Title: Explorations in Numerical Relativity - Lecture 10

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Abstract:

Explorations in Numerical Relativity:

Perimeter Institute
Waterloo, Ontario
April 2 - April 20

Outline and motivation for week's topics:
generalized harmonic formulation, gravitational waves, AMR, parallel computation

Luis Lehner, Perimeter/Guelph
Frans Pretorius, Princeton

Motivation for the topics of this week's lectures

- Gravitational Wave Astrophysics – source simulations
  - compact object mergers
- Studies of dynamical, strong-field gravity
  - Examples:
  - higher dimensional gravity : instability of 5D black strings
  - critical collapse

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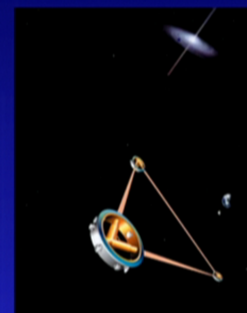
Gravitational Wave Astrophysics

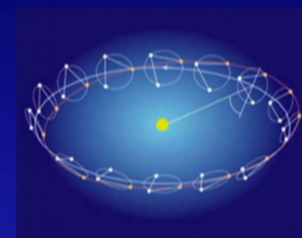
LIGO/VIRGO/GEO/TAMA
ground based laser interferometers

LIGO Livingston

LIGO Hanford

LISA
space-based laser interferometer (hopefully with get funded for a 20?? Launch)



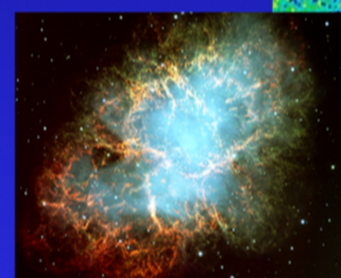


ALLEGRO/NAUTILUS/AURIGA/…
resonant bar detectors

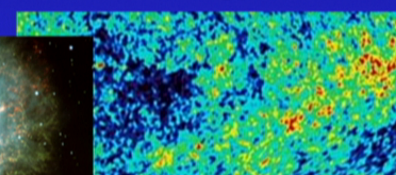
ALLEGRO

AURIGA

Pulsar timing network, CMB anisotropy

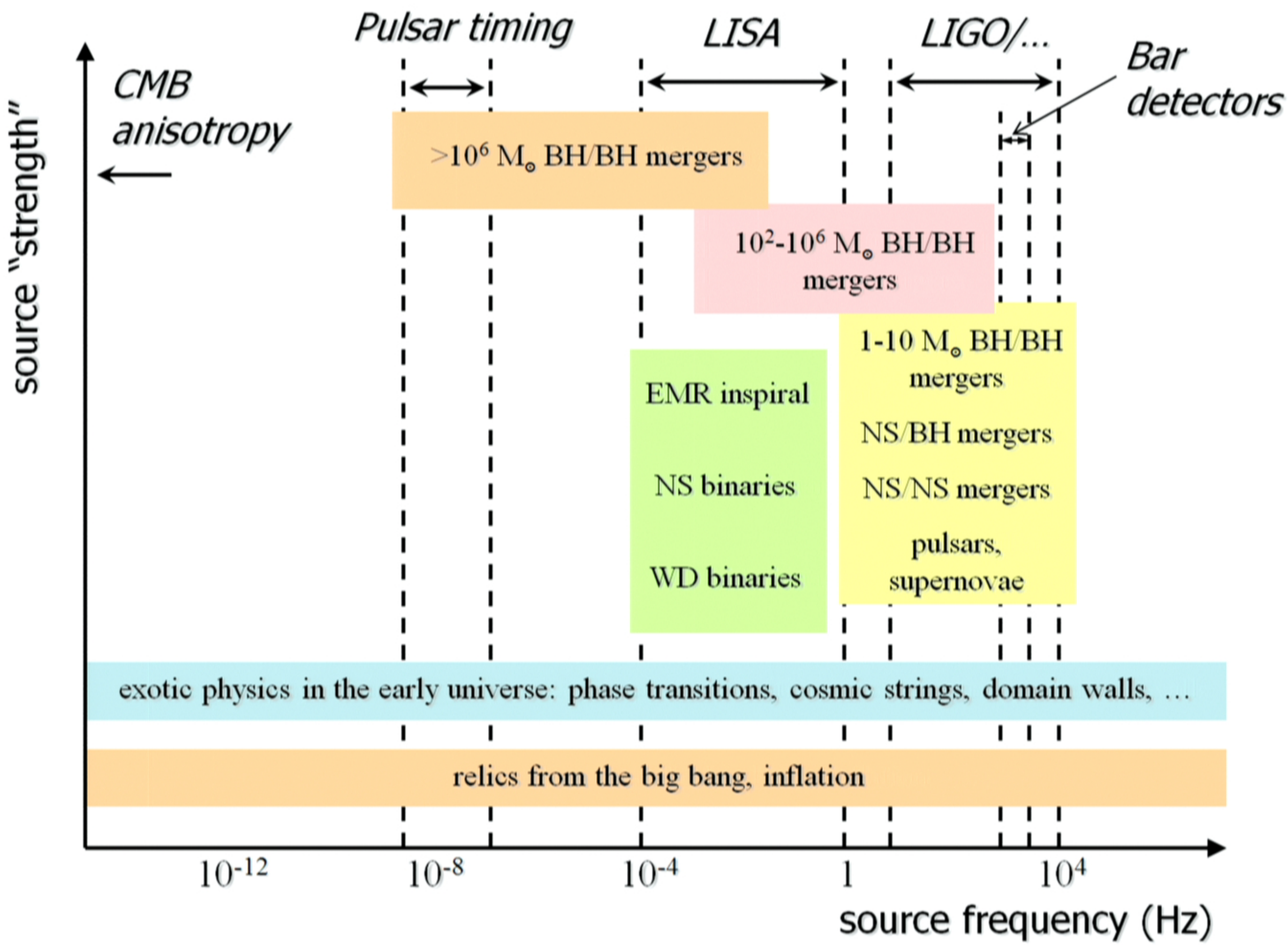


The Crab nebula … a supernovae remnant harboring a pulsar



Segment of the CMB from WMAP

Overview of expected gravitational wave sources



Binary Compact Object Mergers

- Binary black hole (BH), neutron star (NS), and BH-NS mergers
  - expected to be the strongest source of gravitational waves in the universe
  - the binary BH is the "cleanest" system, described exactly by vacuum GR
    - in any astrophysical system, there will be a certain amount of matter (possibly circumbinary disk, ambient gas, CMB photons, …), though for the most part is expected to be very low mass relative to the black holes and hence dynamically insignificant
  - With NS's, microphysics must be approximated, and the sky's the limit as to how complicated this can be
    - neutron star structure (crust, multi-component superfluid core, at nuclear densities with uncertain equations of state, magnetic fields, etc.)
    - radiation and neutrino physics, nuclear processes, magnetic fields, etc. play a role

Binary Compact Object Mergers

- Binary black hole (BH), neutron star (NS), and BH-NS mergers
  - final stages of merger within GR, plus "messy" astrophysics requires numerical solution
    - No symmetries : need a general 3+1 evolution code, which brings several new problems to the fore
      - ADM is only weakly hyperbolic in 3+1 : ***need reformulations***
      - *All* finite-difference/spectral numerical Cauchy solution methods can only satisfy the Einstein constraint equations to within truncation error, and the vast majority (100's) of mathematically well-posed reformulations admit exponentially growing "constraint violating" solutions
        - » given truncation errors, no way the seeds of these modes can be eliminated in a numerical code
      - "nice" gauges, boundary conditions, methods of dealing with BH singul work well in spherical (or axi) symmetry often don't extend to the gene

Binary Compact Object Mergers

- Binary black hole (BH), neutron star (NS), and BH-NS mergers
  - At minimum (vacuum), two orders of magnitude of relevant spatio-temporal length scales that need to be resolved:
    - spatial scales: smallest BH radius ~2M; intermediate orbital radius ~ 20M; largest "wave zone" ~ 200M
    - temporal scales : spatial scales/characteristic speed (1!) ~ spatial scales
  - Including matter (NS/NS, BH/NS), and depending on the kinds of questions one needs to answer (effects of microphysics, EM/neutrino signatures of events, delayed collapse of "hypermassive" neutron stars following NS/NS mergers, formation of accretion disks/jets, etc), can increase the effective dimensionality of the problem, and add many orders of magnitude to the scales that need to be resolved
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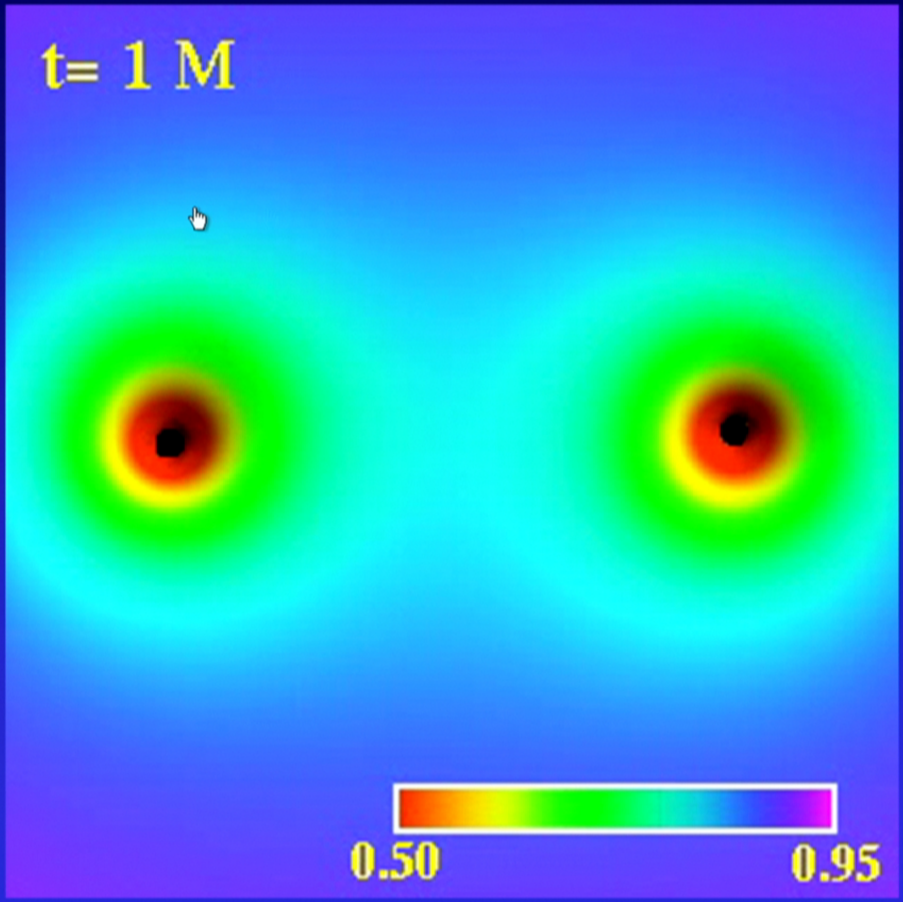
Sample binary BH merger

A. Buonanno, G.B. Cook and F.P.; Phys.Rev.D75:124018,2007

- Code overview:
  - Harmonic evolution with excision and constraint damping
  - 2th order finite difference discretization, equations discretized in second order form
  - Berger and Oliger style adaptive mesh refinement (AMR), parallel evolution
  - Kreiss-Oliger dissipation used for stability
  - Spatially compactified coordinates, i.e. the slices end at spatial infinity, with *Dirichlet* (Minkowski) conditions imposed on the metric there
    - this is purely to have the physically correct outer boundary conditions imposed; the gravitational waves never reach the boundary, and the resolution becomes very coarse approaching it
  - Using Cook-Pfeiffer *[PRD 70 (2004)]* quasi-circular initial data

Lapse Function



Initial black holes are close to non-spinning Schwarzschild black holes; final black hole is a Kerr black hole with spin parameter ~0.7, and ~4% of the total initial rest-mass of the system is emitted in gravitational waves

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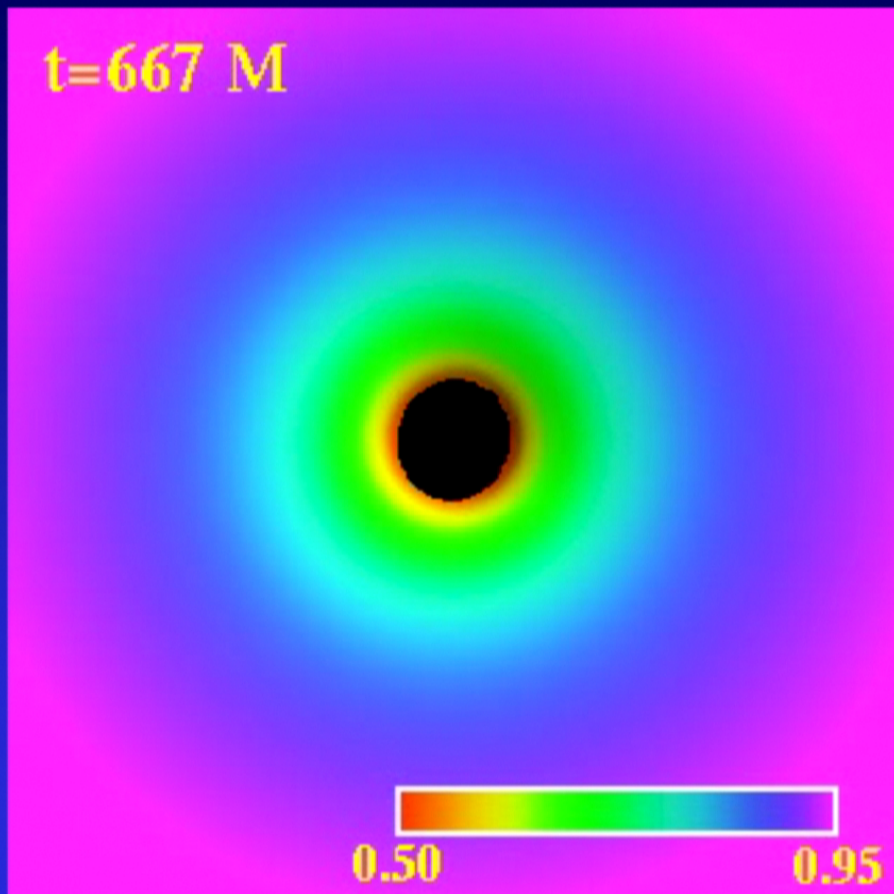

