

Title: Explorations in Numerical Relativity - Lecture 8

Date: Apr 12, 2012 11:30 AM

URL: <http://pirsa.org/12040048>

Abstract:



ideal MHD

$$(e^a = F^{ab} u_b = 0)$$

$$T_{ab} = [\rho_0(1+\varepsilon) + p + b_c b^c] u_a u_b + \left[p + \frac{1}{2} b_c b^c \right] g_{ab} - b_a b_b$$

ideal MHD

$$(e^a = F^{ab} u_b = 0)$$

$$\parallel n^a, \perp n^a$$

$$T_{ab} = [\rho_0(1+\epsilon) + P + b_c b^c] u_a u_b + [P + \frac{1}{2} b_c b^c] g_{ab} - b_a b_b$$

$$\nabla_a T^{ab} = 0$$

$$\nabla_a *F^{ab} = 0$$

$$\frac{d}{dt} \int_{\Sigma} T^{ab} n_a n_b$$

$$L_{ab} = g_{ab} + n_a n_b$$

$$c) U_a U_b + \left[P + \frac{1}{2} b_c b^c \right] g_{ab} - b_a b_b$$

$$I_{ab} = g_{ab} + n_a n_b$$

$$W = -n^a U_a$$

$$v^a = \frac{1}{W} \frac{I^a_b U^b}{}$$

Conservative variables.

$$E \equiv T_{ab} n^a n^b$$

$$S_b = -T_{ac} n^a I_b^c$$

$$(I T)_{cd} = T_{ab} I_c^a I_d^b$$

$$\mathcal{D} = -n^a \partial_a E + KE - \frac{1}{\ell^2} \mathcal{D}_a (x^2 S^a) + (\mathbb{L}T)^{ab} K_{ab}$$

$$\mathcal{D} = -n^a \partial_a E + KE - \frac{1}{\alpha^2} \mathcal{D}_a (\alpha^2 S^a) + (\perp T)^{ab} K_{ab}$$

$$0 = \mathcal{D}_a (*F^{ab} n_b)$$

$$\mathcal{D} = -n^a \partial_a E + KE - \frac{1}{\alpha^2} \partial_a (\alpha^2 S^a) + (\perp T)^{ab} K_{ab}$$

$$0 = \partial_a (*F^{ab} n_b) \rightarrow \nabla \cdot B = 0$$

$$(e^a = F^{ab} u_b = 0)$$

$$\parallel n^a, \perp n^a$$

$$T_{ab} = \left[\rho_0 (1 + \epsilon) + P + b_c b^c \right] u_a u_b + \left[P + \frac{1}{2} b_c b^c \right] g_{ab} - b_a b_b$$

$$\nabla_a T^{ab} = 0$$

$$\nabla_a (*F^{ab}) = 0$$

$$\nabla_a (b_0 u^a) = 0$$

$$\frac{d}{dt} \int_{\Sigma} b_0 u^a$$

$$\perp_{ab} = g_{ab} + n_a n_b$$

$$W = -n^a u_a$$

$$v^a = \frac{1}{W} \perp^a_b u^b$$

$$E \equiv T_{ab} n^a n^b$$

$$S_b = -T_{ab} n^a$$

$$(\perp T)_{cd} = T_{ab} \perp^a_c \perp^b_d$$

$$0 = -n^a \nabla_a E + K E - \frac{1}{\alpha^2} \nabla_a (\alpha^2 S^a) + (\perp T)^{ab} K_{ab}$$

$$0 = \nabla_a (*F^{ab} n_b) \rightarrow \nabla \cdot B = 0$$

$$0 = -n^a \partial_a E + KE - \frac{1}{\alpha^2} \mathcal{D}_a (\alpha^2 S^a) + (LT)^{ab} K_{ab}$$

$$0 = \mathcal{D}_a (*F^{ab} n_b) \rightarrow \nabla \cdot B = 0$$

$$0 = \frac{1}{\alpha} n^a \partial_a (\alpha D) + \frac{1}{\alpha} \mathcal{D}_a (\alpha D n^a) - KW$$

$$0 = -n^a \partial_a E + KE - \frac{1}{\alpha^2} \mathcal{D}_a (\alpha^2 S^a) + (\perp T)^{ab} K_{ab} \rightarrow \partial_0$$

$$0 = \mathcal{D}_a (*F^{ab} n_b) \rightarrow \nabla \cdot B = 0$$

$$0 = \frac{1}{\alpha} n^a \partial_a (\alpha D) + \frac{1}{\alpha} \mathcal{D}_a (\alpha D n^a) - KW$$

$$0 = -n^a \partial_a E + KE - \frac{1}{2} \partial_a (\alpha^2 S^a) + (LT)^{ab} K_{ab} \rightarrow \partial_0 E + \partial_i F^i = \text{"Sources"}$$

$$0 = \partial_a (\star F^{ab} n_b) \rightarrow \nabla \cdot B = 0$$

$$0 = \frac{1}{2} n^a \partial_a (\alpha D) + \frac{1}{2} \partial_a (\alpha D n^a) - KW$$

$$0 = -n^2 \partial_a E + KE - \frac{1}{2} \partial_a (\alpha^2 S^a) + (LT)^{ab} K_{ab} \rightarrow \partial_0 E + \partial_b F^i = \text{"Sources"}$$

$$0 = \partial_a (\star F^{ab} n_b) \rightarrow \nabla \cdot B = 0$$

$$0 = \frac{1}{2} n^a \partial_a (\alpha D) + \frac{1}{2} \partial_a (\alpha D n^a) - KW$$

$$E_a = \frac{1}{2} F_{bc} n^c \quad B_a = \frac{1}{2} \epsilon_{abc} F^{bc}$$

$$E_{abc} = \epsilon_{abc} n^d$$

$$0 = -n^a \partial_a E + KE - \frac{1}{2} \partial_a (\alpha^2 S^a) + (LT)^a K_{ab} \rightarrow \partial_0 E + \partial_i F^i = \text{"Sources"}$$

$$0 = \partial_a (\star F^{ab} n_b) \rightarrow \nabla \cdot B = 0$$

$$0 = \frac{1}{2} n^a \partial_a (\alpha D) + \frac{1}{2} \partial_a (\alpha D n^a) - KW$$

$$E_a = \frac{1}{2} F_{bc} n^c \quad B_a = \frac{1}{2} \epsilon_{abc} F^{bc}$$

$$\epsilon_{abc} = \epsilon_{abcd} n^d$$

$$E_a = \frac{1}{n_d u^d} \underbrace{\epsilon_{abc} u^b \vec{B}^c}_{\sim u \wedge B}$$

