Title: A Luttinger Liquid Core Inside Helium-4 Filled Nanopores

Date: Apr 13, 2012 02:30 PM

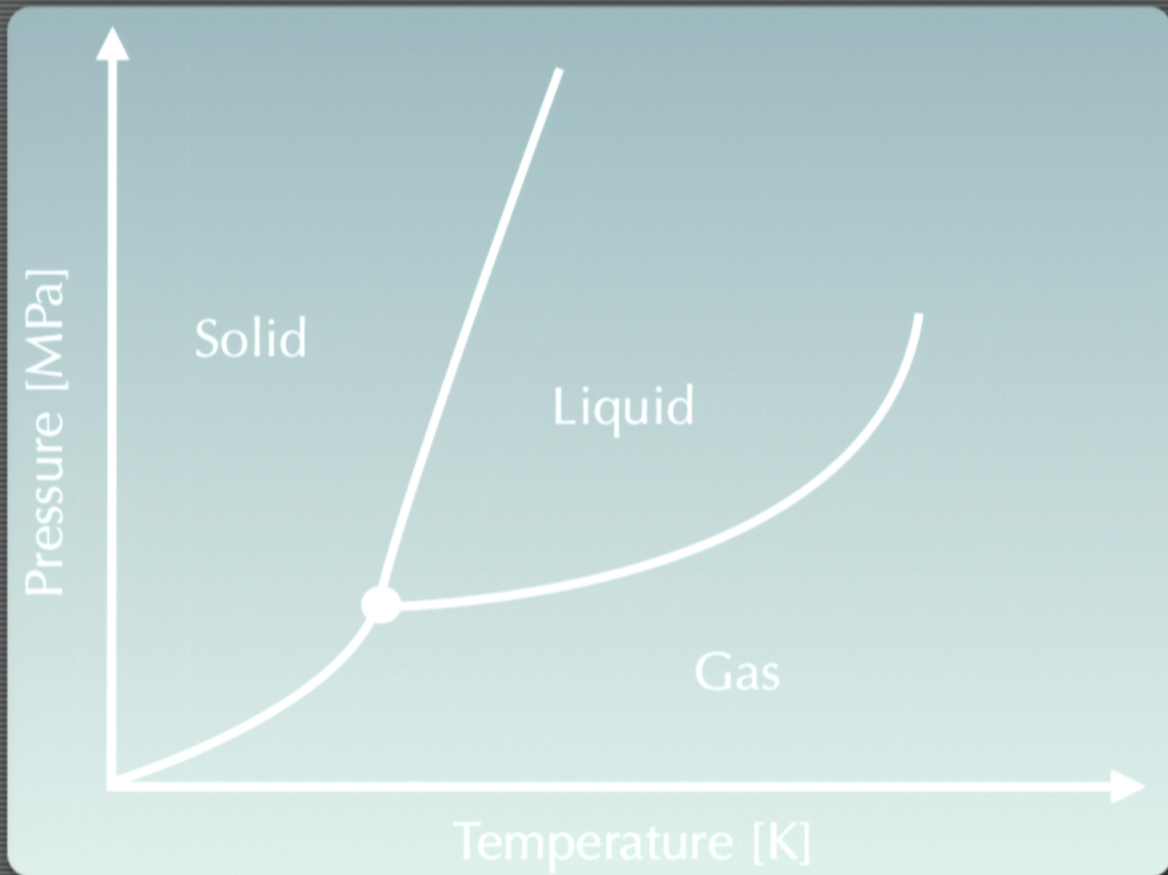
URL: http://pirsa.org/12040061

Abstract: As helium-4 is cooled below 2.17 K in undergoes a phase transition to a state of matter known as a superfluid which supports flow without viscosity. This type of dissipationless transport can be observed by forcing helium to travel through a narrow constriction that the normal liquid could not penetrate. Recent advances in nanofabrication techniques allow for the construction of smooth pores with nanometer radii, that approach the truly one dimensional limit. In one dimension, it is believed that a system of bosons (like helium-4) may have startlingly different behavior than in three dimensions. The one dimensional system is predicted to have a linear hydrodynamic description known as Luttinger liquid theory, where no type of long range order can be sustained. In the limit where the pore radius is small, helium inside the channel would behave as a sort of quasi-supersolid with all correlations decaying as power-laws at zero temperature. We have performed large scale quantum Monte Carlo simulations of helium-4 inside nanopores of varying radii at low temperatures with realistic helium-helium and helium-pore interactions. The results indicate that helium inside the nanopore forms concentric cylindrical layers surrounding a core that can be fully described via Luttinger liquid theory and provides insights towards the exciting possibility of the experimental detection of a Luttinger liquid.

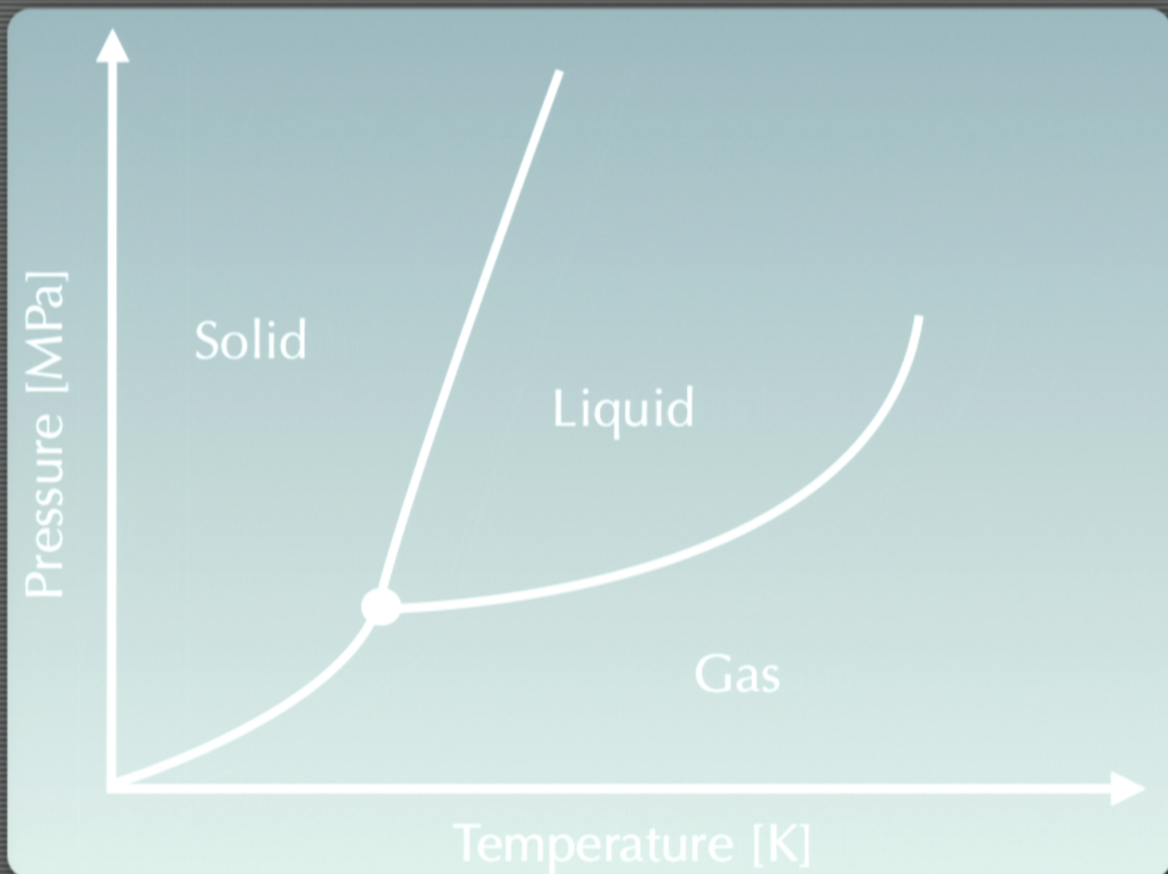
A Luttinger Liquid Core Inside Helium-4 Filled Nanopores

ADRIAN DEL MAESTRO
The University of Vermont

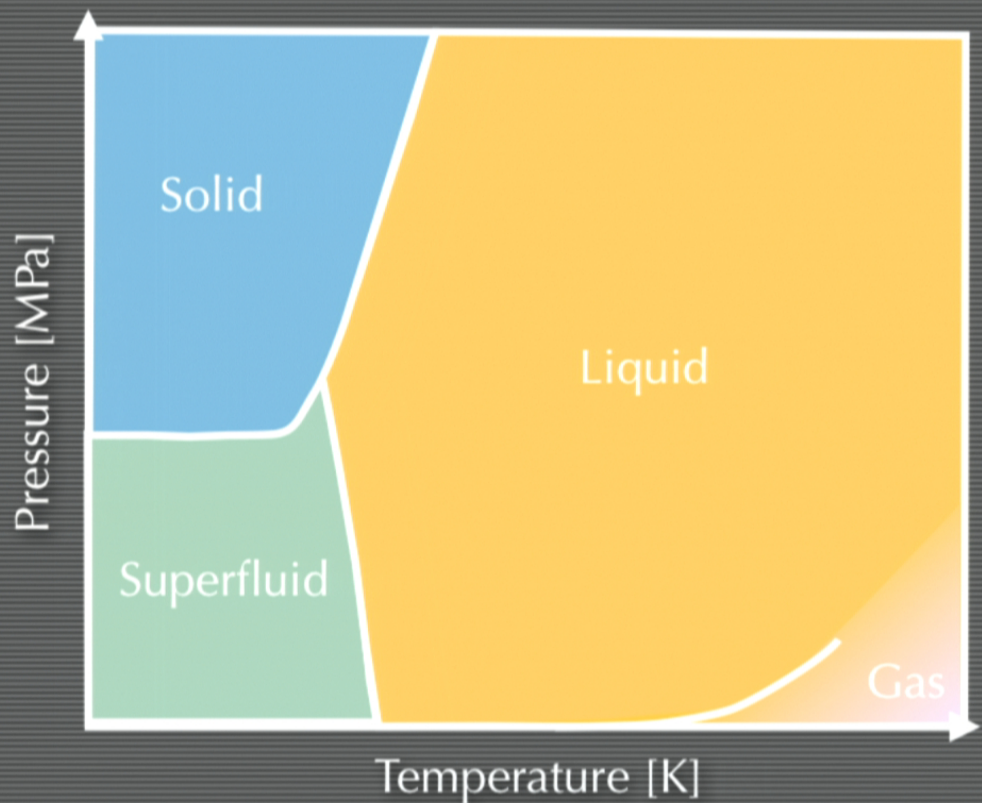
Conventional Phases of Matter



Conventional Phases of Matter



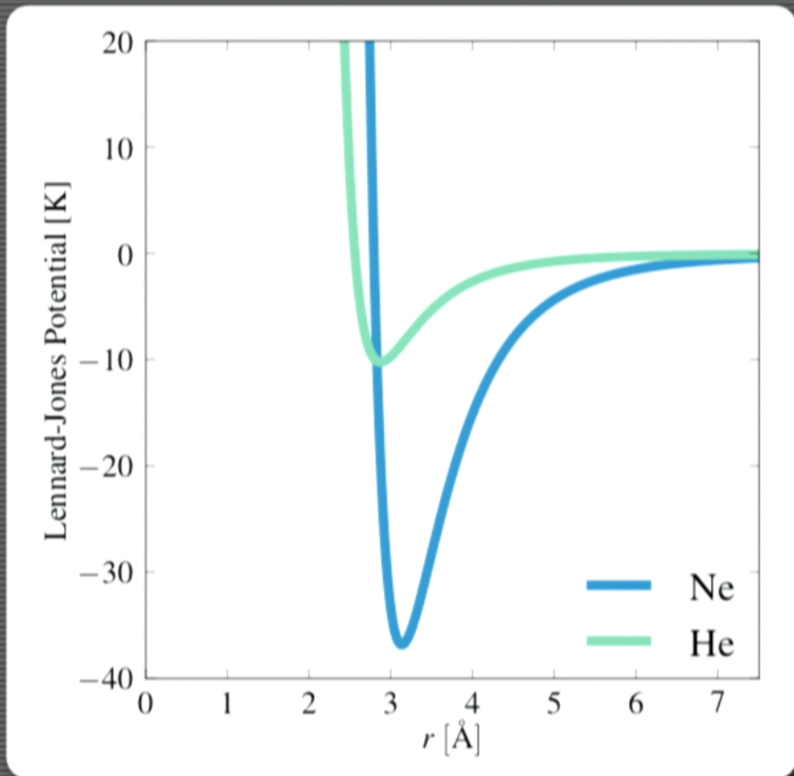
Unconventional Helium-4



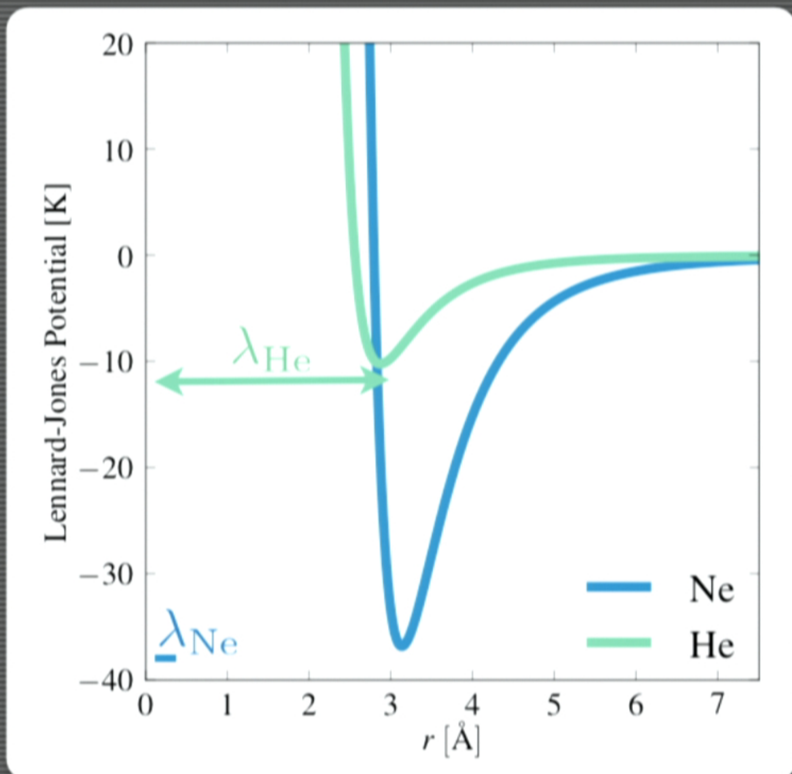
- Superfluid is a fundamentally quantum state of matter
- Flow without viscosity allows superfluid to pass through narrow channels (superleaks)

P. Kapitsa; J. Allen and A.D. Misener (1938)

Why is Helium-4 a Quantum Fluid?

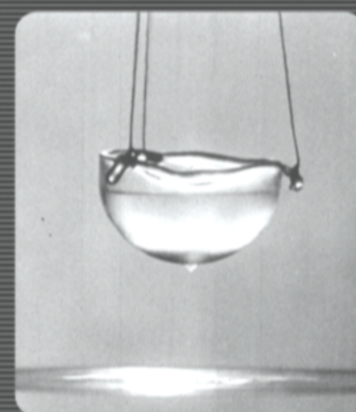


Why is Helium-4 a Quantum Fluid?



$$\lambda_{\mathrm{dB}} = \sqrt{\frac{2\pi\hbar^2}{m k_{\mathrm{B}} T}}$$

- Helium-4 is the only bosonic system with $\lambda \sim r_s$ at low temperature



What happens inside a ^4He superleak that approaches the one dimensional limit?

Enhancement of Fluctuations

- A stable long-range-ordered solid cannot exist in one dimension (even at T = 0).

- For a 1d harmonic system, the intensity of order can be described by the Debye-Waller factor: e^{-W}

$$W \sim \frac{\hbar}{2m\rho} \int \frac{d^d k}{2\pi} \frac{n(\omega_{\vec{k}}) + 1/2}{\omega_{\vec{k}}}$$

Particle Statistics

- Normally, we classify bosons and fermions by the effect of particle permutations on the wave function.