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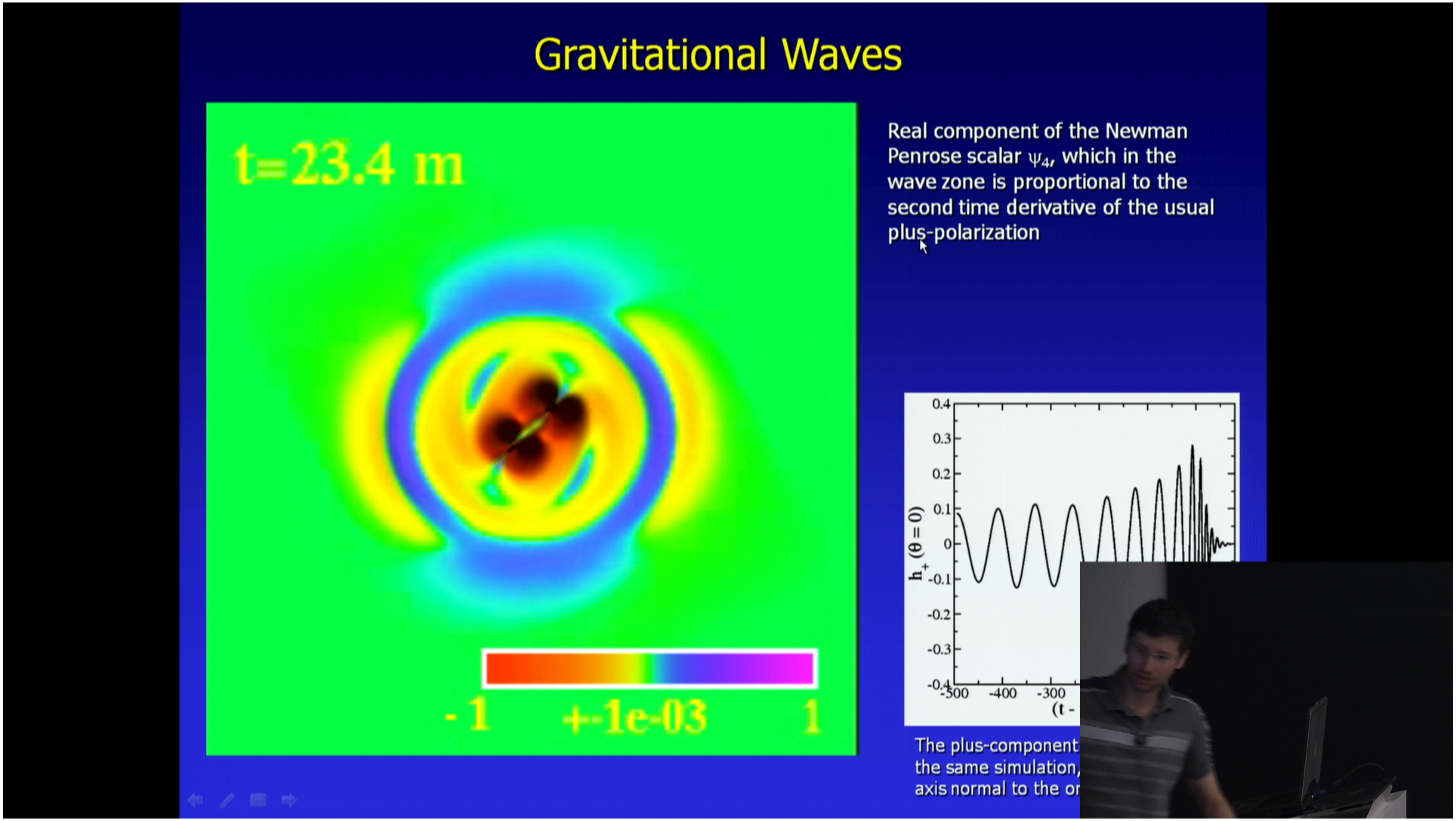
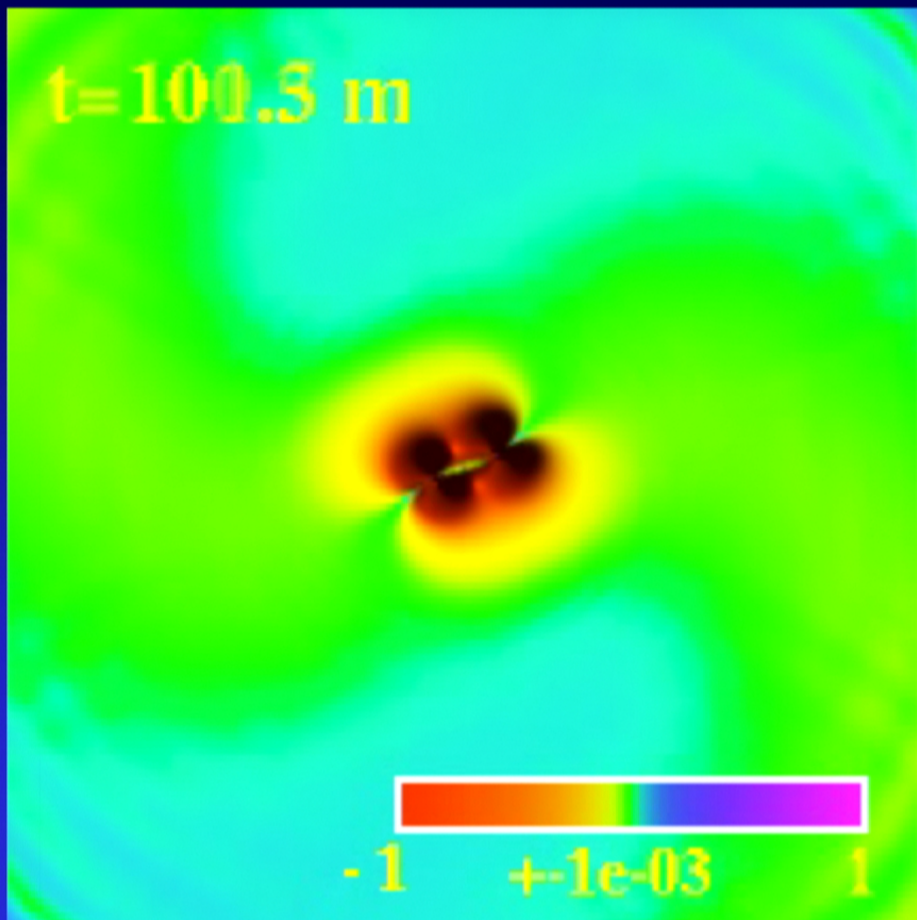

Gravitational Waves
t=23.4 m
-1
+-1e-03
1
Real component of the Newman Penrose scalar ψ4, which in the wave zone is proportional to the second time derivative of the usual plus-polarization
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
h+ (θ = 0)
-500
-400
-300
(t -
The plus-component
the same simulation,
axis normal to the or

Gravitational Waves



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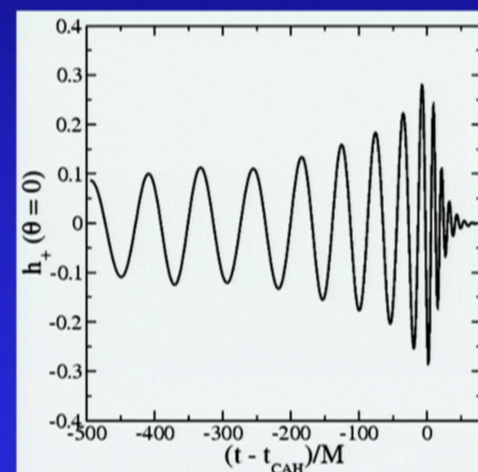
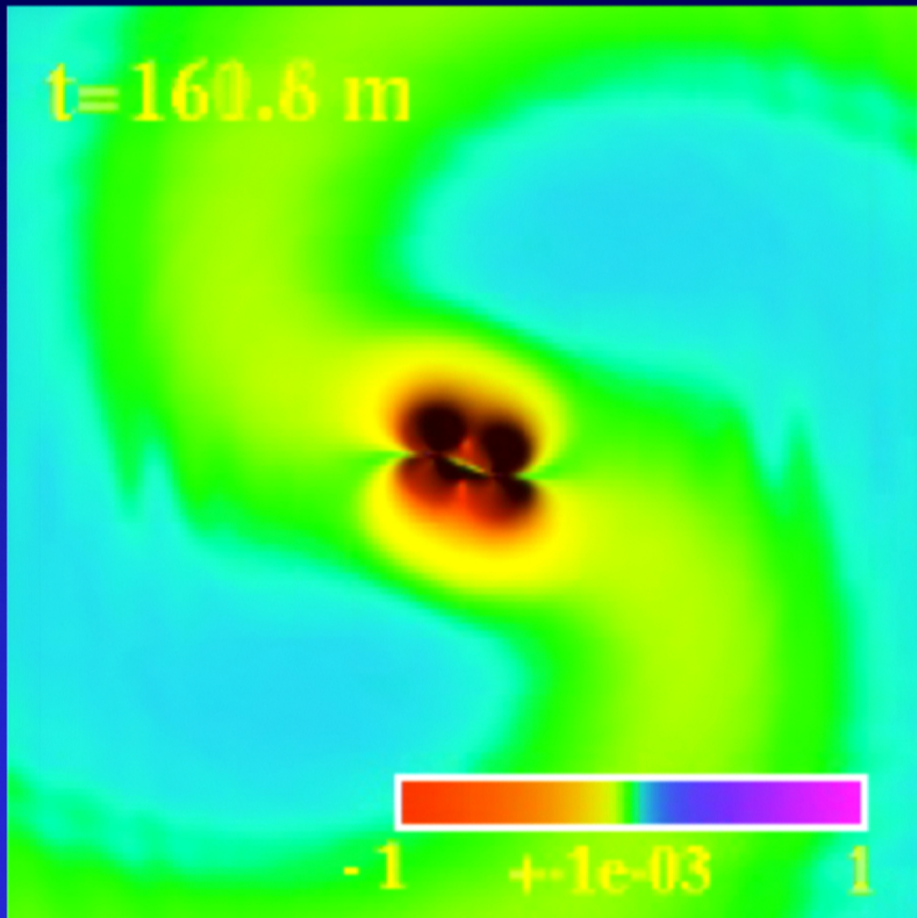


The plus-component of the wave from the same simulation, measured on the axis normal to the orbital plane

Gravitational Waves



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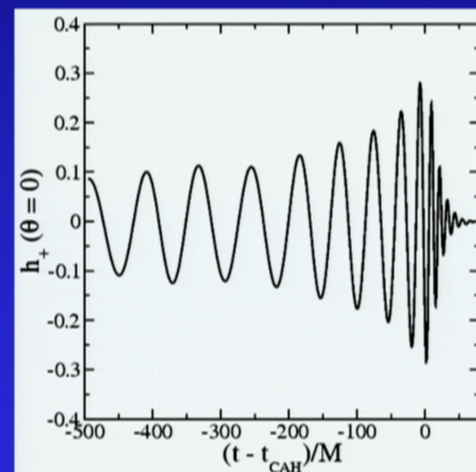

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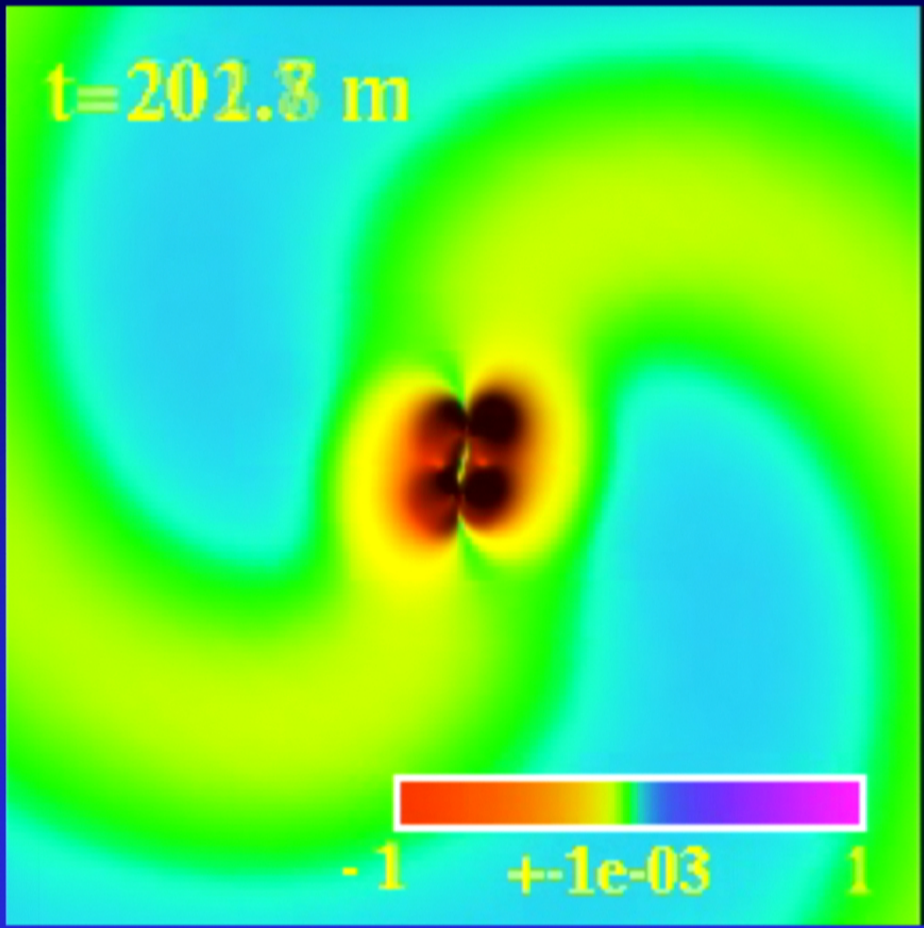

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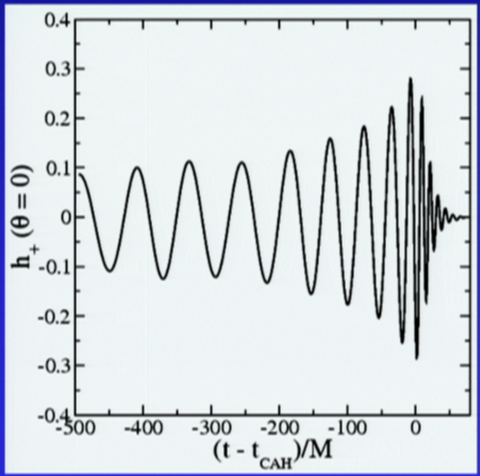

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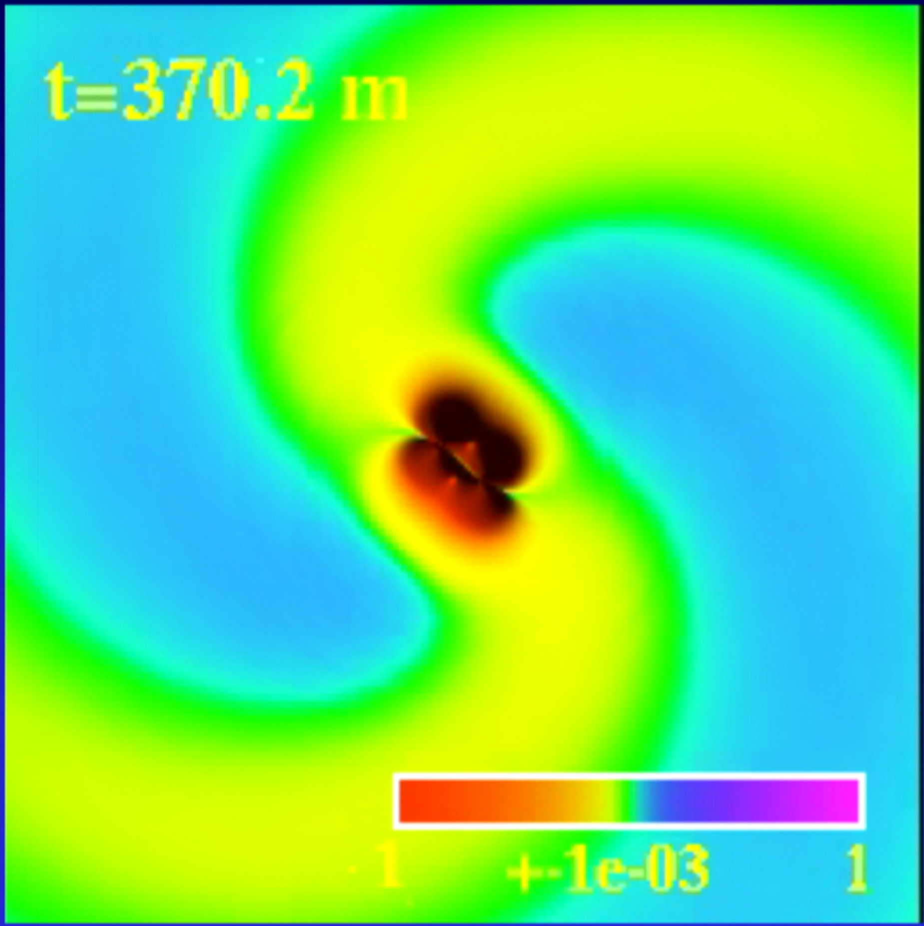

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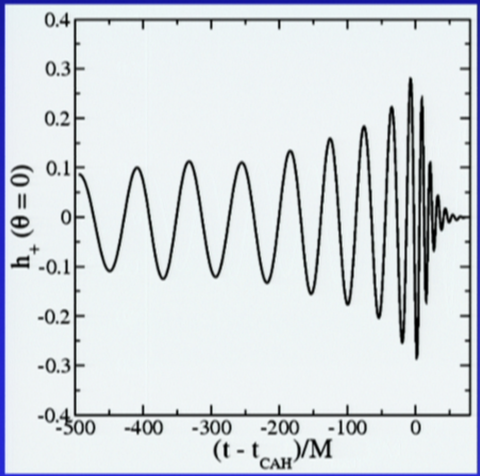