New cons variables

$$D = w s_0$$

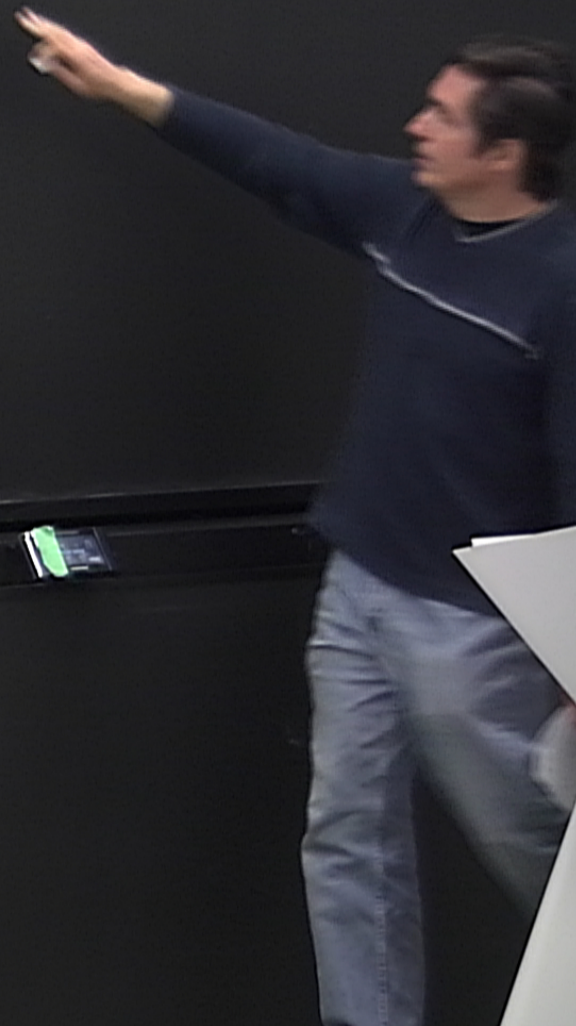
$$B^a = w b^a - u^a (n_c b^c)$$

New cons. variables

$$D = w s_0$$

$$B^a = w b^a - u^a (n \cdot b^c)$$

(Primitive vars. s_0, ϵ
cont'g)



