

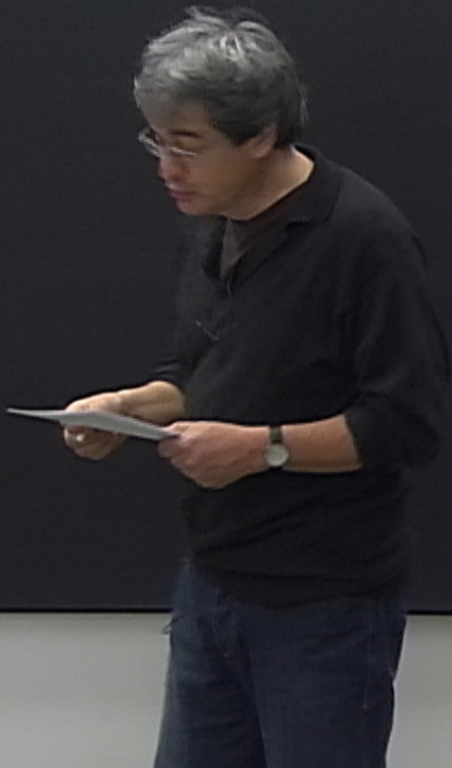
Title: Explorations in Quantum Gravity - Lecture 8

Date: Apr 12, 2012 10:15 AM

URL: <http://pirsa.org/12040029>

Abstract:

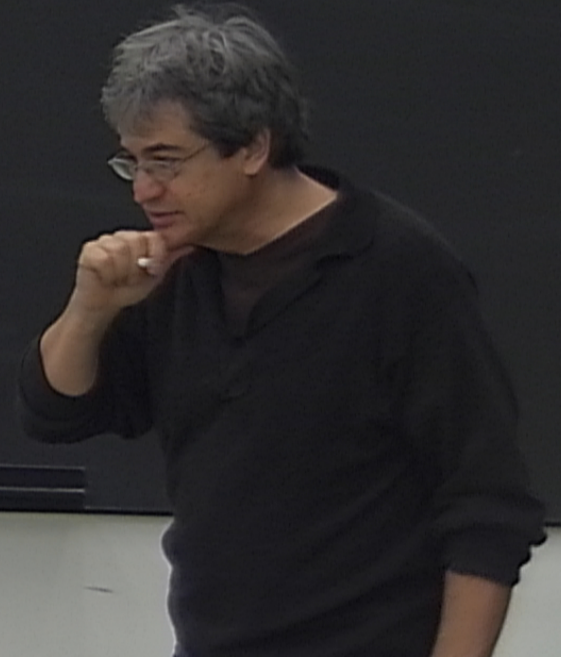
D John Beers



1) John Beers : GAUGE FIELDS, MONDS & PHYSICS -

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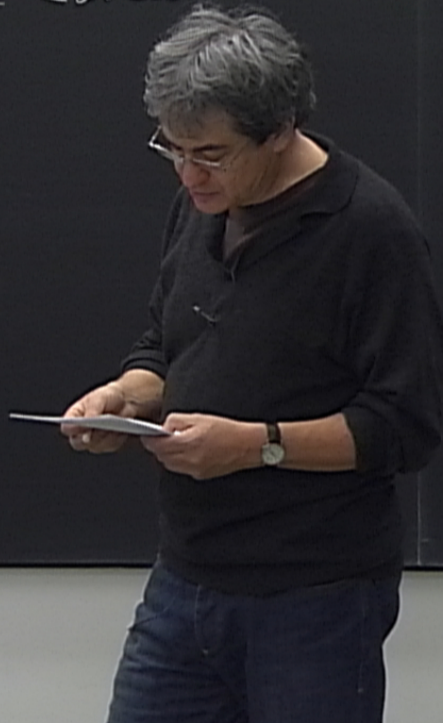
2) Kobayashi Nomizu : FOUNDATIONS OF DIFF. GEOM.



1) John Baez : GAUGE FIELDS, ANOTS & PHYSICS —  $\mathbb{R}$   $\mathbb{C}$   $\mathbb{H}$

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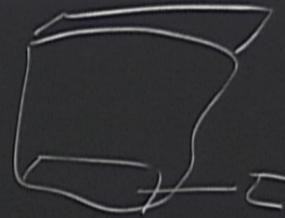
3) Jorge Zuretti : TORSIONAL TOPOLOGICAL INVARIANTS.