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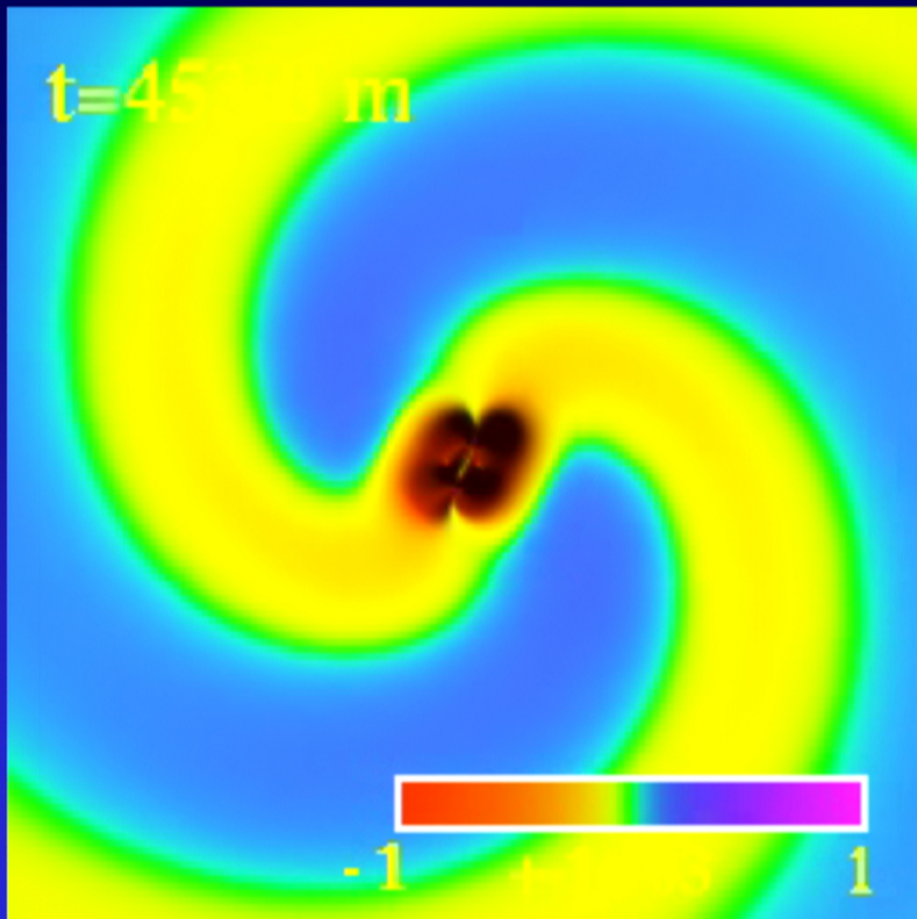

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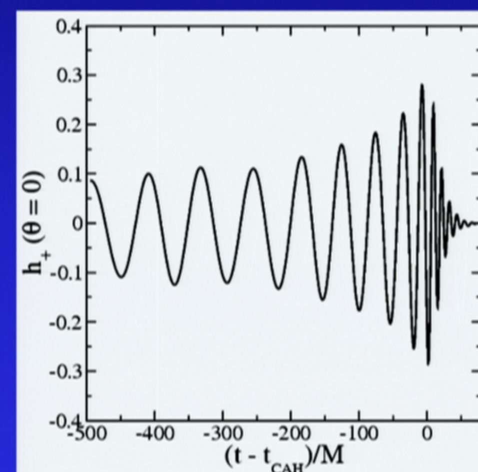

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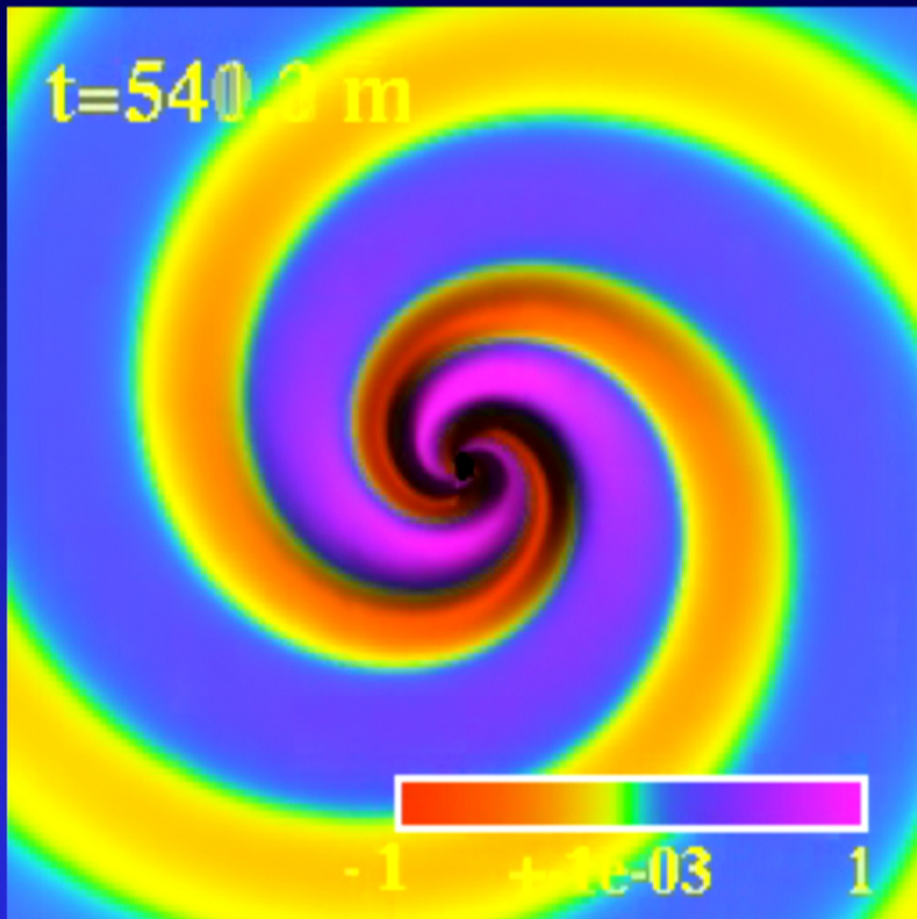

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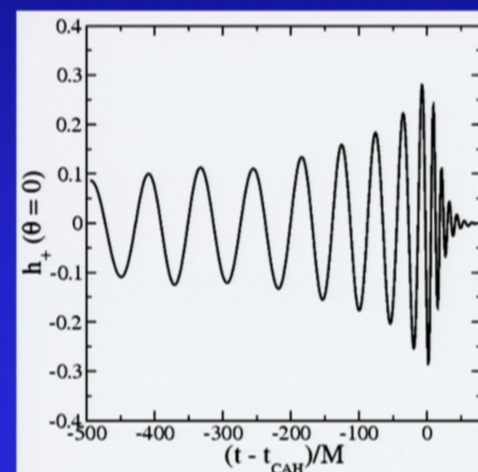

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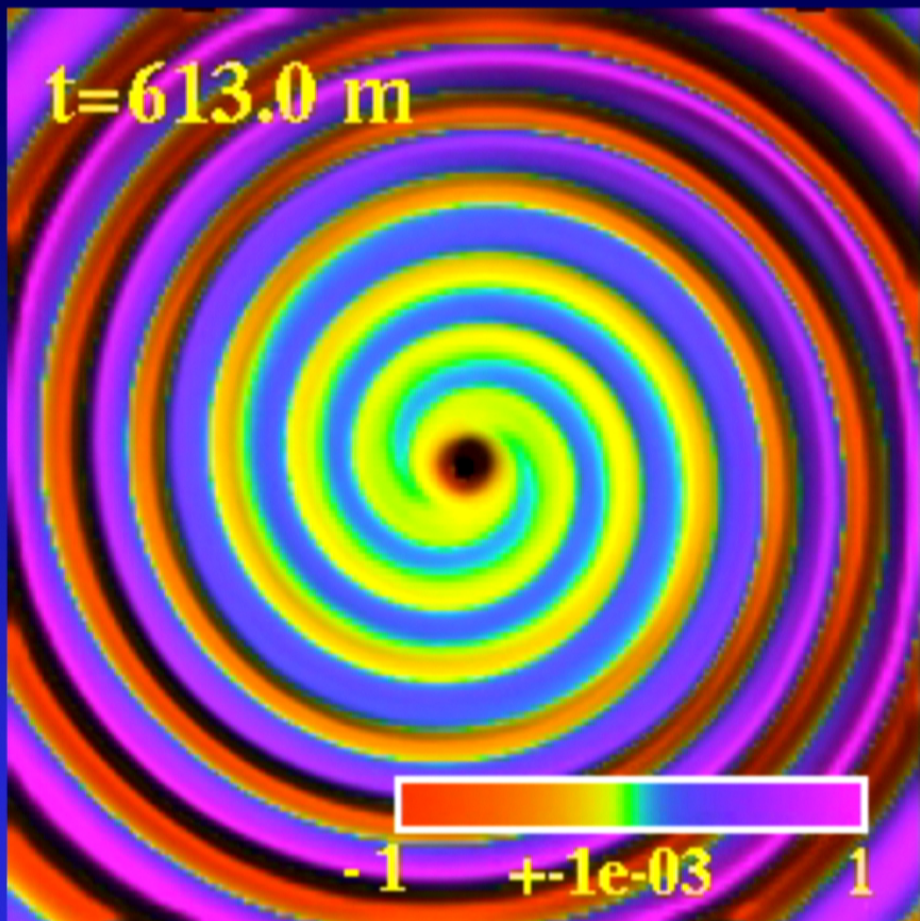

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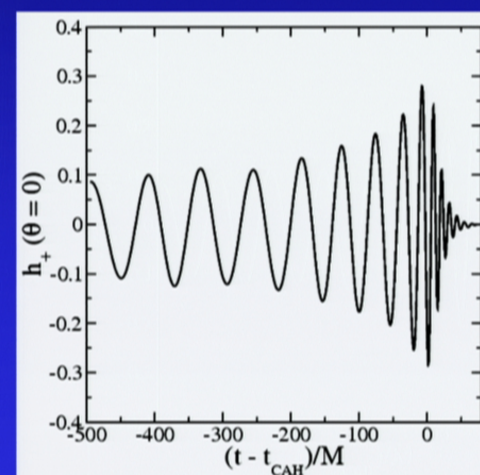

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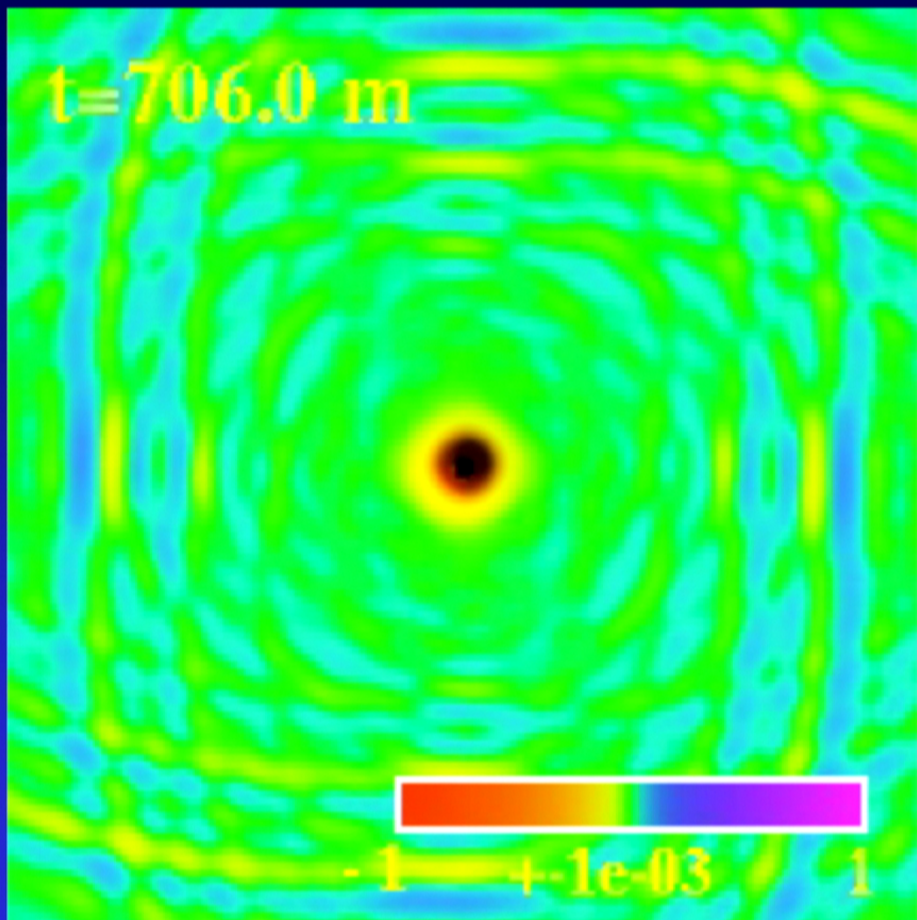

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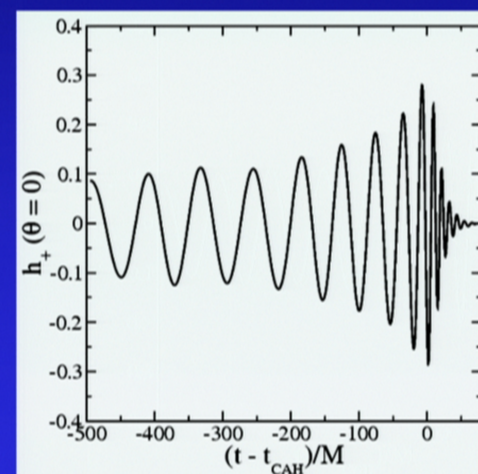

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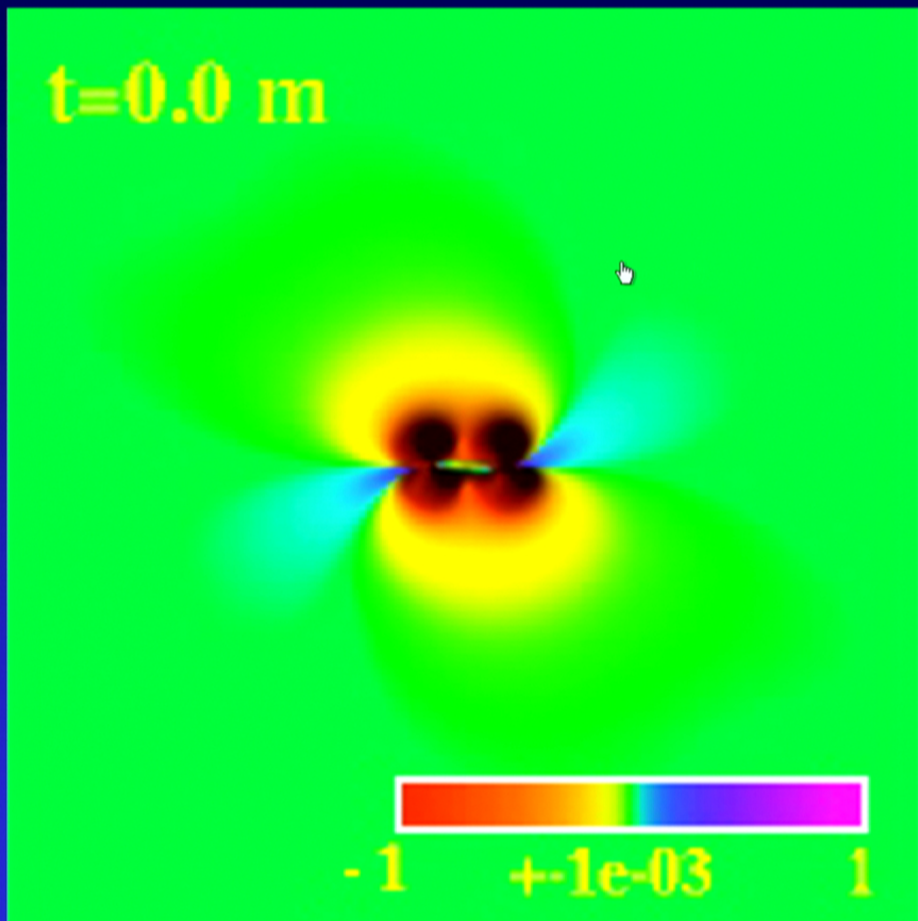

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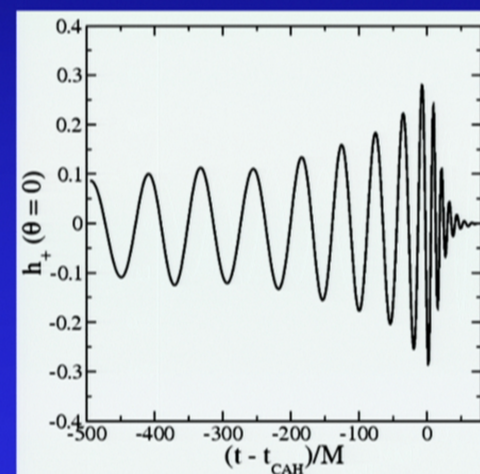

