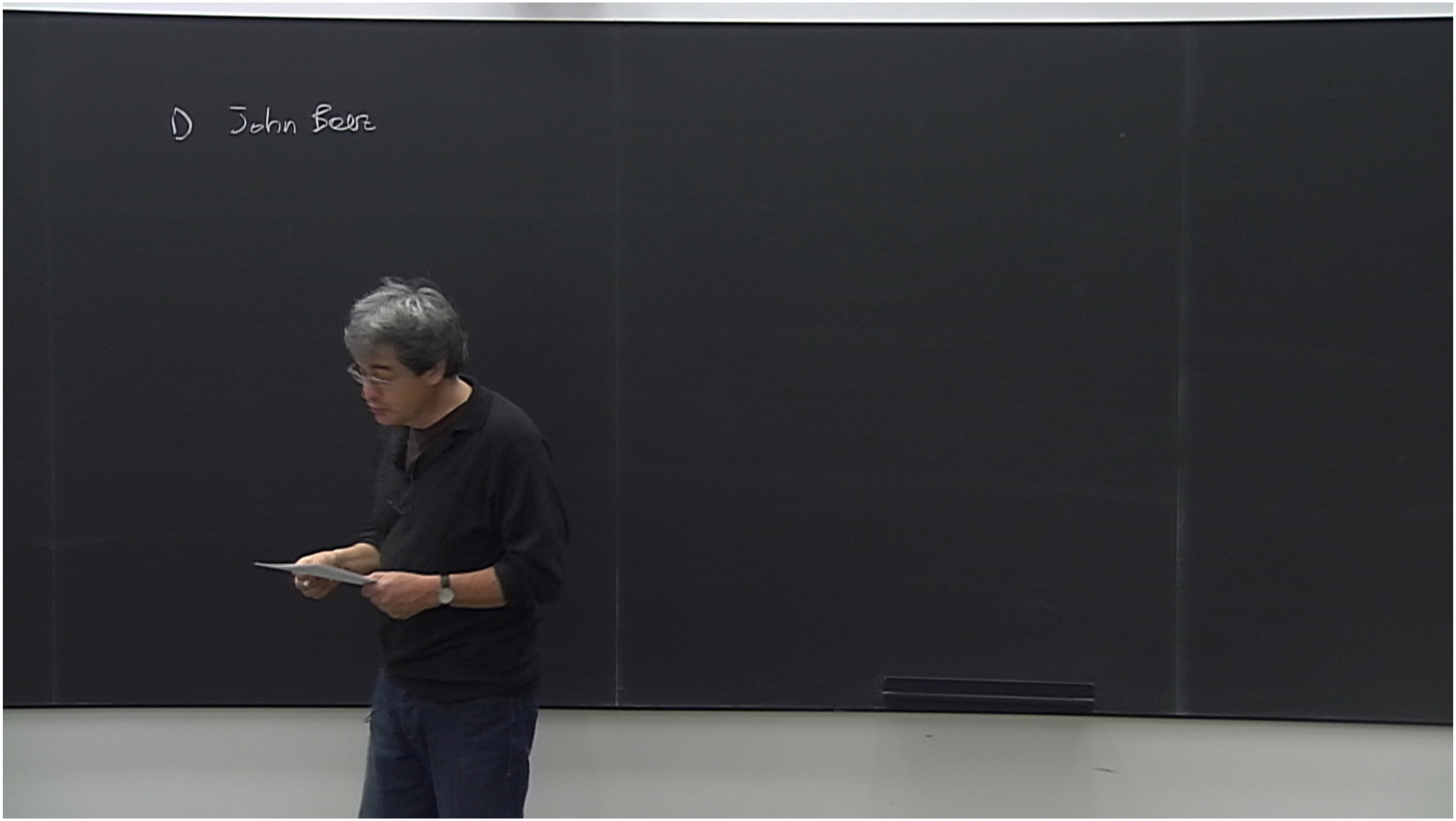
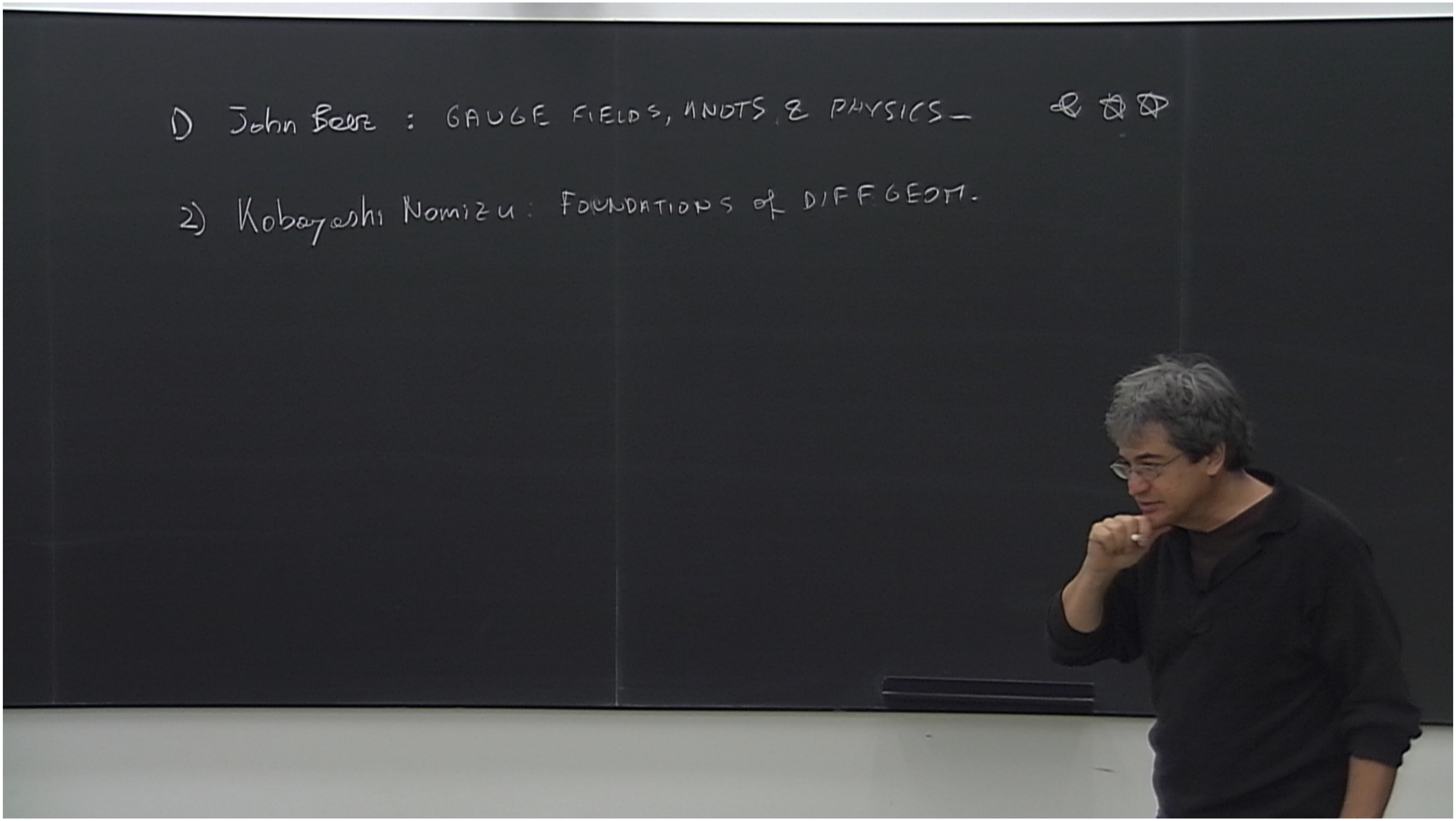
Title: Explorations in Quantum Gravity - Lecture 8

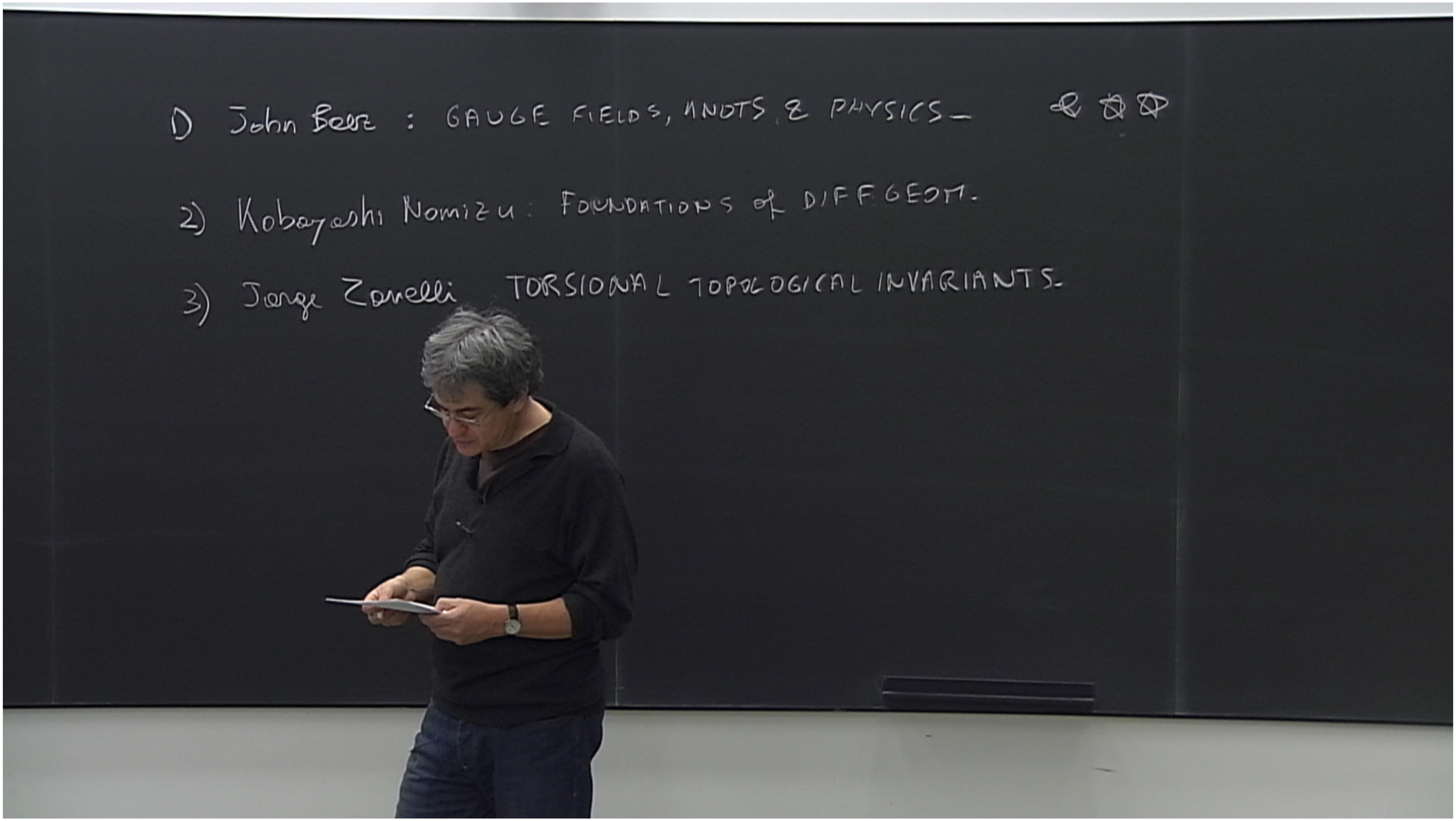
Date: Apr 12, 2012 10:15 AM

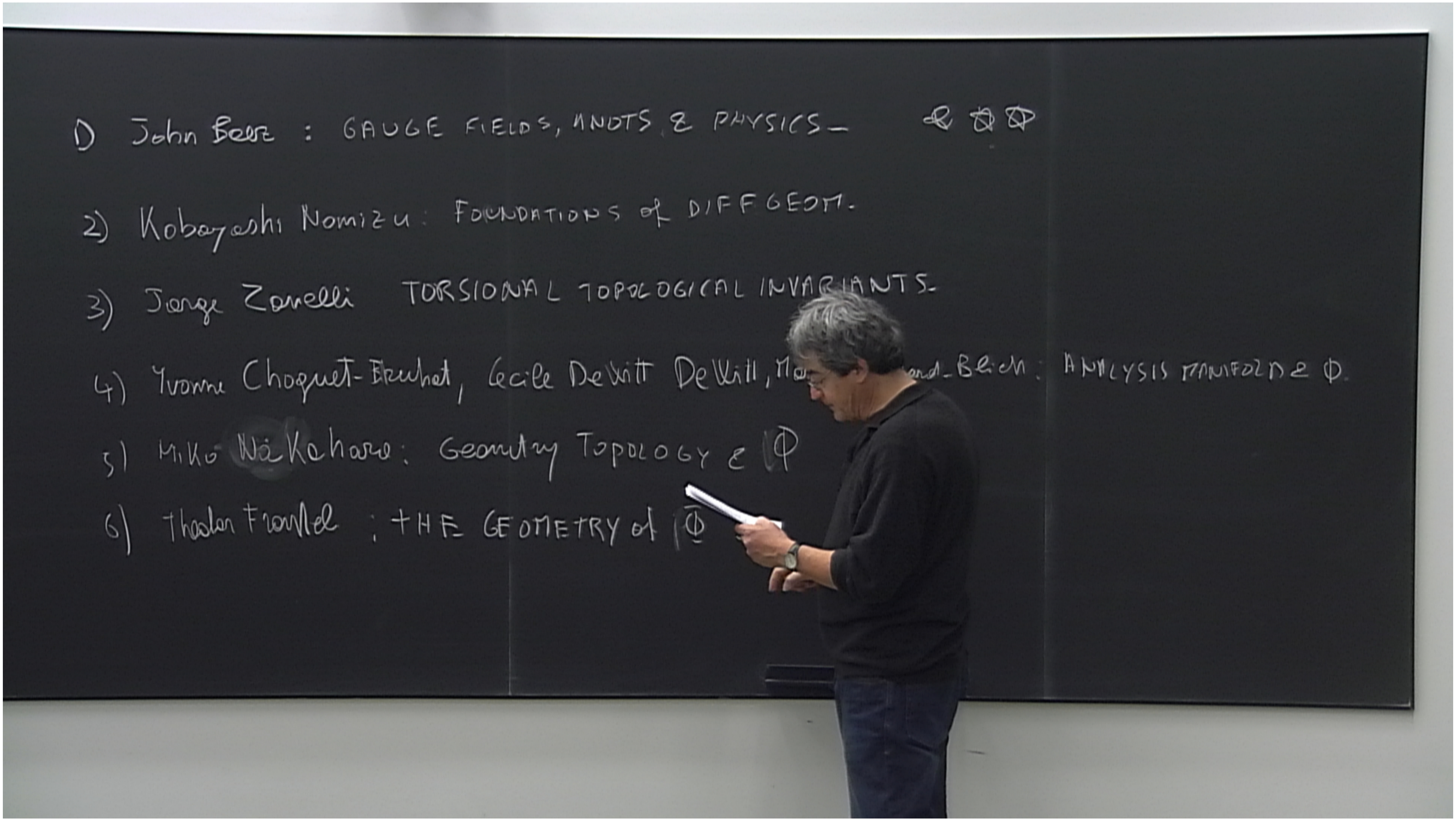
URL: http://pirsa.org/12040029

Abstract:


1) John Baez


1) John Baez : GAUGE FIELDS, KNOTS & PHYSICS –
2) Kobayashi Nomizu : FOUNDATIONS of DIFF GEOM.


1) John Baez : GAUGE FIELDS, KNOTS & PHYSICS
2) Kobayashi Nomizu : FOUNDATIONS of DIFF GEOM.
3) Jorge Zanelli TORSIONAL TOPOLOGICAL INVARIANTS.


1) John Baez : GAUGE FIELDS, KNOTS & PHYSICS
2) Kobayashi Nomizu : FOUNDATIONS of DIFF GEOM.
3) Jorge Zanelli TORSIONAL TOPOLOGICAL INVARIANTS.
4) Yvonne Choquet-Bruhat, Cecile DeWitt DeWitt,
-Bleick : ANALYSIS MANIFOLDS & Φ.
5) Mikio Nakahara : Geometry Topology & Φ
6) Theodore Frankel : THE GEOMETRY of Φ

1) John Baez : GAUGE FIELDS, KNOTS & PHYSICS —

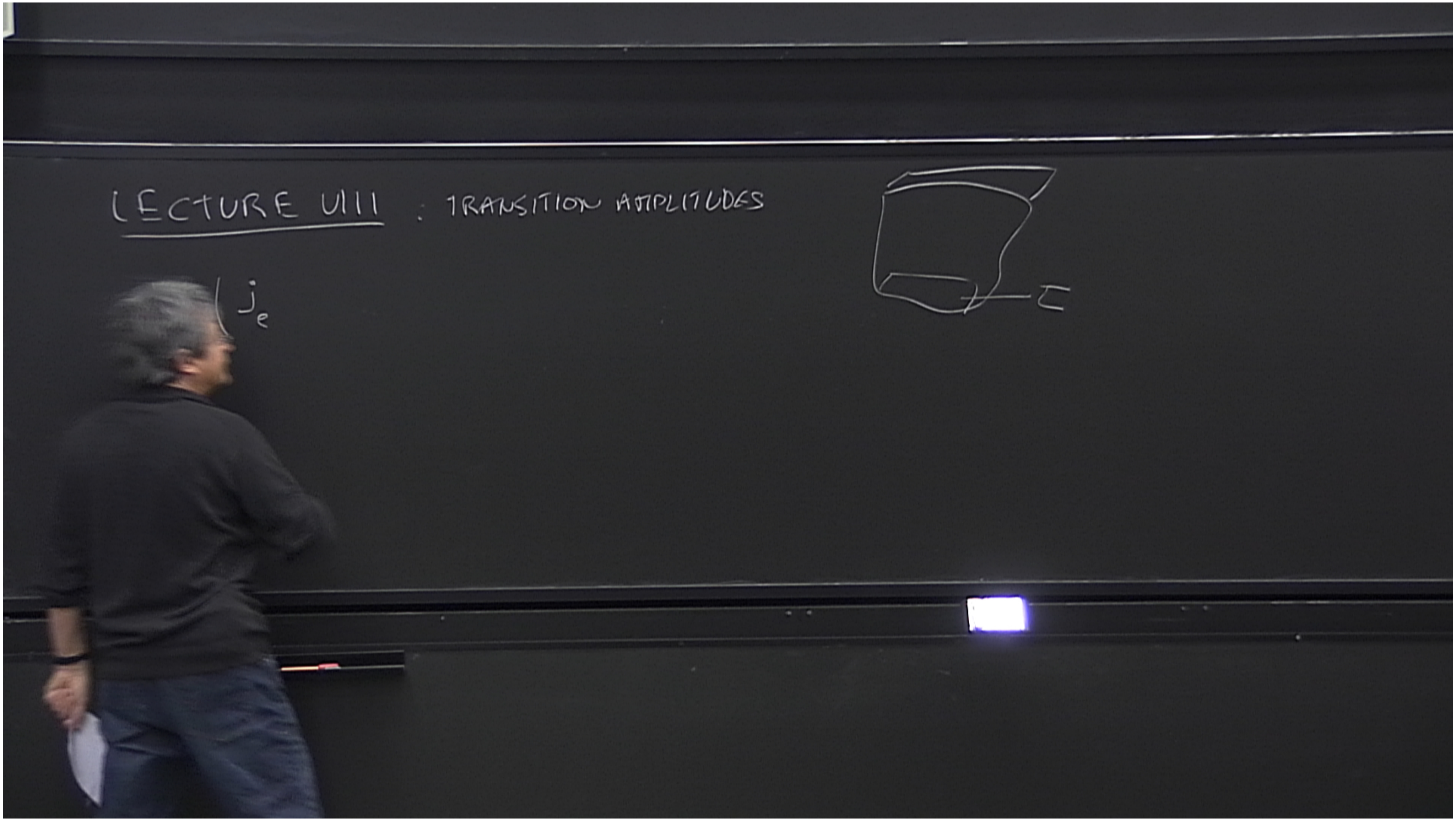
2) Kobayashi Nomizu : FOUNDATIONS of DIFF GEOM.

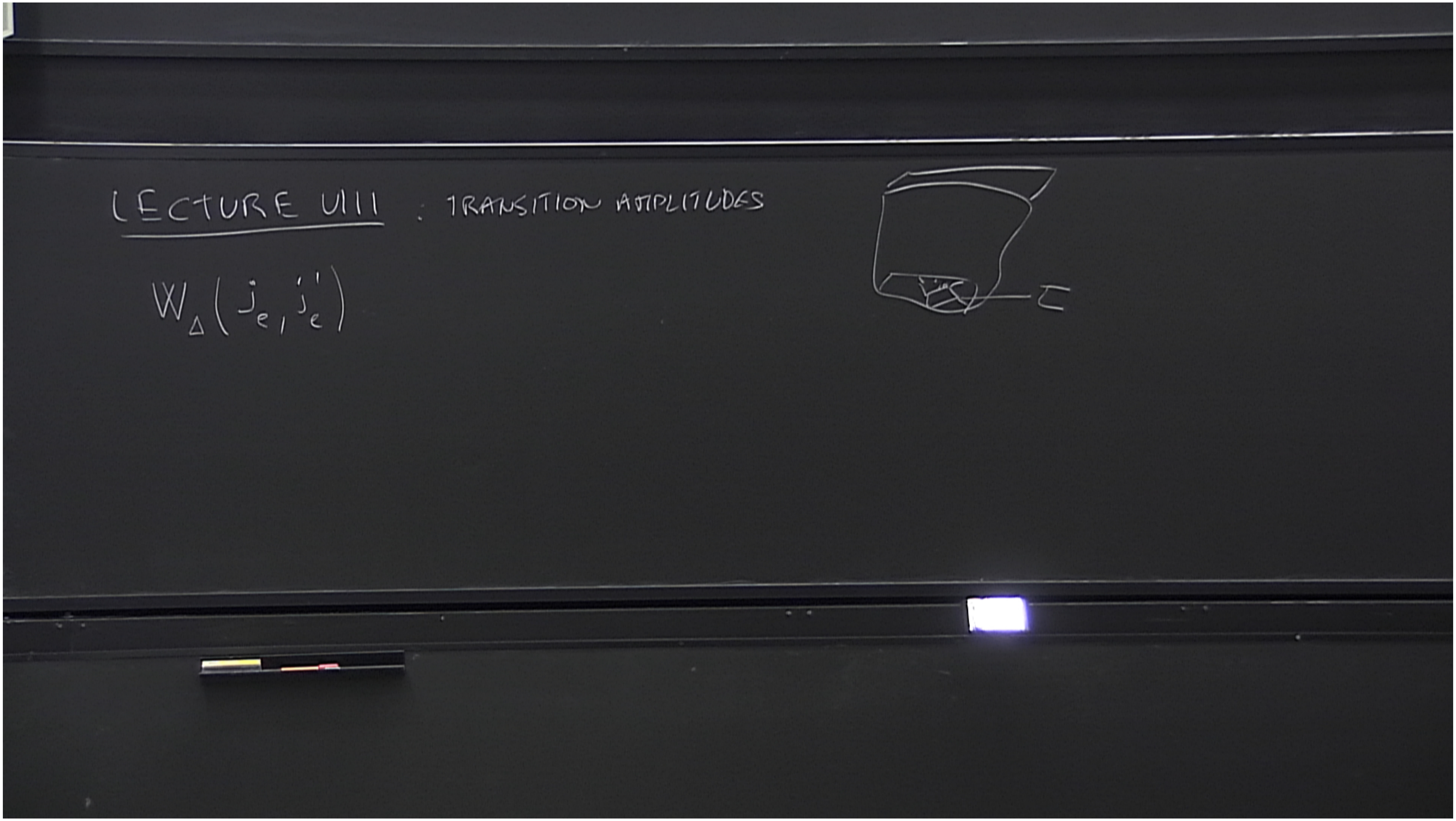
3) Jorge Zanelli TORSIONAL TOPOLOGICAL INVARIANTS.

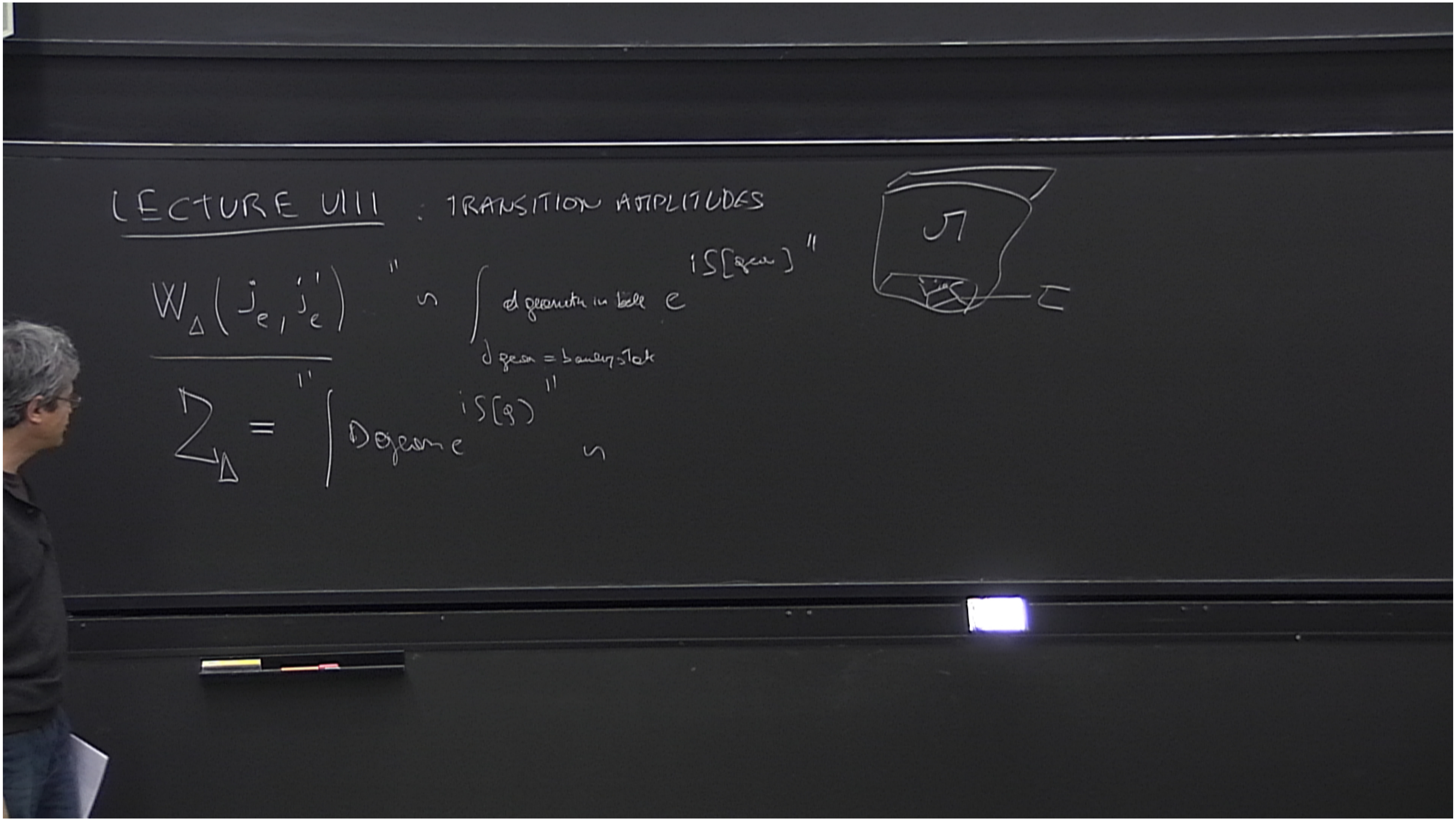
4) Yvonne Choquet-Bruhat, Cecile DeWitt DeWitt, Margaret Dillard-Bleick : ANALYSIS MANIFOLDS & Φ.

