

Title: Explorations in Quantum Gravity - Lecture 14

Date: Apr 20, 2012 10:15 AM

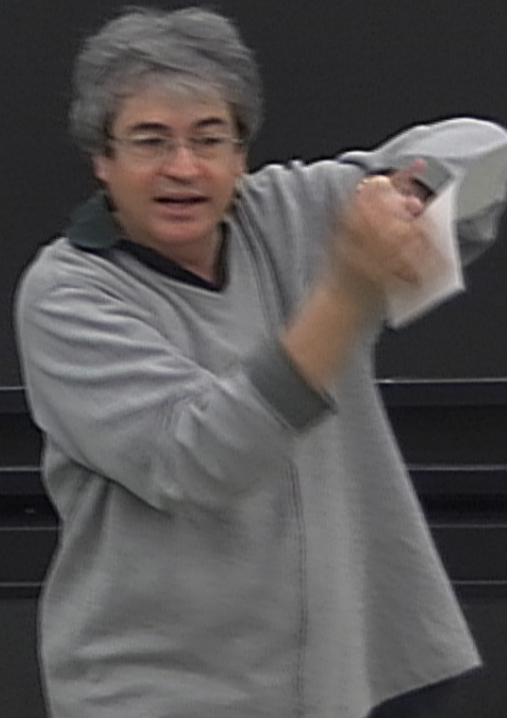
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Abstract:

LECTURE XIV - PHYSICS

KINEMATICS

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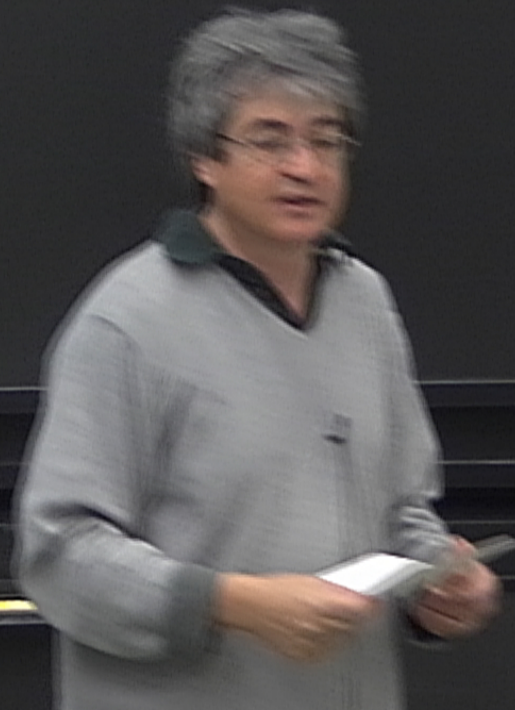


LECTURE XIV

LECTURE XIV - PHYSICS

LECTURE XIV - PHYSICS

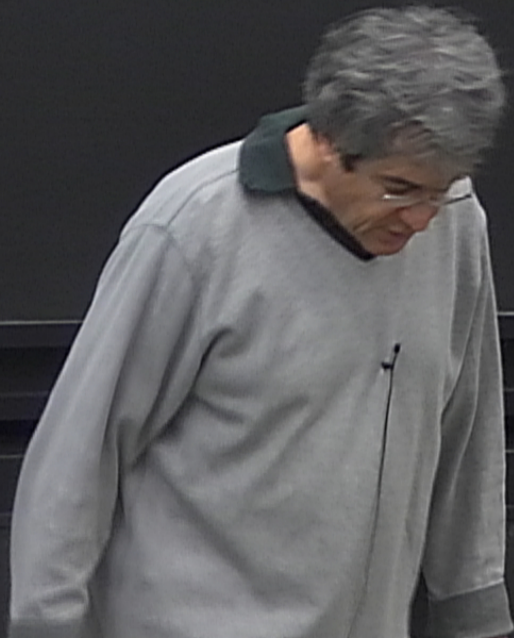
KINEMATICS



LECTURE XIV - PHYSICS

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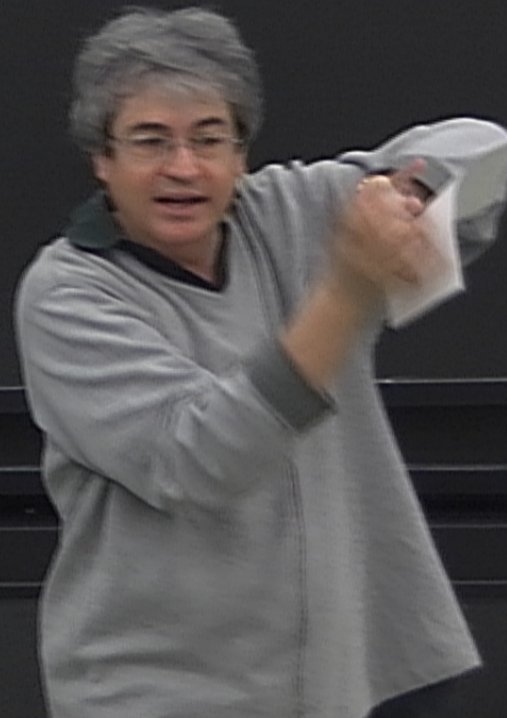
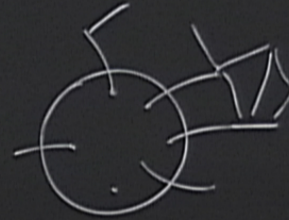
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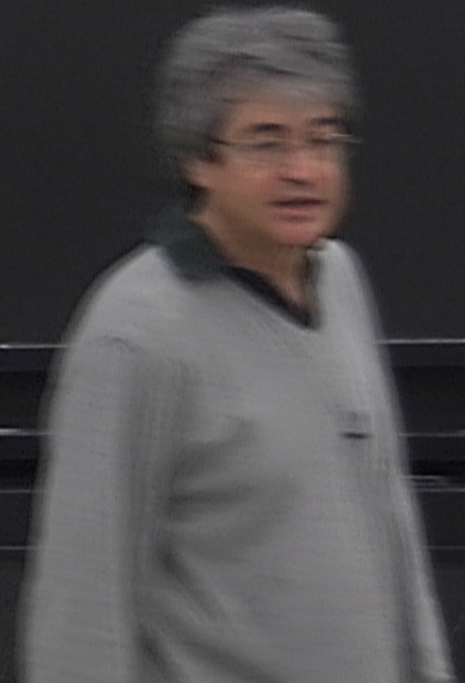


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A # of states

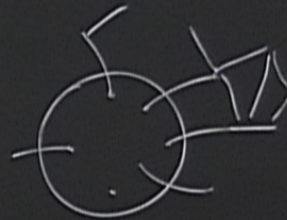


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A # of states

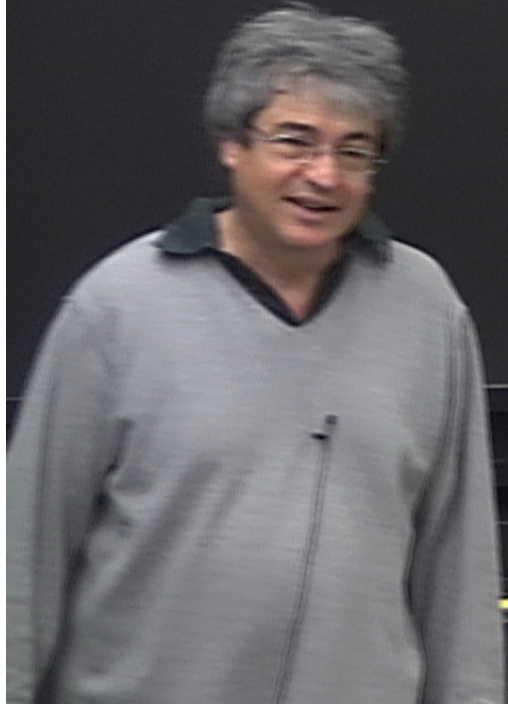
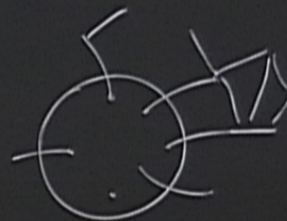


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A # of states



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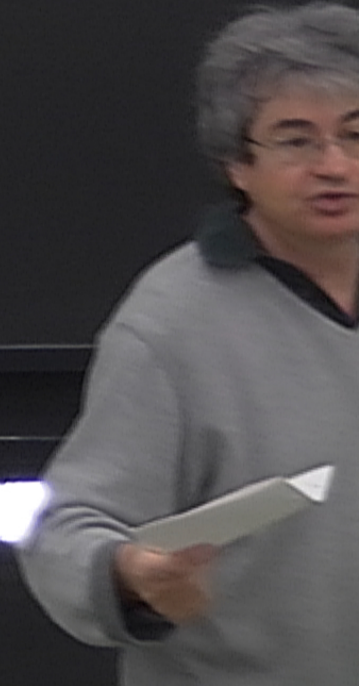
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A # of states