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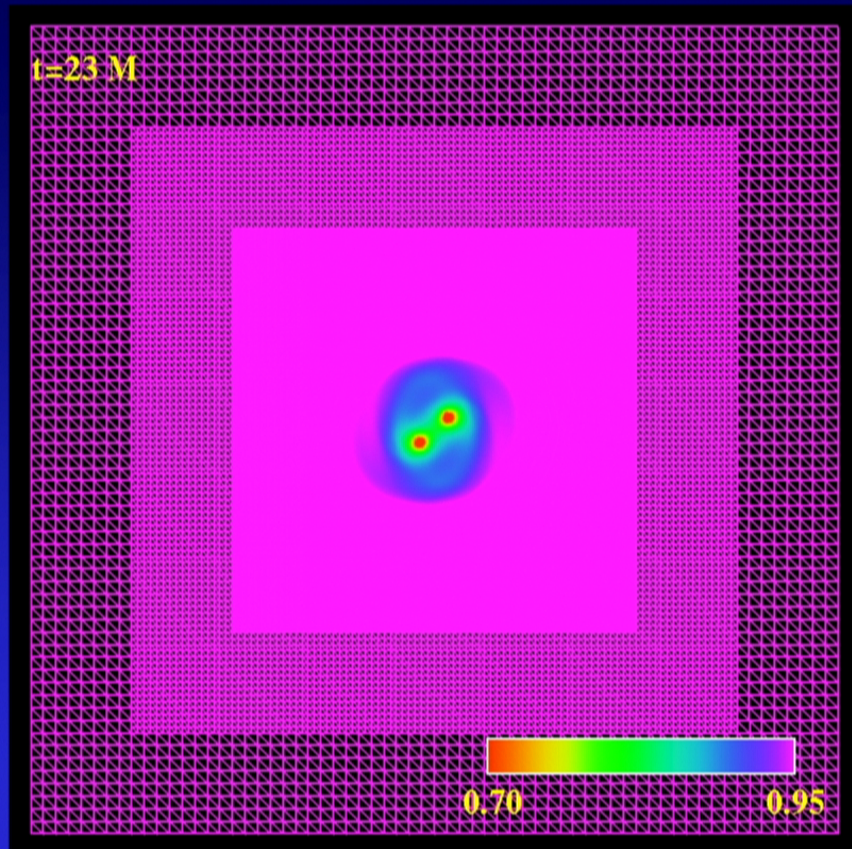
Sample mesh structure



Lapse α, $z=0$ slice

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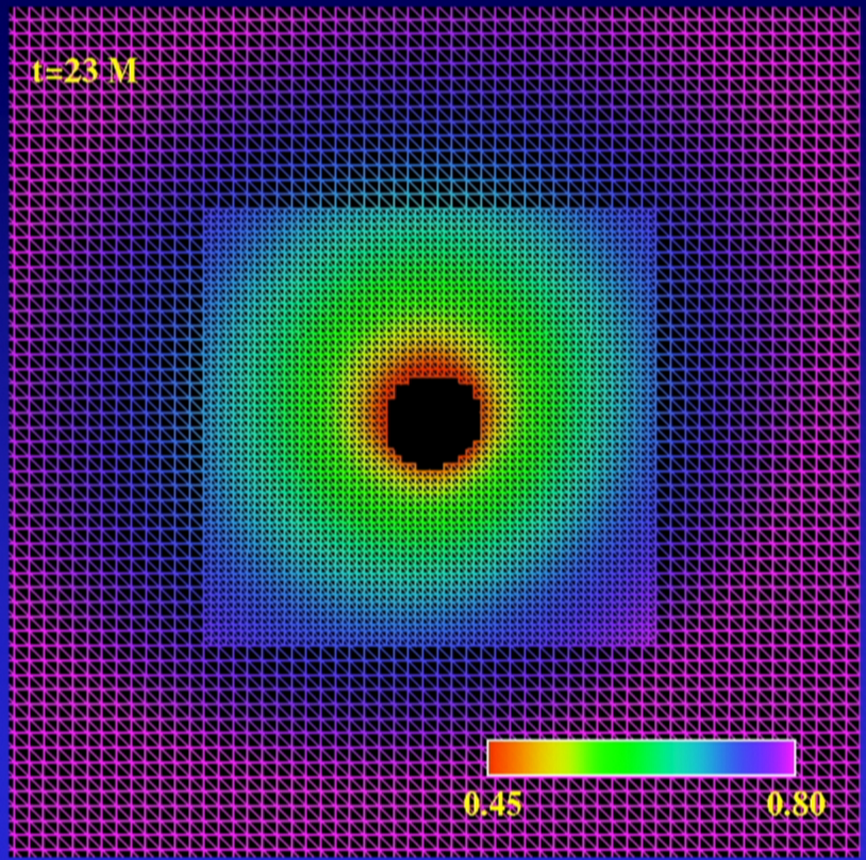

Lapse α, $z=0$ slice, NOTE change of color scale

Motivation: why study higher dimensional gravity?

- If string theory is providing the correct path to a consistent theory of nature valid at Planck scales, the universe is fundamentally higher dimensional
- Even if string theory is not correct, there has recently been a lot of work using the holographic dual correspondences of string theory (AdS/CFT in particular) to describe many aspects of conventional non-gravitational 4D physical processes in terms of higher dimensional gravity
  - superconductors, superfluidity, quark-gluon plasmas, etc.
  - interestingly, the gravitational dual to all the processes studied to date involves *black holes*
- Much interesting geometry in higher dimensional Ricci-flat Lorentzian manifolds, in particular the zoo of "black objects" – black spheres, rings, strings, saturns, drops, …

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Higher dimensional black holes

- As with 4D black holes, much has been learnt via analytical methods (exact solutions, perturbative studies, global methods) about higher dimensional BH's
  - have many properties in common with their 4D counterparts, e.g.
    - can be defined using global (event horizons) or local (isolated horizons) properties of the spacetime
    - contain geometric singularities
    - quasi-stationary processes are governed by the usual laws of black hole mechanics
    - a couple of studies have shown the usual link between gravitational collapse and black hole formation, and critical phenomena at threshold
    - Hawking radiate at the semi-classical level
  - however, a few properties are drastically different, including
    - *no strict uniqueness* of stationary solutions
    - many black objects are *unstable* to perturbations
- Also as with 4D black holes, to study dynamics in the non-linear regime often requires numerical methods, and same basic approaches carry over to higher dimensions

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Black Strings

- Black strings are a particularly simple class of higher dimensional black hole solutions
  - in *N* spacetime dimensions, the metric is *4D Schwarzschild X (N-4)D Euclidean flatspace*; e.g. for N=5, in Schwarzschild coordinates

$$ds^2 = -(1 - 2m/r)dt^2 + \frac{1}{(1 - 2m/r)}dr^2 + r^2 d\Omega^2 + dw^2$$

  - here *m* is interpreted as mass per unit length; a segment of length $\Delta\omega = L$ of the spacetime has asymptotic mass $M = mL$

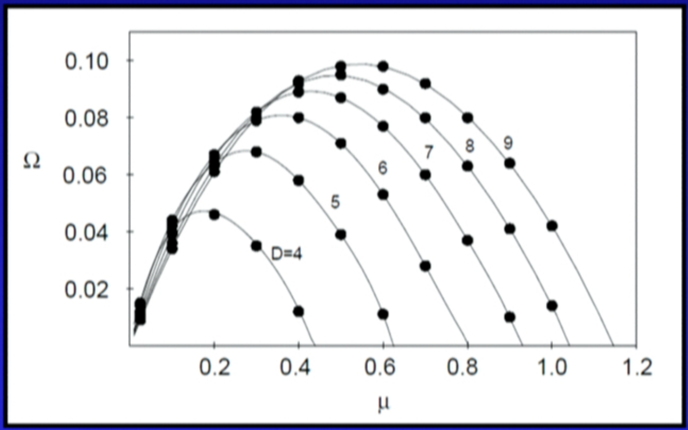
Gregory-Laflamme instability

- Gregory and Laflamme *[PRL 70 (1993)]* first showed that black strings are linearly unstable to long-wavelength perturbations

$$g = g_0 + \delta g \cdot e^{\Omega t/m + i\mu w/m}$$

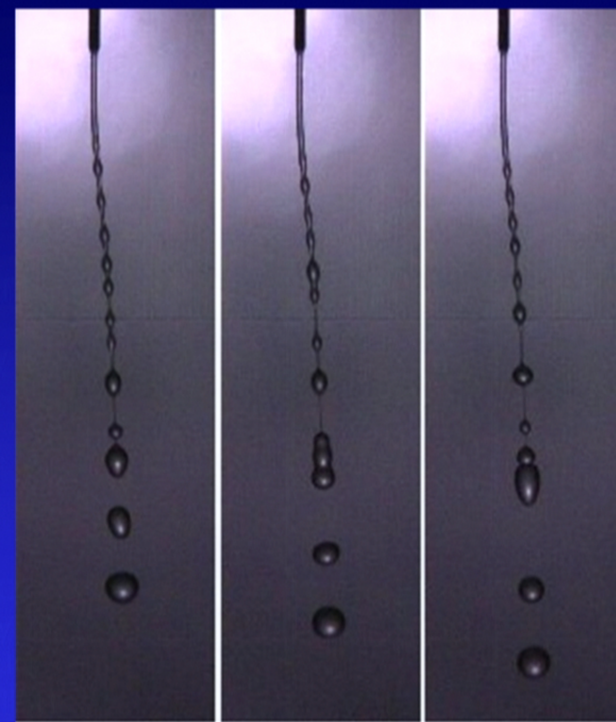
  - Image from R. Gregory and R. Laflamme, Nucl.Phys.B428 (1994)
  - the D=4 curve corresponds to the 5D black string, and the critical wavelength above which modes are unstable is

$$\lambda_c \approx 14.3m$$



End-state of the instability?

- Much debate over end-state of the instability
  - from an entropic argument the string should bifurcate into a sequence of spherical black holes, however this cannot happen without the appearance of a naked singularity $\rightarrow$ a *generic* example of cosmic censorship violation
  - Horowitz and Maeda *[PRL 87, 131301 (2001)]* *proved* that black string horizons cannot shrink to zero cross-sectional radius in *finite* affine time of the generators of the horizon
  - Connections between horizon dynamics and fluid mechanics *suggest* instability is qualitatively akin to the Raleigh-Plateau instability of a thin stream of fluid, which *does* pinch off *[Cardoso and Dias, PRL 96 (2006)]*.



Numerical evolution of 5D black strings

work with L. Lehner, PRL 105 (2010)

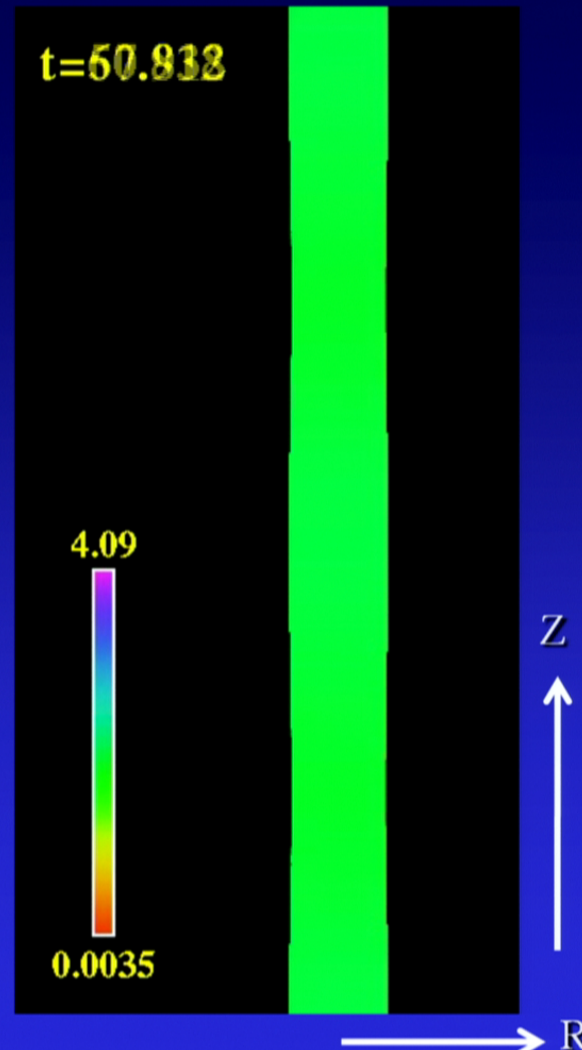
- Code overview:
  - Harmonic evolution with excision and constraint damping; in particular coordinates are harmonic with respect to 5D Cartesian-like coordinates
  - S0(3) symmetry imposed → reduces simulation to 2+1D evolution
  - 4th order finite difference discretization, Runge-Kutta time integration ; 6th order Kreiss Oliger dissipation *[Calabrese et al., CQG 21 (2004)]*
  - Berger and Oliger style adaptive mesh refinement (AMR), parallel evolution
  - Using initial data as constructed in *Choptuik et al. [PRD 68, 044001 (2003)]*, describing a black string perturbed by a small gravitational wave
  - At outer boundary impose *Dirichlet* conditions, with the metric fixed to that of the initial data
    - not physically correct, hence we placed the outer boundary sufficiently far away from the horizon to be out of causal contact with it over the length of the simulation (t~230m)

Embedding Diagram of Apparent Horizon Unstable 5D Black String

- map the geometric 1D shape of each *t=x=y=constant* slice of the apparent horizon to a flat *(R,Z)* Euclidean space; i.e. in parametric form

$$(R,Z) = \left(R(\xi), Z(\xi)\right)$$

- *R(ξ)* is the areal radius of that point on the horizon, and *Z(ξ)* is defined so that the proper length of the curve in the flat space is identical to that of the corresponding curve in the physical geometry
- the movie shows this curve spun around *R=0* to form a surface for visual aid
- color is mapped to *R*
- note that time is "slowing down"



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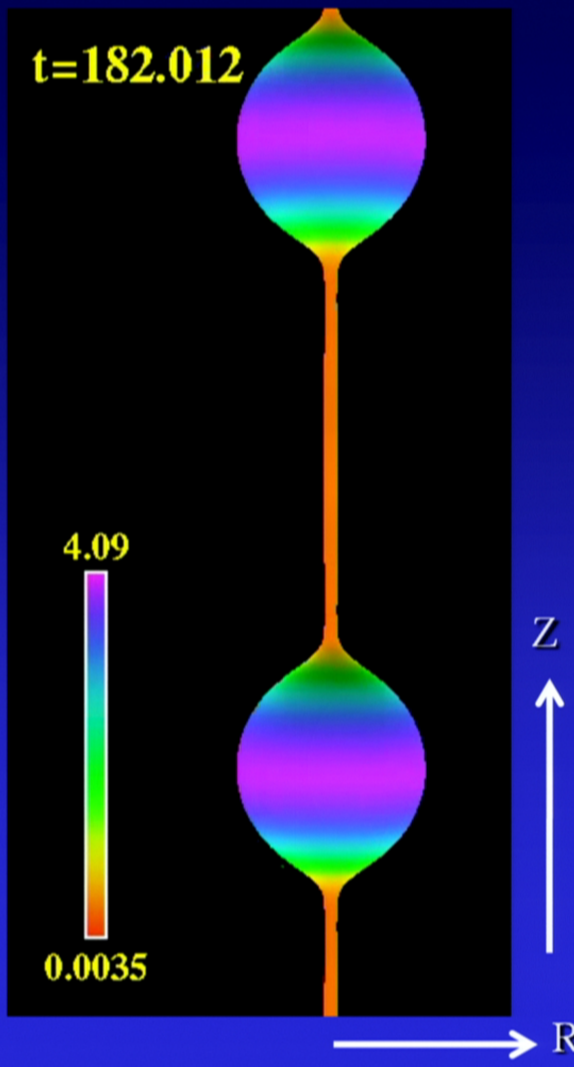