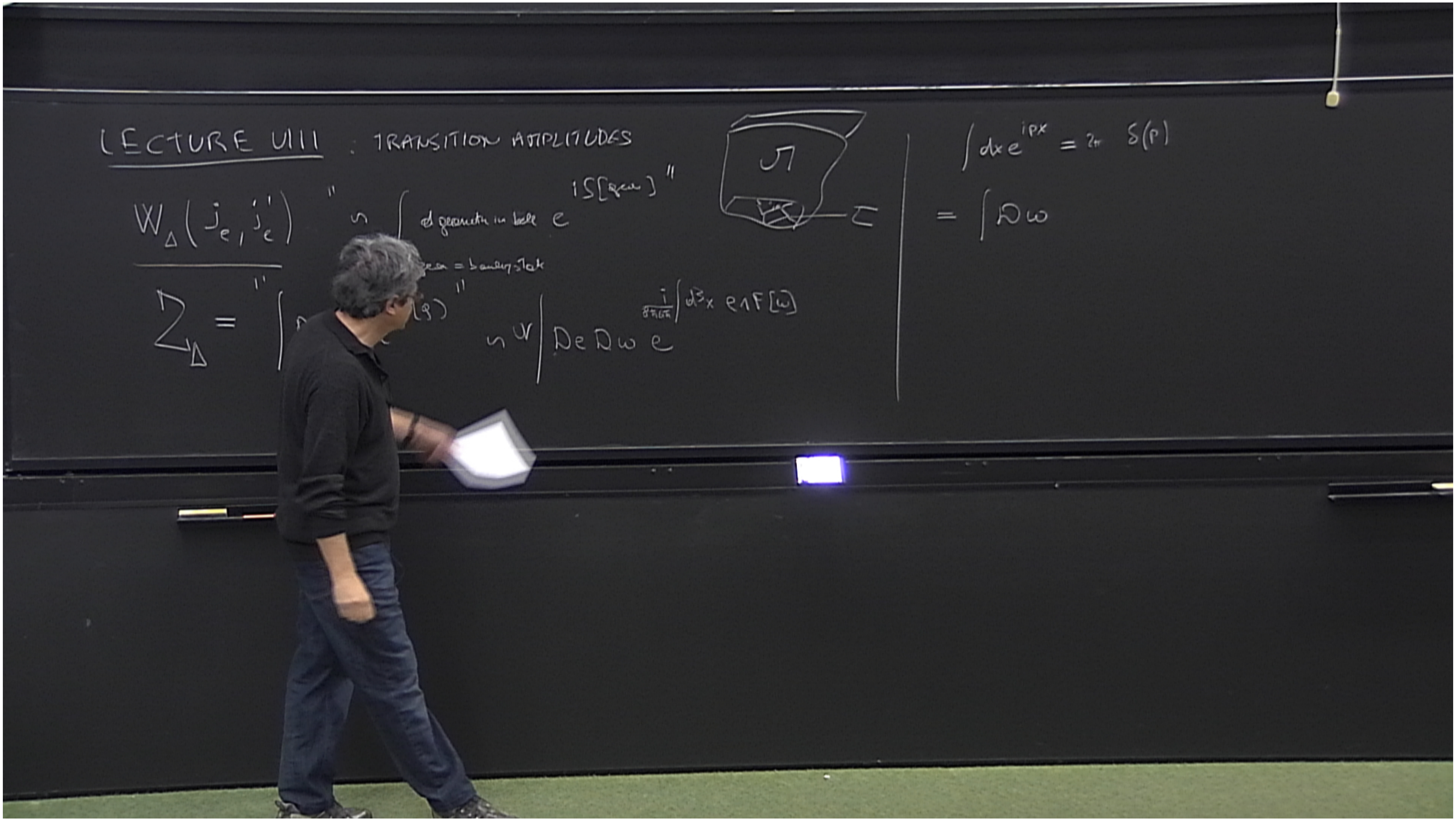
5) Mikio Nakahara : Geometry Topology & Φ

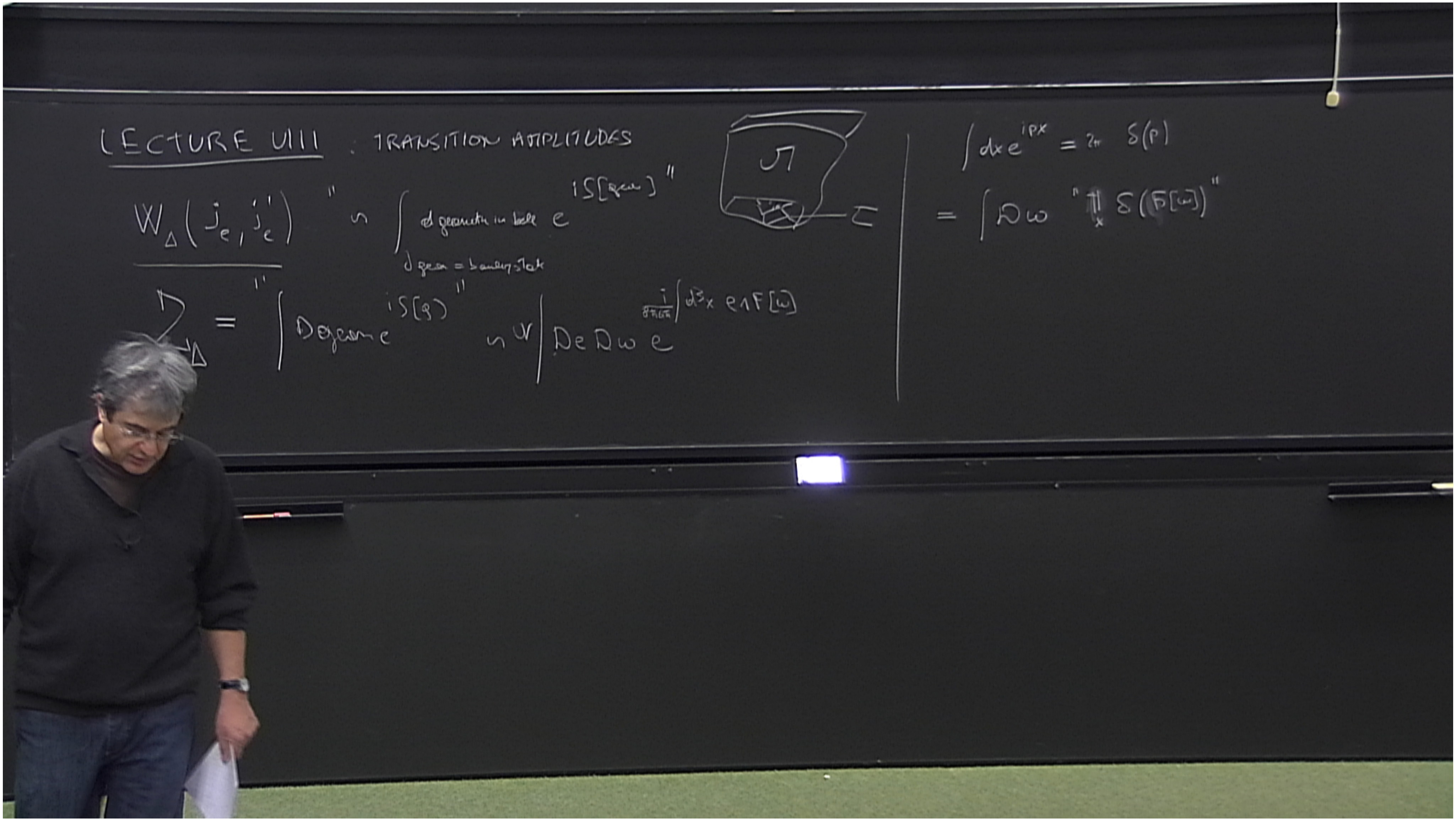
6) Theodore Frankel : THE GEOMETRY of Φ.

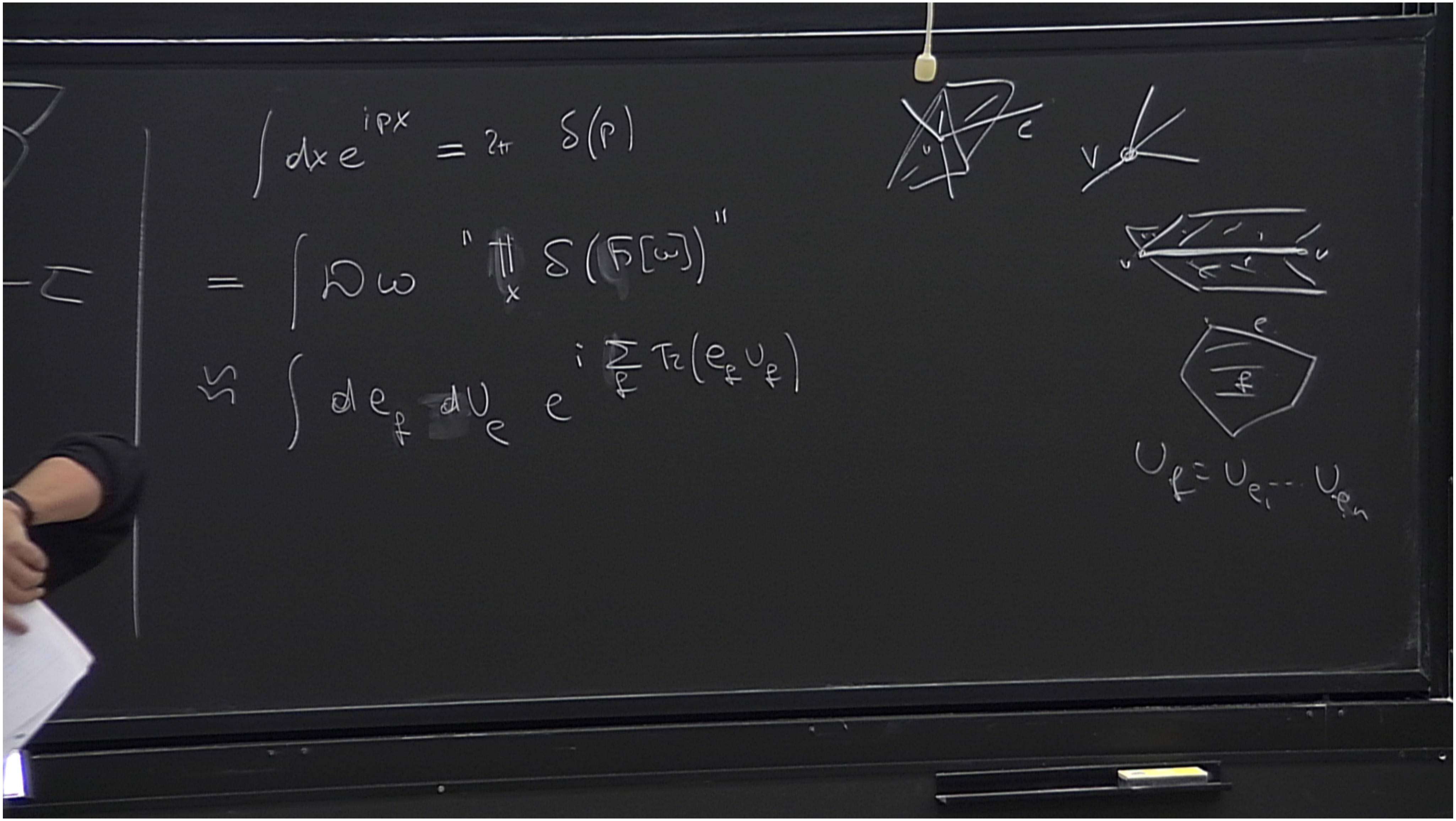

LECTURE VIII : TRANSITION AMPLITUDES
j
e
Σ


LECTURE VIII : TRANSITION AMPLITUDES
$W_\Delta(j_e, j'_e)$
Σ


LECTURE VIII : TRANSITION AMPLITUDES
$W_\Delta(j_e, j'_e)$ " ~ $\int$ d geometry in bulk $e^{iS[geom]}$ "
∂ geom = boundary state
Z_Δ = " $\int$ D geom $e^{iS(g)}$ " ~


LECTURE VIII : TRANSITION AMPLITUDES
Σ


LECTURE VIII : TRANSITION AMPLITUDES
∫dx e^{ipx} = 2π δ(p)

