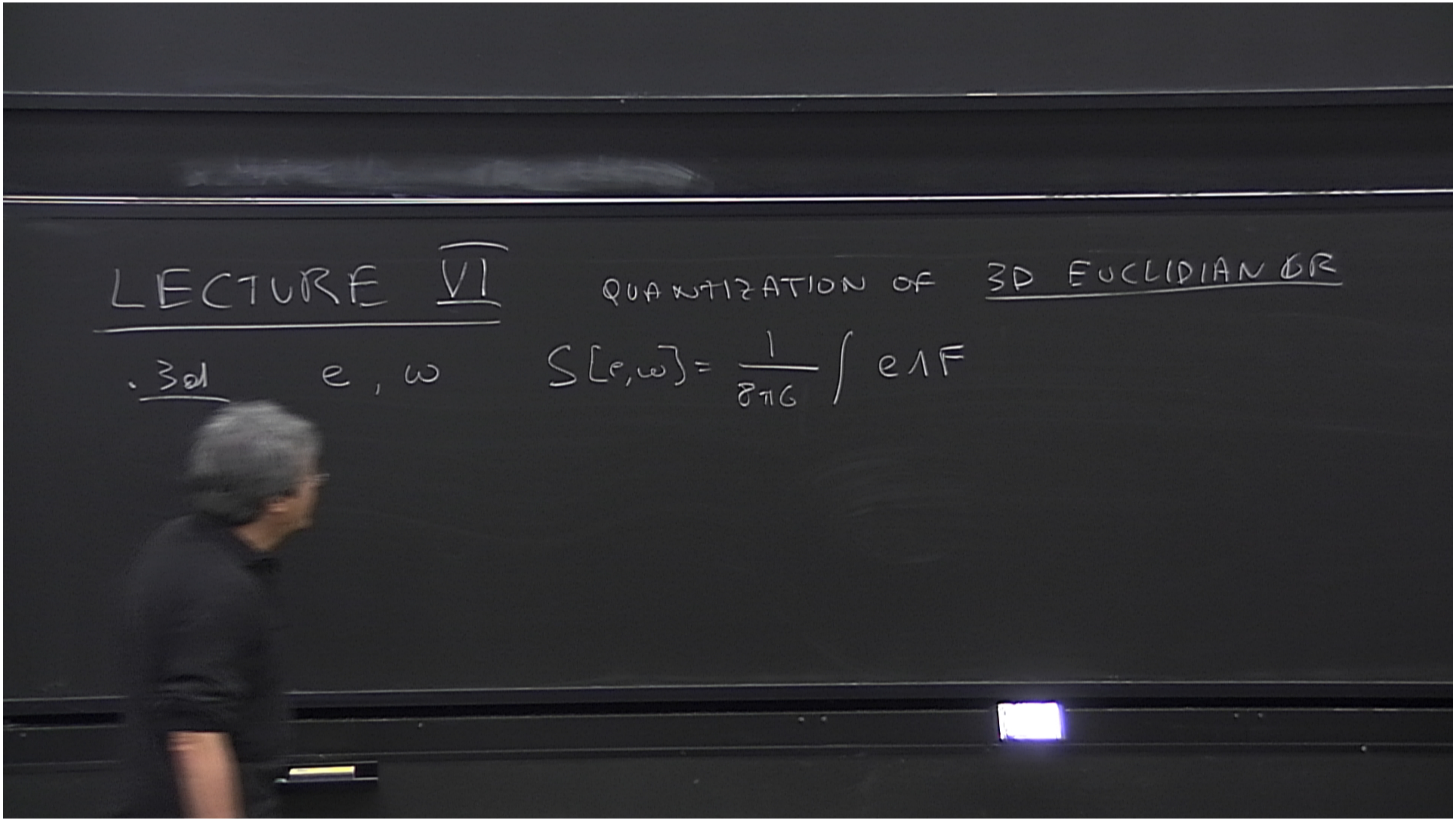
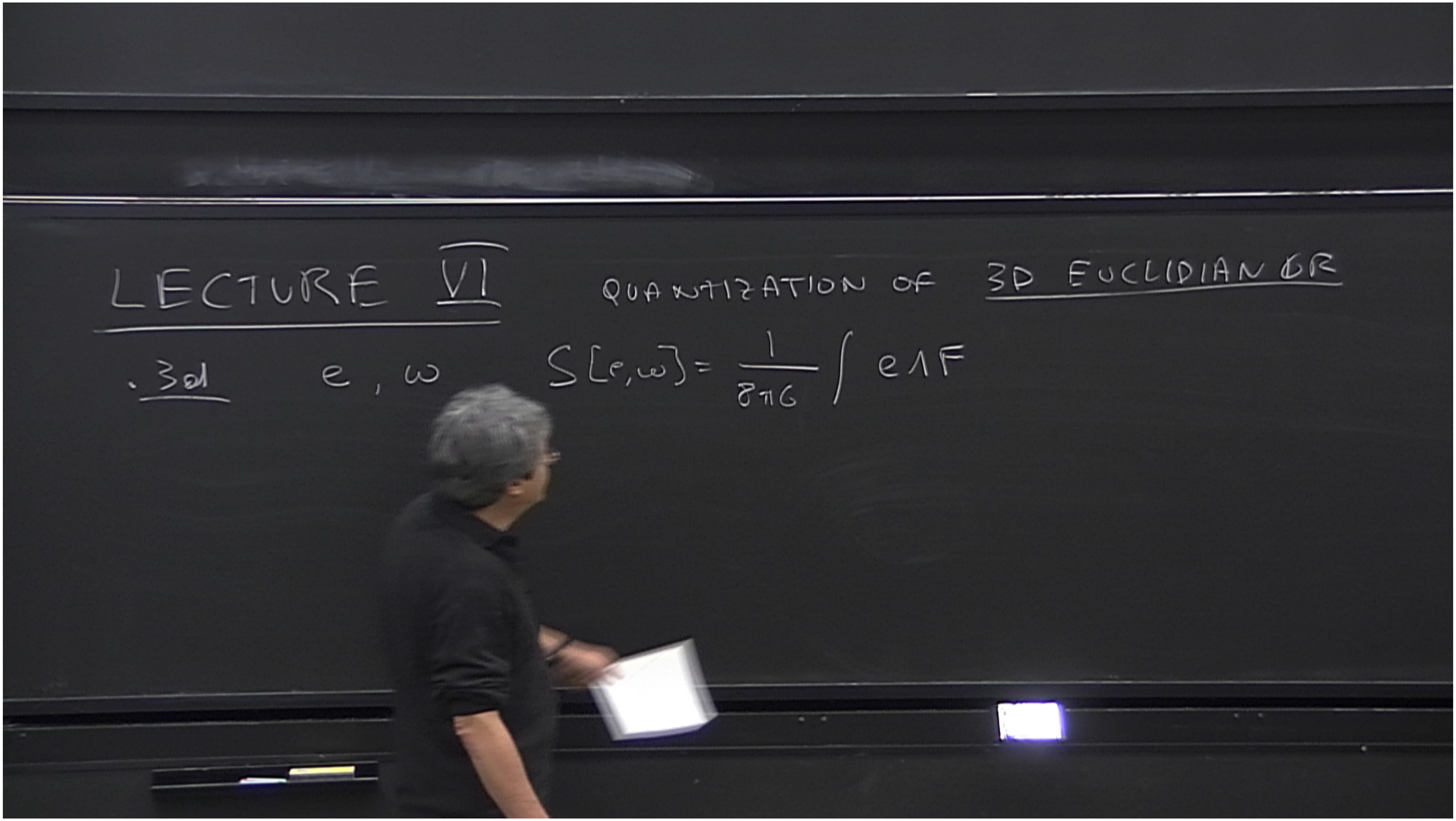
Title: Explorations in Quantum Gravity - Lecture 6

Date: Apr 10, 2012 10:15 AM

URL: http://pirsa.org/12040027

Abstract:


LECTURE VI
QUANTIZATION OF 3D EUCLIDIAN GR
· 3d
e, ω
$S[e,\omega] = \frac{1}{8\pi G} \int e \wedge F$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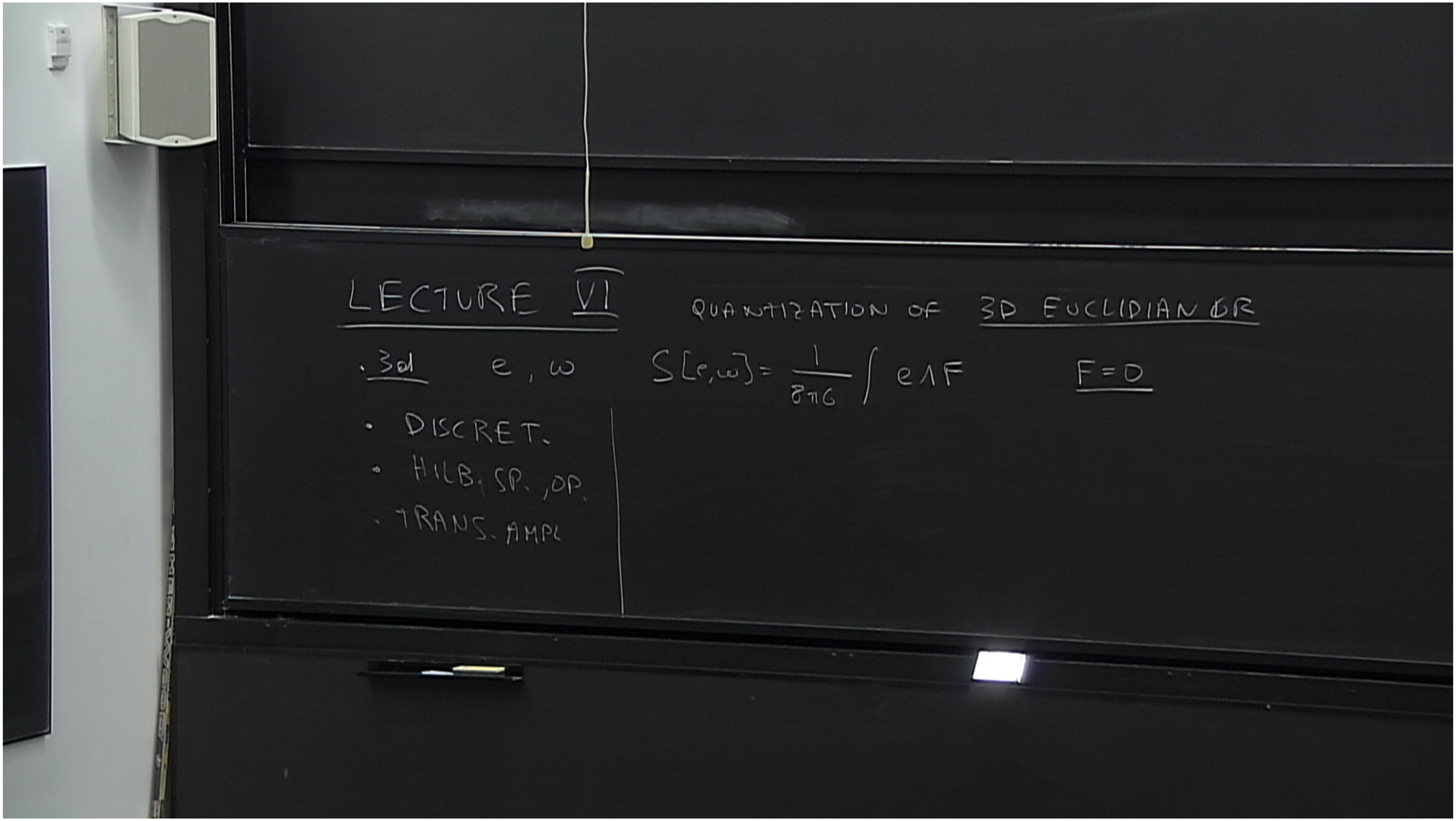
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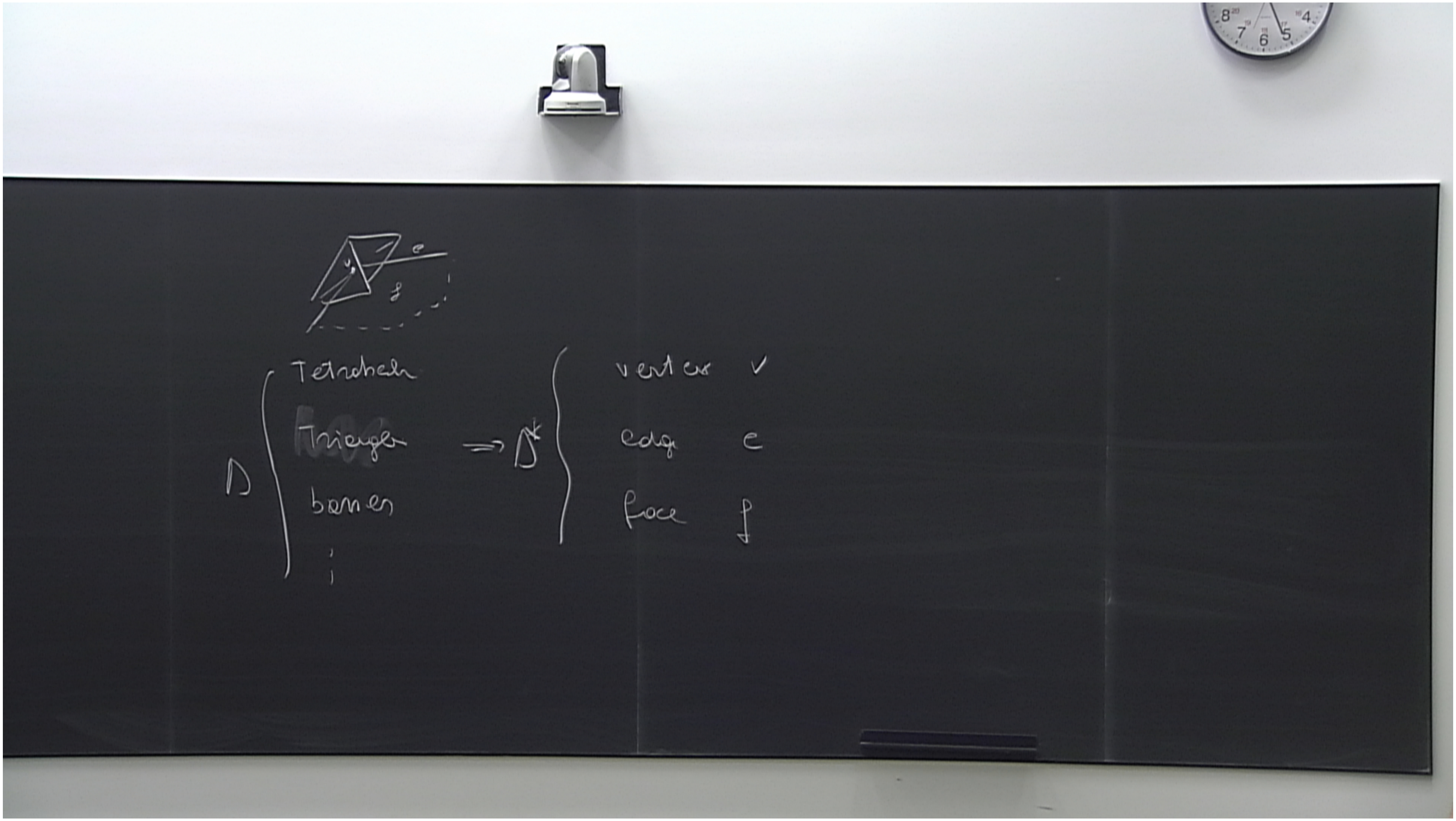
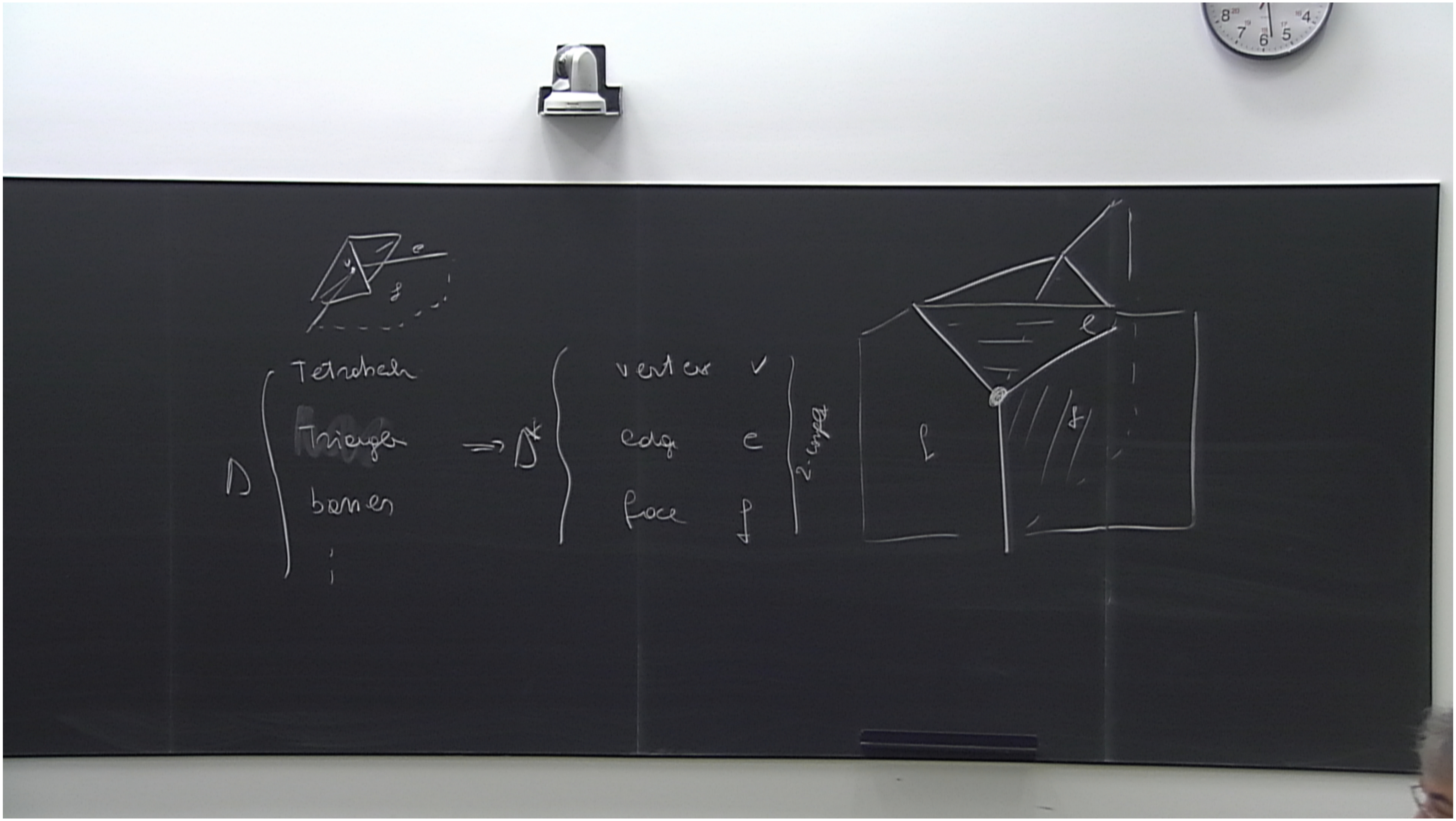
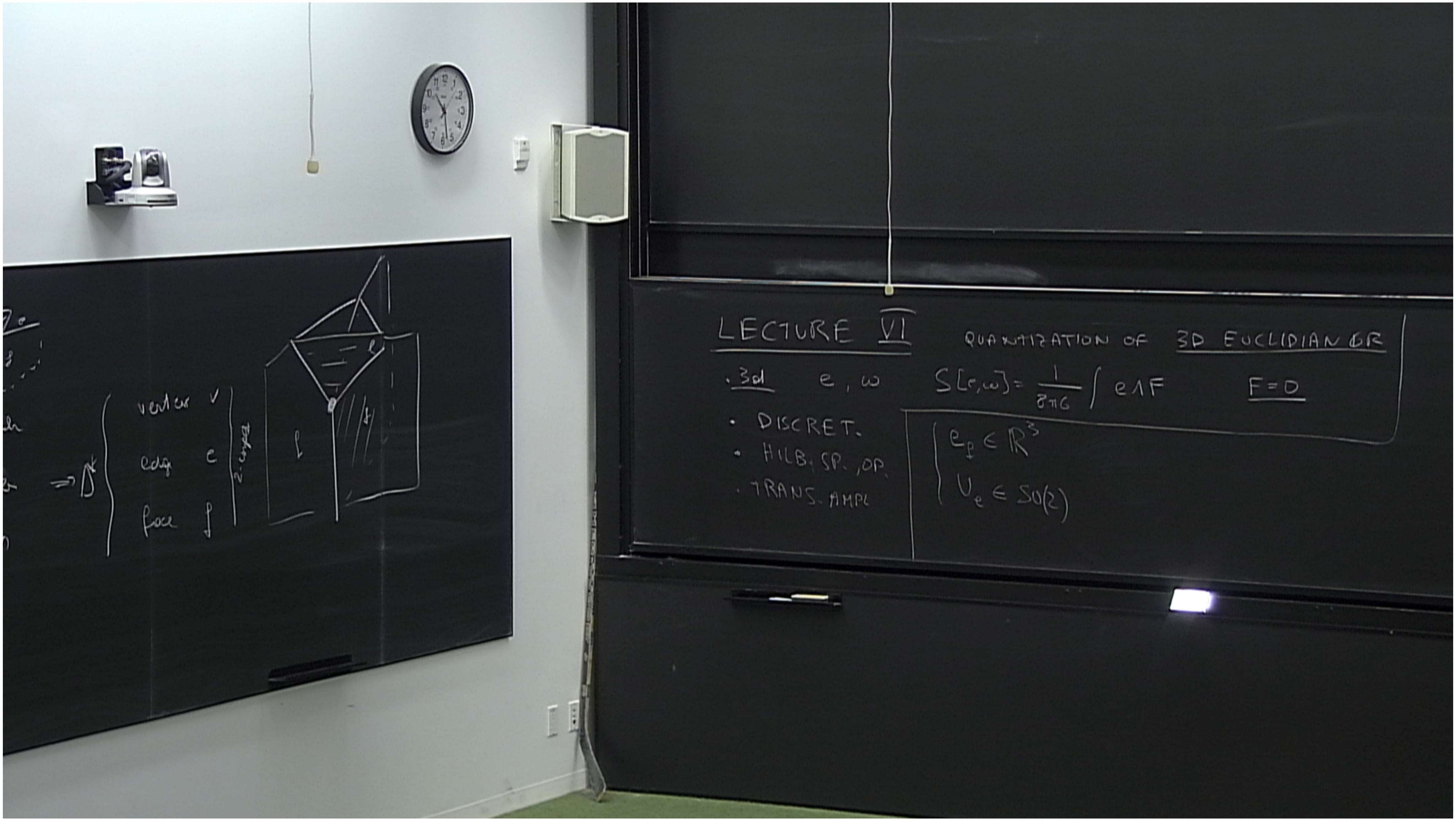
S[e,ω] = 1/8πG ∫ e∧F
F=0
• DISCRET.
• HILB. SP., OP.
. TRANS. AMPL


vertex v
edge e
face f


Tetrahedra
Triangles
$\Delta \Rightarrow \Delta^*$
vertex v
edge e
face f
2-complex