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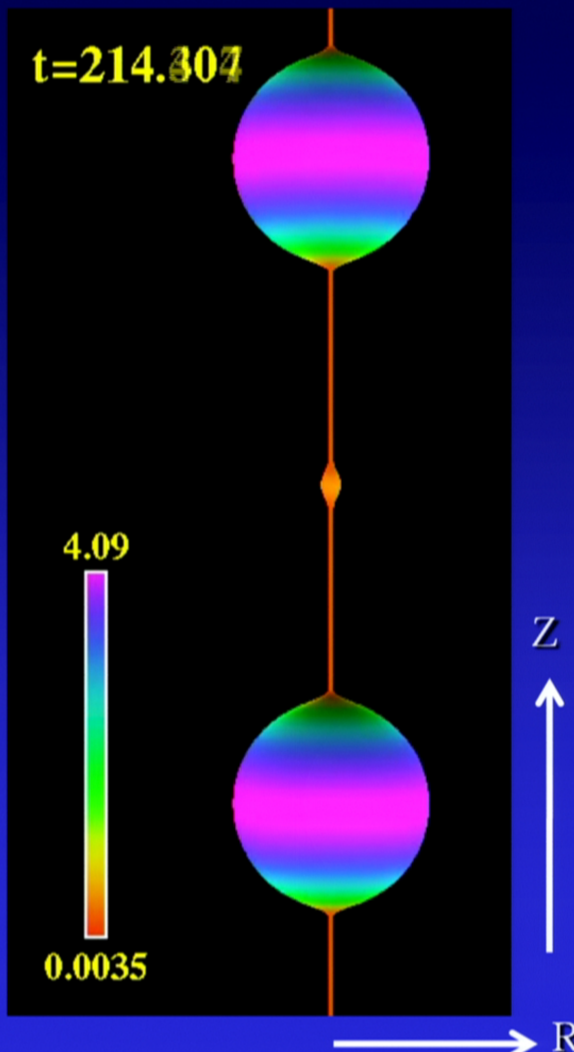
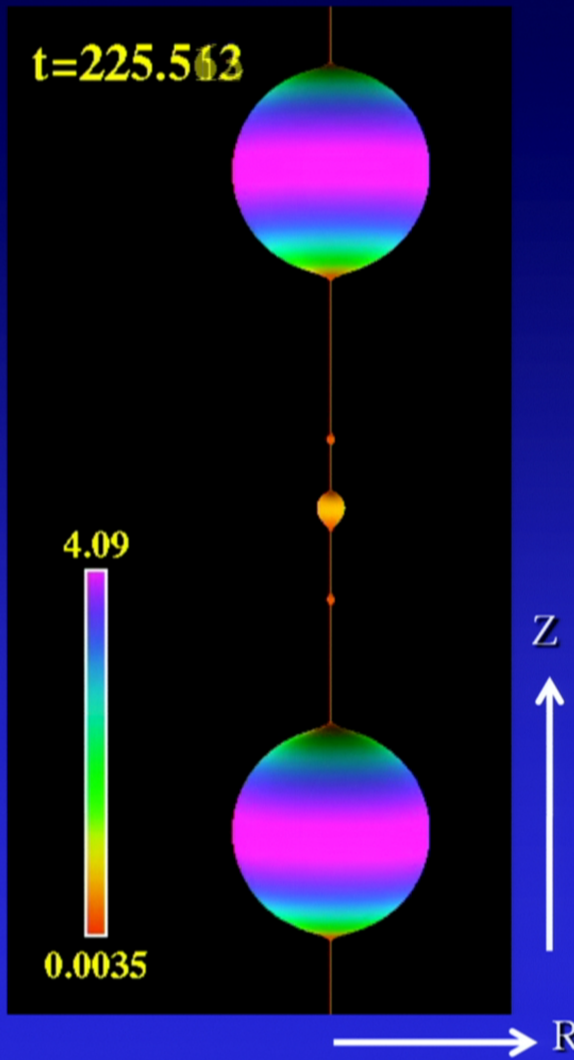


Embedding Diagram of Apparent Horizon Unstable 5D Black String

- map the geometric 1D shape of each $t=x=y=constant$ slice of the apparent horizon to a flat (R,Z) Euclidean space; i.e. in parametric form

$$(R,Z) = \left(R(\xi), Z(\xi)\right)$$

- $R(\xi)$ is the areal radius of that point on the horizon, and $Z(\xi)$ is defined so that the proper length of the curve in the flat space is identical to that of the corresponding curve in the physical geometry
- the movie shows this curve spun around $R=0$ to form a surface for visual aid
- color is mapped to R
- note that time is "slowing down"



Embedding Diagram of Apparent Horizon Unstable 5D Black String

- map the geometric 1D shape of each *t=x=y=constant* slice of the apparent horizon to a flat *(R,Z)* Euclidean space; i.e. in parametric form

$$(R,Z) = \big(R(\xi), Z(\xi)\big)$$

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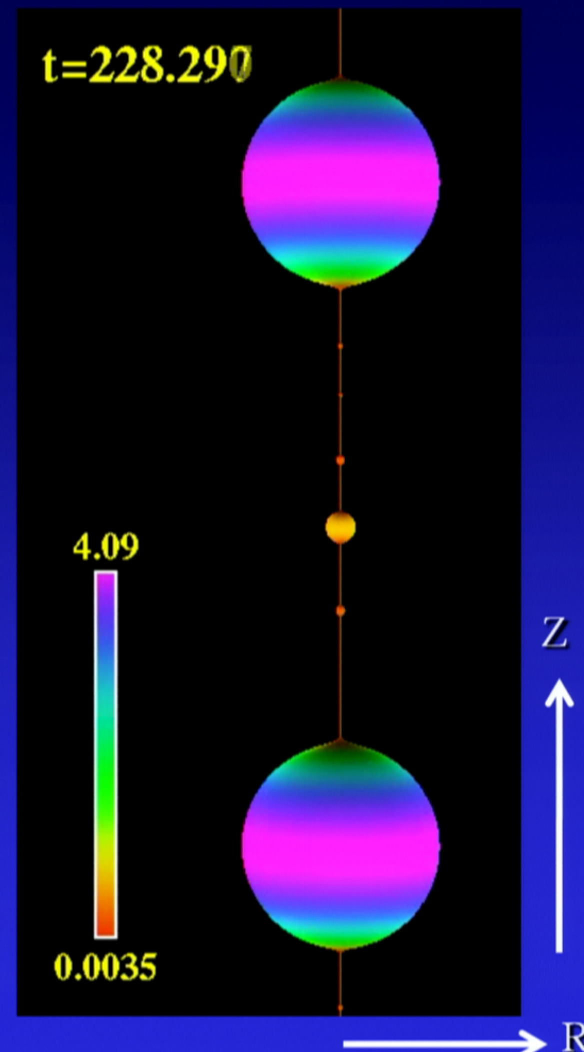

Embedding Diagram of Apparent Horizon Unstable 5D Black String, close-up and in "real time"

- map the geometric 1D shape of each $t=x=y=constant$ slice of the apparent horizon to a flat (R,Z) Euclidean space; i.e. in parametric form

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- the movie shows this curve spun around $R=0$ to form a surface for visual aid
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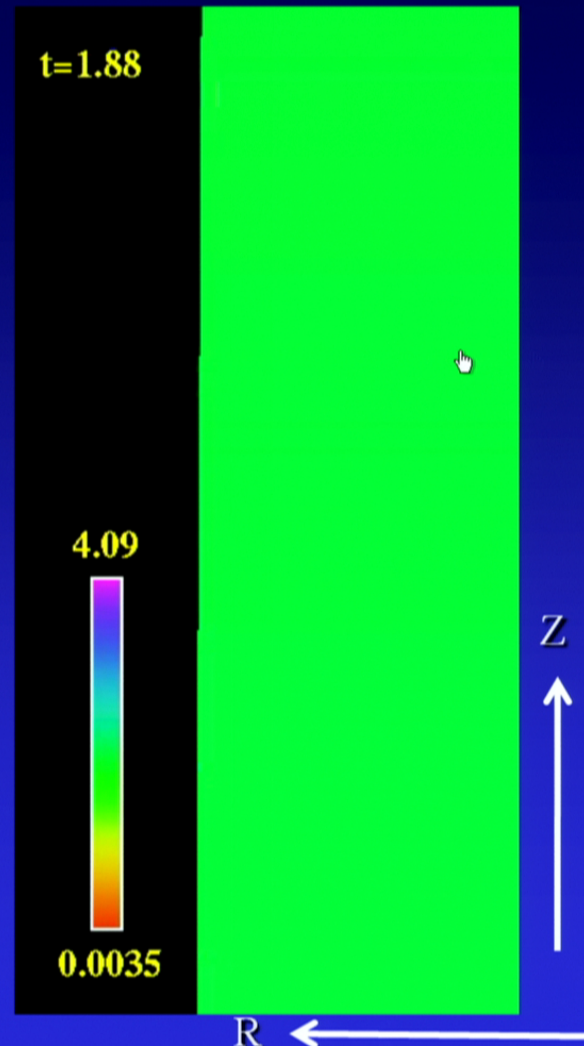

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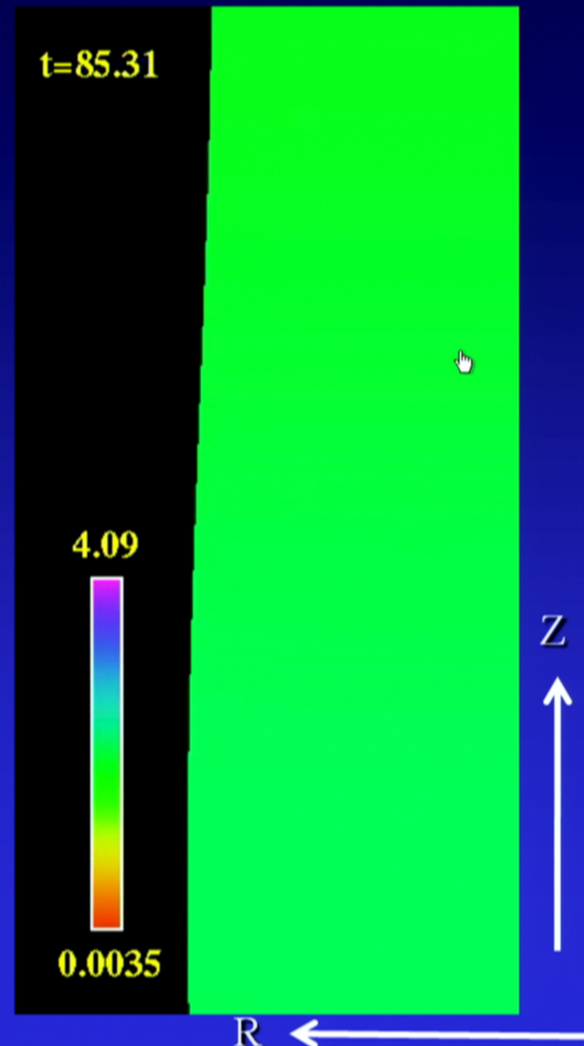

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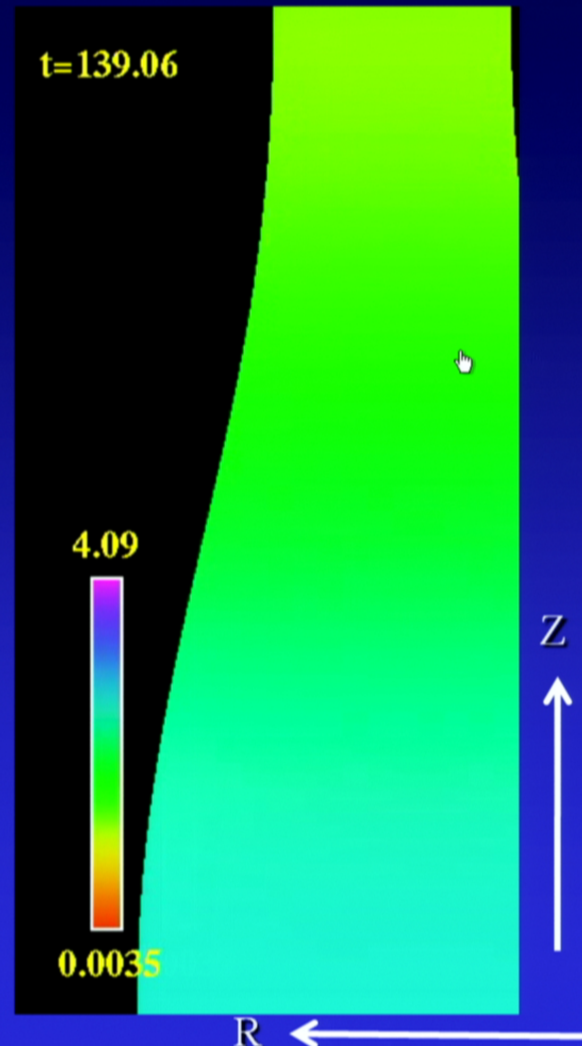

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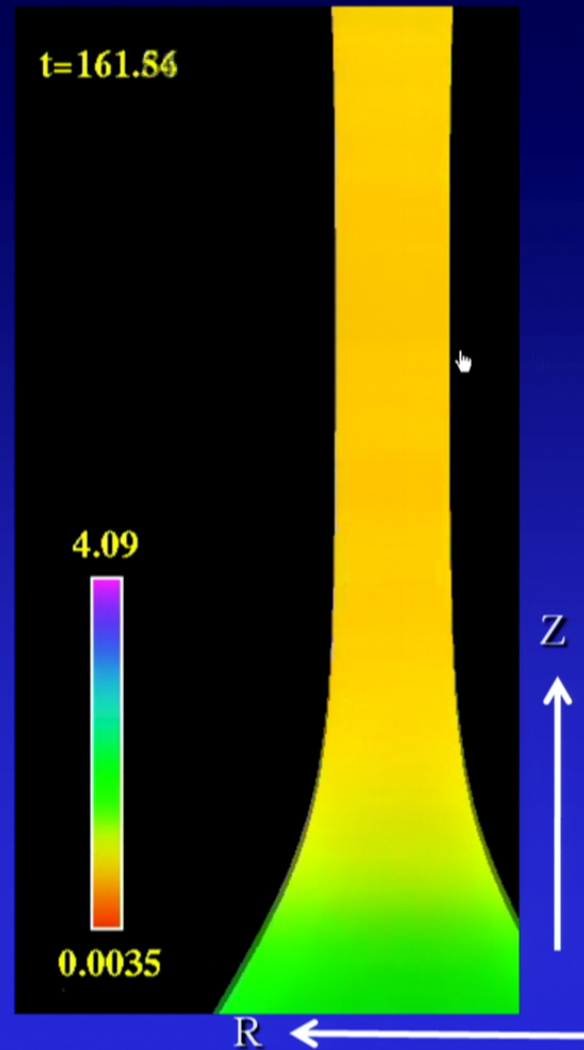

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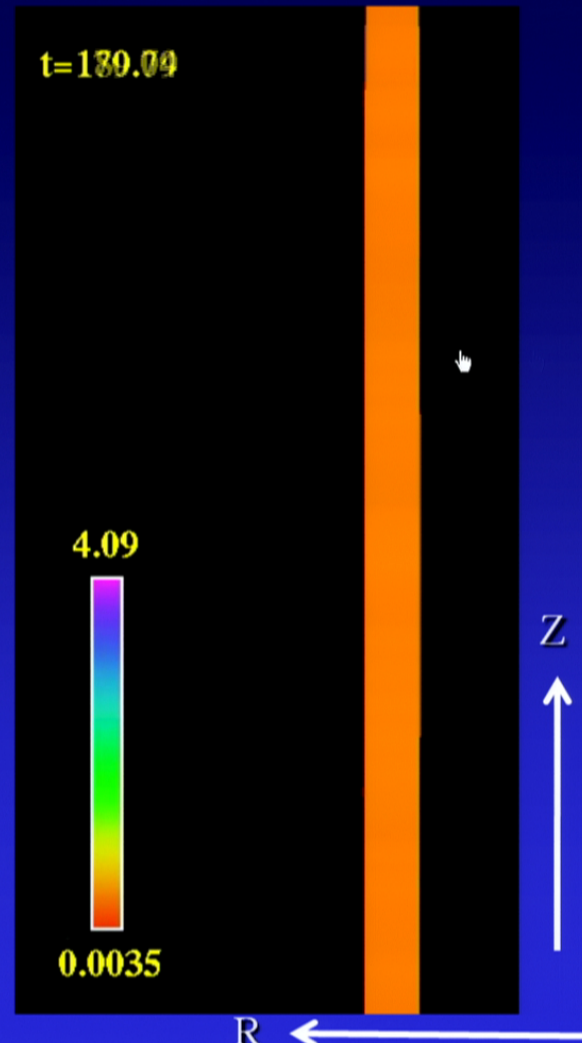

