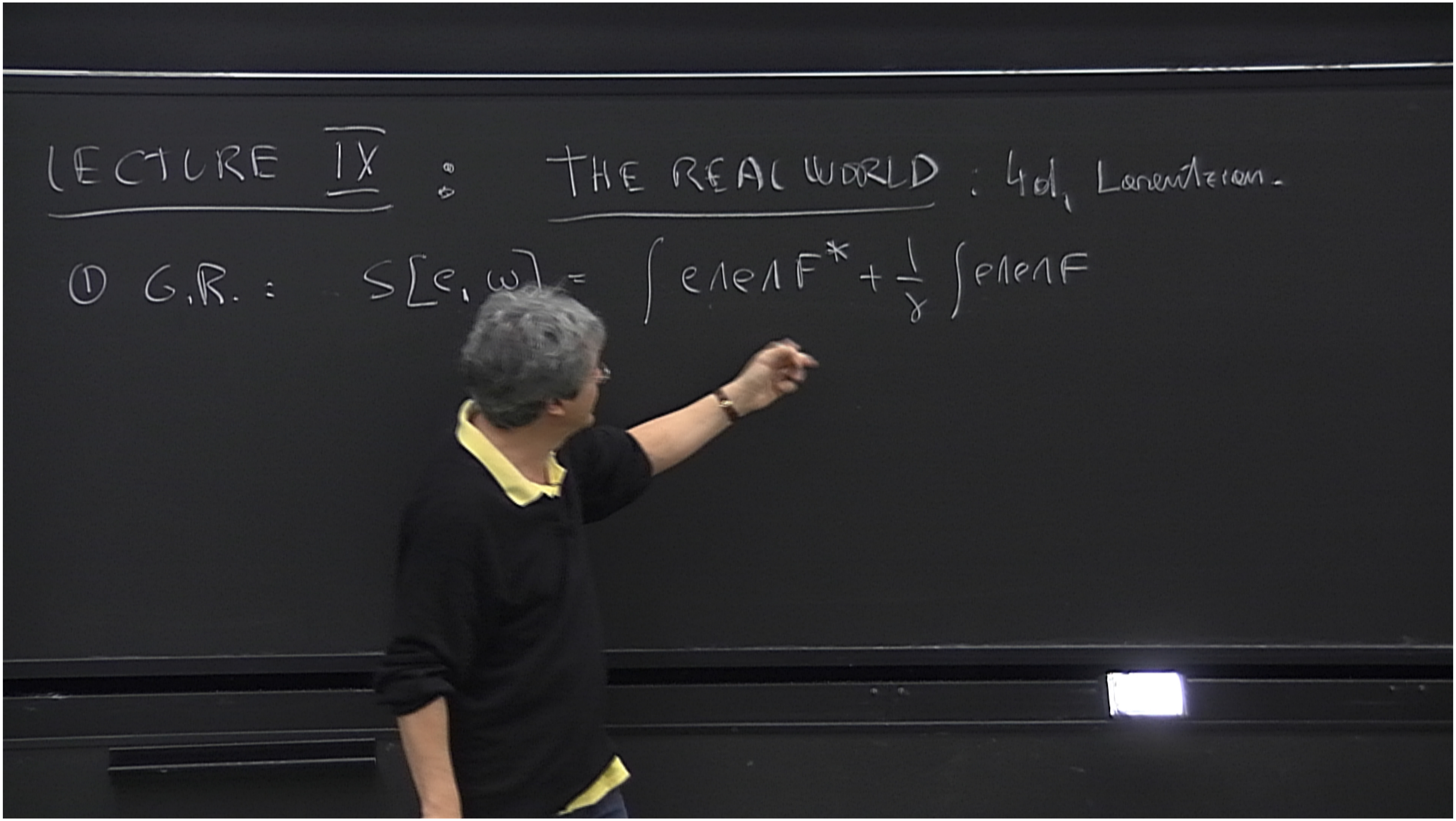
Title: Explorations in Quantum Gravity - Lecture 9

Date: Apr 13, 2012 10:15 AM

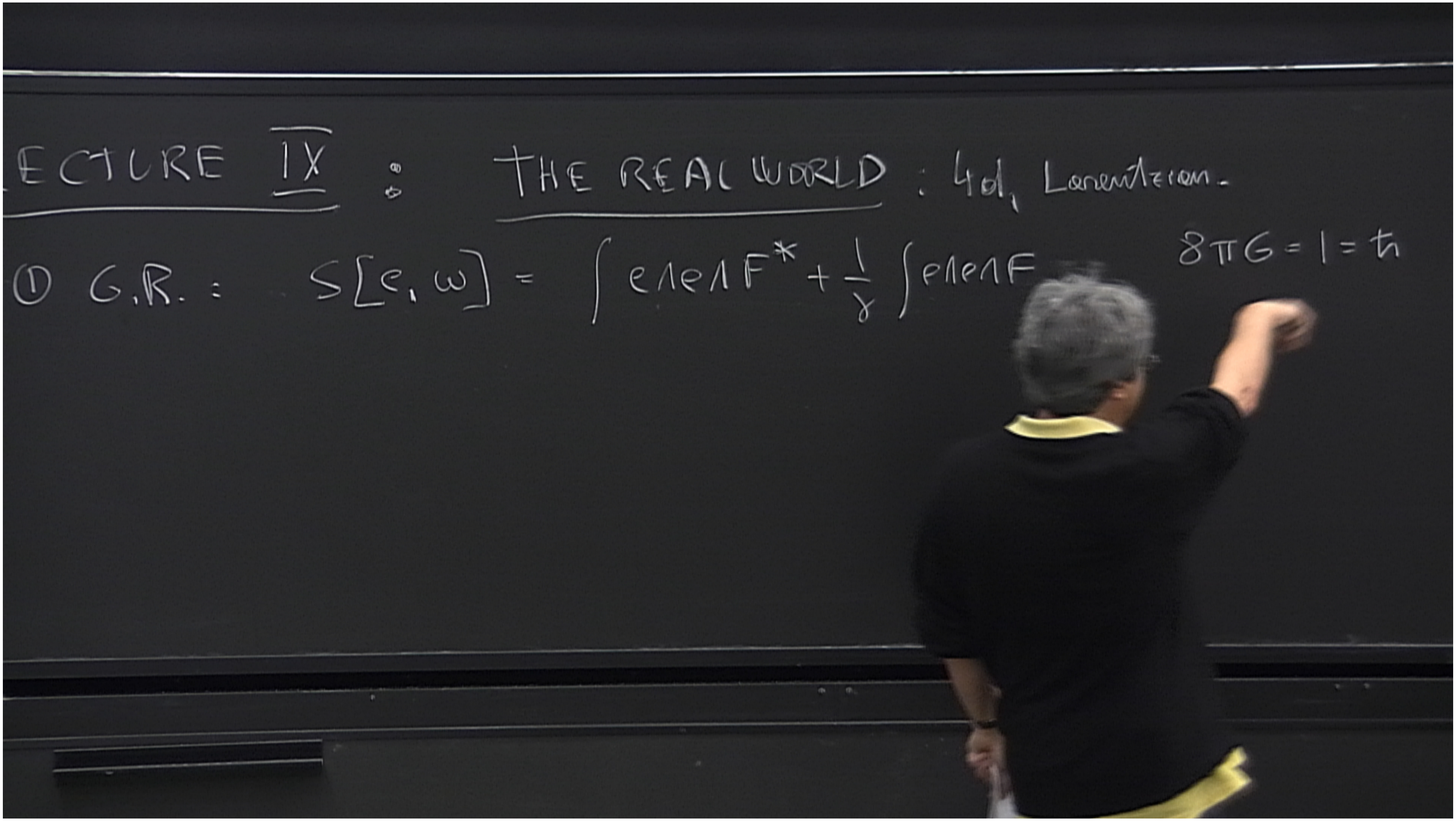
URL: http://pirsa.org/12040030

Abstract:



LECTURE IX : THE REAL WORLD : 4d, Lorentzian.

① G.R. : $S[e, \omega] = \int e \wedge e \wedge F^{*} + \frac{1}{\gamma} \int e \wedge e \wedge F$



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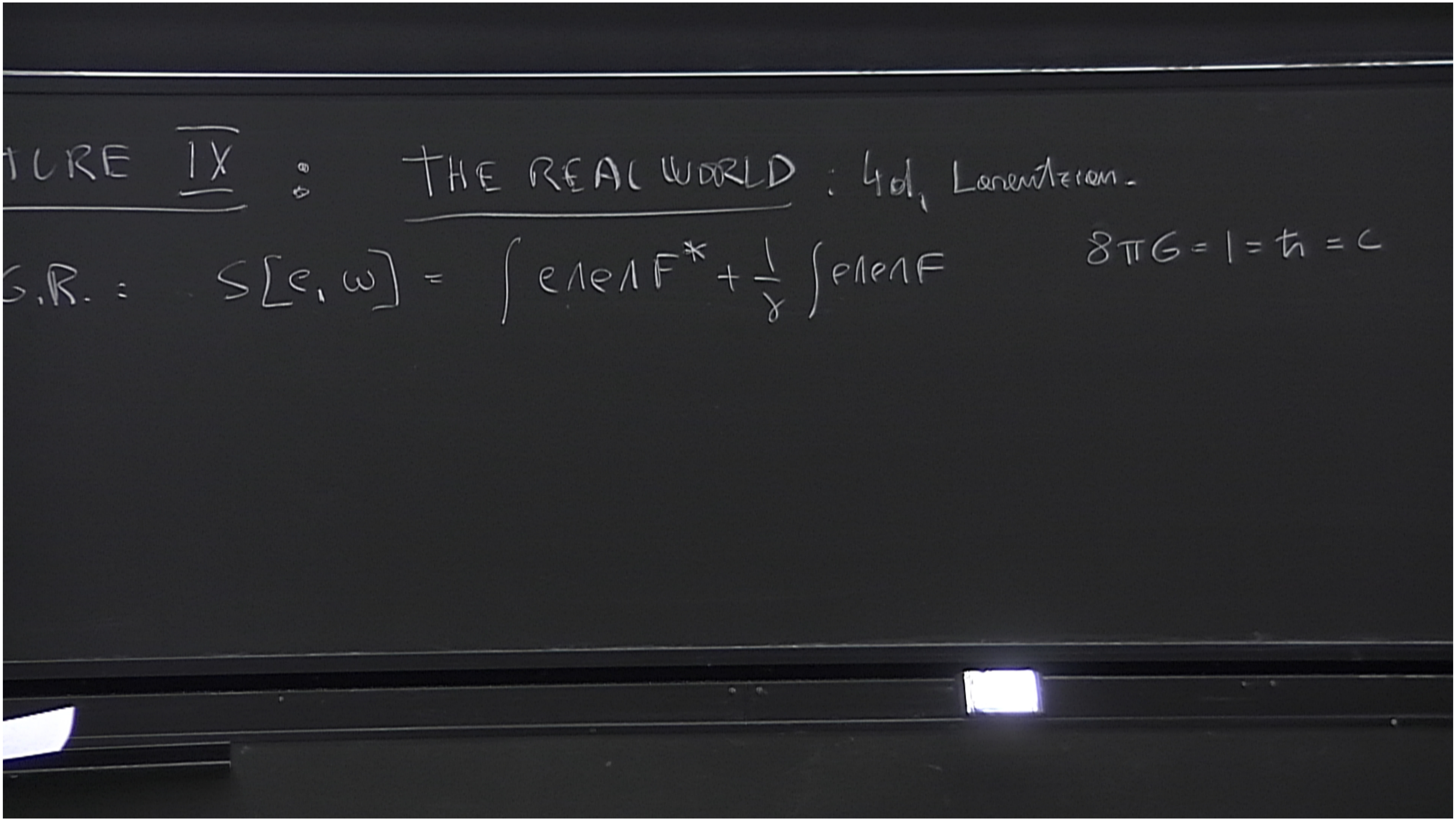
$8\pi G = 1 = \hbar$



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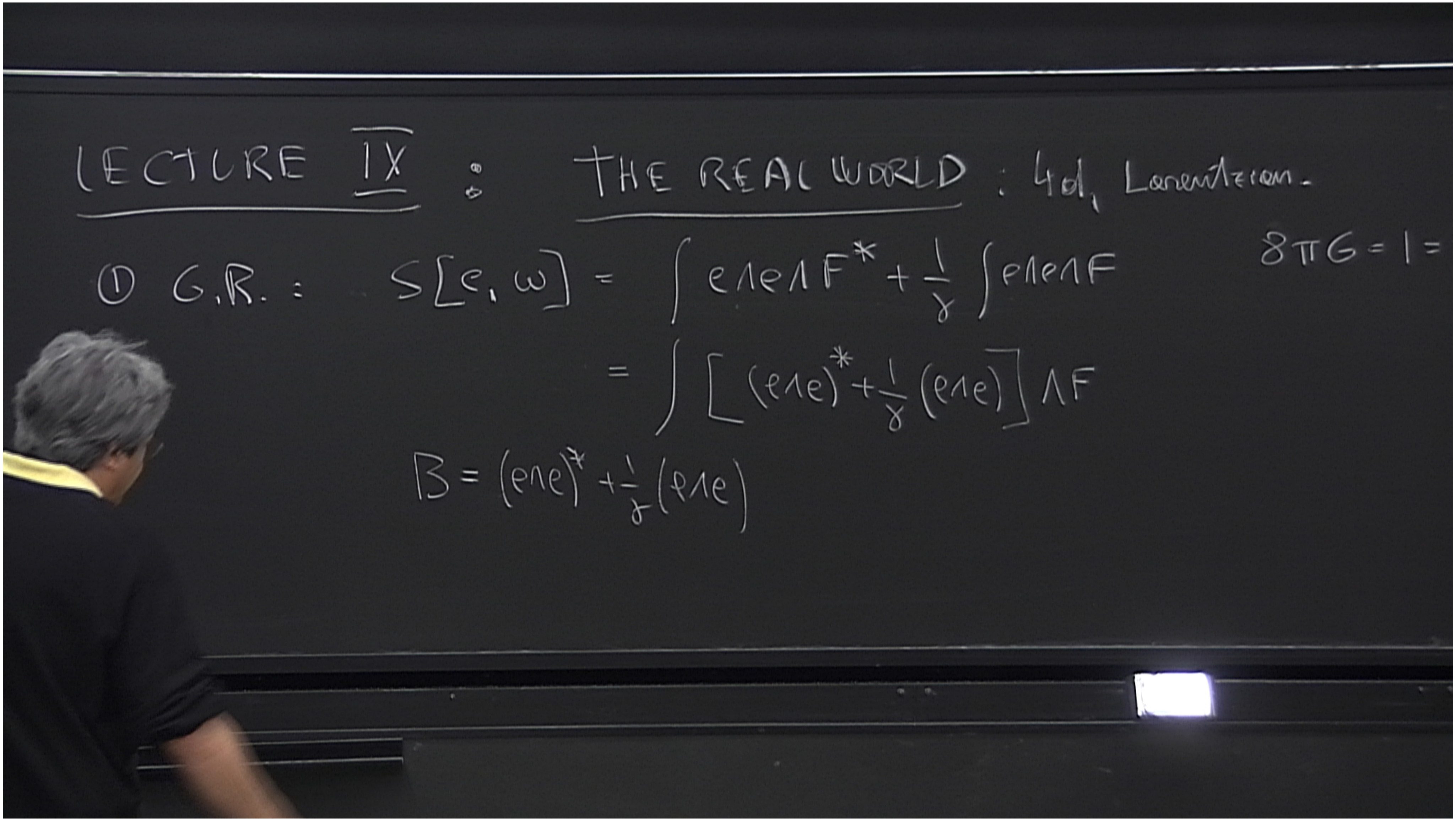


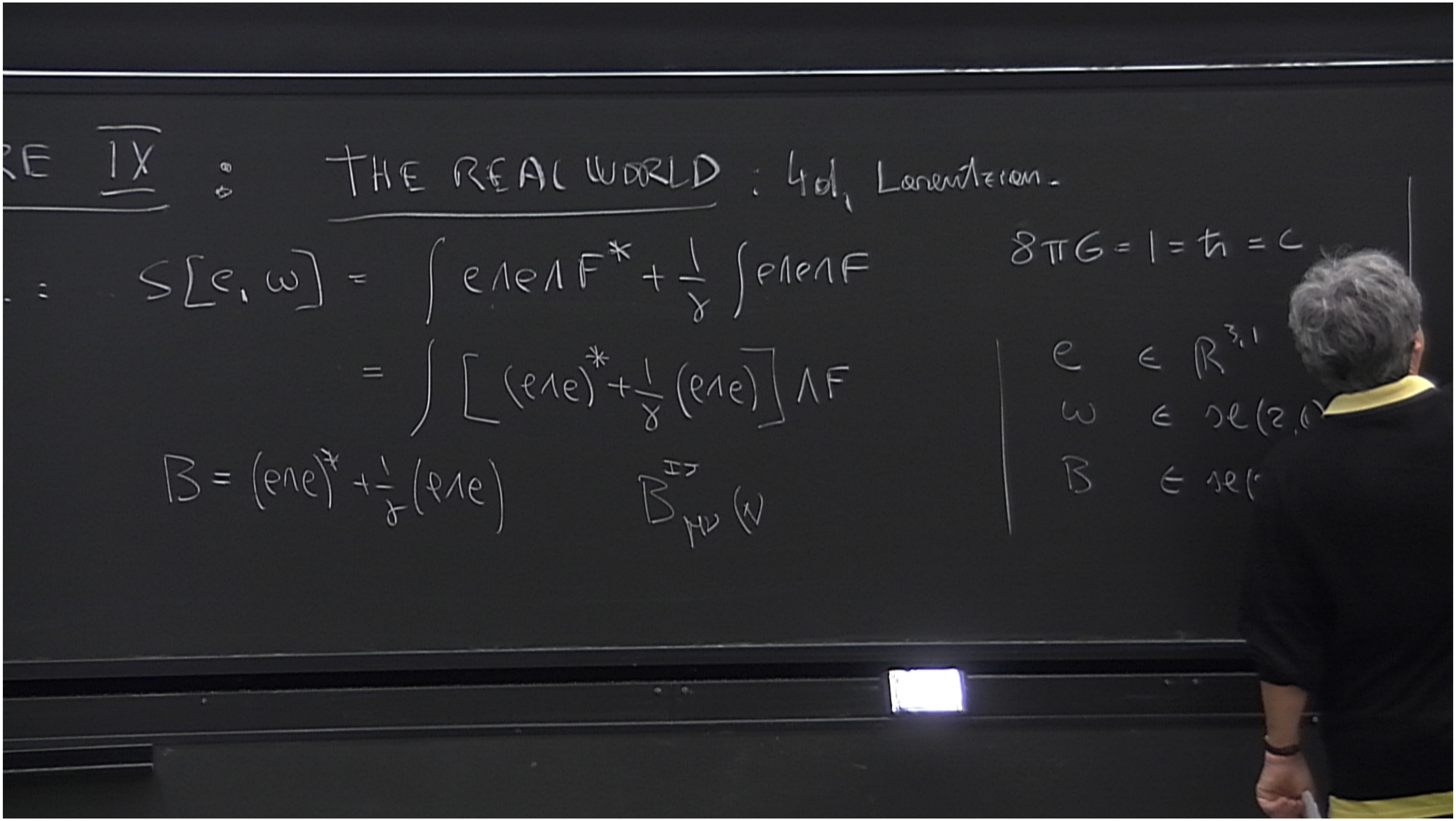
LECTURE IX : THE REAL WORLD : 4d, Lorentzian.

