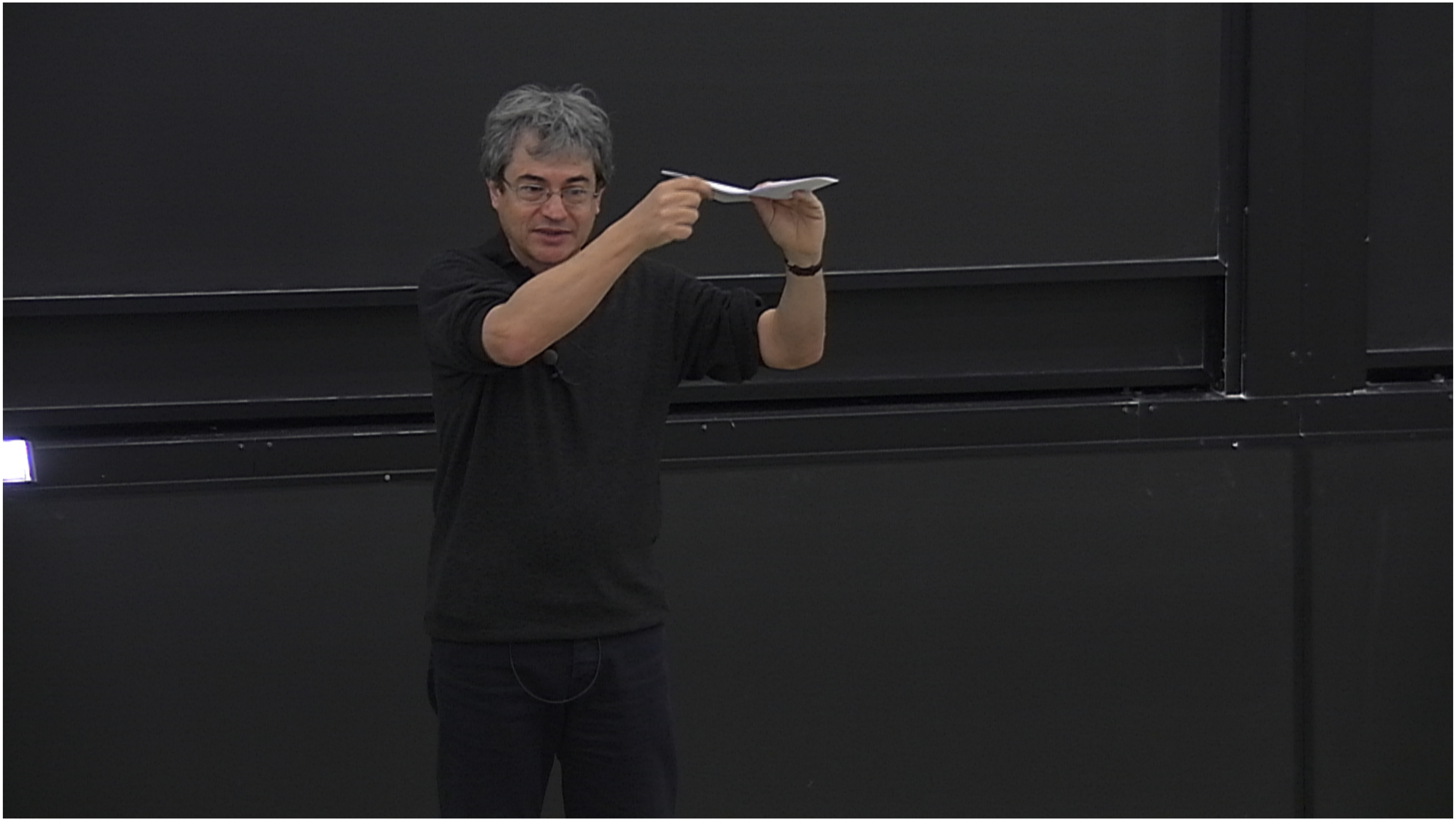
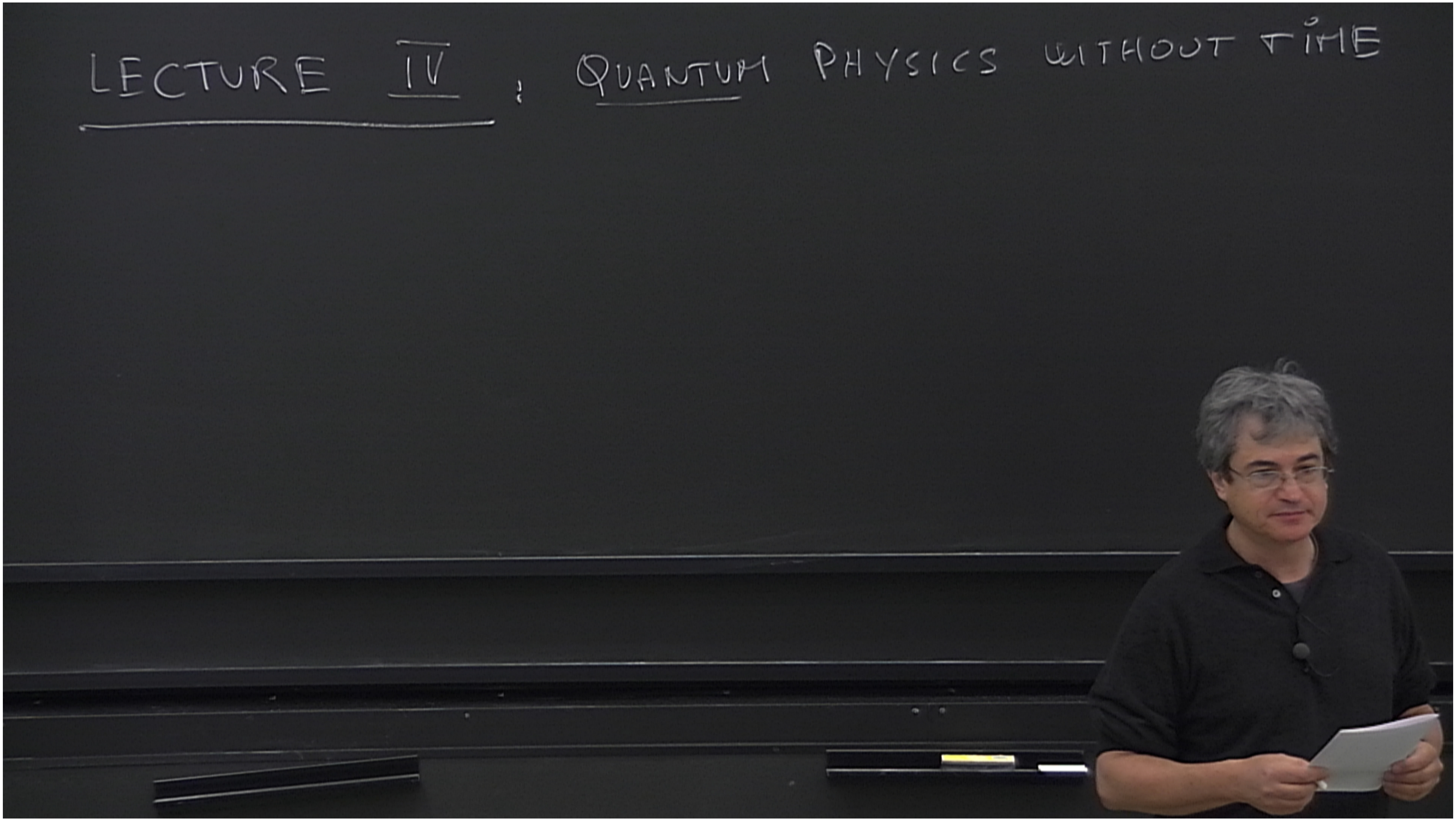
Title: Explorations in Quantum Gravity - Lecture 4

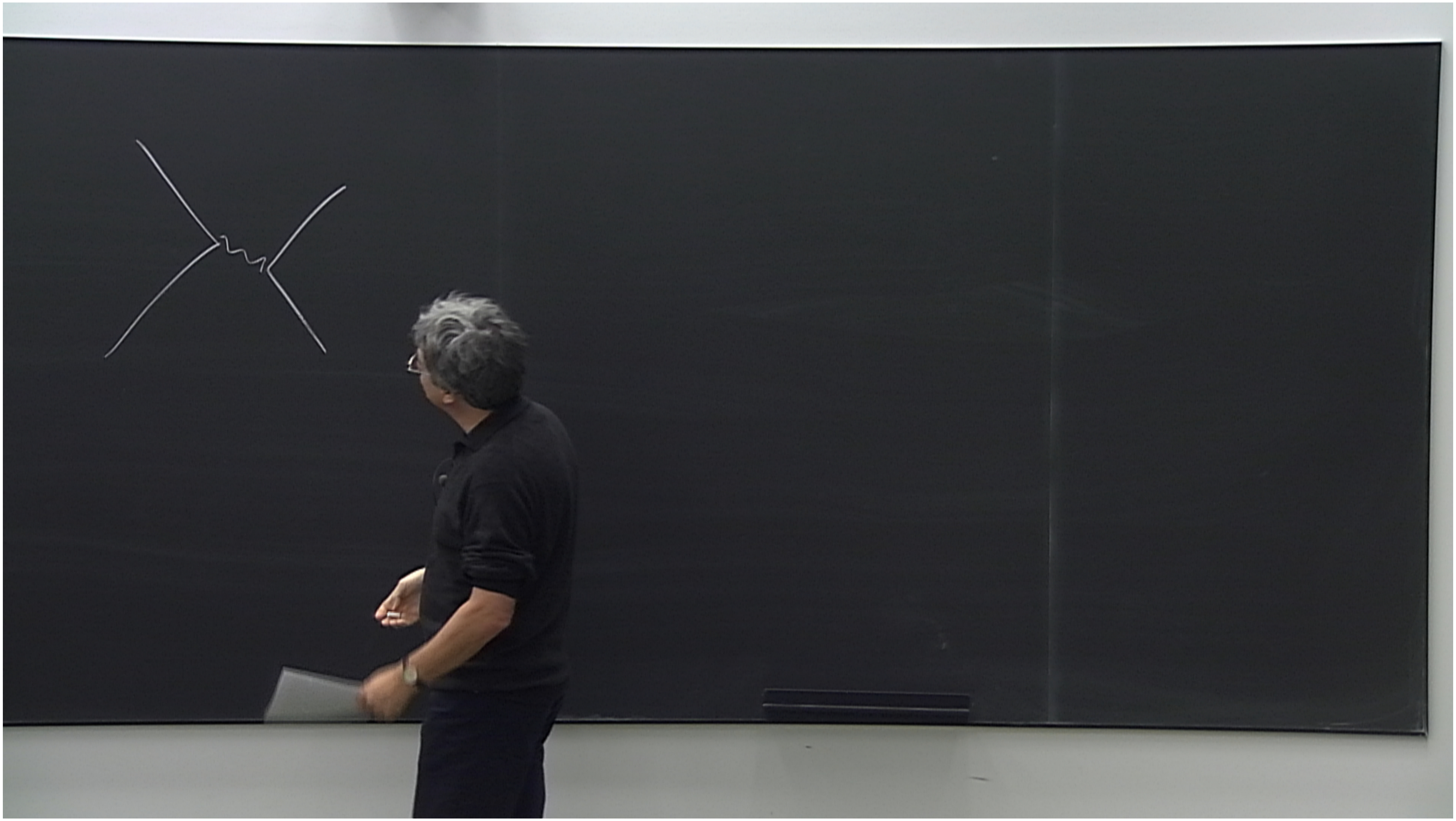
Date: Apr 05, 2012 10:15 AM

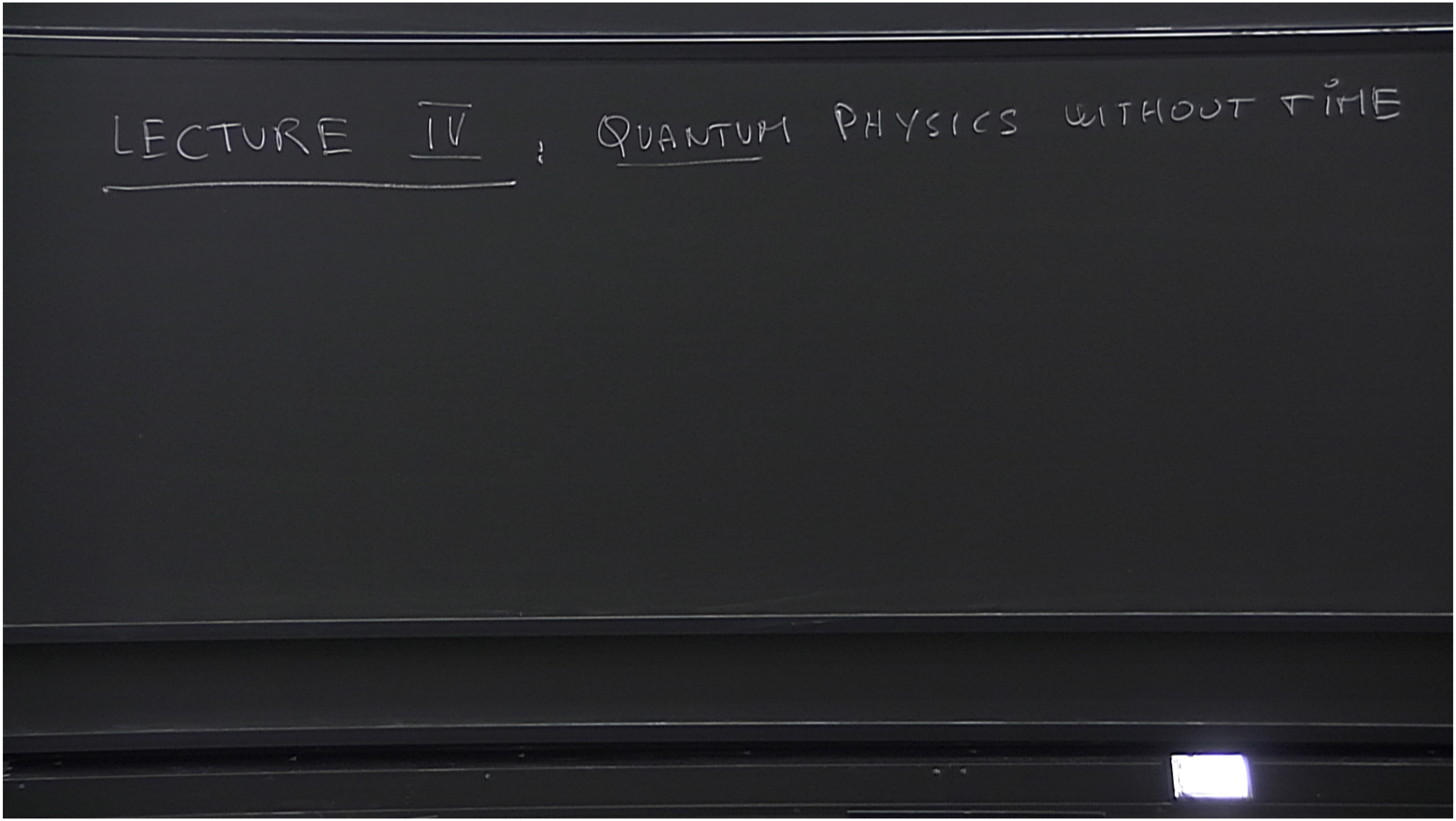
URL: http://pirsa.org/12040022

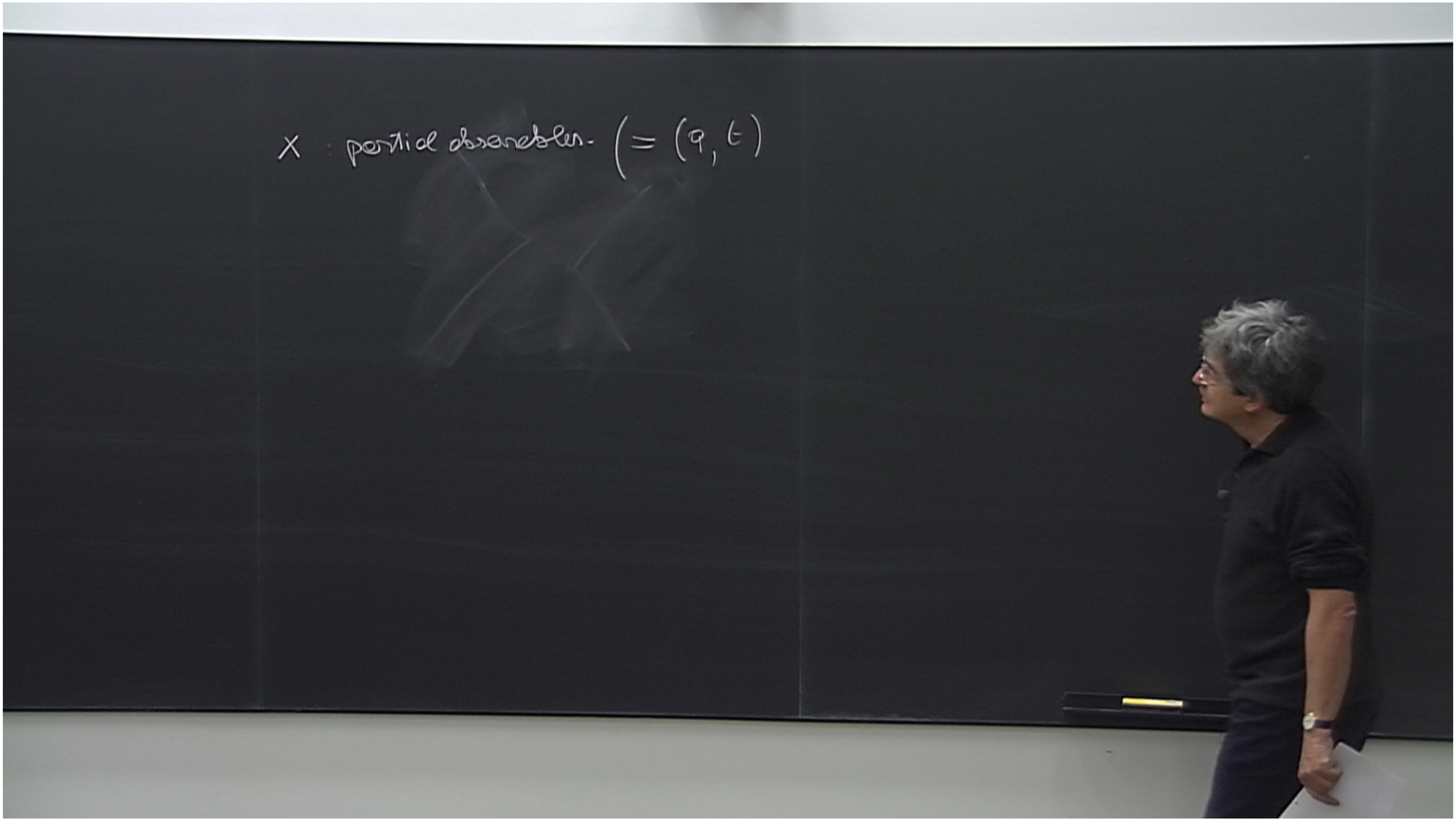
Abstract:

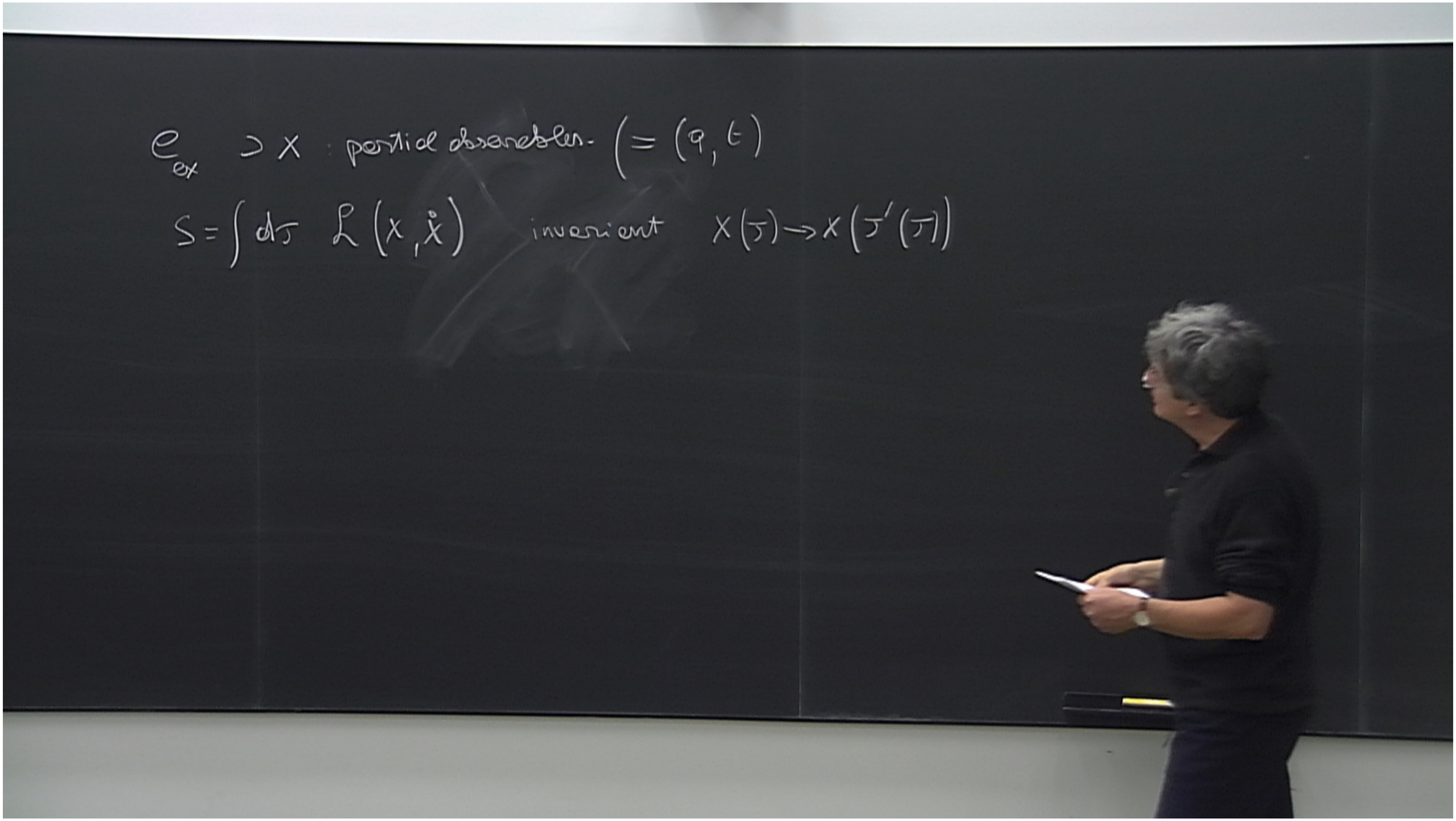


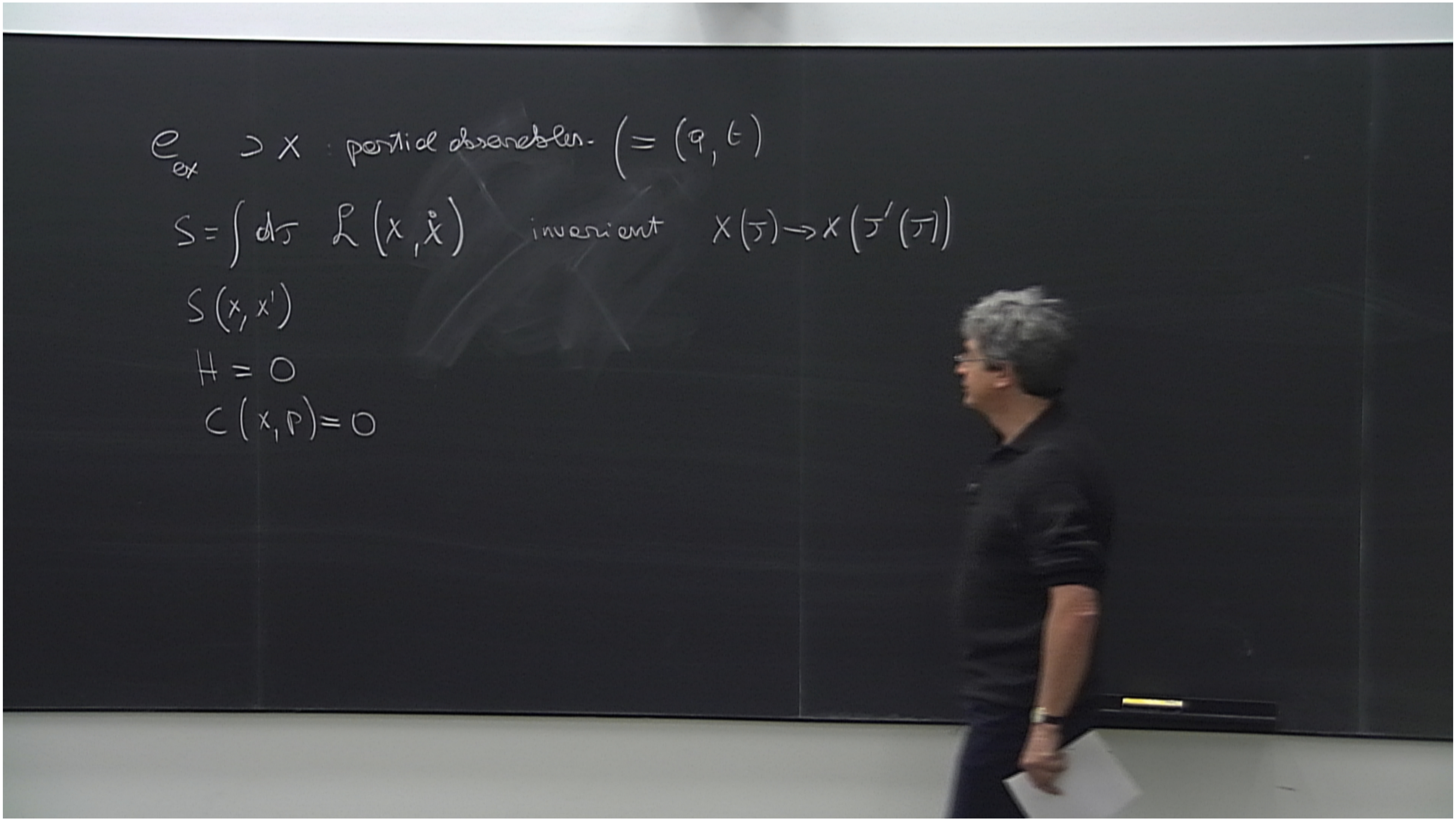

LECTURE IV : QUANTUM PHYSICS WITHOUT TIME

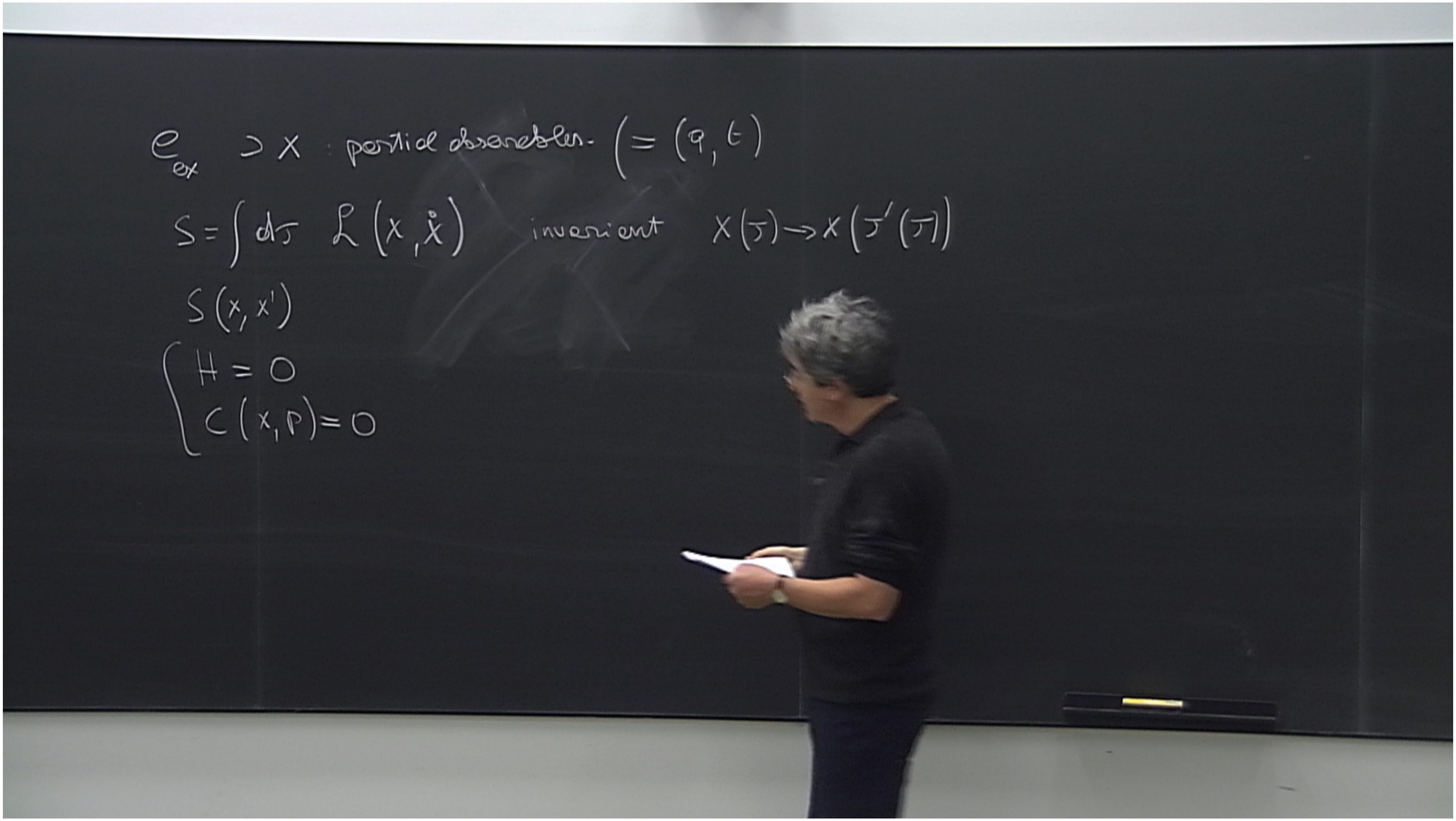



LECTURE IV : QUANTUM PHYSICS WITHOUT TIME


X : partial observables. (= (q, t)