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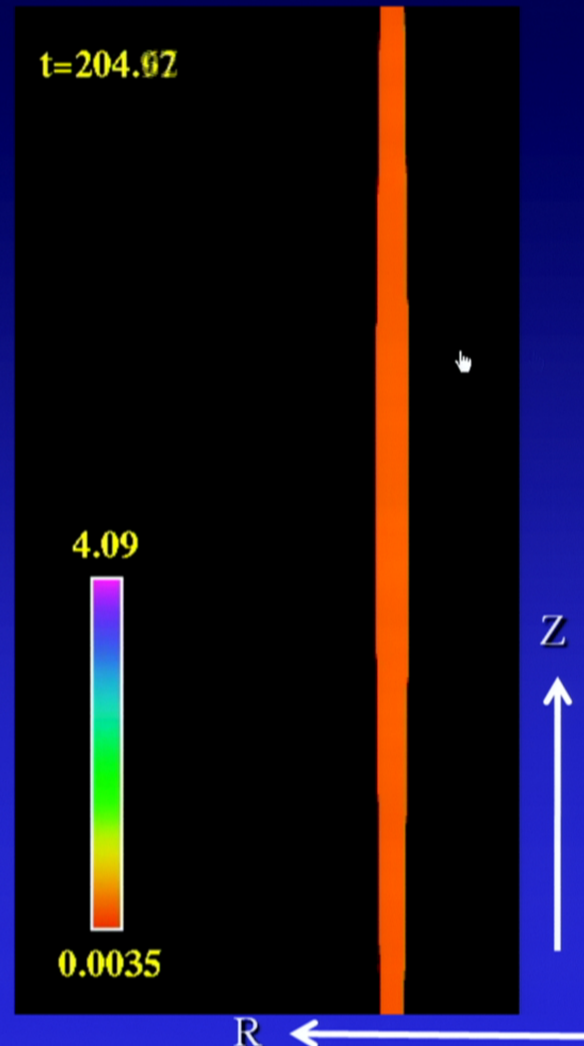

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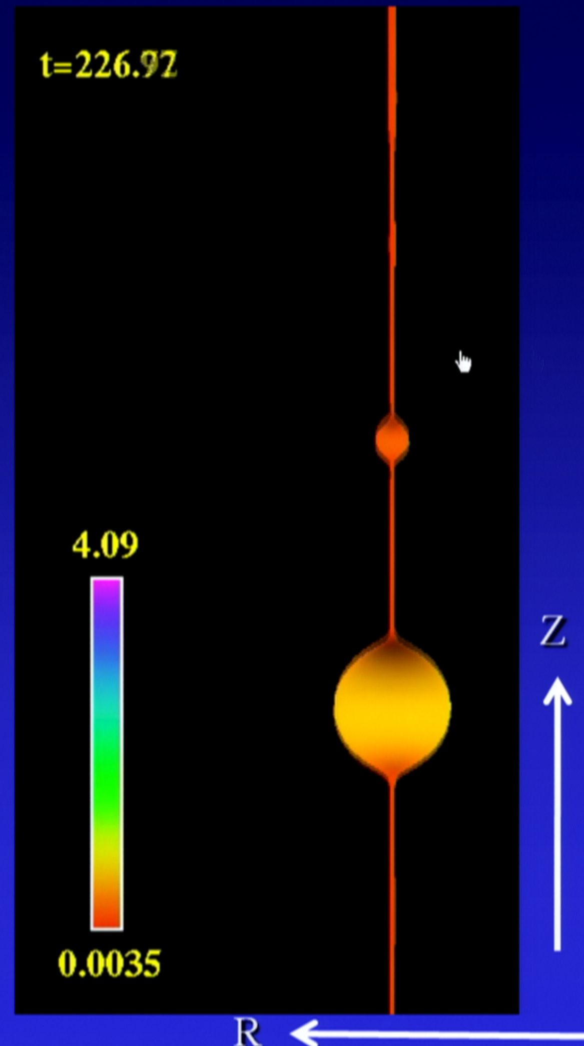


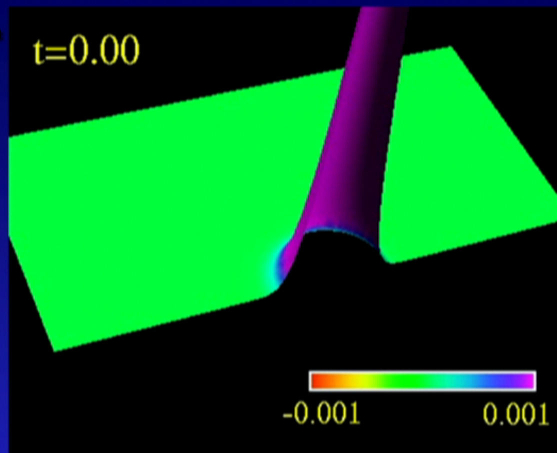
Critical phenomena in gravitational collapse

- Discovered in 1993 by Choptuik, critical phenomena refers to interesting behavior observed at the *threshold of black hole formation* in gravitational collapse
- The question Choptuik was trying to answer was, "can one form black holes of arbitrarily small mass in scalar field collapse?" *(yes!)*
- In the process he discovered behavior that bears striking resemblance to critical phenomena observed at phase transitions in statistical mechanical systems:
  - power law scaling of order parameters (such as the black hole mass M) near threshold
  - universality of the threshold solution
  - scale invariance of the threshold solution
- Rare example in computational physics where a fundamentally new phenomena was discovered via purely numerical methods
  - Even though original example was in spherical symmetry, required AMR to resolve the exponentially rapid develop of features on small length scales

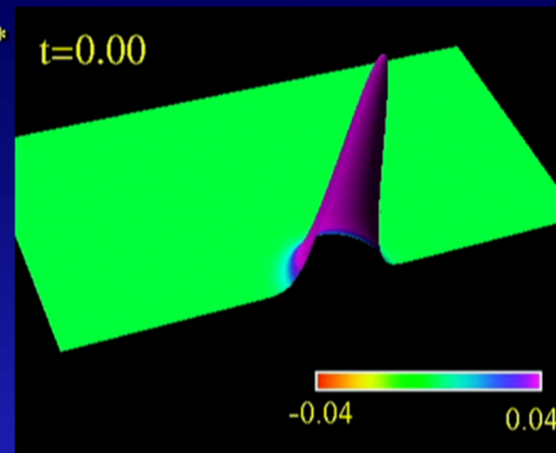
Scalar field gravitational collapse

Axisymmetric simulations, spherical initial data

$p << p^*$

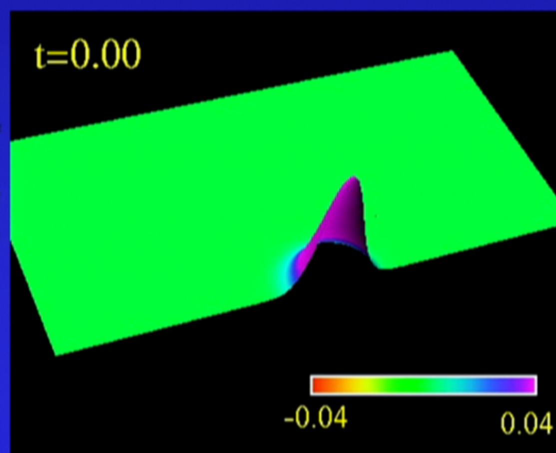


$p >> p^*$

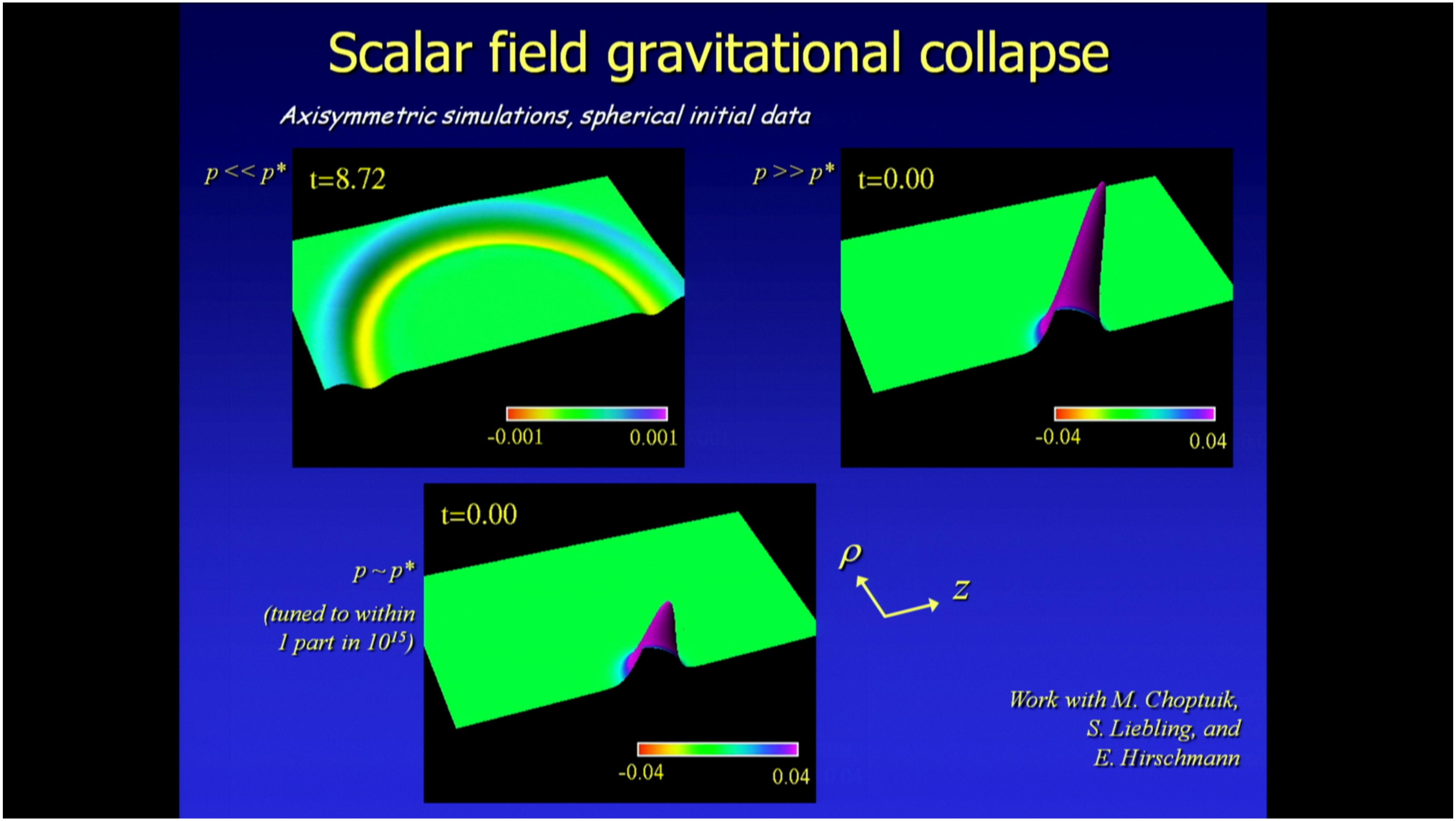


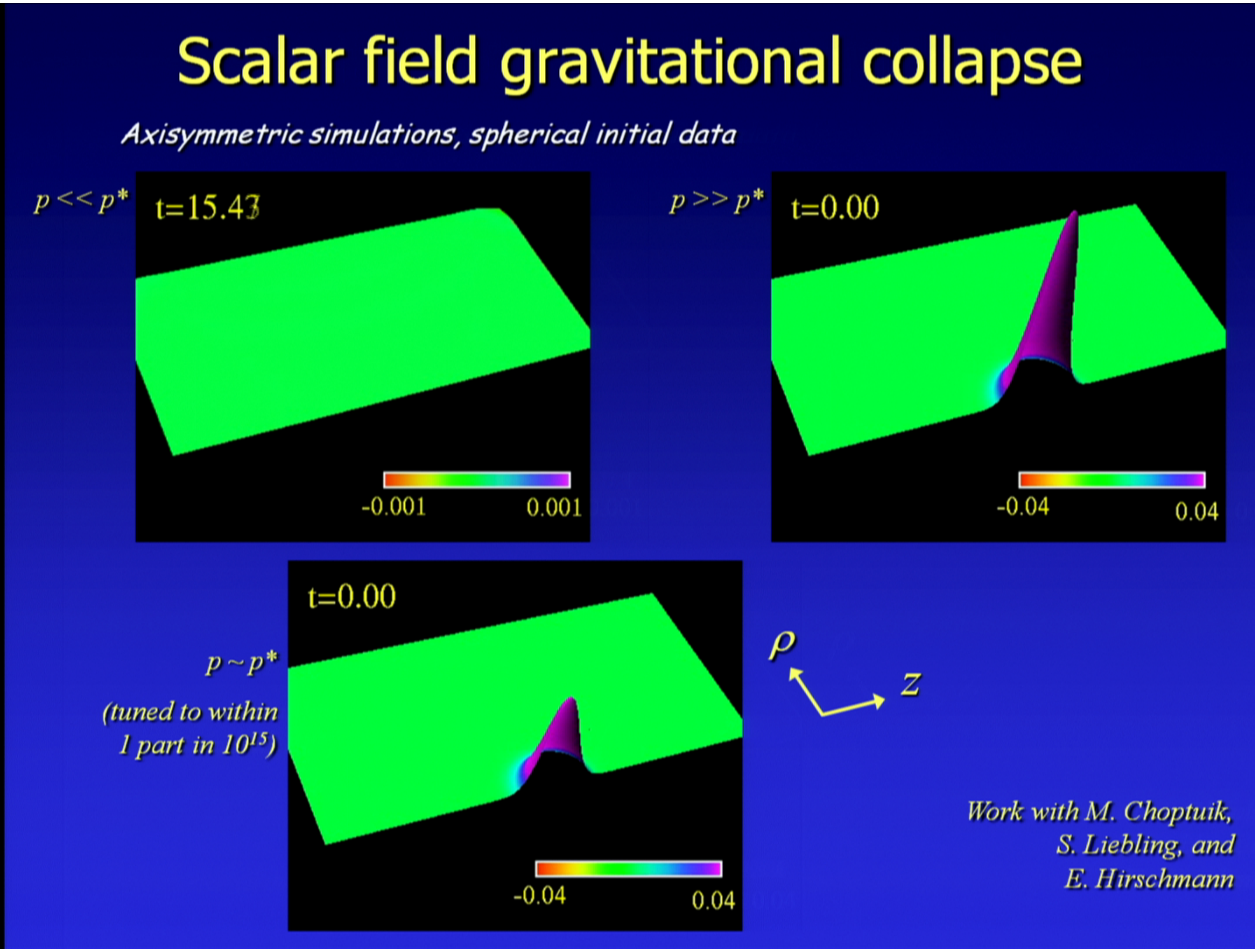
$p \sim p^*$

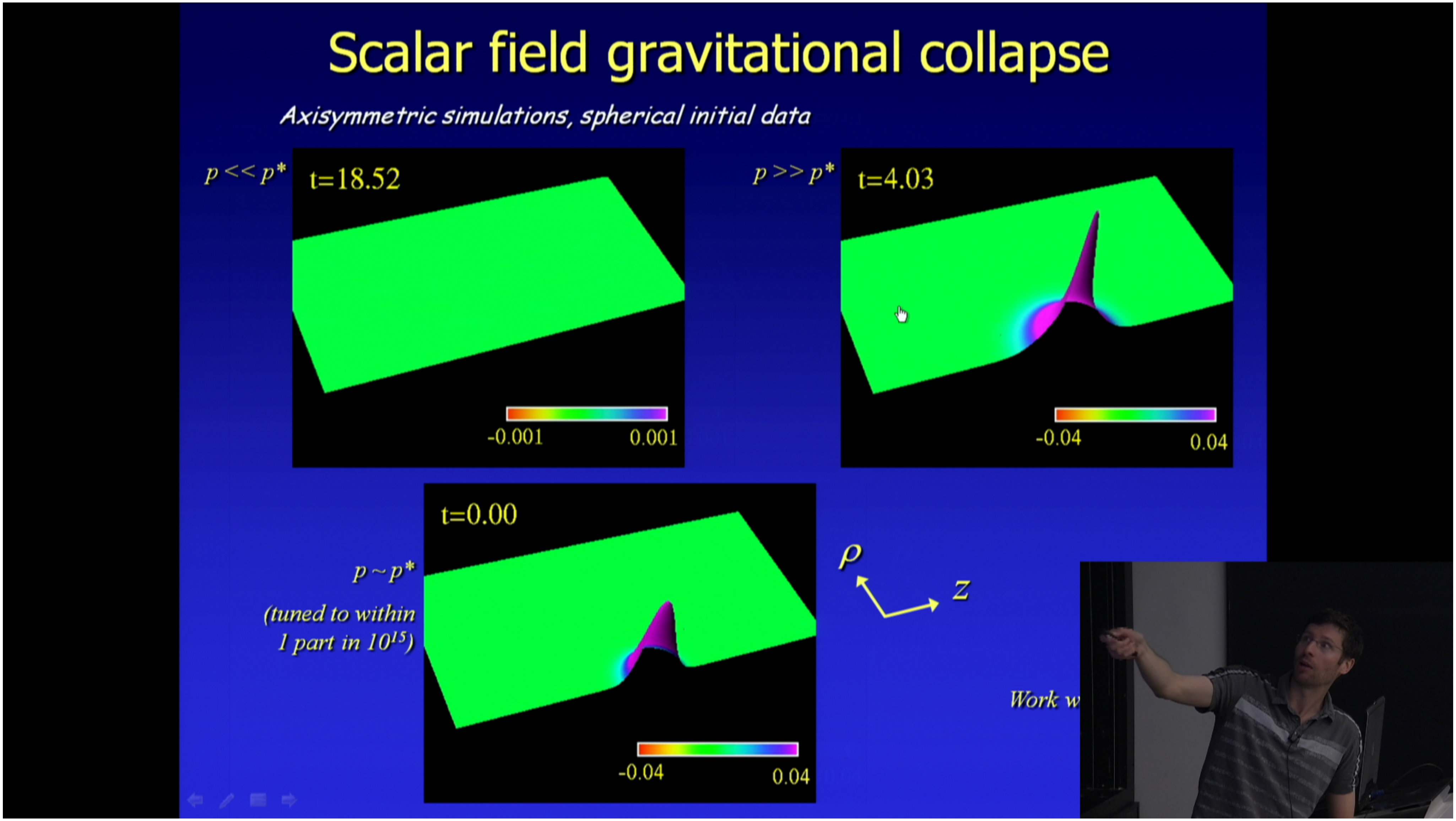
(tuned to within 1 part in 10^{15})