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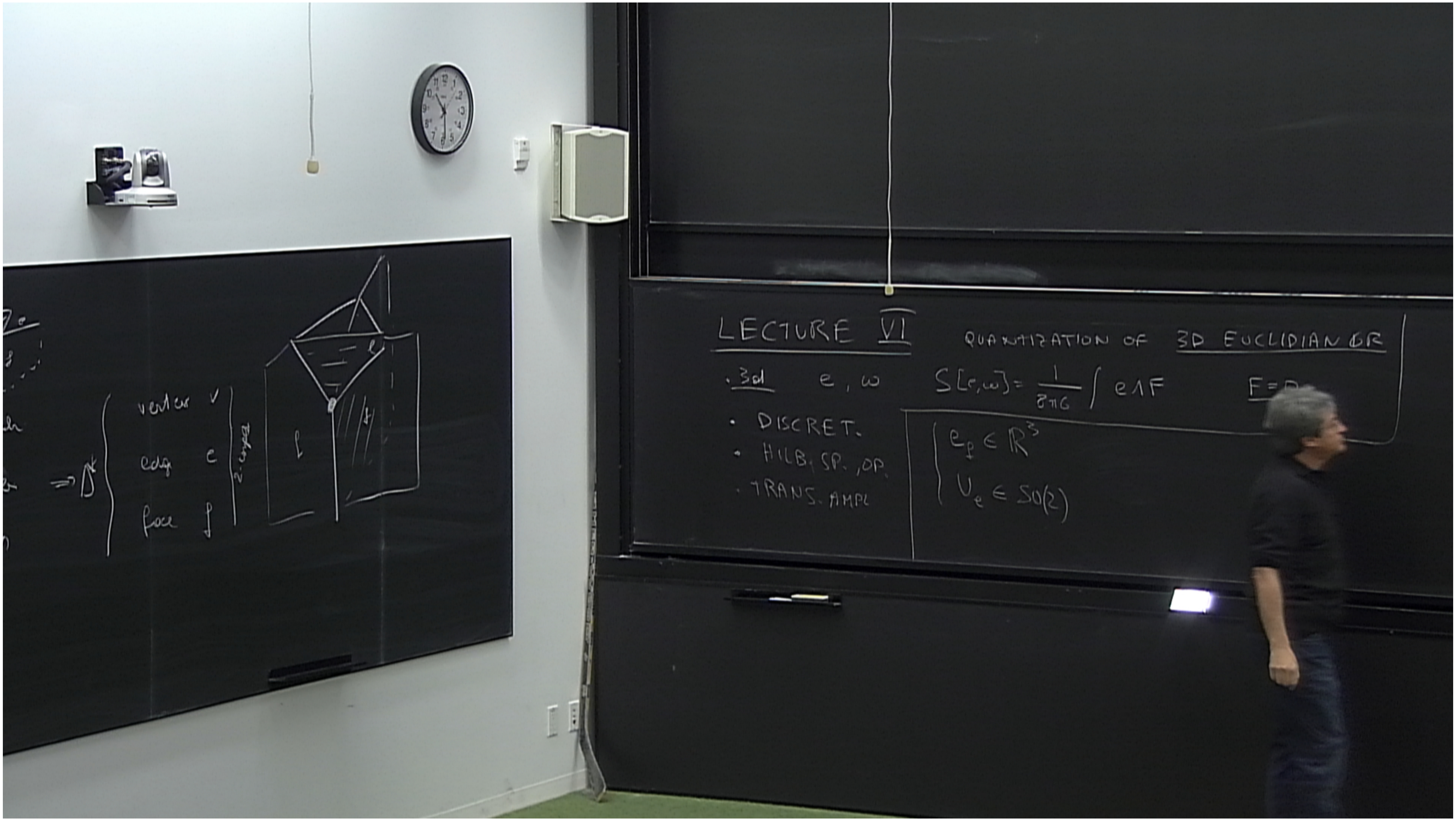
· TRANS. AMPL
$e_f \in \mathbb{R}^3$
$U_e \in SO(2)$
vertex v
edge e
face f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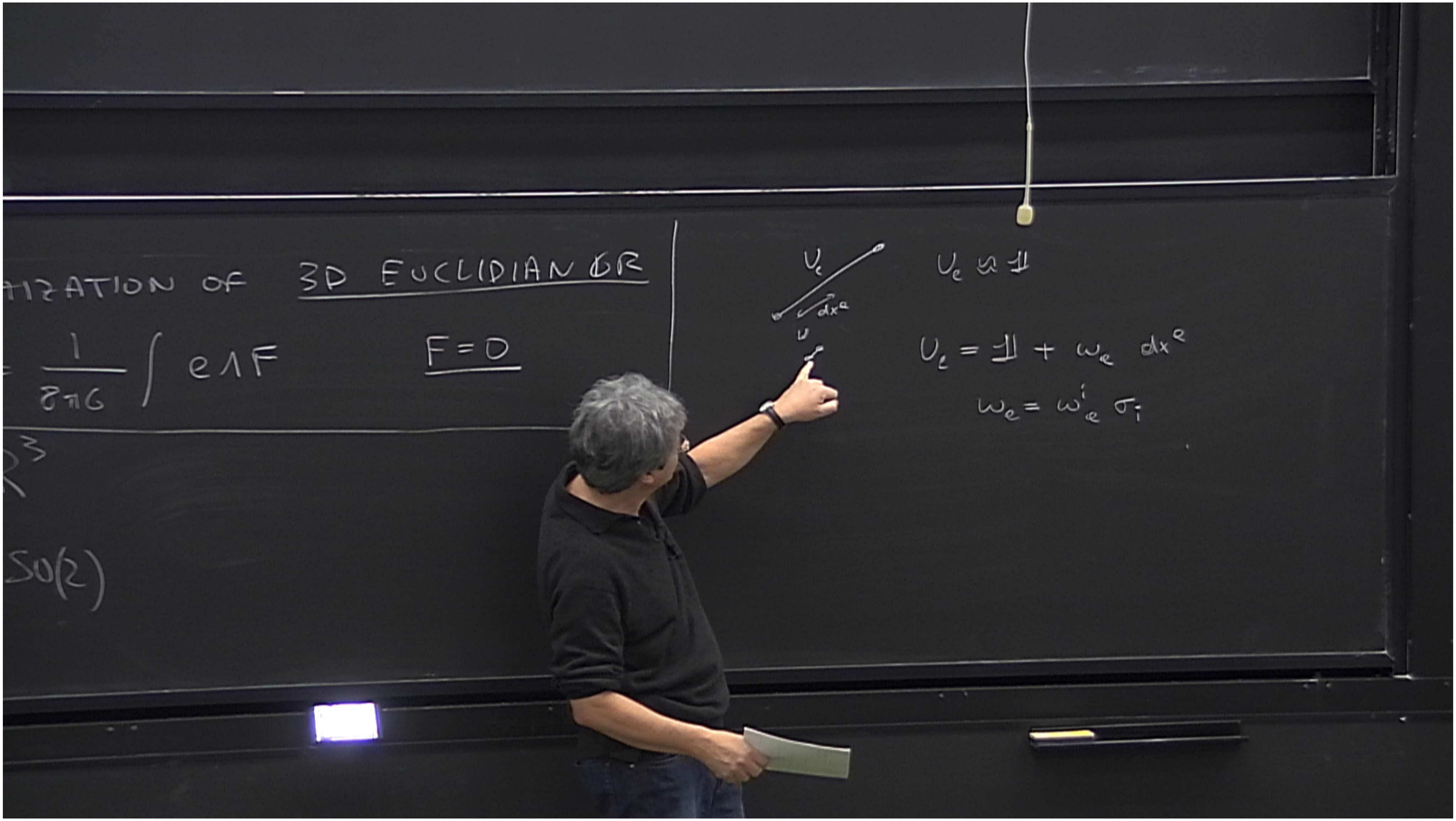
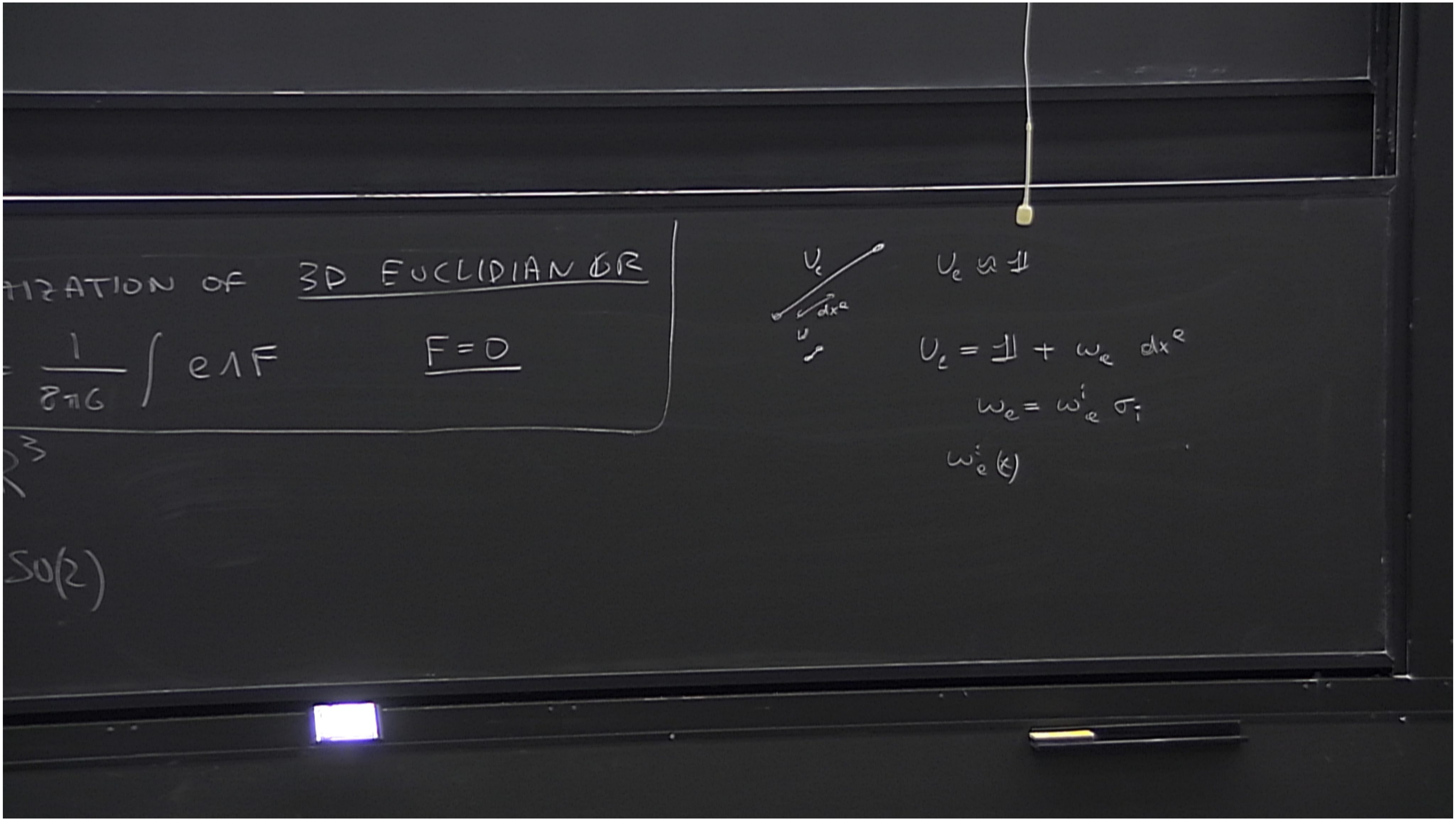
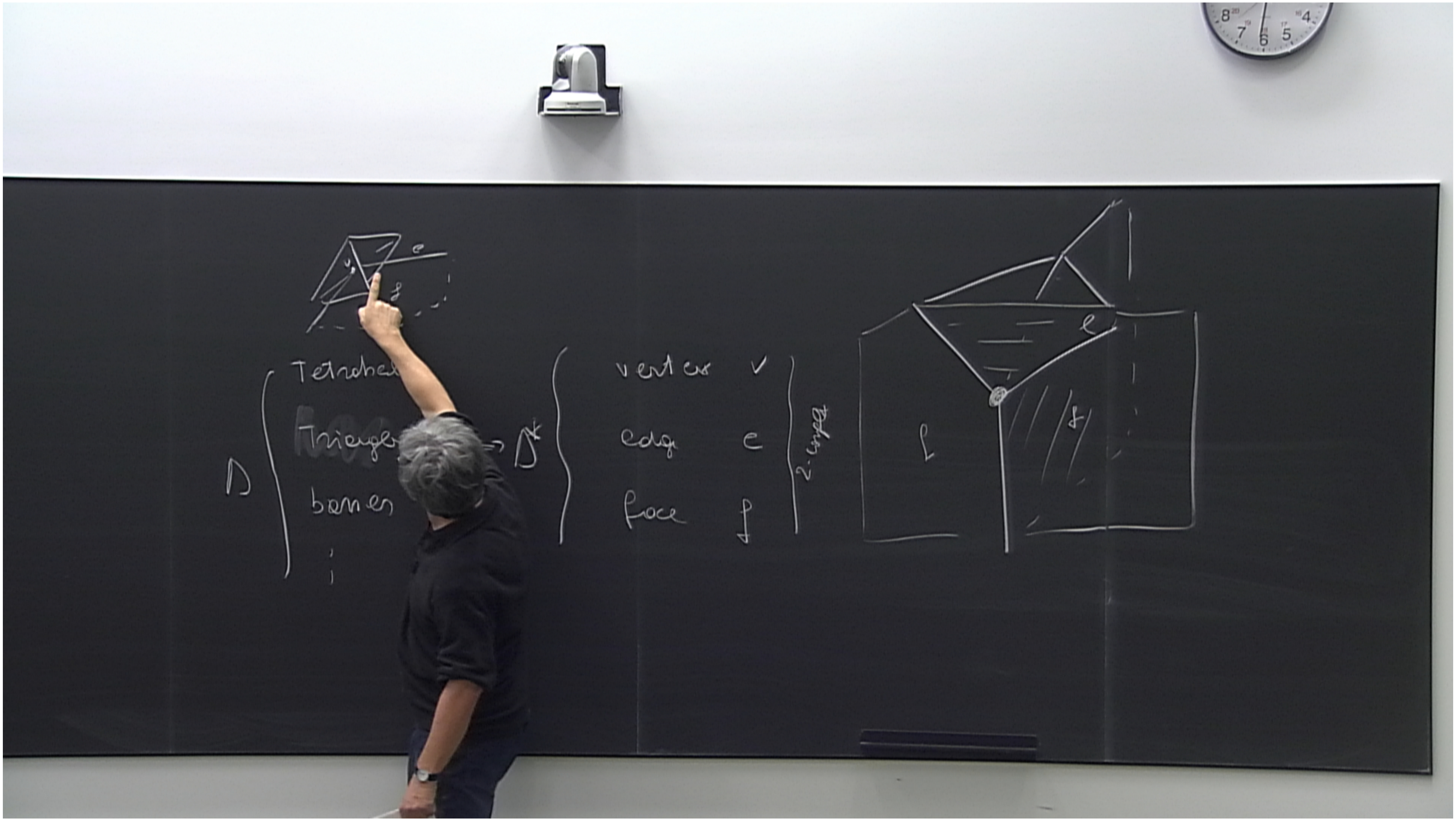
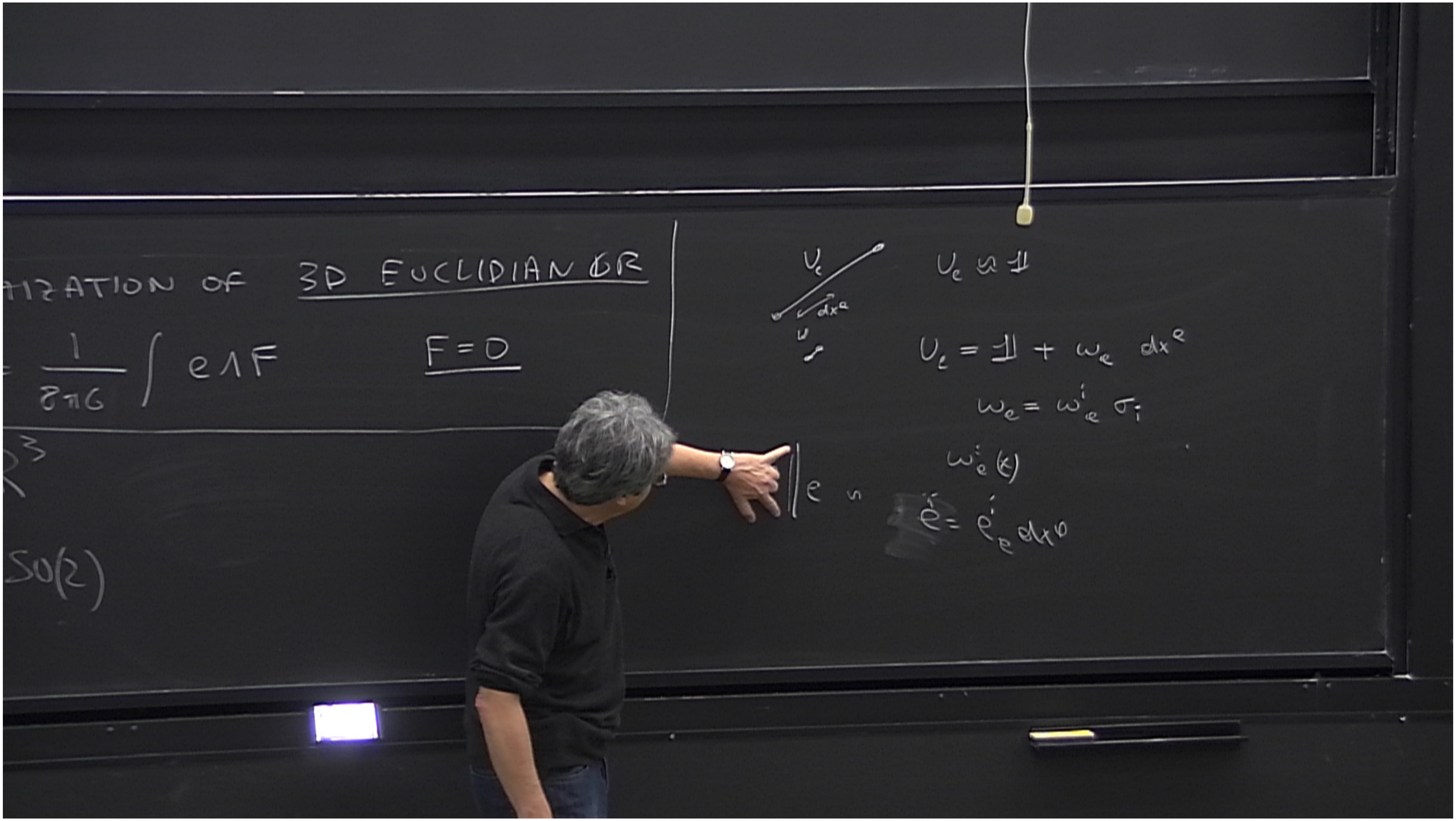

