Title: Transition Amplitudes in Quantum Gravity

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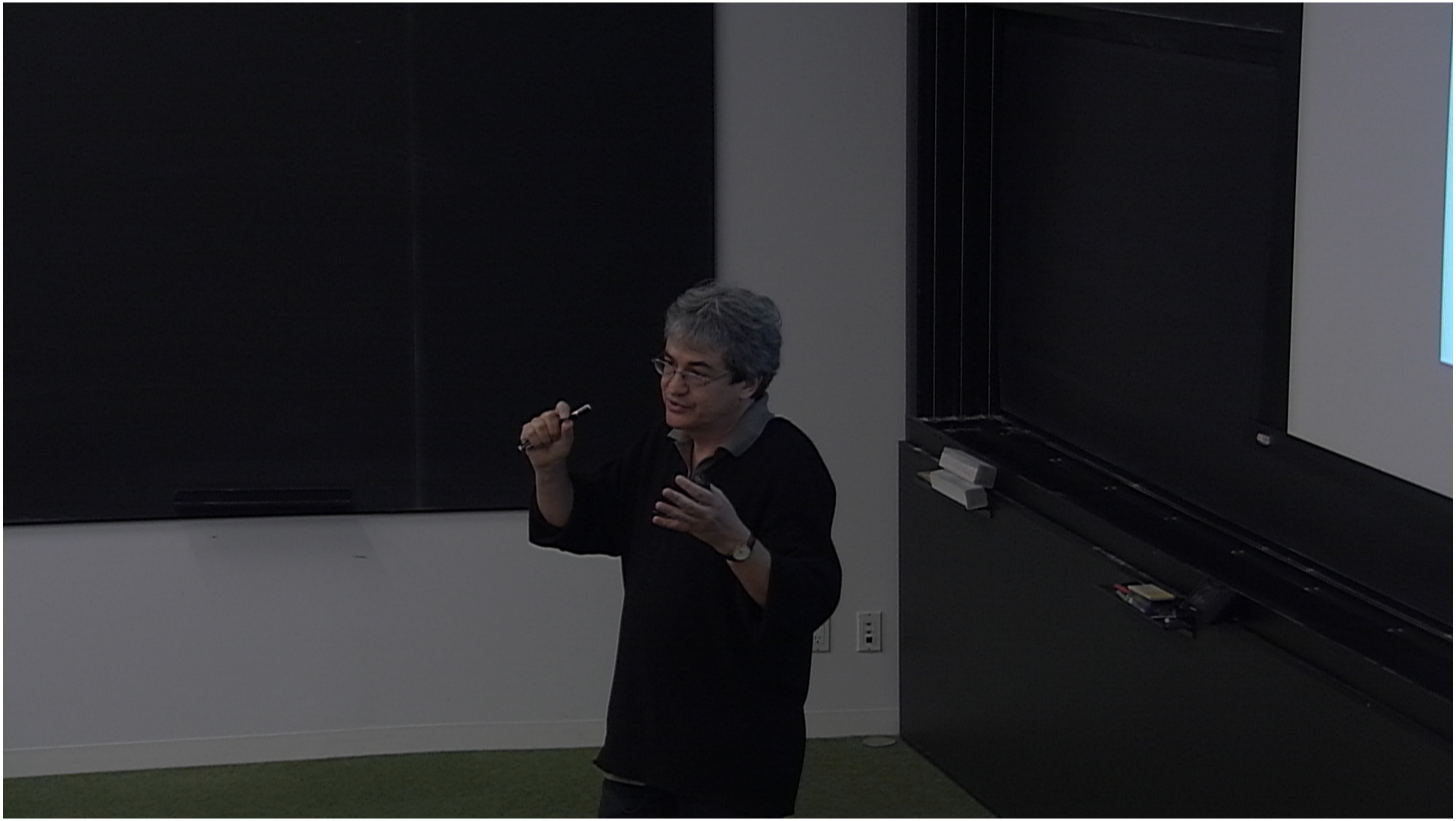
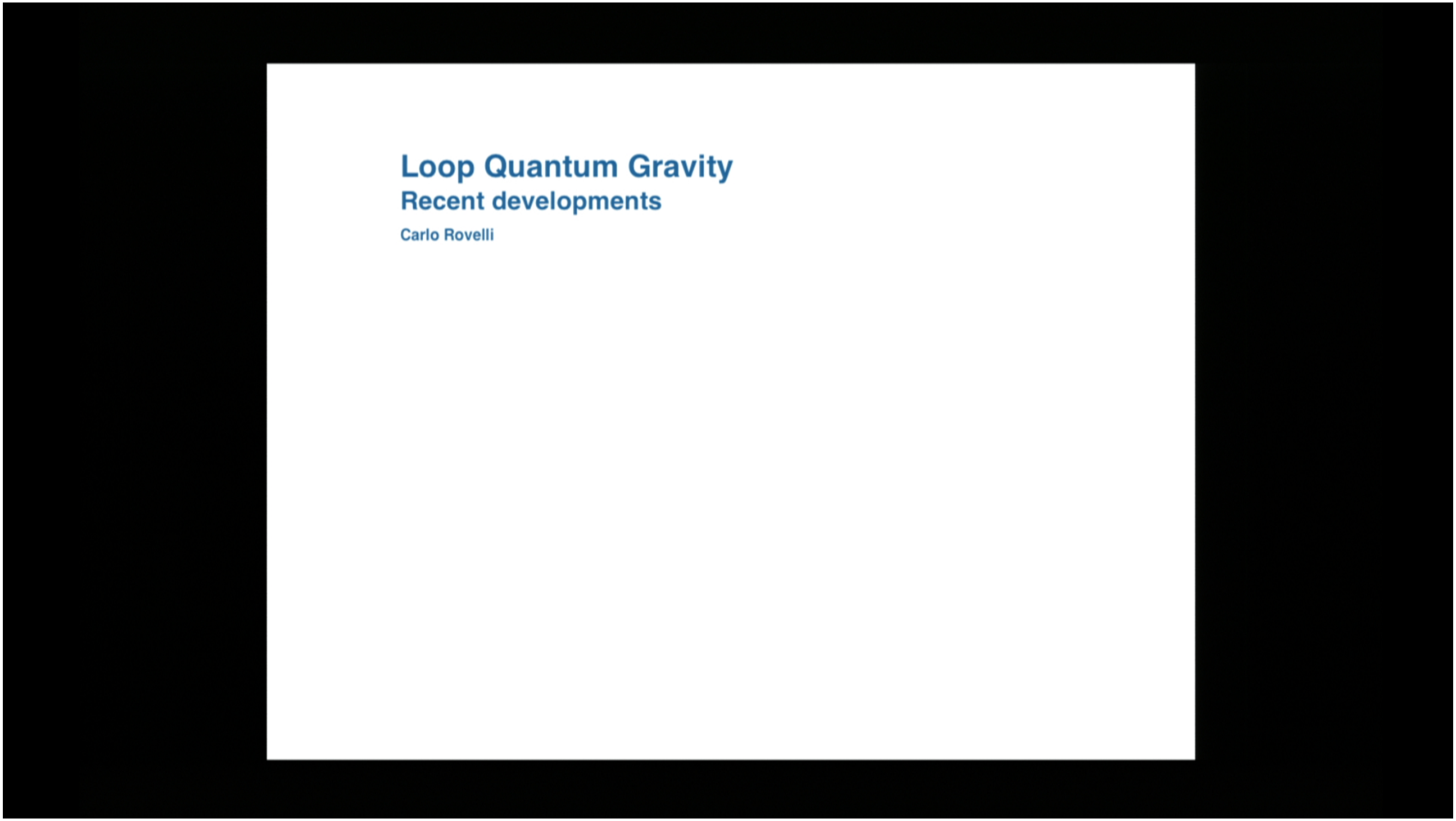
Abstract: The covariant formulation of loop quantum gravity has developed strongly during the last few years. I summarize the current definition of the theory and the results that have been proven. I discuss what is missing towards of the goal of defining a consistent quantum theory whose classical limit is general relativity.





Loop Quantum Gravity

Recent developments

Carlo Rovelli

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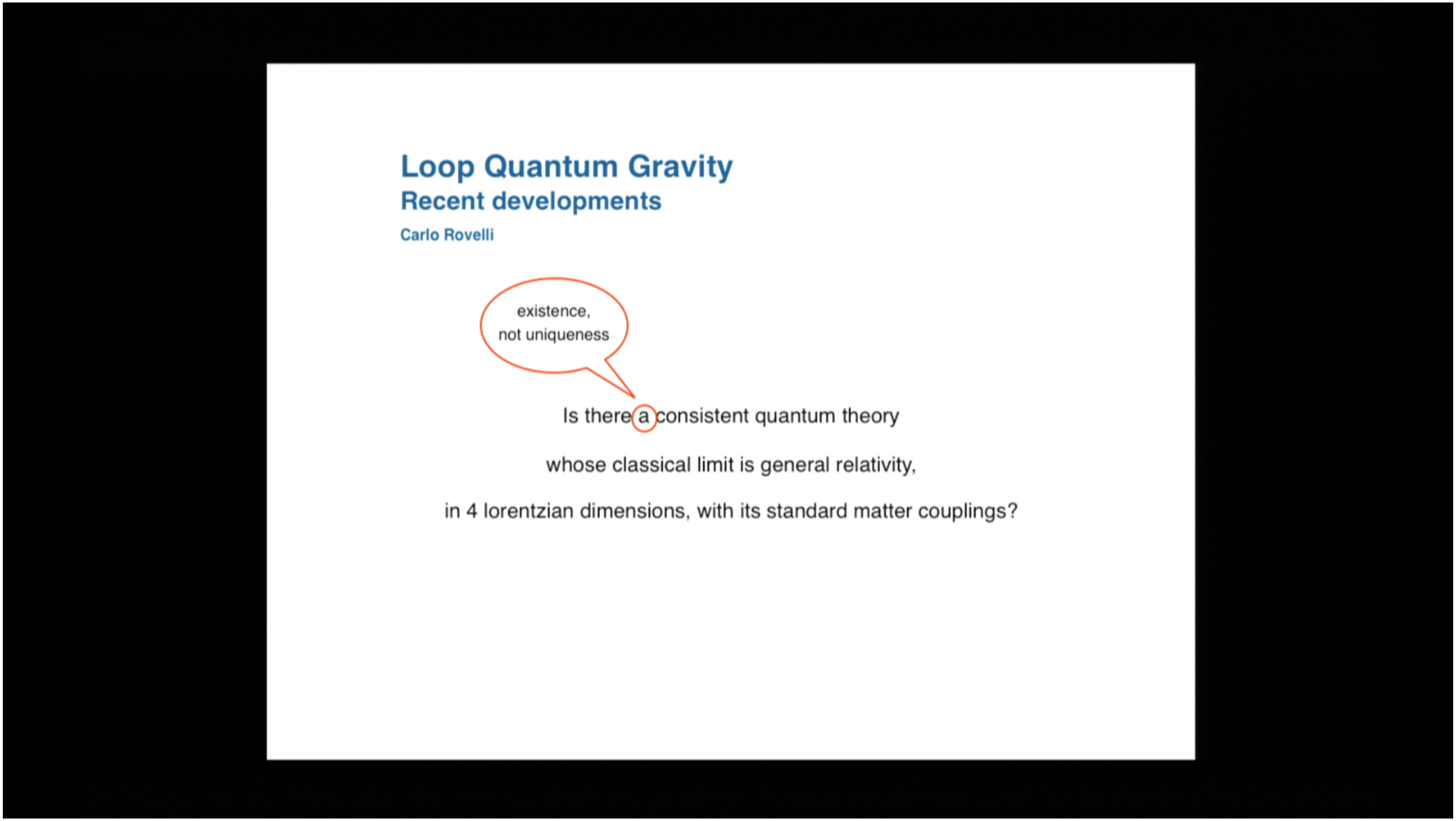
Is there a consistent quantum theory

whose classical limit is general relativity,

in 4 lorentzian dimensions, with its standard matter couplings?



Loop Quantum Gravity

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existence,
not uniqueness

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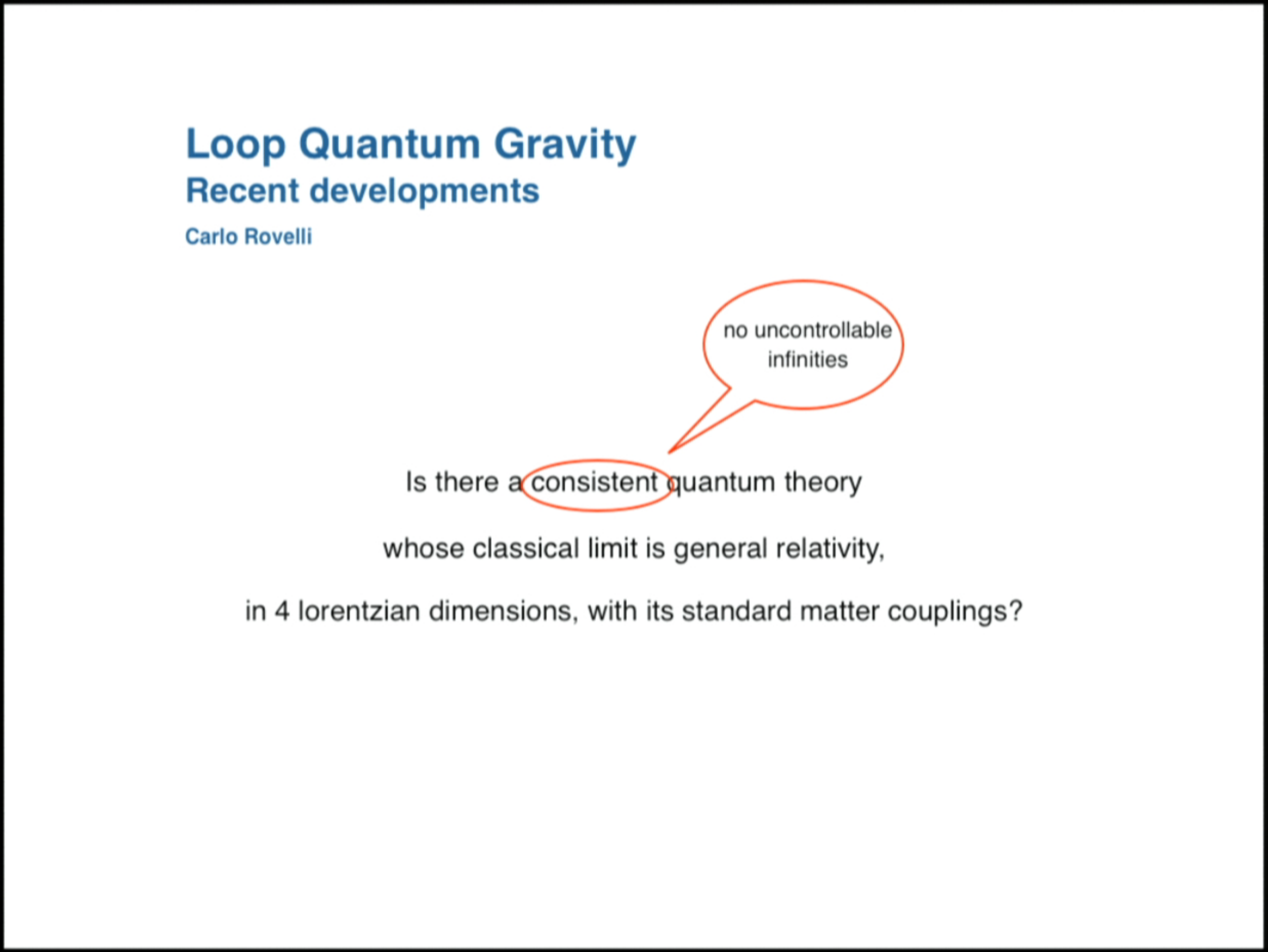