C_ex ⊃ X : partial observables. (= (q, t))
S = ∫ dτ L(x, ẋ) invariant x(τ) → x(τ'(τ))


$C_{ex} \supset X$: partial observables. $(= (q, t)$
$S = \int d\tau \; \mathcal{L}(x, \dot{x})$ invariant $x(\tau) \to x(\tau'(\tau))$
$S(x, x')$
$H = 0$
$C(x, p) = 0$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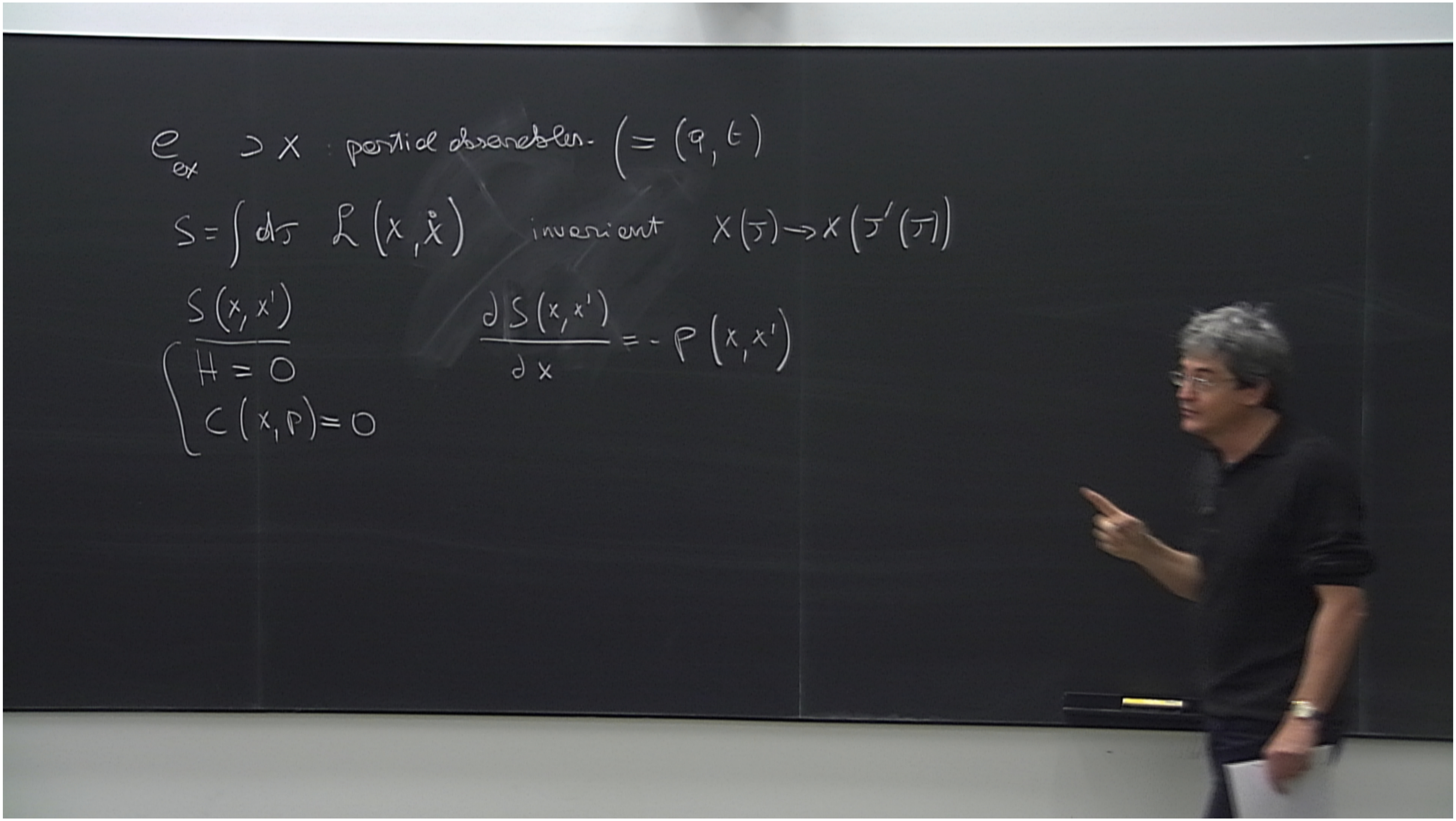
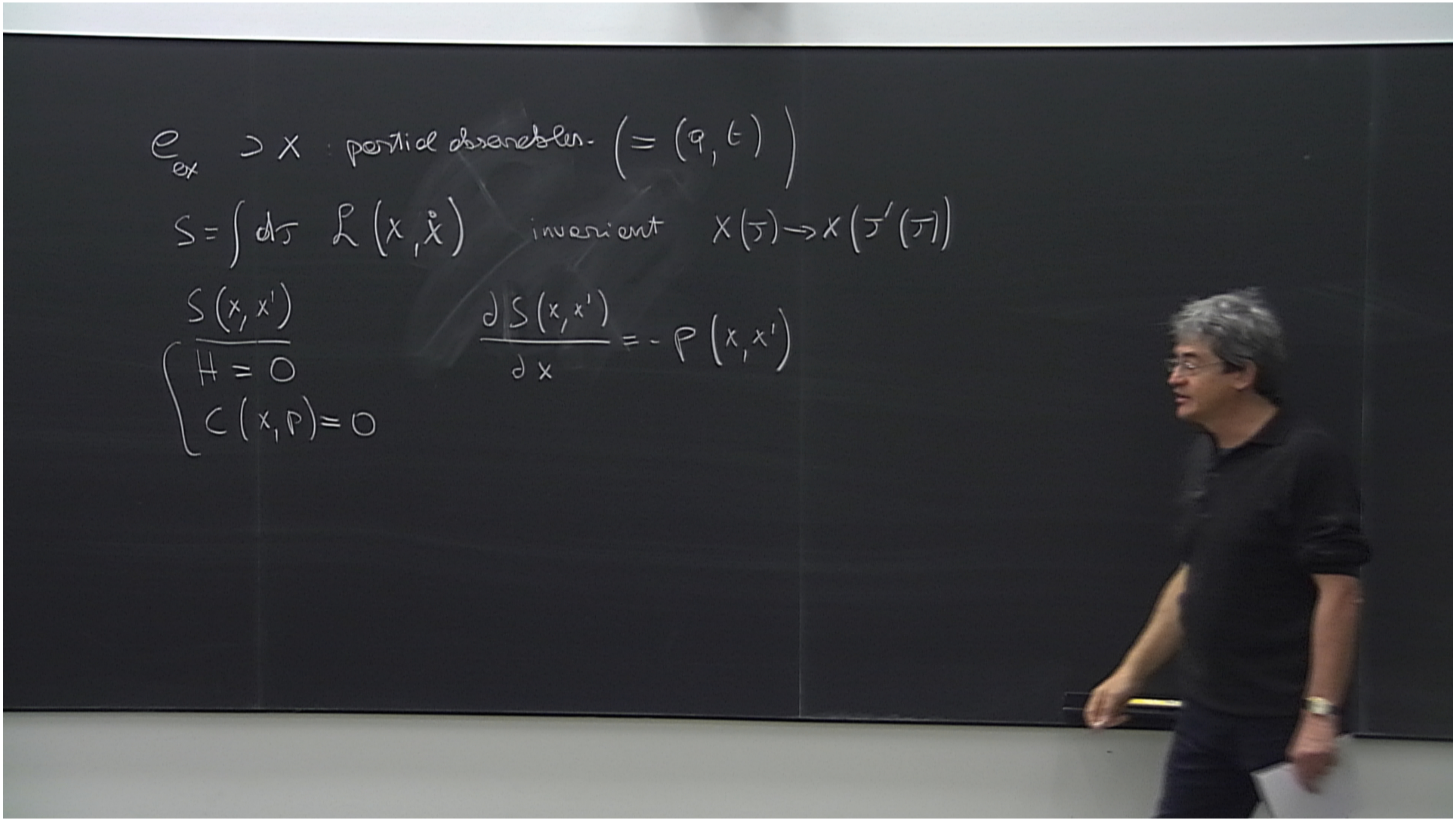

C_ex ⊃ X : partial observables. (= (q, t)
S = ∫ dτ L(x, ẋ)
invariant x(τ) → x(τ'(τ))
S(x, x')
H = 0
C(x, p) = 0
∂S(x, x') / ∂x = - p(x, x')


C_ex ⊃ X : partial observables. (= (q, t))
S = ∫ dτ L(x, ẋ)
invariant x(τ) → x(τ'(τ))
S(x, x')
H = 0
C(x, p) = 0
∂S(x, x') / ∂x = - p(x, x')

$\mathcal{C}_{ex} \supset X$: partial observables. $\left(= (q, t) \right)$

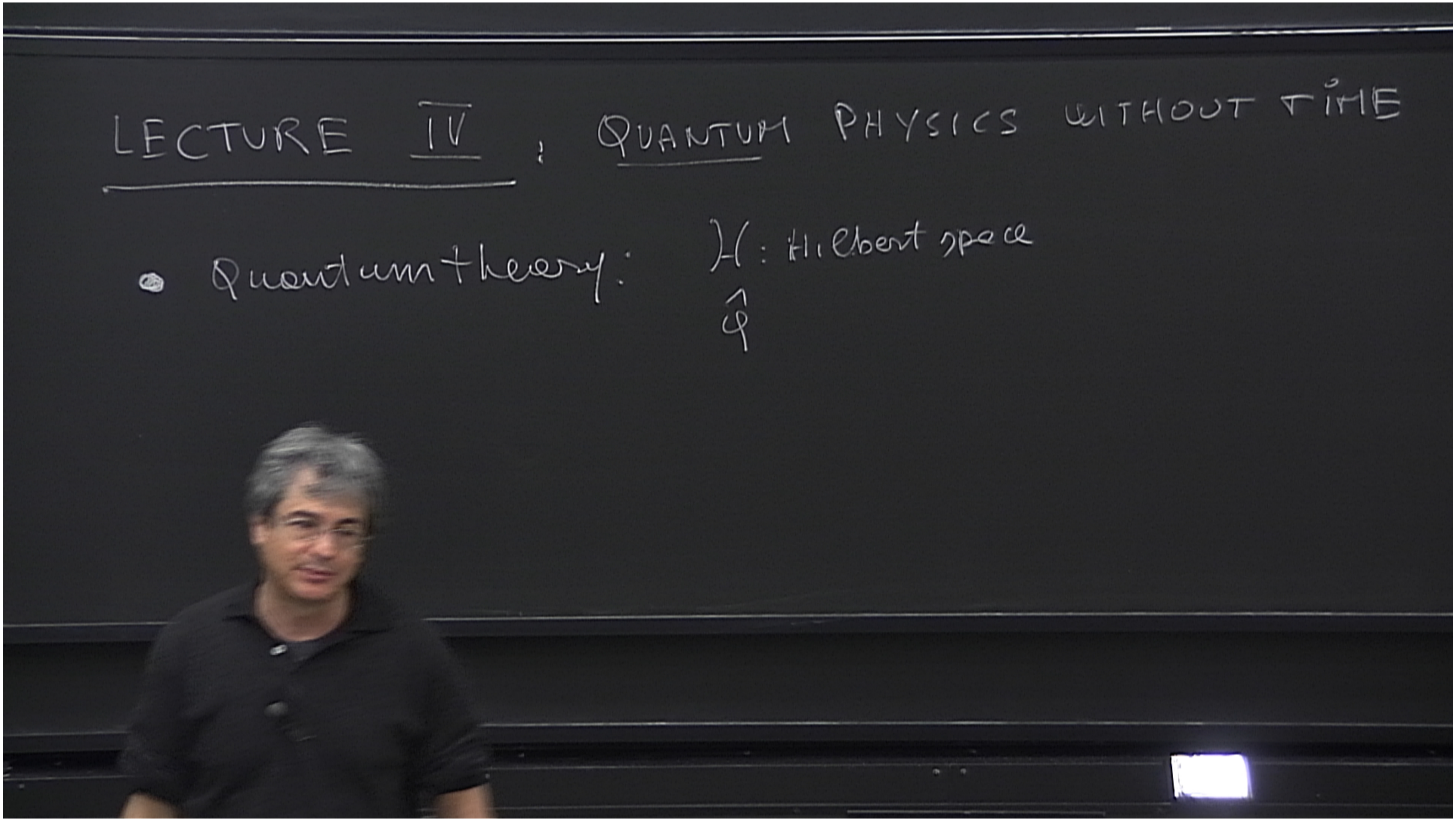
$S = \int d\tau \, \mathcal{L}(x, \dot{x})$ invariant $x(\tau) \to x(\tau'(\tau))$

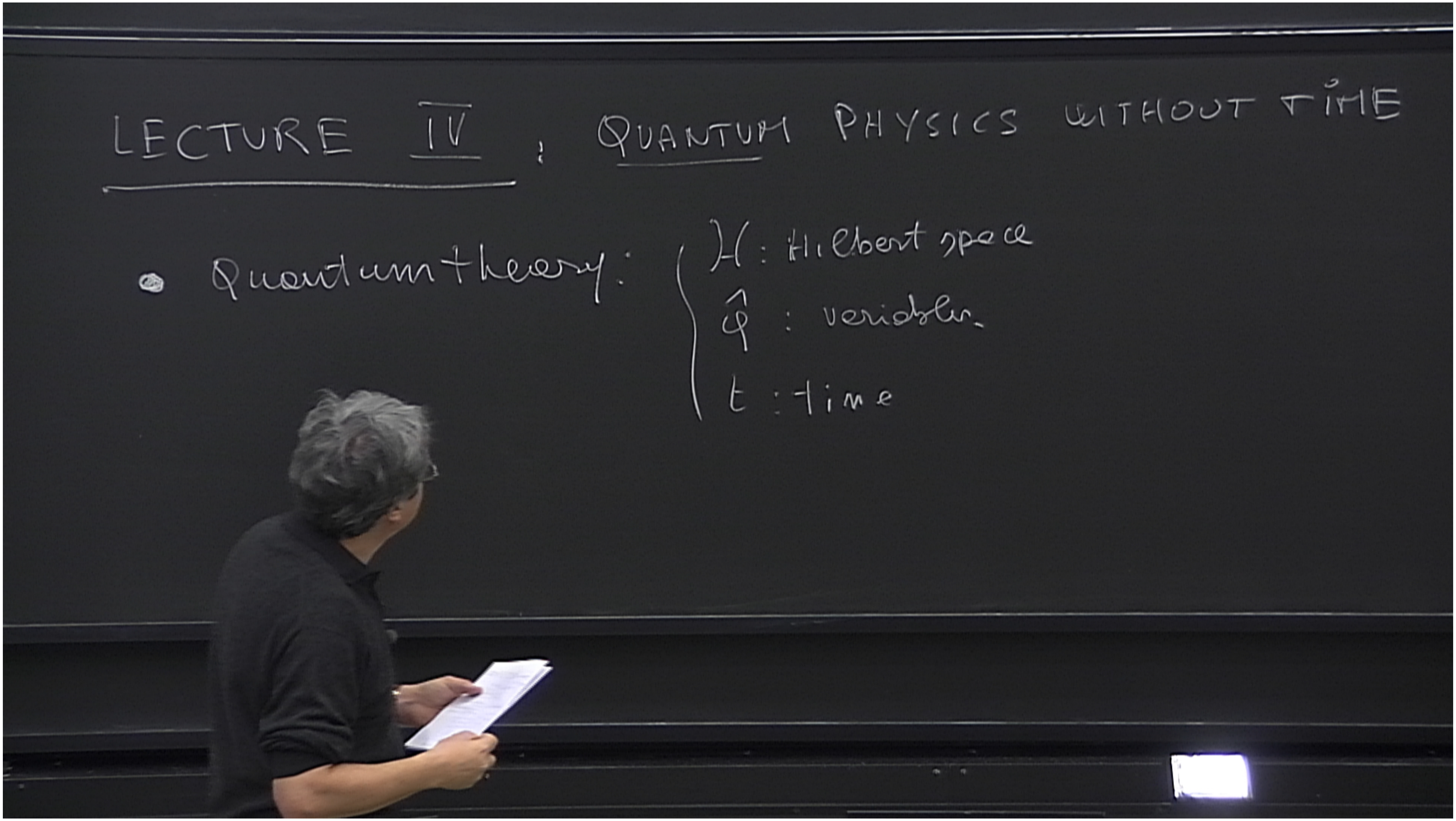
$S(x, x')$

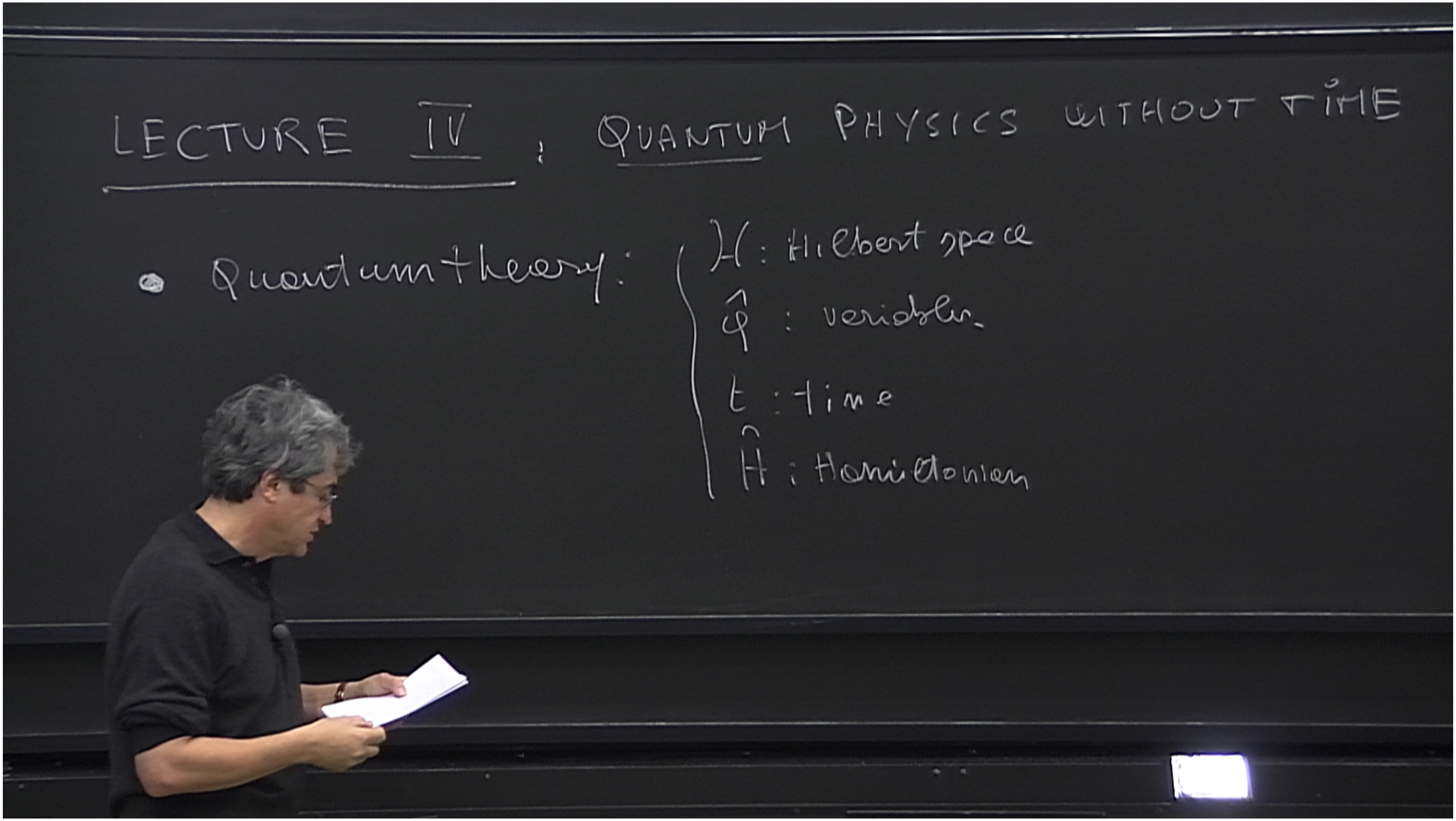
$$\begin{cases} H = 0 \\ C(x, p) = 0 \end{cases}$$

$$\frac{\partial S(x, x')}{\partial x} = -p(x, x')$$

predict : relations between partial observables.


LECTURE IV : QUANTUM PHYSICS WITHOUT TIME
• Quantum theory:
H : Hilbert space
$\hat{q}$


LECTURE IV : QUANTUM PHYSICS WITHOUT TIME
Quantum theory:
H : Hilbert space
q̂ : variables
t : time

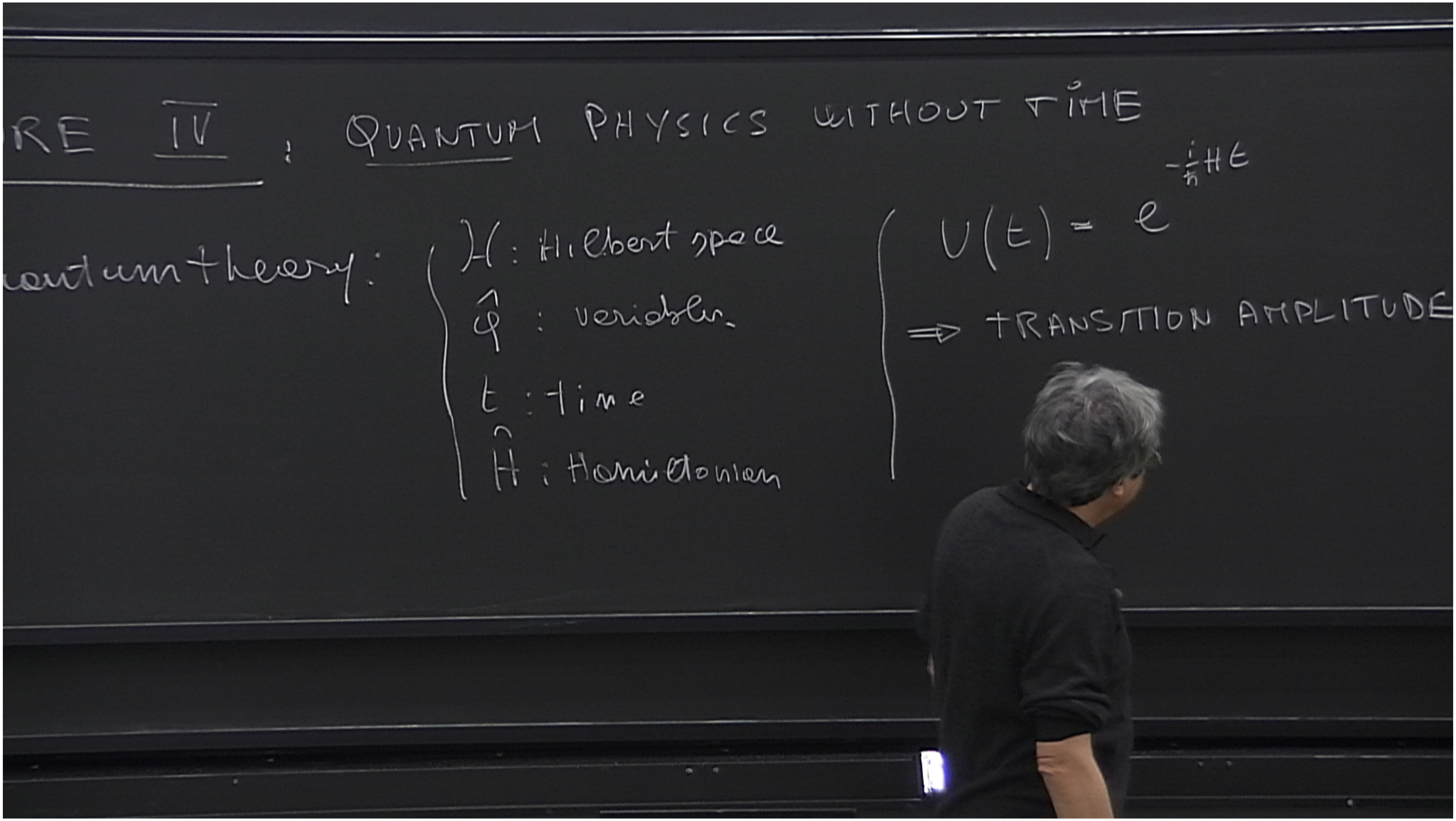

LECTURE IV : QUANTUM PHYSICS WITHOUT TIME
• Quantum theory:
H : Hilbert space
q̂ : variables
t : time
Ĥ : Hamiltonian

