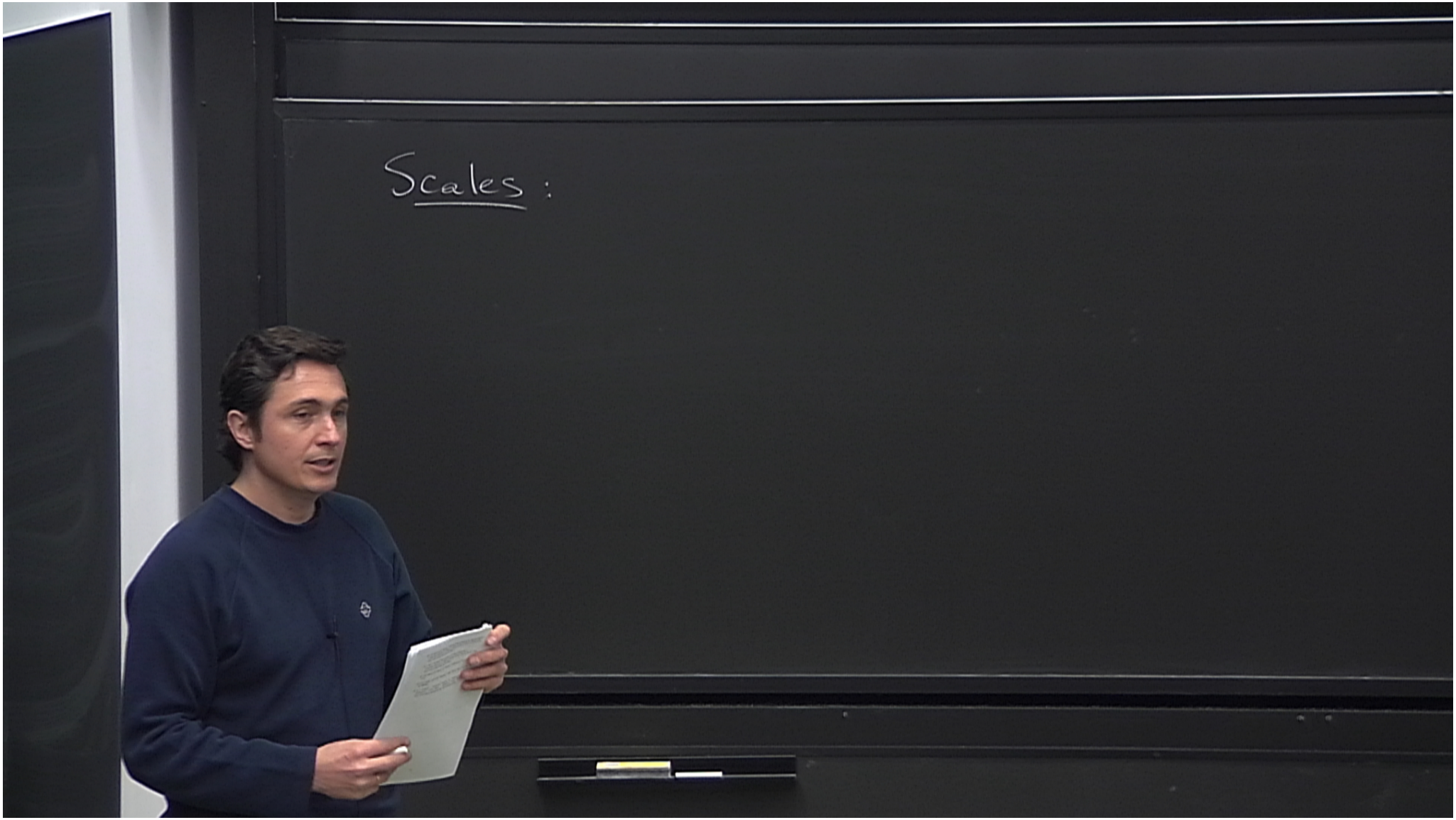
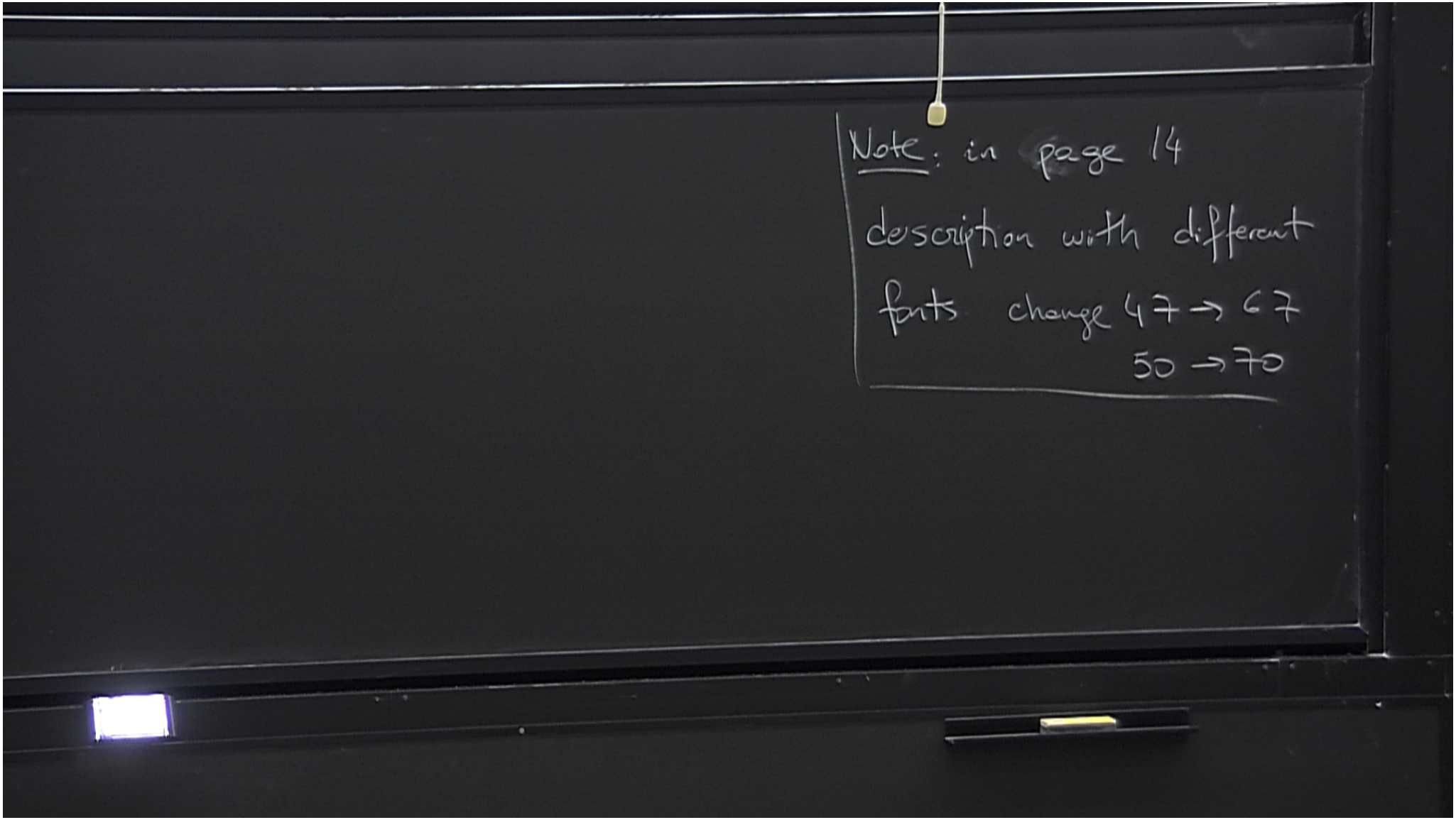
Title: Explorations in Numerical Relativity - Lecture 7

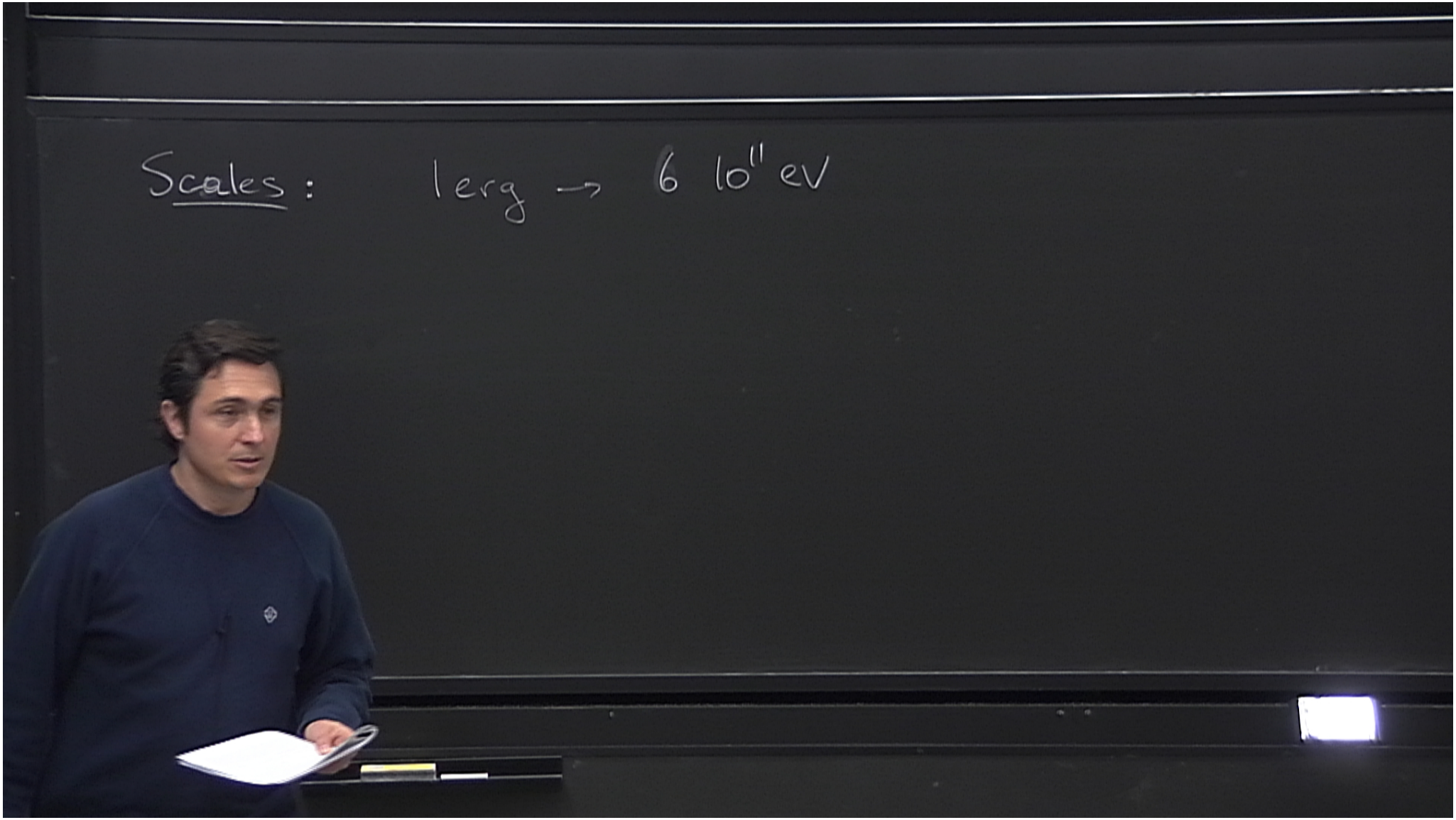
Date: Apr 11, 2012 11:30 AM

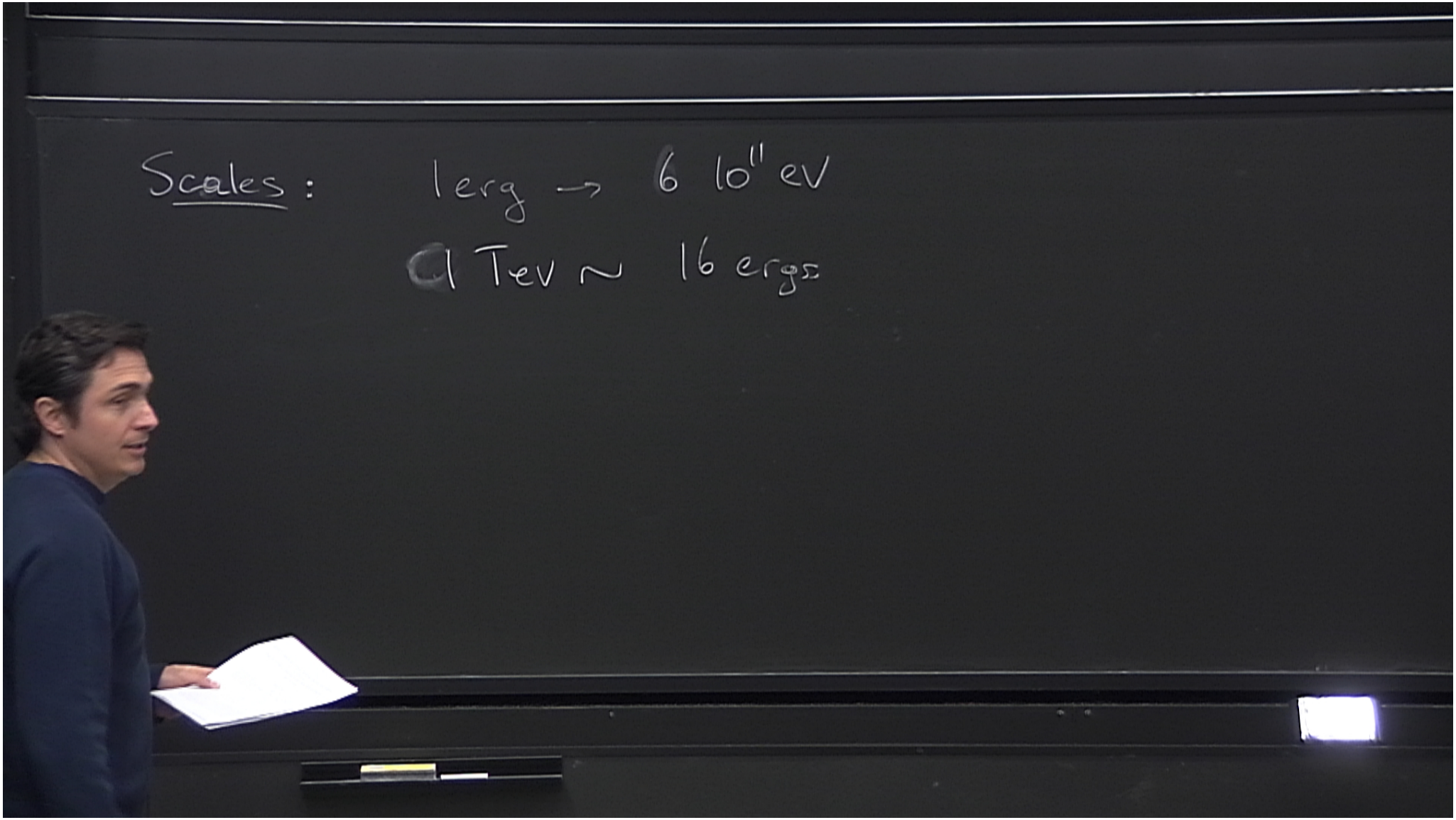
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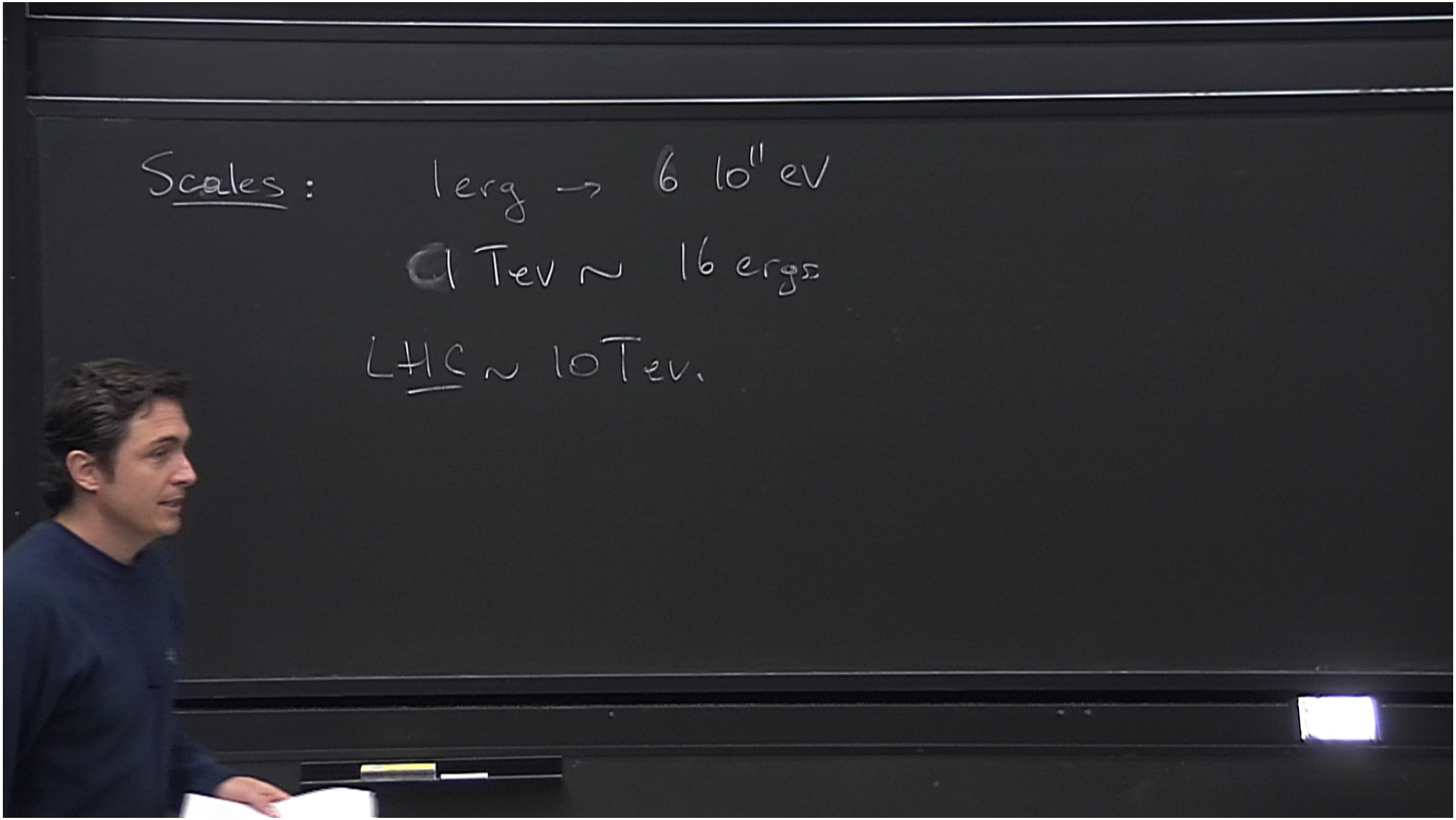
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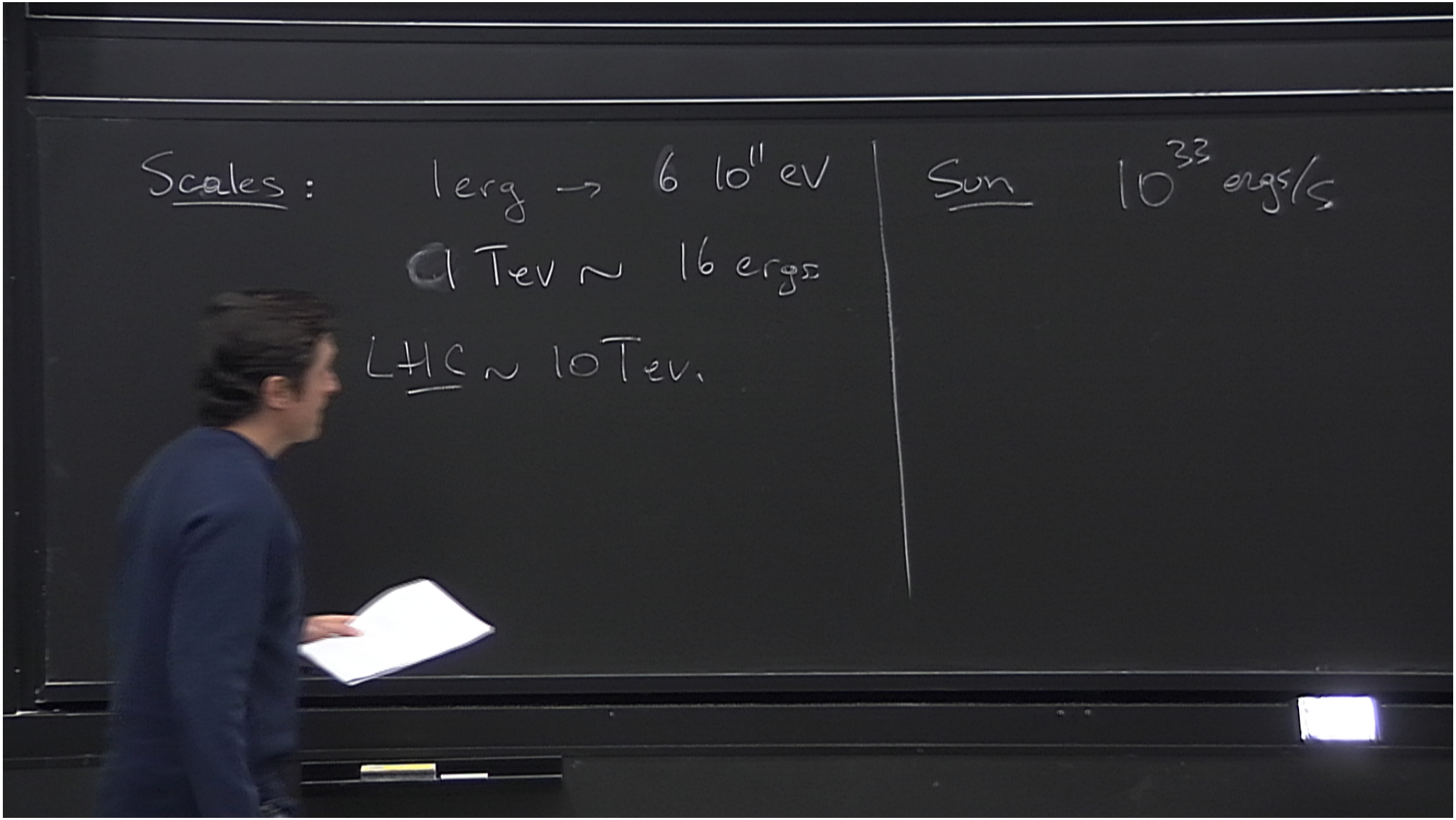

Scales :

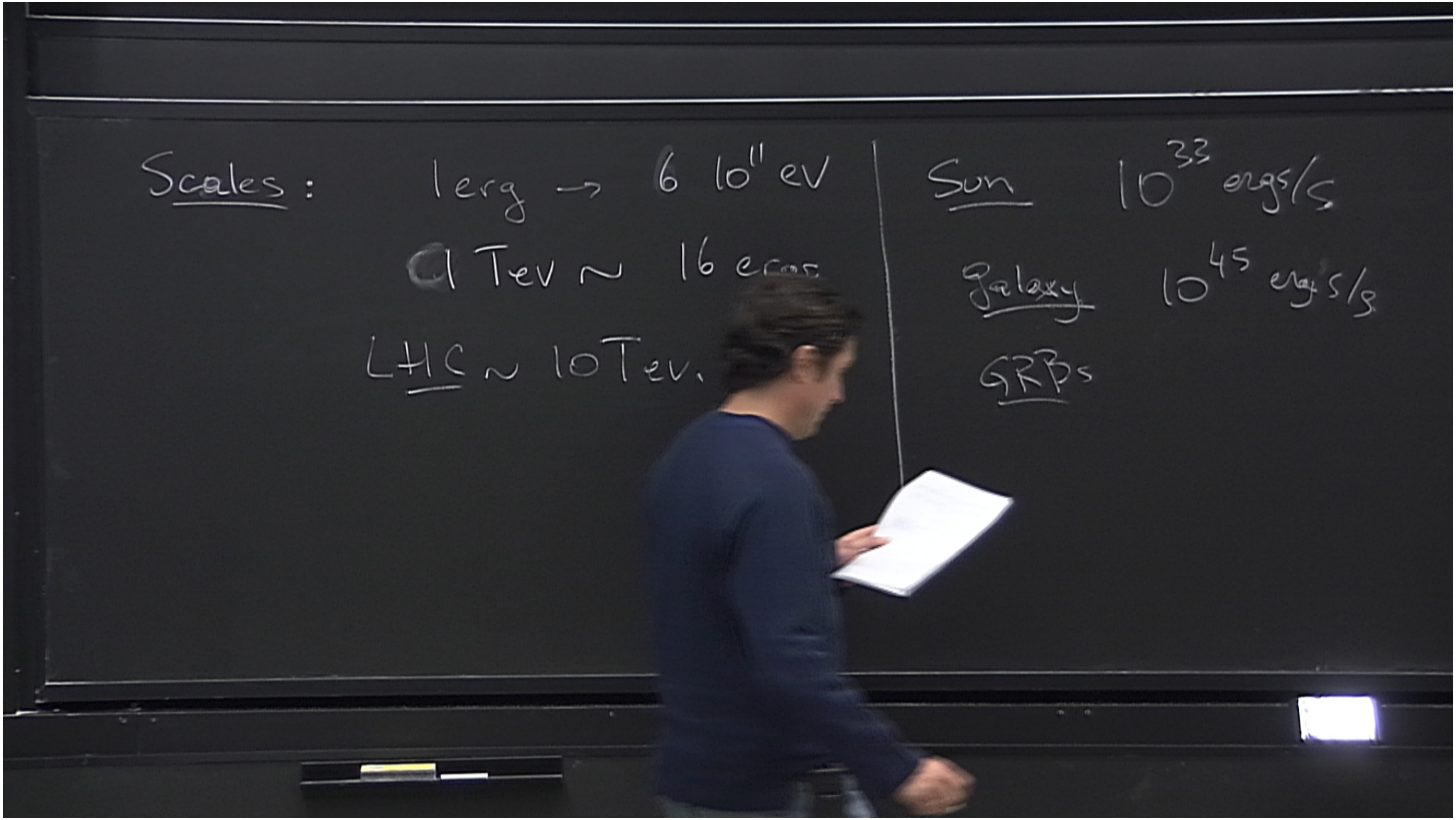

Note: in page 14
description with different
fonts change 47 → 67
50 → 70

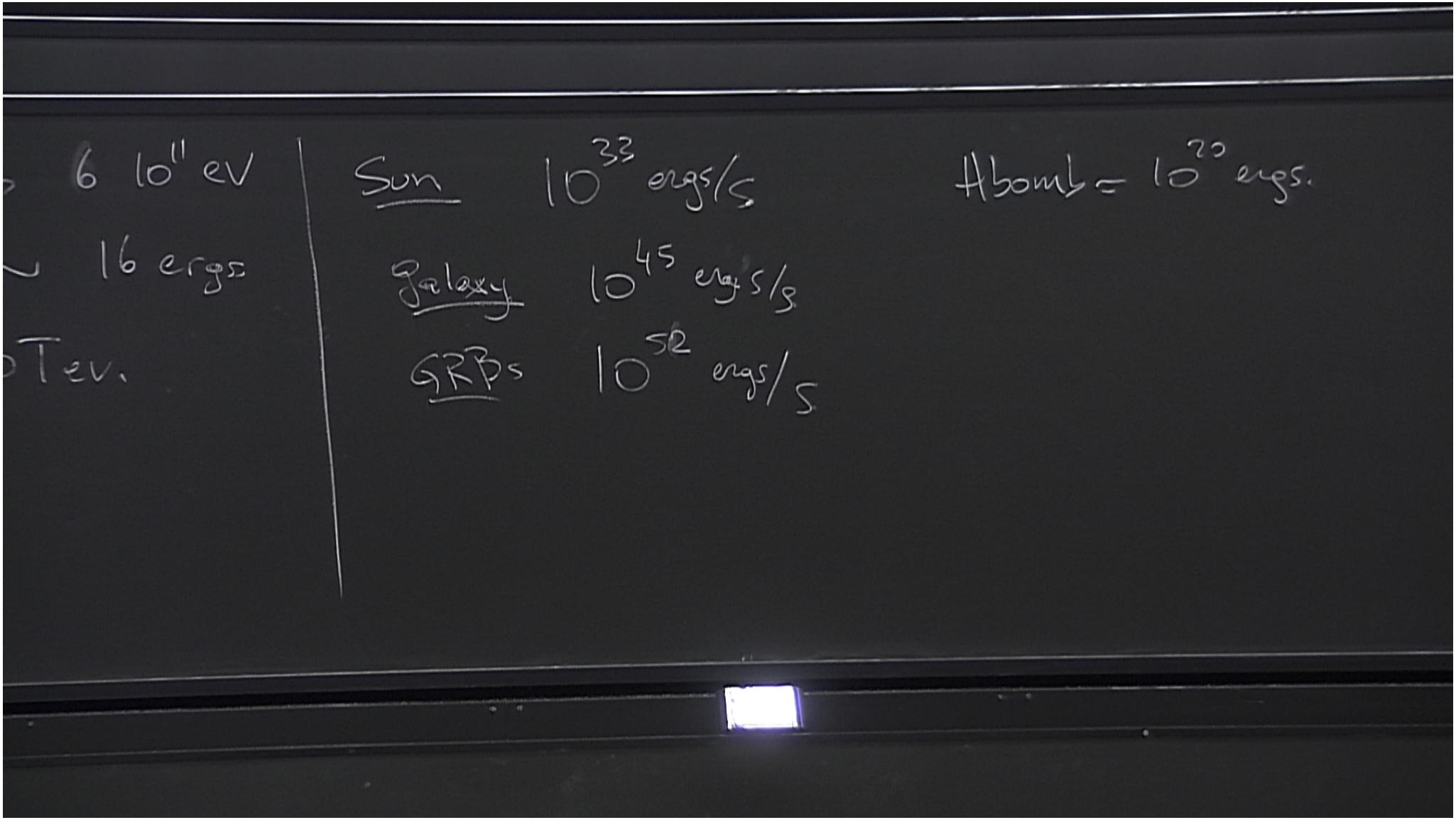

Scales: 1 erg → 6 10^11 eV

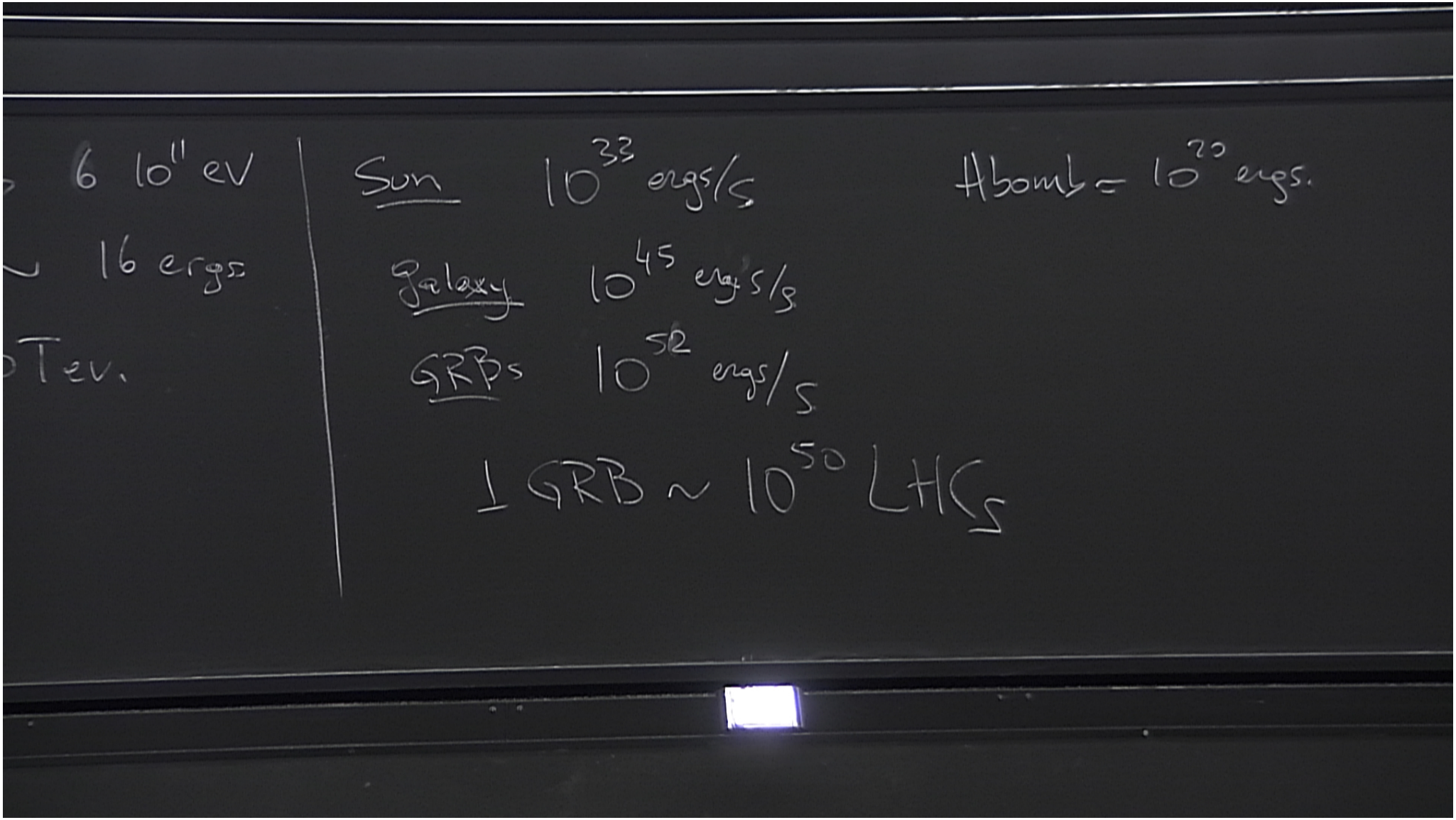

Scales:
1 erg → 6 10^11 eV
1 Tev ~ 16 ergs


Scales: 1 erg → 6 10^11 eV
1 Tev ~ 16 ergs
LHC ~ 10 Tev.


Scales: 1 erg → 6 10^11 eV
1 Tev ~ 16 ergs
LHC ~ 10 Tev.
Sun 10^33 ergs/s


Scales:
1 erg → 6 10^11 eV
1 Tev ~ 1.6 ergs
LHC ~ 10 Tev.
Sun 10^33 ergs/s
Galaxy 10^45 ergs/s
GRBs


6 10^11 eV
16 ergs
Tev.
Sun 10^33 ergs/s
Galaxy 10^45 ergs/s
GRBs 10^52 ergs/s
Abomb = 10^20 ergs.


6 10^{11} eV
~ 16 ergs
Tev.
Sun 10^{33} ergs/s
Galaxy 10^{45} ergs/s
GRBs 10^{52} ergs/s
1 GRB ~ 10^{50} LHCs
Hbomb = 10^{20} ergs.

6 10^{11} eV

~ 16 ergs

Tev.

Sun 10^{33} ergs/s

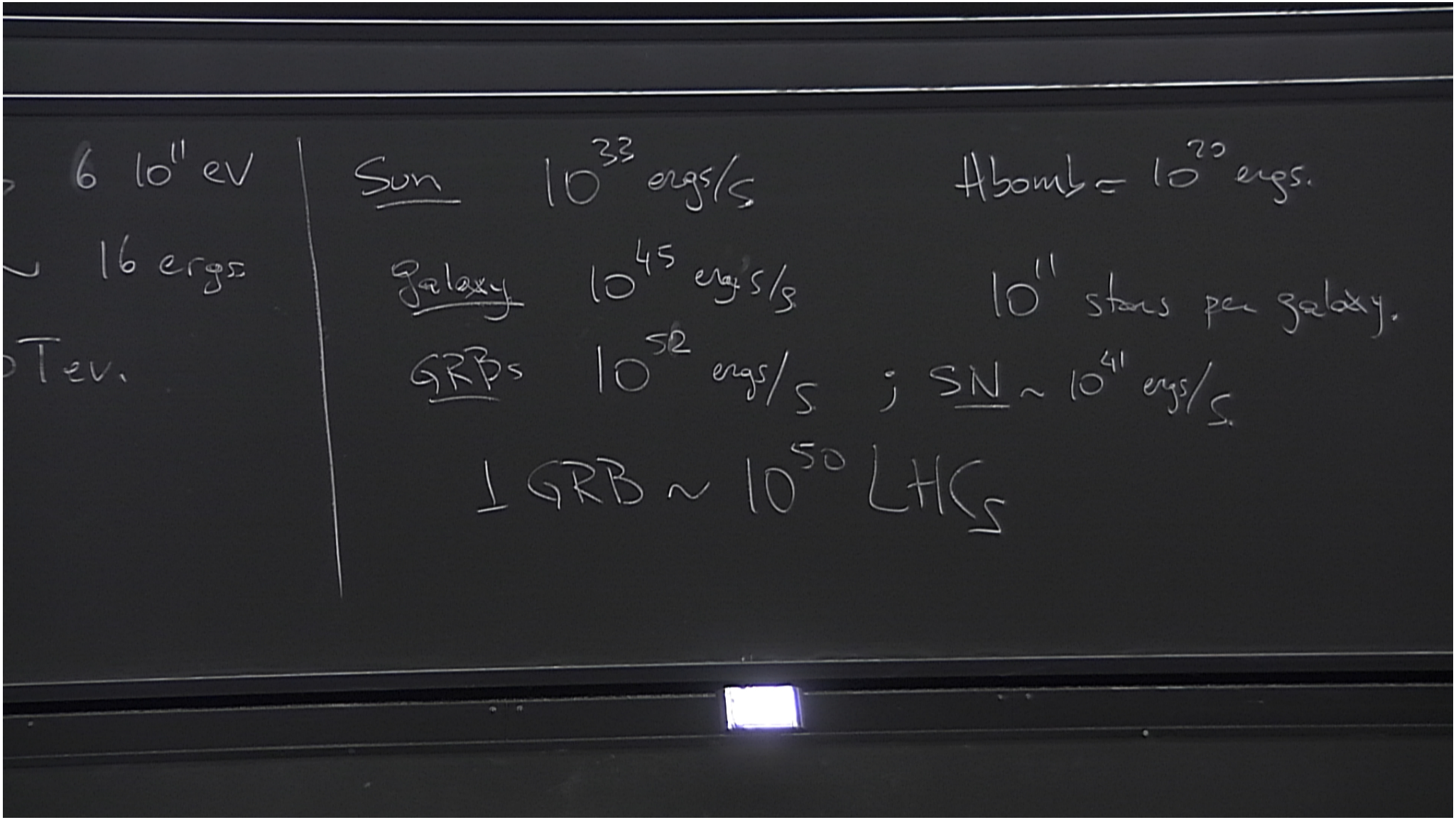
Galaxy 10^{45} ergs/s

GRBs 10^{52} ergs/s

1 GRB ~ 10^{50} LHCs

H bomb = 10^{20} ergs.

10^{11} stars per galaxy.


6 10^11 eV
~ 16 ergs
Tev.
Sun 10^33 ergs/s
Galaxy 10^45 ergs/s
GRBs 10^52 ergs/s ; SN ~ 10^41 ergs/s.
1 GRB ~ 10^50 LHCs
Hbomb = 10^20 ergs.
10^11 stars per galaxy.

ergs/s

45 ergs/s

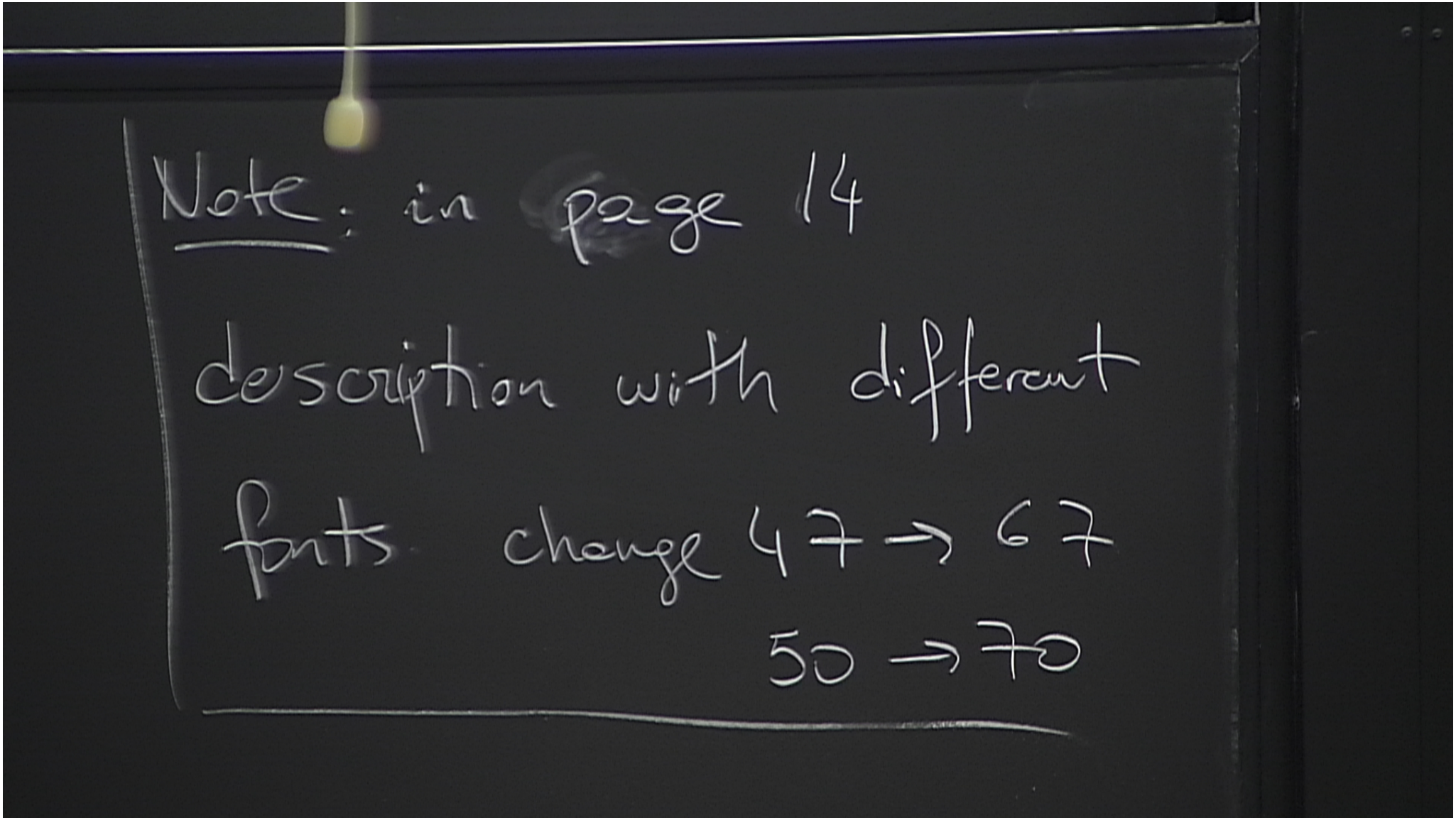
52 ergs/s ; SN ~ 10^{41} ergs/s.