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no uncontrollable infinities

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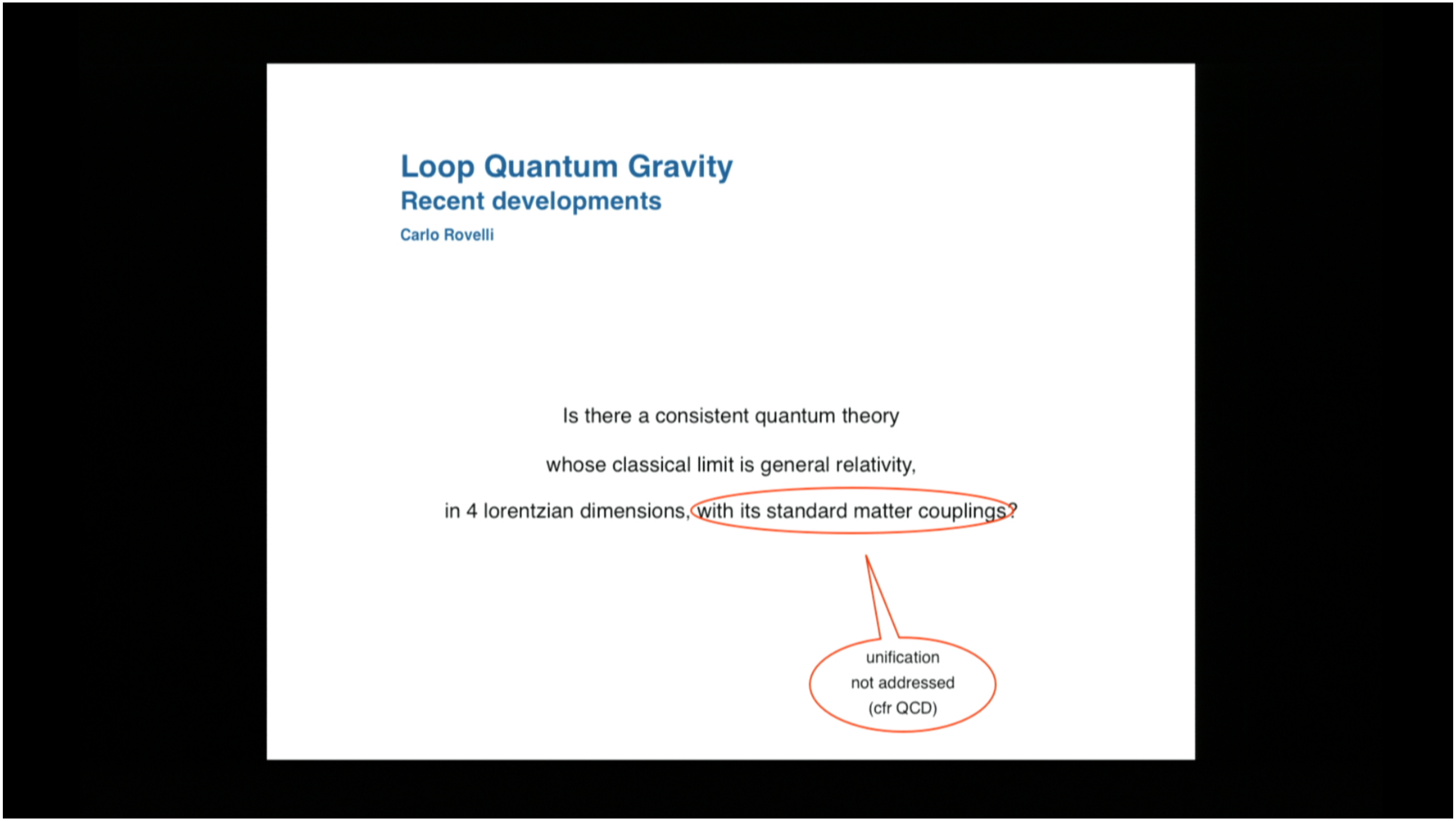

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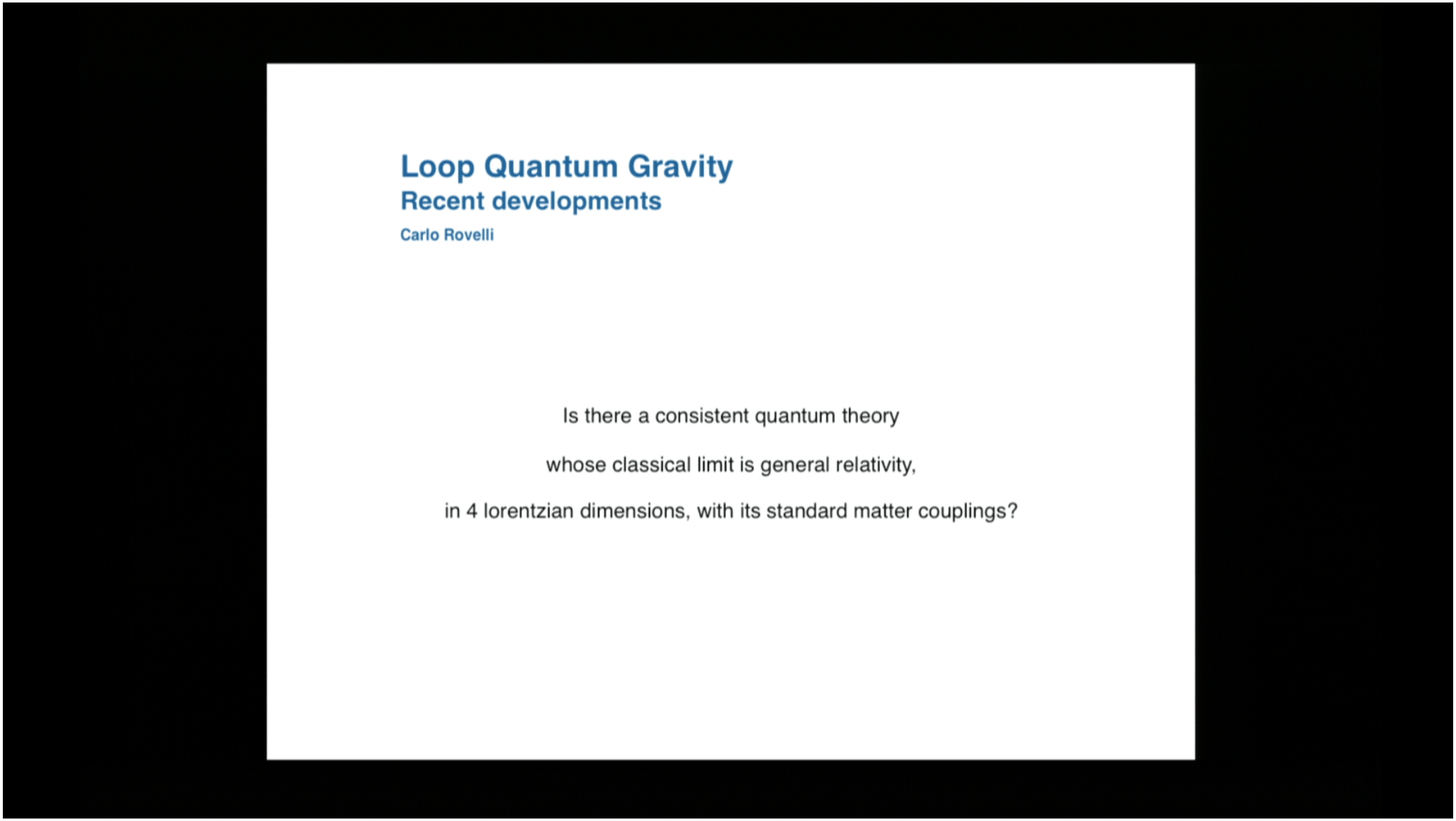
unification
not addressed
(cfr QCD)



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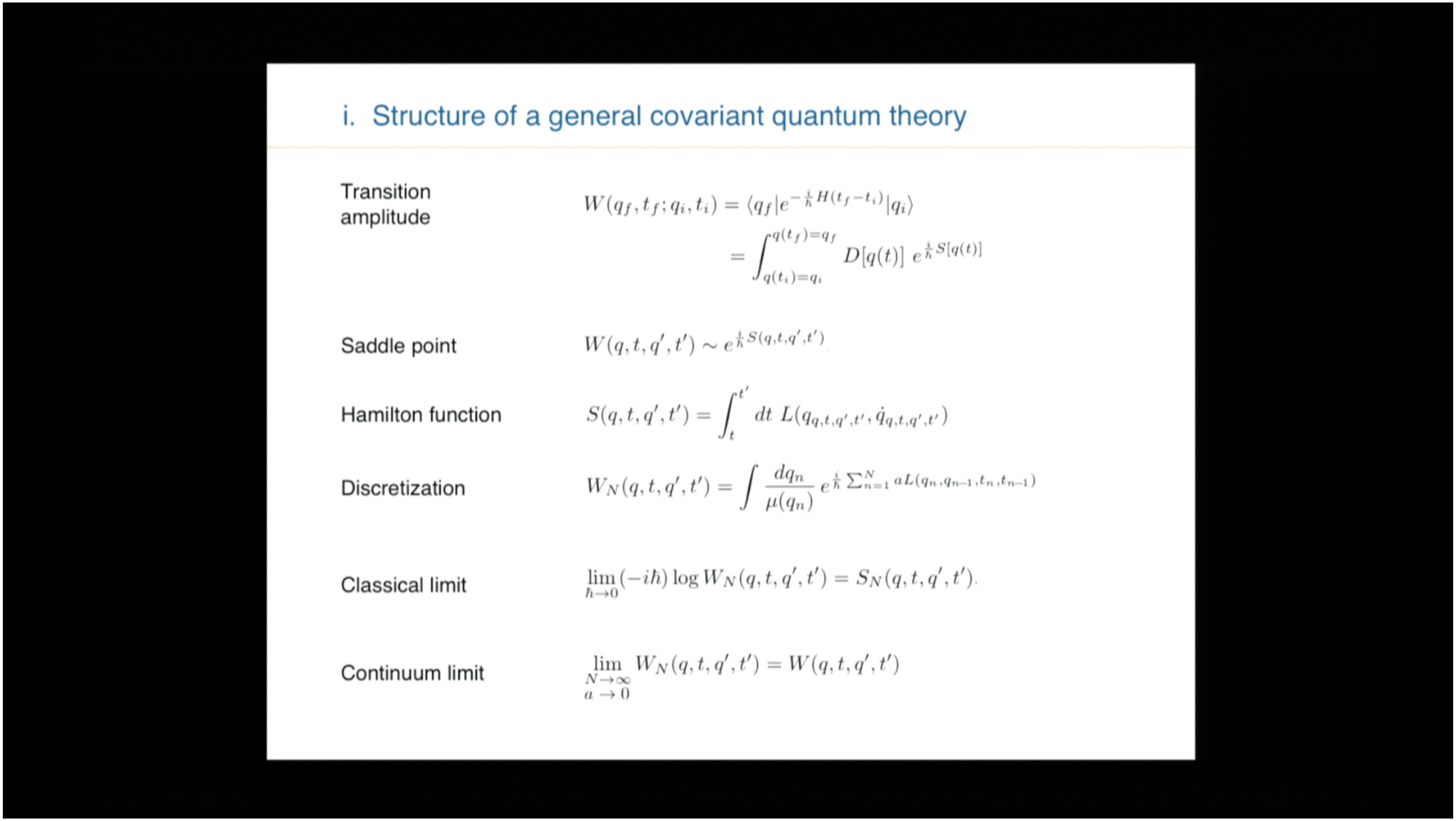

i. Structure of a general covariant quantum theory

Transition amplitude
$$W(q_f,t_f;q_i,t_i) = \langle q_f|e^{-\frac{i}{\hbar}H(t_f-t_i)}|q_i\rangle$$
$$= \int_{q(t_i)=q_i}^{q(t_f)=q_f} D[q(t)]\ e^{\frac{i}{\hbar}S[q(t)]}$$

Saddle point
$$W(q,t,q',t') \sim e^{\frac{i}{\hbar}S(q,t,q',t')}$$

Hamilton function
$$S(q,t,q',t') = \int_t^{t'} dt\ L(q_{q,t,q',t'}, \dot q_{q,t,q',t'})$$

Discretization
$$W_N(q,t,q',t') = \int \frac{dq_n}{\mu(q_n)}\ e^{\frac{i}{\hbar}\sum_{n=1}^N aL(q_n,q_{n-1},t_n,t_{n-1})}$$

Classical limit
$$\lim_{\hbar\to0}(-i\hbar)\log W_N(q,t,q',t') = S_N(q,t,q',t').$$