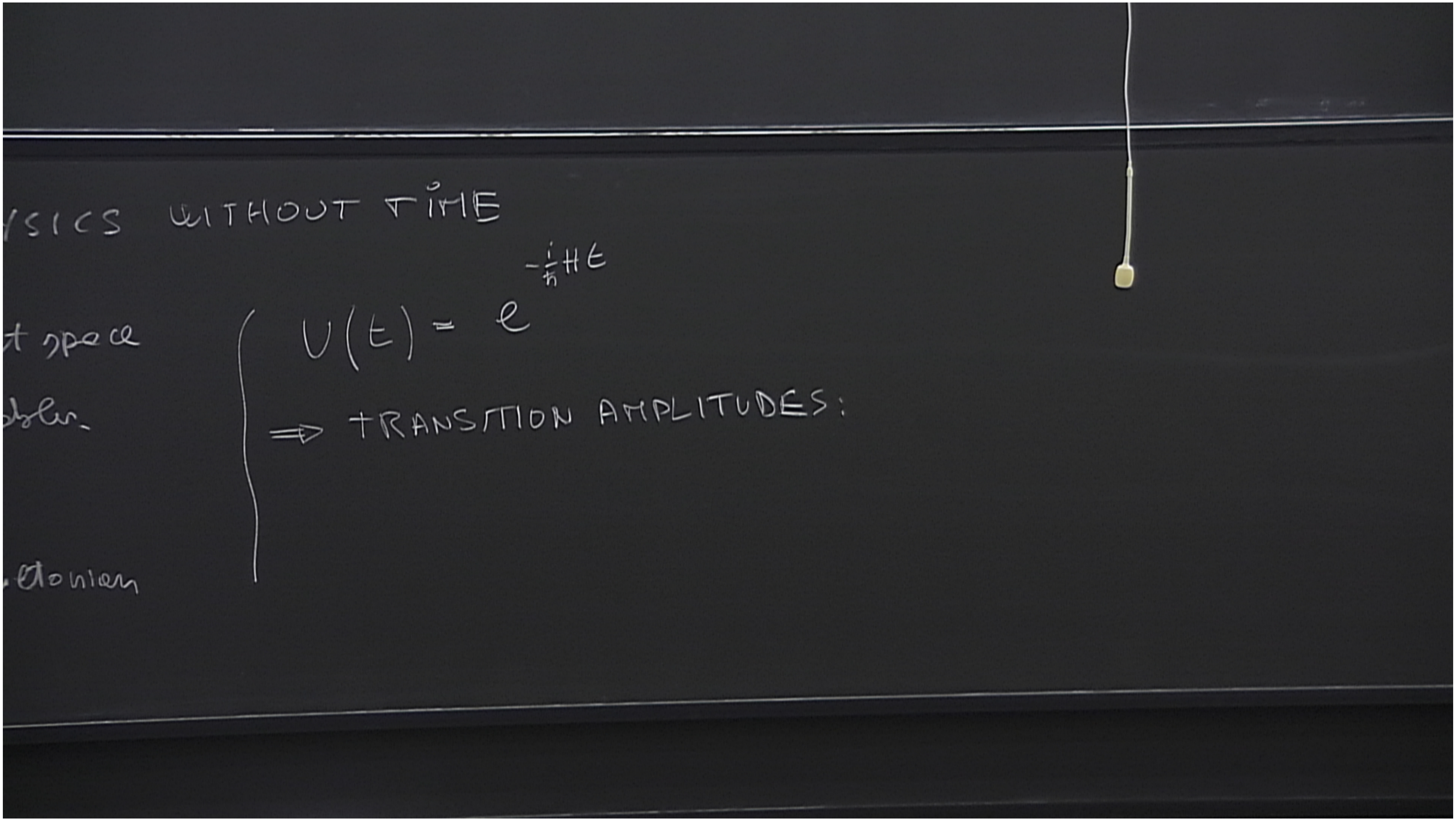
...RE IV : QUANTUM PHYSICS WITHOUT TIME

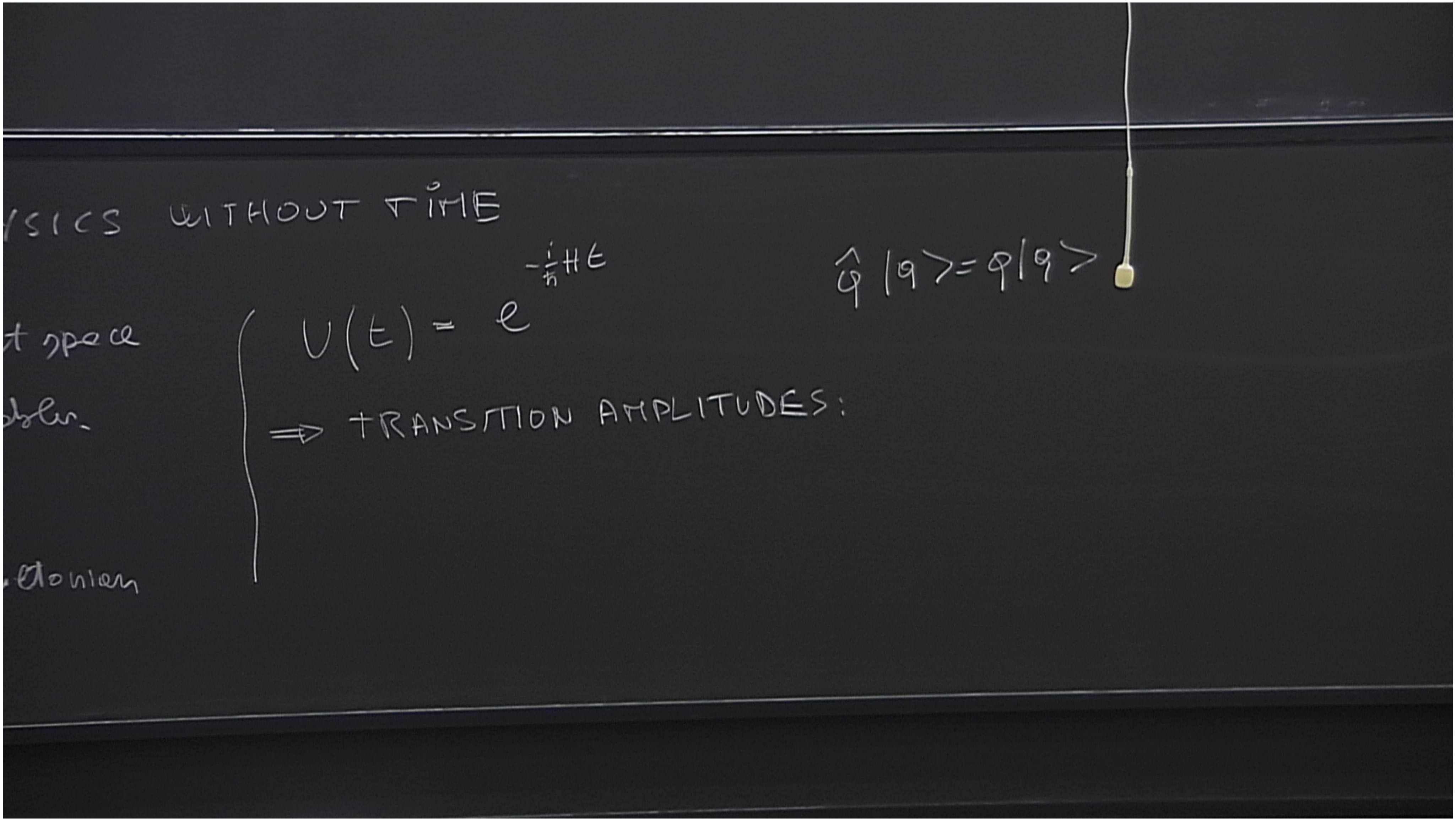
...uantum theory:

- $\mathcal{H}$: Hilbert space
- $\hat{q}$: variables
- t : time
- $\hat{H}$: Hamiltonian

$$U(t) = e^{-\frac{i}{\hbar} H t}$$


RE IV : QUANTUM PHYSICS WITHOUT TIME
uantum theory:
$\mathcal{H}$: Hilbert space
$\hat{q}$: variables
t : time
$\hat{H}$: Hamiltonian
$U(t) = e^{-\frac{i}{\hbar}Ht}$
⇒ TRANSITION AMPLITUDE


YSICS WITHOUT TIME
t space
bles
ltonian
$U(t) = e^{-\frac{i}{\hbar}Ht}$
⟹ TRANSITION AMPLITUDES:

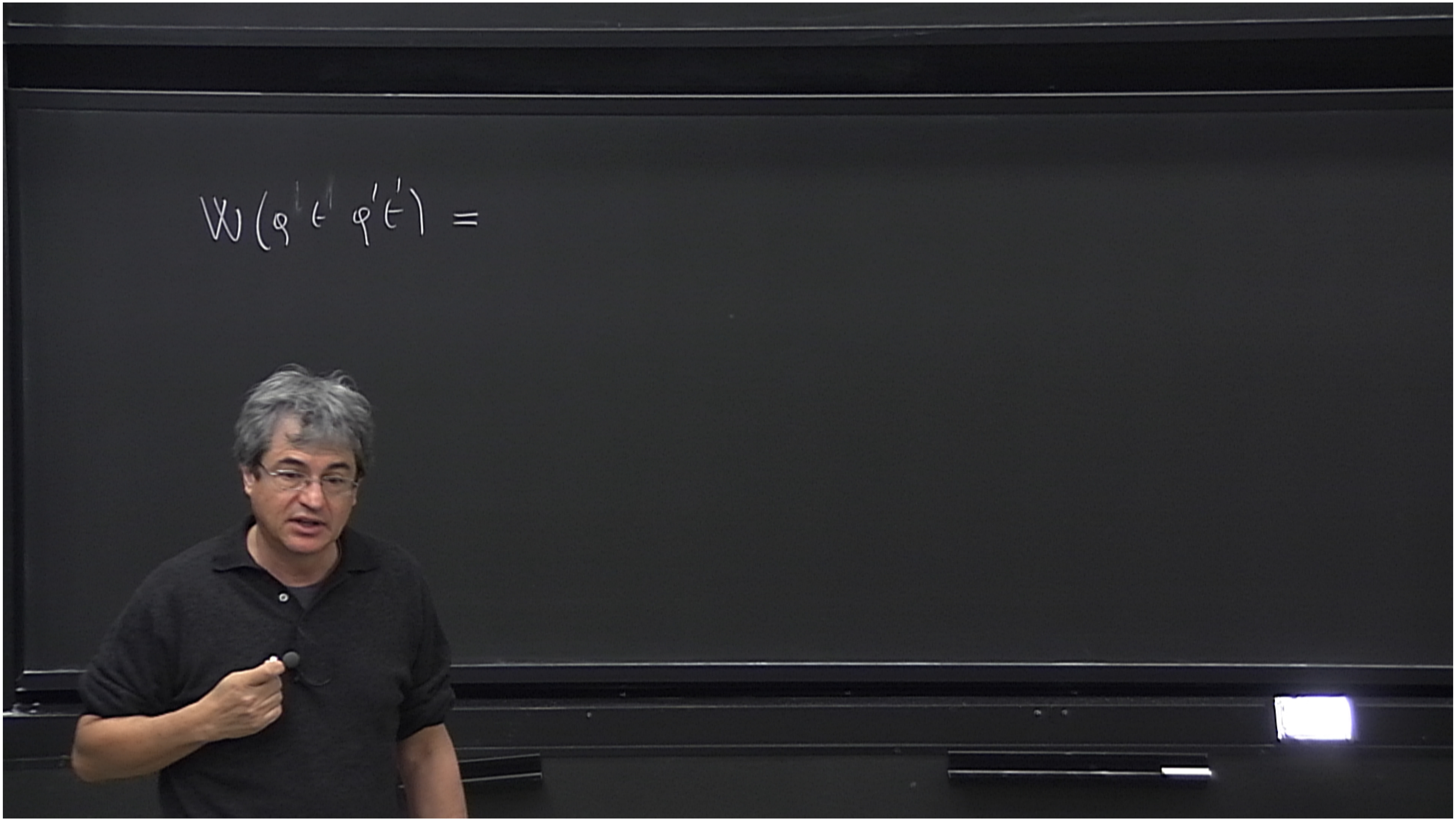

YSICS WITHOUT TIME
t space
U(t) = e^{-i/ħ Ht}
q̂ |q> = q|q>
⇒ TRANSITION AMPLITUDES:

WITHOUT TIME

$$U(t) = e^{-\frac{i}{\hbar}Ht}$$

$$\hat{q}\,|q\rangle = q\,|q\rangle$$

$\Rightarrow$ TRANSITION AMPLITUDES: $W(q,t,q',t') = \langle q'|\, e^{-\frac{i}{\hbar}(t'-t)}\, |q\rangle$

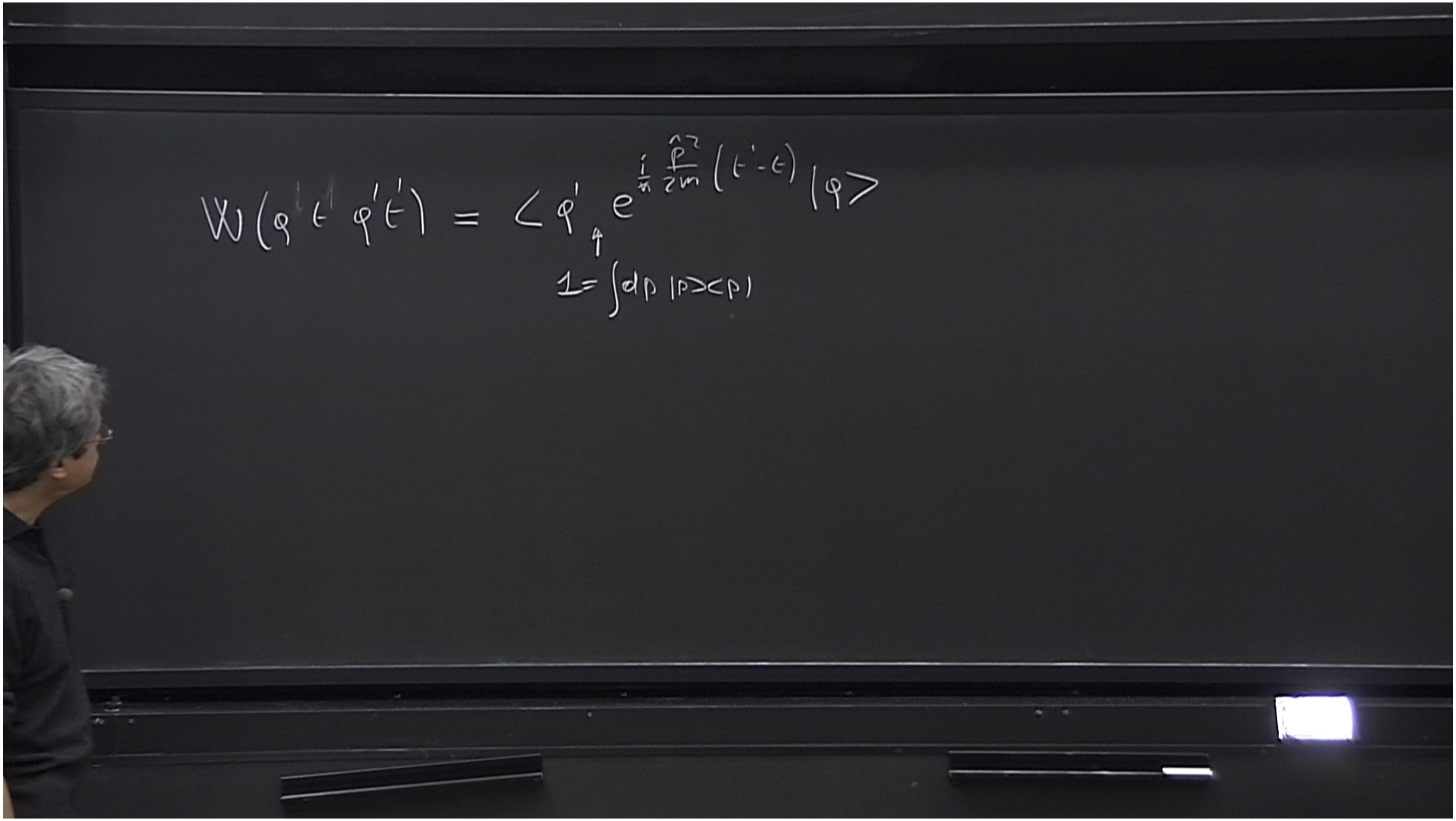

W(φ t φ' t') =

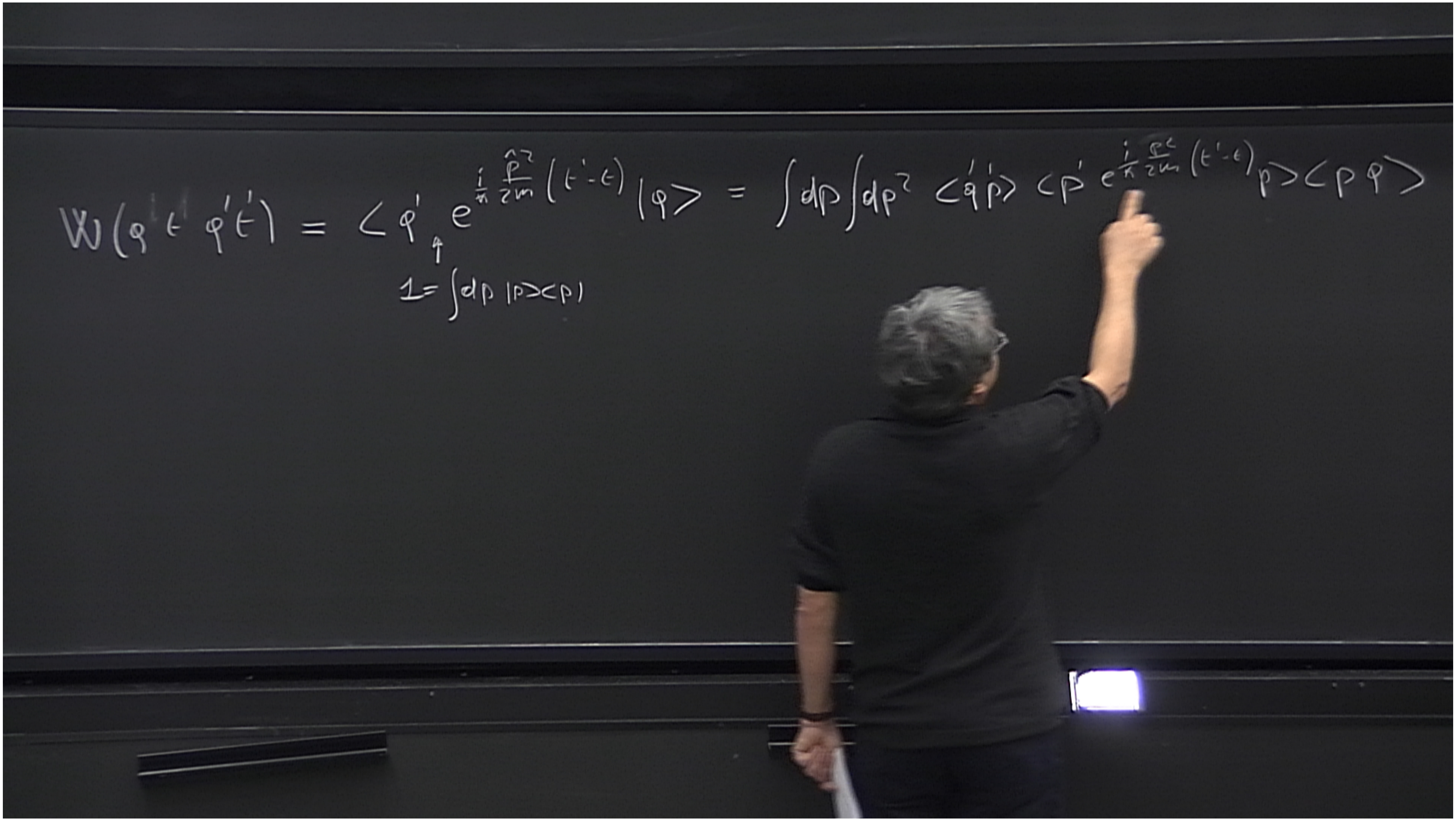
ITHOUT TIME

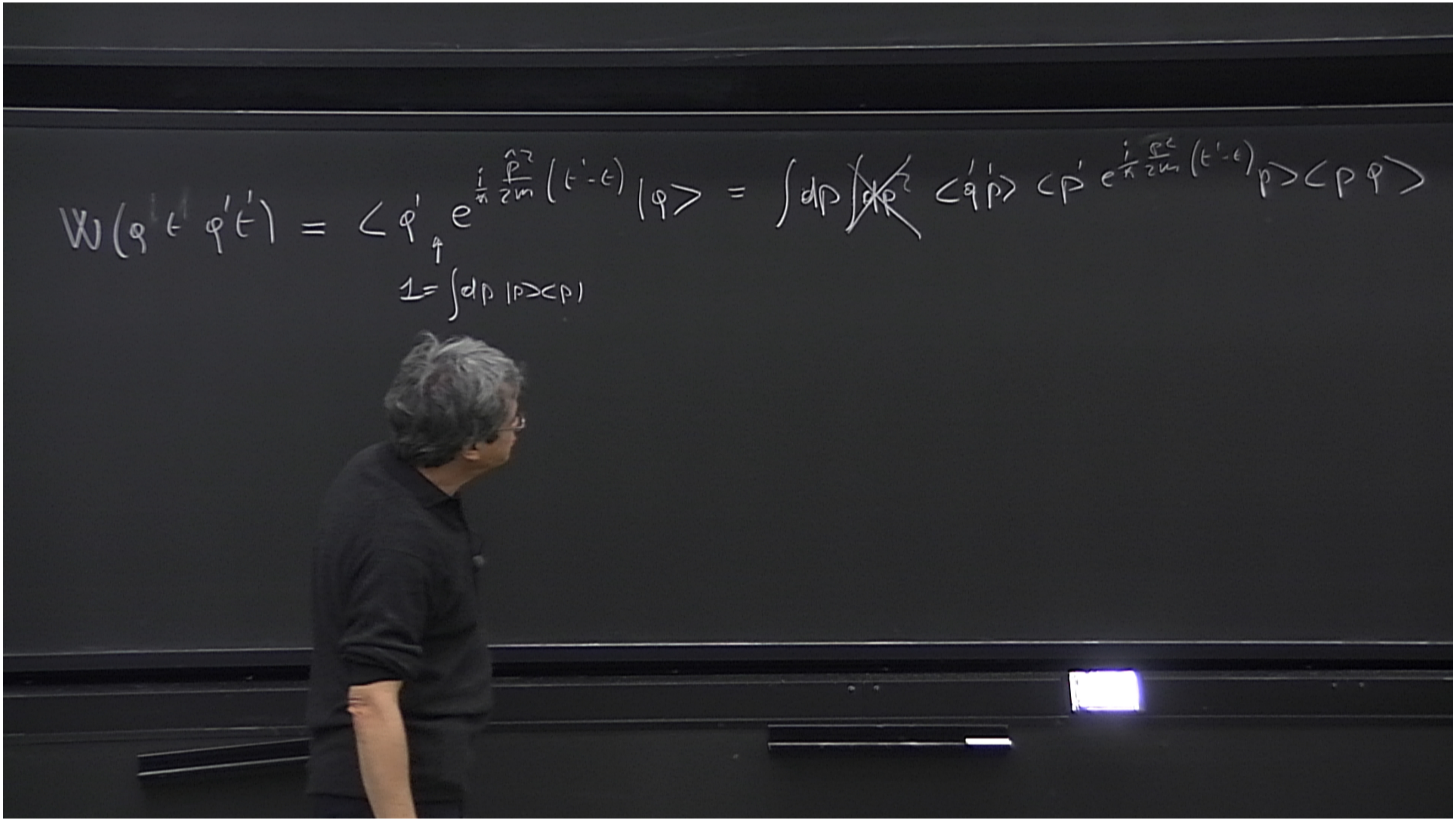
$$U(t) = e^{-\frac{i}{\hbar}Ht}$$

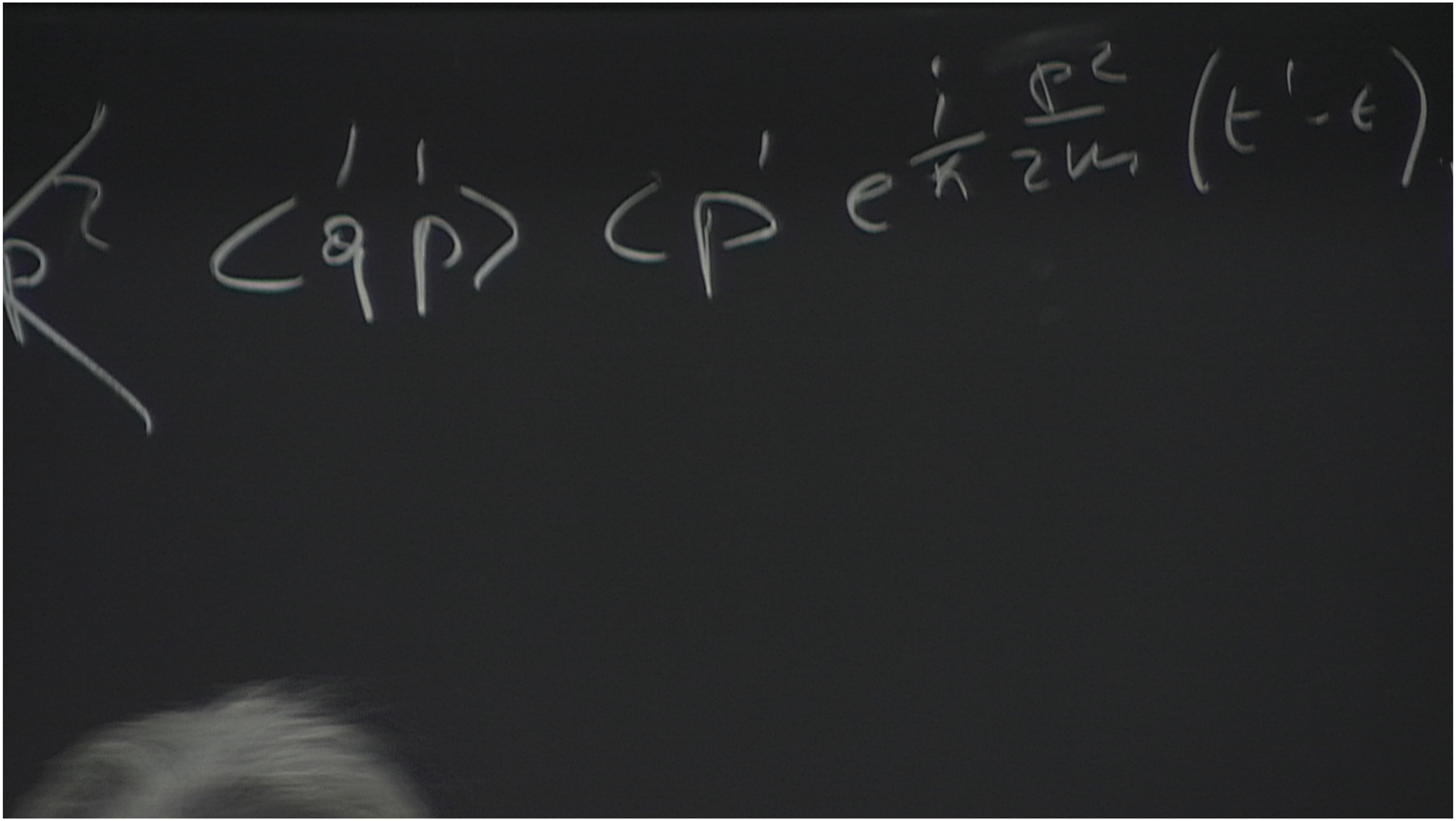
$$\hat{q}\,|q\rangle = q\,|q\rangle$$

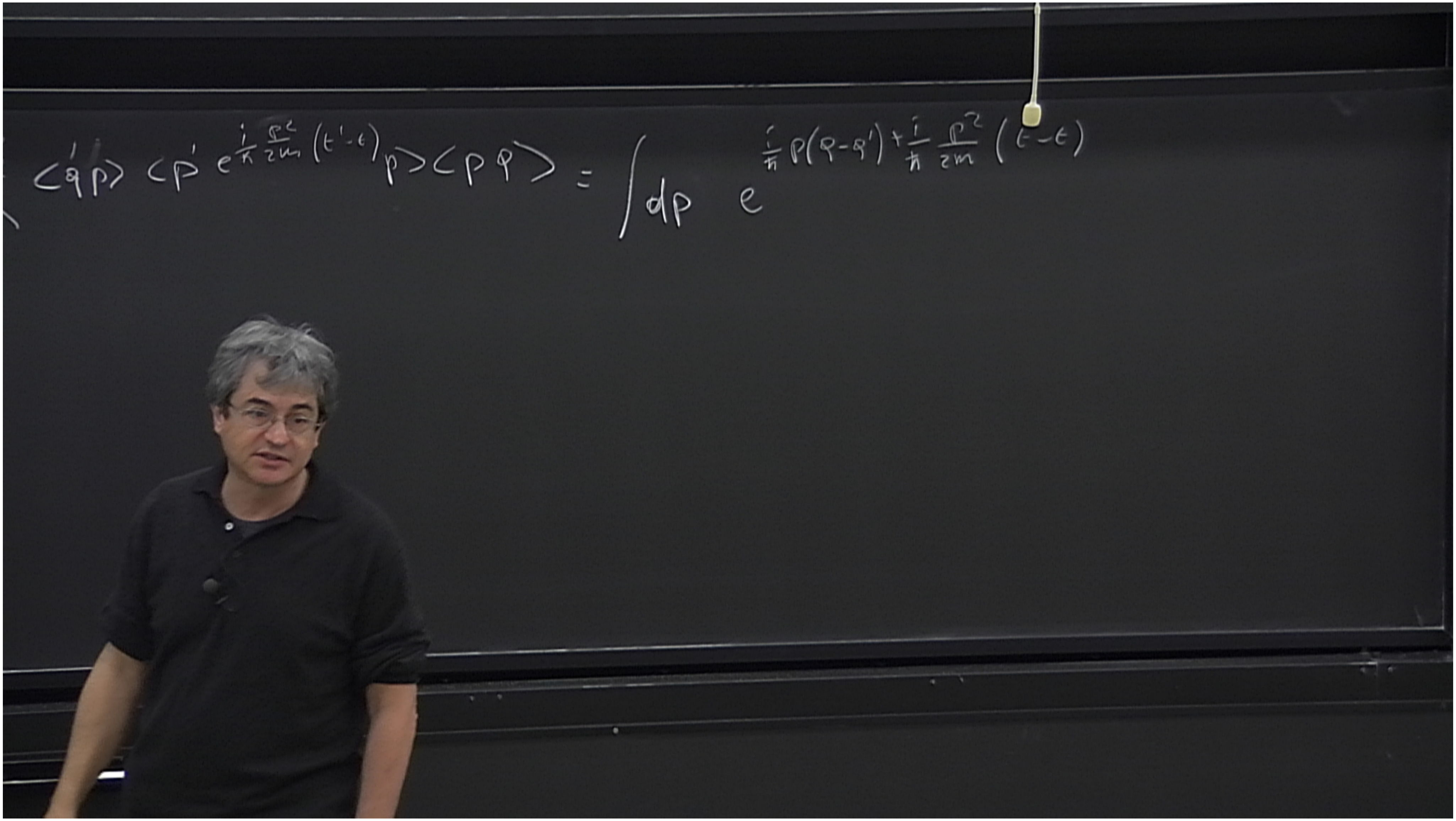
$\Rightarrow$ TRANSITION AMPLITUDES: $W(q,t,q',t') = \langle q'|\, e^{-\frac{i}{\hbar}(t'-t)}\,|q\rangle$