IZATION OF 3D EUCLIDIAN GR
e∧F
8πG
F = 0
SO(2)
U_e ≃ 1
U_e = 1 + ω_e dx^e
ω_e = ω^i_e σ_i

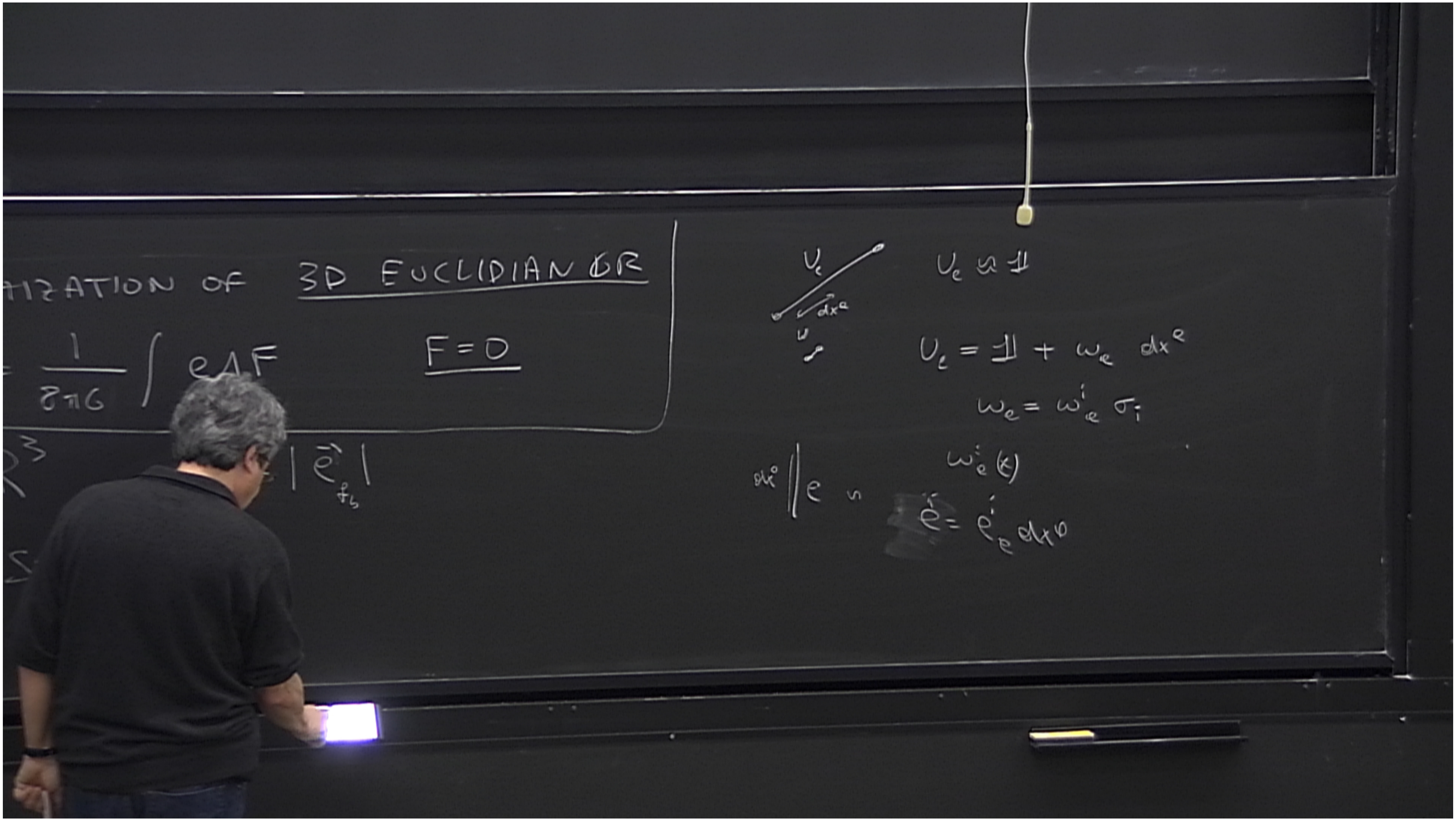

3D EUCLIDIAN GR
$\frac{1}{8\pi G}\int e\wedge F$
$F=0$
$SO(2)$
U_e
dx^e
$U_e \simeq \mathbb{1}$
$U_e = \mathbb{1} + \omega_e \, dx^e$
$\omega_e = \omega^i_e \sigma_i$
$\omega^i_e(x)$