Bosons	Fermions

$$\Psi(\vec{r}_1, \vec{r}_2) = \Psi(\vec{r}_2, \vec{r}_1) \qquad \Psi(\vec{r}_1, \vec{r}_2) = -\Psi(\vec{r}_2, \vec{r}_1)$$

- In strictly 1d, we do not have the *phase space* to permute and particle statistics become unimportant at low densities

Exact and Effective Solutions

- Exact solutions available at:

$T = 0 \quad , \quad L \to \infty$ E. Lieb and W. Liniger, PR **130**, 1616 (1963)

$T > 0 \quad , \quad L \to \infty$ C.N. Yang and C.P. Yang, J. Math. Phys. **10**, 1115 (1969)

- Effective low energy description in terms of the Luttinger liquid theory of harmonic density fluctuations where interactions can be included non-perturbatively.

J.M. Luttinger, J. Math. Phys. **4**, 1154 (1963)

F.D.M. Haldane, PRL **47**, 1840 (1981)

Luttinger Liquid Theory

F.D.M. Haldane, PRL **47**, 1840 (1981)

- Begin with a 1d interacting Bose gas:

$$H = \frac{\hbar^2}{2m}\int dx\,|\partial_x \Psi(x)|^2 + \int dx \int dx' \Psi^\dagger(x)\Psi(x)V(|x-x'|)\Psi^\dagger(x')\Psi(x')$$

$$\rho(x) = \Psi^\dagger(x)\Psi(x) \qquad \Psi^\dagger(x) \sim \sqrt{\partial_x\theta(x)}\mathrm{e}^{i\phi(x)}$$

Luttinger Liquid Theory

F.D.M. Haldane, PRL **47**, 1840 (1981)

- Begin with a 1d interacting Bose gas:

$$H = \frac{\hbar^2}{2m}\int dx\, |\partial_x \Psi(x)|^2 + \int dx \int dx' \Psi^\dagger(x)\Psi(x)V(|x-x'|)\Psi^\dagger(x')\Psi(x')$$

$\rho(x) = \Psi^\dagger(x)\Psi(x) \qquad \Psi^\dagger(x) \sim \sqrt{\partial_x\theta(x)}\mathrm{e}^{i\phi(x)}$

$$H - \mu N = \frac{v}{2\pi}\int dx \left[\frac{1}{K}\left(\partial_x\phi\right)^2 + K\left(\partial_x\theta\right)^2\right]$$

$\frac{v}{K} \sim$ superfluid density $\qquad vK \sim$ compressibility

Luttinger Liquid Theory

F.D.M. Haldane, PRL **47**, 1840 (1981)

- Begin with a 1d interacting Bose gas:

$$H = \frac{\hbar^2}{2m}\int dx\, |\partial_x \Psi(x)|^2 + \int dx \int dx' \Psi^\dagger(x)\Psi(x)V(|x-x'|)\Psi^\dagger(x')\Psi(x')$$

$$\rho(x) = \Psi^\dagger(x)\Psi(x) \qquad \Psi^\dagger(x) \sim \sqrt{\partial_x \theta(x)}\mathrm{e}^{i\phi(x)}$$

$$H - \mu N = \frac{v}{2\pi}\int dx \left[\frac{1}{K}\left(\partial_x \phi\right)^2 + K\left(\partial_x \theta\right)^2\right]$$

$\frac{v}{K} \sim$ superfluid density $\qquad vK \sim$ compressibility

Luttinger Liquid Theory

F.D.M. Haldane, PRL **47**, 1840 (1981)

- Begin with a 1d interacting Bose gas:

$$H = \frac{\hbar^2}{2m}\int dx\, |\partial_x \Psi(x)|^2 + \int dx \int dx' \Psi^\dagger(x)\Psi(x)V(|x-x'|)\Psi^\dagger(x')\Psi(x')$$

$$\rho(x) = \Psi^\dagger(x)\Psi(x) \qquad \Psi^\dagger(x) \sim \sqrt{\partial_x \theta(x)}\mathrm{e}^{i\phi(x)}$$

$$H - \mu N = \frac{v}{2\pi}\int dx \left[\frac{1}{K}\left(\partial_x \phi\right)^2 + K\left(\partial_x \theta\right)^2\right]$$

$\frac{v}{K} \sim$ superfluid density $\qquad vK \sim$ compressibility

superfluid $\longrightarrow$ solid

$K = 0$ $\qquad$ $K = \infty$

Luttinger Liquid Theory

F.D.M. Haldane, PRL **47**, 1840 (1981)

- Begin with a 1d interacting Bose gas:

$$H = \frac{\hbar^2}{2m}\int dx\, |\partial_x \Psi(x)|^2 + \int dx \int dx' \Psi^\dagger(x)\Psi(x)V(|x-x'|)\Psi^\dagger(x')\Psi(x')$$

$$\rho(x) = \Psi^\dagger(x)\Psi(x) \qquad \Psi^\dagger(x) \sim \sqrt{\partial_x \theta(x)}\mathrm{e}^{i\phi(x)}$$

$$H - \mu N = \frac{v}{2\pi}\int dx \left[\frac{1}{K}\left(\partial_x \phi\right)^2 + K\left(\partial_x \theta\right)^2\right]$$

$\frac{v}{K} \sim$ superfluid density $\qquad vK \sim$ compressibility

superfluid $\longrightarrow$ solid

$K = 0$ $\qquad\qquad$ $K = \infty$

Luttinger Liquid Theory

F.D.M. Haldane, PRL **47**, 1840 (1981)

- Begin with a 1d interacting Bose gas:

$$H = \frac{\hbar^2}{2m}\int dx\,|\partial_x \Psi(x)|^2 + \int dx \int dx'\,\Psi^\dagger(x)\Psi(x)V(|x-x'|)\Psi^\dagger(x')\Psi(x')$$

$\rho(x) = \Psi^\dagger(x)\Psi(x) \qquad \Psi^\dagger(x) \sim \sqrt{\partial_x\theta(x)}\mathrm{e}^{i\phi(x)}$

$$H - \mu N = \frac{v}{2\pi}\int dx \left[\frac{1}{K}\left(\partial_x\phi\right)^2 + K\left(\partial_x\theta\right)^2\right]$$

$\frac{v}{K} \sim$ superfluid density $\qquad vK \sim$ compressibility

superfluid → solid

$K = 0$ → $K = \infty$

Luttinger Liquid Theory

F.D.M. Haldane, PRL **47**, 1840 (1981)

- Begin with a 1d interacting Bose gas:

$$H = \frac{\hbar^2}{2m}\int dx\, |\partial_x \Psi(x)|^2 + \int dx \int dx' \Psi^\dagger(x)\Psi(x)V(|x-x'|)\Psi^\dagger(x')\Psi(x')$$

$$\rho(x) = \Psi^\dagger(x)\Psi(x) \qquad \Psi^\dagger(x) \sim \sqrt{\partial_x \theta(x)}\mathrm{e}^{i\phi(x)}$$

$$H - \mu N = \frac{v}{2\pi}\int dx \left[\frac{1}{K}\left(\partial_x \phi\right)^2 + K\left(\partial_x \theta\right)^2\right]$$

$\frac{v}{K} \sim$ superfluid density $vK \sim$ compressibility

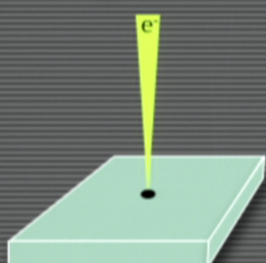
superfluid → solid

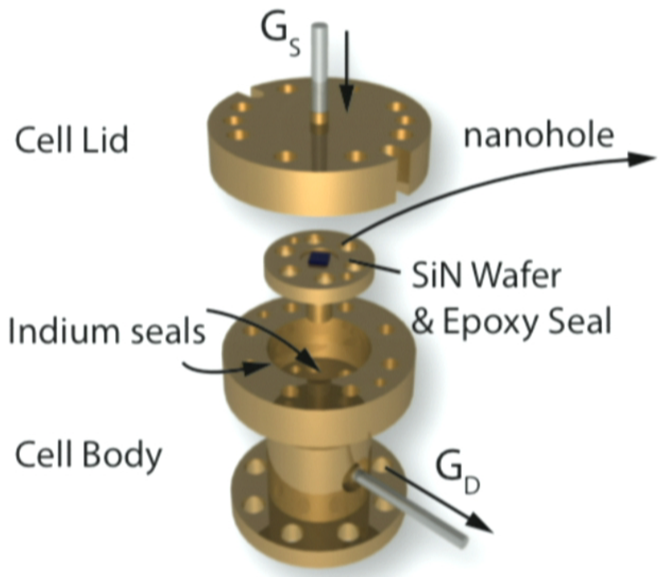
$K = 0$ → $K = \infty$

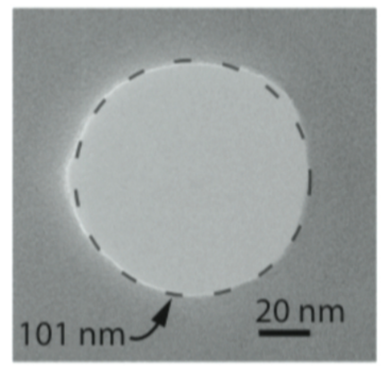
Dimensionally Confined ^{4}He

M. Savard, C. Tremblay-Darveau, and G. Gervais, PRL **103**, 104502 (2009)

pores made via TEM exposure

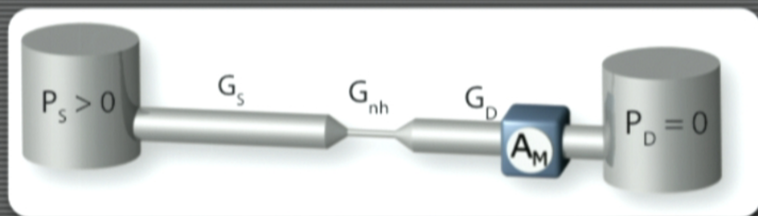






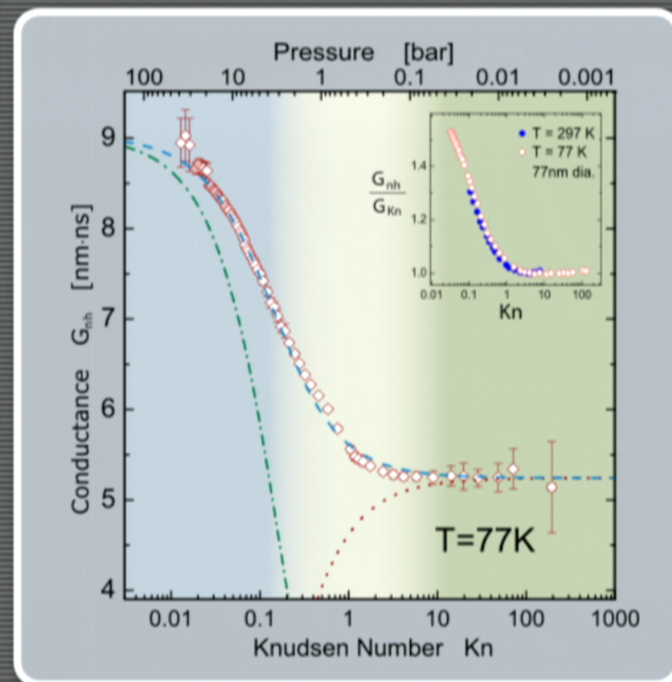
Dimensionally Confined ^{4}He

M. Savard, C. Tremblay-Darveau, and G. Gervais, PRL **103**, 104502 (2009)

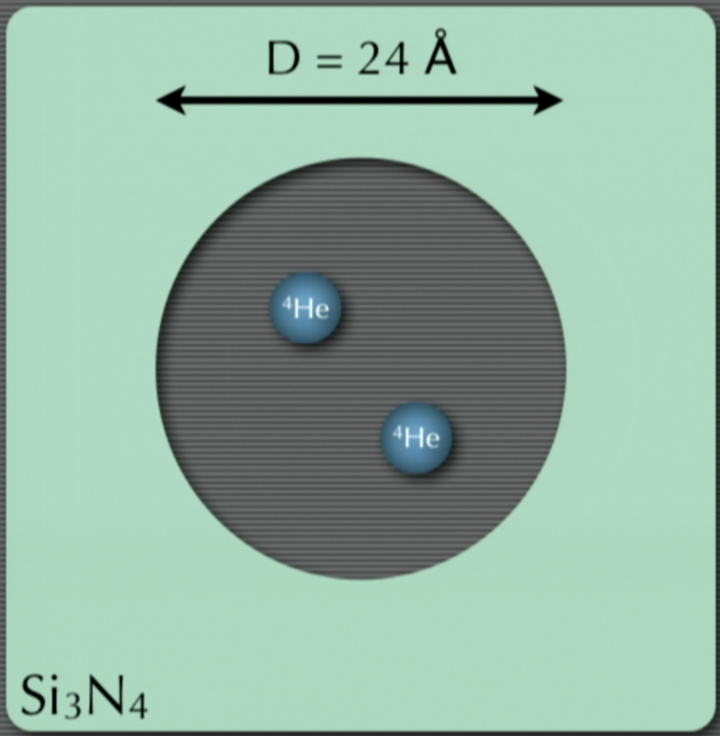


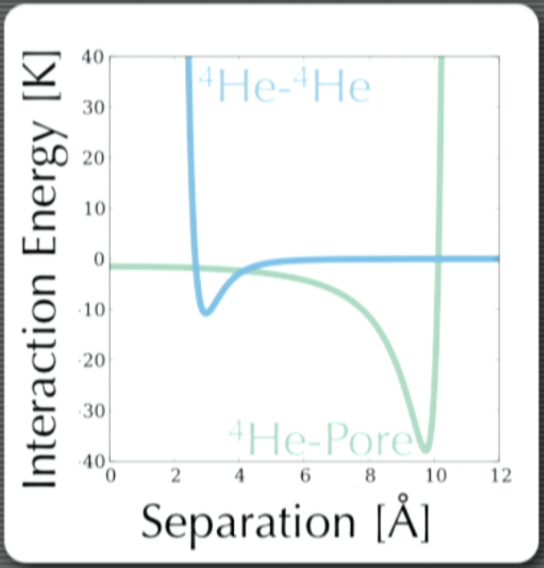
high temperature mass conductance completely understood in terms of Knudsen number