① G.R. : $S[e,\omega] = \int e\wedge e\wedge F^* + \frac{1}{\gamma}\int e\wedge e\wedge F$

$8\pi G = 1 =$

$$= \int \left[(e\wedge e)^* + \frac{1}{\gamma}(e\wedge e)\right]\wedge F$$

$$B = (e\wedge e)^* + \frac{1}{\gamma}(e\wedge e)$$



RE $\overline{\underline{IX}}$: THE REAL WORLD : 4d, Lorentzian.

$$S[e,\omega] = \int e\wedge e\wedge F^{*} + \frac{1}{\gamma}\int e\wedge e\wedge F$$

$8\pi G = 1 = \hbar = c$

$$= \int \left[(e\wedge e)^{*} + \frac{1}{\gamma}(e\wedge e)\right]\wedge F$$

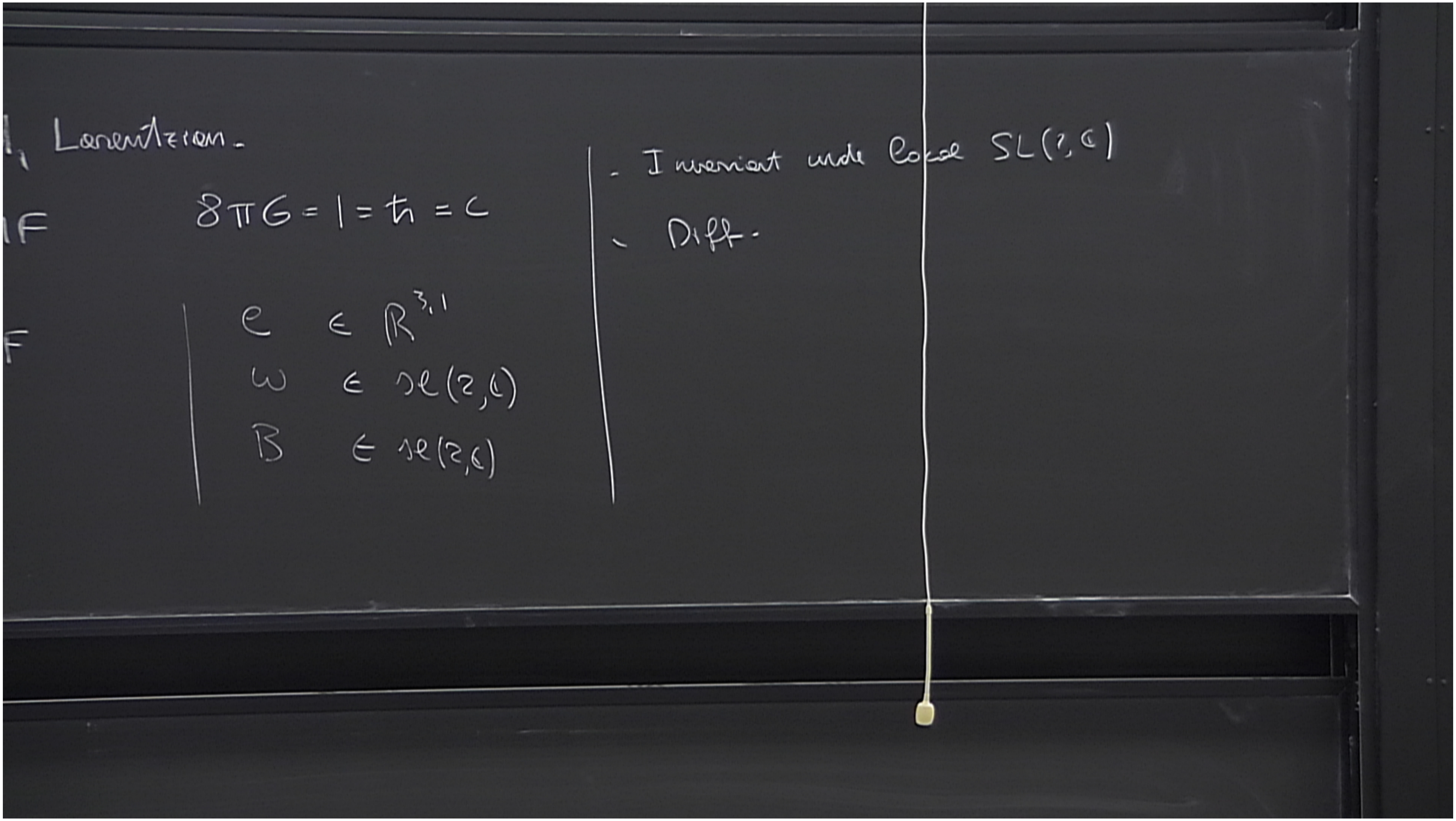
$e \in \mathbb{R}^{3,1}$

$\omega \in \mathfrak{sl}(2,($

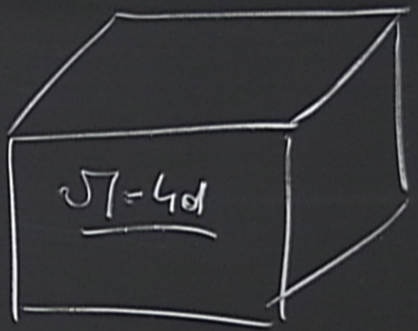
$B \in \mathfrak{sl}($

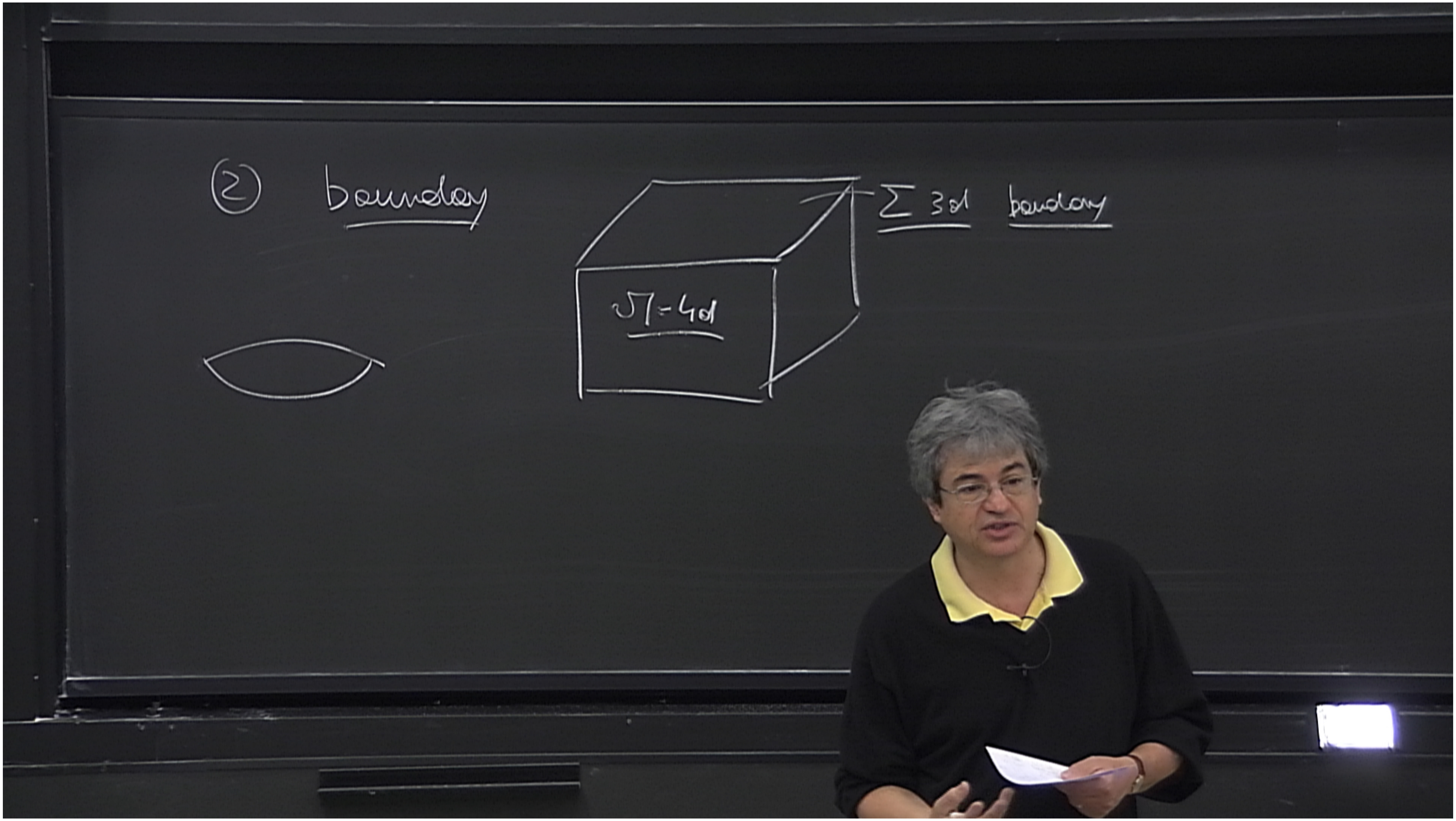
$$B = (e\wedge e)^{*} + \frac{1}{\gamma}(e\wedge e)$$

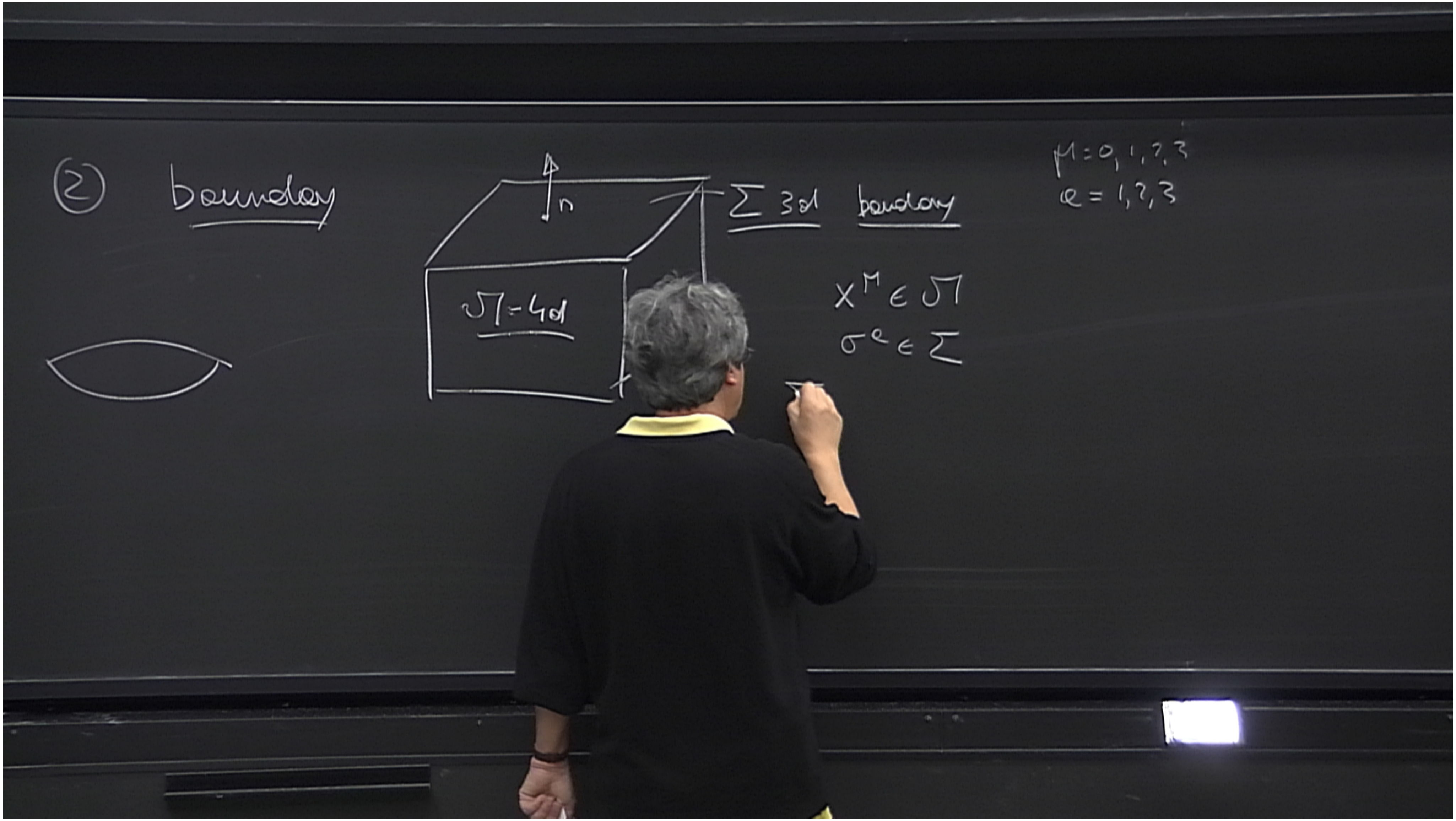
$B^{IJ}_{\mu\nu}(x$



② boundary




② boundary
Σ 3d boundary
Ω = 4d



② boundary

$\mathcal{M} = 4d$

n

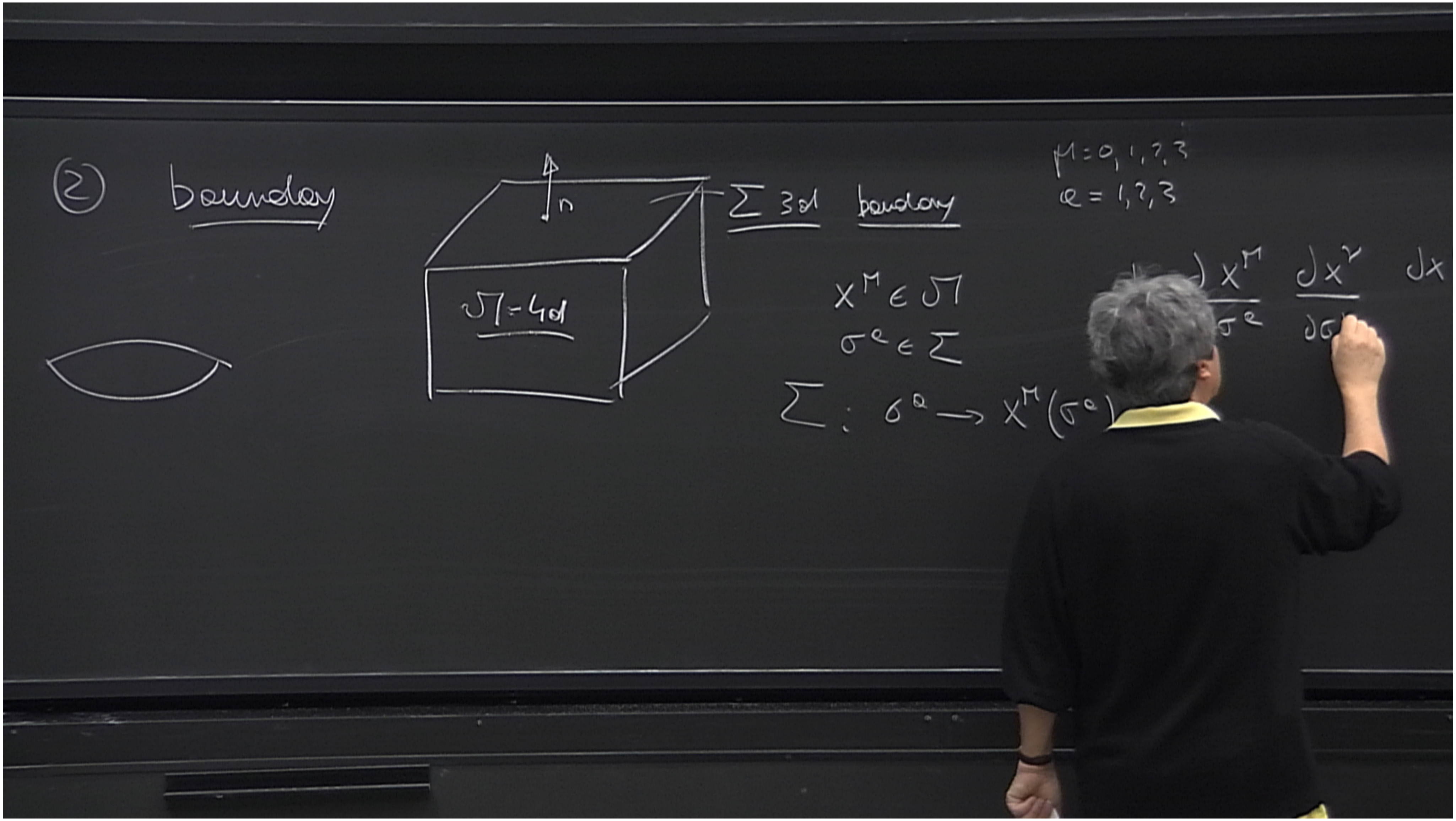
Σ 3d boundary

$\mu = 0, 1, 2, 3$

$a = 1, 2, 3$

$x^\mu \in \mathcal{M}$

$\sigma^a \in \Sigma$



② **boundary**

$\mathcal{M} = 4d$

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Σ 3d **boundary**

$\mu = 0, 1, 2, 3$

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$x^\mu \in \mathcal{M}$

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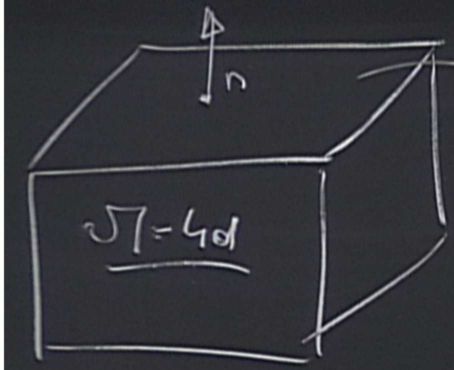
$\Sigma : \sigma^a \longrightarrow x^\mu(\sigma^a)$

$\frac{\partial x^\mu}{\partial \sigma^a} \quad \frac{\partial x^\nu}{\partial \sigma} \quad \partial x$

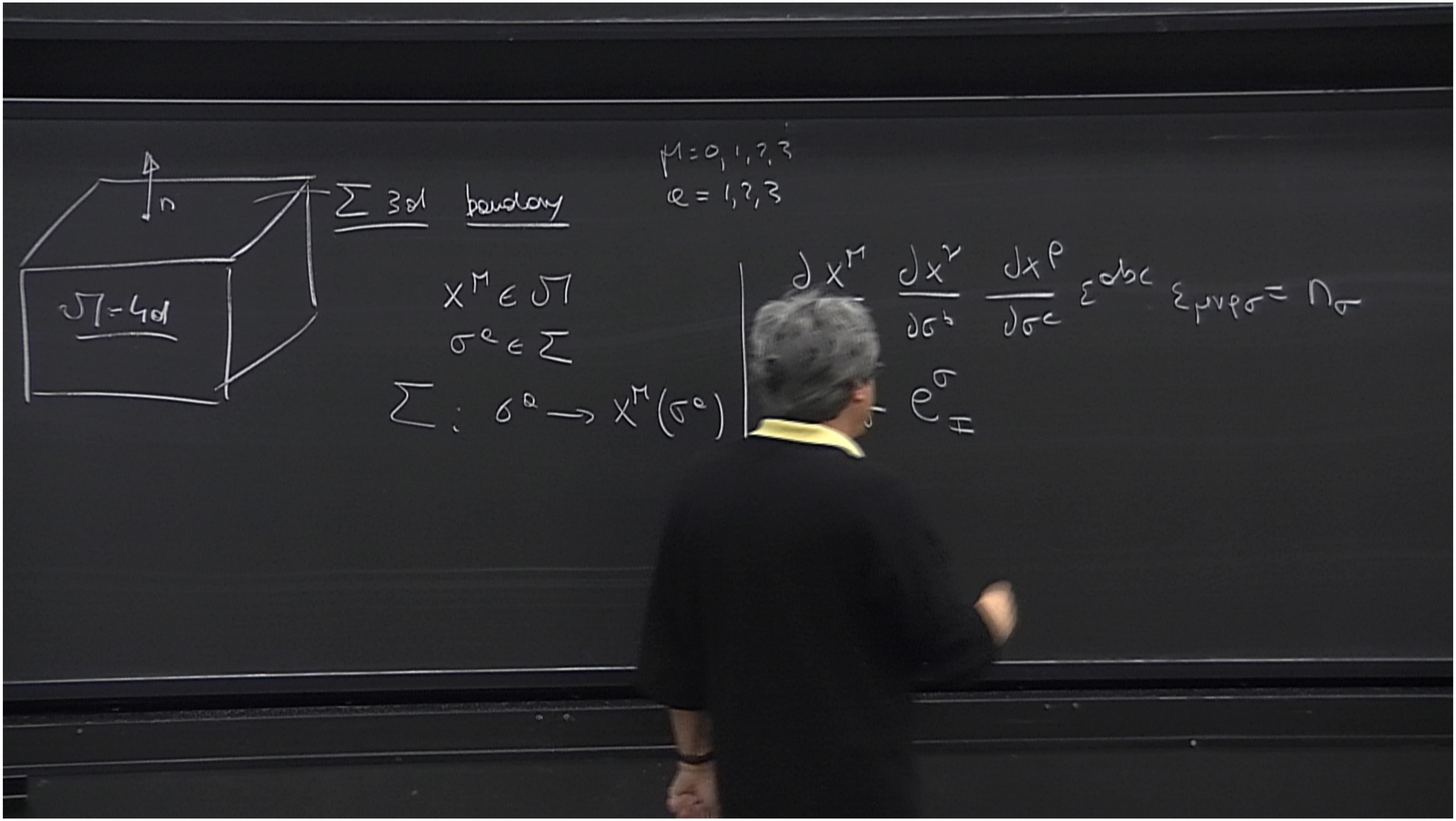


Σ 3d boundary

$\mu = 0, 1, 2, 3$
$a = 1, 2, 3$

$x^\mu \in \partial M$
$\sigma^a \in \Sigma$
$\Sigma : \sigma^a \to x^\mu(\sigma^a)$

$$\frac{\partial x^\mu}{\partial \sigma^a} \frac{\partial x^\nu}{\partial \sigma^b} \frac{\partial x^\rho}{\partial \sigma^c} \varepsilon^{abc} \varepsilon_{\mu\nu\rho\sigma} = n_\sigma$$



