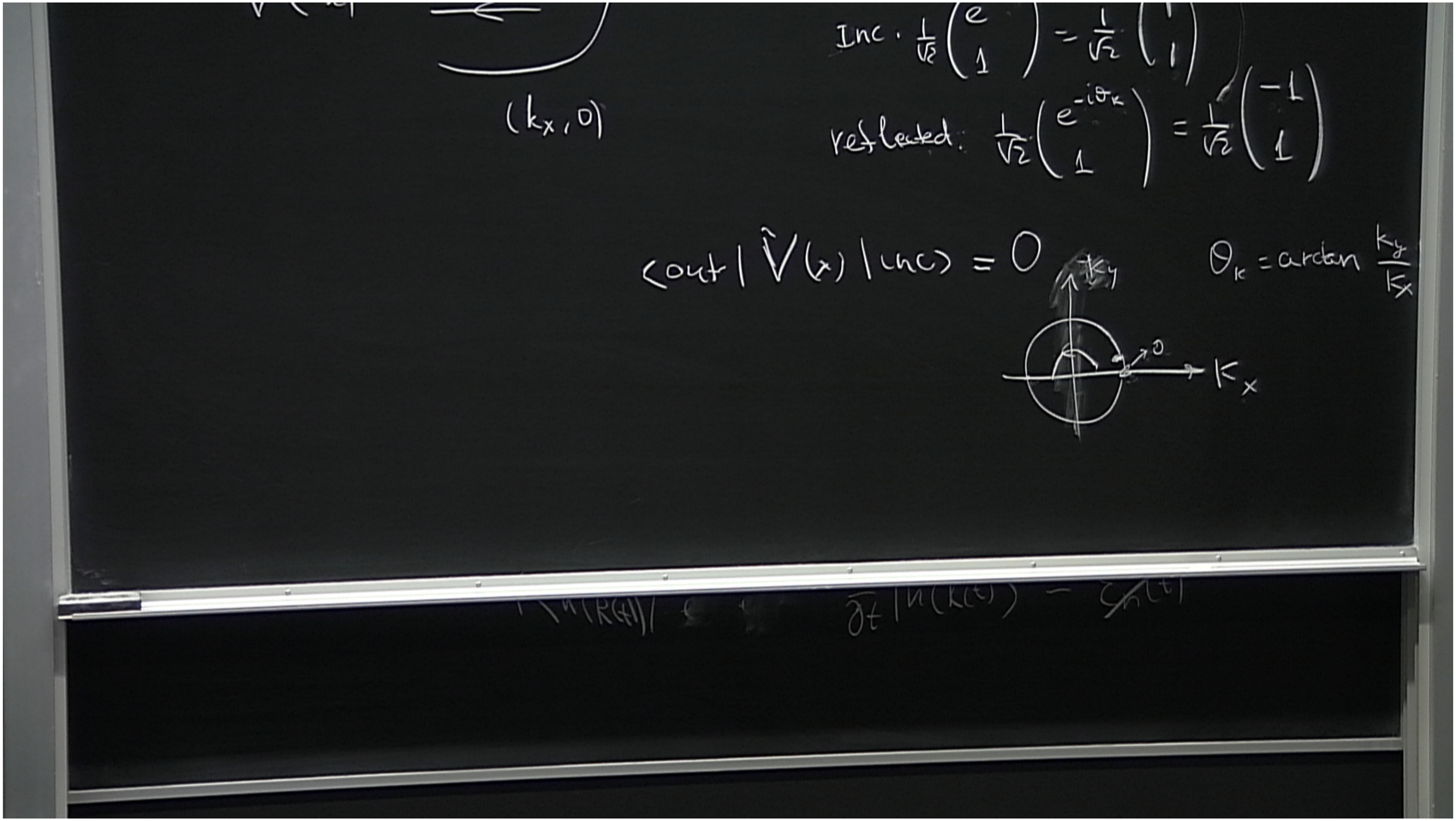

V(x)
(k_x, 0)
V(x) = [1 0 / 0 1]
Inc. 1/√2 (e^{iθ_k}, 1) = 1/√2 (1, 1)
reflected: 1/√2 (e^{-iθ_k}, 1) = 1/√2 (-1, 1)


V(x)
(kx, 0)
V(x) = [1 0; 0 1]
Inc. 1/√2 (e^{iθk}, 1) = 1/√2 (1, 1)
reflected: 1/√2 (e^{-iθk}, 1) = 1/√2 (-1, 1)
<out| V̂(x) |inc> = 0