Continuum limit
$$\lim_{\substack{N\to\infty\\ a\to0}} W_N(q,t,q',t') = W(q,t,q',t')$$

$$\begin{tikzcd}
 & \begin{array}{c}\text{Quantum theory}\\ \textit{Transition amplitude}\\ W(q,t,q',t')\end{array} \arrow[r, "\hbar\to0"] & \begin{array}{c}\text{Classical theory}\\ \textit{Hamilton function}\\ S(q,t,q',t')\end{array} \\
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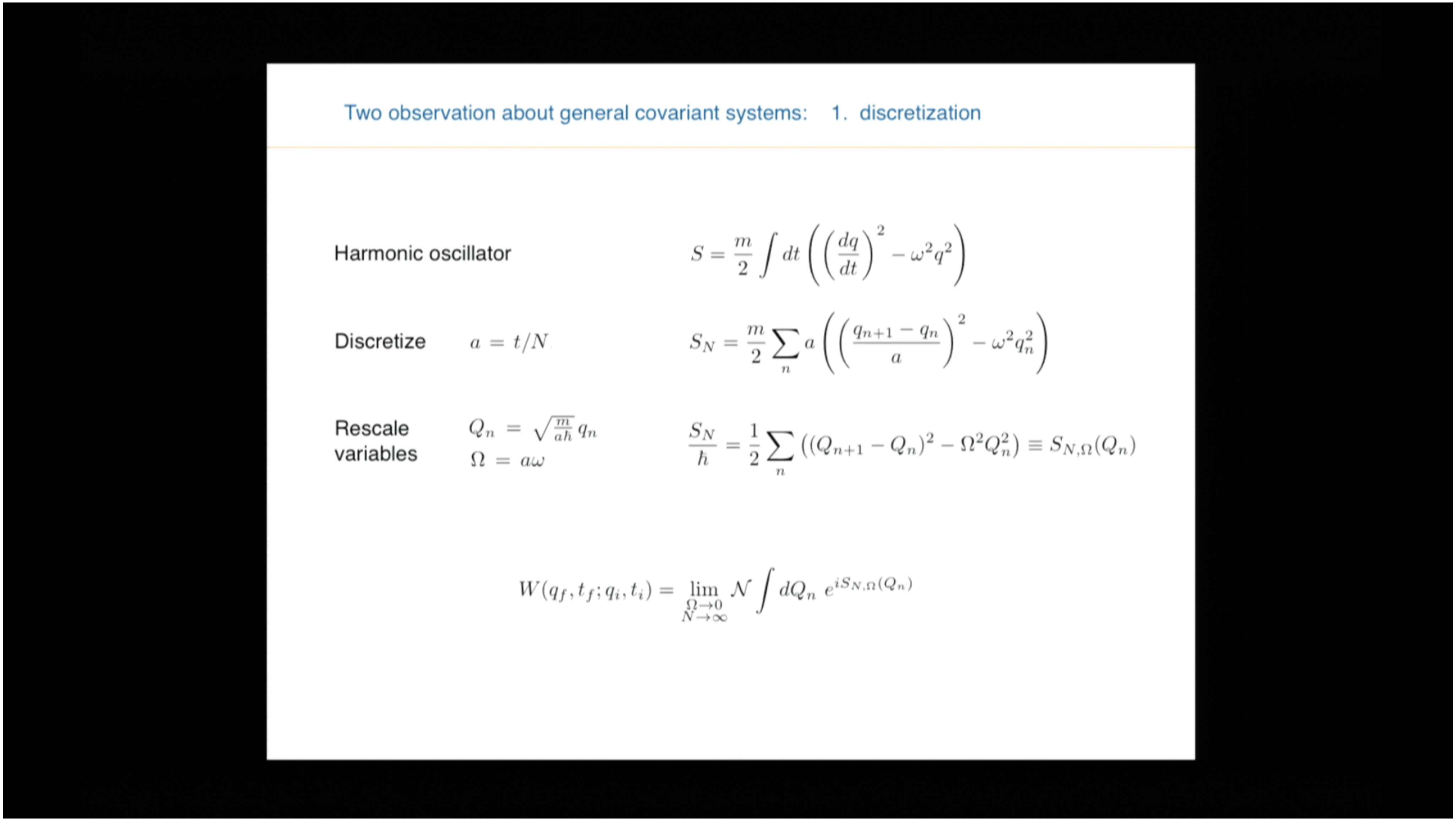

Two observation about general covariant systems: 1. discretization

Harmonic oscillator

$$S = \frac{m}{2}\int dt \left(\left(\frac{dq}{dt} \right)^2 - \omega^2 q^2 \right)$$

Discretize $a = t/N$

$$S_N = \frac{m}{2}\sum_n a \left(\left(\frac{q_{n+1} - q_n}{a} \right)^2 - \omega^2 q_n^2 \right)$$

Rescale variables

$$Q_n = \sqrt{\frac{m}{a\hbar}}\, q_n$$

$$\Omega = a\omega$$

$$\frac{S_N}{\hbar} = \frac{1}{2}\sum_n \left((Q_{n+1} - Q_n)^2 - \Omega^2 Q_n^2 \right) \equiv S_{N,\Omega}(Q_n)$$

$$W(q_f, t_f; q_i, t_i) = \lim_{\substack{\Omega \to 0 \\ N \to \infty}} \mathcal{N} \int dQ_n \; e^{i S_{N,\Omega}(Q_n)}$$

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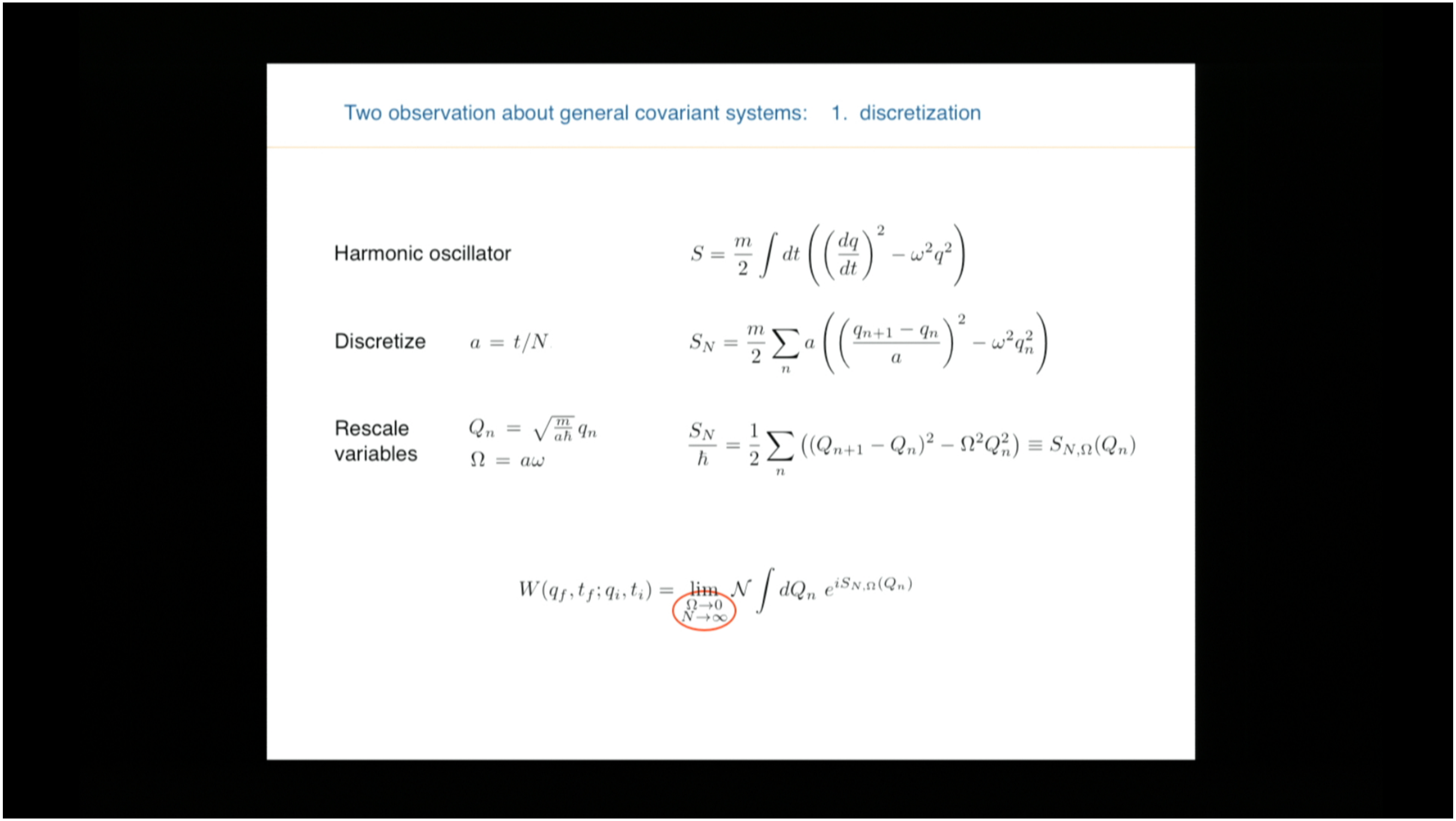

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Parametrized theory
$$S = \frac{m}{2}\int d\tau \left(\frac{\dot{q}^2}{\dot{t}} - \omega^2 \dot{t}\, q^2\right)$$

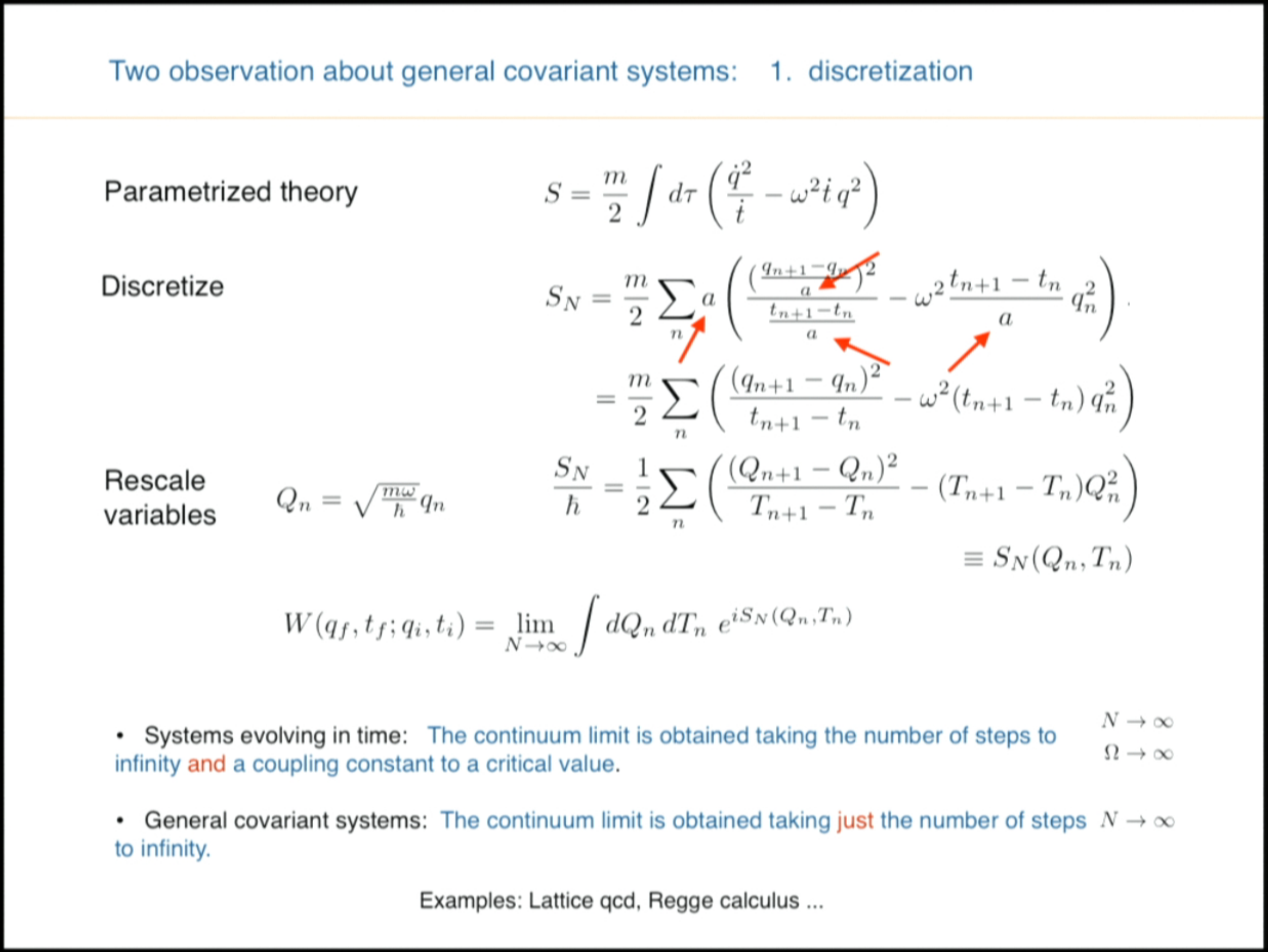
Discretize
$$S_N = \frac{m}{2}\sum_n a \left(\frac{(\frac{q_{n+1}-q_n}{a})^2}{\frac{t_{n+1}-t_n}{a}} - \omega^2 \frac{t_{n+1}-t_n}{a}\, q_n^2\right).$$
$$= \frac{m}{2}\sum_n \left(\frac{(q_{n+1}-q_n)^2}{t_{n+1}-t_n} - \omega^2 (t_{n+1}-t_n)\, q_n^2\right)$$

Rescale variables $Q_n = \sqrt{\frac{m\omega}{\hbar}}q_n$
$$\frac{S_N}{\hbar} = \frac{1}{2}\sum_n \left(\frac{(Q_{n+1}-Q_n)^2}{T_{n+1}-T_n} - (T_{n+1}-T_n)Q_n^2\right)$$
$$\equiv S_N(Q_n, T_n)$$

$$W(q_f, t_f; q_i, t_i) = \lim_{N\to\infty}\int dQ_n\, dT_n\; e^{iS_N(Q_n,T_n)}$$

- Systems evolving in time: The continuum limit is obtained taking the number of steps to infinity and a coupling constant to a critical value. $N \to \infty$ $\Omega \to \infty$
- General covariant systems: The continuum limit is obtained taking just the number of steps to infinity. $N \to \infty$

Examples: Lattice qcd, Regge calculus ...



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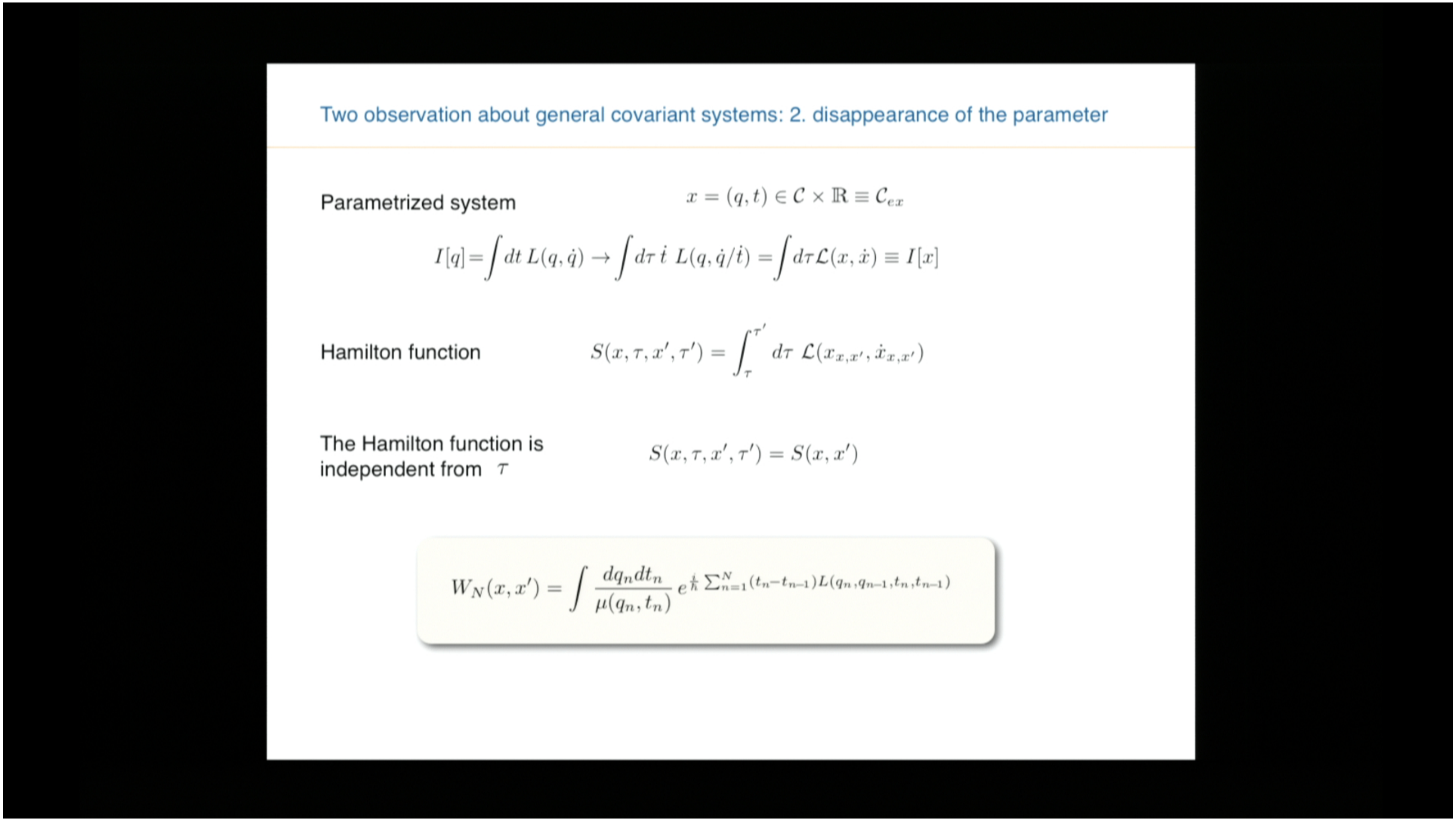