Σ 3d boundary

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$a = 1, 2, 3$

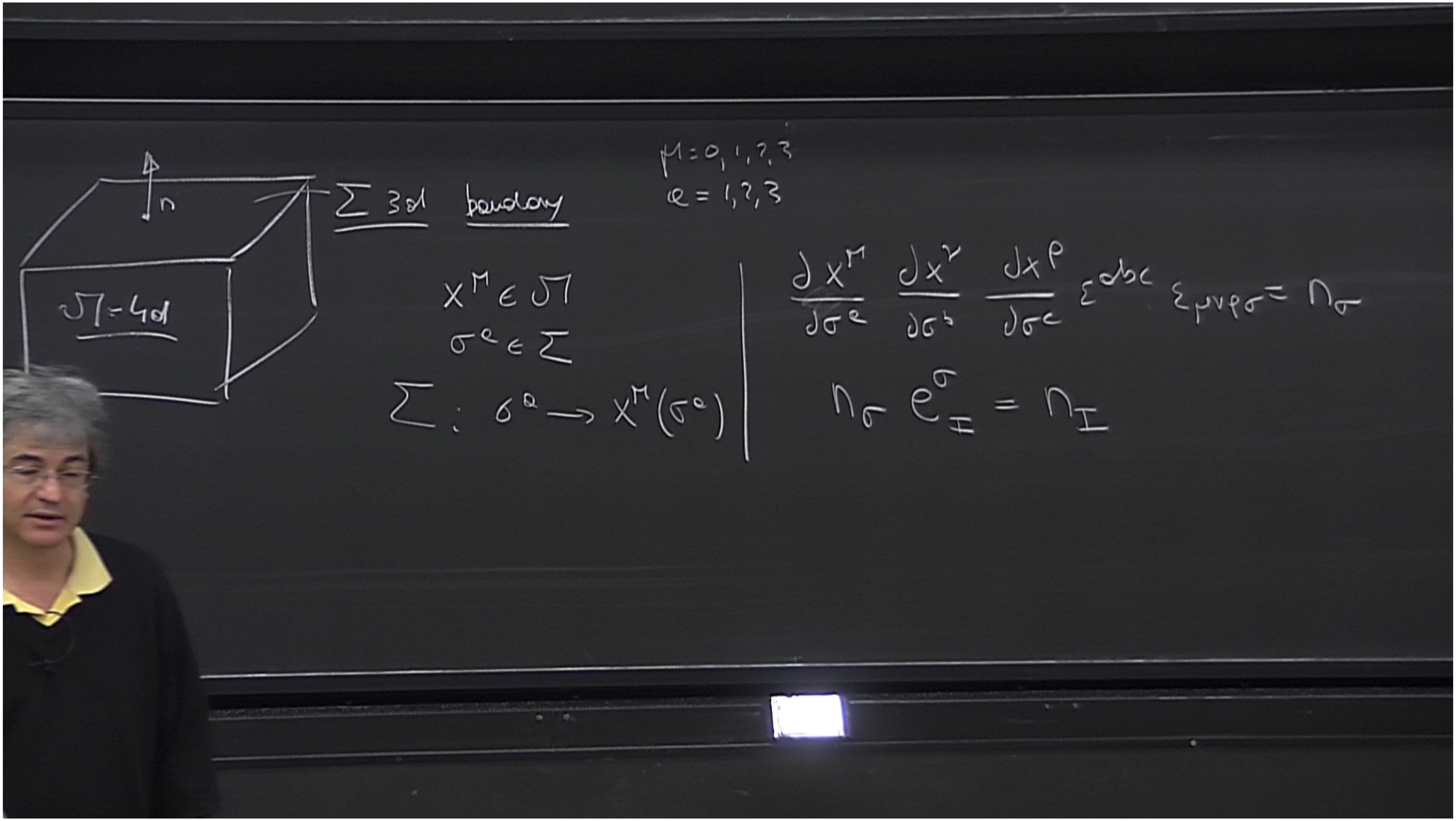
$x^\mu \in \partial M$

$\sigma^a \in \Sigma$

$\Sigma : \sigma^a \longrightarrow x^\mu(\sigma^a)$

$$\partial x^\mu \quad \frac{\partial x^\nu}{\partial \sigma^b} \frac{\partial x^\rho}{\partial \sigma^c} \varepsilon^{abc} \varepsilon_{\mu\nu\rho\sigma} = n_\sigma$$

$$e^\sigma_I$$



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$$n_\sigma e^\sigma_I = n_I$$

$0, 1, 2, 3$

$= 1, 2, 3$

)

$$\frac{\partial x^{\mu}}{\partial \sigma^{a}} \frac{\partial x^{\nu}}{\partial \sigma^{b}} \frac{\partial x^{\rho}}{\partial \sigma^{c}} \varepsilon^{abc} \varepsilon_{\mu\nu\rho\sigma} = n_{\sigma}$$

$$n_{\sigma} e^{\sigma}_{I} = n_{I}$$

$= 0,1,2,3$

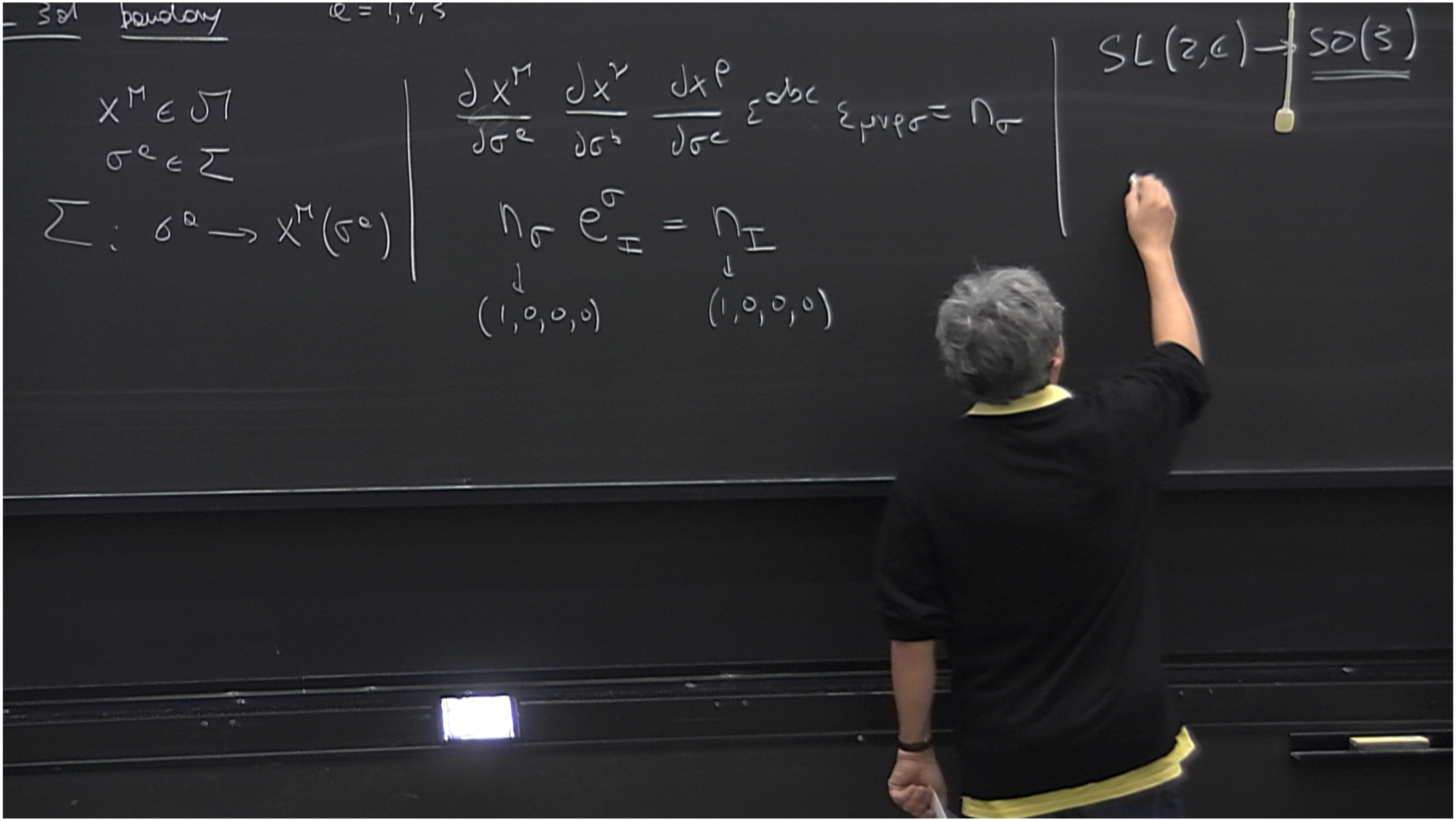
$= 1,2,3$

$e)$

$$\frac{\partial x^\mu}{\partial \sigma^a}\frac{\partial x^\nu}{\partial \sigma^b}\frac{\partial x^\rho}{\partial \sigma^c}\,\epsilon^{abc}\,\epsilon_{\mu\nu\rho\sigma} = n_\sigma$$

$$n_\sigma\, e^\sigma_I = n_I$$

$SL(2,\mathbb{C}) \rightarrow SO(3)$



3d boundary $\quad a = 1,2,3$

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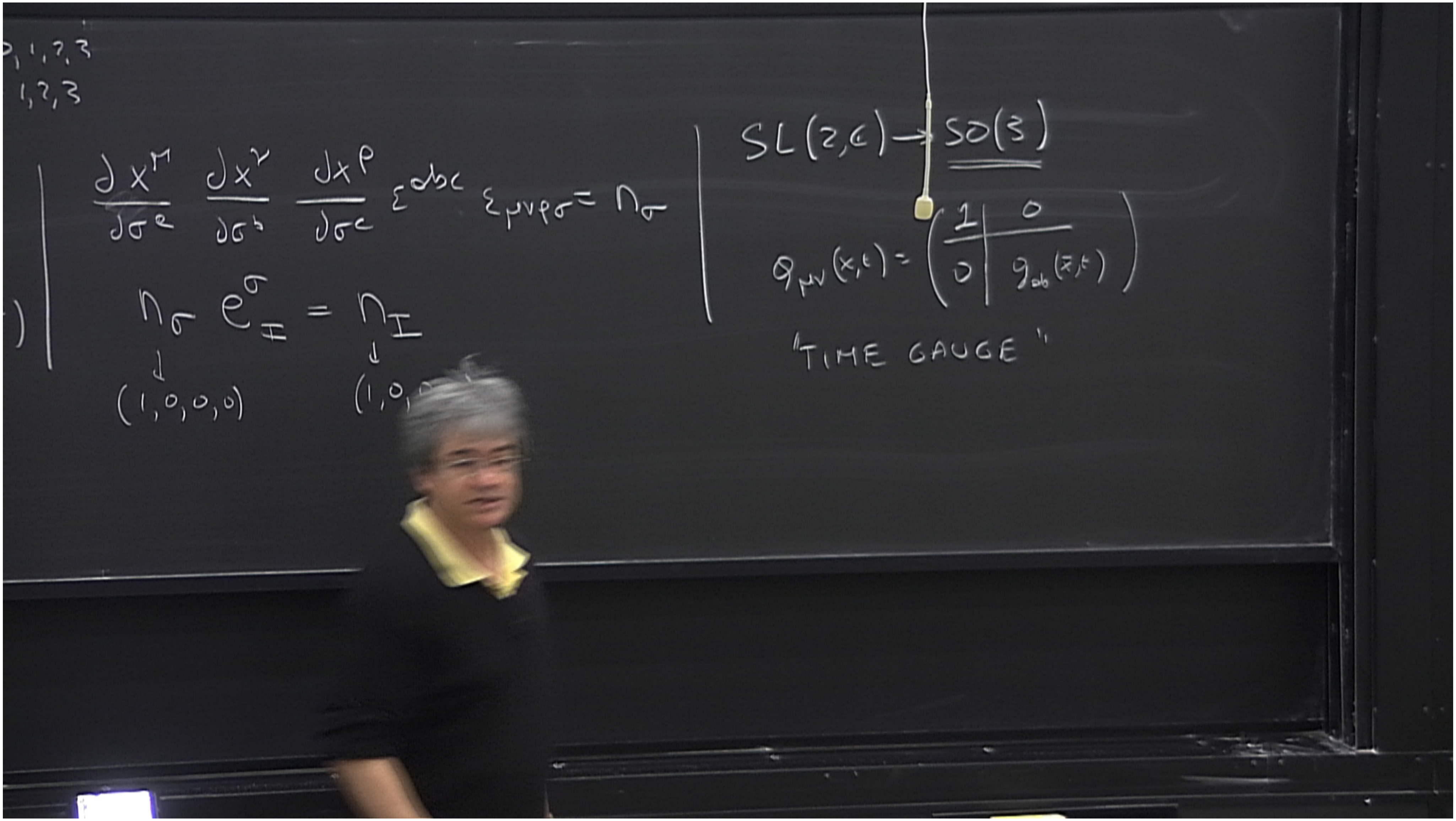
$$\underset{\substack{\downarrow \\ (1,0,0,0)}}{n_\sigma}\, e^\sigma_I = \underset{\substack{\downarrow \\ (1,0,0,0)}}{n_I}$$

$SL(2,\mathbb{C}) \to \underline{SO(3)}$



1, 2, 3

1, 2, 3

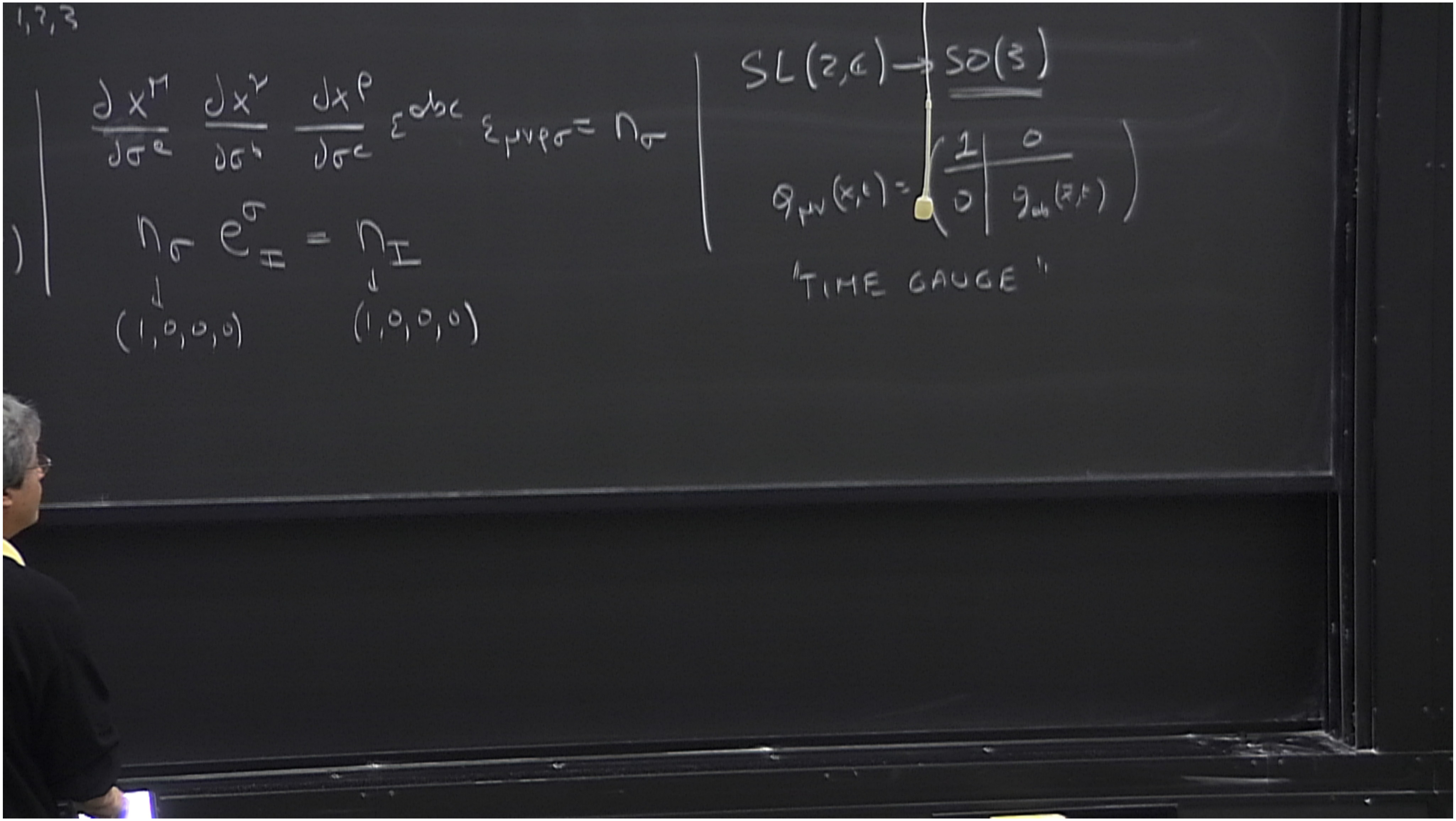
$$\frac{\partial x^\mu}{\partial \sigma^a}\frac{\partial x^\nu}{\partial \sigma^b}\frac{\partial x^\rho}{\partial \sigma^c}\epsilon^{abc}\epsilon_{\mu\nu\rho\sigma} = n_\sigma$$

$$\underset{\underset{(1,0,0,0)}{\downarrow}}{n_\sigma} e^\sigma_I = \underset{\underset{(1,0,}{\downarrow}}{n_I}$$

$$SL(2,\mathbb{C}) \rightarrow \underline{\underline{SO(3)}}$$

$$g_{\mu\nu}(x,t) = \left(\begin{array}{c|c} 1 & 0 \\ \hline 0 & g_{ab}(\vec{x},t) \end{array}\right)$$

"TIME GAUGE"



1, 2, 3

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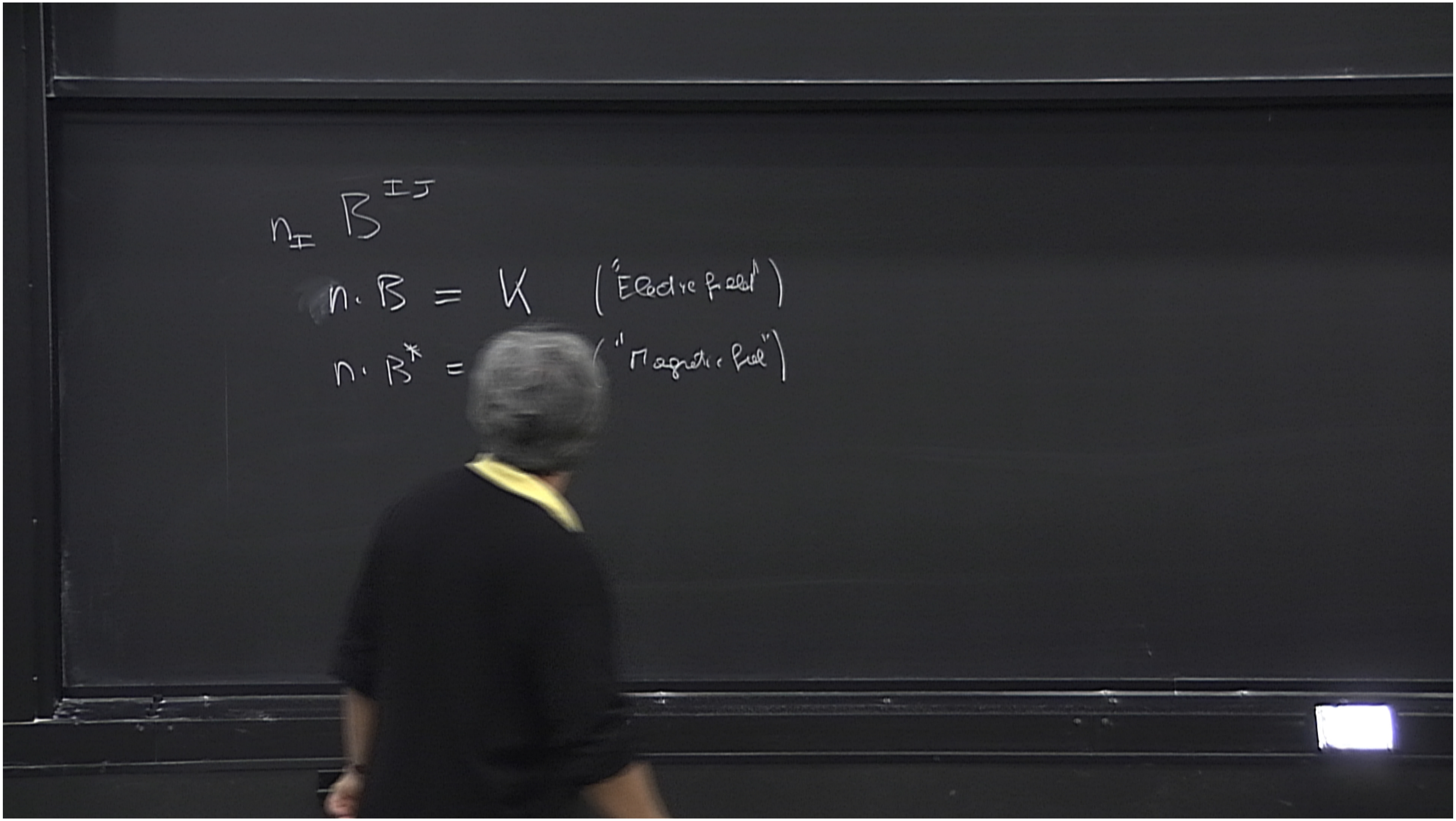
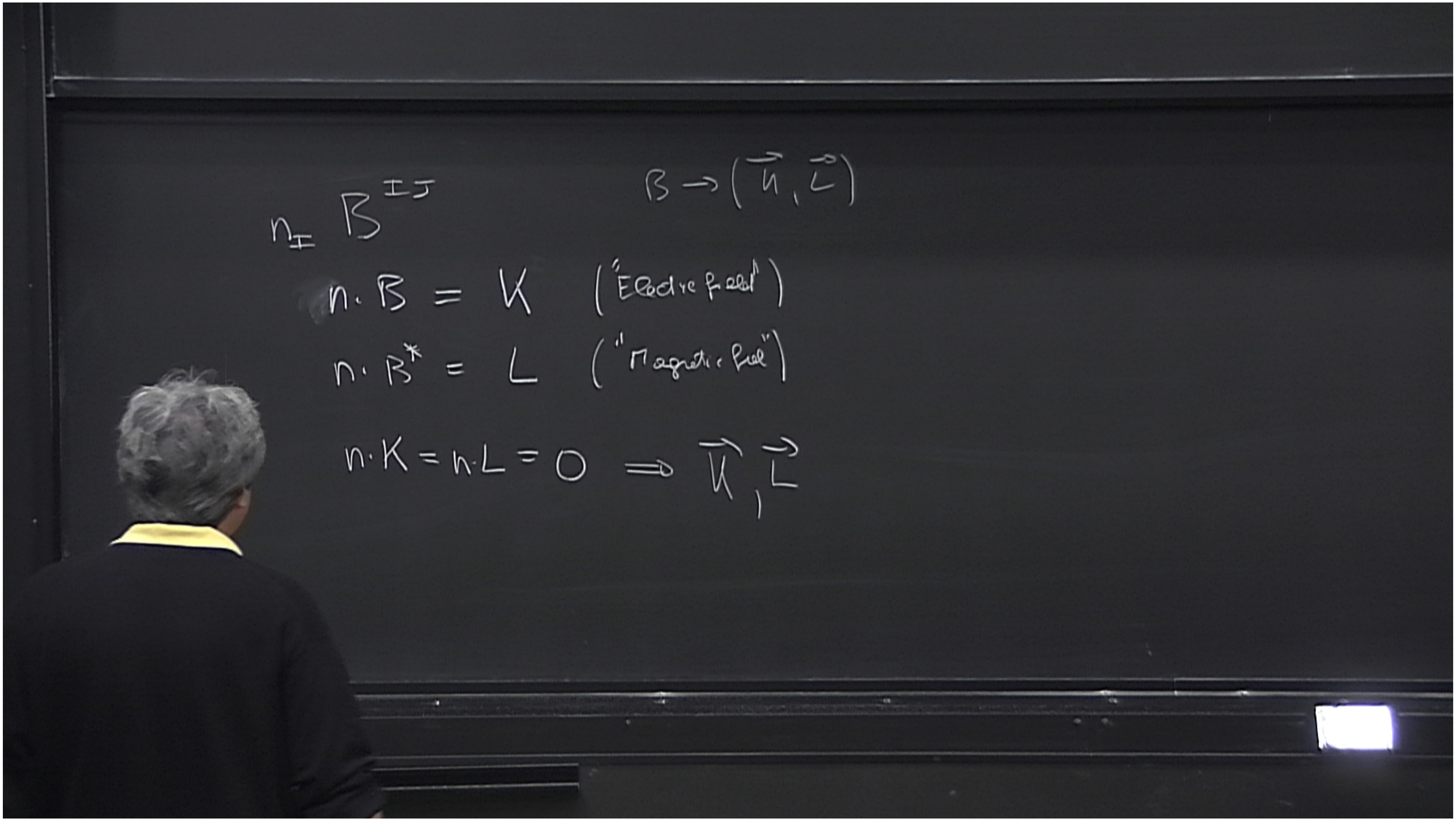