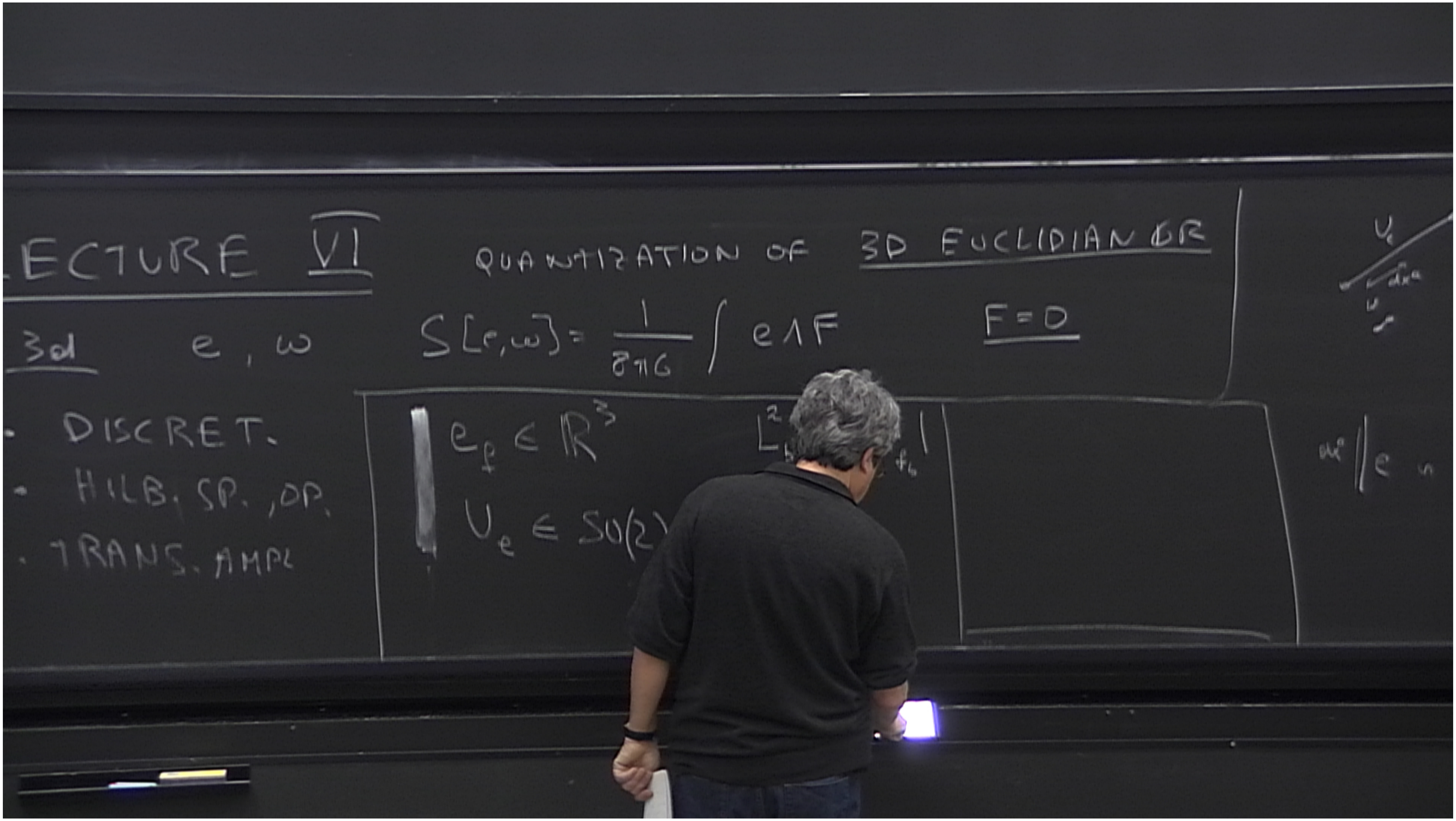


Tetrahe
Triangle
banner
vertex v
edge e
face f
2-cryst



IZATION OF 3D EUCLIDIAN GR
1/8πG ∫ e∧F
F = 0
SO(2)
U_e
dx^a
U_e ∼ 1
U_e = 1 + ω_a dx^a
ω_a = ω^i_a σ_i
ω^i_a (x)
e
e^i = e^i_a dx^a


IZATION OF 3D EUCLIDIAN GR
F = 0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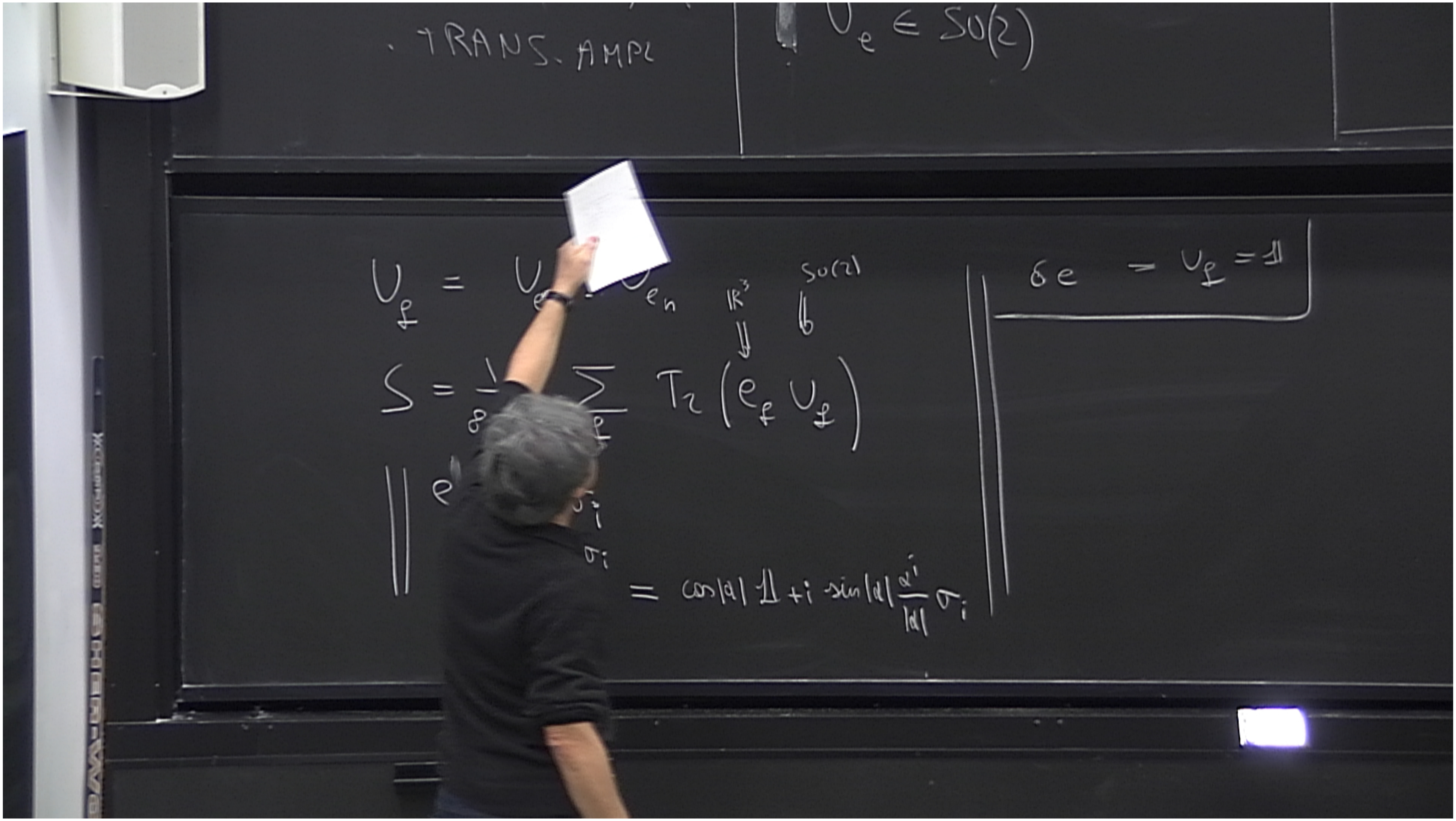
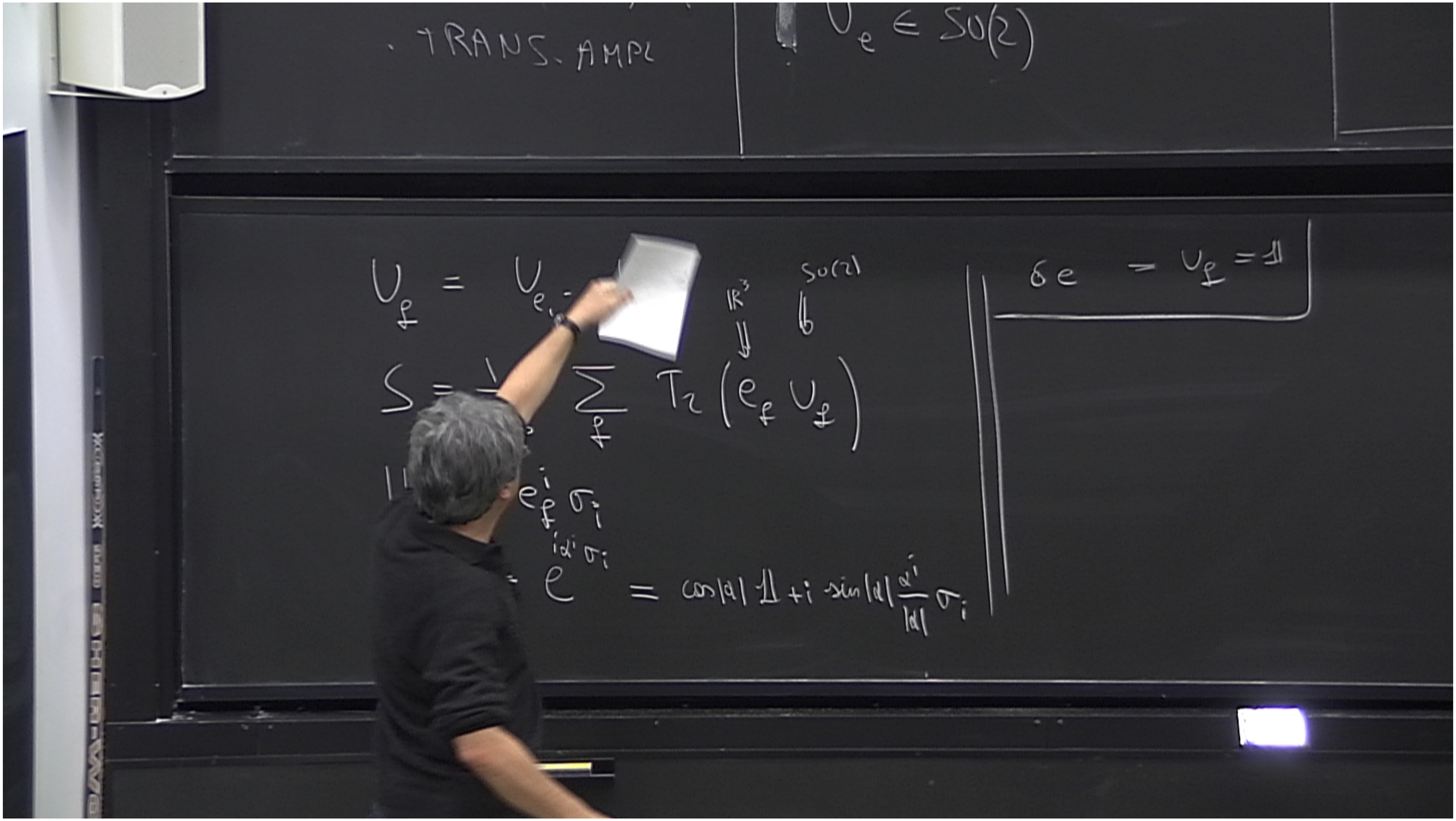
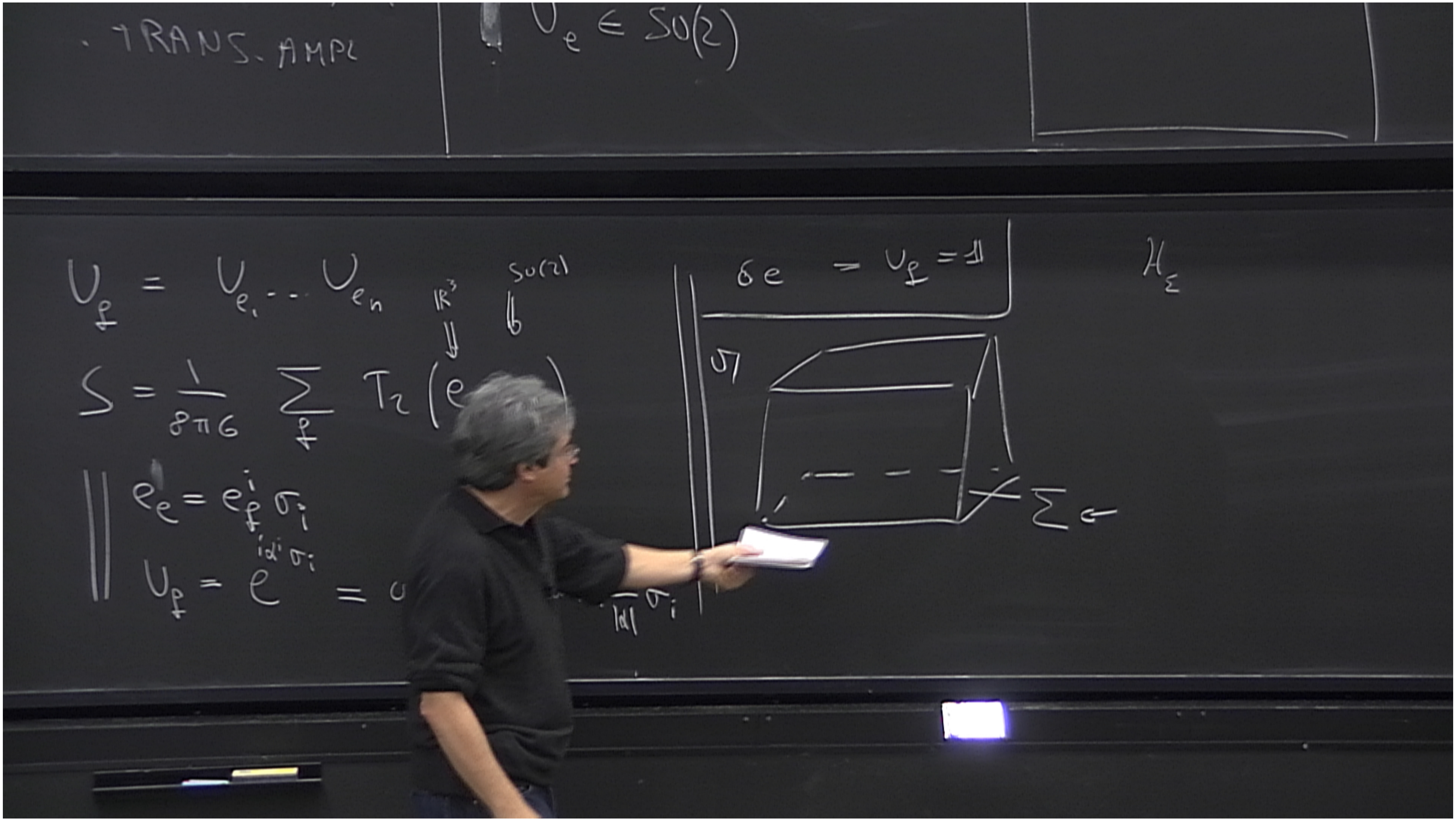
3d e, ω
S[e,ω] = 1/8πG ∫ e∧F
F=0
DISCRET.
HILB. SP., OP.
TRANS. AMPL
e_f ∈ ℝ³
U_e ∈ SU(2)
U_e