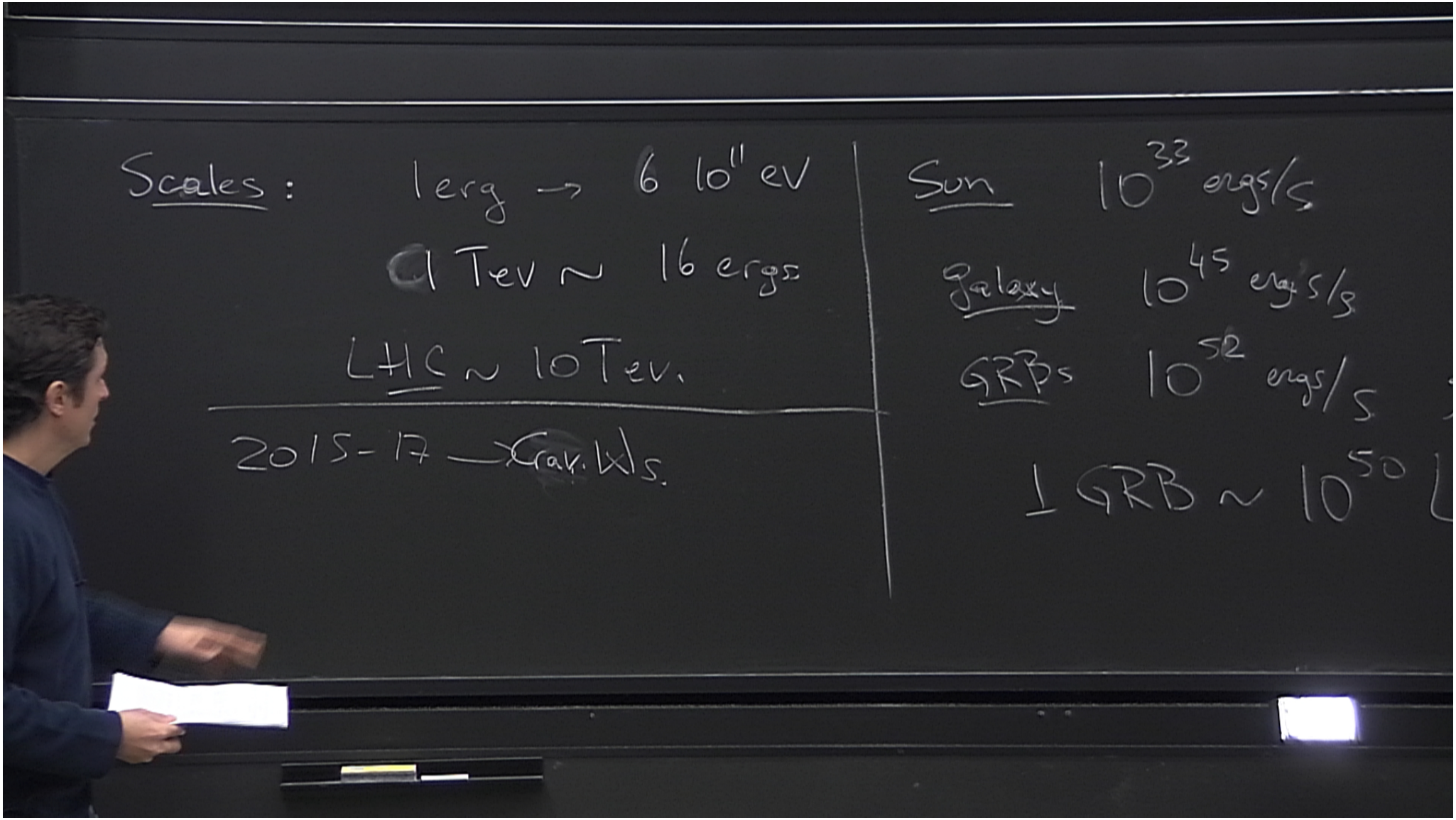
~ 10^{50} LHCs

Abomb ~ 10^{20} ergs.

10^{11} stars per galaxy.

Note: in page 14
description with different
fonts change 47 → 67
50 → 70


Note: in page 14
description with different
fonts change 47 → 67
50 → 70


Scales: 1 erg → 6 10^11 eV
1 Tev ~ 16 ergs
LHC ~ 10 Tev.
2015-17 — Grav. Ws.
Sun 10^33 ergs/s
Galaxy 10^45 ergs/s
GRBs 10^52 ergs/s
1 GRB ~ 10^50

ergs/s

^{45} ergs/s

ergs/s ; SN ~ 10^{41} ergs/s.

~ 10^{50} LHCs

Abomb ~ 10^{20} ergs.

10^{11} stars per galaxy.

Note: in page 14
description with different
fonts change 47 → 67
50 → 70



ergs/s

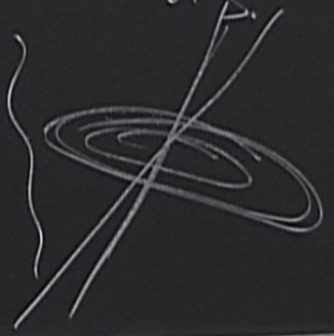
^{45} ergs/s

^{52} ergs/s ; SN ~ 10^{41} ergs/s

~ 10^{50} LHCs

Abomb ~ 10^{20} ergs.

10^{11} stars per galaxy.



Note: in page 14
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50 → 70

ergs/s

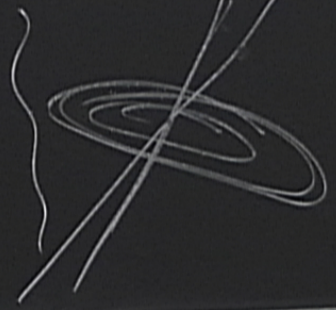
^{45} ergs/s

^{52} ergs/s ; SN ~ 10^{41} ergs/s

~ 10^{50} LHCs

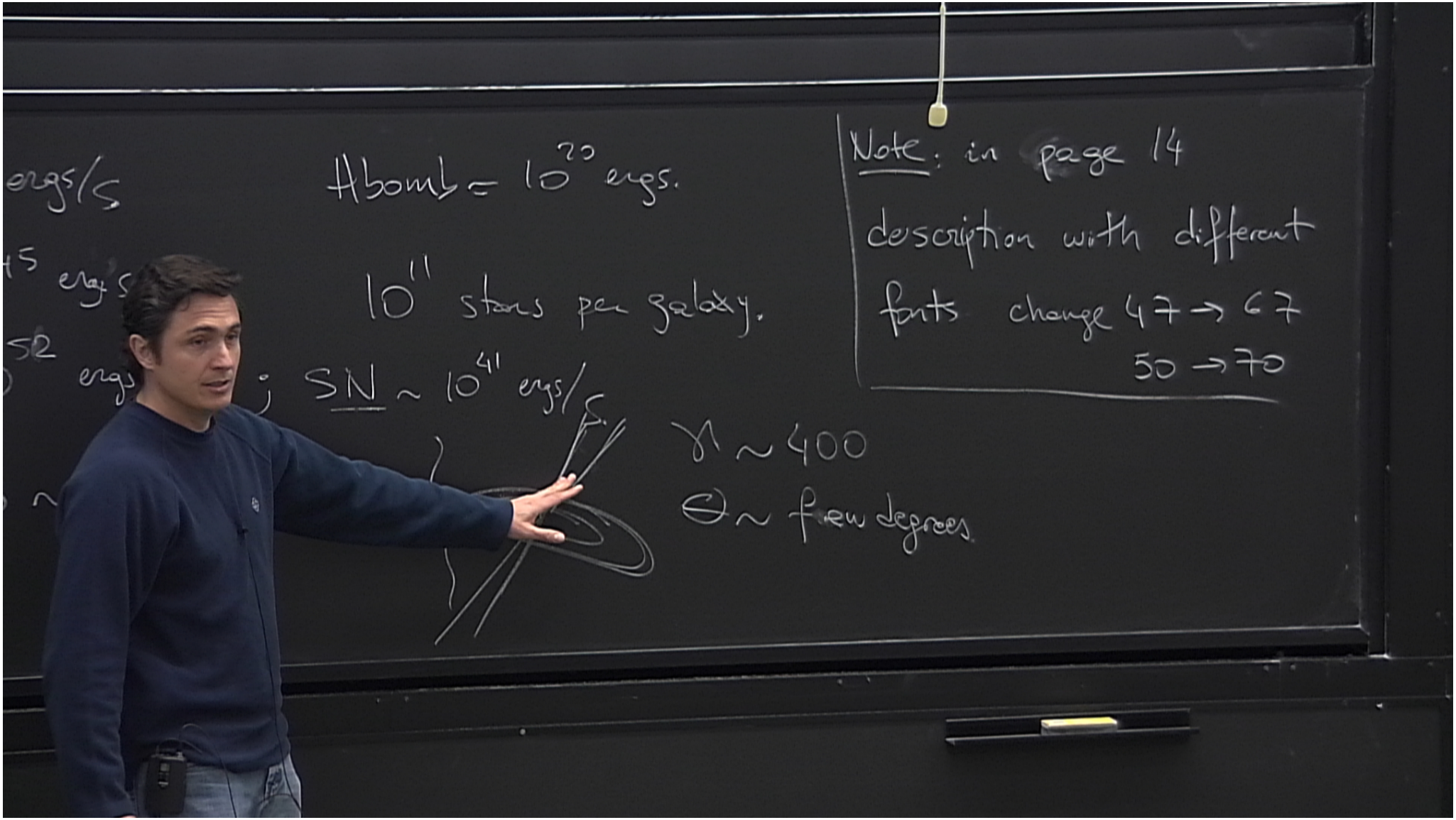
Abomb ~ 10^{20} ergs.

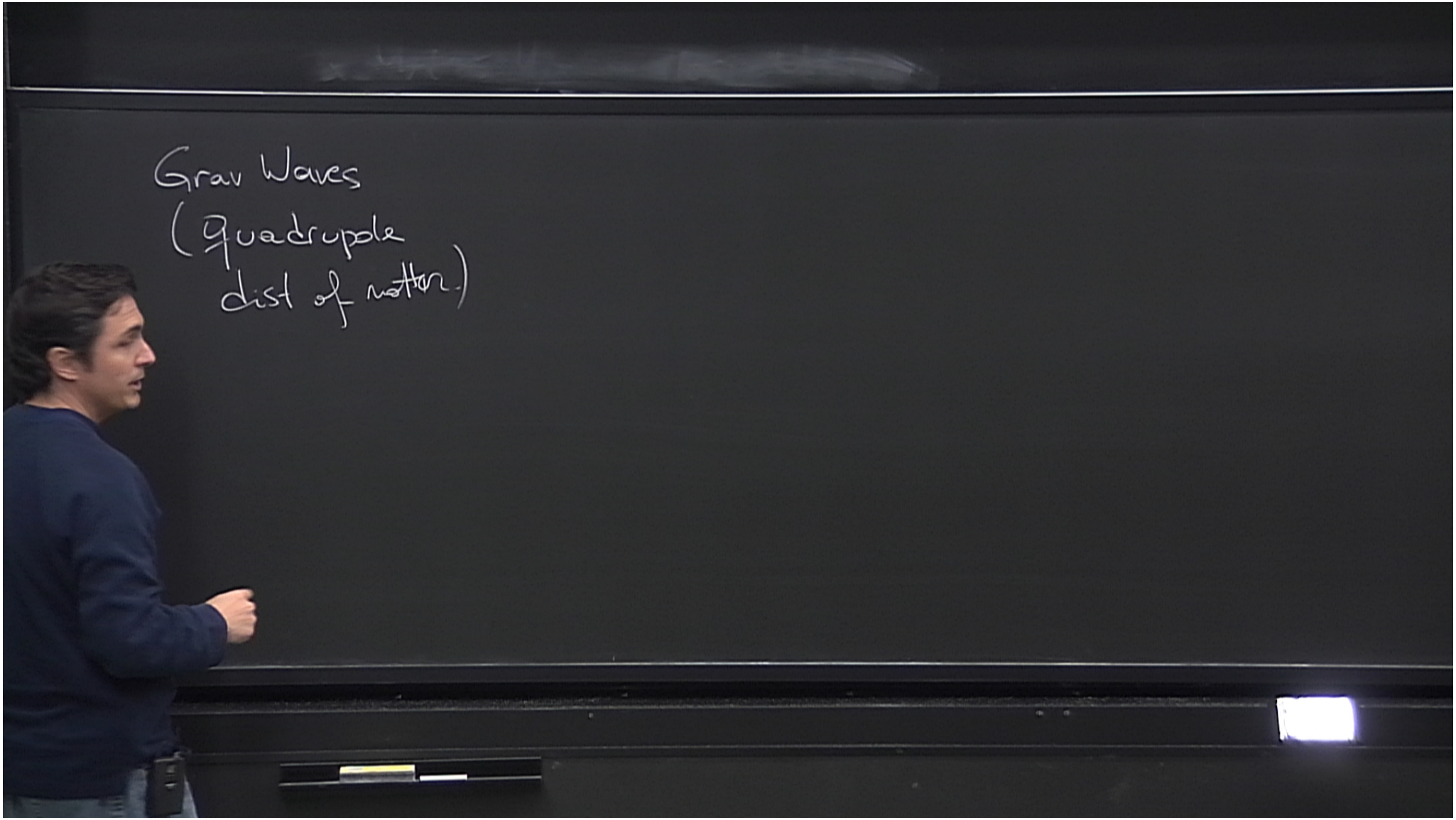
10^{11} stars per galaxy.

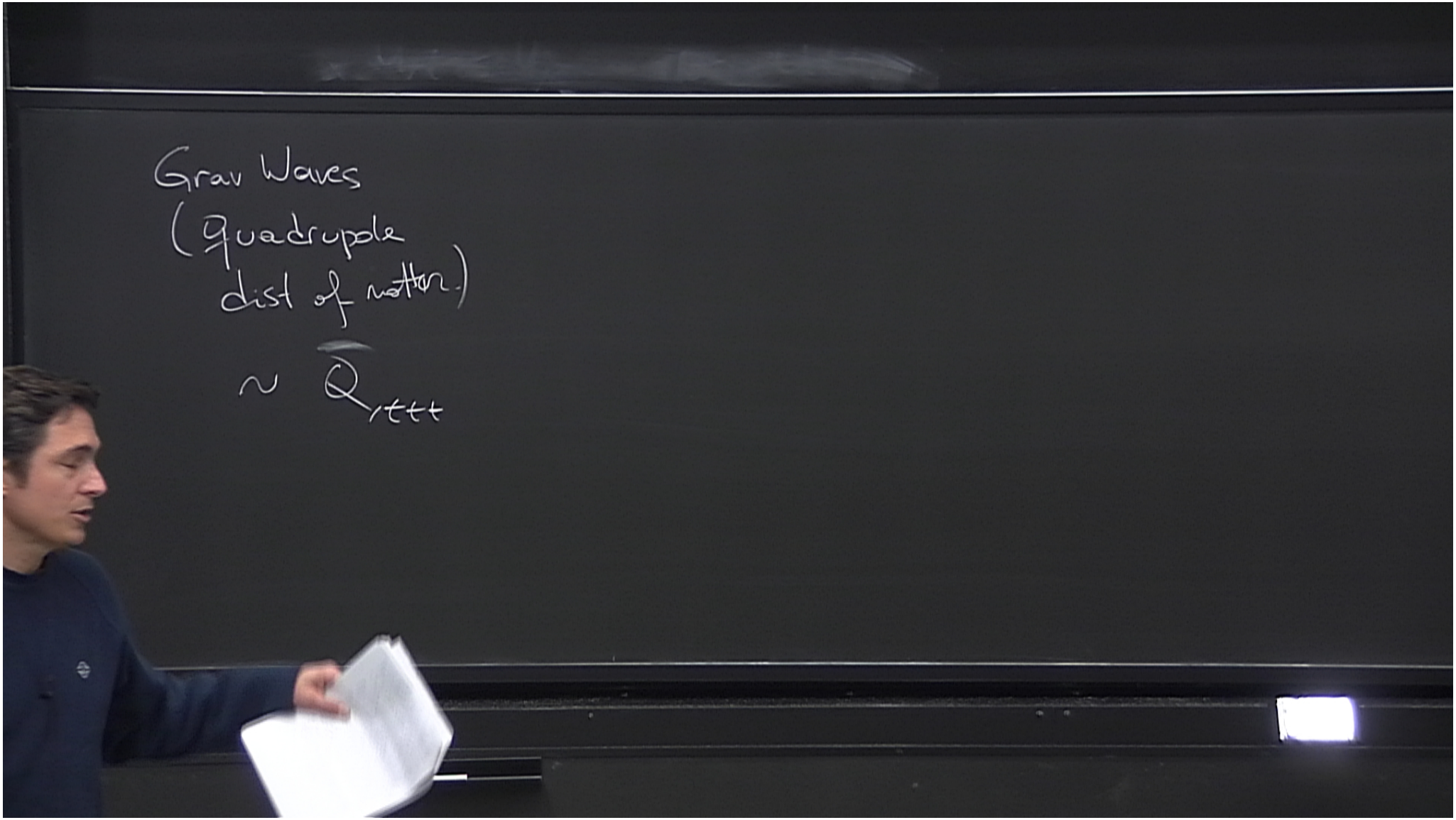


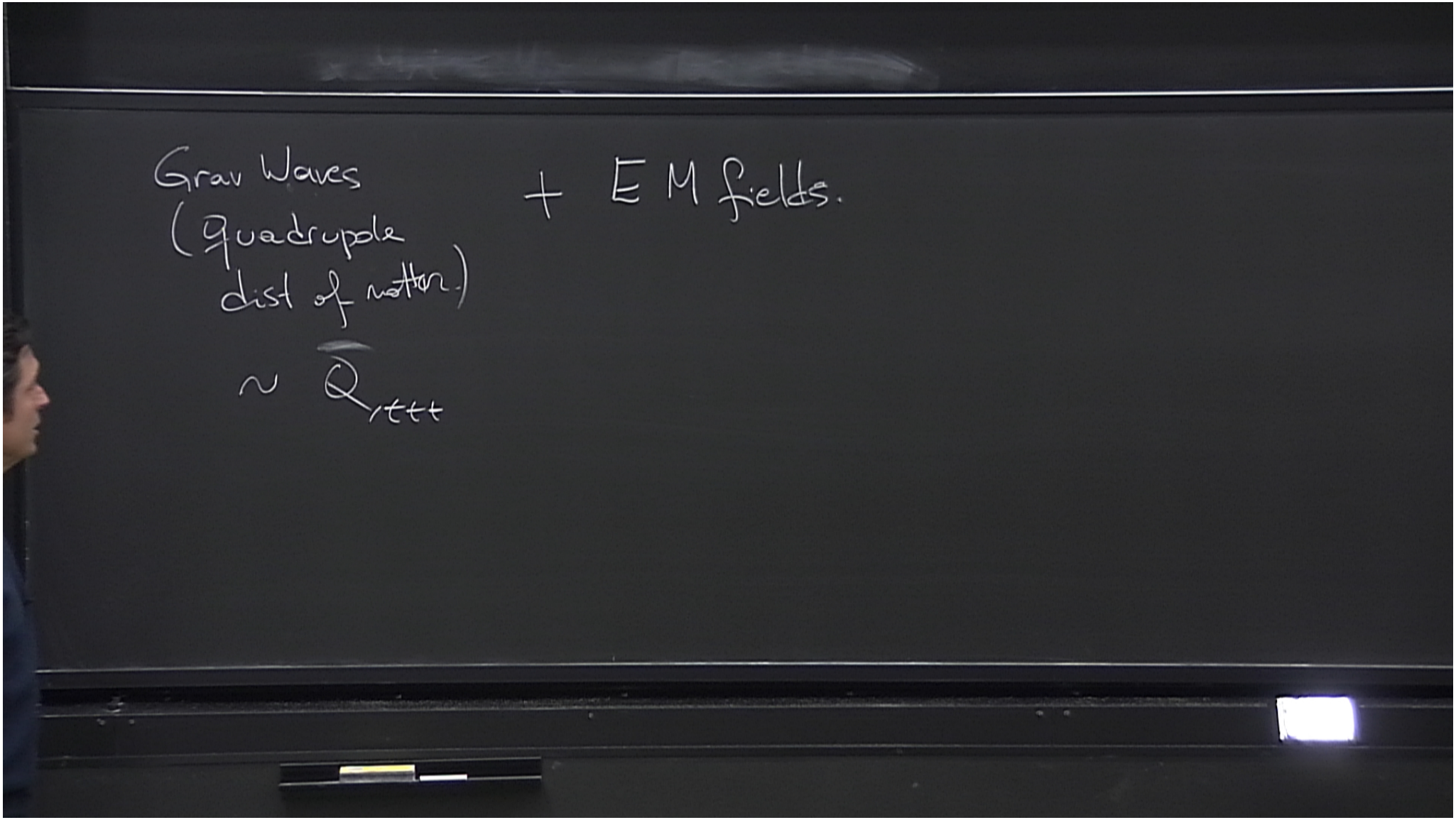
$\gamma \sim 400$

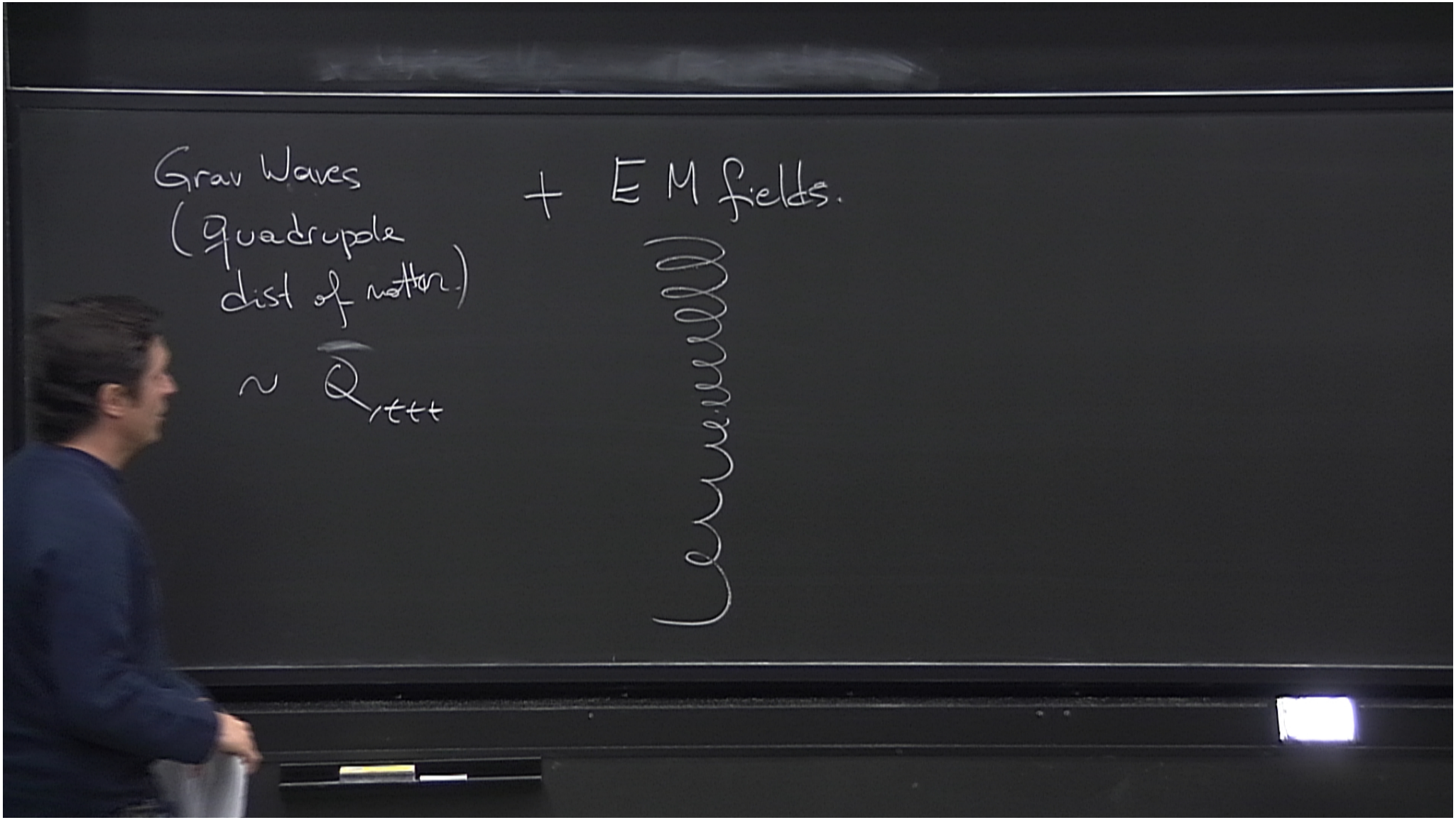
Note: in page 14
description with different
fonts change 47 → 67
50 → 70

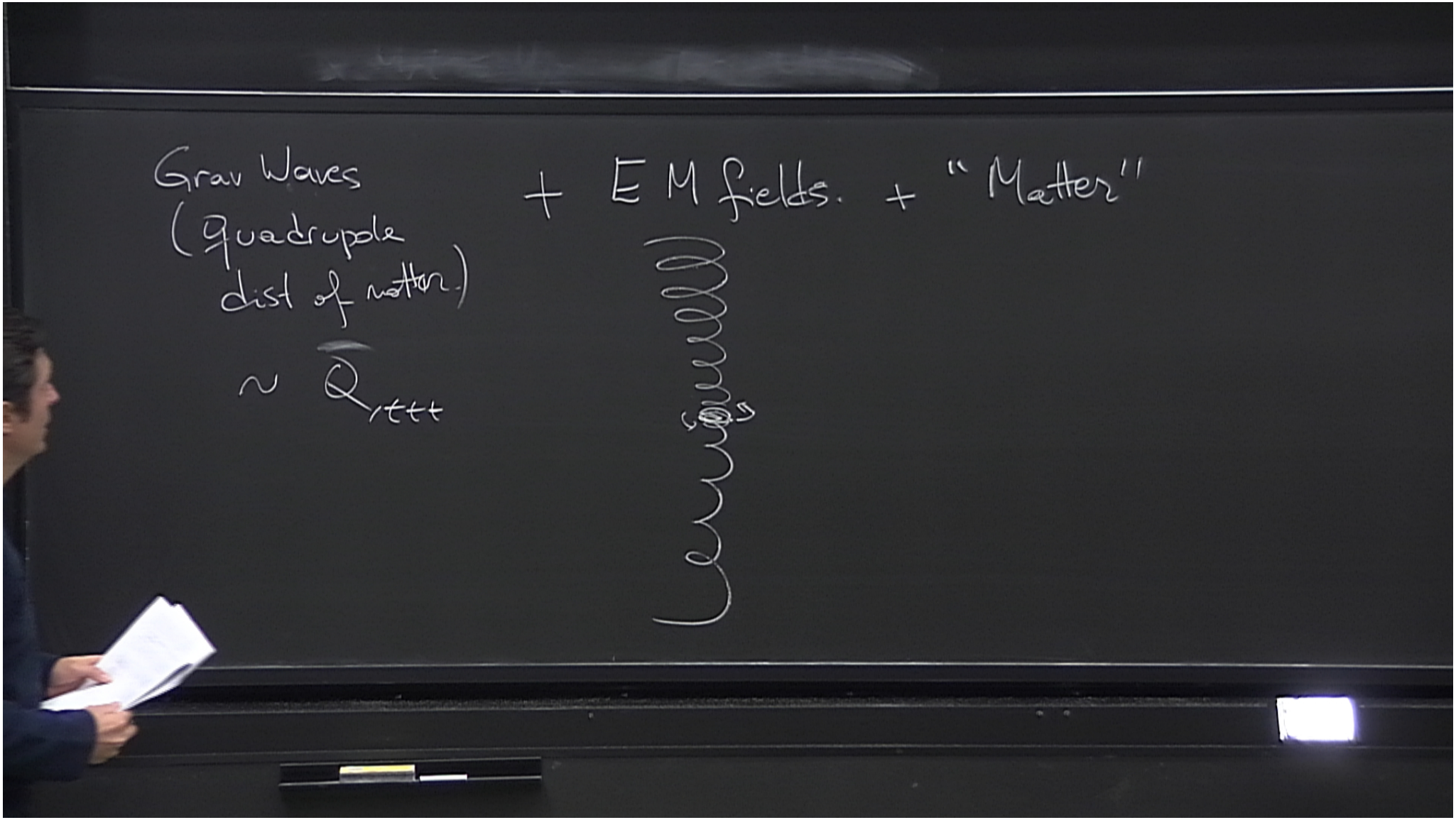

ergs/s
Abomb ~ 10^{20} ergs.
10^{11} stars per galaxy.
SN ~ 10^{41} ergs/
Note: in page 14
description with different
fonts change 47 → 67
50 → 70
γ ~ 400
θ ~ few degrees

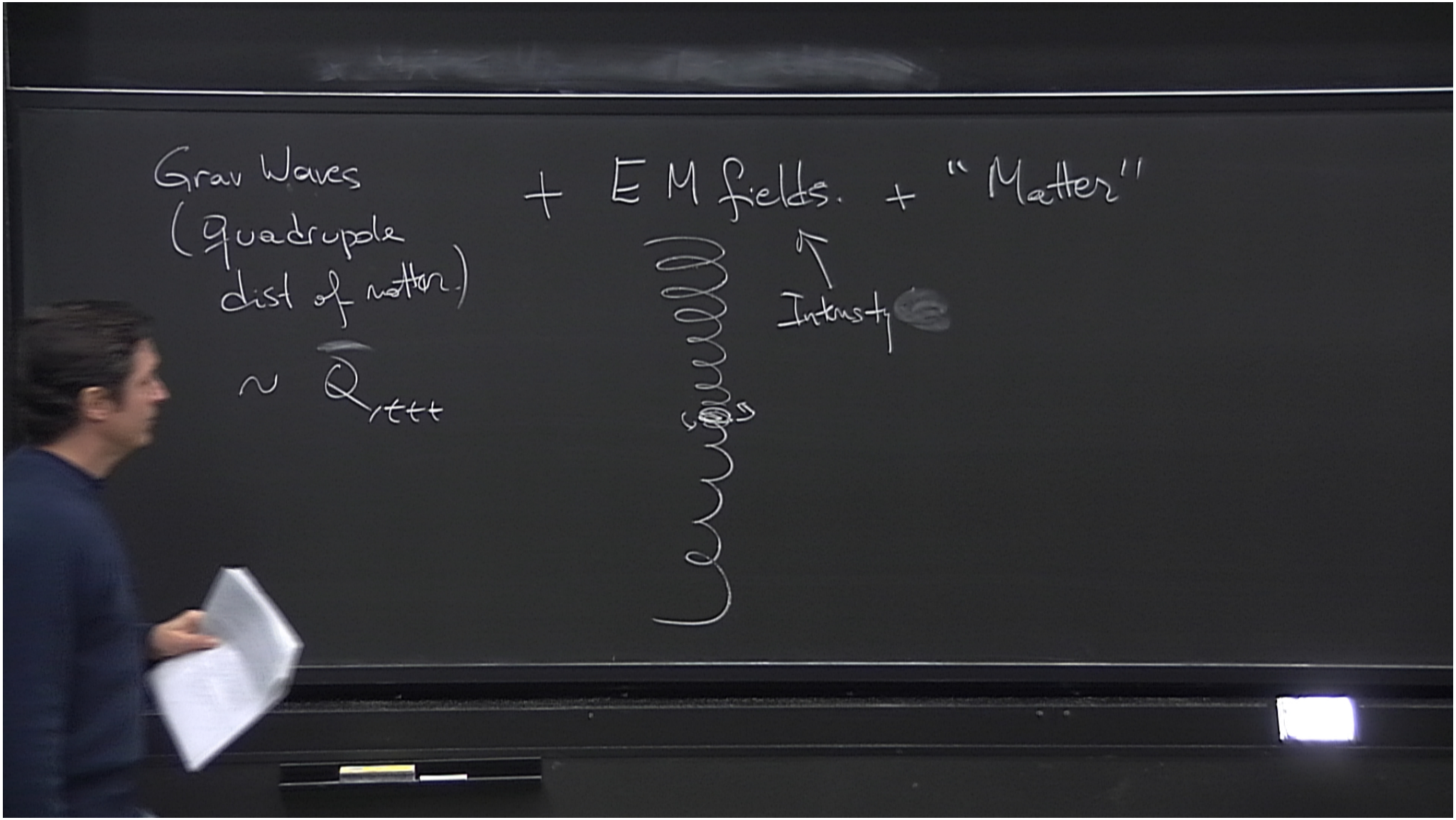

Grav Waves
(quadrupole
dist of matter)


Grav Waves
(quadrupole
dist of matter)
~ $\ddot{Q}_{ijtt}$


Grav Waves
(quadrupole
dist of matter)
+ EM fields.
~ $\ddot{Q}_{ijtt}$


Grav Waves
(quadrupole
dist of matter)
~ $\ddot{Q}_{ijtt}$
+ EM fields.

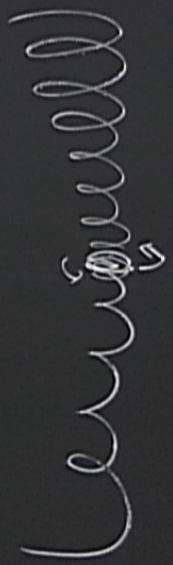

Grav Waves
(quadrupole
dist of matter)
~ $\ddot{Q}_{ijtt}$
+ EM fields. + "Matter"


Grav Waves
(quadrupole
dist of matter)
+ EM fields. + "Matter"
Intnsty

Grav Waves (quadrupole dist of matter)

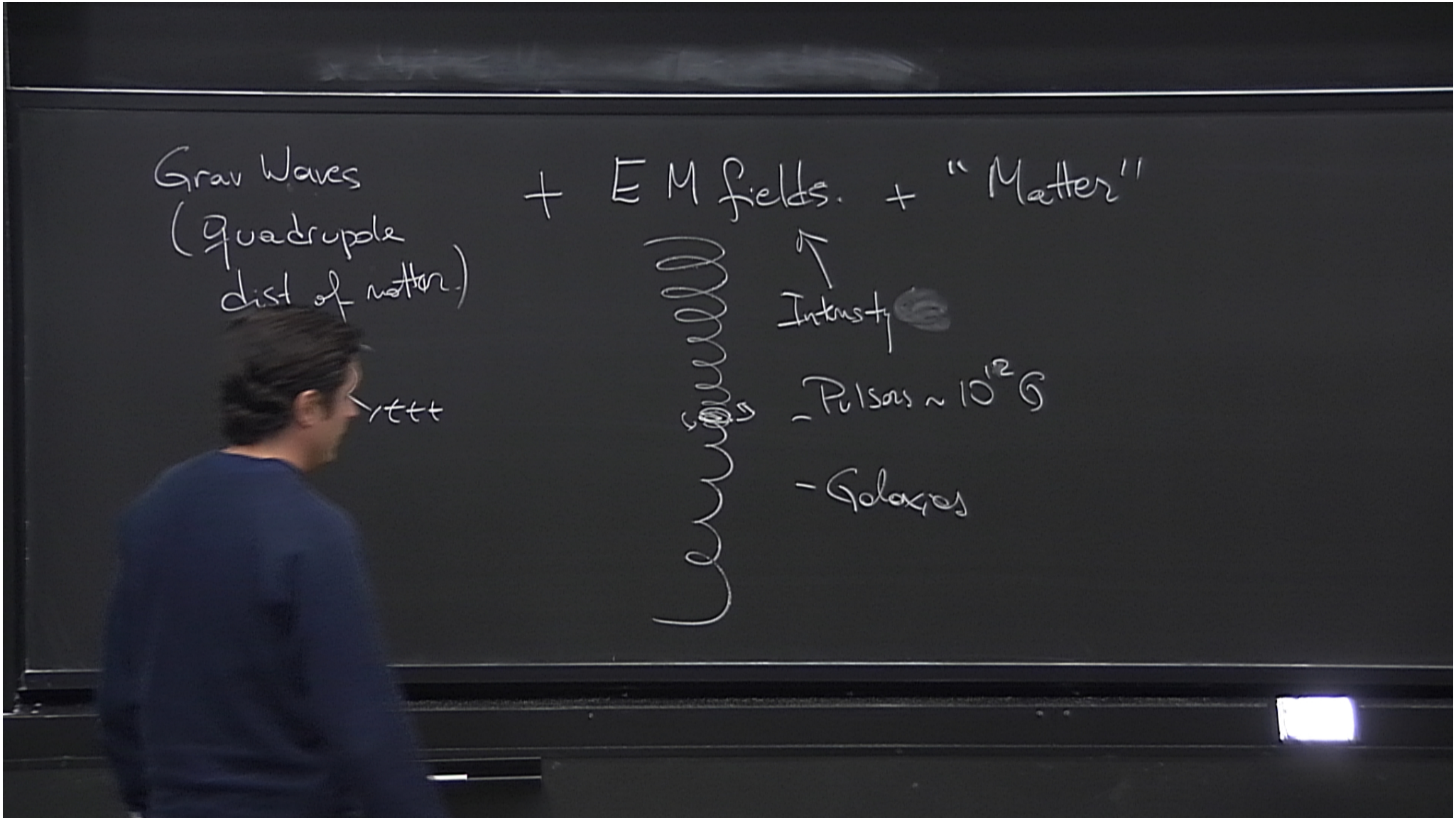
$\sim \ddot{Q}_{ijtt}$

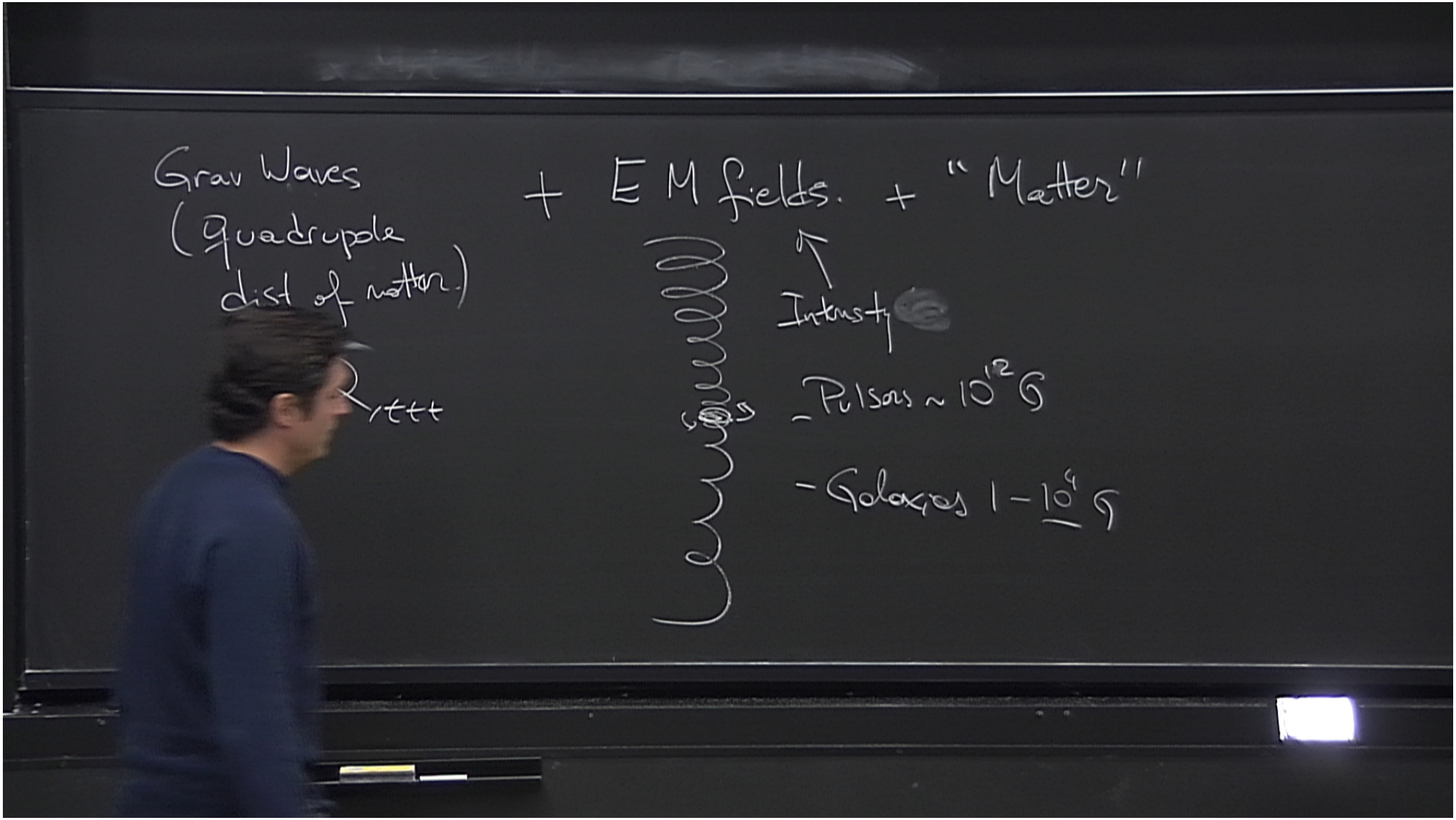
+ EM fields. + "Matter"



Intensity

Pulsars $\sim 10^{12}$ G

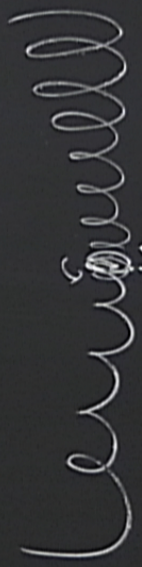

Grav Waves
(quadrupole
dist of matter)
+ EM fields. + "Matter"
Intensity
- Pulsars ~ 10^{12} G
- Galaxies