$n_I \, B^{IJ}$

$n \cdot B = K$ ("Electric field")

$n \cdot B^* =$ ("Magnetic field")



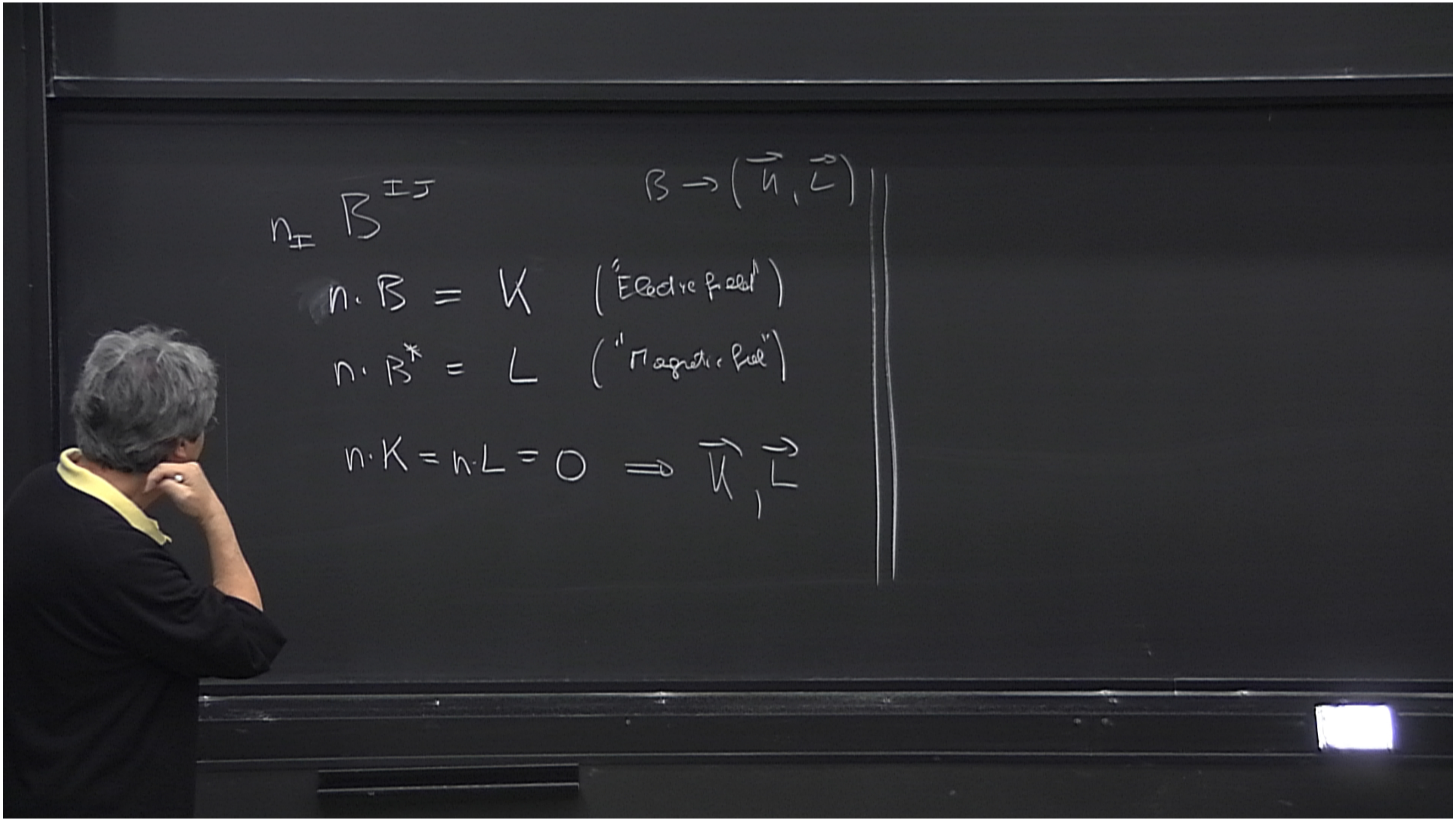
$B \to (\vec{K}, \vec{L})$

$n_I B^{IJ}$

$n \cdot B = K$ ("Electric field")

$n \cdot B^* = L$ ("Magnetic field")

$n \cdot K = n \cdot L = 0 \Rightarrow \vec{K}, \vec{L}$



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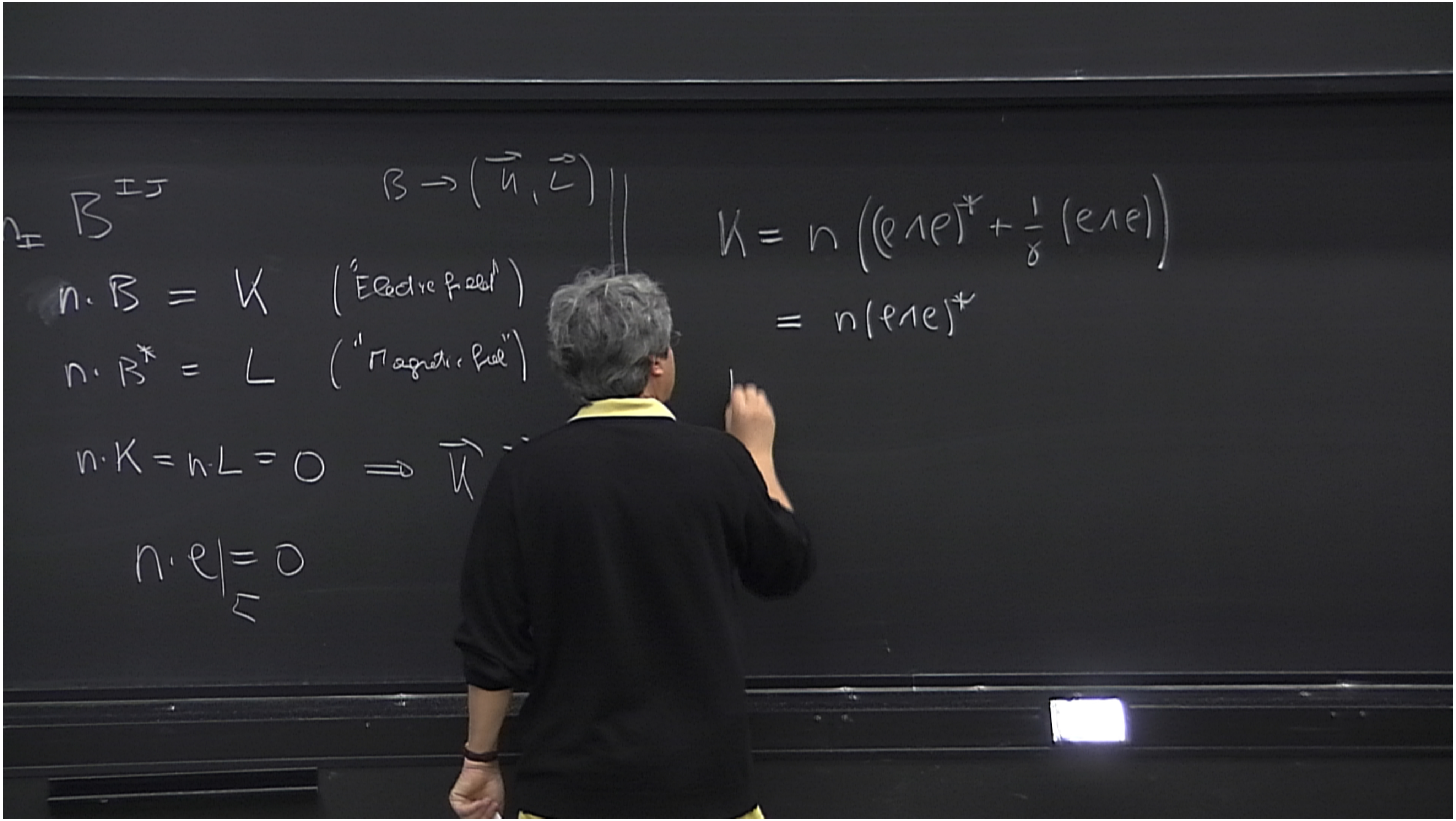

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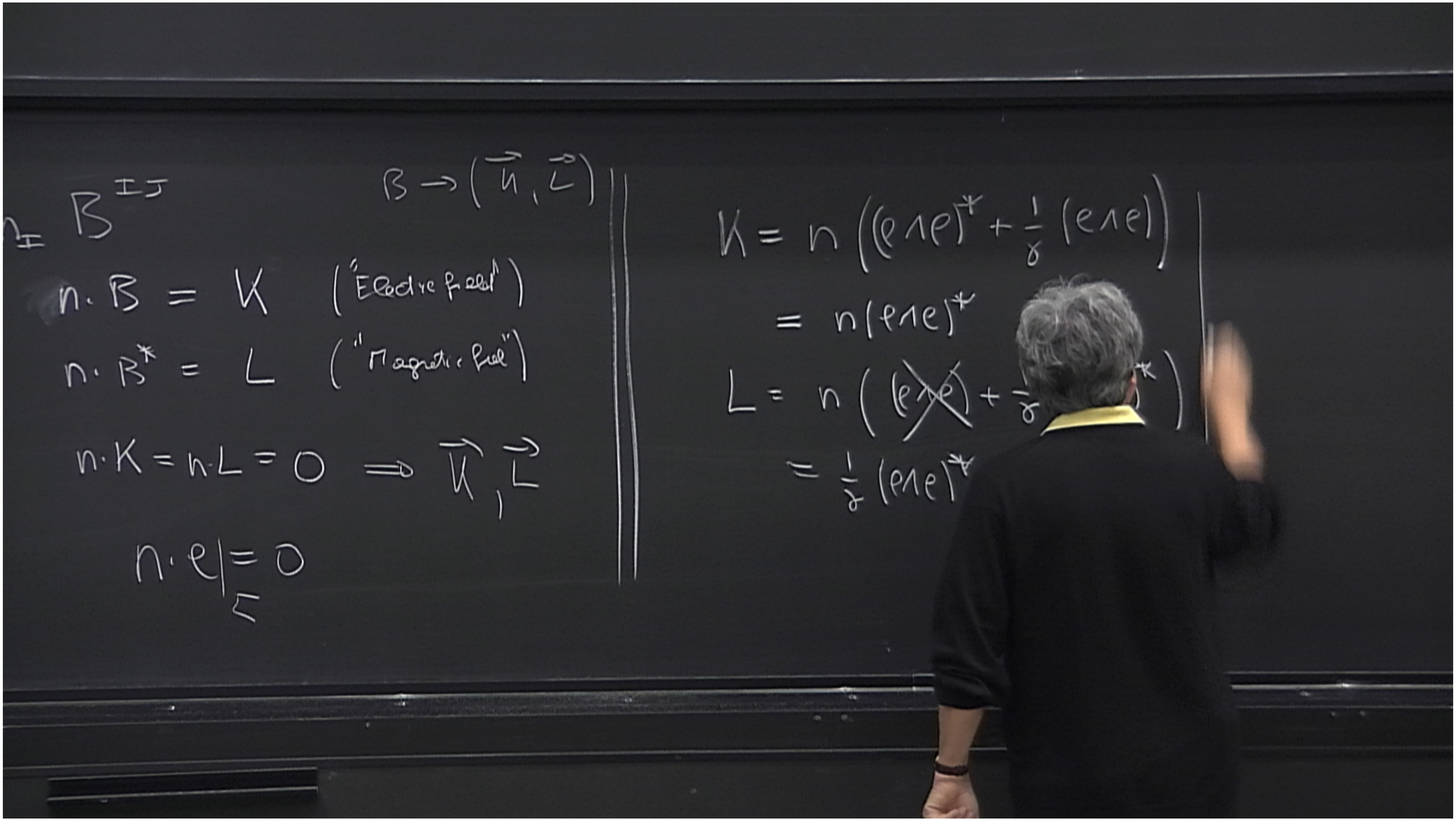
$n \cdot K = n \cdot L = 0 \Longrightarrow \vec{K}$

$n \cdot e|_\Sigma = 0$

$$K = n\left((e \wedge e)^* + \frac{1}{\gamma}(e \wedge e)\right)$$

$$= n(e \wedge e)^*$$



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$n\cdot e|_\Sigma = 0$

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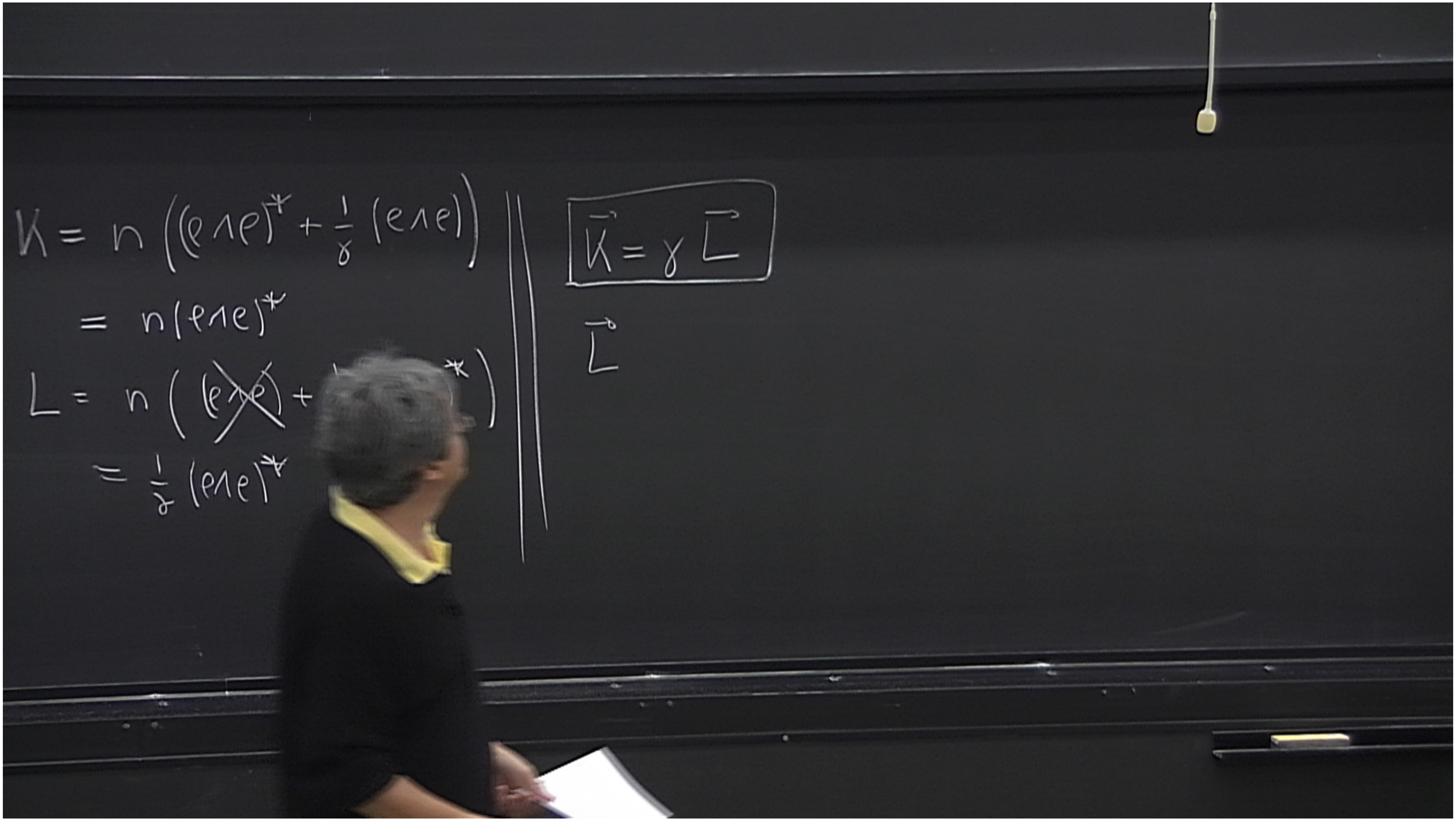
$$= n(e\wedge e)^*$$