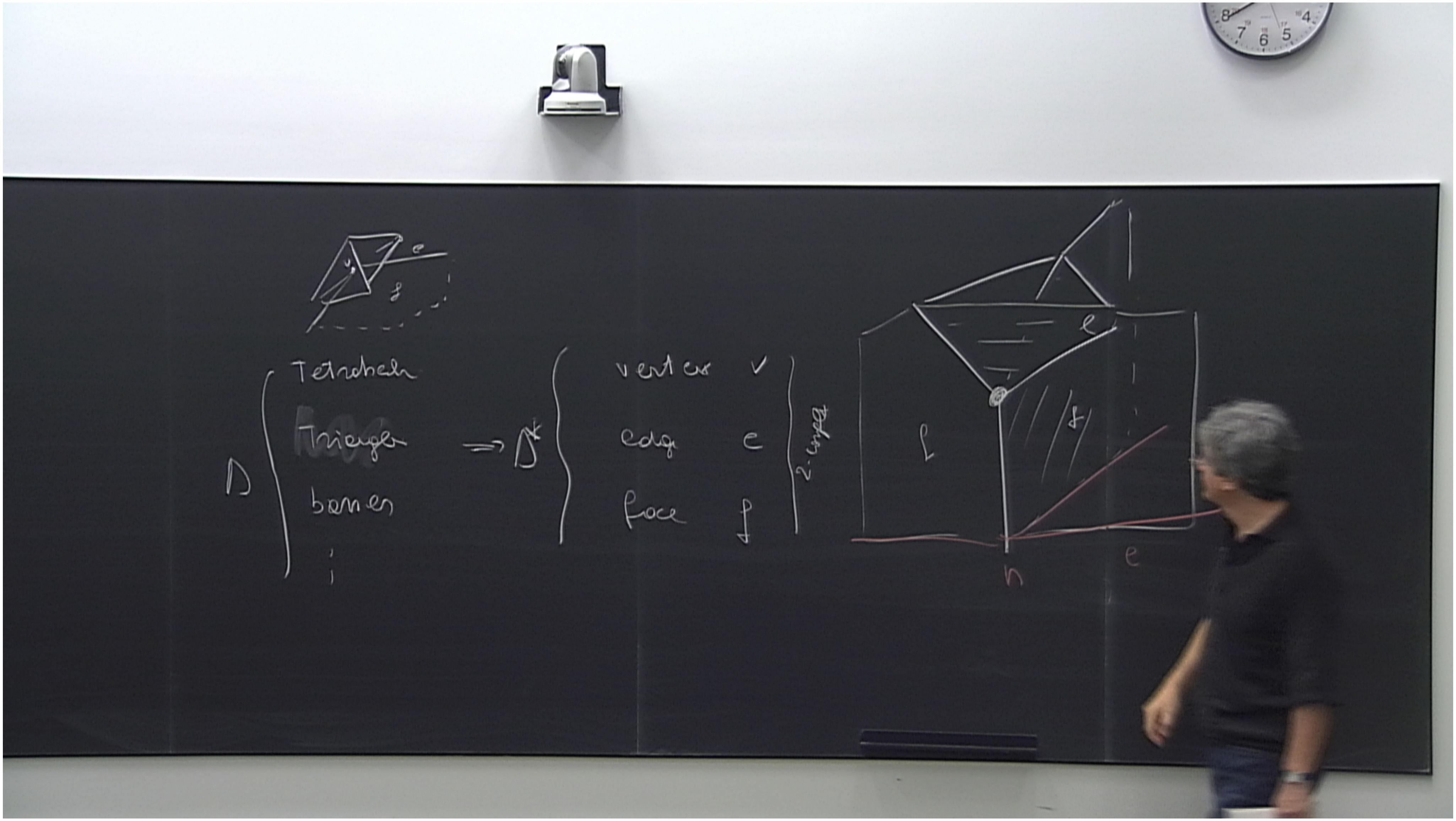
. TRANS. AMPL
$U_e \in SO(2)$
$U_f = U_{e_1} \cdots U_{e_n}$
$\mathbb{R}^3$
SO(2)
$S = \frac{1}{8} \sum_f \mathrm{Tr}(e_f U_f)$
$= \cos|\alpha| \mathbb{1} + i \sin|\alpha| \frac{\alpha^i}{|\alpha|} \sigma_i$
$\delta e \Rightarrow U_f = \mathbb{1}$