Two observation about general covariant systems: 2. disappearance of the parameter

Parametrized system $x = (q, t) \in \mathcal{C} \times \mathbb{R} \equiv \mathcal{C}_{ex}$

$$I[q] = \int dt\, L(q, \dot{q}) \rightarrow \int d\tau\, \dot{t}\ L(q, \dot{q}/\dot{t}) = \int d\tau \mathcal{L}(x, \dot{x}) \equiv I[x]$$

Hamilton function $S(x, \tau, x', \tau') = \int_{\tau}^{\tau'} d\tau\ \mathcal{L}(x_{x,x'}, \dot{x}_{x,x'})$

The Hamilton function is independent from τ $S(x, \tau, x', \tau') = S(x, x')$

$$W_N(x, x') = \int \frac{dq_n dt_n}{\mu(q_n, t_n)}\, e^{\frac{i}{\hbar} \sum_{n=1}^{N} (t_n - t_{n-1}) L(q_n, q_{n-1}, t_n, t_{n-1})}$$

The general structure of a parametrized (general covariant) theory

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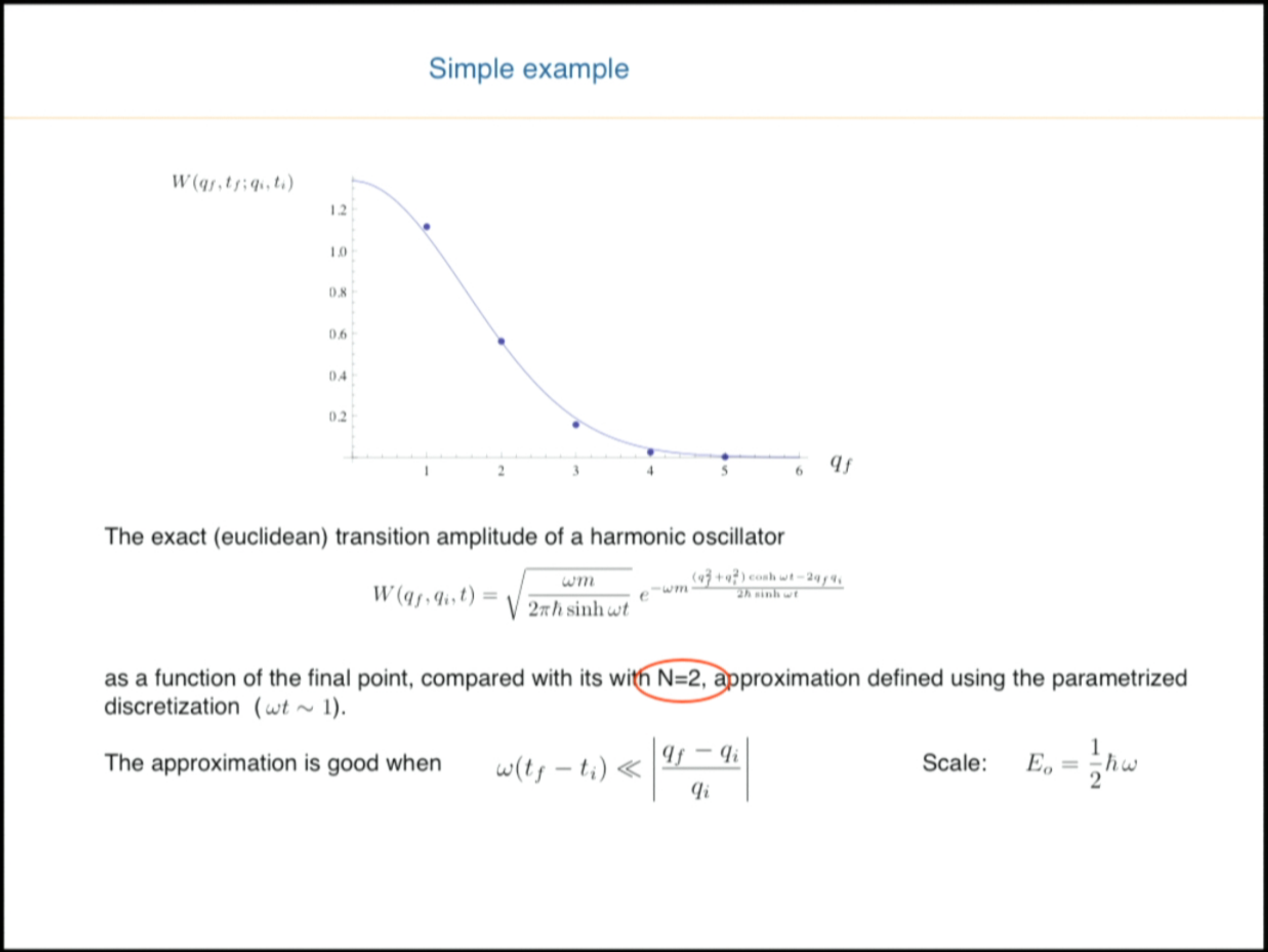


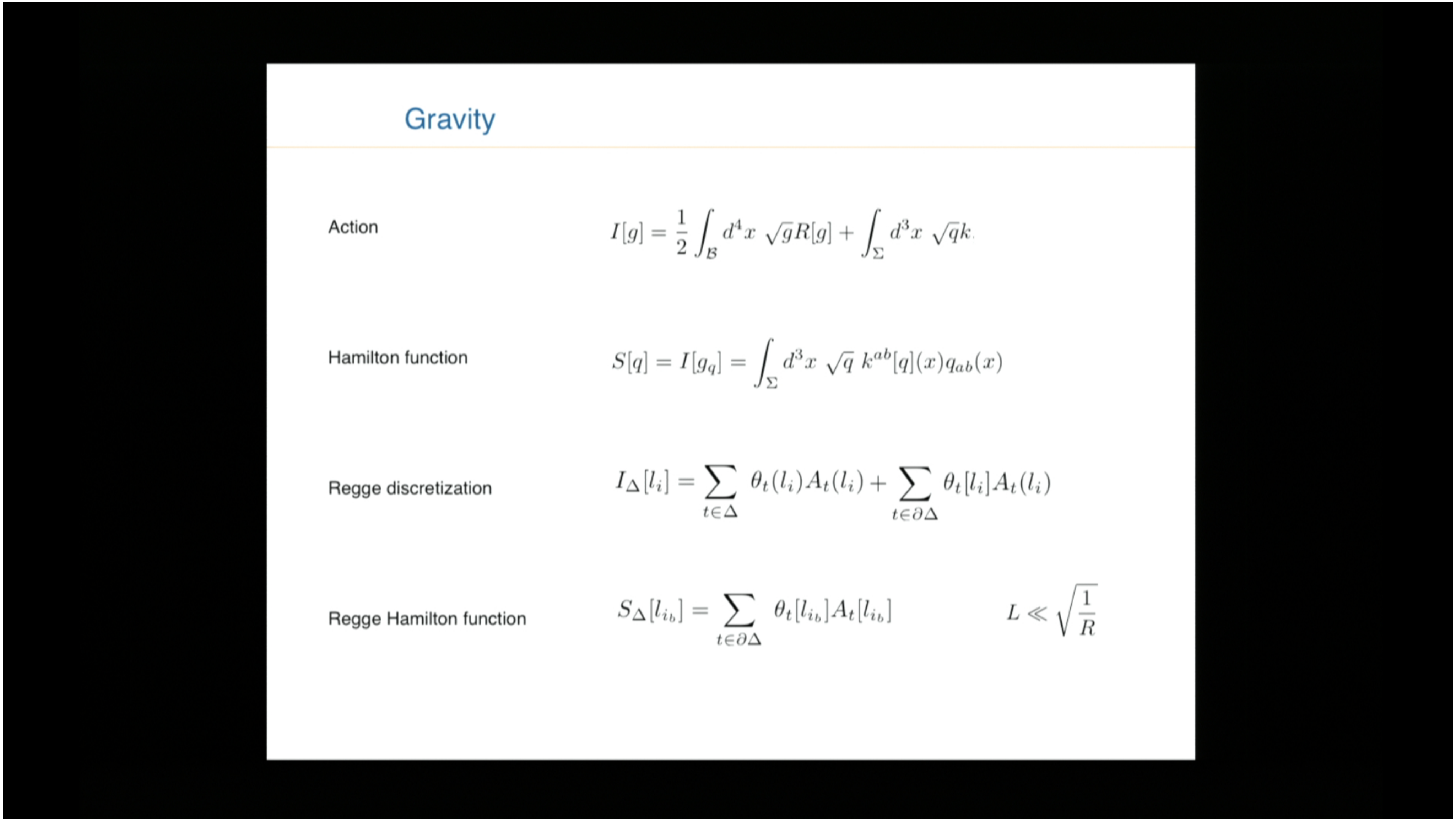
Simple example

The exact (euclidean) transition amplitude of a harmonic oscillator

$$W(q_f,q_i,t)=\sqrt{\frac{\omega m}{2\pi\hbar\sinh\omega t}}\;e^{-\omega m\frac{(q_f^2+q_i^2)\cosh\omega t-2q_fq_i}{2\hbar\sinh\omega t}}$$

as a function of the final point, compared with its with N=2, approximation defined using the parametrized discretization $(\omega t\sim 1)$.

The approximation is good when $\omega(t_f-t_i)\ll\left|\frac{q_f-q_i}{q_i}\right|$ Scale: $E_o=\frac{1}{2}\hbar\omega$



Gravity

Action

$$I[g] = \frac{1}{2}\int_{\mathcal{B}} d^4x \; \sqrt{g}R[g] + \int_{\Sigma} d^3x \; \sqrt{q}k.$$

Hamilton function

$$S[q] = I[g_q] = \int_{\Sigma} d^3x \; \sqrt{q} \; k^{ab}[q](x) q_{ab}(x)$$

Regge discretization

$$I_\Delta[l_i] = \sum_{t\in\Delta} \theta_t(l_i)A_t(l_i) + \sum_{t\in\partial\Delta} \theta_t[l_i]A_t(l_i)$$

Regge Hamilton function

$$S_\Delta[l_{i_b}] = \sum_{t\in\partial\Delta} \theta_t[l_{i_b}]A_t[l_{i_b}] \qquad\qquad L \ll \sqrt{\frac{1}{R}}$$

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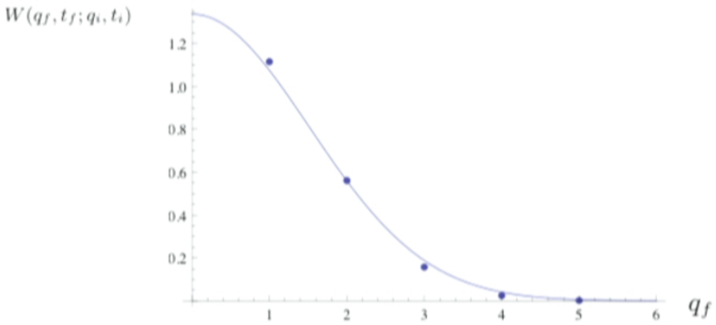

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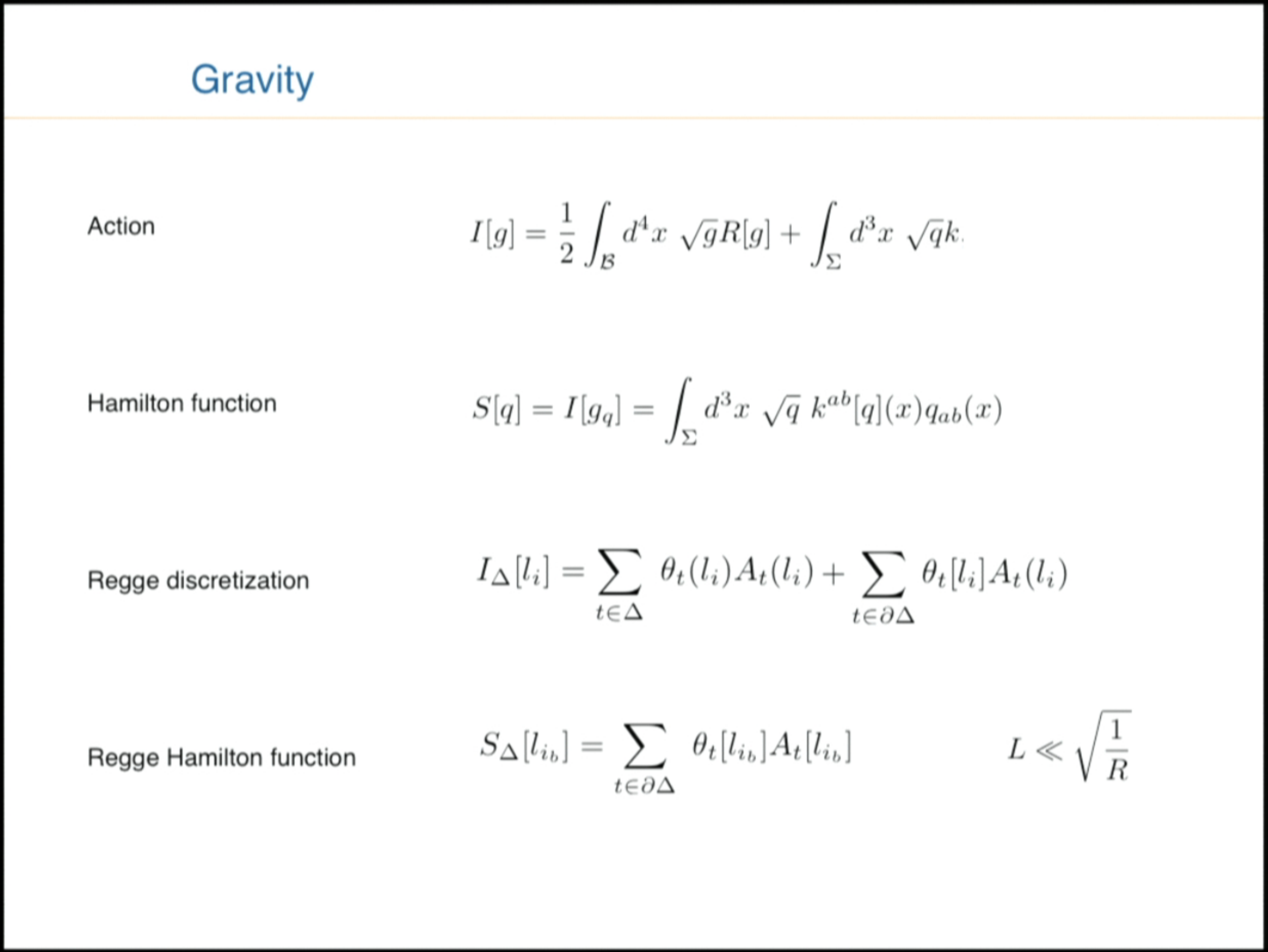