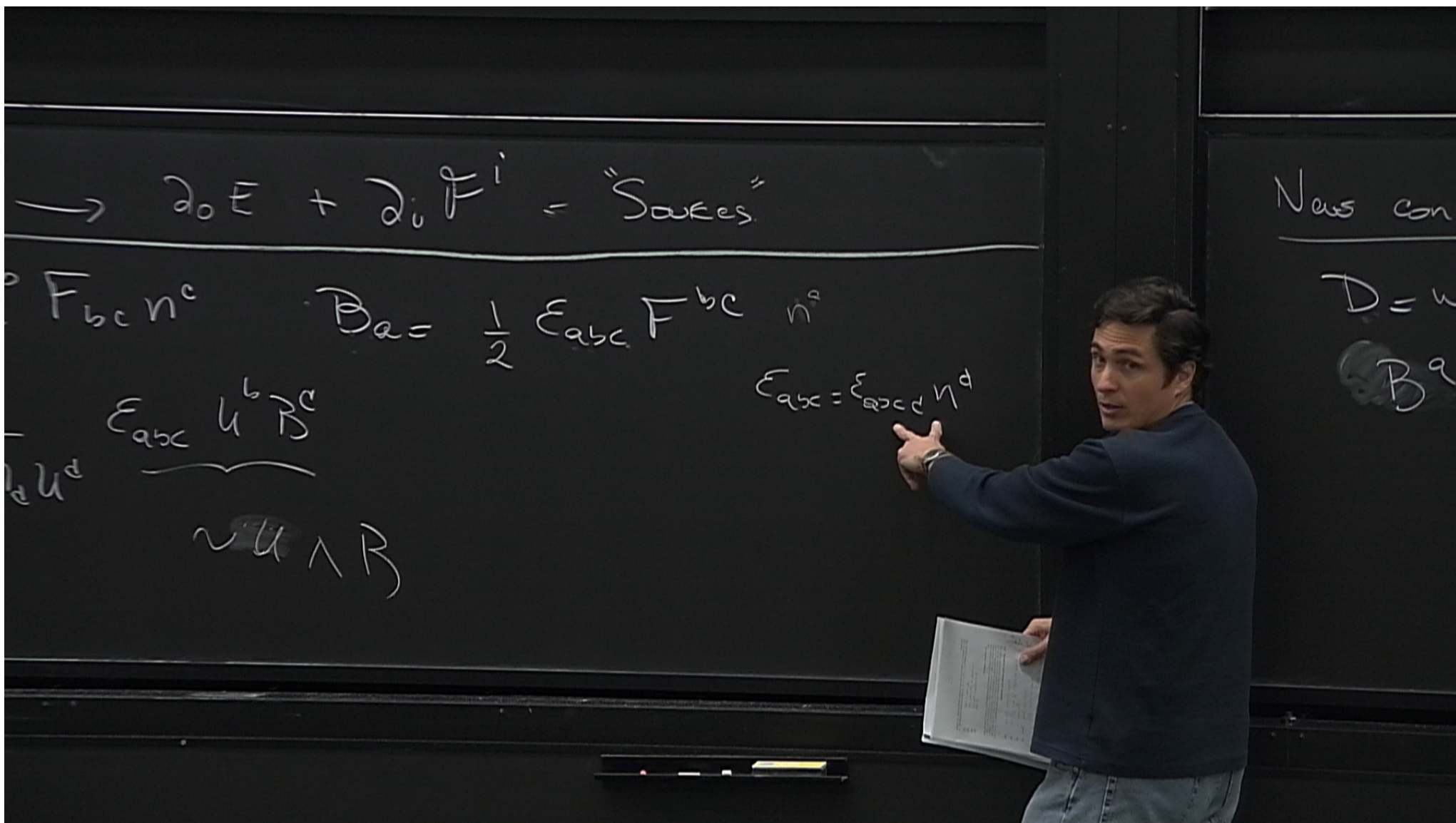
New cons. variables

$$D = w s_0$$

$$B^a = w b^a - u^a (n \cdot b^c)$$

(Primitive vars.
conting)