⊙ LOOP COSMOLOGY (LQC)



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① BLACK HOLES

A # of states



② LOOP COSMOLOGY (LQC)

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} \rho + \frac{\Lambda}{3}$$

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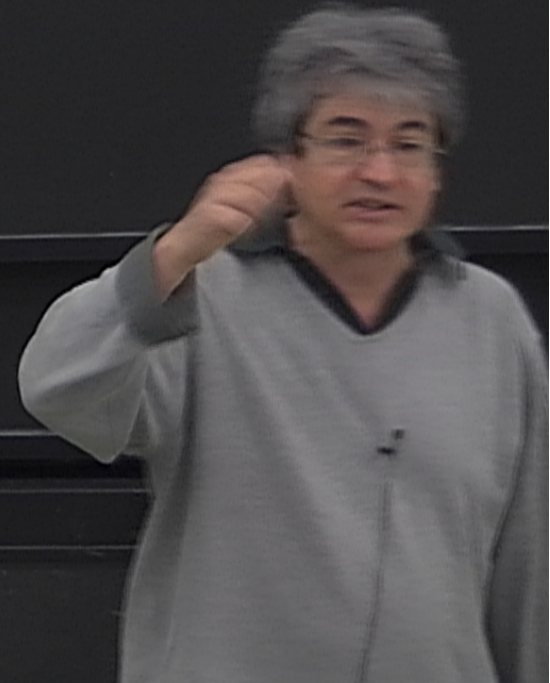
① BLACK HOLES

A # of states



② LOOP COSMOLOGY (LQC)

$$\left(\frac{\dot{p}}{p}\right)^2 = \frac{8\pi G}{3} p \left(1 - \frac{p}{p_c}\right) + \frac{\Lambda}{3}$$



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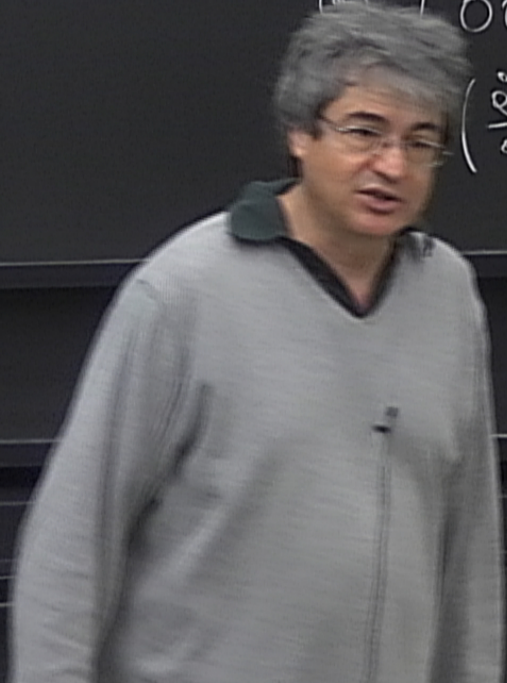
⊙ BLACK HOLES

A # of states



⊙ LOOP COSMOLOGY (LQC)

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} \rho \left(1 - \frac{\rho}{\rho_c}\right) + \frac{\Lambda}{3}$$



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① BLACK HOLES

A # of states



② LOOP COSMOLOGY (LQC)

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} \rho \left(1 - \frac{\rho}{\rho_c}\right) + \frac{\Lambda}{3} \neq \text{BOUNCE}$$

DYNAMICS

BOUNCE

DYNAMICS

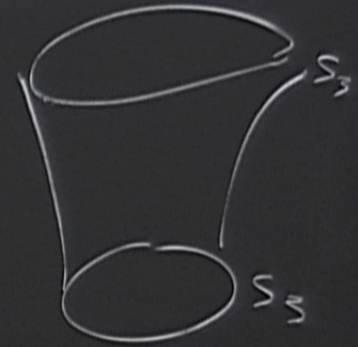
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BOUNCE

DYNAMICS

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Compact SPACE



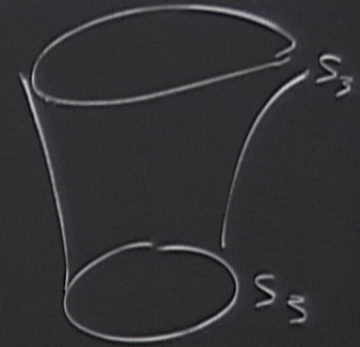
BOUNCE

DYNAMICS

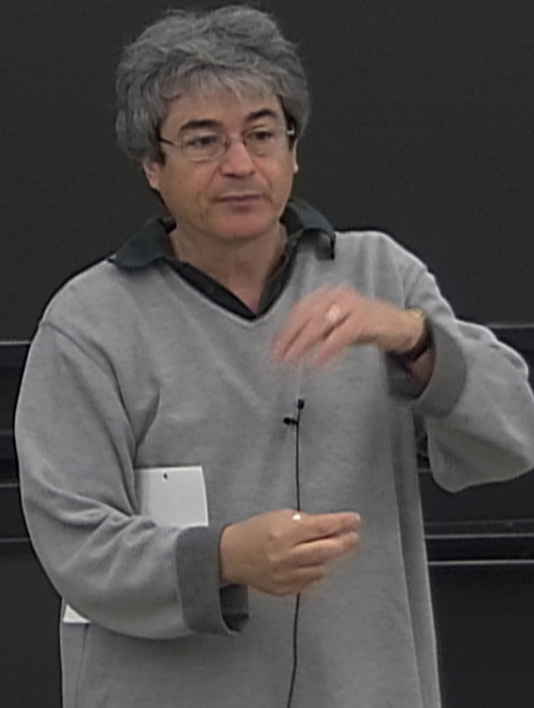
④ SPIN FOLIA COSMOLOGY

compact space

3-sheet \rightarrow Triang.



BOUNCE

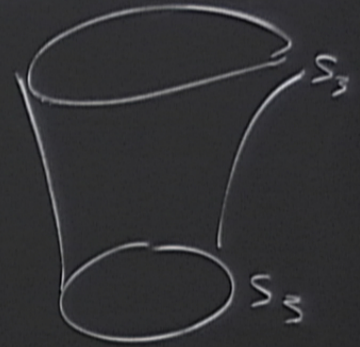
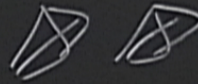


DYNAMICS

④ SPIN FOLIA COSMOLOGY

compact space

3-sphere \rightarrow Triang.

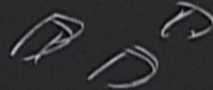


BOUNCE

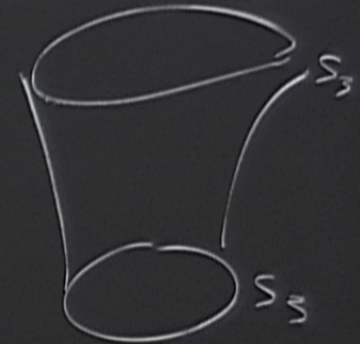
DYNAMICS

④ SPIN FOLIA COHOMOLOGY

3-sphere \rightarrow Triang.



compact space



UNCE

DYNAMICS

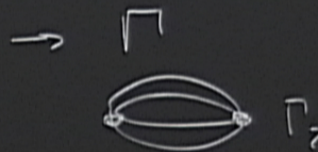
② SPIN FOLIA COHOMOLOGY

3-sheet \rightarrow Triang.



$$H_{\Gamma_2} \cong \mathcal{P}(v_1, \dots, v_4)$$

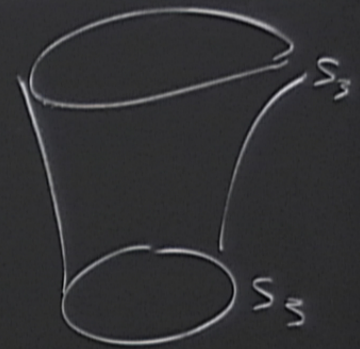
Compact SPACE



Γ_2



Γ_5

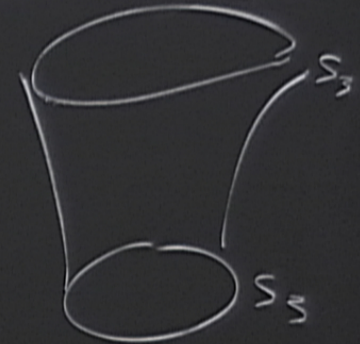


BOUNCE

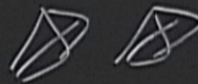
DYNAMICS

② SPIN FOLIATION COSMOLOGY

compact space



3-sphere \rightarrow Triang.



$$H_{\Gamma_2} \ni \psi(u_1, u_4) = \frac{\psi(u_1, u_4)}{a}$$

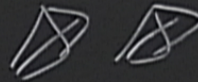
BOUNCE

DYNAMICS

① SPIN FOLIA COHOMOLOGY

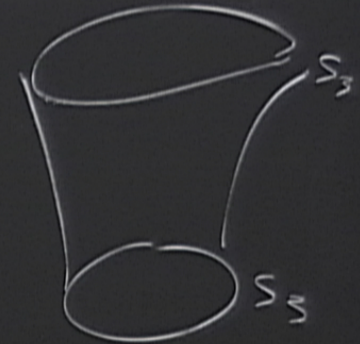
Compact SPACE

3-sheet \rightarrow Triang.