$$K_n = \frac{\text{mean free path}}{\text{pore diameter}}$$

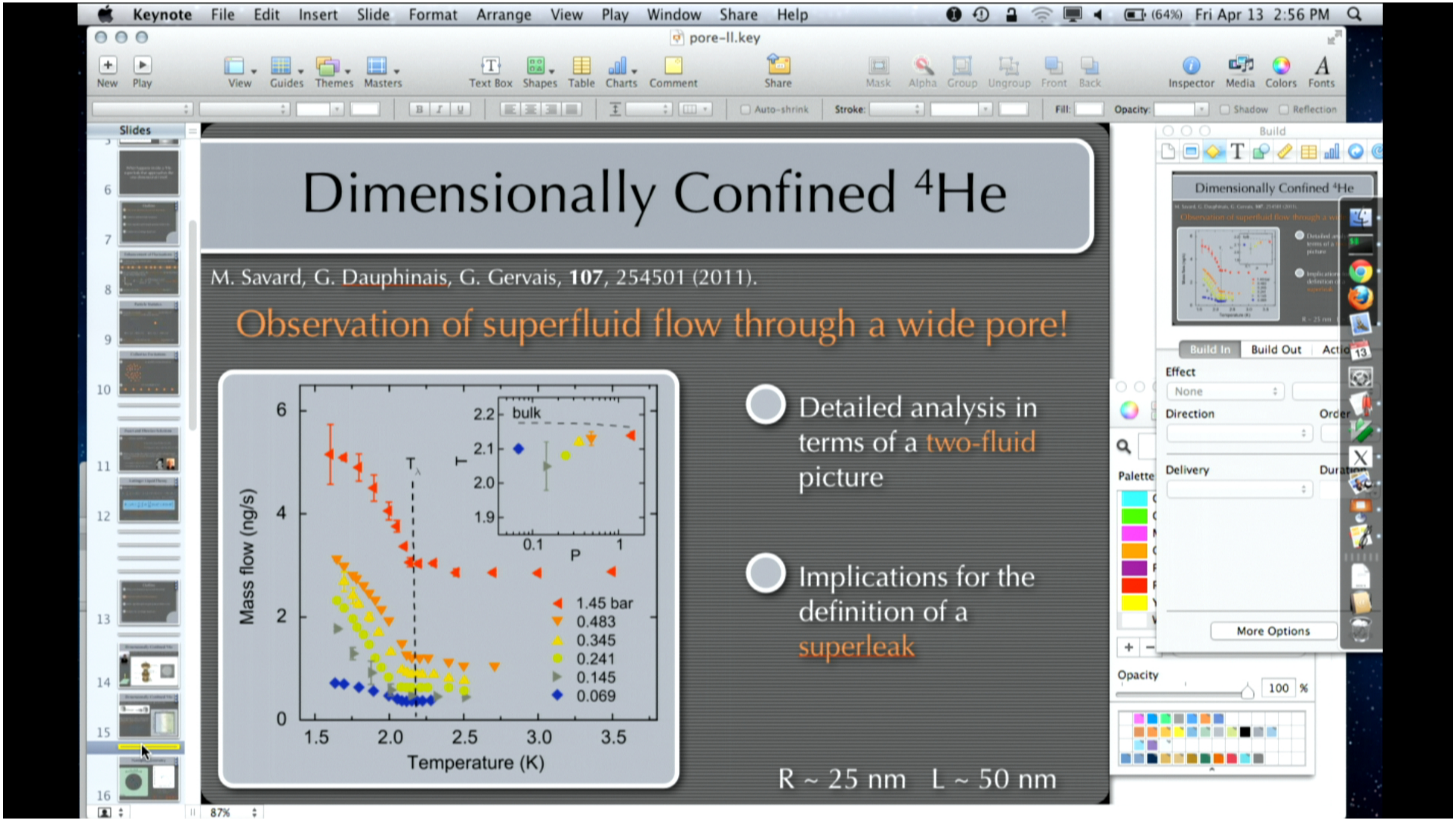


Nanopore Geometry

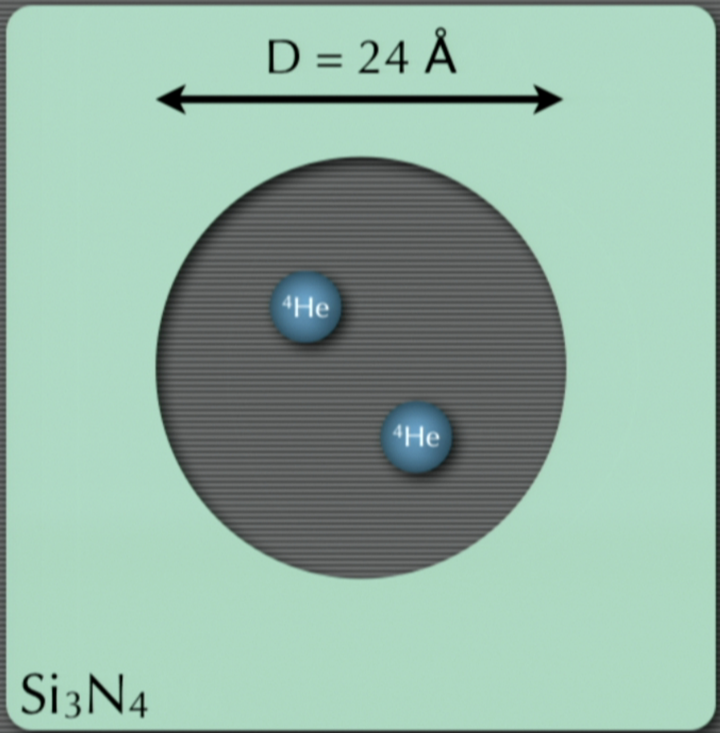


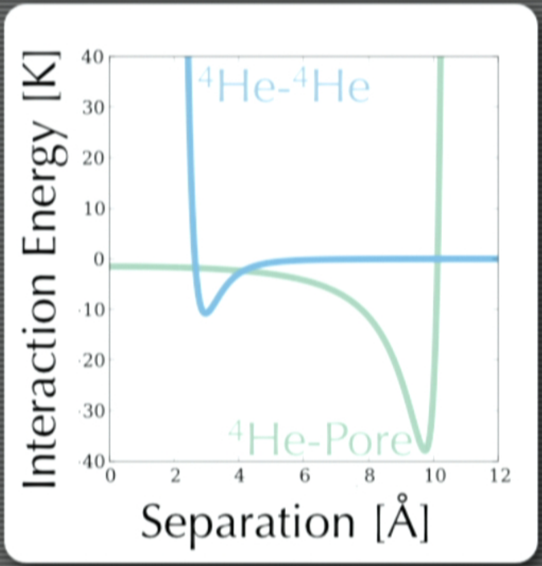


R. A. Aziz *et al.*, J. Chem. Phys. **70**, 4330 (1979)
G. Stan and M. W. Cole, Surf. Sci. **395**, 280 (1998)

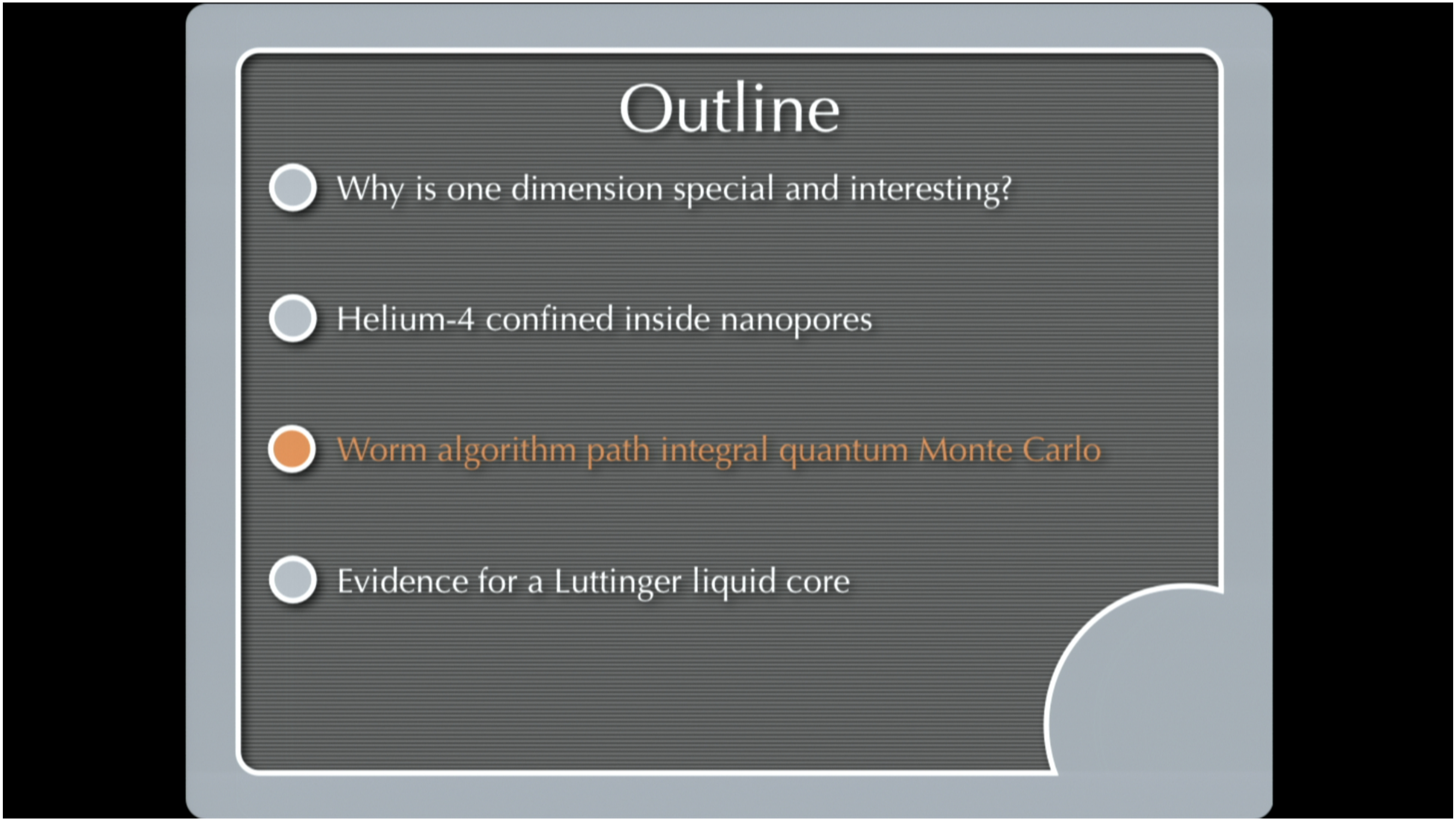

Dimensionally Confined ⁴He
M. Savard, G. Dauphinais, G. Gervais, 107, 254501 (2011).
Observation of superfluid flow through a wide pore!
Mass flow (ng/s)
Temperature (K)
Tλ
bulk
T
P
1.45 bar
0.483
0.345
0.241
0.145
0.069
Detailed analysis in terms of a two-fluid picture
Implications for the definition of a superleak
R ~ 25 nm L ~ 50 nm

Nanopore Geometry





R. A. Aziz *et al.*, J. Chem. Phys. **70**, 4330 (1979)
G. Stan and M. W. Cole, Surf. Sci. **395**, 280 (1998)


Outline
Why is one dimension special and interesting?
Helium-4 confined inside nanopores
Worm algorithm path integral quantum Monte Carlo
Evidence for a Luttinger liquid core

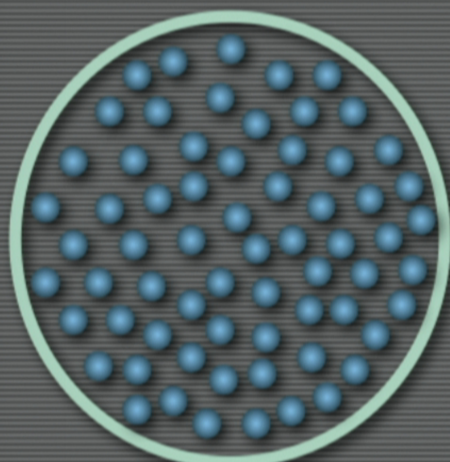
Path Integral Quantum Monte Carlo

(PIMC)

D. M. Ceperly, RMP **67**, 279 (1995)

- Want to study the quantum N-body problem characterized by:

$$\hat{H} = -\frac{\hbar^2}{2m}\sum_i \hat{\nabla}_i^2 + \sum_i \hat{V}_{\mathrm{ext}}(\vec{r}_i) + \sum_{i<j} \hat{V}_{\mathrm{int}}(|\vec{r}_i - \vec{r}_j|)$$



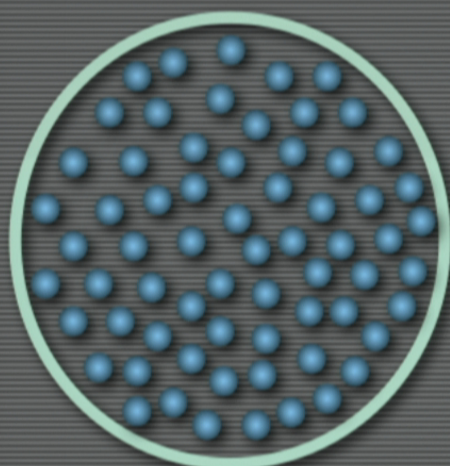
$$|R\rangle = |\vec{r}_1, \ldots \vec{r}_N\rangle$$

Path Integral Quantum Monte Carlo
(PIMC)

D. M. Ceperly, RMP **67**, 279 (1995)

- Want to study the quantum N-body problem characterized by:

$$\hat{H} = -\frac{\hbar^2}{2m}\sum_i \hat{\nabla}_i^2 + \sum_i \hat{V}_{\mathrm{ext}}(\vec{r}_i) + \sum_{i<j} \hat{V}_{\mathrm{int}}(|\vec{r}_i - \vec{r}_j|)$$



- Compute the quantum partition function

$$\begin{aligned}\mathcal{Z} &= \mathrm{Tr}\,\mathrm{e}^{-\beta\hat{H}} \\ &= \int dr_1 \cdots \int dr_N \, \langle r_1, \ldots r_N | \mathrm{e}^{-\beta\hat{H}} | r_1, \ldots, r_N \rangle \\ &= \int \mathcal{D}R \, \langle R | \mathrm{e}^{-\beta\hat{H}} | R \rangle\end{aligned}$$

$$|R\rangle = |\vec{r}_1, \ldots \vec{r}_N\rangle \qquad \beta = \frac{1}{k_{\mathrm{B}}T}$$

- Quantum statistical mechanics is **hard** because:

$$e^{-\beta\hat{H}} = e^{-\beta(\hat{T}+\hat{V})} \neq e^{-\beta\hat{T}} e^{-\beta\hat{V}}$$

Corrections are of order β, so at low T, we make huge errors

- Hamiltonian commutes with itself!

$$e^{-(\beta/2+\beta/2)\hat{H}} = e^{-(\beta/2)\hat{H}} e^{-(\beta/2)\hat{H}}$$

higher temperature

- Performing this convolution M times: $\tau = \frac{\beta}{M}$

$$\mathcal{Z} = \int \mathcal{D}R_0 \cdots \int \mathcal{D}R_{M-1} \langle R_0|e^{-\tau\hat{H}}|R_1\rangle \cdots \langle R_{M-1}|e^{-\tau\hat{H}}|R_0\rangle$$

- Quantum statistical mechanics is **hard** because:

$$e^{-\beta\hat{H}} = e^{-\beta(\hat{T}+\hat{V})} \neq e^{-\beta\hat{T}}e^{-\beta\hat{V}}$$

Corrections are of order β, so at low T, we make huge errors

- Hamiltonian commutes with itself!

$$e^{-(\beta/2+\beta/2)\hat{H}} = e^{-(\beta/2)\hat{H}}e^{-(\beta/2)\hat{H}}$$

higher temperature

- Performing this convolution M times: $\tau = \frac{\beta}{M}$

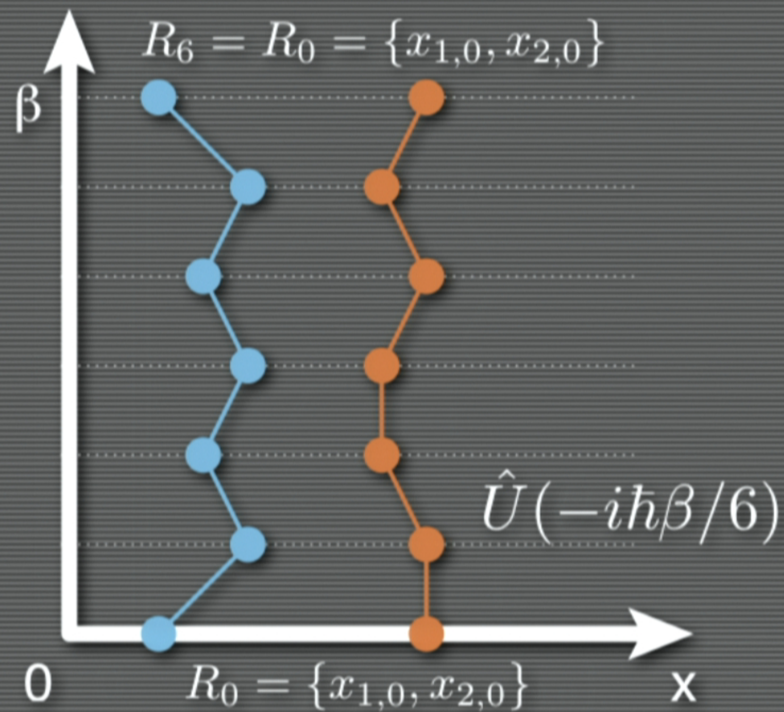
$$\mathcal{Z} = \int \mathcal{D}R_0 \cdots \int \mathcal{D}R_{M-1}\, \langle R_0|e^{-\tau\hat{H}}|R_1\rangle \cdots \langle R_{M-1}|e^{-\tau\hat{H}}|R_0\rangle$$

- Consider a 1d system, consisting of N = 2 particles with M = 6:

$$\mathcal{Z} = \int \mathcal{D}R_0 \cdots \int \mathcal{D}R_5 \, \langle R_0|\mathrm{e}^{-\tau\hat{H}}|R_1\rangle \cdots \langle R_5|\mathrm{e}^{-\tau\hat{H}}|R_0\rangle$$