$s_0, \epsilon, P, u^a, B^i$



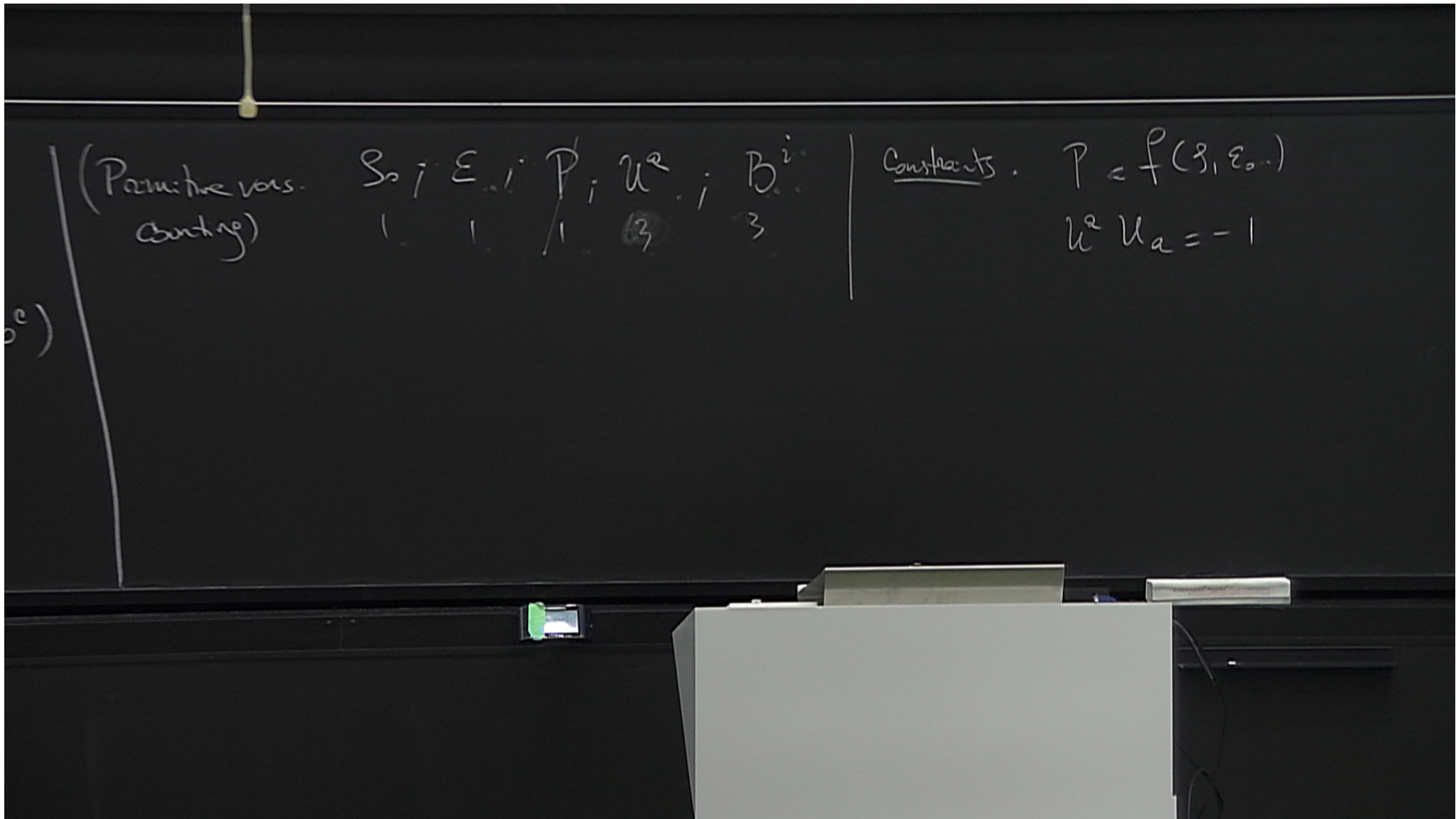
New cons variables

$$D = w s_0$$

$$B^a = w b^a - u^a (n \cdot b^c)$$

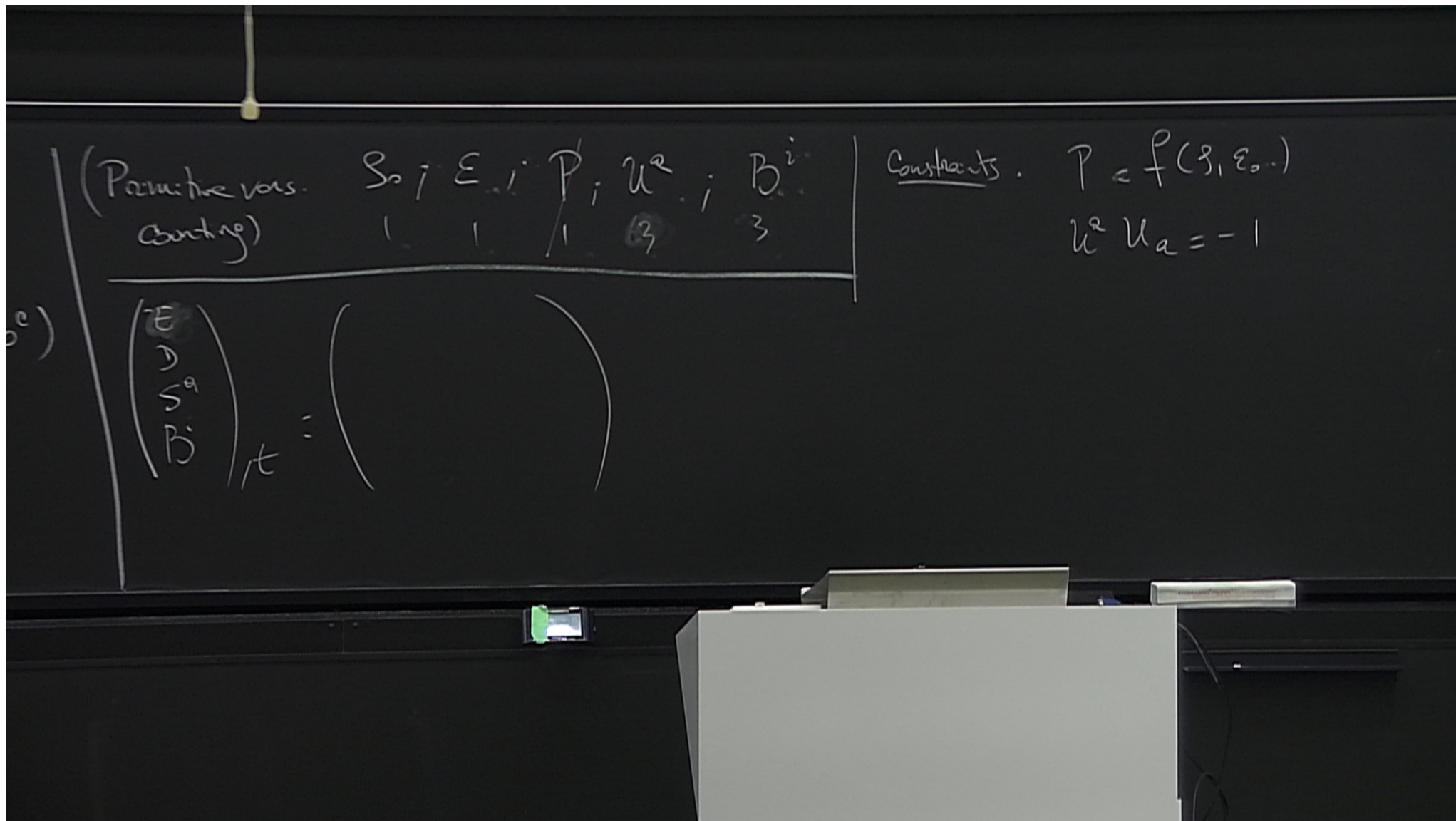
(Primitive vars.
counting)

$s_0; \epsilon; P; u^a; B^i$
1 1 1 4 3



(Primitive vars. counting) $S_0; E; P; u^a; B^i$
 $1 \quad 1 \quad 1 \quad 3 \quad 3$

Constraints. $P = f(\beta, \epsilon_0, \dots)$
 $u^a u_a = -1$



Hyperbolicity

$$u_{it} = M_{it} x$$

↑

$$u_{,t} + \frac{1}{2}(u^2)_{,x} = 0$$

$$\hookrightarrow u_{,t} + u u_{,x} = 0$$

$$= u_0 + \delta u$$

$$+ (u_0 + \delta u) (u_0 + \delta u)_{,x} = 0$$

Hyperbolicity

$$u_{,t} = M u_{,x}$$

$$\left(\frac{d\beta}{d\alpha} \right)$$

Hyperbolicity

$$\begin{cases} u_{,t} + \frac{1}{2}(u^2)_{,x} = 0 \\ u_{,t} + u u_{,x} = 0 \end{cases}$$

$$\tilde{u} = u_0 + \delta u$$

$$(u_0 + \delta u)_{,t} + (u_0 + \delta u)(u_0 + \delta u)_{,x} = 0$$

L.O. $u_{0,t} + u_0 u_{0,x} = 0$

$$\delta u_{,t} + \underline{u_0 \delta u_{,x}} + \underline{\delta u u_{0,x}} = 0$$

$$u_{,t} = M u_{,x}$$

$$\begin{cases} u_{,t} + \frac{1}{2}(u^2)_{,x} = 0 \\ u_{,t} + u u_{,x} = 0 \end{cases}$$

$$\tilde{u} = u_0 + \delta u$$

$$(u_0 + \delta u)_{,t} + (u_0 + \delta u)(u_0 + \delta u)_{,x} = 0$$

L.O.

$$u_{0,t} + u_0 u_{0,x} = 0$$

$$\boxed{\delta u_{,t} + u_0 \delta u_{,x}} + \cancel{(\delta u u_{0,x})} = 0$$

Hyperbolicity

$$u_{,t} = M u_{,x}$$

$$\left(\frac{d\beta}{dt} \right)$$

Hyperbolicity

$$\begin{cases} u_{,t} + \frac{1}{2}(u^2)_{,x} = 0 \\ u_{,t} + u u_{,x} = 0 \end{cases}$$

$$\tilde{u} = u_0 + \delta u$$

$$u_{,t} = M \left(\frac{d}{dt} \right) u_{,x}$$

$$(u_0 + \delta u)_{,t} + (u_0 + \delta u) (u_0 + \delta u)_{,x} = 0$$

L.O.

