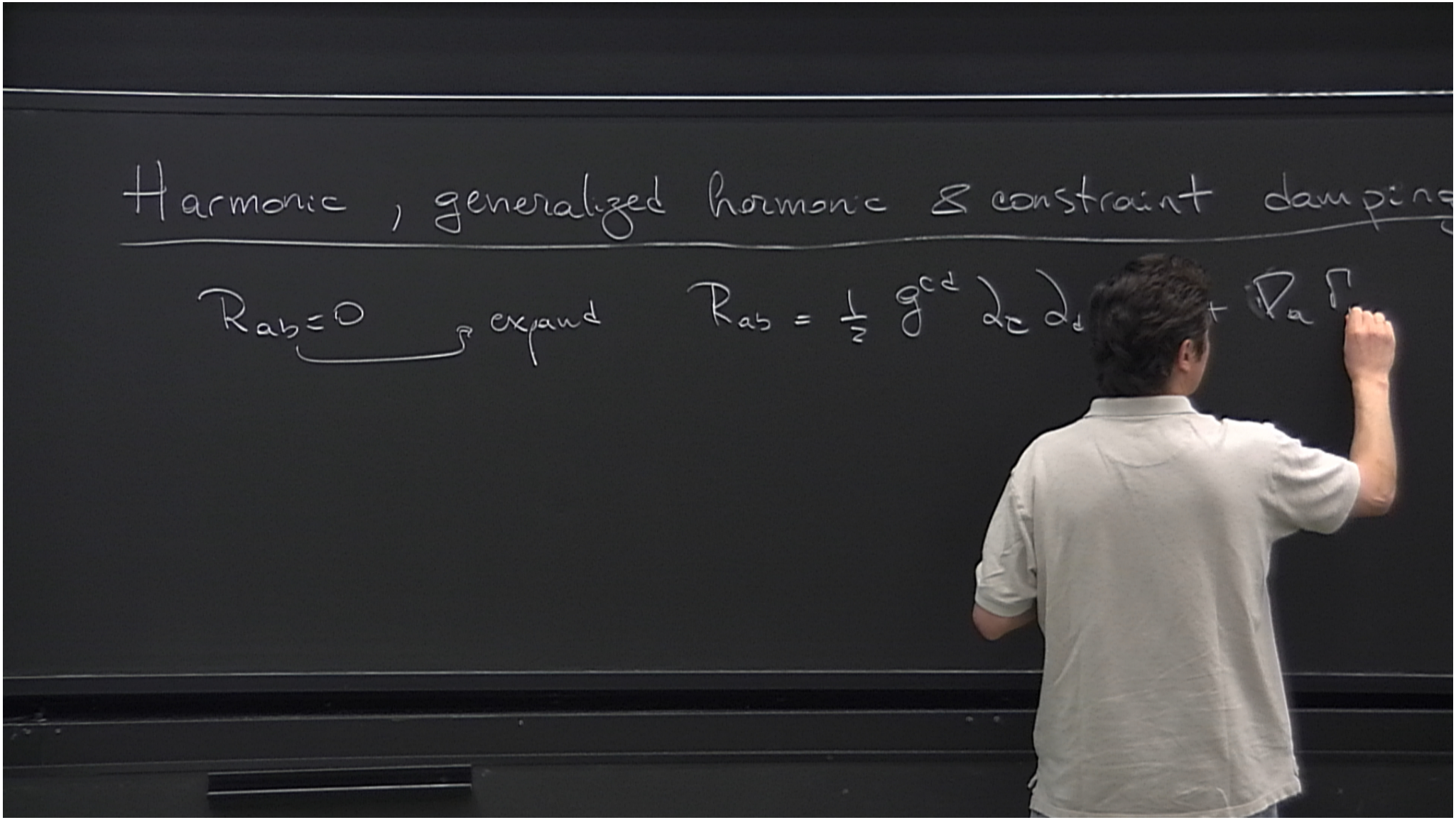
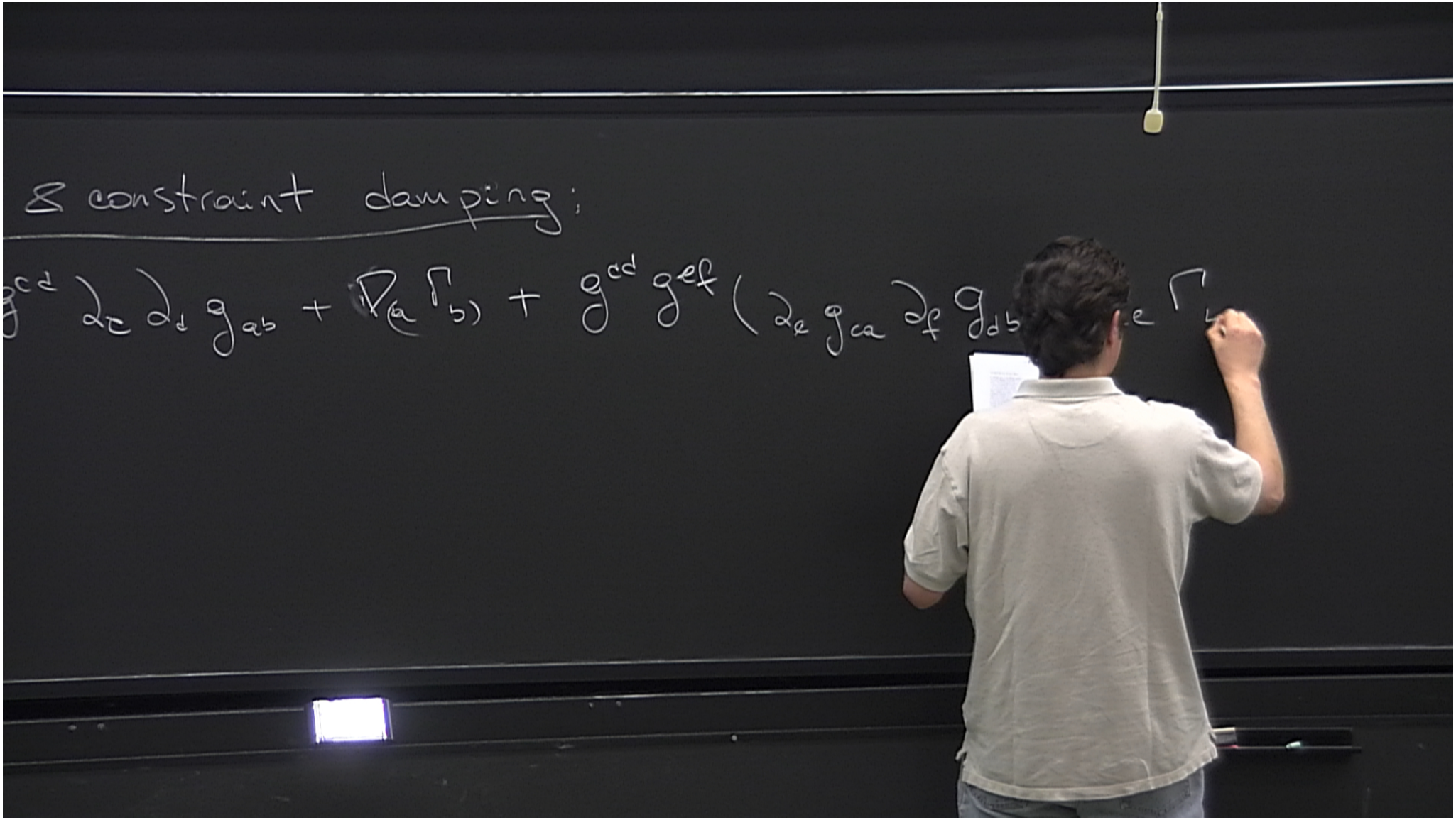
Title: Explorations in Numerical Relativity - Lecture 13

Date: Apr 19, 2012 11:30 AM

URL: http://pirsa.org/12040055

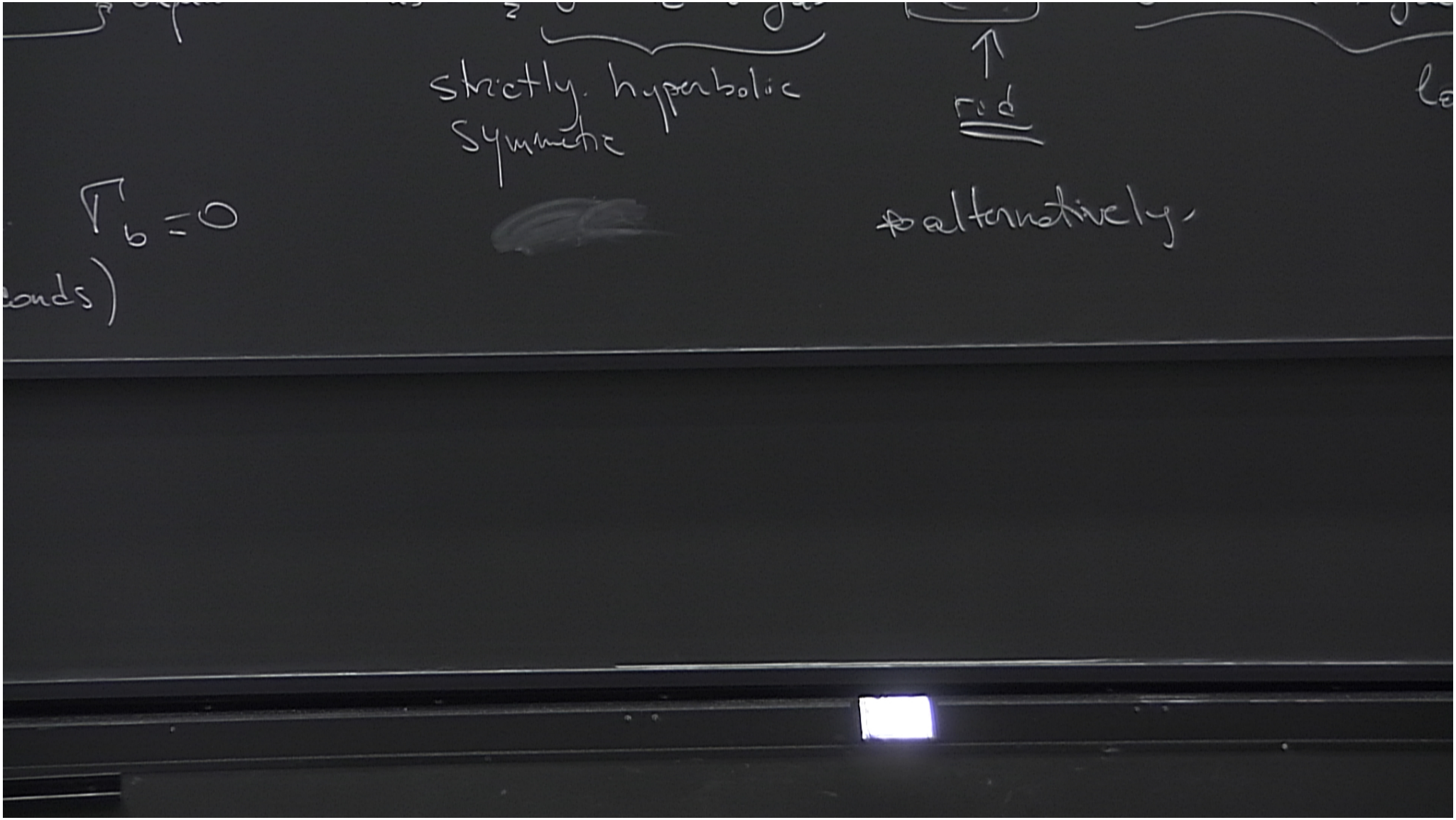
Abstract:

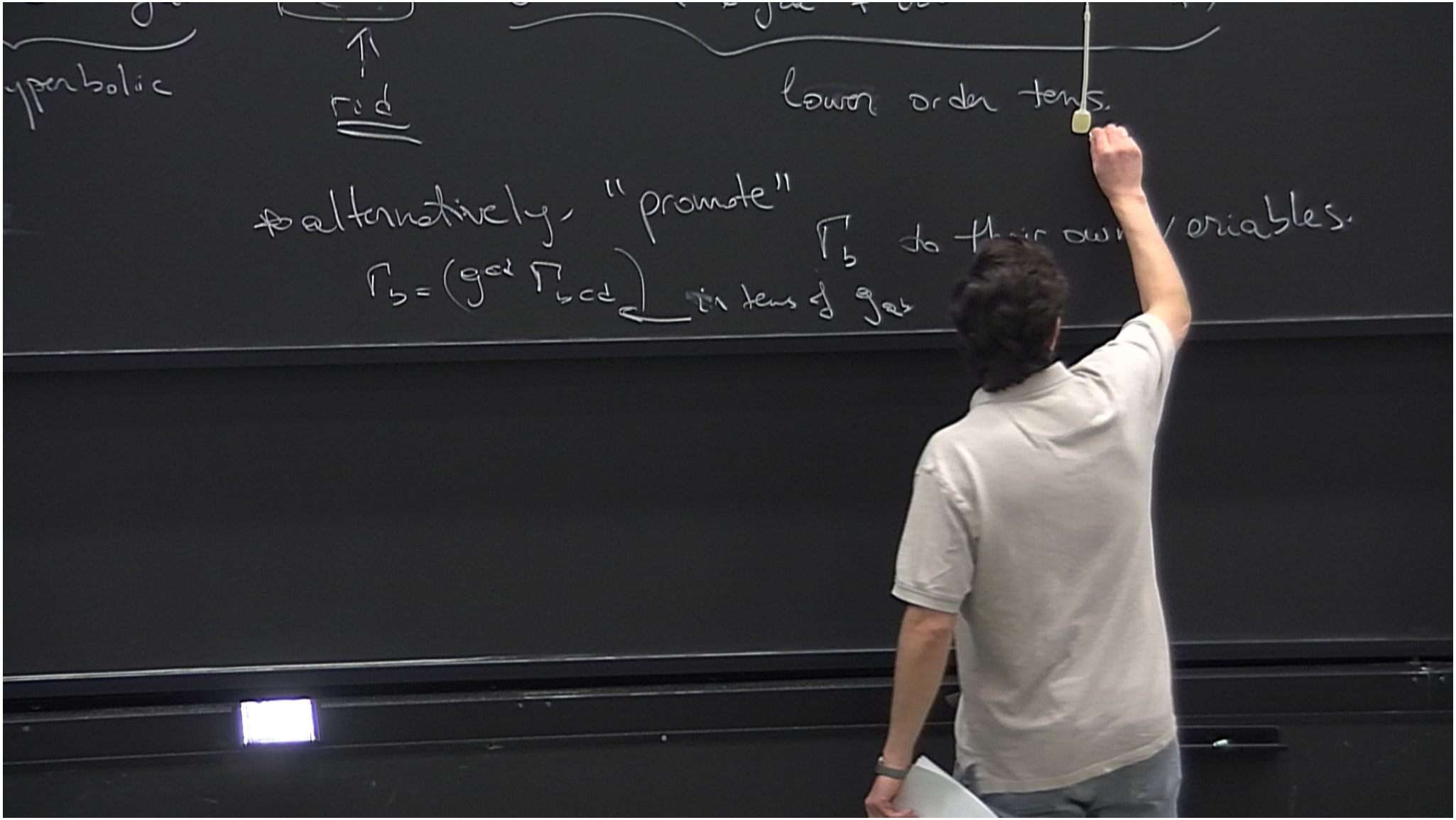

Harmonic , generalized harmonic & constraint damping
$R_{ab}=0$
expand
$R_{ab} = \frac{1}{2} g^{cd} \partial_c \partial_d$
$+ \nabla_a \Gamma$

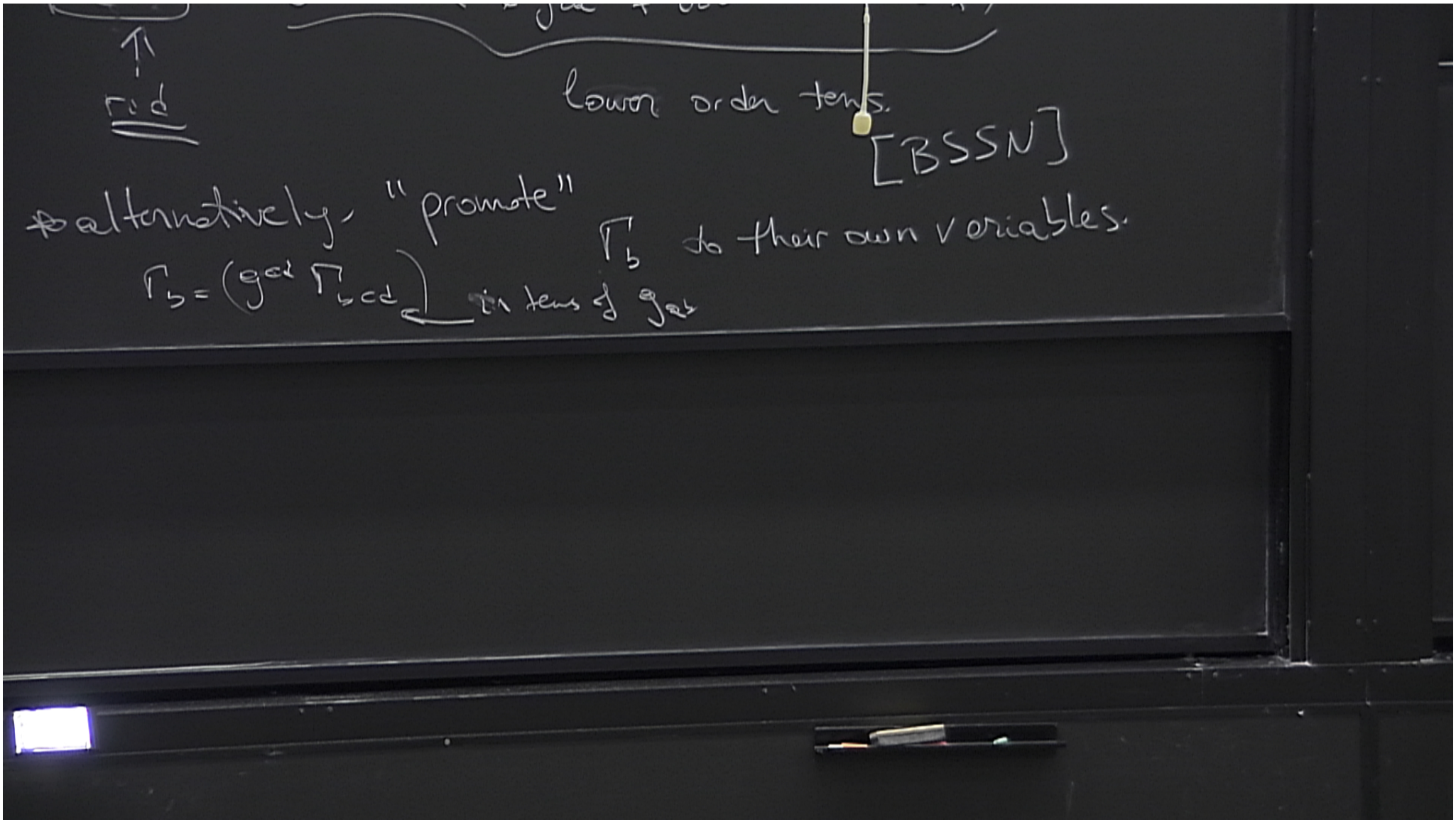

& constraint damping:

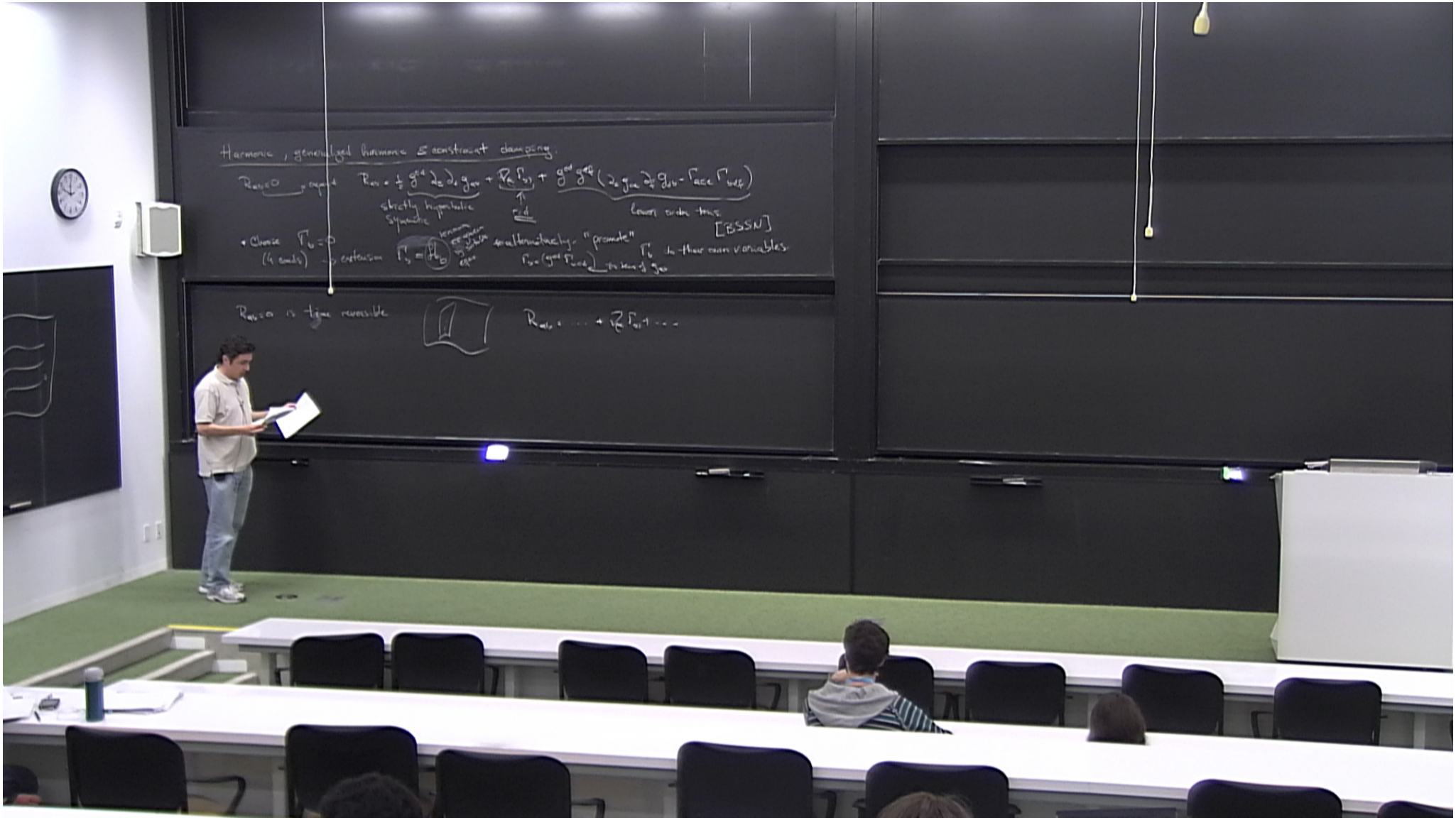
& constraint damping:

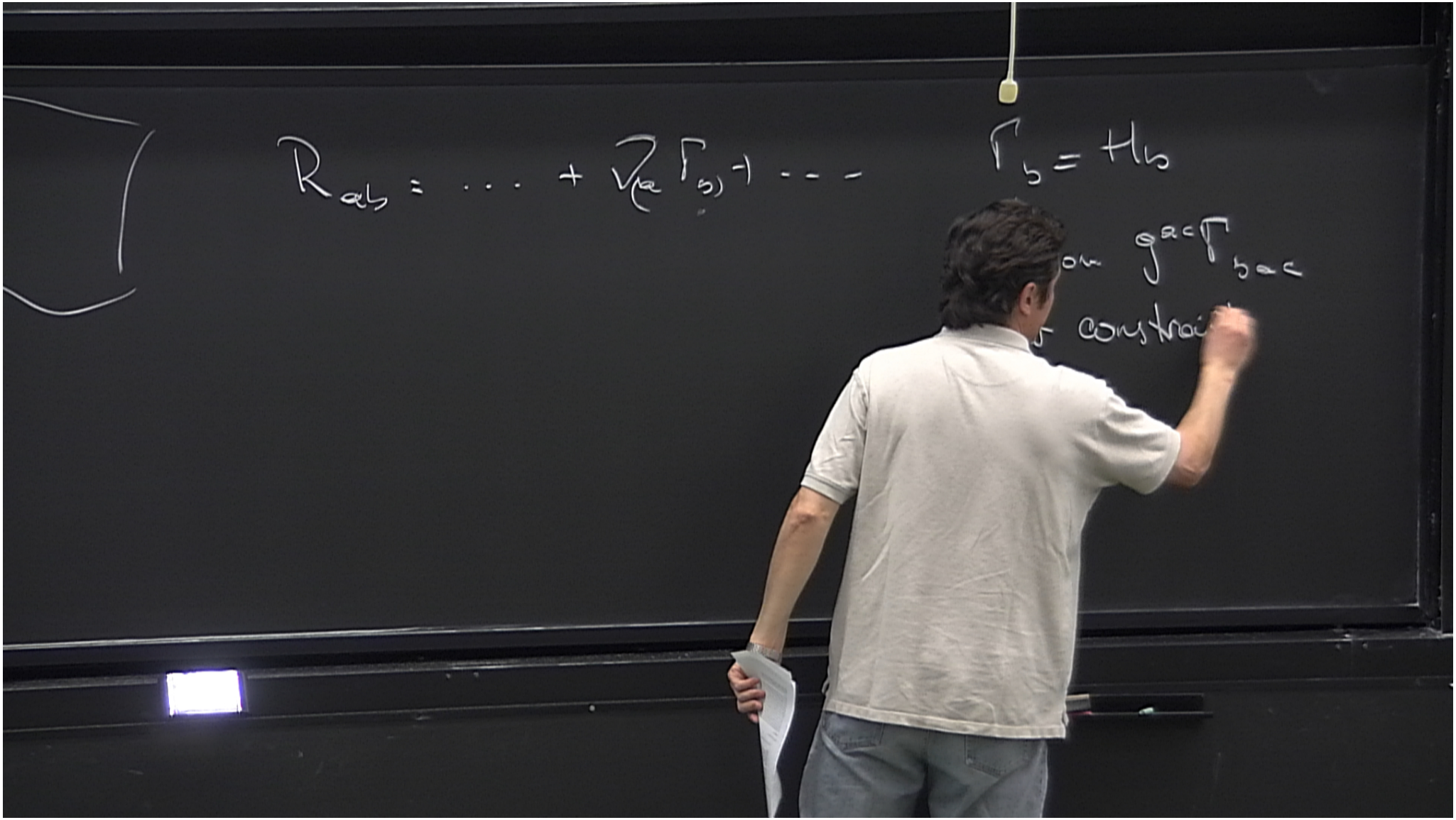
$$g^{cd}\,\partial_c\partial_d\,g_{ab} + \nabla_{(a}\Gamma_{b)} + g^{cd}g^{ef}\underbrace{\left(\partial_e g_{ca}\,\partial_f g_{db} - \Gamma_{ace}\Gamma_{bdf}\right)}_{\text{lower order terms.}}$$

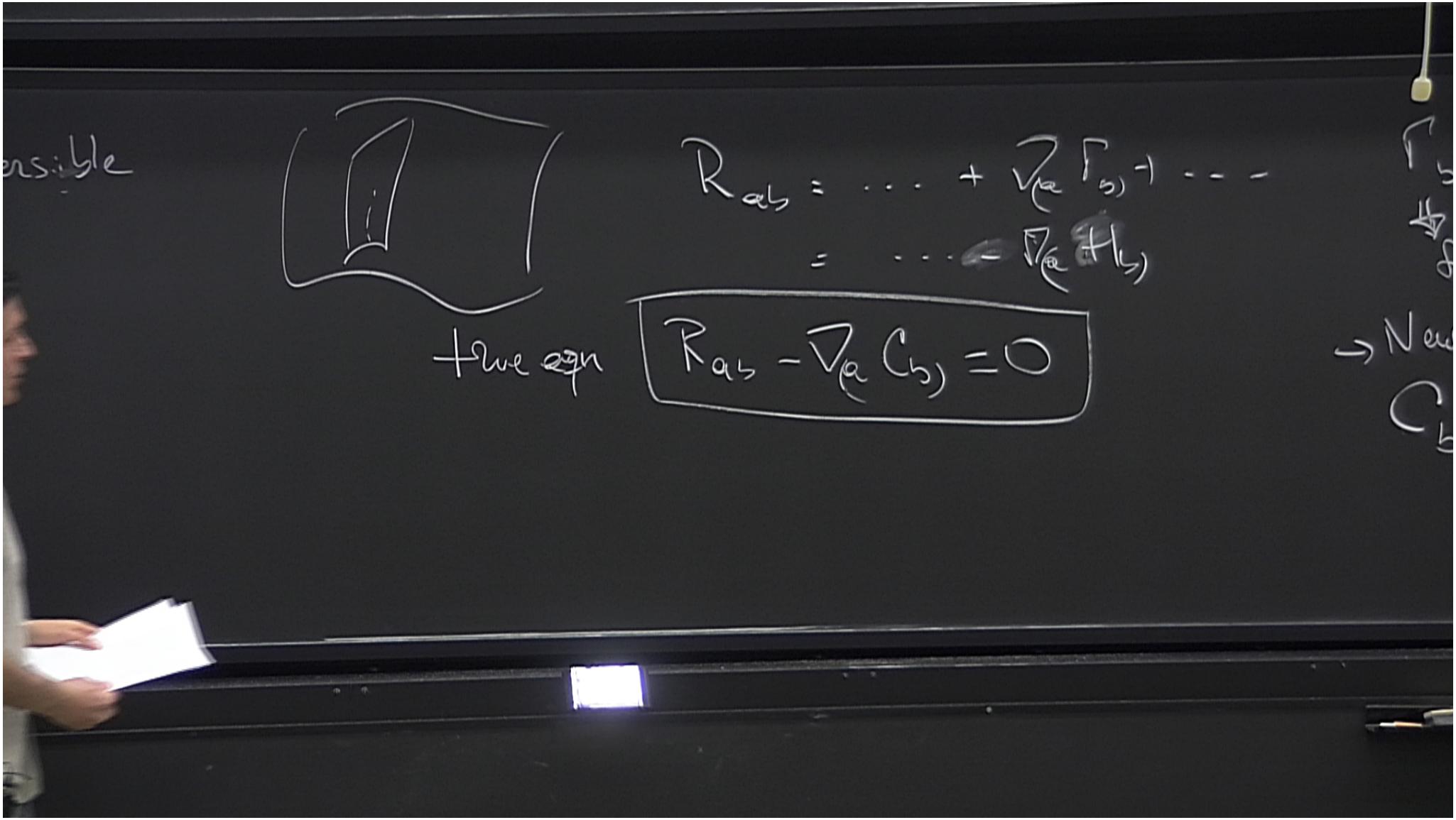

strictly. hyperbolic
symmetric
$\Gamma_b = 0$
* alternatively.