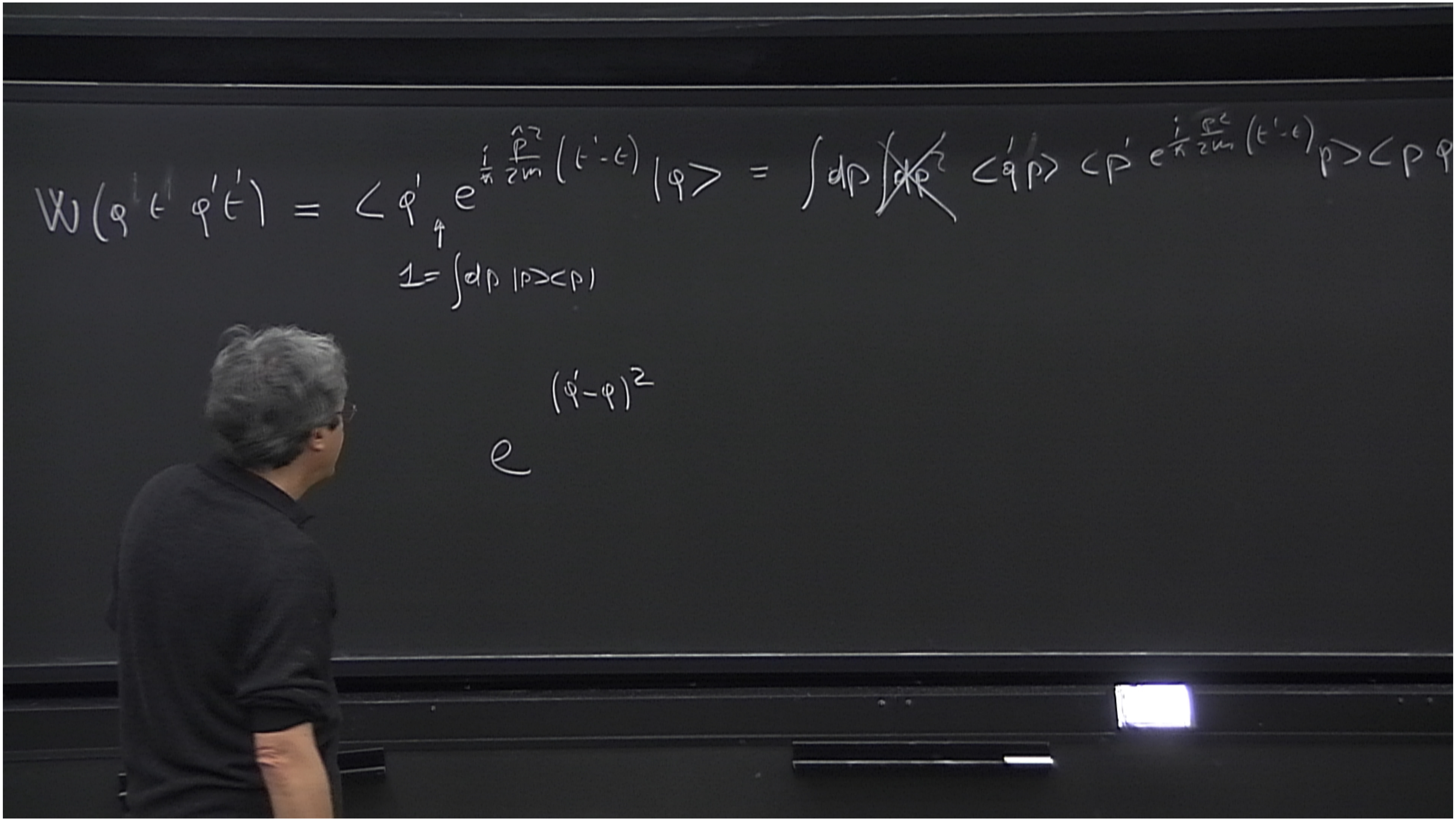

$W(q\,t\,q'\,t') = \langle q' |\, e^{\frac{i}{\hbar}\frac{\hat{p}^2}{2m}(t'-t)} \,| q \rangle$
$\mathbb{1} = \int dp\, |p\rangle\langle p|$

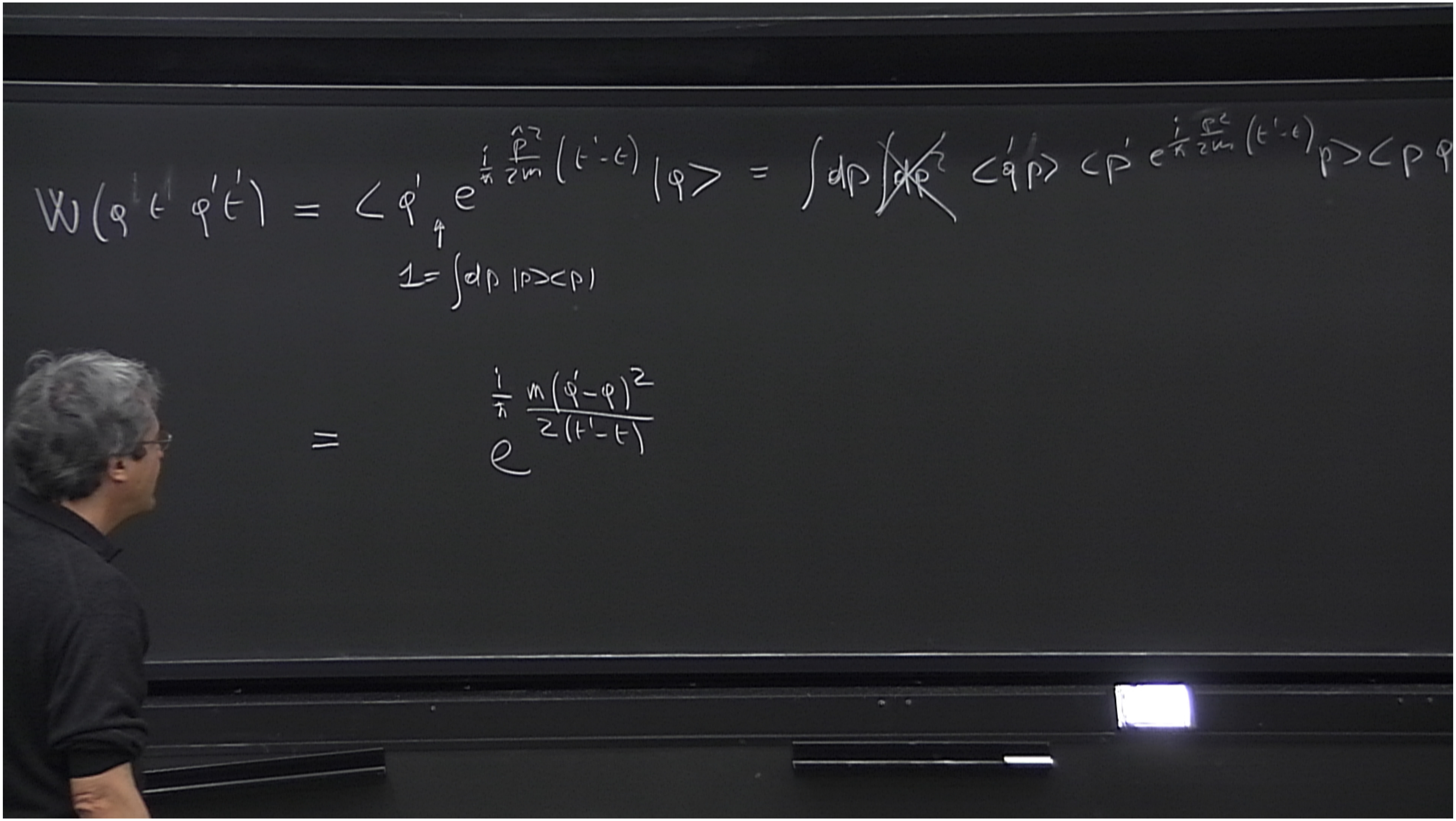

$W(q\,t\,q'\,t') = \langle q' \, e^{\frac{i}{\hbar}\frac{\hat{p}^2}{2m}(t'-t)} |q\rangle = \int dp \int dp' \langle q' p'\rangle \langle p' \, e^{\frac{i}{\hbar}\frac{p^2}{2m}(t'-t)} p\rangle\langle p\, q\rangle$
$1 = \int dp\, |p\rangle\langle p|$

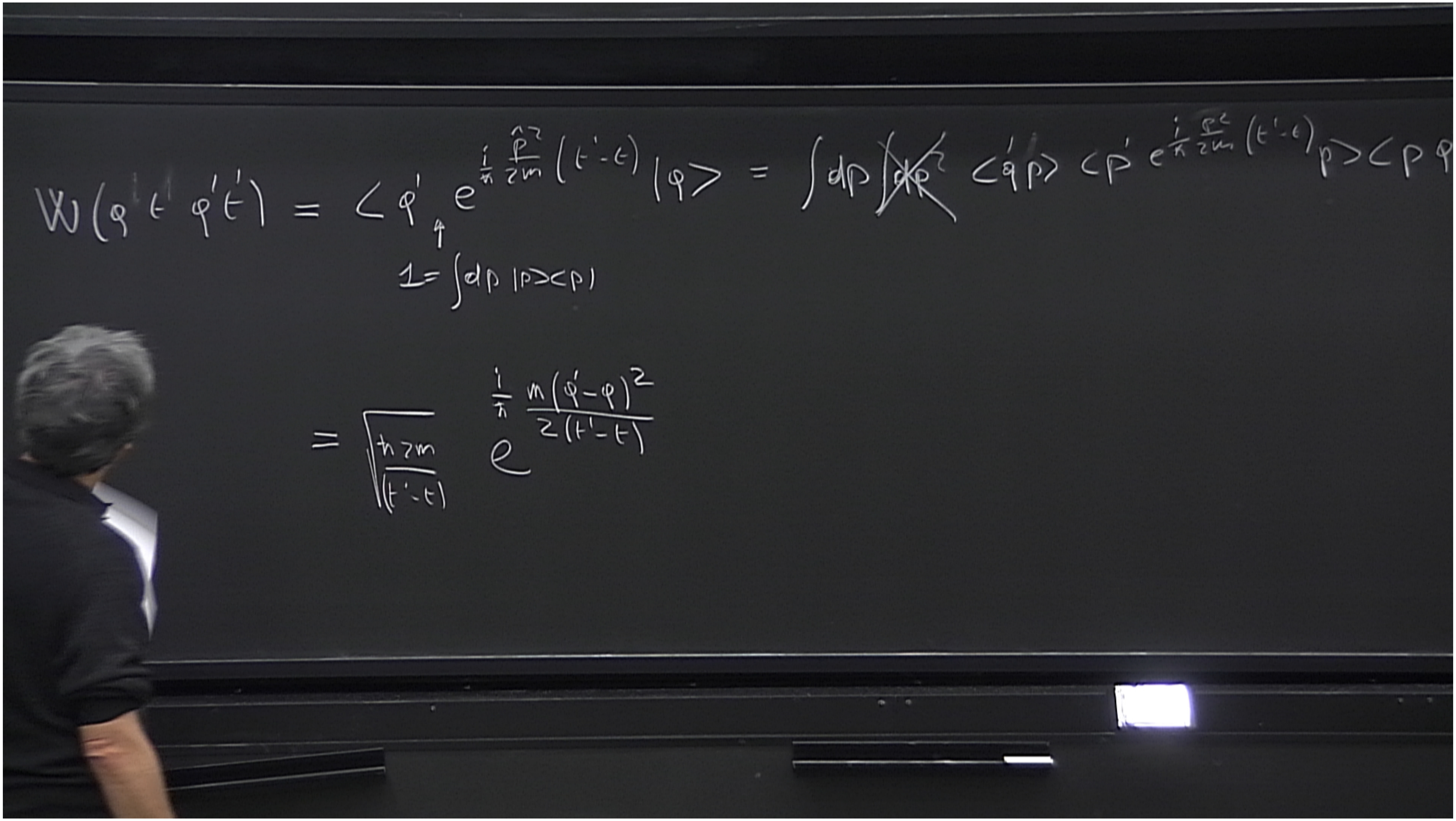

1= ∫dp |p><p|


⟨q'|p'⟩ ⟨p'| e^{i/ħ p²/2m (t' - t)}

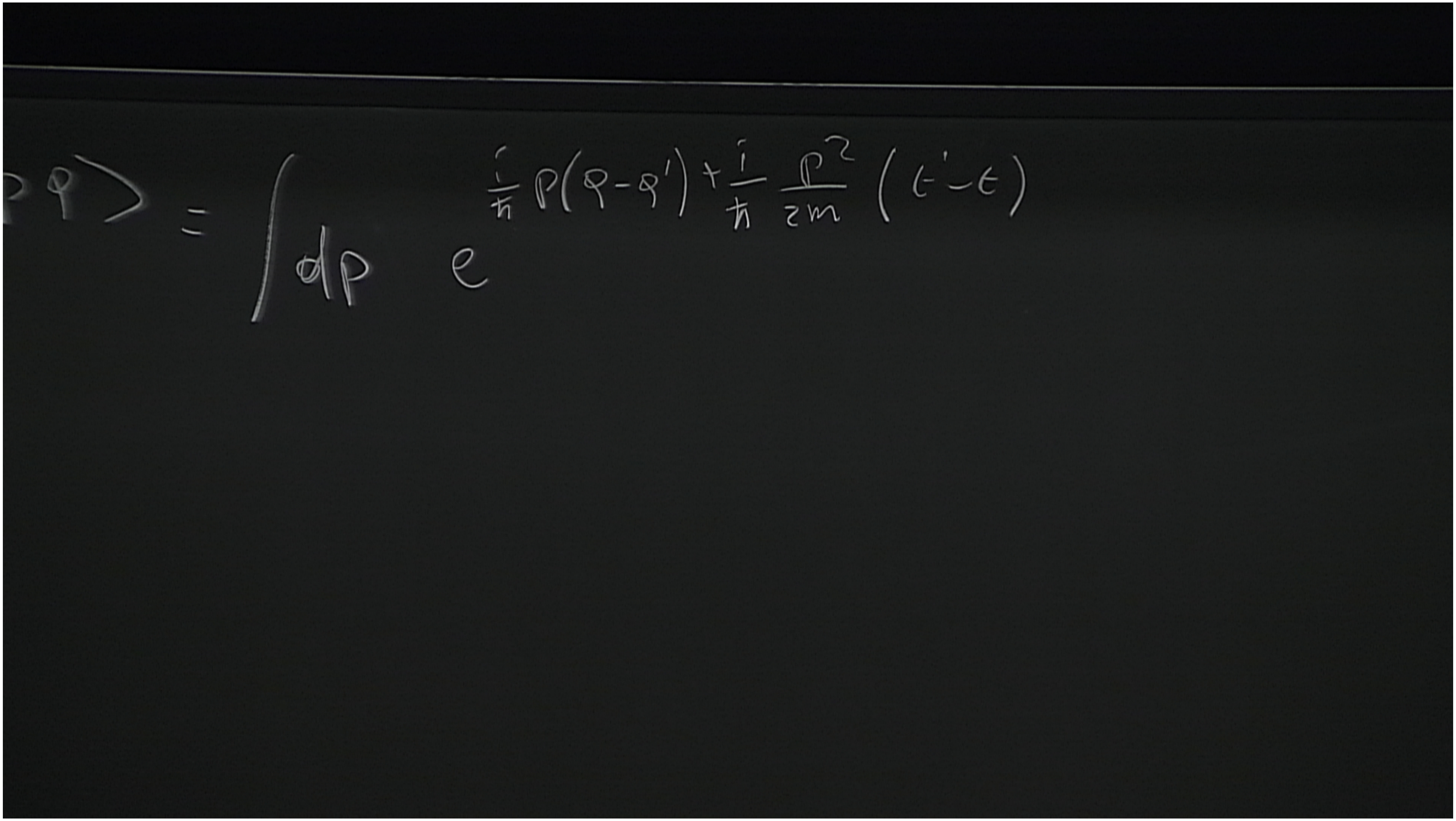

$\langle q' p \rangle \langle p' \, e^{\frac{i}{\hbar}\frac{p^2}{2m}(t'-t)} p \rangle \langle p \, q \rangle = \int dp \; e^{\frac{i}{\hbar} p (q - q') + \frac{i}{\hbar}\frac{p^2}{2m}(t'-t)}$


W(φ t φ' t') = <φ' e^{i/ħ p̂²/2m (t'-t)} |φ> = ∫dp <φ'p> <p' e^{i/ħ p²/2m (t'-t)} p><p φ
1 = ∫dp |p><p|
e^{(φ'-φ)²}


W(φ t φ' t') = <φ' e^{i/ħ p̂²/2m (t'-t)} |φ> = ∫dp <φ'|p> <p| e^{i/ħ p²/2m (t'-t)} |p><p|φ>
1 = ∫dp |p><p|
= e^{i/ħ m(φ'-φ)²/2(t'-t)}



$W(\varphi t\, \varphi' t') = \langle \varphi' | e^{\frac{i}{\hbar}\frac{\hat{p}^2}{2m}(t'-t)} | \varphi \rangle = \int dp\, \langle \varphi' | p \rangle \langle p' | e^{\frac{i}{\hbar}\frac{p^2}{2m}(t'-t)} | p \rangle \langle p\, \varphi$
$1 = \int dp\, |p\rangle\langle p|$
$= \sqrt{\frac{\hbar\, 2m}{(t'-t)}}\; e^{\frac{i}{\hbar}\frac{m(\varphi'-\varphi)^2}{2(t'-t)}}$


$q\rangle = \int dp \; e^{\frac{i}{\hbar} p(q-q') + \frac{i}{\hbar} \frac{p^2}{2m} (t'-t)}$

$$W(\varphi\, t\;\; \varphi' t') = \langle \varphi' |\, e^{\frac{i}{\hbar}\frac{\hat{p}^2}{2m}(t'-t)}\, |\varphi\rangle = \int dp\; \langle \varphi' p\rangle \langle p'|\, e^{\frac{i}{\hbar}\frac{p^2}{2m}(t'-t)}\, |p\rangle \langle p\, \varphi$$

$$1 = \int dp\, |p\rangle\langle p|$$

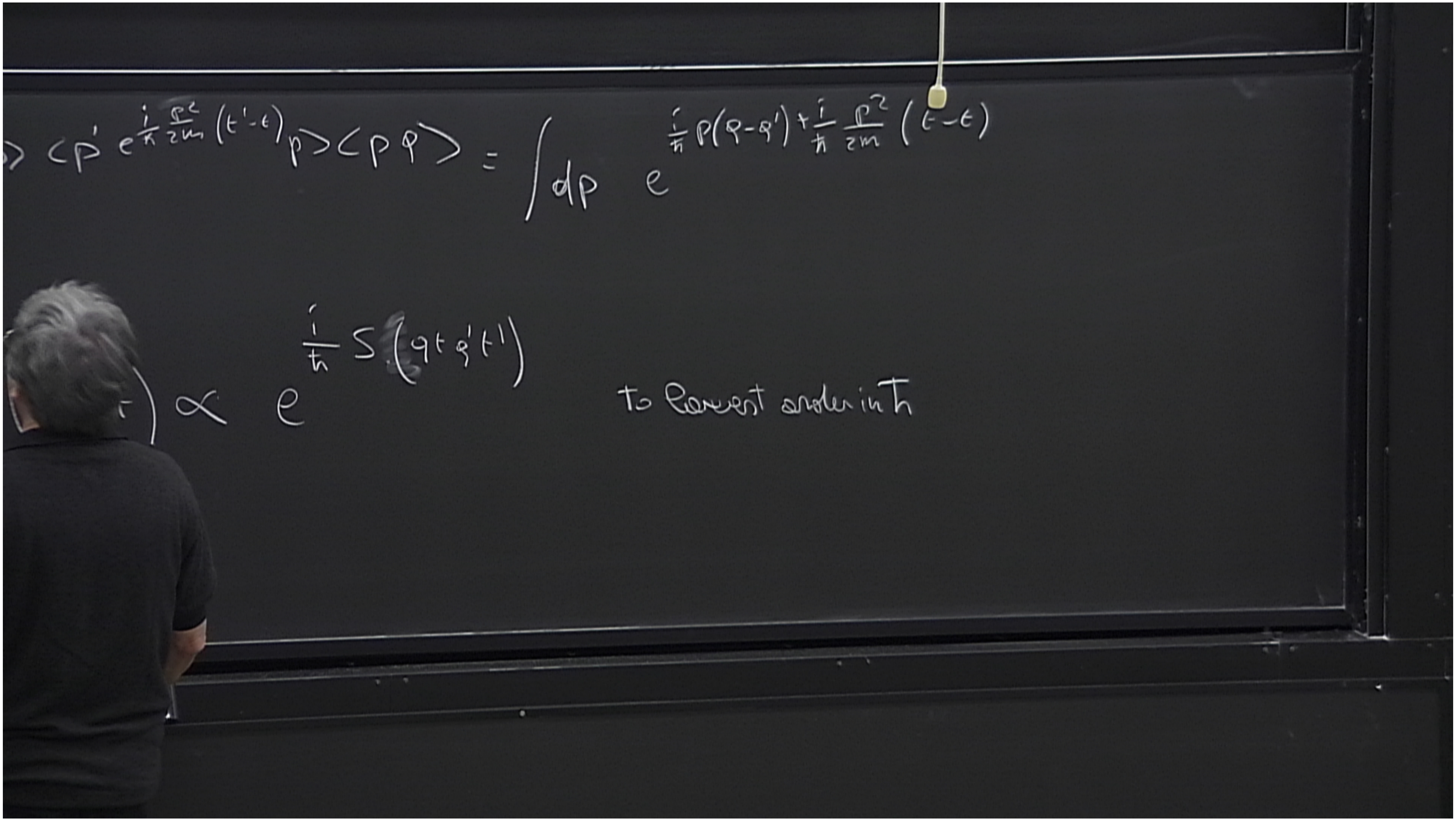
HAMILTON FUNCTION

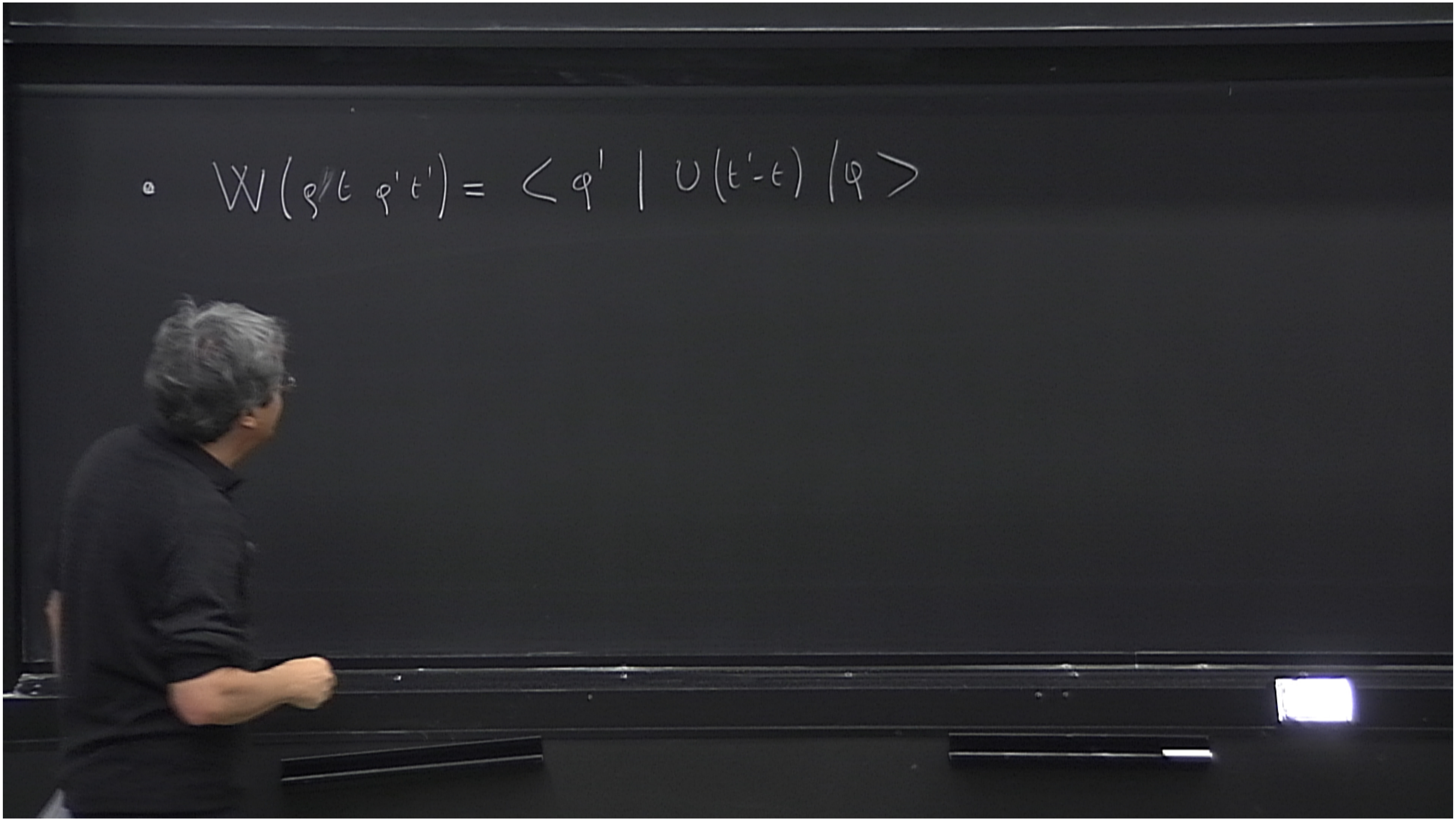
$$= \sqrt{\frac{\hbar\, 2m}{(t'-t)}}\; e^{\frac{i}{\hbar}\frac{m(\varphi'-\varphi)^2}{2(t'-t)}}$$