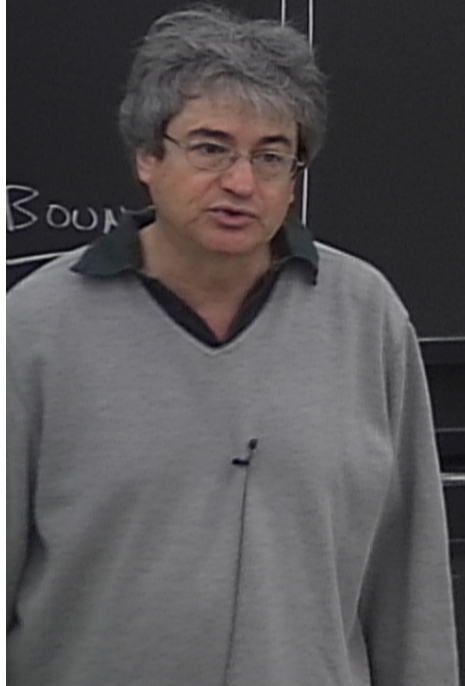
Γ_2

Γ_5



$$H_{\Gamma_2} \ni \psi(u_1, u_2) = \psi(u_1, u_2)_{\mathbb{Q}\mathbb{P}^1}$$

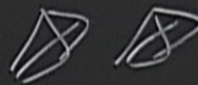
BOUND



DYNAMICS

① SPIN FOLIATION COSMOLOGY

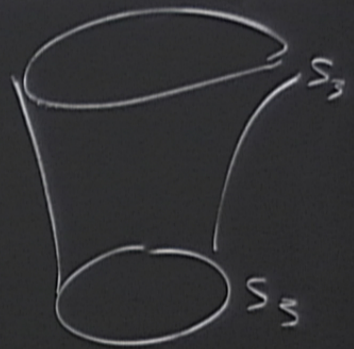
3-sheet \rightarrow Triang.



Γ_2



Γ_5



Γ_{2P}



Γ_1

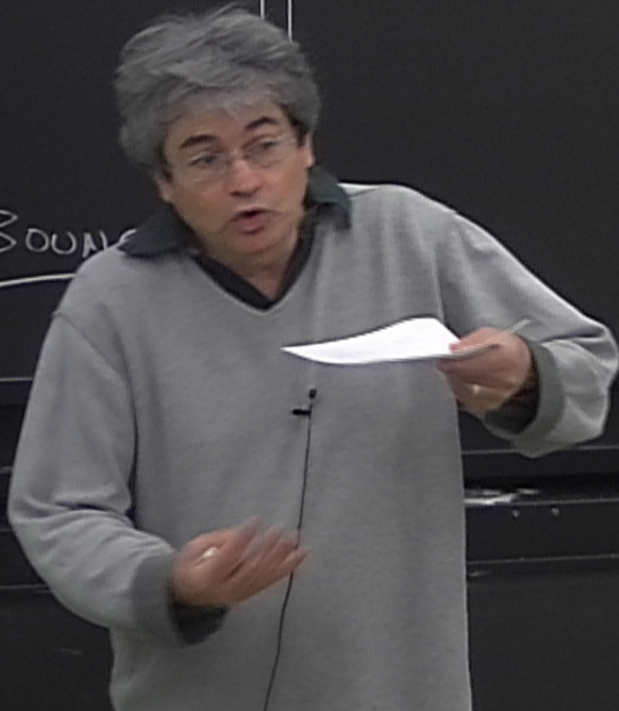
Γ_{2P}



Γ_1

$$H_{\Gamma_2} \ni \psi(u_1, u_4) = \psi(u_1, u_4)_{\mathbb{Q}P_2}$$

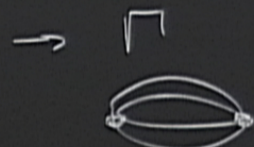
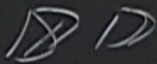
BOUND



DYNAMICS

① SPIN FOLIA COHOMOLOGY

3-sphere \rightarrow Triang.