Grav Waves
(quadrupole
dist of matter)
+ EM fields. + "Matter"
Intensity
- Pulsars ~ 10^{12} G
- Galaxies 1 - 10^{4} G

Grav Waves (quadrupole dist of matter)

$\sim \ddot{Q}_{ijtt}$

+ EM fields. + "Matter"



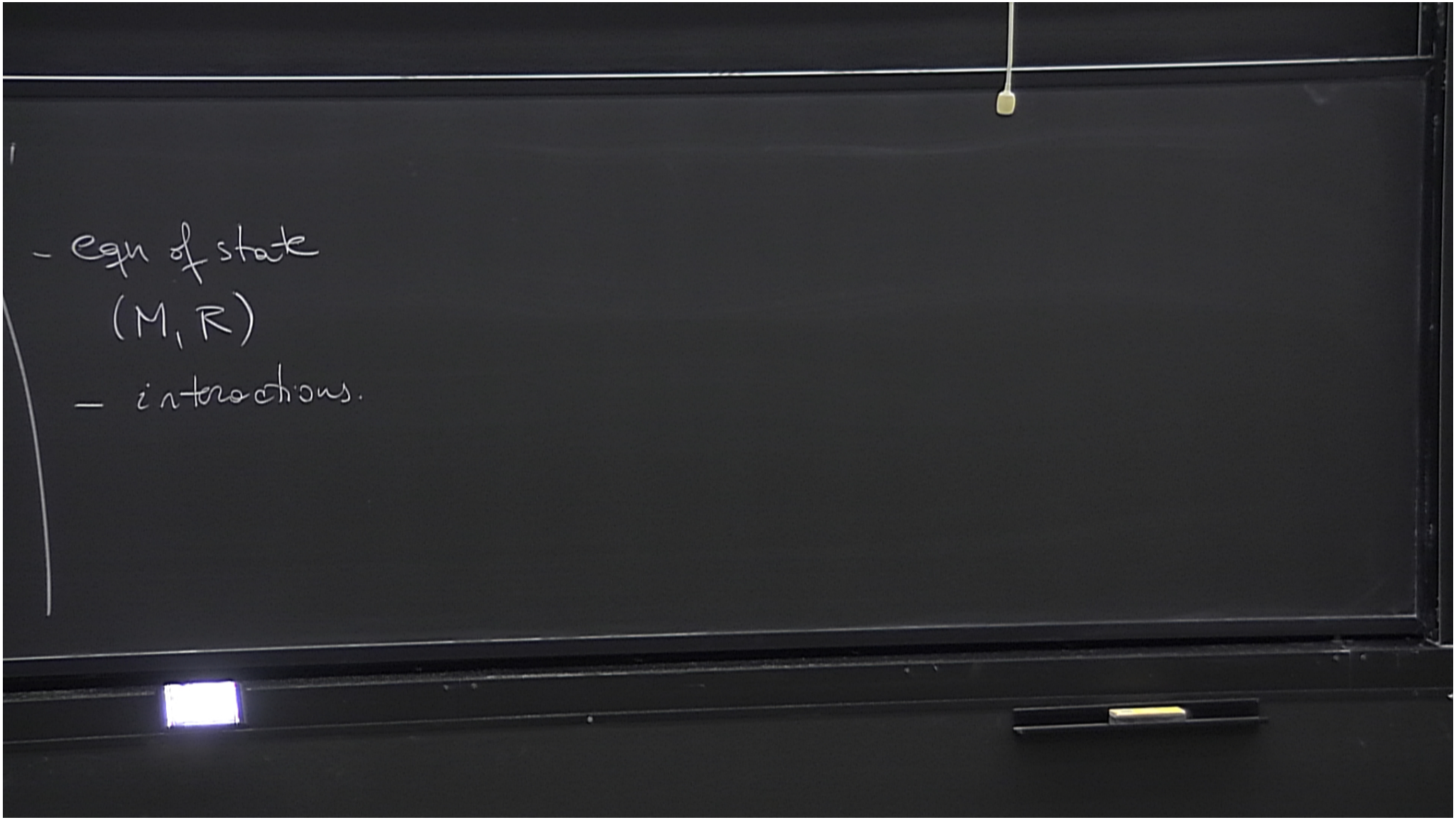
↑ Intensity

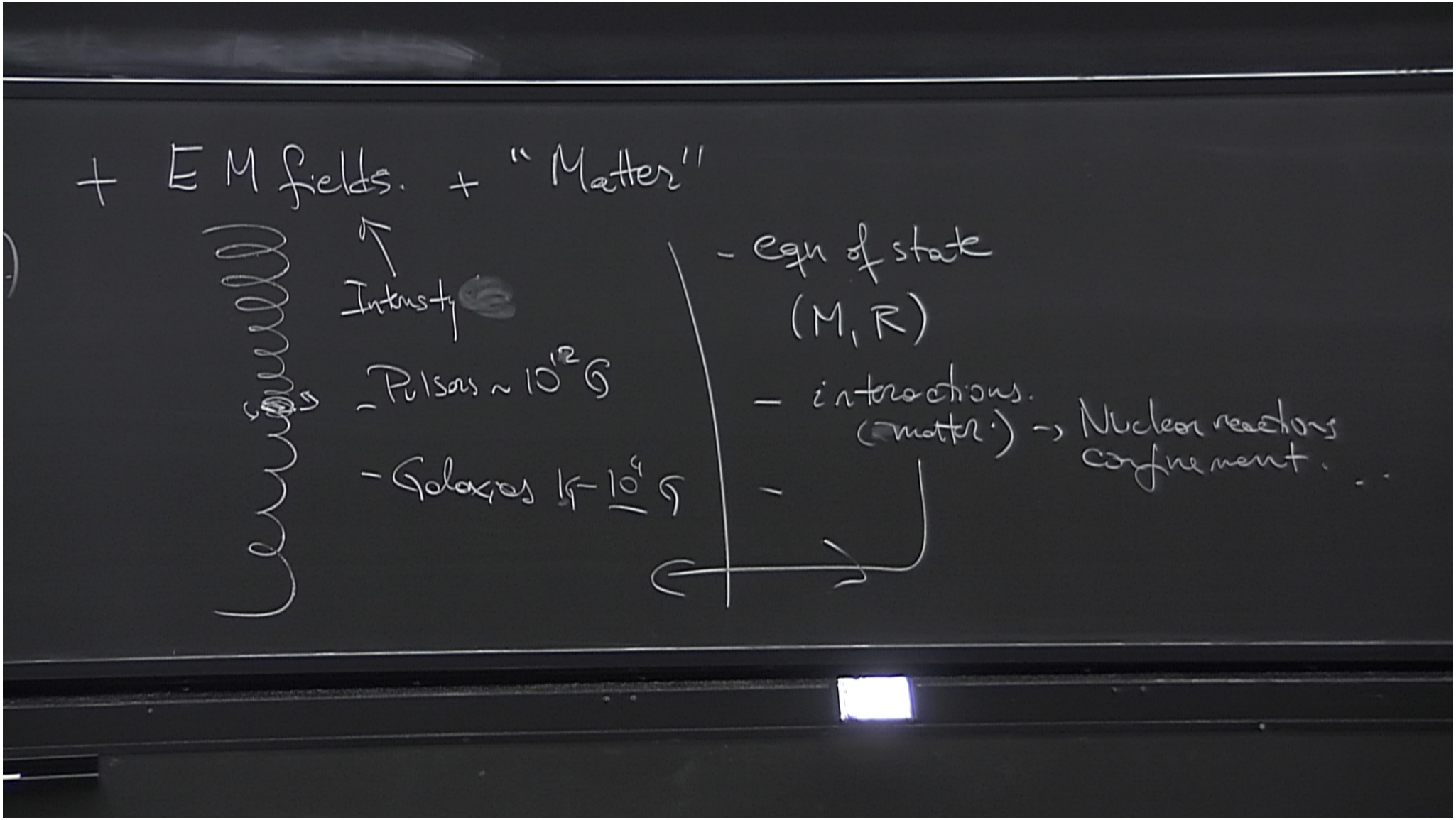
- Pulsars $\sim 10^{12}$ G

- Galaxies $\lesssim 10^{4}$ G

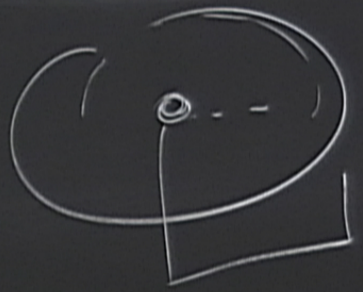
- eqn of state


- eqn of state
(M, R)

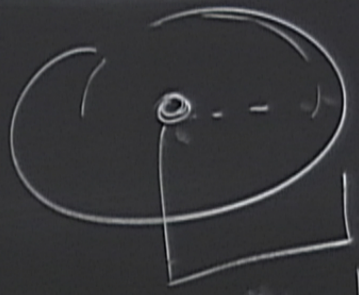

- eqn of state
(M, R)
- interactions.


+ EM fields. + "Matter"
Intensity
- Pulsars ~ 10^{12} G
- Galaxies ~ 10^{6} G
- eqn of state
(M, R)
- interactions.
(matter) -> Nuclear reactions
confinement...

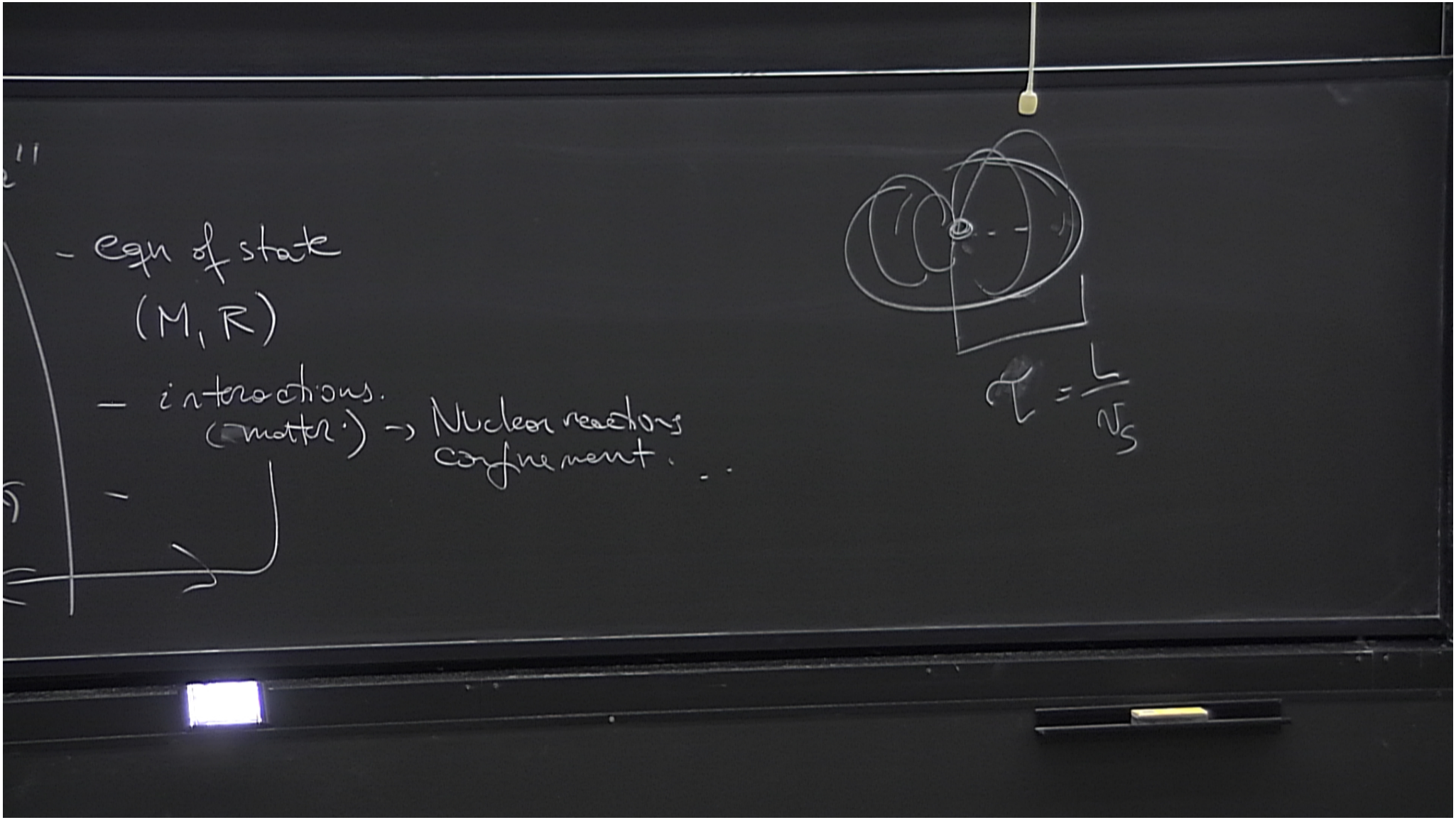
- eqn of state
  (M, R)
- interactions.
  (matter) → Nucleon reactions
  confinement. . .

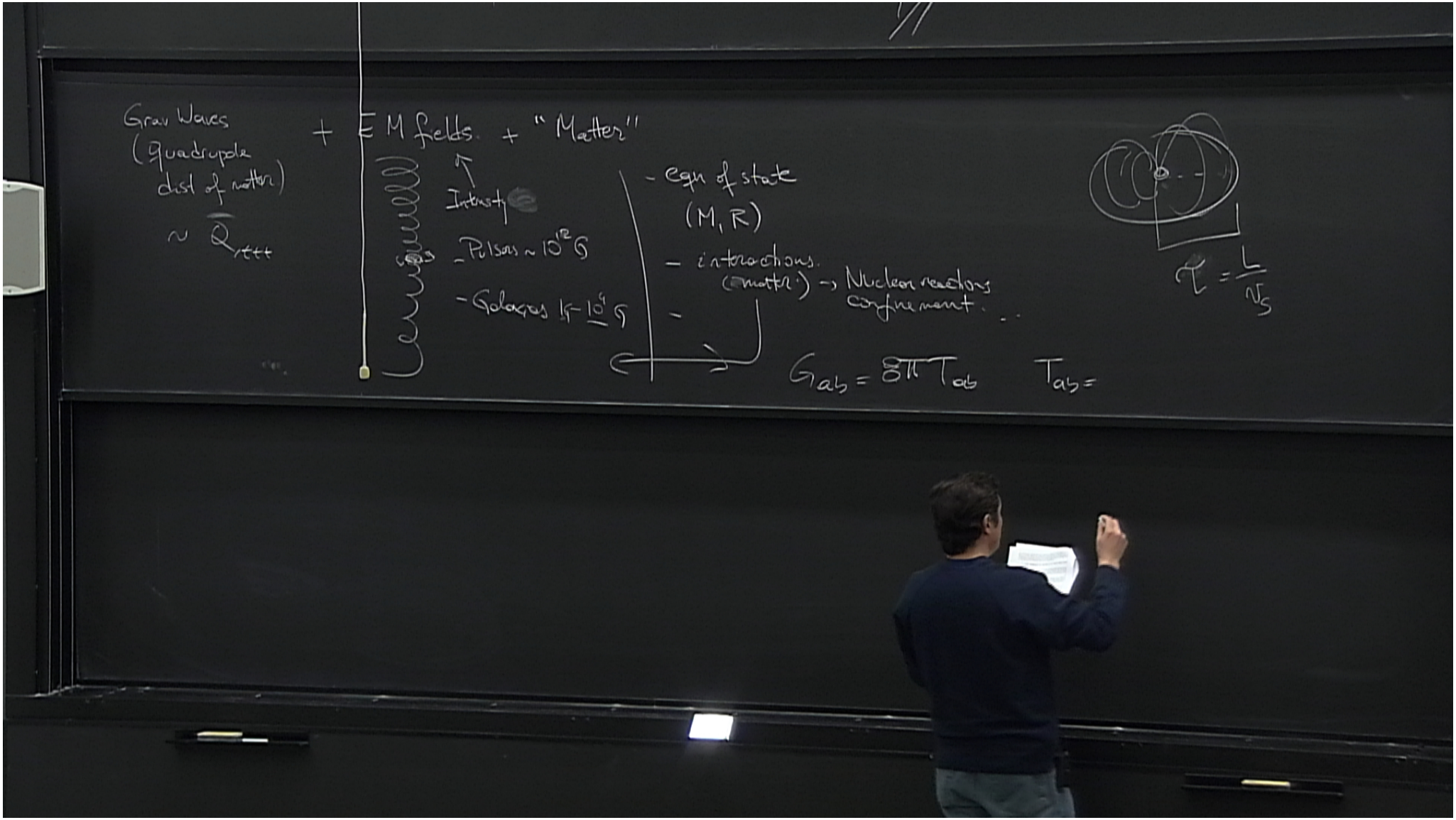


- eqn of state
  (M, R)
- interactions.
  (matter) → Nuclear reactions
  confinement. . .



$\tau = \frac{L}{v_s}$

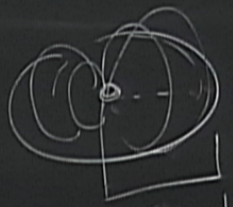

- eqn of state
(M, R)
- interactions.
(matter) → Nuclear reactions
confinement. . .
$\tau = \frac{L}{v_s}$


Grav Waves
(quadrupole
dist of matter)
~ Q_{ttt}
+
EM fields
+
"Matter"
Intensity
- Pulsars ~ 10^2 G
- Galaxies ~ 10^6 G
- eqn of state
(M, R)
- interactions
(matter) -> Nuclear reactions
confinement
G_ab = 8π T_ab
T_ab =
τ = L/v_s

Grav Waves (quadrupole dist of matter) $\sim \ddot{Q}_{ijtt}$ + EM fields + "Matter"

- Intrinsic
- Pulsars $\sim 10^2$ G
- Galaxies $\lesssim 10^5$ G

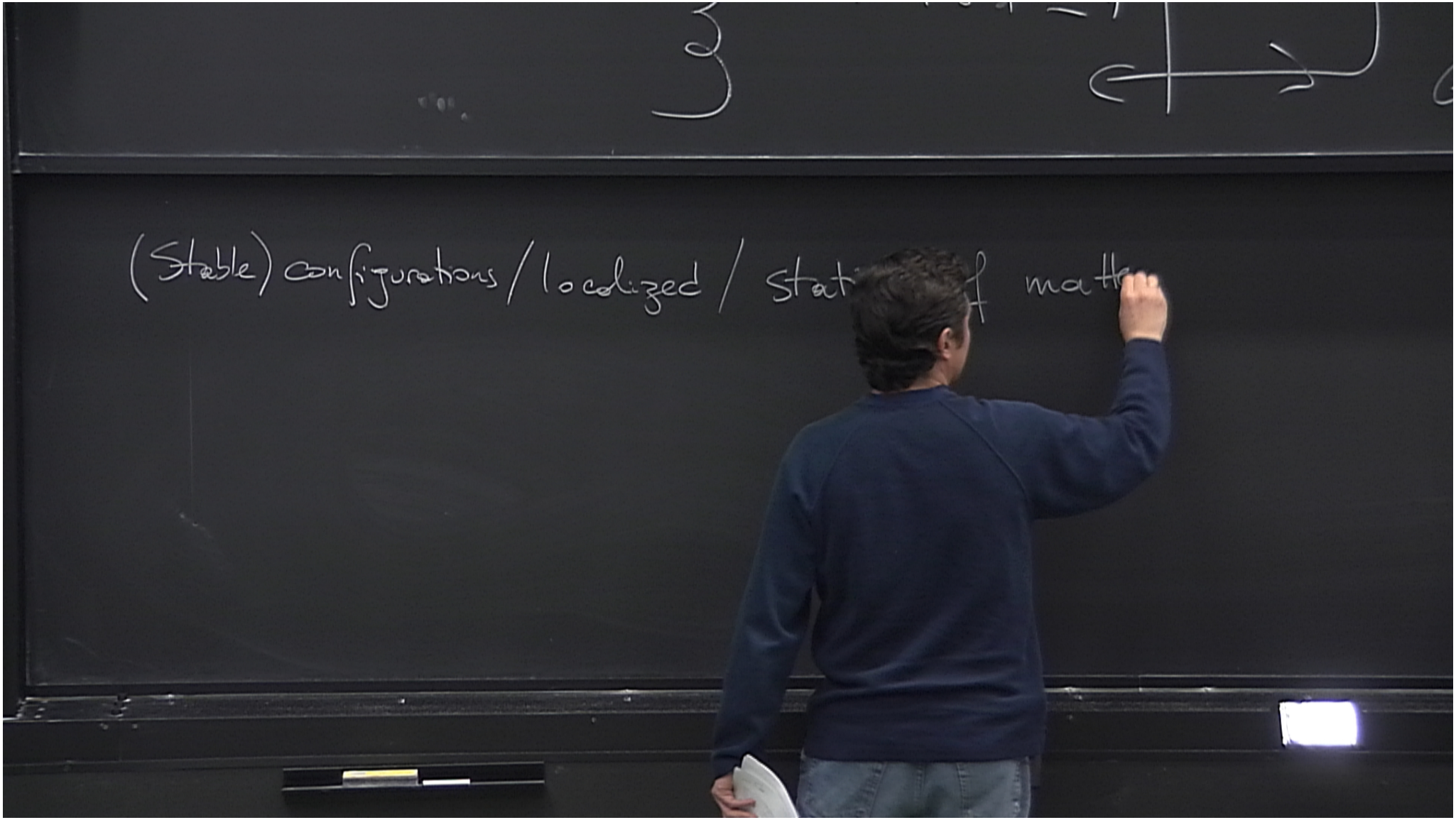
- eqn of state (M, R)
- interactions (matter) → Nuclear reactions confinement
-



$\tau = \frac{L}{v_s}$

$G_{ab} = 8\pi T_{ab}$ $\quad T_{ab} = (\rho_0(1+\varepsilon)+P)u_a u_b + P g_{ab}$.

St


(Stable) configurations / localized / stat

$G_{ab} = 8\pi T_{ab}$ $T_{ab} = (\rho_0(1+\varepsilon)+p)u_a u_b + p g_{ab}$.

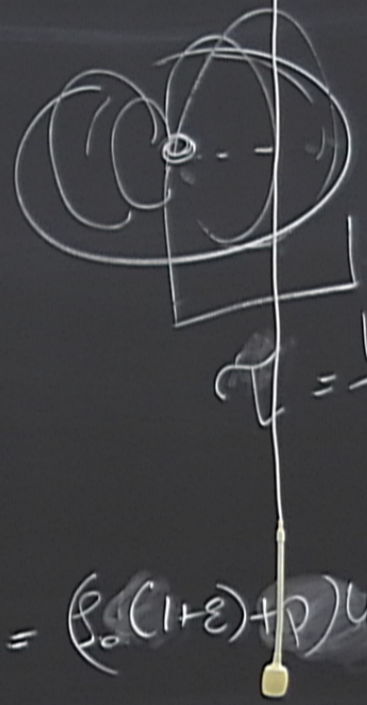
→ possible stars?

eqn of state

(M, R)

– interactions.

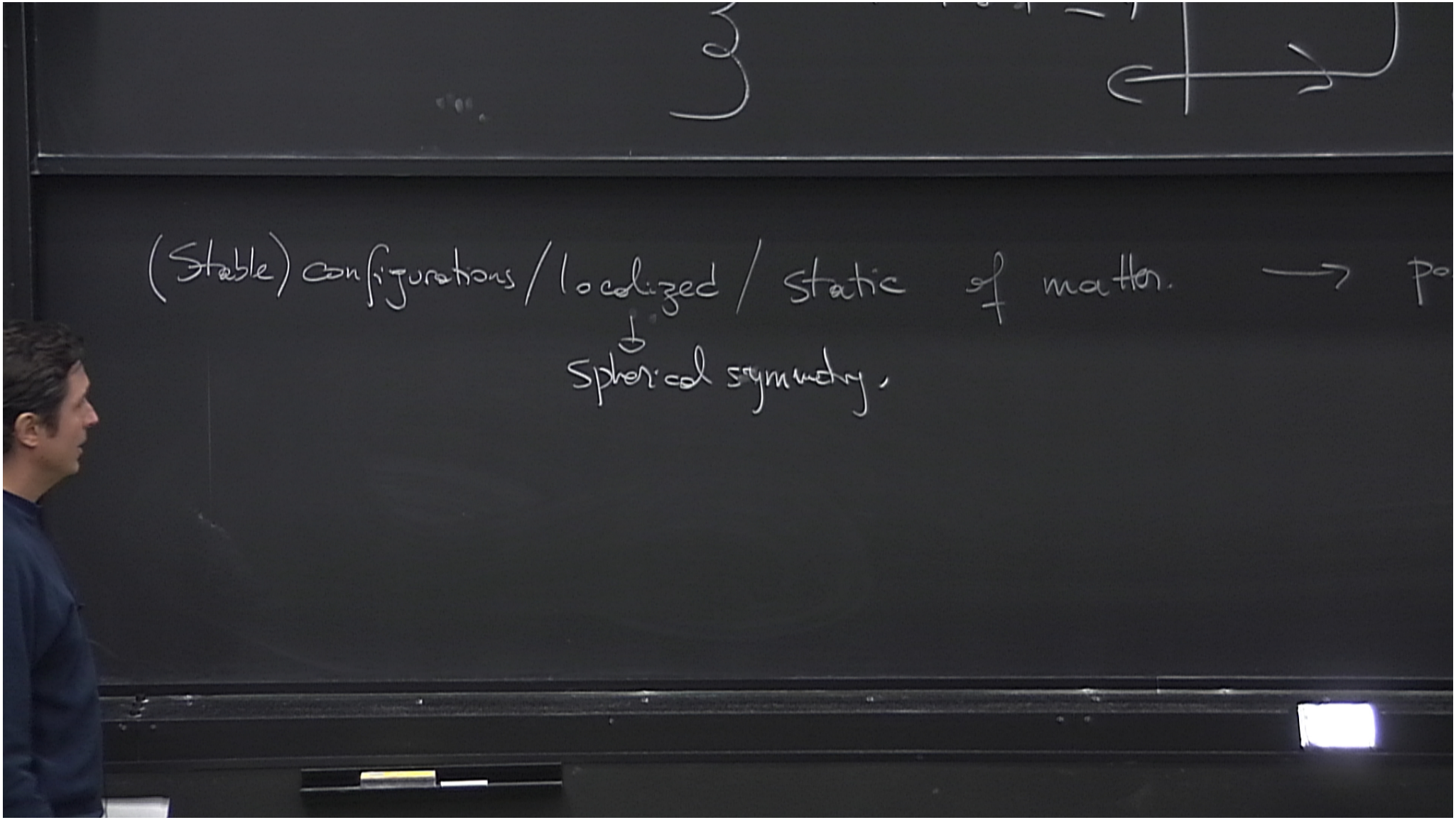
(matter) → Nuclear reactions
confinement. . .

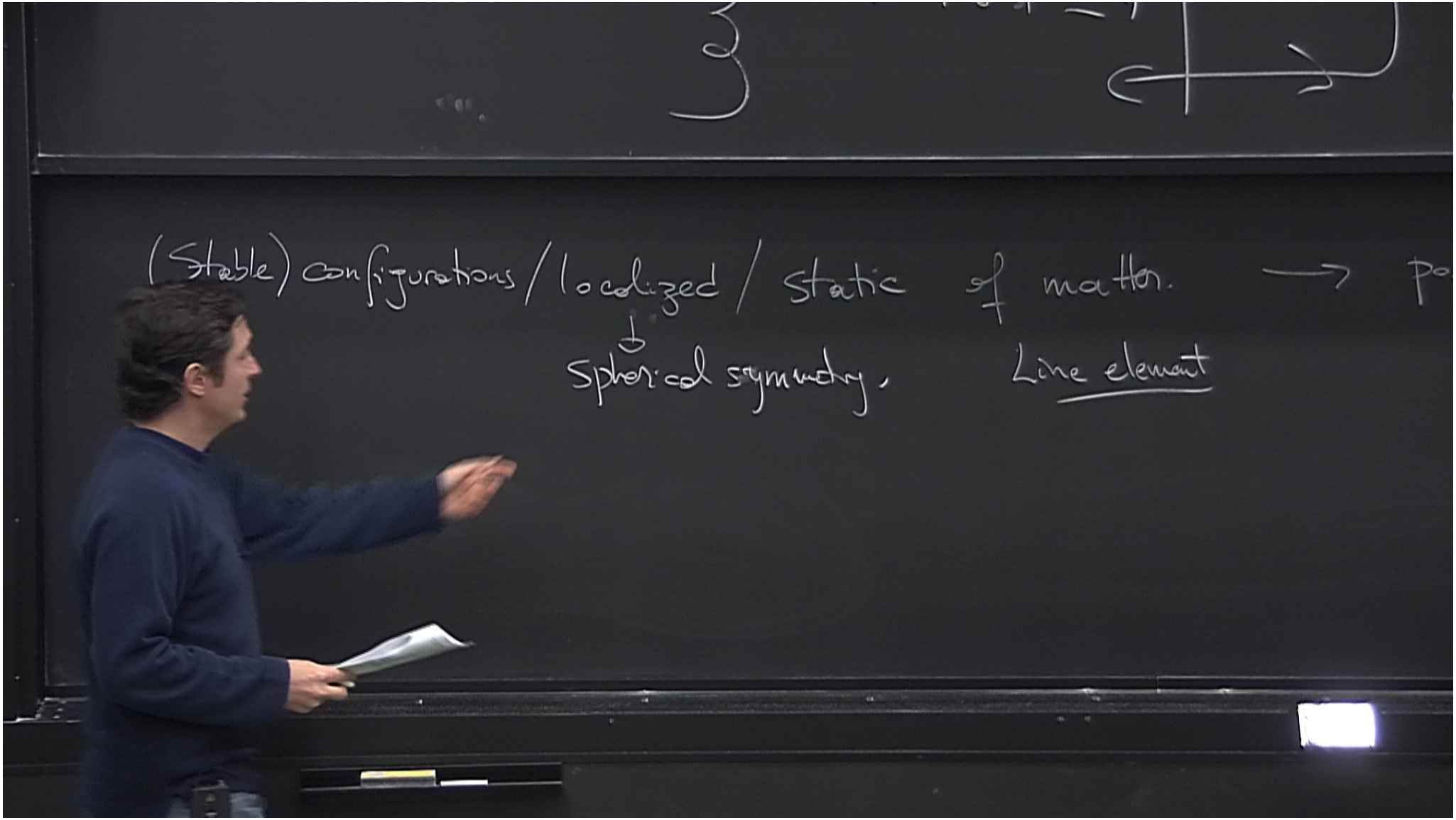


$\tau = \frac{L}{v_s}$

$G_{ab} = 8\pi T_{ab}$ $\quad T_{ab} = (\rho_0(1+\varepsilon)+p)u_a u_b + p g_{ab}$.

→ possible stars?


(Stable) configurations / localized / static of matter. → Po
Spherical symmetry.


(Stable) configurations / localized / static of matter. → Po
Spherical symmetry.
Line element

$G_{ab} = 8\pi T_{ab}$ $\quad T_{ab} = (\rho_0(1+\varepsilon) + p) u_a u_b + p g_{ab}$.

→ possible stars?

$ds^2 = -B(r)dt^2 + A(r)dr^2 + r^2 d\Omega^2$.

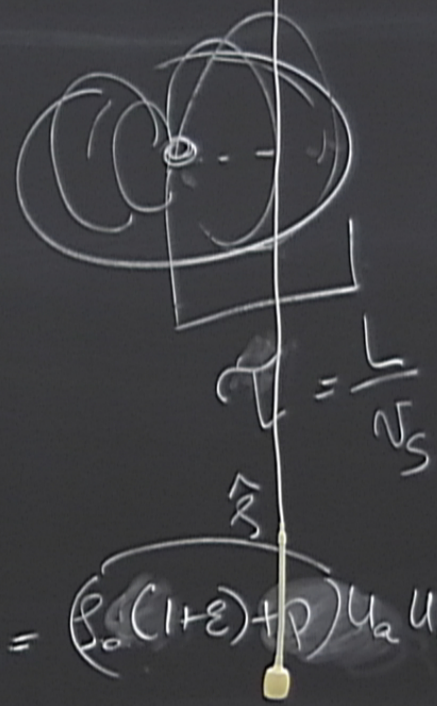
eqn of state

(M, R)

– interactions.

(matter) → Nuclear reactions

confinement. . .



$\tau = \frac{L}{v_s}$

$G_{ab} = 8\pi T_{ab}$ $\quad T_{ab} = (\rho_0(1+\varepsilon)+p)u_a u_b + p g_{ab}$.

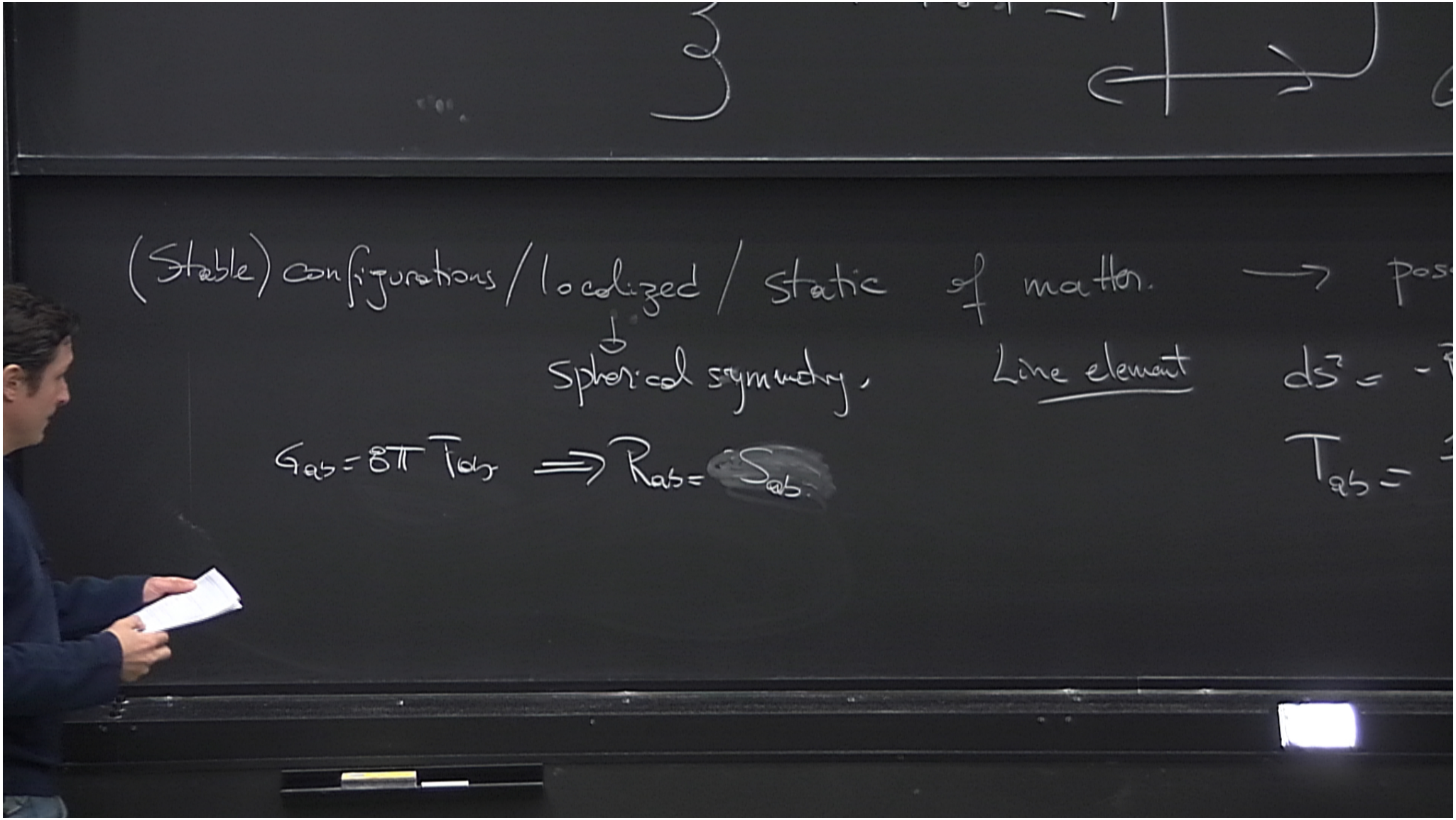
→ possible stars?

$G_{ab} = 8\pi T_{ab}$ $\quad T_{ab} = (\rho_0(1+\varepsilon)+p) u_a u_b + p g_{ab}$.

→ possible stars?

$ds^2 = -B(r)dt^2 + A(r)dr^2 + r^2 d\Omega^2$.

$T_{ab} = \hat{\rho}(r) u_a u_b + p g_{ab}$


(Stable) configurations / localized / static of matter. → pos
spherical symmetry.
Line element
$ds^2 = -$
$G_{ab} = 8\pi T_{ab} \Rightarrow R_{ab} = S_{ab}$
$T_{ab} =$

$G_{ab} = 8\pi T_{ab}$ $\quad T_{ab} = (\rho_0(1+\varepsilon)+p)u_a u_b + p g_{ab}$.

→ possible stars?

$ds^2 = -B(r)dt^2 + A(r)dr^2 + r^2 d\Omega^2$.

$T_{ab} = \hat{\rho}(r) u_a u_b + p g_{ab}$ $\quad u_a = (+,0,0,0)$



$G_{ab} = 8\pi T_{ab}$ $\quad T_{ab} = (\rho_0(1+\varepsilon) + p) u_a u_b + p g_{ab}$.

→ possible stars?

$u_a u^a = -1$

$ds^2 = -B(r) dt^2 + A(r) dr^2 + r^2 d\Omega^2$.

$T_{ab} = \hat{\rho}(r) u_a u_b + p g_{ab}$ $\quad u_a = (+, 0, 0, 0)$

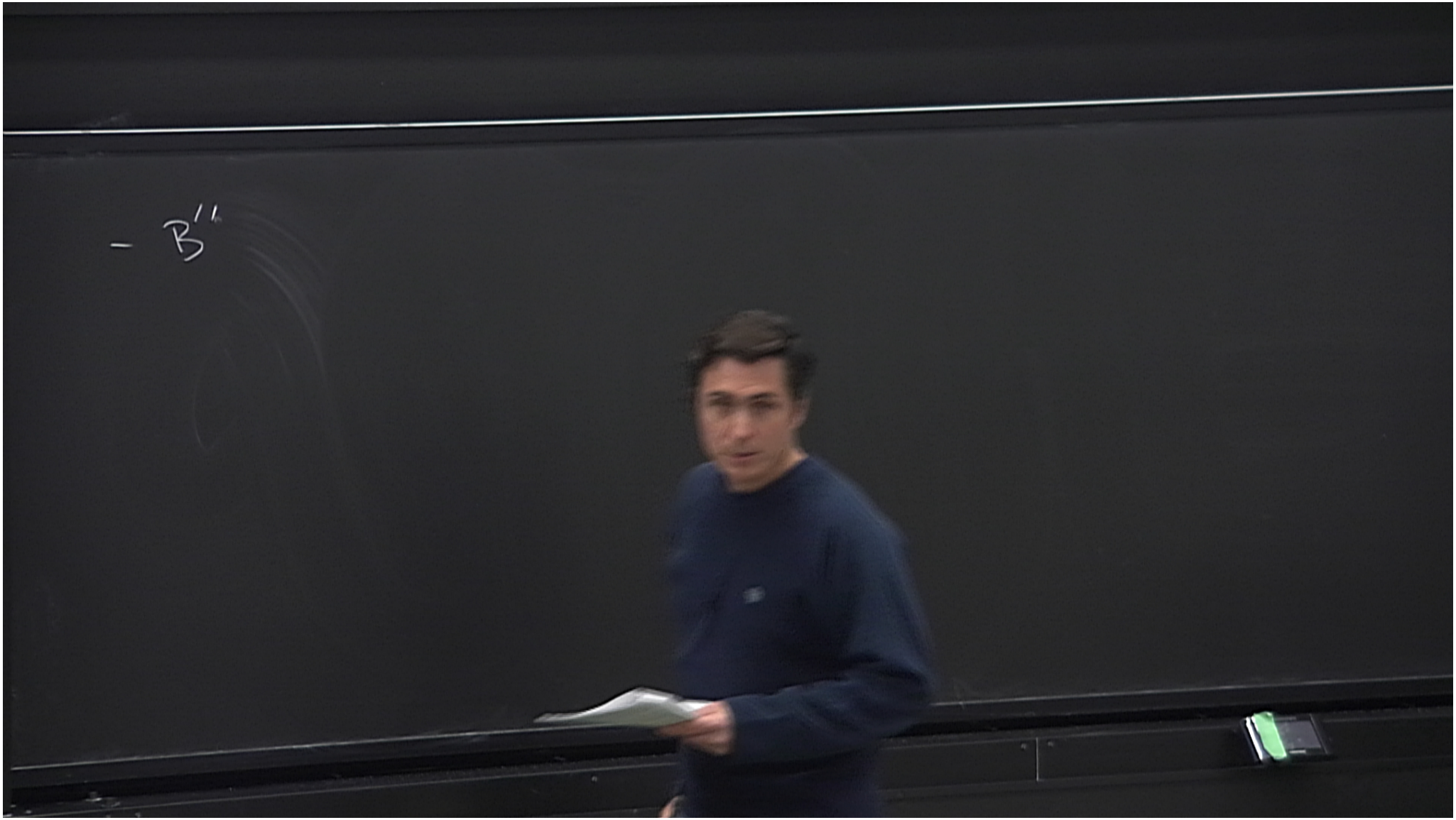
$G_{ab} = 8\pi T_{ab}$ $\quad T_{ab} = (\rho_0(1+\varepsilon) + p) u_a u_b + p g_{ab}$.

→ possible stars?

$u_a u^a = -1$

$ds^2 = -B(r) dt^2 + A(r) dr^2 + r^2 d\Omega^2$.