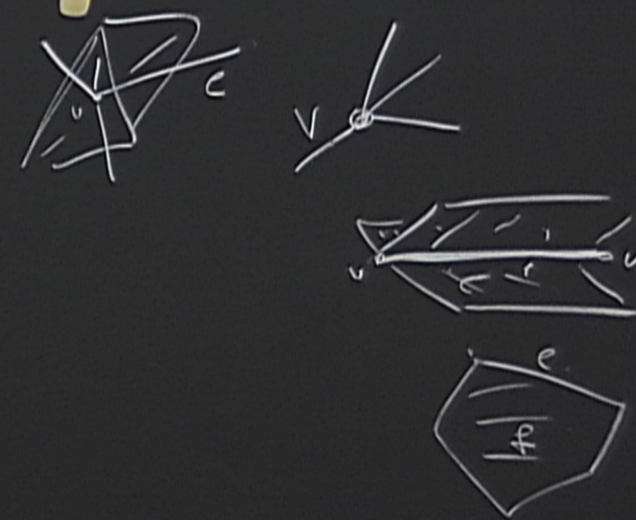

$\int dx\, e^{ipx} = 2\pi\, \delta(p)$
$= \int \mathcal{D}\omega \text{ "} \prod_x \delta(F[\omega]) \text{"}$
$\simeq \int de_f\, dU_e\, e^{i \sum_f \mathrm{Tr}(e_f U_f)}$
e
v
v
e
f
$U_f = U_{e_1} \cdots U_{e_n}$

$$\int dx\, e^{ipx} = 2\pi\, \delta(p)$$

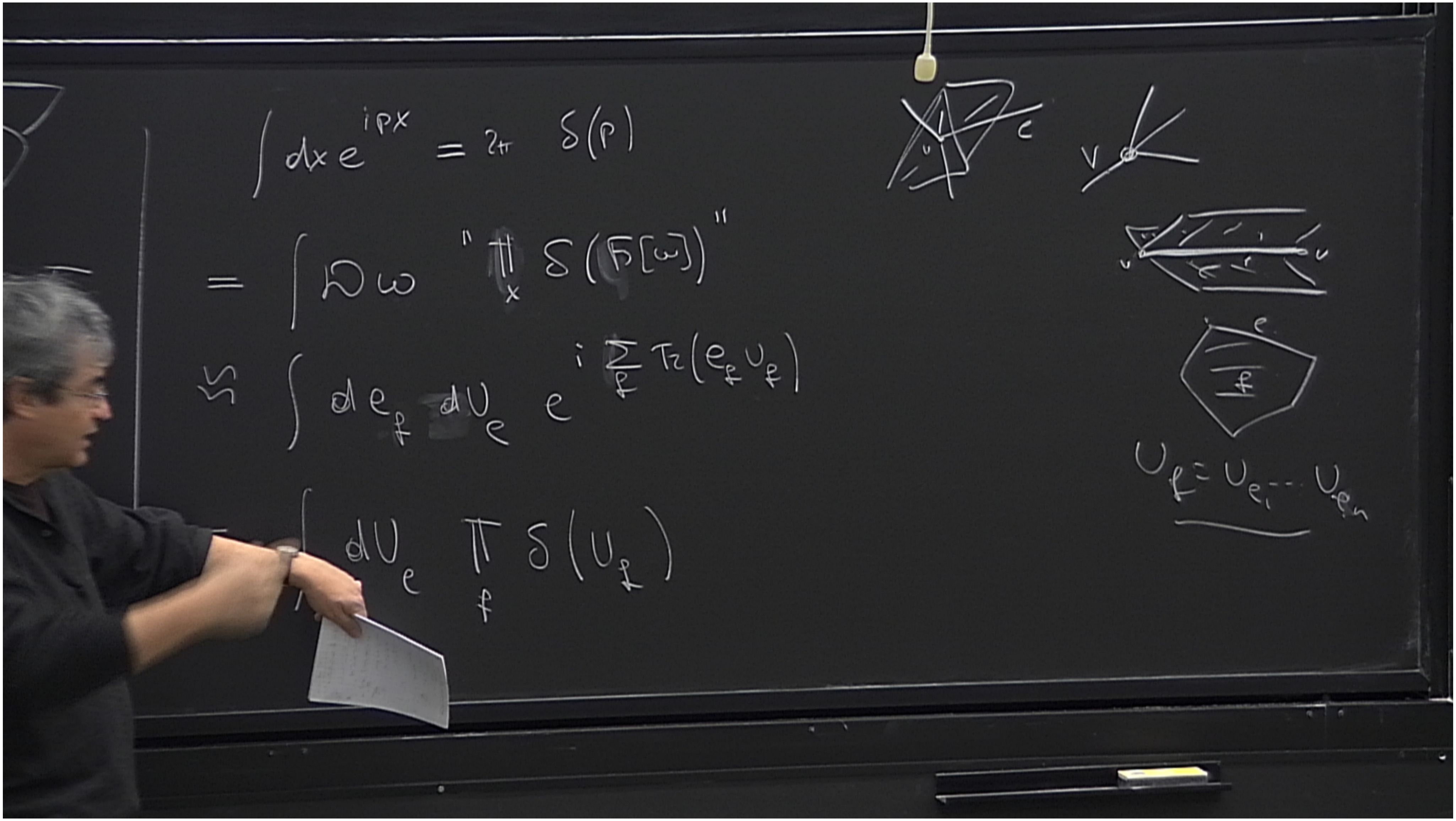
$$= \int \mathcal{D}\omega \;\; \text{"}\prod_x \delta(F[\omega])\text{"}$$

$$= \int de_f\, dU_e\; e^{i\sum_f \mathrm{Tr}(e_f U_f)}$$

$$=$$



$$U_f = U_{e_1} \cdots U_{e_n}$$



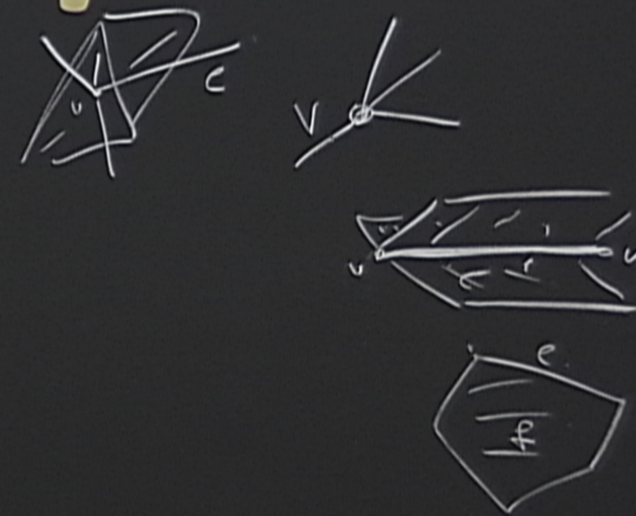
$\int dx\, e^{ipx} = 2\pi\, \delta(p)$
$= \int D\omega \ \text{"} \prod_x \delta(F[\omega]) \text{"}$
$\simeq \int de_f\, dU_e\, e^{i \sum_f \mathrm{Tr}(e_f U_f)}$
$= \int dU_e \prod_f \delta(U_f)$
e
v
v
v
e
f
$U_f = U_{e_1} \cdots U_{e_n}$

$$\int dx\, e^{ipx} = 2\pi\, \delta(p)$$

$$= \int \mathcal{D}\omega \;\text{"}\prod_x \delta(F[\omega])\text{"}$$

$$\simeq \int de_f\, dU_e\; e^{i\sum_f \mathrm{Tr}(e_f U_f)}$$

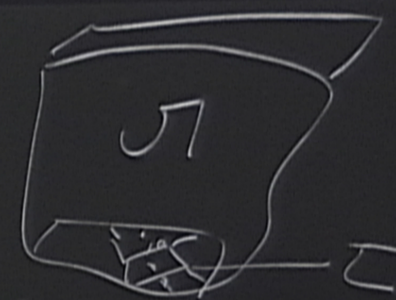
$$\boxed{Z_\Delta = \int dU_e \prod_f \delta(U_f)}$$



$$\underline{U_f = U_{e_1} \cdots U_{e_n}}$$

LECTURE VIII : TRANSITION AMPLITUDES

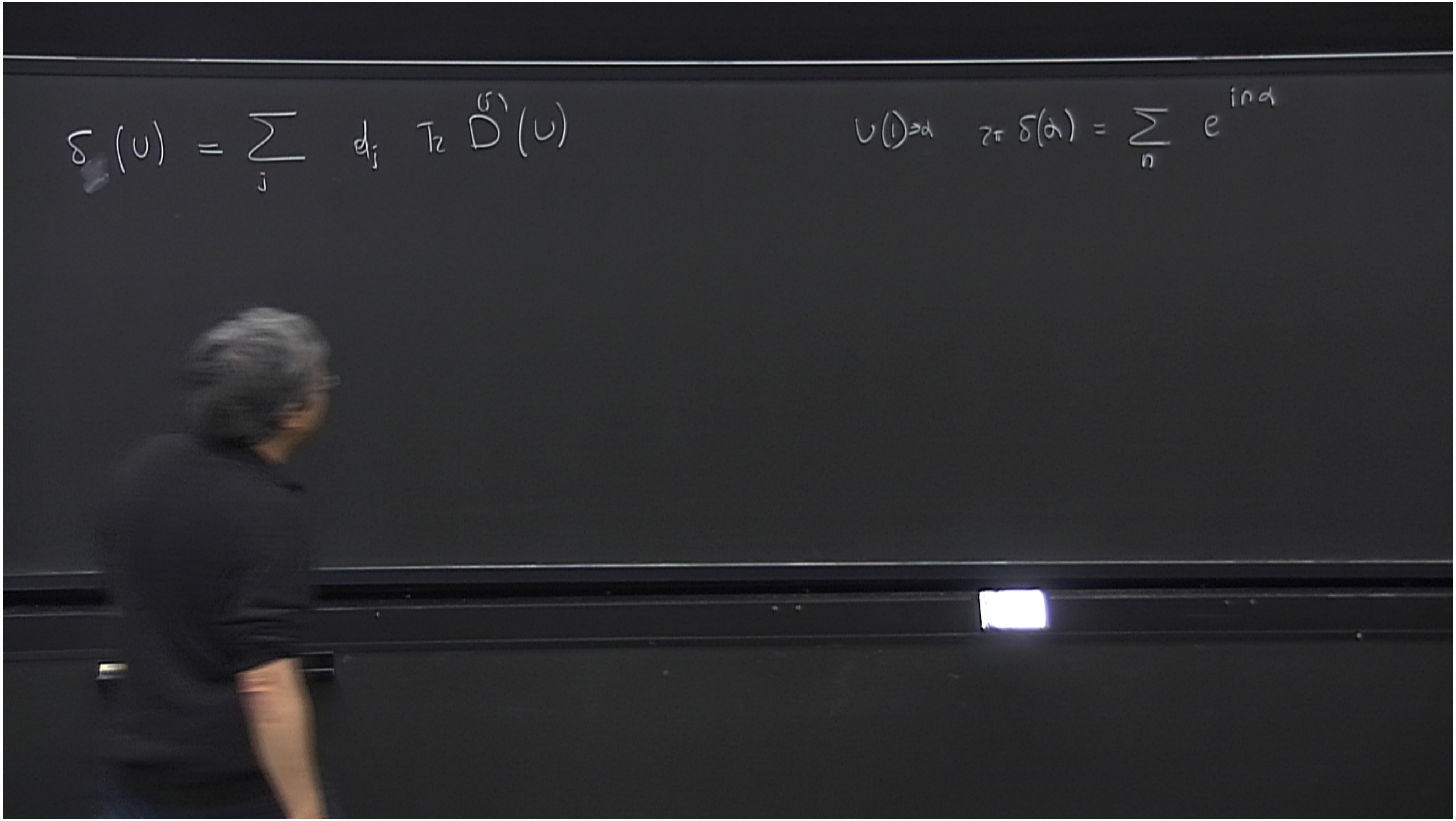
$$W_\Delta(j_e, j'_e) \;\text{"}\!\sim \int_{\partial\,\text{geom} = \text{boundary state}} d\,\text{geometry in bulk}\; e^{iS[\text{geom}]}\text{"}$$