Work with M. Choptuik, S. Liebling, and E. Hirschmann


Scalar field gravitational collapse
Axisymmetric simulations, spherical initial data
p << p*
t=8.72
-0.001
0.001
p >> p*
t=0.00
-0.04
0.04
t=0.00
p ~ p*
(tuned to within
1 part in 10^15)
-0.04
0.04
ρ
z
Work with M. Choptuik,
S. Liebling, and
E. Hirschmann

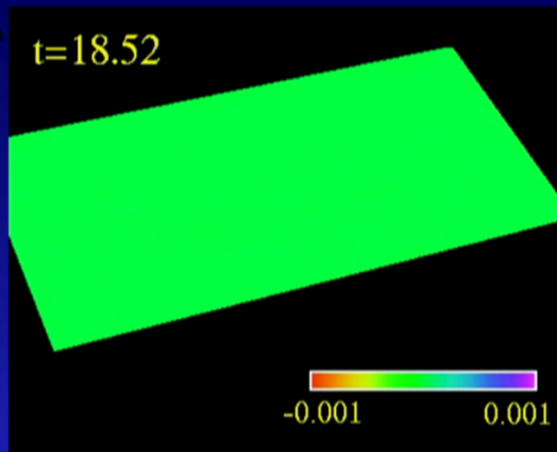

Scalar field gravitational collapse
Axisymmetric simulations, spherical initial data
p << p*
t=15.43
-0.001
0.001
p >> p*
t=0.00
-0.04
0.04
t=0.00
p ~ p*
(tuned to within
1 part in 10^15)
-0.04
0.04
ρ
z
Work with M. Choptuik,
S. Liebling, and
E. Hirschmann


Scalar field gravitational collapse
Axisymmetric simulations, spherical initial data
p << p*
t=18.52
-0.001
0.001
p >> p*
t=4.03
-0.04
0.04
t=0.00
p ~ p*
(tuned to within
1 part in 10^15)
-0.04
0.04
ρ
z
Work w

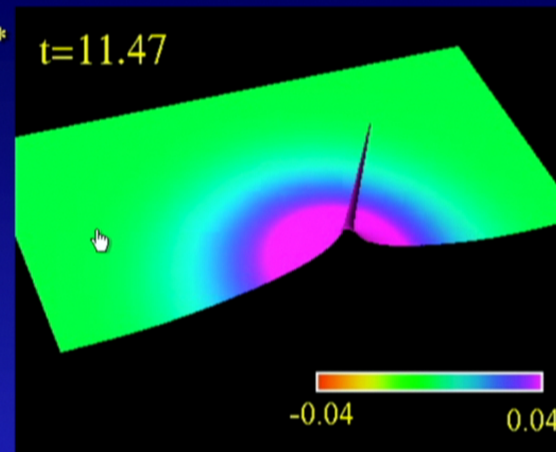
Scalar field gravitational collapse

Axisymmetric simulations, spherical initial data

$p << p^*$

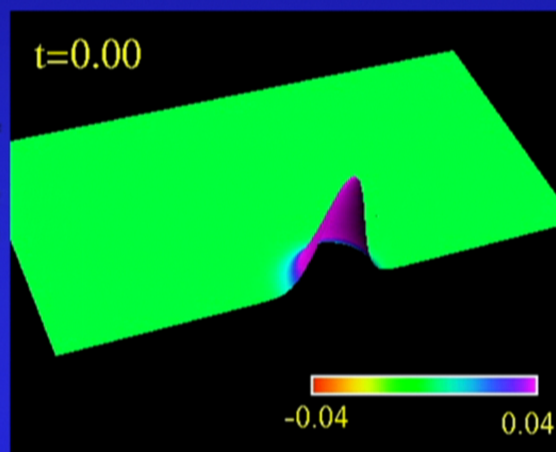


$p >> p^*$



$p \sim p^*$

(tuned to within 1 part in 10^{15})

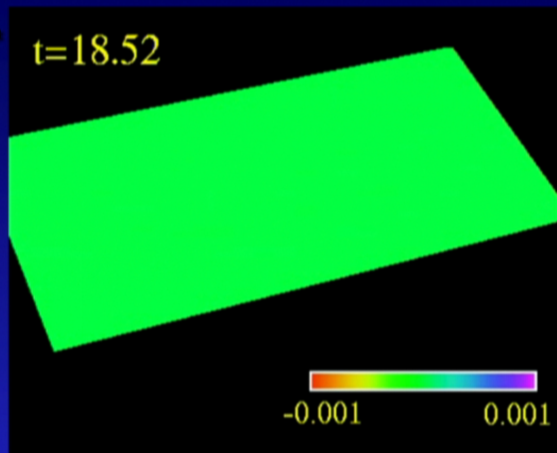


Work with M. Choptuik, S. Liebling, and E. Hirschmann

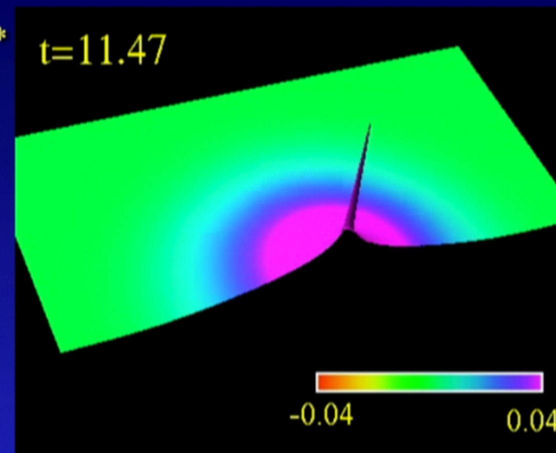
Scalar field gravitational collapse

Axisymmetric simulations, spherical initial data

$p << p^*$

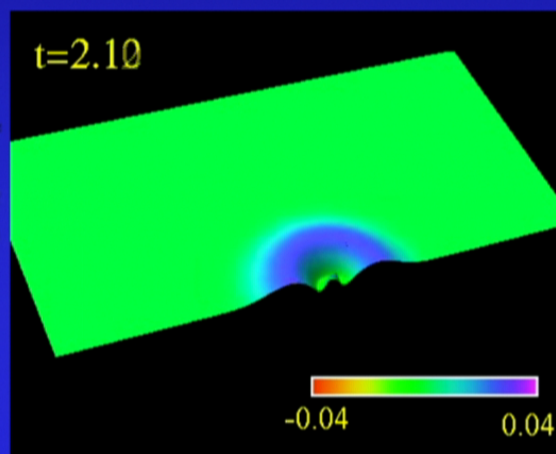


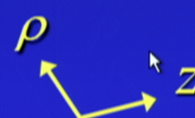
$p >> p^*$



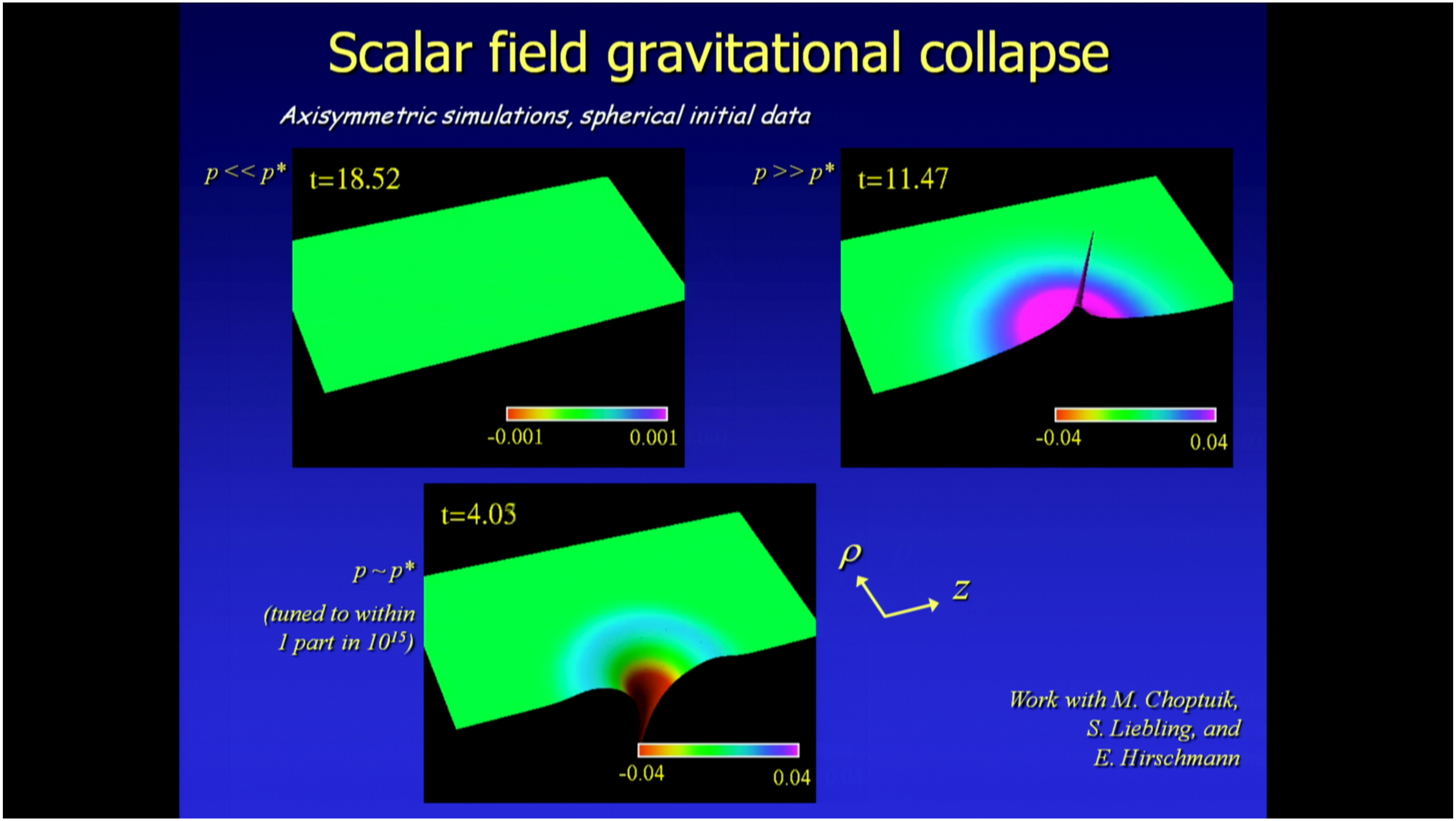
$p \sim p^*$

(tuned to within 1 part in 10^{15})





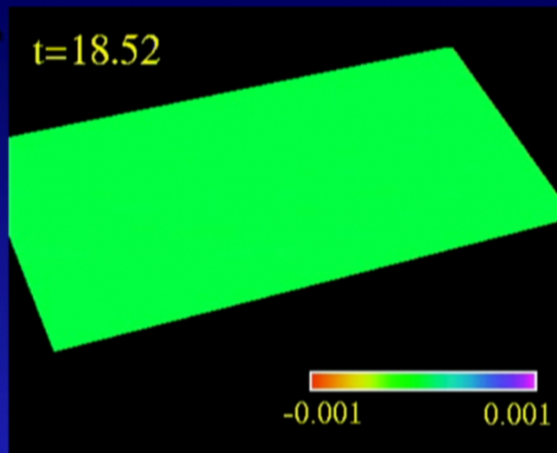
Work with M. Choptuik, S. Liebling, and E. Hirschmann


Scalar field gravitational collapse
Axisymmetric simulations, spherical initial data
p << p*
t=18.52
-0.001
0.001
p >> p*
t=11.47
-0.04
0.04
t=4.03
p ~ p*
(tuned to within
1 part in 10^15)
-0.04
0.04
ρ
z
Work with M. Choptuik,
S. Liebling, and
E. Hirschmann

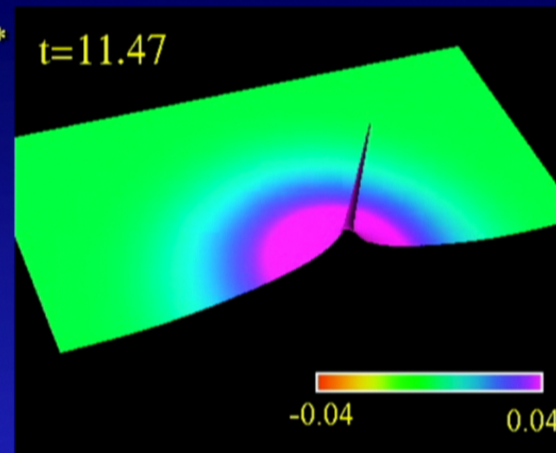
Scalar field gravitational collapse

Axisymmetric simulations, spherical initial data

$p << p^*$

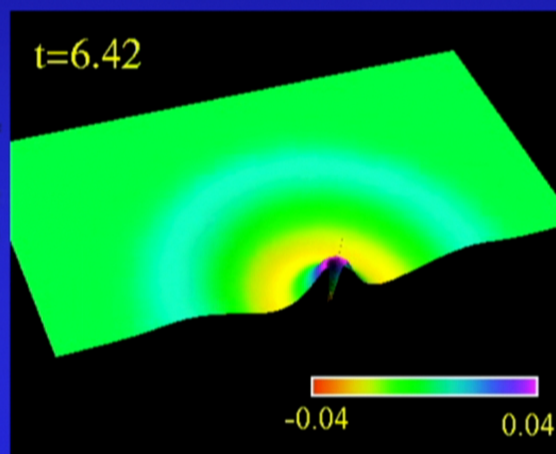


$p >> p^*$



$p \sim p^*$

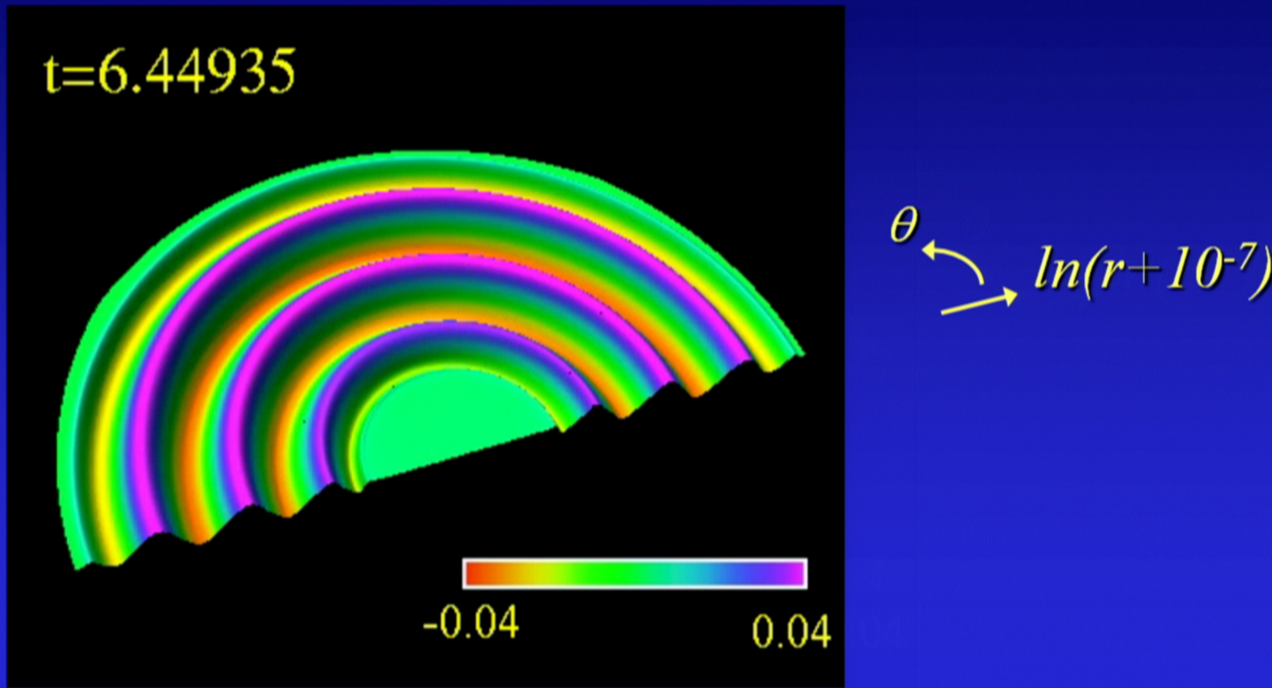
(tuned to within 1 part in 10^{15})