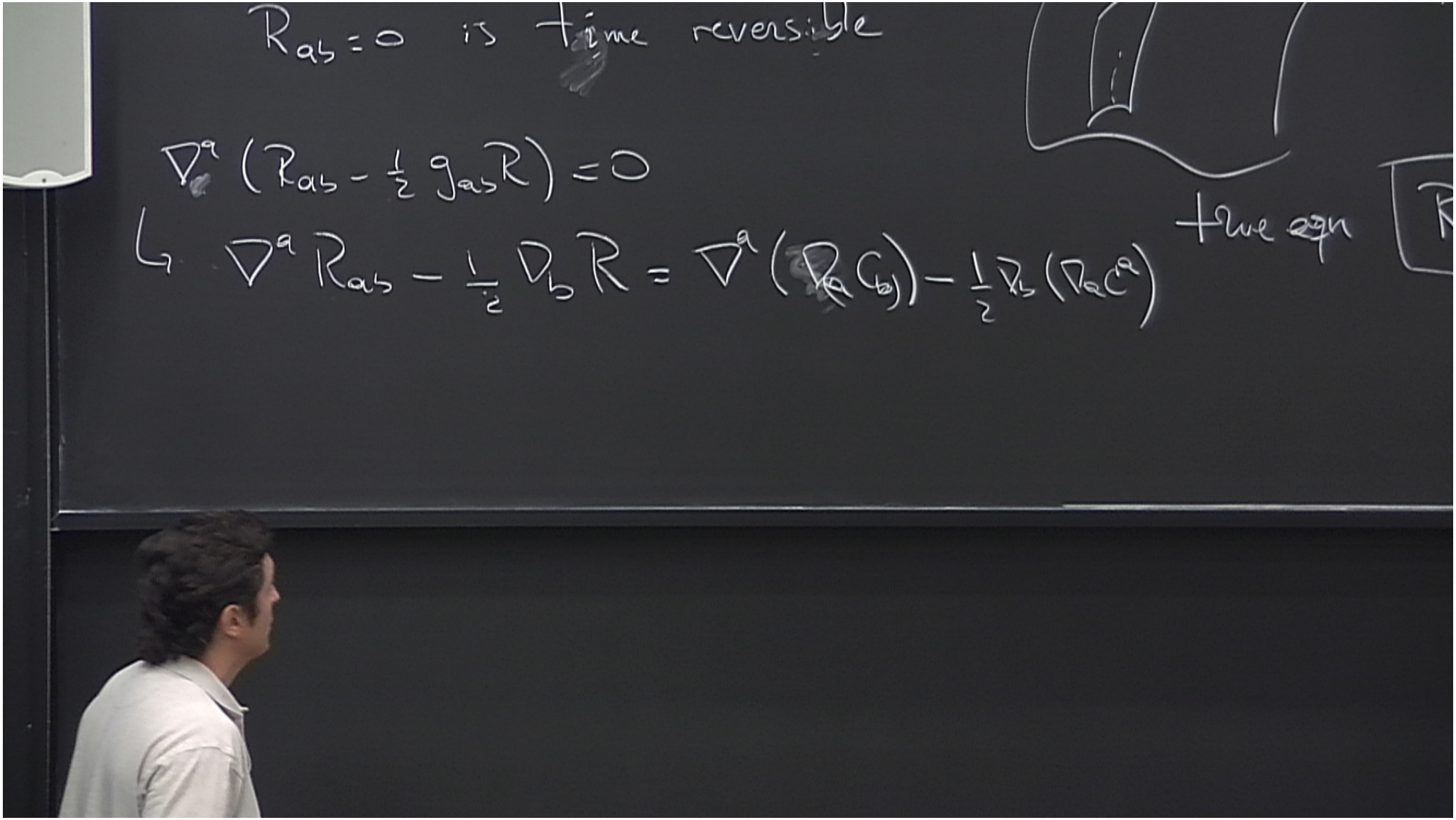

yperbolic
rid
lower order tens.
* alternatively, "promote"
Γ_b do th ir ow variables.
$\Gamma_b = (g^{ad}\Gamma_{bcd})$ in tems of g_{ab}

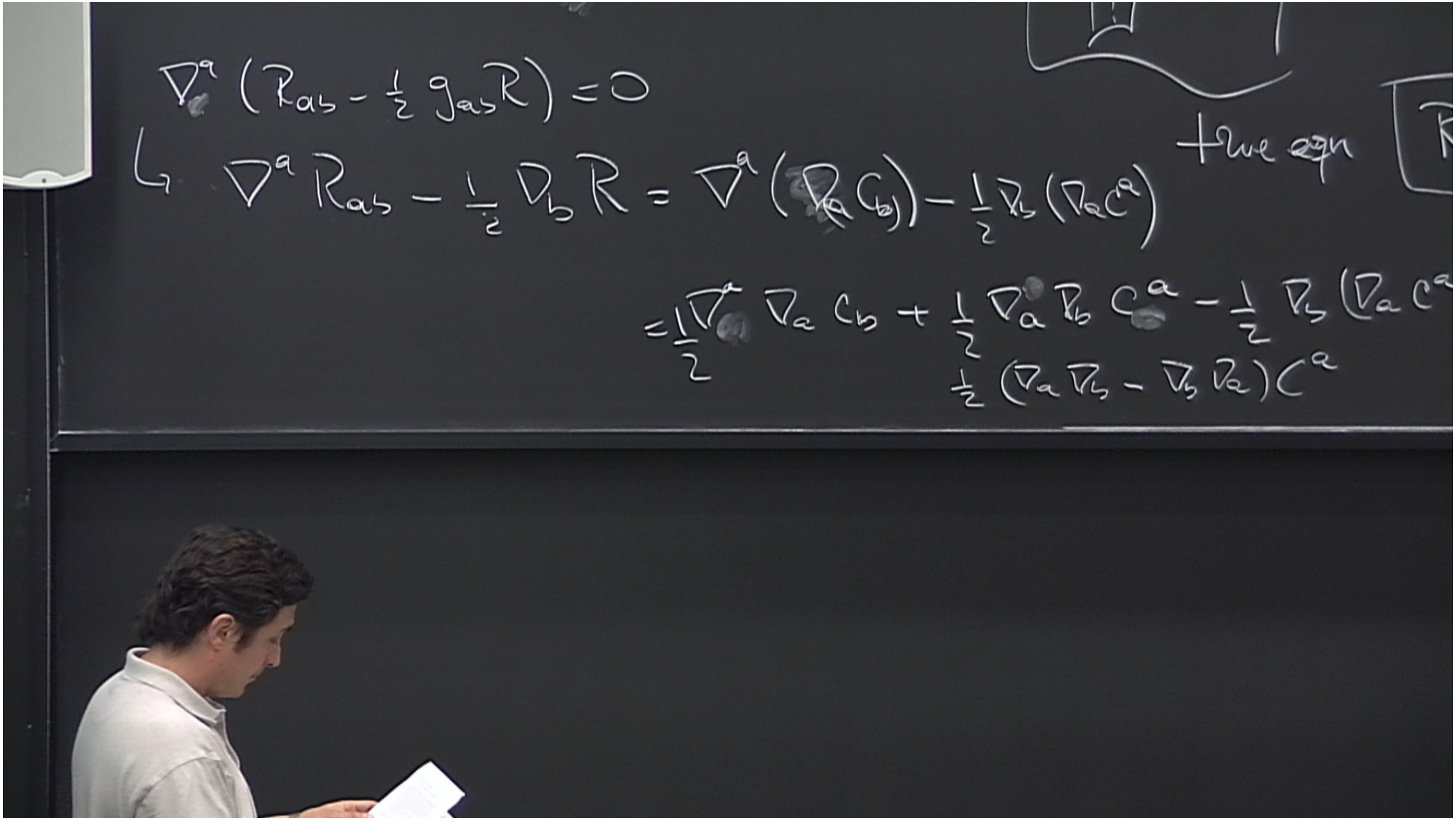

r:d
lower order terms.
[BSSN]
* alternatively: "promote" Γ_b to their own variables.
$\Gamma_b = (g^{cd}\Gamma_{bcd})$ in terms of g_{ab}

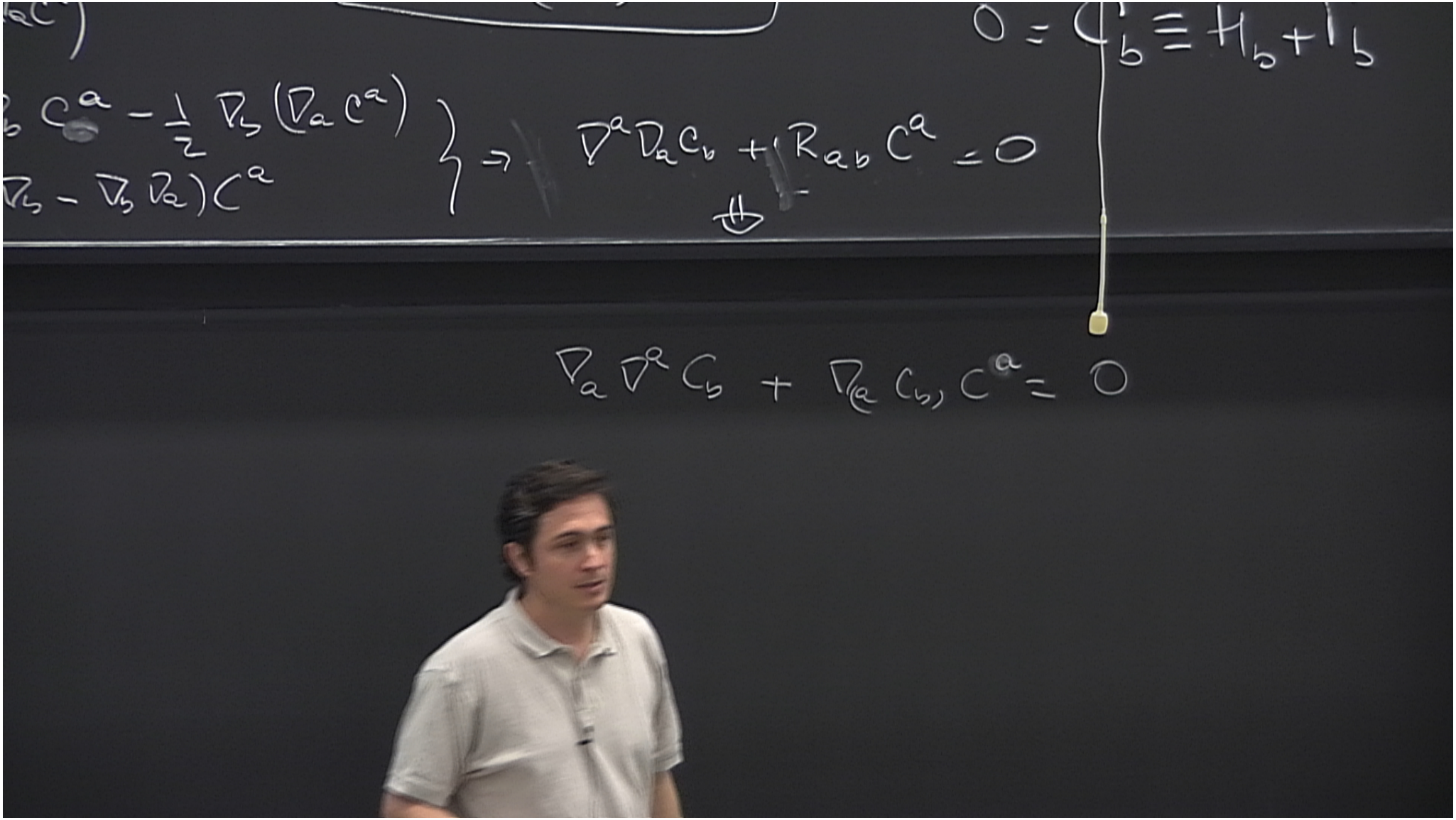

Harmonic, generalized harmonic & constraint damping.
[BSSN]
Ricci=0 is time reversible

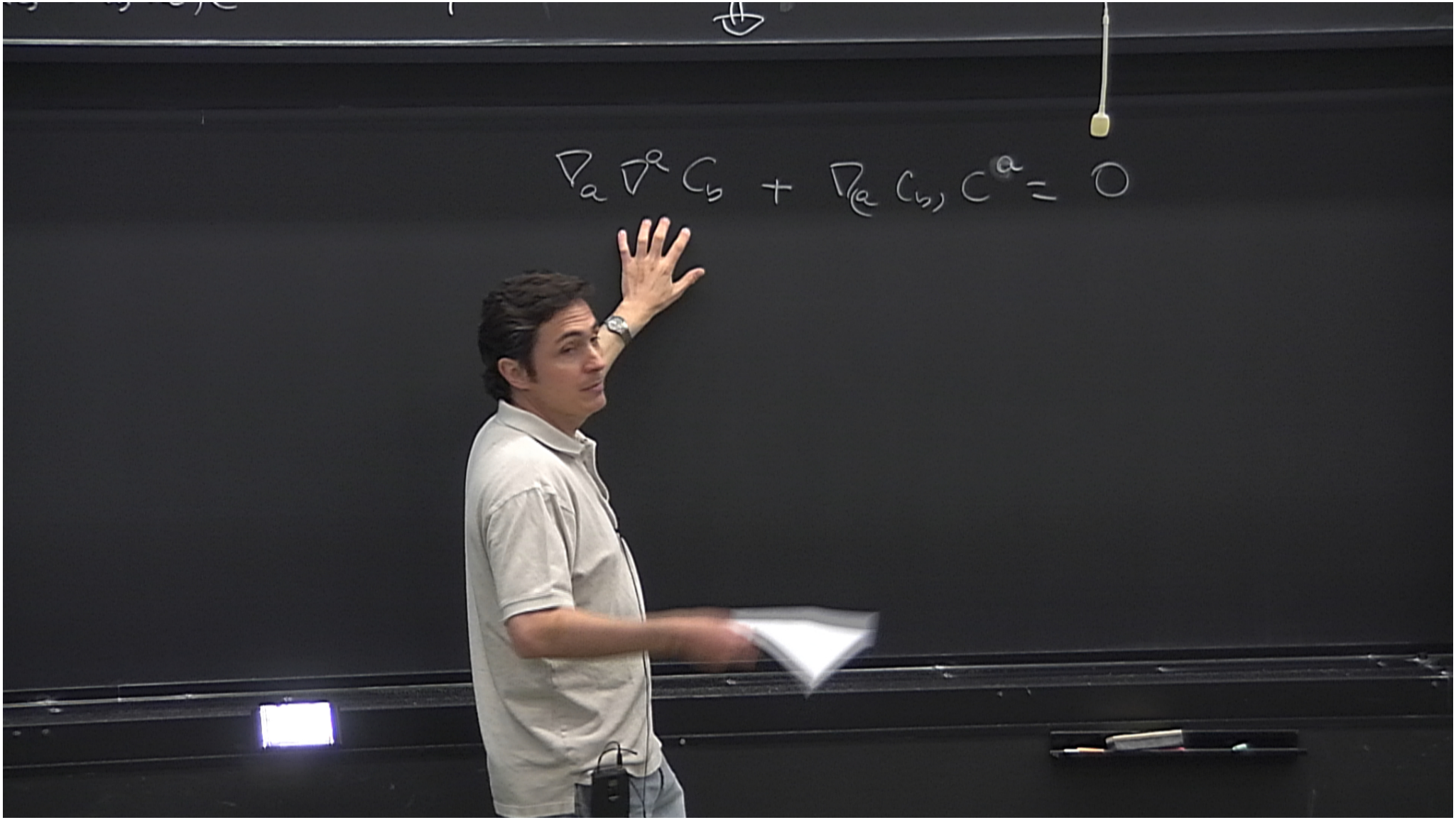

$R_{ab} = \ldots + \partial_{[a}\Gamma_{b]} - 1 \ldots$
$\Gamma_b = H_b$
$g^{ac}\Gamma_{bac}$

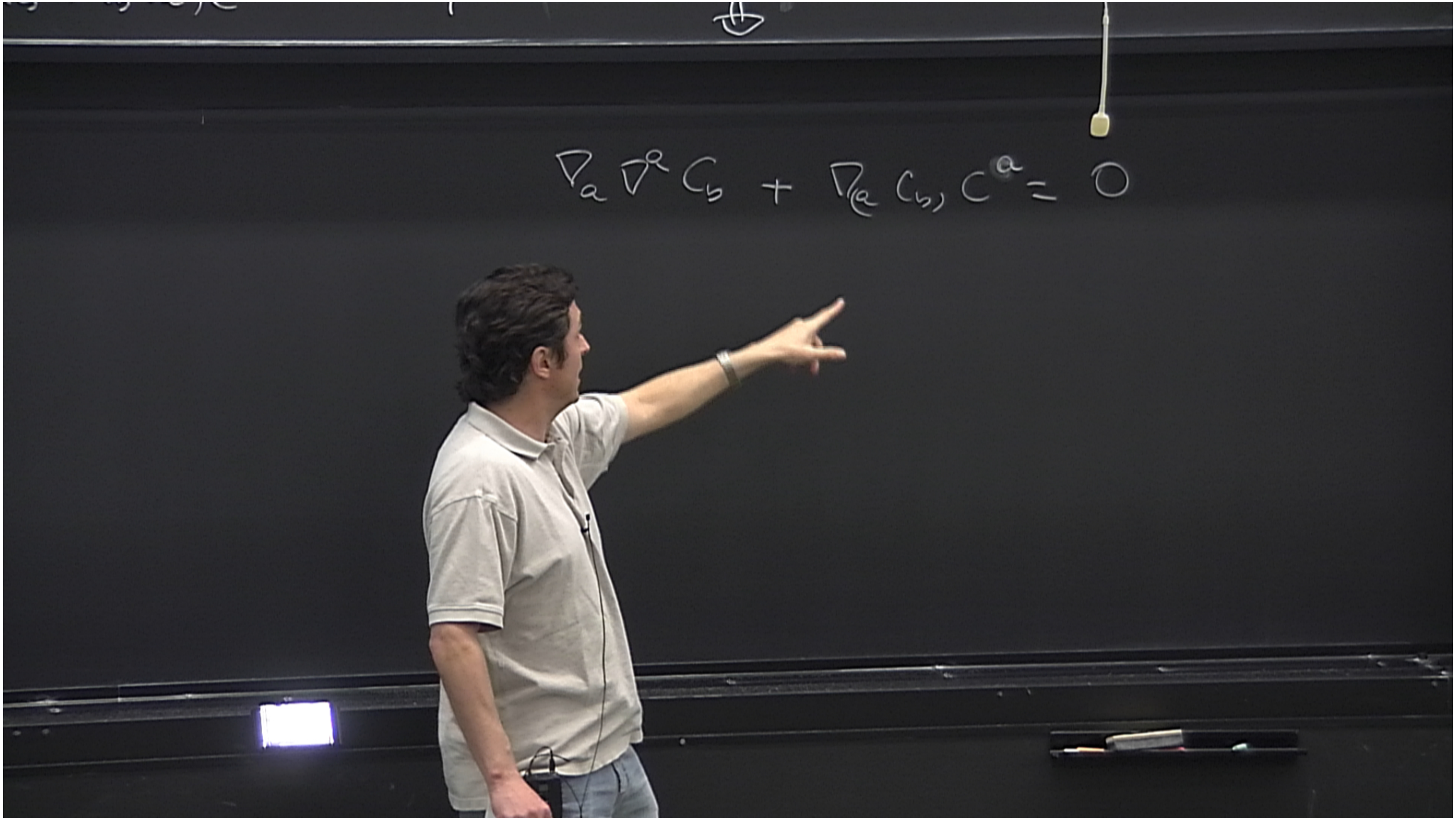

ersible
$R_{ab} = \cdots + \nabla_{(a}\Gamma_{b)} - \cdots$
$= \cdots - \nabla_{(a}\Gamma_{b)}$
true eqn
$R_{ab} - \nabla_{(a}C_{b)} = 0$
→ New
C_b

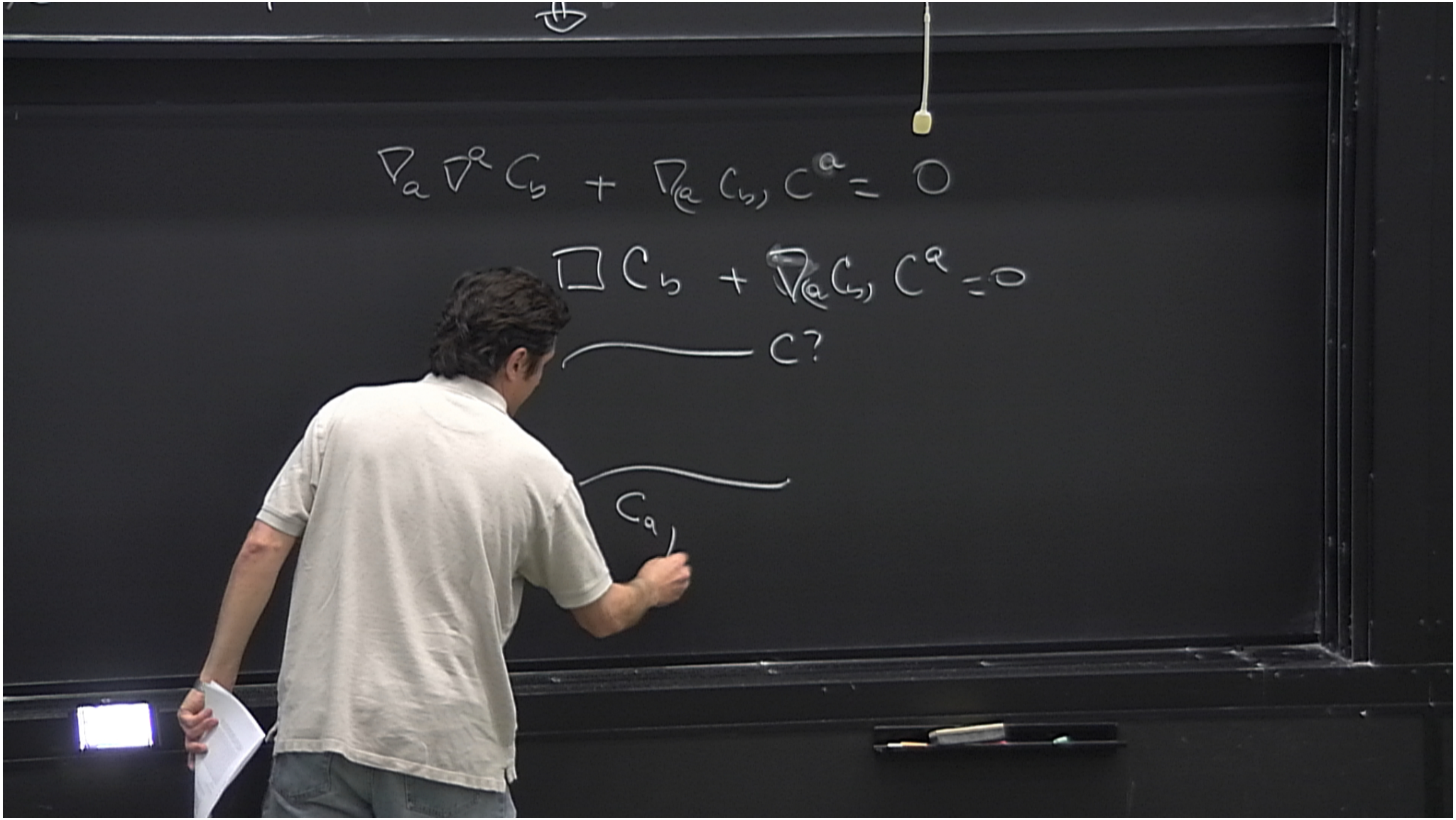

$R_{ab} = 0$ is time reversible
$\nabla^a (R_{ab} - \frac{1}{2} g_{ab} R) = 0$
$\hookrightarrow \nabla^a R_{ab} - \frac{1}{2} \nabla_b R = \nabla^a (R_{ac} C_b) - \frac{1}{2} R_b (R_{ac} C^a)$
true eqn

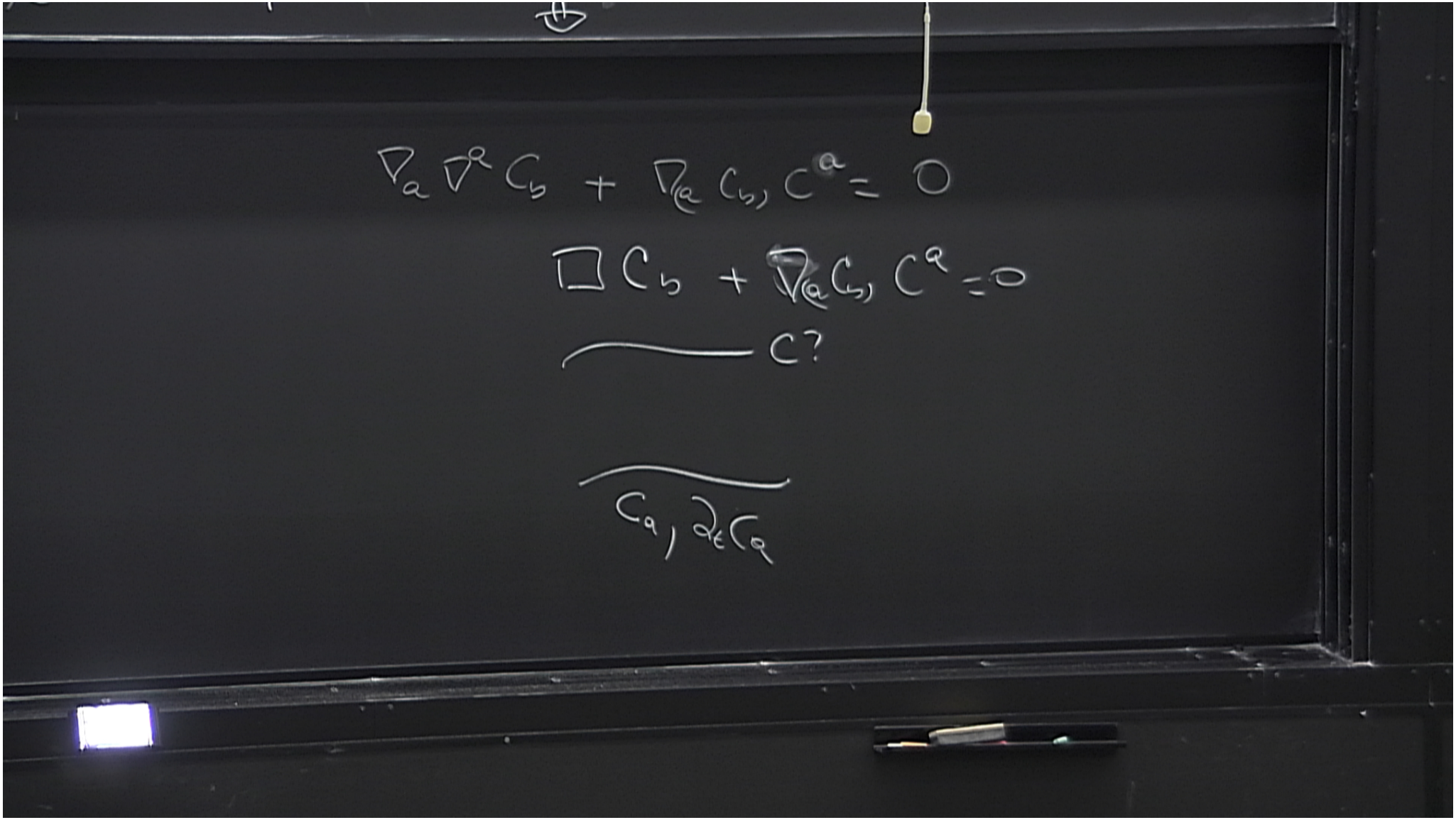

$\nabla^a (R_{ab} - \frac{1}{2} g_{ab} R) = 0$
$\hookrightarrow \nabla^a R_{ab} - \frac{1}{2} \nabla_b R = \nabla^a (\nabla_a C_b) - \frac{1}{2} \nabla_b (\nabla_a C^a)$
+ two eqn
$= \frac{1}{2} \nabla^a \nabla_a C_b + \frac{1}{2} \nabla_a \nabla_b C^a - \frac{1}{2} \nabla_b (\nabla_a C^a$
$\frac{1}{2} (\nabla_a \nabla_b - \nabla_b \nabla_a) C^a$

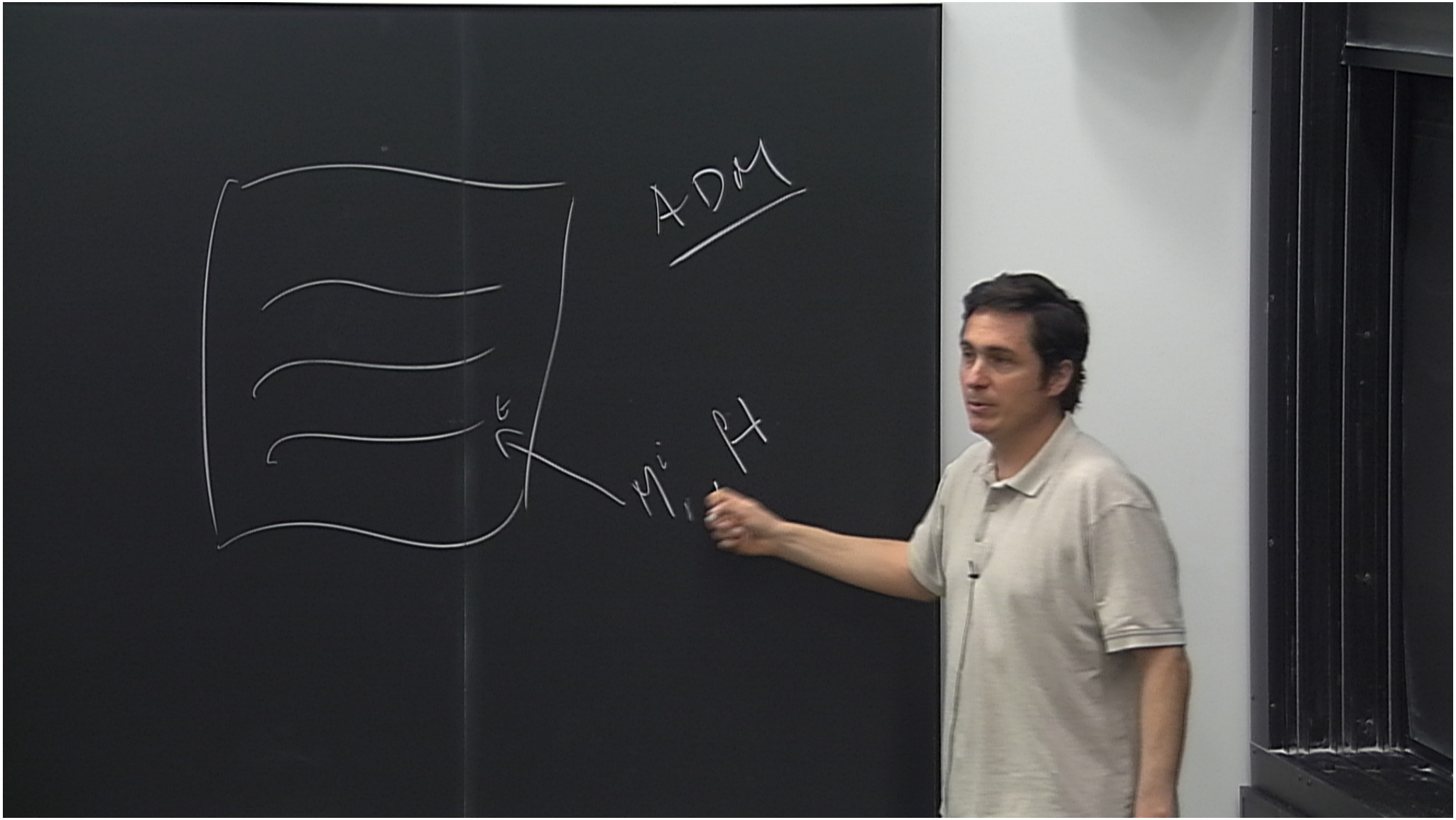

$C^a - \frac{1}{2} R_b (\nabla_a C^a)$
$\nabla_b - \nabla_b \nabla_a) C^a$
$\Rightarrow \nabla^a \nabla_a C_b + R_{ab} C^a = 0$
$0 = C_b \equiv H_b + I_b$
$\nabla_a \nabla^a C_b + \nabla_{(a} C_{b)} C^a = 0$