$\tau = \beta/6$

$|R_m\rangle = |x_{1,m}, x_{2,m}\rangle$

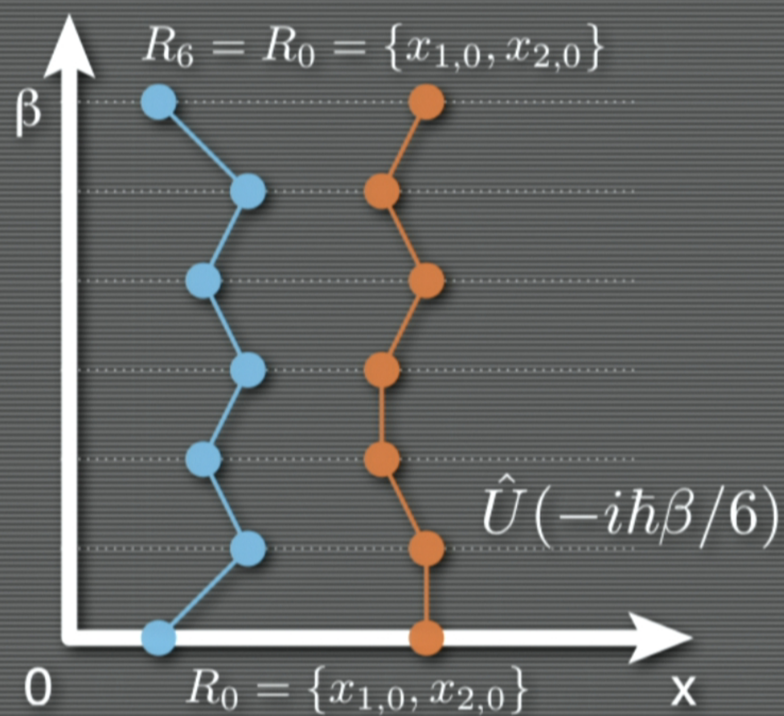


Consider a 1d system, consisting of N = 2 particles with M = 6:

$$\mathcal{Z} = \int \mathcal{D}R_0 \cdots \int \mathcal{D}R_5 \, \langle R_0 | e^{-\tau \hat{H}} | R_1 \rangle \cdots \langle R_5 | e^{-\tau \hat{H}} | R_0 \rangle$$

$$\tau = \beta/6$$

$$|R_m\rangle = |x_{1,m}, x_{2,m}\rangle$$

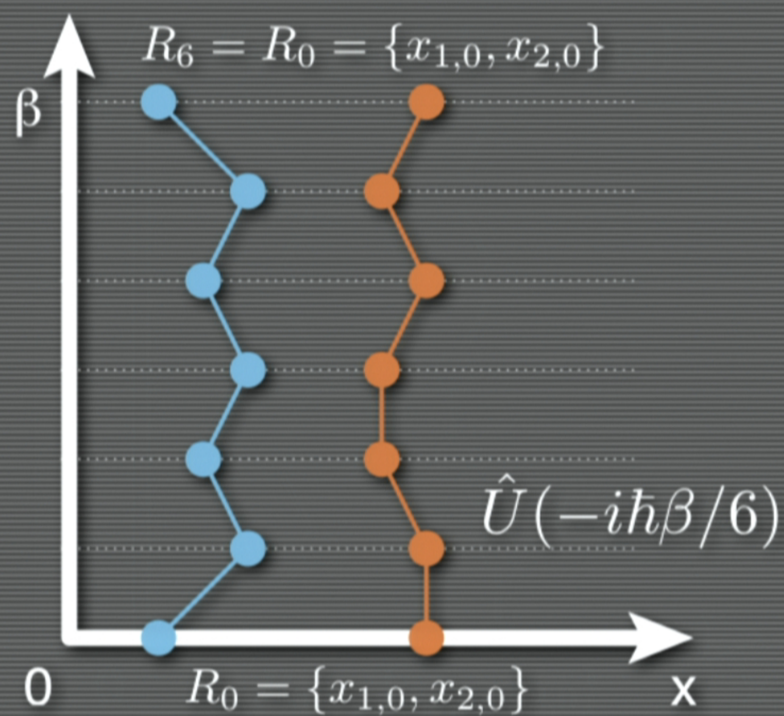


Consider a 1d system, consisting of N = 2 particles with M = 6:

$$\mathcal{Z} = \int \mathcal{D}R_0 \cdots \int \mathcal{D}R_5 \, \langle R_0|\mathrm{e}^{-\tau\hat{H}}|R_1\rangle \cdots \langle R_5|\mathrm{e}^{-\tau\hat{H}}|R_0\rangle$$

$\tau = \beta/6$

$|R_m\rangle = |x_{1,m}, x_{2,m}\rangle$

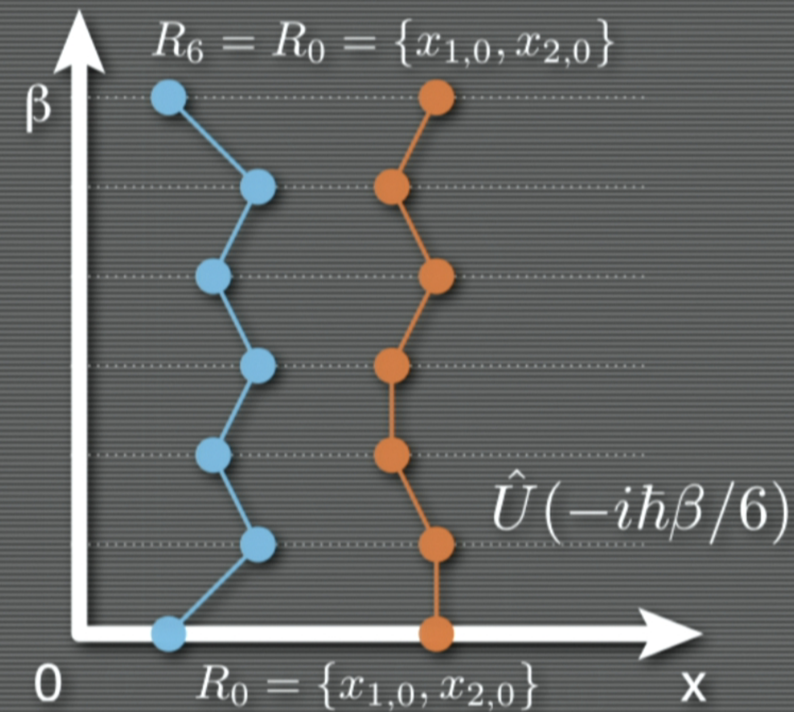


Consider a 1d system, consisting of N = 2 particles with M = 6:

$$\mathcal{Z} = \int \mathcal{D}R_0 \cdots \int \mathcal{D}R_5 \, \langle R_0 | \mathrm{e}^{-\tau \hat{H}} | R_1 \rangle \cdots \langle R_5 | \mathrm{e}^{-\tau \hat{H}} | R_0 \rangle$$

$\tau = \beta/6$

$|R_m\rangle = |x_{1,m}, x_{2,m}\rangle$

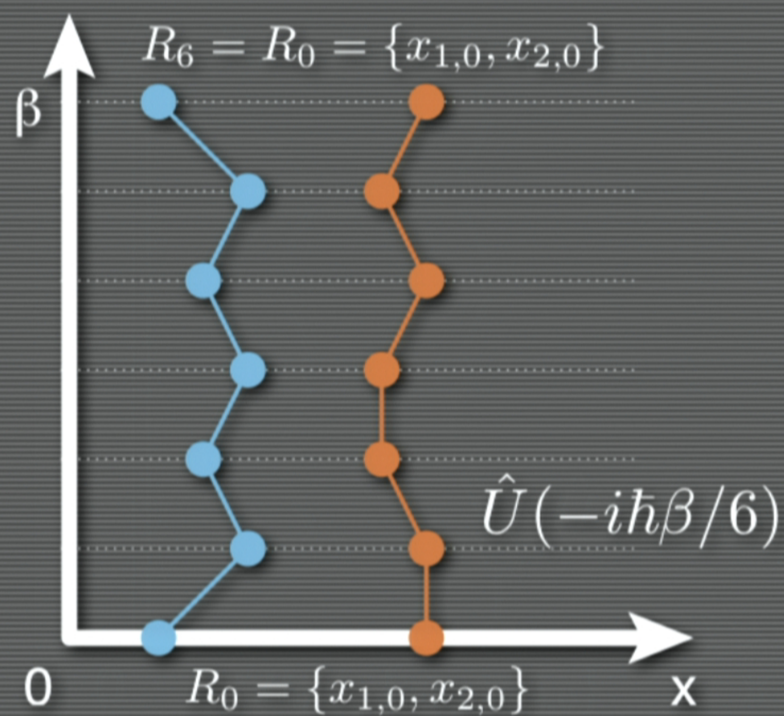


Consider a 1d system, consisting of $N = 2$ particles with $M = 6$:

$$\mathcal{Z} = \int \mathcal{D}R_0 \cdots \int \mathcal{D}R_5 \,\langle R_0|\mathrm{e}^{-\tau\hat{H}}|R_1\rangle \cdots \langle R_5|\mathrm{e}^{-\tau\hat{H}}|R_0\rangle$$

$$\tau = \beta/6$$

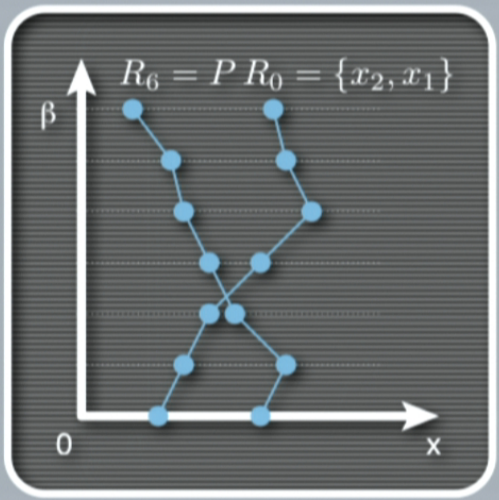
$$|R_m\rangle = |x_{1,m}, x_{2,m}\rangle$$



mapping between quantum particles and classical ring polymers

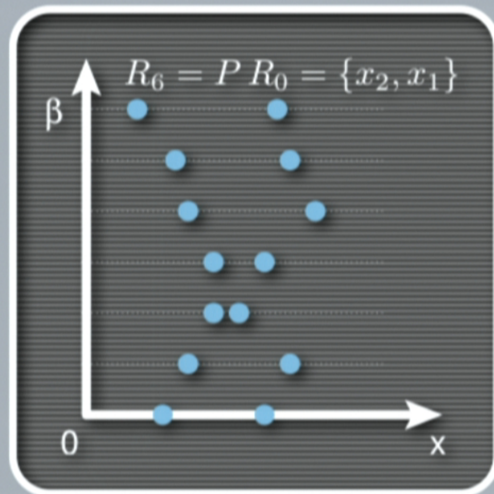
D. Chandler and P.G. Wolynes, J. Chem. Phys. 74, 4078 (1981)

identical bosons can become involved in multi-particle exchange cycles



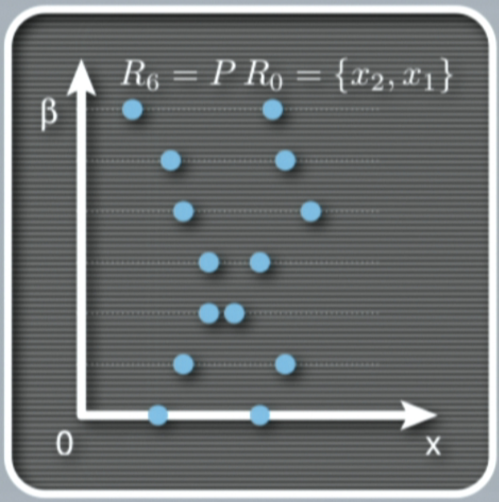
- A system of identical particles can reduce its kinetic energy by linking up the worldlines or quantum trajectories of neighboring bosons.
- As a macroscopic number of particle worldlines link up, Bose-Einstein condensation can occur.

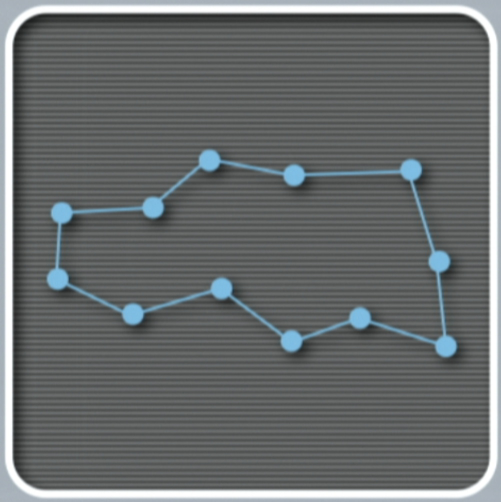
identical bosons can become involved in multi-particle exchange cycles



- A system of identical particles can reduce its kinetic energy by linking up the worldlines or quantum trajectories of neighboring bosons.
- As a macroscopic number of particle worldlines link up, Bose-Einstein condensation can occur.

identical bosons can become involved in multi-particle exchange cycles





- A system of identical particles can reduce its kinetic energy by linking up the worldlines or quantum trajectories of neighboring bosons.
- As a macroscopic number of particle worldlines link up, Bose-Einstein condensation can occur.

$$\mathcal{Z} = \int \mathcal{D}R_0 \cdots \int \mathcal{D}R_{M-1} \, \langle R_0 | \mathrm{e}^{-\tau \hat{H}} | R_1 \rangle \cdots \langle R_{M-1} | \mathrm{e}^{-\tau \hat{H}} | R_0 \rangle$$

- We can approximate the high-temperature transition amplitudes to high order in τ (exact in the limit $M \to \infty$):

D. M. Ceperly, RMP **67**, 279 (1995)

$$\mathcal{Z} = \int \mathcal{D}R_0 \cdots \int \mathcal{D}R_{M-1} \langle R_0|\mathrm{e}^{-\tau \hat{H}}|R_1\rangle \cdots \langle R_{M-1}|\mathrm{e}^{-\tau \hat{H}}|R_0\rangle$$

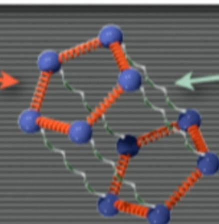
- We can approximate the high-temperature transition amplitudes to high order in τ (exact in the limit $M \to \infty$):

$$\mathcal{Z} \sim \frac{1}{N!} \sum_{P} \prod_{m=0}^{M-1} \int \mathcal{D}R_m \, \exp\left[-\frac{(R_m - R_{m+1})^2}{4\lambda\tau} - \tau U(R_m)\right]$$

$$R_M = P R_0$$

$$\lambda \equiv \frac{\hbar^2}{2m}$$

kinetic: connects time steps



potential: connects particles

Image courtesy of Carl McBride

D. M. Ceperly, RMP **67**, 279 (1995)

$$\mathcal{Z} = \int \mathcal{D}R_0 \cdots \int \mathcal{D}R_{M-1} \langle R_0 | e^{-\tau \hat{H}} | R_1 \rangle \cdots \langle R_{M-1} | e^{-\tau \hat{H}} | R_0 \rangle$$