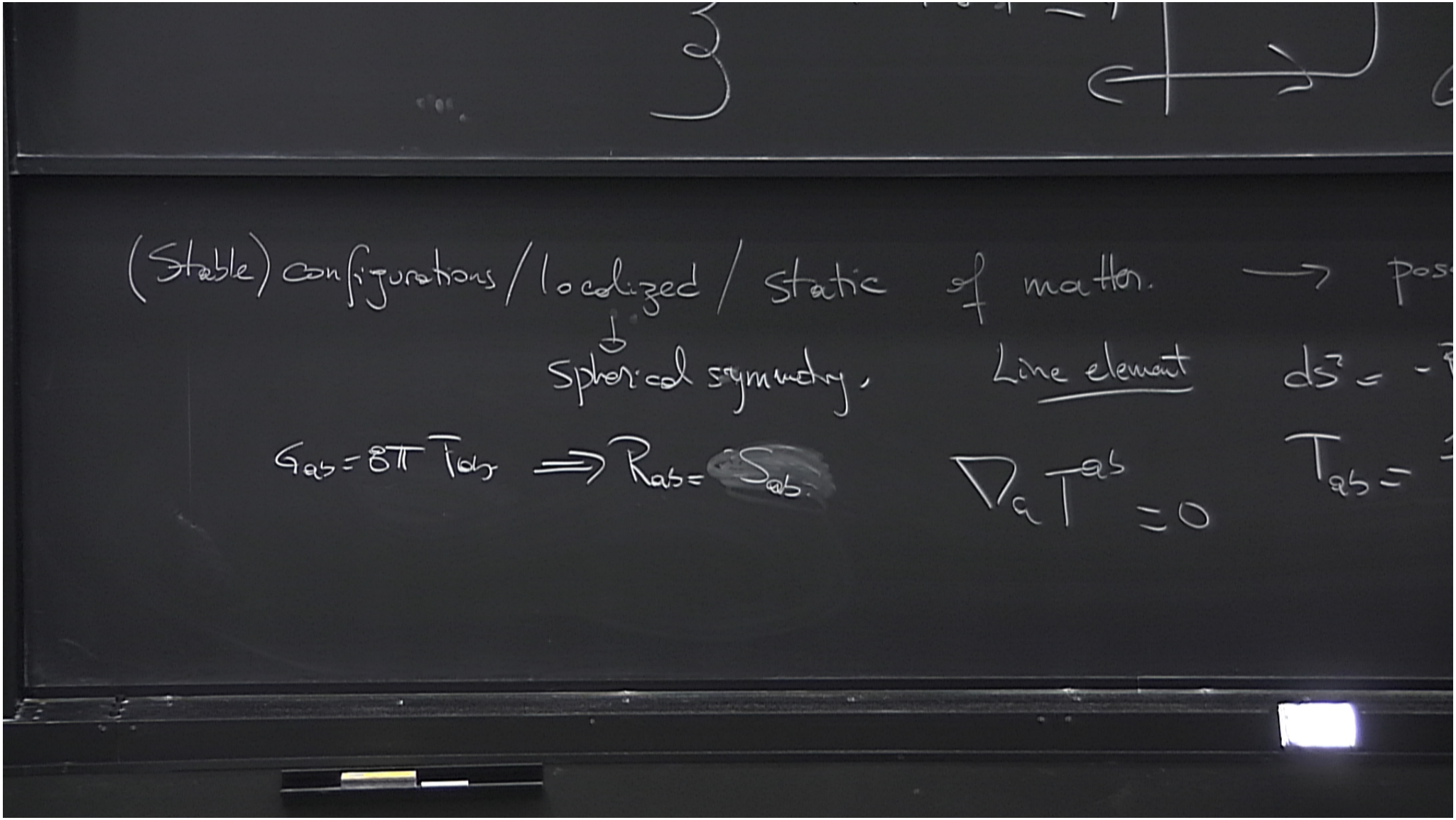
$T_{ab} = \hat{\rho}(r) u_a u_b + p g_{ab}$ $\quad u_a = (-\sqrt{B}, 0, 0, 0)$


- B''

1) $\frac{B''}{2B} - \frac{B'}{4B}\left(\frac{A'}{A} + \frac{B'}{B}\right) - \frac{A'}{rA} = -4\pi(\hat{s} - p)\,A$

2) $-1 + \frac{r}{2A}\left(-\frac{A'}{A} + \frac{B'}{B}\right) + \frac{1}{A} = -4\pi(\hat{s} - p)\,r^2$

3) $-\frac{B''}{2A} + \frac{B'}{4A}\left(\frac{A'}{A} + \frac{B'}{B}\right) - \frac{B'}{rA} = -4\pi(\hat{s} + 3p)\,B$

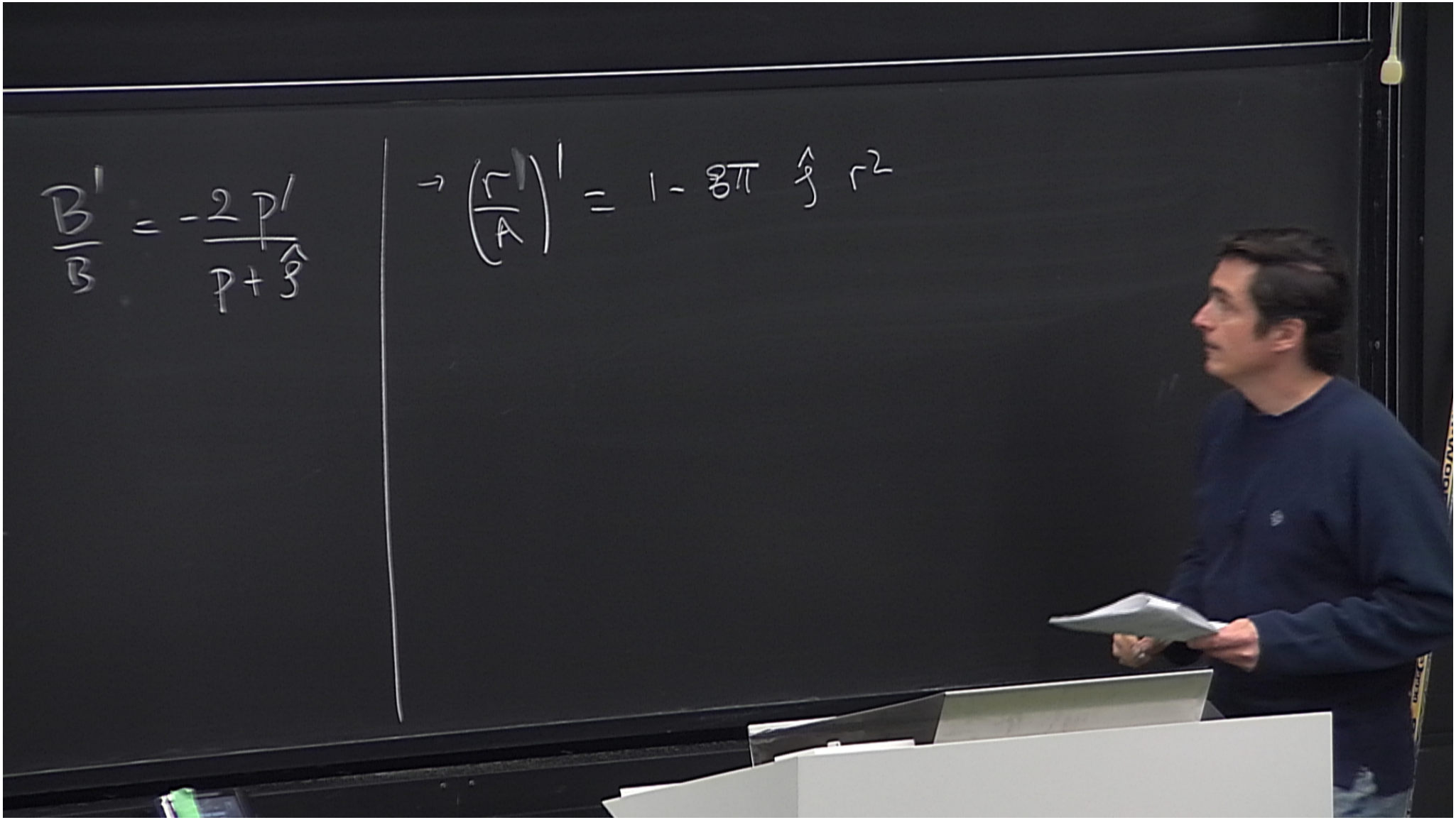

(Stable) configurations / localized / static of matter. → pos
Spherical symmetry.
Line element
$G_{ab} = 8\pi T_{ab} \Rightarrow R_{ab} =$
$\nabla_a T^{ab} = 0$
$T_{ab} =$

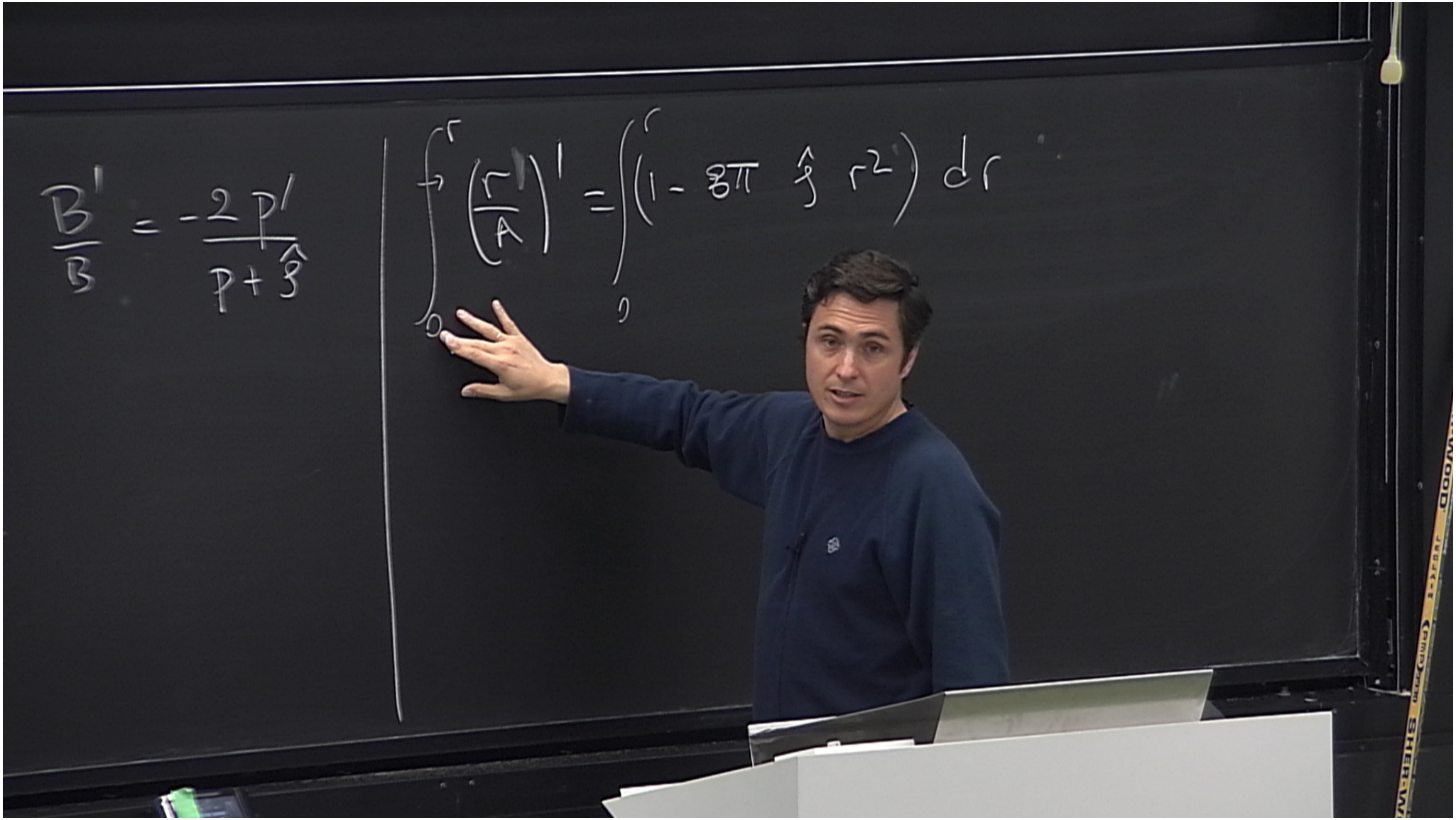
1) $\frac{B''}{2B} - \frac{B'}{4B}\left(\frac{A'}{A} + \frac{B'}{B}\right) - \frac{A'}{rA} = -4\pi(\hat{s} - p)A$

2) $-1 + \frac{r}{2A}\left(-\frac{A'}{A} + \frac{B'}{B}\right) + \frac{1}{A} = -4\pi(\hat{s} - p)r^2$

3) $-\frac{B''}{2A} + \frac{B'}{4A}\left(\frac{A'}{A} + \frac{B'}{B}\right) - \frac{B'}{rA} = -4\pi(\hat{s} + 3p)B$

4) $\frac{B'}{B} = \frac{-2p'}{p + \hat{s}}$


$\frac{B'}{B} = \frac{-2p'}{p+\hat{\beta}}$
$\rightarrow \left(\frac{r'}{A}\right)' = 1 - 8\pi \hat{\beta} r^2$


$\frac{B'}{B} = -\frac{2p'}{p+\hat{\beta}}$
$\int_0^r \left(\frac{r}{A}\right)' = \int_0^r \left(1 - 8\pi \hat{\beta} r^2\right) dr$

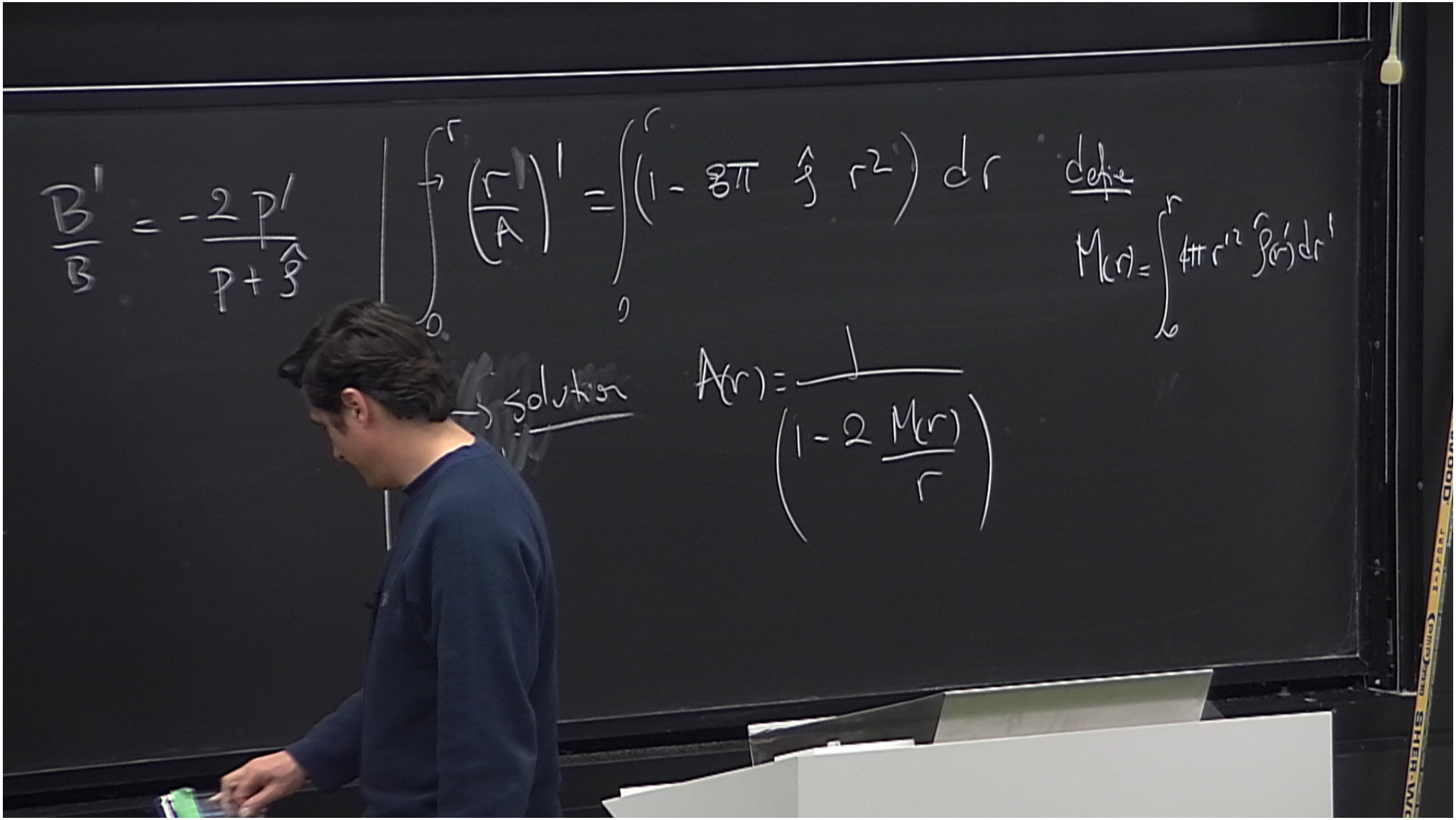
$$\frac{B'}{B} = \frac{-2p'}{p+\hat{\beta}}$$

$$\int_0^r \left(\frac{r}{A}\right)' = \int_0^r \left(1 - 8\pi \hat{\rho} r^2\right) dr$$

$$\frac{r}{A(r)} =$$

define

$$M(r) = \int_0^r 4\pi r'^2 \rho(r') dr'$$


$\frac{B'}{B} = -\frac{2p'}{p+\hat{\rho}}$
$\int_0^r \left(\frac{r}{A}\right)' = \int_0^r \left(1 - 8\pi \hat{\rho} r^2\right) dr$
define
$M(r) = \int_0^r 4\pi r'^2 \hat{\rho}(r') dr'$
→ solution
$A(r) = \frac{1}{\left(1 - 2\frac{M(r)}{r}\right)}$

$$\frac{B'}{B} = -\frac{2p'}{p+\hat{\rho}}$$

$$\int_0^r \left(\frac{r}{A}\right)' = \int_0^r \left(1 - 8\pi\,\hat{\rho}\, r^2\right) dr$$

define

$$M(r) = \int_0^r 4\pi r'^2 \hat{\rho}(r')\, dr'$$

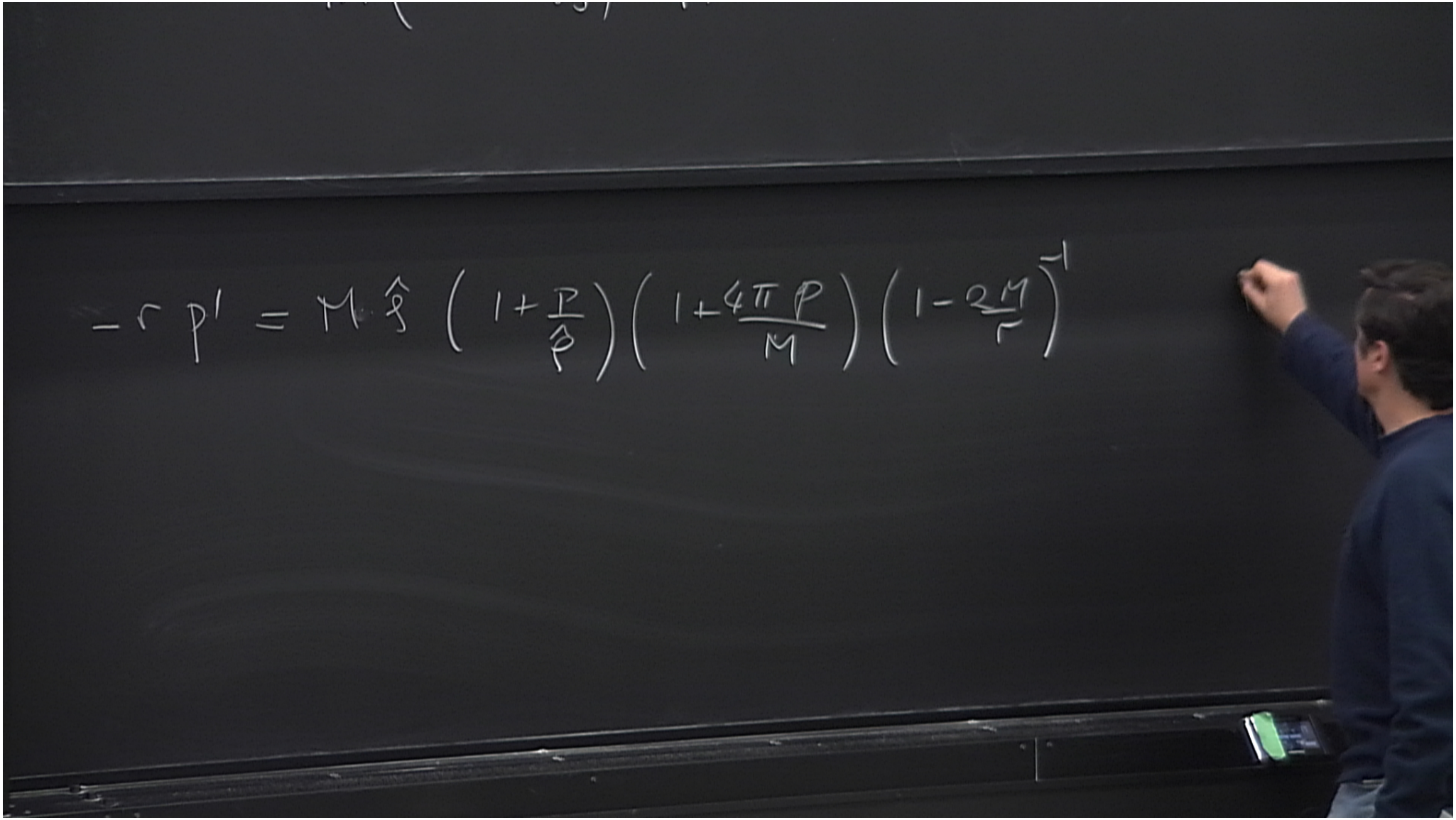
$\rightarrow$ solution

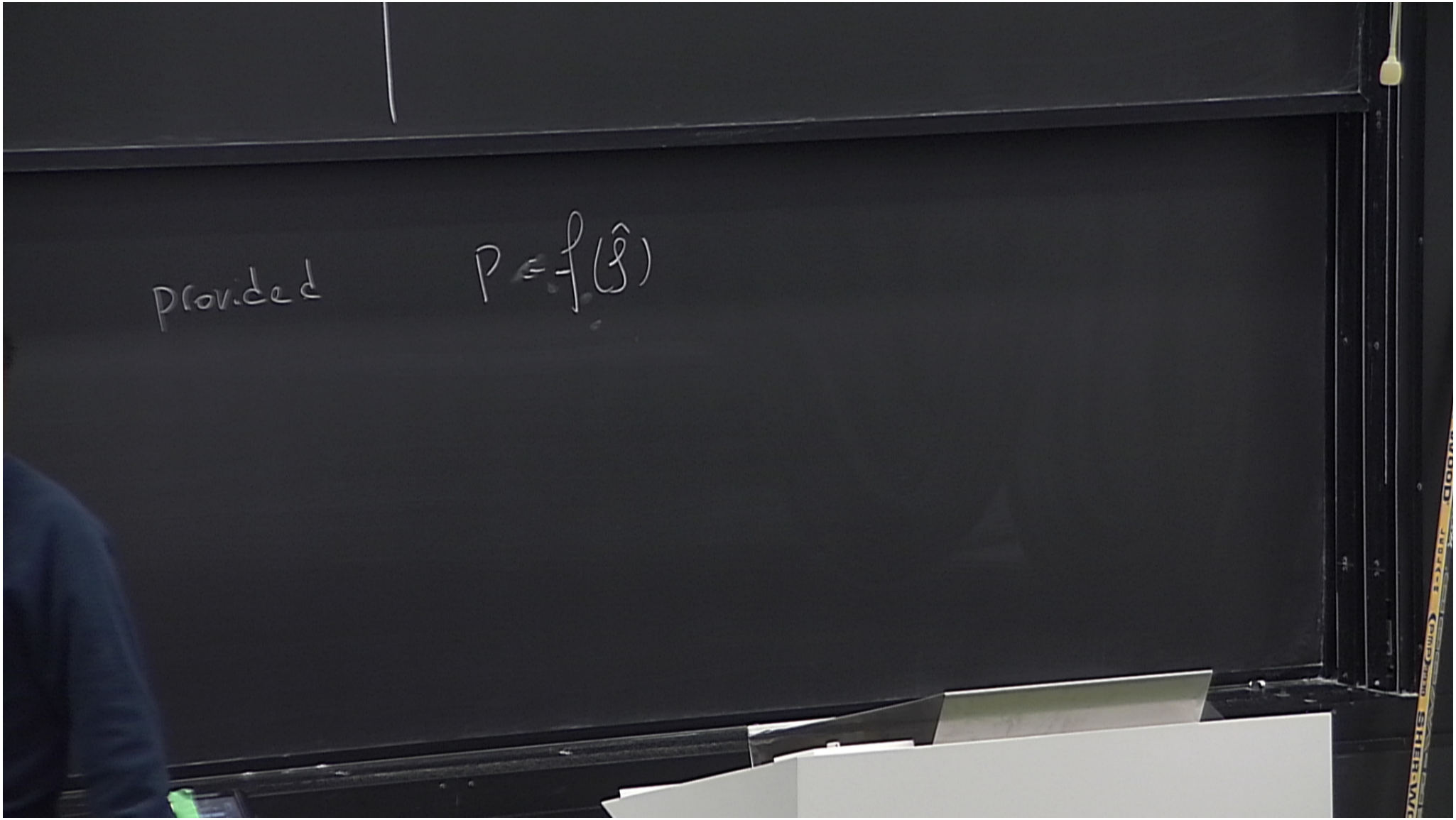
$$A(r) = \frac{1}{\left(1 - 2\frac{M(r)}{r}\right)}$$

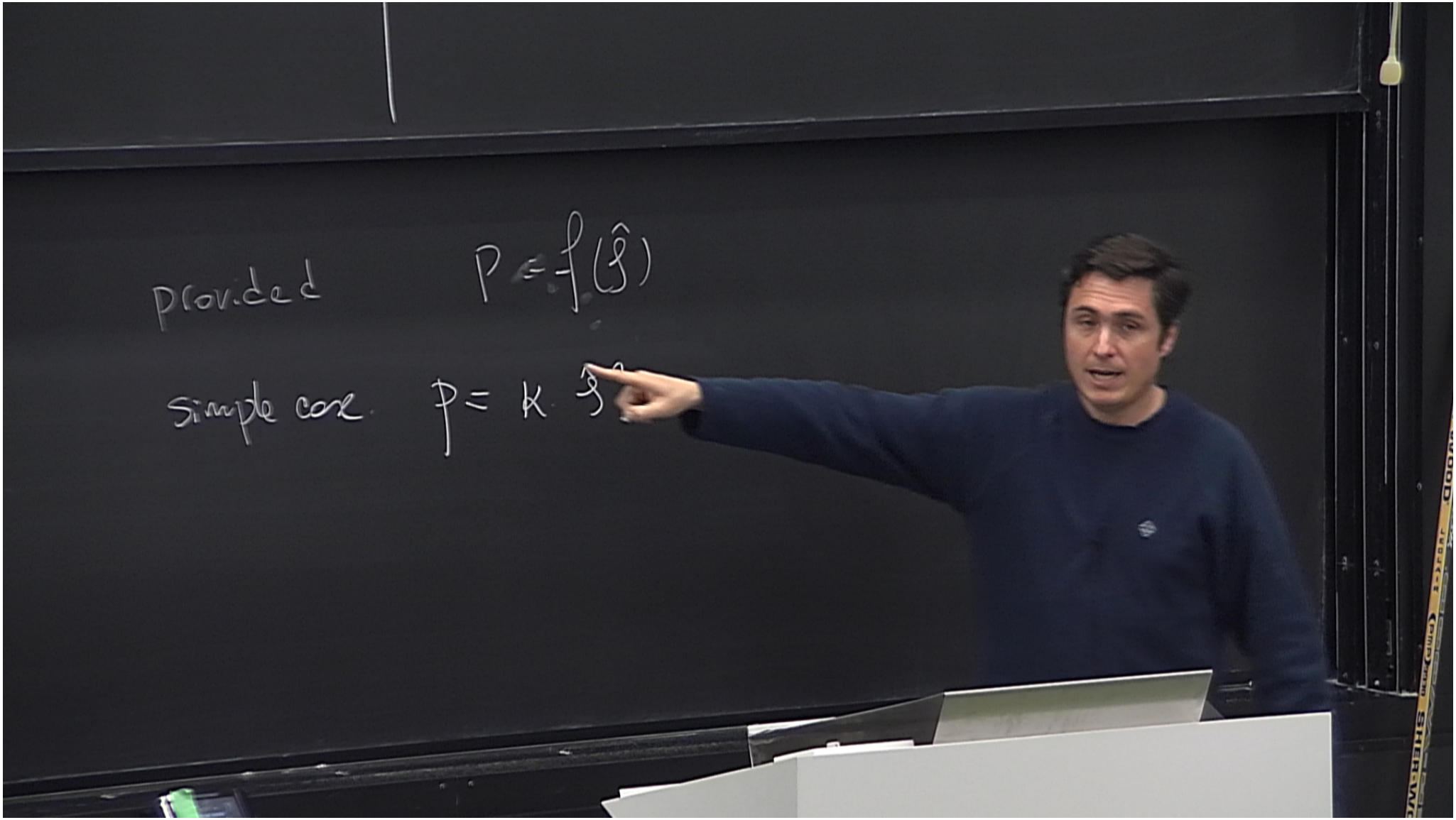


$$-r p' = M \hat{s} \left(1 + \frac{P}{\hat{\rho}}\right)\left(1 + \frac{4\pi P}{M}\right)\left(1 - \frac{2M}{r}\right)^{-1}$$

$$-r\,p' = M\,\hat{\rho}\left(1+\frac{P}{\hat{\rho}}\right)\left(1+\frac{4\pi P}{M}\right)\left(1-\frac{2M}{r}\right)^{-1}$$


$-r p' = M \hat{s} \left(1 + \frac{P}{\hat{\rho}}\right)\left(1 + \frac{4\pi \hat{\rho}}{M}\right)\left(1 - \frac{2M}{r}\right)^{-1}$


Provided
P = f(θ)

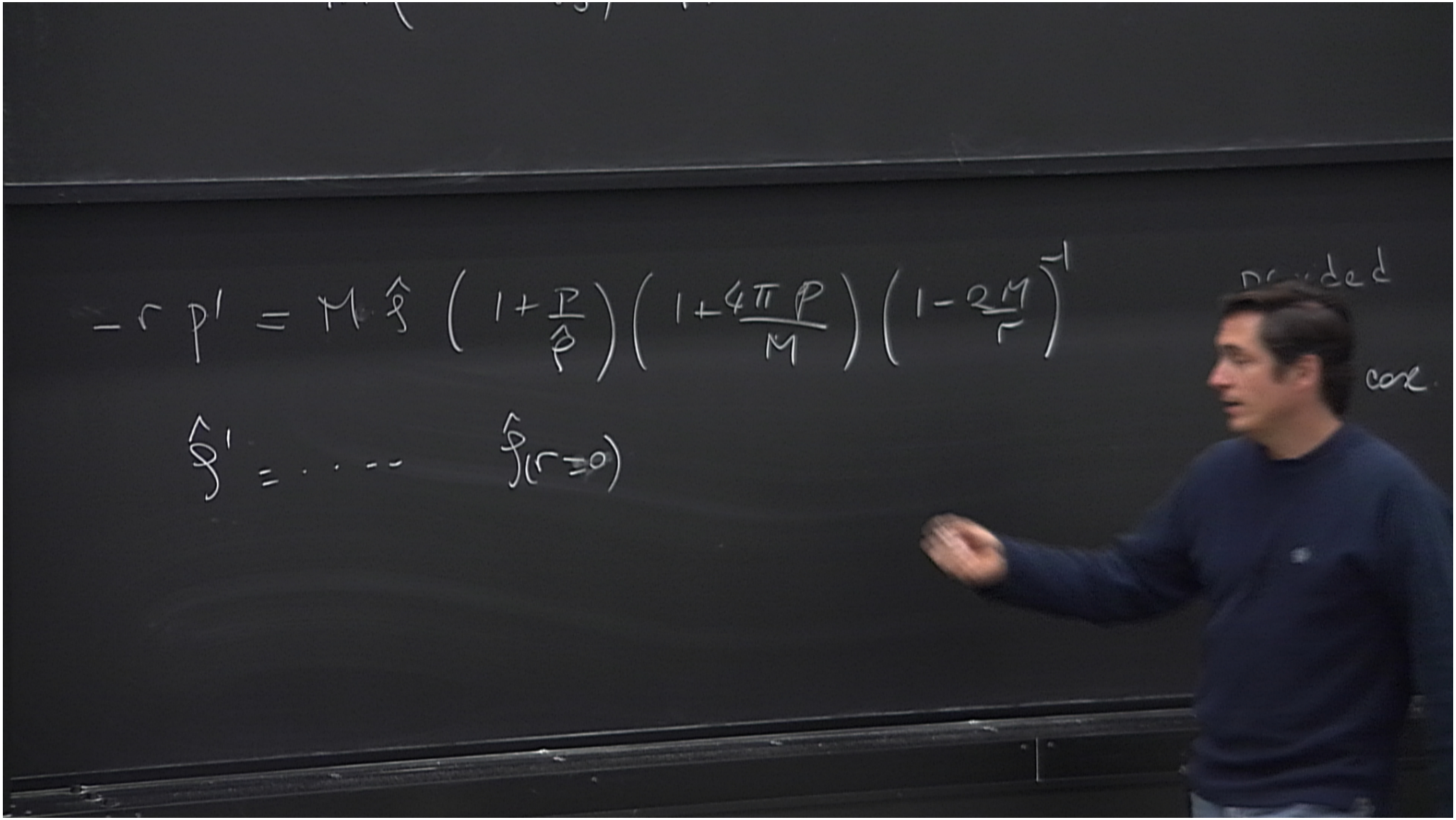

provided
simple case

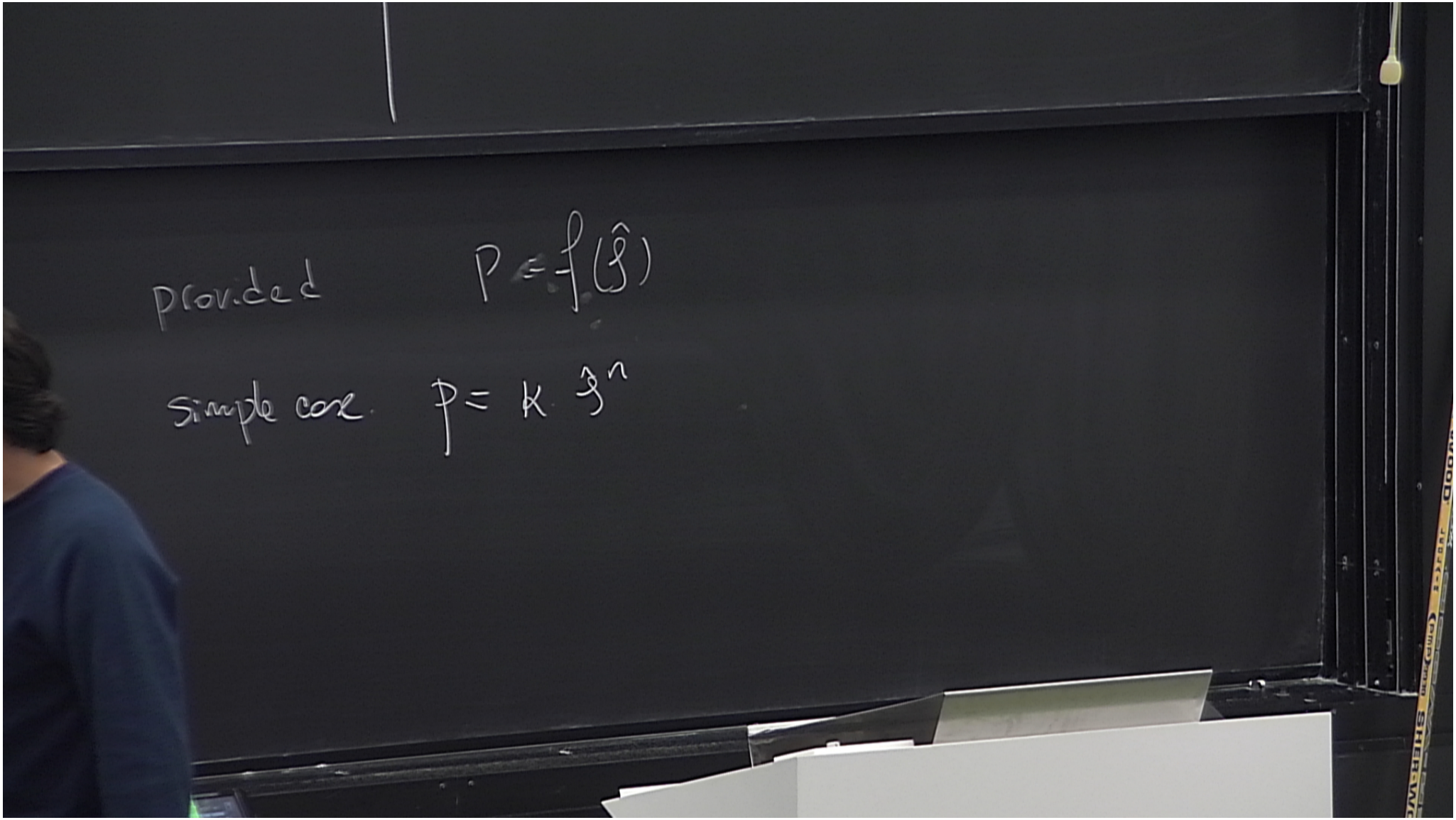
$$-r\,p' = M\,\hat{s}\left(1+\frac{P}{\hat{\rho}}\right)\left(1+\frac{4\pi P}{M}\right)\left(1-\frac{2M}{r}\right)^{-1}$$