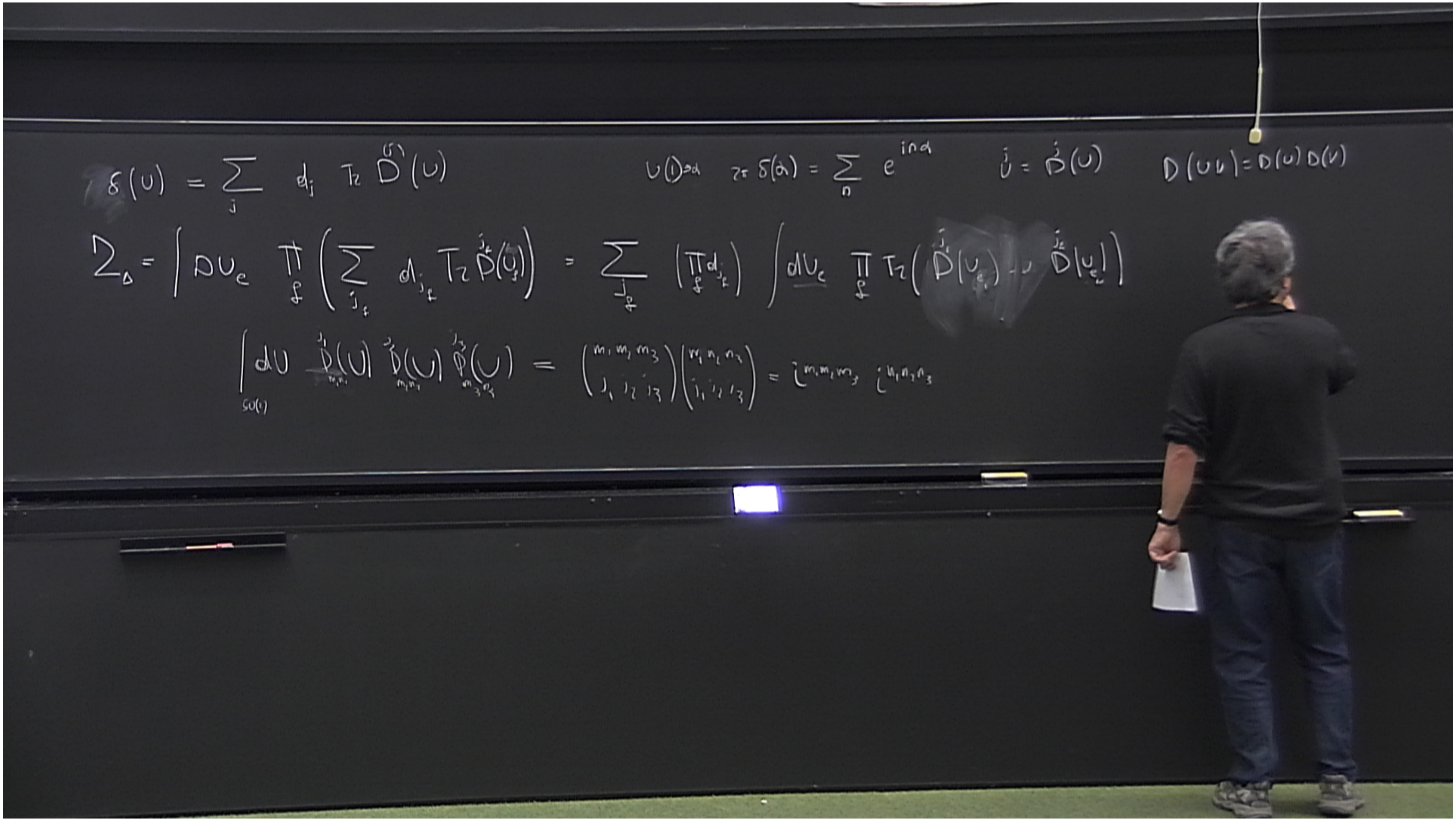


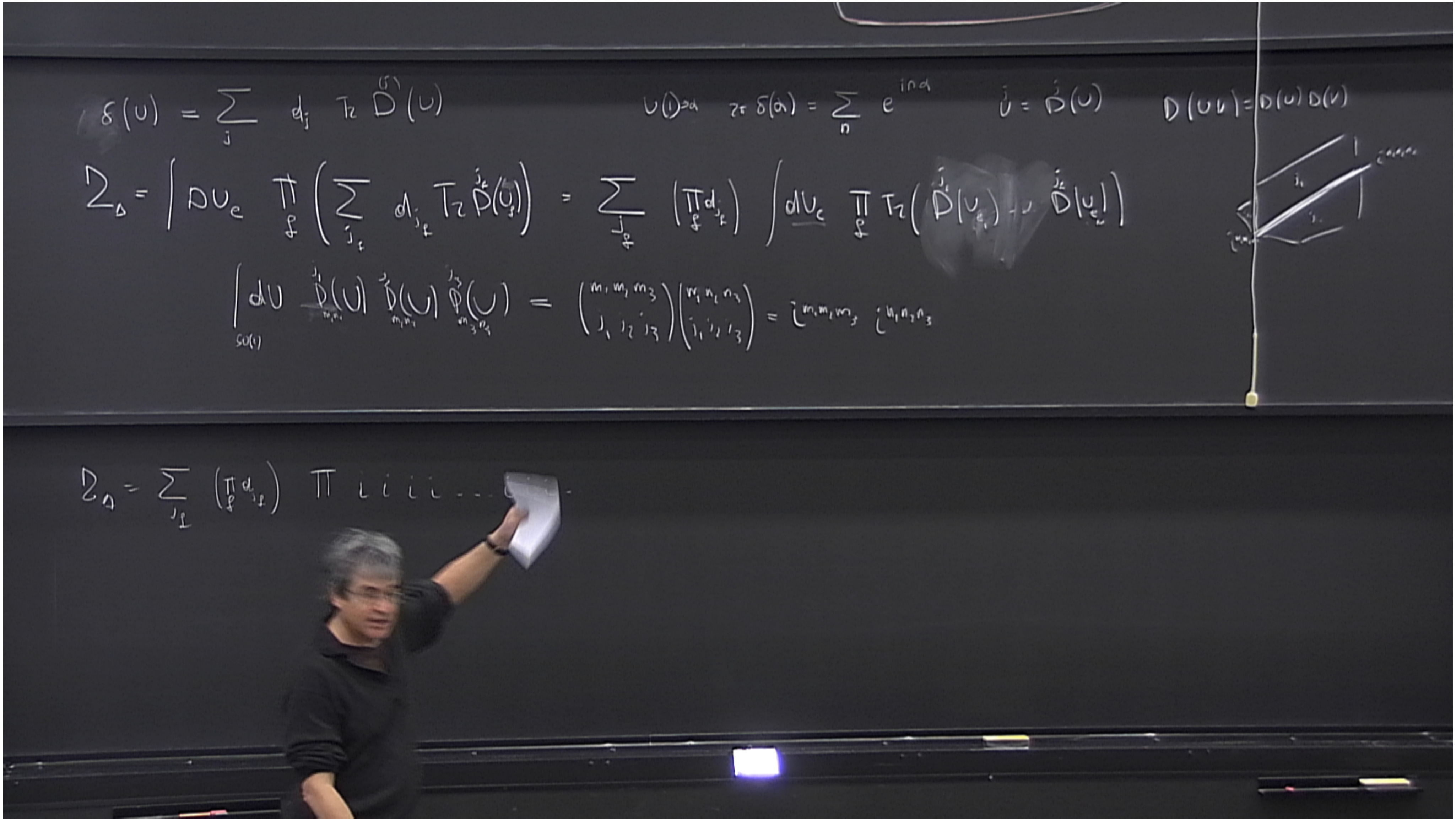
$$\sum_\Delta = \text{"}\int D\,\text{geom}\; e^{iS(g)}\text{"} \sim \int De\,D\omega\; e^{\frac{i}{8\pi G\hbar}\int d^3x\; e\wedge F[\omega]}$$

$$\delta(U) = \sum_j d_j \, \mathrm{Tr}\, D^{(j)}(U)$$

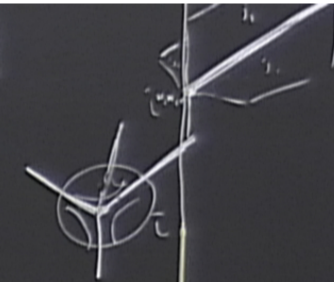
$U(1) \ni \alpha \qquad \delta(\alpha) = \sum_n$


$\delta(U) = \sum_j d_j \, \mathrm{Tr}\, D^{(j)}(U)$
$U(1) \ni \alpha \quad 2\pi\,\delta(\alpha) = \sum_n e^{in\alpha}$


$\delta(U) = \sum_j d_j \, \mathrm{Tr}\, D^{(j)}(U)$
$U(1) \ni \alpha \quad 2\pi\,\delta(\alpha) = \sum_n e^{in\alpha}$
$j \mapsto D^{j}(U)$
$D(UV) = D(U)\,D(V)$
$Z_\Delta = \int DU_e \prod_f \left(\sum_{j_f} d_{j_f} \mathrm{Tr}\, D^{j_f}(U_f) \right) = \sum_{j_f} \left(\prod_f d_{j_f} \right) \int dU_e \prod_f \mathrm{Tr} \left(D^{j_1}(U_{e_1}) \cdots D^{j_2}(U_{e_2}) \right)$
$\int_{SU(2)} dU \, D^{j_1}_{m_1 n_1}(U) \, D^{j_2}_{m_2 n_2}(U) \, D^{j_3}_{m_3 n_3}(U) = \begin{pmatrix} m_1 & m_2 & m_3 \\ j_1 & j_2 & j_3 \end{pmatrix} \begin{pmatrix} n_1 & n_2 & n_3 \\ j_1 & j_2 & j_3 \end{pmatrix} = i^{m_1 m_2 m_3} \, i^{n_1 n_2 n_3}$


$\delta(U) = \sum_j d_j \, Tr \, D^{(j)}(U)$
$U(1) \ni \alpha \quad 2\pi\,\delta(\alpha) = \sum_n e^{in\alpha}$
$j = D^{j}(U)$
$D(UV) = D(U)\,D(V)$
$Z_\Delta = \int DU_e \prod_f \left(\sum_{j_f} d_{j_f} Tr\, D^{j_f}(U_f) \right) = \sum_{j_f} \left(\prod_f d_{j_f} \right) \int dU_e \prod_f Tr\left(D^{j_1}(U_{e_1}) \cdots D^{j_2}(U_{e_2}) \right)$
$\int_{SU(2)} dU \, D^{j_1}_{m_1 n_1}(U) \, D^{j_2}_{m_2 n_2}(U) \, D^{j_3}_{m_3 n_3}(U) = \begin{pmatrix} m_1 & m_2 & m_3 \\ j_1 & j_2 & j_3 \end{pmatrix} \begin{pmatrix} n_1 & n_2 & n_3 \\ j_1 & j_2 & j_3 \end{pmatrix} = i^{m_1 m_2 m_3} \, i^{n_1 n_2 n_3}$
$Z_\Delta = \sum_{j_f} \left(\prod_f d_{j_f} \right) \prod i \; i \; i \; i \ldots$

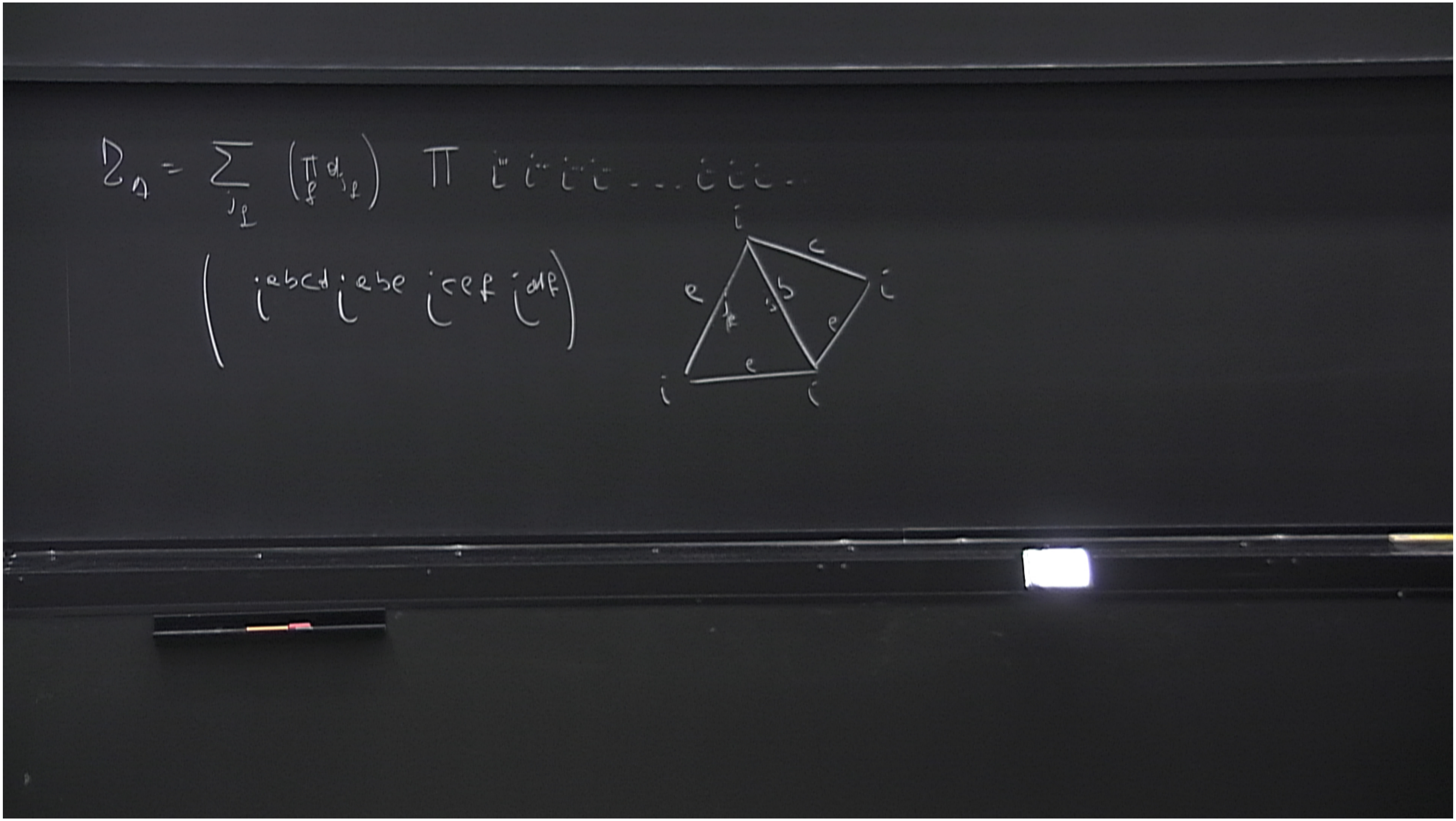
$$Z_\Delta = \int DU_e \prod_f \left(\sum_{j_f} d_{j_f} \mathrm{Tr}\, \overset{j_f}{D}(U_f)\right) = \sum_{j_f} \left(\prod_f d_{j_f}\right) \int dU_e \prod_f \mathrm{Tr}\left(\overset{j_f}{D}(U_{e_1}) \cdots \overset{j_f}{D}(U_{e_n})\right)$$