$L = n\Big(\cancel{(e\wedge e)} + \frac{}{\gamma}$ [illegible] ${}^{*}\Big)$

$$= \frac{1}{\gamma}(e\wedge e)^*$$



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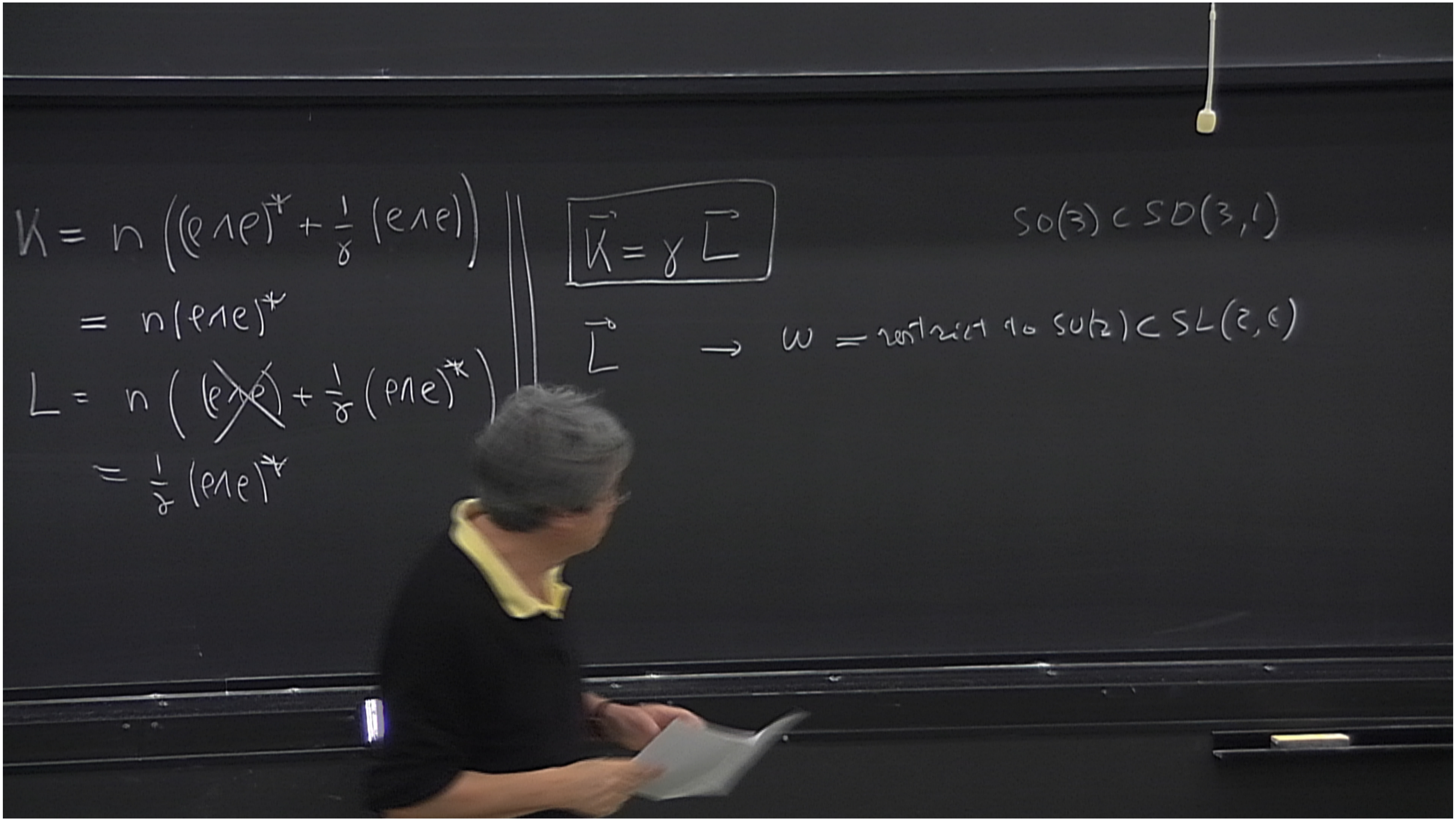
$L = n\Big(\cancel{(e\wedge e)} +$ [illegible] $^{*}\Big)$

$$= \frac{1}{\gamma}(e\wedge e)^*$$

$$\boxed{\vec{K} = \gamma \vec{L}}$$

$$\vec{L}$$



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$$= n(e\wedge e)^{*}$$

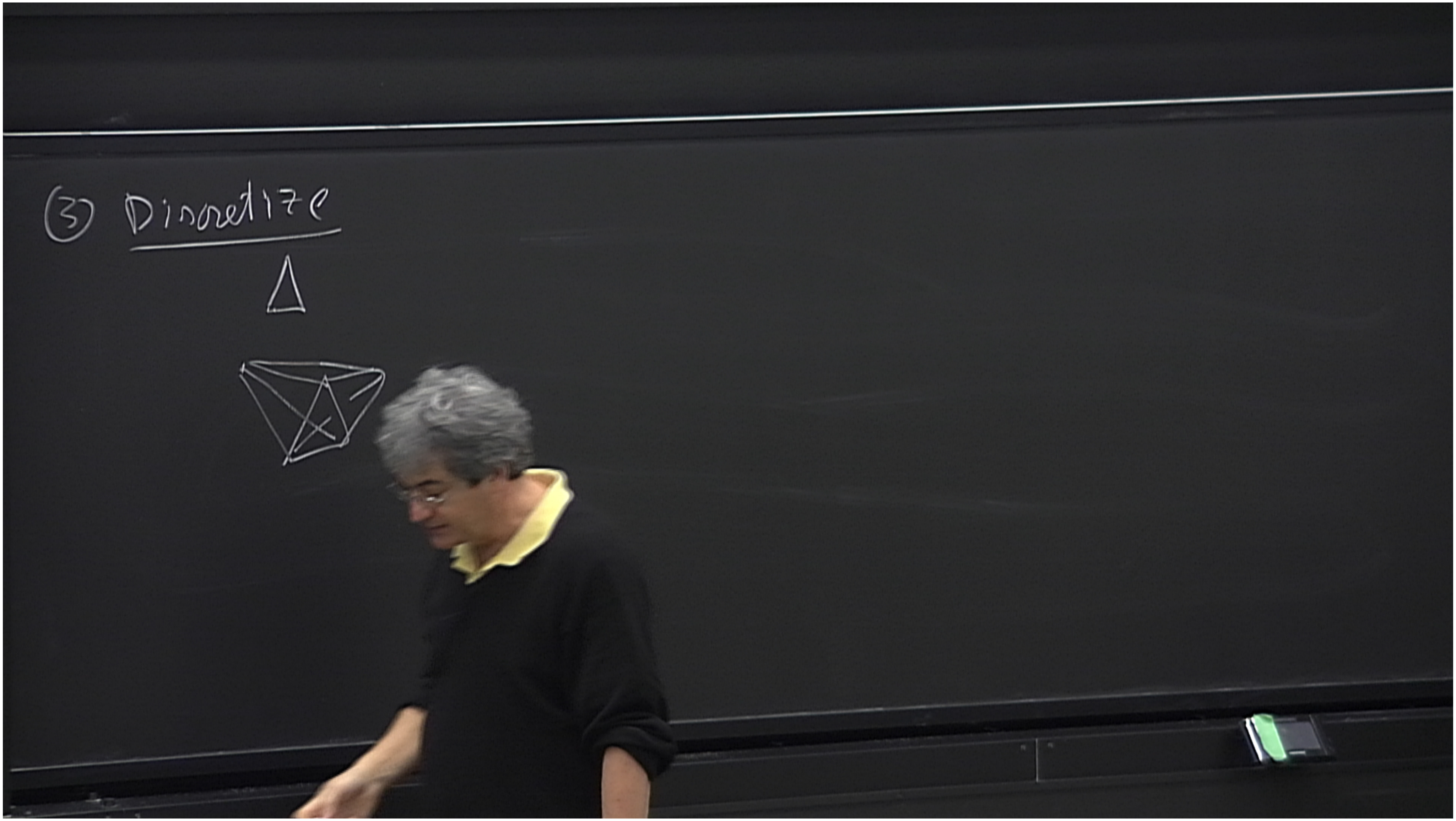
$$L = n\left(\cancel{(e\wedge e)} + \frac{1}{\gamma}(e\wedge e)^{*}\right)$$

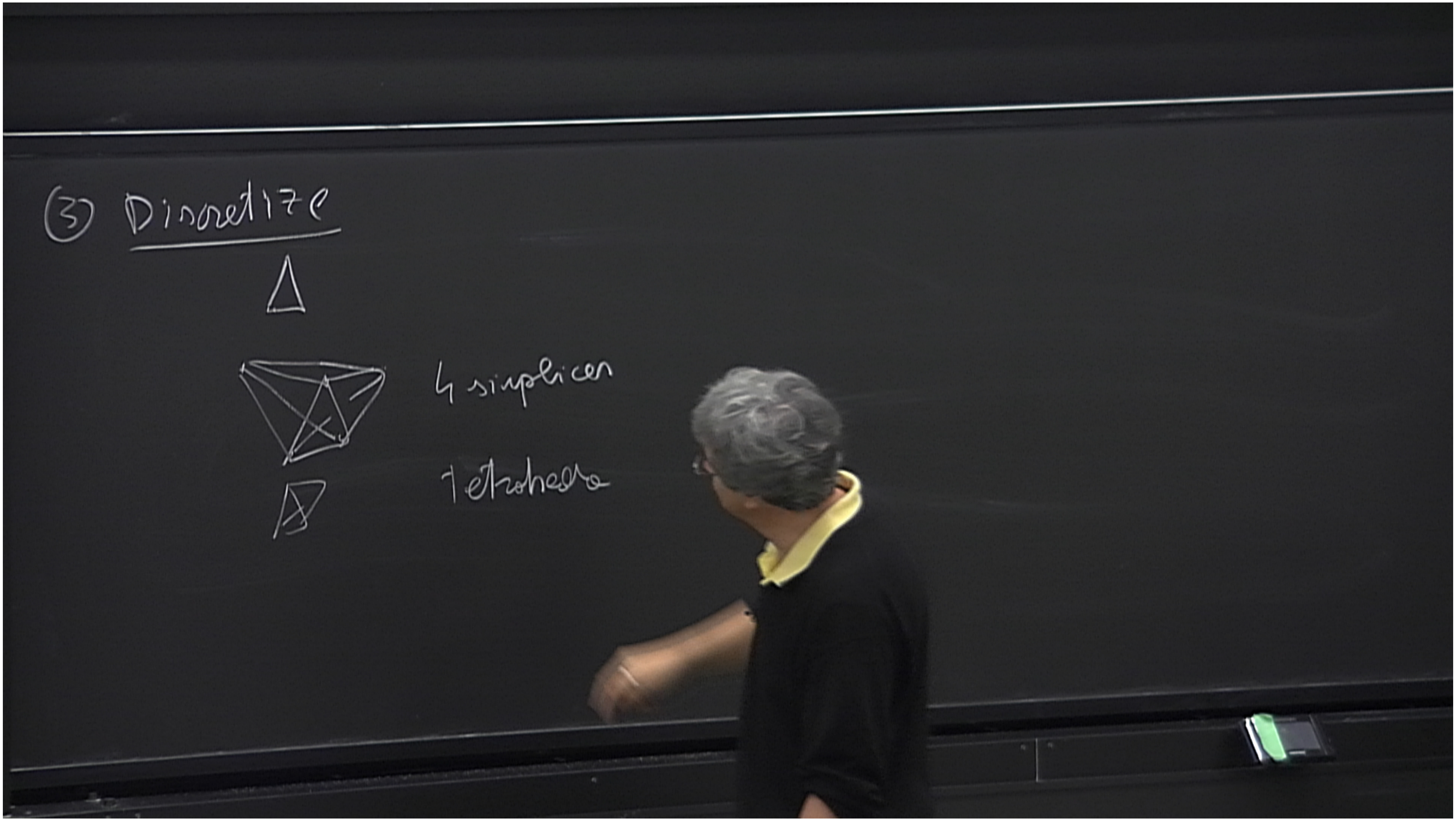
$$= \frac{1}{\gamma}(e\wedge e)^{*}$$

$$\boxed{\vec{K} = \gamma \vec{L}}$$

$$SO(3) \subset SO(3,1)$$

$\vec{L} \longrightarrow \omega =$ restrict to $SU(2) \subset SL(2,\mathbb{C})$

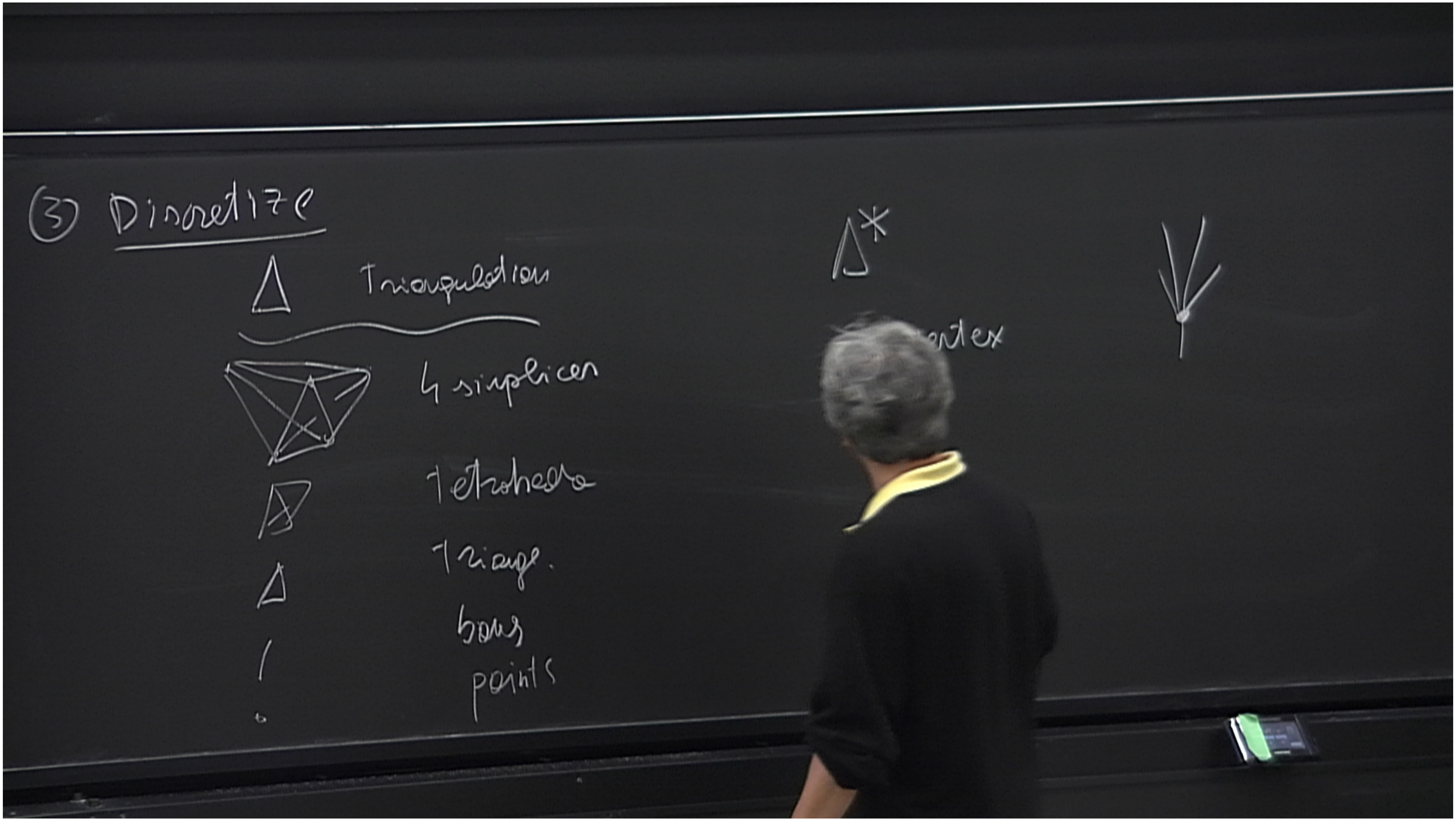

③ Discretize



③ Discretize

4 simplices

Tetrahedra



③ Discretize

Δ Triangulation

4 simplices

Tetrahedra

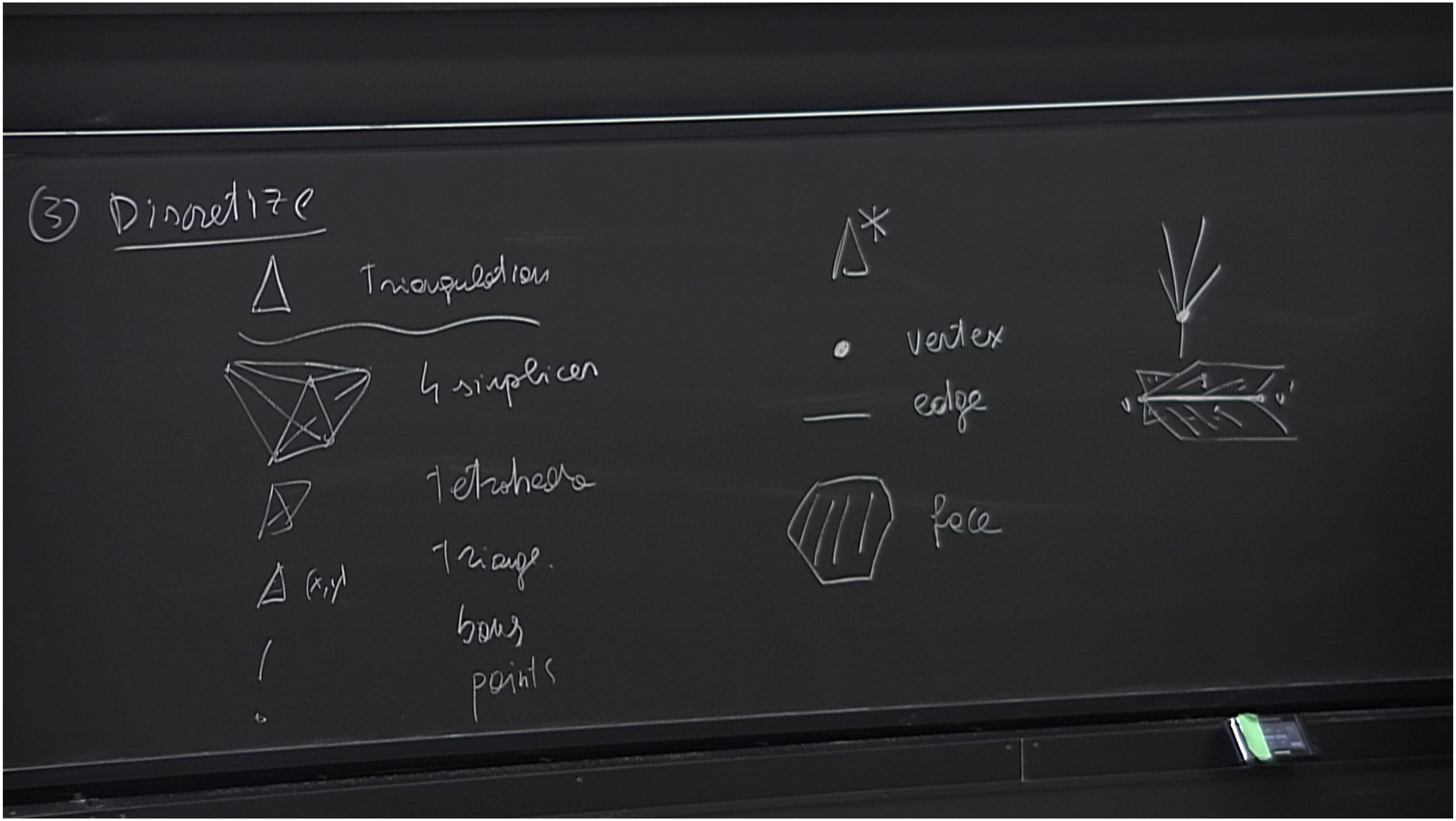
Triangl.

bons

points

Δ^*

ertex



③ Discretize

Δ Triangulation

4 simplices

Tetrahedra

Triangl. (x,y)