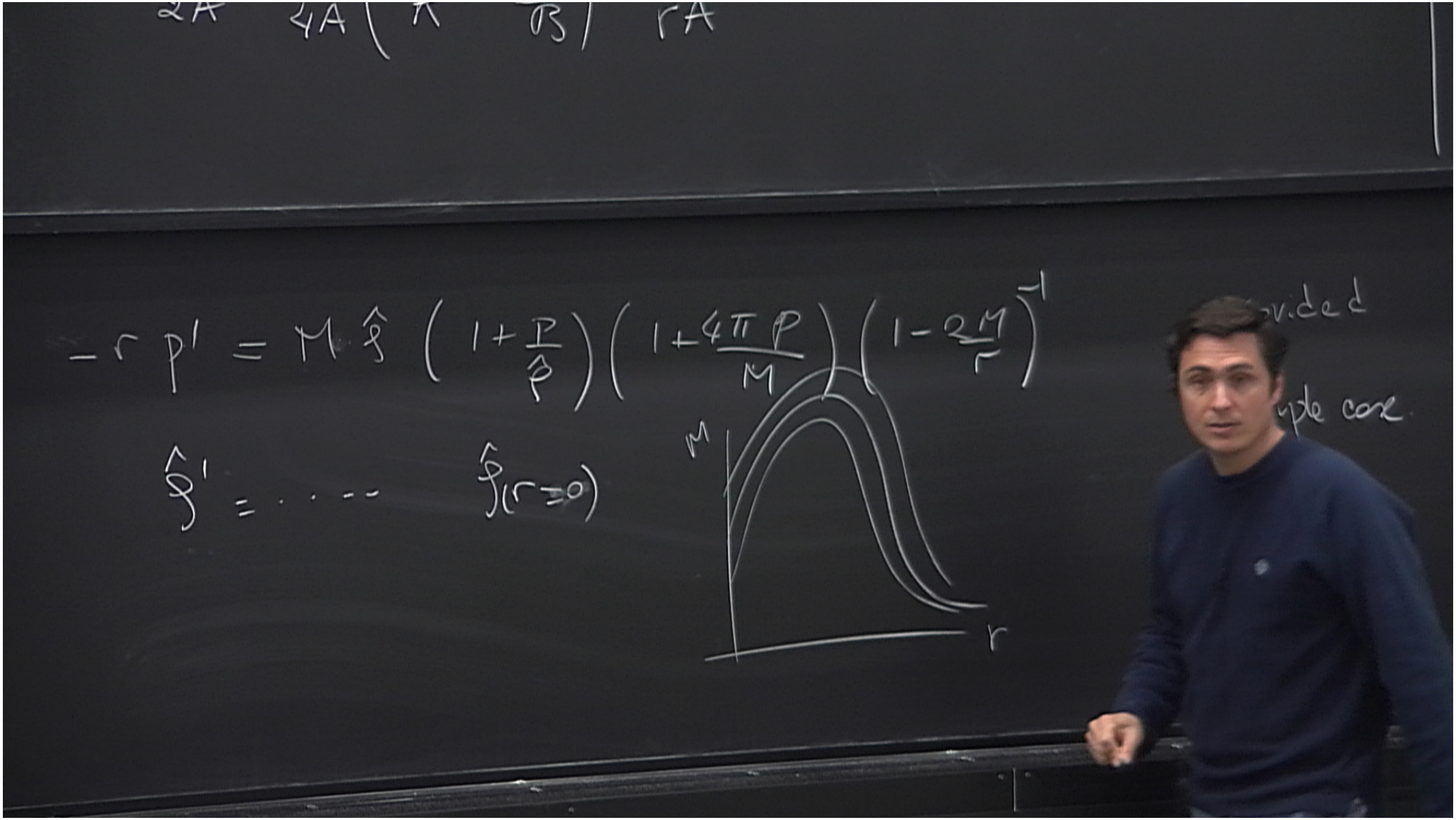
provided

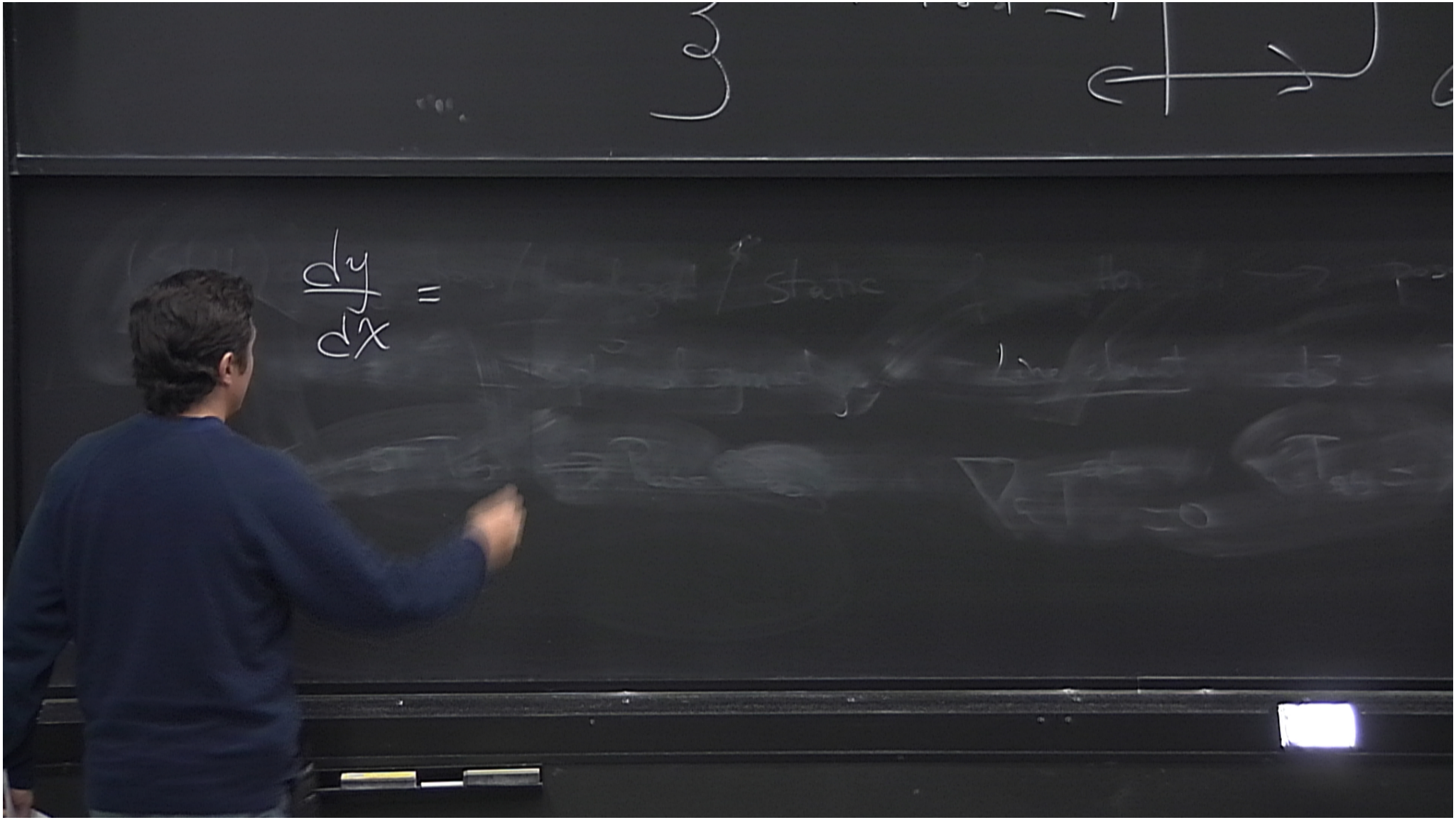
simple core

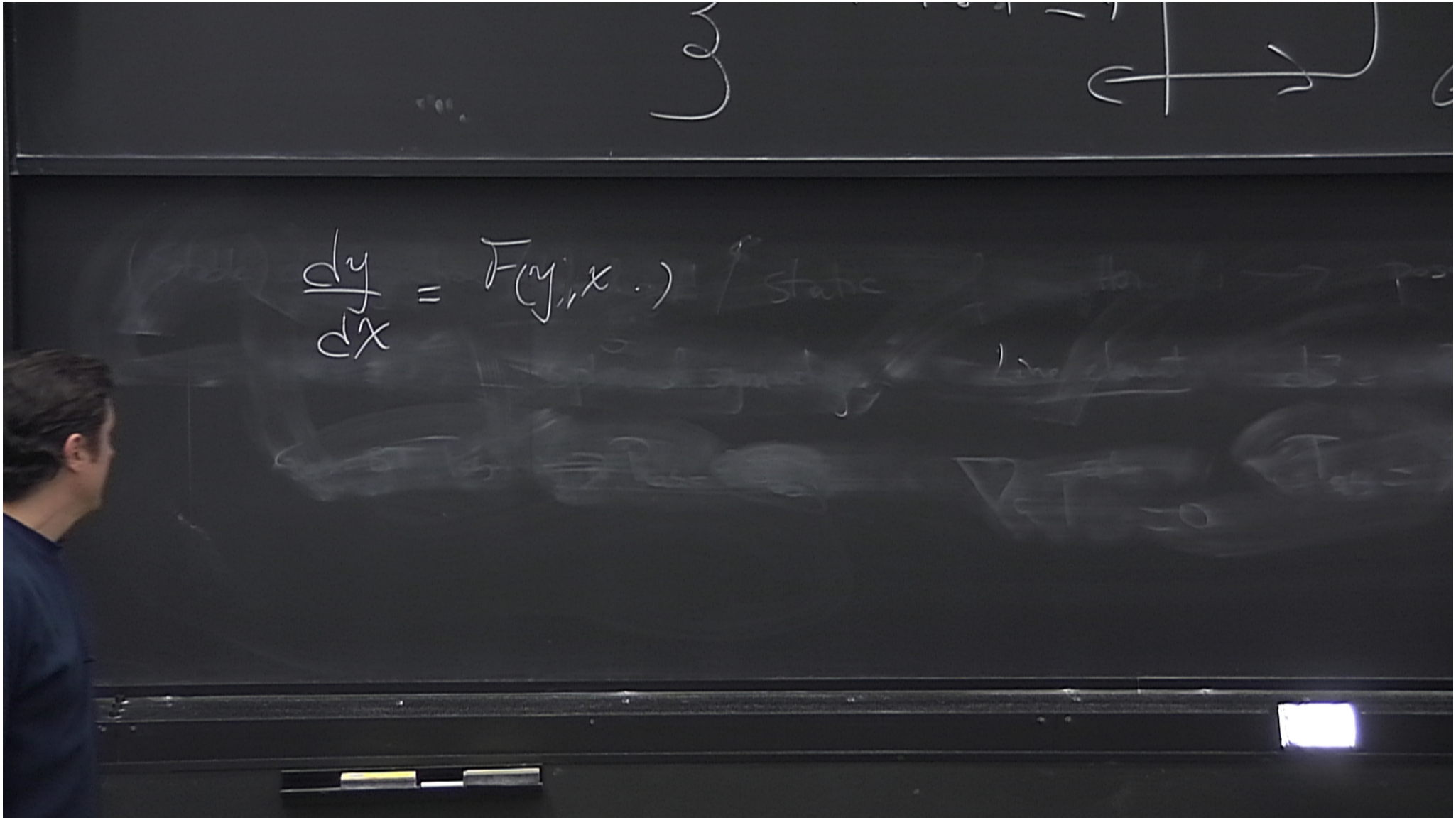
$$\hat{s}' = \cdots$$


$-r\,p' = M\,\hat{\rho}\left(1+\frac{p}{\hat{\rho}}\right)\left(1+\frac{4\pi p}{M}\right)\left(1-\frac{2M}{r}\right)^{-1}$
$\hat{\rho}' = \ldots -$
$\hat{\rho}(r=0)$
core

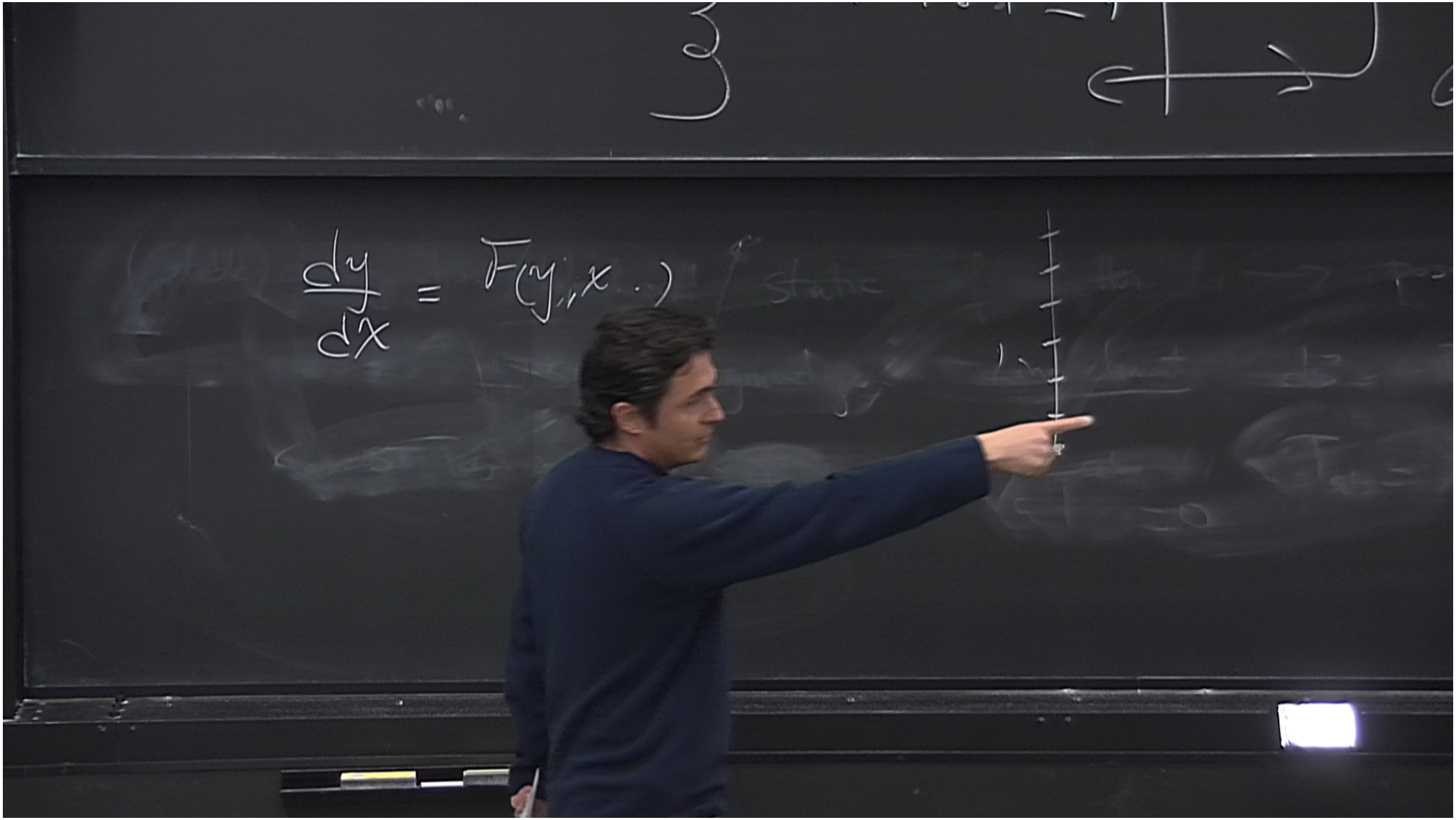

provided $P = f(\hat{s})$
simple case $P = K \hat{s}^n$

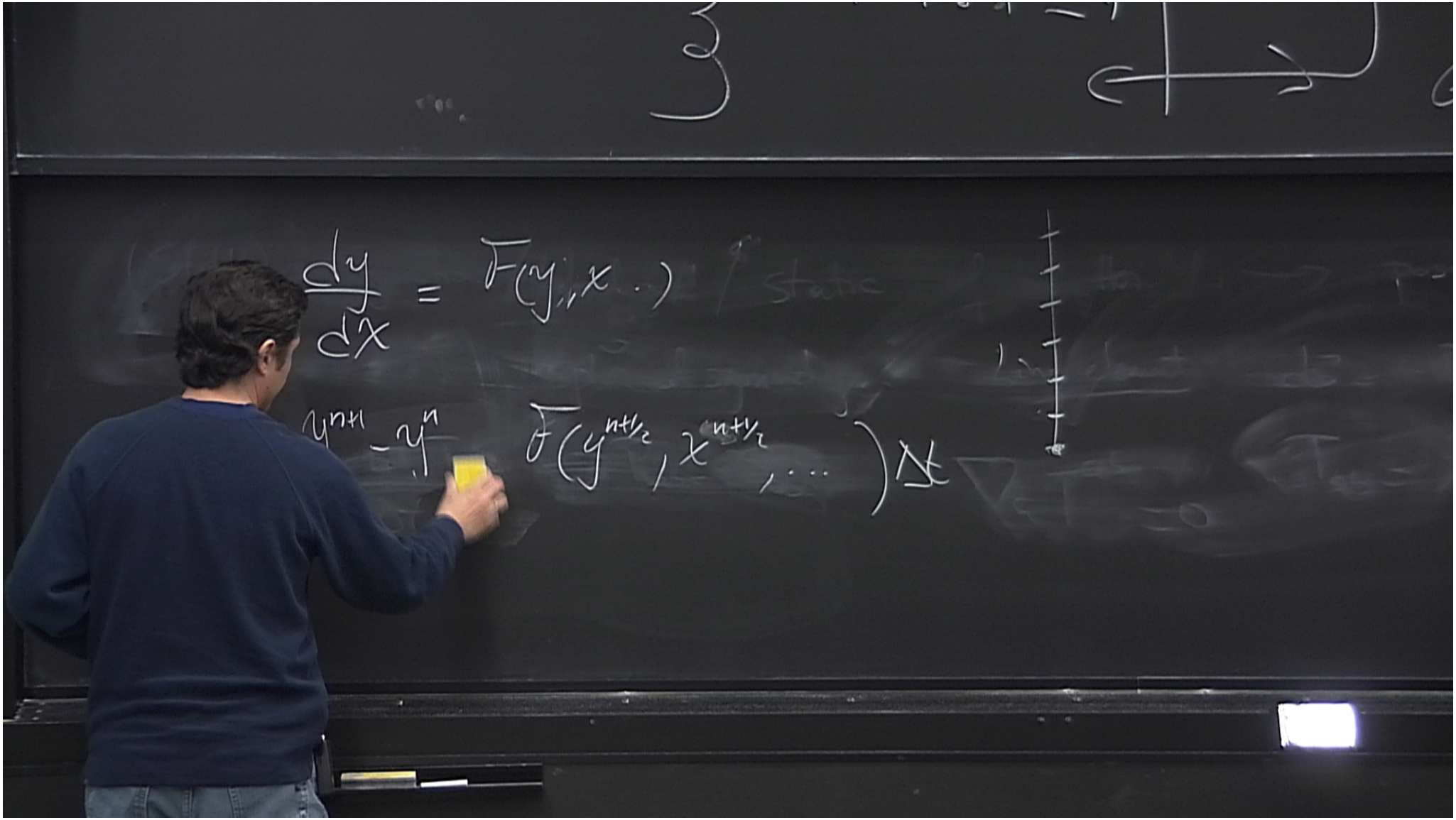

$-r p' = M \hat{\rho} \left(1+\frac{p}{\hat{\rho}}\right)\left(1+\frac{4\pi p}{M}\right)\left(1-\frac{2M}{r}\right)^{-1}$
$\hat{\rho}' = \ldots \quad \hat{\rho}(r=0)$
M
r
vided
pte core

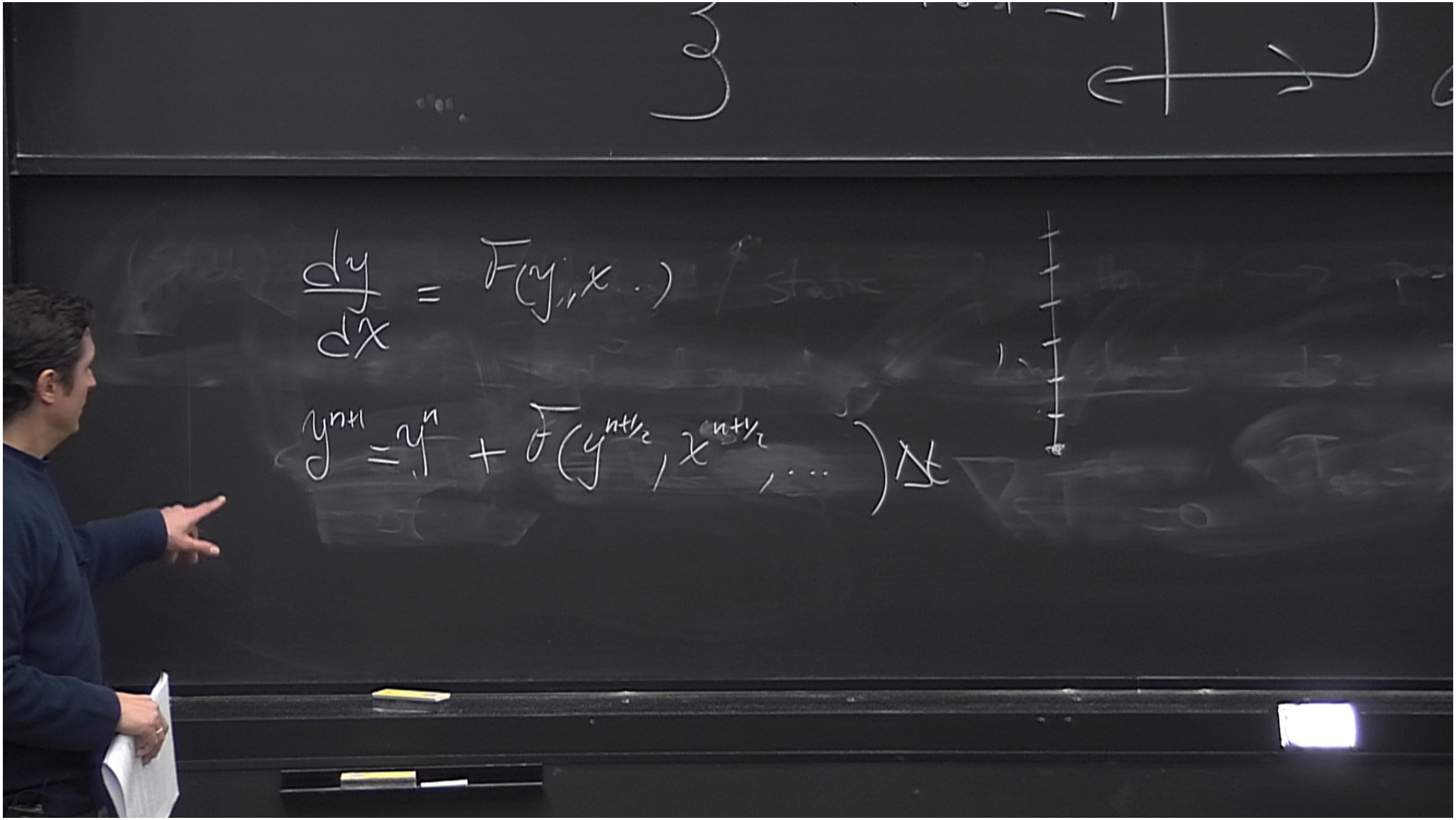

$\frac{dy}{dx} =$

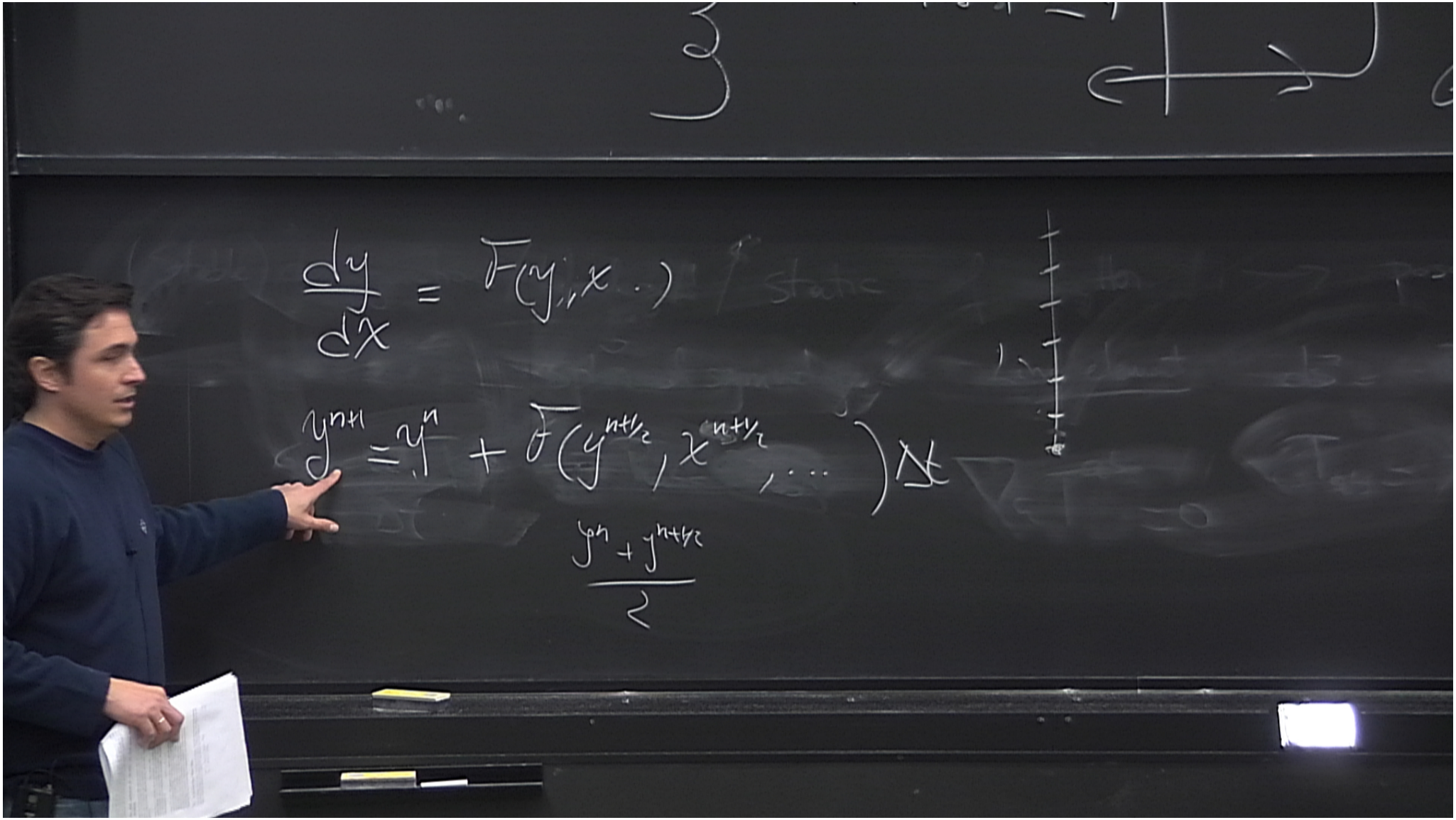

$\frac{dy}{dx} = F(y, x)$

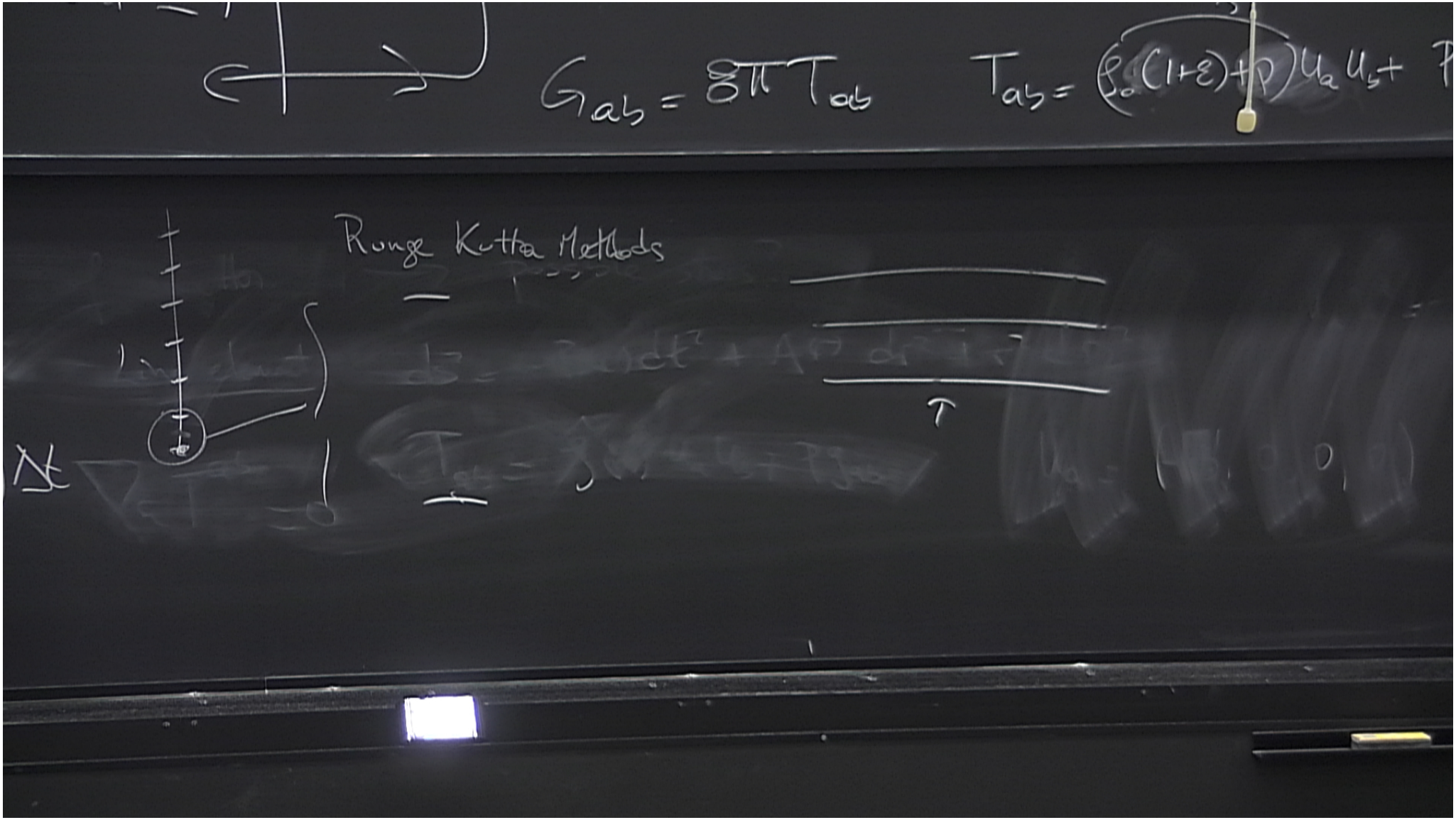

$G_{ab} = 8\pi T_{ab}$
$T_{ab} = (\rho_0(1+\varepsilon)+p)u_a u_b + p g_{ab}$.
T

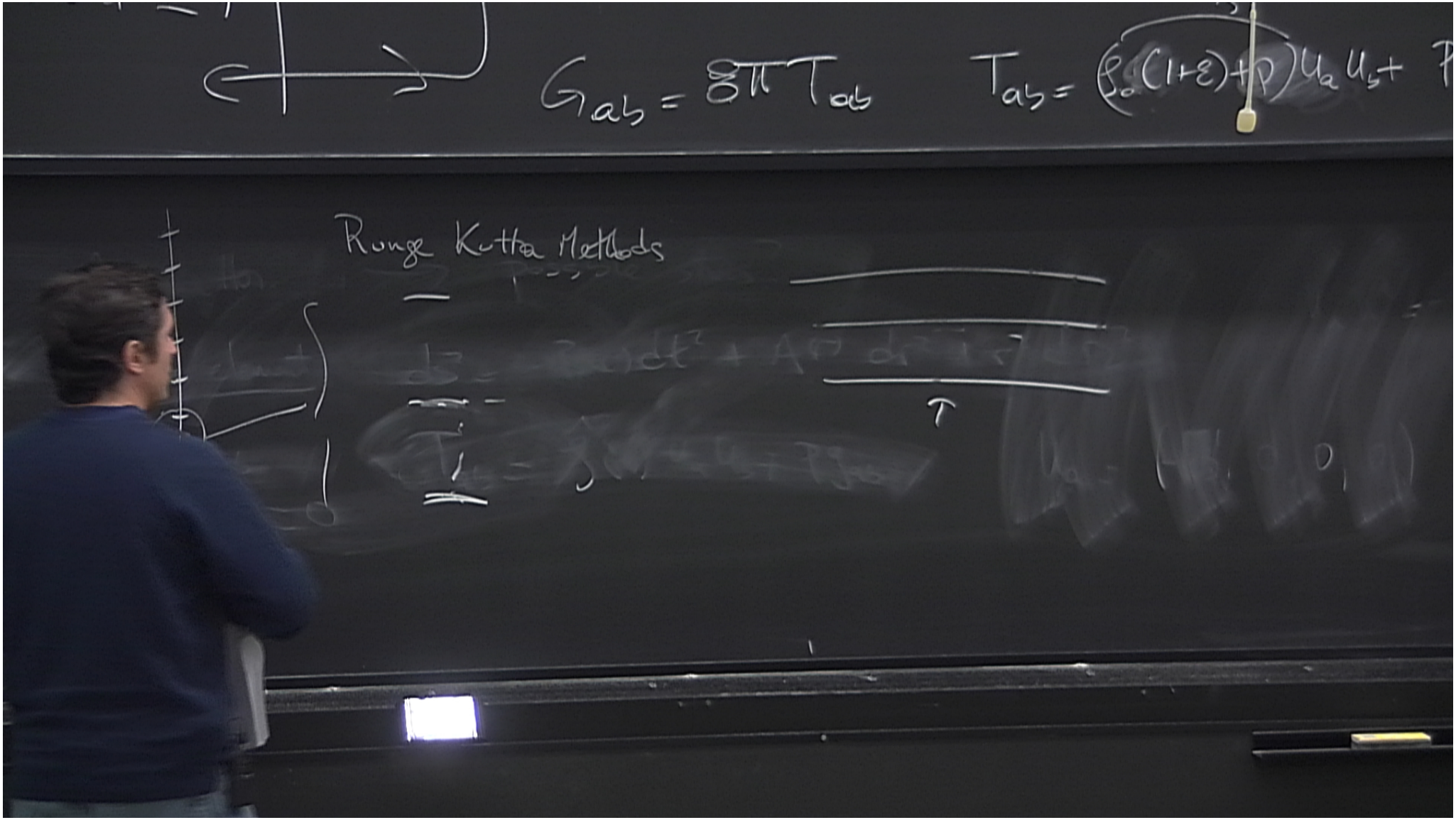

3
$\frac{dy}{dx} = F(y, x)$

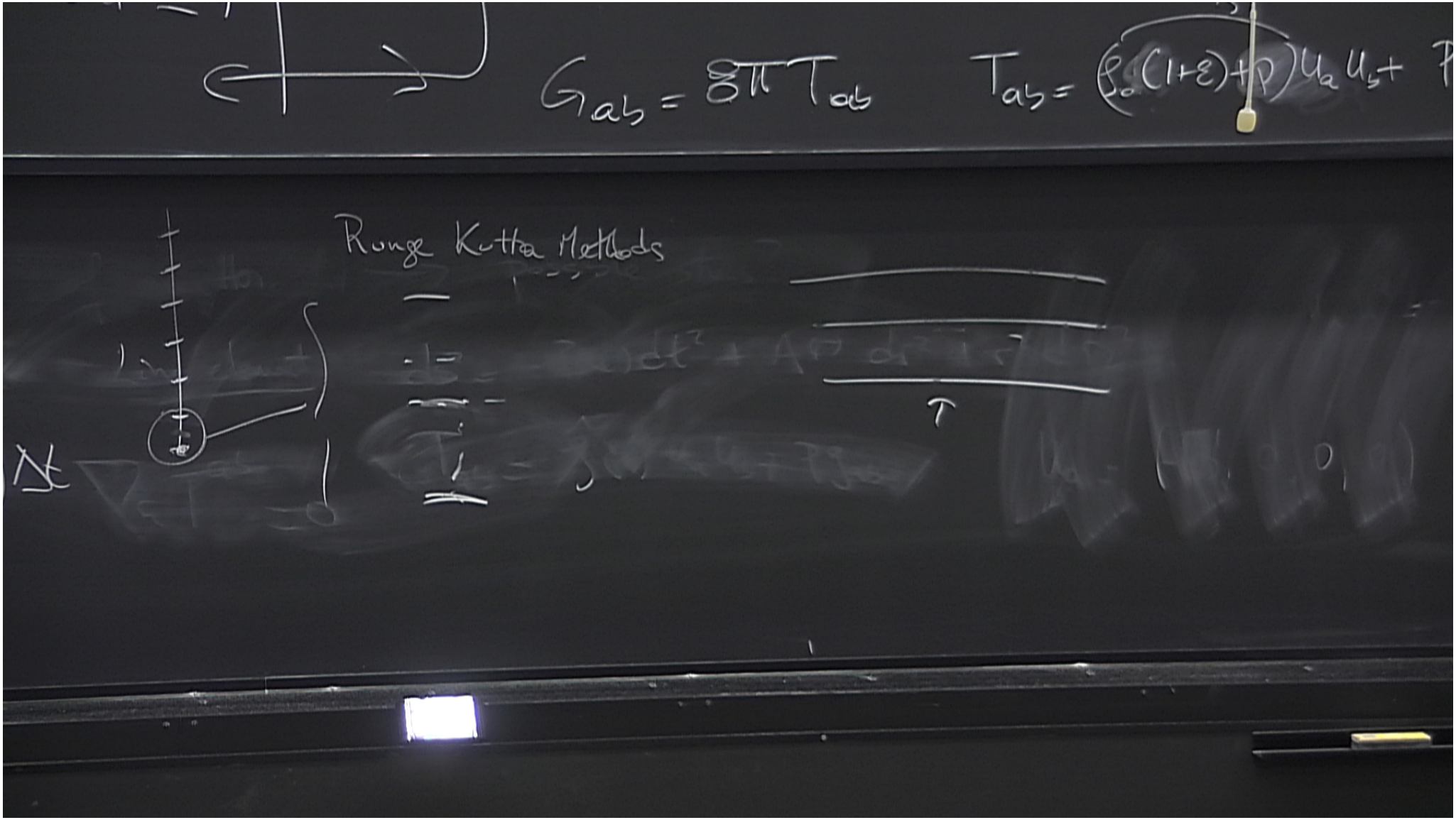

3
$\frac{dy}{dx} = F(y, x, \ldots)$
$y^{n+1} - y^n = F(y^{n+1/2}, x^{n+1/2}, \ldots)\Delta t$

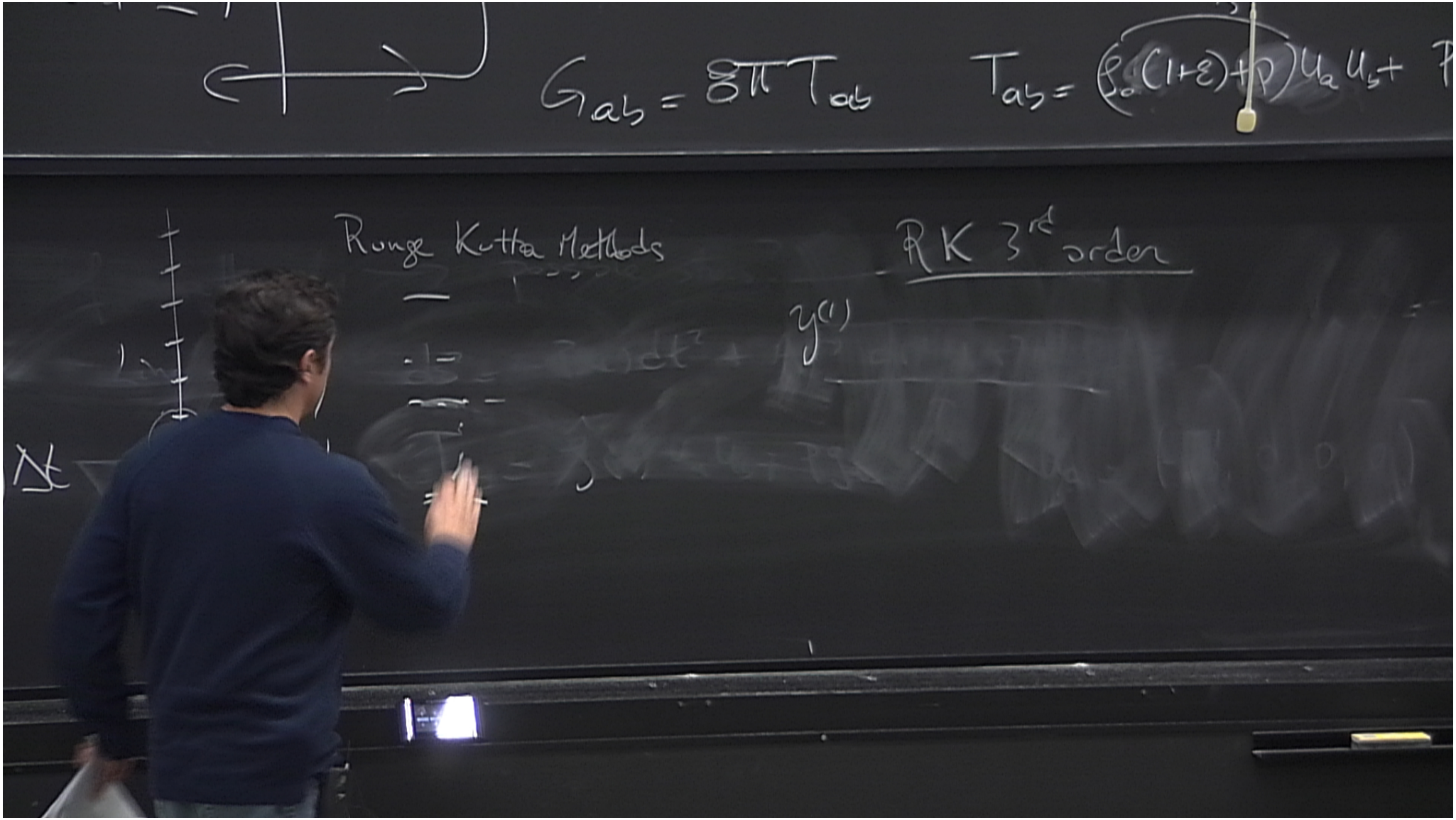

3
$\frac{dy}{dx} = F(y, x \ldots)$
$y^{n+1} = y^n + F(y^{n+1/2}, x^{n+1/2}, \ldots)\Delta t$

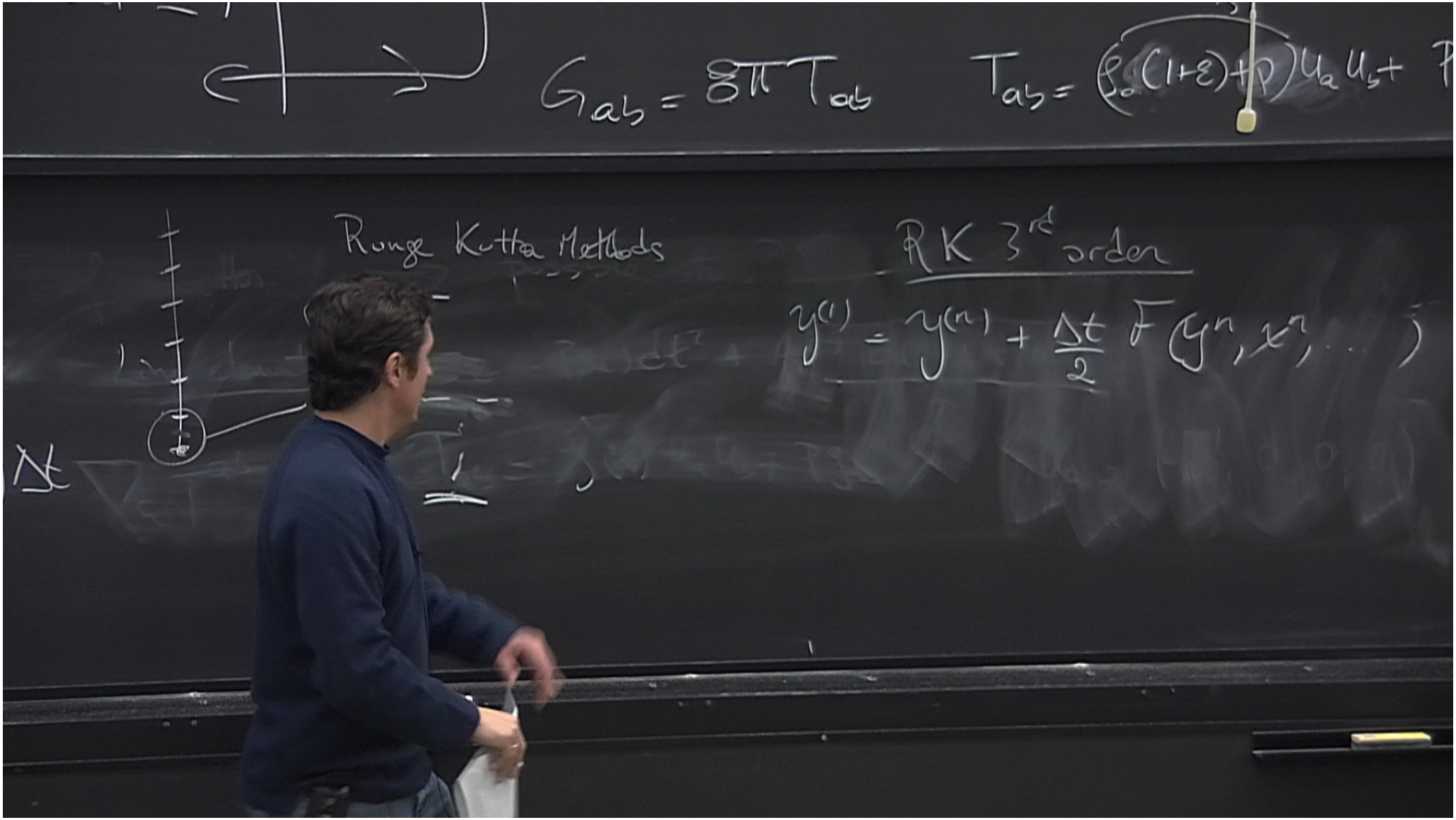

3
$\frac{dy}{dx} = F(y, x, \ldots)$
$y^{n+1} = y^n + F(y^{n+1/2}, x^{n+1/2}, \ldots)\Delta t$
$\frac{y^n + y^{n+1/2}}{2}$