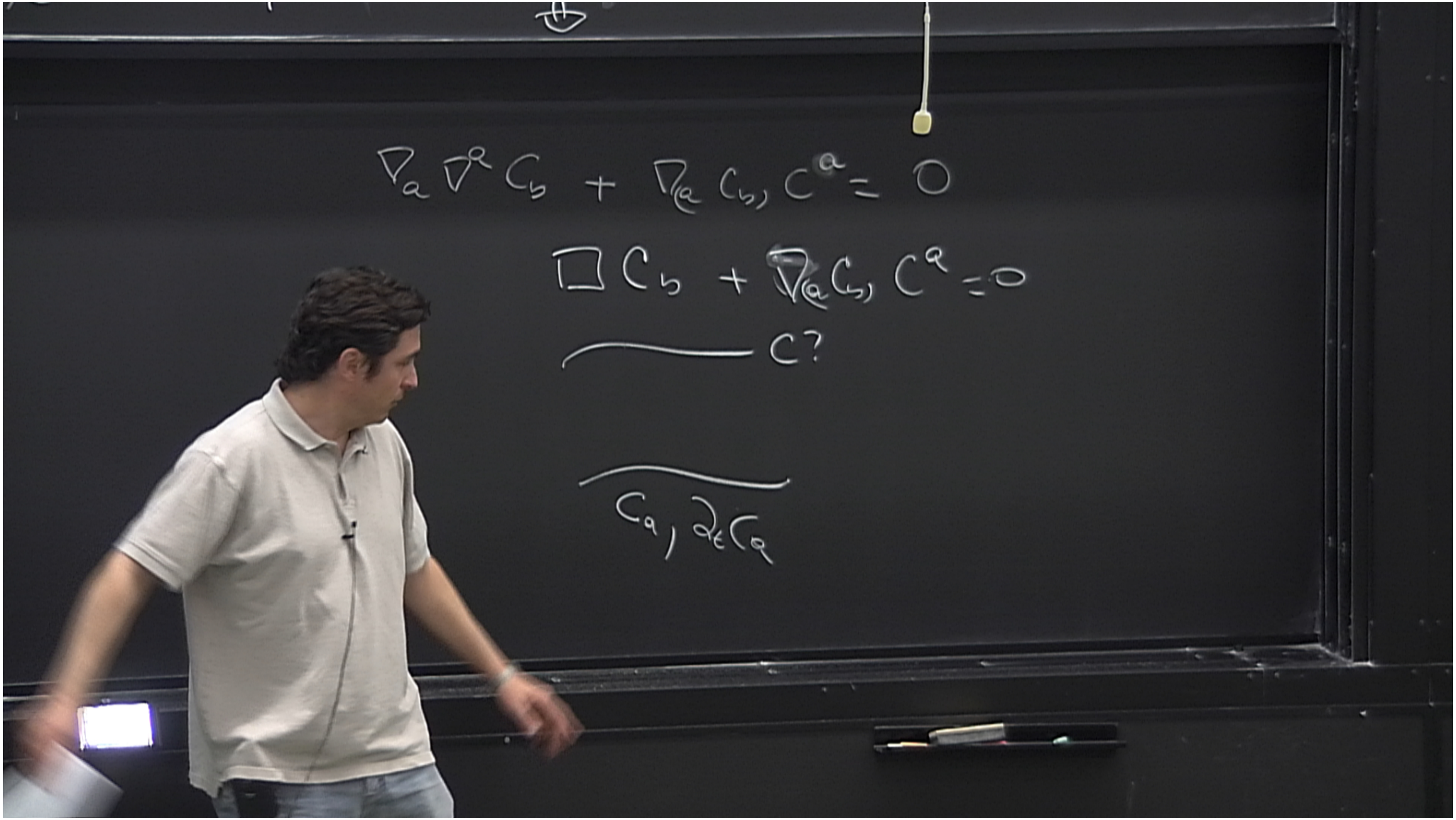

$\nabla_a \nabla^a C_b + \nabla_{(a} C_{b)} C^a = 0$

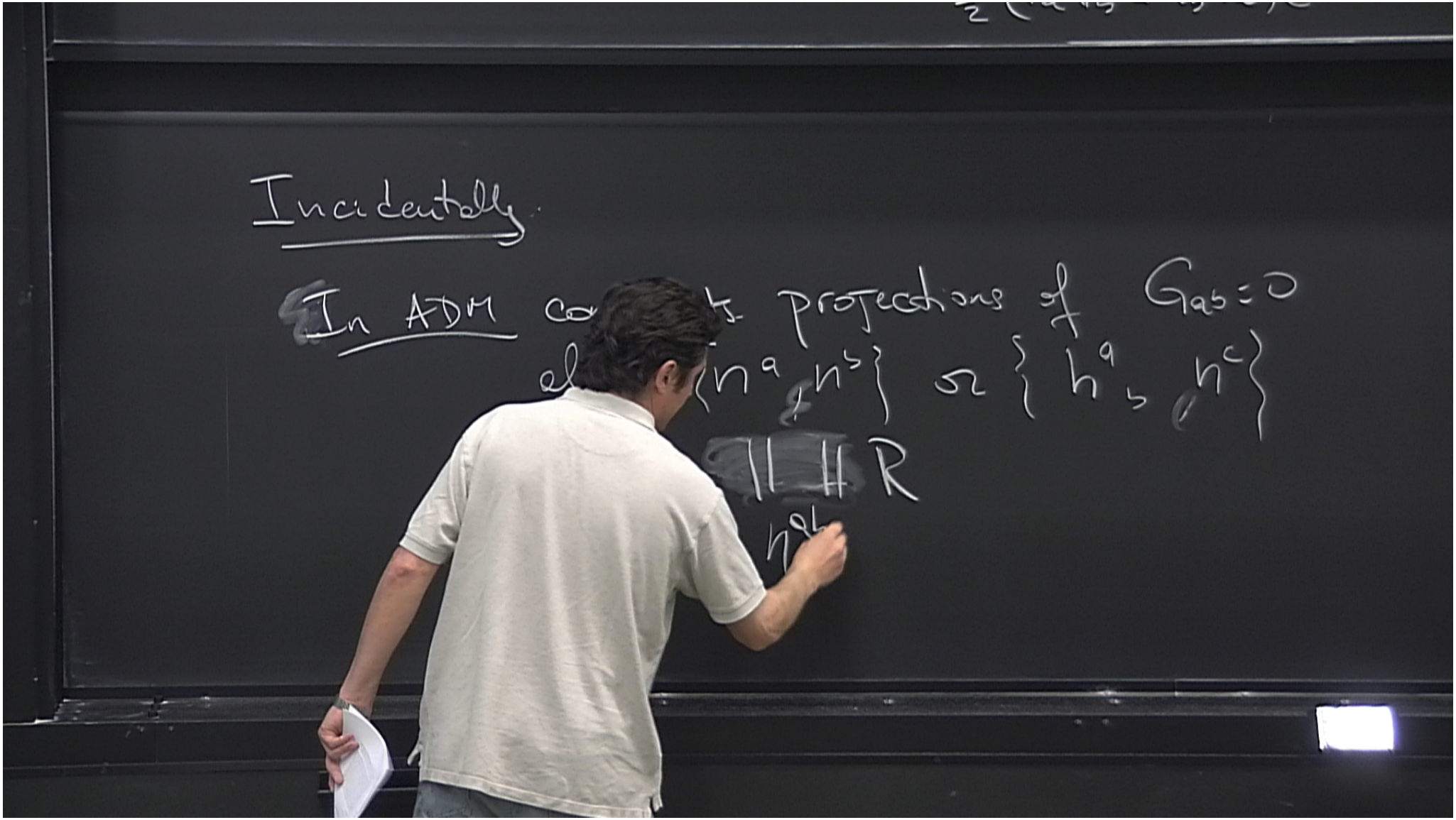

$\nabla_a \nabla^a C_b + \nabla_{(a} C_{b)} C^a = 0$

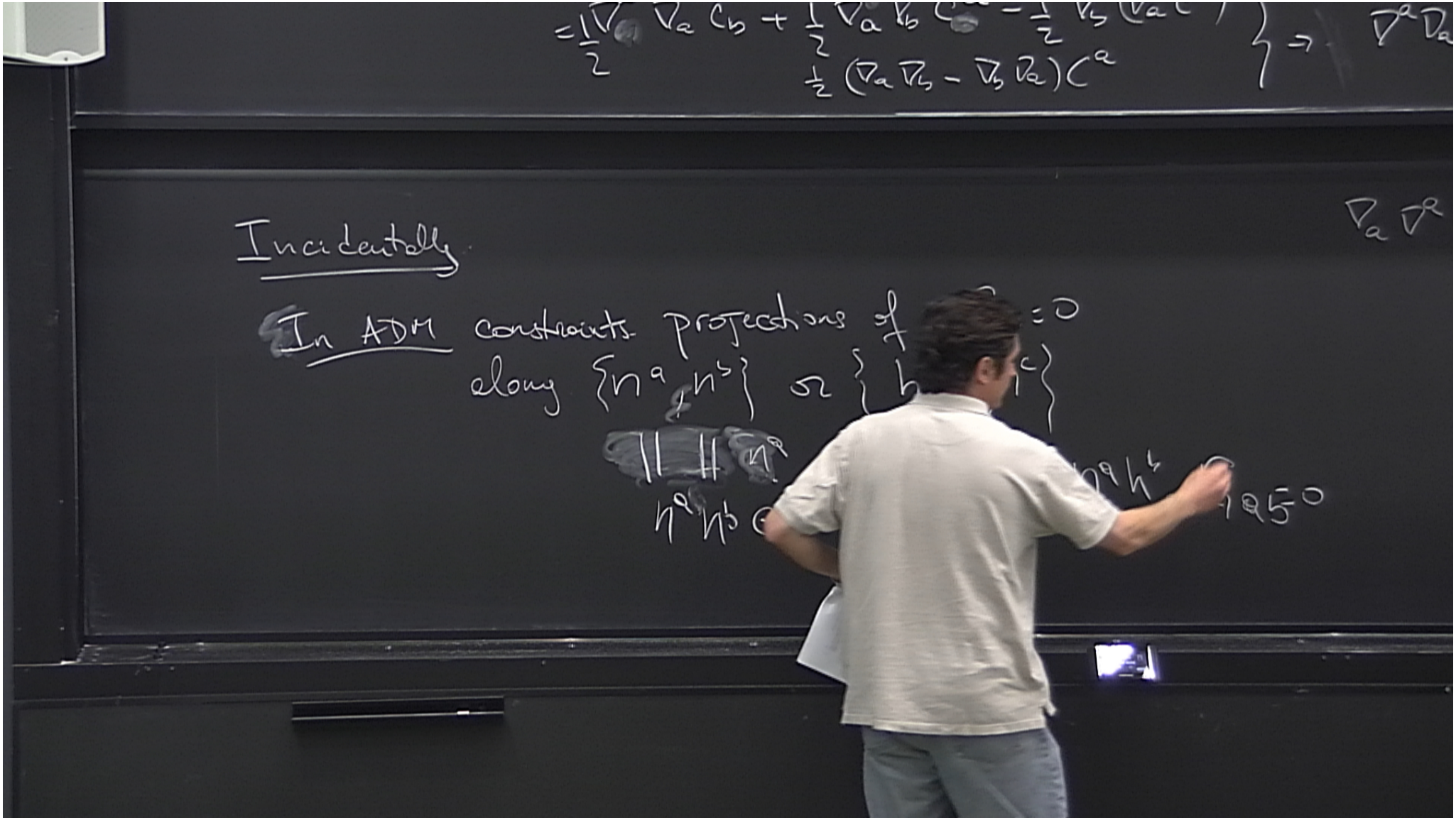

$\nabla_a \nabla^a C_b + \nabla_{(a} C_{b)} C^a = 0$
$\Box C_b + \nabla_{(a} C_{b)} C^a = 0$
C?
C_a

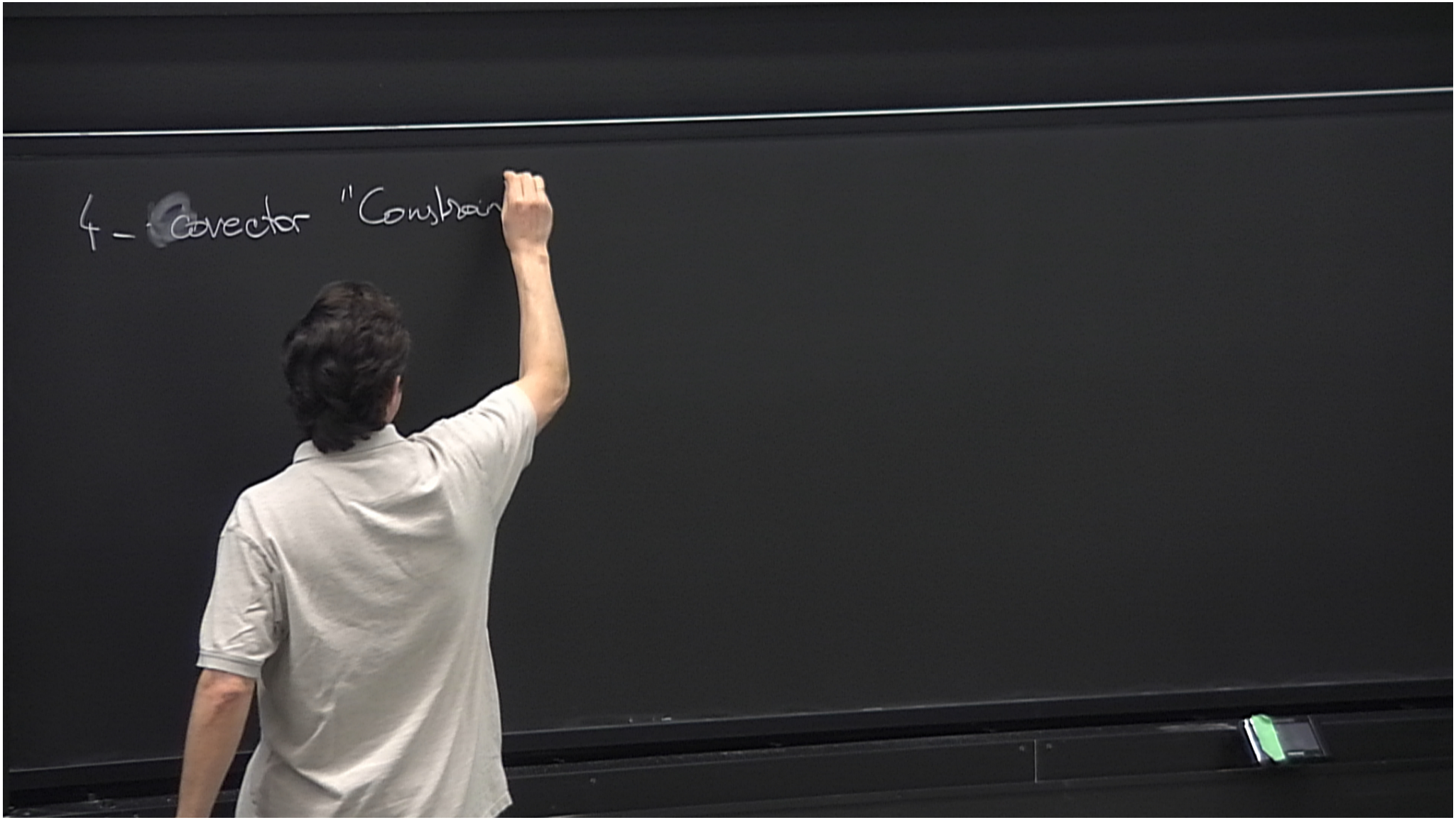

$\nabla_a \nabla^a C_b + \nabla_{(a} C_{b)} C^a = 0$
$\Box C_b + \nabla_{(a} C_{b)} C^a = 0$
C?
$C_a, \partial_t C_a$


ADM


$\nabla_a \nabla^a C_b + \nabla_{(a} C_{b)} C^a = 0$
$\Box C_b + \nabla_{(a} C_{b)} C^a = 0$
$C?$
$C_a, \partial_t C_a$

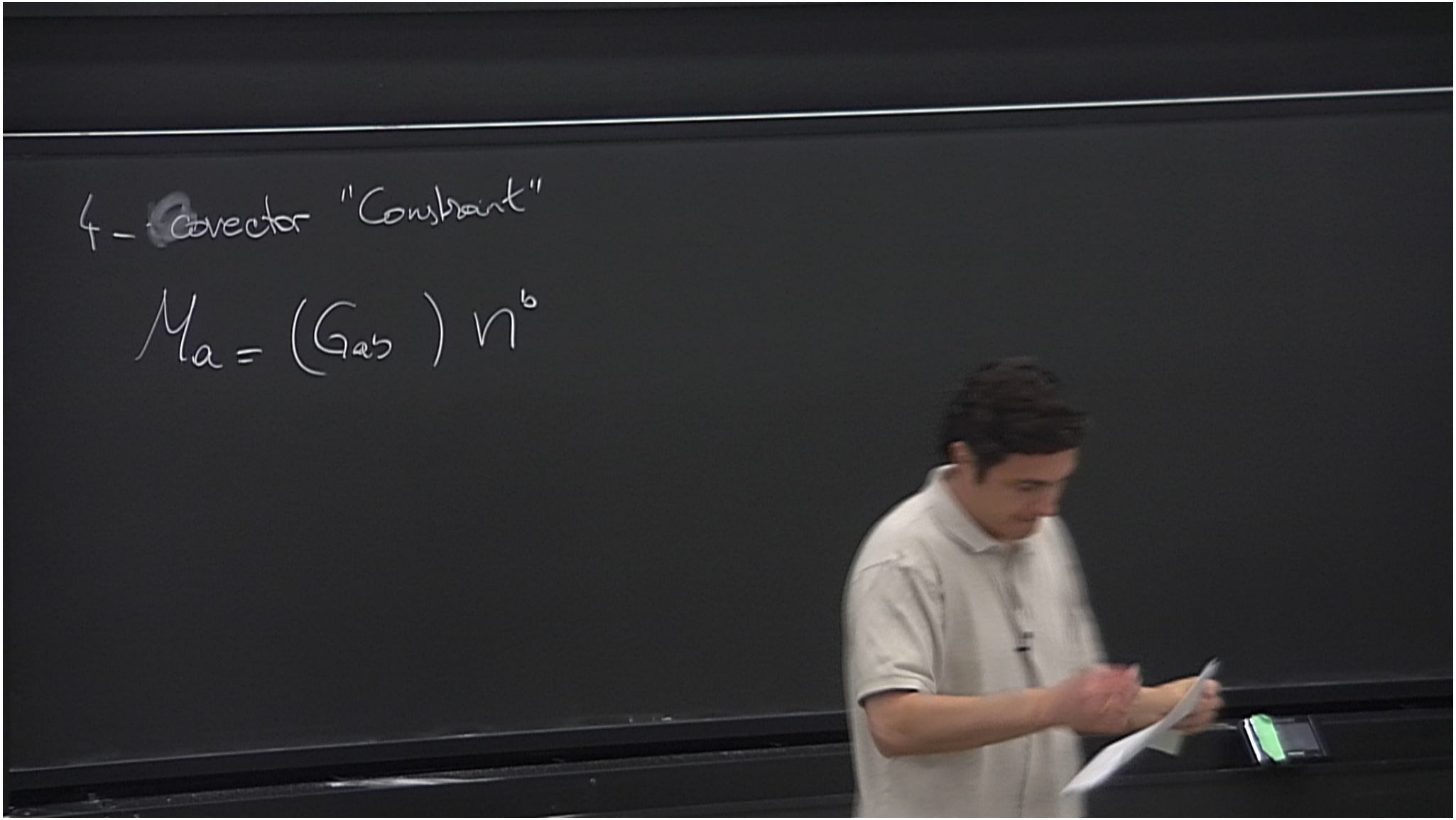

Incidentally
In ADM
R

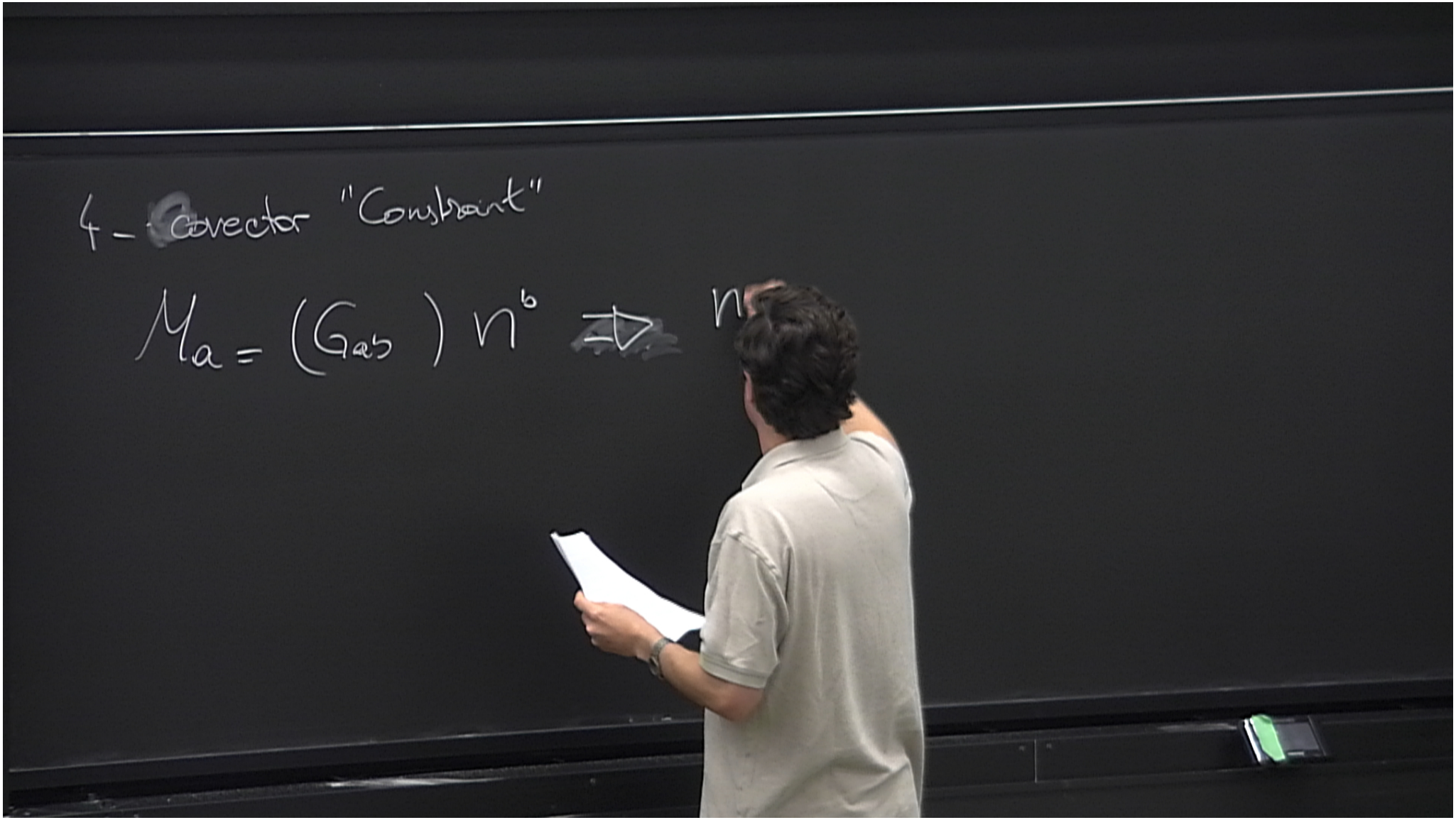

Incidentally
In ADM constraints projections of
along

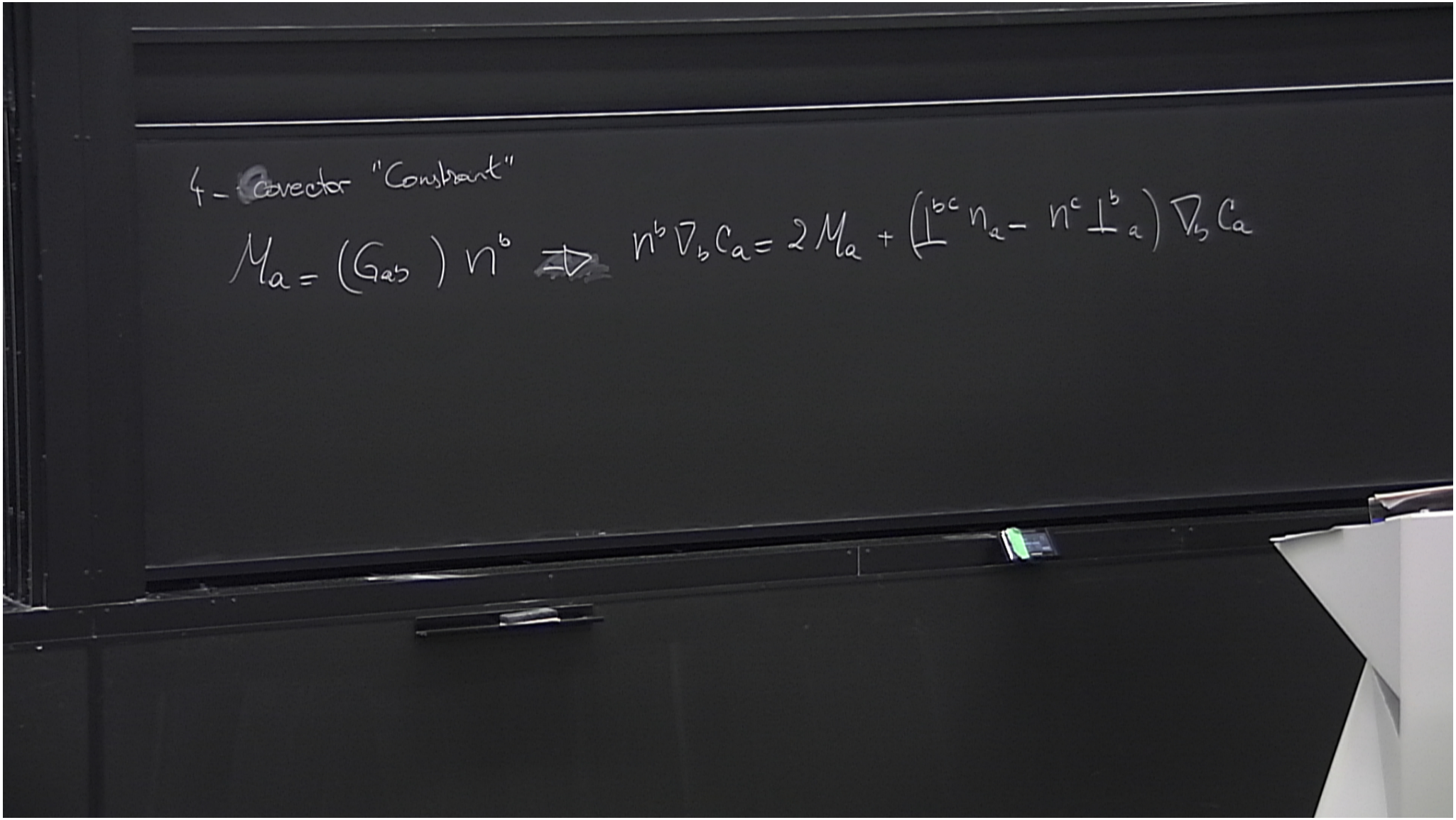

4 - Covector "Constrain

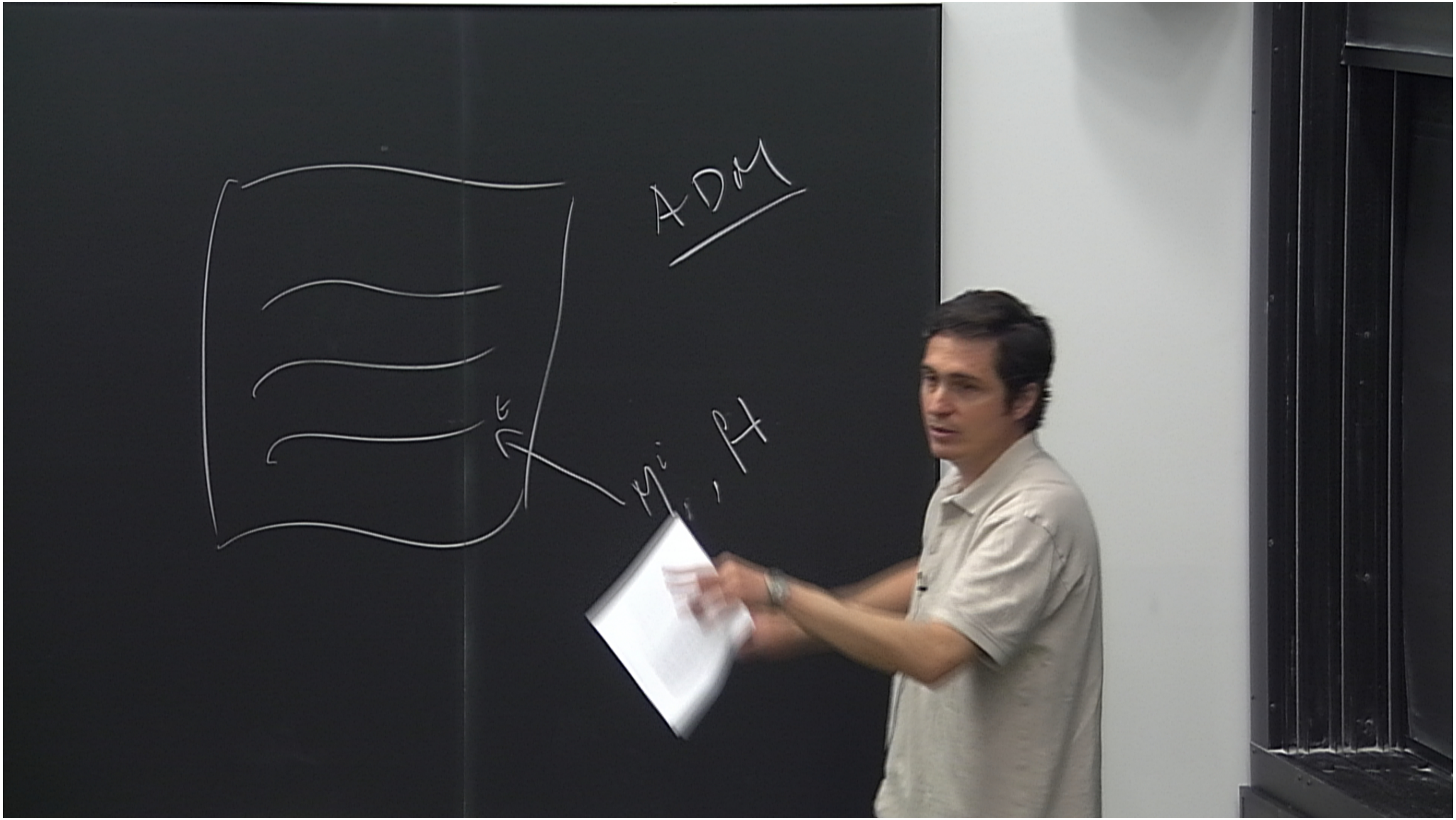
4 - Covector "Constraint"