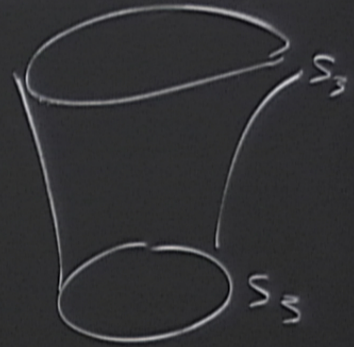


Γ_2

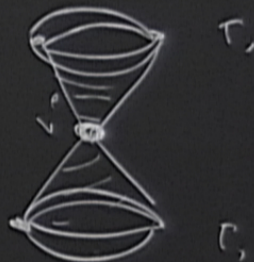
Γ_5

$$H_{\Gamma_2} \ni \psi(u_1, u_4) = \psi(u_1, u_4)_{\mathbb{R}P^2}$$

Compact SPACES



$\Gamma_{\mathbb{R}P^2}$



$\Gamma_{\mathbb{R}P^2}$

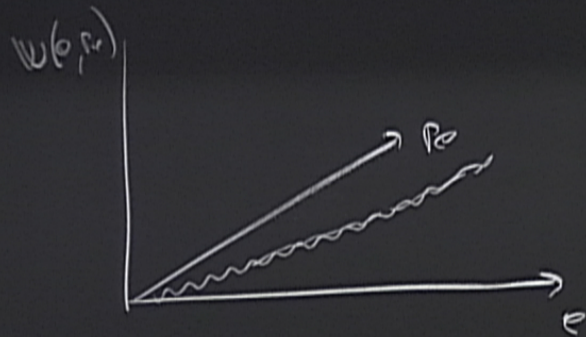
$$W_e (u_1, u_2; u'_1, u'_2)$$

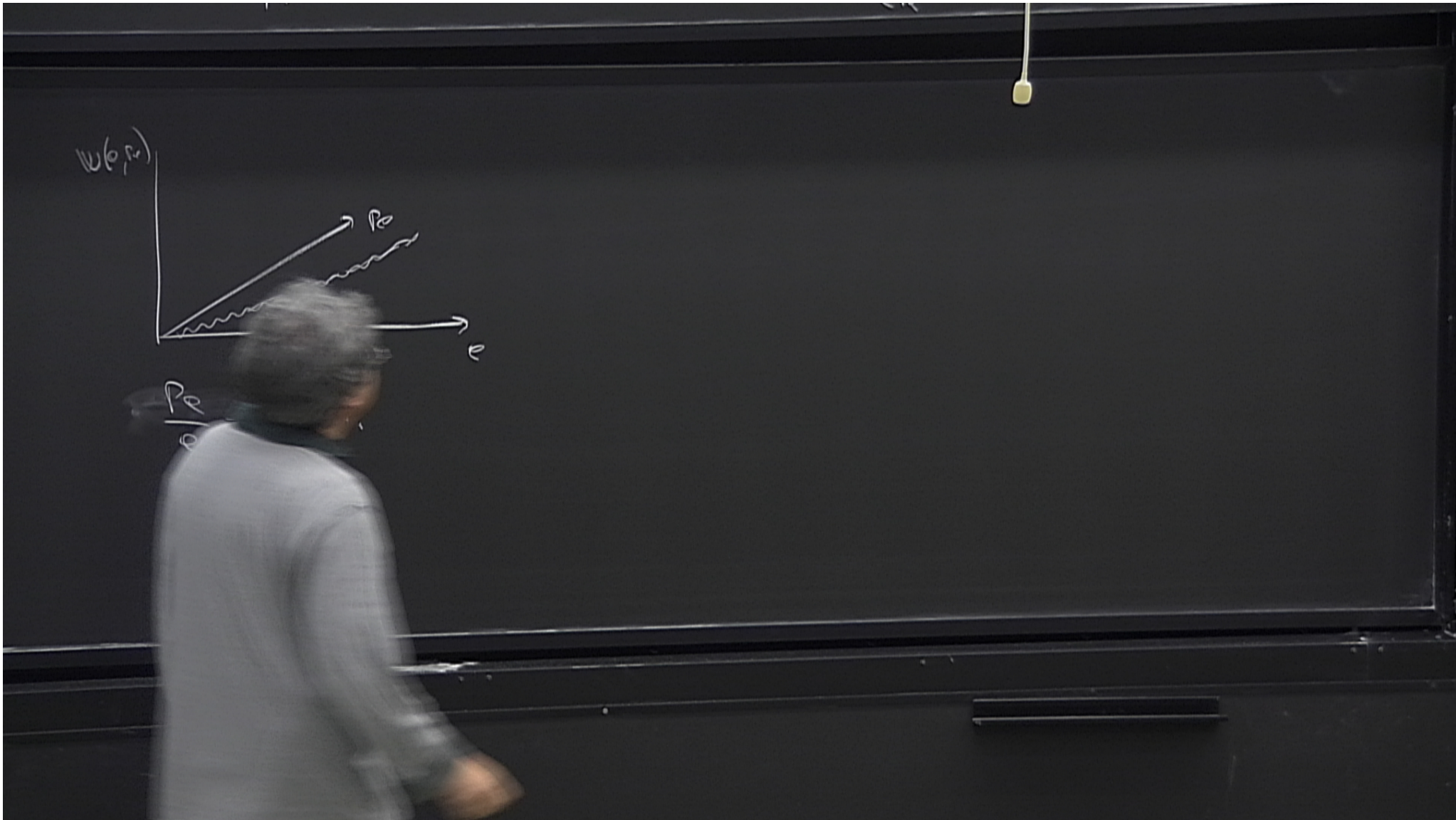
$$W_e(u_1, u_2; u'_1, u'_2) =$$

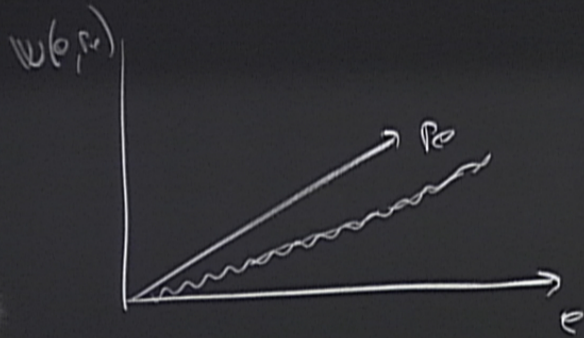
$$W(e p_e, e' p'_e) = \int du du' \overline{\psi_{ep_e}(u)} W(u, u') \psi_{e'p'_e}(u')$$

$$W_e(u_1, u_2; u'_1, u'_2) =$$

$$\begin{aligned} W(q, p_e; q', p'_e) &= \int du du' \overline{\psi_{ep_e}(u)} W(u, u') \psi_{e'p'_e}(u') \\ &= W(q, p_e) W(q', p'_e) \end{aligned}$$

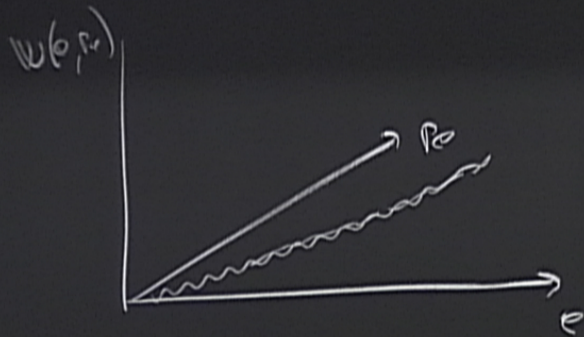






- No matter
- cosmological constant.

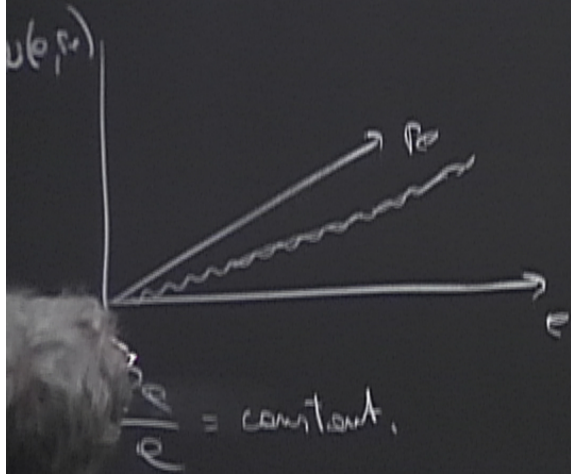
$$\frac{P_0}{p} = \text{constant.}$$



- No matter
- cosmological constant

$$\frac{\rho_0}{r} = \text{constant}$$

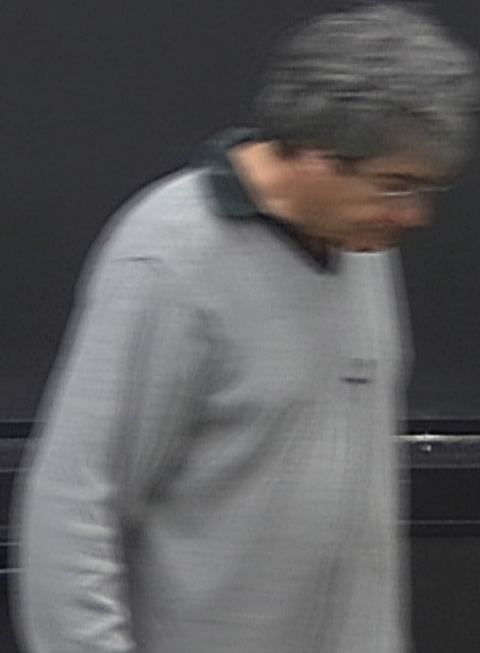
$$\frac{\rho}{\rho_0} = \text{const.} = \sqrt{\frac{r}{r_0}}$$



- No matter
- cosmological constant

$$p/p_0 = \text{const.} = \sqrt{\frac{\Lambda}{3M^2}}$$

⑥ n-point functions



⑥ n-point functions

$$W(x_1, \dots, x_n) = \langle 0 | \varphi(x_1) \dots \varphi(x_n) | 0 \rangle$$

① n-point functions

$$W(x_1, \dots, x_n) = \langle 0 | \varphi(x_1) \dots \varphi(x_n) | 0 \rangle$$

How are transition amplit. & n-point func. related?

$q(t)$

$$W(q, q'; t, t') = \langle q | e^{iH(t-t')} | q' \rangle$$

eq?

$q(t)$

$$W(q, q') = \langle q | e^{iH(t-t')} | q' \rangle$$

$$W(t, t') = \langle 0 | \hat{q}(t) \hat{p}(t') | 0 \rangle$$

$q(t)$

$$W(q, q') = \langle q | e^{iH(t-t')} | q' \rangle$$

$$W(t, t') = \langle 0 | \hat{q}(t) \hat{p}(t') | 0 \rangle$$

$$= \langle 0 | e^{i t H} \hat{p} e^{-i t H} e^{i t' H} \hat{q} e^{-i t' H} | 0 \rangle$$

$$\begin{aligned}
 q(t) \\
 W(q(t), q'(t)) &= \langle q | e^{iH(t-t')} | q' \rangle \\
 W &= \langle 0 | \hat{q}(t) \hat{p}(t') | 0 \rangle \\
 &= \langle 0 | e^{i\tau H} \hat{p} e^{-i\tau H} e^{i\tau' H} \hat{q} e^{-i\tau' H} | 0 \rangle \\
 &\quad (\hat{q} \hat{p} \hat{p} \hat{q})
 \end{aligned}$$

$$q(t)$$

$$W(q(t), q'(t)) = \langle q | e^{iH(t-t')} | q' \rangle$$

$$W(t, t') = \langle 0 | \hat{q}(t) \hat{p}(t') | 0 \rangle$$

$$= \langle 0 | e^{i\int_t^H} \hat{p} e^{-i\int_t^H} e^{i\int_{t'}^H} \hat{p} e^{-i\int_{t'}^H} | 0 \rangle$$

$$= \int dq dp'$$

$$q(t)$$

$$W(q, t; q', t') = \langle q | e^{iH(t-t')} | q' \rangle$$

$$W(t, t') = \langle 0 | \hat{q}(t) \hat{p}(t') | 0 \rangle$$

$$= \langle 0 | e^{i\int^t H} \hat{p} e^{-i\int^t H} e^{i\int^{t'} H} \hat{p} e^{-i\int^{t'} H} | 0 \rangle$$

$$= \int dq dp' \langle 0 | q \rangle q W(q, t; p', t')$$

$$q(t)$$

$$W(q, q', t, t') = \langle q | e^{iH(t-t')} | q' \rangle$$

$$W(t, t') = \langle 0 | \hat{q}(t) \hat{p}(t') | 0 \rangle$$

$$= \langle 0 | e^{iH(t-t')} \hat{q}(t) e^{-iH(t-t')} e^{iH(t-t')} \hat{p}(t') e^{-iH(t-t')} | 0 \rangle$$

$$= \int dq dp' \langle 0 | q \rangle q W(q, p', t, t') p' \langle p' | 0 \rangle$$

$$q(t)$$

$$W(q, q', t, t') = \langle q | e^{iH(t-t')} | q' \rangle$$

$$W(t, t') = \langle 0 | \hat{q}(t) \hat{p}(t') | 0 \rangle$$

$$= \langle 0 | e^{iH(t-t')} \hat{p}(t') e^{-iH(t-t')} e^{iH(t-t')} \hat{q}(t) e^{-iH(t-t')} | 0 \rangle$$

$$= \int dq dq' \langle 0 | q \rangle q W(q, q', t, t') q' \langle q' | 0 \rangle$$

$$= \int dq dq' W$$

$$q(t)$$

$$W(q, q', t) = \langle q | e^{iH(t-t')} | q' \rangle$$

$$W(t, t') = \langle 0 | \hat{\varphi}(t) \hat{\varphi}(t') | 0 \rangle$$

$$= \langle 0 | e^{i\int H} \hat{\varphi} e^{-i\int H} e^{i\int H} \hat{\varphi} e^{-i\int H} | 0 \rangle$$

$$= \int dq dq' \langle 0 | q \rangle q W(q, q', t) q' \langle q' | 0 \rangle$$

$$= \int dq dq' W(q, q', t) \bar{\varphi}(q) \varphi(q')$$

(c) n-point functions

- trans. Amplitude
- field operators
- \downarrow STATE

$$W(x_1, \dots, x_n) = \langle 0 | \varphi(x_1) \dots \varphi(x_n) | 0 \rangle$$

how are transition amp & n-point func. related?