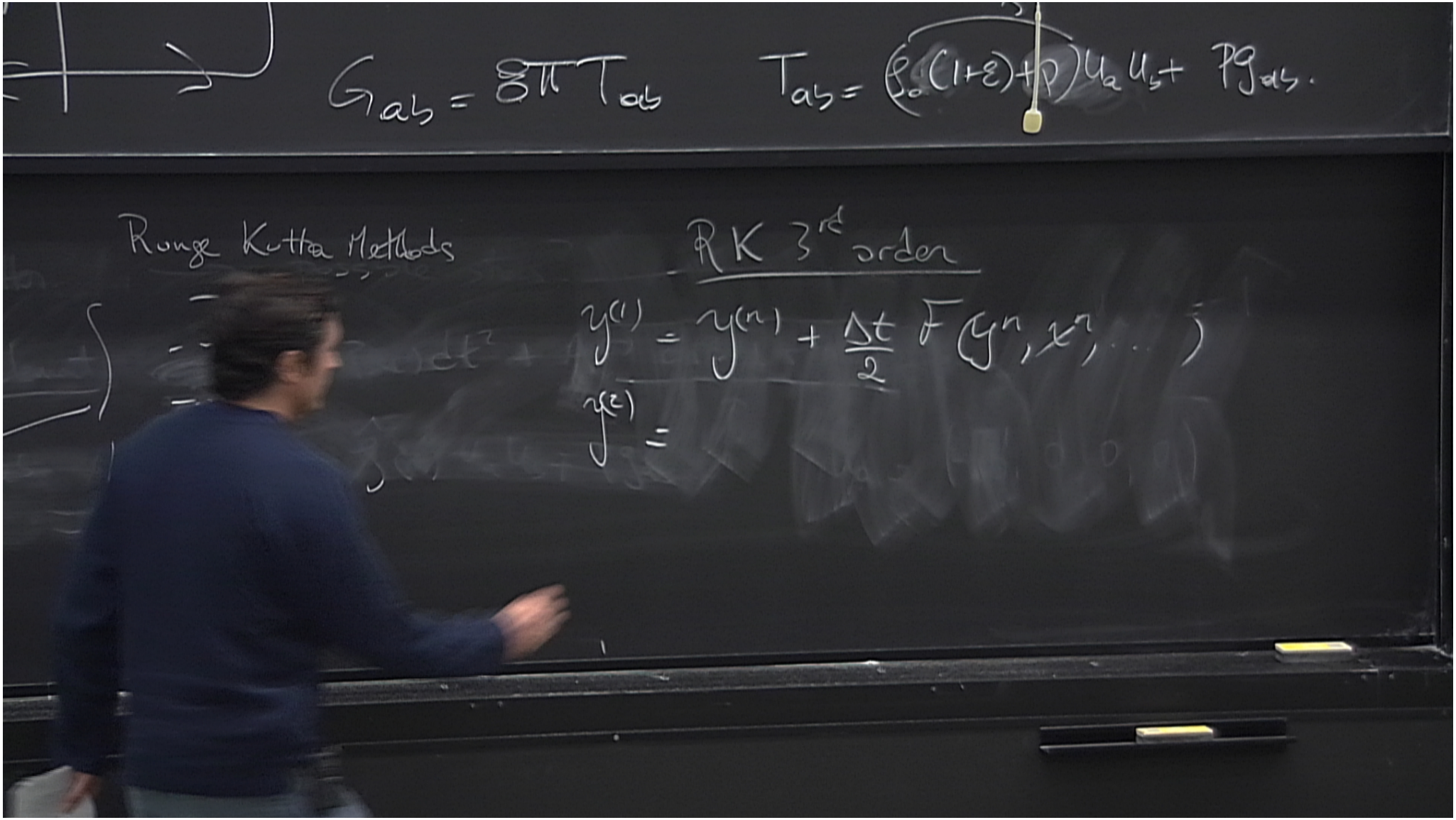

$G_{ab} = 8\pi T_{ab}$
$T_{ab} = (\rho_0(1+\varepsilon)+p)u_a u_b +$
Runge Kutta Methods
Δt
T

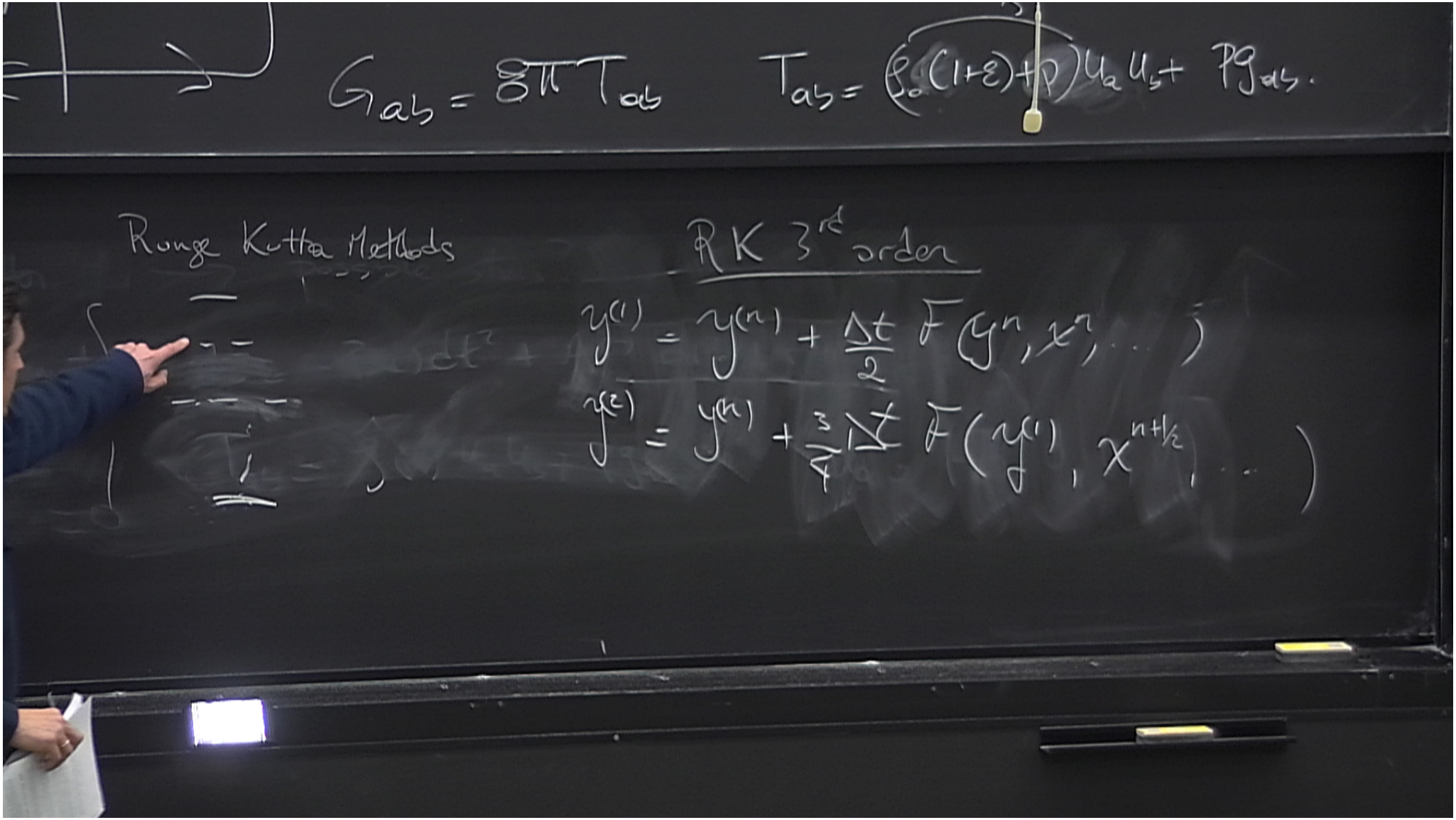

$G_{ab} = 8\pi T_{ab}$
$T_{ab} = (\rho_0(1+\varepsilon)+p)u_a u_b +$
Runge Kutta Methods


$G_{ab} = 8\pi T_{ab}$
$T_{ab} = (\rho_0(1+\varepsilon)+p)u_a u_b +$
Runge Kutta Methods
Δt


$G_{ab} = 8\pi T_{ab}$
$T_{ab} = (\rho_0(1+\varepsilon)+p)u_a u_b +$
Runge Kutta Methods
RK 3rd order
$y(t)$
Δt


$G_{ab} = 8\pi T_{ab}$
$T_{ab} = (\rho_0(1+\varepsilon)+p)u_a u_b +$
Runge Kutta Methods
RK 3rd order
$y^{(1)} = y^{(n)} + \frac{\Delta t}{2} F(y^n, t^n, \ldots)$
Δt


$G_{ab} = 8\pi T_{ab}$
$T_{ab} = (\rho_0(1+\varepsilon)+p)u_a u_b + p g_{ab}$.
Runge Kutta Methods
RK 3rd order
$y^{(1)} = y^{(n)} + \frac{\Delta t}{2} F(y^n, x^n, \ldots)$
$y^{(2)} =$


$G_{ab} = 8\pi T_{ab}$
$T_{ab} = (\rho_0(1+\varepsilon)+p)u_a u_b + p g_{ab}$.
Runge Kutta Methods
RK 3rd order
$y^{(1)} = y^{(n)} + \frac{\Delta t}{2} F(y^n, x^n, \ldots)$
$y^{(2)} = y^{(n)} + \frac{3}{4}\Delta t\, F(y^{(1)}, x^{n+1/2}, \ldots)$

$G_{ab} = 8\pi T_{ab}$ $\quad T_{ab} = (\rho_0(1+\varepsilon)+p)u_a u_b + p g_{ab}$.

Runge Kutta Methods

RK 3rd order

$$y^{(1)} = y^{(n)} + \frac{\Delta t}{2} F(y^n, x^n, \dots)$$

$$y^{(2)} = y^{(n)} + \frac{3}{4}\Delta t\, F(y^{(1)}, x^{n+1/2}, \dots)$$

$$y^{n+1} = y^n + \frac{\Delta t}{9}\left(2F(y^n; t^n) + 3F(y^{(1)}; x^{n+1/2}) + 4F(y^{(2)}, x^{n+3/4})\right)$$

$G_{ab} = 8\pi T_{ab}$ $\quad T_{ab} = (\rho_0(1+\varepsilon)+p)u_a u_b + p g_{ab}$.

Runge Kutta Methods

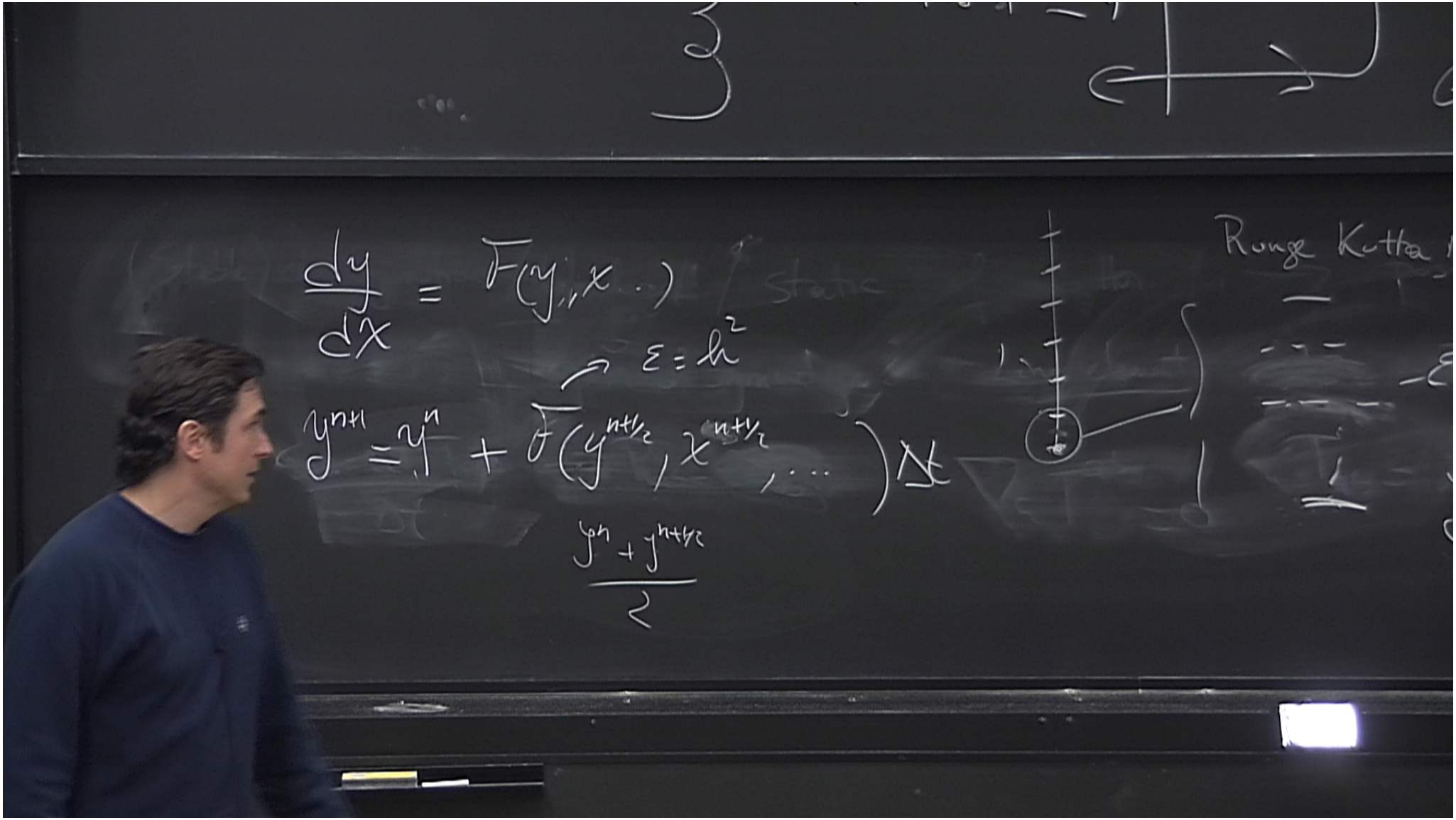
$\sim \varepsilon \sim h^3$

$y^{n+1} = y^n + \frac{\Delta t}{9}\left(2F(y^n, t^n) + 3F(y^{(1)}, x^{n+1/2}) + 4F(y^{(2)}, x^{n+3/4})\right)$

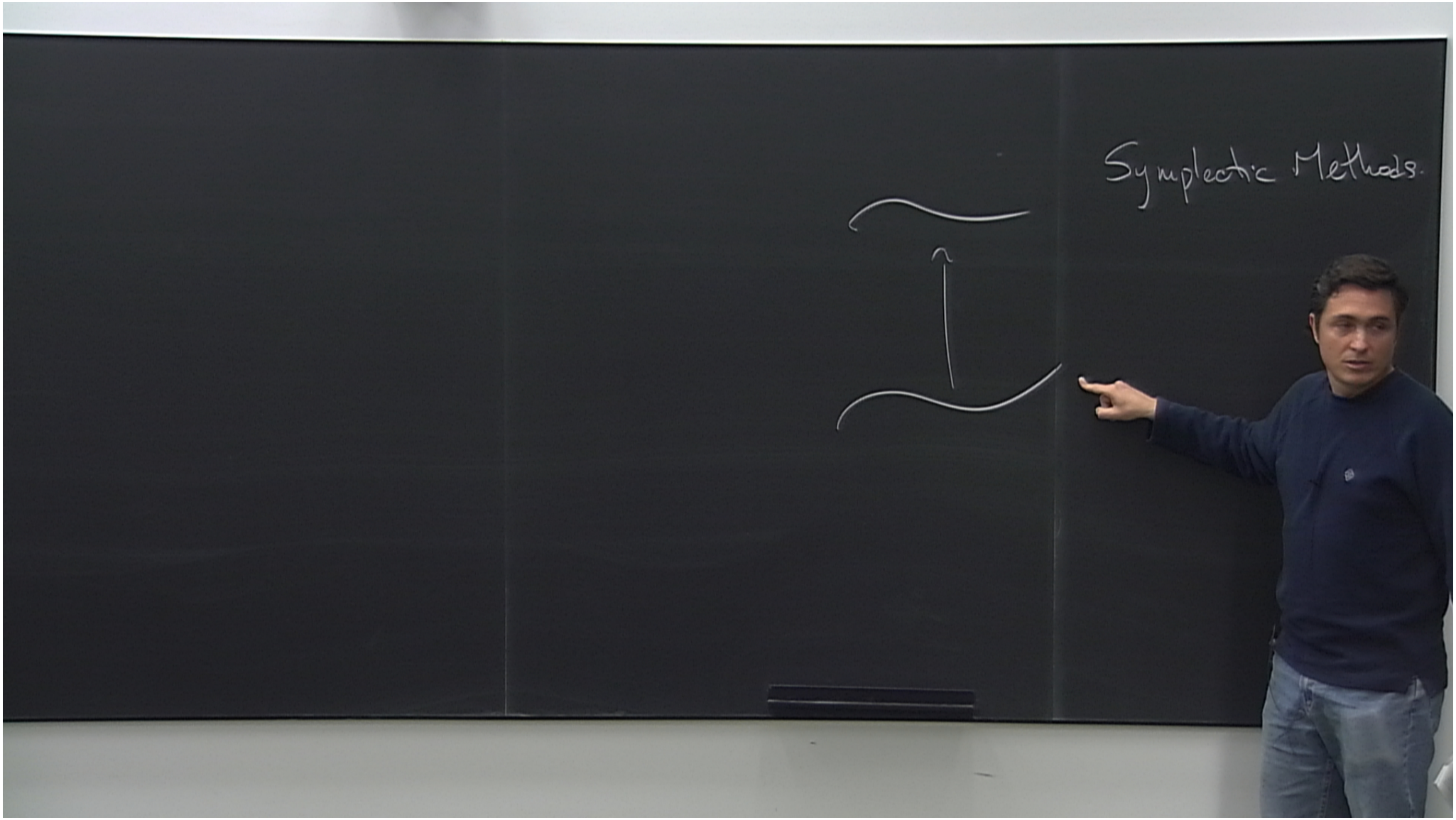
RK 3rd order

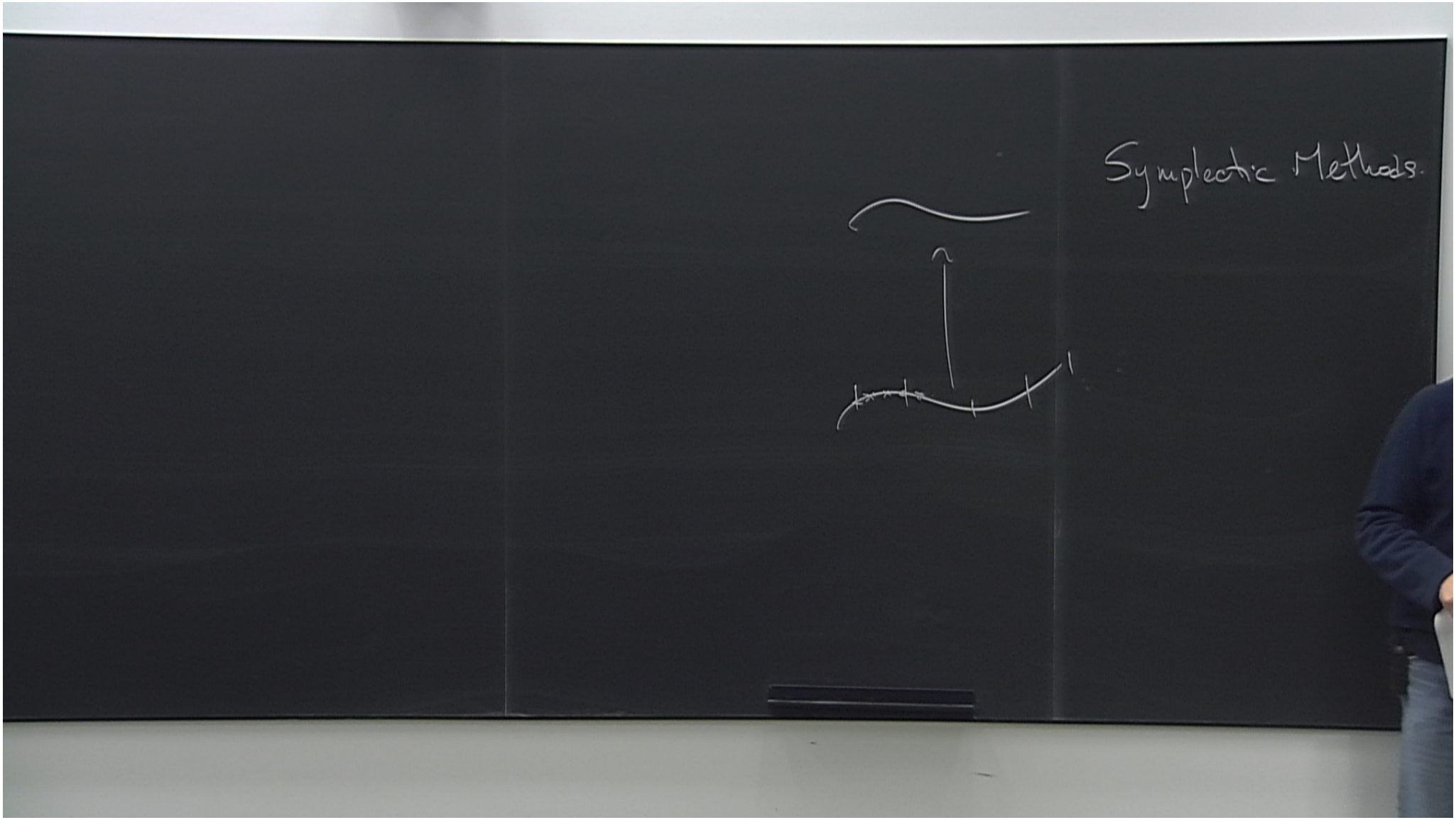
$y^{(1)} = y^{(n)} + \frac{\Delta t}{2} F(y^n, x^n, \ldots)$

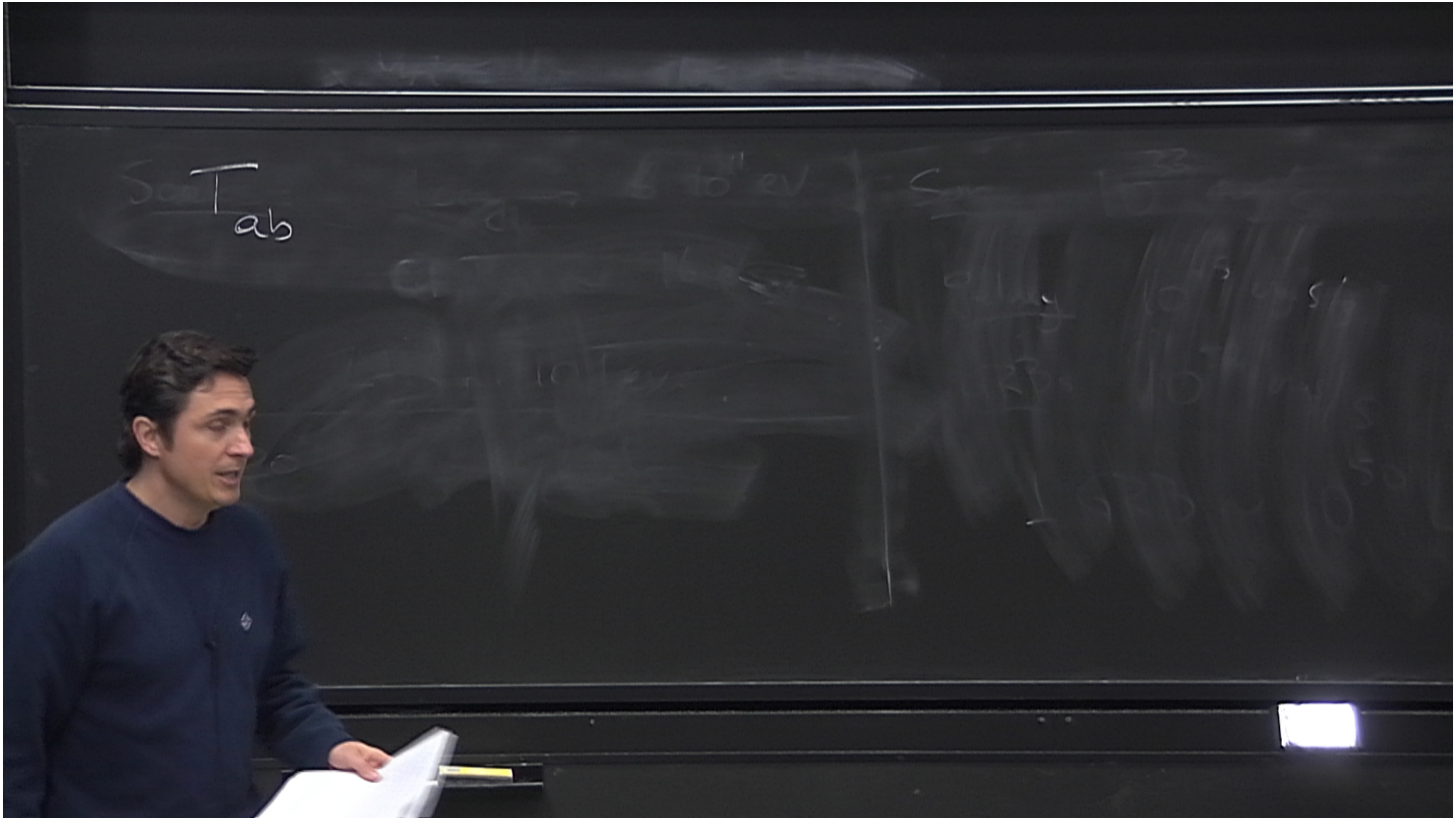
$y^{(2)} = y^{(n)} + \frac{3}{4}\Delta t\, F(y^{(1)}, x^{n+1/2}, \ldots)$

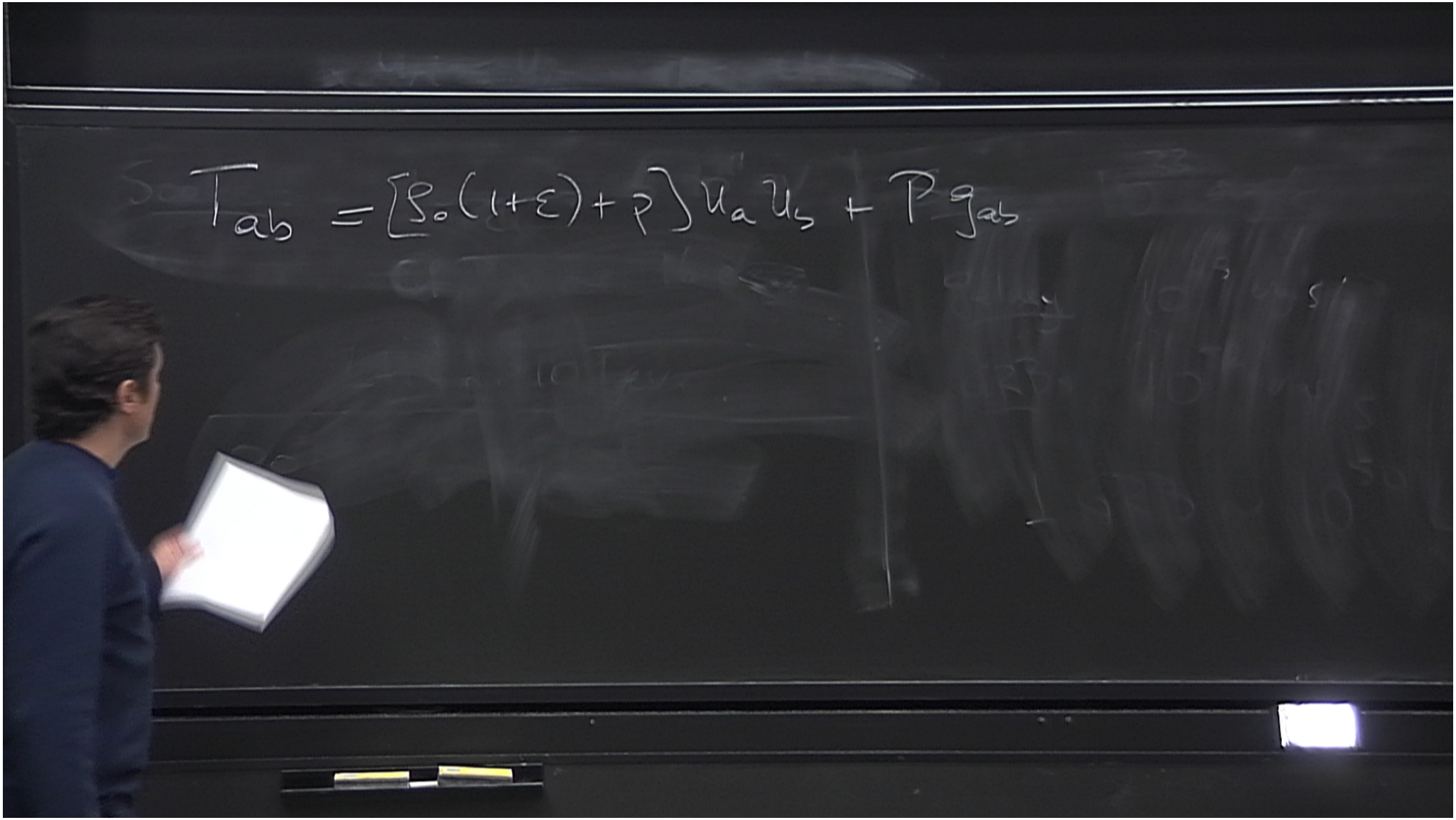

$\frac{dy}{dx} = F(y, x \ldots)$
$\varepsilon = h^2$
$y^{n+1} = y^n + \bar{F}(y^{n+1/2}, x^{n+1/2}, \ldots)\Delta t$
$\frac{y^n + y^{n+1/2}}{2}$
Runge Kutta

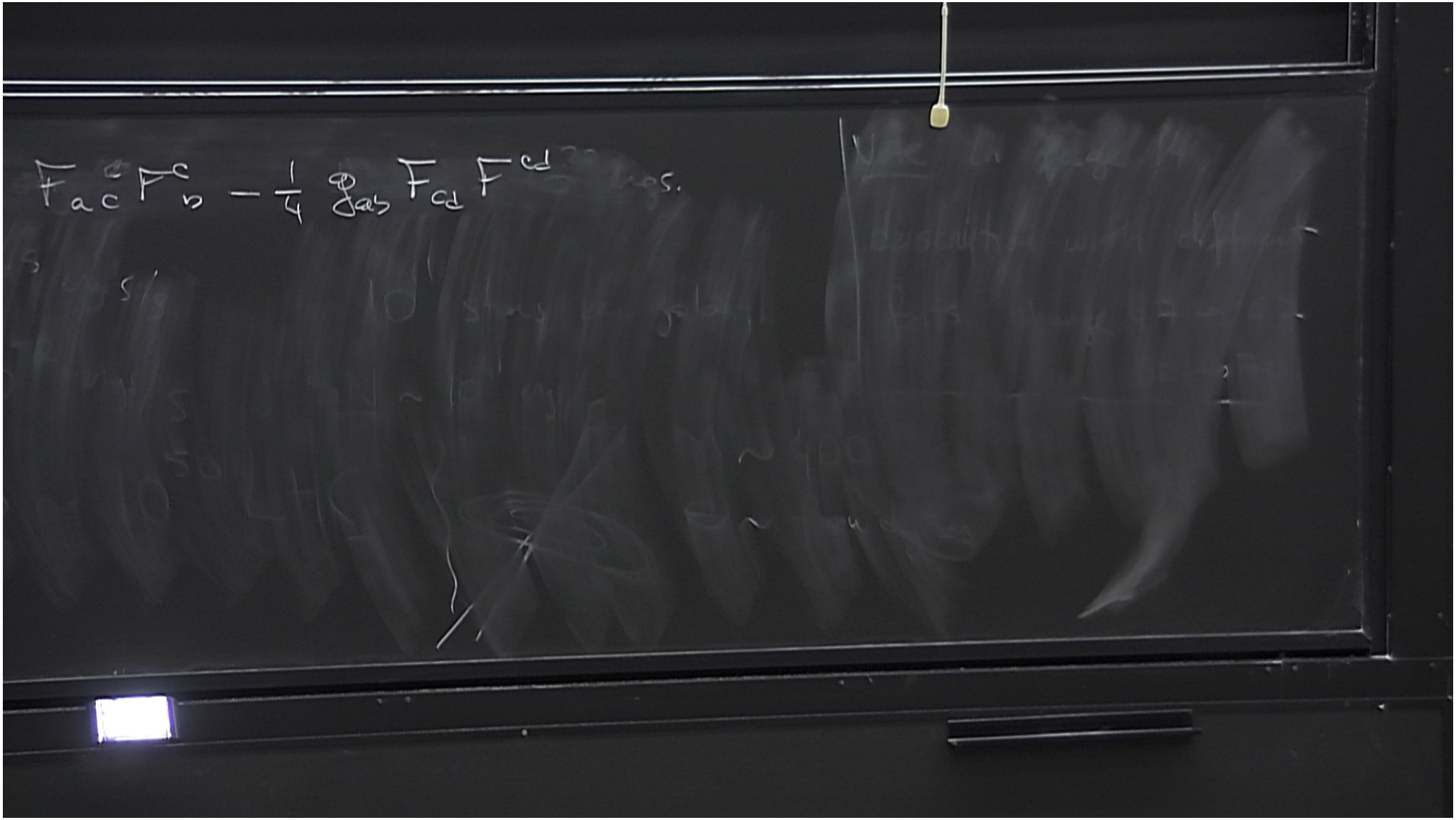

Symplectic Methods.

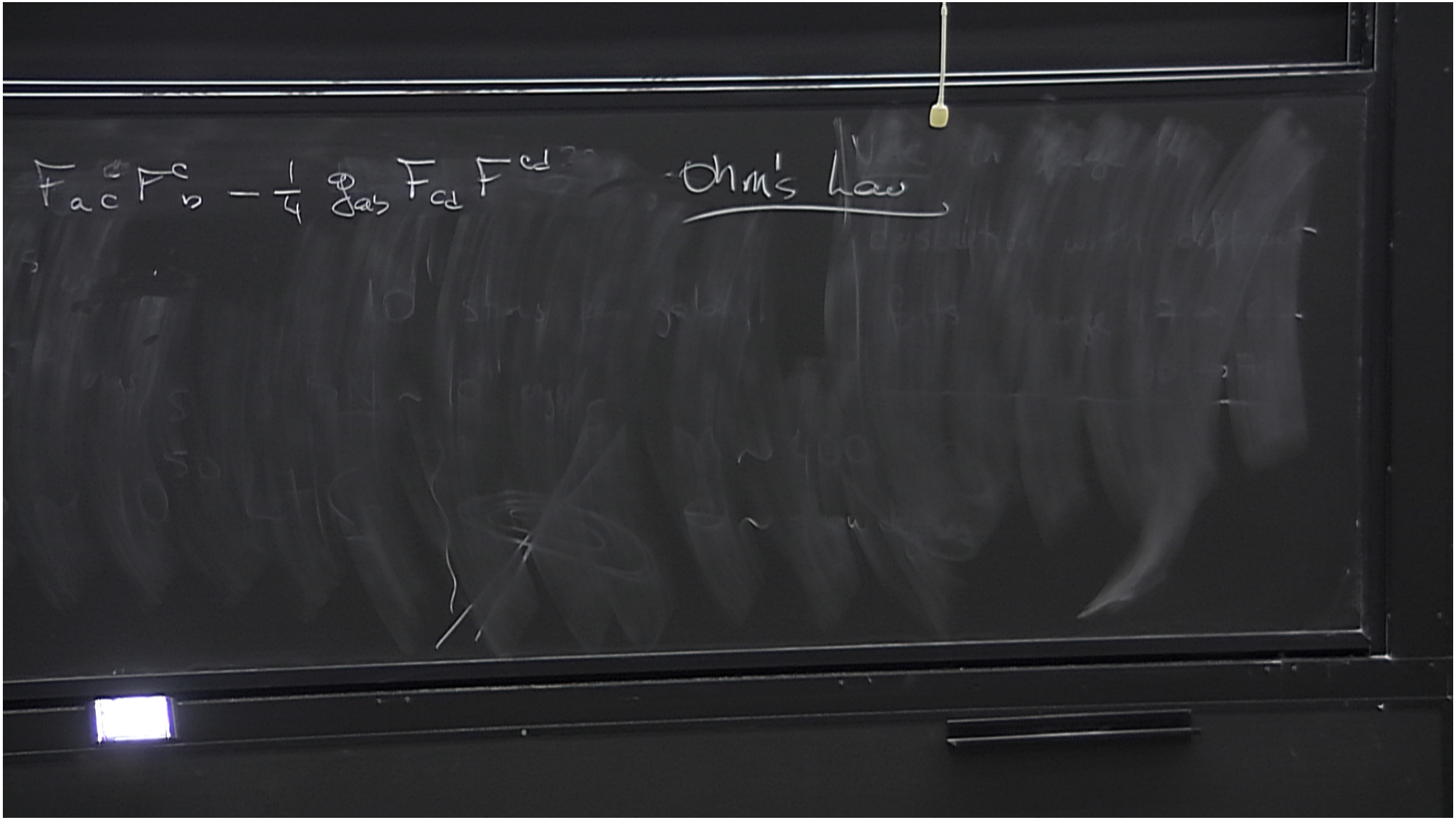

Symplectic Methods.

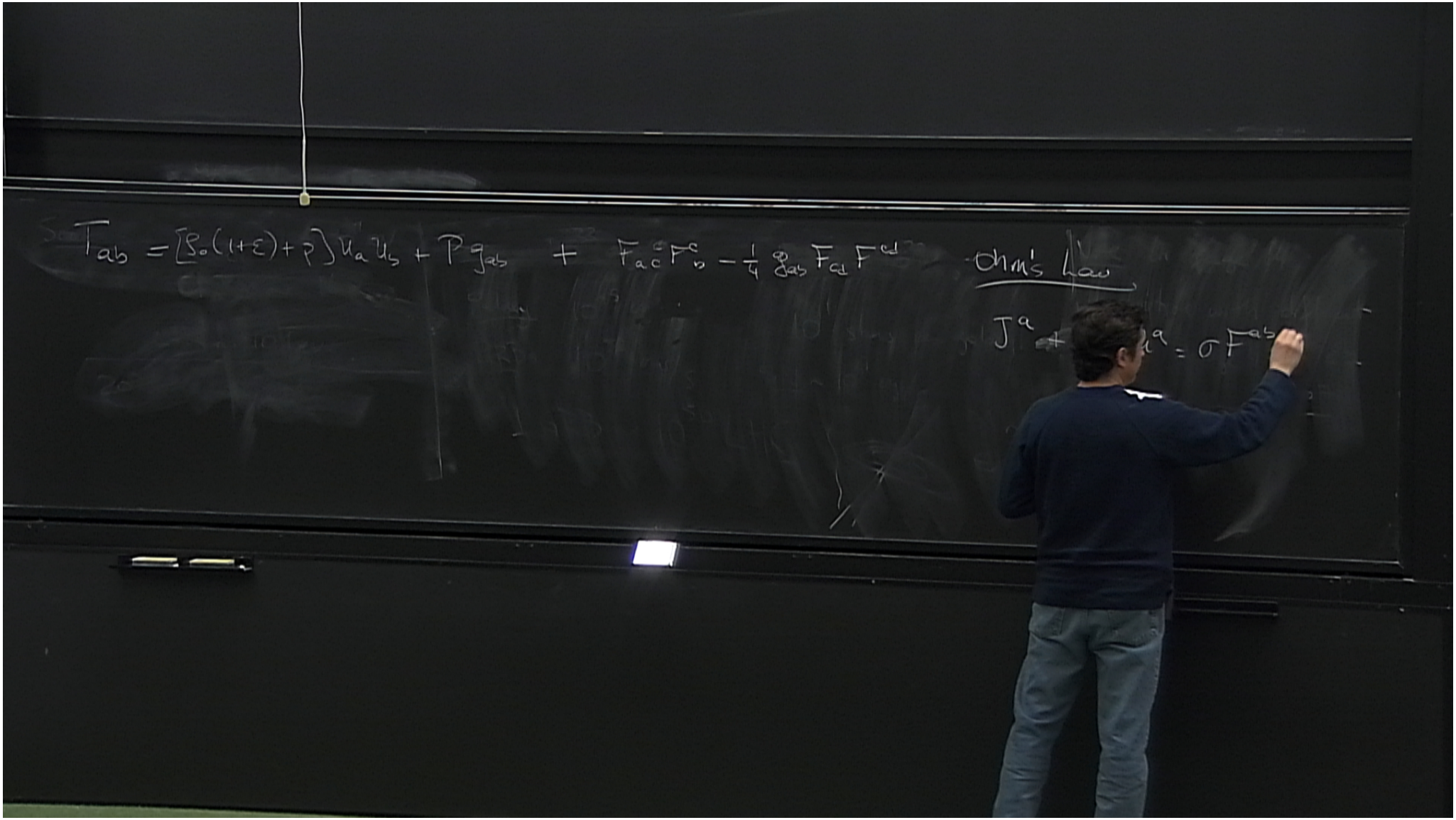

Symplectic Methods.