$$\begin{aligned} & u_{0,t} + u_0 u_{0,x} = 0 \\ & \boxed{\delta u_{,t} + u_0 \delta u_{,x}} + \cancel{(\delta u u_{0,x})} = 0 \end{aligned}$$

$$\begin{cases} u_{,t} + \frac{1}{2}(u^2)_{,x} = 0 \\ u_{,t} + u u_{,x} = 0 \end{cases}$$

$$\tilde{u} = u_0 + \delta u$$

$$(u_0 + \delta u)_{,t} + (u_0 + \delta u)(u_0 + \delta u)_{,x} = 0$$

L.O.

$$u_{0,t} + u_0 u_{0,x} = 0$$

$$\boxed{\delta u_{,t} + u_0 \delta u_{,x}} + \cancel{\delta u u_{0,x}} = 0$$

Hyperbolicity

$$u_{,t} = M u_{,x} \\ \left(\frac{d\beta}{dt} \right)$$

Eigenvalues
($\alpha = 1, \beta = 0$)

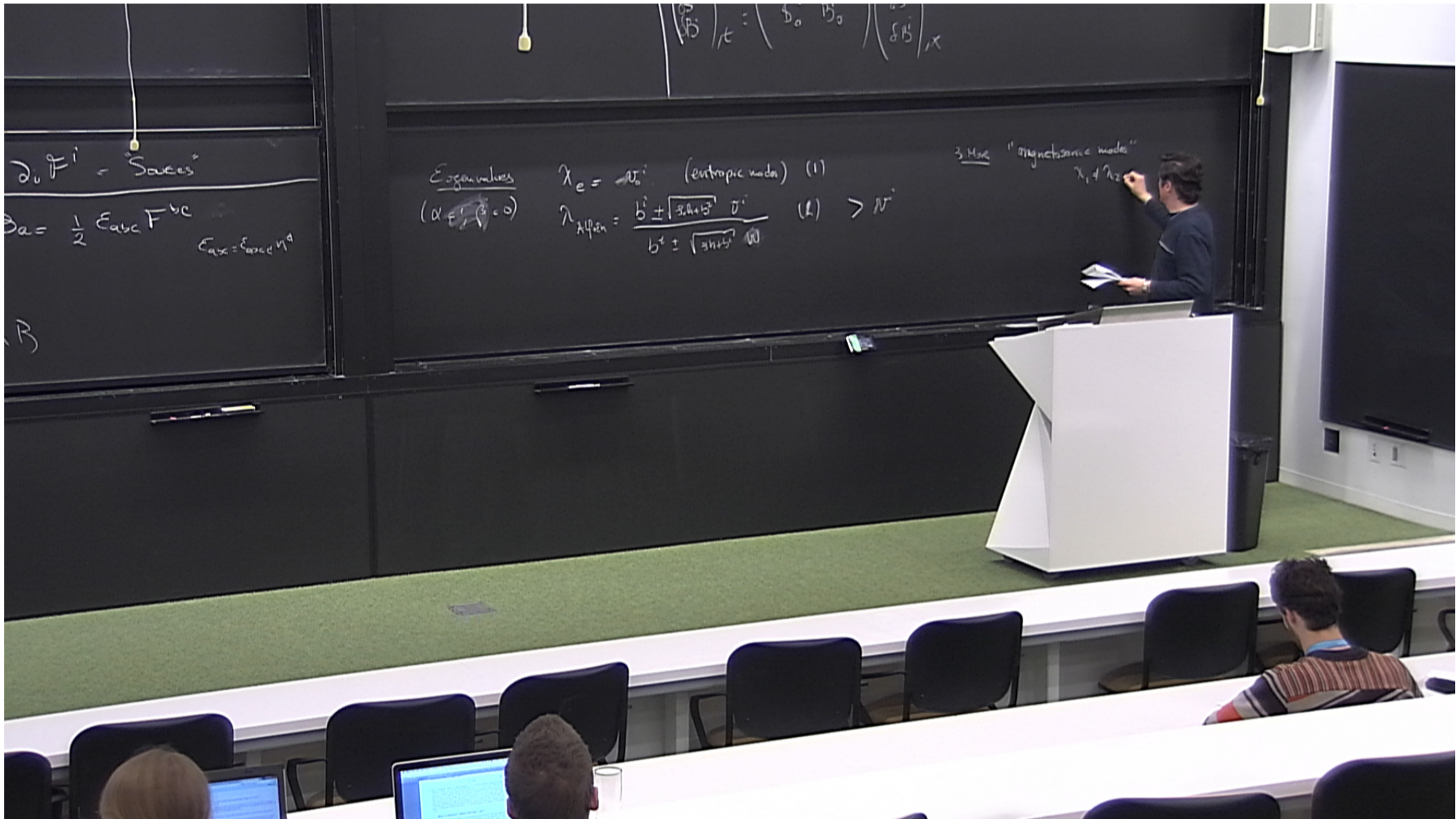
$\chi_e = v_i$ (entropic modes)

$\lambda_{\text{Alfven}} = \frac{v_i}{s_0}$

Eigenvalues
($\alpha = 1, \beta^i = 0$)

$$\chi_e = v_0^i \quad (\text{entropic modes}) \quad (1)$$

$$\lambda_{\text{Alfven}} = \frac{b^i \pm \sqrt{g_{0h} + b^2} v^i}{b^t \pm \sqrt{g_{0h} + b^2} W} \quad (2) \quad > v^i$$



$$D = W b^2 - u^2 (n_e b^2)$$

$$\begin{pmatrix} \delta D \\ \delta S_a \\ \delta B_i \end{pmatrix} = \begin{pmatrix} E_0, D_0 \\ \delta S_a & B_0 \end{pmatrix} \begin{pmatrix} \delta D \\ \delta S^2 \\ \delta B_i \end{pmatrix}_{,X}$$

Eigenvalues
($\alpha = 1, \beta = 0$)

$$\lambda_e = v_0^i \quad (\text{entropic modes}) \quad (1)$$

$$\lambda_{\text{Alfven}} = \frac{b^i \pm \sqrt{g_{ab} b^2} v^i}{b^2 \pm \sqrt{g_{ab} b^2} W} \quad (2) > v^i$$

3 More "magnetosonic modes"
 $\lambda_1 \neq \lambda_2 \neq \lambda_3$

$$\begin{pmatrix} u \\ w \end{pmatrix}_t = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} u \\ w \end{pmatrix}, x$$

weekly hyperbolic

$$\dot{E} = (\nabla \cdot \vec{B}) + \kappa \vec{B} \cdot (\nabla \vec{B})$$

$$(D_{nB}) + \lambda \vec{B} (D_{IB})$$

"8th wave approach"

$$\vec{E} = (\nabla \cdot \vec{B}) + \kappa \vec{B} \quad (\nabla \cdot \vec{B})$$

Astrophys - Newt. Level.