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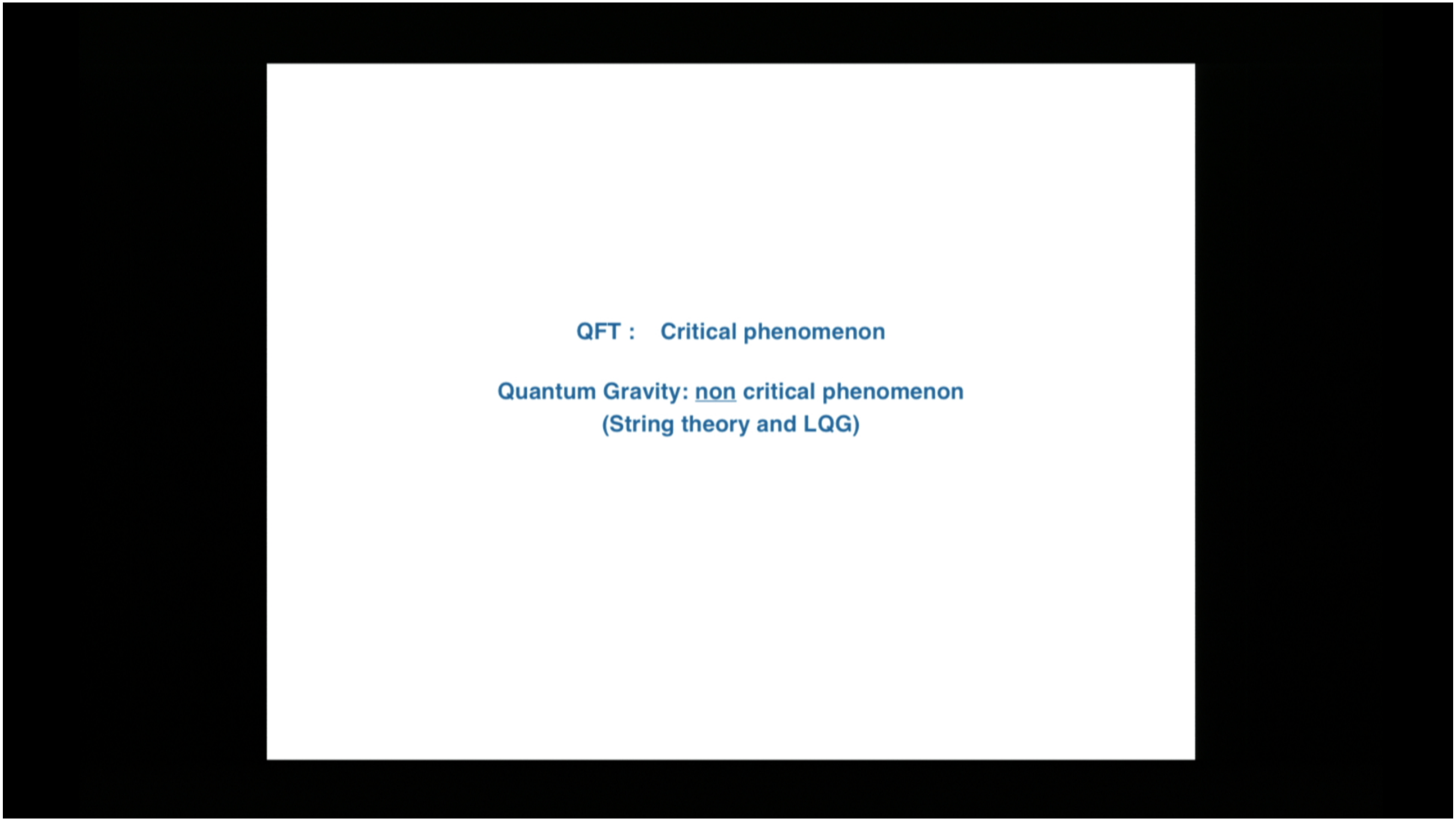
\end{tikzcd}$$

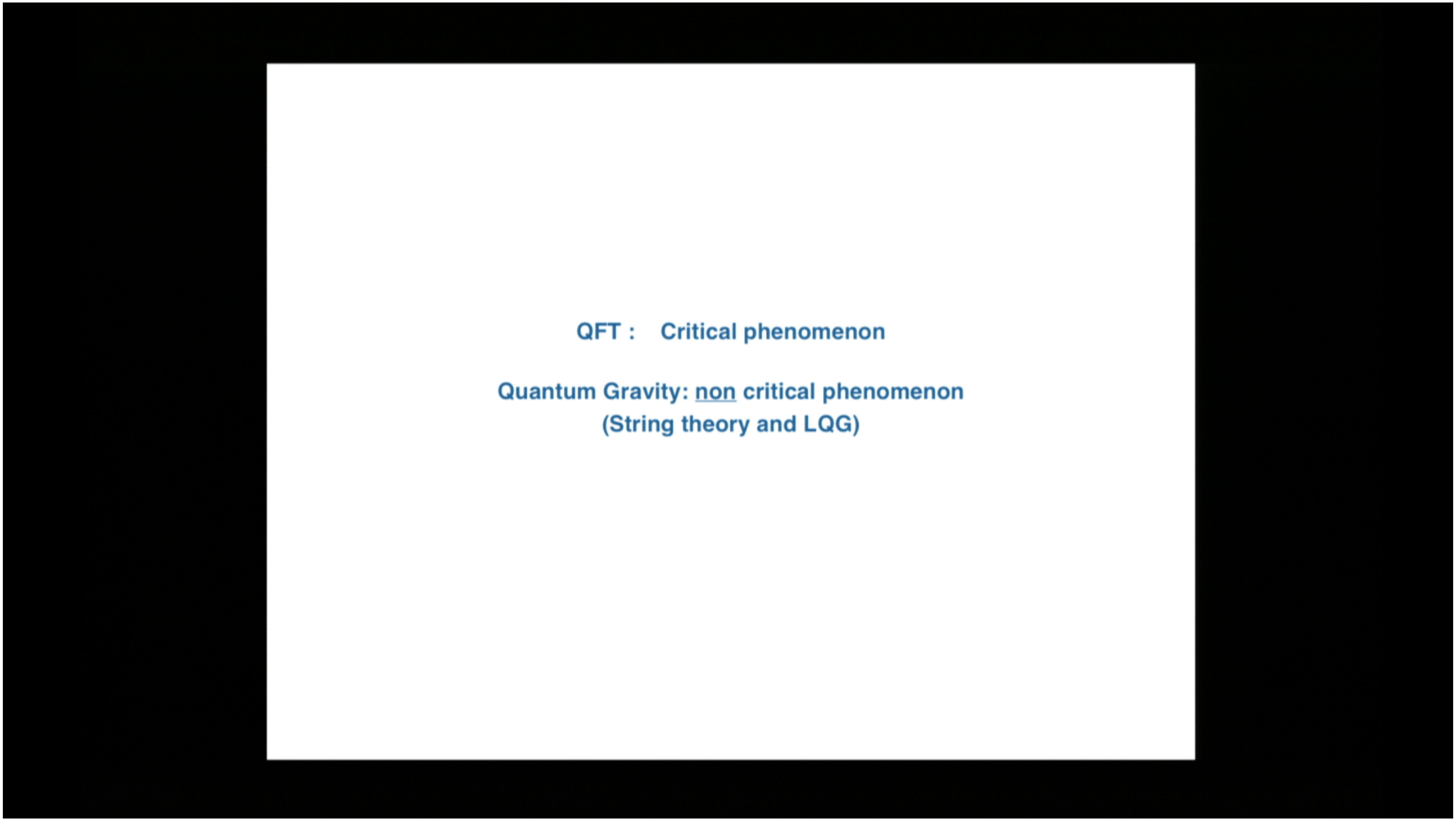
Notice:

- No critical point
- No infinite renormalization
- Physical scale: $L^2_{Planck} = \hbar G$
- Cfr: condensed matter away from critical points
- Same hint from string theory: finite UV scale and no infinities.



QFT : Critical phenomenon

Quantum Gravity: non critical phenomenon
(String theory and LQG)



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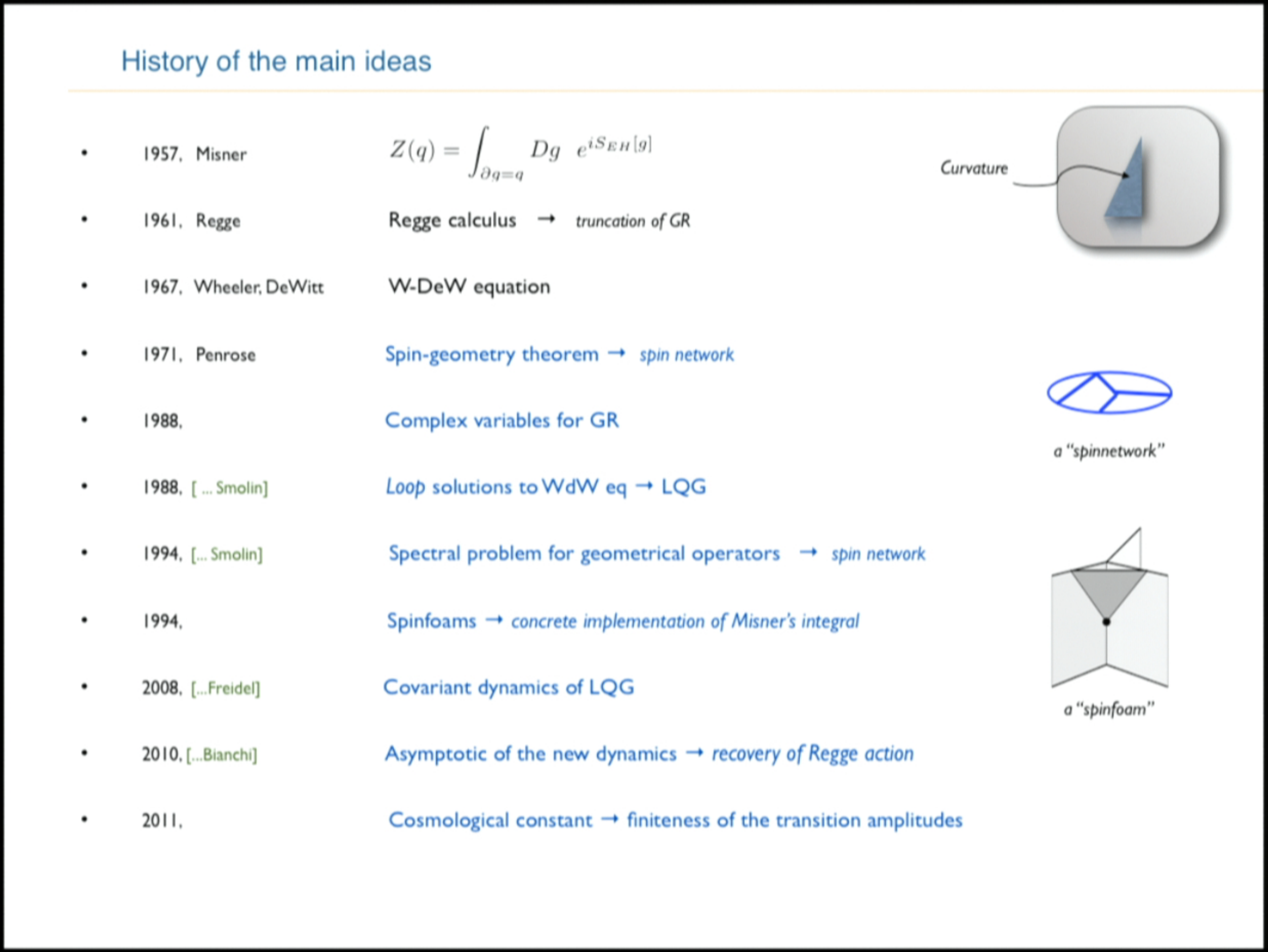

History of the main ideas

1957, Misner	$Z(q) = \int_{\partial g = q} Dg \quad e^{iS_{EH}[g]}$
1961, Regge	Regge calculus → *truncation of GR*
1967, Wheeler, DeWitt	W-DeW equation
1971, Penrose	Spin-geometry theorem → *spin network*
1988,	Complex variables for GR
1988, [... Smolin]	*Loop* solutions to WdW eq → LQG
1994, [... Smolin]	Spectral problem for geometrical operators → *spin network*
1994,	Spinfoams → *concrete implementation of Misner's integral*
2008, [...Freidel]	Covariant dynamics of LQG
2010, [...Bianchi]	Asymptotic of the new dynamics → *recovery of Regge action*
2011,	Cosmological constant → finiteness of the transition amplitudes

a "spinnetwork"

a "spinfoam"