- 1) John Baez : GAUGE FIELDS, KNOTS & PHYSICS —  $\mathbb{R} \otimes \mathbb{C} \otimes \mathbb{H}$
- 2) Kobayashi Nomizu : FOUNDATIONS OF DIFF GEOM.
- 3) Jorge Zurelli TORSIONAL TOPOLOGICAL INVARIANTS.
- 4) Yvonne Choquet-Bruhat, Cecile DeWitt DeWitt, Michael Brinkmann : ANALYSIS MANIFOLDS  $\mathbb{R} \otimes \mathbb{C}$ .
- 5) Mikio Nakahara : GEOMETRY TOPOLOGY &  $\mathbb{C}$
- 6) Tholen Frenkel : THE GEOMETRY OF  $\mathbb{C}$

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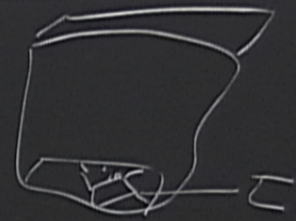
LECTURE VIII : TRANSITION AMPLITUDES



$j_e$

LECTURE VIII : TRANSITION AMPLITUDES

$$W_{\Delta}(j_e, j'_e)$$



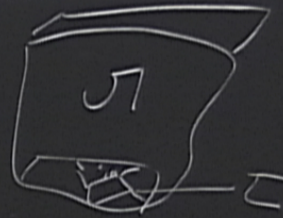


LECTURE VIII : TRANSITION AMPLITUDES

$$W_{\Delta}(j_e, j'_e) \approx \int d\text{geom in ball } e \quad iS[\varphi_{\text{geom}}]$$

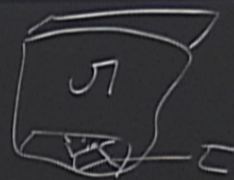
$d\text{geom} = \text{boundary state}$

$$\sum_{\Delta} = \int \text{Deform } e \quad iS(\varphi)$$



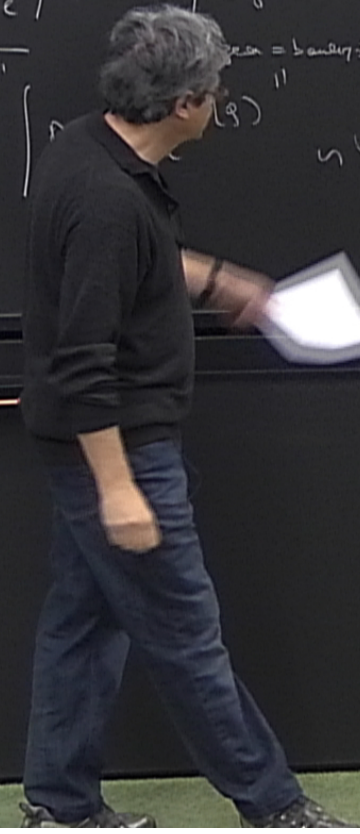
LECTURE VIII : TRANSITION AMPLITUDES

$W_{\Delta}(j_e, j'_e)$  "  $iS[\varphi_{cl}]$  "



$\int dx e^{ipx} = 2\pi \delta(p)$   
 $= \int D\omega$

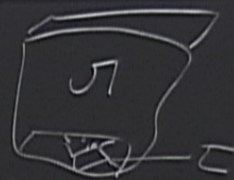
$\sum_{\Delta} = \int \dots$  "  $\frac{i}{8\pi G^2} \int d^3x e^{iF[\omega]}$  "



LECTURE VIII : TRANSITION AMPLITUDES

$$W_{\Delta}(j_e, j_e') \sim \int d\text{geometry in ball } e^{iS[\varphi_{\text{geom}}]}$$