bones

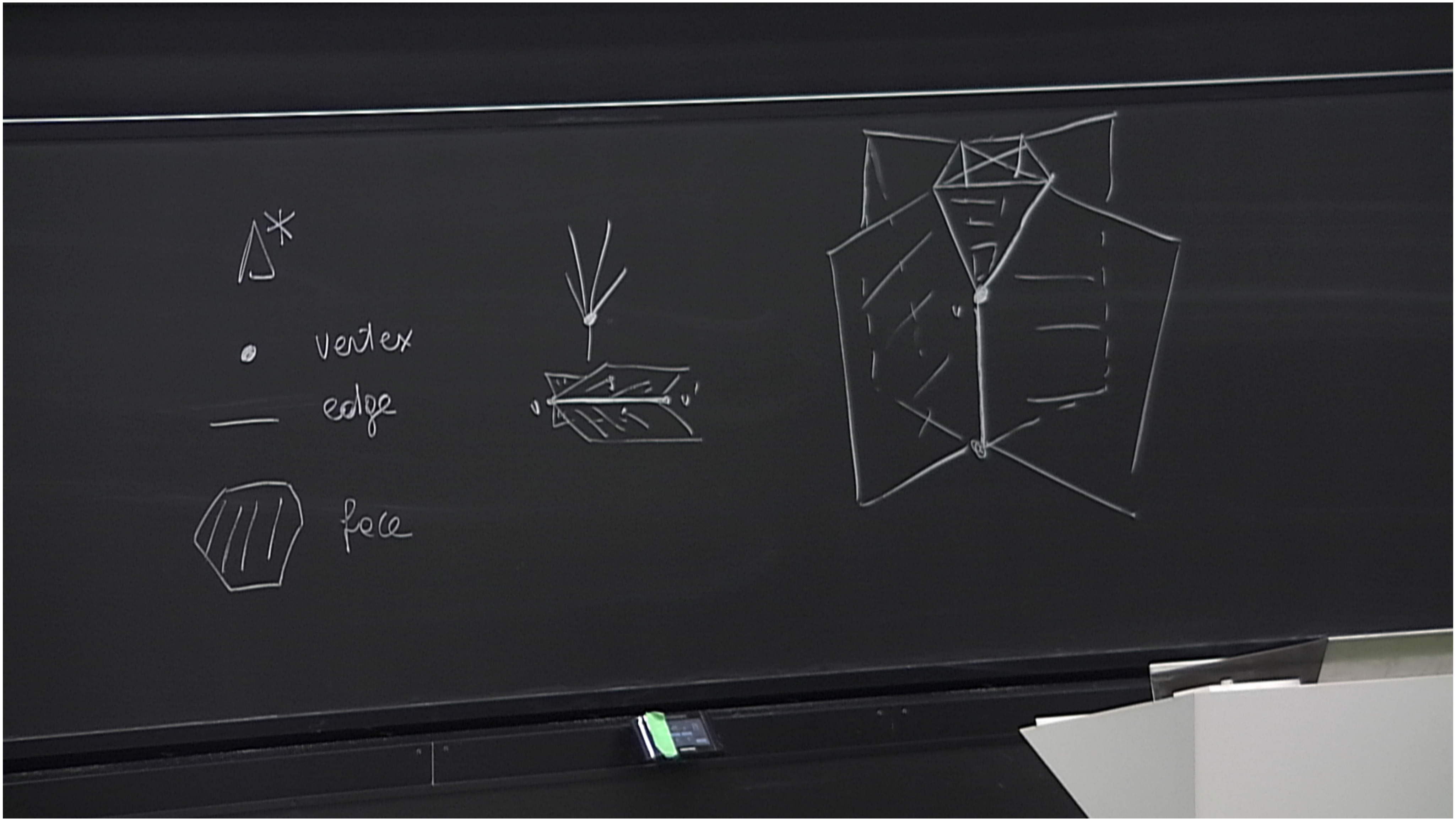
points

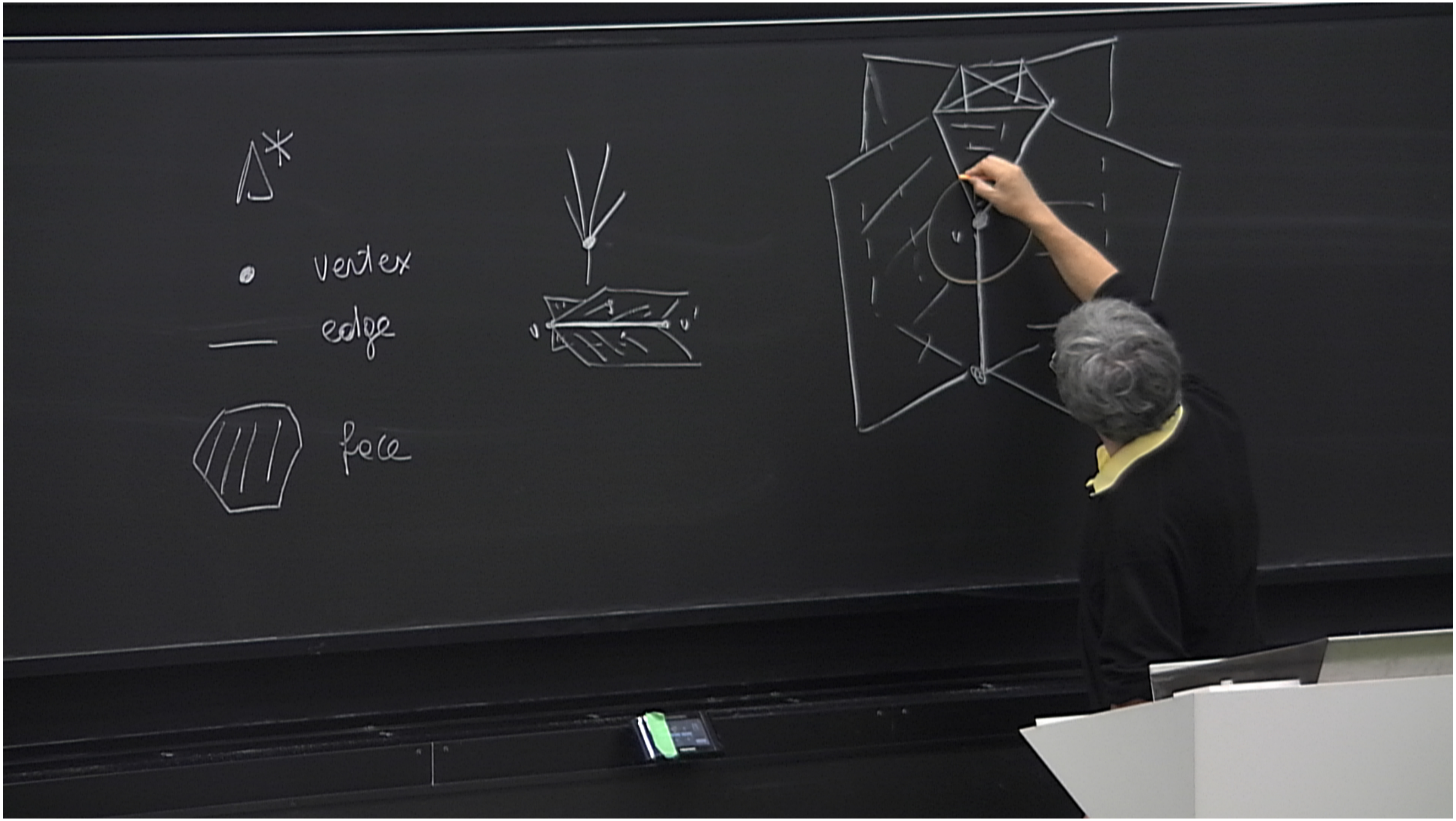
Δ^*

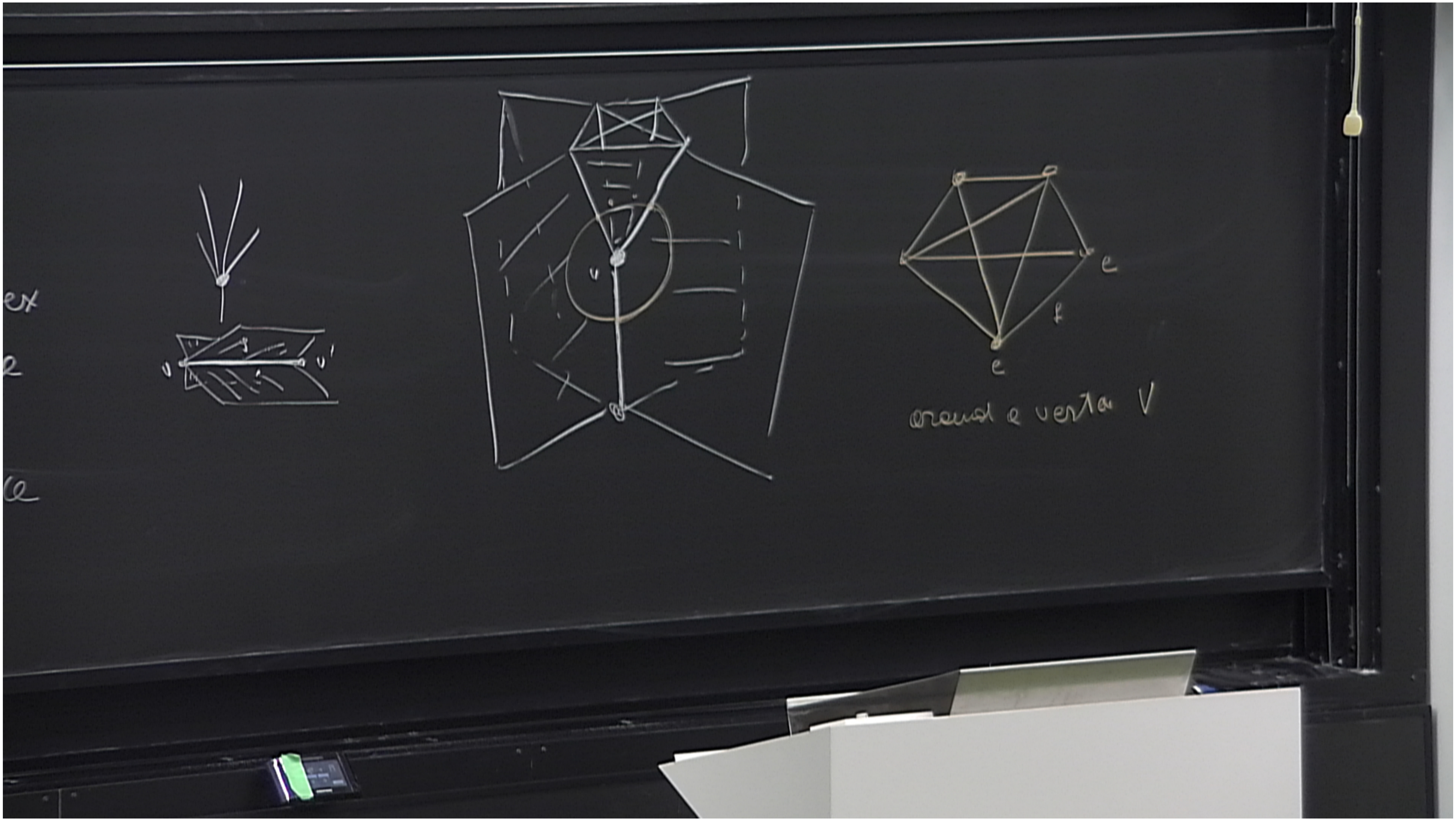
vertex

edge

face


Δ*
vertex
edge
face
v
v'


Δ^*
vertex
edge
face
v
v'
v

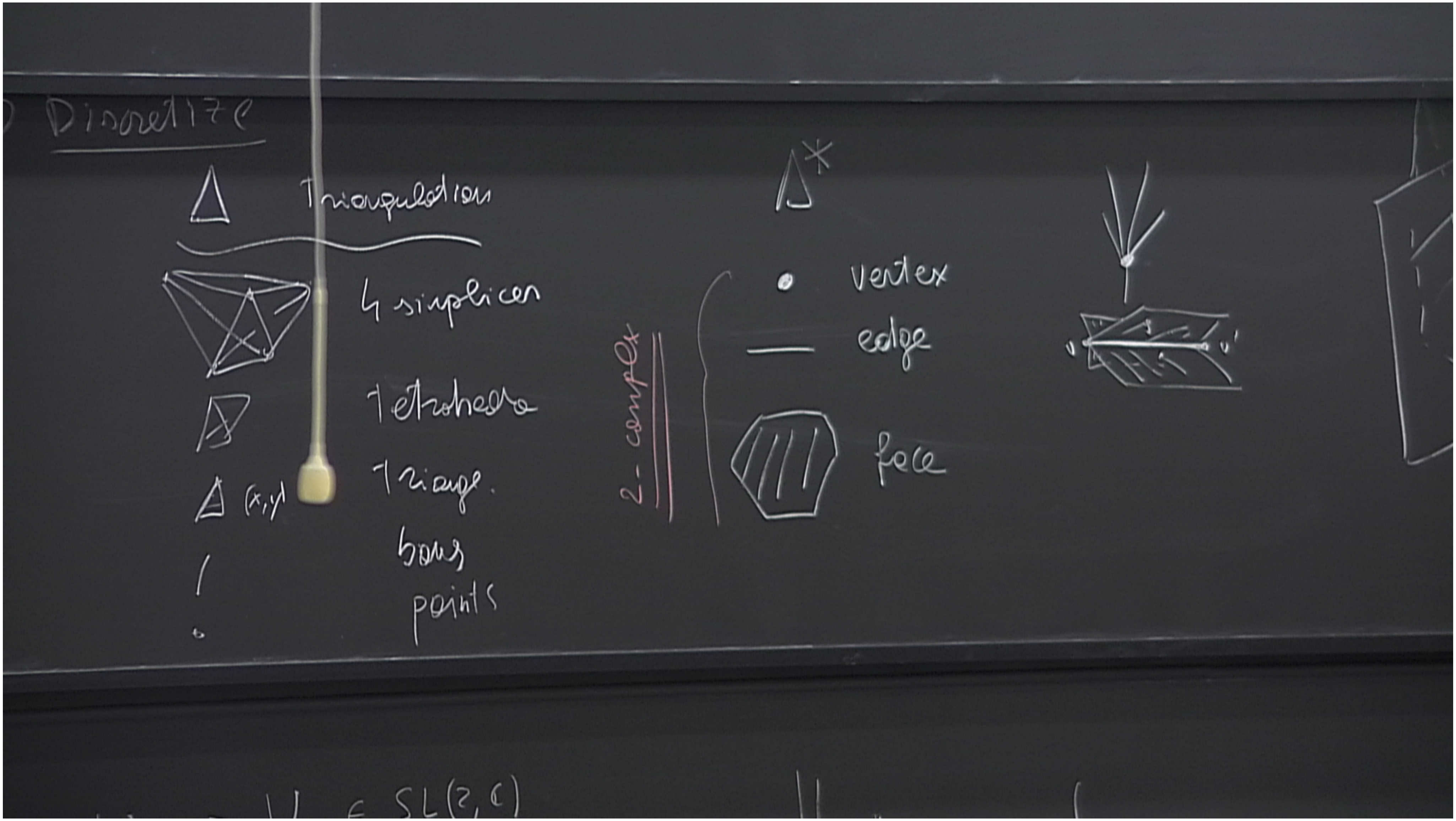


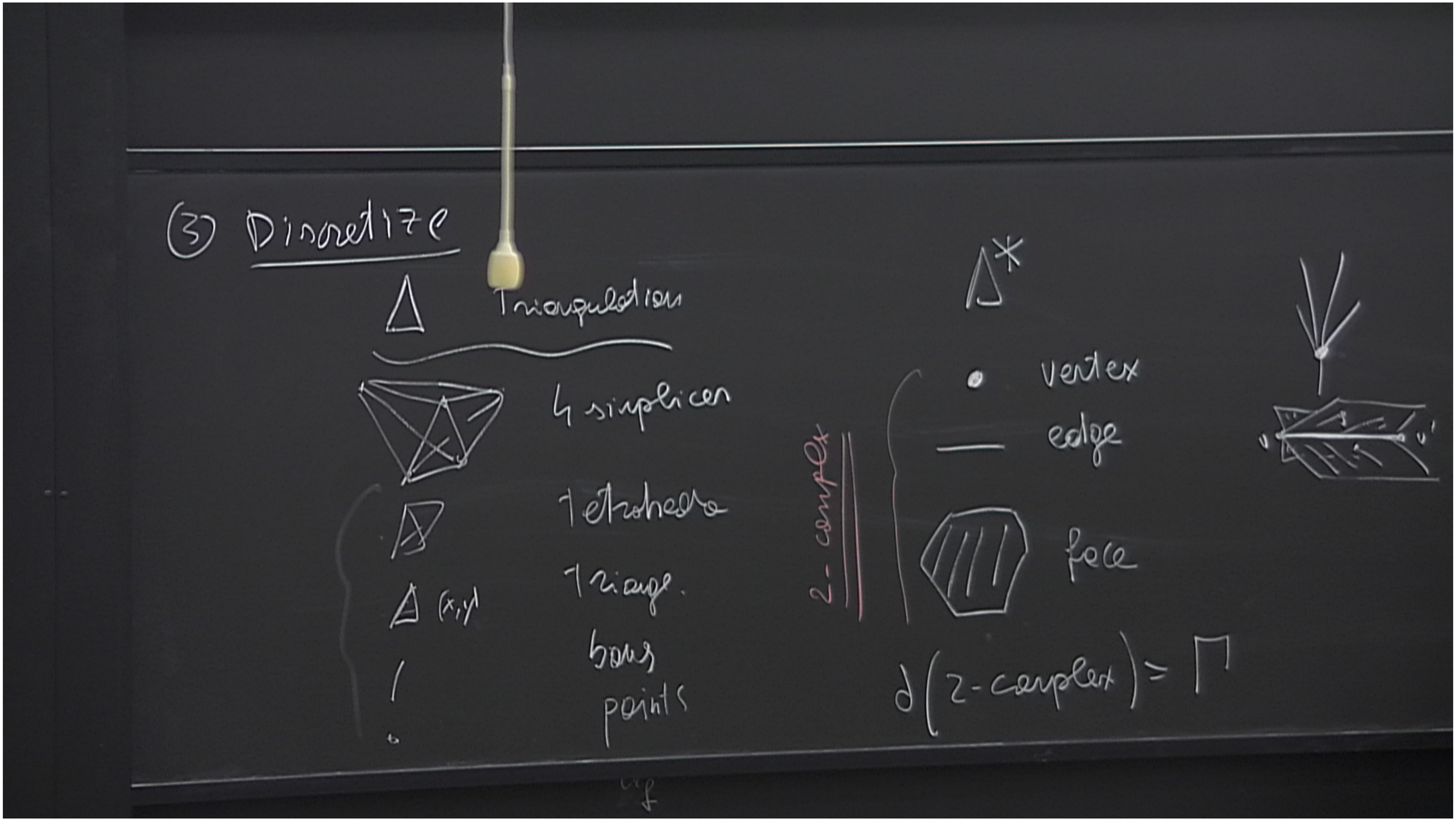
ex
e
ce
v
v'
v
e
f
e
around a vertex V

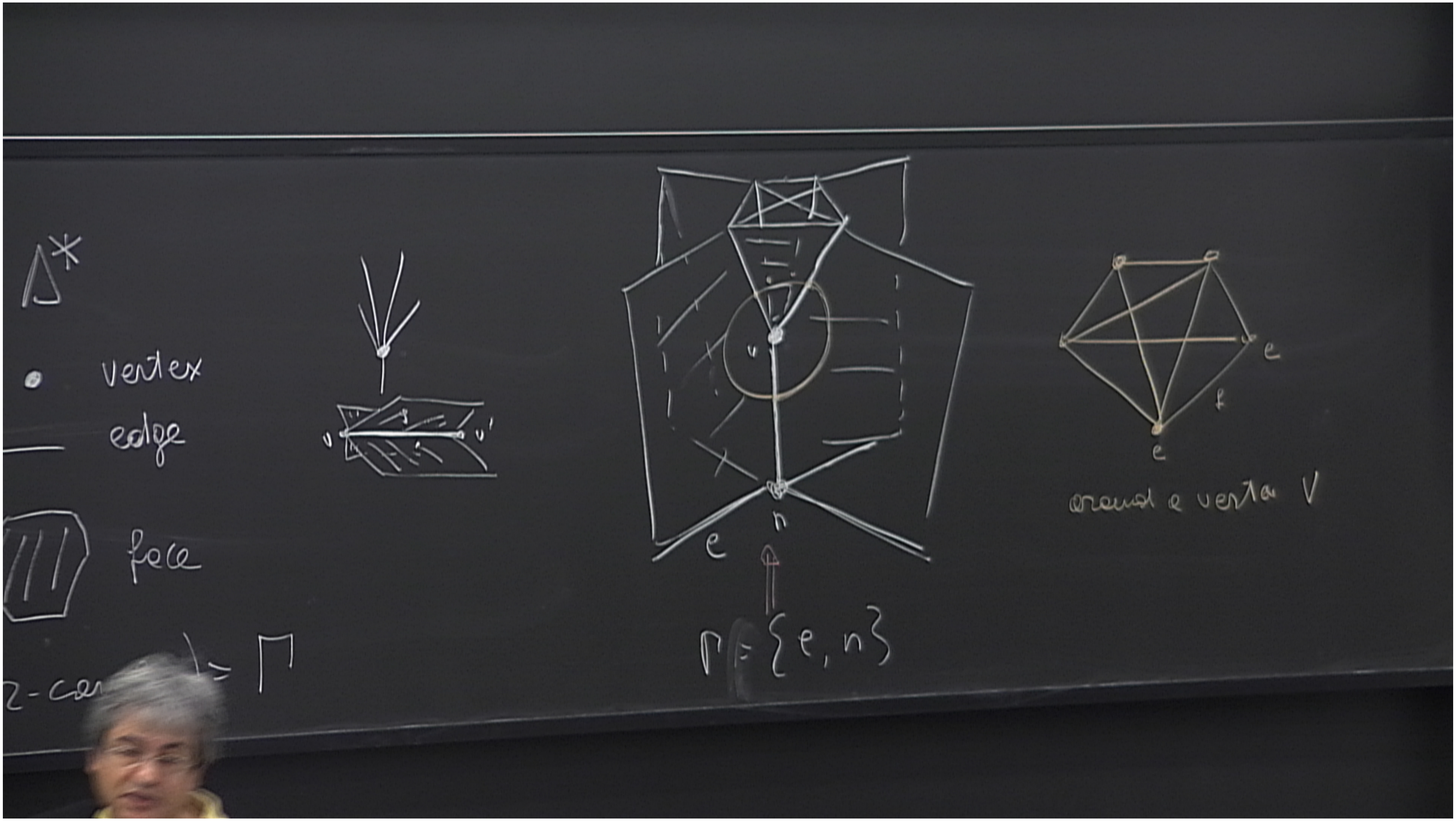
!