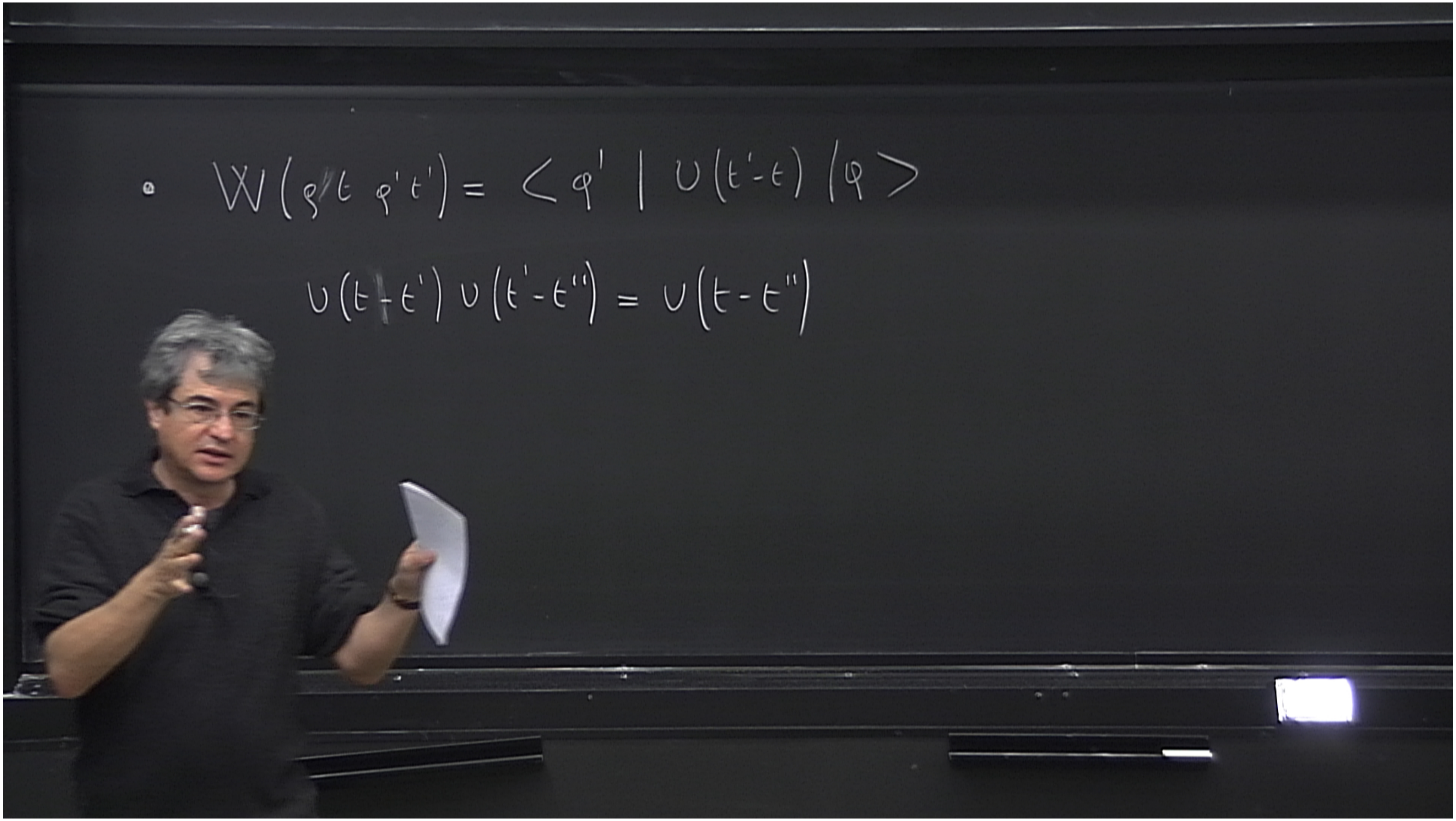
$$\int dp \, \cancel{dp'} \, \langle q' p \rangle \langle p' \, e^{\frac{i}{\hbar}\frac{p^2}{2m}(t'-t)} \, p \rangle \langle p \, q \rangle = \int dp \; e^{\frac{i}{\hbar} p (q - q') + \frac{i}{\hbar}\frac{p^2}{2m}(t'-t)}$$

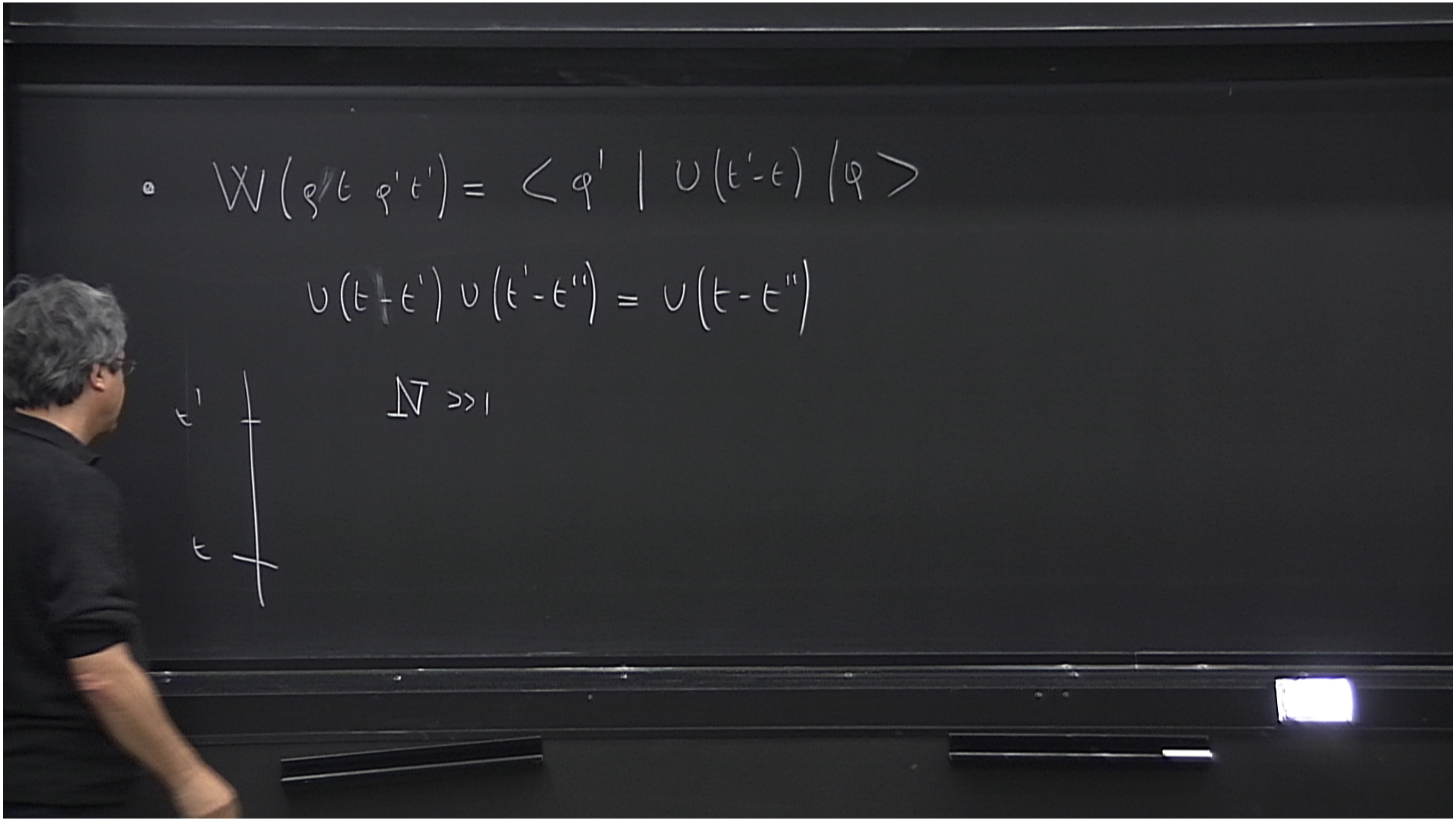
$$\Psi(q t \, q' t') \propto e^{\frac{i}{\hbar} S(q t \, q' t')}$$

to lowest order in $\hbar$

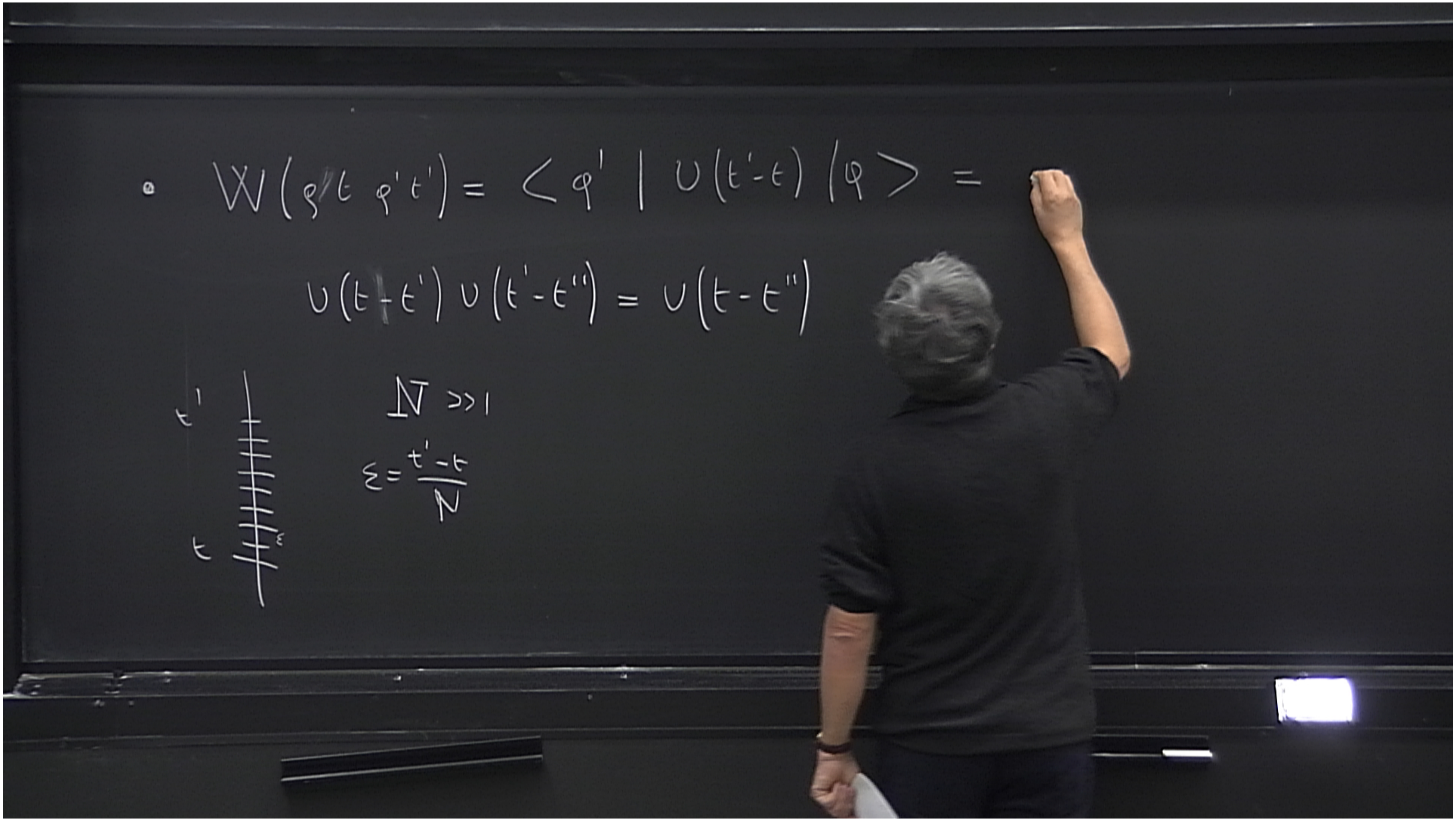

$\rangle \langle p' e^{\frac{i}{\hbar}\frac{p^2}{2m}(t'-t)} p \rangle \langle p\, q \rangle = \int dp \; e^{\frac{i}{\hbar}p(q-q') + \frac{i}{\hbar}\frac{p^2}{2m}(t'-t)}$
$t) \propto e^{\frac{i}{\hbar}S(q t\, q' t')}$
to lowest order in $\hbar$

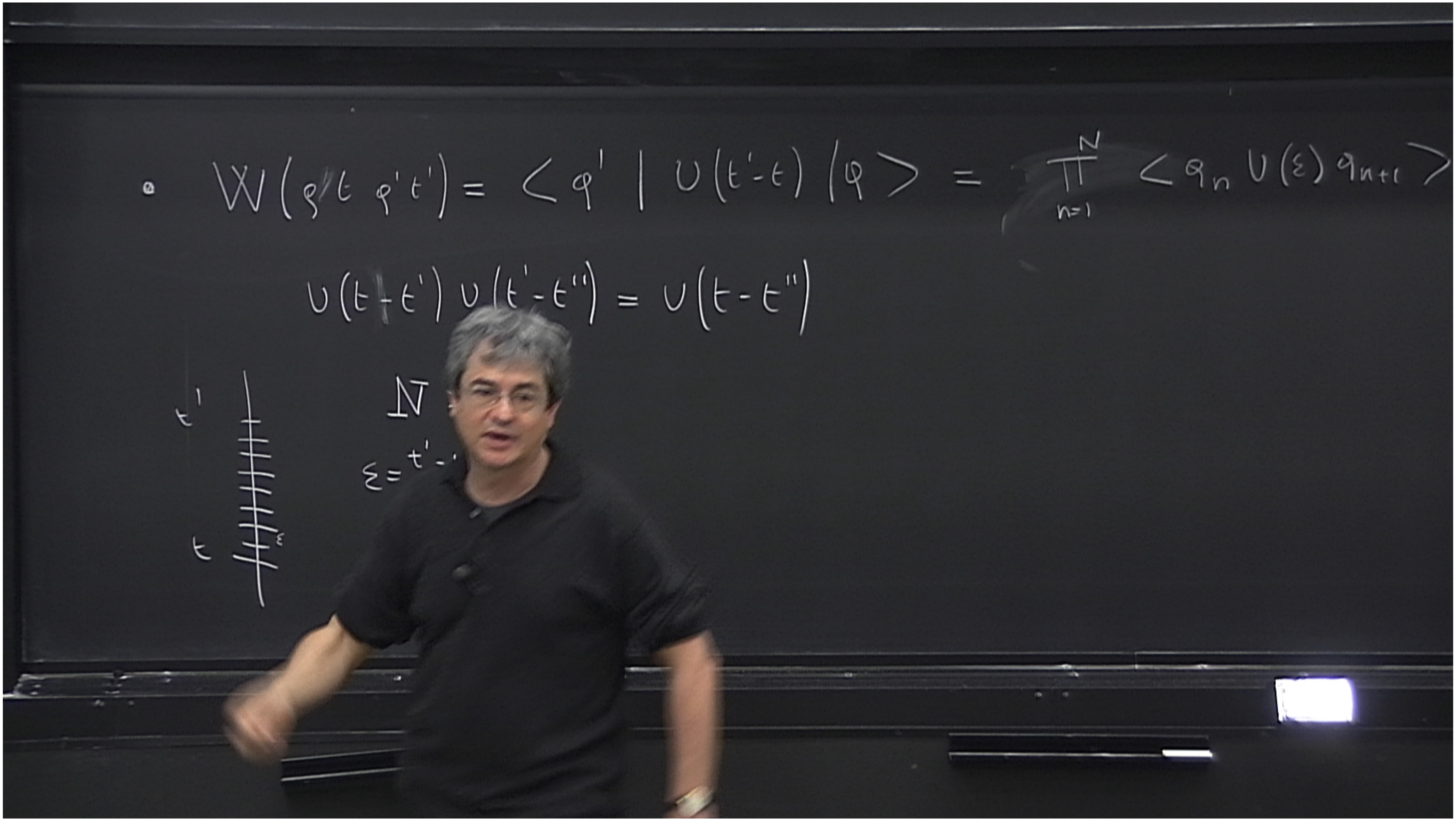

$W(\varphi t\, \varphi' t') = \langle \varphi' | U(t'-t) | \varphi \rangle$

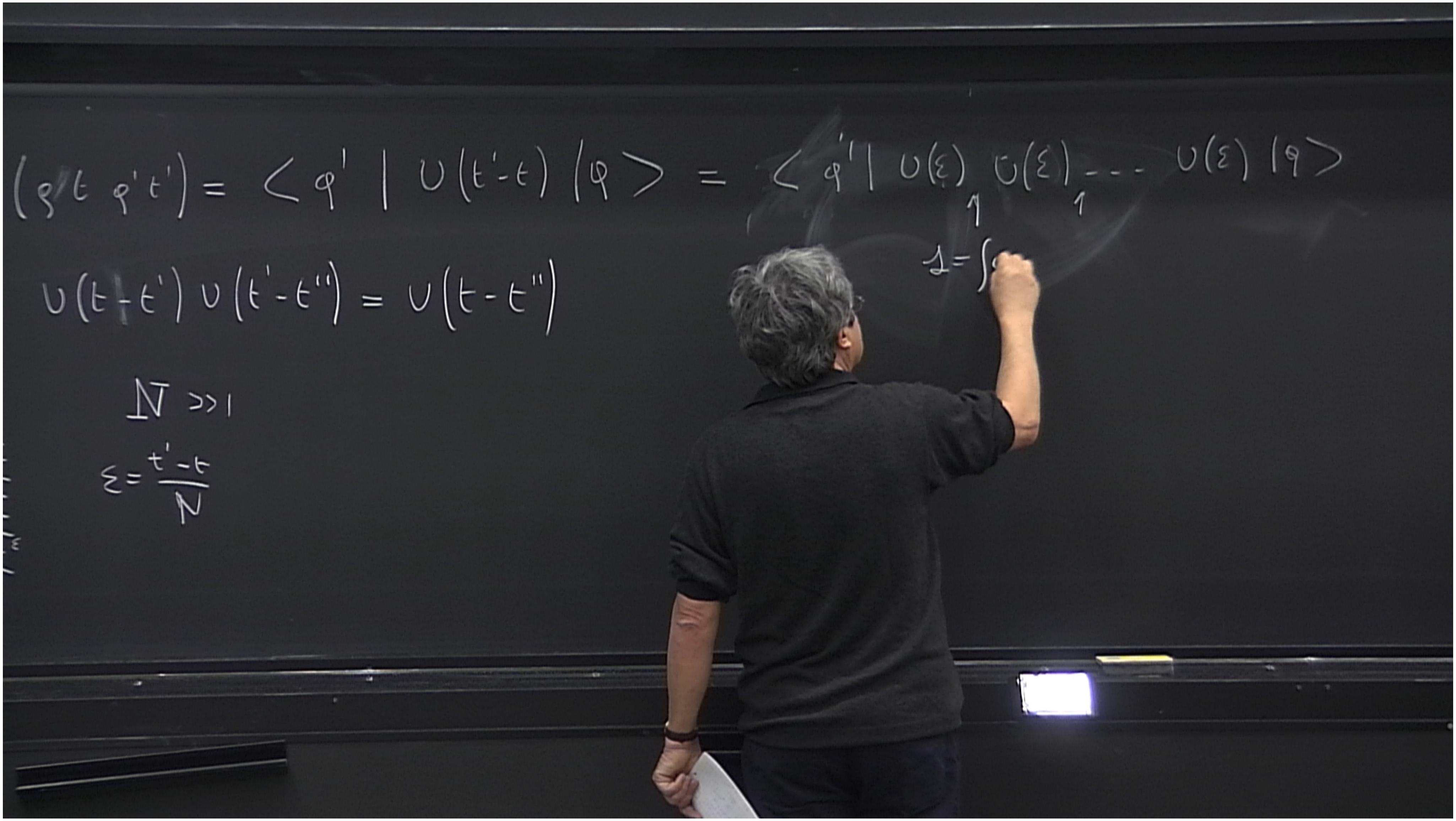

$W(q\,t\,q'\,t') = \langle q' | U(t'-t) | q \rangle$
$U(t-t')\,U(t'-t'') = U(t-t'')$

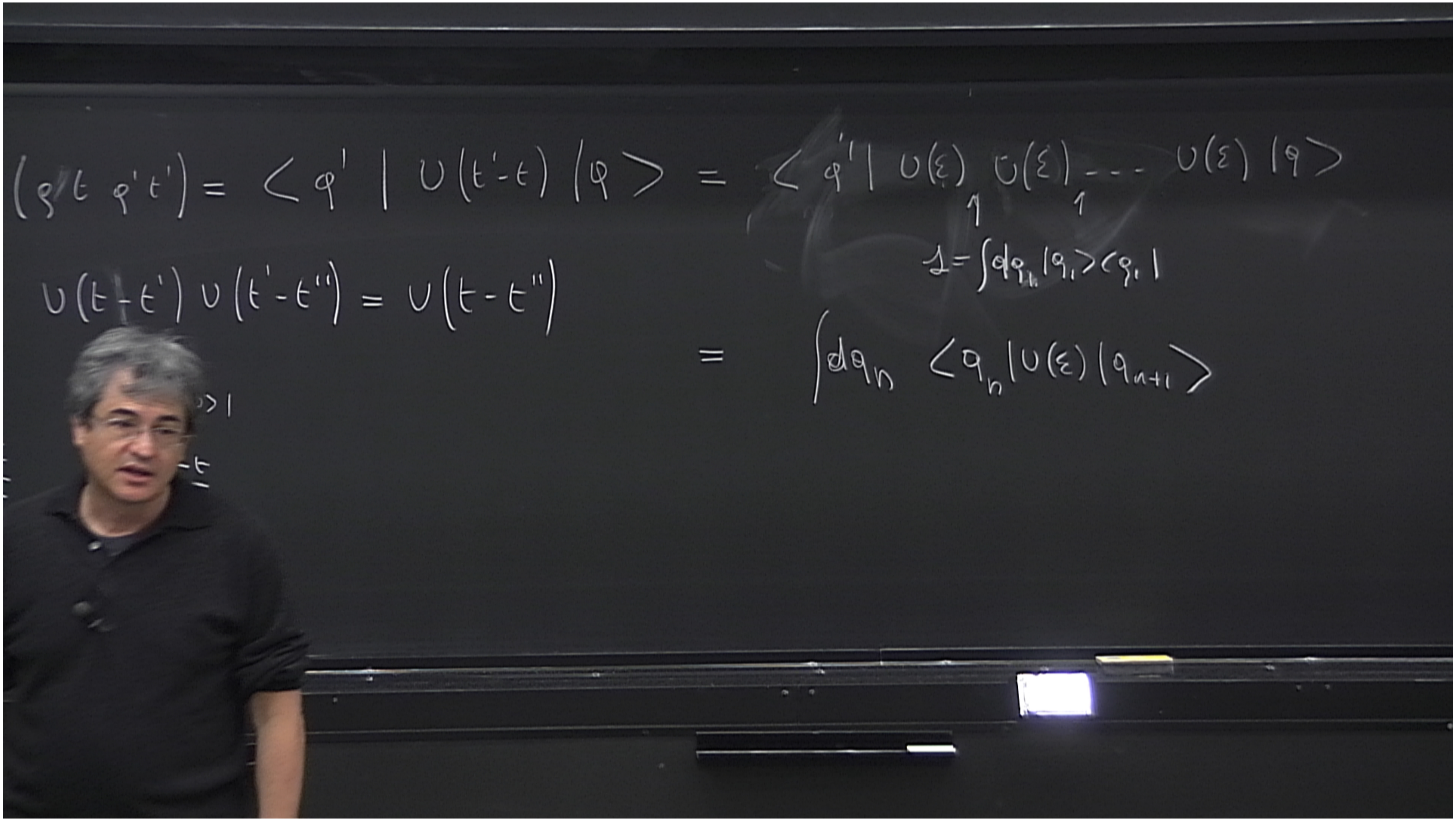


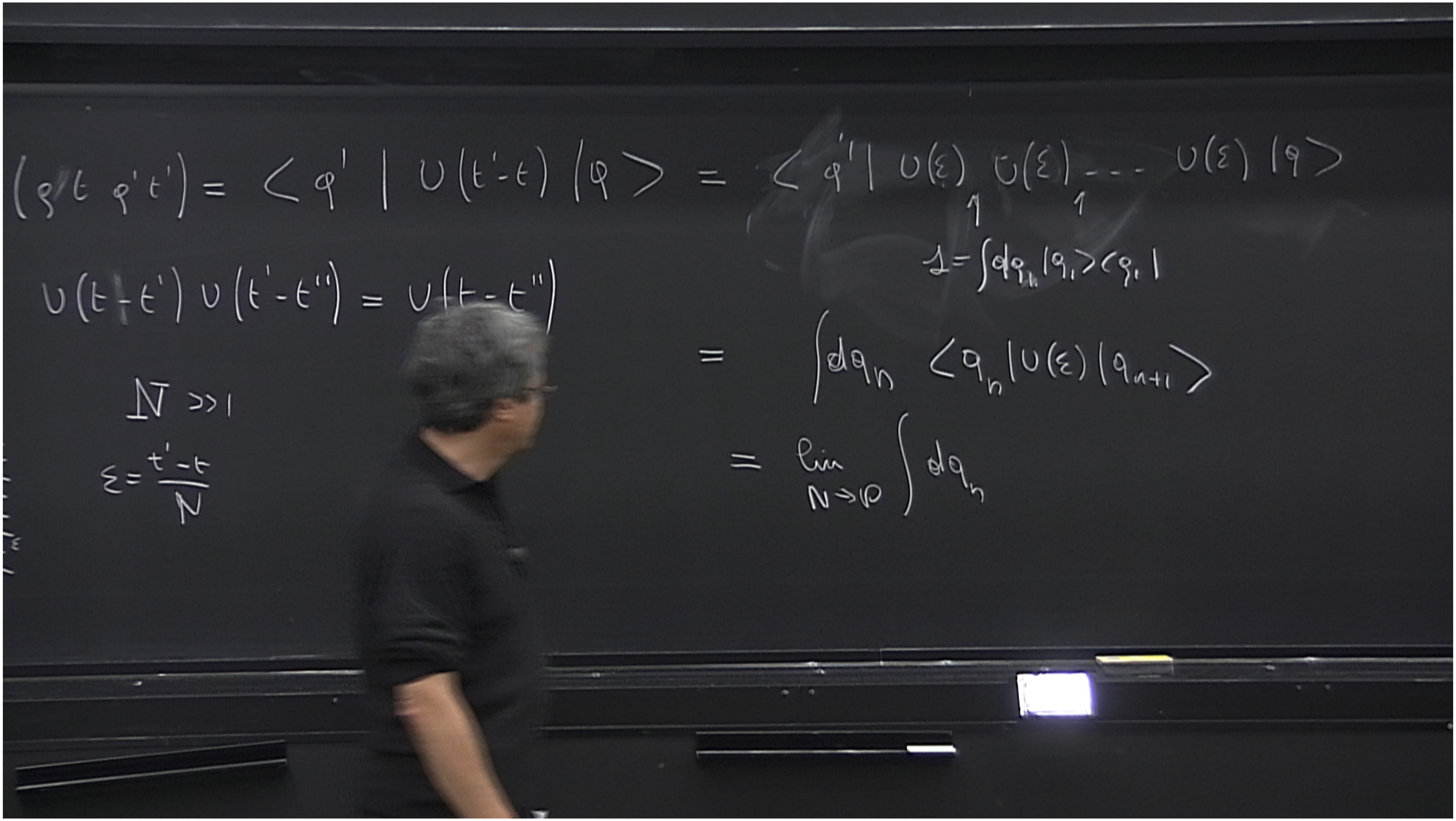
$W(q\,t\;q'\,t') = \langle q' | U(t'-t) | q \rangle$
$U(t-t')\,U(t'-t'') = U(t-t'')$
t'
t
$N \gg 1$