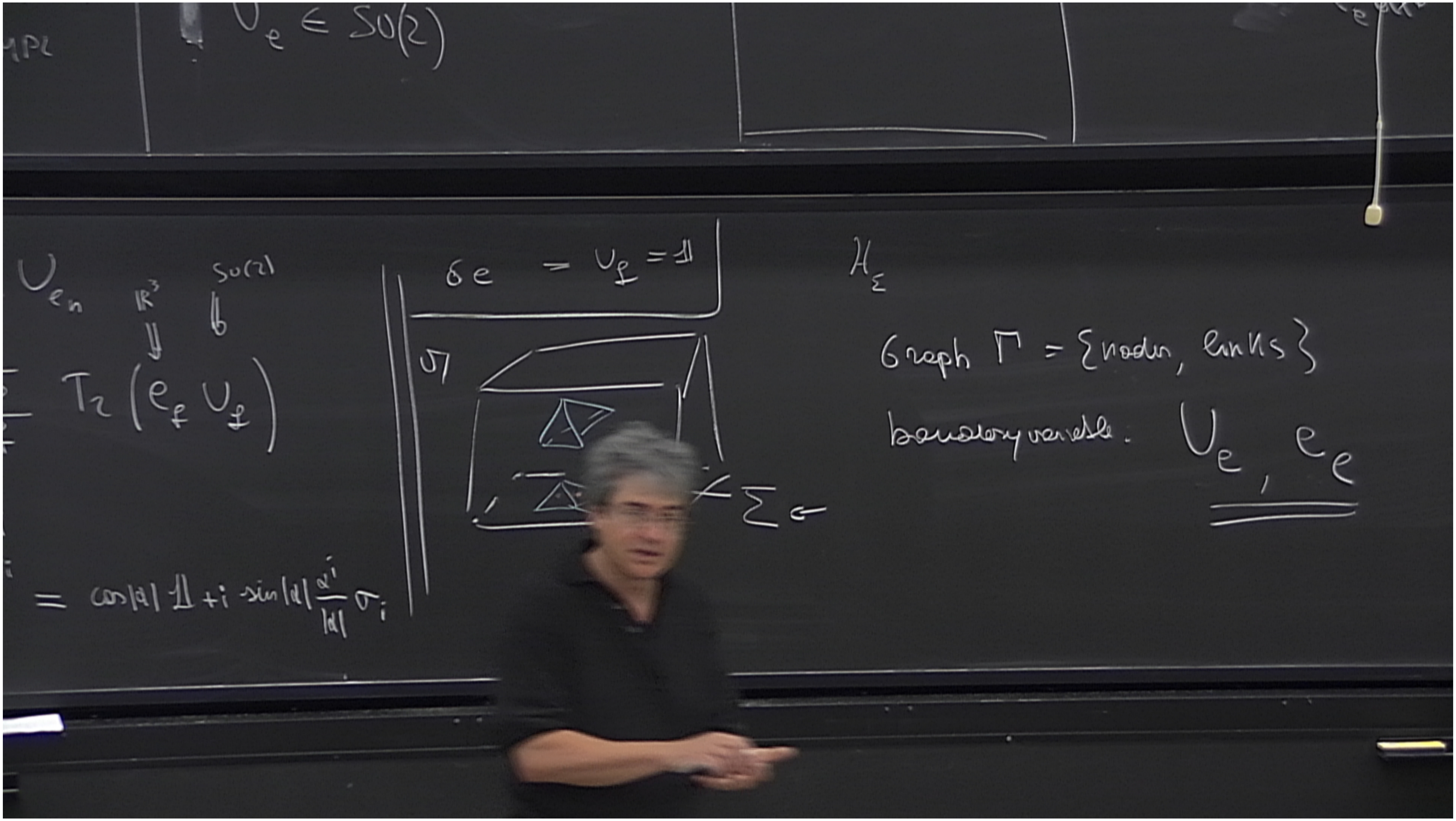


· TRANS. AMPL
$U_e \in SO(2)$
$U_f = U_e \ldots$
$\mathbb{R}^3 \quad SO(2)$
$S = \ldots \sum_f \mathrm{Tr}\,(e_f\, U_f)$
$e_f^i\, \sigma_i$
$e^{i\alpha^i \sigma_i} = \cos|\alpha|\,\mathbb{1} + i \sin|\alpha| \frac{\alpha^i}{|\alpha|}\sigma_i$
$\delta e \Rightarrow U_f = \mathbb{1}$



· TRANS. AMPL
U_e ∈ SU(2)
U_f = U_e1 ... U_en
SU(2)
S = 1/8πG Σ_f Tr (e
e_e = e^i_f σ_i
U_f = e^{iα^i σ_i}
σe ⇒ U_f = 1
σ
Σ
H_Σ


Tetrahedra
Triangles
branes
vertices v
edge e
face f
2-complex


$U_e \in SO(2)$
$\sigma e = U_f = 1\!\!1$
$\mathcal{H}_\Sigma$
Graph Γ = {nodes, links}
U_e, e_e
Σ
$T_2(e_f U_f)$
$= \cos|\alpha|\, 1\!\!1 + i \sin|\alpha| \frac{\alpha^i}{|\alpha|} \sigma_i$

$$U_f = e^{i\alpha^i\sigma_i} = \cos|\alpha|\,\mathbb{1} + i\sin|\alpha|\frac{\alpha^i}{|\alpha|}\sigma_i$$

$$\mathcal{H}_\Gamma \ni \psi(U_e)$$

L = # of links in the boundary graph

$$\mathcal{H}_\Gamma = L_2\left[SU(2)^L\right]$$

$$\vec{E}_e$$

$$U_\ell = e^{i\alpha^i\sigma_i} = \cos|\alpha|\,\mathbb{1} + i\,\sin|\alpha|\,\frac{\alpha^i}{|\alpha|}\sigma_i$$

$$\mathcal{H}_\Gamma \ni \psi(U_\ell) \qquad L = \#\text{ of links in the boundary graph}$$

$$\mathcal{H}_\Gamma = L_2\left[SU(2)^L\right]$$

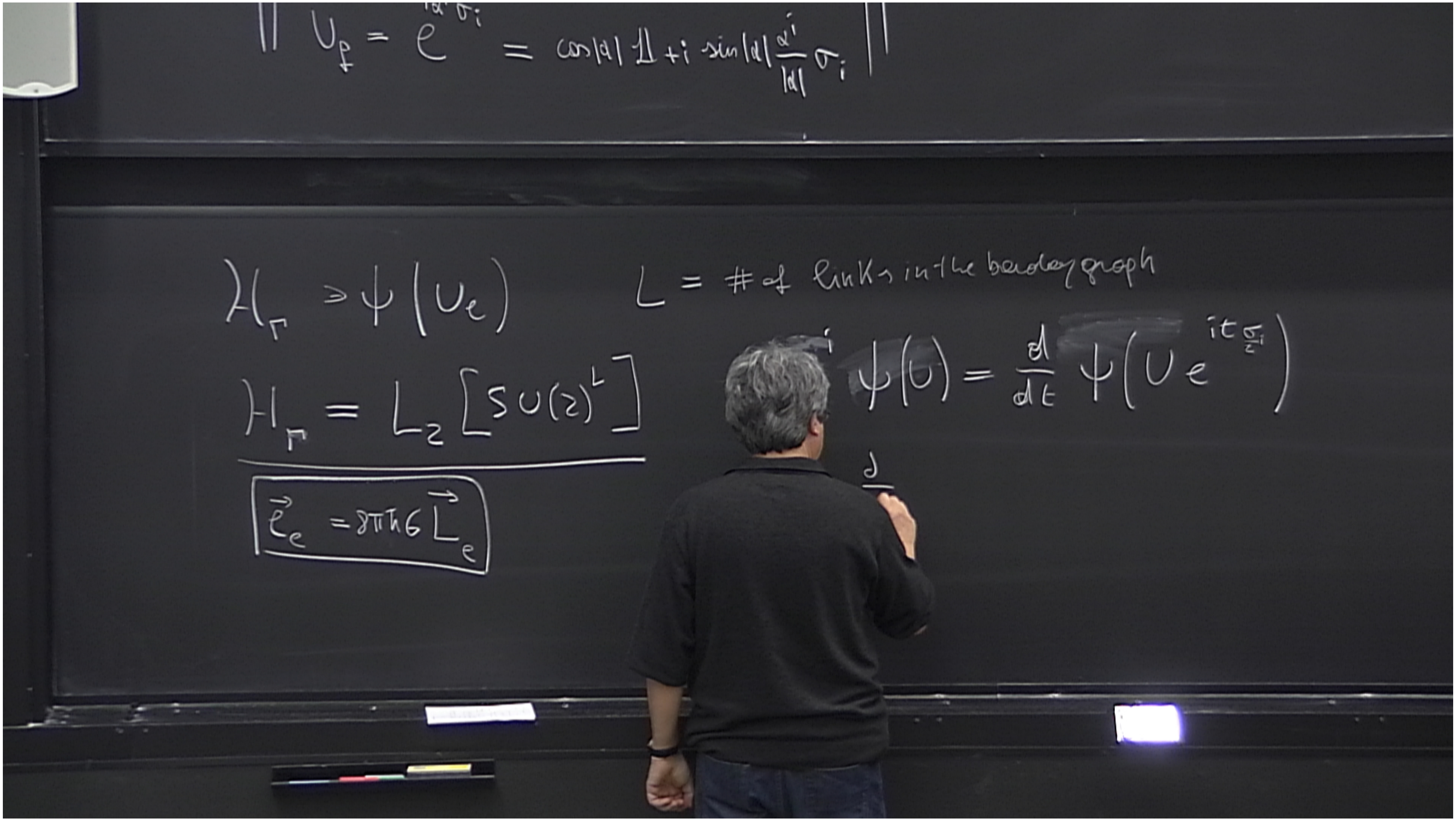
$$\vec{E}_e$$

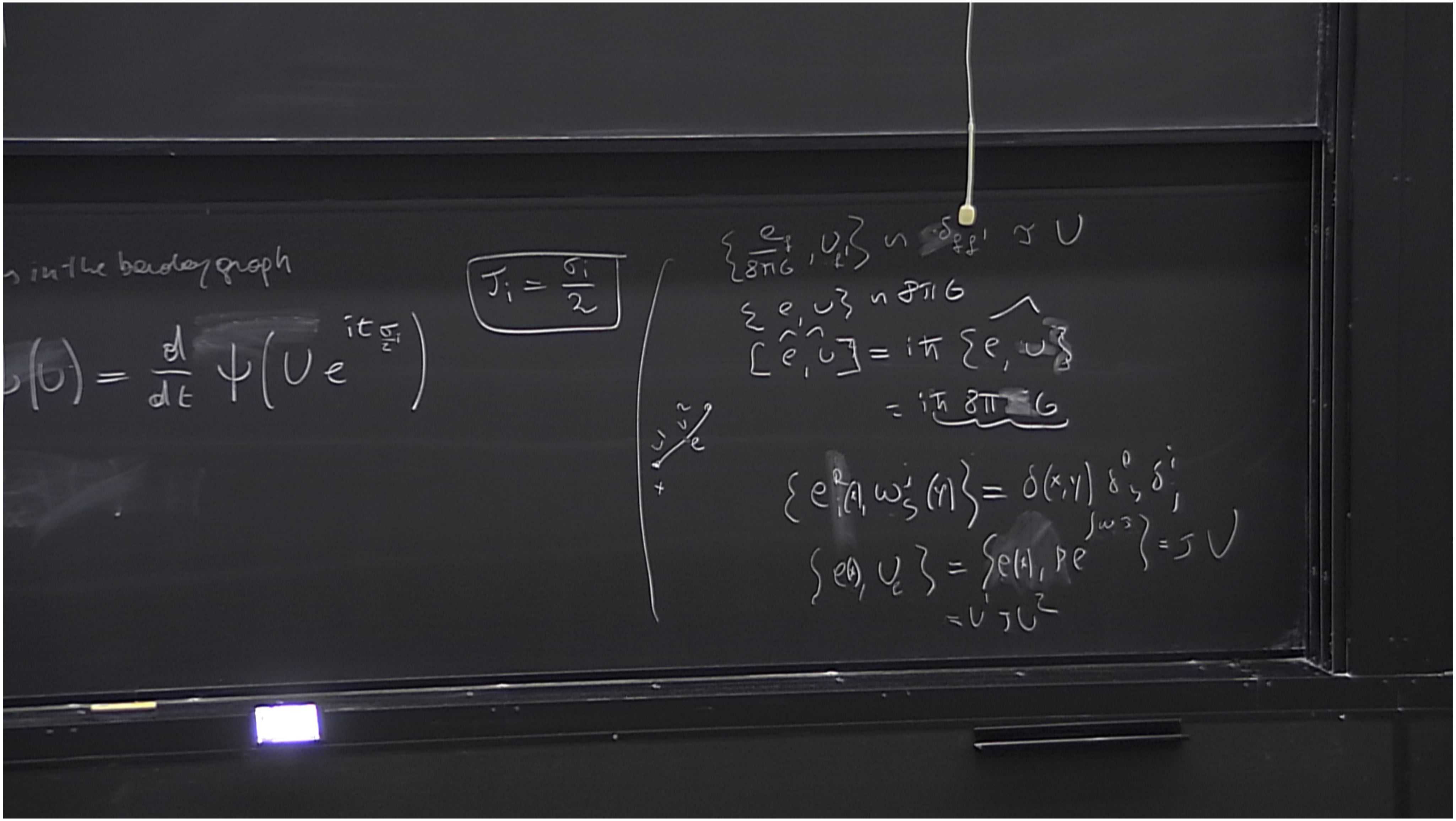
$$U_f = e^{i\alpha^i \sigma_i} = \cos|\alpha|\,\mathbb{1} + i \sin|\alpha| \frac{\alpha^i}{|\alpha|}\sigma_i$$