$d\varphi_{\text{geom}} = \text{boundary field}$



$$\sum_{\Delta} = \int \text{De geom } e^{iS[\varphi]} \sim \int \text{De } \int \frac{i}{8\pi G} d^3x e^{iF[\omega]}$$

$$\int dx e^{iPx} = 2\pi \delta(P)$$

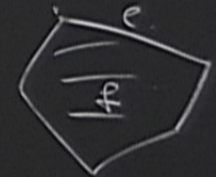
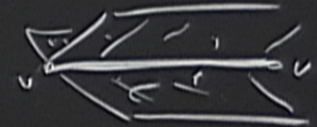
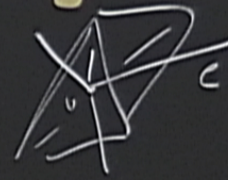
$$= \int D\omega \int_x \delta(F[\omega])$$



$$\int dx e^{ipx} = 2\pi \delta(p)$$

$$= \int D\omega \prod_x \delta(F[\omega])$$

$$\int \prod_f de_f dV_e e^{i \sum_f T_2(e_f V_e)}$$



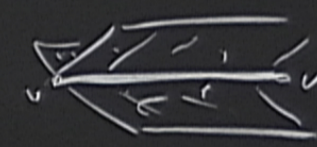
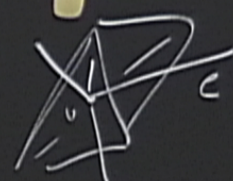
$$U_f = U_{e_1} \dots U_{e_n}$$

$$\int dx e^{ipx} = 2\pi \delta(p)$$

$$= \int D\omega \prod_x \delta(F[\omega])$$

$$= \int \prod_f de_f dV_e e^{i \sum_f T_f(e_f, V_e)}$$

=



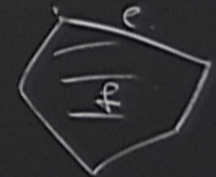
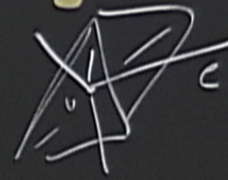
$$U_f = U_{e_1} \dots U_{e_n}$$

$$\int dx e^{ipx} = 2\pi \delta(p)$$

$$= \int D\omega \prod_x \delta(F[\omega])$$

$$\approx \int \prod_f de_f dU_e e^{i \sum_f T_f(e_f U_e)}$$

$$\int \prod_e dU_e \prod_f \delta(U_f)$$



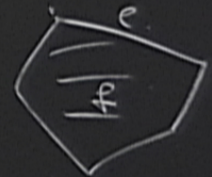
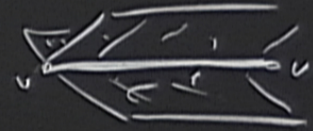
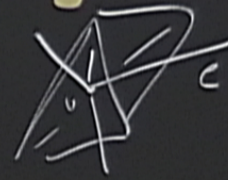
$$U_f = U_{e_1} \dots U_{e_n}$$

$$\int dx e^{iPx} = 2\pi \delta(P)$$

$$= \int D\omega \prod_x \delta(F[\omega])$$

$$\approx \int \prod_f de_f dU_e e^{i \sum_f T_f(e_f U_e)}$$

$$Z_{\Delta} = \int \prod_e dU_e \prod_f \delta(U_f)$$

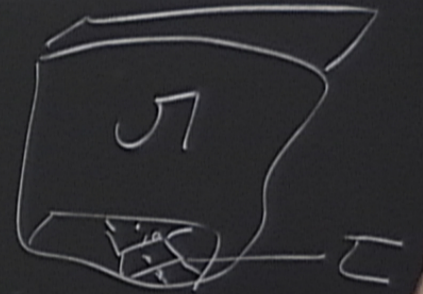


$$U_f = U_{e_1} \dots U_{e_n}$$

# LECTURE VIII : TRANSITION AMPLITUDES

$$W_{\Delta}(j_e, j'_e) \approx \int d\text{geom in ball } e^{iS[q, \omega]}$$

$d\text{geom} = \text{boundary state}$



$$\sum_{\Delta} = \int \text{Degeom } e^{iS[q]} \approx \int \frac{i}{8\pi G \hbar} d^3x e^{iF[\omega]}$$



$$\delta(u) = \sum_j d_j T_2 \overset{G_1}{D}(u)$$

$u(1) \approx d$

$$\delta(\alpha) = \sum_{\alpha}$$

$$\delta(u) = \sum_j d_j T_2 \overset{(\sigma)}{D}(u)$$

$$u(1) \approx d \quad 2\pi \delta(\alpha) = \sum_n e^{in\alpha}$$

$$g(u) = \sum_j d_j T_2 \vec{D}^j(u)$$

$$u(1) \rightarrow a \quad z = g(a) = \sum_n e^{in a}$$

$$\dot{u} = \dot{g}(u)$$

$$D(uv) = D(u)D(v)$$

$$\Sigma_{\Delta} = \int DU_c \prod_f \left( \sum_{j_f} d_{j_f} T_2 \vec{D}^{j_f}(u) \right) = \sum_{\{j_f\}} \left( \prod_f d_{j_f} \right) \int DU_c \prod_f T_2 \left( \vec{D}^{j_f}(u) \right)$$