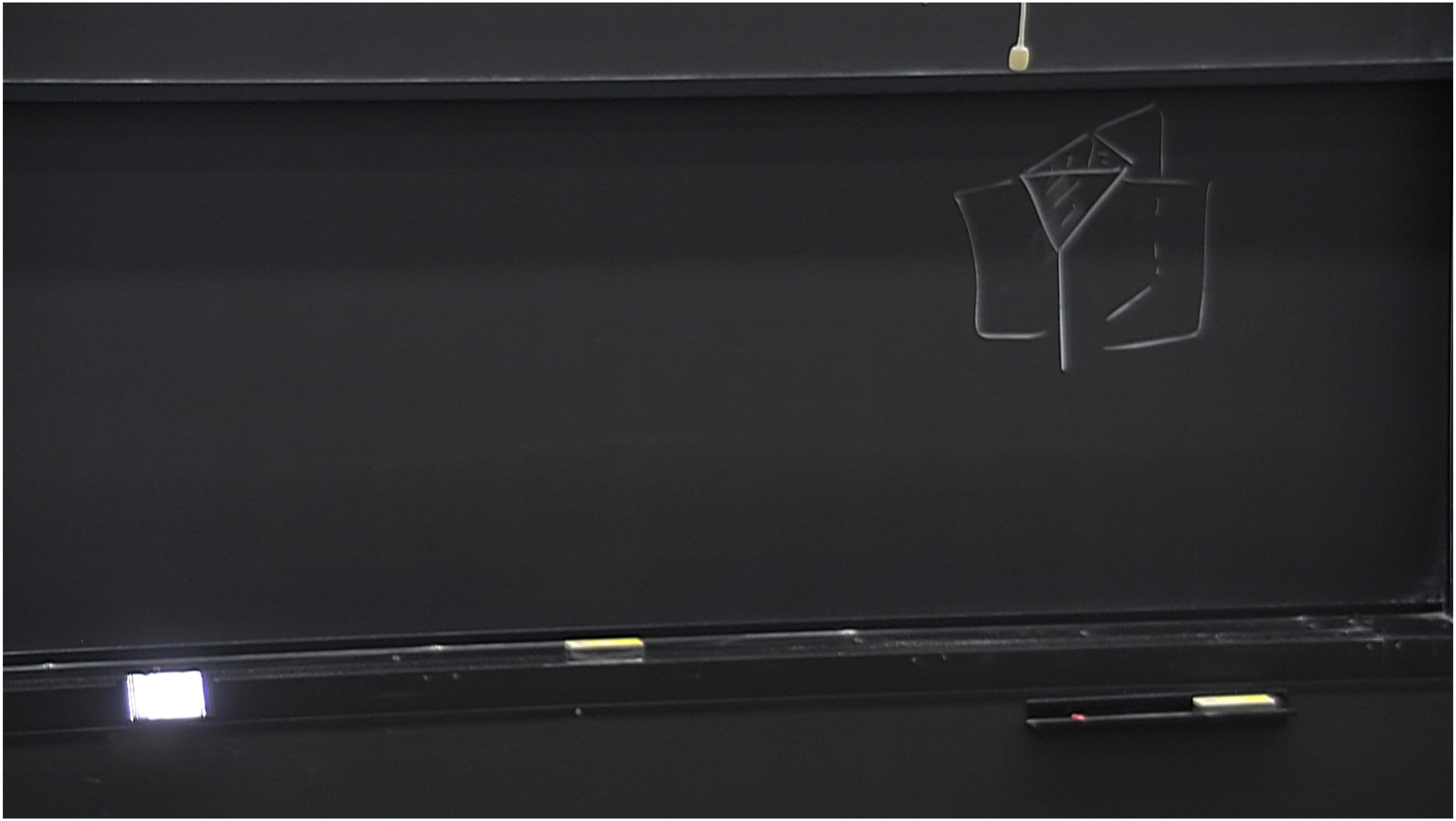
$$\int_{SU(2)} dU \, \overset{j_1}{D}_{m_1 n_1}(U)\, \overset{j_2}{D}_{m_2 n_2}(U)\, \overset{j_3}{D}_{m_3 n_3}(U) = \begin{pmatrix} m_1 & m_2 & m_3 \\ j_1 & j_2 & j_3 \end{pmatrix}\begin{pmatrix} n_1 & n_2 & n_3 \\ j_1 & j_2 & j_3 \end{pmatrix} = i^{m_1 m_2 m_3}\, i^{n_1 n_2 n_3}$$

$$Z_\Delta = \sum_{j_f} \left(\prod_f d_{j_f}\right) \prod i\, i\, i\, i \ldots i\, i\, i.$$

$$\left(i^{abcd}\, i^{ebe}\, i^{cef}\, i^{dif} \right)$$

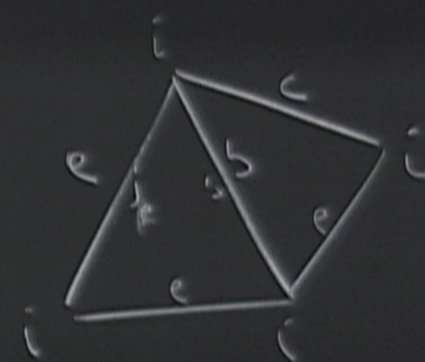


$Z_\Delta = \sum_{j_f} \left(\prod_f d_{j_f}\right) \prod i\, i\, i\, i \ldots i\, i\, i$
$\left(i^{abcd}\, i^{abe}\, i^{cef}\, i^{def} \right)$

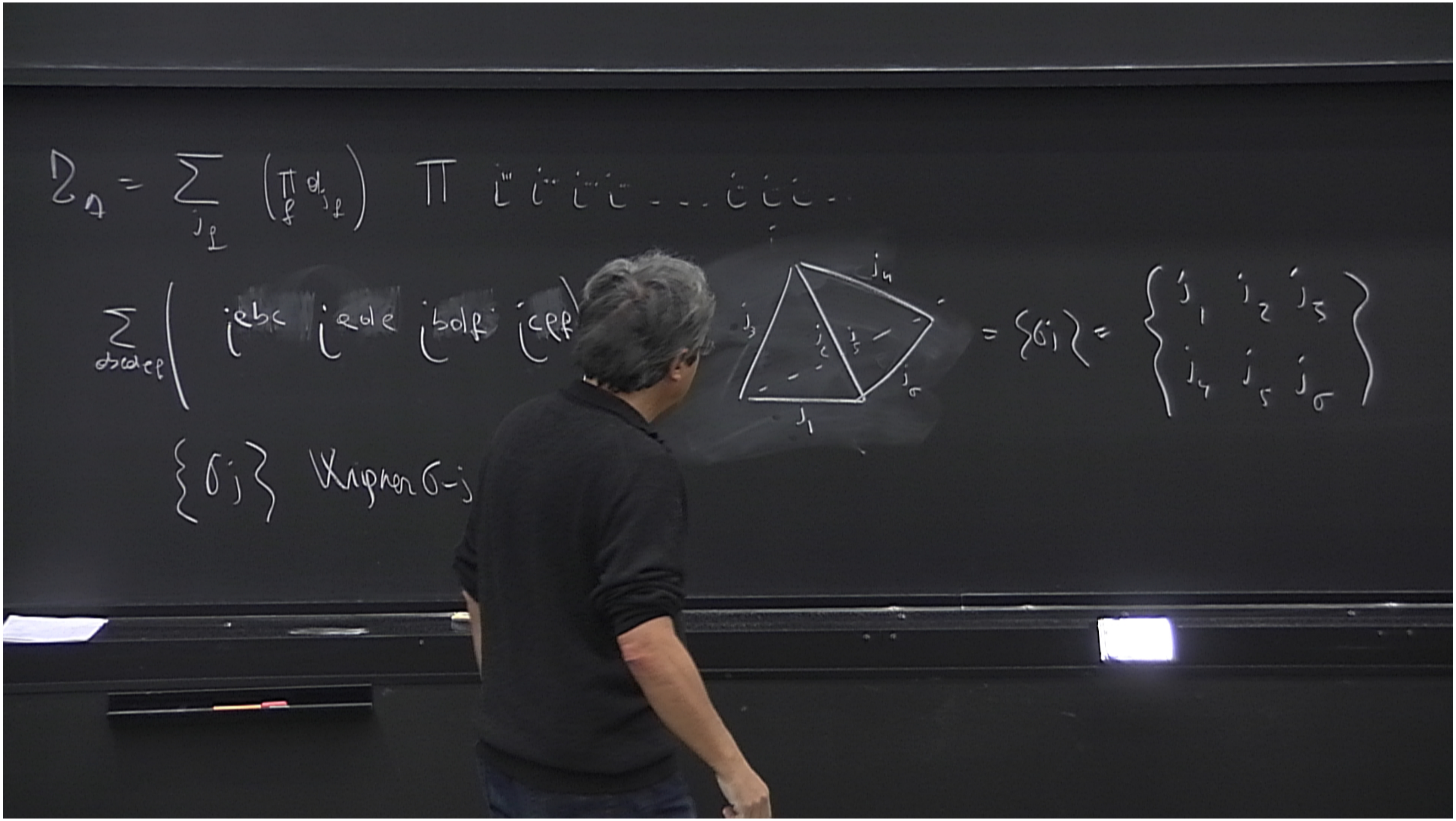


$$Z_\Delta = \sum_{j_\ell} \left(\prod_\ell d_{j_\ell} \right) \prod i^{\cdots} i^{\cdots} i^{\cdots} i^{\cdots} \ldots i\, i\, i \ldots$$

$$\sum_{abcdef} \left(i^{abcd}\, i^{abe}\, i^{cef}\, i^{daf} \right)$$

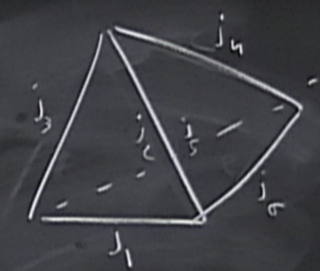


$\{6j\}$ Wigner 6-j symbol.



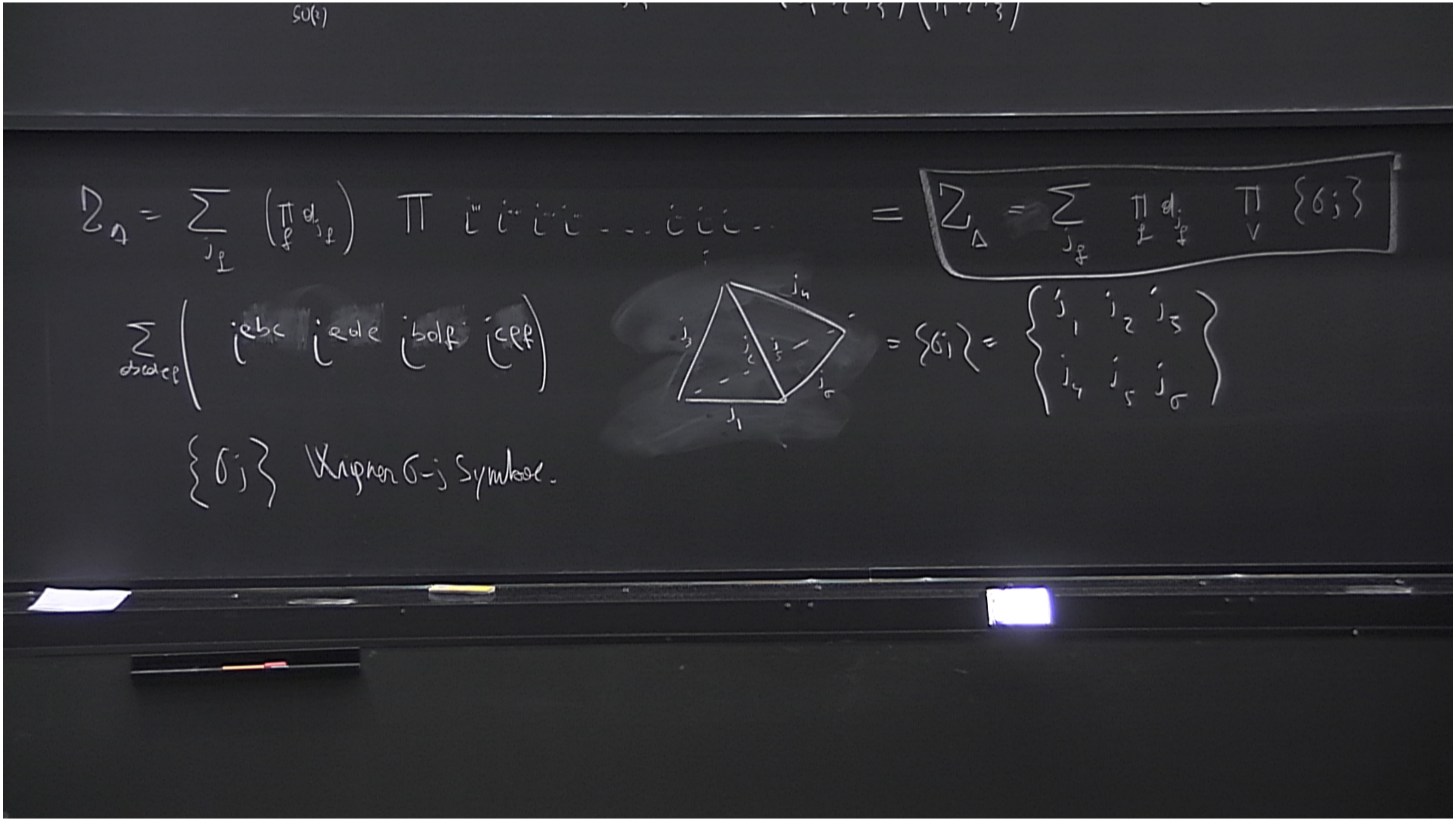
$$Z_\Delta = \sum_{j_f} \left(\prod_f d_{j_f}\right) \prod i^{\cdots} i^{\cdots} i^{\cdots} i \ldots i\, i\, i \ldots$$