$$\begin{aligned}
 & q(t) \\
 W(q(t), q'(t')) &= \langle q | e^{iS} | q' \rangle \\
 W(t, t') &= \langle 0 | \dots | 0 \rangle \\
 &= \langle 0 | \dots | 0 \rangle \\
 &= \int \mathcal{D}p \mathcal{D}q
 \end{aligned}$$

(c) n-point functions

- trans. Amplitude
- field operators
- \downarrow STATE

$$W(x_1, \dots, x_n) = \langle 0 | \varphi(x_1) \dots \varphi(x_n) | 0 \rangle$$

How are transition amp & n-point func. related?

$$\hookrightarrow \int \mathcal{D}\varphi \varphi(x_1) \dots \varphi(x_n) e^{iS[\varphi]}$$

$q(t)$

$$W(q(t), q'(t')) = \langle q | e^{iS} | q' \rangle$$

$$W(t, t') = \langle 0 | e^{iS} | 0 \rangle$$

$$= \langle 0 | e^{iS} | 0 \rangle$$

$$= \int \mathcal{D}q \mathcal{D}q'$$

$$\langle \varphi' | e^{iH(t-t')} | \varphi \rangle$$

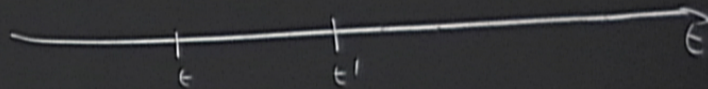
$$= \langle 0 | \hat{\varphi}(t) \hat{\varphi}(t') | 0 \rangle$$

$$= \langle 0 | e^{i\int_t^t \hat{\varphi} H \hat{\varphi}} e^{-i\int_t^{t'} \hat{\varphi} H \hat{\varphi}} e^{i\int_{t'}^{t'} \hat{\varphi} H \hat{\varphi}} | 0 \rangle$$

$$= \int d\varphi d\varphi' \langle 0 | \varphi \rangle \varphi W(\varphi, t; \varphi', t') \varphi' \langle \varphi' | 0 \rangle$$

$$= \int d\varphi d\varphi' W(\varphi, t; \varphi', t') \varphi \varphi' \psi(\varphi) \psi(\varphi')$$

$$= \int \mathcal{D}\varphi \varphi(t) \varphi(t') e^{iS[\varphi]}$$



$$\langle \varphi'(t') | \varphi(t) \rangle = \langle q | e^{iH(t-t')} | \varphi' \rangle$$

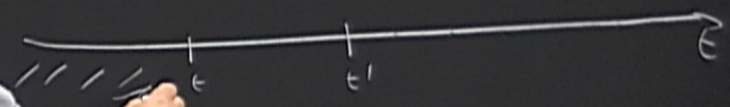
$$= \langle 0 | \hat{\varphi}(t) \hat{\varphi}(t') | 0 \rangle$$

$$= \langle 0 | e^{i\int_{t'}^t H} \hat{\varphi} e^{-i\int_t^{t'} H} e^{i\int_{t'}^t H} \hat{\varphi} e^{-i\int_t^{t'} H} | 0 \rangle$$

$$= \int dq dq' \langle 0 | q \rangle q W$$

$$= \int dq dq' W(q, t; q', t') \varphi(q) \varphi'(q')$$

$$= \int \mathcal{D}[\varphi] \varphi(t) \varphi(t') e^{iS[\varphi]}$$



$$\psi'(t') = \langle q | e^{iH(t-t')} | \psi' \rangle$$

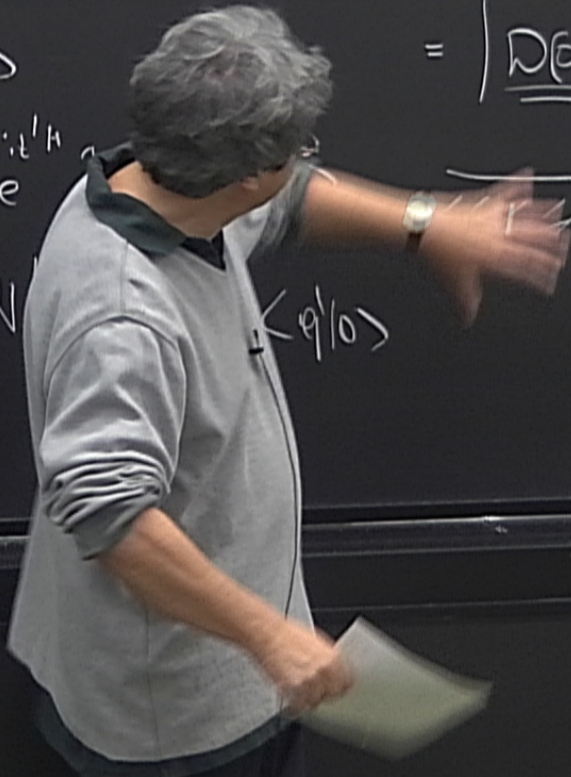
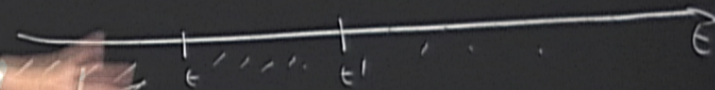
$$\psi(t) = \langle 0 | \hat{\varphi}(t) \hat{\varphi}(t') | 0 \rangle$$

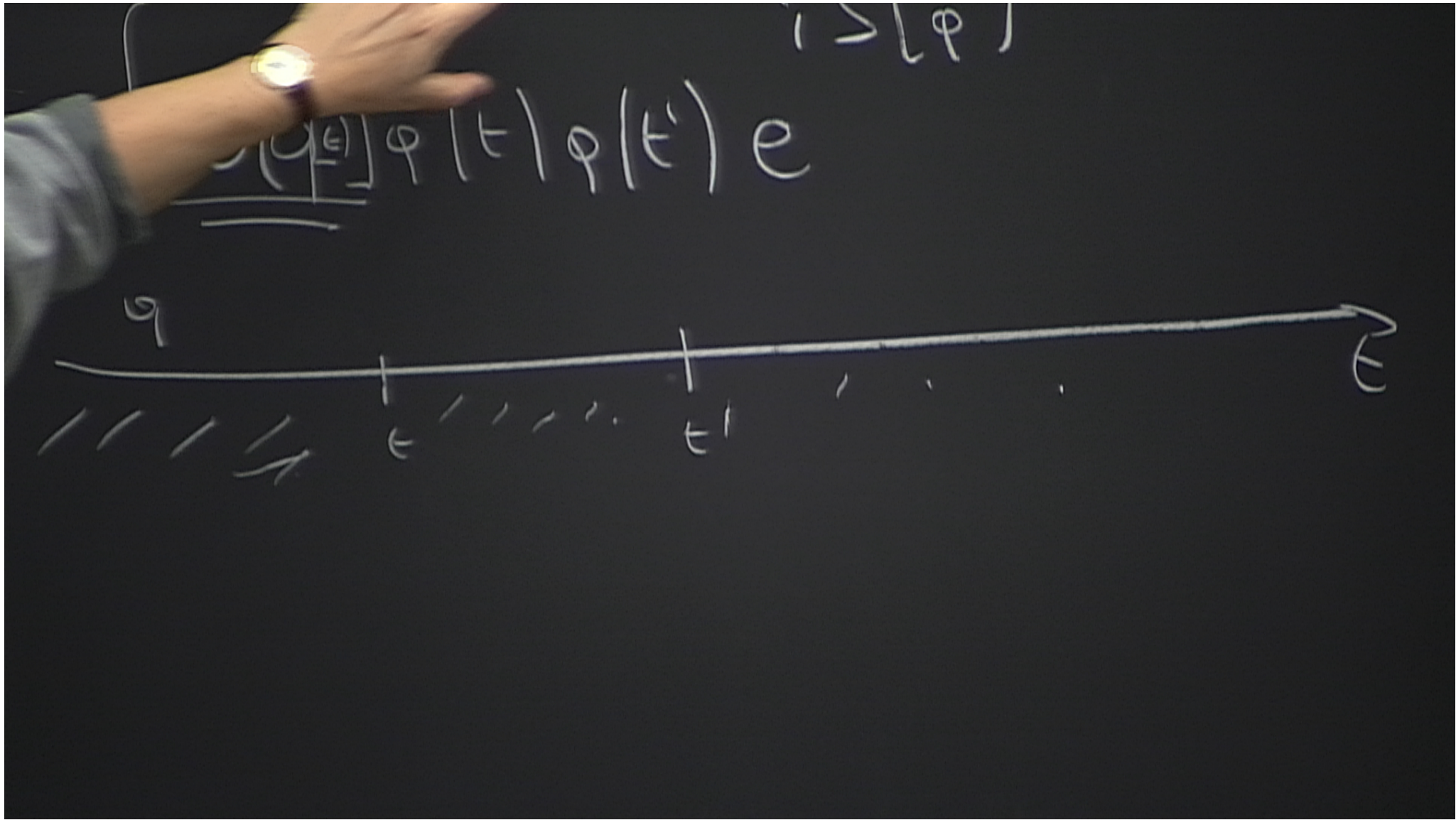
$$= \langle 0 | e^{i\lambda H} \hat{\varphi} e^{-itH} e^{it'H} | 0 \rangle$$