$$\int DU \vec{D}^{j_1}(u) \vec{D}^{j_2}(u) \vec{D}^{j_3}(u) = \begin{pmatrix} m_1 & m_2 & m_3 \\ j_1 & j_2 & j_3 \end{pmatrix} \begin{pmatrix} n_1 & n_2 & n_3 \\ i_1 & i_2 & i_3 \end{pmatrix} = \begin{bmatrix} m_1 & m_2 & m_3 \\ j_1 & j_2 & j_3 \end{bmatrix} \begin{bmatrix} n_1 & n_2 & n_3 \\ i_1 & i_2 & i_3 \end{bmatrix}$$

$$\delta(u) = \sum_j d_j T_2 \dot{D}^j(u)$$

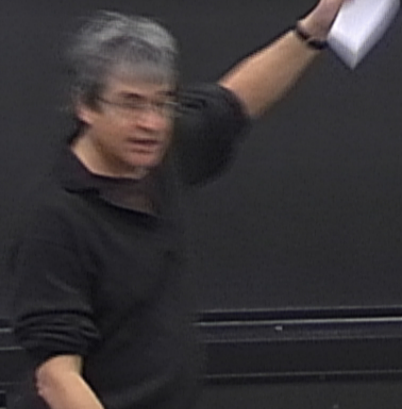
$$u(1) \approx \alpha \quad \alpha = \delta(\alpha) = \sum_n e^{in\alpha} \quad \dot{u} = \dot{D}(u) \quad D(uv) = D(u)D(v)$$

$$\Sigma_\Delta = \int dU_e \prod_f \left( \sum_{j_f} d_{j_f} T_2 \dot{D}^{j_f}(u_f) \right) = \sum_{j_f} \left( \prod_f d_{j_f} \right) \int dU_e \prod_f T_2 \left( \dot{D}^{j_f}(u_{e_f}) \right)$$

$$\int_{SU(2)} dU \dot{D}^{j_1}(u) \dot{D}^{j_2}(u) \dot{D}^{j_3}(u) = \begin{pmatrix} m_1 & m_2 & m_3 \\ j_1 & j_2 & j_3 \end{pmatrix} \begin{pmatrix} n_1 & n_2 & n_3 \\ j_1 & j_2 & j_3 \end{pmatrix} = \begin{bmatrix} m_1 & m_2 & m_3 \\ j_1 & j_2 & j_3 \end{bmatrix} \begin{bmatrix} n_1 & n_2 & n_3 \\ j_1 & j_2 & j_3 \end{bmatrix}$$

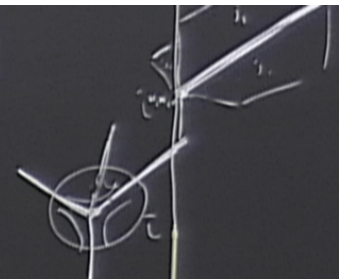


$$\Sigma_\Delta = \sum_{j_f} \left( \prod_f d_{j_f} \right) \prod L_i \dots$$



$$\Sigma_D = \int DU_c \prod_f \left( \sum_{j_f} d_{j_f} T_2 \hat{D}(U_f) \right) = \sum_{j_f} \left( \prod_f d_{j_f} \right) \int DU_c \prod_f T_2 \left( \hat{D}(U_{f_1}) \dots \hat{D}(U_{f_n}) \right)$$

$$\int_{SU(2)} dU \hat{D}(U) \hat{D}(U) \hat{D}(U) = \begin{pmatrix} m_1 & m_2 & m_3 \\ j_1 & j_2 & j_3 \end{pmatrix} \begin{pmatrix} n_1 & n_2 & n_3 \\ i_1 & i_2 & i_3 \end{pmatrix} = \begin{pmatrix} m_1 & m_2 & m_3 \\ j_1 & j_2 & j_3 \end{pmatrix} \begin{pmatrix} n_1 & n_2 & n_3 \\ i_1 & i_2 & i_3 \end{pmatrix}$$