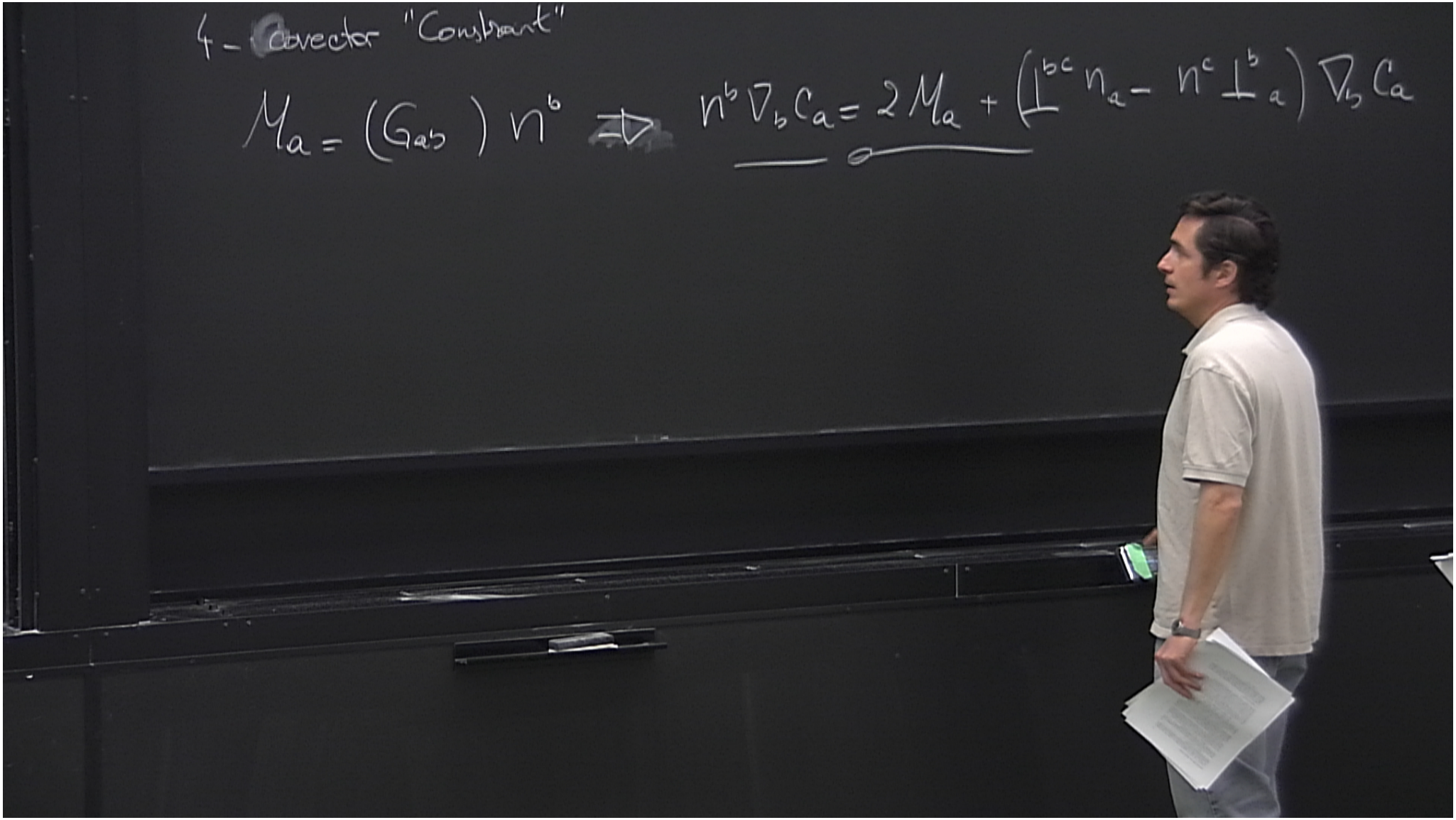
$$M_a = (G_{ab})\, n^b$$

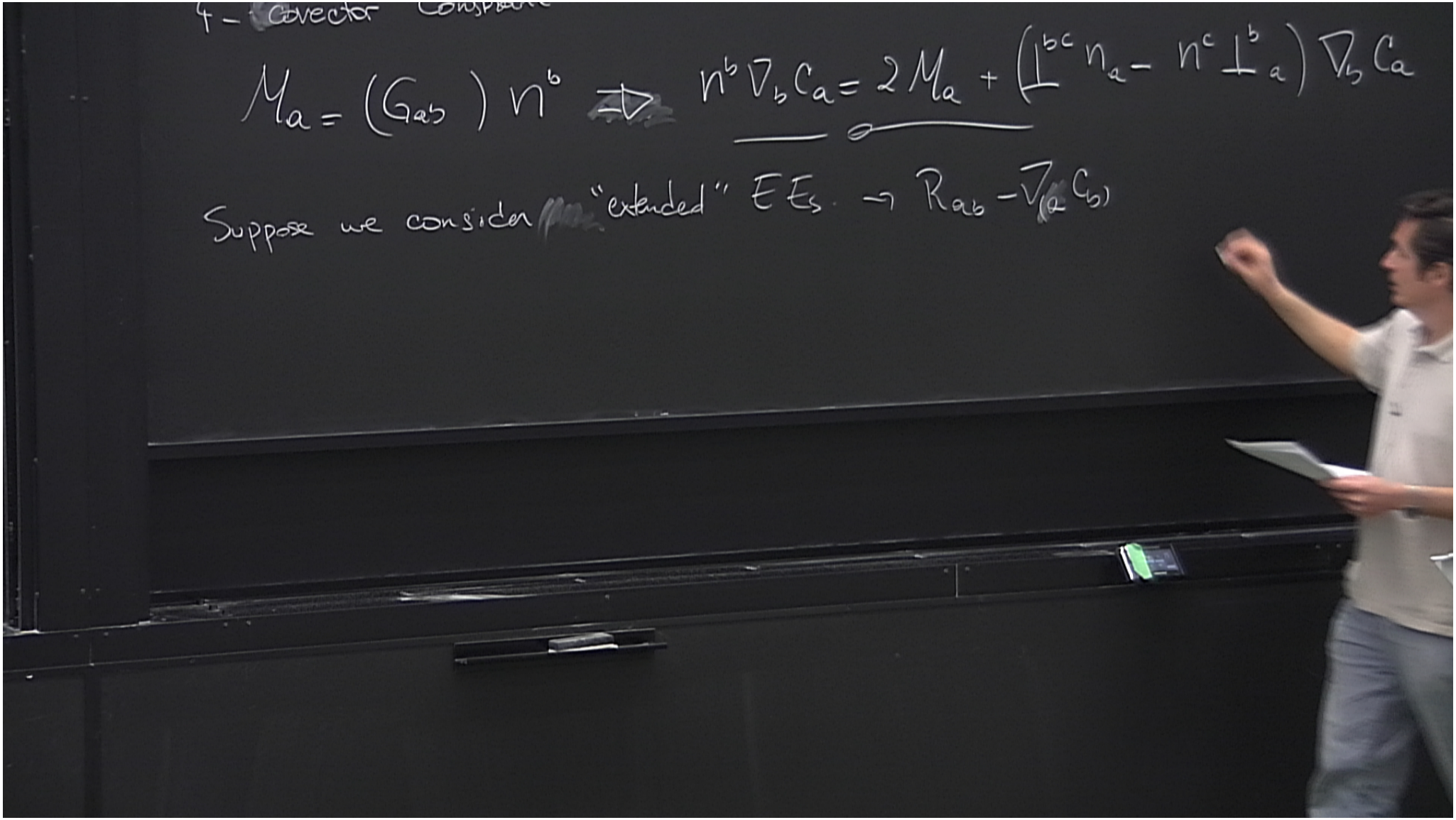

4 - Covector "Constraint"
$M_a = (G_{ab}) n^b$

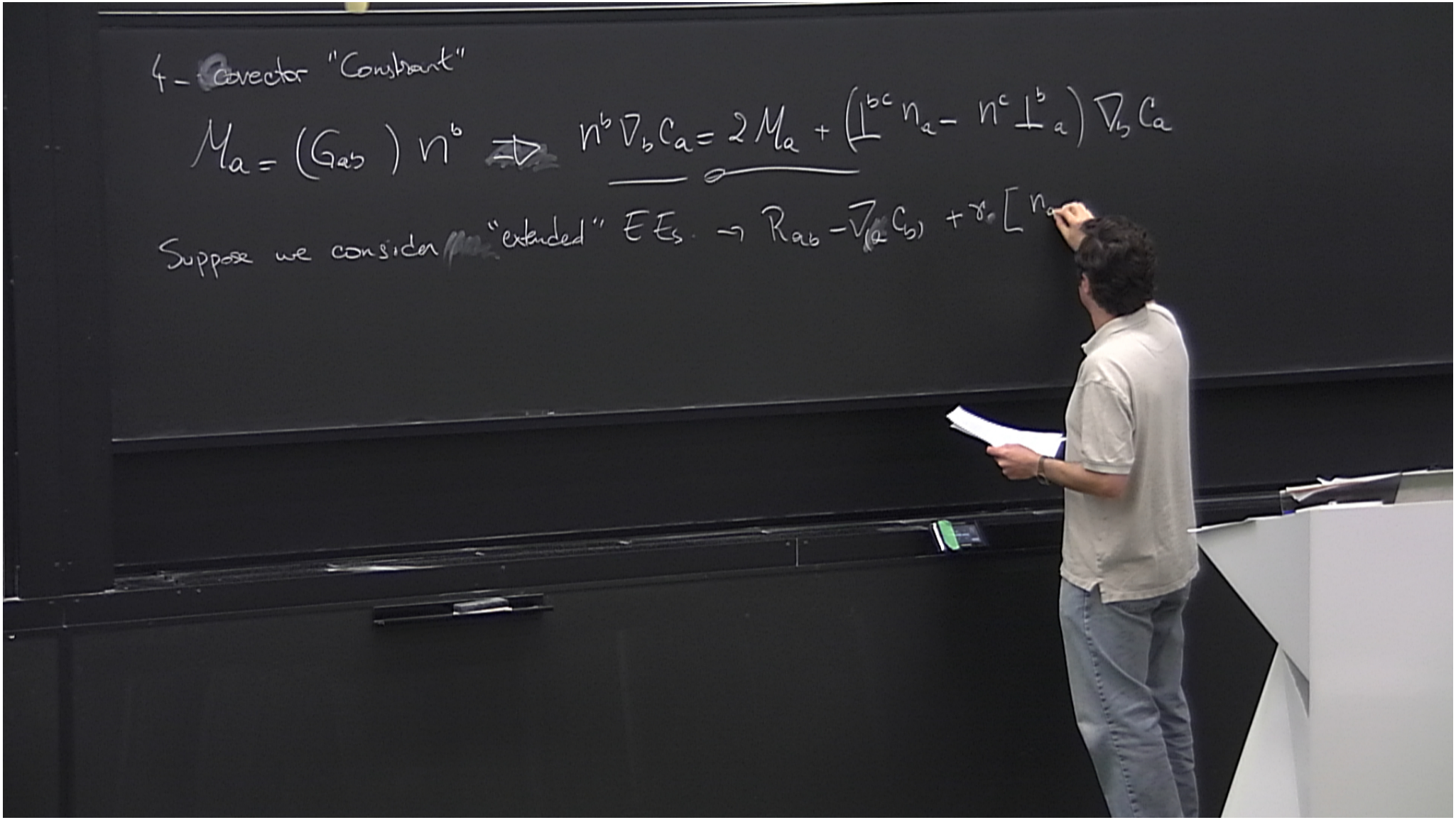

4 - Covector "Constraint"
$M_a = (G_{ab}) n^b \Rightarrow$

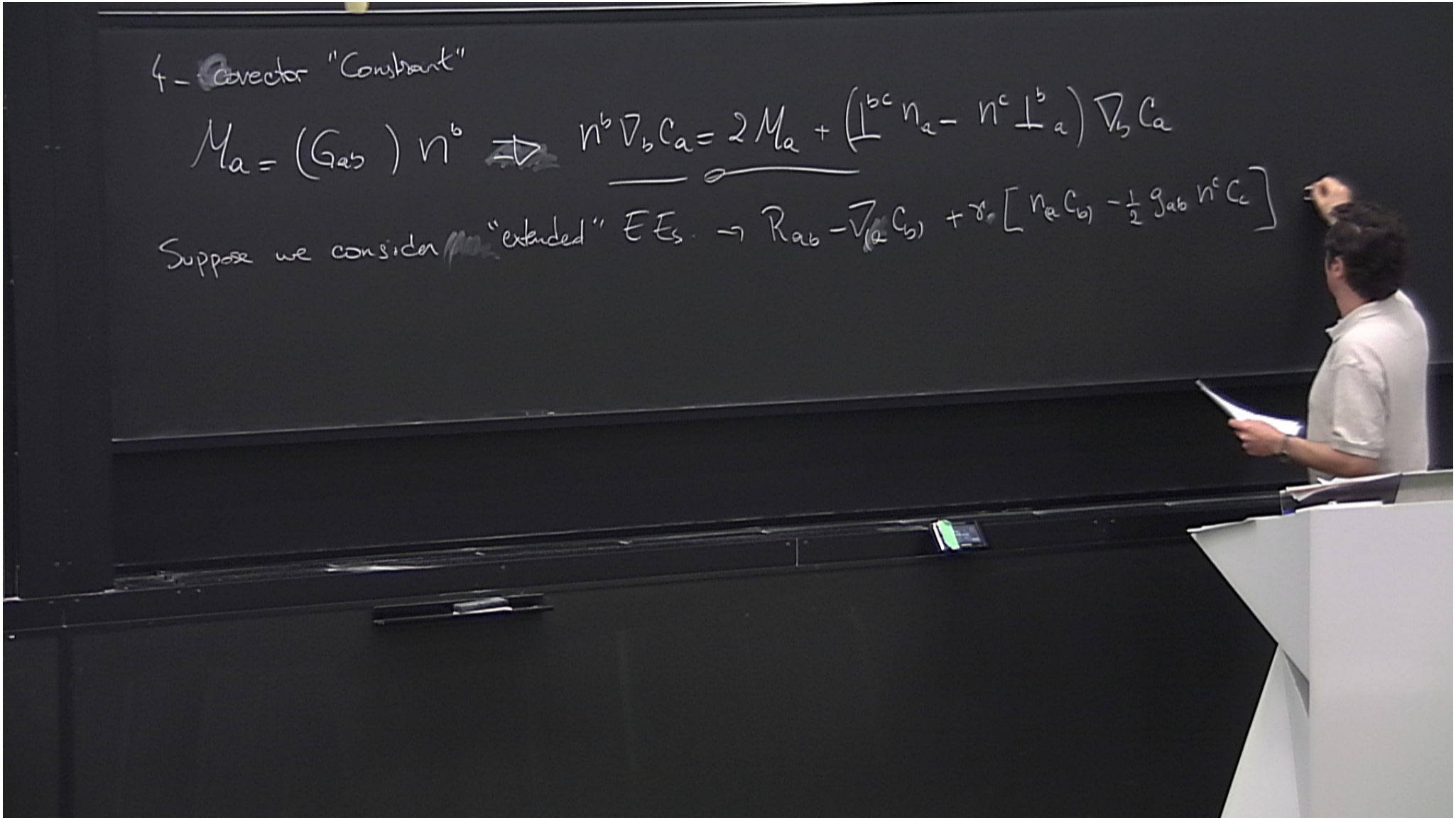

4- Covector "Constraint"
$M_a = (G_{ab})\, n^b \Rightarrow n^b \nabla_b C_a = 2 M_a + \left(\perp^{bc} n_a - n^c \perp^b_a\right) \nabla_b C_a$

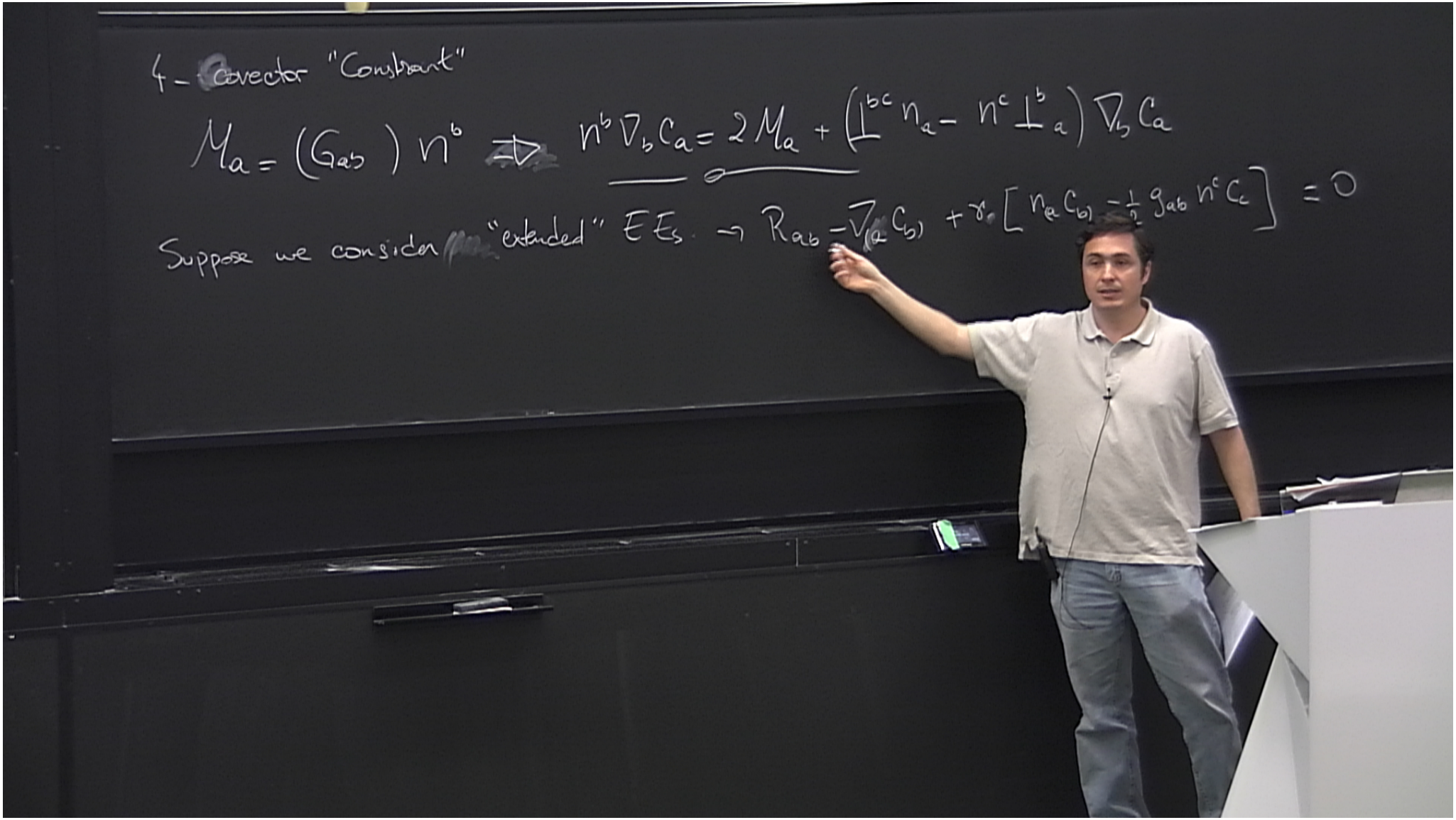

ADM

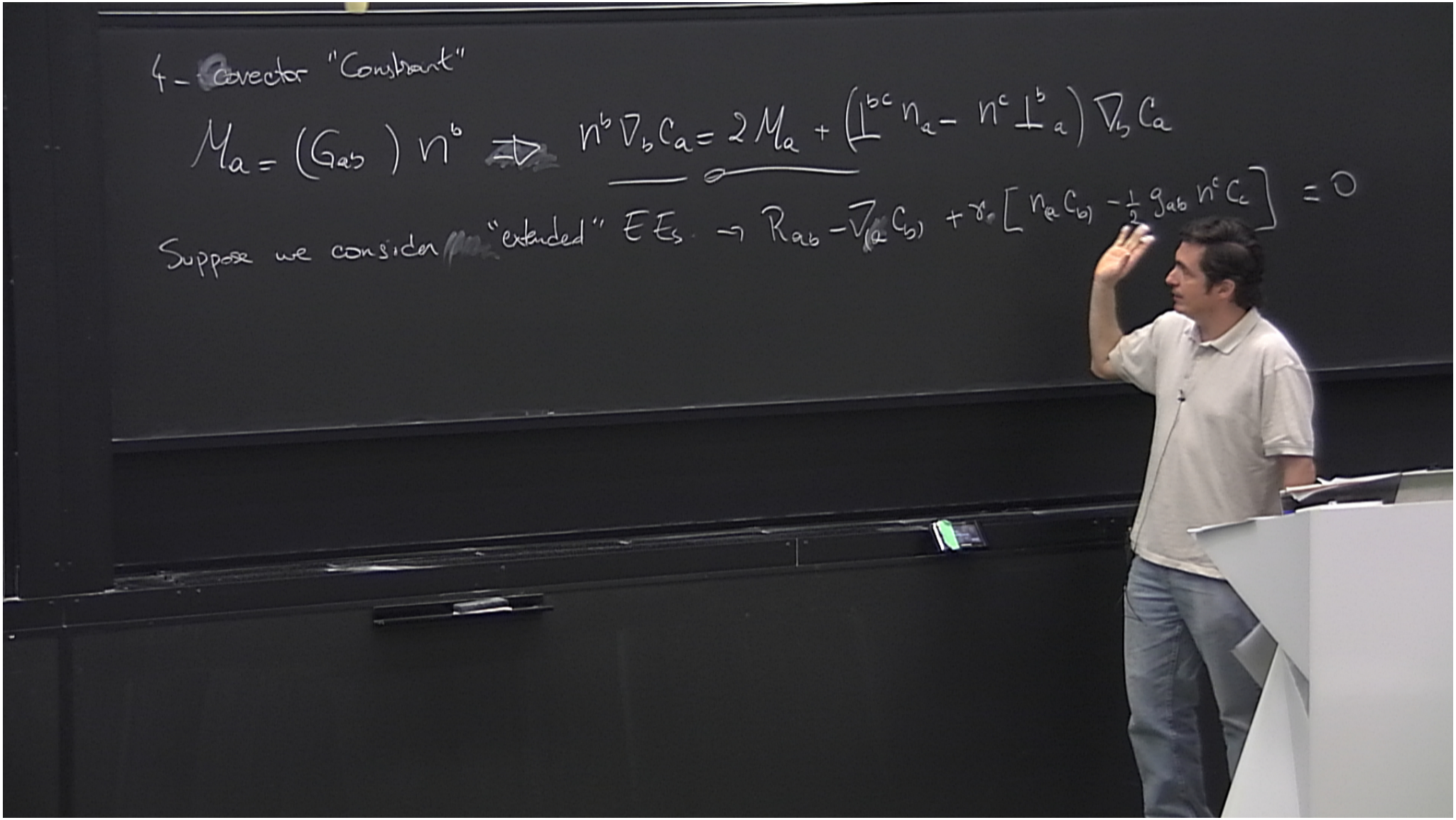

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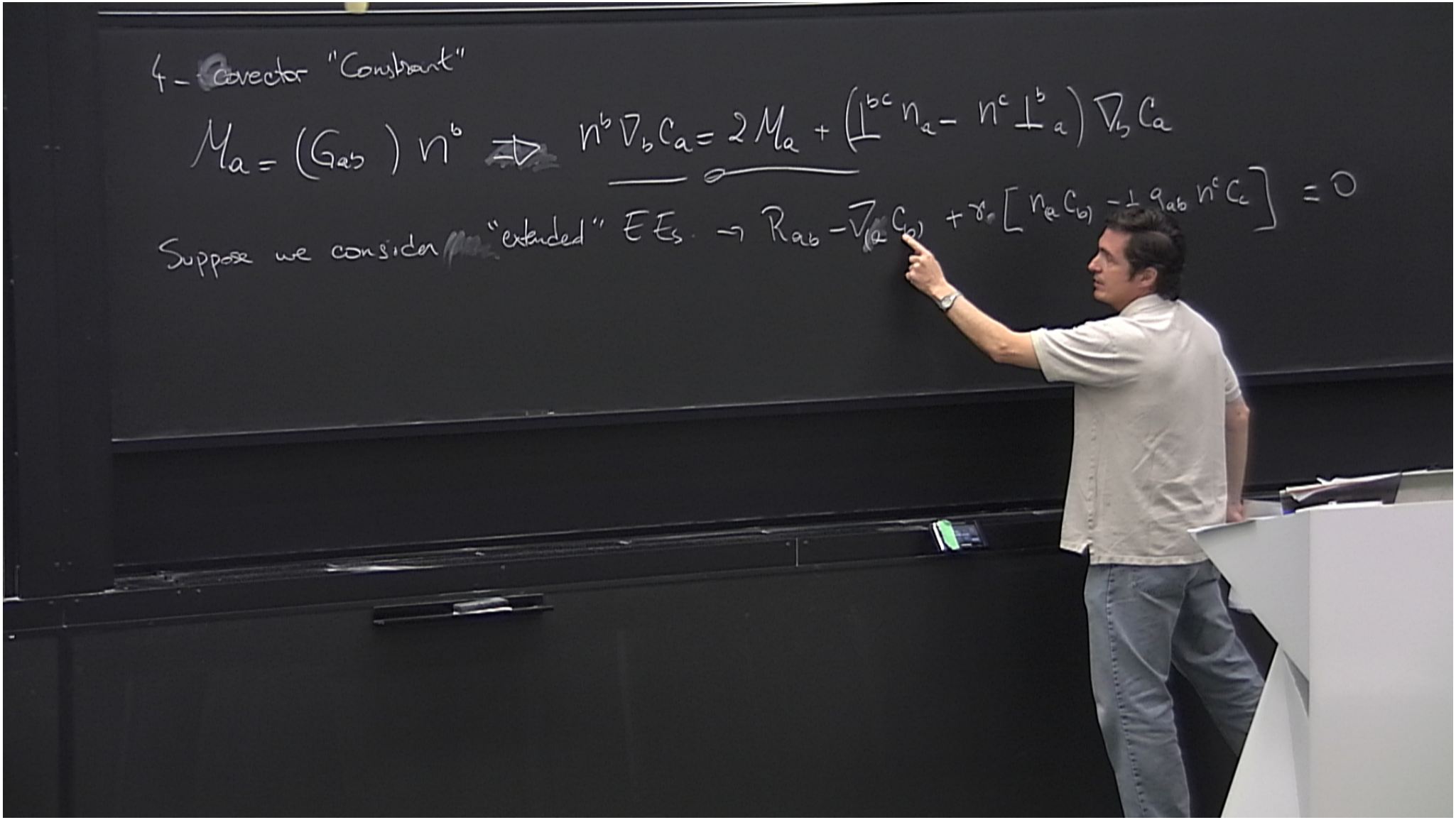

Covector
$M_a = (G_{ab})\, n^b \Rightarrow n^b \nabla_b C_a = 2M_a + (\perp^{bc} n_a - n^c \perp^b_a) \nabla_b C_a$
Suppose we consider "extended" EEs. $\rightarrow R_{ab} - \nabla_{(a} C_{b)}$

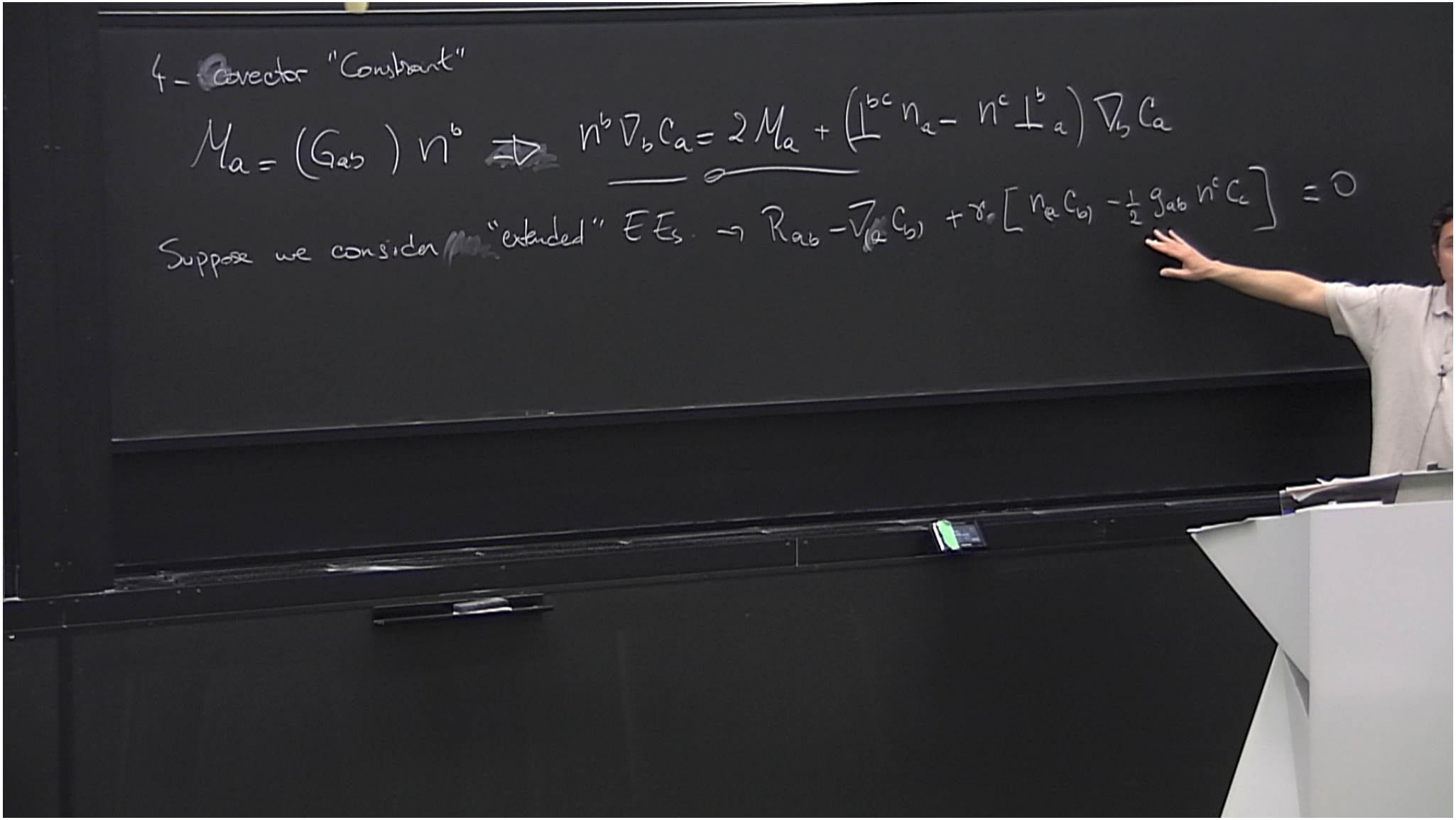

4 - covector "Constraint"
$M_a = (G_{ab})\, n^b \Rightarrow n^b \nabla_b C_a = 2 M_a + (\perp^{bc} n_a - n^c \perp^b{}_a)\, \nabla_b C_a$
Suppose we consider "extended" EEs. $\rightarrow R_{ab} - \nabla_{(a} C_{b)} + \gamma_0 [n_a$


4 - covector "Constraint"
$M_a = (G_{ab})\, n^b \Rightarrow n^b \nabla_b C_a = 2 M_a + (\perp^{bc} n_a - n^c \perp^b{}_a)\, \nabla_b C_a$
Suppose we consider "extended" EEs $\rightarrow R_{ab} - \nabla_{(a} C_{b)} + \gamma_{(a} [\, n_{a} C_{b)} - \frac{1}{2} g_{ab} n^c C_c \,]$


4 - covector "Constraint"
$M_a = (G_{ab})\, n^b \Rightarrow n^b \nabla_b C_a = 2 M_a + (\perp^{bc} n_a - n^c \perp^b{}_a)\, \nabla_b C_a$
Suppose we consider "extended" EEs. $\rightarrow R_{ab} - \nabla_{(a} C_{b)} + \gamma_{ab} \left[n_{(a} C_{b)} - \frac{1}{2} g_{ab} n^c C_c \right] = 0$


4 - covector "Constraint"
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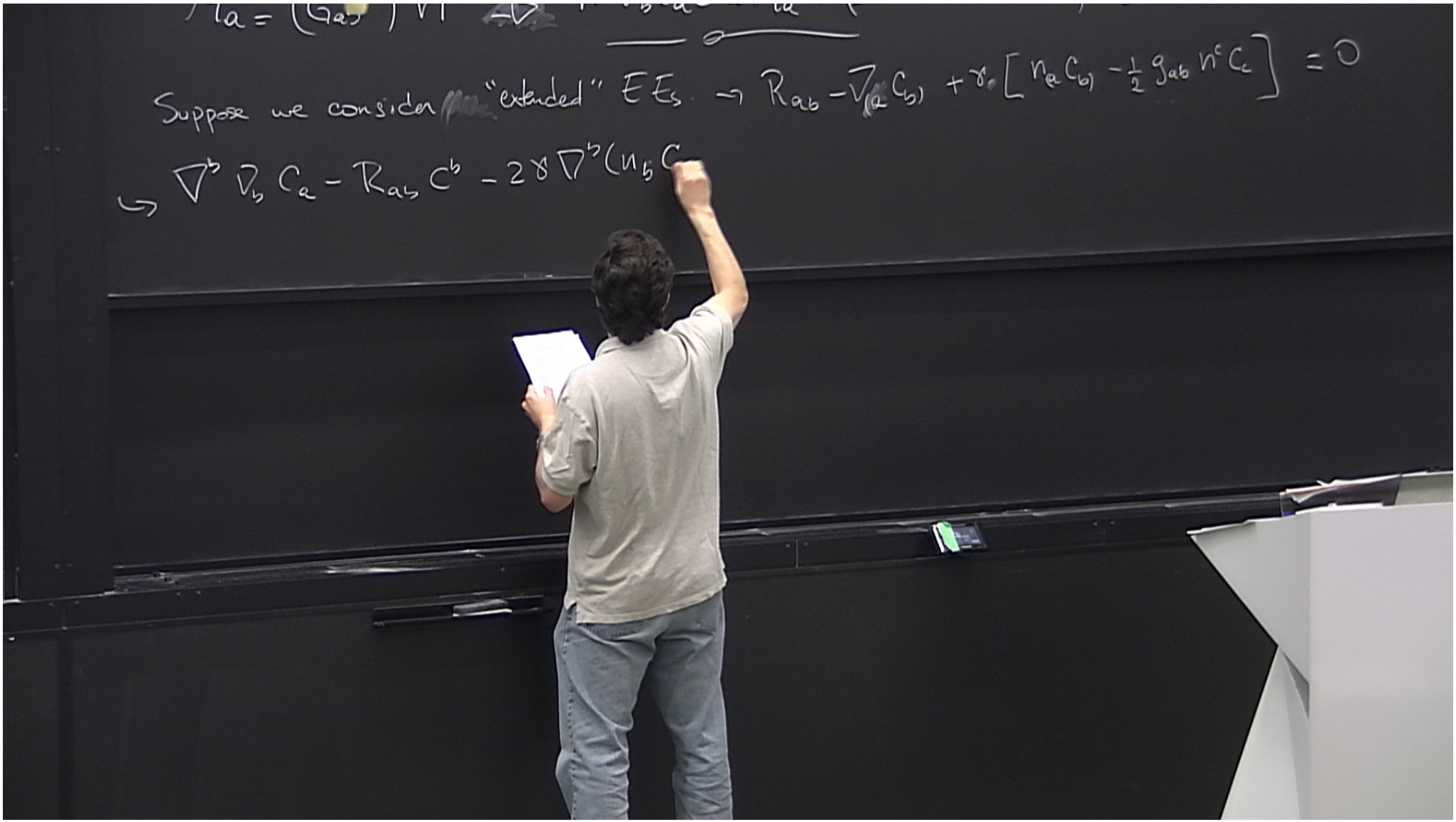

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$M_a = (G_{ab})\, n^b \Rightarrow n^b \nabla_b C_a = 2 M_a + \left(\perp^{bc} n_a - n^c \perp^b{}_a\right) \nabla_b C_a$
Suppose we consider "extended" EEs. $\rightarrow R_{ab} - \nabla_{(a} C_{b)} + \gamma_{ab} \left[n_{(a} C_{b)} - \perp q_{ab} n^c C_c \right] = 0$

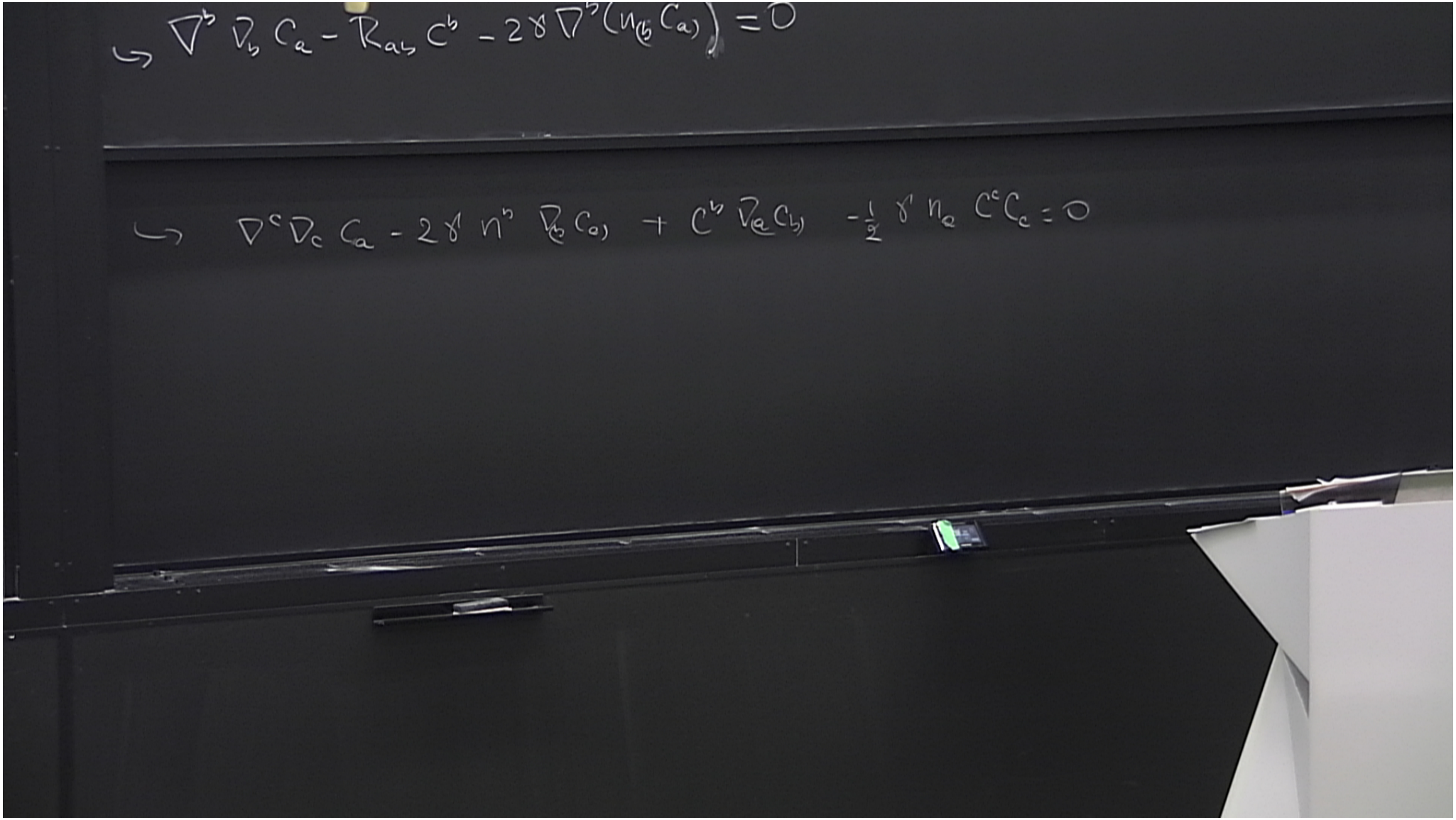

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$M_a = (G_{ab})\, n^b \Rightarrow n^b \nabla_b C_a = 2 M_a + (\perp^{bc} n_a - n^c \perp^b{}_a)\, \nabla_b C_a$
Suppose we consider "extended" EEs $\rightarrow R_{ab} - \nabla_{(a} C_{b)} + \gamma_{ab} \left[n_{(a} C_{b)} - \frac{1}{2} g_{ab} n^c C_c \right] = 0$