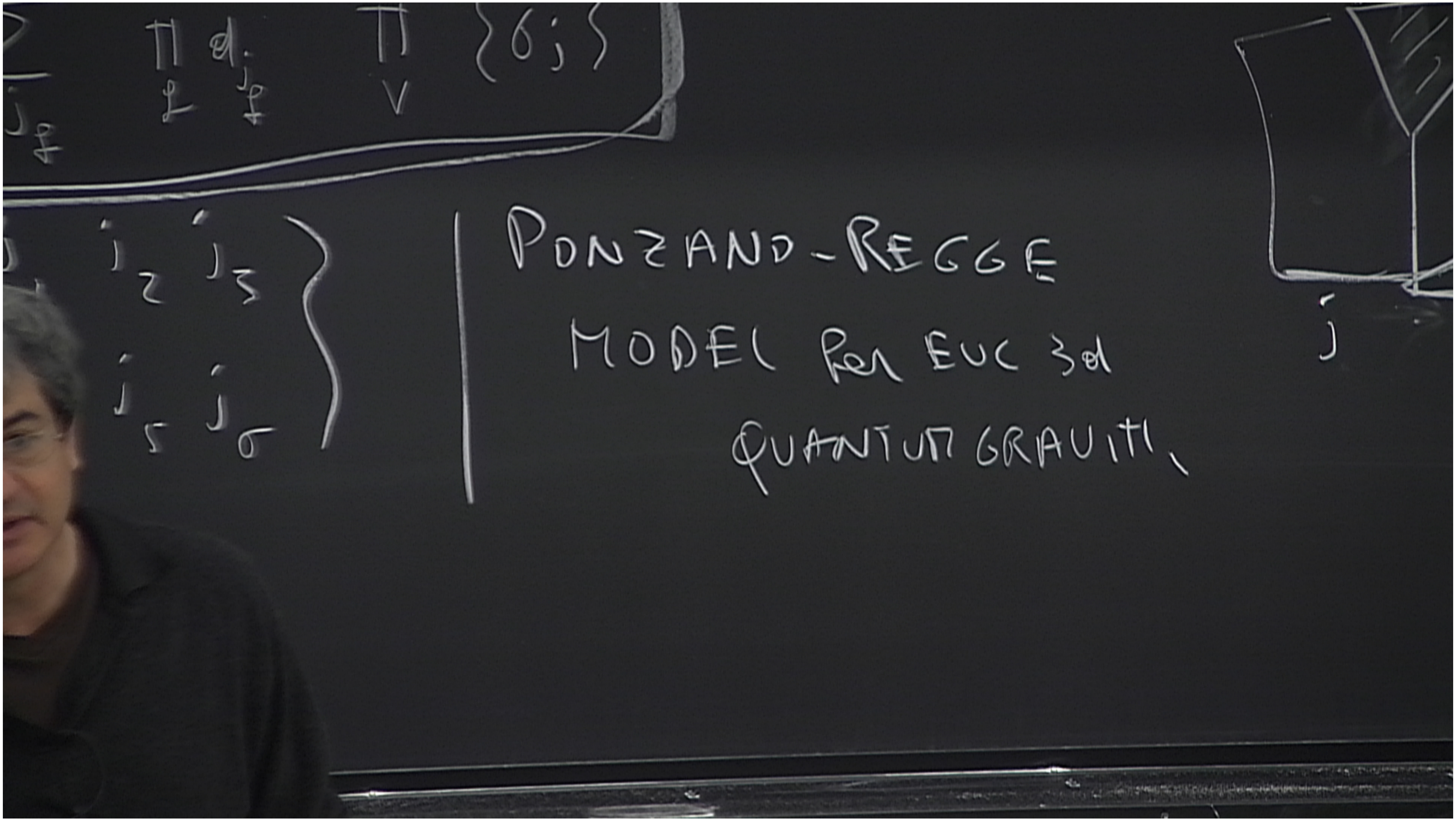
$$\sum_{abcdef} \left(i^{abc}\, i^{ade}\, i^{bdf}\, i^{cef} \right)$$



$$= \{6j\} = \begin{Bmatrix} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \end{Bmatrix}$$

$\{6j\}$ Wigner 6-j Symbol.


SU(2)
Z_Δ = Σ_{j_f} (Π_f d_{j_f}) Π i i i i . . . i i i . . = Z_Δ = Σ_{j_f} Π_f d_{j_f} Π_v {6j}
Σ_{abcdef} (i^{ebc} i^{ade} i^{bdf} i^{cef})
j_1 j_2 j_3 j_4 j_5 j_6
= {6j} = { j_1 j_2 j_3 ; j_4 j_5 j_6 }
{6j} Wigner 6-j Symbol.

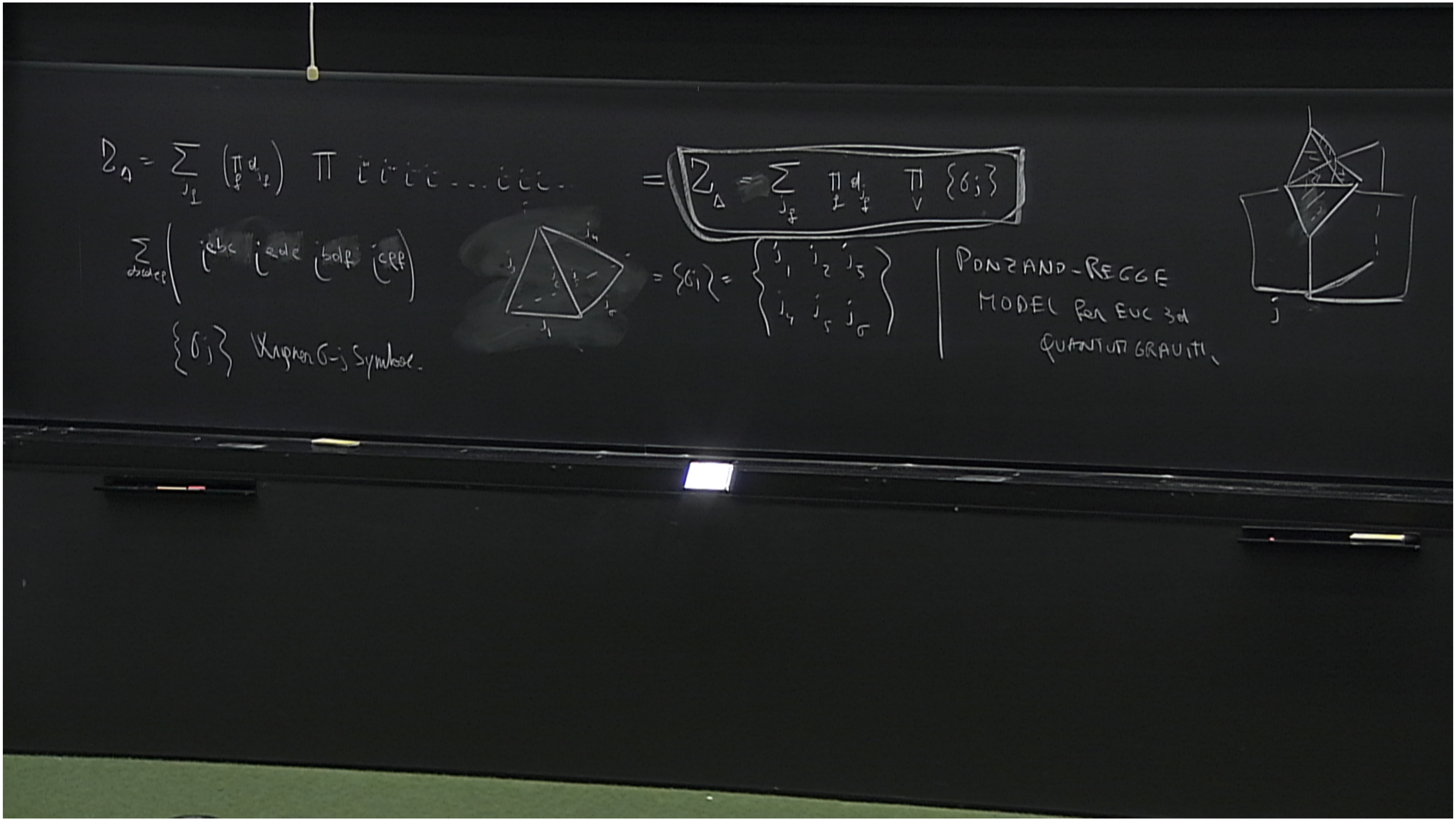

PONZANO-REGGE
MODEL FOR EUC 3d
QUANTUM GRAVITY.

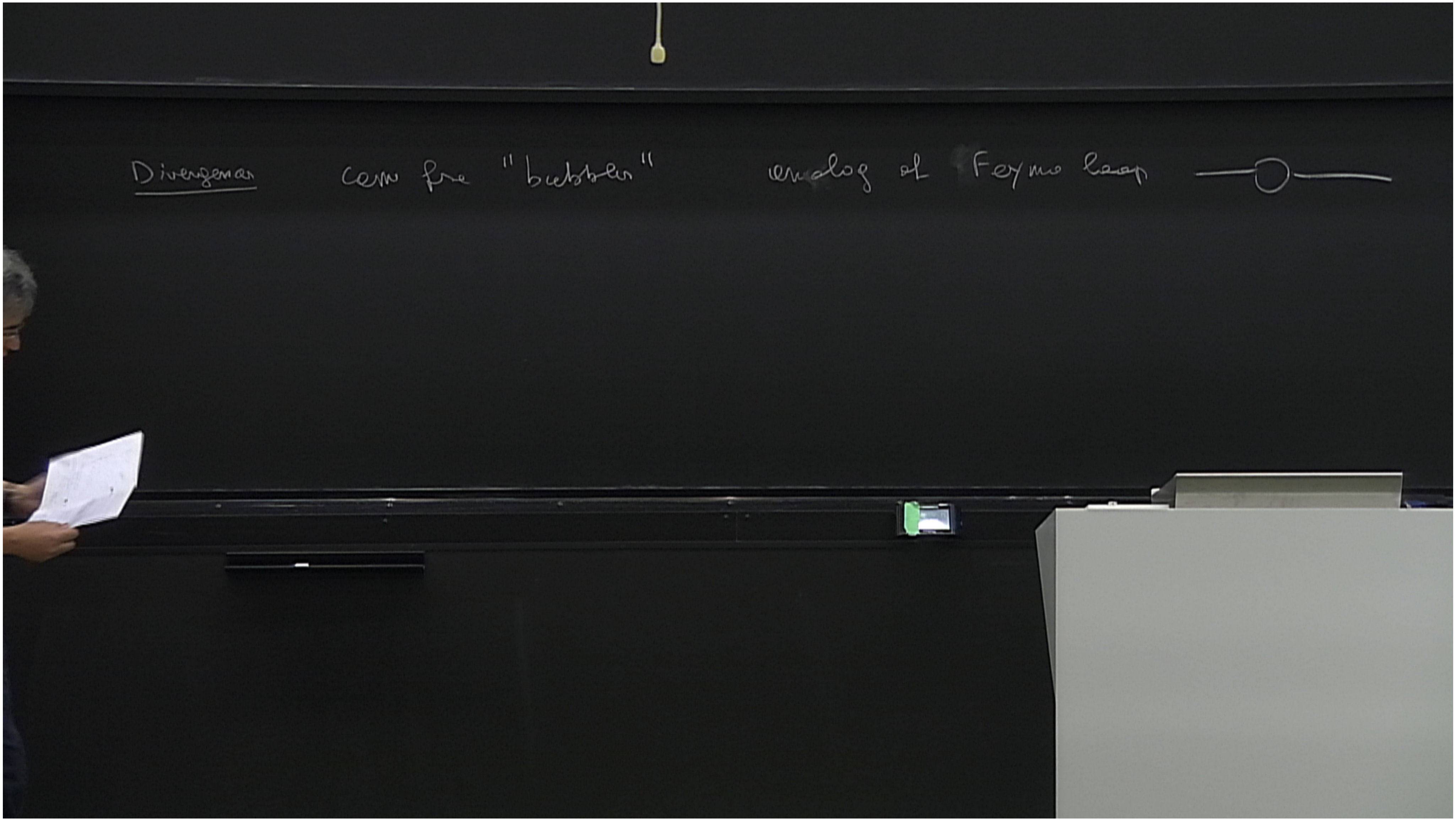
$$\{6j\} \underset{\text{large spin } j}{\sim} \frac{1}{\sqrt{12\pi V}} \left(e^{iS[\text{Tetrahed.}]} + e^{-iS[\text{Tetr.}]} + \frac{\pi}{4} \right)$$