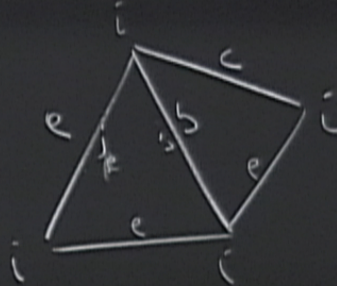


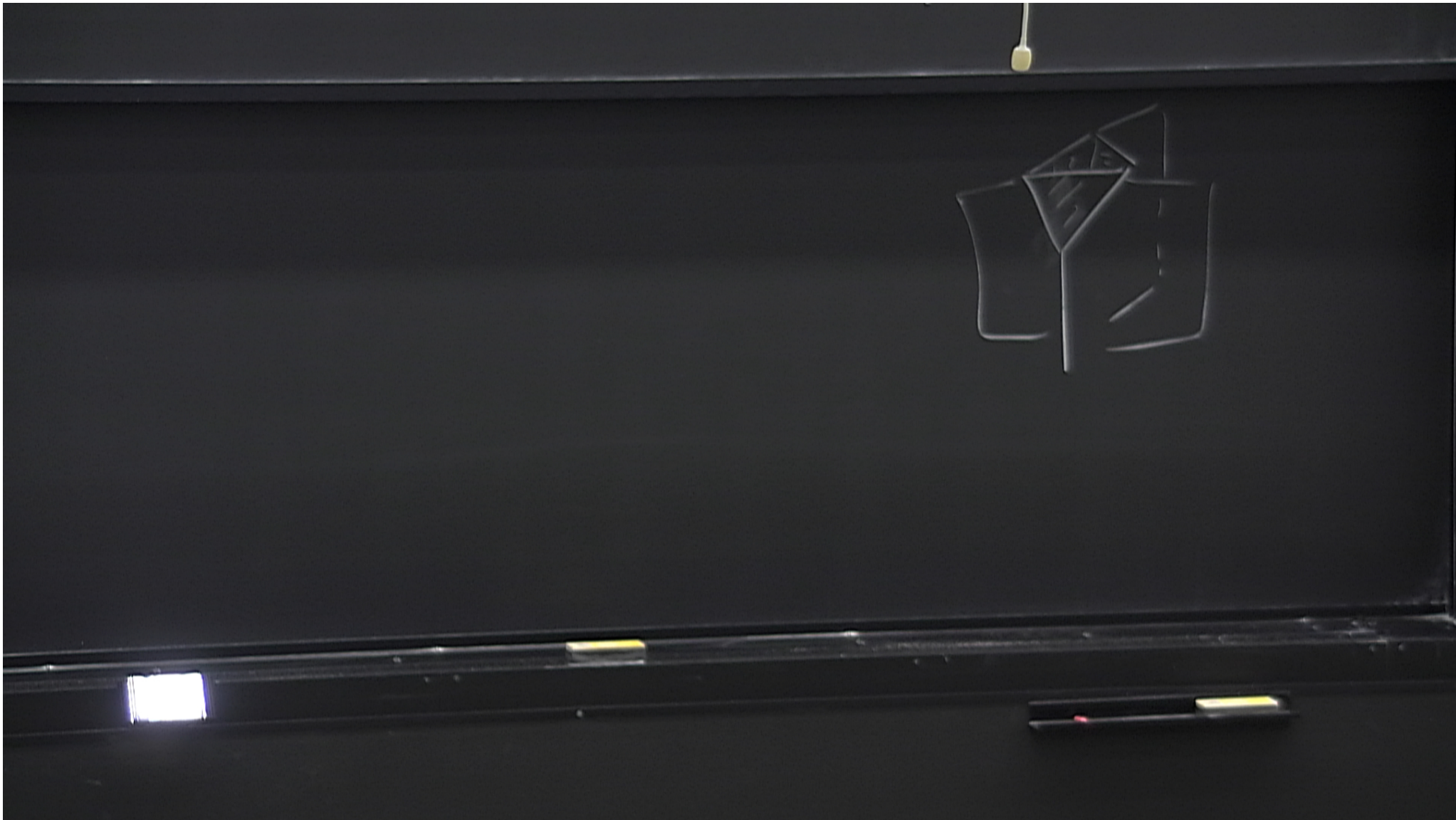
$$\Sigma_A = \sum_{j_f} \left( \prod_f d_{j_f} \right) \prod \dots$$

$$\begin{pmatrix} a & b & c & d \\ e & f & g & h \end{pmatrix}$$

$$\sum_A = \sum_{i_f} \left( \prod_f d_{i_f} \right) \prod i \dots i \dots i \dots$$

$$\left( \begin{matrix} i e b c d \\ i e b e \\ i c e f \\ i d f \end{matrix} \right)$$