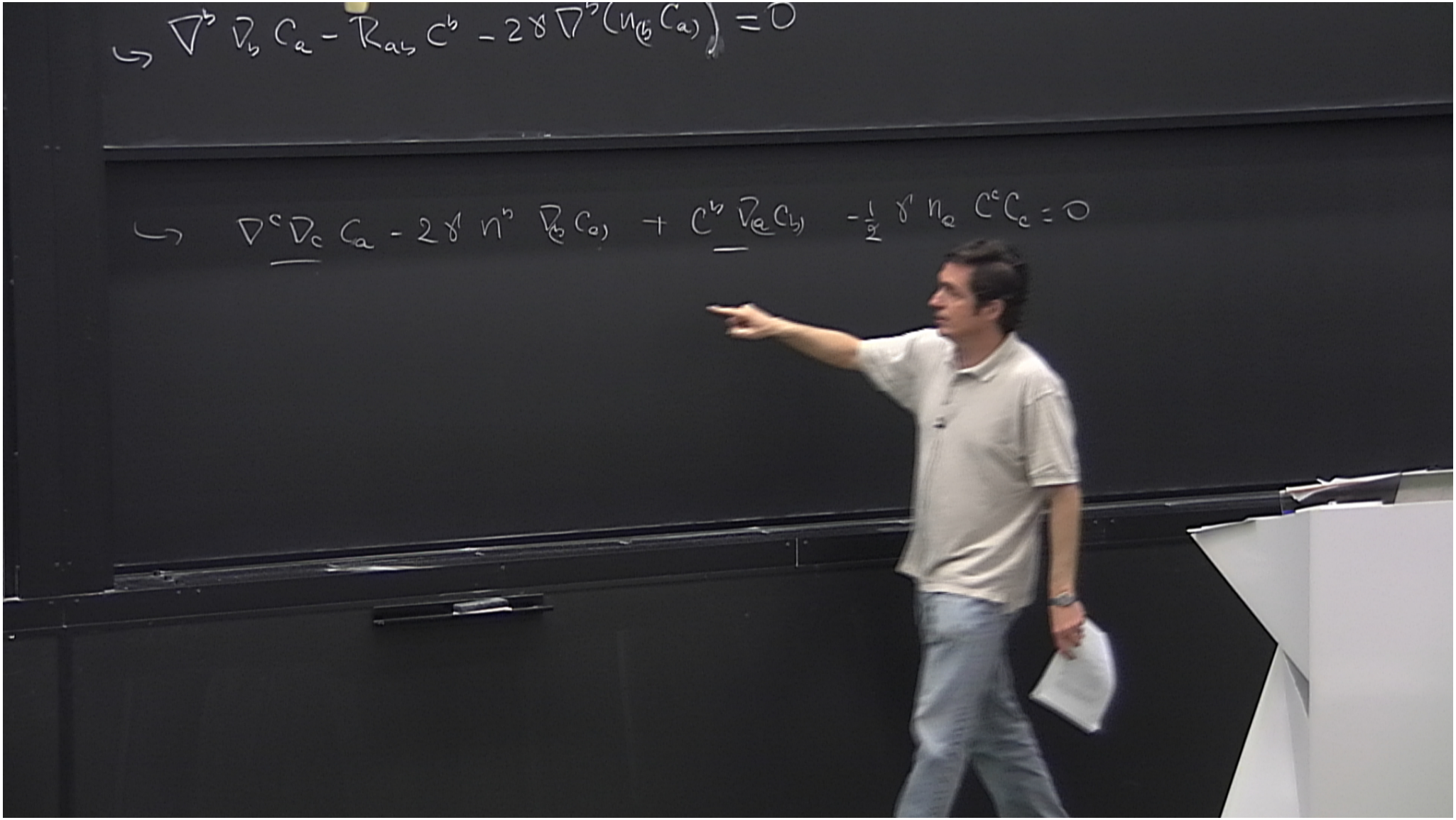
4 - covector "Constraint"

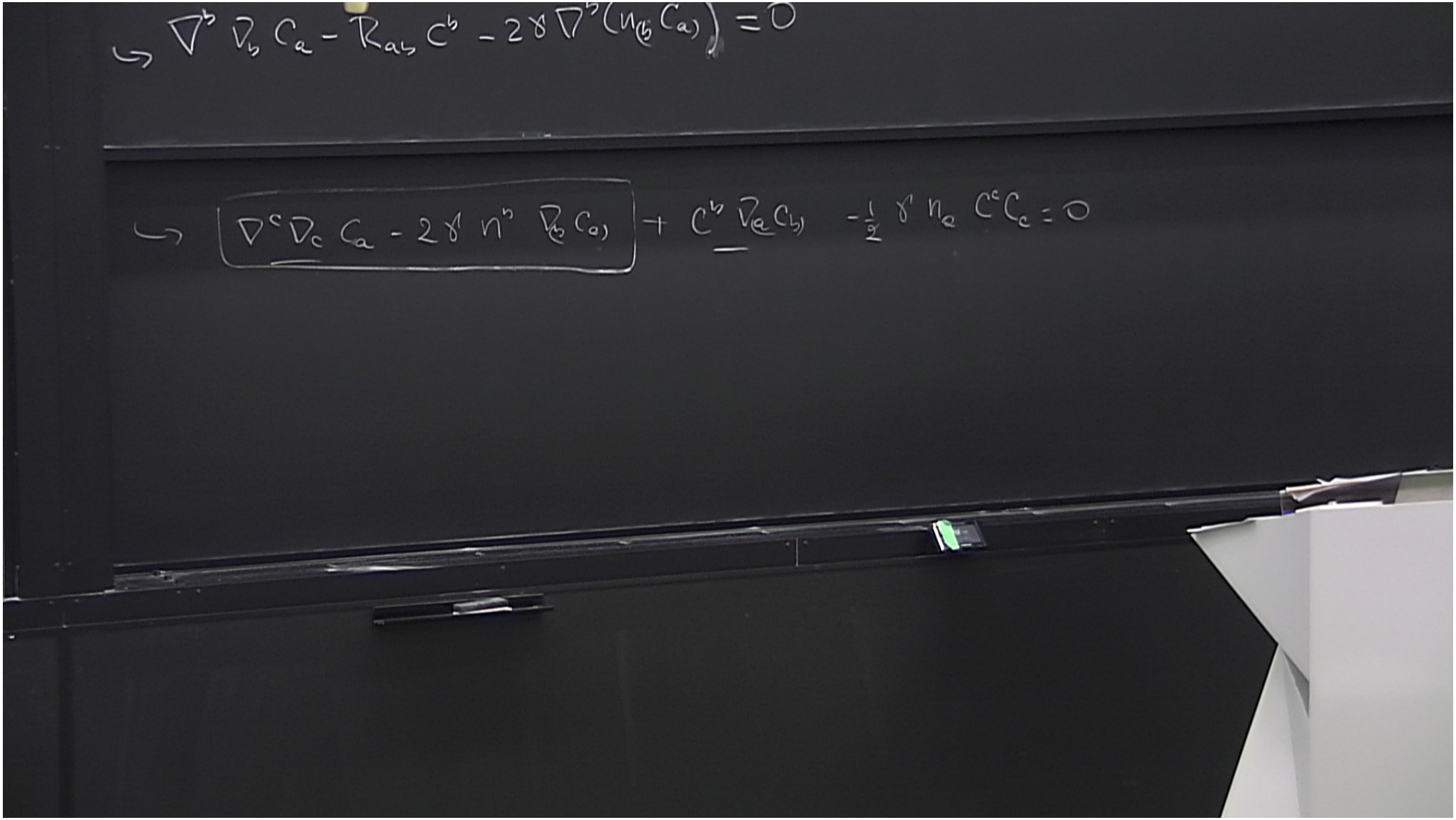
$$M_a = (G_{ab})\, n^b \Rightarrow n^b \nabla_b C_a = 2 M_a + \left(\perp^{bc} n_a - n^c \perp^b{}_a\right) \nabla_b C_a$$

Suppose we consider "extended" EEs. $\rightarrow R_{ab} - \nabla_{(a} C_{b)} + \gamma_{ab}\left[n_{(a} C_{b)} - \frac{1}{2} g_{ab} n^c C_c\right] = 0$


Suppose we consider "extended" EEs → $R_{ab} - \nabla_{(a} C_{b)} + \gamma_s \left[n_{(a} C_{b)} - \frac{1}{2} g_{ab} n^c C_c \right] = 0$
↳ $\nabla^b \nabla_b C_a - R_{ab} C^b - 2\gamma \nabla^b (n_b C$


$\hookrightarrow \nabla^b D_b C_a - R_{ab} C^b - 2\gamma \nabla^b (n_{(b} C_{a)}) = 0$
$\hookrightarrow \nabla^c D_c C_a - 2\gamma n^b D_{(b} C_{a)} + C^b D_{(a} C_{b)} - \frac{1}{2}\gamma n_a C^c C_c = 0$

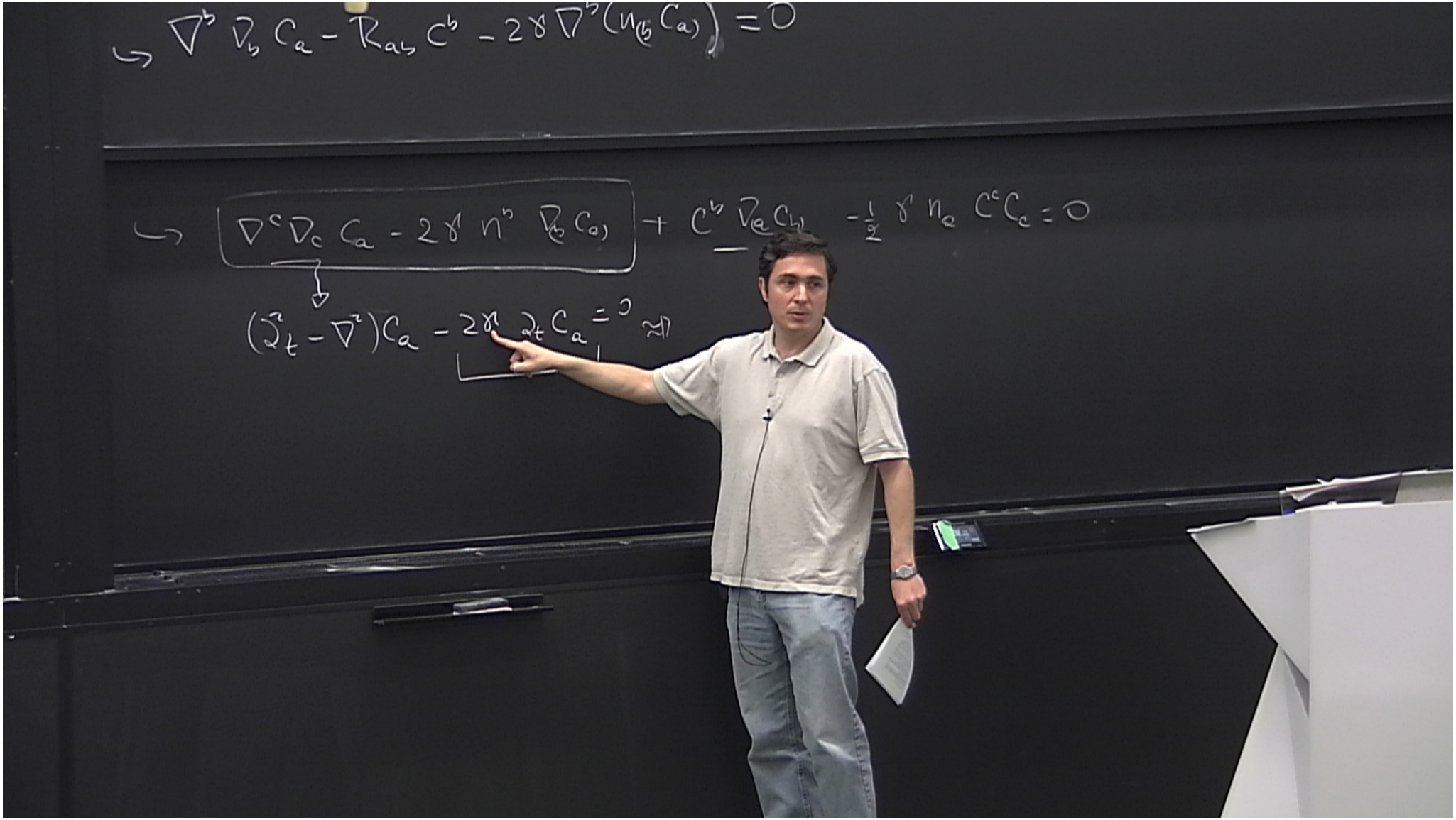

$\hookrightarrow \nabla^b D_b C_a - R_{ab} C^b - 2\gamma \nabla^b (n_{(b} C_{a)}) = 0$
$\hookrightarrow \nabla^c D_c C_a - 2\gamma n^b D_{(b} C_{a)} + C^b D_{(a} C_{b)} - \frac{1}{2}\gamma n_a C^c C_c = 0$

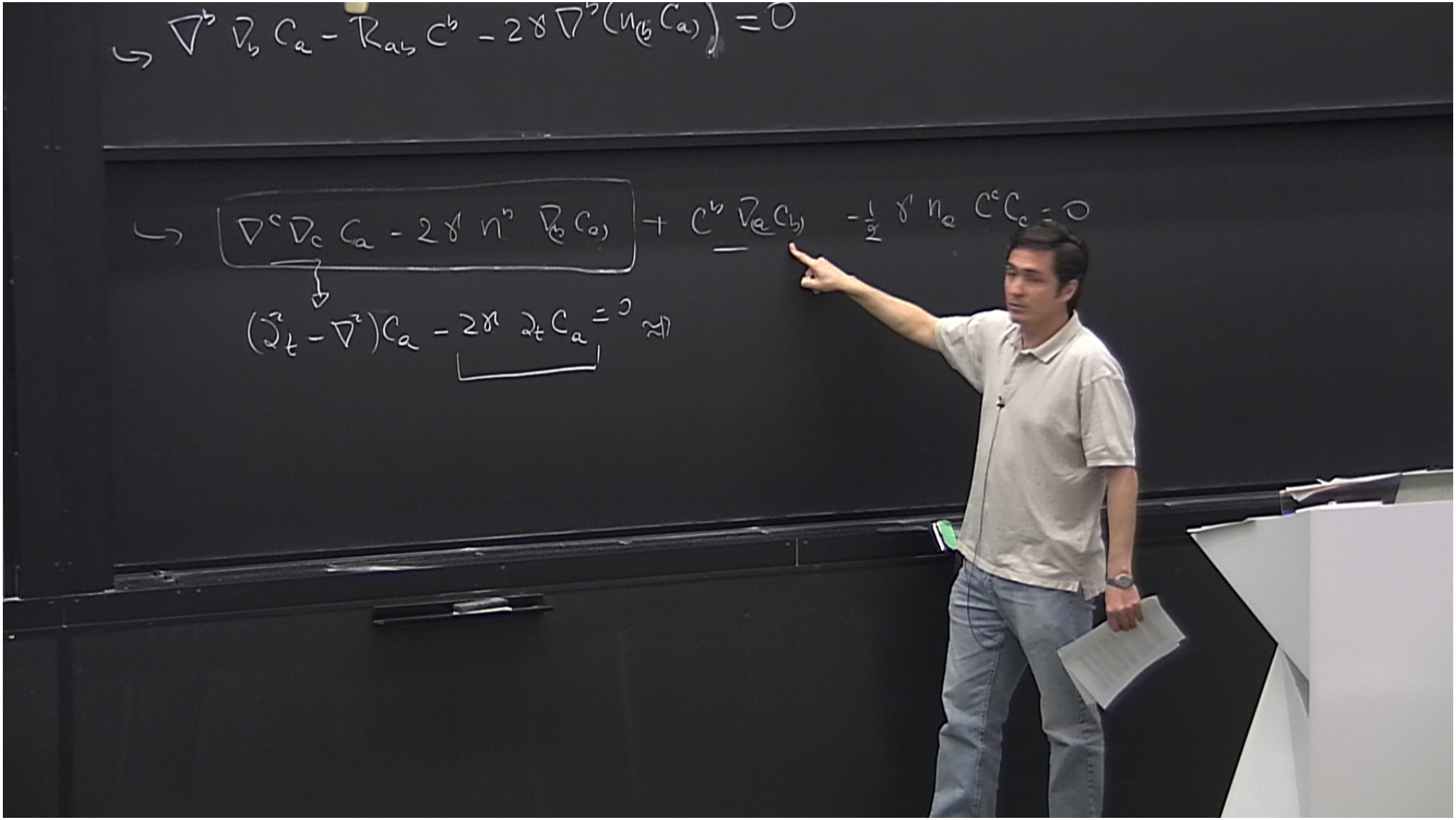

$\hookrightarrow \nabla^b D_b C_a - R_{ab} C^b - 2\gamma \nabla^b (n_{(b} C_{a)}) = 0$
$\hookrightarrow \boxed{\nabla^c D_c C_a - 2\gamma n^b D_{(b} C_{a)}} + C^b D_{(a} C_{b)} - \frac{1}{2}\gamma n_a C^c C_c = 0$

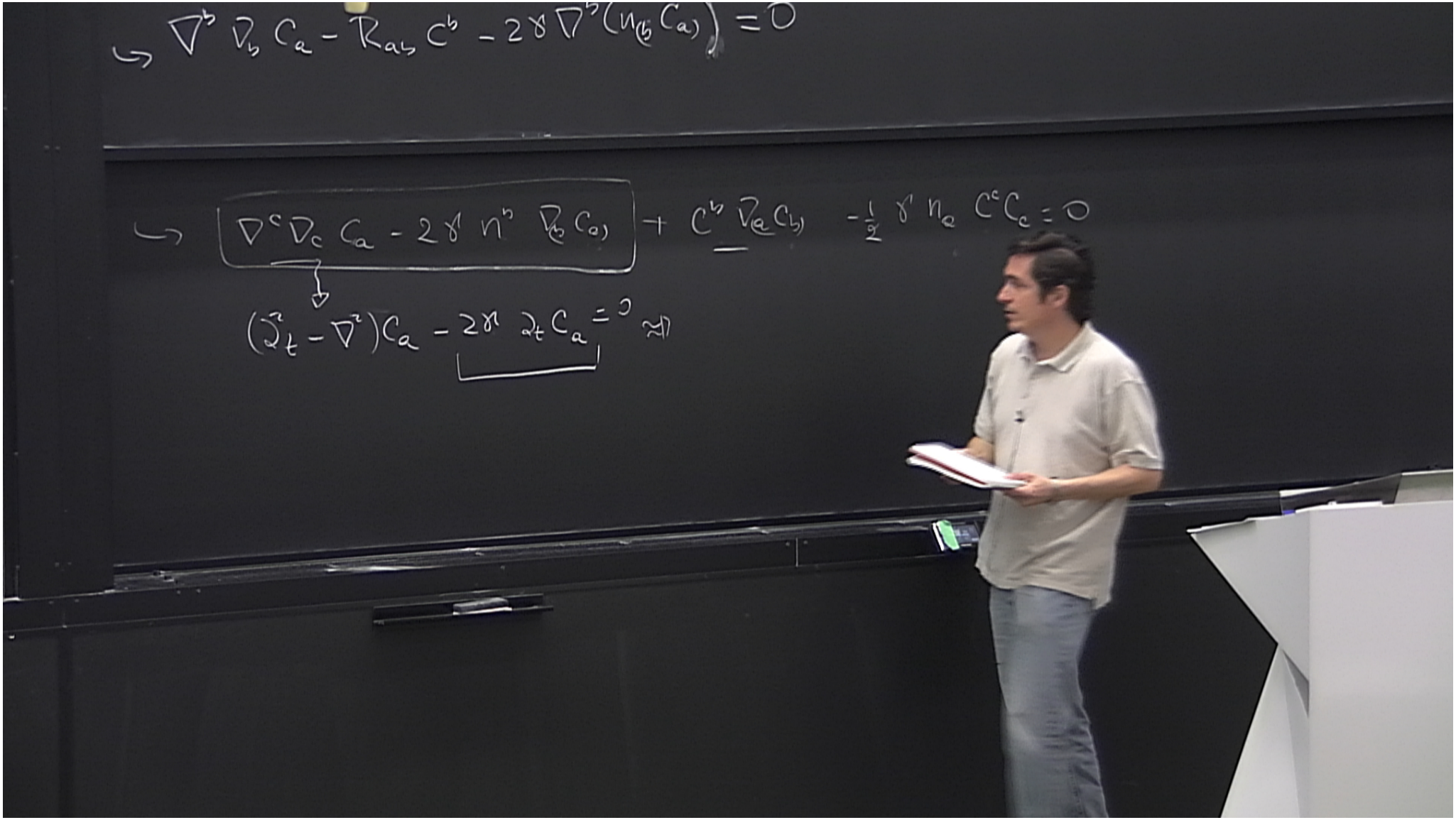
$$\hookrightarrow \nabla^b \nabla_b C_a - R_{ab} C^b - 2\gamma \nabla^b (n_{(b} C_{a)}) = 0$$

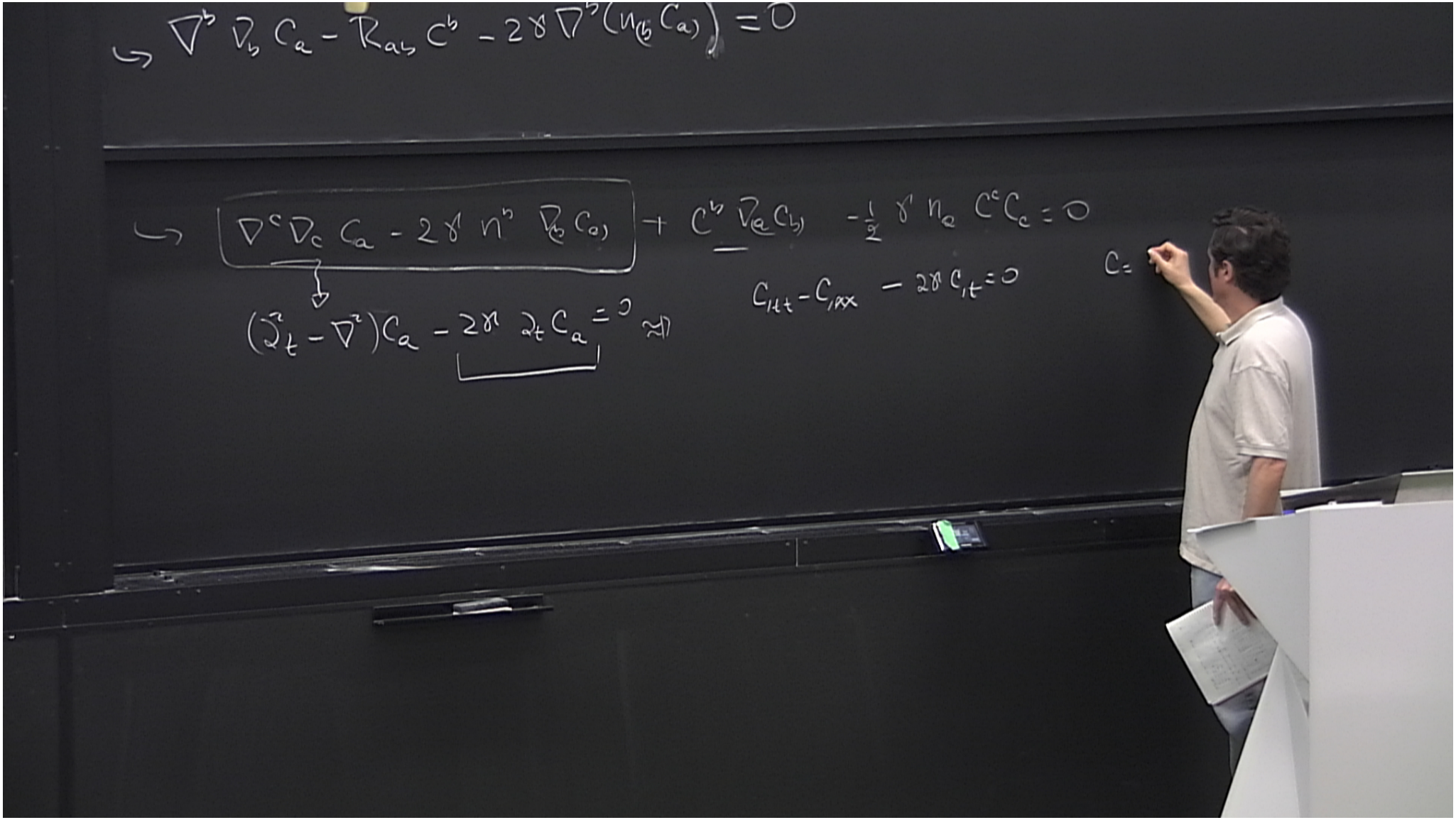
$$\hookrightarrow \boxed{\nabla^c \nabla_c C_a - 2\gamma n^b \nabla_{(b} C_{a)}} + C^b \nabla_{[a} C_{b]} - \frac{1}{2}\gamma n_a C^c C_c = 0$$

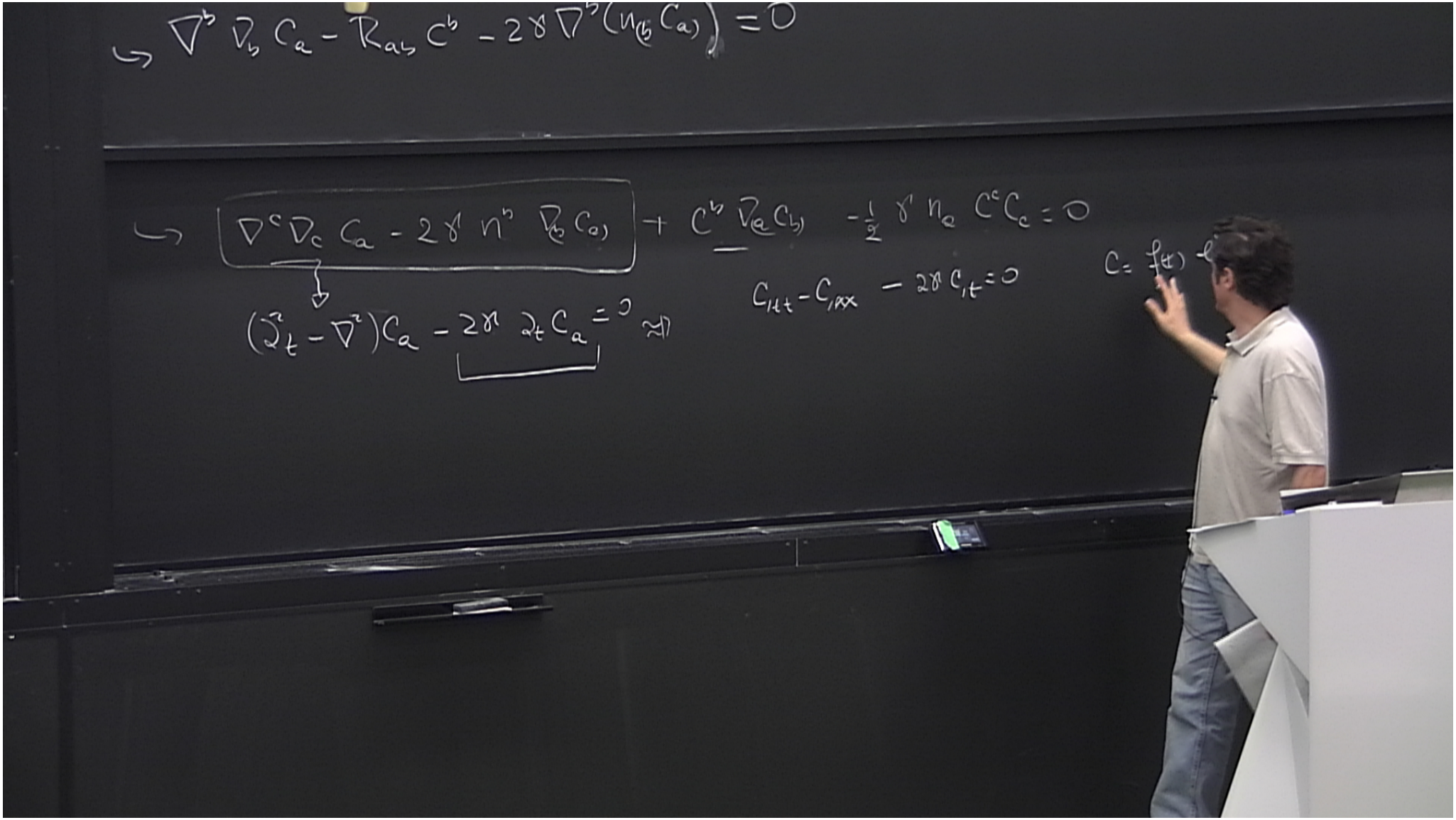
$$(\partial_t^2 - \nabla^2) C_a$$

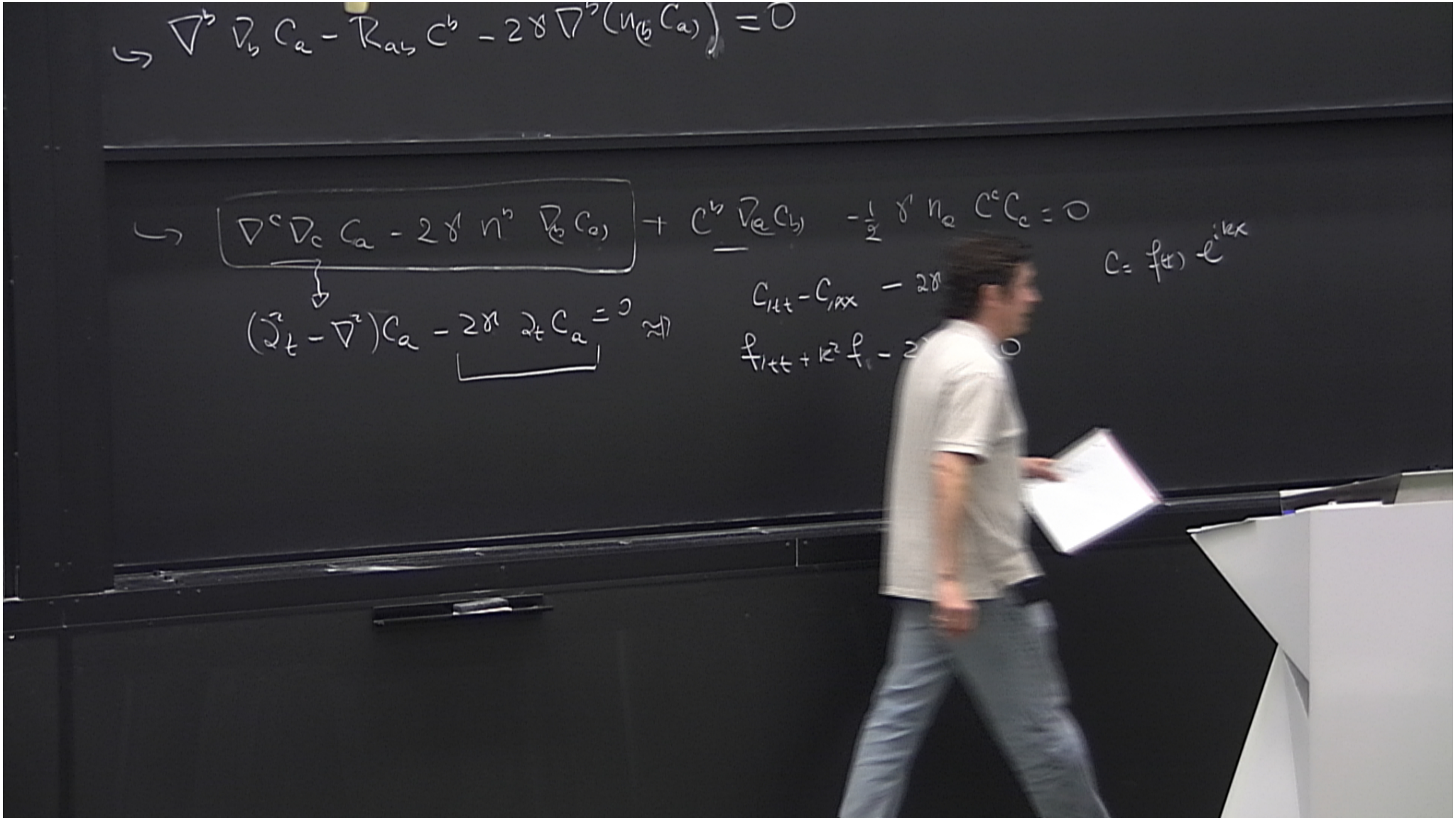

$\hookrightarrow \nabla^b D_b C_a - R_{ab} C^b - 2\gamma \nabla^b (n_{(b} C_{a)}) = 0$
$\hookrightarrow \nabla^c D_c C_a - 2\gamma n^b D_{(b} C_{a)} + C^b D_a C_b - \frac{1}{2}\gamma n_a C^c C_c = 0$
$(\partial_t^2 - \nabla^2) C_a - 2\gamma \partial_t C_a = 0$