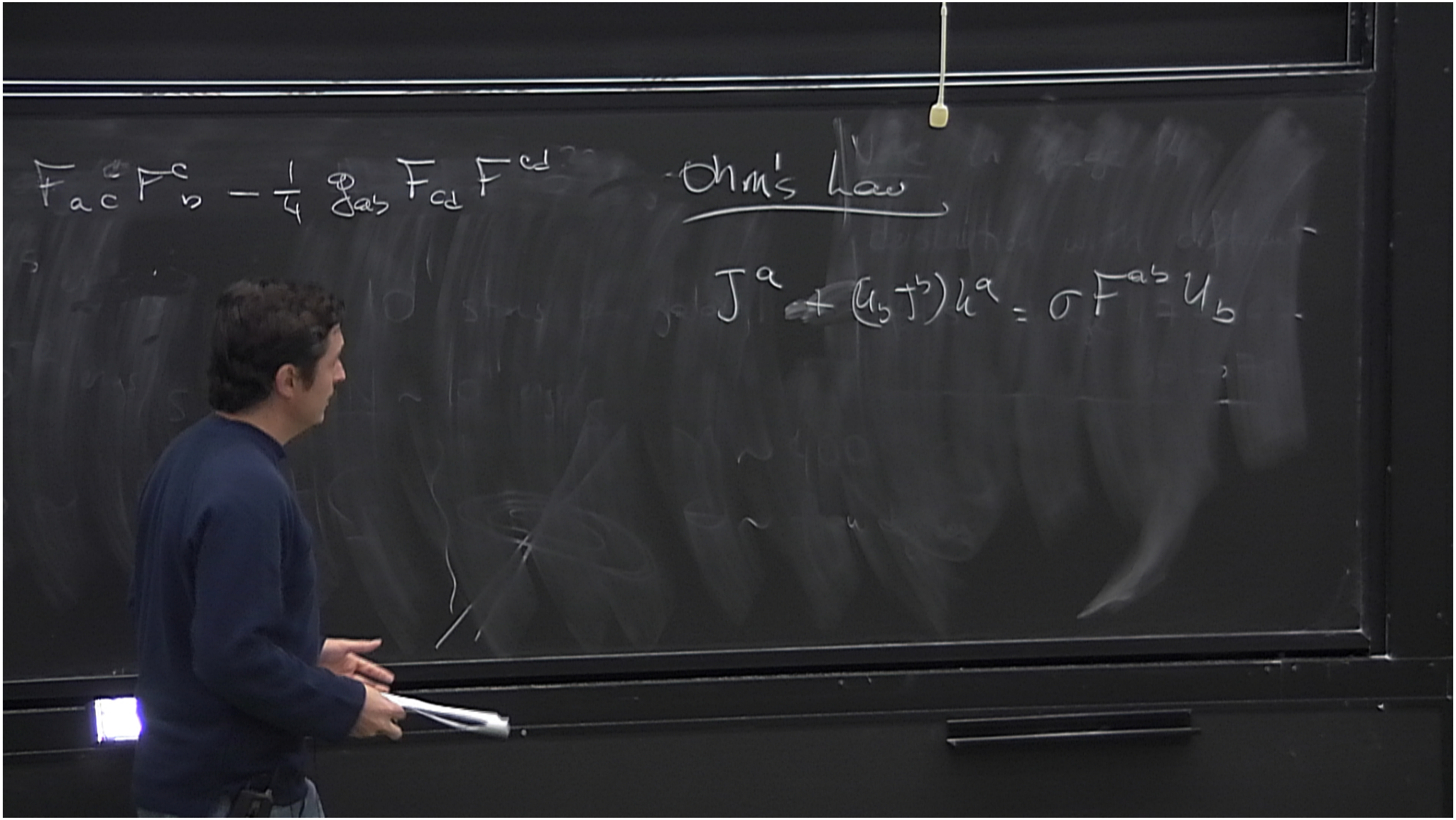

Tab


$T_{ab} = [\rho_0(1+\varepsilon) + p] u_a u_b + P g_{ab}$


$F_{ac}^{c} F^{cb} - \frac{1}{4} g_{as} F_{cd} F^{cd}$


$F_{ac} F^{c}_{b} - \frac{1}{4} g_{ab} F_{cd} F^{cd}$
Ohm's law


$T_{ab} = [\rho_0(1+\varepsilon)+p]u_a u_b + p g_{ab} + F_{ac}F^{c}_{b} - \frac{1}{4} g_{ab} F_{cd} F^{cd}$
Ohm's Law
$J^a + \ldots = \sigma F^{ab}$


$F_{ac}F^{c}_{b} - \frac{1}{4} g_{ab} F_{cd} F^{cd}$
Ohm's law
$J^a + (u_b J^b) u^a = \sigma F^{ab} u_b$

$F_{ac} F^{c}_{\ b} - \frac{1}{4} g_{ab} F_{cd} F^{cd}$

Ohm's law

$J^a + (u_b J^b) u^a = \sigma F^{ab} u_b$

σ ↑ conductivity

$F_{ac}F^{c}_{b} - \frac{1}{4} g_{ab} F_{cd} F^{cd}$

Ohm's law

$J^a + (u_b J^b) u^a = \sigma F^{ab} u_b$

↑ conductivity



$F_{ac} F^{c}_{\ b} - \frac{1}{4} g_{ab} F_{cd} F^{cd}$

Ohm's law

$J^a + (u_b J^b) u^a = \sigma F^{ab} u_b$

σ: conductivity

$\sigma = \infty$

$F_{ac}F^{c}_{b} - \frac{1}{4} g_{ab} F_{cd} F^{cd}$

Ohm's law

$J^a + (u_b J^b) u^a = \sigma F^{ab} u_b$

conductivity

$\sigma \to \infty$

$\sigma \to 0$

$F_{ac}F^{c}_{b} - \frac{1}{4} g_{ab} F_{cd} F^{cd}$

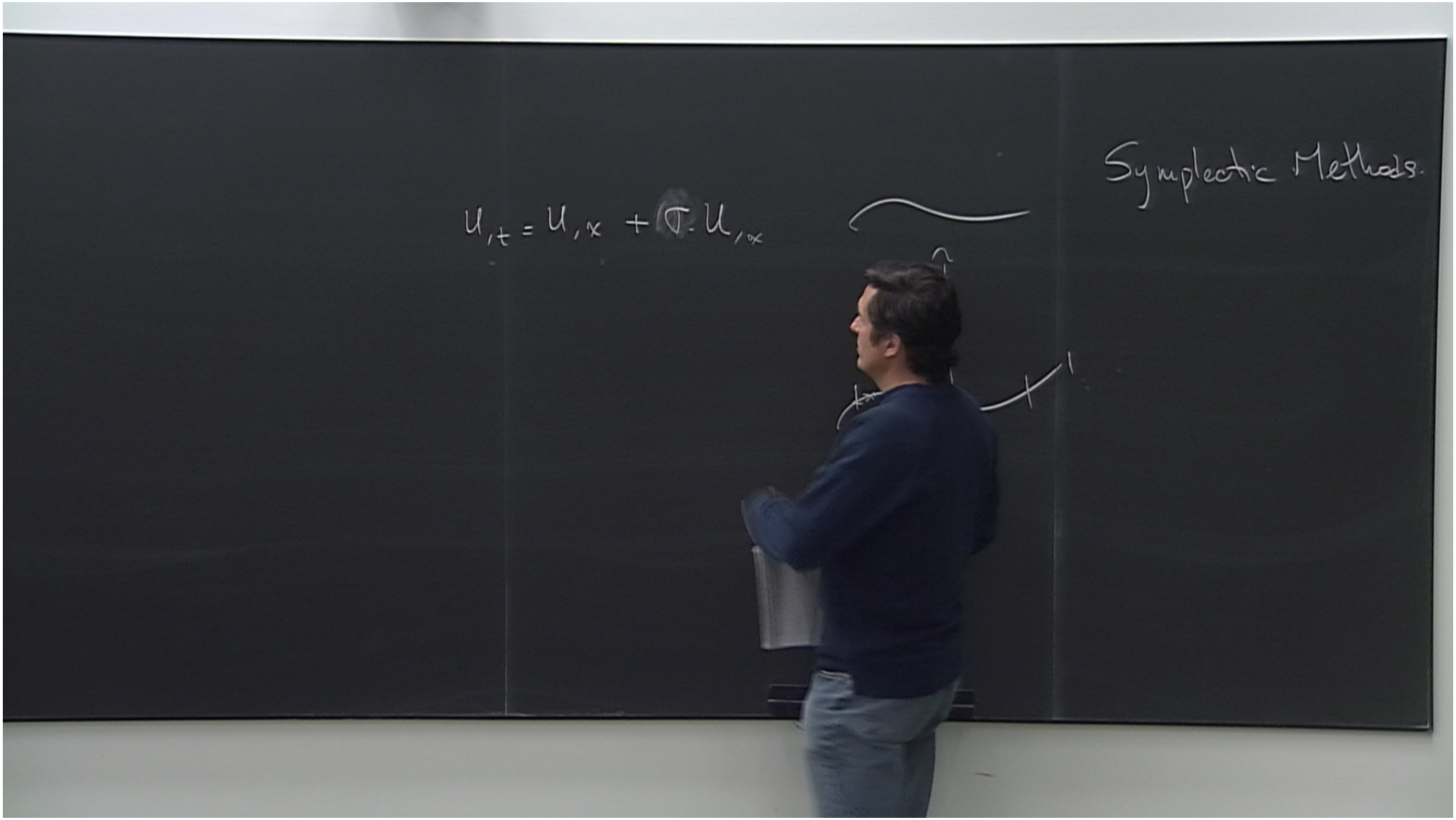
Ohm's law

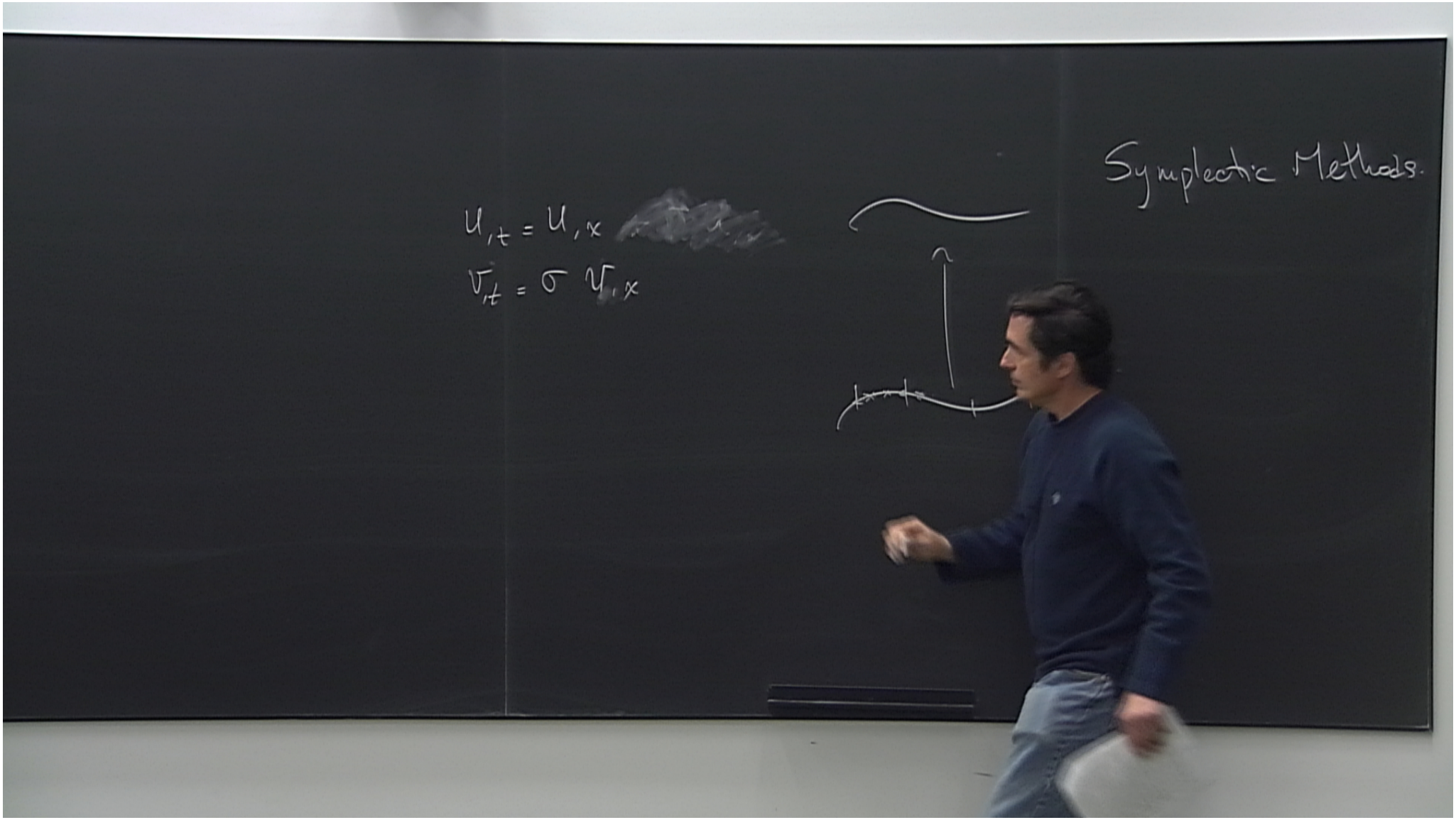
$J^a + (u_b J^b) u^a = \sigma F^{ab} u_b$

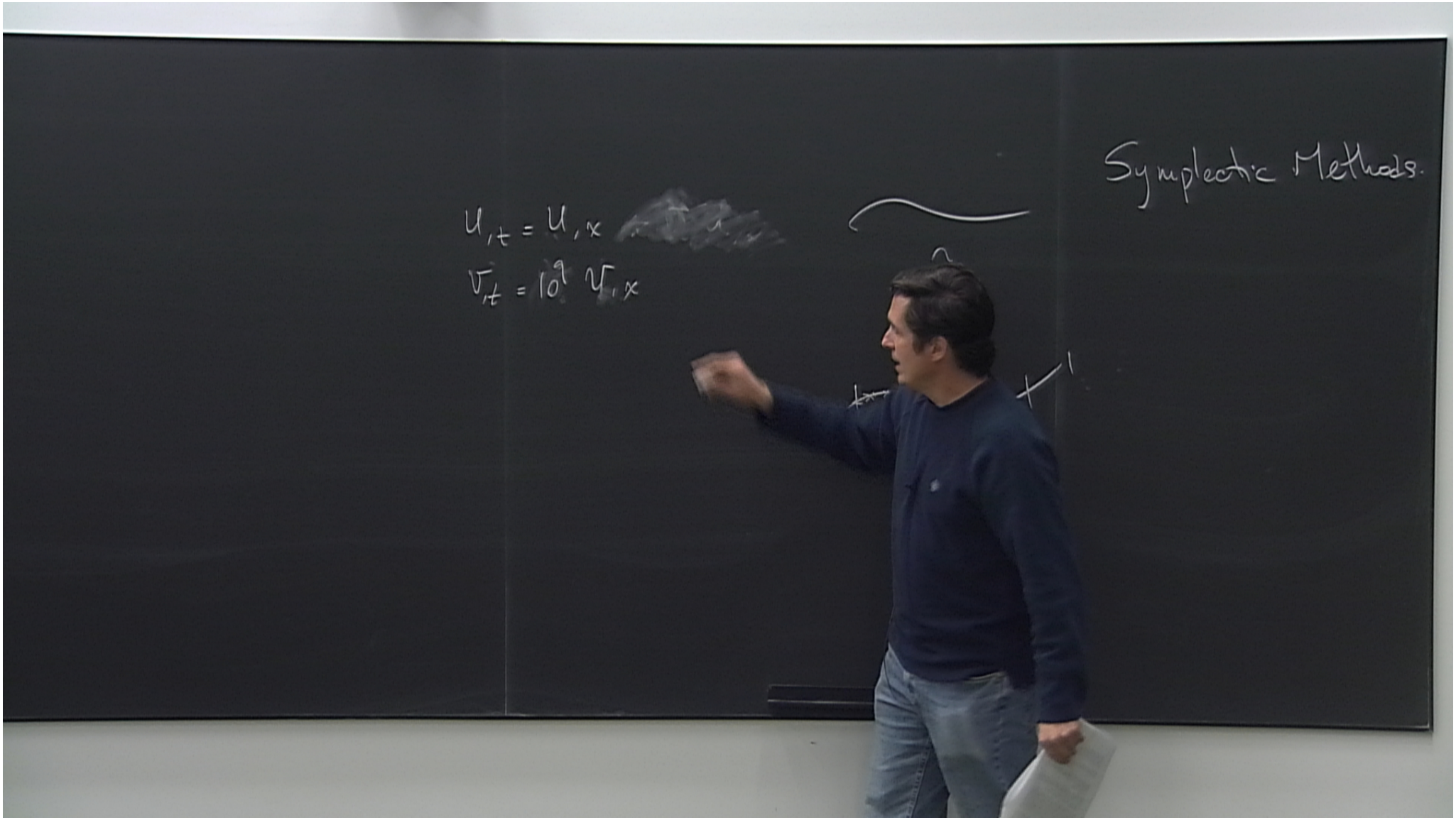
conductivity

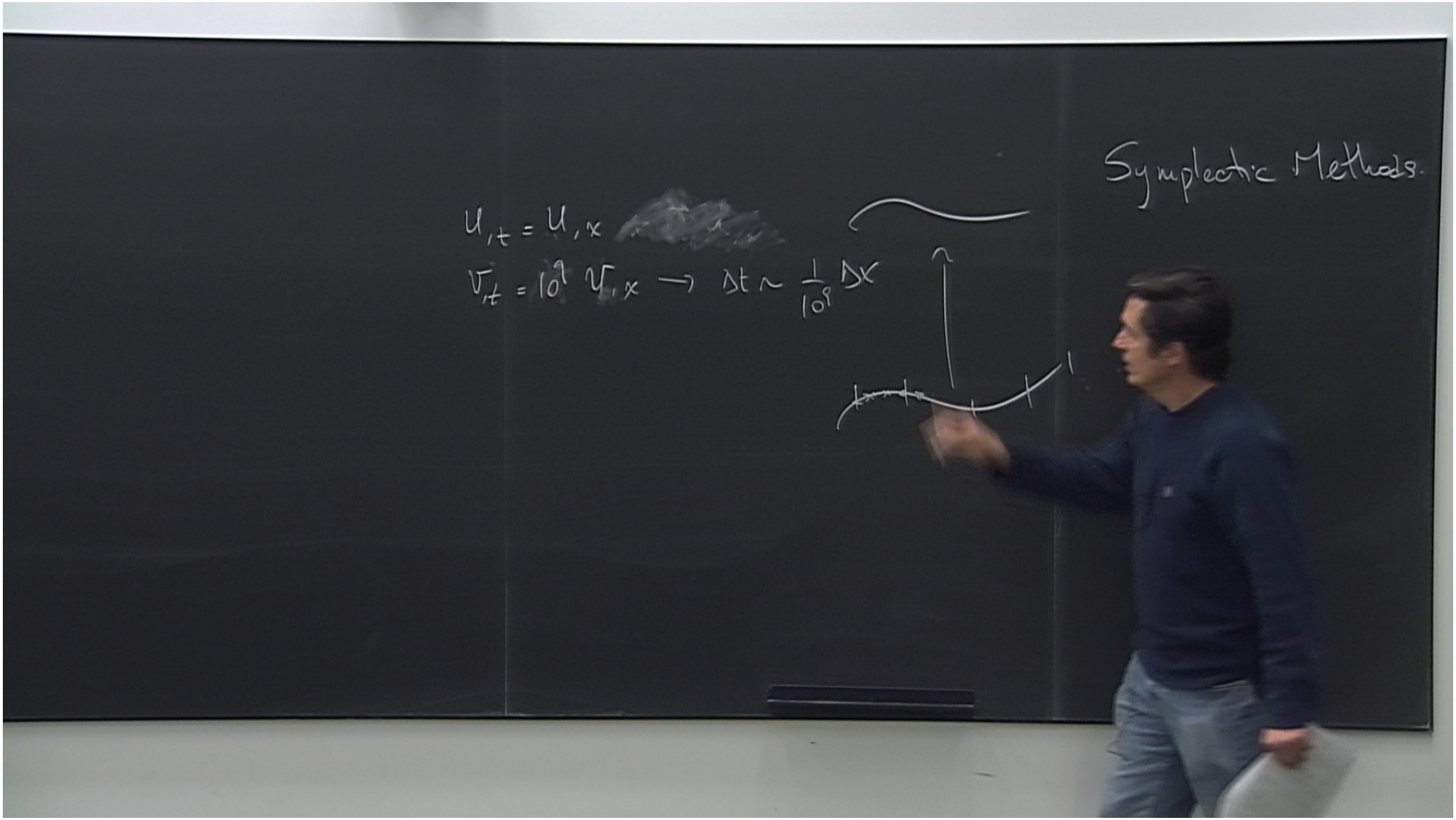
$\sigma \to \infty$

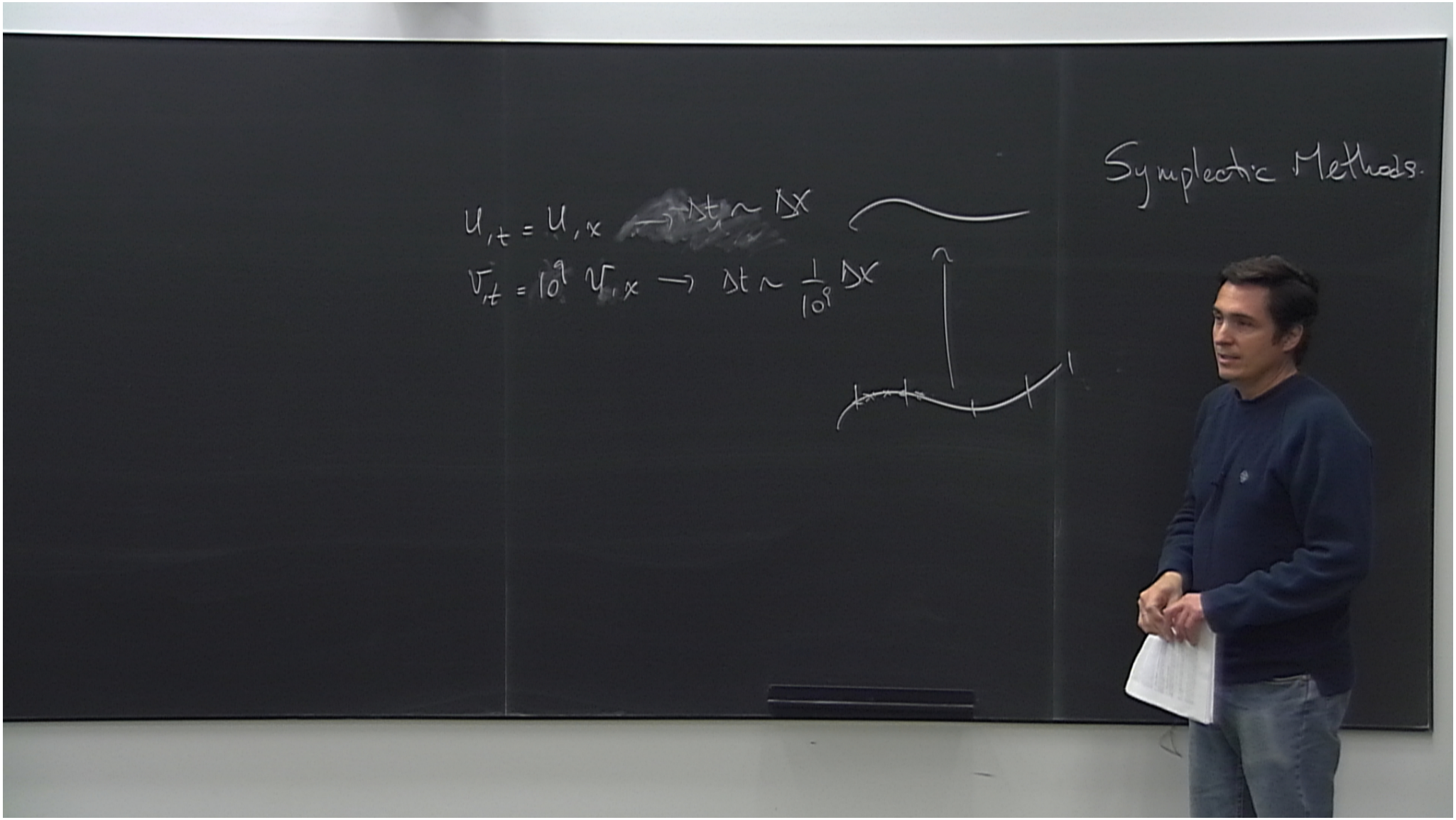
$\sigma \to 0$

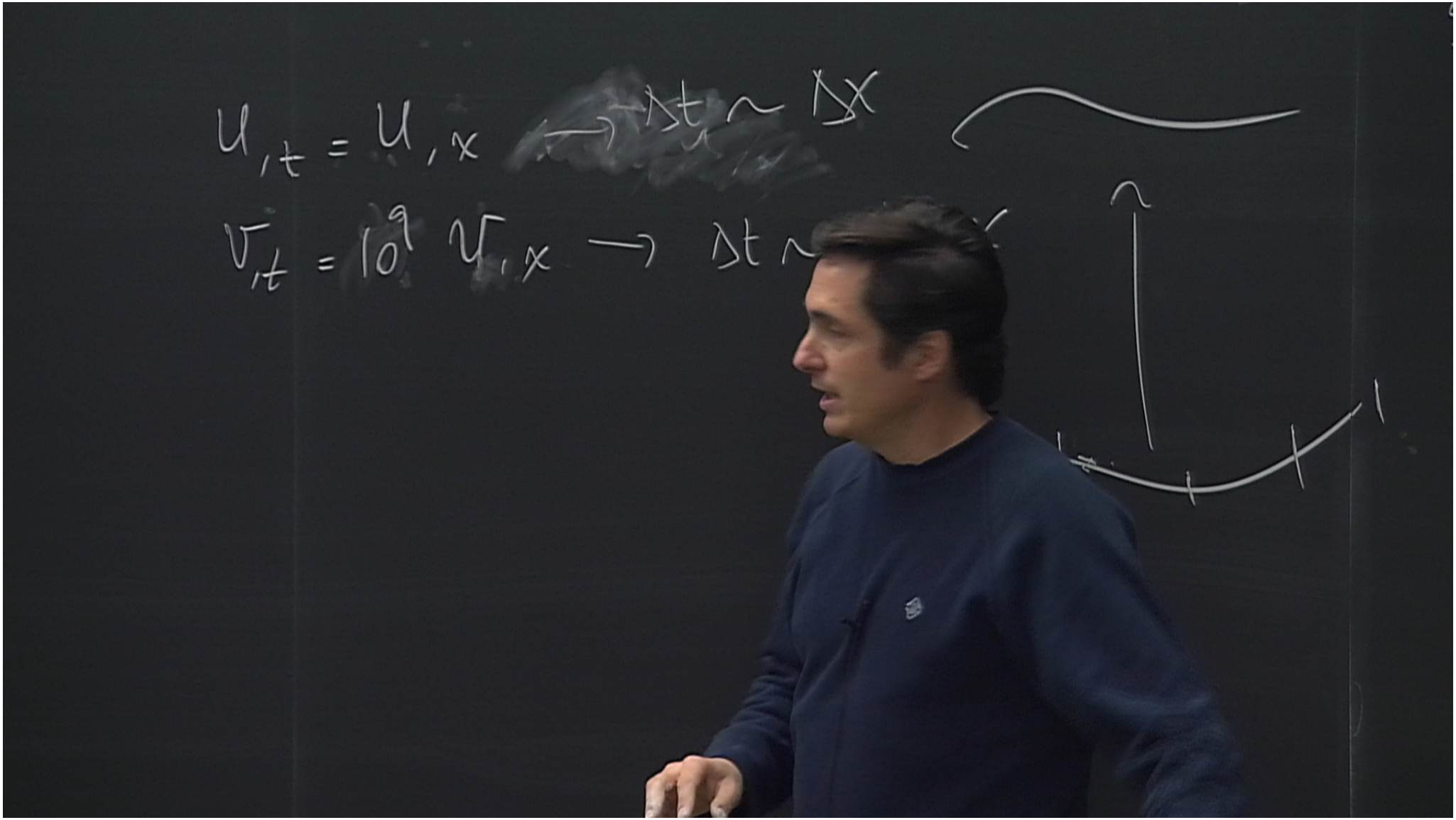

Symplectic Methods.
$u_{,t} = u_{,x} + \sigma \cdot u_{,xx}$


Symplectic Methods.
$u_{,t} = u_{,x}$
$v_{,t} = \sigma\, v_{,x}$


Symplectic Methods.
$u_{,t} = u_{,x}$
$v_{,t} = 10^9 v_{,x}$

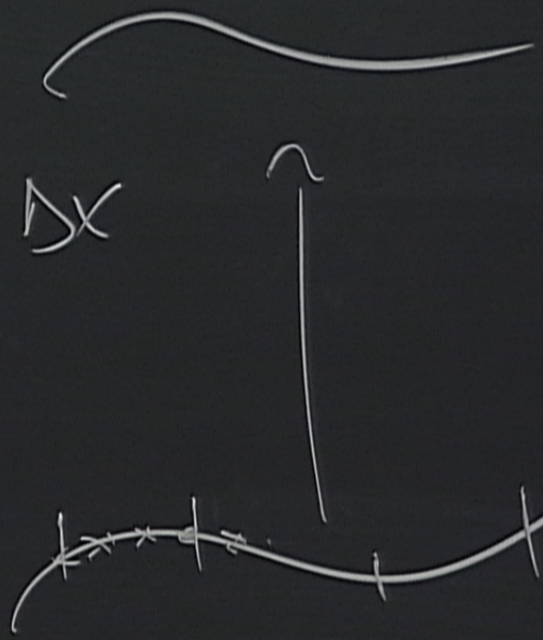

Symplectic Methods.
$u_{,t} = u_{,x}$
$v_{,t} = 10^9 v_{,x} \rightarrow \Delta t \sim \frac{1}{10^9} \Delta x$

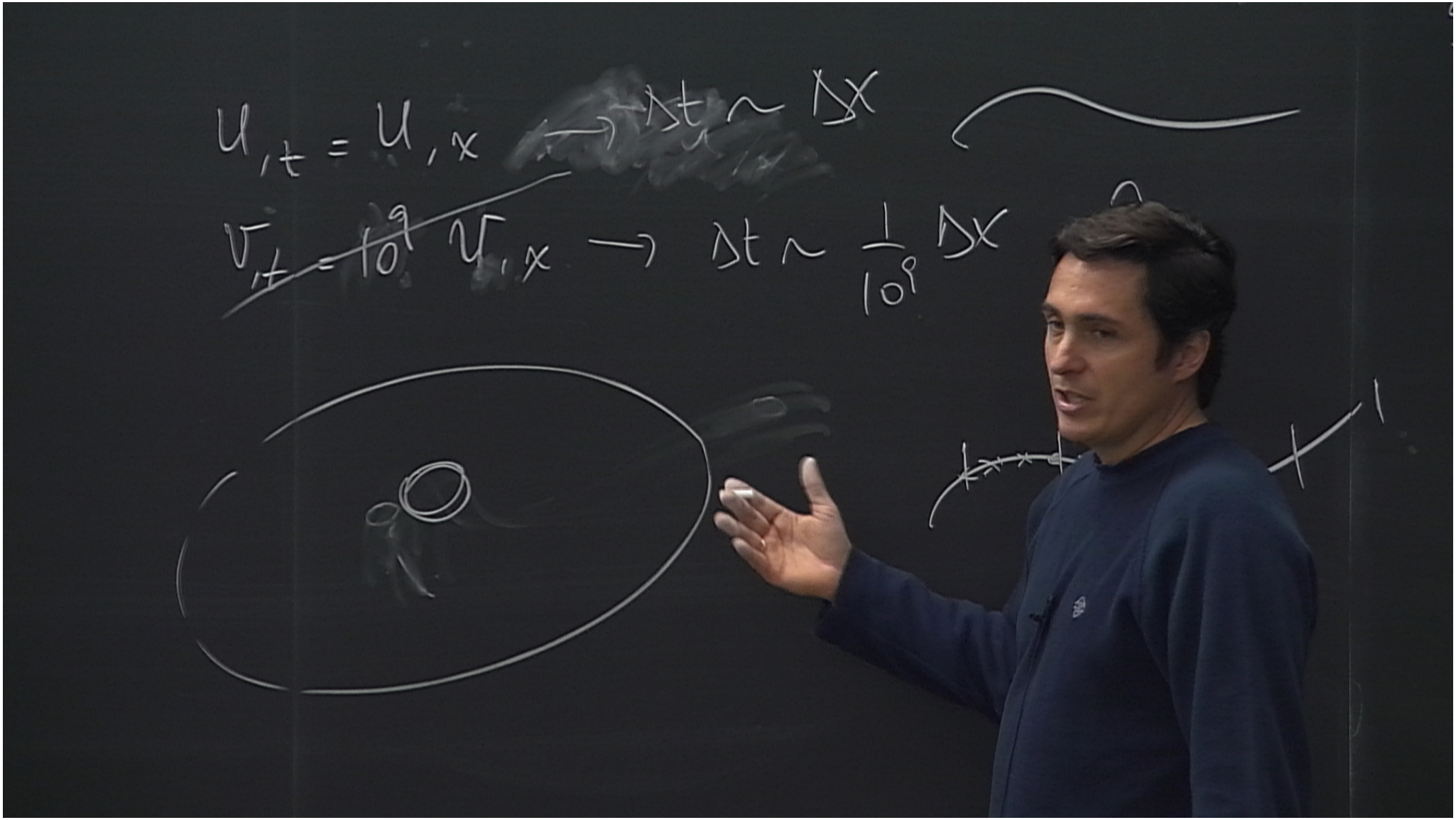

Symplectic Methods.
$u_{,t} = u_{,x} \longrightarrow \Delta t \sim \Delta x$
$v_{,t} = 10^9 v_{,x} \longrightarrow \Delta t \sim \frac{1}{10^9} \Delta x$

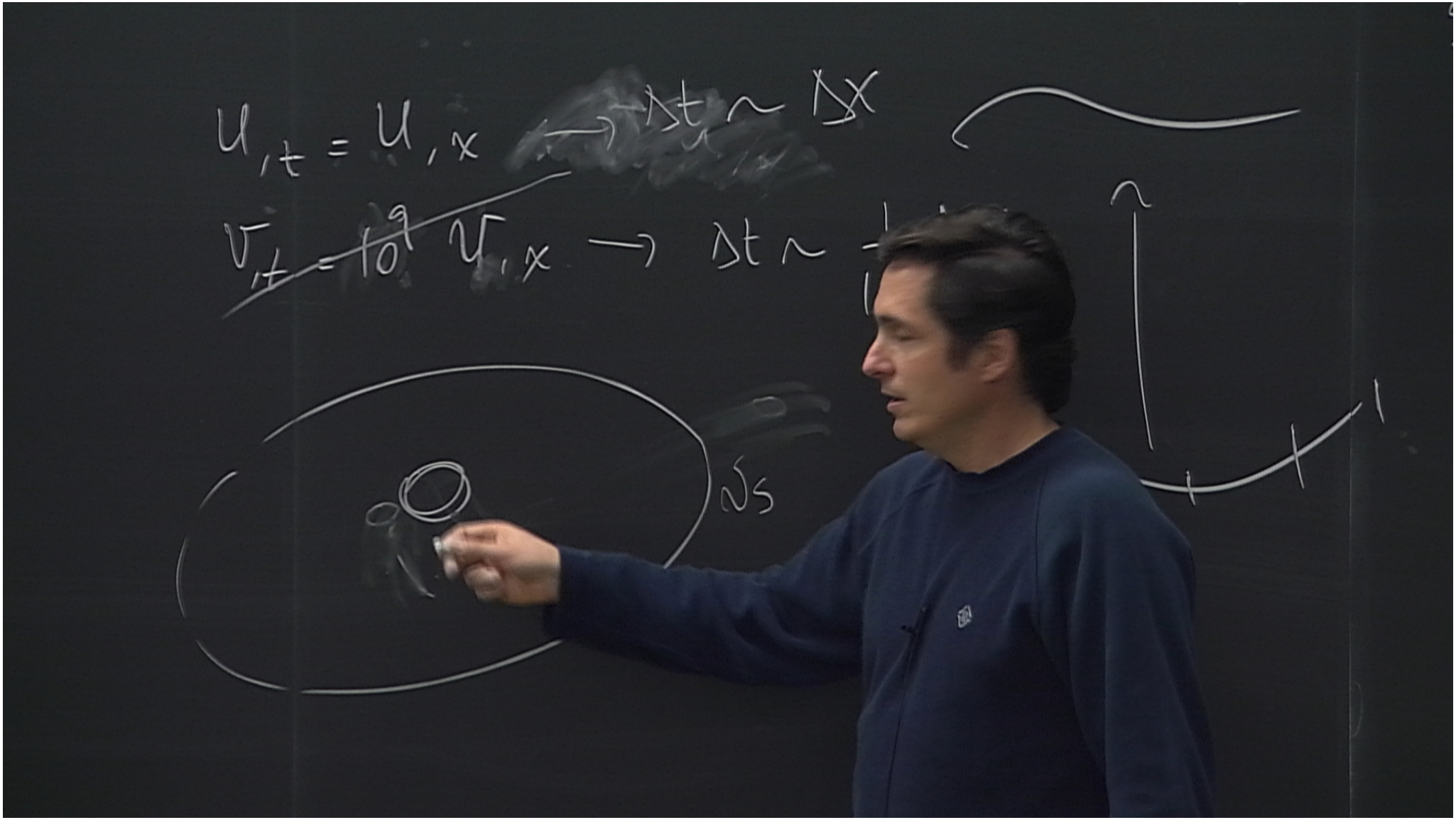

$u_{,t} = u_{,x} \rightarrow \Delta t \sim \Delta x$
$v_{,t} = 10^9 v_{,x} \rightarrow \Delta t \sim$

$u_{,t} = u_{,x} \rightarrow \Delta t \sim \Delta x$

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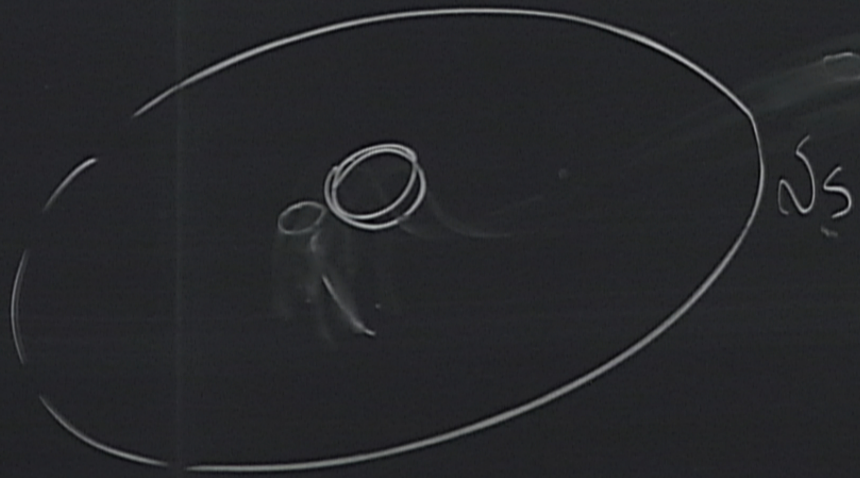


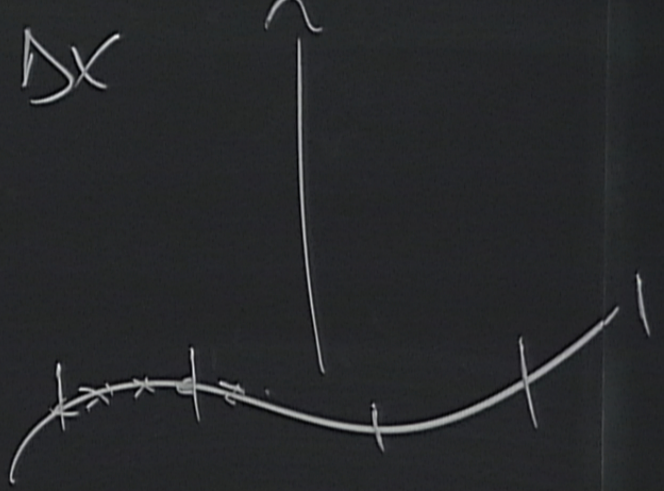

$u_{,t} = u_{,x} \rightarrow \Delta t \sim \Delta x$
$v_{,t} = 10^9 v_{,x} \rightarrow \Delta t \sim \frac{1}{10^9} \Delta x$

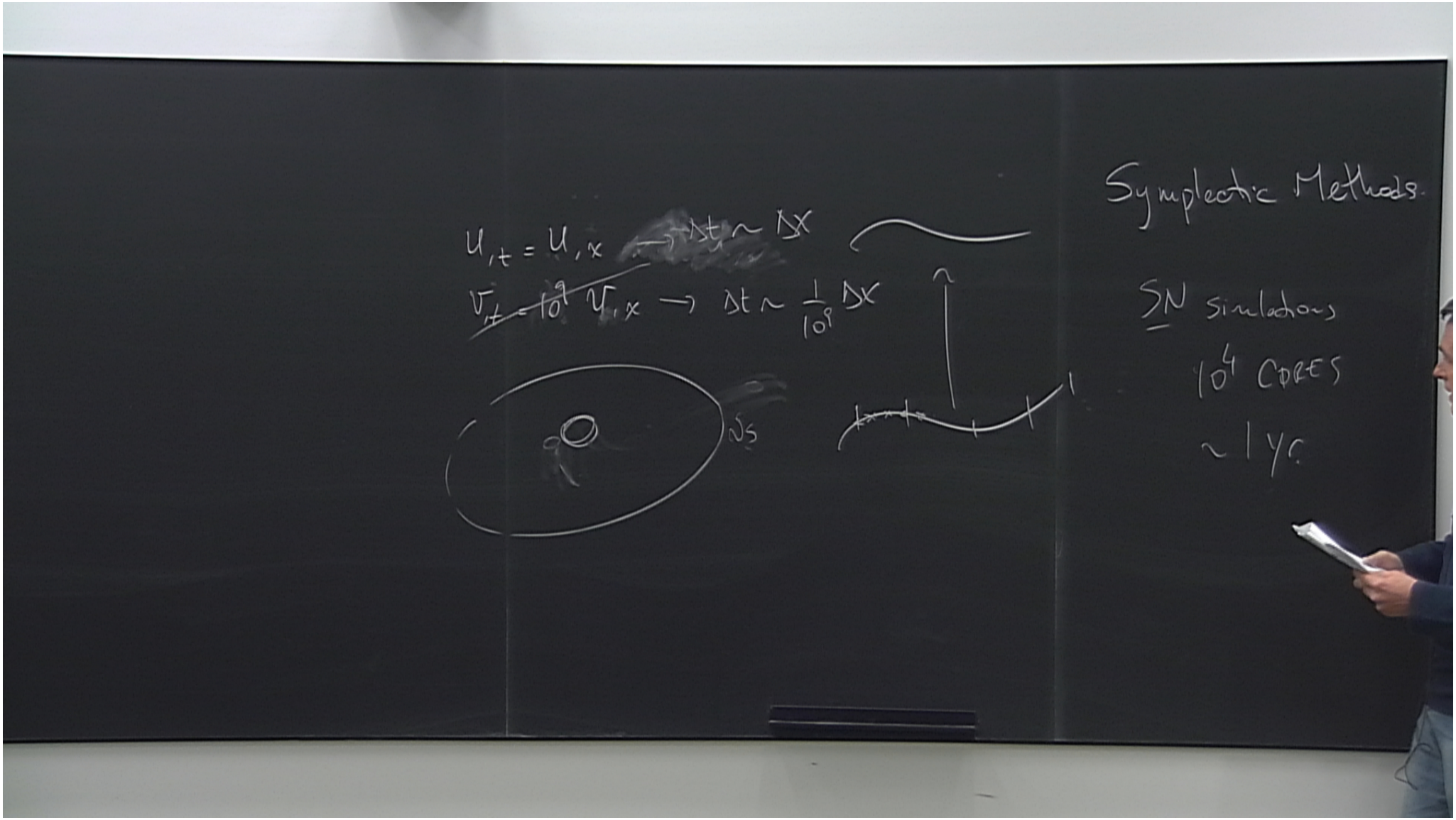

$u_{,t} = u_{,x} \rightarrow \Delta t \sim \Delta x$
$v_{,t} = 10^9 v_{,x} \rightarrow \Delta t \sim$
$\mathcal{N}_S$