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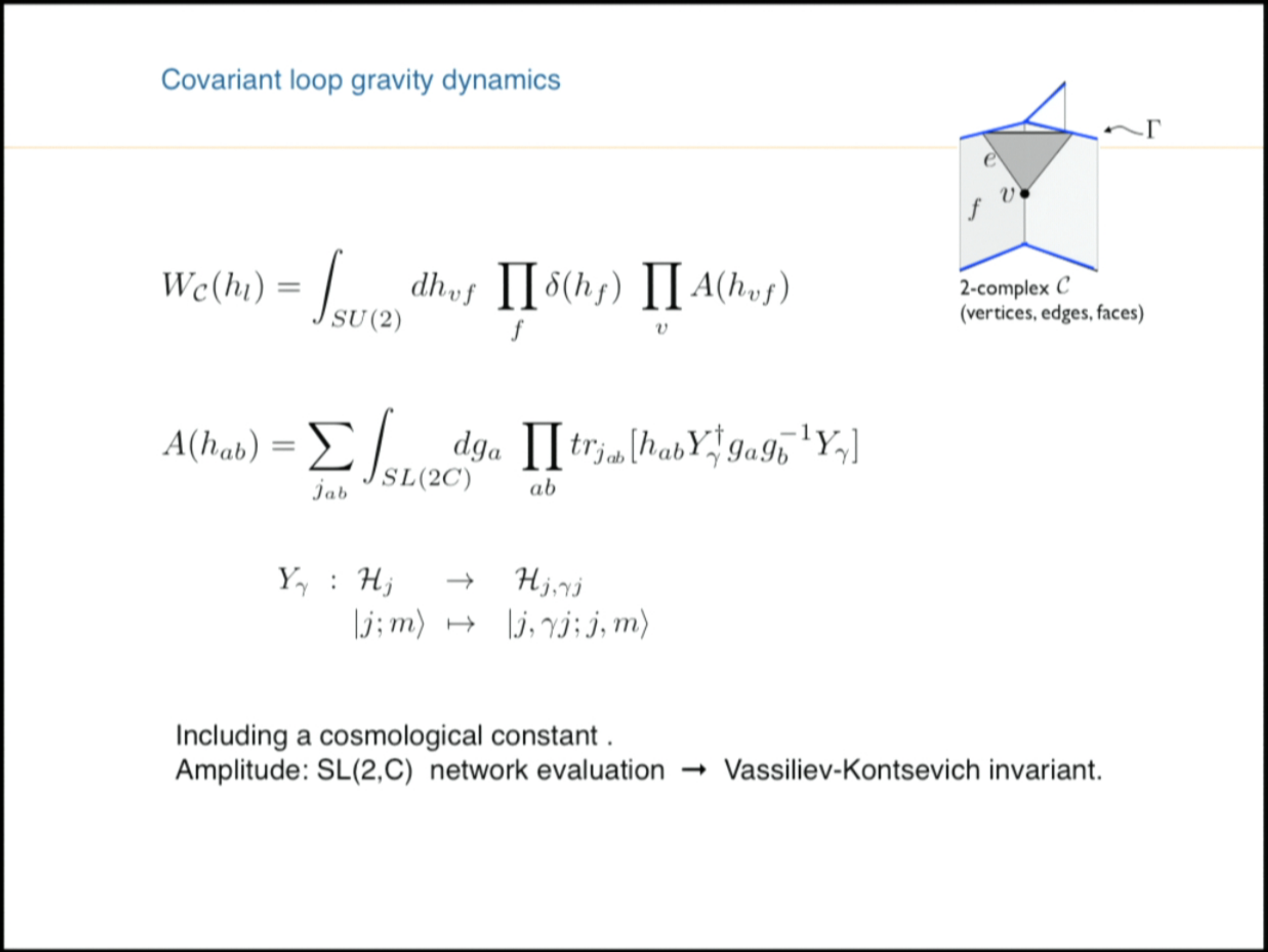


Covariant loop gravity dynamics

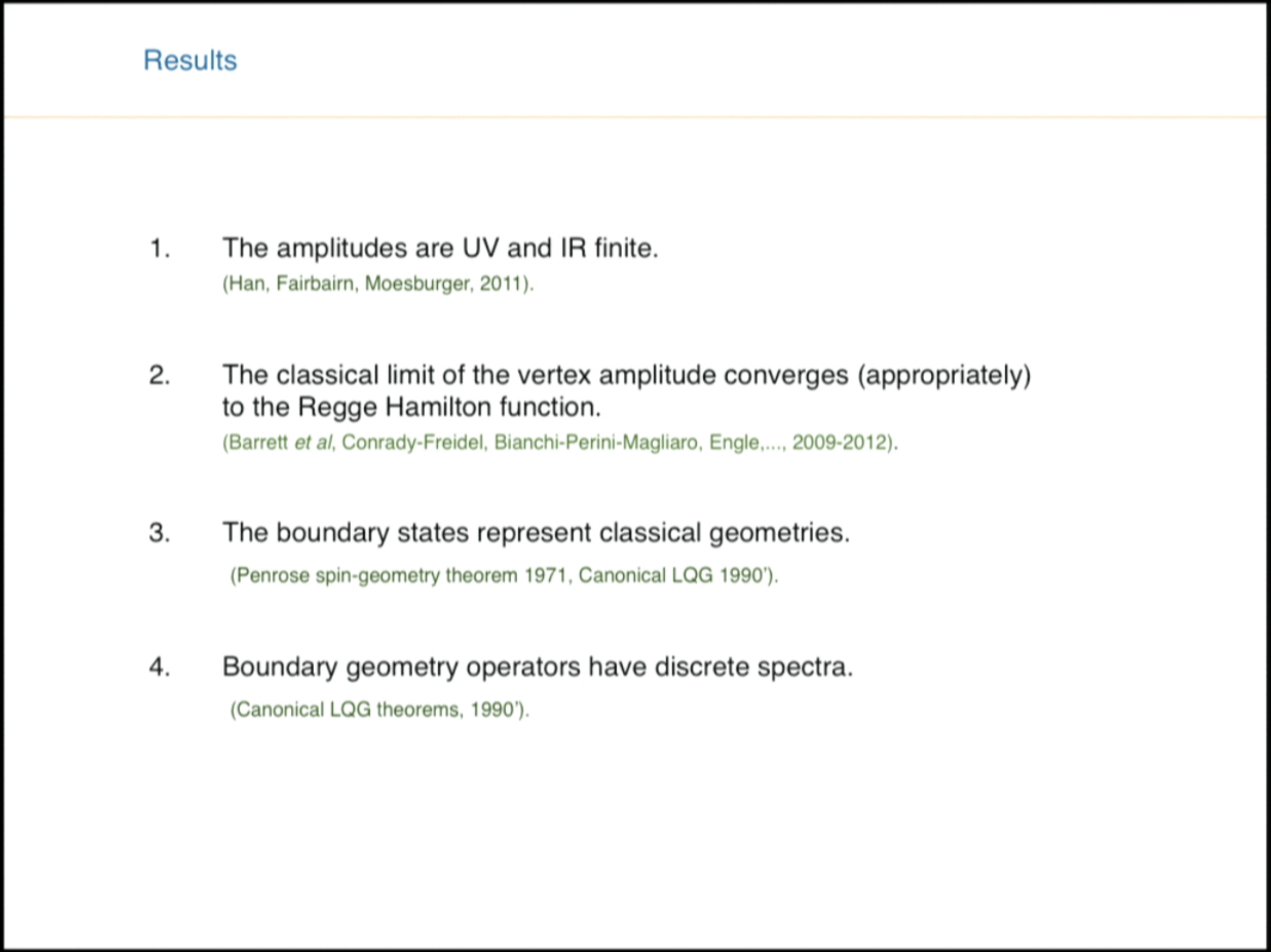
2-complex $\mathcal{C}$
(vertices, edges, faces)

$$W_{\mathcal{C}}(h_l) = \int_{SU(2)} dh_{vf} \prod_f \delta(h_f) \prod_v A(h_{vf})$$

$$A(h_{ab}) = \sum_{j_{ab}} \int_{SL(2C)} dg_a \prod_{ab} tr_{j_{ab}}[h_{ab} Y_\gamma^\dagger g_a g_b^{-1} Y_\gamma]$$

$$\begin{array}{rccc} Y_\gamma : & \mathcal{H}_j & \to & \mathcal{H}_{j,\gamma j} \\ & |j;m\rangle & \mapsto & |j,\gamma j; j, m\rangle \end{array}$$

Including a cosmological constant .
Amplitude: SL(2,C) network evaluation → Vassiliev-Kontsevich invariant.



Results

1. The amplitudes are UV and IR finite.
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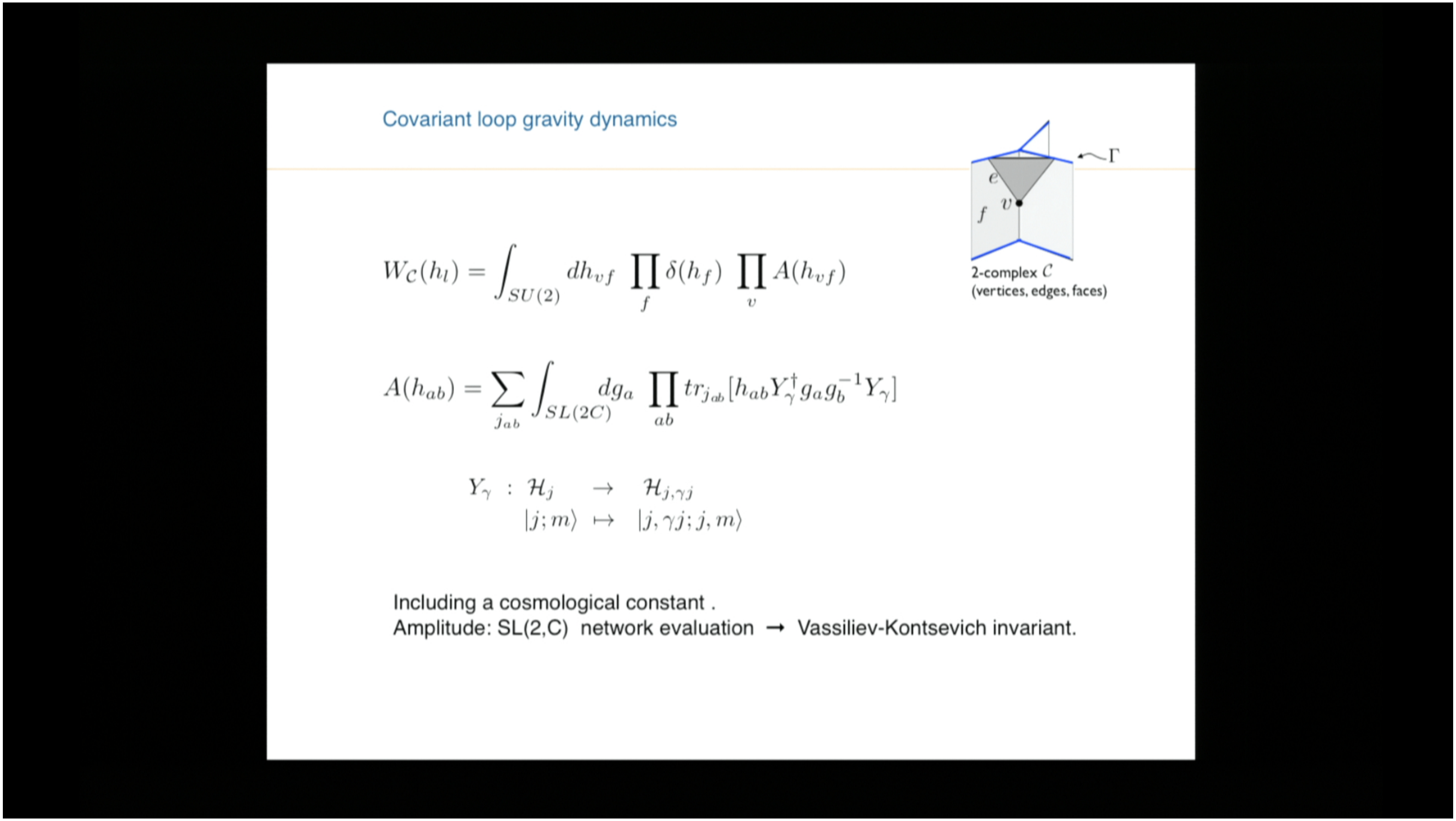

Covariant loop gravity dynamics

2-complex $\mathcal{C}$
(vertices, edges, faces)

$$W_{\mathcal{C}}(h_l) = \int_{SU(2)} dh_{vf} \prod_f \delta(h_f) \prod_v A(h_{vf})$$

$$A(h_{ab}) = \sum_{j_{ab}} \int_{SL(2C)} dg_a \prod_{ab} tr_{j_{ab}}[h_{ab} Y_\gamma^\dagger g_a g_b^{-1} Y_\gamma]$$

$$\begin{array}{rcccl} Y_\gamma & : & \mathcal{H}_j & \rightarrow & \mathcal{H}_{j,\gamma j} \\ & & |j;m\rangle & \mapsto & |j,\gamma j; j, m\rangle \end{array}$$

Including a cosmological constant .
Amplitude: SL(2,C) network evaluation → Vassiliev-Kontsevich invariant.

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Continuous limit $\uparrow$

Classical limit $\longrightarrow$

Notice:

- No critical point
- No infinite renormalization
- Physical scale: $L^2_{Planck} = \hbar G$
- Cfr: condensed matter away from critical points
- Same hint from string theory: finite UV scale and no infinities.

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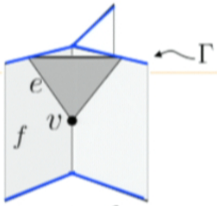

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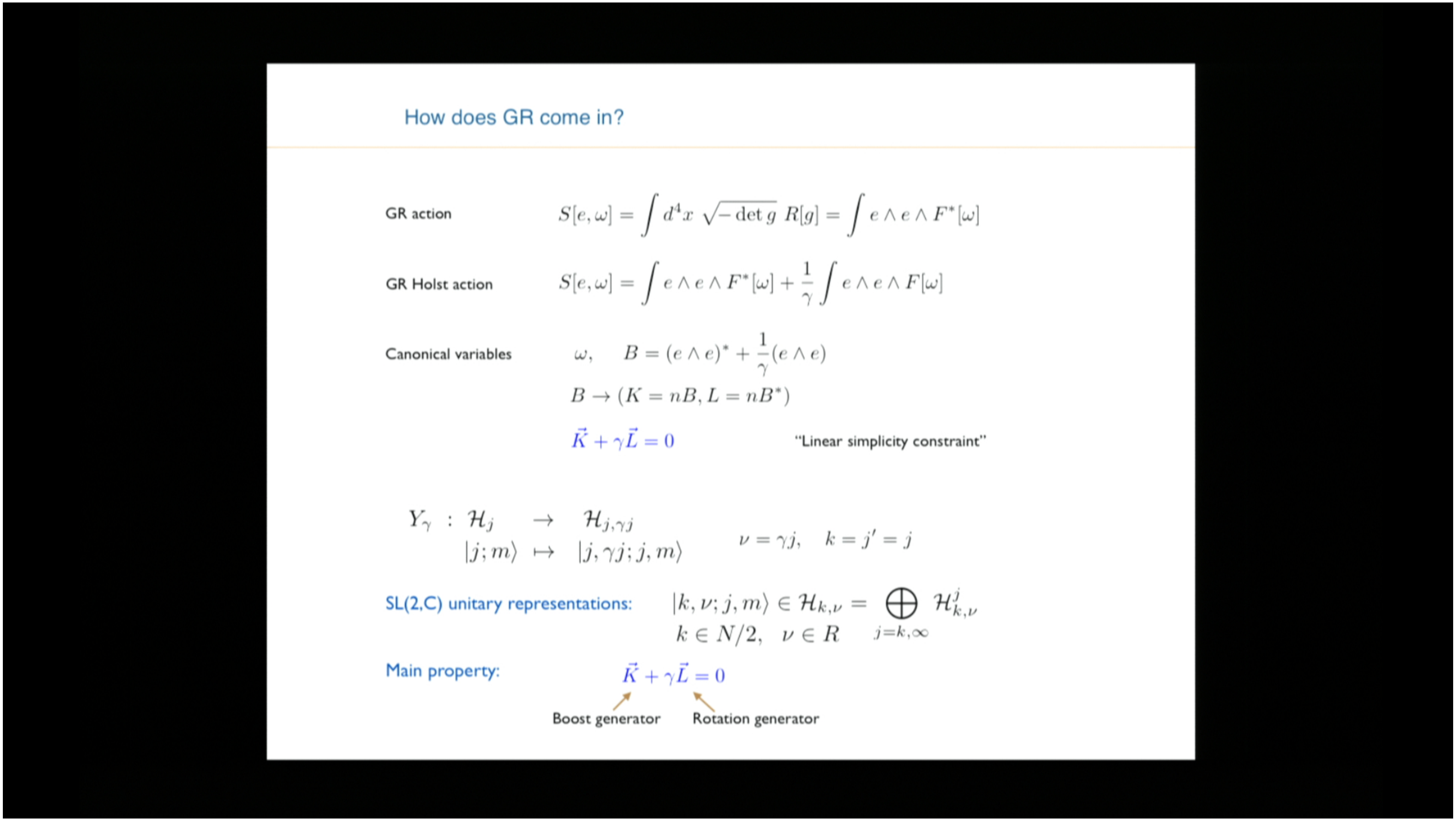


How does GR come in?