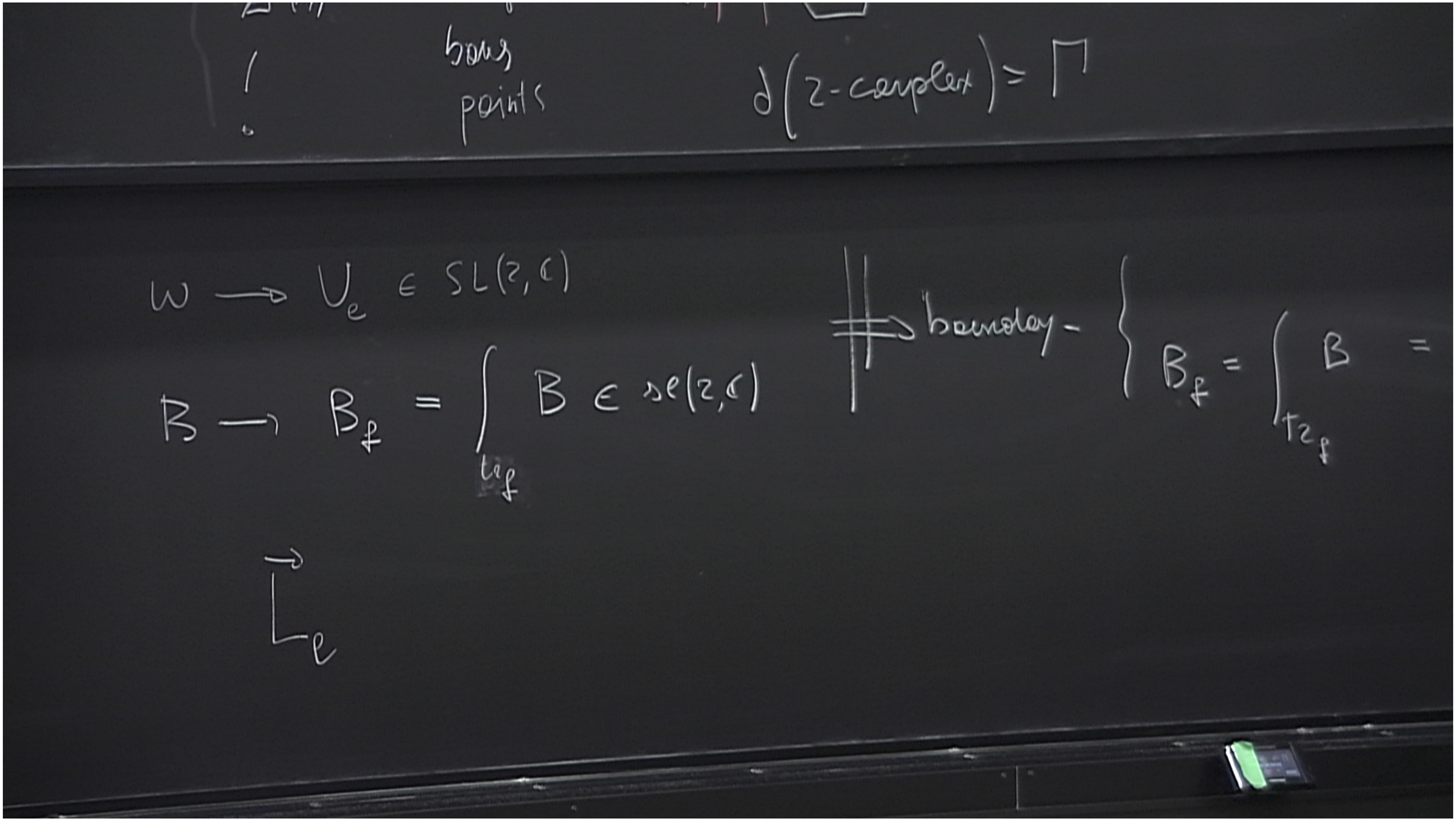
some
points

$$\omega \longrightarrow U_e \in SL(2,\mathbb{C})$$


Discretize
Triangulation
4 simplices
Tetrahedra
Triangle
(x,y)
bones
points
Δ*
vertex
edge
face
2-complex
v
v'
V ∈ SL(2,C)


③ Discretize
Δ Triangulation
4 simplices
Tetrahedra
Triangle.
(x,y)
bones
points
Δ*
vertex
edge
face
2-complex
∂(2-complex) = Γ
v
v'


Δ*
vertex
edge
face
2-ce… = Γ
v
v'
v
n
e
Γ = {e, n}
e
f
e
around a vertex V



bons points

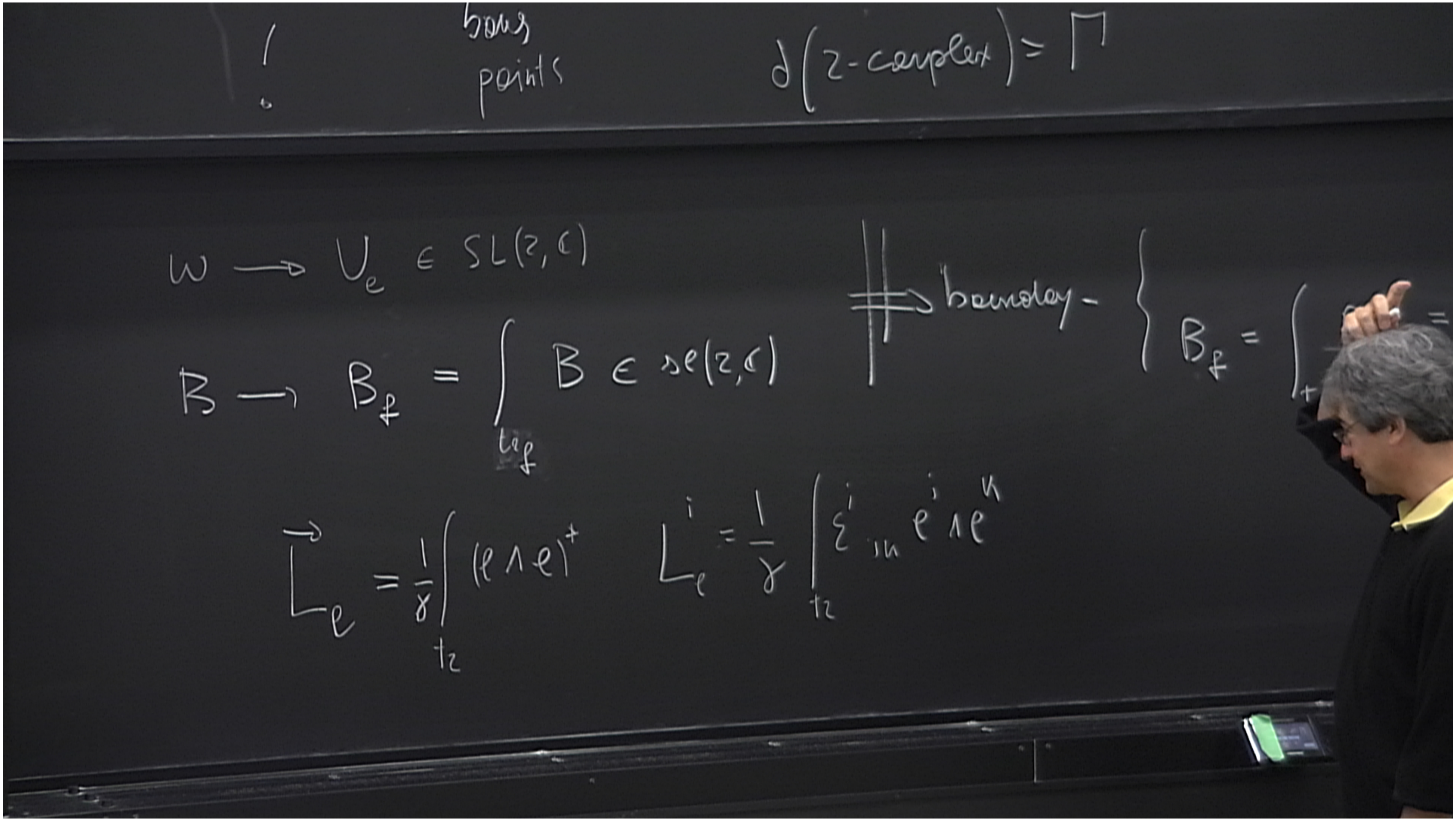
$\partial(\text{2-complex}) = \Gamma$

$\omega \longrightarrow U_e \in SL(2,\mathbb{C})$

$B \longrightarrow B_f = \int_{t_f} B \in sl(2,\mathbb{C})$

$\not\Longrightarrow$ boundary - $\Big\{ B_f = \int_{t2_f} B =$

$\vec{L}_e$



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bours points

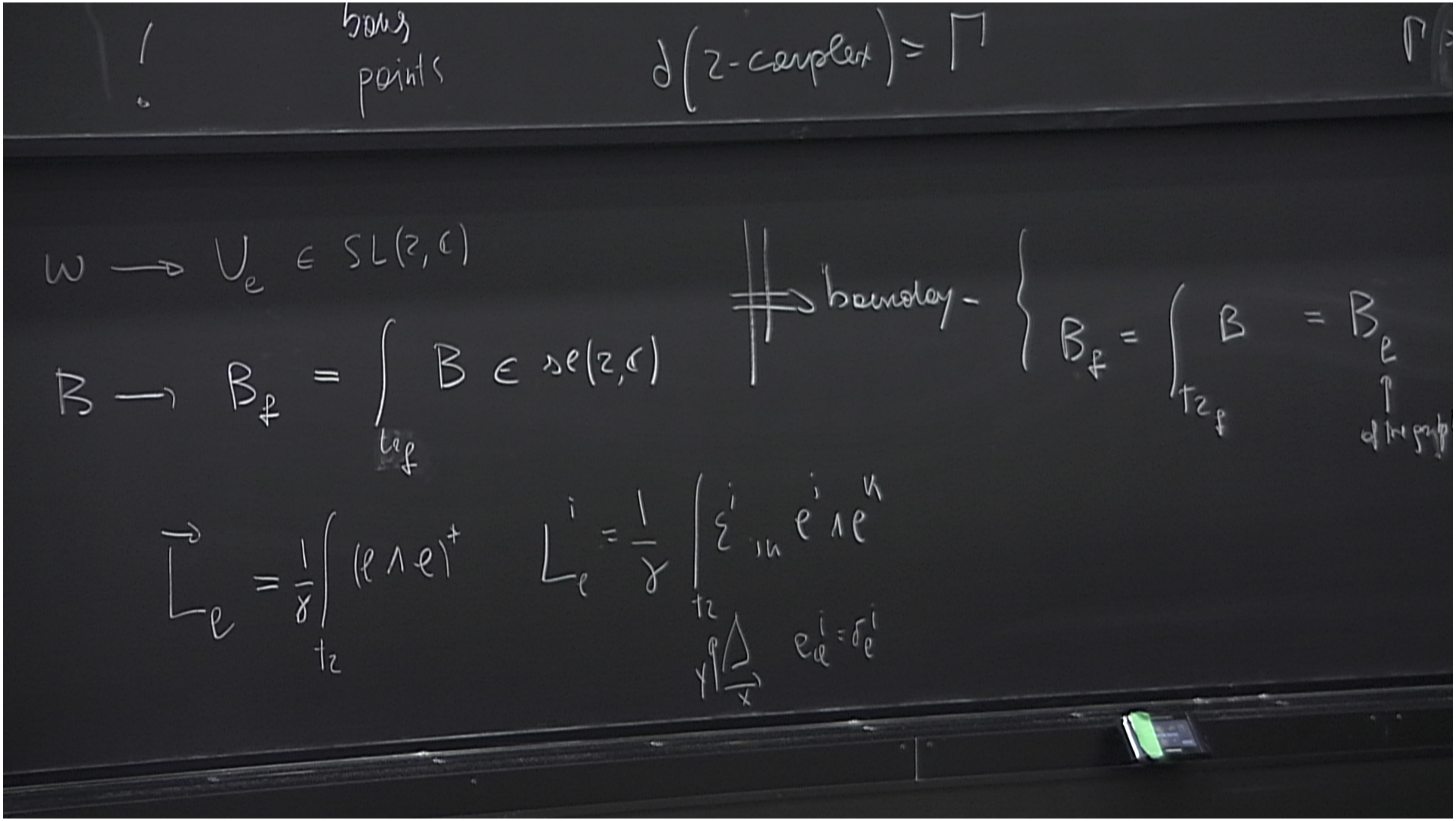
$\partial(\text{2-complex}) = \Gamma$

$\omega \longrightarrow U_e \in SL(2,\mathbb{C})$

$B \longrightarrow B_f = \int_{t_f} B \in sl(2,\mathbb{C})$

$\not\Longrightarrow$ boundary - $\left\{ B_f = \int_t \right.$

$\vec{L}_e = \frac{1}{\gamma}\int_{t_2} (e \wedge e)^+ \quad L^i_e = \frac{1}{\gamma}\int_{t_2} \epsilon^i{}_{jk}\, e^j \wedge e^k$



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bons points

$\partial(2\text{-complex}) = \Gamma$

Γ

$\omega \longrightarrow U_e \in SL(2,\mathbb{C})$

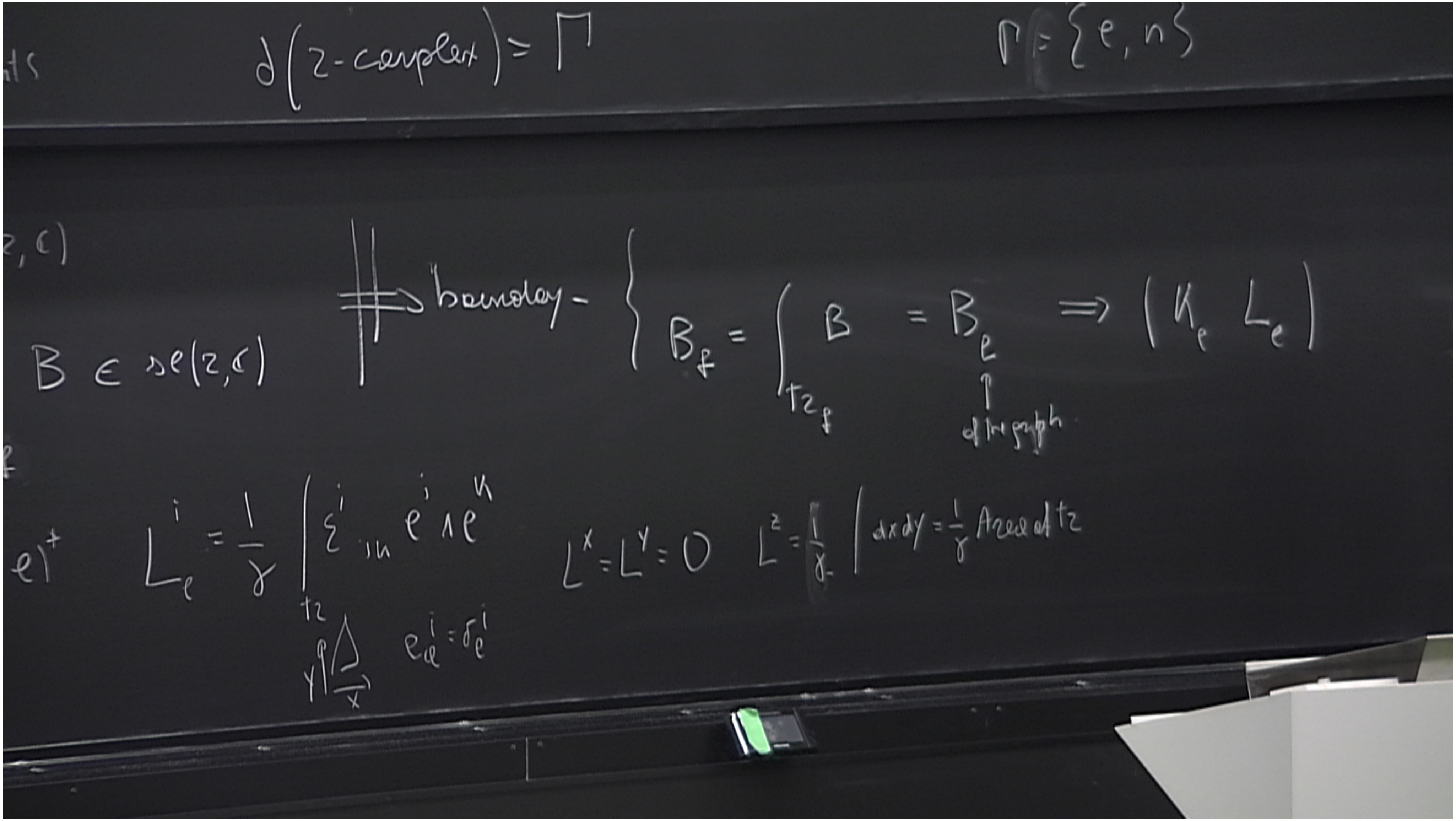
$B \longrightarrow B_f = \int_{t_{\ell f}} B \in sl(2,\mathbb{C})$

$\not\Rightarrow$ boundary - $\left\{ B_f = \int_{t_{2f}} B = B_\ell \right.$

$\uparrow$ of the graph

$\vec{L}_\ell = \frac{1}{\gamma}\int_{t_2} (e \wedge e)^+ \qquad L^i_\ell = \frac{1}{\gamma}\int_{t_2} \varepsilon^i{}_{jk}\, e^j \wedge e^k$

$e^i_\ell = \delta^i_\ell$



nts $\partial(2\text{-complex}) = \Gamma$ $\Gamma = \{e, n\}$

$2, \mathbb{C})$

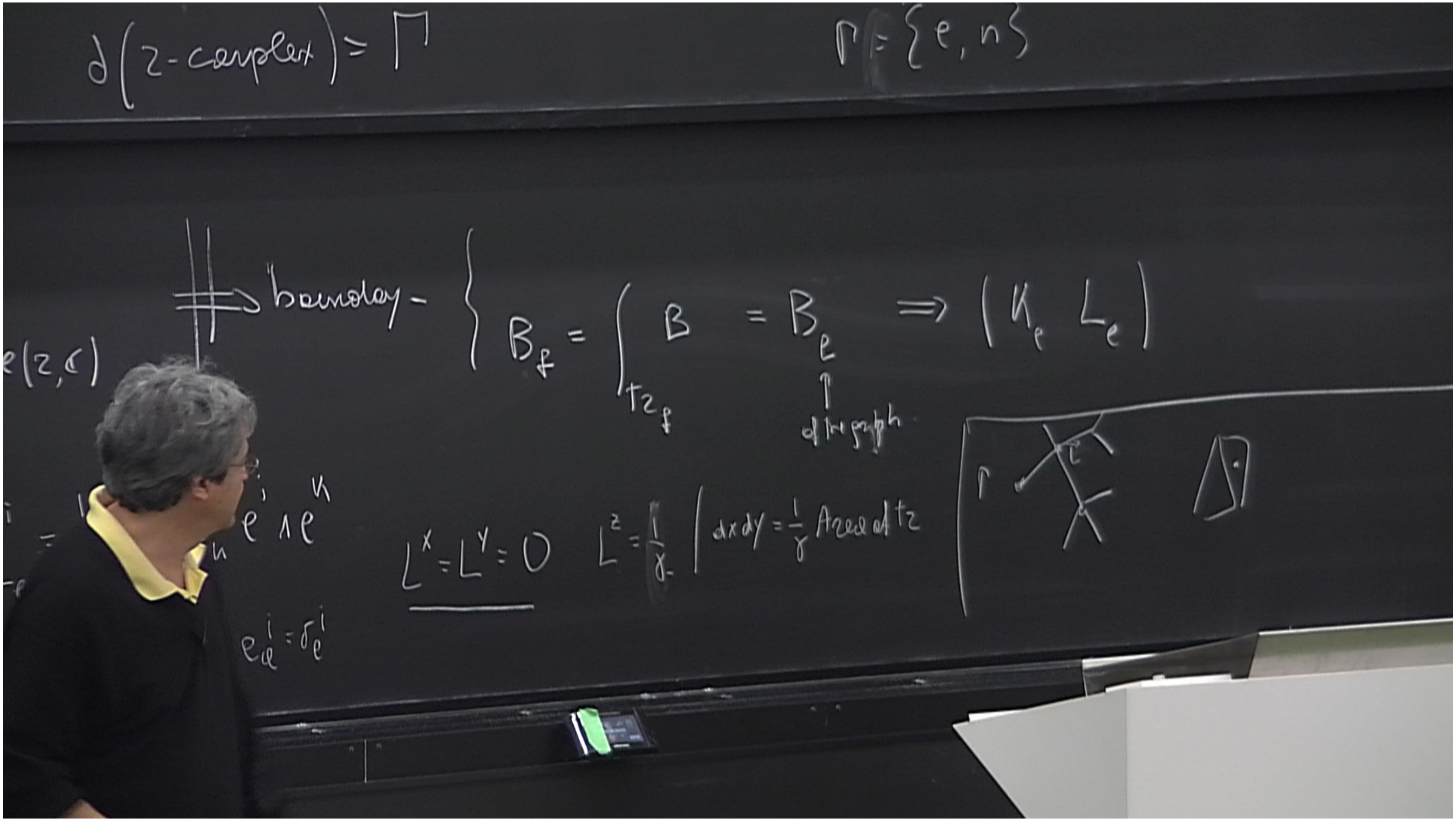
$\#\!\Rightarrow$ boundary – $\Big\{$ $B_f = \int_{t2_f} B = \underset{\substack{\uparrow \\ \text{of the graph}}}{B_e} \Longrightarrow (K_e \;\; L_e)$

$B \in sl(2,\mathbb{C})$

f

$e)^+$ $L^i_e = \frac{1}{\gamma}\int_{t2} \varepsilon^i{}_{jk}\, e^j \wedge e^k$ $L^x = L^y = 0$ $L^z = \frac{1}{\gamma}\int dx\,dy = \frac{1}{\gamma}$ Area of t2

$e^i_e = \sigma^i_e$


∂(2-complex) = Γ
Γ = {e, n}
# boundary -
B_f = ∫_{t2_f} B = B_e
of the graph
⇒ (K_e L_e)
e(2,σ)
e^j ∧ e^k
L^x = L^y = 0
L^z = 1/γ ∫ dx dy = 1/γ Area of t2
e_e^i = σ_e^i
Γ
e

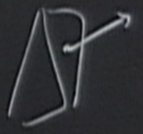
$\text{plex}) = \Gamma$ $\qquad \Gamma = \{e, n\}$

$\to$ boundary - $\left\{ \begin{array}{l} U_e \in SU(2) \\ B_{\underset{e}{f}} = \int_{t_{2_f}} B = \underset{\substack{\uparrow \\ \text{of the graph}}}{B_e} \Longrightarrow (K_e \ L_e) \end{array} \right.$