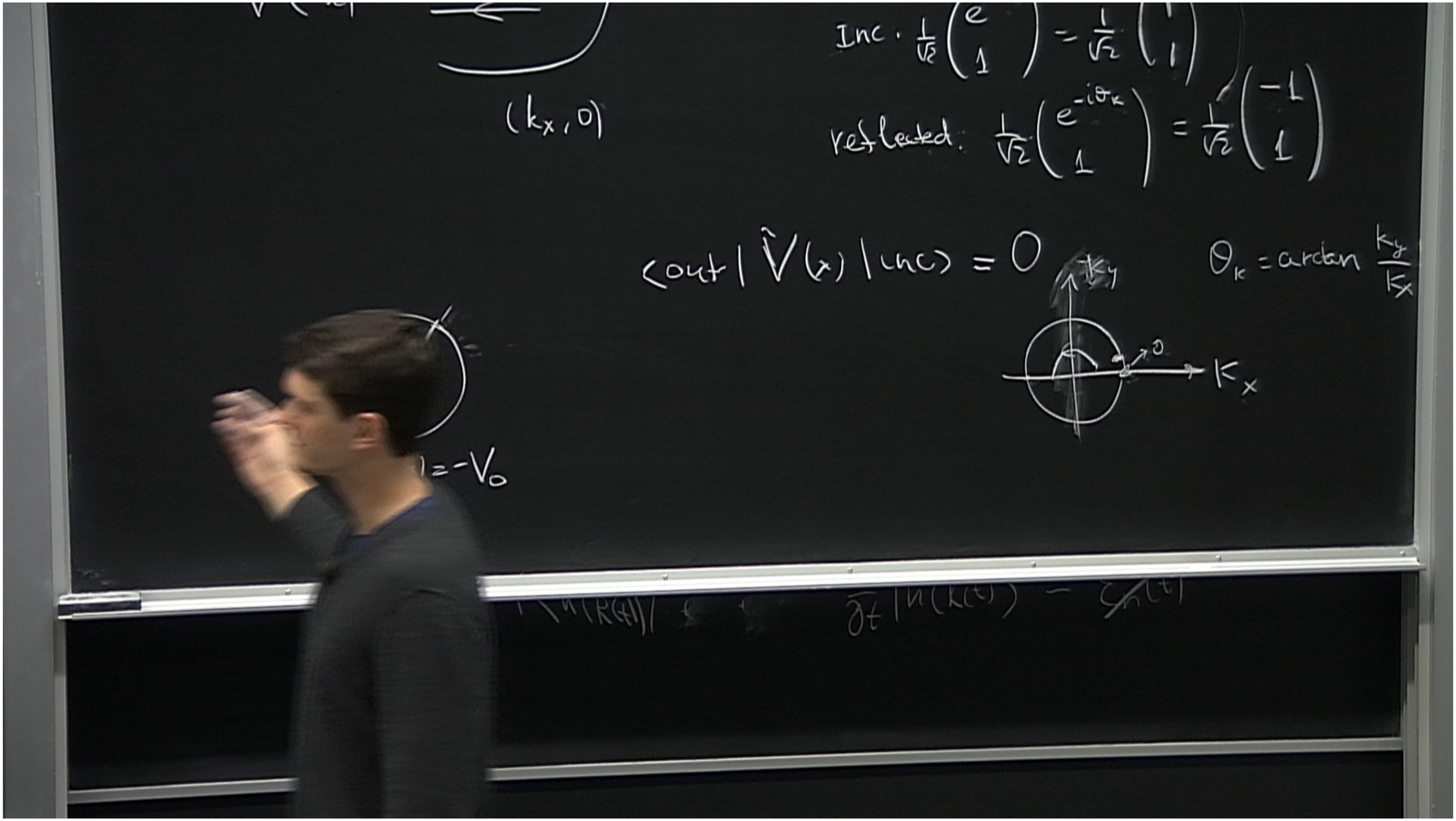

V(x)
(k_x, 0)
$V(x) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
Inc. $\frac{1}{\sqrt{2}}\begin{pmatrix} e^{i\theta_k} \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$
reflected. $\frac{1}{\sqrt{2}}\begin{pmatrix} e^{-i\theta_k} \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}}$
$\langle out | \hat{V}(x) | inc \rangle = 0$