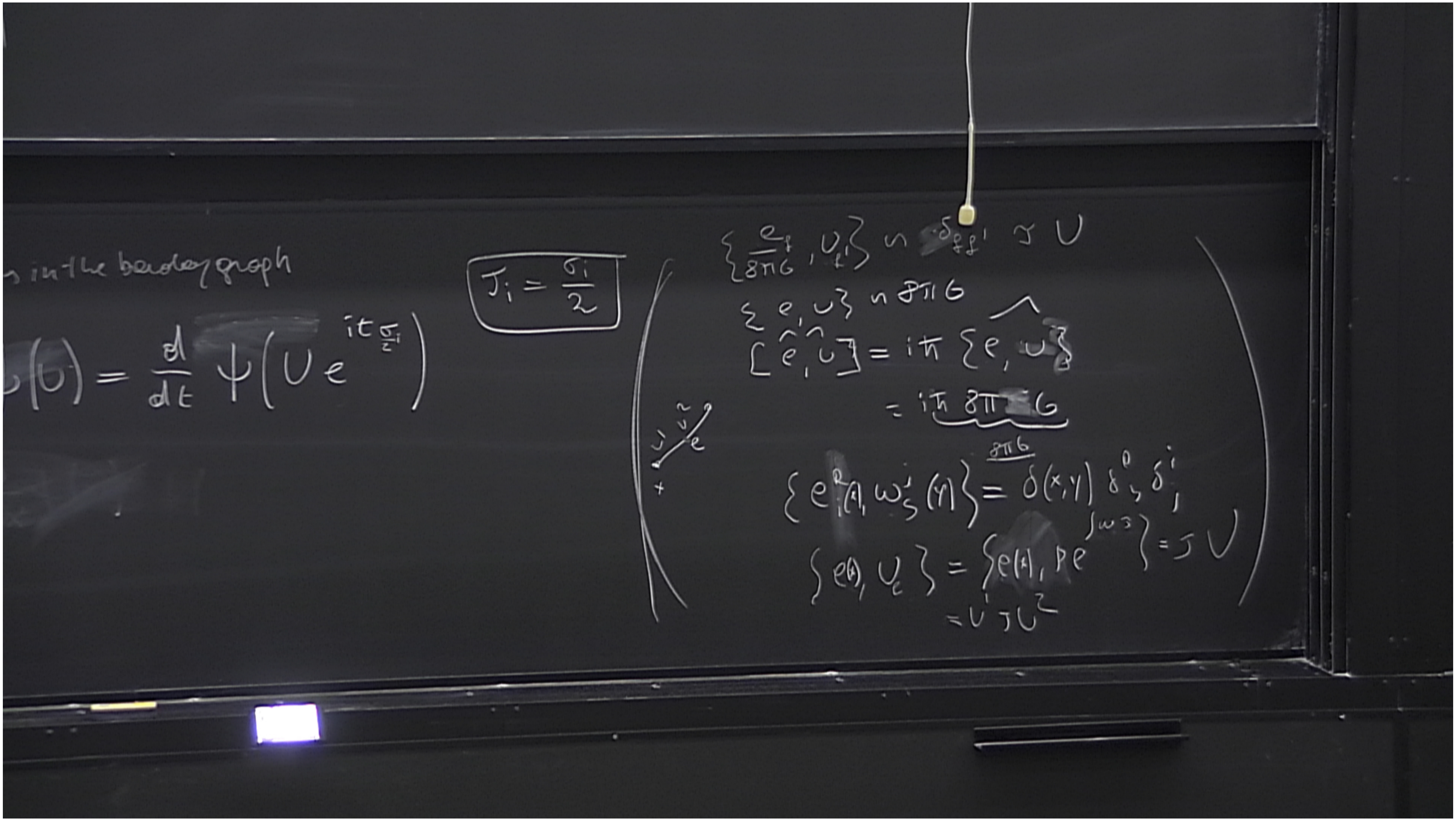
$$\mathcal{H}_\Gamma \ni \psi(U_e) \qquad L = \text{\# of links in the boundary graph}$$

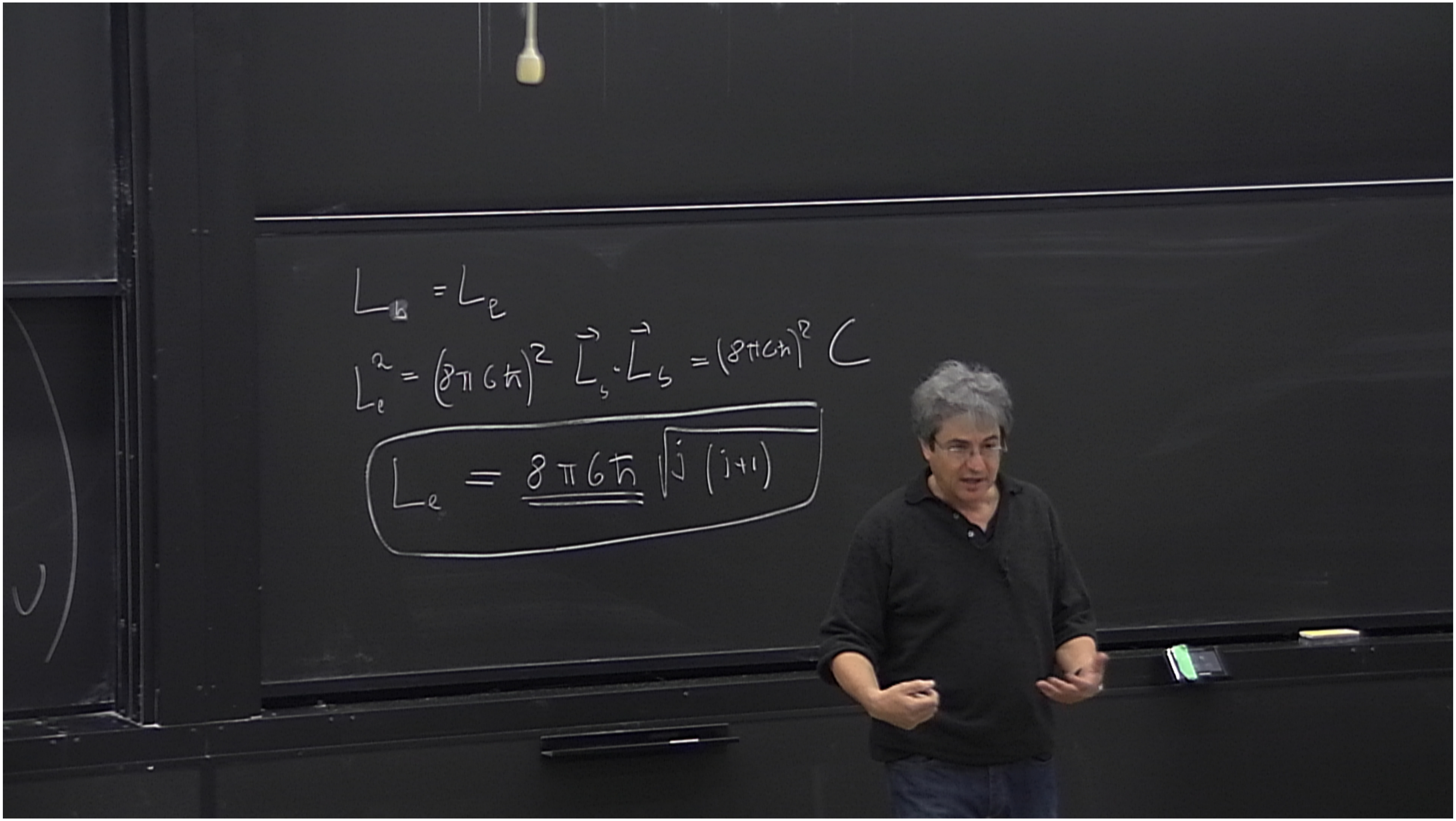
$$\mathcal{H}_\Gamma = L_2\left[SU(2)^L\right] \qquad L^i \psi(U) = \frac{d}{dt}\psi\left(U e^{it\frac{\sigma_i}{2}}\right)$$

$$\vec{\ell}_e = \vec{L}_e$$


$U_f = e^{i\alpha^i \sigma_i} = \cos|\alpha|\,\mathbb{1} + i \sin|\alpha| \frac{\alpha^i}{|\alpha|} \sigma_i$
$\mathcal{H}_\Gamma \ni \psi(U_e)$
L = # of links in the boundary graph
$\psi(U) = \frac{d}{dt} \psi(U e^{it\frac{\sigma_i}{2}})$
$\mathcal{H}_\Gamma = L_2[SU(2)^L]$
$\vec{E}_e = 8\pi\hbar G \vec{L}_e$
$\frac{d}{\ }$


in the boundary graph
$J_i = \frac{\sigma_i}{2}$
$\{e, U\} \sim 8\pi G$
$[\hat{e}, \hat{U}] = i\hbar \{e, U\}$
$= i\hbar\, 8\pi G$
$\{e, U_e\} = \{e(x), P e^{\int \omega \tau}\} = \tau U$
$= U^1 \tau U^2$


s in the boundary graph
$(U) = \frac{d}{dt} \psi\left(U e^{it\frac{\sigma_i}{2}}\right)$
$J_i = \frac{\sigma_i}{2}$
$\left\{\frac{e_\ell}{8\pi G}, U_{\ell'}\right\} \sim \delta_{\ell\ell'} \, J U$
$\{e, U\} \sim 8\pi G$
$[\hat{e}, \hat{U}] = i\hbar \, \widehat{\{e, U\}}$
$= i\hbar \, \underbrace{8\pi G}_{8\pi G}$
$\{e^p_i(x), \omega^j_s(y)\} = \delta(x,y) \, \delta^p_s \, \delta^j_i$
$\{e(x), U_\ell\} = \{e(x), P e^{\int \omega J}\} = J U$
$= U^1 J U^2$
U
e
x


$L_b = L_e$
$L_e^2 = (8\pi G\hbar)^2 \vec{L}_s \cdot \vec{L}_s = (8\pi G\hbar)^2 C$
$L_e = 8\pi G\hbar \sqrt{j(j+1)}$

$$e_e = e^i_f \, \sigma_i$$

$$U_f = e^{i\alpha^i \sigma_i} = \cos|\alpha| \, \mathbb{1} + i \sin|\alpha| \frac{\alpha^i}{|\alpha|} \sigma_i$$

$\Sigma \leftarrow$

$$\mathcal{H}_\Gamma \ni \psi(U_e)$$

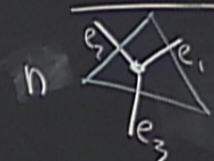
L = # of links in the boundary graph

$$\boxed{J_i = \frac{\sigma_i}{2}}$$

$$L^i \psi(U) = \frac{d}{dt} \psi\left(U e^{it\frac{\sigma_i}{2}}\right)$$

$$\mathcal{H}_\Gamma = L_2\left[SU(2)^L\right]$$

$$\boxed{\vec{e}_e = 8\pi\hbar G \, \vec{L}_e}$$



$$\boxed{\vec{C}_n = \vec{L}_{e_1} + \vec{L}_{e_2} + \vec{L}_{e_3} = 0}$$