- We can approximate the high-temperature transition amplitudes to high order in τ (exact in the limit M → ∞):

$$\mathcal{Z} \sim \frac{1}{N!} \sum_P \prod_{m=0}^{M-1} \int \mathcal{D}R_m \ \exp\left[-\frac{(R_m - R_{m+1})^2}{4\lambda\tau} - \tau U(R_m) \right]$$

$$R_M = PR_0$$

$$\lambda \equiv \frac{\hbar^2}{2m}$$

kinetic: connects time steps

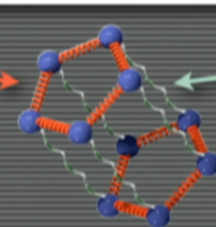


Image courtesy of Carl McBride

potential: connects particles

D. M. Ceperly, RMP **67**, 279 (1995)

Continuous Space Worm Algorithm

N. Prokof'ev et al. PLA **238**, 253 (1998)   M. Boninsegni et al. PRE **74**, 036701 (2006)

- In the conventional PIMC scheme, the computational effort required to study systems of N indistinguishable particles scales prohibitively with N
- Worm algorithm operates in an extended configuration space containing both closed and open world line configurations

$$\mathcal{Z}_W = \sum_{N=0}^{\infty} \mathcal{Z}(N) e^{\beta\mu N} + \mathcal{Z}'$$

diagonal (periodic in β) configurations present in conventional PIMC

off-diagonal worm configurations contribute to the one particle Green function

Continuous Space Worm Algorithm

N. Prokof'ev et al. PLA **238**, 253 (1998) M. Boninsegni et al. PRE **74**, 036701 (2006)

- In the conventional PIMC scheme, the computational effort required to study systems of N indistinguishable particles scales prohibitively with N
- Worm algorithm operates in an extended configuration space containing both closed and open world line configurations

$$\mathcal{Z}_W = \sum_{N=0}^{\infty} \mathcal{Z}(N) \mathrm{e}^{\beta\mu N} + \mathcal{Z}'$$

diagonal (periodic in β) configurations present in conventional PIMC

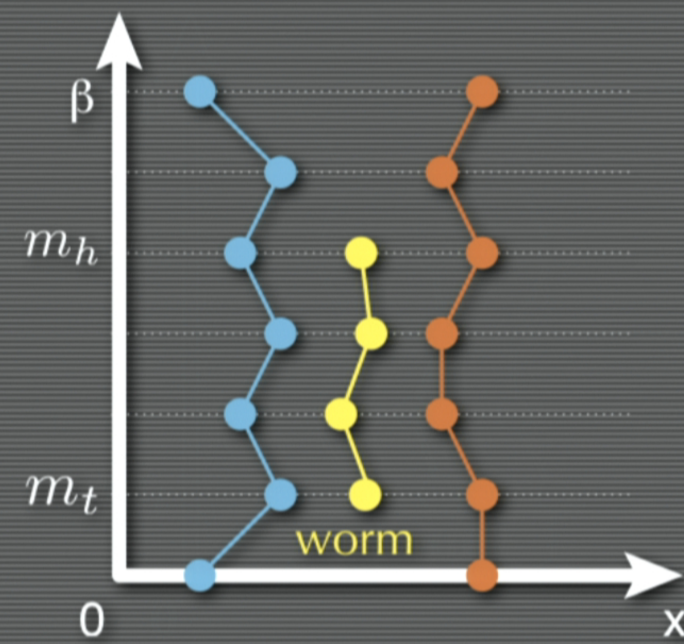
off-diagonal worm configurations contribute to the one particle Green function

N. Prokof'ev et al. PLA **238**, 253 (1998) M. Boninsegni et al. PRE **74**, 036701 (2006)

$$\mathcal{Z}' = C \sum_{m_h, m_t} \int d\vec{r}_h d\vec{r}_t g[\vec{r}_h, \vec{r}_t, \tau(m_h - m_t)]$$

$$\langle T\{\psi(\vec{r}_h, m_h\tau)\psi^\dagger(\vec{r}_t, m_t\tau)\}\rangle \equiv \frac{g[\vec{r}_h, \vec{r}_t, \tau(m_h - m_t)]}{\mathcal{Z}}$$

- Can simulate the grand canonical ensemble at fixed chemical potential μ
- Diagonal observables are only measured when no worms are present

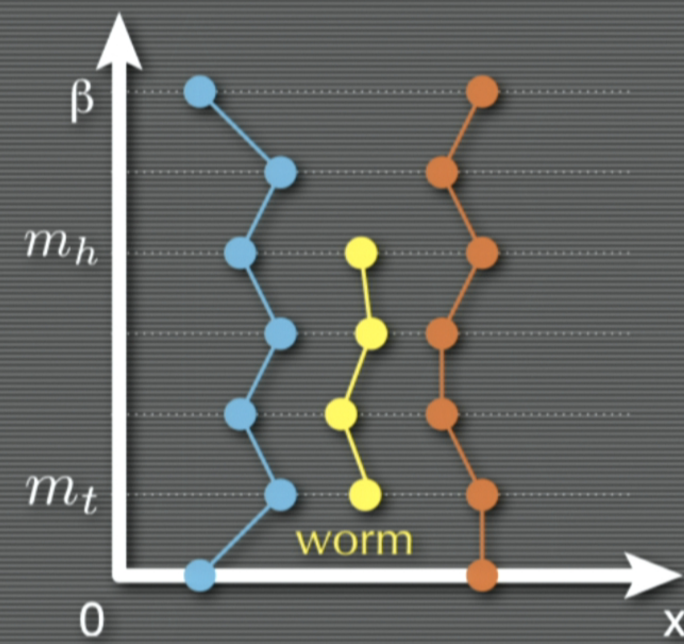


N. Prokof'ev et al. PLA **238**, 253 (1998)   M. Boninsegni et al. PRE **74**, 036701 (2006)

$$\mathcal{Z}' = C \sum_{m_h, m_t} \int d\vec{r}_h d\vec{r}_t g[\vec{r}_h, \vec{r}_t, \tau(m_h - m_t)]$$

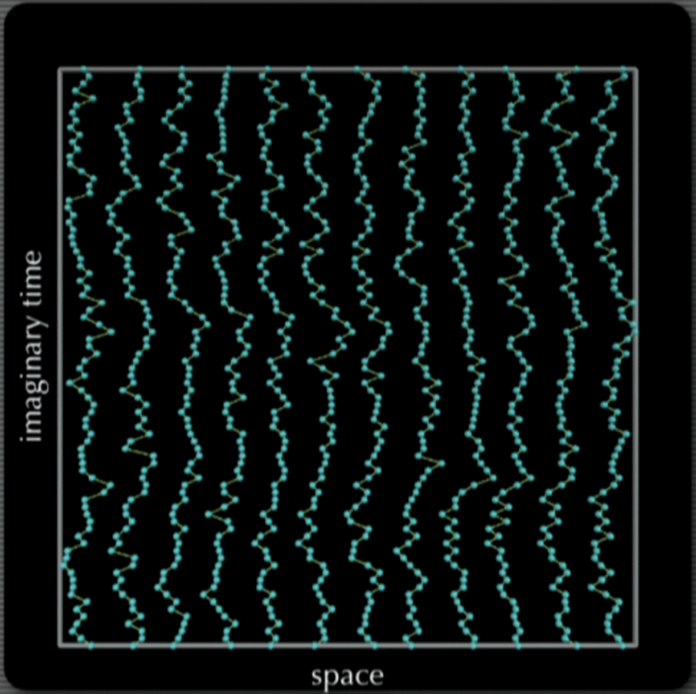
$$\langle T\{\psi(\vec{r}_h, m_h\tau)\psi^\dagger(\vec{r}_t, m_t\tau)\}\rangle \equiv \frac{g[\vec{r}_h, \vec{r}_t, \tau(m_h - m_t)]}{\mathcal{Z}}$$

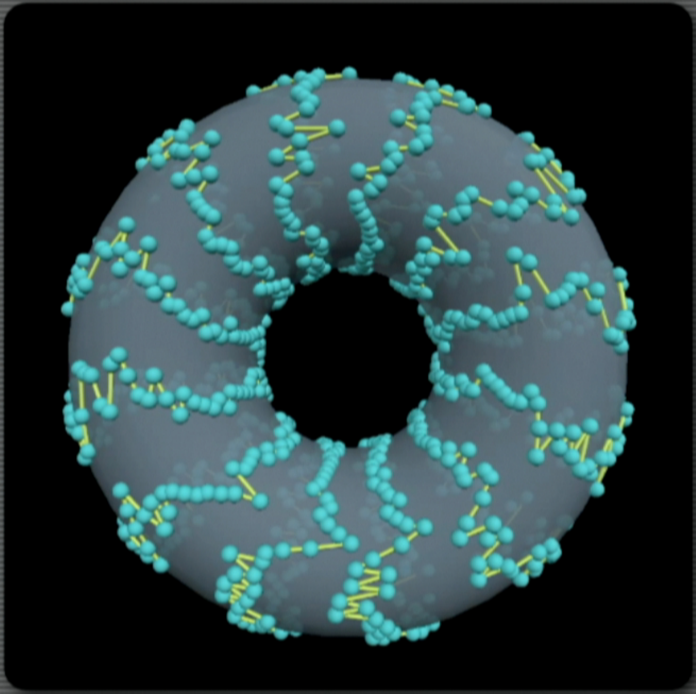
- Can simulate the grand canonical ensemble at fixed chemical potential μ
- Diagonal observables are only measured when no worms are present



1d Helium-4 Example Simulation

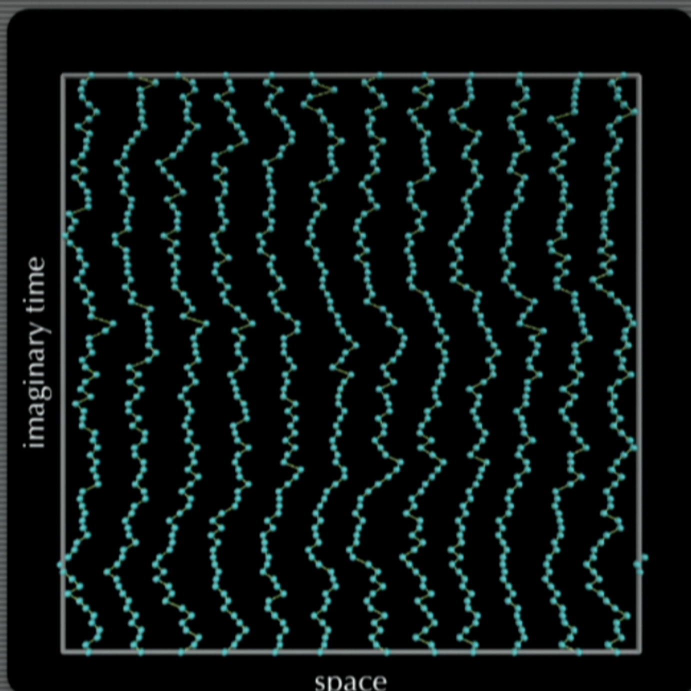
$T = 0.25\ K$ $\mu = 85\ K$ $L = 40\ Å$ $\tau = 0.05$

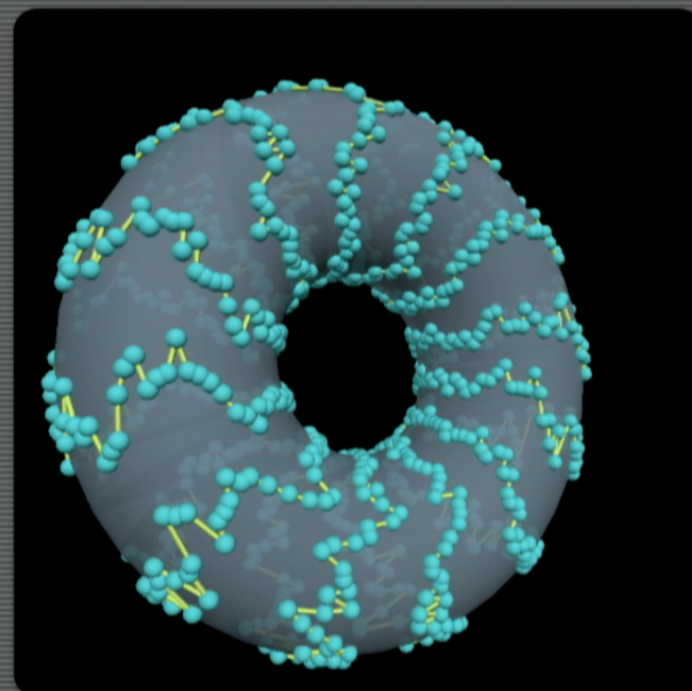




1d Helium-4 Example Simulation

$T = 0.25$ K $\mu = 85$ K $L = 40$ Å $\tau = 0.05$





Observables from PIMC

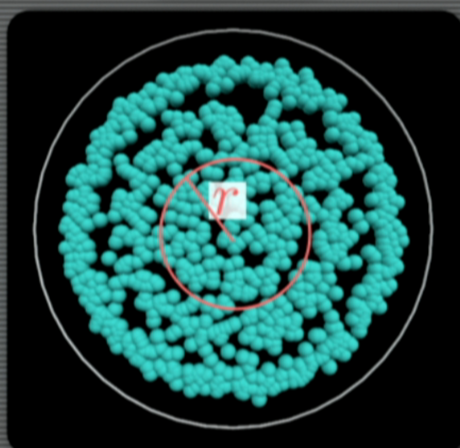
- Can easily measure any observable that can be written in the position basis. We will focus on two that provide insights into the behavior of helium inside the nanopore:

Observables from PIMC

- Can easily measure any observable that can be written in the position basis. We will focus on two that provide insights into the behavior of helium inside the nanopore:

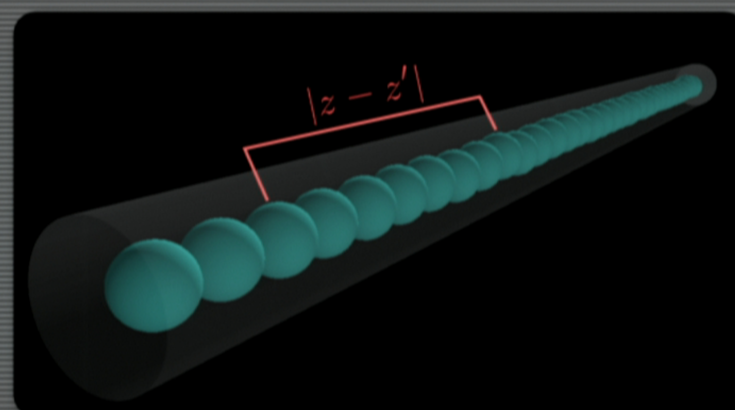
Radial Density

$\rho(r)$



Pair Correlation Function

$\rho(z)\rho(z')$

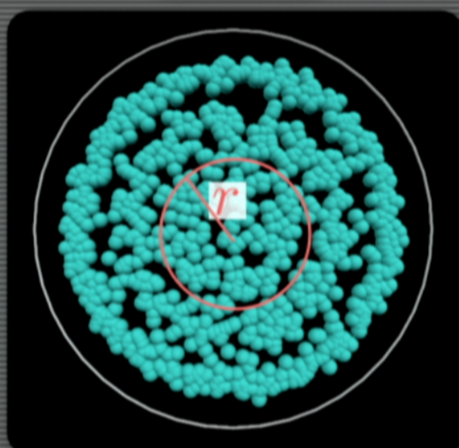


Observables from PIMC

- Can easily measure any observable that can be written in the position basis. We will focus on two that provide insights into the behavior of helium inside the nanopore:

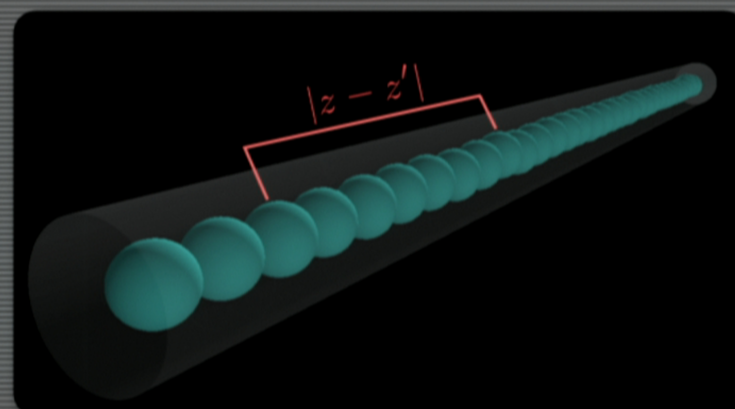
Radial Density

$\rho(r)$



Pair Correlation Function

$\rho(z)\rho(z')$

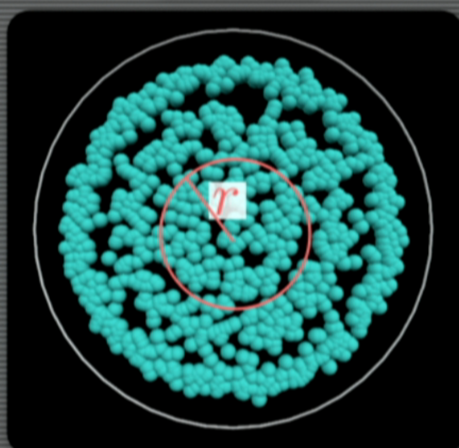


Observables from PIMC

- Can easily measure any observable that can be written in the position basis. We will focus on two that provide insights into the behavior of helium inside the nanopore:

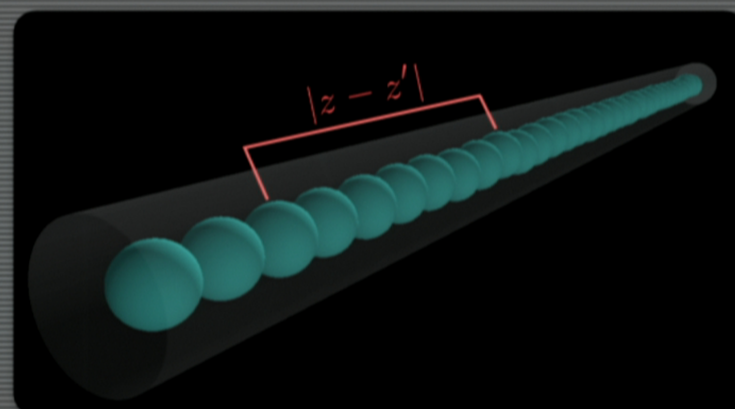
Radial Density

$\rho(r)$



Pair Correlation Function

$\rho(z)\rho(z')$



Outline

- Why is one dimension special and interesting?
- Helium-4 confined inside nanopores
- Worm algorithm path integral quantum Monte Carlo
- Evidence for a Luttinger liquid core

  A.D., arXiv:1201.4869 (2012)
  A.D., M. Boninsegni, I. Affleck, Phys. Rev. Lett. (2011)

Details of the ^{4}He Nanopores

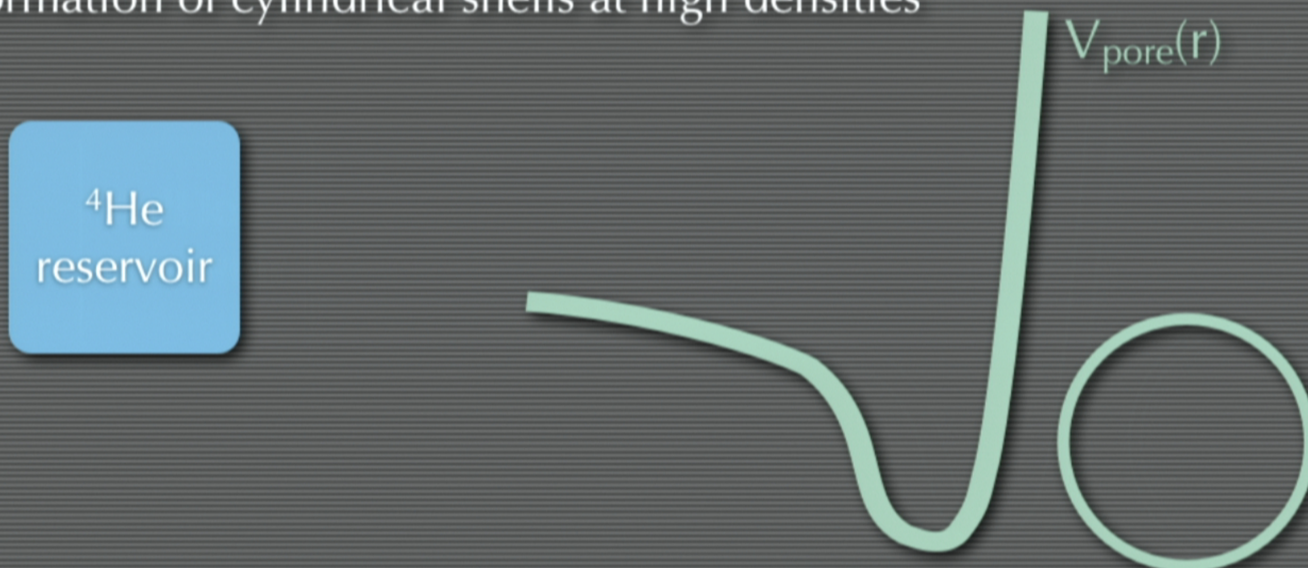
dimension	3
interaction	Aziz + Pore
number of atoms	1000+
chemical potential	-7.2 K (SVP)
length	50-200 Å
radius	2.9 - 12 Å
temperature	0.5 - 2.0 K
resources	~20 CPU Years

Details of the ^{4}He Nanopores

dimension	3
interaction	Aziz + Pore
number of atoms	1000+
chemical potential	-7.2 K (SVP)
length	50-200 Å
radius	2.9 - 12 Å
temperature	0.5 - 2.0 K
resources	~20 CPU Years

Formation of Cylindrical Shells

- The presence of the helium-pore potential leads to the formation of cylindrical shells at high densities

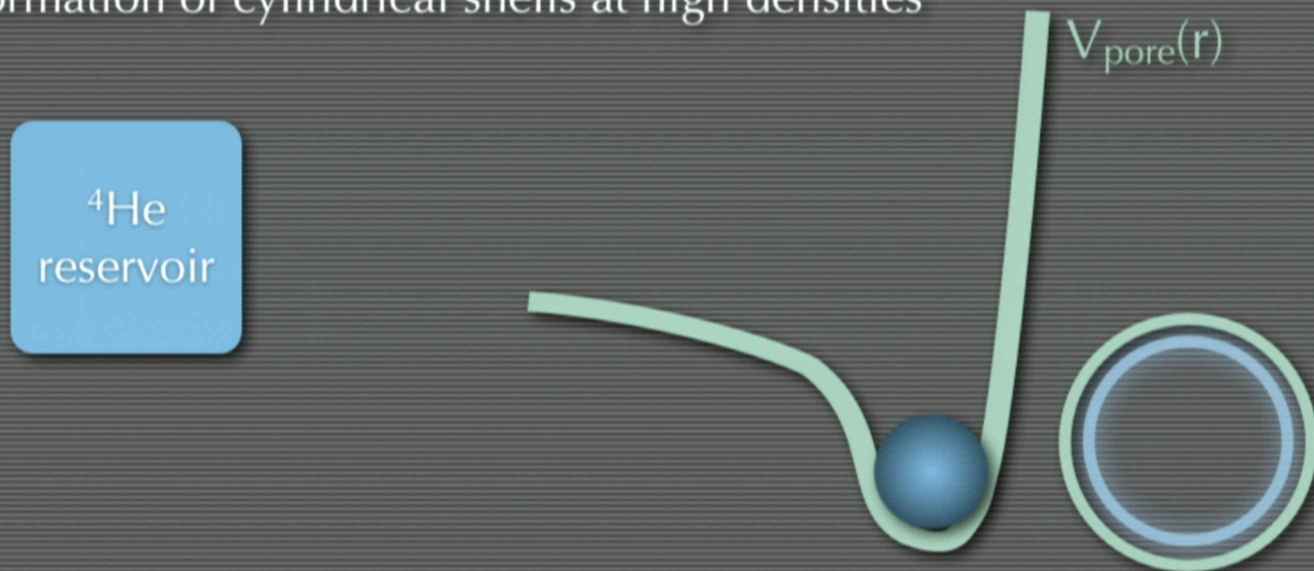


S.M. Gatica, G. Stan, M.M. Calbi, J.K. Johnson and M.W. Cole, J. Low T. Phys. **120**, 337 (2000)
M.W. Cole, V.H. Crespi, G. Stan, C. Ebner, J. M. Hartman, S. Moroni, and M. Boninsegni, Phys. Rev. Lett. **84**, 3883 (2000)
M.C. Gordillo, J. Boronat, and J. Casulleras, Phys. Rev. B. **61**, R878 (2000); **76**, 193402 (2007)

E. Hernandez, J. Low. T. Phys. **1**, (2010)
M. Rossi, D.E. Galli and L. Reatto, Phys. Rev. B **72**, 064516 (2005); J.Low Temp. Phys. **146**, 95 (2007)
N.M. Urban, M.W. Cole and E.S. Hernandez, arXiv:cond-mat/0702228 (2007)

Formation of Cylindrical Shells

- The presence of the helium-pore potential leads to the formation of cylindrical shells at high densities

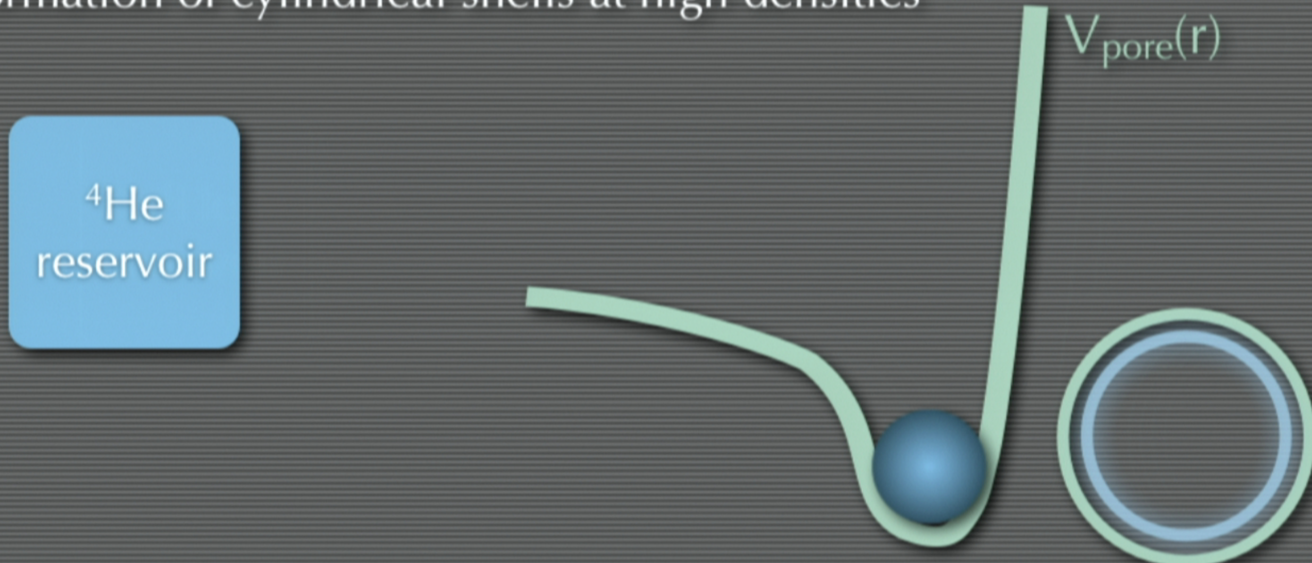


S.M. Gatica, G. Stan, M.M. Calbi, J.K. Johnson and M.W. Cole, J. Low T. Phys. **120**, 337 (2000)
M.W. Cole, V.H. Crespi, G. Stan, C. Ebner, J. M. Hartman, S. Moroni, and M. Boninsegni, Phys. Rev. Lett. **84**, 3883 (2000)
M.C. Gordillo, J. Boronat, and J. Casulleras, Phys. Rev. B. **61**, R878 (2000); **76**, 193402 (2007)

E. Hernandez, J. Low. T. Phys. **1**, (2010)
M. Rossi, D.E. Galli and L. Reatto, Phys. Rev. B **72**, 064516 (2005); J.Low Temp. Phys. **146**, 95 (2007)
N.M. Urban, M.W. Cole and E.S. Hernandez, arXiv:cond-mat/0702228 (2007)

Formation of Cylindrical Shells

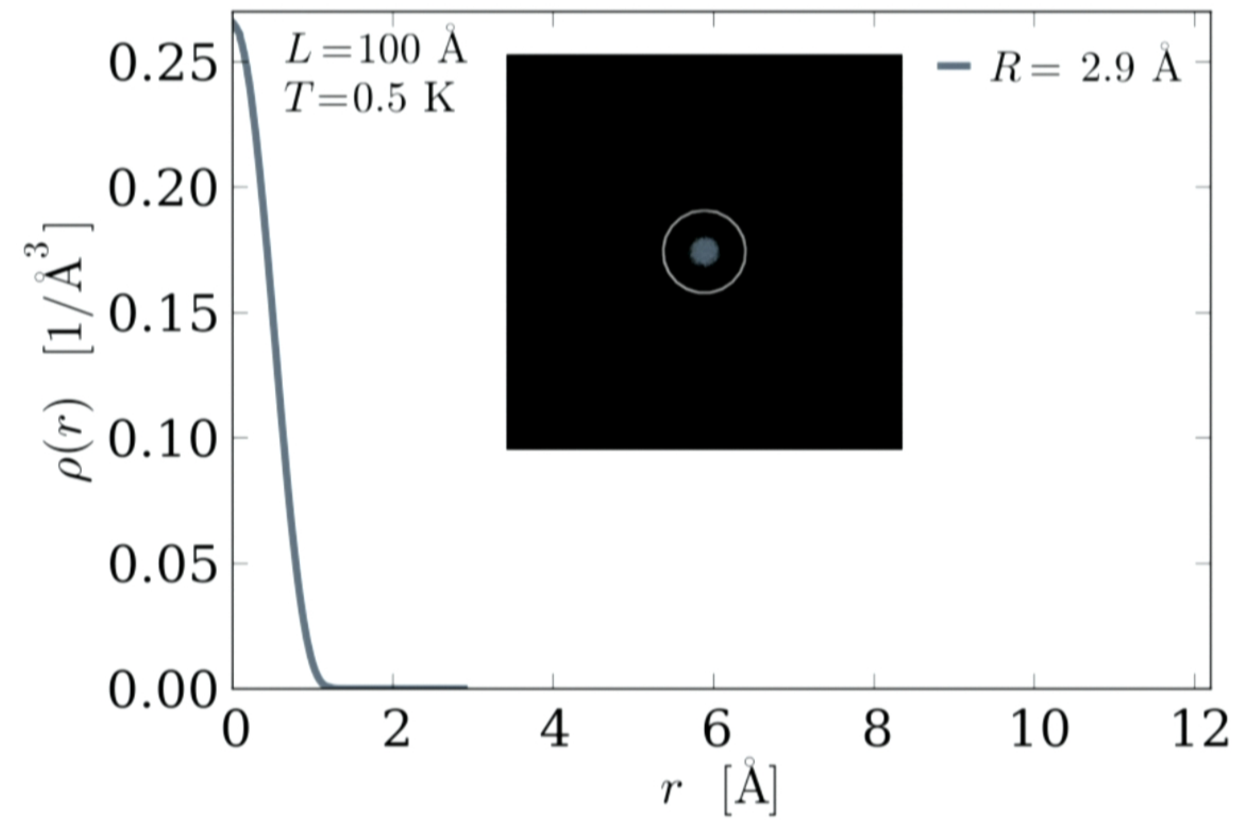
- The presence of the helium-pore potential leads to the formation of cylindrical shells at high densities



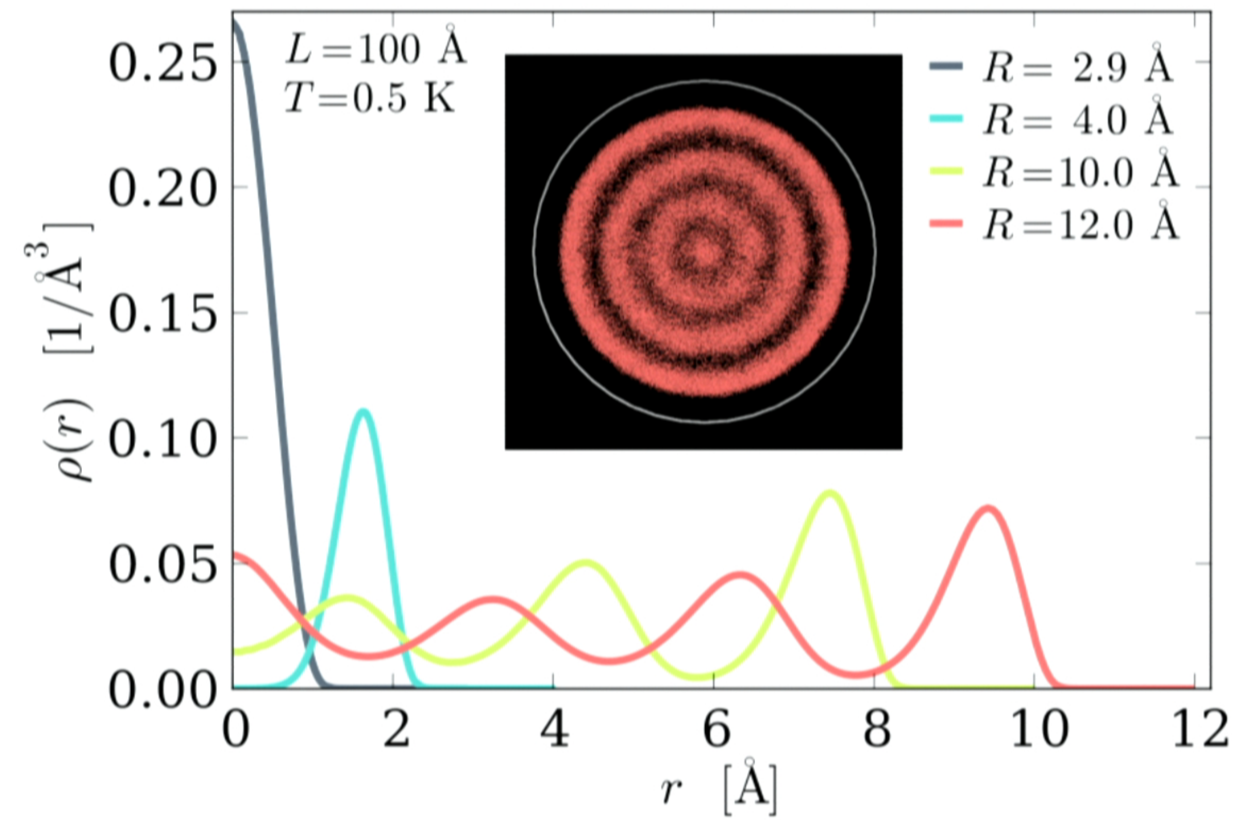
S.M. Gatica, G. Stan, M.M. Calbi, J.K. Johnson and M.W. Cole, J. Low T. Phys. **120**, 337 (2000)
M.W. Cole, V.H. Crespi, G. Stan, C. Ebner, J. M. Hartman, S. Moroni, and M. Boninsegni, Phys. Rev. Lett. **84**, 3883 (2000)
M.C. Gordillo, J. Boronat, and J. Casulleras, Phys. Rev. B. **61**, R878 (2000); **76**, 193402 (2007)