Work with M. Choptuik, S. Liebling, and E. Hirschmann

The scalar field threshold solution

Same near critical solution, transformed to spherical polar coordinates, and using logarithmic radial and time coordinates



Properties of scalar field critical collapse

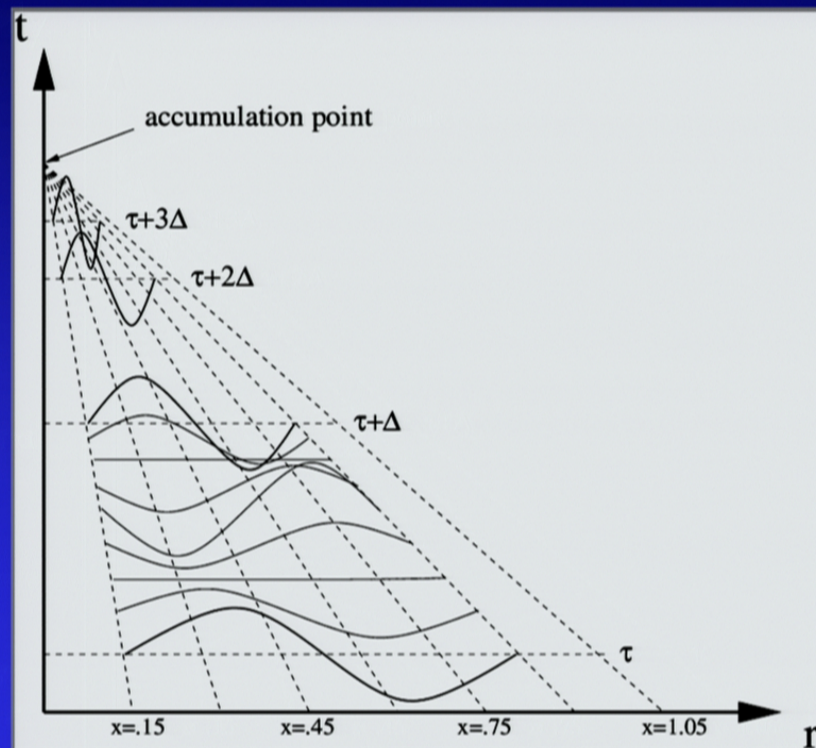
- the critical solution (scalar field and spacetime geometry) is spherically symmetric and scale invariant — specifically it is *discretely self-similar*

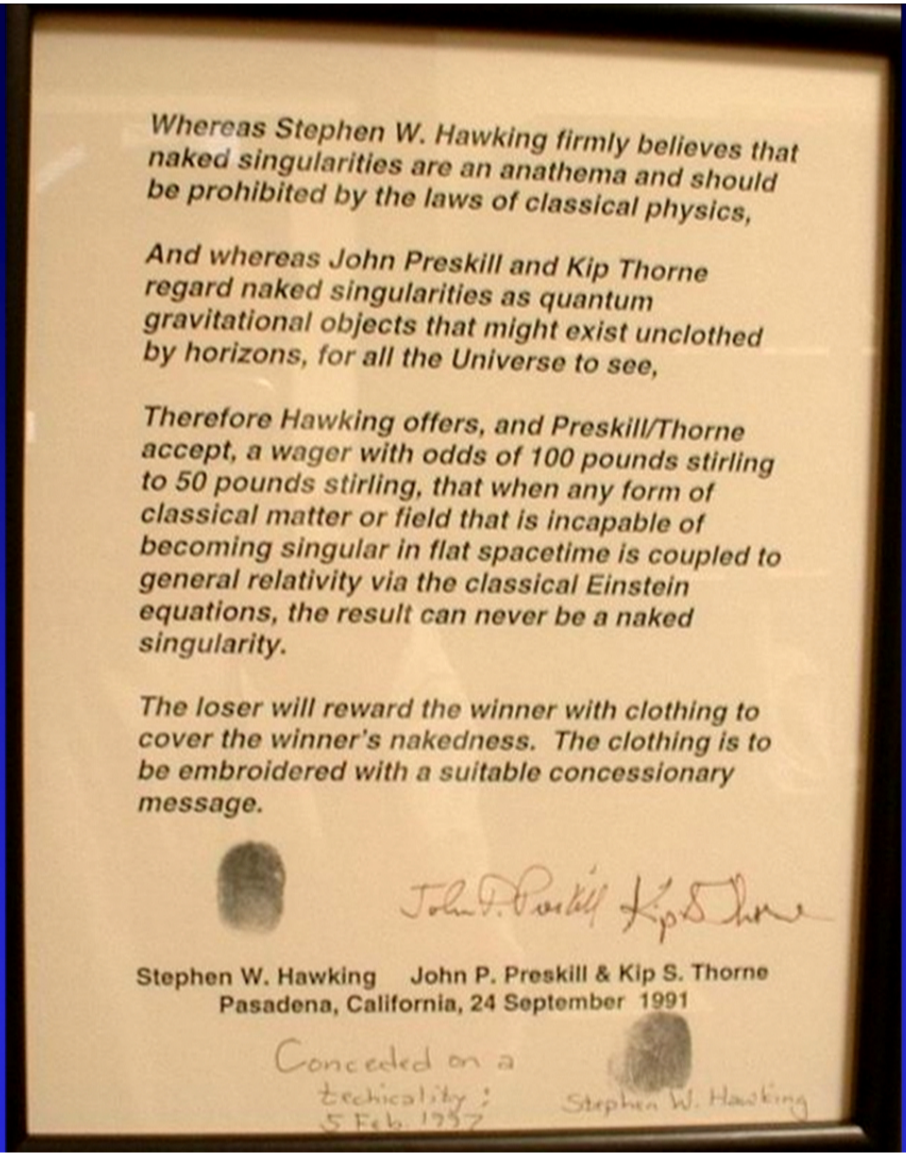
- example of a discretely self similar function $f(x, \tau)$
  - $f(x, t)$ is periodic in time τ with *echoing period* Δ
  - τ is related to the proper time t measured by a central observer (at radius $r=0$) via

    $\tau=-\ln(-t)$

  - x is a *dimensionless* variable, related to r and t via

    $x=r/t$





Whereas Stephen W. Hawking firmly believes that naked singularities are an anathema and should be prohibited by the laws of classical physics,

And whereas John Preskill and Kip Thorne regard naked singularities as quantum gravitational objects that might exist unclothed by horizons, for all the Universe to see,

Therefore Hawking offers, and Preskill/Thorne accept, a wager with odds of 100 pounds stirling to 50 pounds stirling, that when any form of classical matter or field that is incapable of becoming singular in flat spacetime is coupled to general relativity via the classical Einstein equations, the result can never be a naked singularity.

The loser will reward the winner with clothing to cover the winner's nakedness. The clothing is to be embroidered with a suitable concessionary message.

John P. Preskill Kip S. Thorne

Stephen W. Hawking John P. Preskill & Kip S. Thorne
Pasadena, California, 24 September 1991

Conceded on a technicality; 5 Feb. 1997

Stephen W. Hawking

Beyond spherical symmetry

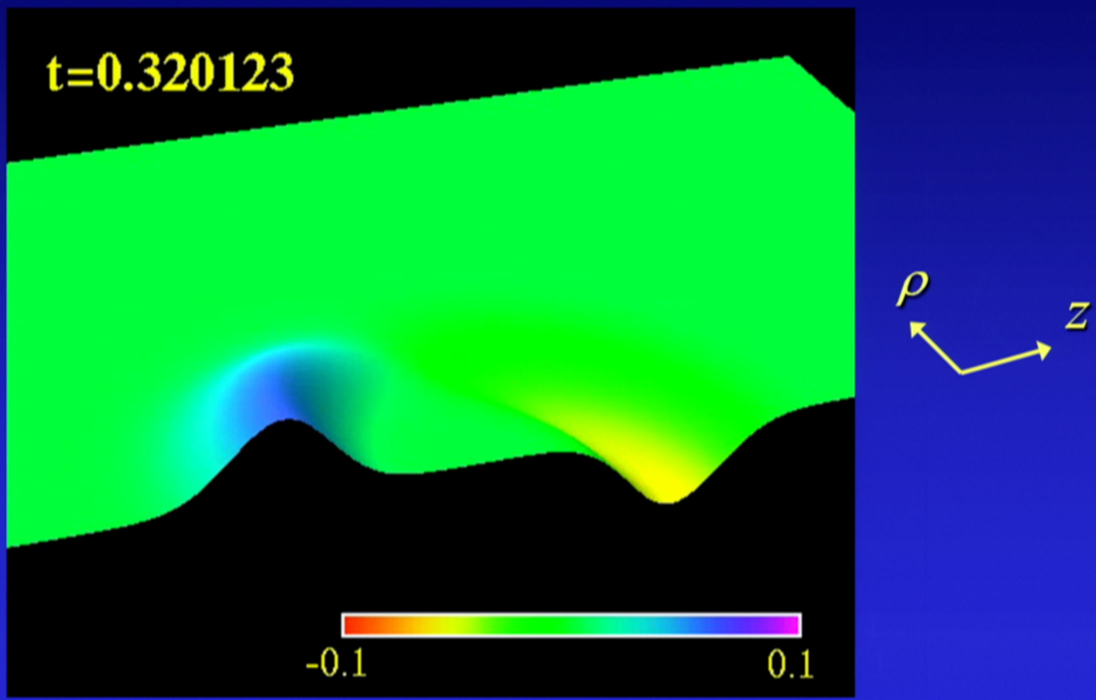
- Example in next slide was a (failed) attempt to "break" the universality conjecture
  - 2D, axisymmetric GR code, solved as a *constrained* evolution
    - 4 elliptic equations (3 constraints, + maximal slicing condition, solved via multigrid), 3 hyperbolics (2^{nd} order finite difference + KO dissipation)
    - ADM *based*, using (2+1)+1 decomposition of metric, and explicitly enforcing maximal slicing, which prevents hyperbolicity problems of the 3+1 version of ADM from arising
      - first "+1" is symmetry reduction, then a 2+1 space+time decomposition within the reduced manifold
      - Maximal slicing imposed, together with conformal flatness of the 2D spatial slices
  - single CPU, takes from minutes (far from threshold) to days (close to threshold) for a solution

Evolution of plane anti-symmetric initial data

Initial data that is reflection anti-symmetric about z=0 (a conserved symmetry)

$p \sim p^*$

(tuned to within 1 part in 10^{15})



Evolution of plane anti-symmetric initial data

Initial data that is reflection anti-symmetric about z=0 (a conserved symmetry)

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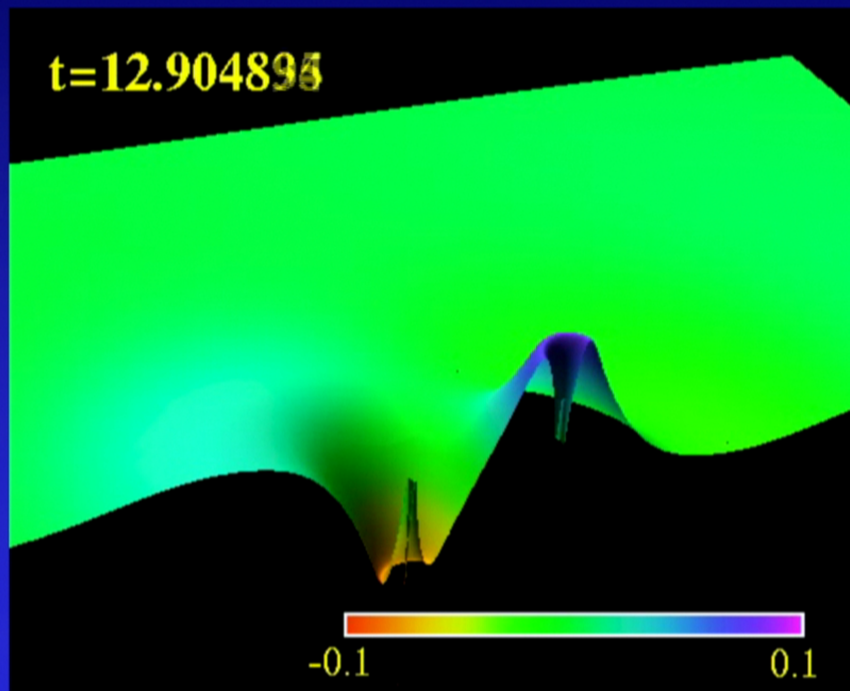
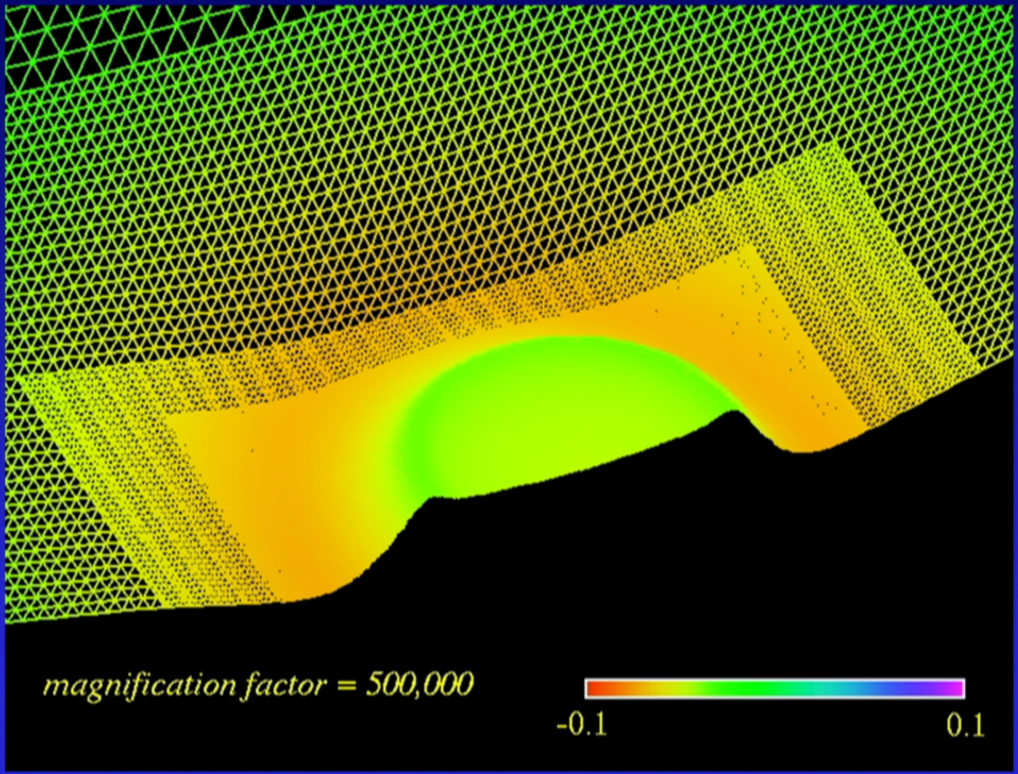


Aside: AMR grid hierarchy sample

Last frame from the previous animation



25, 2:1 refined levels

(2:1 coarsened in figure)