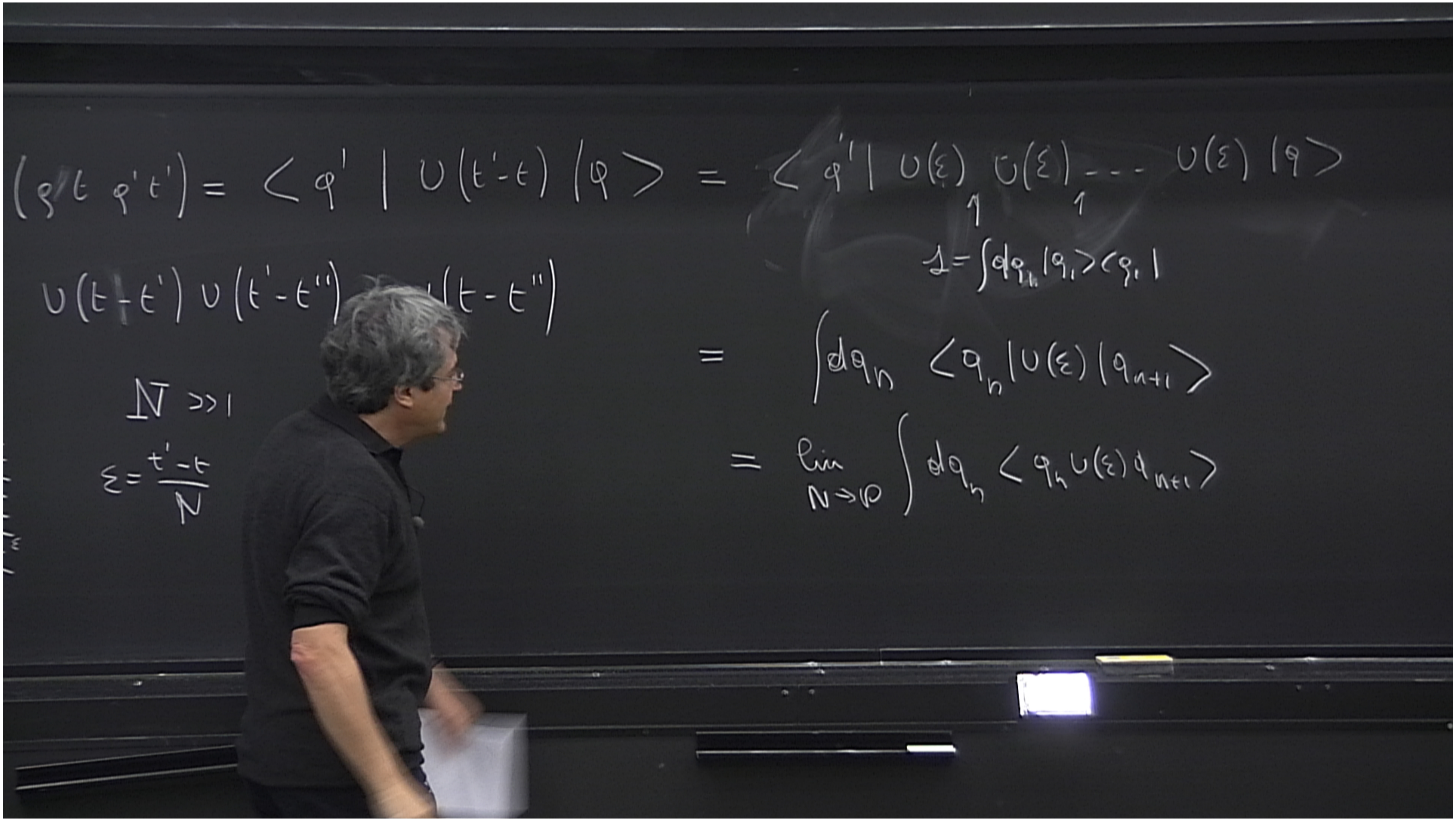

W(q t q' t') = < q' | U(t'-t) | q > =
U(t-t') U(t'-t'') = U(t-t'')
t'
t
ε
N >> 1
ε = (t'-t)/N

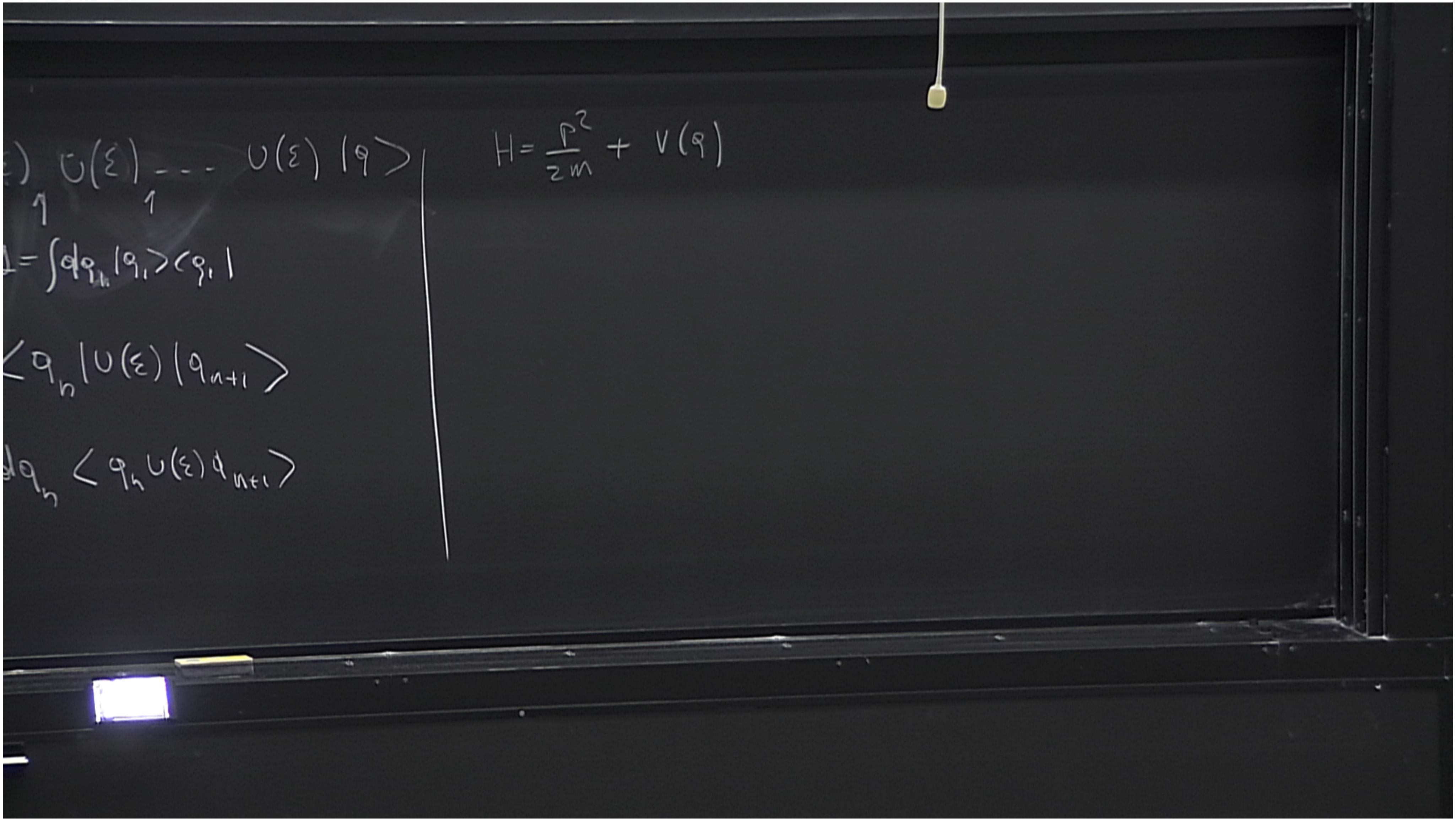

W(q t q' t') = < q' | U(t'-t) | q > = ∏_{n=1}^{N} < q_n U(ε) q_{n+1} >
U(t-t') U(t'-t'') = U(t-t'')
t'
t
ε
N
ε = t' -

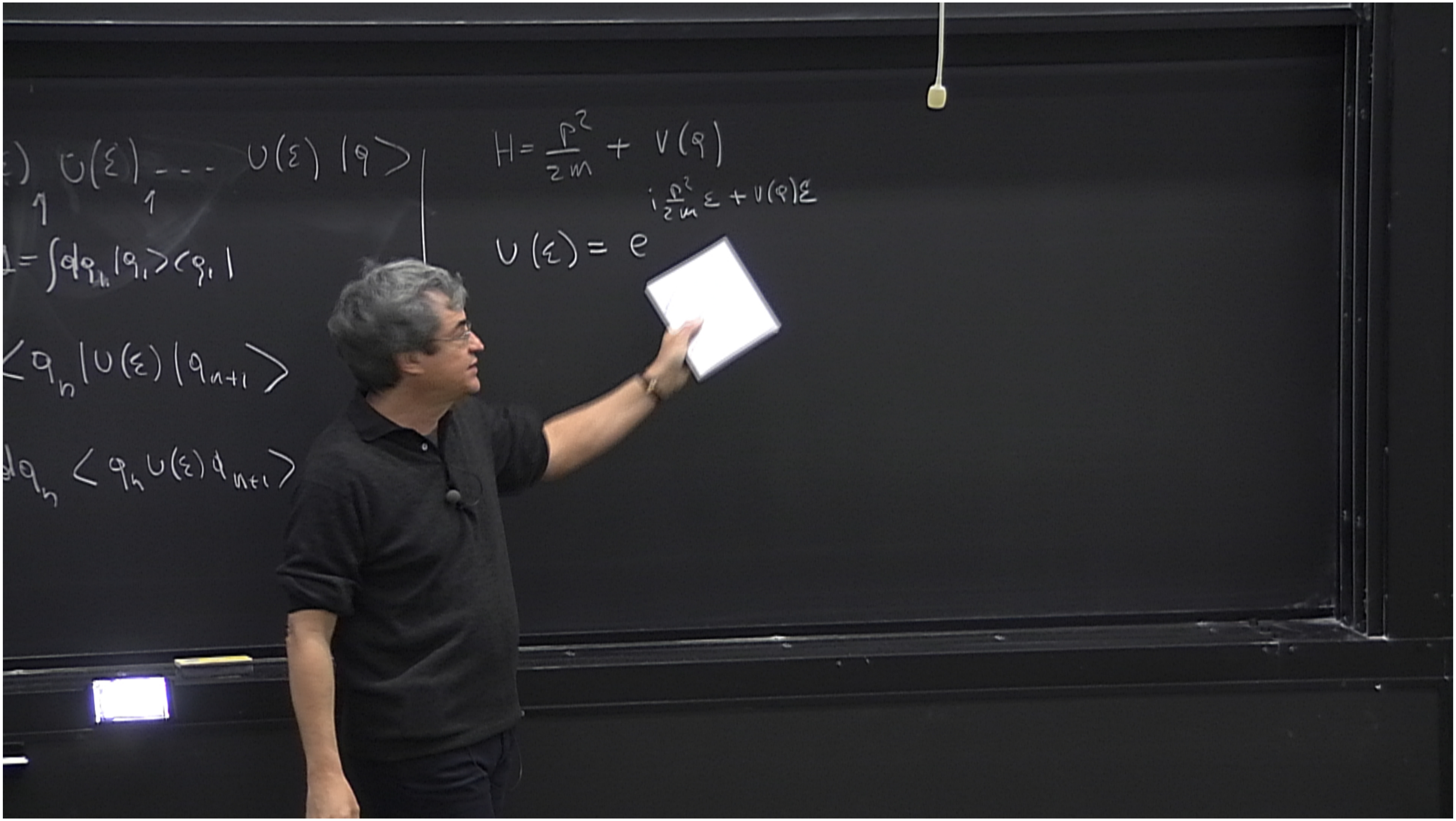

(q t q' t') = < q' | U(t'-t) | q > = < q' | U(ε) U(ε) ... U(ε) | q >
U(t-t') U(t'-t'') = U(t-t'')
N >> 1
ε = (t'-t)/N

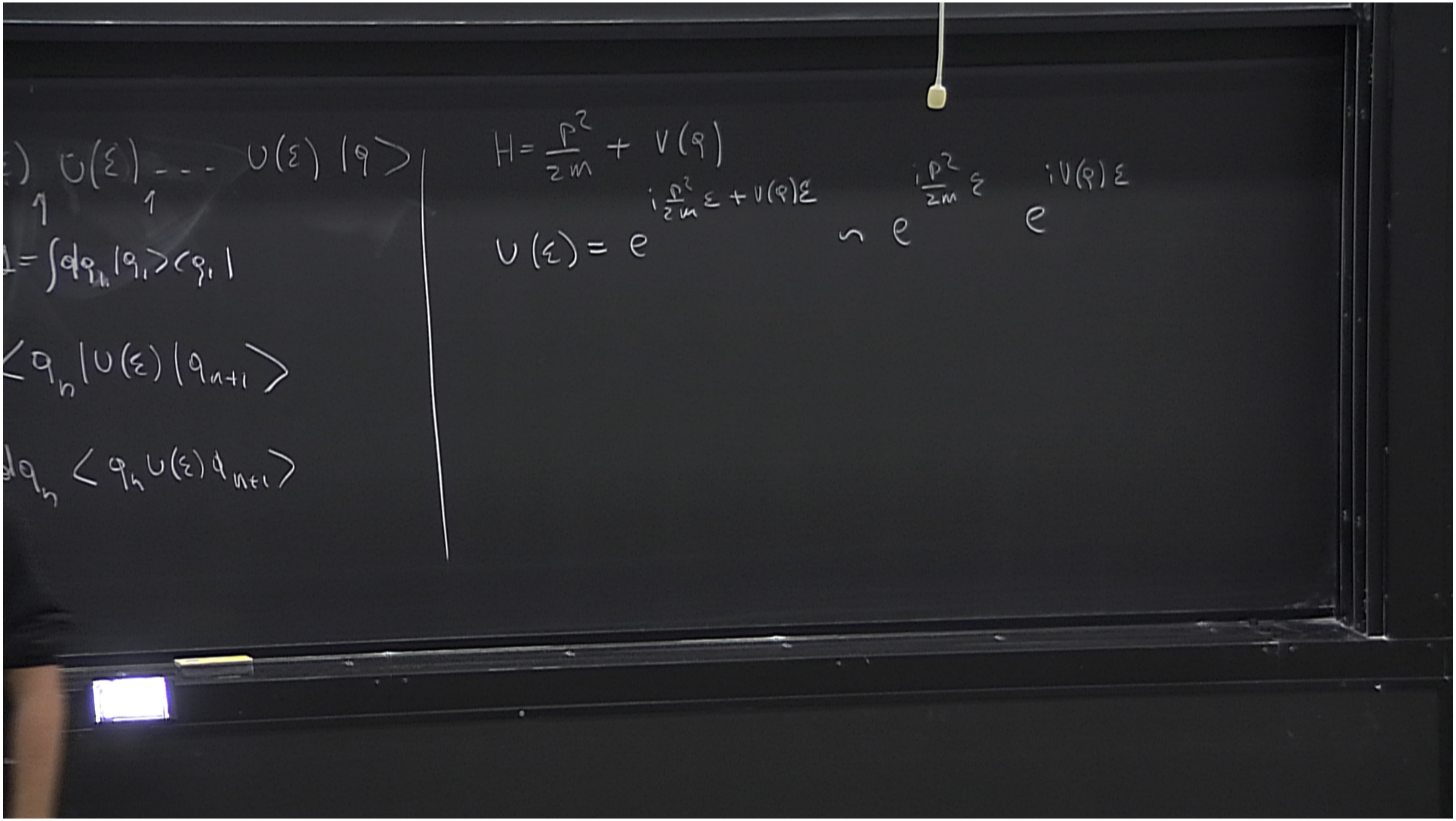

(q t q' t') = < q' | U(t'-t) | q > = < q' | U(ε) U(ε) ... U(ε) | q >
1 = ∫dq₁ |q₁><q₁|
U(t-t') U(t'-t'') = U(t-t'')
= ∫dqₙ < qₙ | U(ε) | qₙ₊₁ >

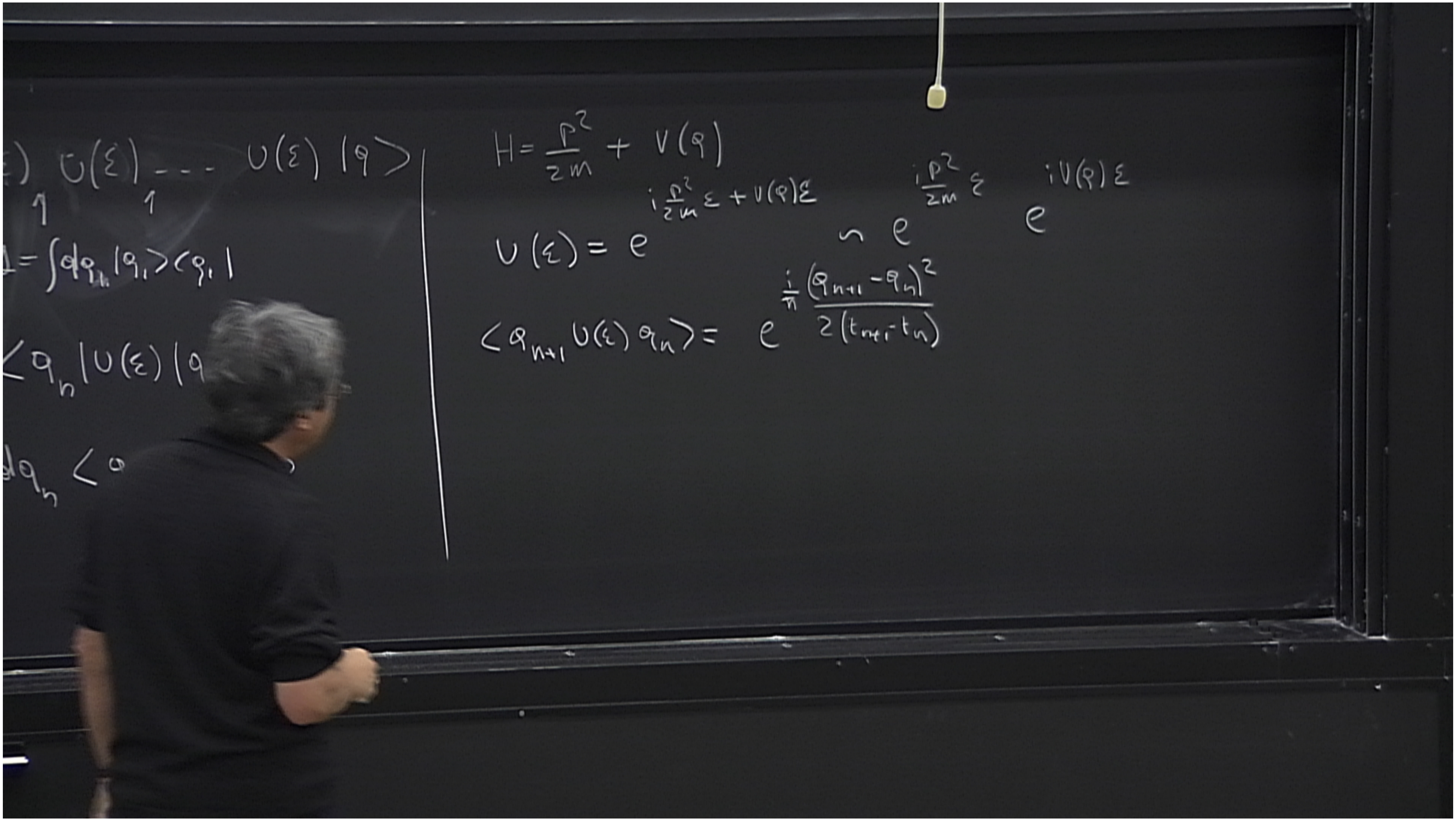

(q t q' t') = < q' | U(t' - t) | q > = < q' | U(ε) U(ε) ... U(ε) | q >
1 = ∫dq_n |q_n>< q_n |
U(t - t') U(t' - t'') = U(t - t'')
= ∫dq_n < q_n | U(ε) | q_{n+1} >
N >> 1
= lim N→∞ ∫dq_n
ε = (t' - t)/N

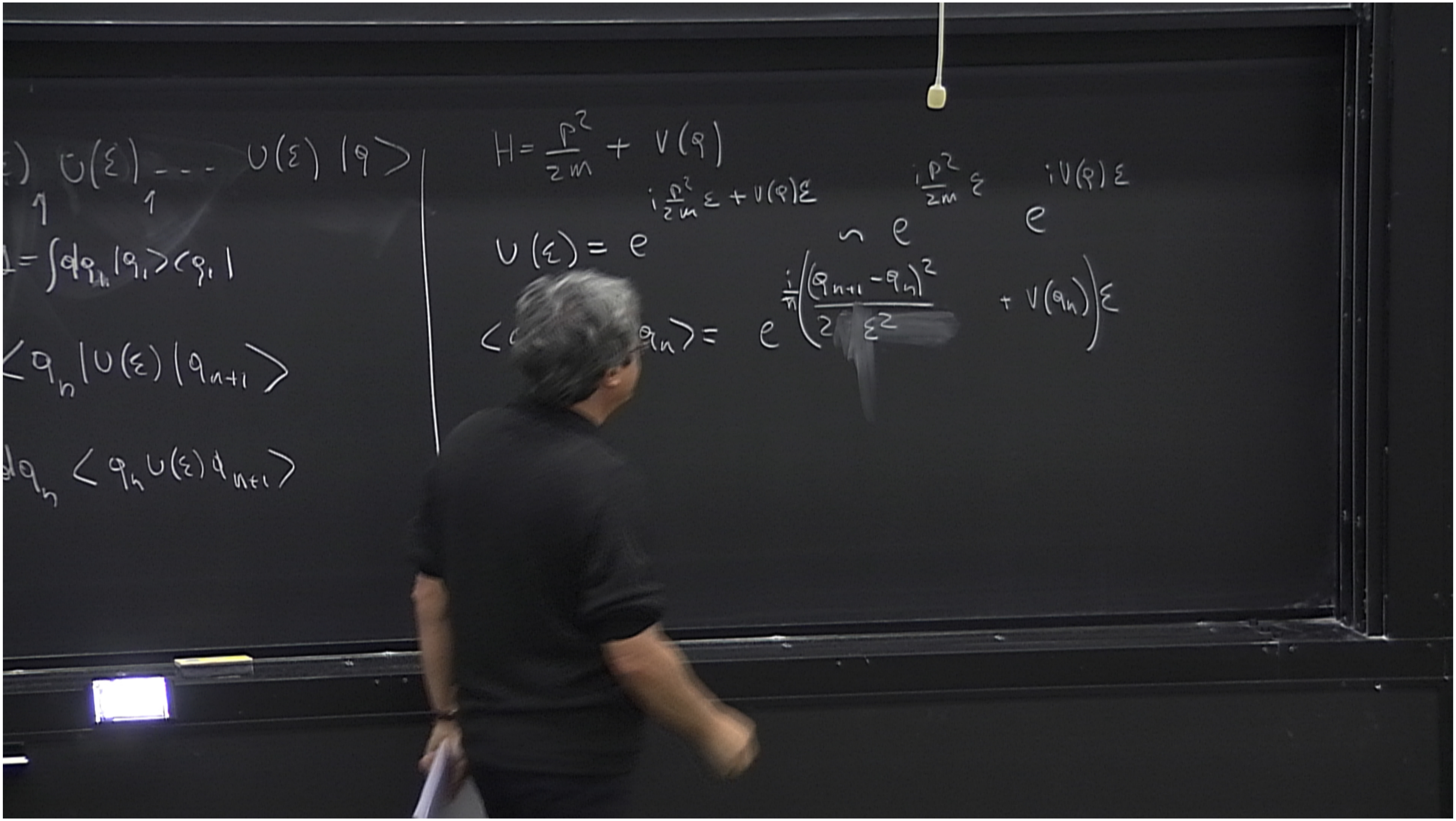

(q t q' t') = < q' | U(t'-t) | q > = < q' | U(ε) U(ε) - - - U(ε) | q >
1 = ∫dq_n |q_n><q_n|
U(t-t') U(t'-t'')
(t-t'')
= ∫dq_n <q_n | U(ε) | q_{n+1}>
N >> 1
ε = (t'-t)/N
= lim N→∞ ∫dq_n <q_n U(ε) q_{n+1}>

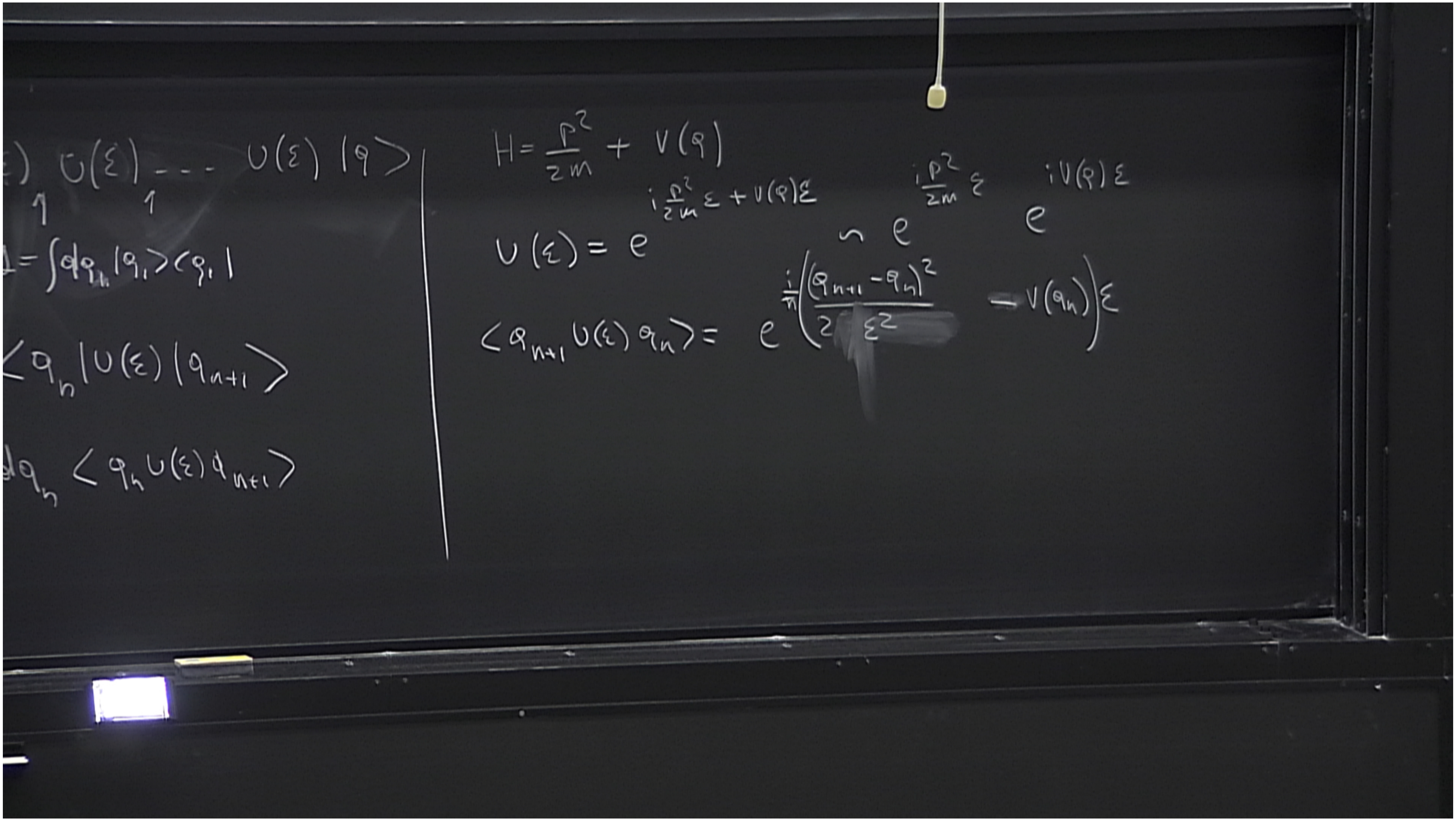

) U(ε) ... U(ε) |q>
1 = ∫dq_n |q_n><q_n|
<q_n|U(ε)|q_{n+1}>
dq_n <q_n U(ε) q_{n+1}>
H = p^2/2m + V(q)