E. Hernandez, J. Low. T. Phys. **1**, (2010)
M. Rossi, D.E. Galli and L. Reatto, Phys. Rev. B **72**, 064516 (2005); J.Low Temp. Phys. **146**, 95 (2007)
N.M. Urban, M.W. Cole and E.S. Hernandez, arXiv:cond-mat/0702228 (2007)

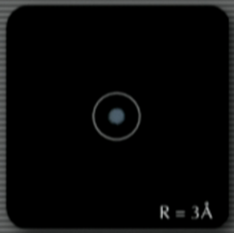
Radial Density Inside the Nanopore



Radial Density Inside the Nanopore

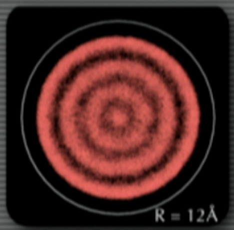


- For R ~ 3 x integer we observe a 1d core of helium with high linear density



- For R < 5 Å, we observe no evidence of superfluidity down to T = 0.5 K

- For R > 10 Å, a finite superfluid fraction exists below T = 1.0 K and helium atoms are exchanged between the outer shells



- For R ~ 3 x integer we observe a 1d core of helium with high linear density
- For $R < 5$ Å, we observe no evidence of superfluidity down to $T = 0.5$ K
- For $R > 10$ Å, a finite superfluid fraction exists below $T = 1.0$ K and helium atoms are exchanged between the outer shells

Predictions from LL Theory

- The utility of the linear hydrodynamic description is that even in the presence of interactions, the resulting theory is quadratic!

$$H - \mu N = \frac{v}{2\pi} \int dz \left[\frac{1}{K} \left(\partial_z \phi\right)^2 + K \left(\partial_z \theta\right)^2 \right]$$

Predictions from LL Theory

- The utility of the linear hydrodynamic description is that even in the presence of interactions, the resulting theory is quadratic!

$$H - \mu N = \frac{v}{2\pi} \int dz \left[\frac{1}{K} \left(\partial_z \phi \right)^2 + K \left(\partial_z \theta \right)^2 \right]$$

Predictions from LL Theory

- The utility of the linear hydrodynamic description is that even in the presence of interactions, the resulting theory is quadratic!

$$H - \mu N = \frac{v}{2\pi} \int dz \left[\frac{1}{K} \left(\partial_z \phi\right)^2 + K \left(\partial_z \theta\right)^2 \right]$$

- The details of the high-energy microscopic system (m,V,μ) are contained in the Luttinger velocity and Luttinger parameter

Axial Pair Correlation Function

$$\langle \rho(z)\rho(0)\rangle = \rho_0^2 + \frac{1}{2\pi^2 K}\frac{d^2}{dz^2}\ln\theta_1(\pi z/L, e^{-\pi v/LT})$$

$$+\mathcal{A}\cos(2\pi\rho_0 z)\left[\frac{2\eta(iv/LT)\mathrm{e}^{-\pi v/6LT}}{\theta_1(\pi z/L, e^{-\pi v/LT})}\right]^{2/K}$$

Axial Pair Correlation Function

$$\langle \rho(z)\rho(0)\rangle = \rho_0^2 + \frac{1}{2\pi^2 K}\frac{d^2}{dz^2}\ln\theta_1(\pi z/L, e^{-\pi v/LT})$$
$$+\mathcal{A}\cos(2\pi\rho_0 z)\left[\frac{2\eta(iv/LT)\mathrm{e}^{-\pi v/6LT}}{\theta_1(\pi z/L, e^{-\pi v/LT})}\right]^{2/K}$$

- Looks complicated, but in the thermodynamic limit: $\frac{L}{Tv} \to \infty$

$$\langle \rho(z)\rho(0)\rangle \to \rho_0^2 - \frac{1}{2\pi^2 K z^2} + \mathcal{A}\frac{\cos(2\pi\rho_0 z)}{z^{2/K}}$$

F.D.M. Haldane, PRL **47**, 1840 (1981)

Axial Pair Correlation Function

$$\langle \rho(z)\rho(0)\rangle = \rho_0^2 + \frac{1}{2\pi^2 K}\frac{d^2}{dz^2}\ln\theta_1(\pi z/L, e^{-\pi v/LT})$$

$$+\mathcal{A}\cos(2\pi\rho_0 z)\left[\frac{2\eta(iv/LT)\mathrm{e}^{-\pi v/6LT}}{\theta_1(\pi z/L, e^{-\pi v/LT})}\right]^{2/K}$$

Axial Pair Correlation Function

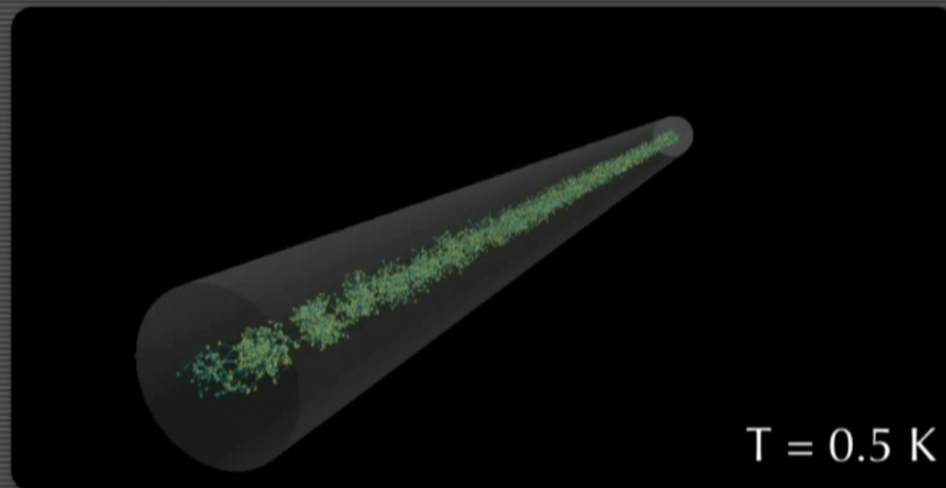
$$\langle \rho(z)\rho(0)\rangle = \rho_0^2 + \frac{1}{2\pi^2 K}\frac{d^2}{dz^2}\ln\theta_1(\pi z/L, e^{-\pi v/LT})$$

$$+\mathcal{A}\cos(2\pi\rho_0 z)\left[\frac{2\eta(iv/LT)\mathrm{e}^{-\pi v/6LT}}{\theta_1(\pi z/L, e^{-\pi v/LT})}\right]^{2/K}$$

Axial Pair Correlation Function

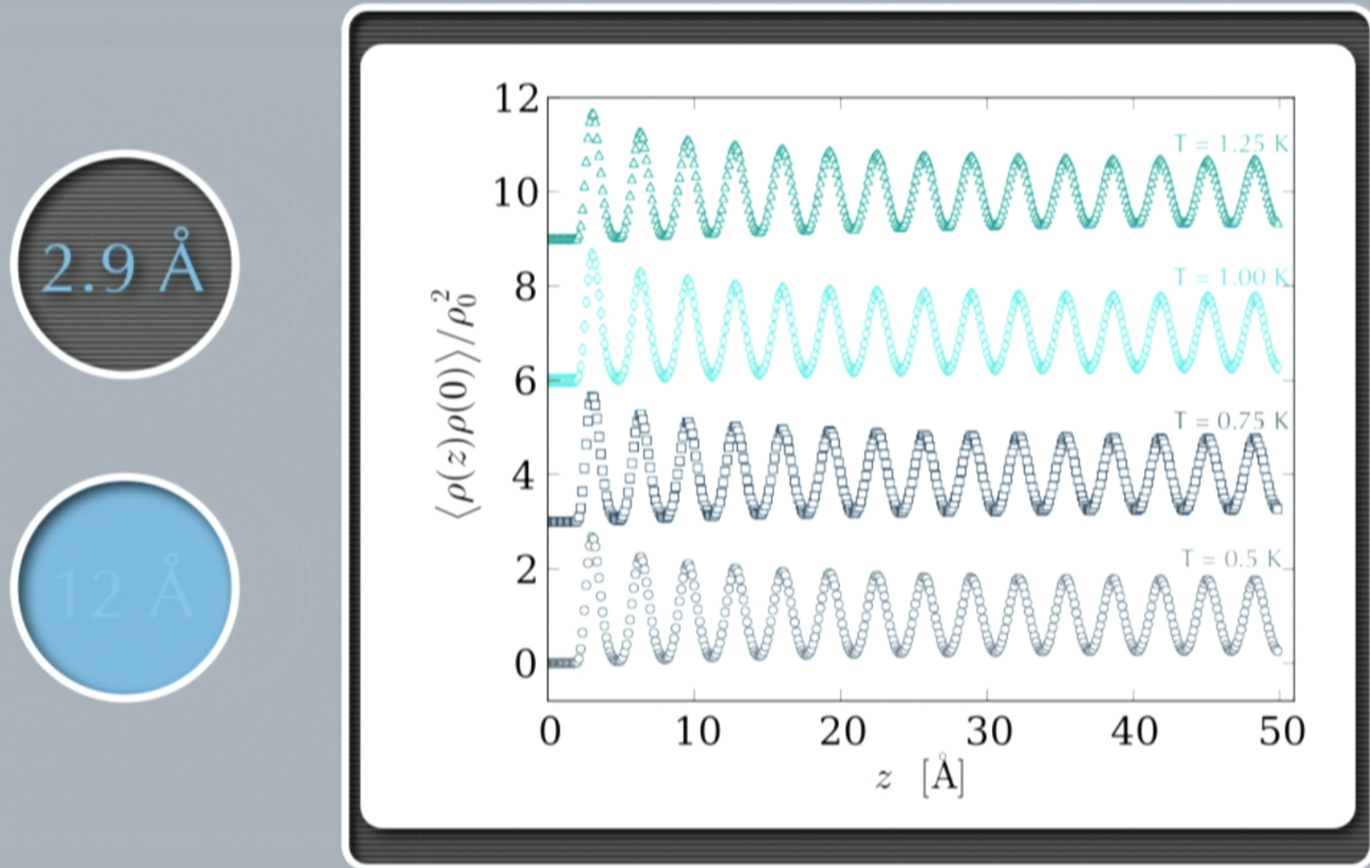
2.9 Å

12 Å



$$\langle \rho(z)\rho(0)\rangle = \rho_0^2 + \frac{1}{2\pi^2 K}\frac{d^2}{dz^2}\ln\theta_1(\pi z/L, e^{-\pi v/LT}) + \mathcal{A}\cos(2\pi\rho_0 z)\left[\frac{2\eta(iv/LT)\mathrm{e}^{-\pi v/6LT}}{\theta_1(\pi z/L, e^{-\pi v/LT})}\right]^{2/K}$$