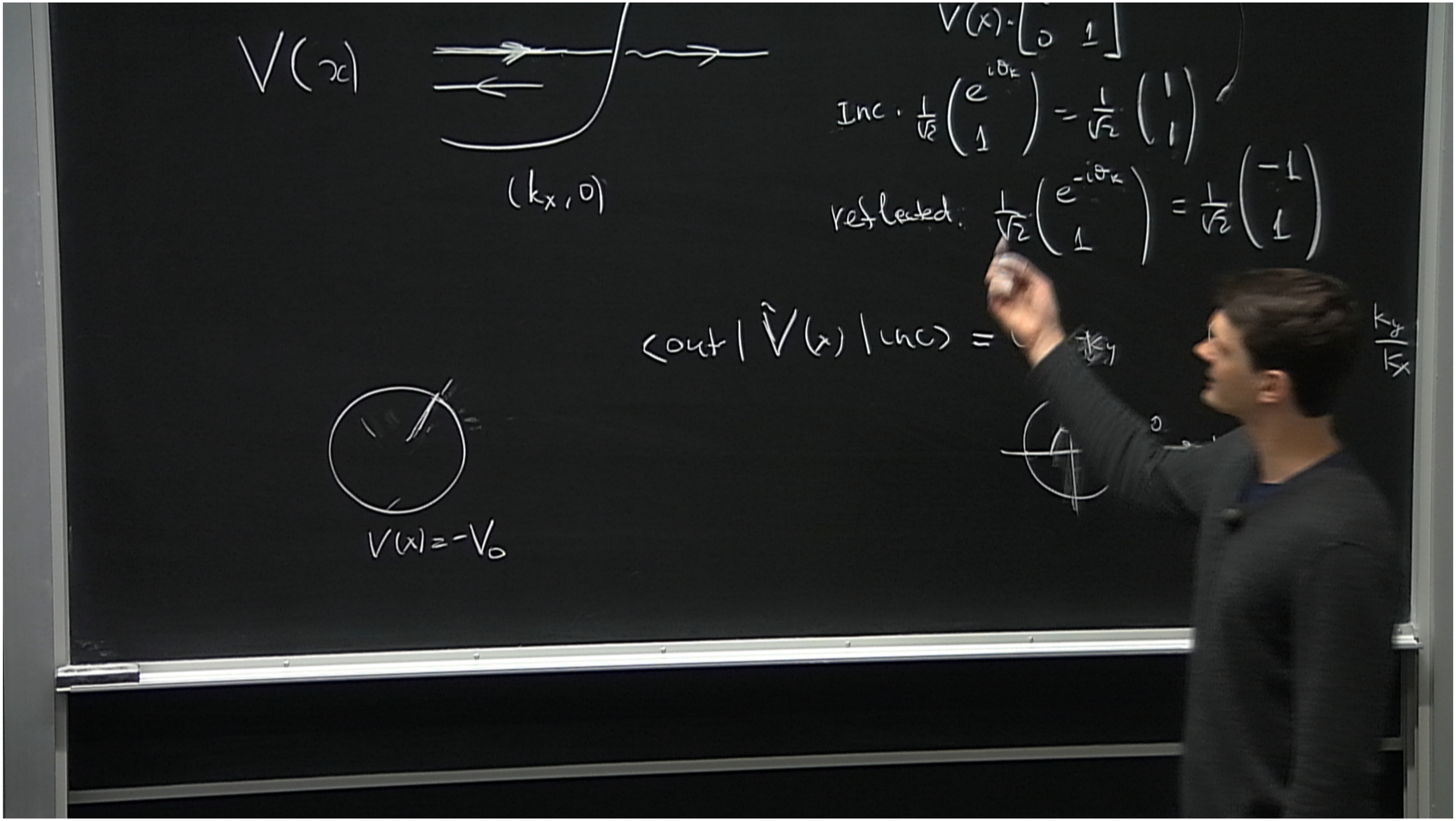

V(x)
(kx, 0)
Inc.
reflected:
<out| V(x) |inc> = 0
ky
kx

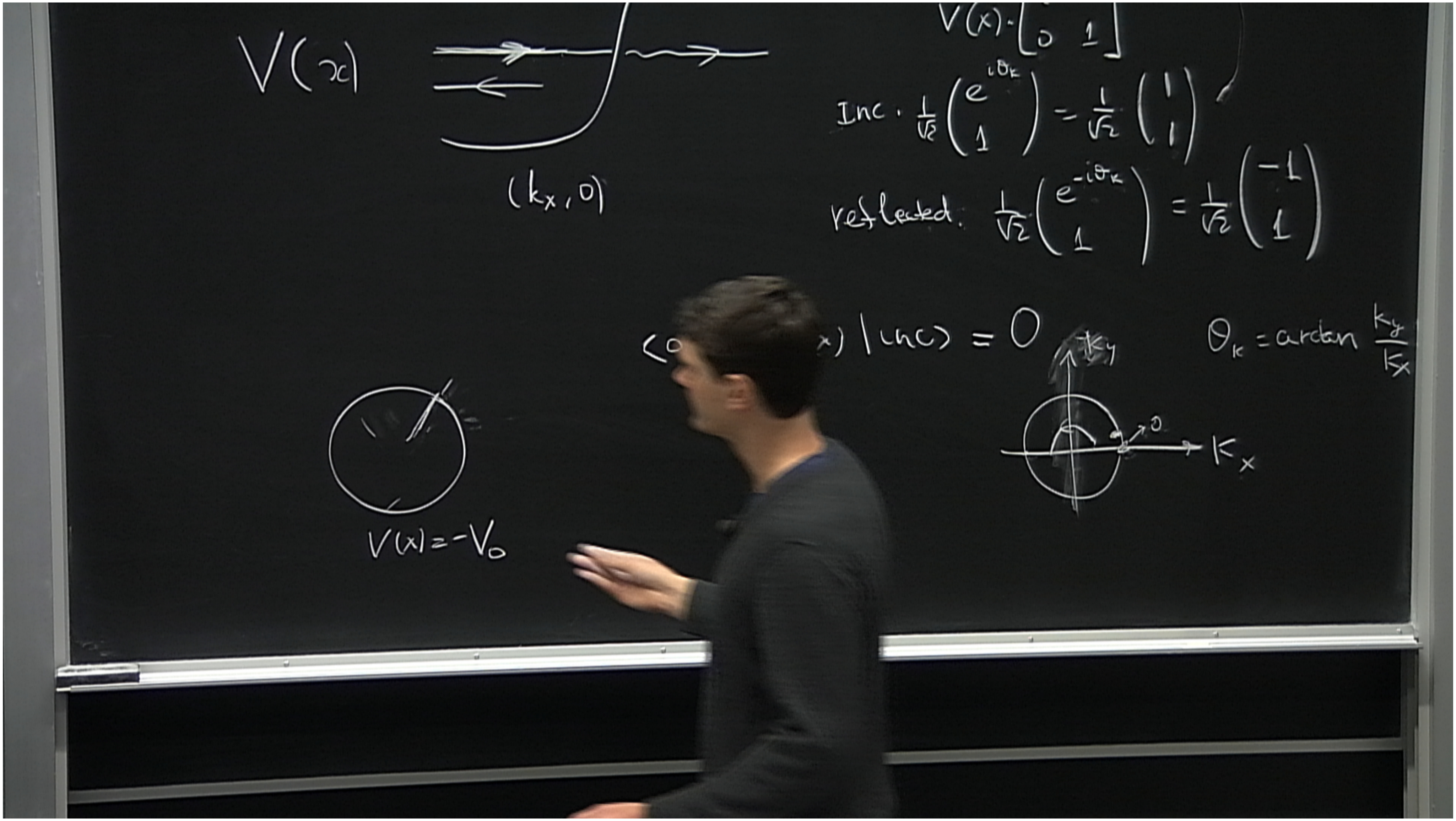

V(x)
(kx, 0)
V(x) = [1 0; 0 1]
Inc.
reflected:
<out| V̂(x) |inc> = 0
ky
kx
θk

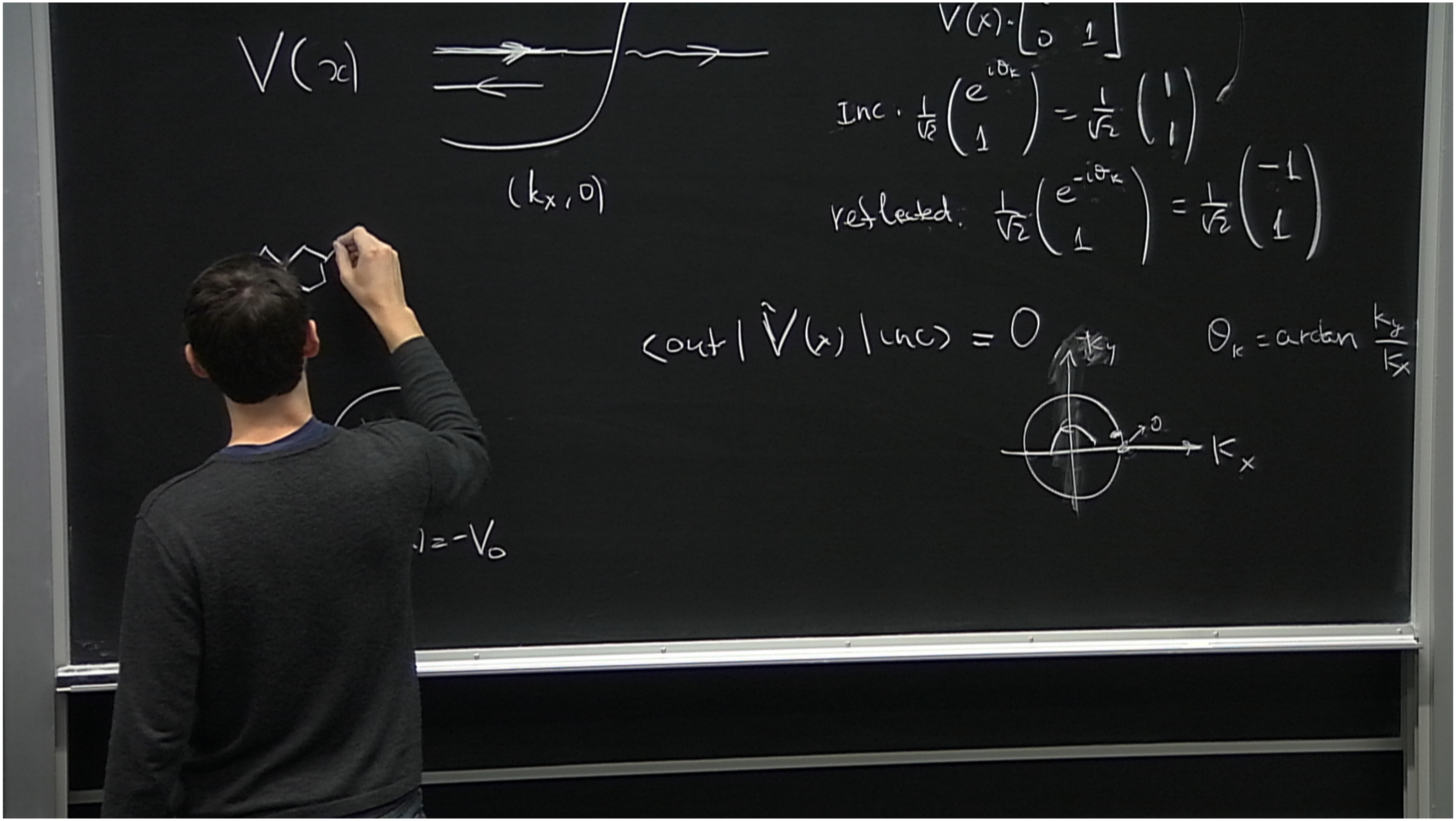

Inc. 1/√2 (e, 1) = 1/√2 (1, 1)
(kx, 0)
reflected: 1/√2 (e^-iθk, 1) = 1/√2 (-1, 1)
<out| V̂(x) |inc> = 0
θk = arctan ky/kx
ky
kx

Inc. 1/√2 (e 1) = 1/√2 (1 1)
(kx, 0)
reflected: 1/√2 (e^-iθk 1) = 1/√2 (-1 1)
<out| V̂(x) |inc> = 0
θk = arctan ky/kx
ky
kx
V(x) = -V0