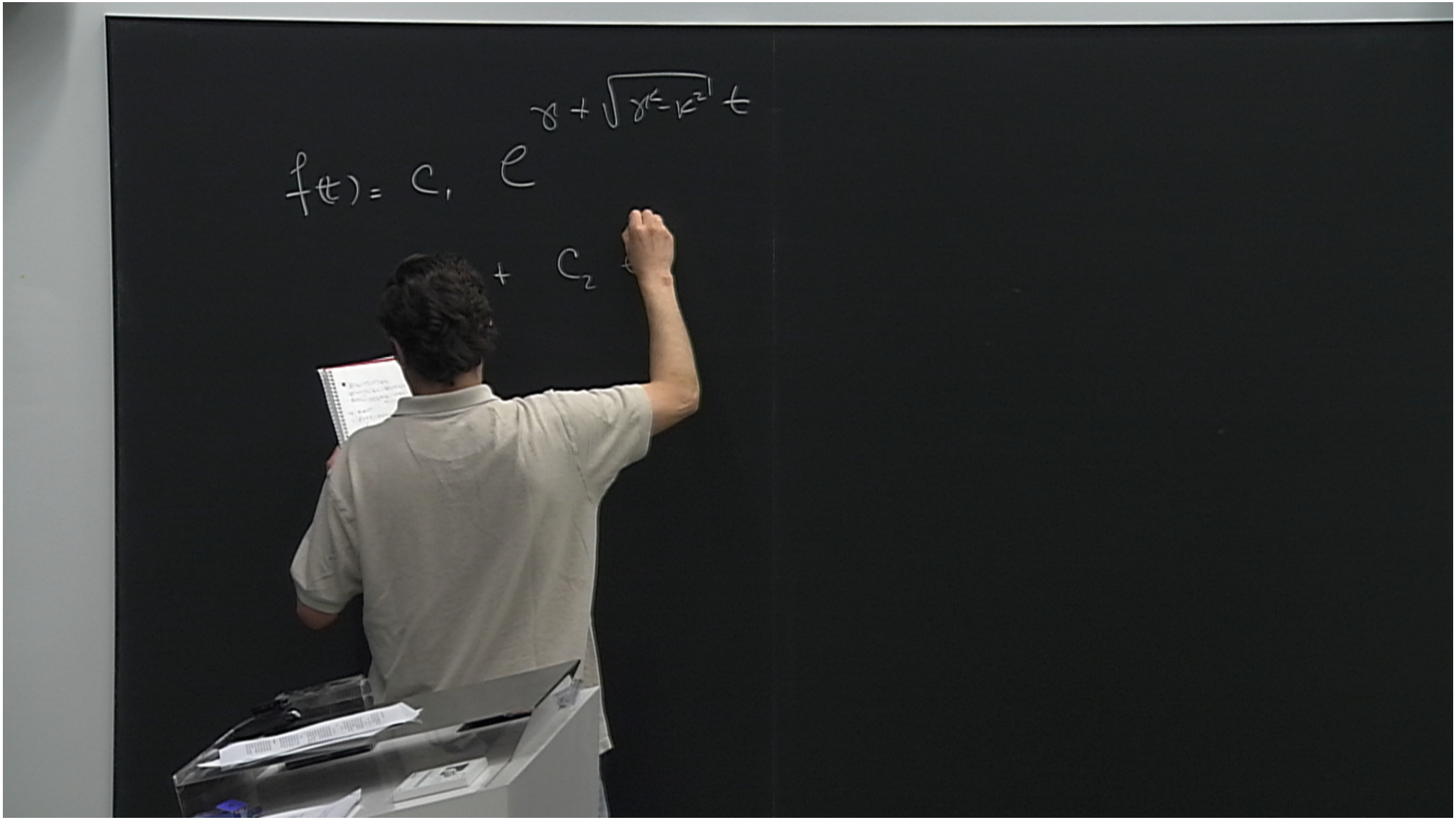

$\hookrightarrow \nabla^b D_b C_a - R_{ab} C^b - 2\gamma \nabla^b (n_{(b} C_{a)}) = 0$
$\hookrightarrow \boxed{\nabla^c D_c C_a - 2\gamma n^b D_b C_a} + C^b D_a C_b - \frac{1}{2}\gamma n_a C^c C_c = 0$
$(\partial_t^2 - \nabla^2) C_a - 2\gamma \partial_t C_a = 0$ ≈

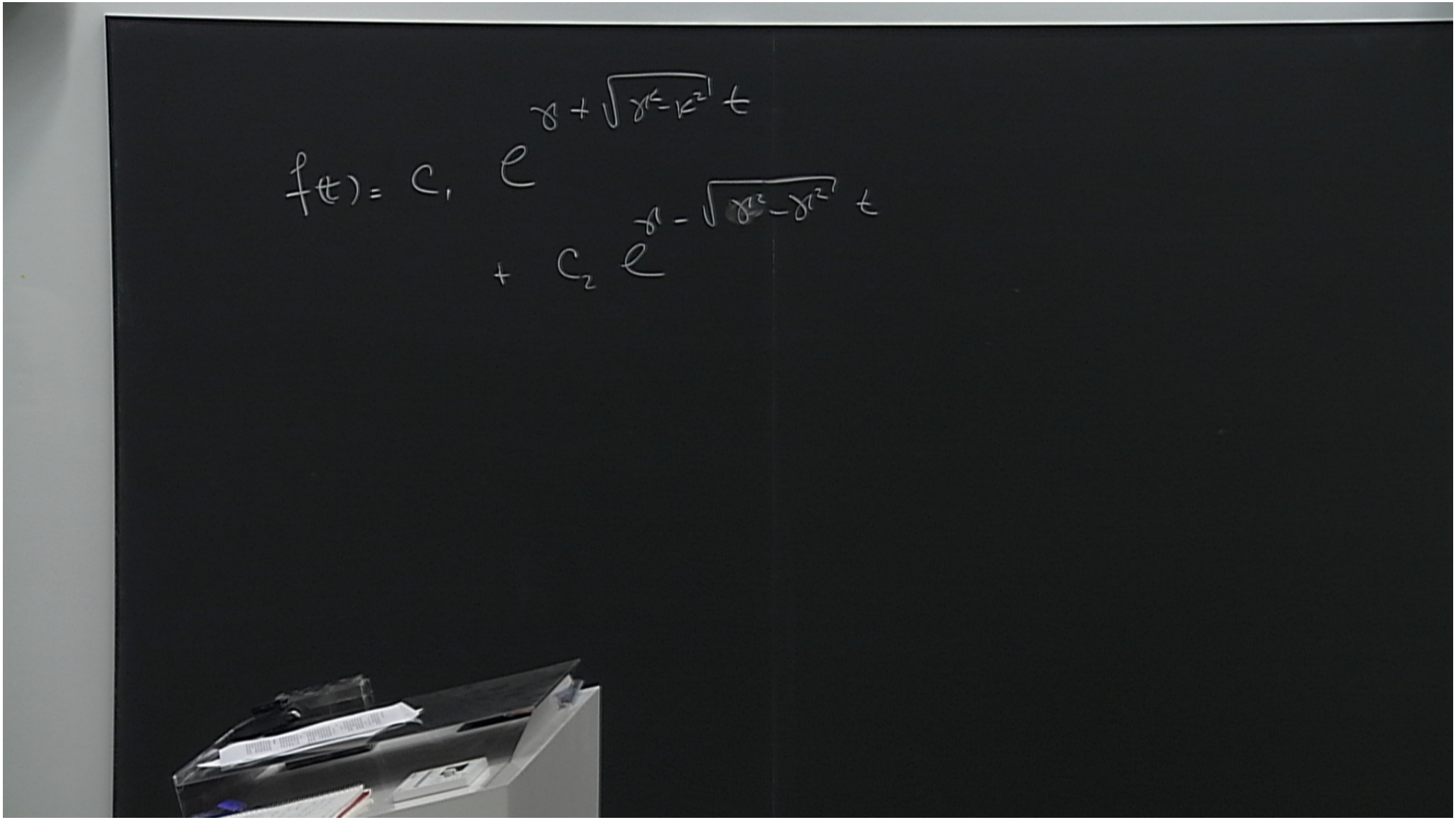

$\hookrightarrow \nabla^b D_b C_a - R_{ab} C^b - 2\gamma \nabla^b (n_{(b} C_{a)}) = 0$
$\hookrightarrow \nabla^c D_c C_a - 2\gamma n^b D_{(b} C_{a)} + C^b D_{(a} C_{b)} - \frac{1}{2}\gamma n_a C^c C_c = 0$
$(\partial_t^2 - \nabla^2) C_a - 2\gamma \partial_t C_a = 0$

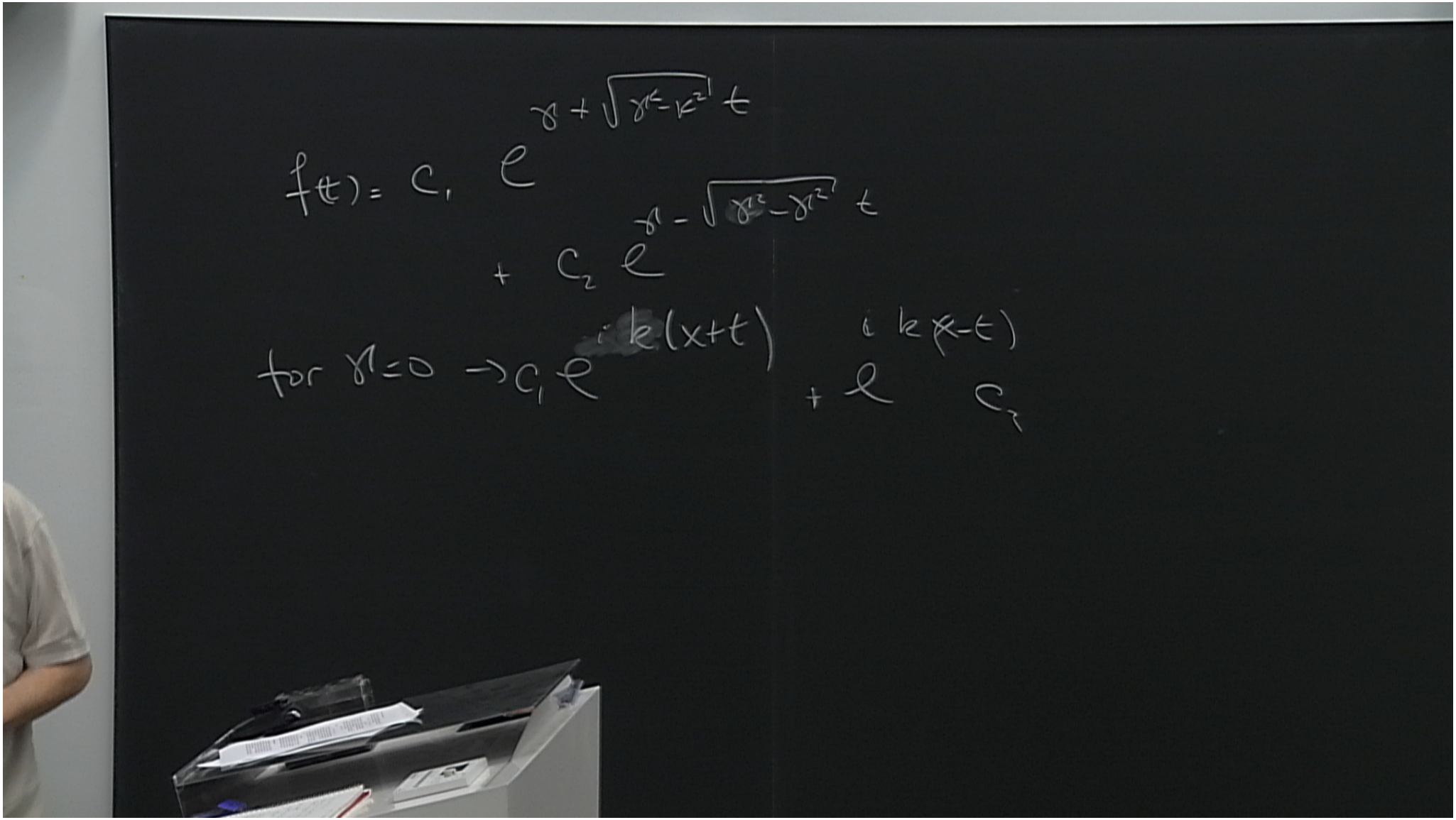

$\hookrightarrow \nabla^b D_b C_a - R_{ab} C^b - 2\gamma \nabla^b (n_{(b} C_{a)}) = 0$
$\hookrightarrow \nabla^c D_c C_a - 2\gamma n^b D_{(b} C_{a)} + C^b D_{(a} C_{b)} - \frac{1}{2}\gamma n_a C^c C_c = 0$
$(\partial_t^2 - \nabla^2) C_a - 2\gamma \partial_t C_a = 0 \rightsquigarrow$
$C_{i,tt} - C_{i,xx} - 2\gamma C_{i,t} = 0$

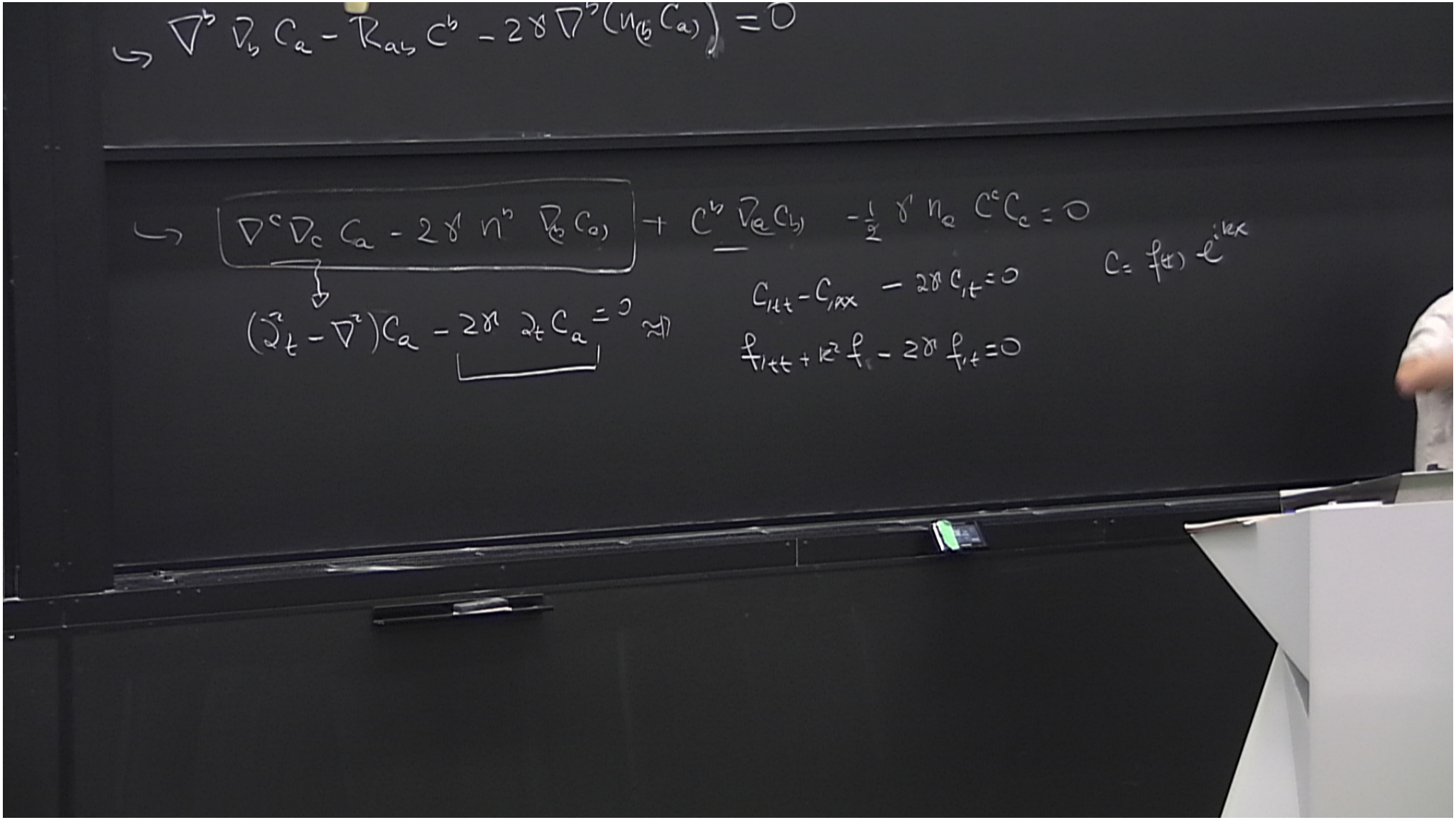

$\hookrightarrow \nabla^b D_b C_a - R_{ab} C^b - 2\gamma \nabla^b (n_{(b} C_{a)}) = 0$
$\hookrightarrow \nabla^c D_c C_a - 2\gamma n^b D_{(b} C_{a)} + C^b D_{(a} C_{b)} - \frac{1}{2}\gamma n_a C^c C_c = 0$
$(\partial_t^2 - \nabla^2) C_a - 2\gamma \partial_t C_a = 0 \rightsquigarrow$
$C_{,tt} - C_{,xx} - 2\gamma C_{,t} = 0$
$C = f(t)$

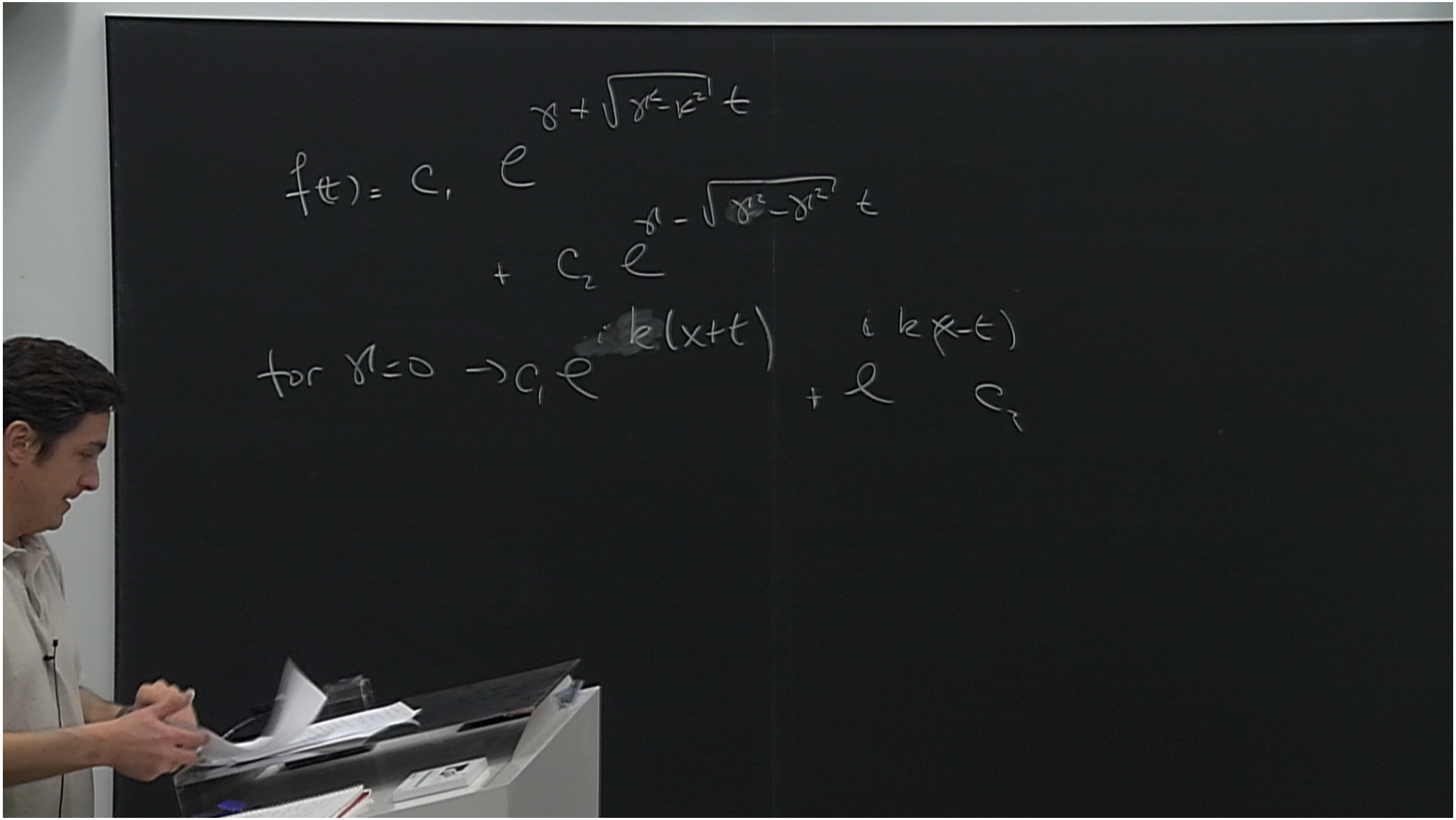

$\hookrightarrow \nabla^b D_b C_a - R_{ab} C^b - 2\gamma \nabla^b (n_{(b} C_{a)}) = 0$
$\hookrightarrow \nabla^c D_c C_a - 2\gamma n^b D_{(b} C_{a)} + C^b D_{(a} C_{b)} - \frac{1}{2}\gamma n_a C^c C_c = 0$
$(\partial_t^2 - \nabla^2) C_a - 2\gamma \partial_t C_a = 0 \Rightarrow$
$C_{1tt} - C_{1xx} - 2\gamma$
$f_{1tt} + k^2 f_1 - 2$
$C = f(t) e^{ikx}$

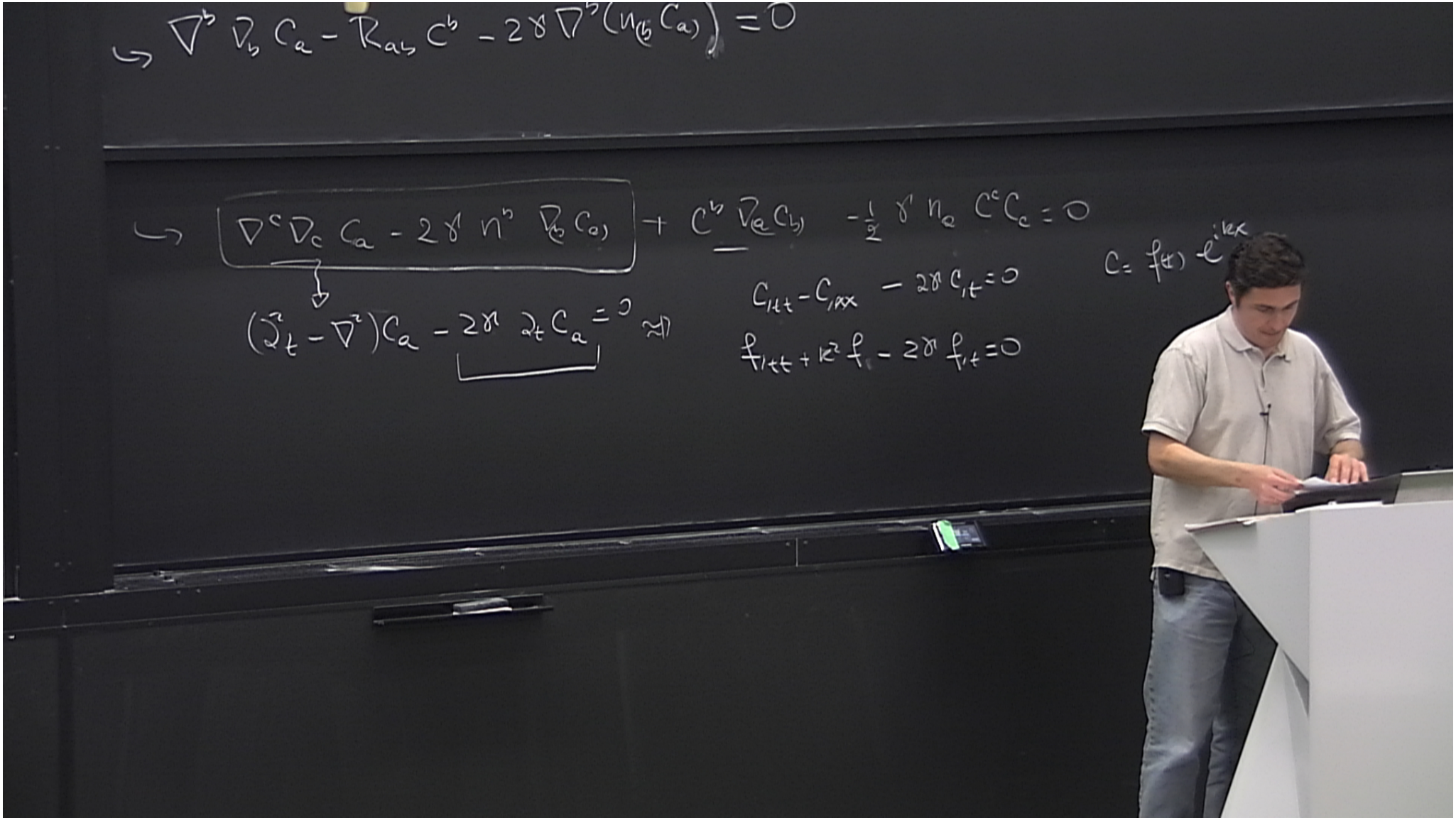

$f(t) = c_1 e^{\gamma + \sqrt{\gamma^2 - \kappa^2}\, t}$
$+ c_2$

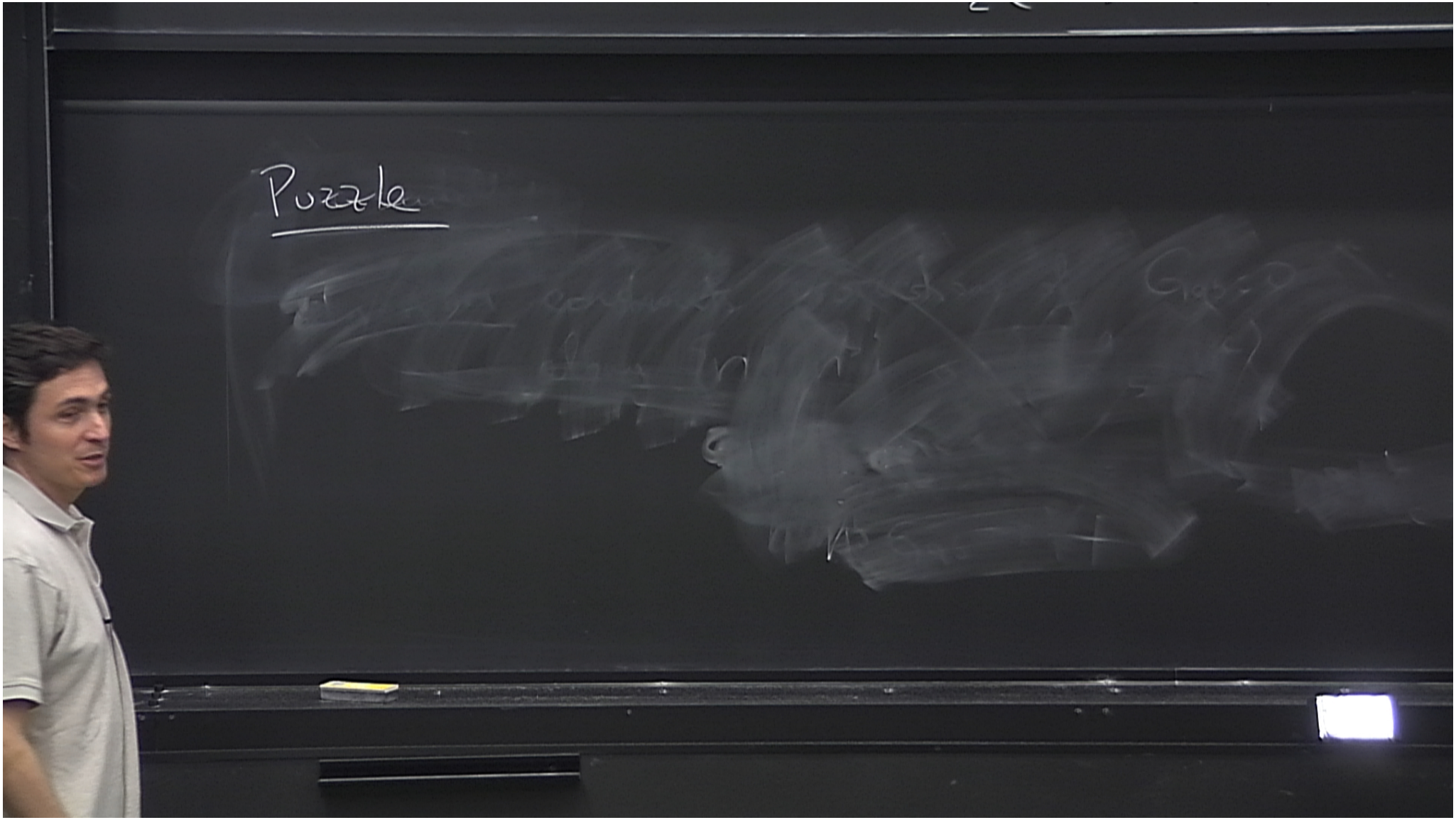

$f(t) = c_1 e^{\gamma + \sqrt{\gamma^2 - \kappa^2}\, t}$
$+ c_2 e^{\gamma - \sqrt{\gamma^2 - \gamma^2}\, t}$

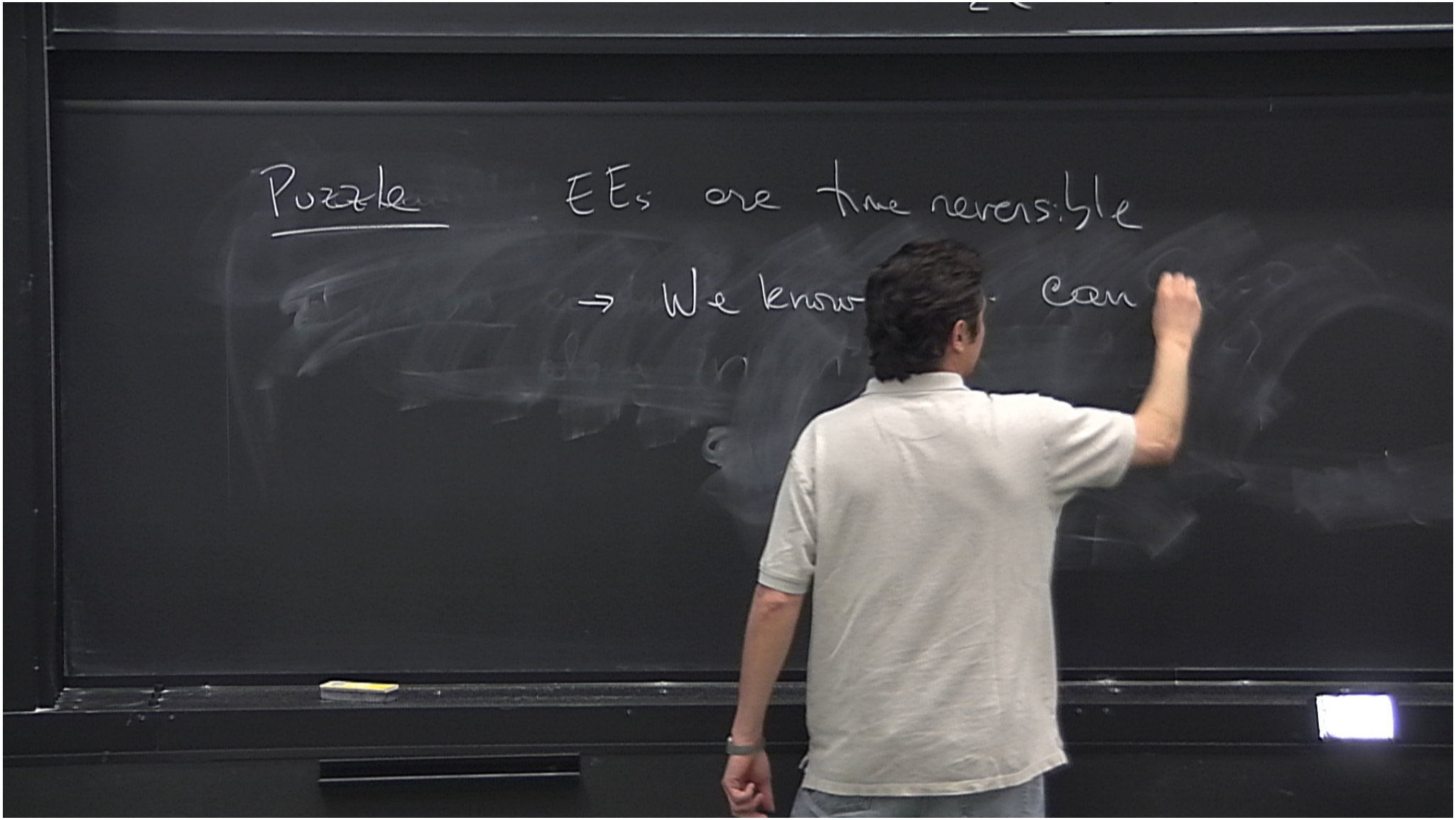

$f(t) = c_1 e^{\gamma + \sqrt{\gamma^2 - \kappa^2}\, t}$
$+ c_2 e^{\gamma - \sqrt{\gamma^2 - \gamma^2}\, t}$
for $\gamma = 0 \rightarrow c_1 e^{ik(x+t)}$
$+ e$
c_2