→ "8th wave approach"

→ conserved enrgy.

→ "ang. Mom."

$$\left. \begin{array}{l} \rightarrow \text{conserved enrgy.} \\ \rightarrow \text{"ang. Mom."} \end{array} \right\} \rightarrow \partial_t \mathcal{K} + \partial_i F^i = 0$$

$$\dot{E} = (\nabla \cdot \vec{B}) + \kappa \vec{B} \cdot (\nabla \vec{B})$$

Astrophys Newt. Level.

→ "Bth wave approach"

→ conserved energy

→ "ang. Mom."

$$\left. \begin{array}{l} \rightarrow \text{conserved energy} \\ \rightarrow \text{"ang. Mom."} \end{array} \right\} \rightarrow \partial_t \mathcal{E} + \partial_i \mathcal{F}^i = -(\nabla \cdot \vec{B})$$

$$\vec{E} = (\nabla \cdot \vec{B}) + \lambda \vec{B} (\nabla \cdot \vec{B})$$

Astrophys - Newt. Level.

→ "Bth wave approach"

→ conserved enrgy.

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$$\left. \begin{array}{l} \rightarrow \text{conserved enrgy.} \\ \rightarrow \text{"ang. Mom."} \end{array} \right\} \rightarrow \partial_t \mathcal{U} + \partial_i F^i = (\nabla \cdot \vec{B})$$

Eigenvalues

$(\alpha = 1, \beta = 0)$

(I) all $|\lambda| < c$

$\lambda_e = \omega_0^i$ (entropic modes) (1)

$\lambda_{\text{Alfven}} = \frac{b^i \pm \sqrt{3(h+b^2)} \tilde{v}^i}{b^2 \pm \sqrt{3(h+b^2)} \omega}$ (2) $> v^i$

(II) Where is the missing eigenvector!

3 More "constant transport"

degenerate due to $h^2, 10^{-14}$

$\nabla B = 0$

$$E = (\nabla \cdot \vec{B}) + \kappa \vec{B} \cdot (\nabla \vec{B})$$



Astrophys

Newt. Level.

→ "Bth wave approach"

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→ "ang. Mom."

$$\left. \begin{array}{l} \rightarrow \text{conserved energy} \\ \rightarrow \text{"ang. Mom."} \end{array} \right\} \rightarrow \partial_t \mathcal{L} + \partial_i F^i = -(\nabla \cdot \vec{B})$$

$$0 = -n^a \partial_a E + KE - \frac{1}{2} D_a (\alpha^2 S^a) + (11) K_{ab} \rightarrow d_0 E + d_0 V = \text{powers}$$

$$0 = D_a (\star F^{ab} n_b) \rightarrow \nabla \cdot B = 0$$

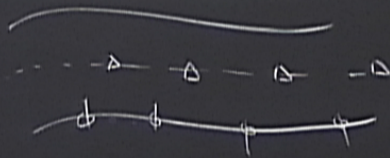
$$0 = \frac{1}{2} n^a \partial_a (\alpha D) + \frac{1}{2} D_a (\alpha D n^a) - KW$$

$$E_a = \frac{1}{2} F_{bc} n^c \quad B_a = \frac{1}{2} \epsilon_{abc} F^{bc}$$

$$E_a = \frac{1}{n_d u^d} \underbrace{\epsilon_{abc} u^b B^c}_{\sim u \wedge B}$$

$$E_{abc} = E_{bcd} n^d$$

$$E = (\nabla \cdot B) + \star \vec{B} (D B)$$



Astrophys Newt. Level.

→ "8th wave approach"

→ conserved enrg.

→ "ang. Mom."

$$\left. \begin{array}{l} \rightarrow \text{conserved enrg.} \\ \rightarrow \text{"ang. Mom."} \end{array} \right\} \rightarrow \partial_t u + \partial_i F^i = \star (\nabla B)$$

$$0 = -n^a \partial_a E + KE - \frac{1}{2} \partial_a (\alpha^2 S^a) + (11) K_{ab} \rightarrow d_0 E + d_0 V = \text{powers}$$

$$0 = D_a (\star F^{ab} n_b) \rightarrow \nabla \cdot B = 0$$

$$0 = \frac{1}{2} n^a \partial_a (\alpha D) + \frac{1}{2} D_a (\alpha D n^a) - KW$$

$$E_a = \frac{1}{2} F_{bc} n^c \quad B_a = \frac{1}{2} \epsilon_{abc} F^{bc}$$

$$E_a = \frac{1}{n_d u^d} \underbrace{\epsilon_{abc} u^b B^c}_{\sim u \wedge B}$$

$$\epsilon_{abc} = \epsilon_{abcd} n^d$$

Ohm's Law

$$J_a + (J^b u_b) u_a = \sigma F_{ab} u^b$$

$$0 = -n^a \partial_a E + KE - \frac{1}{2} \partial_a (\alpha^2 S^a) + (II) K_{ab} \rightarrow d_0 E + d_0 I = \text{powers}$$