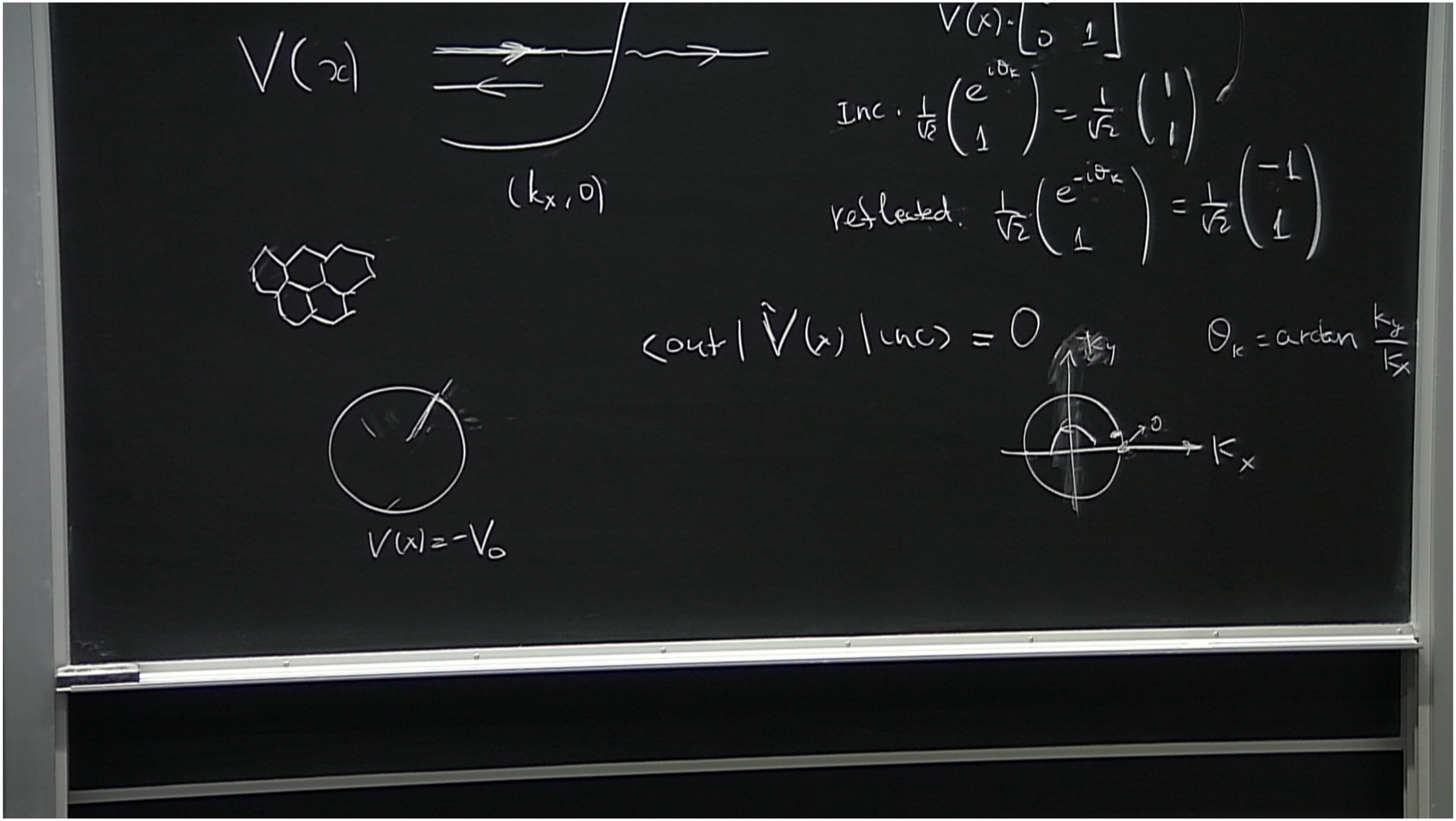

Inc. $\frac{1}{\sqrt{2}}\begin{pmatrix} e \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$
reflected: $\frac{1}{\sqrt{2}}\begin{pmatrix} e^{-i\theta_k} \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} -1 \\ 1 \end{pmatrix}$
$(k_x, 0)$
$\langle out | \hat{V}(x) | inc \rangle = 0$
$\theta_k = \arctan \frac{k_y}{k_x}$
k_y
k_x
$= -V_0$