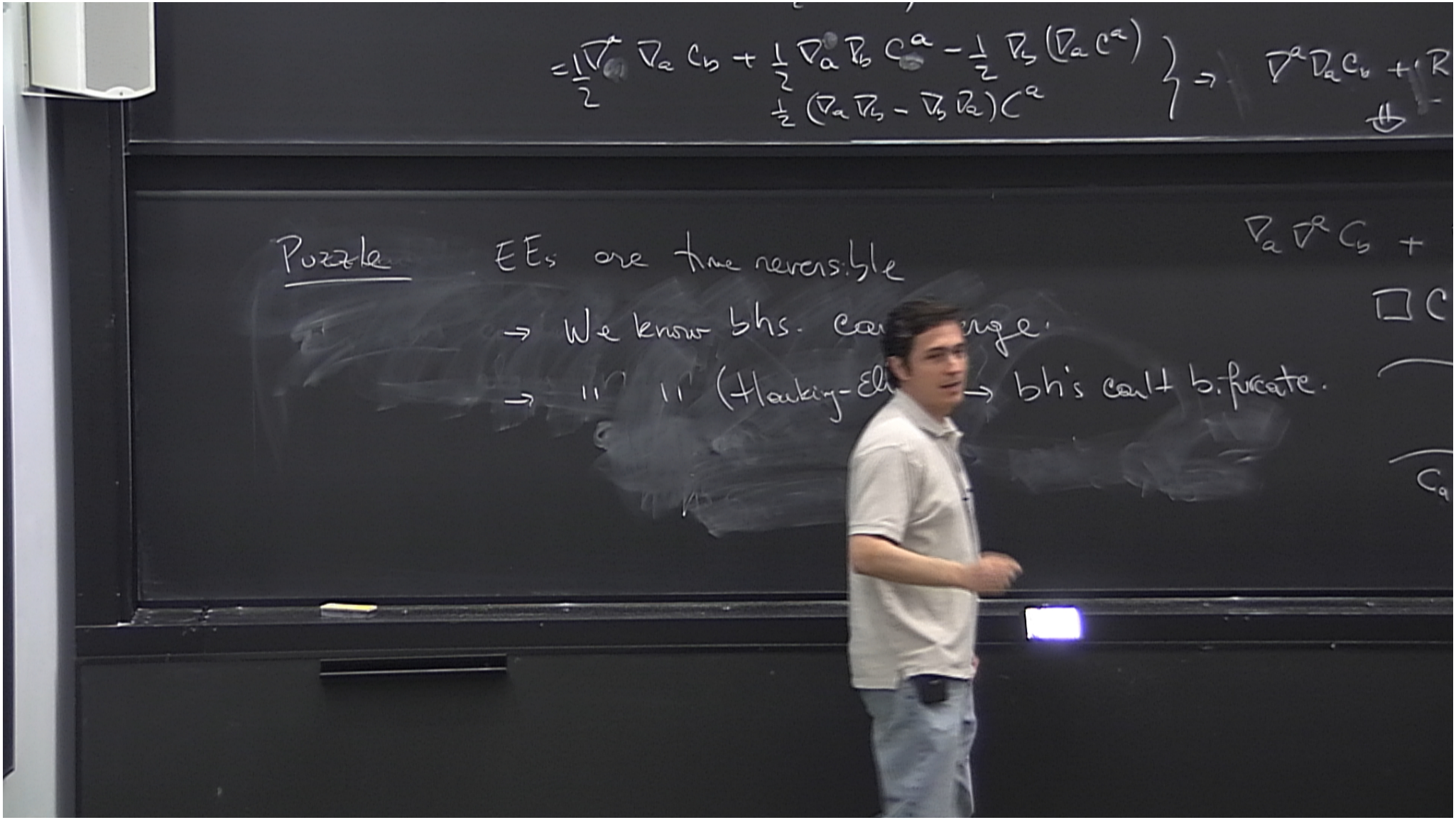

$\hookrightarrow \nabla^b D_b C_a - R_{ab} C^b - 2\gamma \nabla^b (n_{(b} C_{a)}) = 0$
$\hookrightarrow \nabla^c D_c C_a - 2\gamma n^b D_{(b} C_{a)} + C^b D_{(a} C_{b)} - \frac{1}{2}\gamma n_a C^c C_c = 0$
$(\partial_t^2 - \nabla^2) C_a - 2\gamma \partial_t C_a = 0 \rightsquigarrow$
$C_{,tt} - C_{,xx} - 2\gamma C_{,t} = 0$
$C = f(t) e^{ikx}$
$f_{,tt} + k^2 f - 2\gamma f_{,t} = 0$

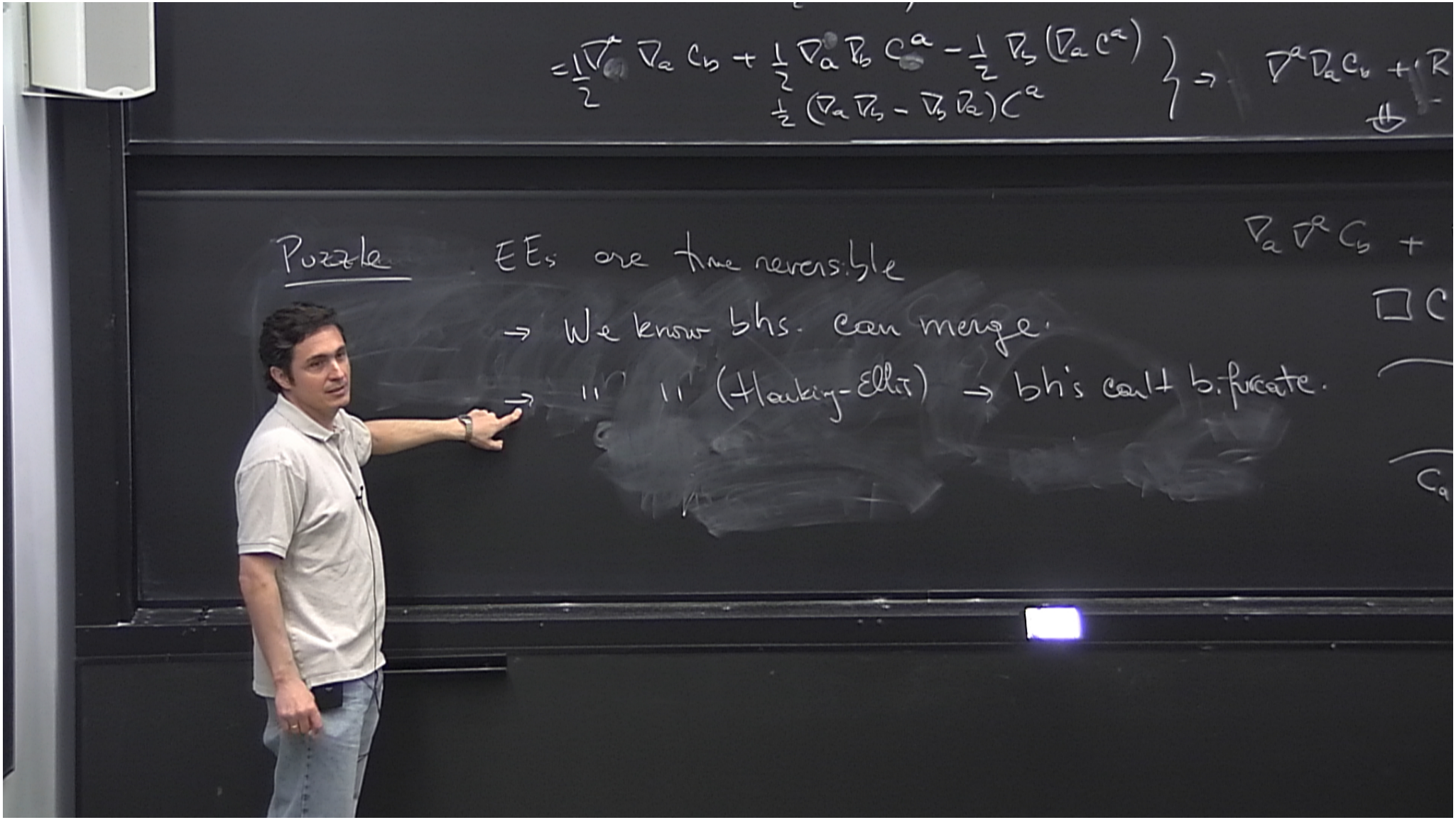

$f(t) = c_1 e^{\gamma + \sqrt{\gamma^2 - \kappa^2}\, t}$
$+ c_2 e^{\gamma - \sqrt{\gamma^2 - \gamma^2}\, t}$
for $\gamma = 0 \rightarrow c_1 e^{ik(x+t)}$
$+ e^{ik(x-t)} c_2$


$\hookrightarrow \nabla^b D_b C_a - R_{ab} C^b - 2\gamma \nabla^b (n_{(b} C_{a)}) = 0$
$\hookrightarrow \nabla^c D_c C_a - 2\gamma n^b D_{(b} C_{a)} + C^b D_{(a} C_{b)} - \frac{1}{2}\gamma n_a C^c C_c = 0$
$(\partial_t^2 - \nabla^2) C_a - 2\gamma \partial_t C_a = 0 \rightsquigarrow$
$C_{,tt} - C_{,xx} - 2\gamma C_{,t} = 0$
$f_{,tt} + k^2 f - 2\gamma f_{,t} = 0$
$C = f(t) e^{ikx}$


Puzzle


Puzzle
EEs are time reversible
→ We know


$= \frac{1}{2}\nabla^a \nabla_a C_b + \frac{1}{2}\nabla_a B_b C^a - \frac{1}{2}\nabla_b(\nabla_a C^a)$
$\frac{1}{2}(\nabla_a \nabla_b - \nabla_b \nabla_a) C^a$
$\Rightarrow \nabla^a \nabla_a C_b + R$
Puzzle
EEs are time reversible
$\rightarrow$ We know bhs. ca
$\rightarrow$ " " (Hawking-El
$\rightarrow$ bh's can't bifurcate.
$\nabla_a \nabla^a C_b +$
$\Box C$
C_a


$= \frac{1}{2}\nabla_a \nabla_a C_b + \frac{1}{2}\nabla_a R_b C^a - \frac{1}{2}\nabla_b(\nabla_a C^a)$
$\frac{1}{2}(\nabla_a \nabla_b - \nabla_b \nabla_a) C^a$
$\Rightarrow \nabla^a \nabla_a C_b + R$
Puzzle
EEs are time reversible
$\rightarrow$ We know bhs. can merge.
$\rightarrow$ " " (Hawking-Ellis) $\rightarrow$ bh's can't bifurcate.
$\nabla_a \nabla^a C_b +$
$\Box C$
C_a

$= \frac{1}{2}\nabla^a \nabla_a C_b + \frac{1}{2}\nabla_a R_b C^a - \frac{1}{2}\nabla_b(\nabla_a C^a)$
$\frac{1}{2}(\nabla_a \nabla_b - \nabla_b \nabla_a) C^a$
$\Big\} \Rightarrow \nabla^a \nabla_a C_b + R$

$\nabla_a \nabla^a C_b +$

$\square C$

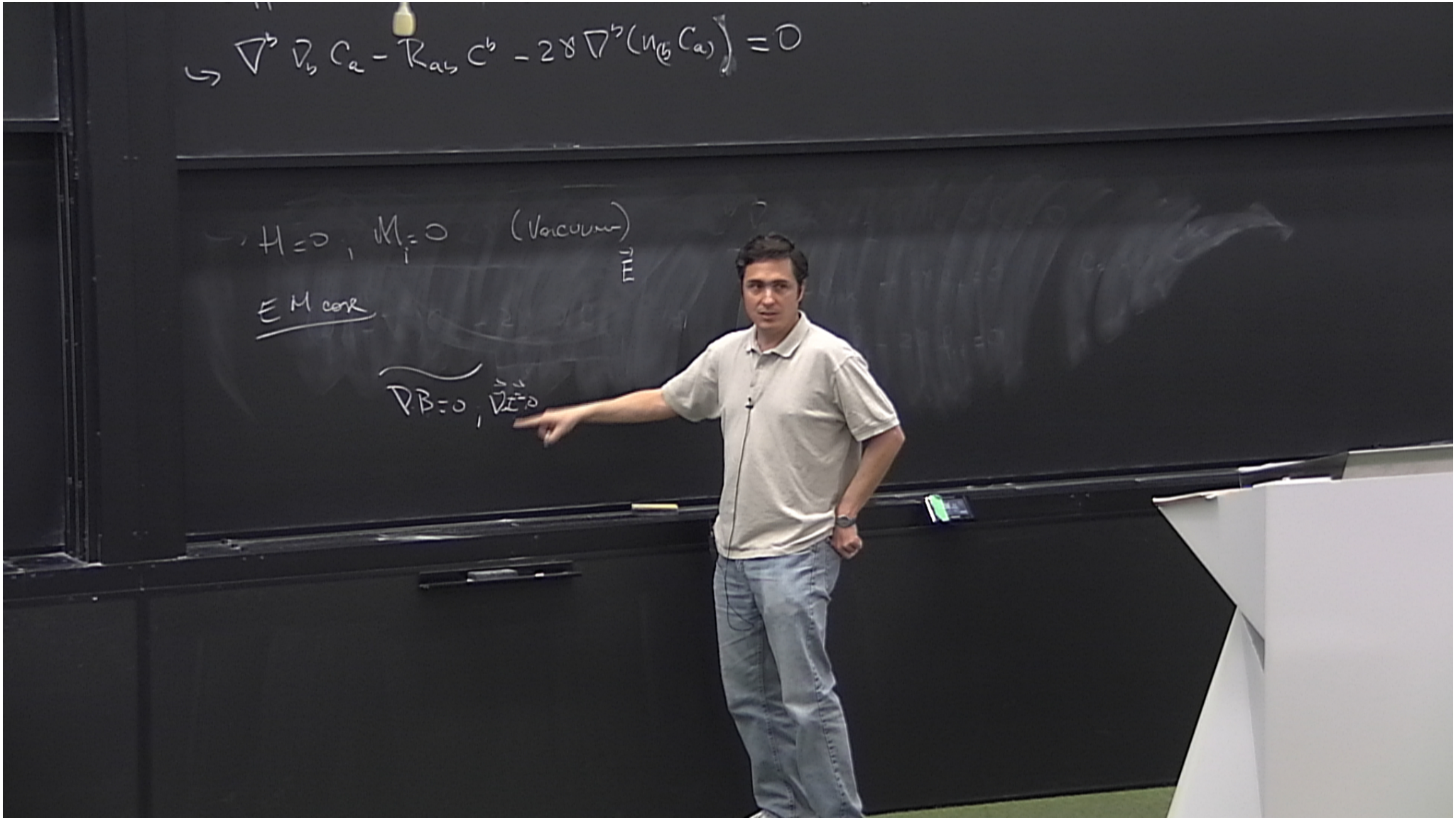
C_a

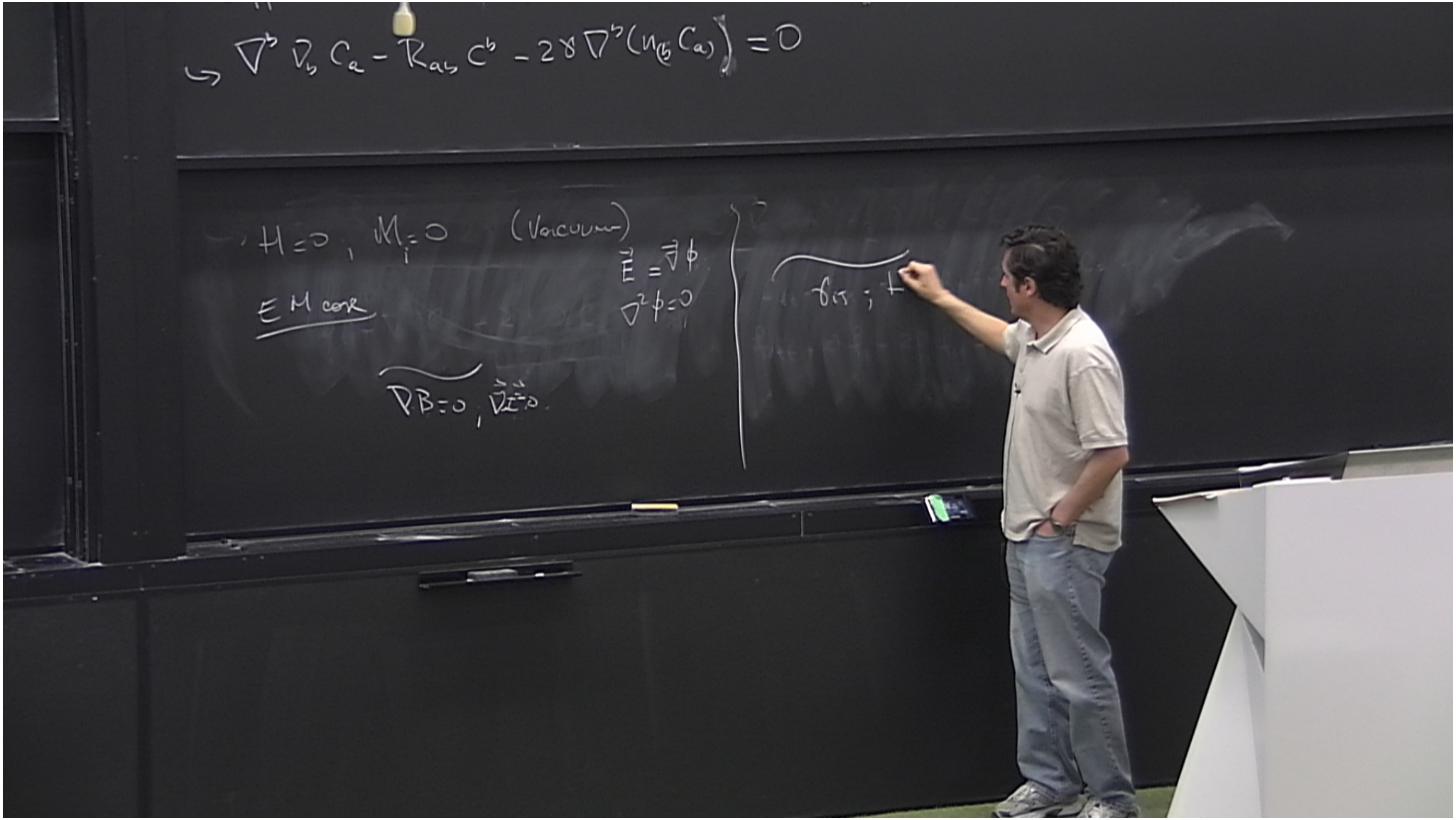
Puzzle EEs are time reversible

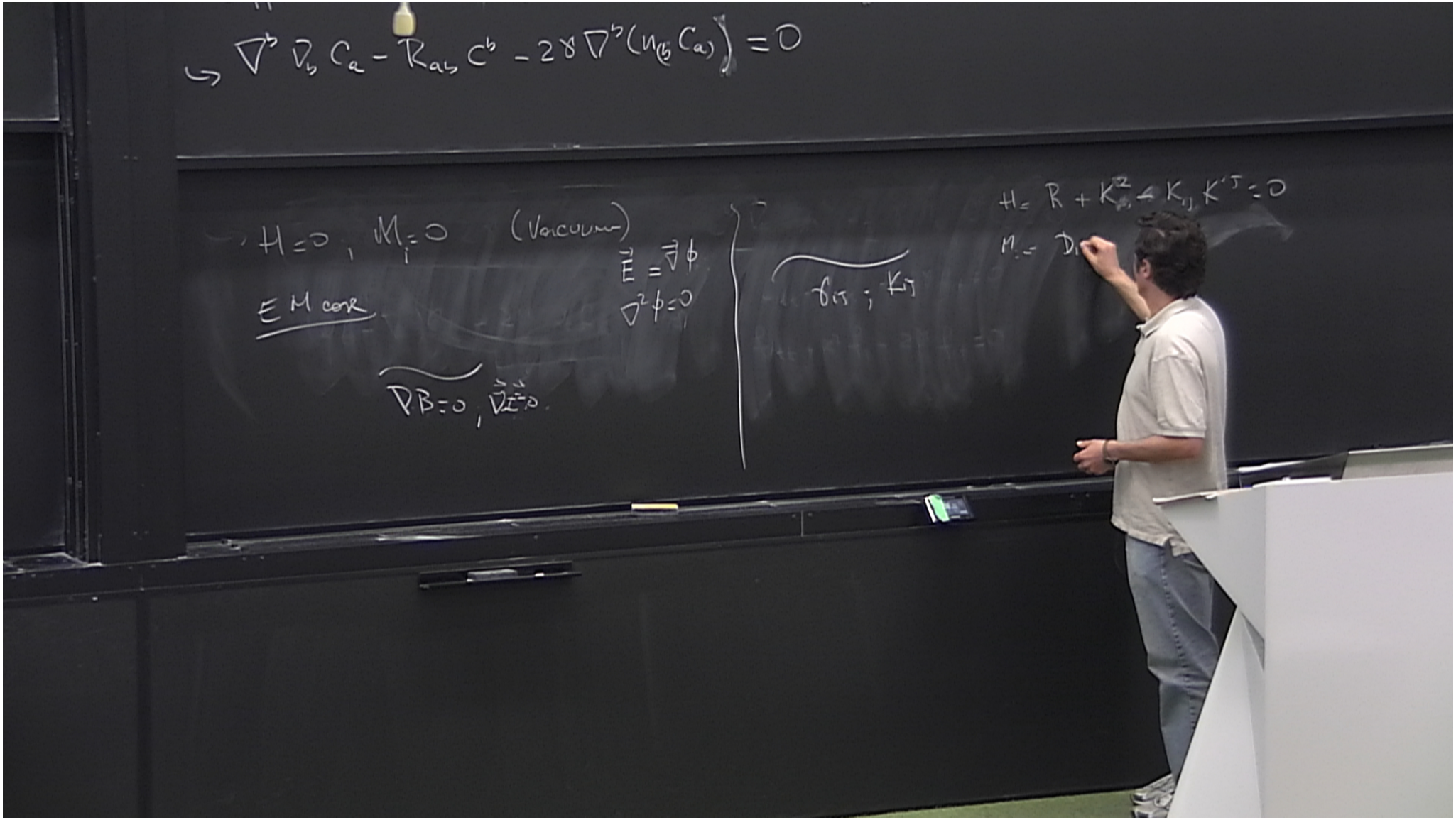
(a) → We know bhs. can merge.

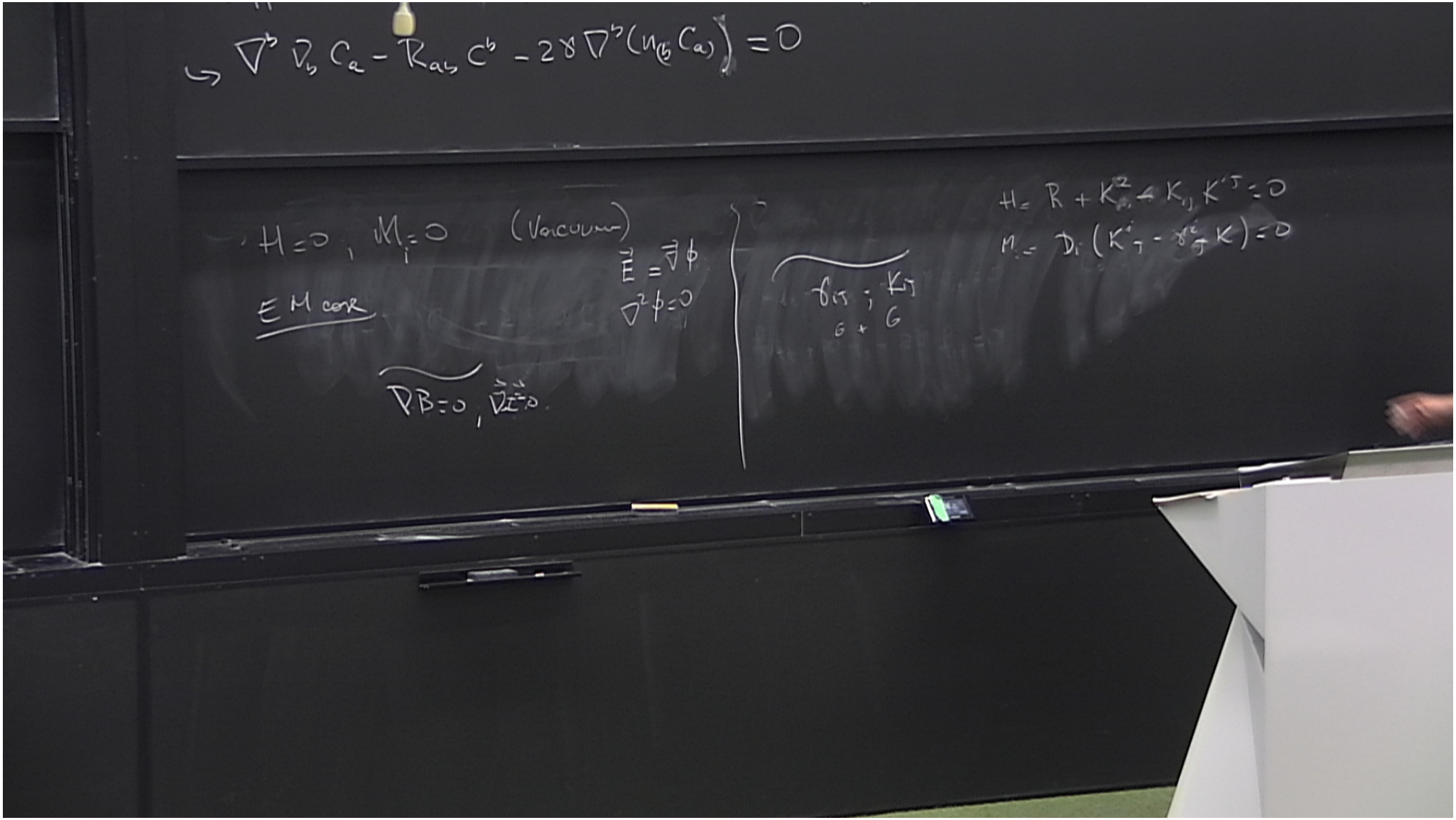
→ " " (Hawking-Ellis) → bh's can't bifurcate.

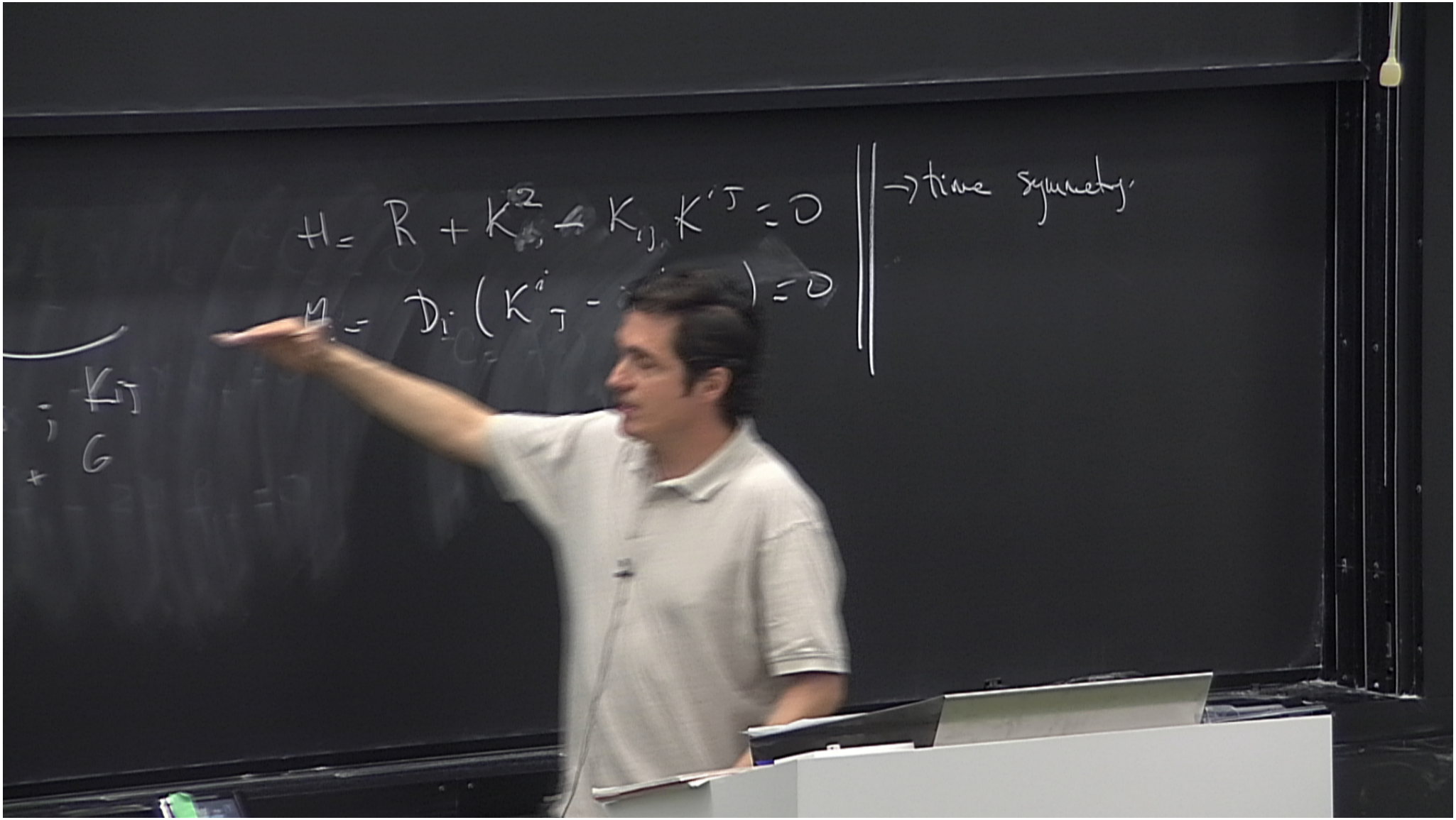
(b) ⇝ Simple argument based solely in Principle of equivalence

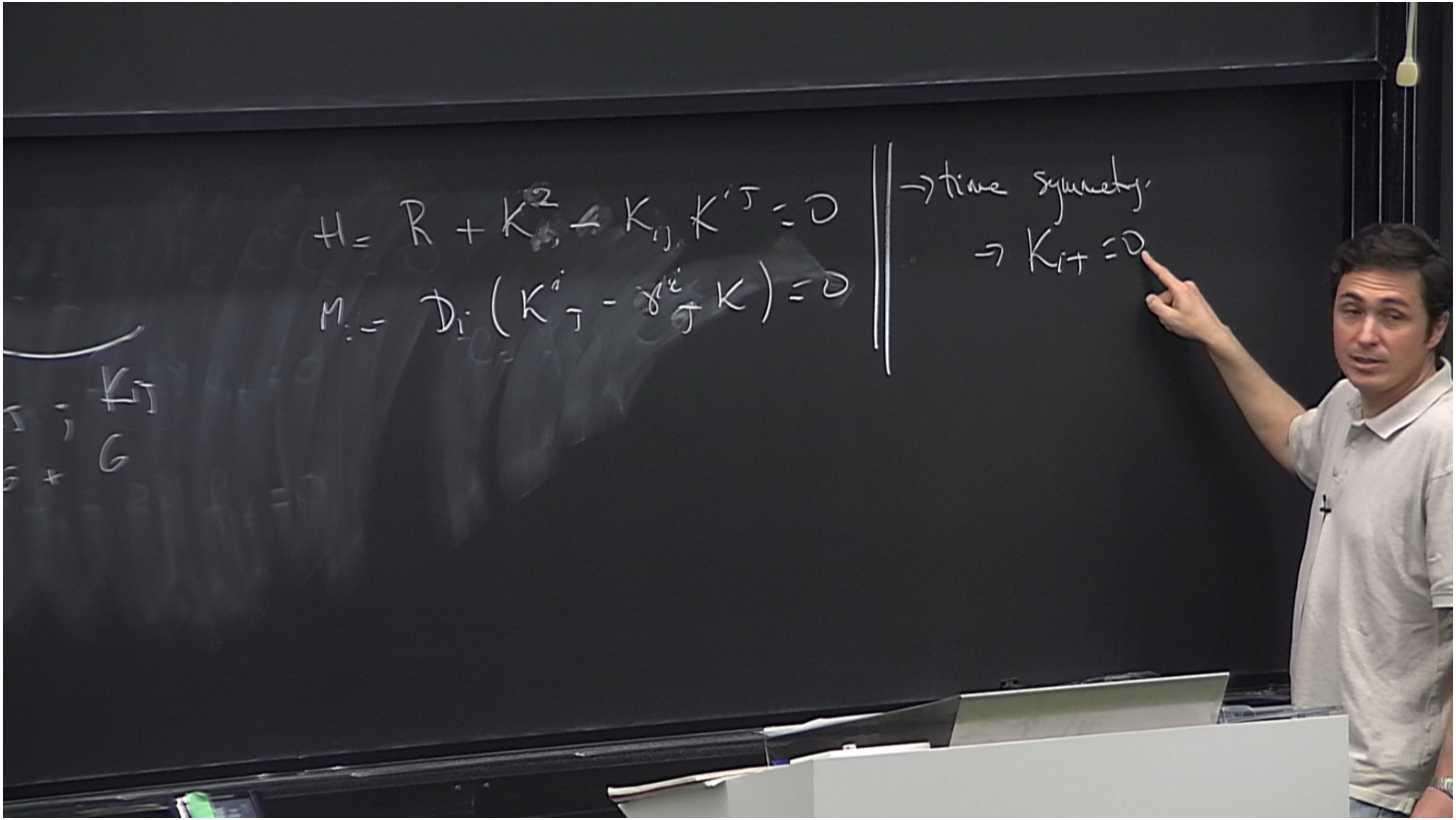

$\hookrightarrow \nabla^b B_b C_a - R_{ab} C^b - 2\gamma \nabla^b (u_{(b} C_{a)}) = 0$
$H = 0$, $M_i = 0$ (Vacuum)
EM conk
$\nabla B = 0$, $\dot{\vec{E}} = 0$