GR action $$S[e,\omega] = \int d^4x \sqrt{-\det g}\; R[g] = \int e \wedge e \wedge F^*[\omega]$$

GR Holst action $$S[e,\omega] = \int e \wedge e \wedge F^*[\omega] + \frac{1}{\gamma} \int e \wedge e \wedge F[\omega]$$

Canonical variables $$\omega, \qquad B = (e \wedge e)^* + \frac{1}{\gamma}(e \wedge e)$$

$$B \rightarrow (K = nB, L = nB^*)$$

$$\vec{K} + \gamma \vec{L} = 0$$

"Linear simplicity constraint"

$$\begin{array}{rcl} Y_\gamma \;:\; \mathcal{H}_j & \rightarrow & \mathcal{H}_{j,\gamma j} \\ |j;m\rangle & \mapsto & |j,\gamma j; j, m\rangle \end{array} \qquad \nu = \gamma j, \quad k = j' = j$$

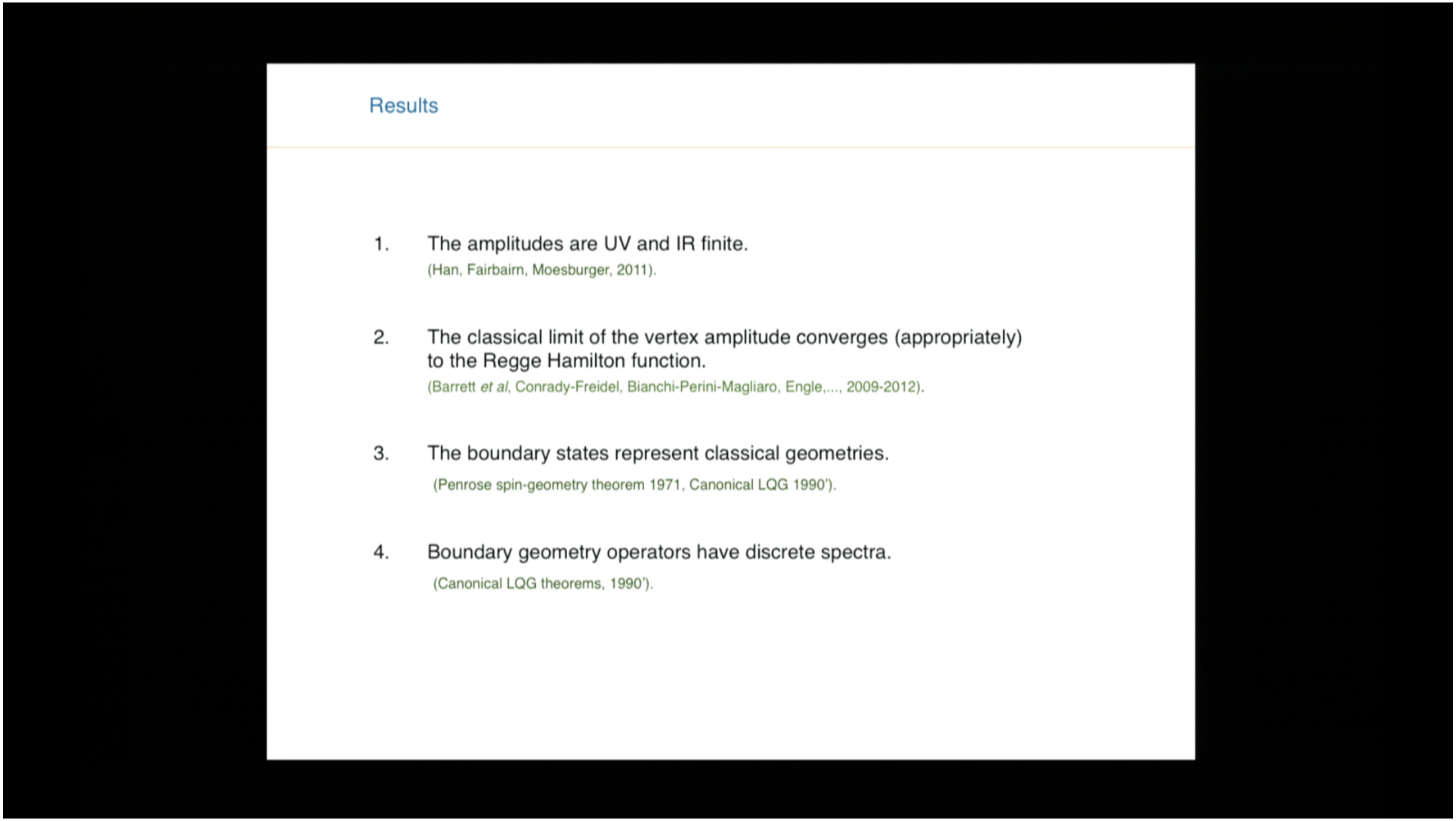
SL(2,C) unitary representations: $$|k,\nu;j,m\rangle \in \mathcal{H}_{k,\nu} = \bigoplus_{j=k,\infty} \mathcal{H}^j_{k,\nu}$$

$$k \in N/2, \quad \nu \in R$$

Main property: $$\vec{K} + \gamma \vec{L} = 0$$

Boost generator

Rotation generator



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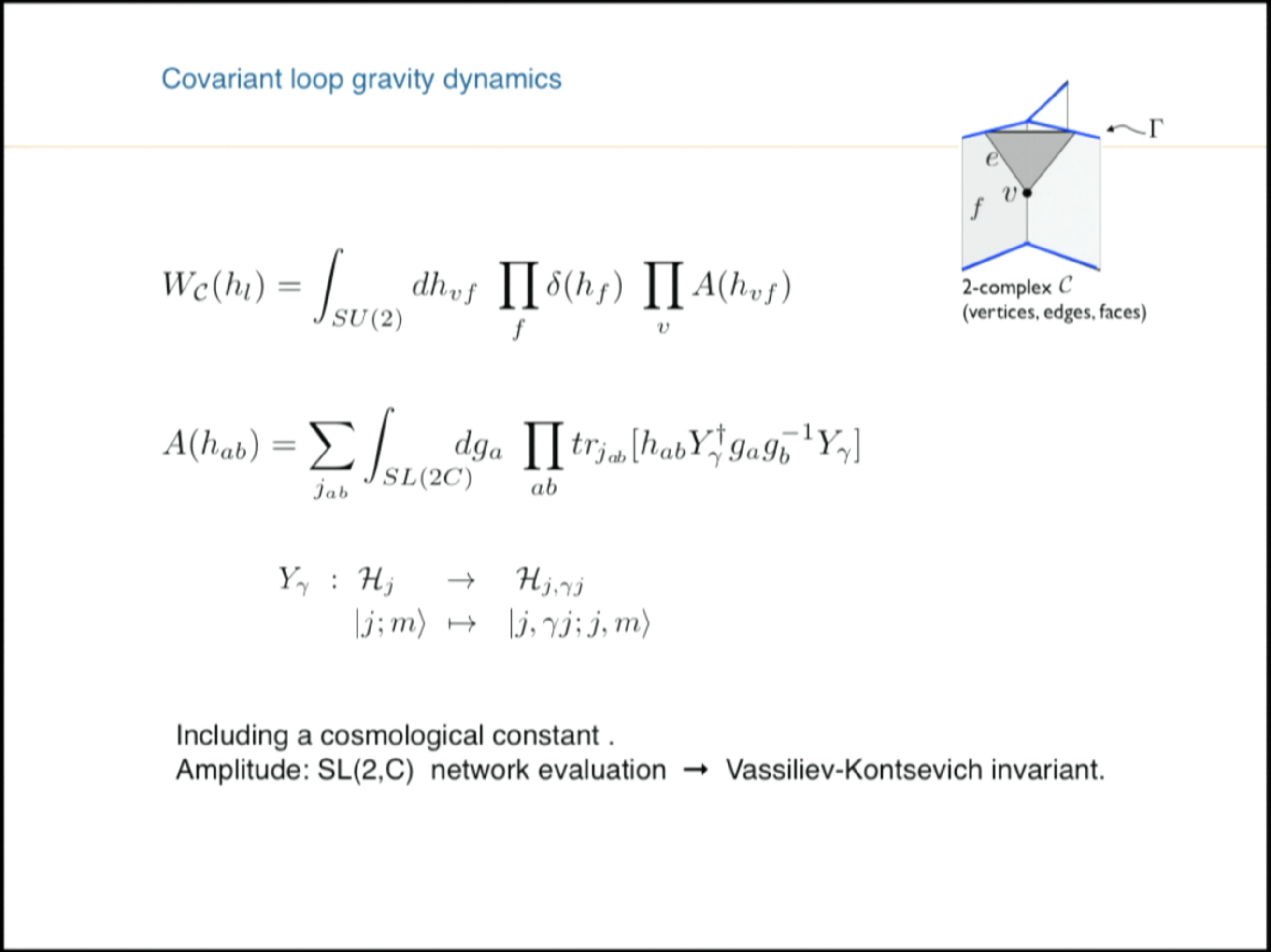

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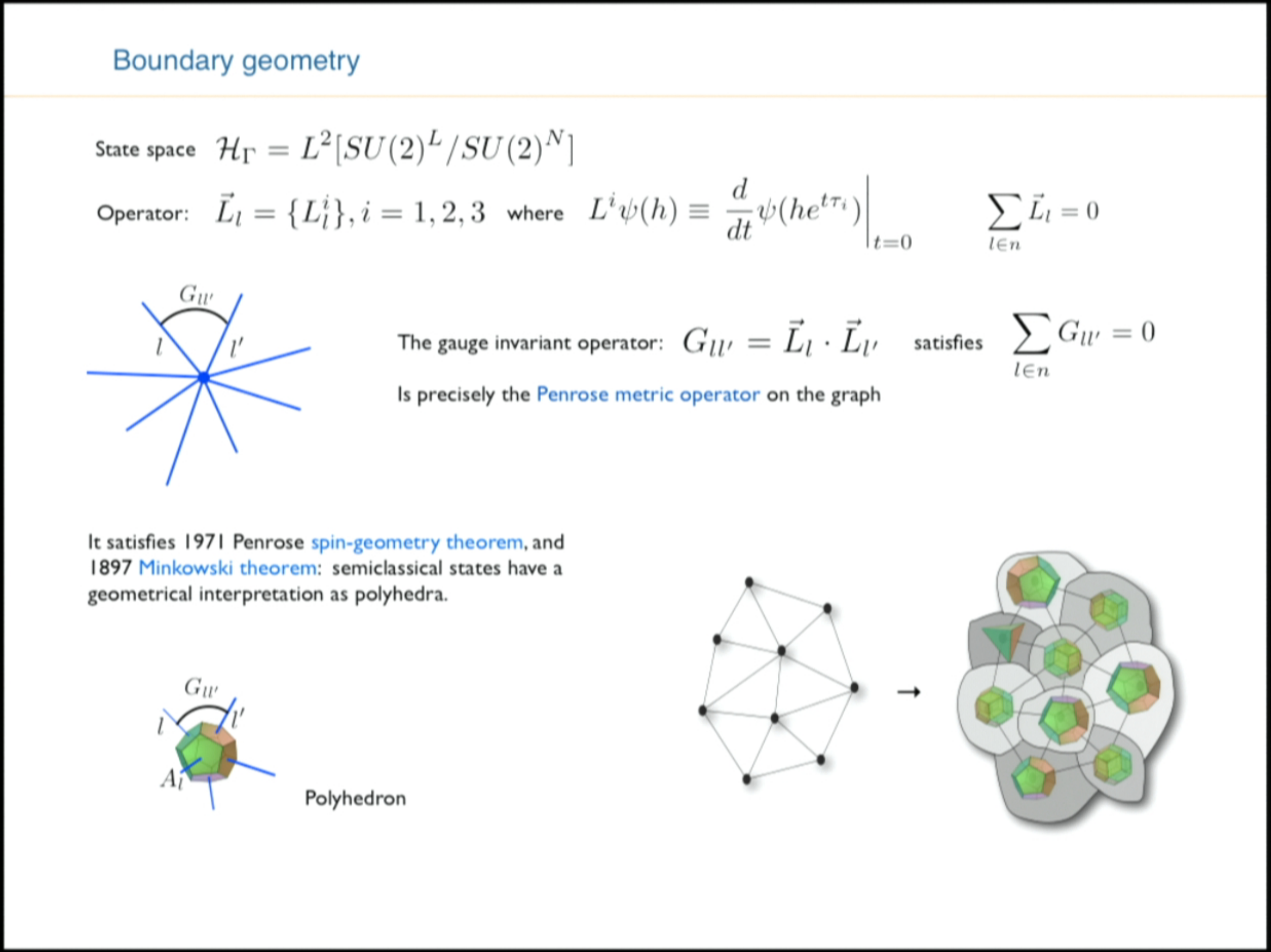


Boundary geometry

State space $\mathcal{H}_\Gamma = L^2[SU(2)^L/SU(2)^N]$

Operator: $\vec{L}_l = \{L^i_l\}, i = 1,2,3$ where $L^i\psi(h) \equiv \frac{d}{dt}\psi(he^{t\tau_i})\Big|_{t=0}$ $\sum_{l\in n}\vec{L}_l = 0$

The gauge invariant operator: $G_{ll'} = \vec{L}_l \cdot \vec{L}_{l'}$ satisfies $\sum_{l\in n} G_{ll'} = 0$

Is precisely the Penrose metric operator on the graph

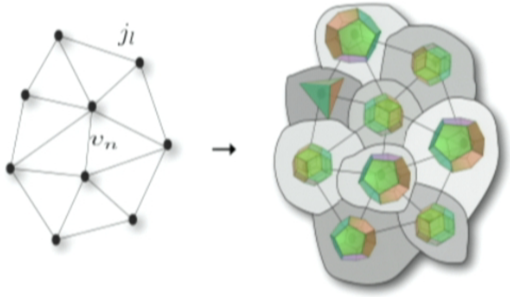
It satisfies 1971 Penrose spin-geometry theorem, and 1897 Minkowski theorem: semiclassical states have a geometrical interpretation as polyhedra.