$$0 = D_a (\star F^{ab} n_b) \rightarrow \nabla \cdot B = 0$$

$$0 = \frac{1}{2} n^a \partial_a (\alpha D) + \frac{1}{2} D_a (\alpha D n^a) - KW$$

$$E_a = \frac{1}{2} F_{bc} n^c \quad B_a = \frac{1}{2} \epsilon_{abc} F^{bc}$$

$$E_a = \frac{1}{n_d u^d} \underbrace{\epsilon_{abc} u^b B^c}_{\sim u \wedge B}$$

$$\epsilon_{abc} = \epsilon_{abcd} n^d$$

Ohm's Law

$$J_a + (J^b u_b) = \sigma F_{ab} u^b \rightarrow J^i = \sigma W \cdot [\vec{E} + \vec{v} \wedge \vec{B} - (\vec{E} \cdot \vec{v}) \vec{v}] + q \vec{v}$$

$$0 = -n^a \partial_a E + KE - \frac{1}{2} \partial_a (\alpha^2 S^a) + (11) K_{ab} \rightarrow d_0 E + d_0 V = \text{powers}$$

$$0 = D_a (\star F^{ab} n_b) \rightarrow \nabla \cdot B = 0$$

$$0 = \frac{1}{2} n^a \partial_a (\alpha D) + \frac{1}{2} D_a (\alpha D n^a) - KW$$

$$E_a = \frac{1}{2} F_{bc} n^c \quad B_a = \frac{1}{2} \epsilon_{abc} F^{bc}$$

$$E_a = \frac{1}{n_a u^d} \underbrace{\epsilon_{abc} u^b B^c}_{\sim u \wedge B}$$

$$E_{abc} = \epsilon_{abc} n^d$$

Ohm's Law

$$J_a + (J^b u_b) u_a = \sigma F_{ab} u^b \rightarrow \vec{J} = \sigma W [\vec{E} + \vec{v} \wedge \vec{B} - (\vec{E} \cdot \vec{v}) \vec{v}] + q \vec{v}$$

(1) observe $\sigma \rightarrow \infty$

$$0 = -n^a \partial_a E + K E - \frac{1}{2} \partial_a (\alpha^2 S^a) + (11) K_{ab} \rightarrow d_0 E + d_0 V = \text{sources}$$

$$0 = D_a (\star F^{ab} n_b) \rightarrow \nabla \cdot B = 0$$

$$0 = \frac{1}{2} n^a \partial_a (\alpha D) + \frac{1}{2} D_a (\alpha D n^a) - K W$$

$$E_a = \frac{1}{2} F_{bc} n^c \quad B_a = \frac{1}{2} \epsilon_{abc} F^{bc}$$

$$E_a = \frac{1}{n_a u^d} \underbrace{\epsilon_{abc} u^b B^c}_{\sim u \wedge B}$$

$$\epsilon_{abc} = \epsilon_{abcd} n^d$$

Ohm's Law

$$J_a + (J^b u_b) u_a = \sigma F_{ab} u^b \rightarrow \vec{J} = \sigma W \cdot \underbrace{[\vec{E} + \vec{v} \wedge \vec{B} - (\vec{E} \cdot \vec{v}) \vec{v}]}_{\substack{0 \\ \text{(1) observe } \sigma \rightarrow \infty}} + q \vec{v}$$

$$0 = -n^a \partial_a E + K E - \frac{1}{2} \partial_a (\alpha^2 S^a) + (11) K_{ab} \rightarrow d_0 E + d_0 V = \text{pieces}$$

$$0 = D_a (\star F^{ab} n_b) \rightarrow \nabla \cdot B = 0$$

$$0 = \frac{1}{2} n^a \partial_a (\alpha D) + \frac{1}{2} D_a (\alpha D n^a) - K W$$

$$E_a = \frac{1}{2} F_{bc} n^c \quad B_a = \frac{1}{2} \epsilon_{abc} F^{bc}$$

$$E_a = \frac{1}{n_d u^d} \underbrace{\epsilon_{abc} u^b B^c}_{\sim u \wedge B}$$

$$E_{abc} = \epsilon_{abcd} n^d$$

Ohm's Law

$$J_a + (J^b u_b) u_a = \sigma F_{ab} u^b \rightarrow \vec{J} = \sigma W \cdot \left[\vec{E} + \vec{v} \wedge \vec{B} - (\vec{E} \cdot \vec{v}) \vec{v} \right] + q \vec{v}$$

(1) observe $\sigma \rightarrow \infty$

$$\vec{v} \rightarrow \vec{0} \quad \vec{E} \perp \vec{v} \quad (\vec{E} \cdot \vec{v}) (1 - v^2) = 0$$

$$0 = -n^a \partial_a E + KE - \frac{1}{2} \partial_a (\alpha^2 S^a) + (11) K_{ab} \rightarrow d_0 E + d_0 V = \text{powers}$$

$$0 = D_a (\star F^{ab} n_b) \rightarrow \nabla \cdot B = 0$$

$$0 = \frac{1}{2} n^a \partial_a (\alpha D) + \frac{1}{2} D_a (\alpha D n^a) - KW$$

$$E_a = \frac{1}{2} F_{bc} n^c \quad B_a = \frac{1}{2} \epsilon_{abc} F^{bc}$$

$$E_a = \frac{1}{n_d u^d} \underbrace{\epsilon_{abc} u^b B^c}_{\sim u \wedge B}$$

$$\epsilon_{abc} = \epsilon_{abc} n^d$$

Ohm's Law

$$J_a + (J^b u_b) = \sigma F_{ab} u^b \rightarrow \vec{J} = \sigma W \cdot [\vec{E} + \vec{v} \wedge \vec{B} - (\vec{E} \cdot \vec{v}) \vec{v}] + q \vec{v}$$

(1) observe $\sigma \rightarrow \infty$

$$\begin{aligned} & \vec{v} \rightarrow \vec{0} \\ & \vec{E} \perp \vec{v} \\ & \vec{E} \cdot \vec{v} = 0 \\ & \vec{E} = -\vec{v} \wedge \vec{B} \end{aligned}$$