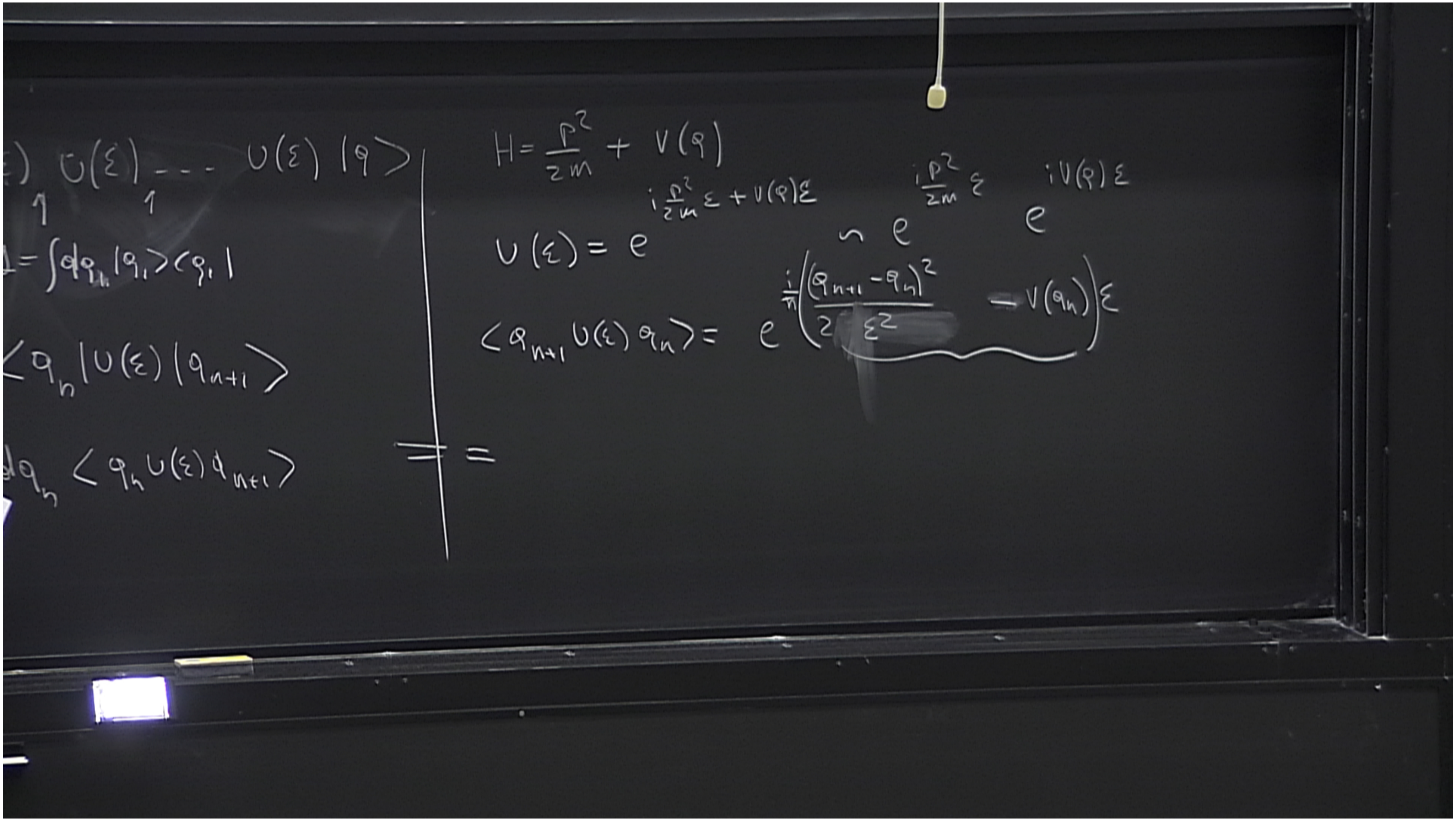

H = p²/2m + V(q)
U(ε) = e^{i p²/2m ε + V(q)ε}

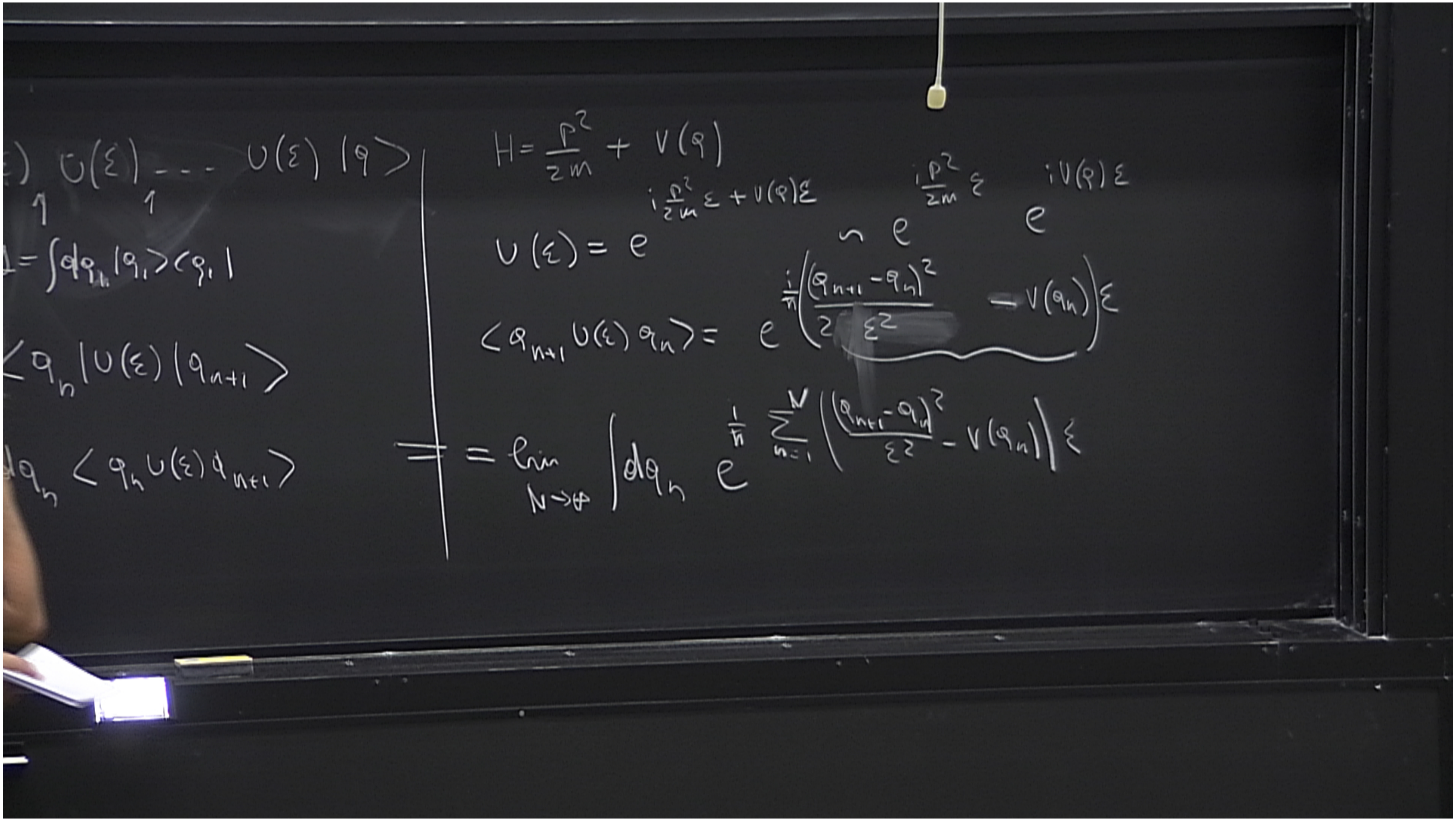

ε) U(ε) - - - U(ε) |q>
1 = ∫dq_n |q_n><q_n|
<q_n|U(ε)|q_{n+1}>
dq_n <q_n U(ε) q_{n+1}>
H = p^2/2m + V(q)
U(ε) = e^{i p^2/2m ε + V(q)ε} ~ e^{i p^2/2m ε} e^{i V(q) ε}

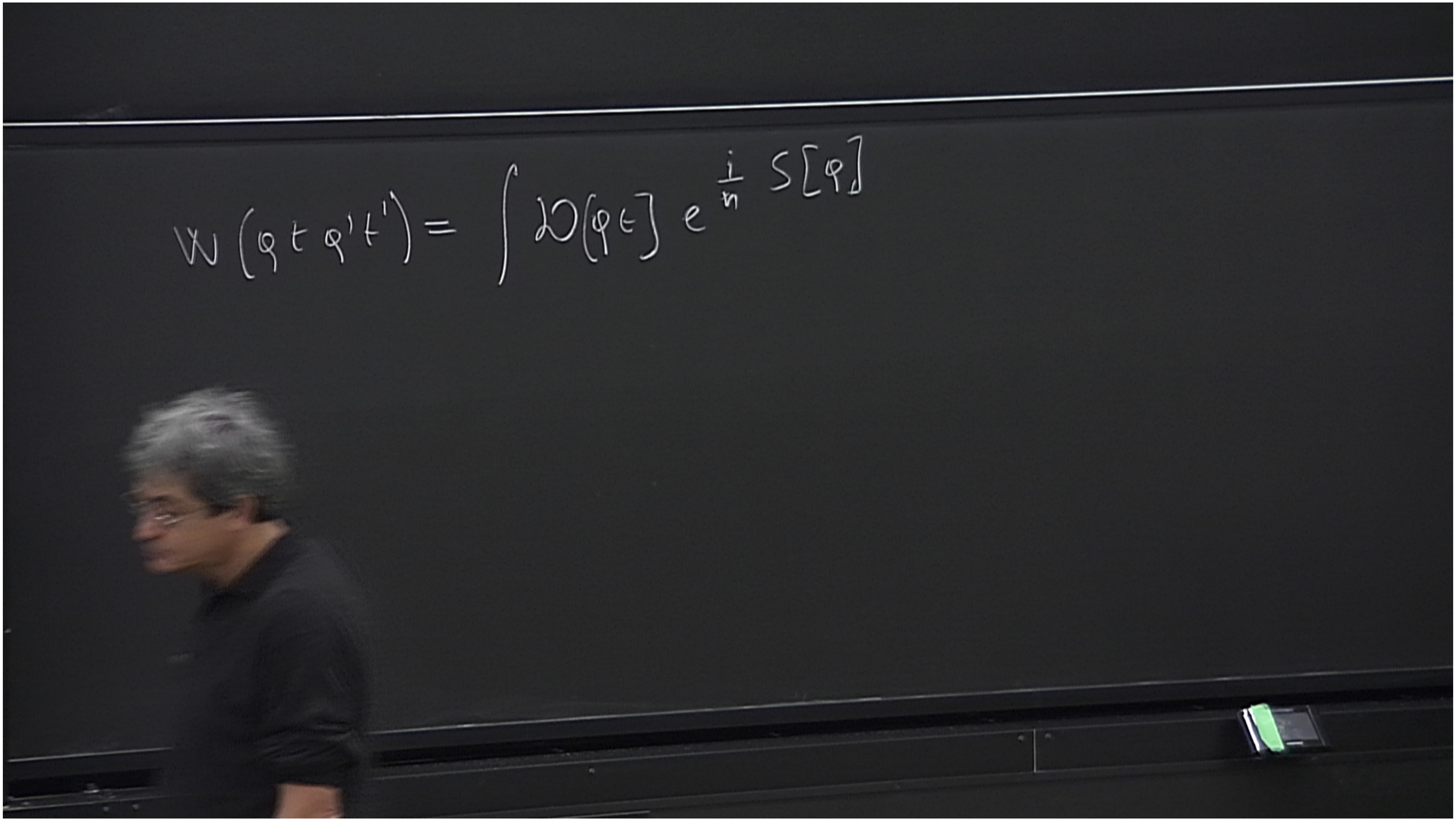

ε) U(ε) --- U(ε) |q>
1 = ∫dq_n |q_n><q_n|
<q_n|U(ε)|q
dq_n <q
H = p^2/2m + V(q)
U(ε) = e^{i p^2/2m ε + V(q)ε} ~ e^{i p^2/2m ε} e^{iV(q)ε}
<q_{n+1} U(ε) q_n> = e^{i/ħ (q_{n+1} - q_n)^2 / 2(t_{n+1} - t_n)}

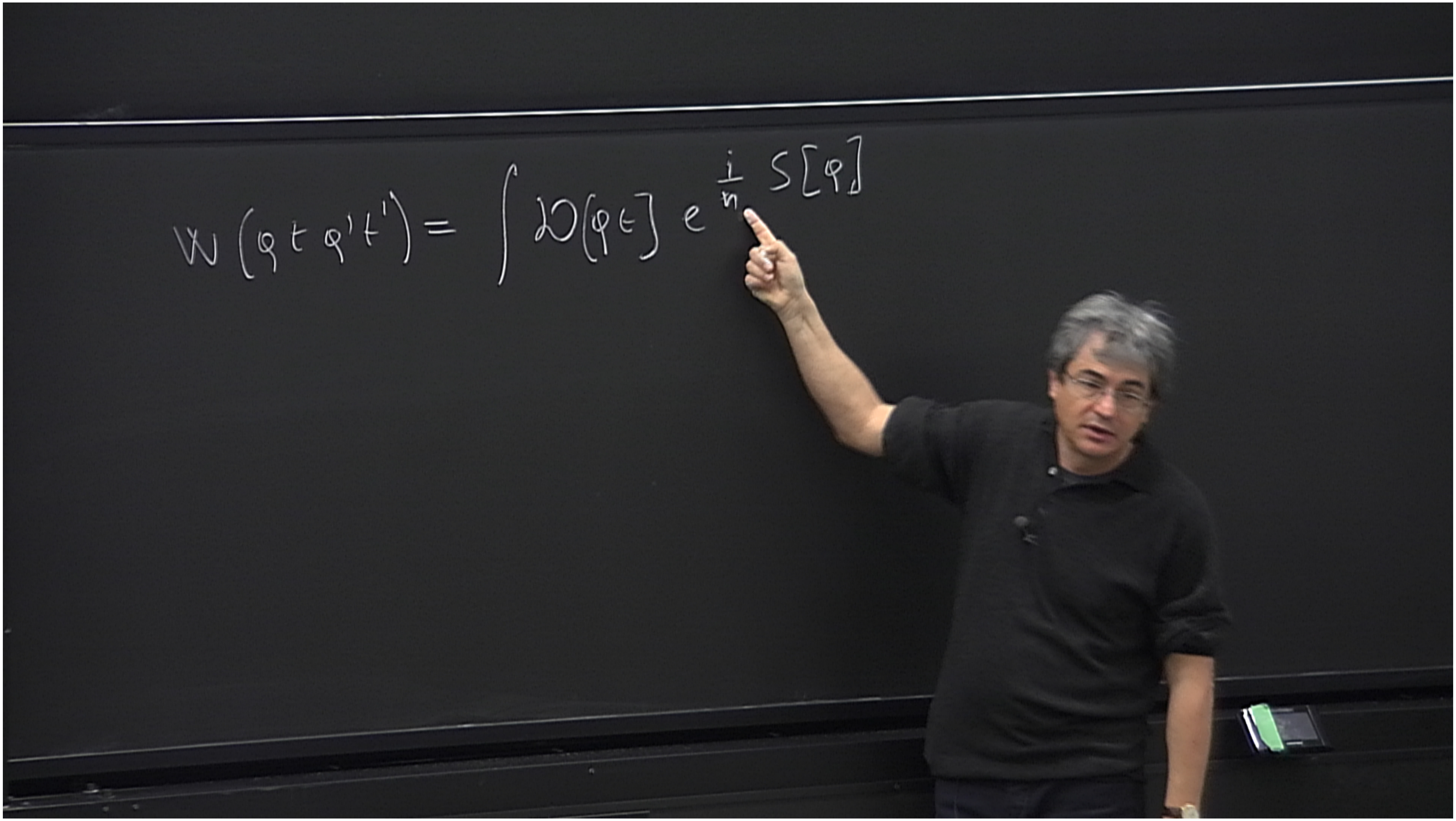

U(ε) ---- U(ε) |q>
1 = ∫dq_n |q_n><q_n|
<q_n|U(ε)|q_{n+1}>
dq_n <q_n U(ε) q_{n+1}>
H = p²/2m + V(q)
U(ε) = e^{i p²/2m ε + V(q)ε} ~ e^{i p²/2m ε} e^{i V(q) ε}
+ V(q_n))ε

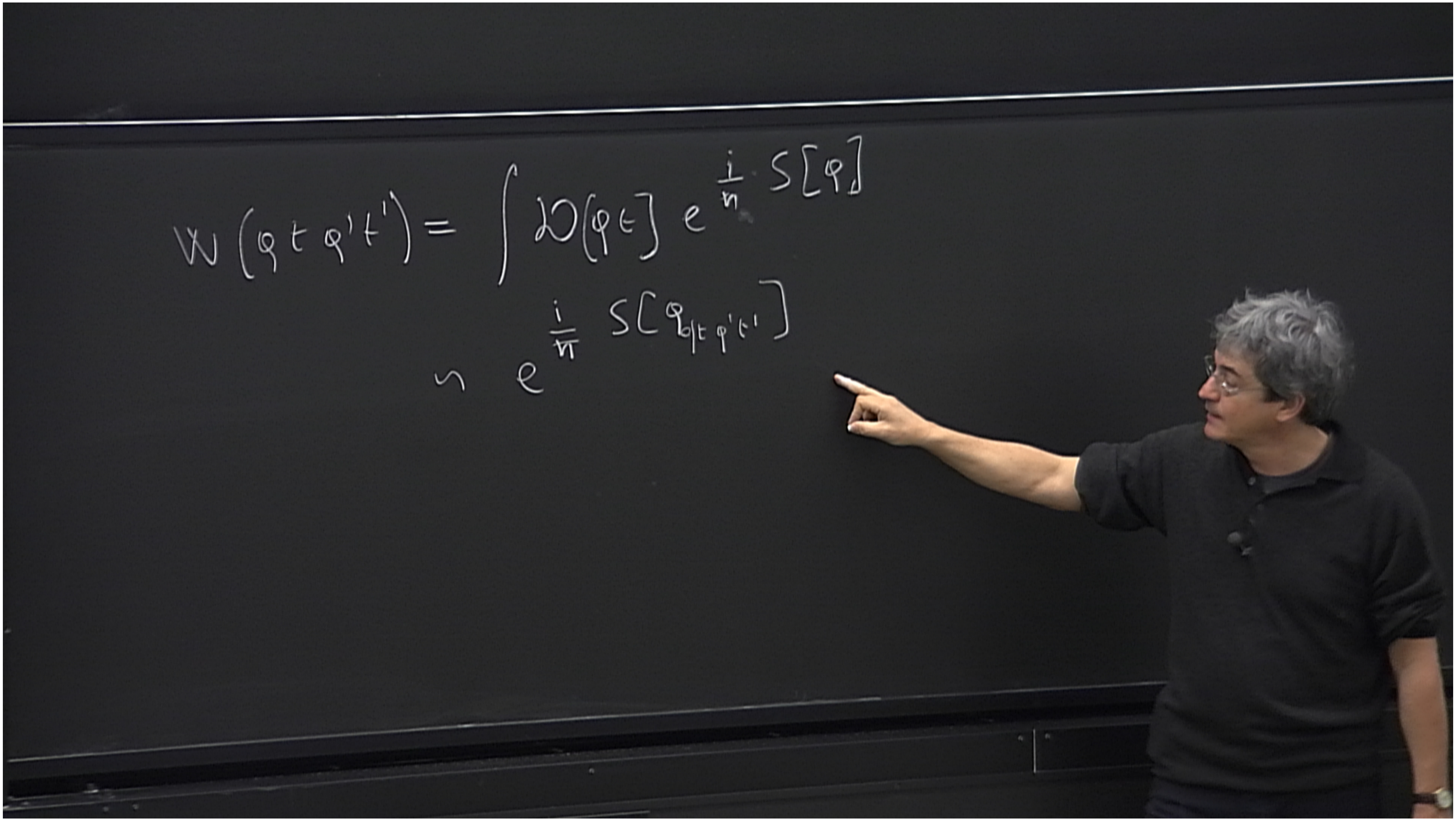

) U(ε) ... U(ε) |q>
1 = ∫dq_n |q_n><q_n|
<q_n|U(ε)|q_{n+1}>
dq_n <q_n U(ε) q_{n+1}>
H = p^2/2m + V(q)
U(ε) = e^{i p^2/2m ε + V(q)ε} ~ e^{i p^2/2m ε} e^{i V(q) ε}
<q_{n+1} U(ε) q_n> = e^{i/ħ ((q_{n+1} - q_n)^2 / 2 ε^2 - V(q_n)) ε}

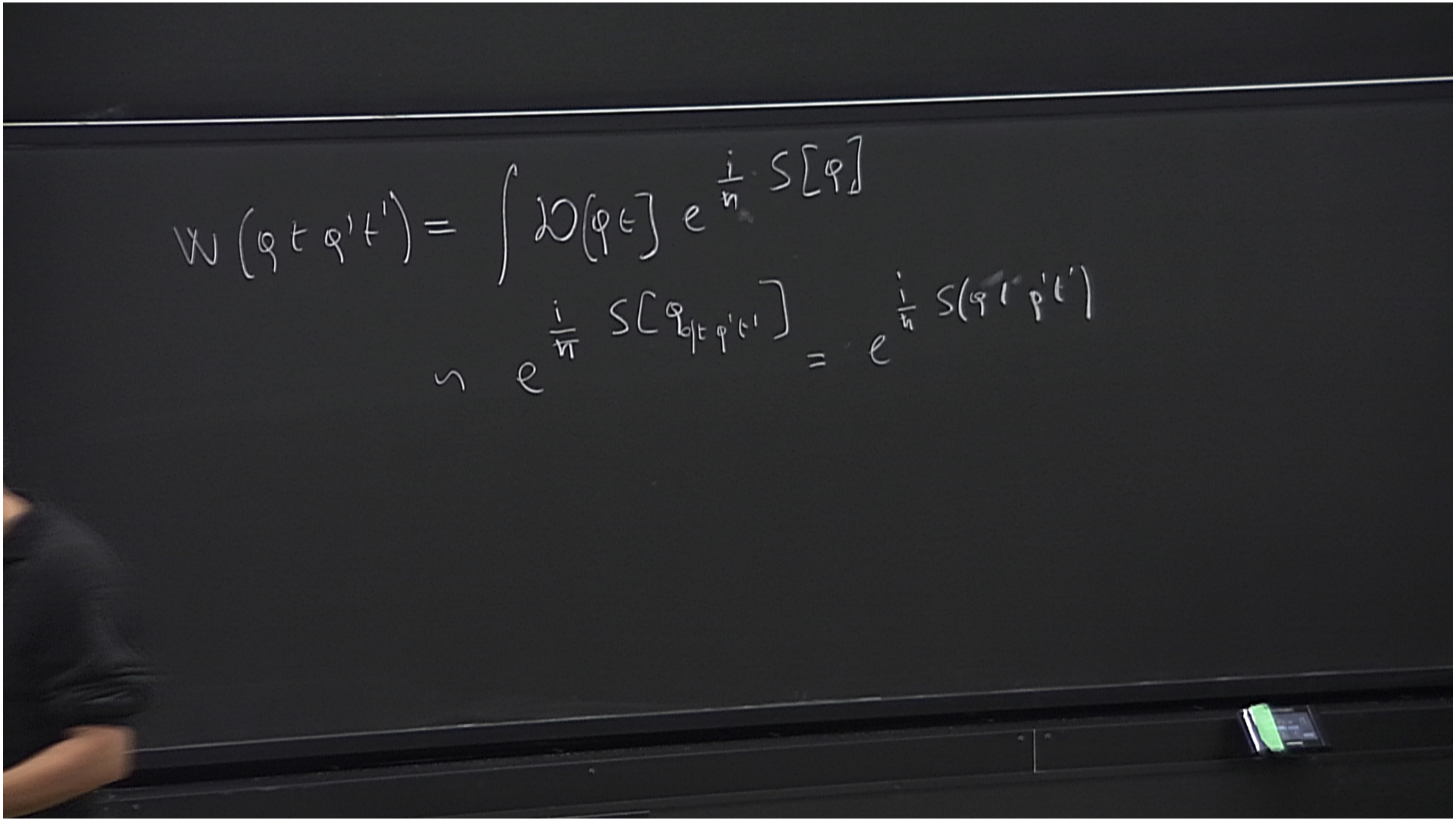

$H = \frac{p^2}{2m} + V(q)$
$U(\varepsilon) = e^{i\frac{p^2}{2m}\varepsilon + V(q)\varepsilon} \sim e^{i\frac{p^2}{2m}\varepsilon} e^{iV(q)\varepsilon}$
$\langle q_{n+1} U(\varepsilon) q_n \rangle = e^{\frac{i}{\hbar}\left(\frac{(q_{n+1}-q_n)^2}{2\varepsilon^2} - V(q_n)\right)\varepsilon}$
$U(\varepsilon) \ldots U(\varepsilon) |q\rangle$
$1 = \int dq_n |q_n\rangle\langle q_n|$
$\langle q_n | U(\varepsilon) | q_{n+1} \rangle$
$dq_n \langle q_n U(\varepsilon) q_{n+1} \rangle$