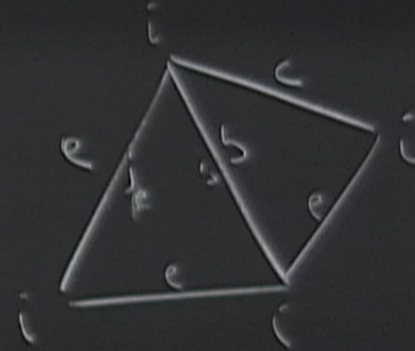




$$\Sigma_A = \sum_{j \in I} \binom{\Pi d_{ij}}{f} \Pi \dots \dots \dots$$

$$\sum_{abcd \in P} \binom{abcd \cdot ebe \cdot cef \cdot daf}{f}$$

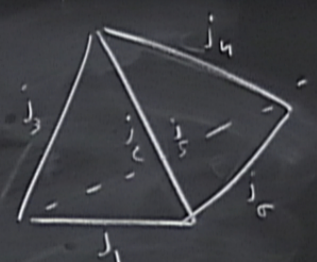
$\{ \sigma_j \}$  Kapranov  $\sigma$ -j Symbole.





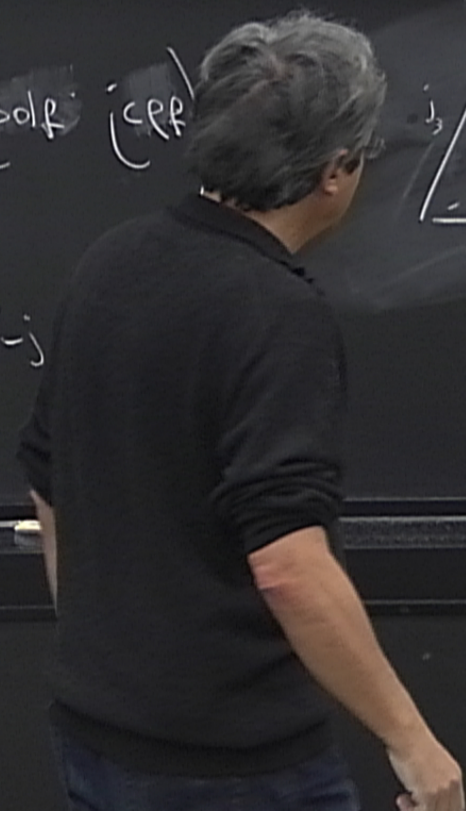
$$\Sigma_A = \sum_{j_f} \binom{d_{j_f}}{f} \prod i^{\dots} i^{\dots} \dots$$

$$\sum_{abc, cde, bdf, cef} (j_{bc} j_{cd} j_{db} j_{cf})$$



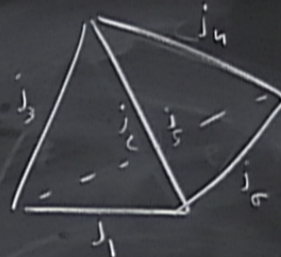
$$= \{ \sigma_i \} = \left\{ \begin{matrix} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \end{matrix} \right\}$$

$\{ \sigma_i \}$  Kugeln  $\sigma_i$



$$\Sigma_A = \sum_{j_f} \left( \prod_f d_{j_f} \right) \prod i \dots i \dots i \dots$$

$$\sum_{abcd \dots} \left( i_{abc} i_{ade} i_{bdf} i_{cpe} \right)$$



$$= \{ \sigma_i \} = \left\{ \begin{array}{ccc} i_1 & i_2 & i_3 \\ i_4 & i_5 & i_6 \end{array} \right\}$$

$\{ \sigma_i \}$  Krupner  $\sigma$ -j Symbole.