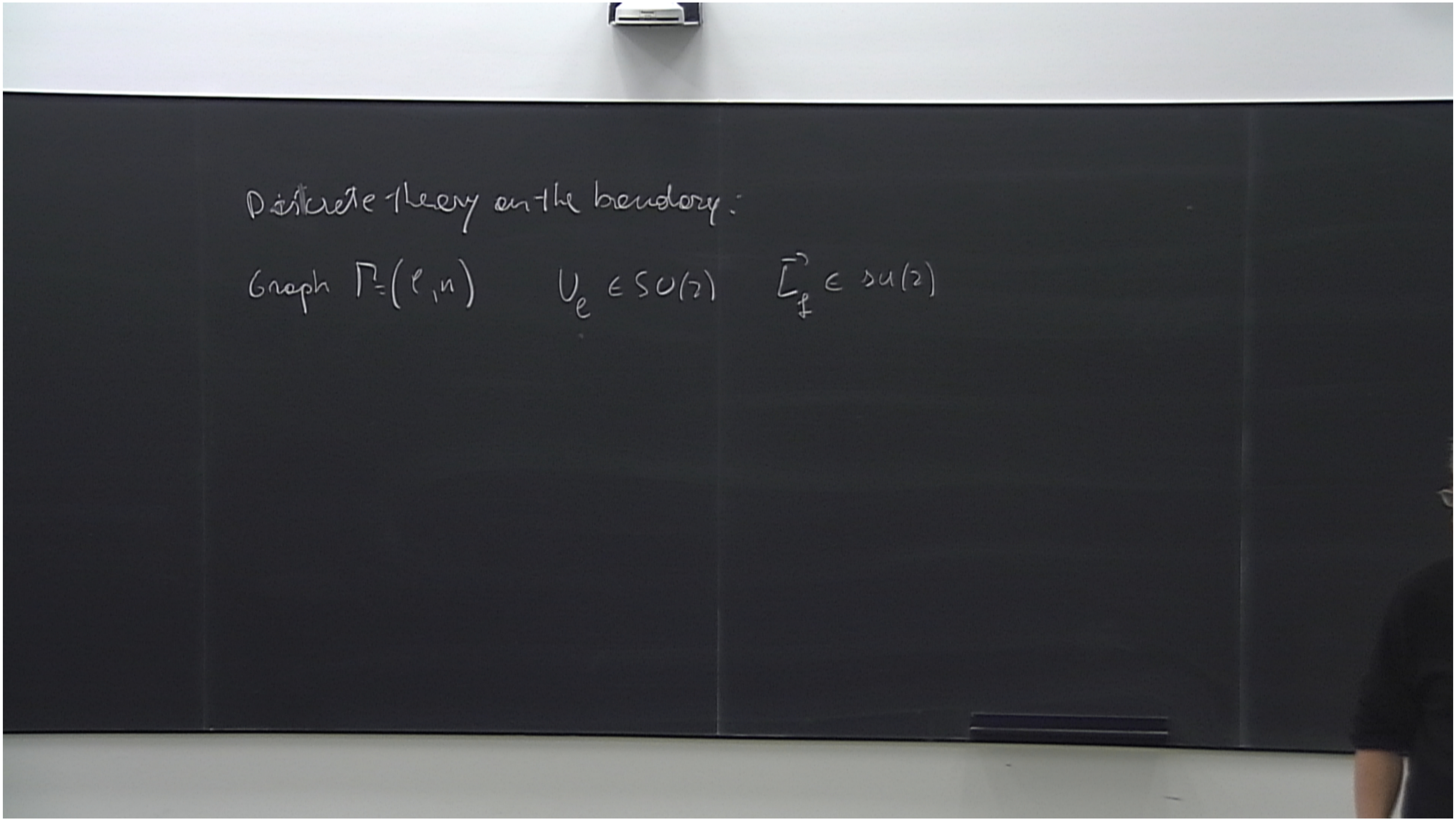
$e^i \wedge e^k$

$\underline{L^x = L^y = 0} \quad L^z = \frac{1}{\gamma} \int dx\,dy = \frac{1}{\gamma}$ Area of t_2

$e_e^i = \delta_e^i$

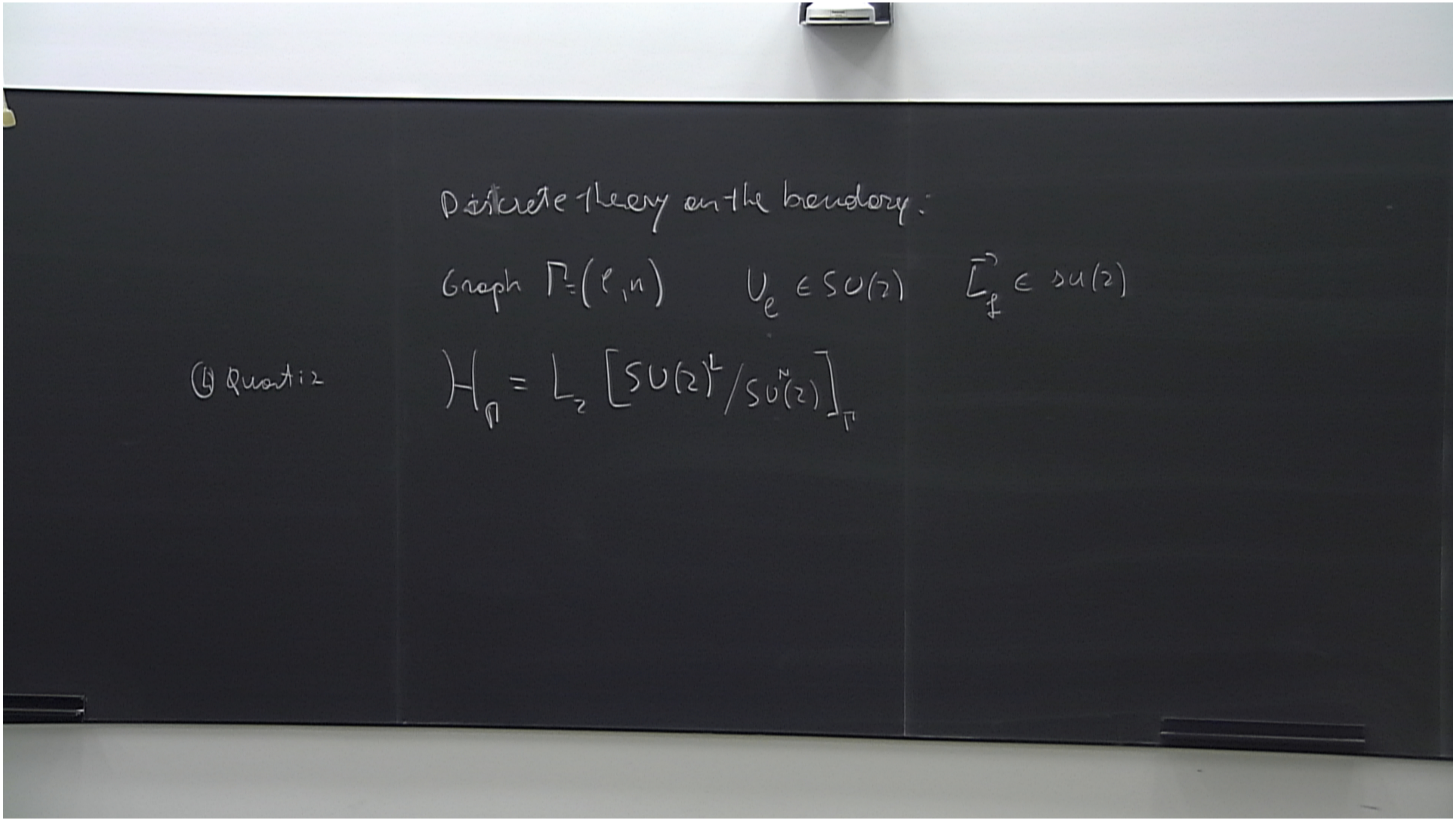


$|\vec{L}| = \frac{1}{\gamma}$ Area of the triangle



Discrete theory on the boundary:

Graph $\Gamma = (e, n)$ $\quad U_e \in SU(2) \quad \vec{L}_f \in su(2)$

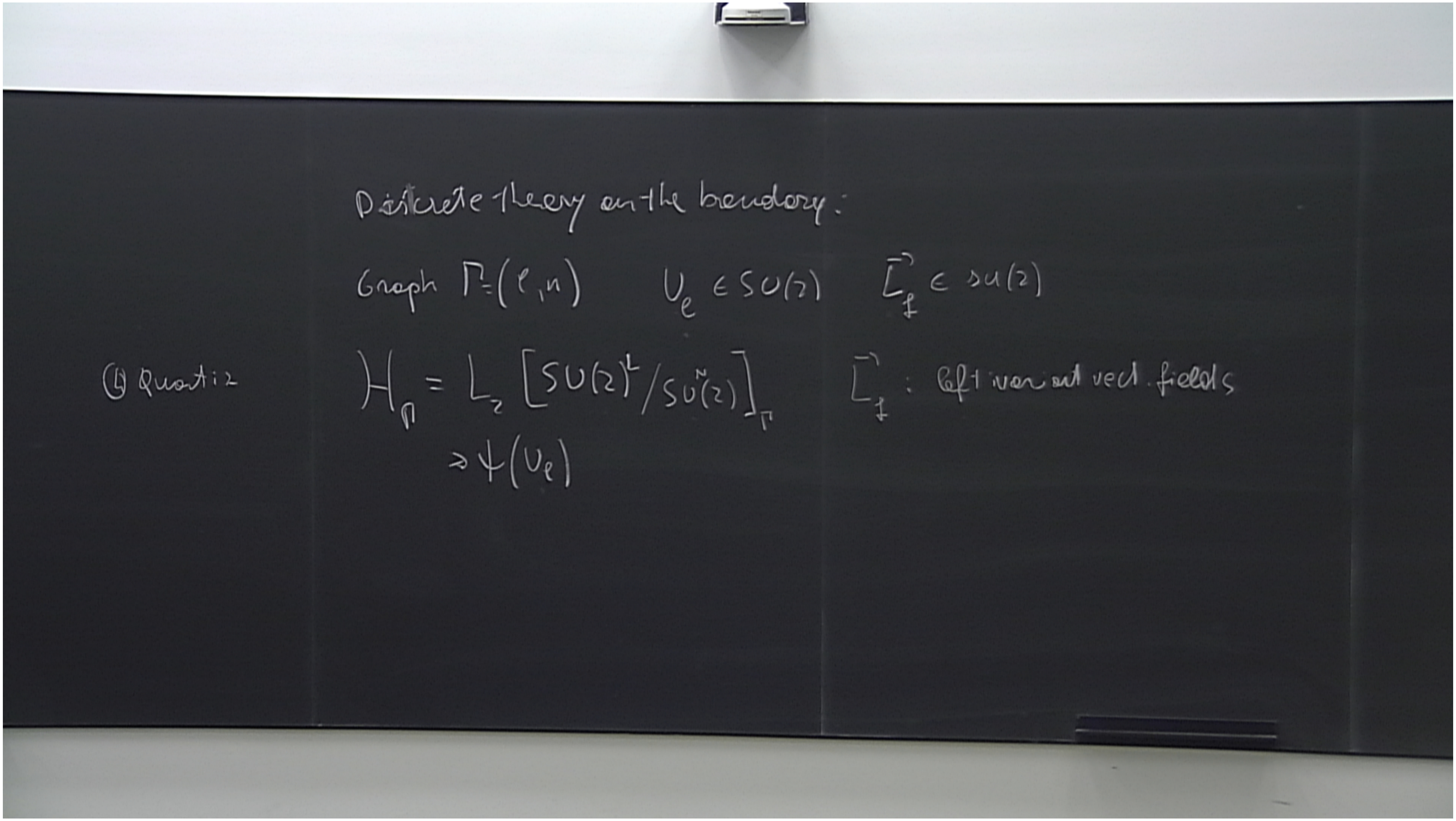


Discrete theory on the boundary:

Graph $\Gamma = (\ell, n)$ $\quad U_\ell \in SU(2)$ $\quad \vec{L}_\ell \in su(2)$

① Quantiz

$$\mathcal{H}_\Gamma = L_2\left[SU(2)^L / SU^N(2)\right]_\Gamma$$



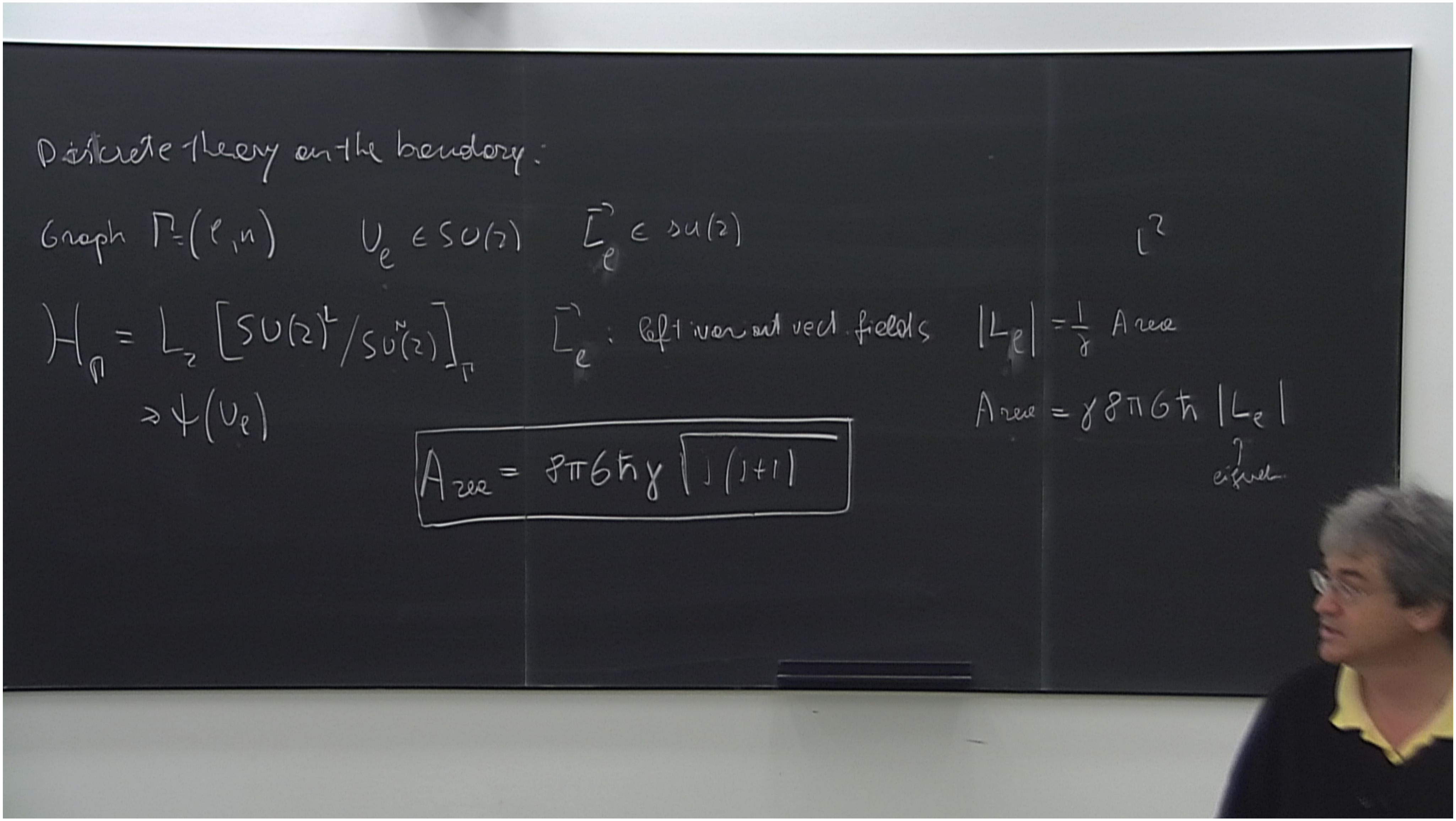
Discrete theory on the boundary:

Graph $\Gamma = (\ell, n)$ $U_\ell \in SU(2)$ $\vec{L}_f \in su(2)$

① Quantiz

$$\mathcal{H}_\Gamma = L_2\left[SU(2)^L / SU^N(2)\right]_\Gamma \ni \psi(U_\ell)$$

$\vec{L}_f$: left invariant vect. fields



Discrete theory on the boundary:

Graph $\Gamma=(\ell,n)$ $\quad U_e \in SU(2)$ $\quad \vec{L}_e \in su(2)$ $\quad L^2$

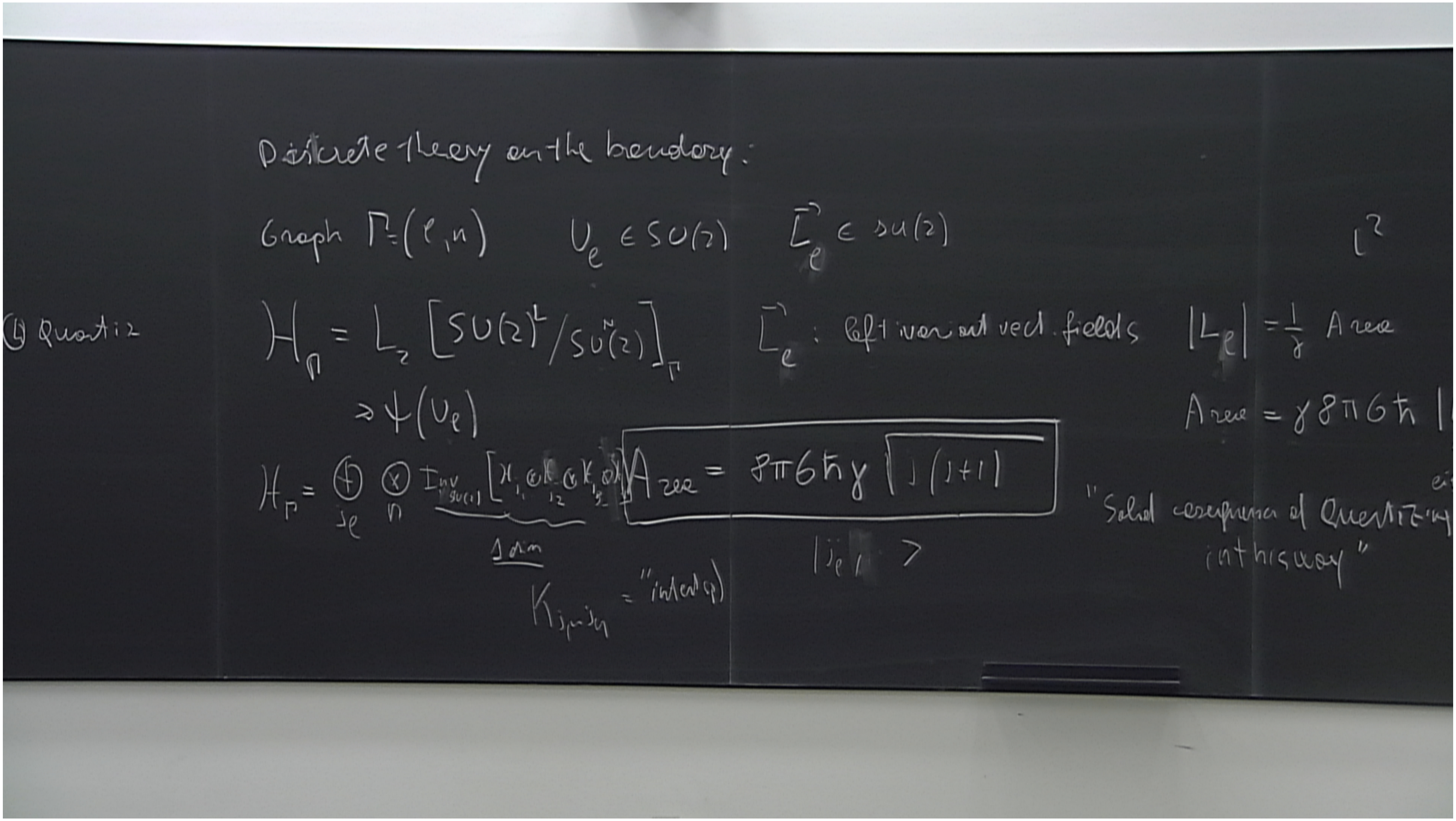
$\mathcal{H}_\Gamma = L_2\left[SU(2)^L/SU^N(2)\right]_\Gamma$ $\quad \vec{L}_e$: left invariant vect. fields $\quad |L_e| = \frac{1}{\gamma}\,$Area

$\ni \psi(U_\ell)$

Area $= \gamma 8\pi G\hbar\, |L_e|$

eigenval.

$$\boxed{A_{rea} = 8\pi G\hbar\gamma\sqrt{j(j+1)}}$$



Discrete theory on the boundary:

Graph $\Gamma=(\ell,n)$ $\quad U_\ell \in SU(2)$ $\quad \vec{L}_\ell \in su(2)$ $\quad L^2$

① Quantize

$$\mathcal{H}_\Gamma = L_2\left[SU(2)^L/SU^N(2)\right]_\Gamma$$

$$\ni \psi(U_\ell)$$

$\vec{L}_\ell$: left invariant vect. fields $\quad |L_\ell| = \frac{1}{\gamma}$ Area

Area $= \gamma 8\pi G\hbar\, |$

$$\mathcal{H}_\Gamma = \bigoplus_{j_\ell} \bigotimes_{n} \underbrace{\mathrm{Inv}_{SU(2)}\left[\mathcal{H}_{j_1}\otimes\mathcal{H}_{j_2}\otimes\mathcal{H}_{j_3}\otimes\right]}_{\underline{1\,\dim}}$$

$K_{j_1\ldots j_4}$ = "intertw)

$$\boxed{\text{Area} = 8\pi G\hbar\gamma\sqrt{j(j+1)}}$$

$|j_\ell, \quad \rangle$

"Solid consequence of Quantizing in this way"

ei