$$= \int dq dq' \langle 0 | q \rangle q W(q, q') \langle q' | 0 \rangle$$

$$= \int dq dq' W(q, q') \psi(q) \psi'(q')$$

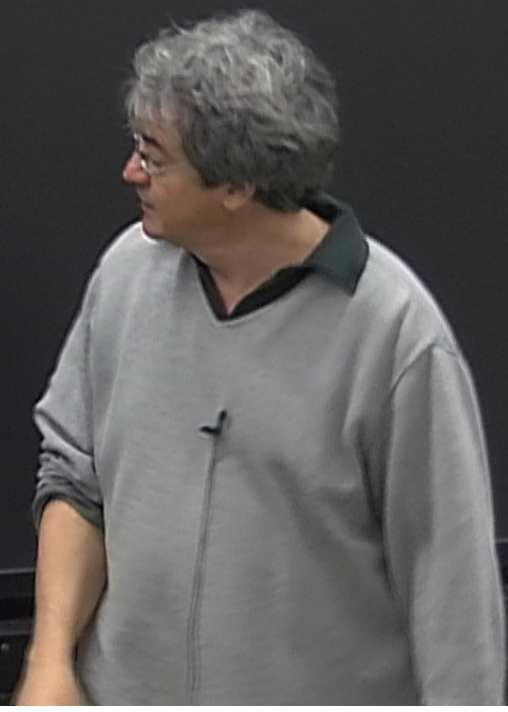
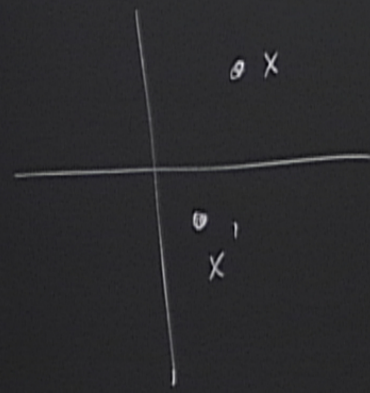
$$= \int \mathcal{D}[\varphi] \varphi(t) \varphi(t') e^{iS[\varphi]}$$





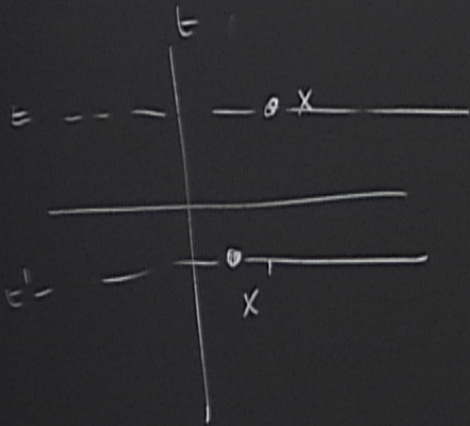
Fields

$$\langle 0 | \varphi(x) \varphi(x') | 0 \rangle$$



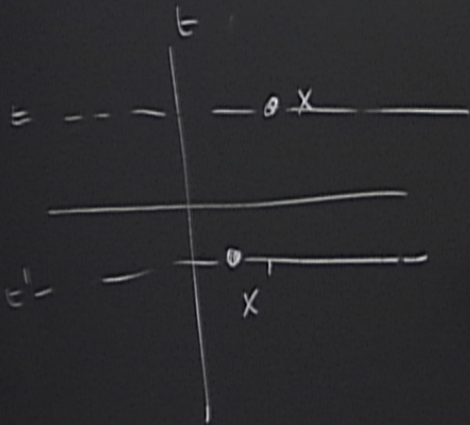
Fields

$$\langle 0 | \varphi(x) \varphi(x') | 0 \rangle = \int D\varphi(\vec{x}) D\varphi'(\vec{x}')$$



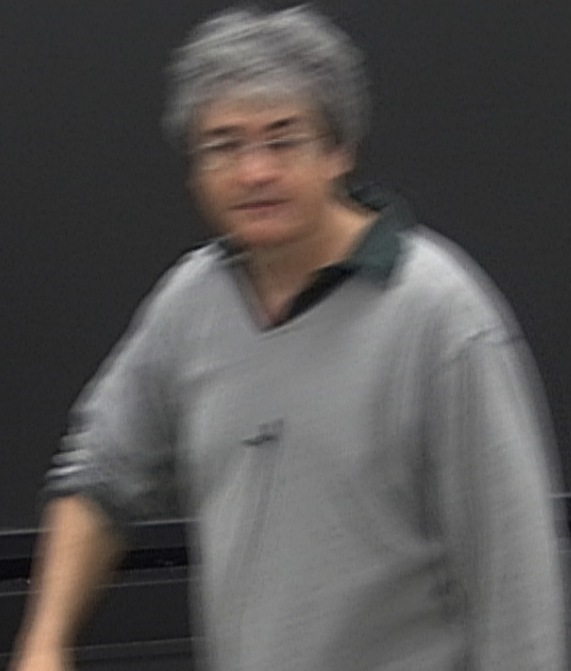
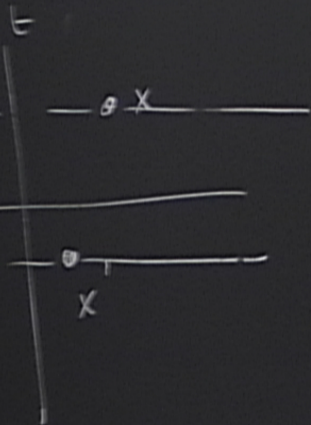
Fields

$$\langle 0 | \varphi(x) \varphi(x') | 0 \rangle = \int D\varphi(\vec{x}) D\varphi'(\vec{x}') W[\varphi(\vec{x}, t), \varphi'(\vec{x}', t')] \varphi(\vec{x}) \varphi'(\vec{x}')$$



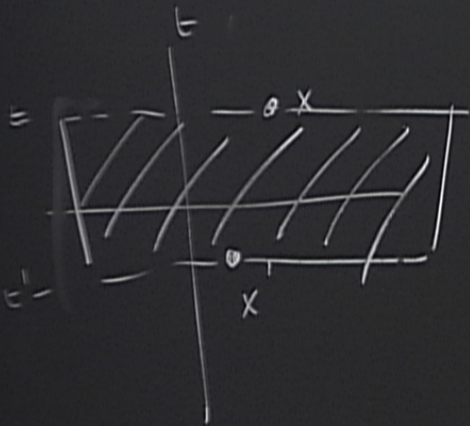
Fields

$$\langle 0 | \varphi(x) \varphi(x') | 0 \rangle = \int D\varphi(\vec{x}) D\varphi'(\vec{x}') W[\varphi(\vec{x}, t), \varphi'(\vec{x}', t')] \varphi(\vec{x}) \varphi'(\vec{x}') \overline{\psi}_0[\varphi'] \psi_0[\varphi]$$