SU(2)

$$\Sigma_{\Delta} = \sum_{j_f} \left( \prod_f d_{j_f} \right) \prod_i i^{\dots} i^{\dots} \dots$$

$$= \boxed{\Sigma_{\Delta} = \sum_{j_f} \prod_f d_{j_f} \prod_v \{ \sigma_j \}}$$

$$\sum_{\text{dual cell}} \left( j_{bc} \ j_{cd} \ j_{df} \ j_{cf} \right)$$



$$= \{ \sigma_j \} = \begin{Bmatrix} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \end{Bmatrix}$$

$\{ \sigma_j \}$  Krupner  $\sigma$ -j Symbole.

$\Pi$   $d$   $\Pi$   $\{0; \}$   
 $\frac{1}{2}$   $\frac{1}{2}$   
 $v$



$j_2$   $j_3$   
 $j_5$   $j_6$

PONZANO-REGGE  
MODEL FOR EUCLIDEAN 3D  
QUANTUM GRAVITY

$$\{G_j\} \xrightarrow{\text{log spin}} \frac{1}{\sqrt{12\pi V}} \left( e^{iS[\text{tetrahed.}]} + e^{-iS[\text{tetrah.}]} + \frac{\mathbb{F}}{4} \right)$$

Regge Calculus: discretized GR on  $\Delta$

$$\int_{\mathcal{T}} = \int e \wedge F$$

$$S_{\text{EH}} = \int \sqrt{g} R$$

$$S_{\mathcal{T}} = \int \text{det } e R[e]$$

$$S = \int |\text{det } e| R[e]$$

$$Z_{\Delta} = \sum_{j_f} \left( \prod_f d_{j_f} \right) \prod \dots$$

$$= \sum_{\Delta} \sum_{j_f} \prod_f d_{j_f} \prod_V \{G_j\}$$

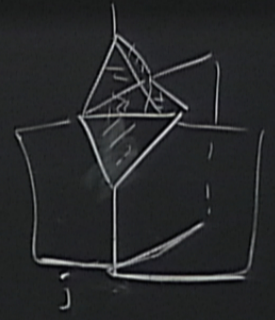
$$\sum_{abcd} \left( \begin{matrix} abc & aqc & bcp & cfp \end{matrix} \right)$$



$$= \{G_j\} = \left\{ \begin{matrix} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \end{matrix} \right\}$$

$\{G_j\}$  Kronecker  $\delta_{ij}$  Symbol.

PONZANO-REGGE  
MODEL FOR EUC 3d  
QUANTUM GRAVITY.



Divergeman

com for "bubbles"

analog of Feynman loop

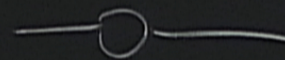




Divergences

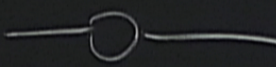
come from "bubbles"

analog of Feynman loop



• Turaev-Viro

$SU(2) \rightarrow \underline{SU(2)}_q$  quantum group


Divergences come from "bubbles" analog of Feynman loop 

• Turaev-Viro  $SU(2) \rightarrow \underline{SU(2)}_q$  quantum group -  $d_q(j)$   $\{\sigma_i\}_q$

$$\left| \sum_{\mathcal{I}_F} \prod_L d_q(j_F) \prod_V \{\sigma_i\}_q \right.$$

X

Poincaré series:  $P_{\text{graph}} = \frac{1}{2}n + \frac{1}{2}$


Dimensional case for "bubbles" analog of Feynman loop 

• Turaev-Viro  $SU(2) \rightarrow SU(2)_q$  quantum group  $d_q(j)$   $\{ \sigma_i \}_q$   $q^n = 1$   $j = 0, \dots, j_{\text{max}}$

$$\sum_{\Delta} \prod_L d_q(l_L) \prod_V \{ \sigma_i \}_q < \infty$$



Point-Point analog:  $\text{Egen} = \frac{1}{2}n + \frac{1}{2}$

Dirac-geman com fra "bubster" analog of Feynman loop 

• Turau-Viro  $SU(2) \rightarrow SU(2)_q$  quantum group  $d_q(j)$   $\{b_i\}_q$   $\frac{q^n = 1}{iS(n+1)}$   $j = 0, \dots, j_{\max}$   $iN(n+1)^2$   $q \in \mathbb{C}$

$$\sum_{\Delta} = \sum_{\mathbb{F}} \prod_L d_q(b_L) \prod_V \{a_i\}_q < \infty$$

$\{b_i\}_q \in \mathbb{C}$