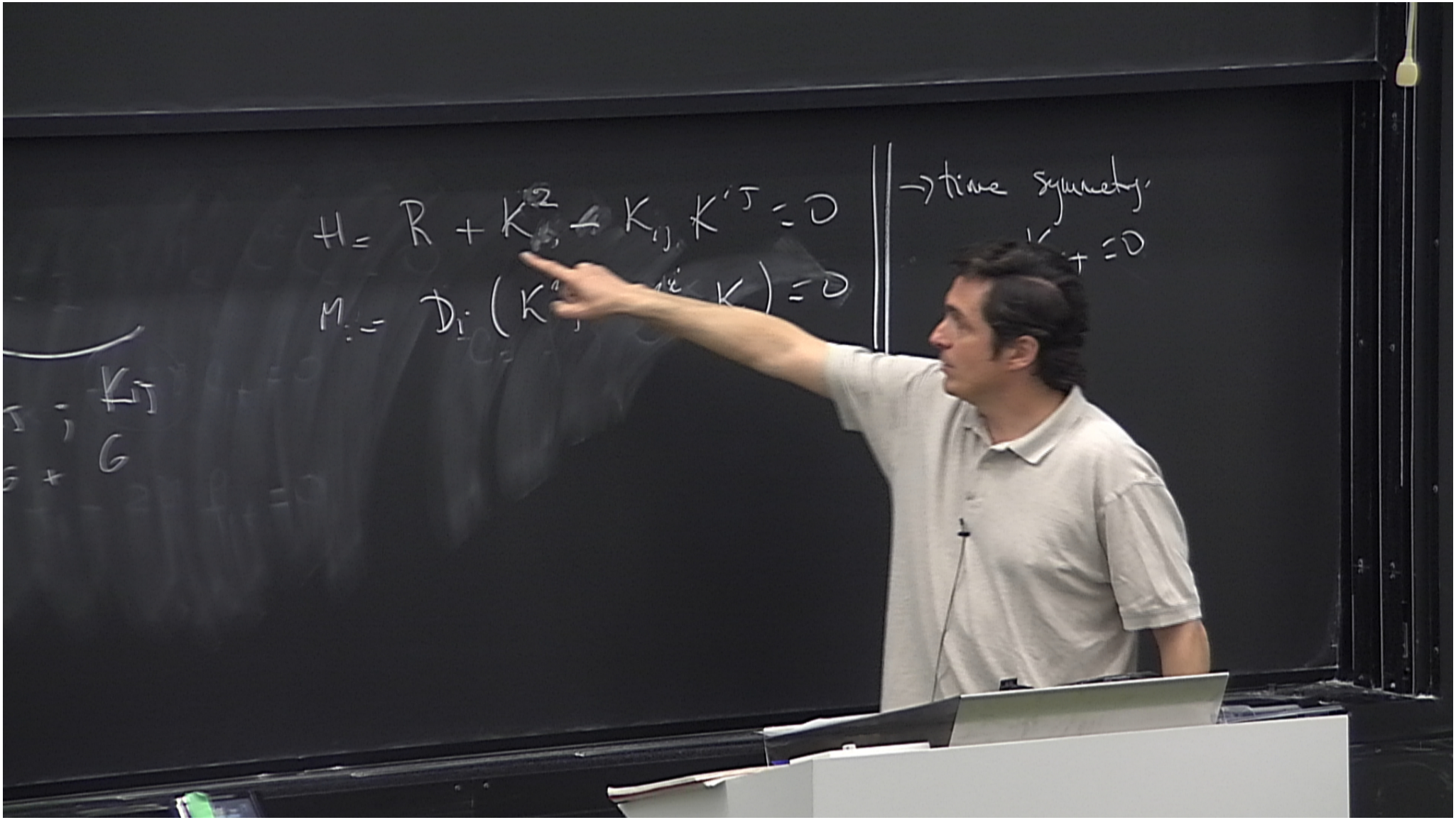

$\hookrightarrow \nabla^b D_b C_a - R_{ab} C^b - 2\gamma \nabla^b (u_{[b} C_{a]}) = 0$
$H=0$, $M_i=0$ (Vacuum)
EM cons
$\vec{E} = \vec{\nabla}\phi$
$\nabla^2\phi = 0$
$\nabla \cdot B = 0$, $\vec{\nabla} \cdot \vec{E} = 0$

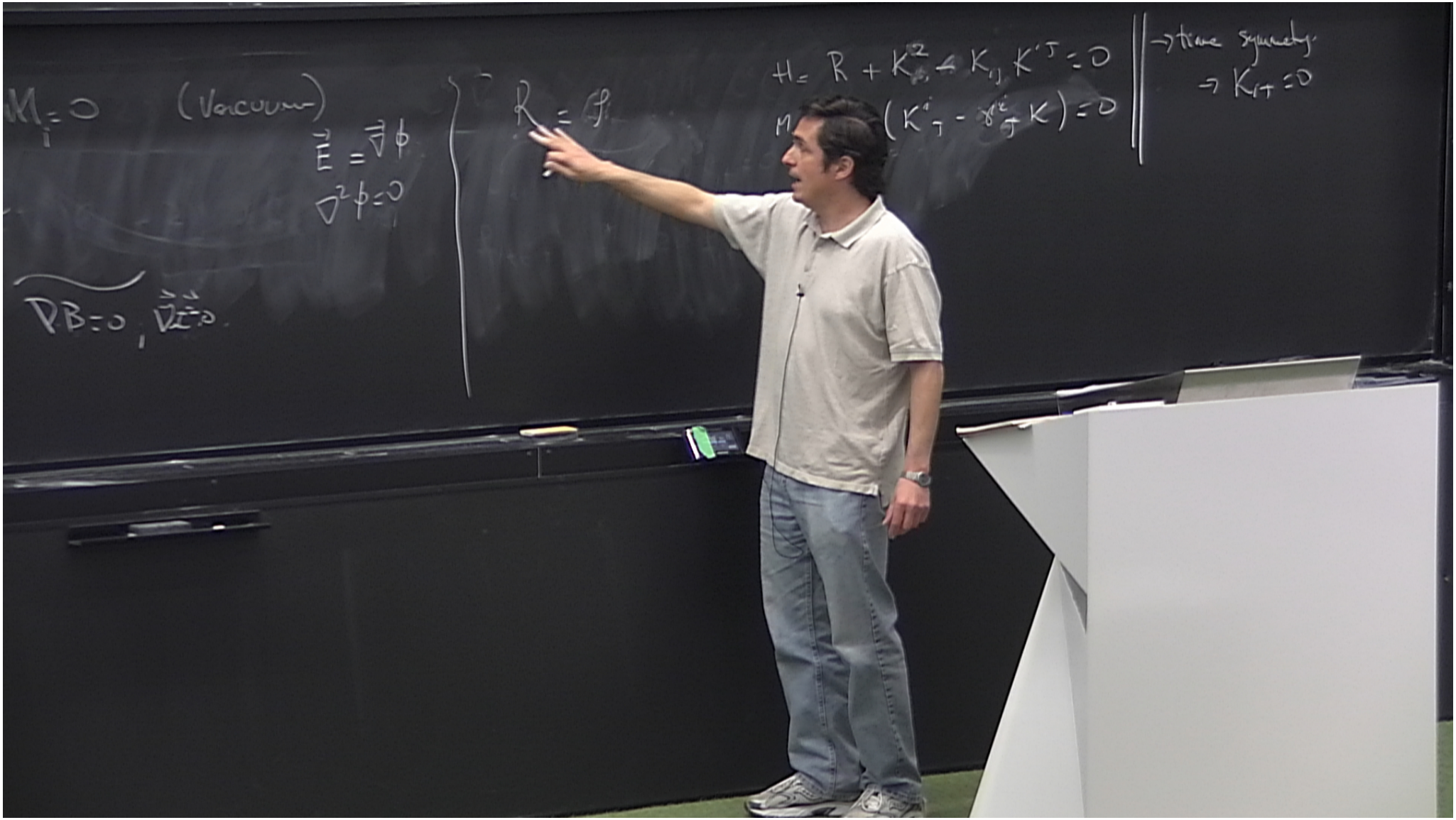

$\hookrightarrow \nabla^b D_b C_a - R_{ab} C^b - 2\gamma \nabla^b (u_{(b} C_{a)}) = 0$
$H = 0$, $M_i = 0$ (Vacuum)
EM cons
$\vec{E} = \vec{\nabla}\phi$
$\nabla^2 \phi = 0$
$\vec{\nabla} \cdot \vec{B} = 0$, $\vec{\nabla} \cdot \vec{E} = 0$
h_{ij} ; K_{ij}
$H = R + K^2 - K_{ij} K^{ij} = 0$
$M_i = D_j$

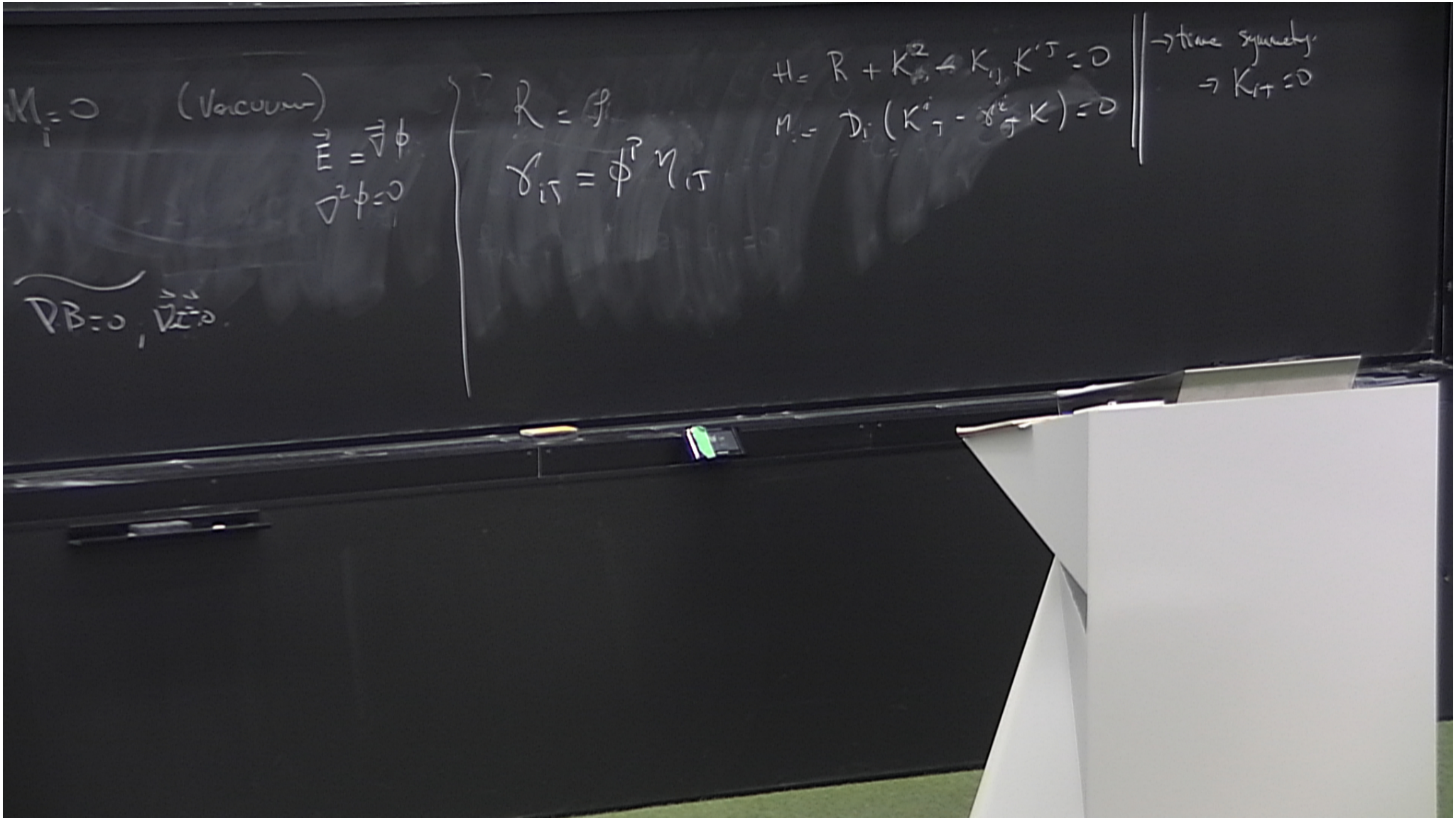

$\hookrightarrow \nabla^b D_b C_a - R_{ab} C^b - 2\gamma \nabla^b (u_{(b} C_{a)}) = 0$
$H = 0$, $M_i = 0$ (Vacuum)
EM cons
$\vec{E} = \vec{\nabla}\phi$
$\nabla^2 \phi = 0$
$\vec{\nabla} B = 0$, $\vec{\nabla} \vec{E} = 0$
γ_{ij} ; K_{ij}
$G + G$
$H = R + K^2 - K_{ij} K^{ij} = 0$
$M_i = D_j (K^j_i - \gamma^j_i K) = 0$

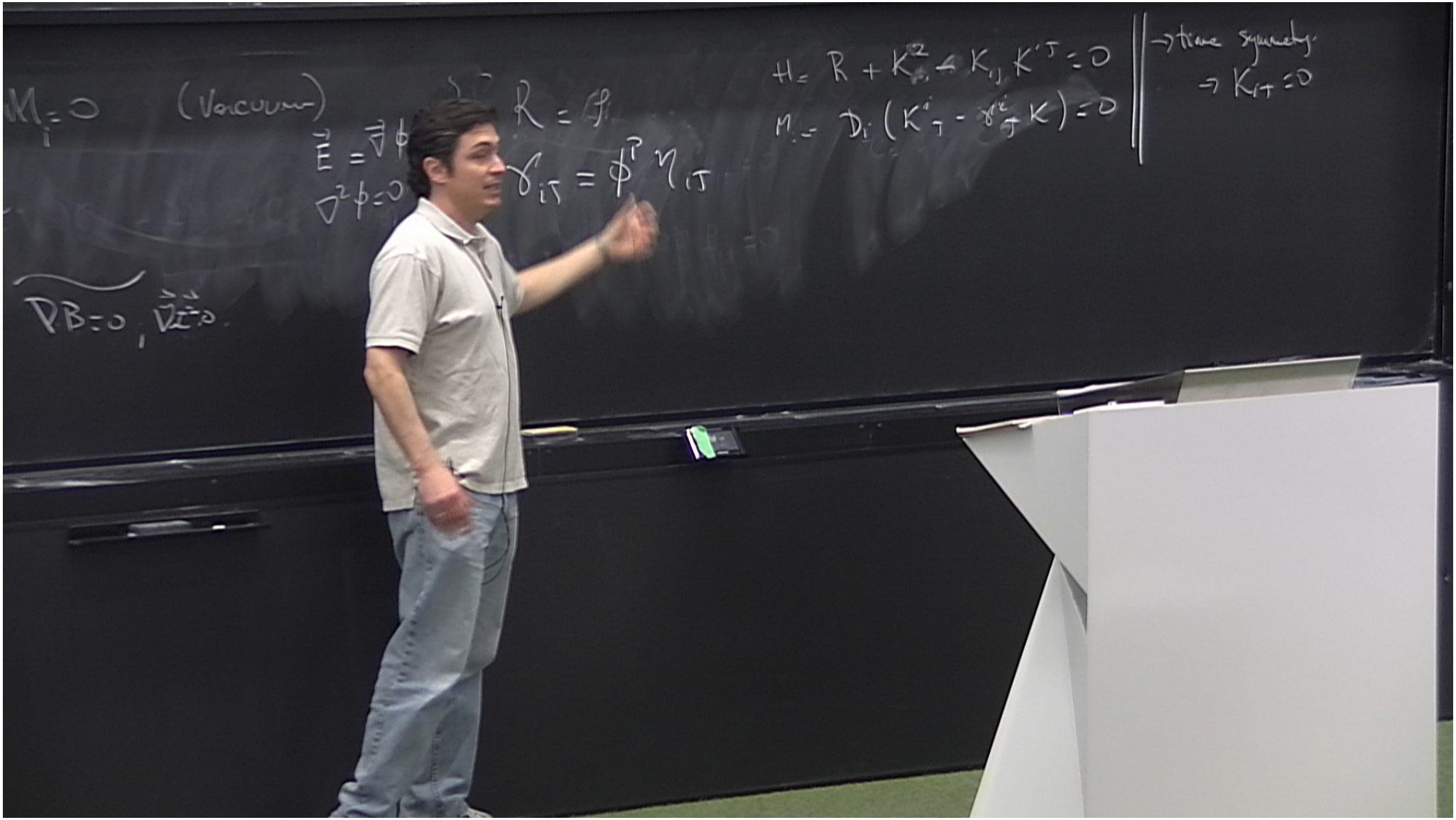

H = R + K² − K_ij K^ij = 0
M = D_i (K^i_j −
→ time symmetry.

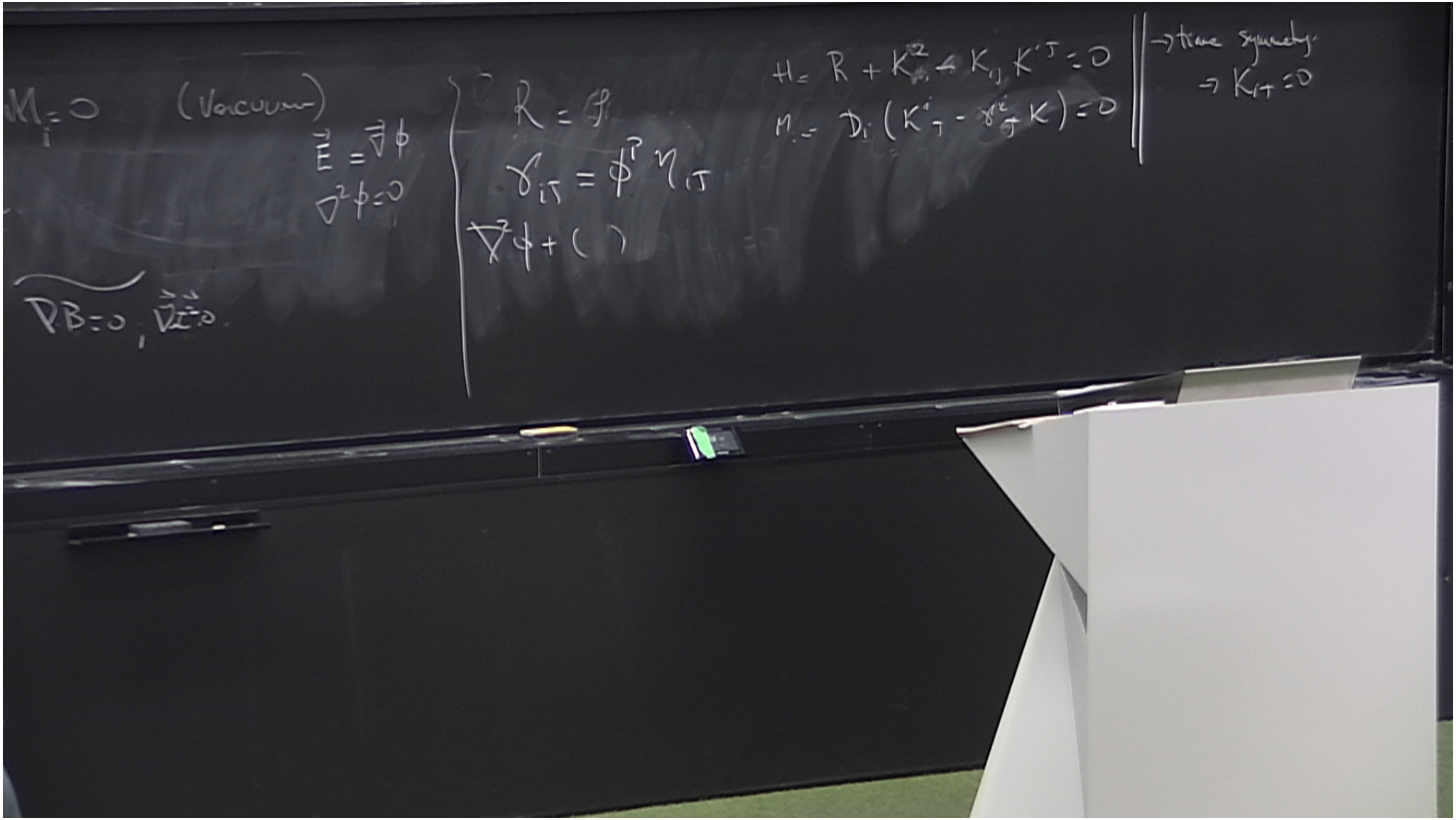

$H = R + K^2 - K_{ij} K^{ij} = 0$
$M_i = D_j (K^j_i - \gamma^{jk}_{ik} K) = 0$
→ time symmetry
$\Rightarrow K_{it} = 0$

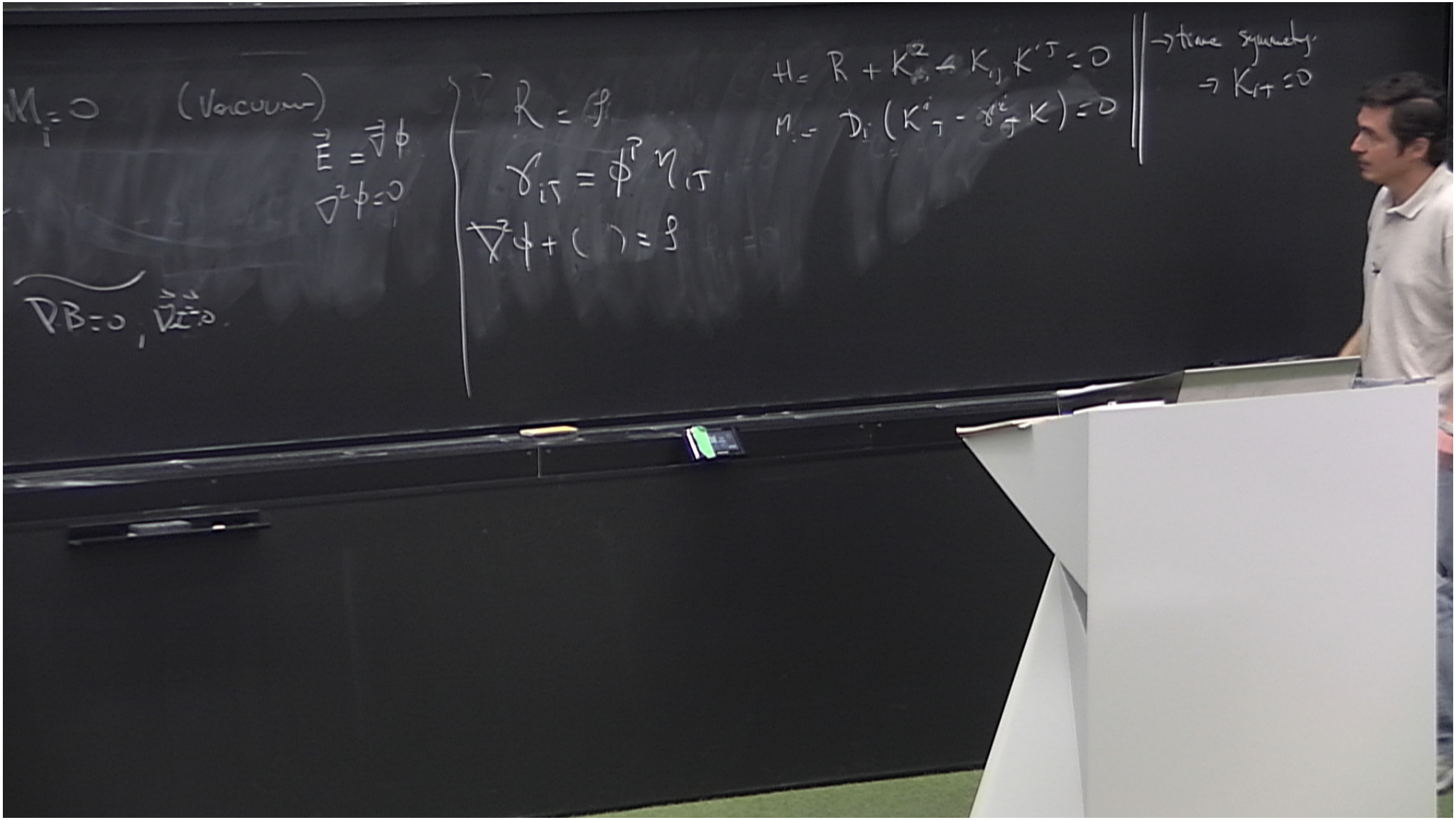

H = R + K² − K_ij K'^ij = 0
M_i = D_j (K^j_i − δ^j_i K) = 0
→ time symmetry.

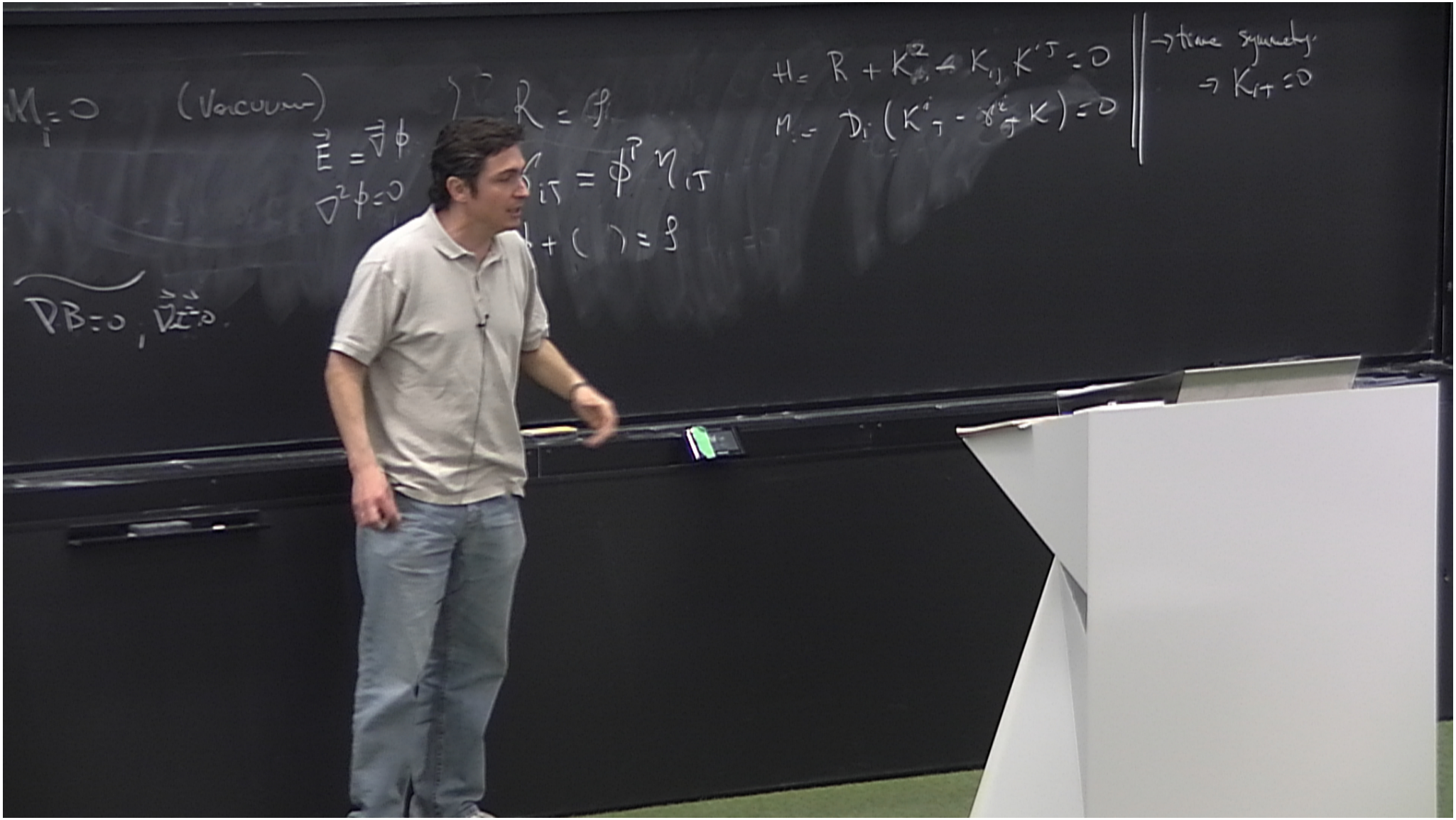

(Vacuum)
$\vec{E} = \vec{\nabla}\phi$
$\nabla^2\phi = 0$
$\vec{\nabla}\cdot\vec{B} = 0$, $\vec{\nabla}\times\vec{E} = 0$
$R = 0$
$H = R + K^2 - K_{ij}K^{ij} = 0$
$M_i = (K^j_{\ j} - \gamma^{jk}_{\ \ j}K) = 0$
→ time symmetry
→ $K_{ij} = 0$

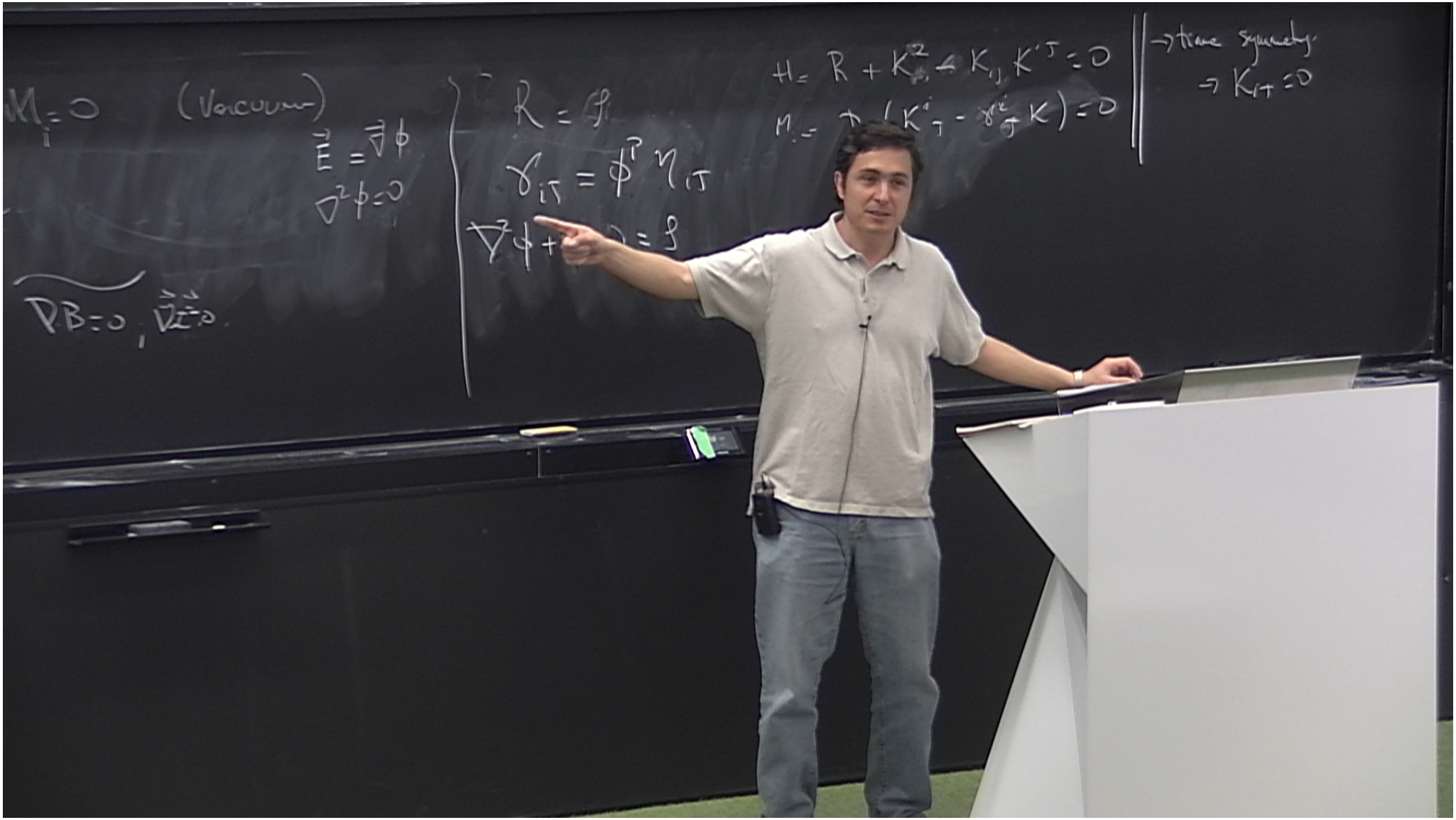

$M_i = 0$ (Vacuum)
$\vec{E} = \vec{\nabla}\phi$
$\nabla^2\phi = 0$
$R = 0$
$\gamma_{ij} = \phi^2 \eta_{ij}$
$H = R + K^2 - K_{ij}K^{ij} = 0$
$M_i = D_j(K^j_i - \gamma^j_i K) = 0$
→time symmetry
→ $K_{ij} = 0$
$\vec{B} = 0$, $\vec{E} = 0$


$M_i = 0$ (Vacuum)
$\vec{E} = \vec{\nabla}\phi$
$\nabla^2\phi = 0$
$R = 0$
$\gamma_{ij} = \phi^p \eta_{ij}$
$H = R + K^2 - K_{ij}K^{ij} = 0$
$M_i = D_j(K^j_i - \gamma^j_i K) = 0$
→ time symmetry
→ $K_{ij} = 0$
$\vec{B} = 0$, $\vec{E} = 0$


$M_i = 0$ (Vacuum)
$\vec{E} = \vec{\nabla}\phi$
$\nabla^2\phi = 0$
$R = 0$
$\gamma_{ij} = \phi^2 \eta_{ij}$
$\nabla^2\phi + (\ ) = 0$
$H = R + K^2 - K_{ij} K^{ij} = 0$
$M_i = D_j (K^j_i - \gamma^j_i K) = 0$
$\vec{B} = 0$, $\vec{E} = 0$
→ time symmetry
→ $K_{ij} = 0$


$M_i = 0$ (Vacuum)
$\vec{E} = \vec{\nabla}\phi$
$\nabla^2\phi = 0$
$R = 0$
$\gamma_{ij} = \phi^p \eta_{ij}$
$\nabla^2\phi + (\ ) = 0$
$H = R + K^2 - K_{ij}K^{ij} = 0$
$M_i = D_j(K^j_i - \gamma^j_i K) = 0$
→ time symmetry
→ $K_{ij} = 0$
$\vec{B} = 0$, $\vec{E} = 0$


(Vacuum)
$\vec{E} = \vec{\nabla}\phi$
$\nabla^2\phi = 0$
$\vec{\nabla}\cdot\vec{B} = 0$, $\vec{\nabla}\cdot\vec{E} = 0$
$H = R + K^2 - K_{ij}K^{ij} = 0$
$M_i = D_j(K^j_i - \gamma^j_i K) = 0$
→ time symmetry
→ $K_{ij} = 0$


(Vacuum)
$\vec{E} = \vec{\nabla}\phi$
$\nabla^2\phi = 0$
$R = 0$
$\gamma_{ij} = \phi^2 \eta_{ij}$
$H = R + K^2 - K_{ij} K^{ij} = 0$
$M_i = \nabla_j (K^j_i - \gamma^j_i K) = 0$
$\vec{B} = 0$, $\vec{E} = 0$
→ time symmetry
→ $K_{ij} = 0$