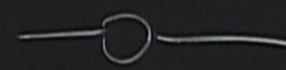
Regge Calculus: discrete GR on Δ-

$$S_T = \int e \wedge F \qquad S_{EH} = \int \sqrt{g}\; R$$

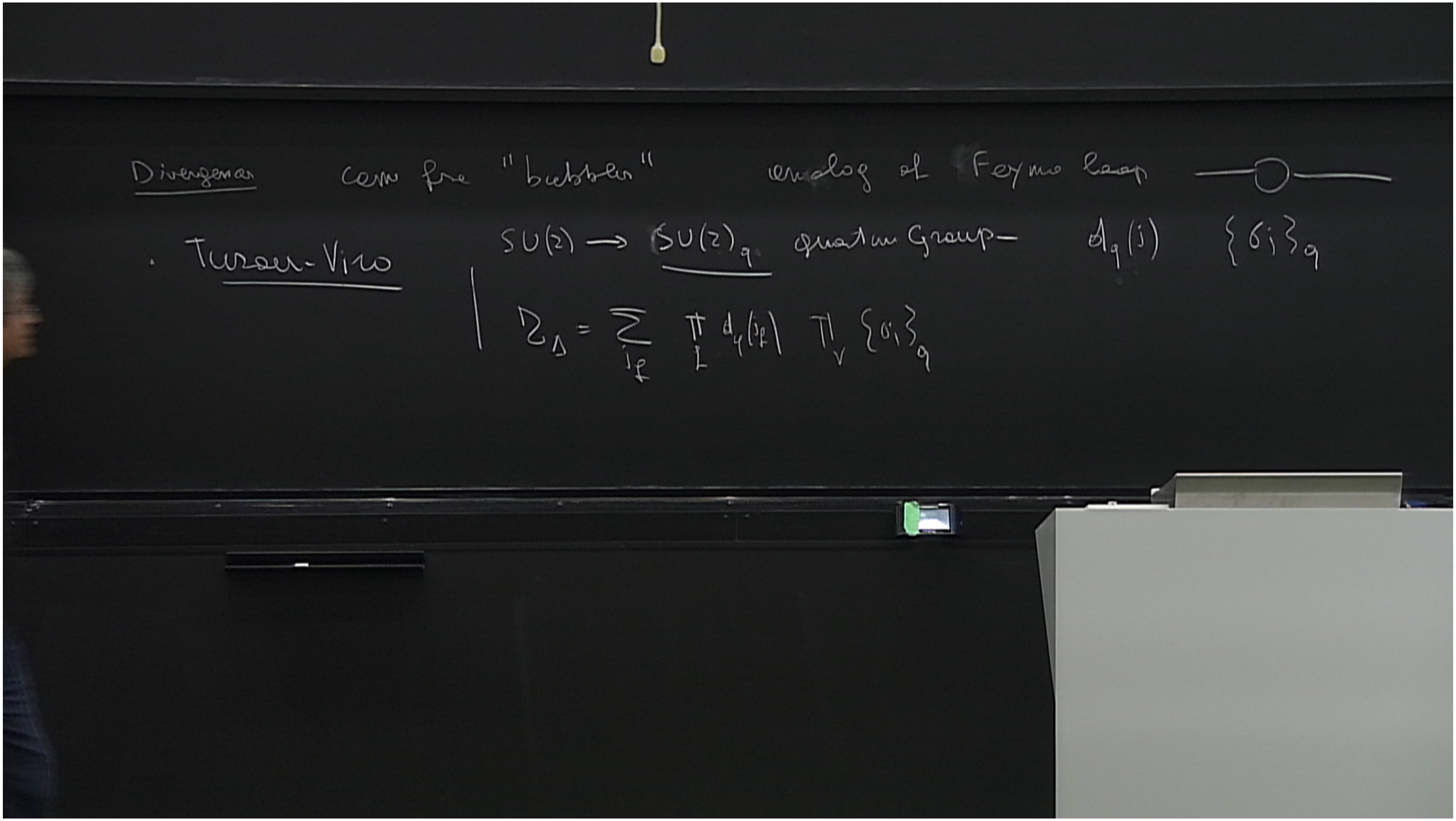
$$S_T = \int \det e \;\; R[e] \qquad S = \int |\det e| \; R[e]$$

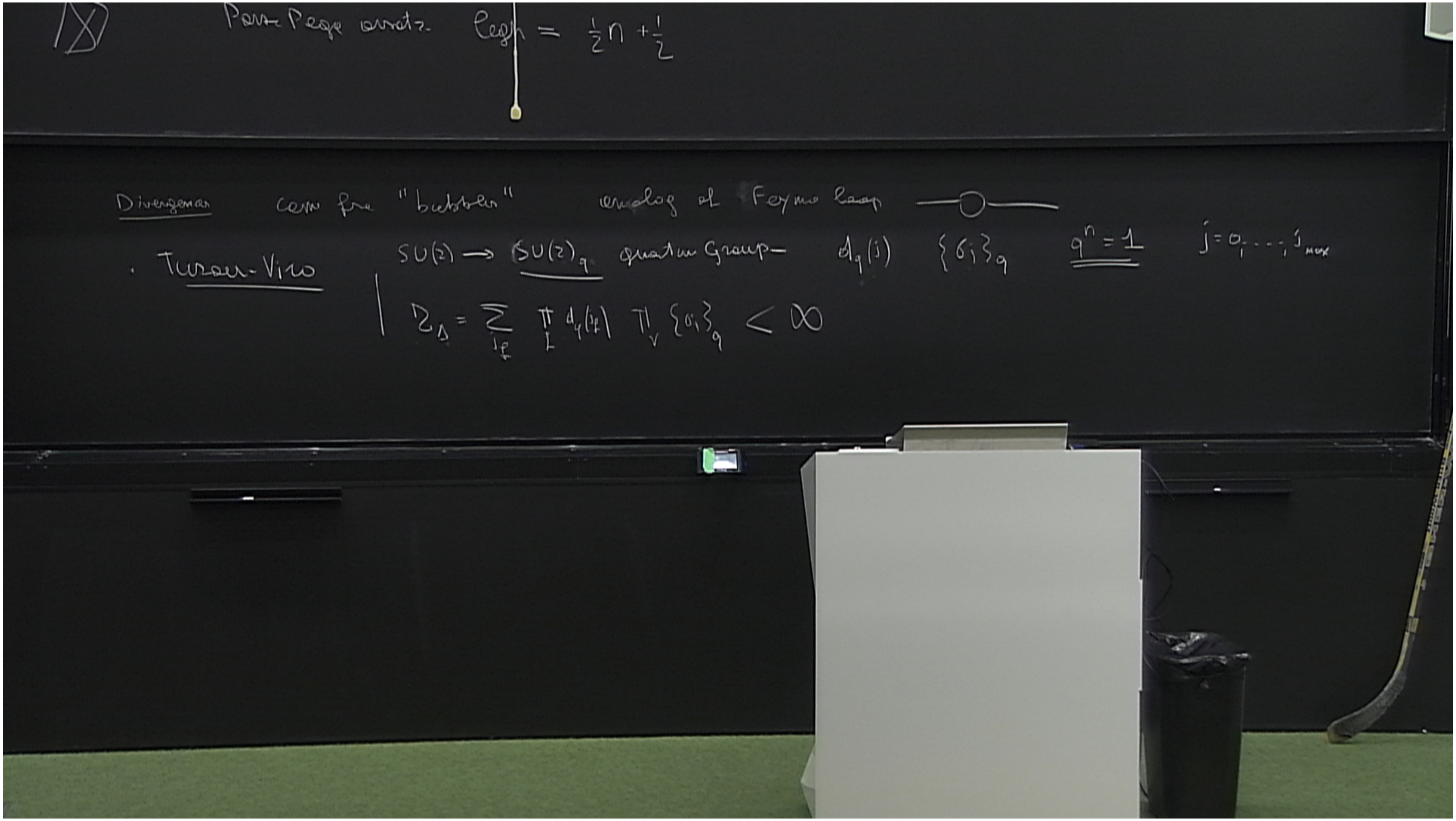

$Z_\Delta = \sum_{j_f} \prod_f d_{j_f} \prod_v \{6j\}$
$\{6j\} = \begin{Bmatrix} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \end{Bmatrix}$
$\{6j\}$ Wigner 6-j Symbol.
PONZANO-REGGE
MODEL FOR EUC 3d
QUANTUM GRAVITY

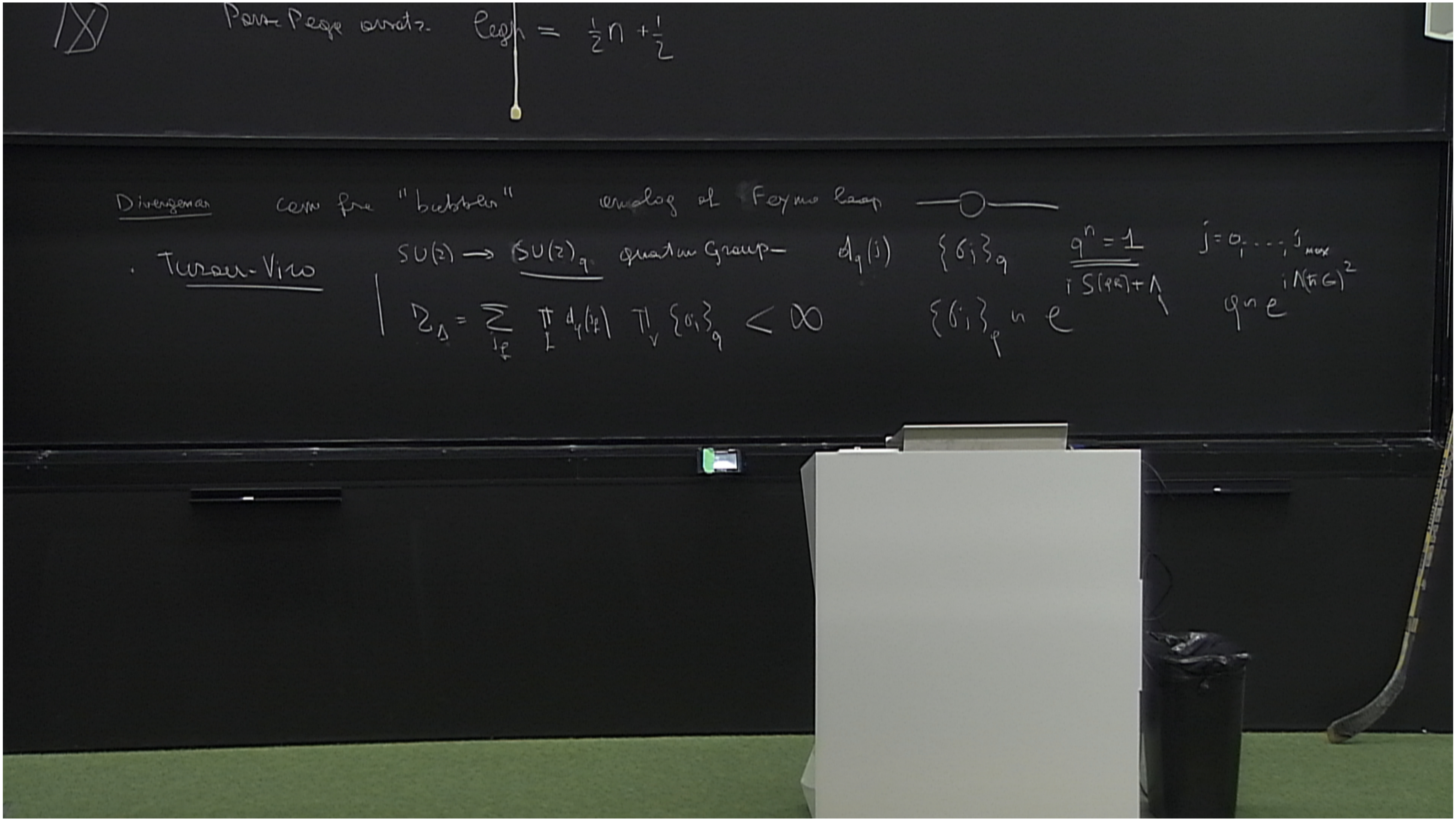

Divergences
come from "bubbles"
analog of Feynman loop

Divergences come from "bubbles" analog of Feynman loop 

· Turaev-Viro $SU(2) \longrightarrow SU(2)_q$ quantum Group


Divergences come from "bubbles" analog of Feynman loop
. Turaev-Viro
SU(2) → SU(2)_q quantum Group — d_q(j) {6j}_q
Z_Δ = Σ_{j_f} Π_f d_q(j_f) Π_v {6j}_q


Ponzano Regge ansatz. $\ell_{eff} = \frac{1}{2}n + \frac{1}{2}$
Divergences come from "bubbles" analog of Feynman loop
· Turaev-Viro $SU(2) \longrightarrow SU(2)_q$ quantum Group – $d_q(j)$ $\{6j\}_q$ $q^n = 1$ $j = 0, \ldots, j_{max}$
$Z_\Delta = \sum_{j_f} \prod_f d_q(j_f) \prod_v \{6j\}_q < \infty$


Ponzano-Regge ansatz. ℓ_eff = ½ n + ½
Divergences come from "bubbles" analog of Feynman loop
· Turaev-Viro
SU(2) → SU(2)_q quantum Group
d_q(j)
{6j}_q
q^n = 1
j = 0, ..., j_max
Z_Δ = Σ_{j_e} Π_e d_q(j_e) Π_v {6j}_q < ∞
{6j}_q ~ e^{i S(Regge) + Λ}
q ~ e^{i Λ(ħG)^2}