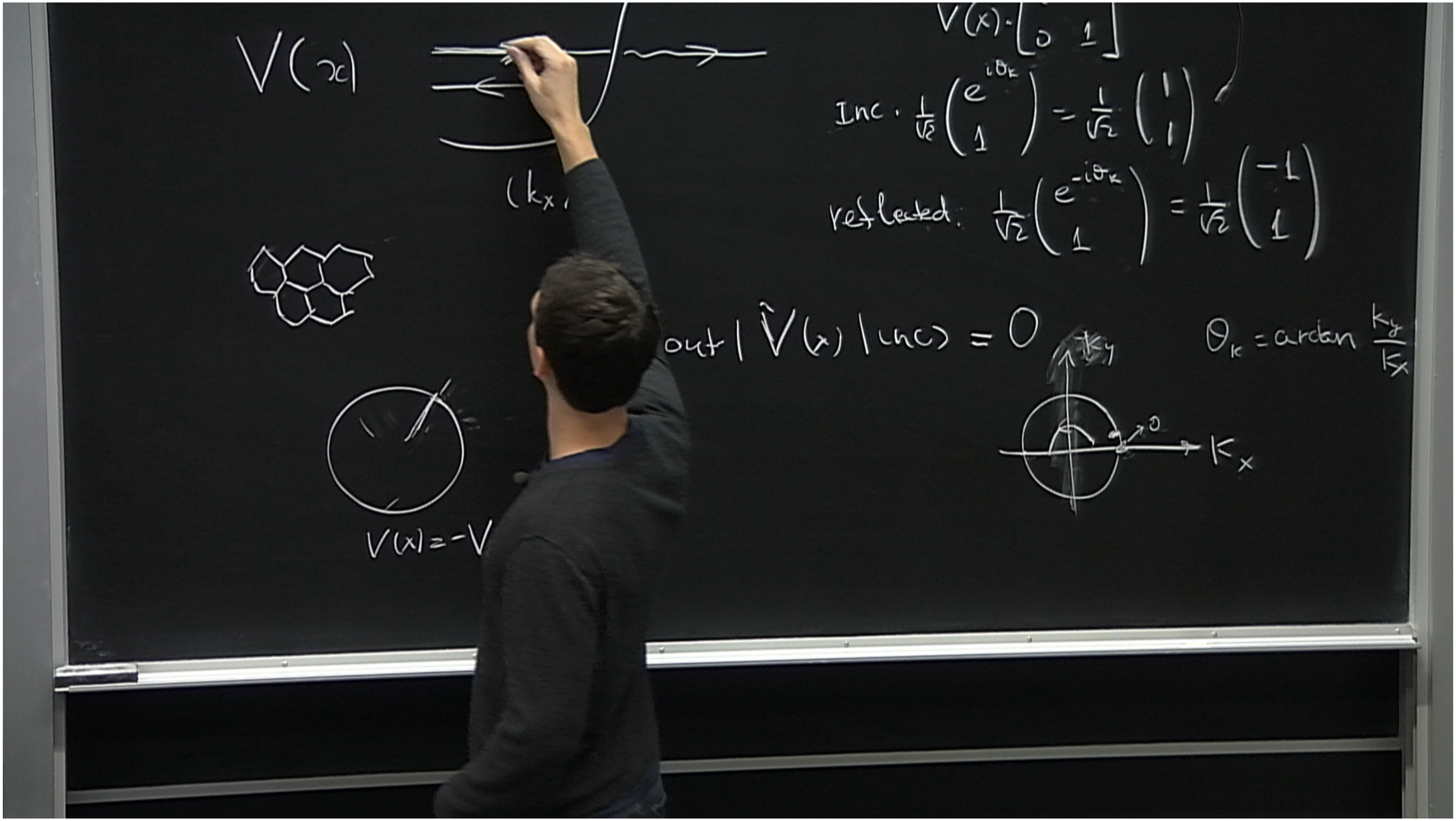

V(x)
(kx, 0)
Inc. 1/√2 (e^{iθk}, 1) = 1/√2 (1, 1)
reflected: 1/√2 (e^{-iθk}, 1) = 1/√2 (-1, 1)
<out| V̂(x) |inc> =
V(x) = -V0
ky/kx