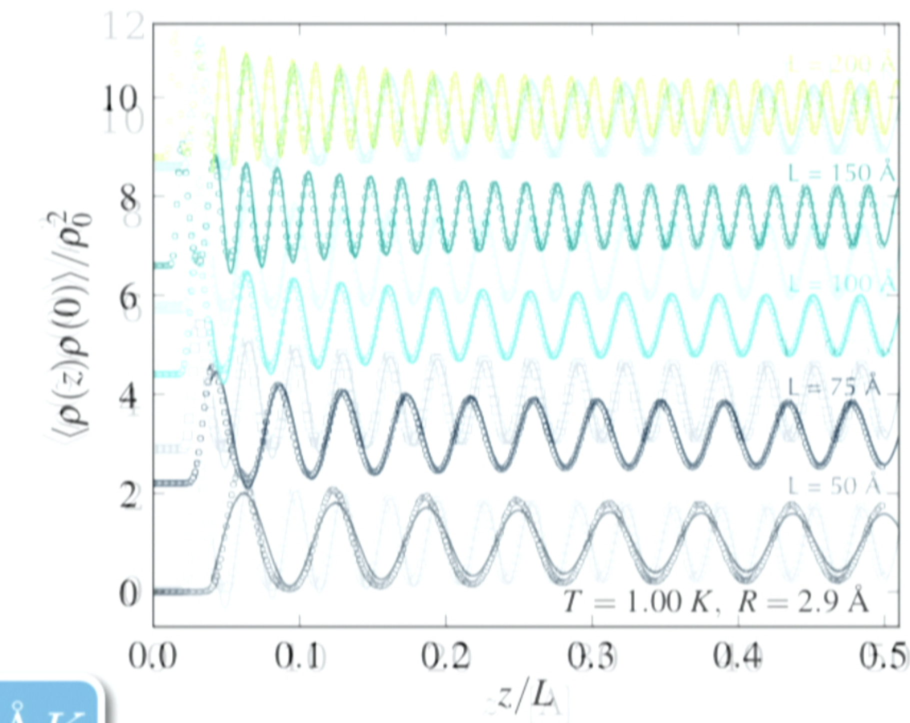
Axial Pair Correlation Function



Axial Pair Correlation Function

2.9 Å

12 Å



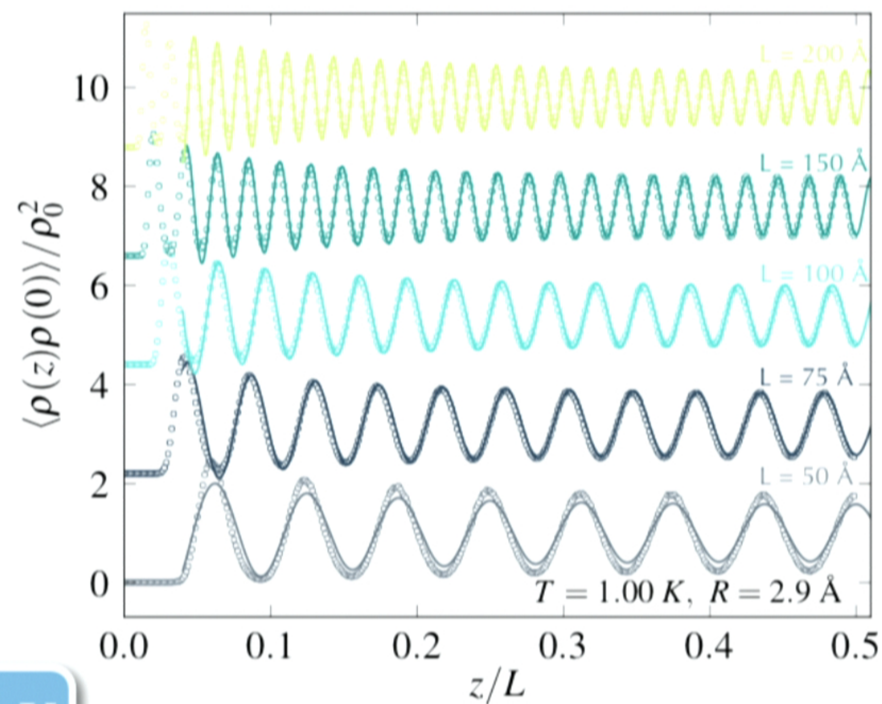
$v = 70(3)\ \text{Å}K$

$K = 6.0(2)$

Axial Pair Correlation Function

2.9 Å

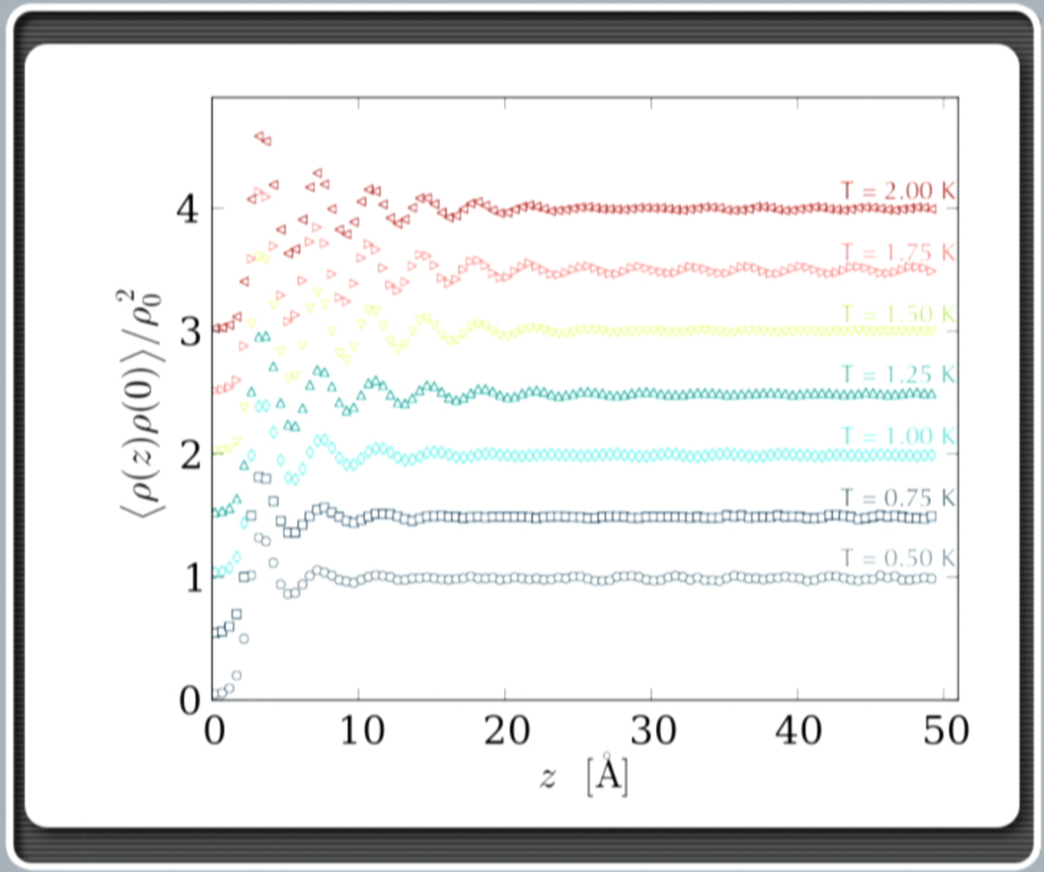
12 Å

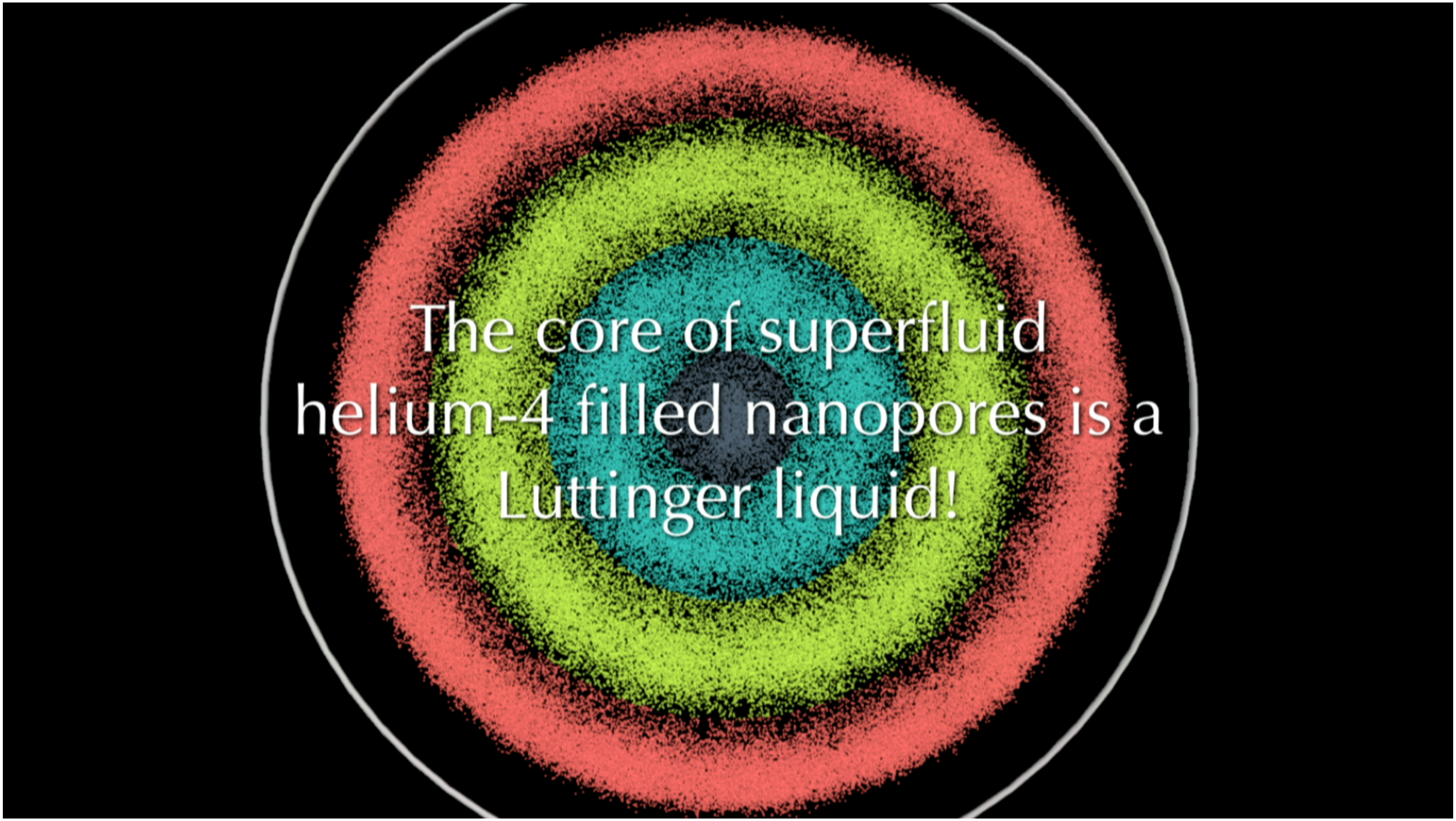


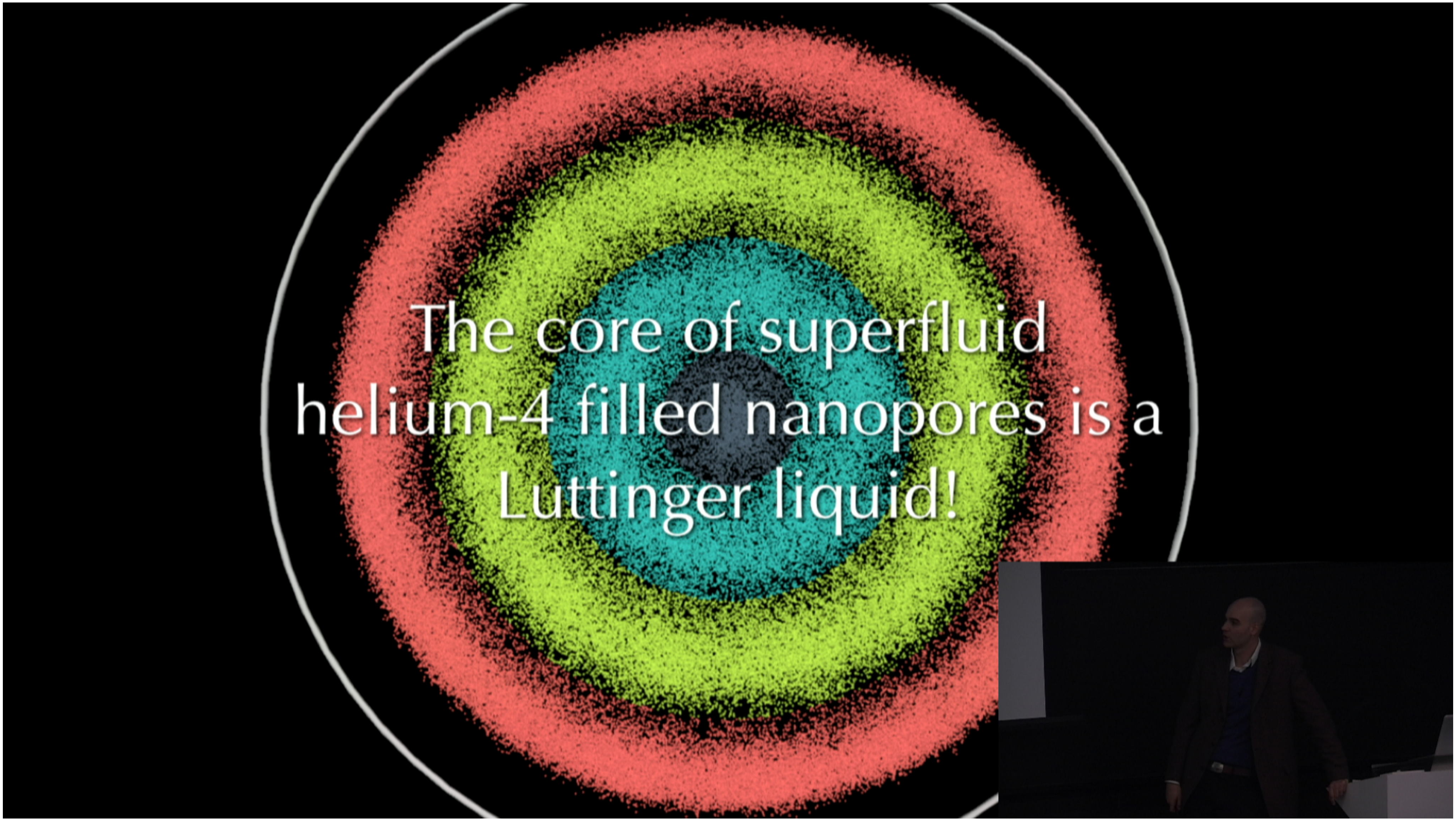
$v = 70(3)$ ÅK

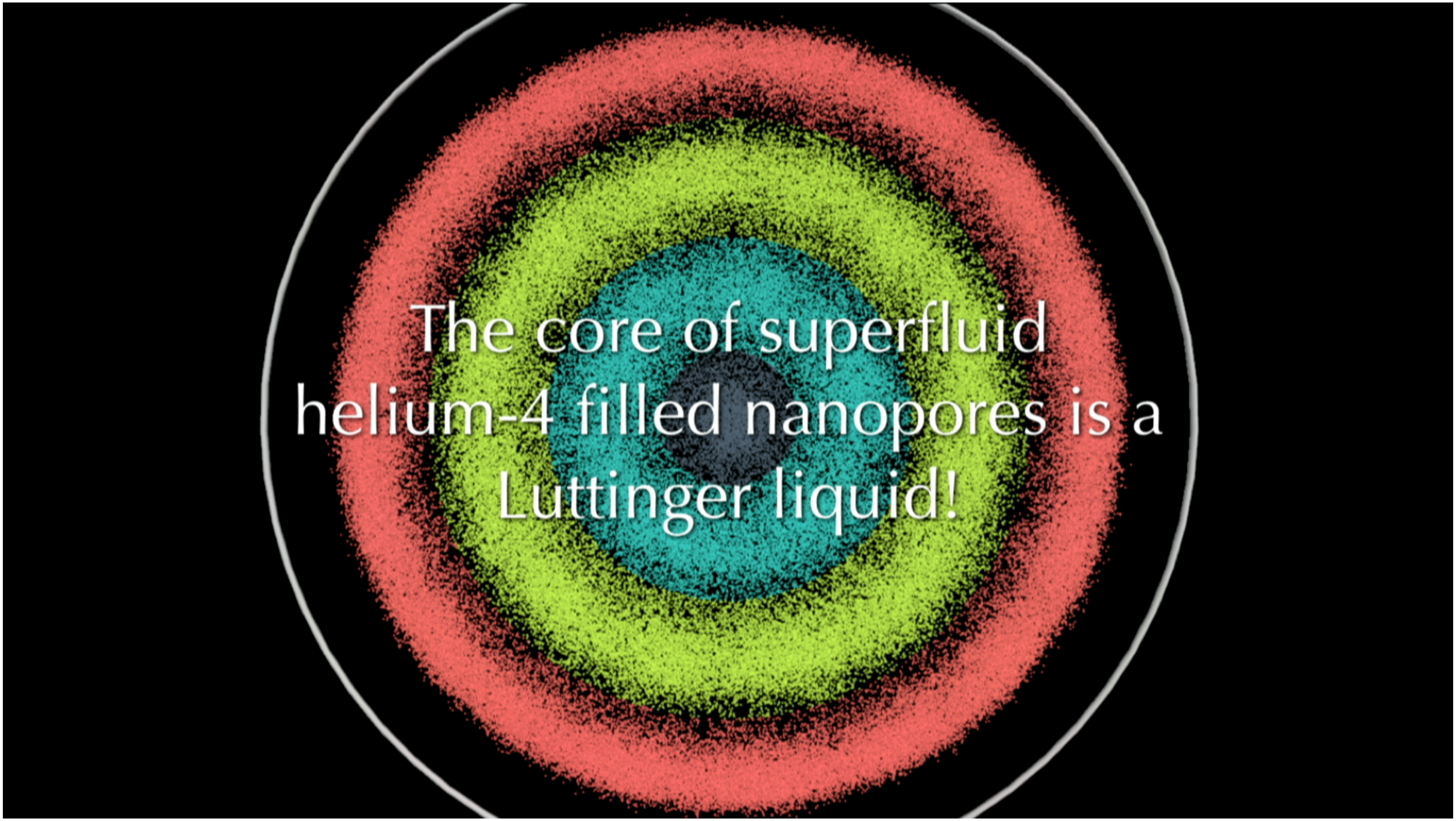
$K = 6.0(2)$

Axial Pair Correlation Function




The core of superfluid
helium-4 filled nanopores is a
Luttinger liquid!


The core of superfluid
helium-4 filled nanopores is a
Luttinger liquid!


The core of superfluid
helium-4 filled nanopores is a
Luttinger liquid!

Implications for Experiments

Implications for Experiments

- Known from renormalization group arguments that:

periodic substrate is irrelevant for:	$K < 1/2$
disorder is irrelevant for:	$K < 2/3$

- Observation of power-law decay of Bragg peaks for helium confined in porous glasses?

F. Albergamo, J. Bossy, J. V. Pearce, H. Schober, and H.R. Glyde, Phys. Rev. B **76**, 064503 (2007)

Conclusions

- Low dimensions provide an arena where exact solutions exist for strongly interacting quantum systems.
- PIMC is one of the only exact methods for studying the interacting many-body problem in the continuum.
- Helium-4 inside narrow pores forms a series of concentric shells, and the core of the pores is well described by Luttinger liquid theory!
- Just scratching the surface of what can be investigated ...

Conclusions

- Low dimensions provide an arena where exact solutions exist for strongly interacting quantum systems.
- PIMC is one of the only exact methods for studying the interacting many-body problem in the continuum.
- Helium-4 inside narrow pores forms a series of concentric shells, and the core of the pores is well described by Luttinger liquid theory!
- Just scratching the surface of what can be investigated ...