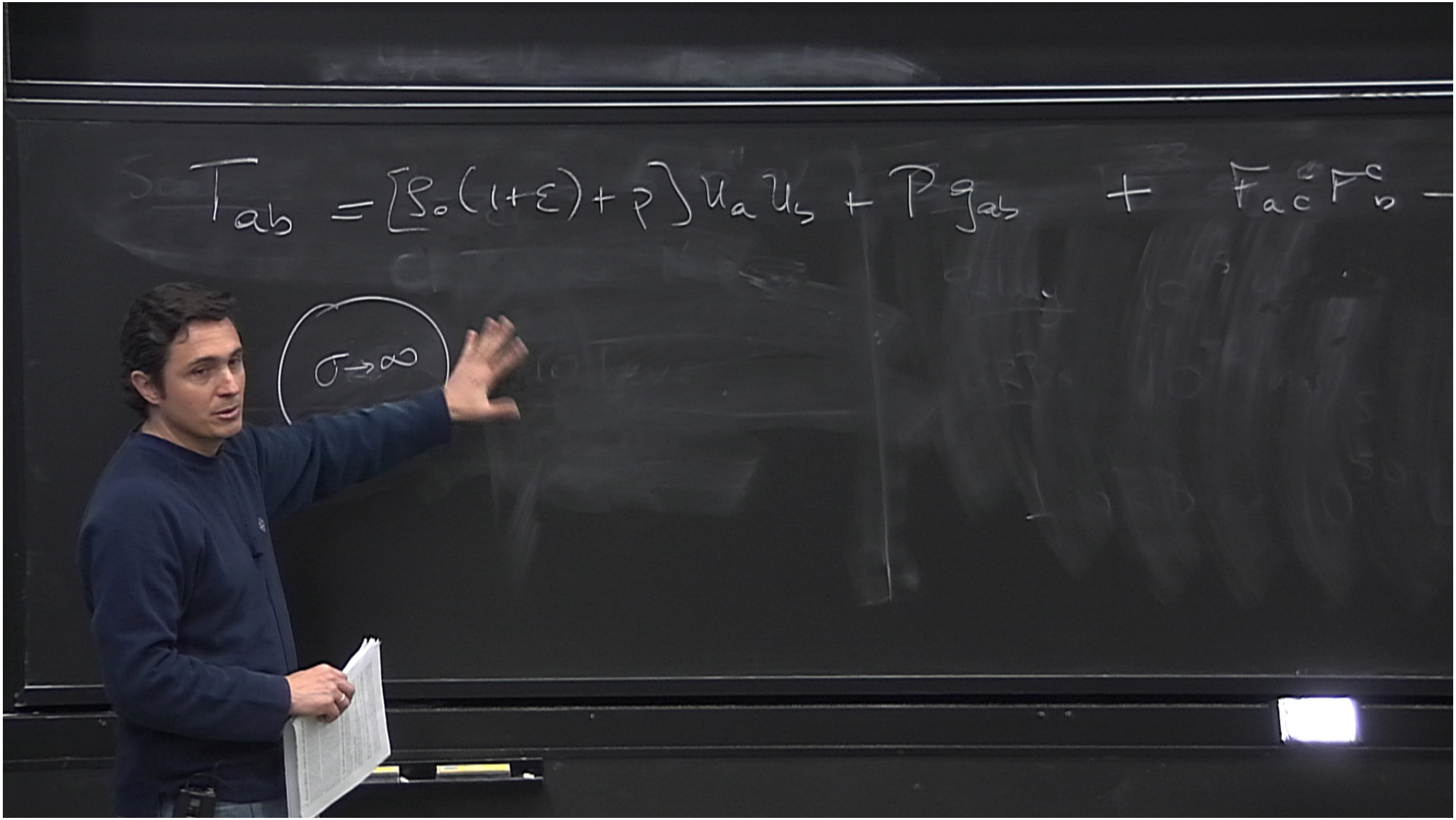
$u_{,t} = u_{,x} \rightarrow \Delta t \sim \Delta x$

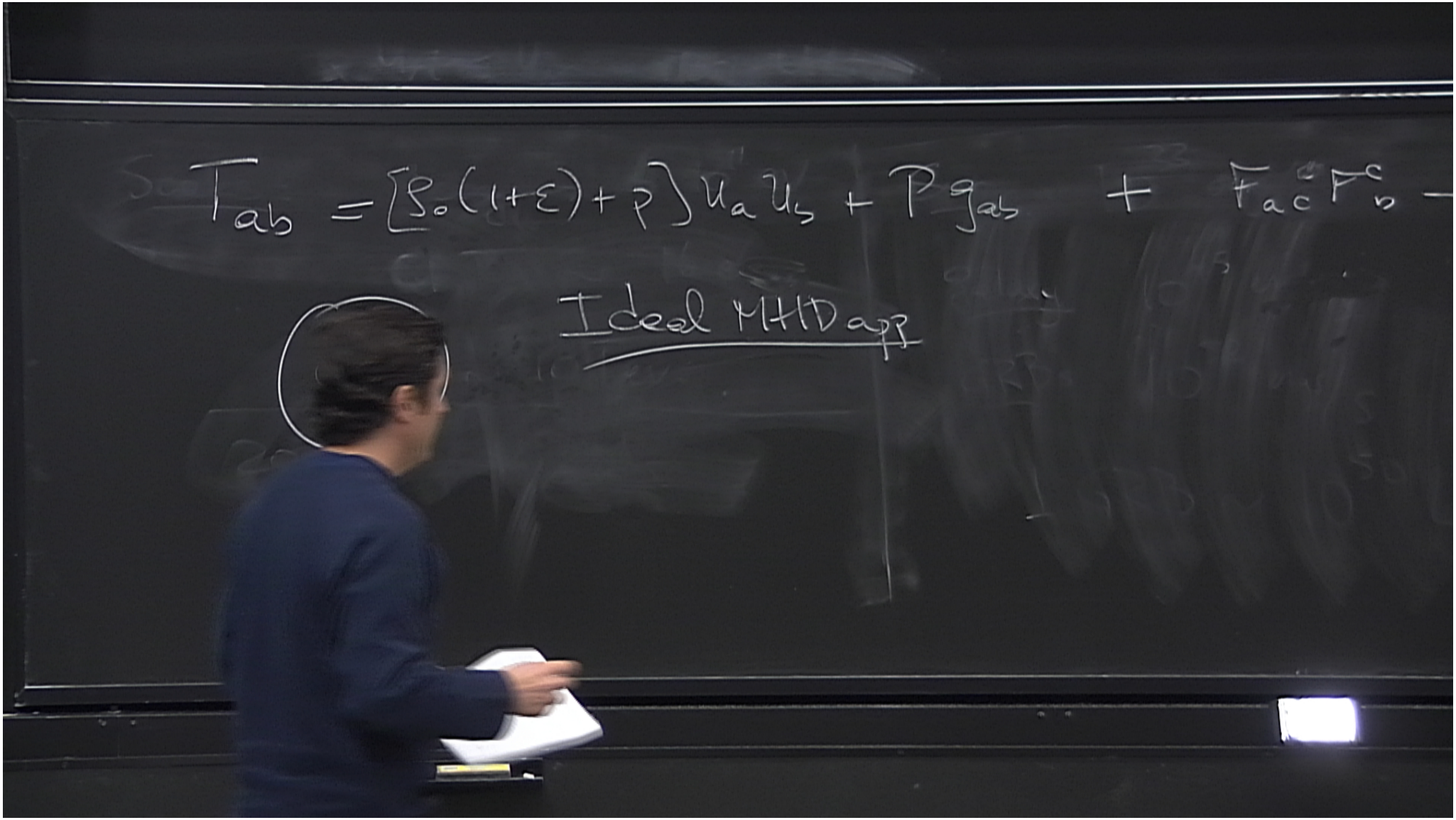
$v_{,t} = 10^9 v_{,x} \rightarrow \Delta t \sim \frac{1}{10^9} \Delta x$






$U_{,t} = U_{,x} \rightarrow \Delta t \sim \Delta x$
$V_{,t} = 10^9 V_{,x} \rightarrow \Delta t \sim \frac{1}{10^9} \Delta x$
Ω_S
Symplectic Methods.
SN simulations
10^4 CORES
~ 1 yr.

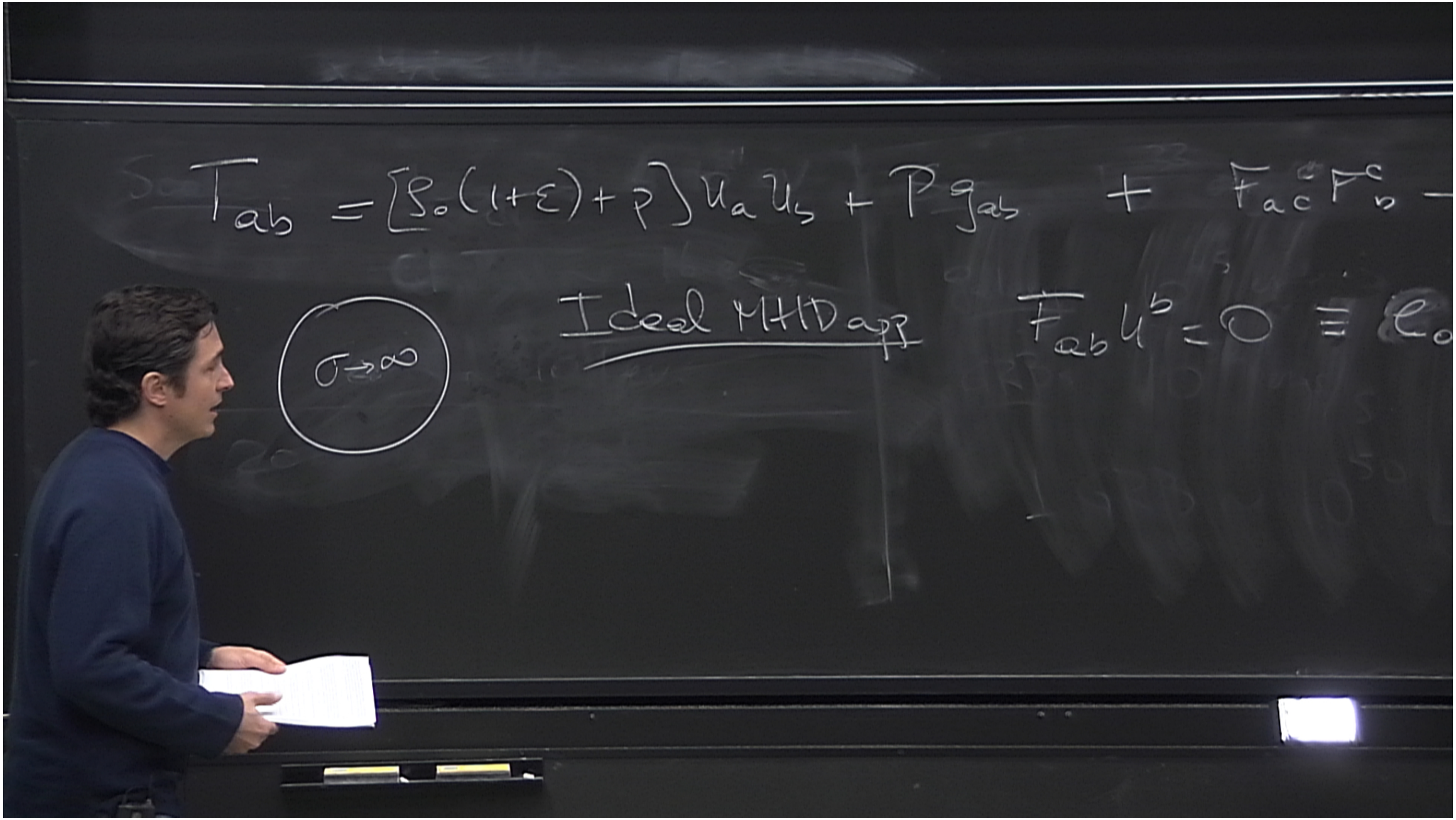

$T_{ab} = [\rho_0(1+\varepsilon) + p] u_a u_b + P g_{ab} + F_{ac} F^c_b -$
$\sigma \to \infty$

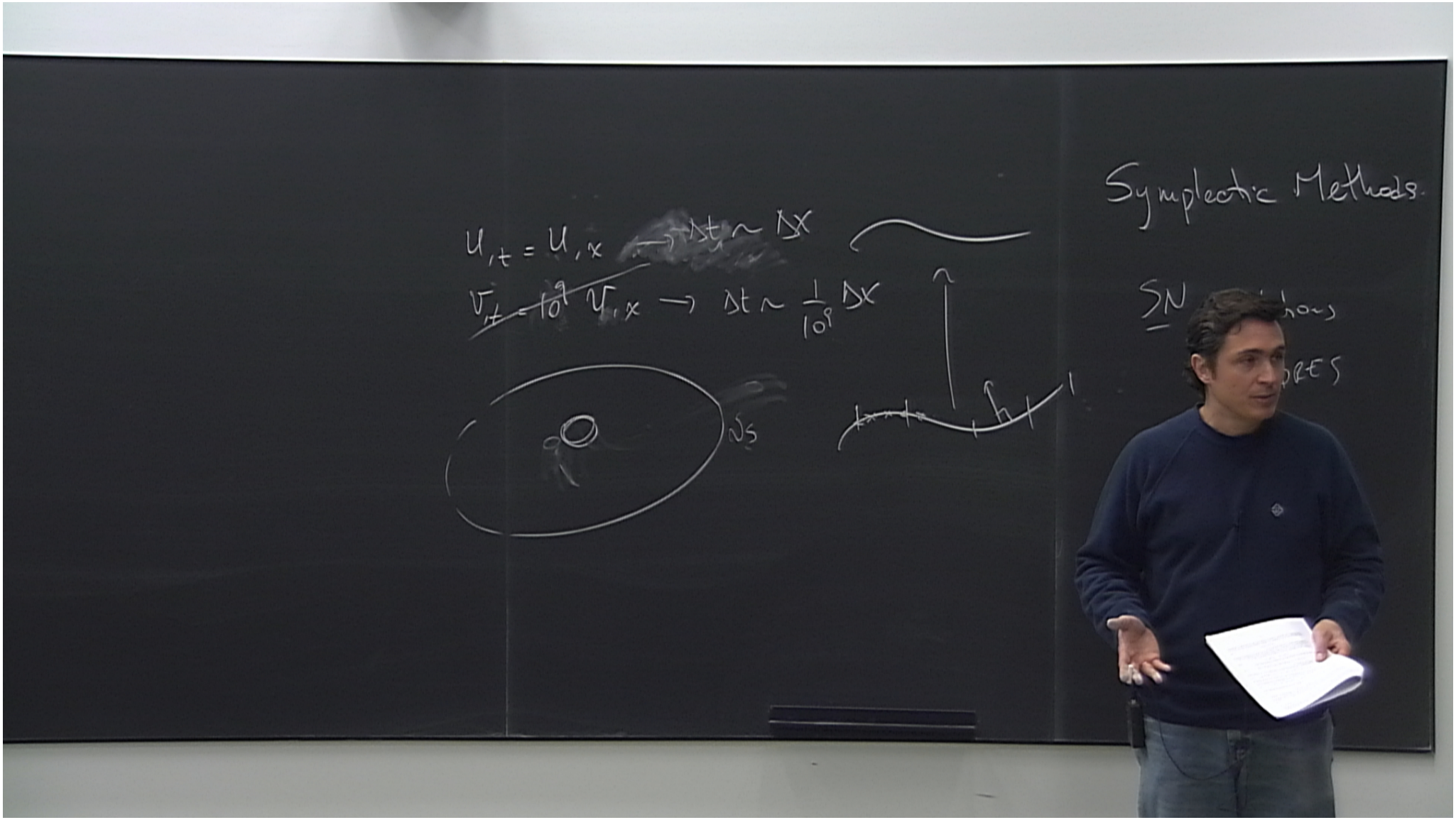

$T_{ab} = [\rho_0(1+\varepsilon) + p] u_a u_b + P g_{ab} + F_{ac} F^c_b -$
Ideal MHD app

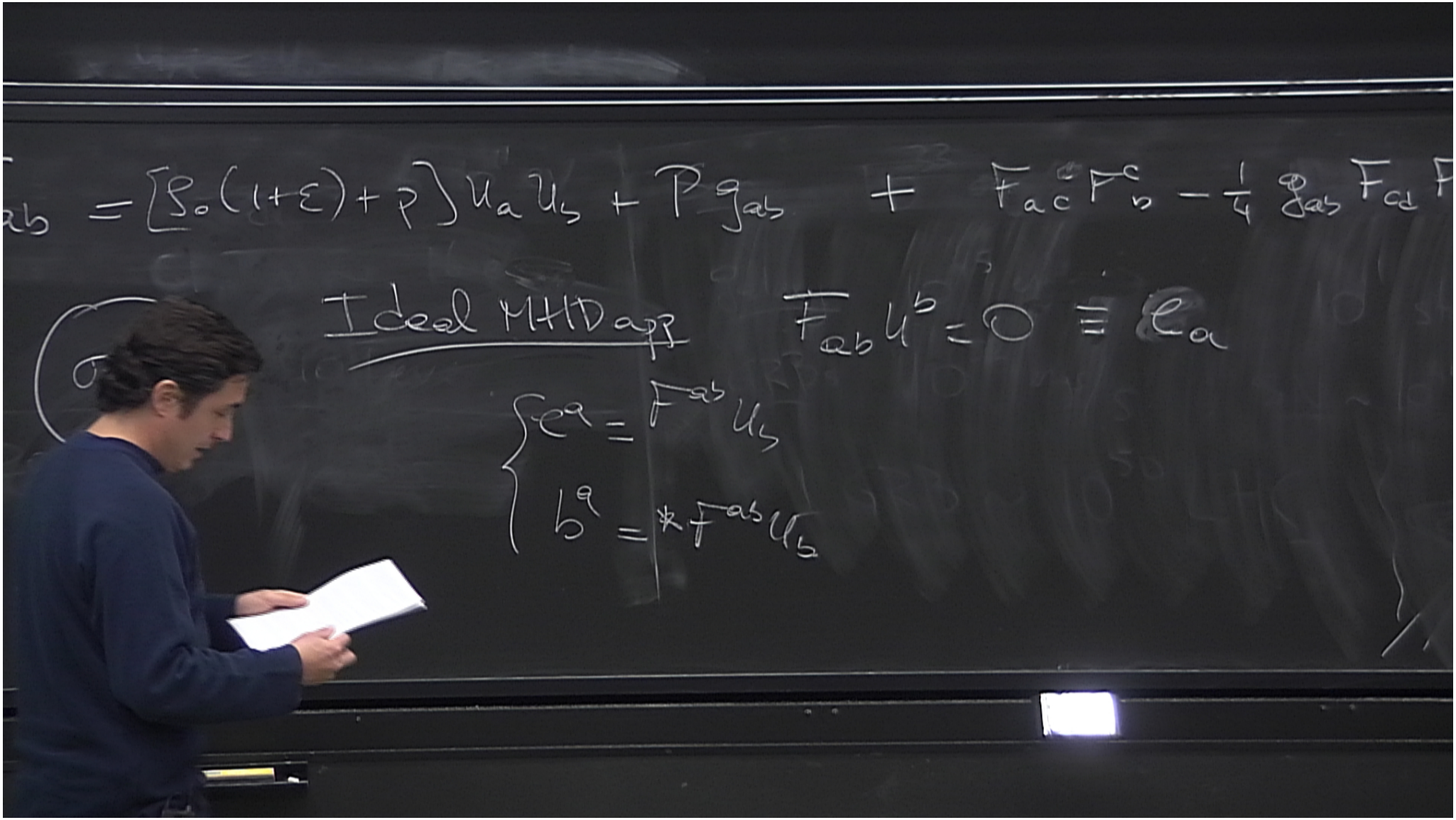
$$T_{ab} = [\rho_0(1+\varepsilon) + p] u_a u_b + P g_{ab} + F_{ac} F^c_{\ b} - \frac{1}{4} g_{ab} F$$

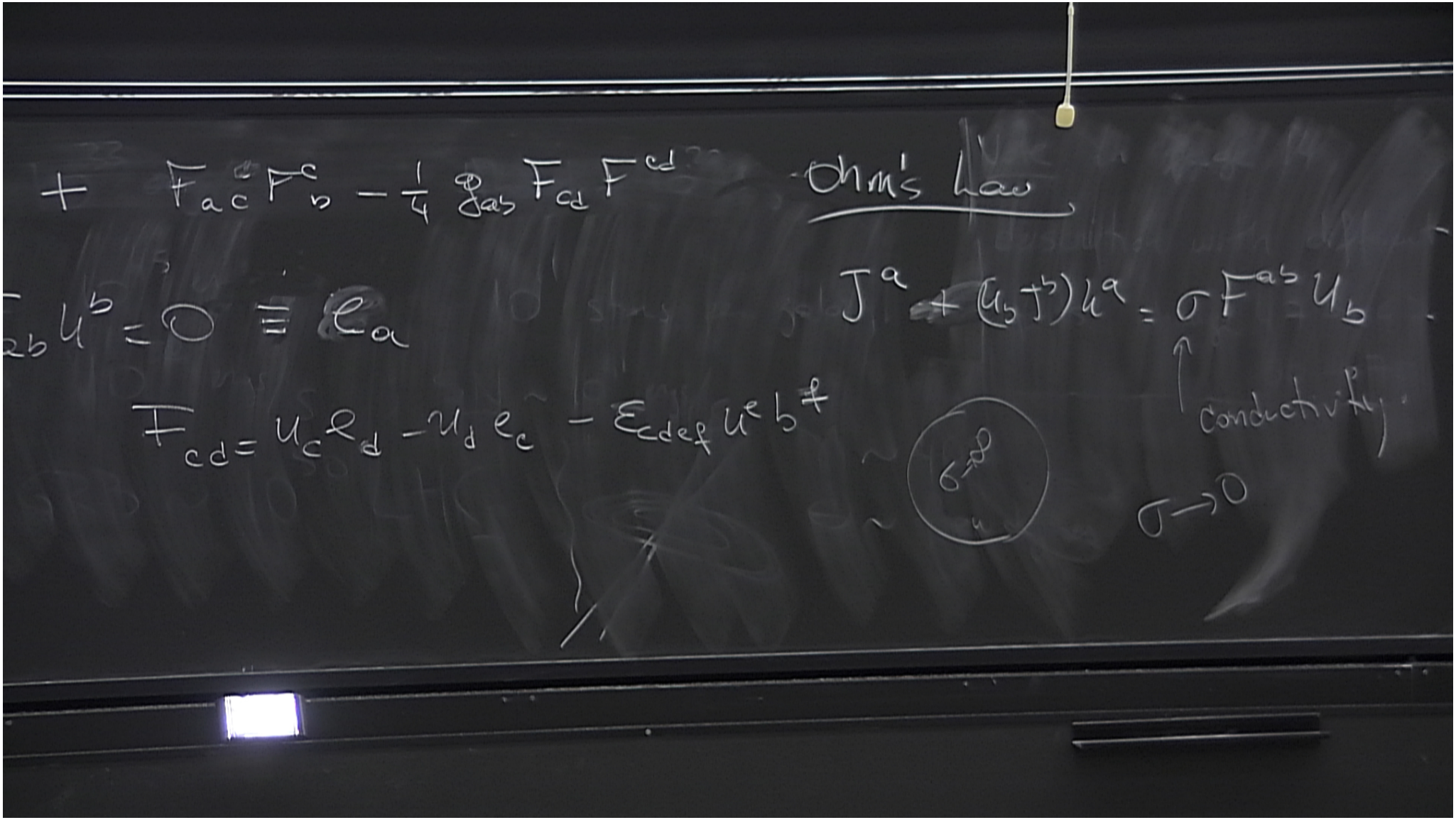
$\sigma \to \infty$

Ideal MHD app $\quad F_{ab} u^b = 0 \equiv E_a$


$T_{ab} = [\rho_0(1+\varepsilon) + p] u_a u_b + P g_{ab} + F_{ac} F^c_b -$
$\sigma \to \infty$
Ideal MHD appr
$F_{ab} u^b = 0 \equiv E_a$


Symplectic Methods.
$u_{,t} = u_{,x} \rightarrow \Delta t \sim \Delta x$
$v_{,t} = 10^9 v_{,x} \rightarrow \Delta t \sim \frac{1}{10^9} \Delta x$
S^1


$T_{ab} = [\rho_0(1+\varepsilon)+p]u_a u_b + p g_{ab} + F_{ac}F^c_b - \frac{1}{4} g_{ab} F_{cd} F$
Ideal MHD app
$F_{ab}u^b = 0 \equiv \mathcal{E}_a$
$e^a = F^{ab}u_b$
$b^a = {}^*F^{ab}u_b$


+ $F_{ac}F^{c}_{b} - \frac{1}{4} g_{ab} F_{cd} F^{cd}$
Ohm's law
$J^a + (u_b J^b) u^a = \sigma F^{ab} u_b$
conductivity
$\sigma \to \infty$
$\sigma \to 0$
$F_{cd} = u_c \ell_d - u_d \ell_c - \varepsilon_{cdef} u^e b^f$

$+ \quad F_{ac} F^{c}_{\ b} - \frac{1}{4} g_{ab} F_{cd} F^{cd}$

Ohm's Law

$F_{ab} u^b = 0 \equiv e_a$

$J^a + (u_b J^b) u^a = \sigma F^{ab} u_b$

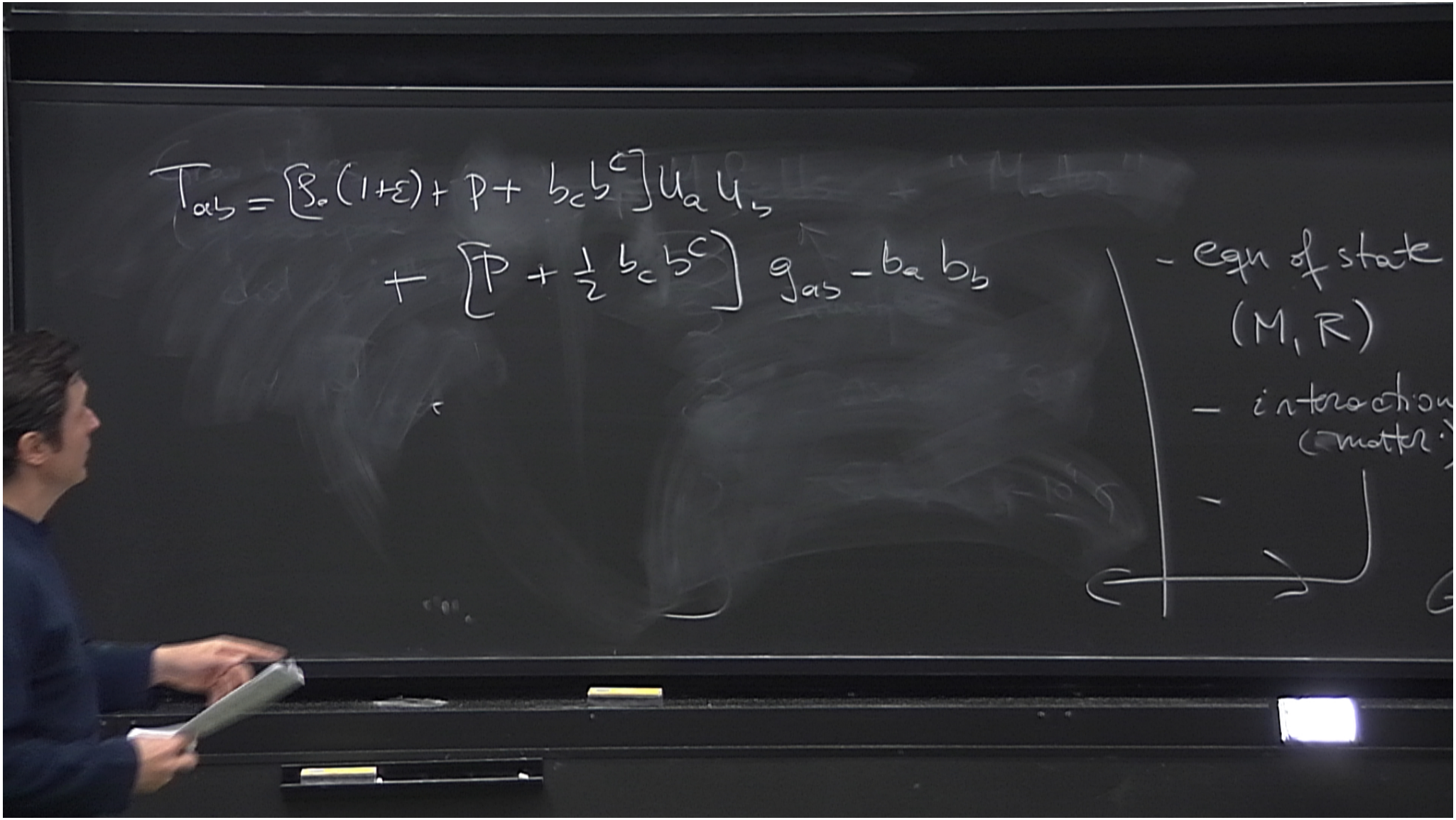
σ: conductivity

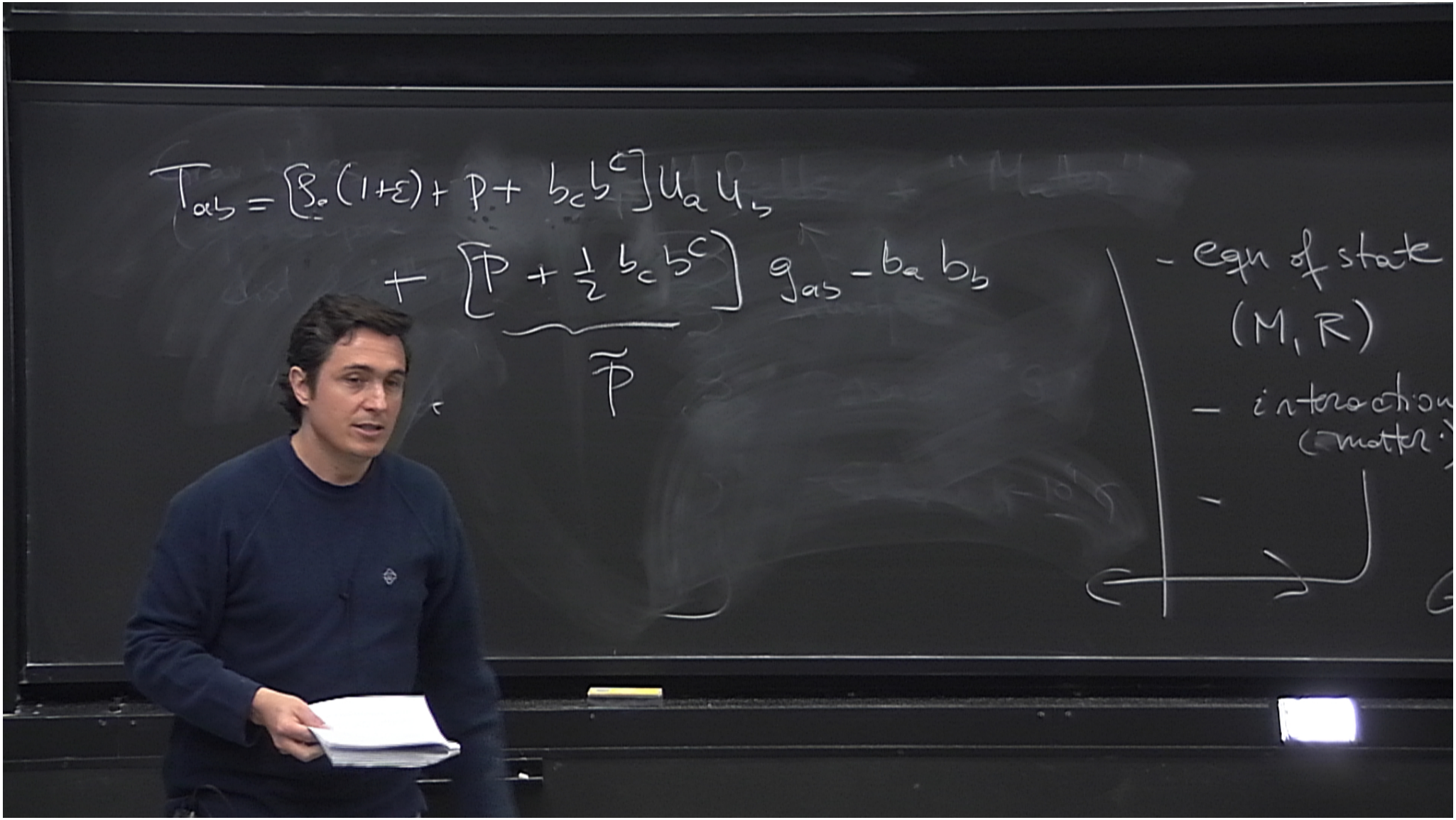
$F_{cd} = u_c e_d - u_d e_c - \varepsilon_{cdef} u^e b^f$

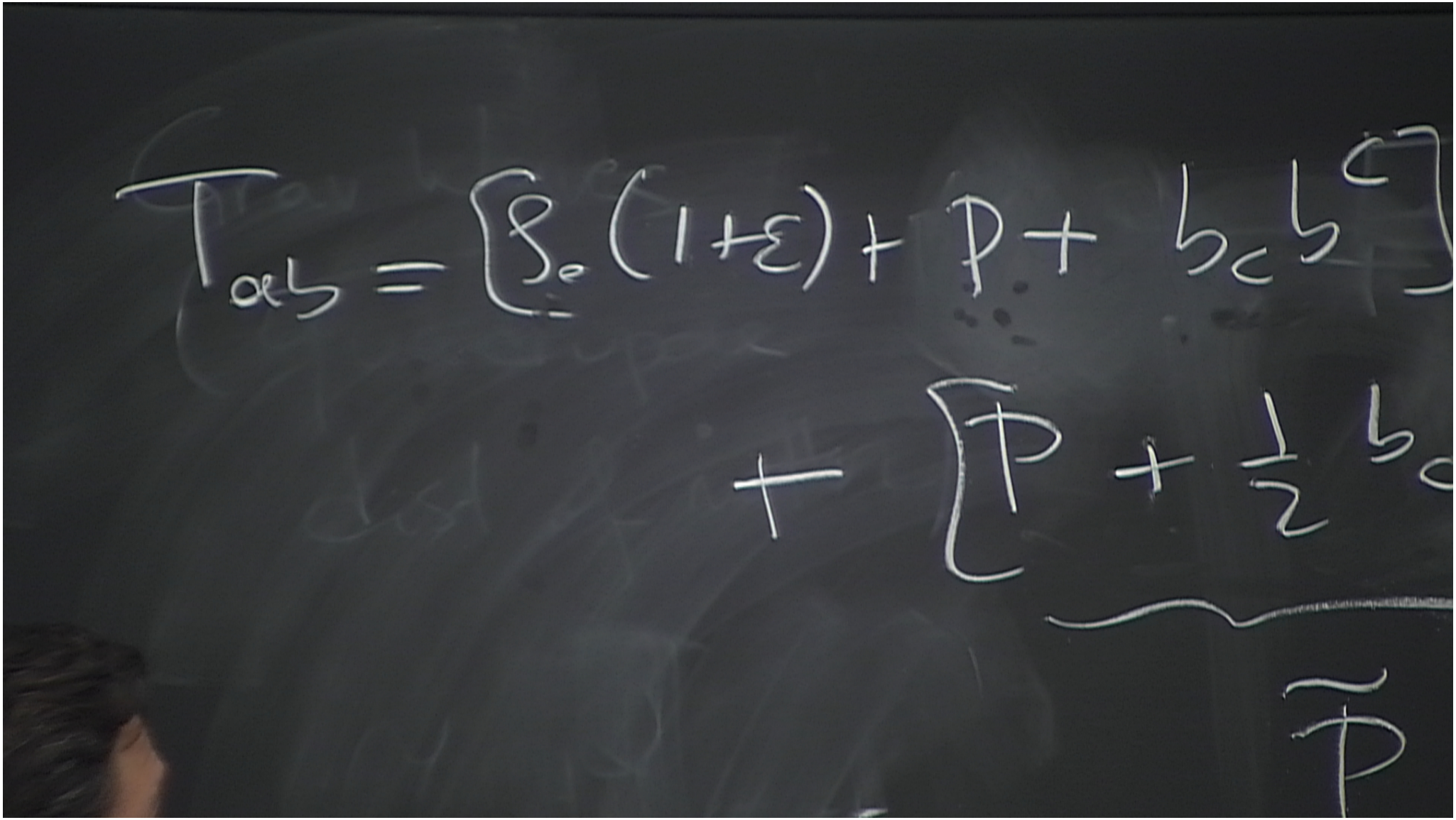
$*F_{cd} = u_c b_d - u_d b_c + \varepsilon_{cdef} u^e e^f$

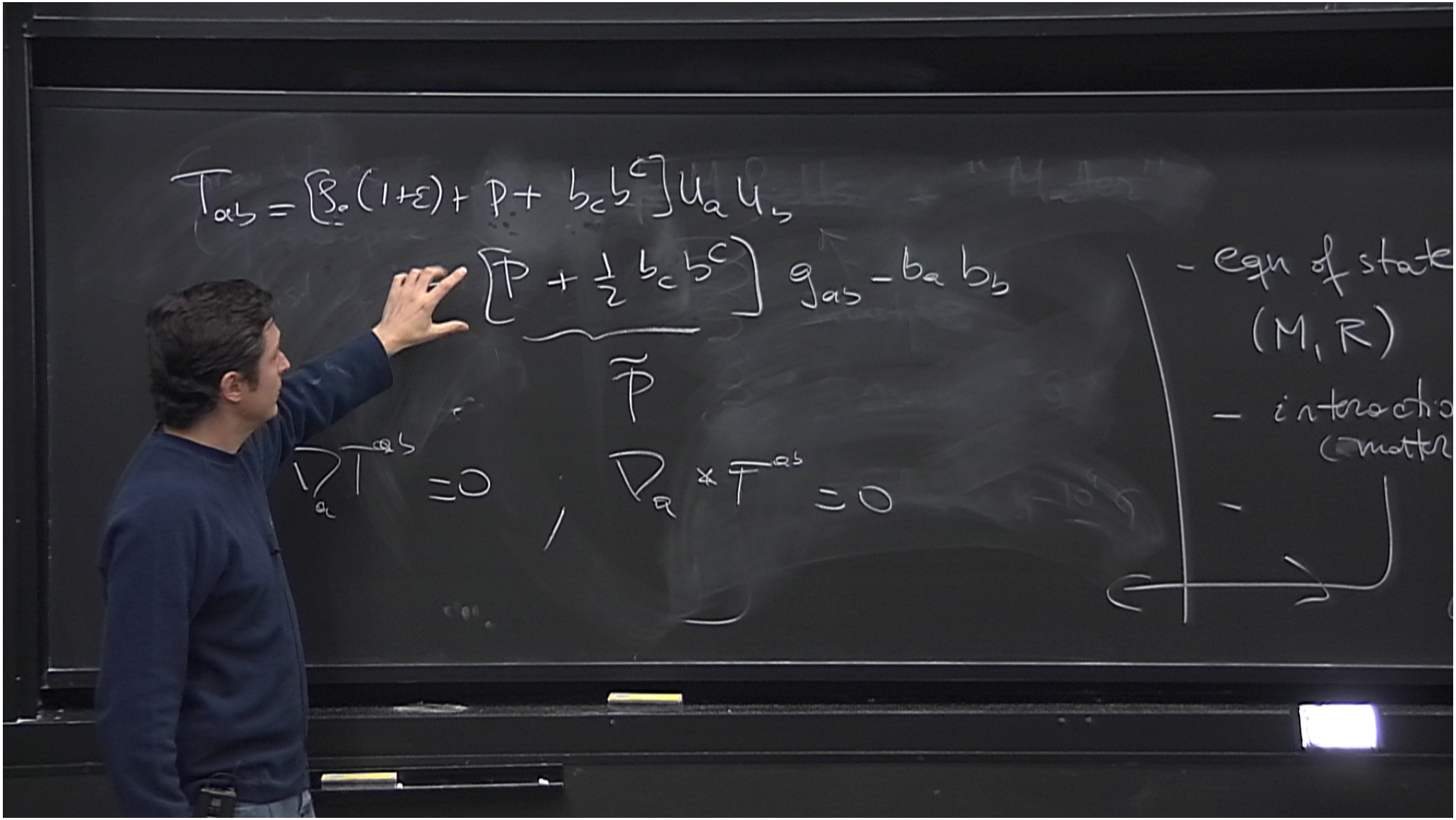
$\sigma \to \infty$ ~

$\sigma \to 0$


$T_{ab} = [\rho_0(1+\varepsilon) + p + b_c b^c] u_a u_b$
$+ [p + \frac{1}{2} b_c b^c] g_{ab} - b_a b_b$
- eqn of state
(M, R)
- interaction
(matter)


$T_{ab} = [\rho_0(1+\varepsilon) + P + b_c b^c] u_a u_b$
$+ [\underbrace{P + \tfrac{1}{2} b_c b^c}_{\tilde{P}}] g_{ab} - b_a b_b$
- eqn of state
(M, R)
- interaction
(matter)


$T_{ab} = [\rho_e(1+\varepsilon) + p + u_a u_b]$
$+ [\bar{p} + \frac{1}{2} u_c$
$\bar{p}$


$T_{ab} = [\rho_0(1+\varepsilon) + p + b_c b^c] u_a u_b$
$[p + \frac{1}{2} b_c b^c] g_{ab} - b_a b_b$
$\tilde{p}$
$\nabla_a T^{ab} = 0$
$\nabla_a {}^*F^{ab} = 0$
- eqn of state
(M, R)
- interactio
(matter
-

$$T_{ab} = [\rho_0(1+\varepsilon) + p + b_c b^c] u_a u_b + [\underbrace{p + \tfrac{1}{2} b_c b^c}_{\tilde{p}}] g_{ab} - b_a b_b$$

$$\nabla_a T^{ab} = 0 \quad ; \quad \nabla_a * F^{ab} = 0$$

- eqn of state
(M, R)
- interactio
(matter
-