Maxwell's eqns

$$\vec{\nabla} \cdot \vec{E} = \rho$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\partial_t \vec{E} - \vec{\nabla} \times \vec{B} = -\vec{J}$$

$$\partial_t \vec{B} + \vec{\nabla} \times \vec{E} = 0$$

$$\left. \begin{array}{l} \partial_t \vec{E} - \vec{\nabla} \times \vec{B} = -\vec{J} \\ \partial_t \vec{B} + \vec{\nabla} \times \vec{E} = 0 \end{array} \right\} (\partial_t \vec{B} - \vec{\nabla} \times \vec{E}) = \sigma (\partial_t \vec{B} - \vec{\nabla} \times (\vec{v} \times \vec{B}))$$

Maxwell's eqns

$$\vec{\nabla} \cdot \vec{E} = \rho$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\partial_t \vec{E} - \vec{\nabla} \times \vec{B} = -\vec{J}$$

$$\partial_t \vec{B} + \vec{\nabla} \times \vec{E} = 0$$

$$\left. \begin{array}{l} \partial_t \vec{E} - \vec{\nabla} \times \vec{B} = -\vec{J} \\ \partial_t \vec{B} + \vec{\nabla} \times \vec{E} = 0 \end{array} \right\} (\partial_t \vec{B} - \vec{\nabla} \times \vec{E}) = \sigma (\partial_t \vec{B} - \vec{\nabla} \times \vec{E})$$

L

Maxwell's eqs

$$\vec{\nabla} \cdot \vec{E} = \rho$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\partial_t \vec{E} - \vec{\nabla} \times \vec{B} = -\vec{J}$$

$$\partial_t \vec{B} + \vec{\nabla} \times \vec{E} = 0$$

$$\left. \begin{array}{l} \partial_t \vec{E} - \vec{\nabla} \times \vec{B} = -\vec{J} \\ \partial_t \vec{B} + \vec{\nabla} \times \vec{E} = 0 \end{array} \right\} (\partial_t \vec{B} - \vec{\nabla} \times \vec{E}) = \sigma (\partial_t \vec{B} - \vec{\nabla} \times \vec{E})$$

Maxwell's eqns

$E =$

$$\vec{\nabla} \cdot \vec{E} = \rho$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\partial_t \vec{E} - \vec{\nabla} \times \vec{B} = -\vec{J}$$

$$\partial_t \vec{B} + \vec{\nabla} \times \vec{E} = 0$$

$$\left. \begin{array}{l} \partial_t \vec{E} - \vec{\nabla} \times \vec{B} = -\vec{J} \\ \partial_t \vec{B} + \vec{\nabla} \times \vec{E} = 0 \end{array} \right\} (\partial_t \vec{B} - \vec{\nabla} \times \vec{E}) = \sigma (\partial_t \vec{B} - \vec{\nabla} \times (\vec{v} \times \vec{B}))$$

L

Maxwell's eqns

$$\partial_t E \approx 0$$

$$\vec{\nabla} \cdot \vec{E} = \rho$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\partial_t \vec{E} - \vec{\nabla} \times \vec{B} = -\vec{J}$$

$$\partial_t \vec{B} + \vec{\nabla} \times \vec{E} = 0$$

$$\left. \begin{array}{l} \partial_t \vec{E} - \vec{\nabla} \times \vec{B} = -\vec{J} \\ \partial_t \vec{B} + \vec{\nabla} \times \vec{E} = 0 \end{array} \right\} (\partial_t \vec{B} - \vec{\nabla} \times \vec{B}) = \sigma (\partial_t \vec{B} + \vec{\nabla} \times \vec{B})$$

Maxwell's eqns

$$\vec{\nabla} \cdot \vec{E} = \rho$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\partial_t \vec{E} - \vec{\nabla} \times \vec{B} = -\vec{J}$$

$$\partial_t \vec{B} + \vec{\nabla} \times \vec{E} = 0$$

$$\partial_t E \approx 0 \approx \partial_t B$$

$$\left. \begin{array}{l} \partial_t \vec{E} - \vec{\nabla} \times \vec{B} = -\vec{J} \\ \partial_t \vec{B} + \vec{\nabla} \times \vec{E} = 0 \end{array} \right\} \left(\partial_t \vec{B} - \vec{\nabla}^2 \vec{B} - (\partial_t \vec{B} - \vec{\nabla} \cdot (\vec{v} \cdot \vec{B})) \right)$$

Maxwell's eqns

$$\vec{\nabla} \cdot \vec{E} = \rho$$

$$\vec{\nabla} \times \vec{B} = 0$$

$$\partial_t \vec{E} - \vec{\nabla} \times \vec{B} = -\vec{J}$$

$$\partial_t \vec{B} + \vec{\nabla} \times \vec{E} = 0$$

$$\partial_t E \approx 0 \approx \partial_t B$$

$$\left(\frac{\partial_t \vec{B} - \vec{\nabla} \times \vec{E}}{\sigma} \right) = 0 \quad \left(\partial_t \vec{B} - \vec{\nabla} \times (\nu \nabla \vec{B}) \right)$$

Maxwell's eqns

$$\vec{\nabla} \cdot \vec{E} = \rho$$

$$\vec{\nabla} \times \vec{B} = 0$$

$$\partial_t \vec{E} - \vec{\nabla} \times \vec{B} = -\vec{J}$$

$$\partial_t \vec{B} + \vec{\nabla} \times \vec{E} = 0$$

$$\partial_t \vec{E} \approx 0 \approx \partial_t \vec{B}$$

$$\left(\underbrace{\partial_t \vec{B} - \vec{\nabla} \times \vec{E}}_{=0} \right) = 0 \left(\underbrace{\partial_t \vec{B} - \vec{\nabla} \times (\vec{v} \times \vec{B})}_{=0} \right)$$

Force-Free approximation