Fields

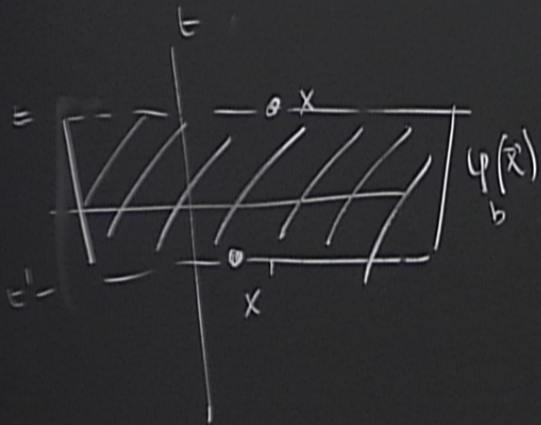
$$\langle 0 | \varphi(x) \varphi(x') | 0 \rangle = \int D\varphi(\vec{x}) D\varphi'(\vec{x}') W[\varphi(\vec{x}, t), \varphi'(\vec{x}', t')] \varphi(\vec{x}) \varphi'(\vec{x}') \overline{\psi}[\varphi'] \psi$$



Fields

$$\langle 0 | \varphi(x) \varphi(x') | 0 \rangle = \int D\varphi(\vec{x}) D\varphi'(\vec{x}') W[\varphi(\vec{x}), t, \varphi'(\vec{x}', t')] \varphi(\vec{x}) \varphi'(\vec{x}') \psi_0[\varphi] \psi_0[\varphi']$$

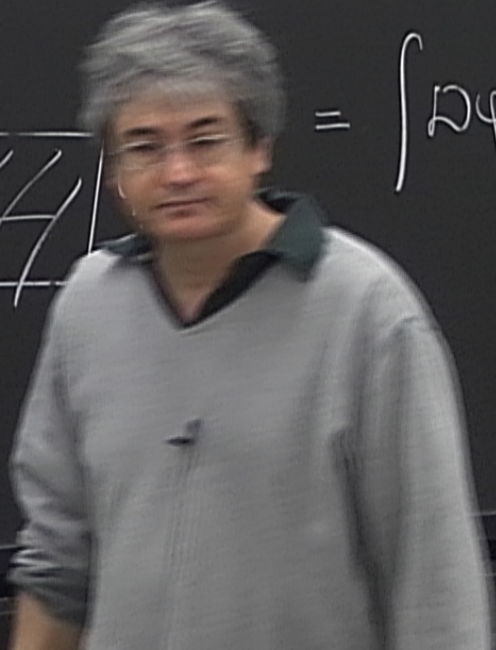
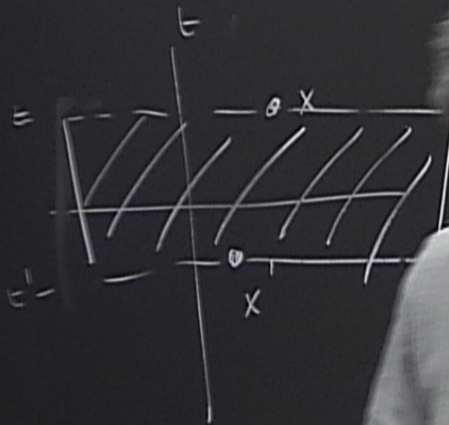
$$= \int D\varphi_b W[\varphi_b] \varphi(\vec{x}) \varphi(\vec{x}')$$



Fields

$$\langle 0 | \varphi(x) \varphi(x') | 0 \rangle = \int D\varphi(\vec{x}) D\varphi'(\vec{x}') W[\varphi(\vec{x}, t), \varphi'(\vec{x}', t')] \varphi(\vec{x}) \varphi'(\vec{x}') \psi_0[\varphi] \psi_0[\varphi']$$

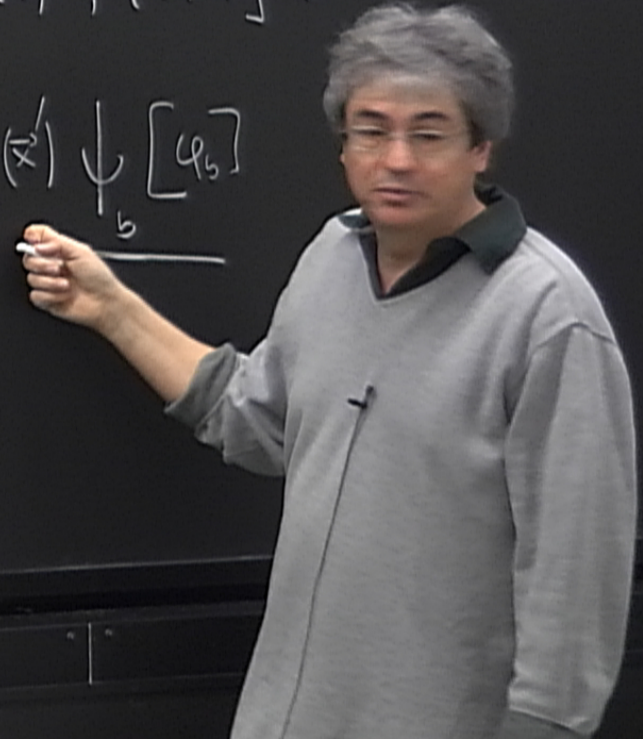
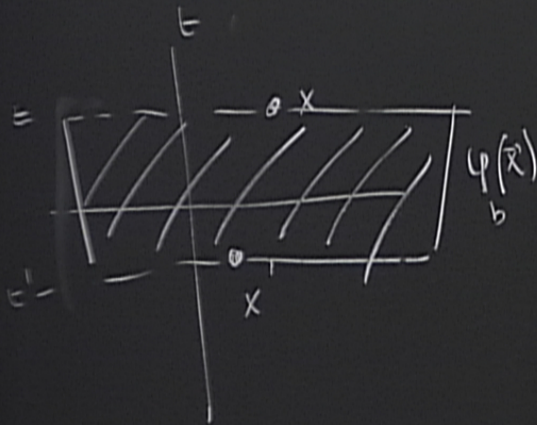
$$= \int D\varphi_b W[\varphi_b] \varphi(\vec{x}) \varphi(\vec{x}') \psi_b[\varphi_b]$$



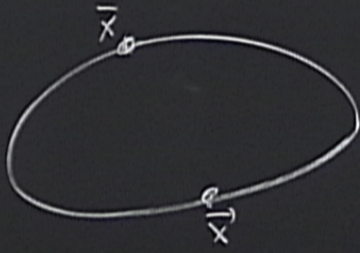
Fields

$$\langle 0 | \varphi(x) \varphi(x') | 0 \rangle = \int D\varphi(\vec{x}) D\varphi'(\vec{x}') W[\varphi(\vec{x}), t, \varphi'(\vec{x}', t')] \varphi(\vec{x}) \varphi'(\vec{x}') \psi_b[\varphi] \psi_b[\varphi']$$

$$= \int D\varphi_b \underline{W[\varphi_b]} \varphi(\vec{x}) \varphi(\vec{x}') \psi_b[\varphi_b]$$

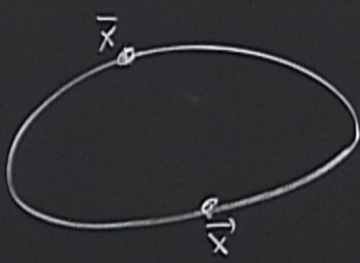


$$e) \varphi(x) \varphi'(x) \overline{\psi_0[\varphi]} \psi_0[\varphi]$$



$$P \quad \vec{E}_e \quad \int_P \vec{E}_0$$

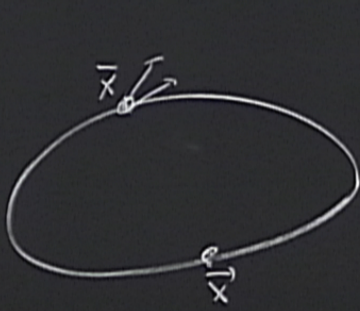
$$\varphi(x) \varphi'(x) \overline{\psi}[\varphi] \psi[\varphi]$$



$$P \vec{E}_e \rightarrow \mathbb{I}_0$$

$$\langle W \vec{E} \vec{E} \mathbb{I}_0 \rangle$$

$$\psi(x) \psi'(x) \psi_0[\psi] \psi_0[\psi]$$



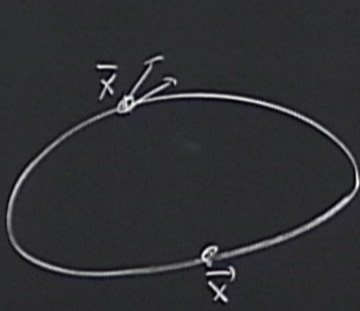
$$P \vec{E}_e \rightarrow \mathbb{F}_0$$

$$\langle W | \vec{E}_e \vec{E}_e | \mathbb{F}_0 \rangle$$

$$\langle W | \vec{E}_e \vec{E}_e \vec{E}_e \vec{E}_e | \psi_0 \rangle$$

$$\langle 0 | \rho_{\omega_5}(x) \rho_{\omega_9}(y) | 0 \rangle = \underline{\underline{W_{\text{dotted}}(x, y)}}$$

$$\psi(x) \psi'(x) \psi_0[\psi] \psi_0[\psi]$$



$$P \vec{E}_e \rightarrow \mathbb{F}_0$$

$$\langle W | \vec{E}_e \vec{E}_e | \mathbb{F}_0 \rangle$$

$$\langle W | \vec{E}_e \vec{E}_e \vec{E}_e \vec{E}_e | \psi_0 \rangle$$

$$\langle 0 | \rho_{as}(x) \rho_{ca}(y) | 0 \rangle = \underline{\underline{W_{dotted}(x,y)}}$$

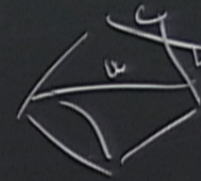
What yet to do?