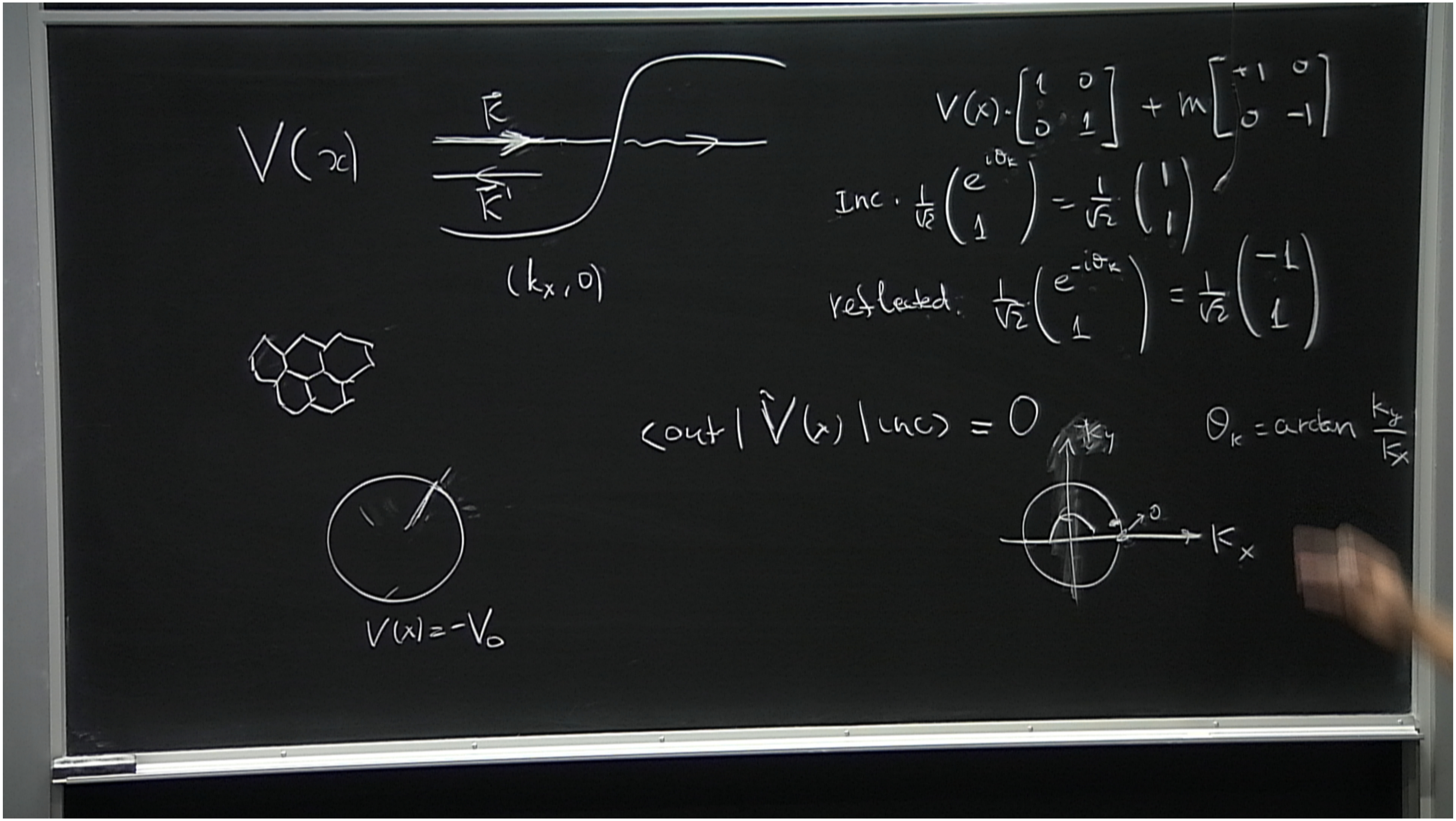

V(x)
(kx, 0)
V(x)=-V0
Inc.
reflected:
θk = arctan ky/kx
ky
kx

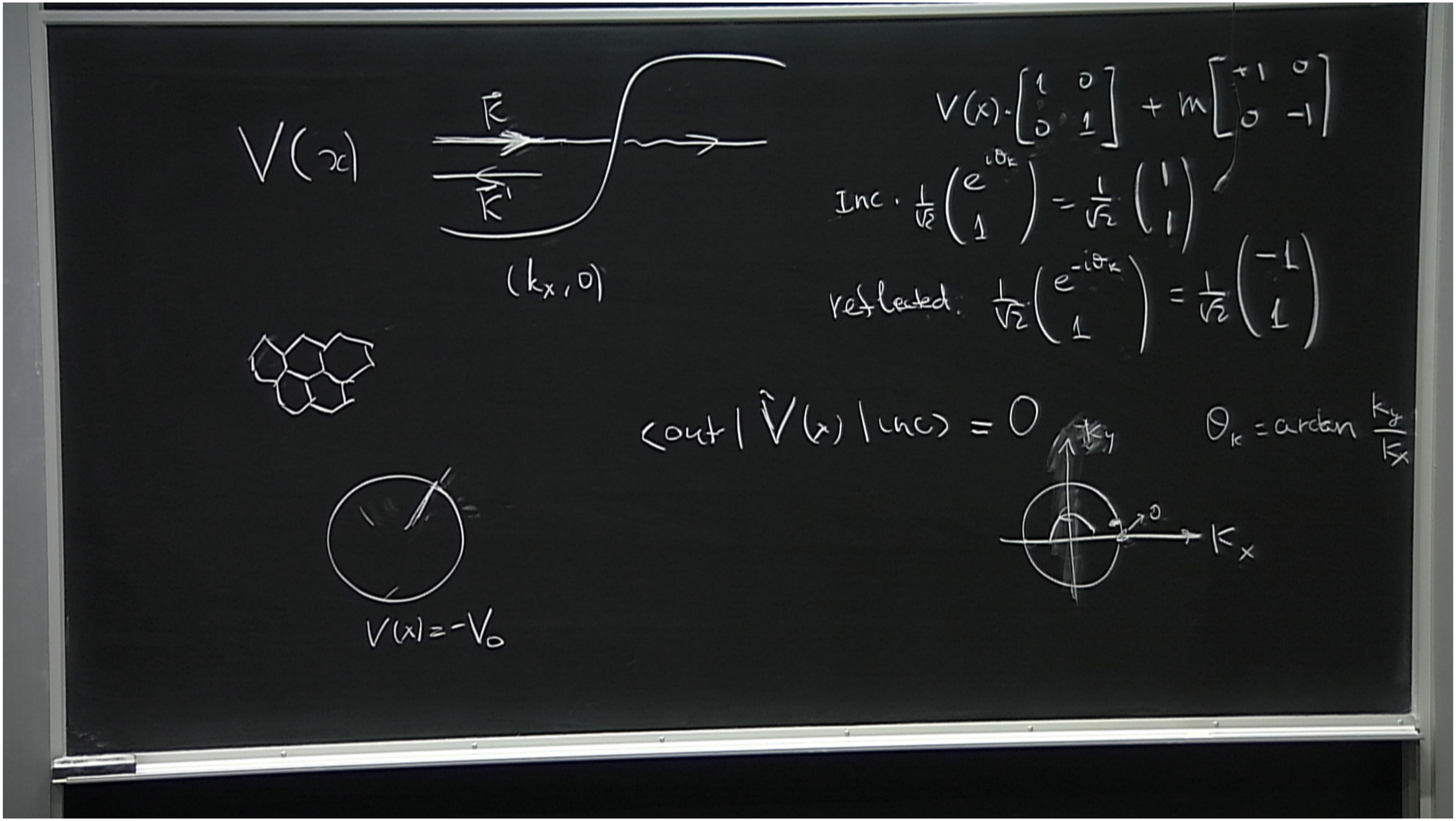


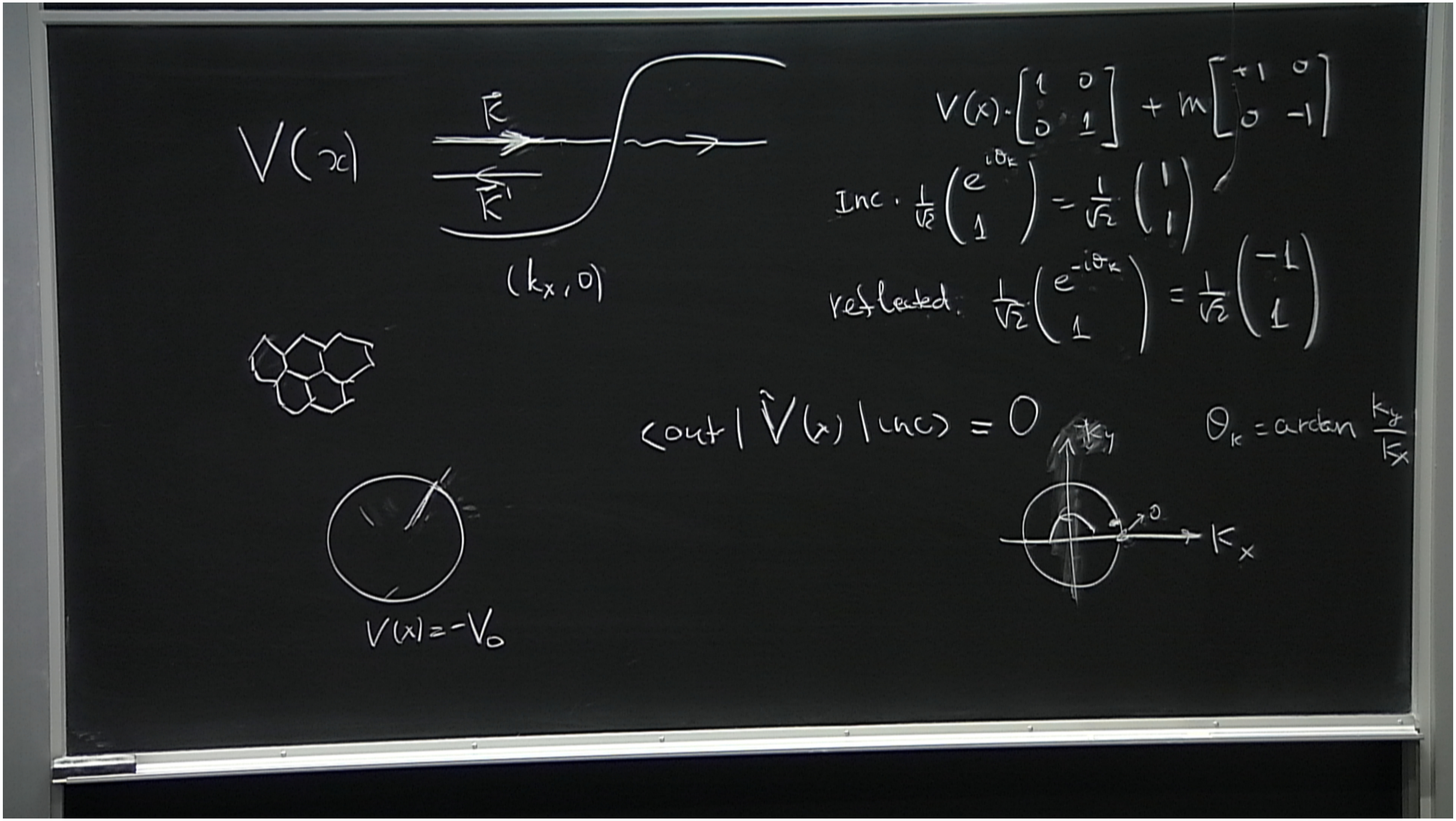
V(x)
(kx, 0)
Inc. 1/√2 (e^iθk, 1) = 1/√2 (1, 1)
reflected: 1/√2 (e^-iθk, 1) = 1/√2 (-1, 1)
<out| V̂(x) |inc> = 0
θk = arctan ky/kx
ky
Kx
=-V0


V(x)
(kx, 0)
V(x) = -V0
Inc. 1/√2 (e^(iθk), 1) = 1/√2 (1, 1)
reflected: 1/√2 (e^(-iθk), 1) = 1/√2 (-1, 1)
<out| V̂(x) |inc> = 0
θk = arctan ky/kx
ky
Kx


V(x)
(kx,
V(x) = -V
Inc. $\frac{1}{\sqrt{2}}\begin{pmatrix} e^{i\theta_k} \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$
reflected: $\frac{1}{\sqrt{2}}\begin{pmatrix} e^{-i\theta_k} \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} -1 \\ 1 \end{pmatrix}$
$\langle out | \hat{V}(x) | inc \rangle = 0$
$\theta_k = \arctan \frac{k_y}{k_x}$
k_y
k_x


V(x)
K
K'
(kx, 0)
V(x) = -V0
Inc.
reflected
<out | V(x) | inc> = 0
ky
kx
θk = arctan ky/kx


V(x)
K
K'
(kx, 0)
V(x) = -V0
V(x)·[1 0; 0 1] + m[+1 0; 0 -1]
Inc. 1/√2 (e^{iθk}, 1) = 1/√2 (1, 1)
reflected: 1/√2 (e^{-iθk}, 1) = 1/√2 (-1, 1)
<out| V̂(x) |inc> = 0
θk = arctan ky/kx
ky
Kx


V(x)
(kx, 0)
V(x) = −V0
Inc.
reflected
⟨out | V̂(x) | inc⟩ = 0
θk = arctan ky/kx
ky
kx