Polyhedron

Boundary geometry

area $A_l^2 = G_{ll}$ volume $V_n^2 = \frac{2}{9}\, \vec{L}_{l_1} \cdot (\vec{L}_{l_2} \times \vec{L}_{l_3})$

- Area and volume (A_l, V_n) form a complete set of commuting observables and have discrete spectra
- → basis $|\Gamma, j_l, v_n\rangle$



Geometry is quantized:

(i) eigenvalues are discrete
(ii) the operators do not commute
(iii) a generic state is a quantum superposition

→ coherent states theory

Nodes: discrete quanta of volume ("quanta of space") with quantum number v_n .

Links: discrete quanta of area, with quantum number j_l .

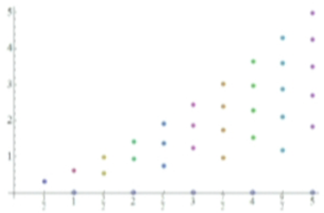
→ States describe quantum geometries:

not quantum states in spacetime

but rather quantum states of spacetime

Boundary geometry

- Area eigenvalues $A = 8\pi\gamma\hbar G \sqrt{j_l(j_l+1)}$
- There is an area gap $a_o = 8\pi\gamma\hbar G \frac{\sqrt{3}}{2}$ and the volume eigenvalues are finite and discrete:



- Using this basis, the amplitude reads $Z_{\mathcal{C}} = \sum_{j_f, v_e} N_{\{j_f\}} \prod_f (2j_f+1) \prod_v A_v(j_f, v_e)$

→ UV finiteness of the transition amplitudes

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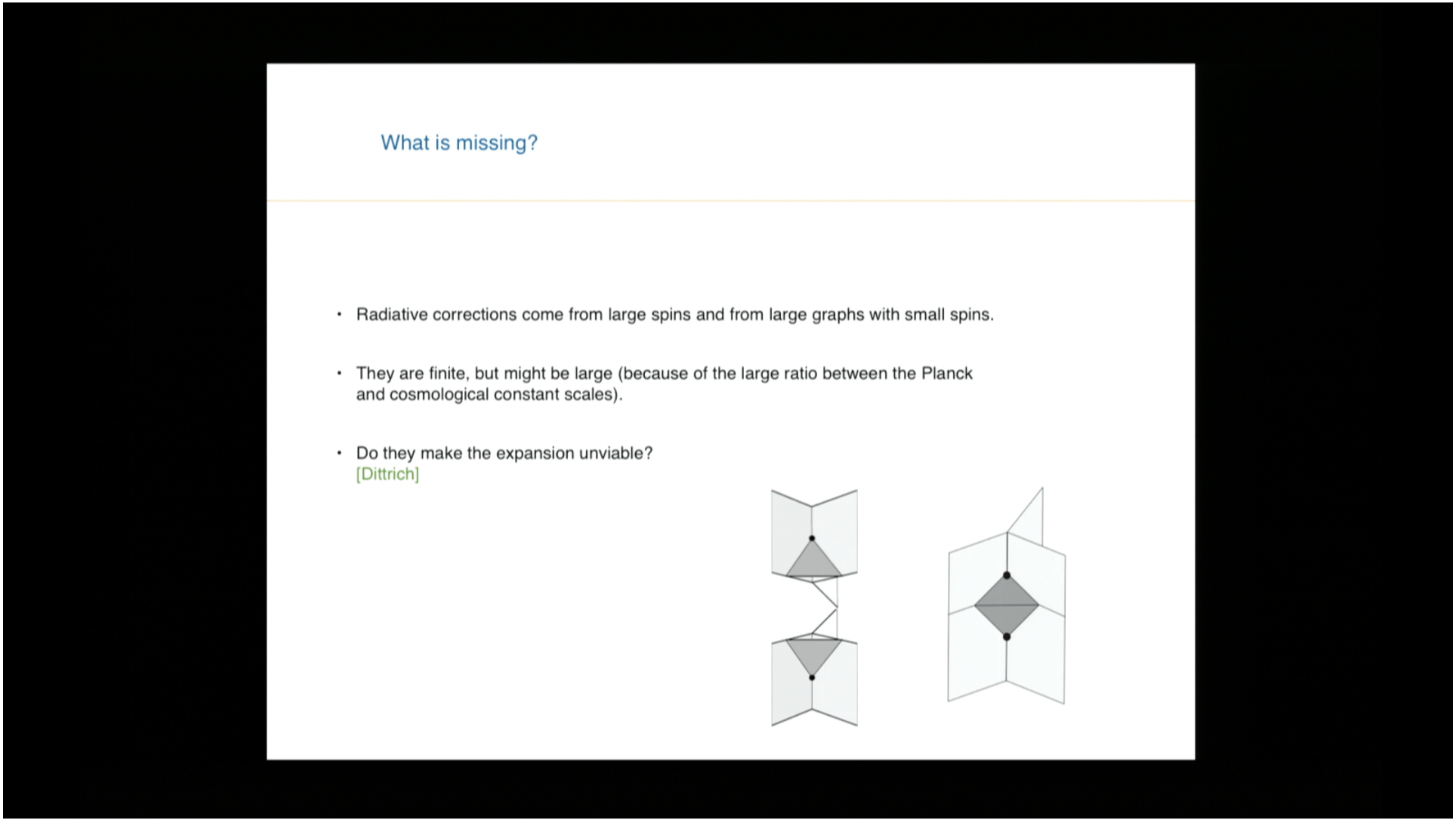
Continuous limit ↑

Classical limit →

Regime of validity of the expansion: $L_{Planck} \ll L \ll \sqrt{\frac{1}{R}}$



What is missing?

- Radiative corrections come from large spins and from large graphs with small spins.
- They are finite, but might be large (because of the large ratio between the Planck and cosmological constant scales).
- Do they make the expansion unviable?
  [Dittrich]

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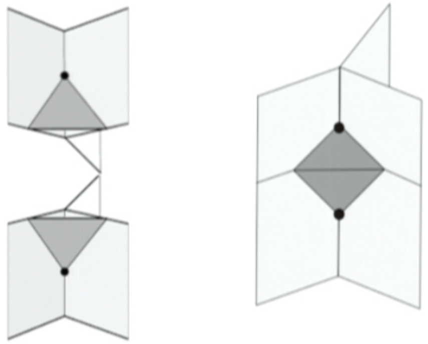
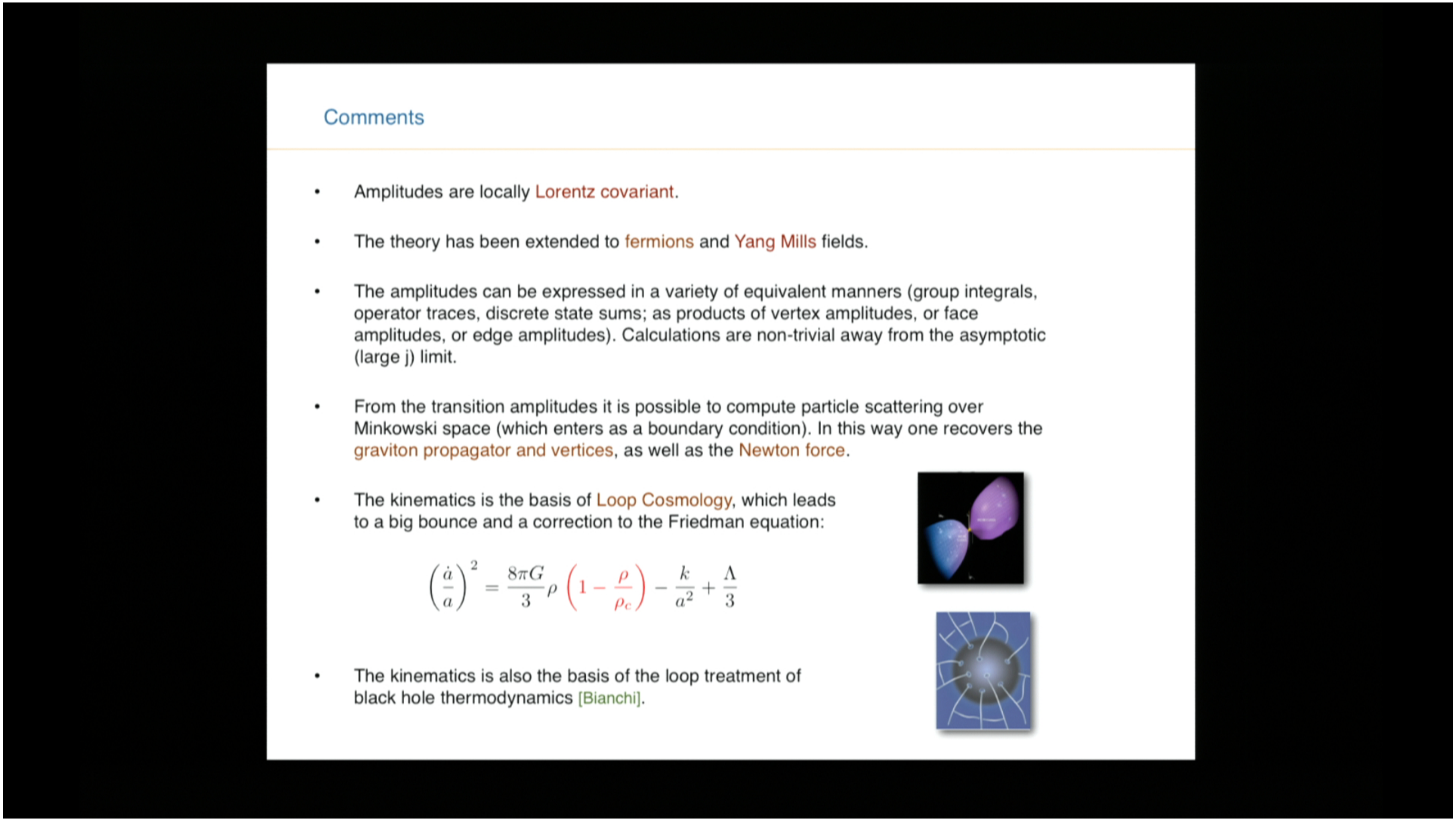


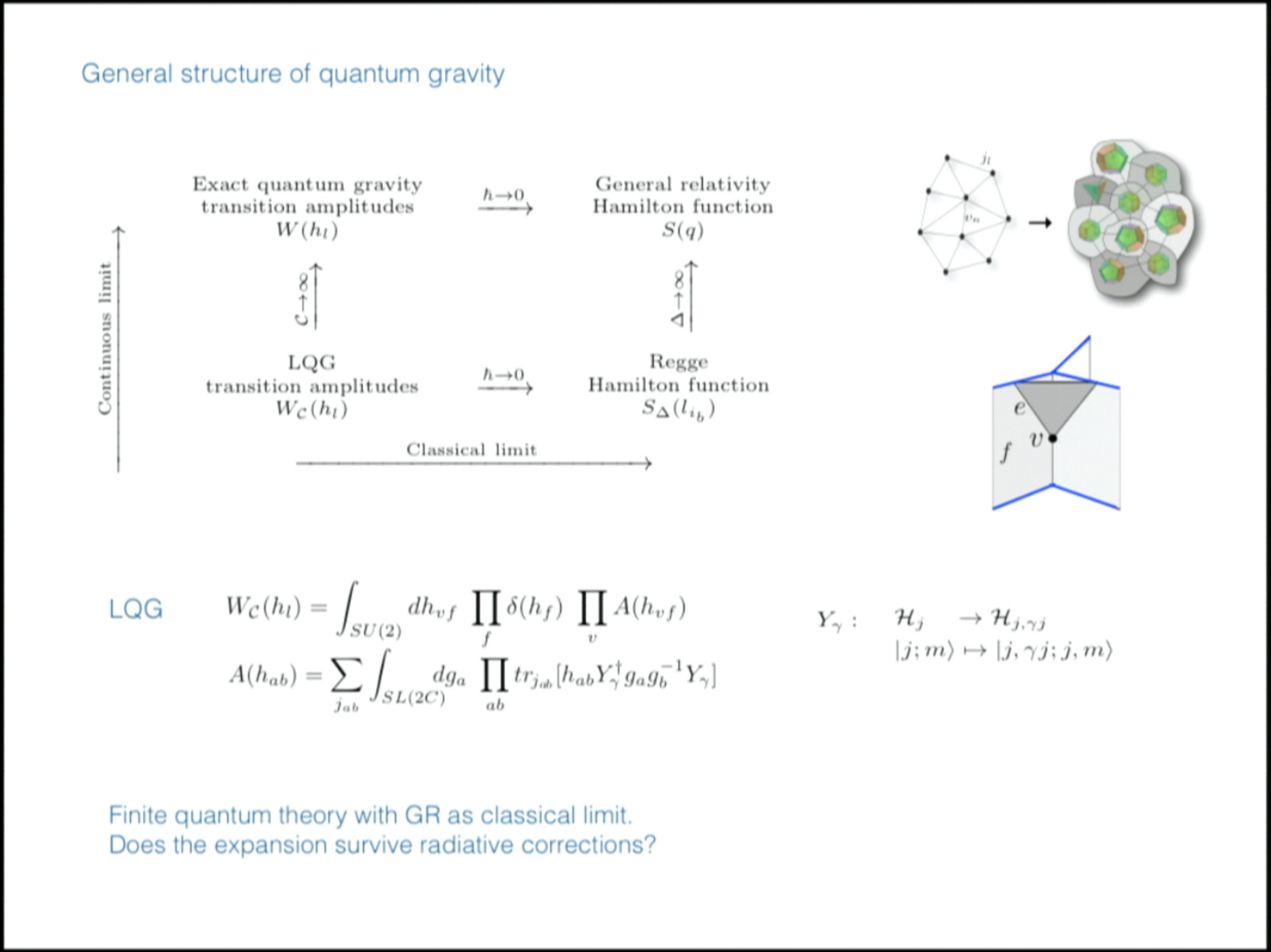


Comments

- Amplitudes are locally Lorentz covariant.
- The theory has been extended to fermions and Yang Mills fields.
- The amplitudes can be expressed in a variety of equivalent manners (group integrals, operator traces, discrete state sums; as products of vertex amplitudes, or face amplitudes, or edge amplitudes). Calculations are non-trivial away from the asymptotic (large j) limit.
- From the transition amplitudes it is possible to compute particle scattering over Minkowski space (which enters as a boundary condition). In this way one recovers the graviton propagator and vertices, as well as the Newton force.
- The kinematics is the basis of Loop Cosmology, which leads to a big bounce and a correction to the Friedman equation:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho\left(1-\frac{\rho}{\rho_c}\right) - \frac{k}{a^2} + \frac{\Lambda}{3}$$

- The kinematics is also the basis of the loop treatment of black hole thermodynamics [Bianchi].



General structure of quantum gravity

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Finite quantum theory with GR as classical limit.
Does the expansion survive radiative corrections?

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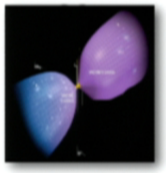

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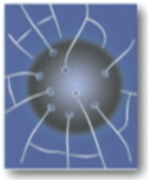

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