• Radiative corrections

What yet to do?

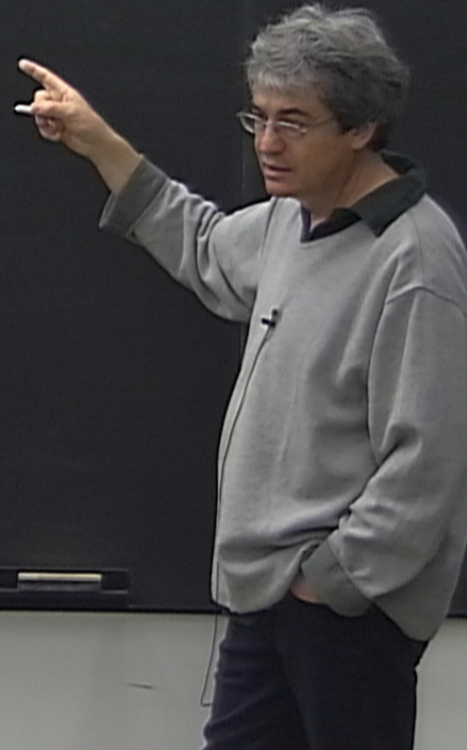
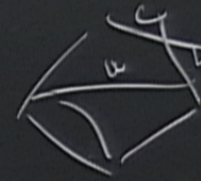
• Radiative corrections

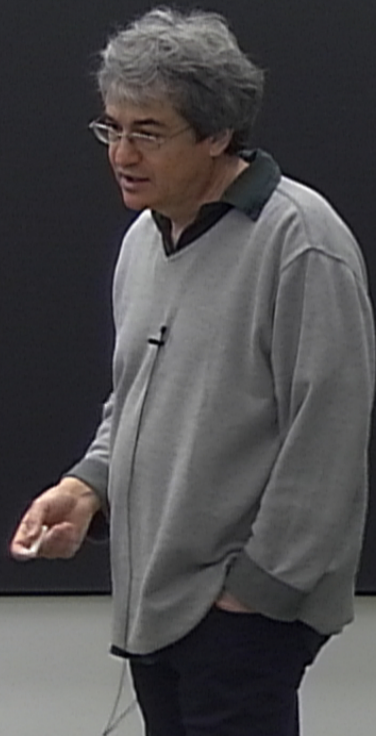
$$\psi = T_2(uv\alpha)$$



$$\psi = T_2(UV\alpha)$$

$$U = \hat{D}(U)$$



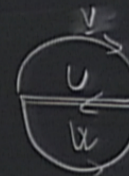


$$\psi = T_2(uv\alpha)$$

$$v = \overset{\cdot}{D}(u)$$

$$\psi(uv\alpha) = T_2(u) T_2(v\alpha)$$

$$= \overset{\cdot}{D}(u) \overset{\cdot}{D}(v) \overset{\cdot}{D}(\alpha)$$



$$\overset{\cdot}{D}(u) \overset{\cdot}{D}(v) =$$

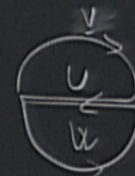
$$\frac{1}{2} \otimes \frac{1}{2} = 1 \oplus 0$$

$$\psi = T_2(uv\alpha)$$

$$u = \overset{\cdot}{D}(v)$$

$$\psi(uv\alpha) = T_2(uv) T_2(v\alpha)$$

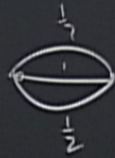
$$= \overset{\cdot}{D}(u) \overset{\cdot}{D}(v) \overset{\cdot}{D}(\alpha)$$



$$\overset{\cdot}{D}(u) \overset{\cdot}{D}(v) =$$

$$\frac{1}{2} \otimes \frac{1}{2} = 1 \oplus 0$$

H_1

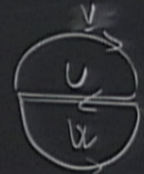
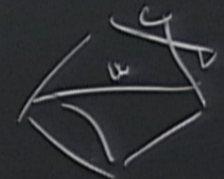


$$\psi = T_2(uv\alpha)$$

$$u = \overset{1}{D}(u)$$

$$= T_2(uv) T_2(u\alpha)$$

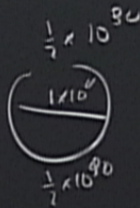
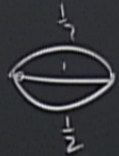
$$= \overset{1}{D}(u) \overset{1}{D}(v) \overset{1}{D}(\alpha)$$



$$\overset{1}{D}(u) \overset{1}{D}(v) =$$

$$\frac{1}{2} \otimes \frac{1}{2} = 1 \oplus 0$$

H_5

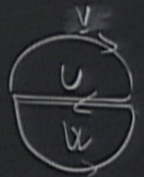
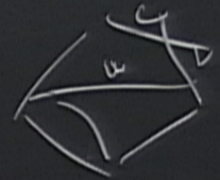


$$\psi = T_2(uv\alpha)$$

$$U = \dot{D}(u)$$

$$= T_2(uv) T_2(u\alpha)$$

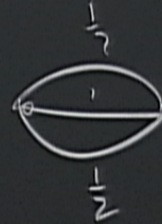
$$= \dot{D}(u) \dot{D}(v) \dot{D}(\alpha)$$



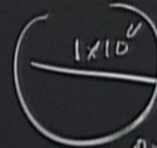
$$\dot{D}(u) \dot{D}(v) =$$

$$\frac{1}{2} \otimes \frac{1}{2} = 7 \oplus 0$$

$H_{\mathbb{P}^1}$



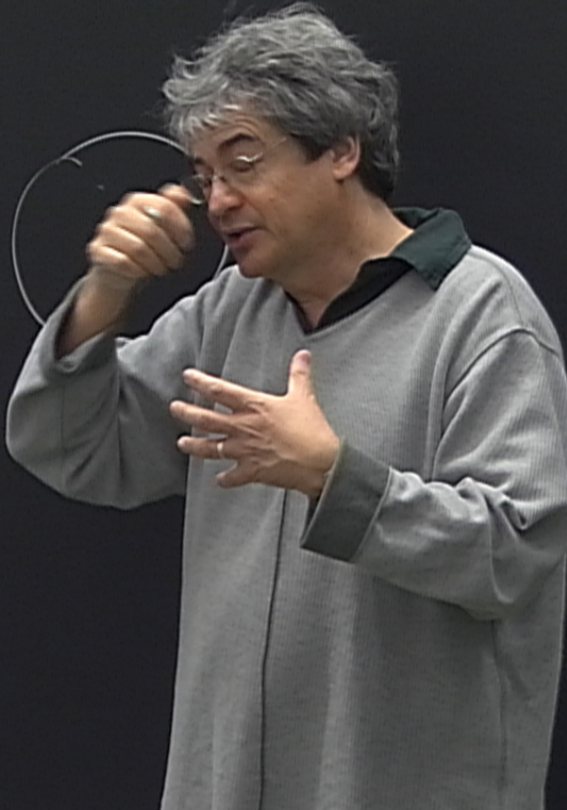
$$\frac{1}{2} \times 10^{30}$$



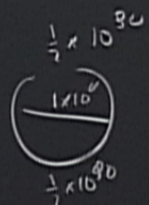
$$\frac{1}{2} \times 10^{30}$$

$\downarrow =$

$\downarrow (u, v, \alpha)$



H_{Γ}

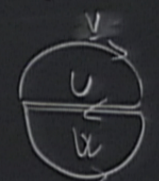


$$\psi = T_2(uv\alpha)$$

$$v = \frac{1}{2} \dot{D}(u)$$

$$\psi(uv) = T_2(uv) T_2(uv)$$

$$\dot{D}(u) \dot{D}(v) \dot{D}(\alpha)$$



$$\dot{D}(u) \dot{D}(v) =$$

$$\frac{1}{2} \otimes \frac{1}{2} = 1 \otimes 0$$

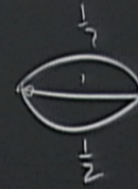
λ

H^L

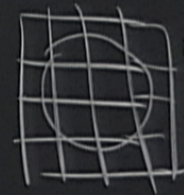


λ

I



$\frac{1}{2} \times 10^{30}$
 1×10^{30}
 $\frac{1}{2} \times 10^{30}$



$N > N_0$