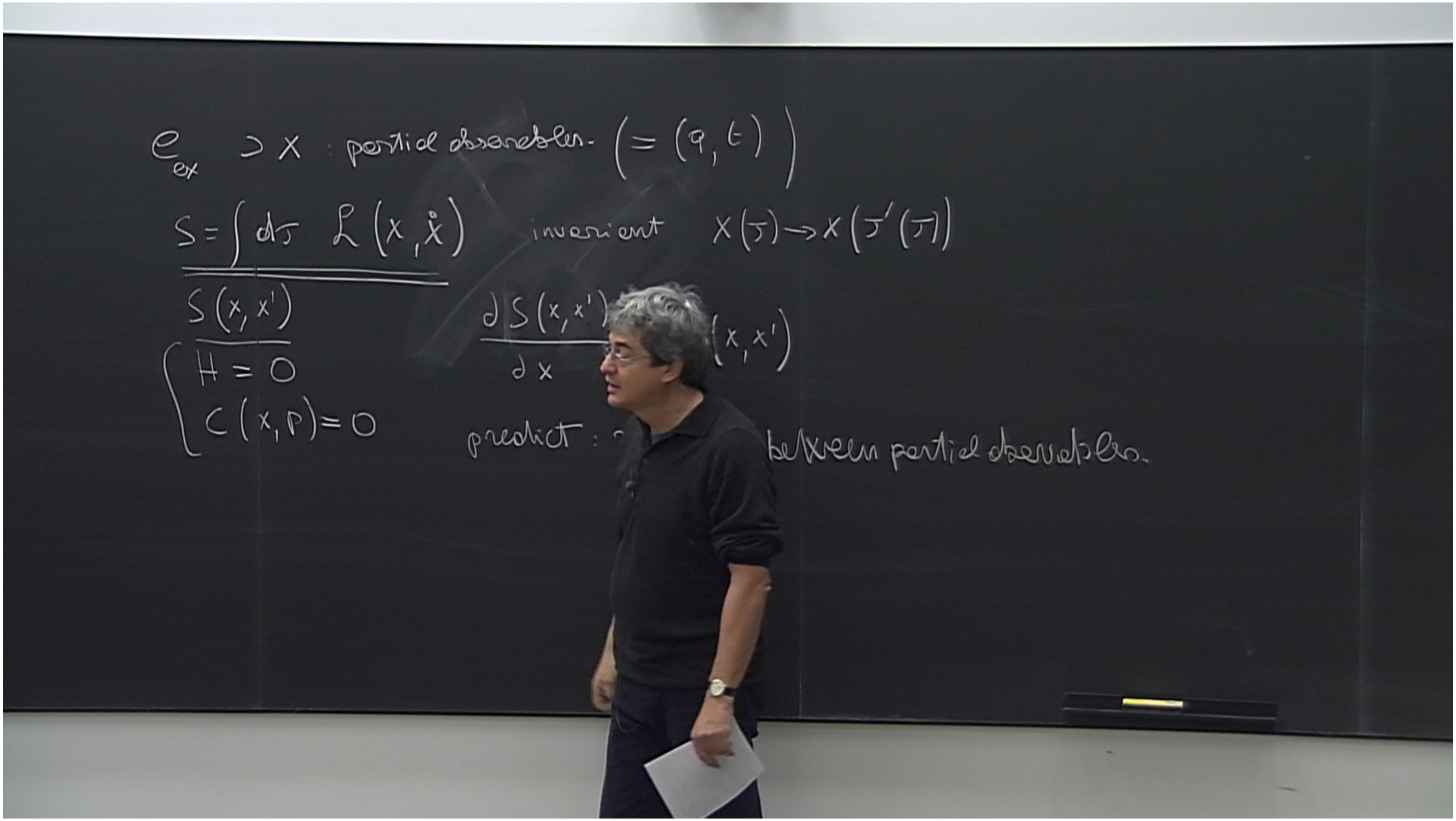

H = p^2/2m + V(q)
U(ε) = e^{i p^2/2m ε + V(q)ε} ~ e^{i p^2/2m ε} e^{i V(q) ε}
1 = ∫dq_n |q_1><q_1|
<q_{n+1} U(ε) q_n> = e^{i/ħ ((q_{n+1} - q_n)^2 / 2 ε^2 - V(q_n)) ε}
<q_n | U(ε) | q_{n+1}>
dq_n <q_n U(ε) q_{n+1}>
= lim_{N→∞} ∫dq_n e^{i/ħ Σ_{n=1}^{N} ((q_{n+1} - q_n)^2 / ε^2 - V(q_n)) ε}

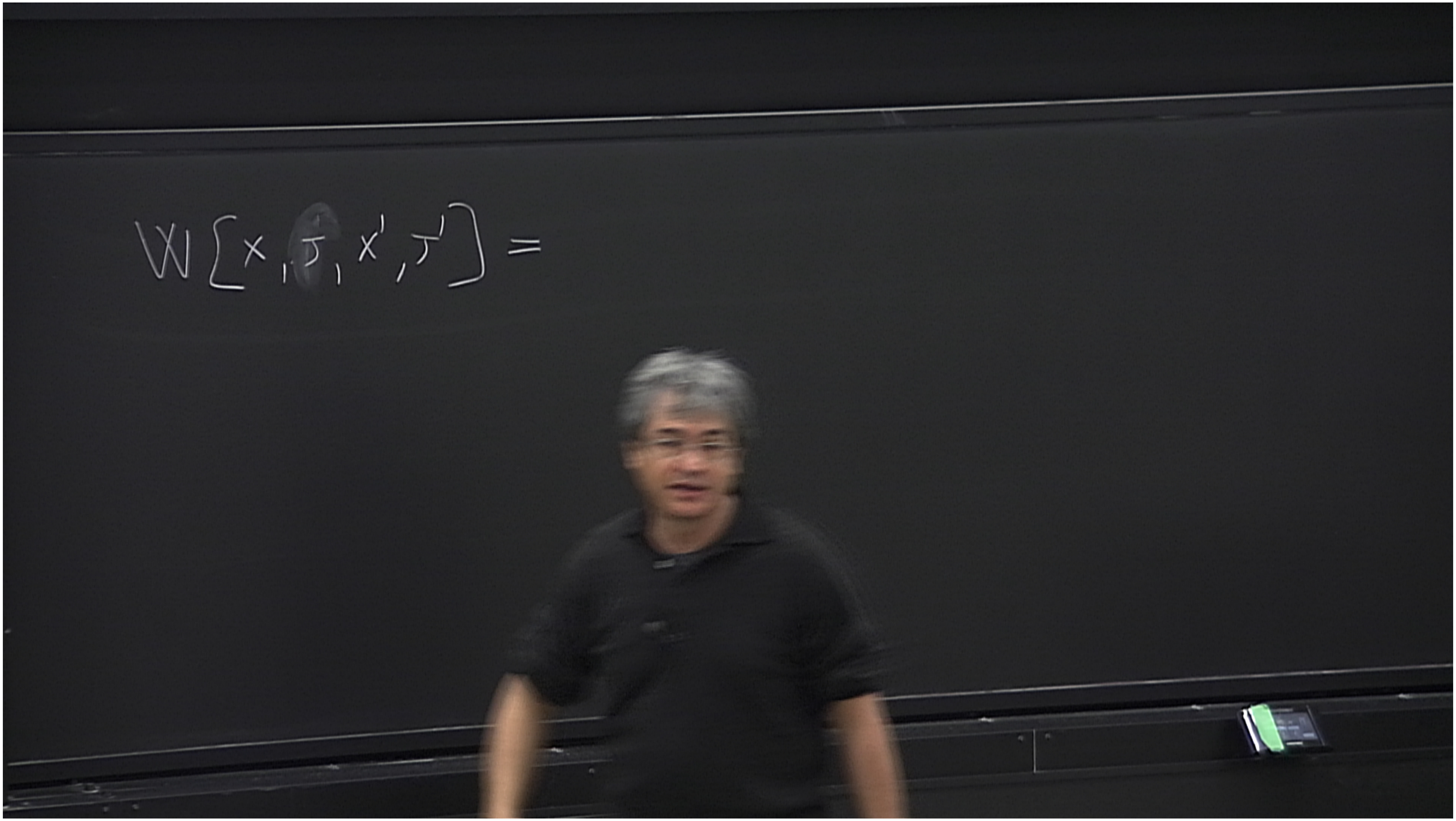

$W(\varphi t \varphi' t') = \int \mathcal{D}[\varphi t]\, e^{\frac{i}{\hbar} S[\varphi]}$

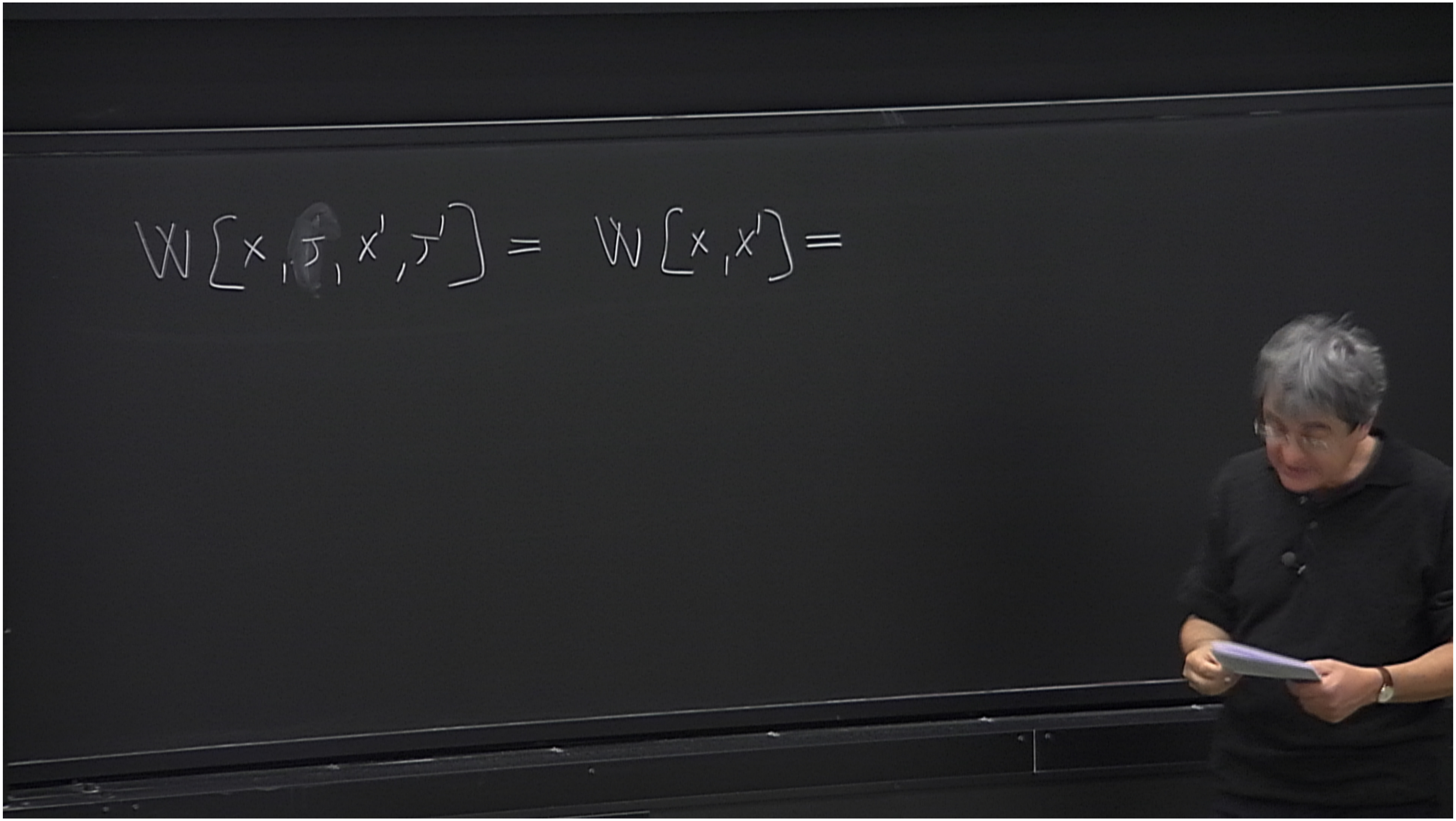

$W(q\,t\,q'\,t') = \int \mathcal{D}[q\,t]\; e^{\frac{i}{\hbar} S[q]}$


$W(q\,t\,q'\,t') = \int \mathcal{D}[q\,t]\; e^{\frac{i}{\hbar} S[q]}$
$\sim e^{\frac{i}{\hbar} S[q_{q\,t\,q'\,t'}]}$


$W(q\,t\,q'\,t') = \int \mathcal{D}[q\,t]\, e^{\frac{i}{\hbar} S[q]}$
$\sim e^{\frac{i}{\hbar} S[q_{q t q' t'}]} = e^{\frac{i}{\hbar} S(q\,t\,q'\,t')}$

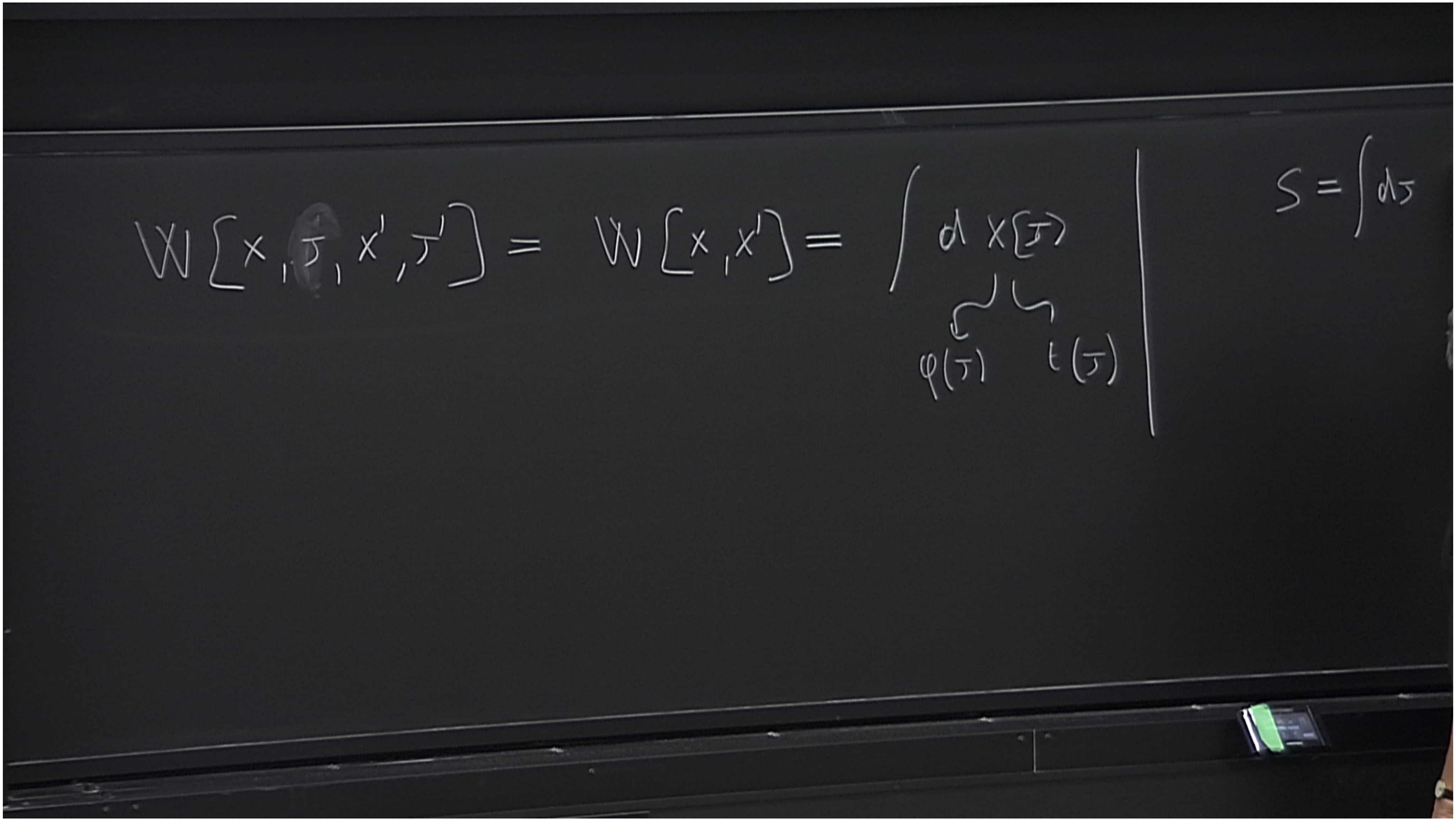

C_ex ⊃ X : partial observables. (= (q, t))
S = ∫ dτ L(x, ẋ)
invariant x(τ) → x(τ'(τ))
S(x, x')
H = 0
C(x, p) = 0
∂S(x, x') / ∂x
(x, x')
predict :
between partial observables.

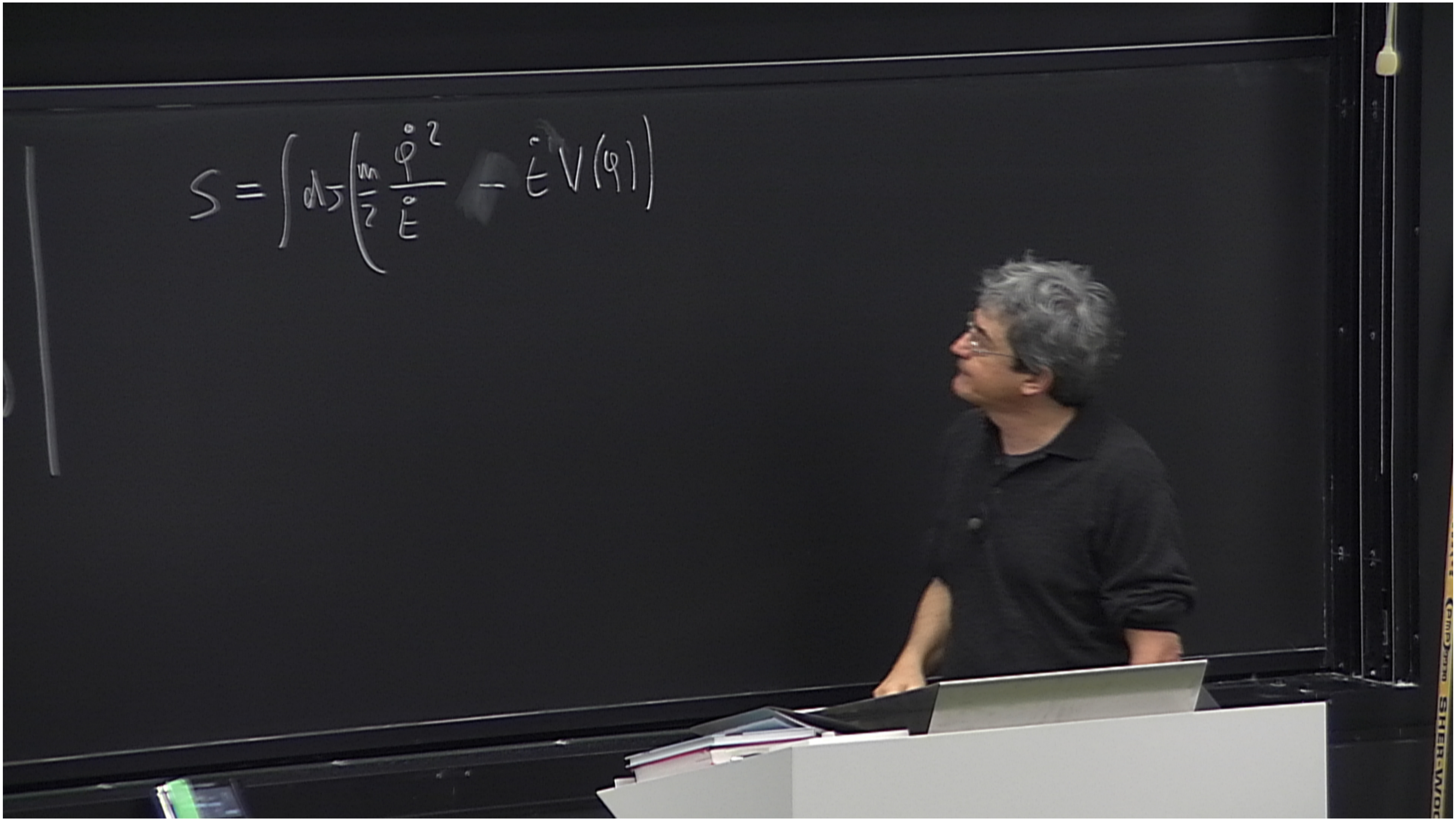

W[x, τ, x', τ'] =

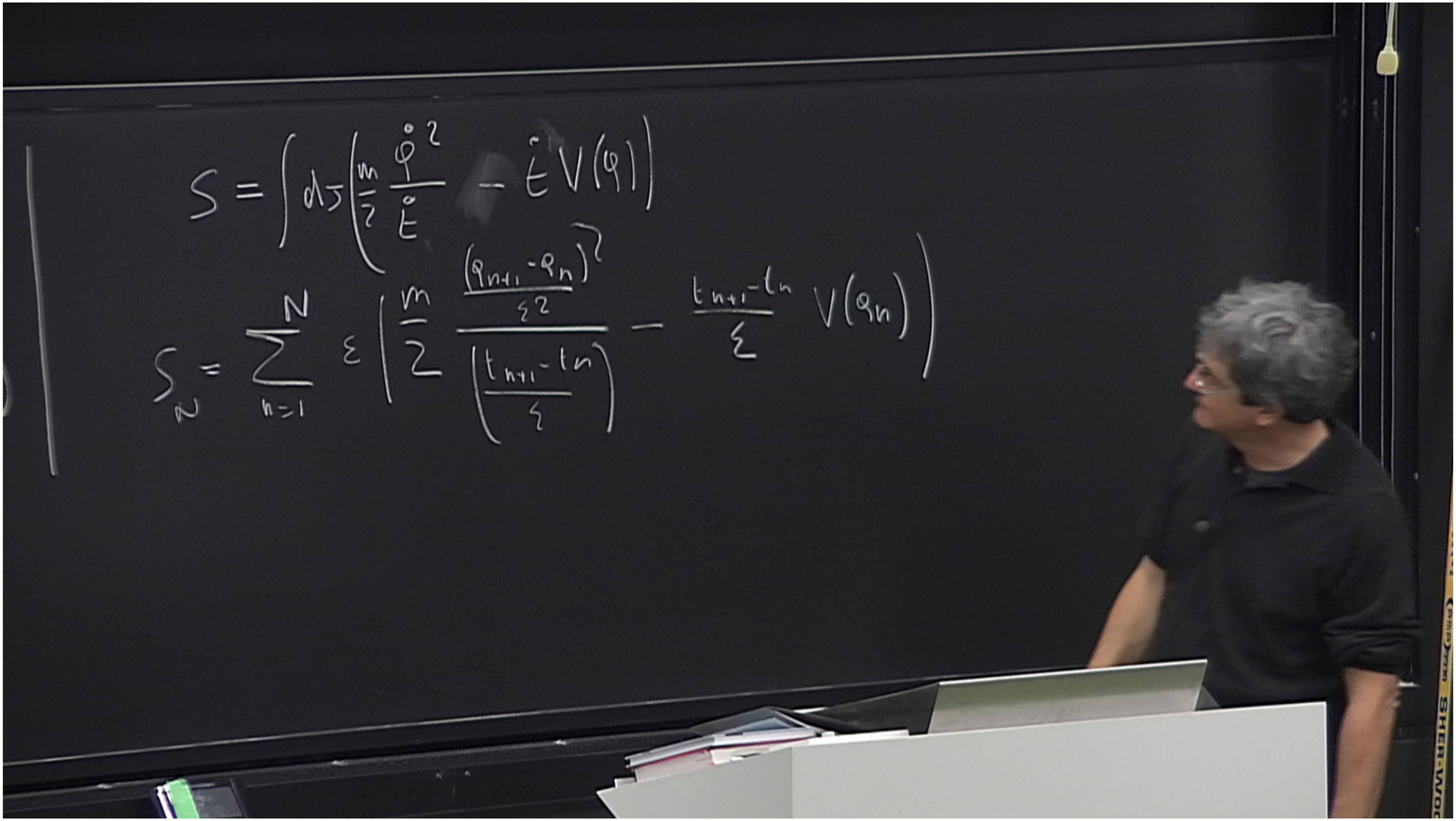

W[x, σ, x', σ'] = W[x, x'] =

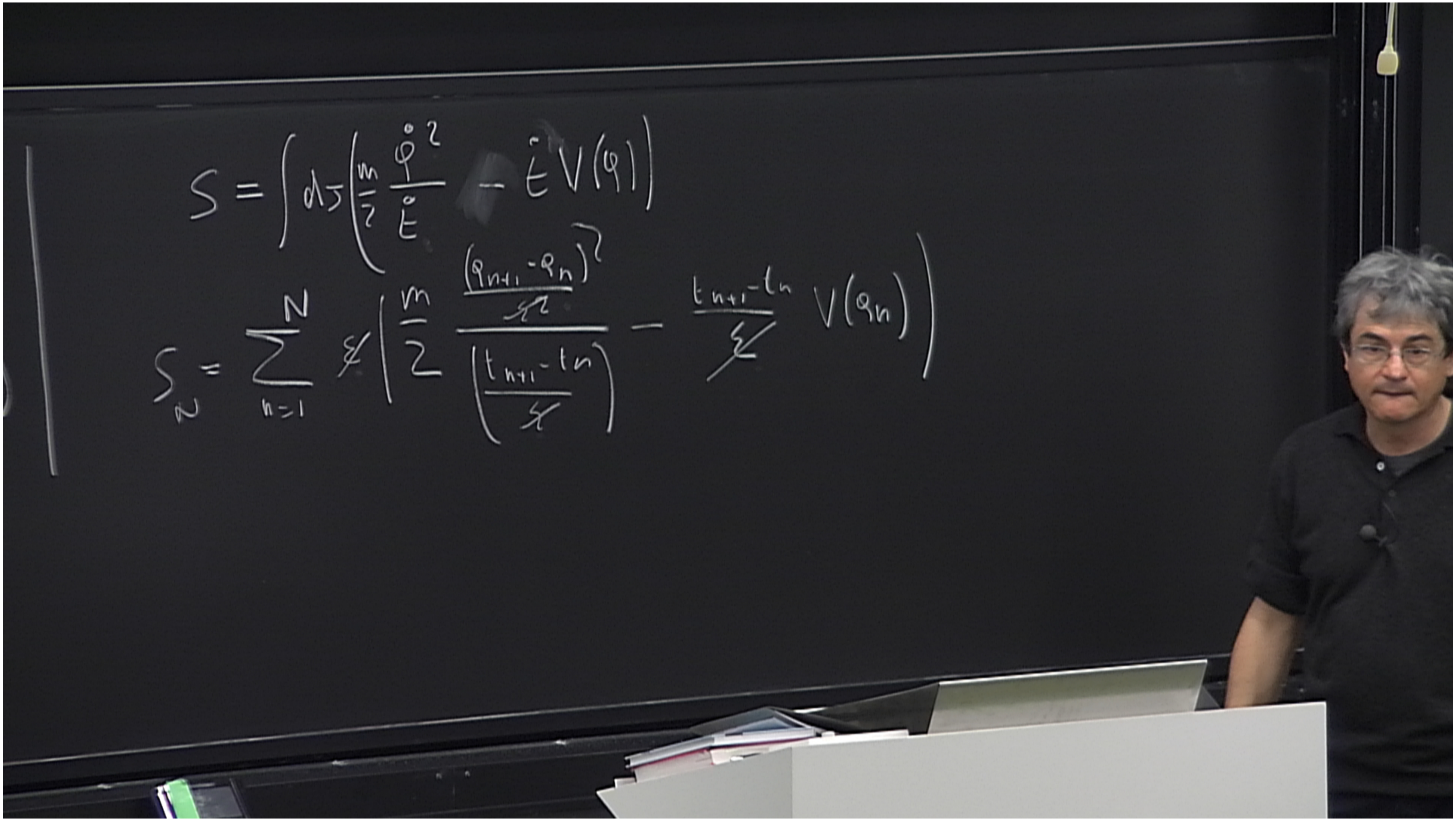
$$W[x,\tau,x',\tau'] = W[x,x'] = \int dx[\tau)$$

$\varphi(\tau)$ $\quad t(\tau)$


W[x, τ, x', τ'] = W[x, x'] = ∫ d x[τ)
φ(τ) t(τ)
S = ∫ dτ


S = ∫ dτ (m/2 φ̇²/ṫ − ṫ V(φ))

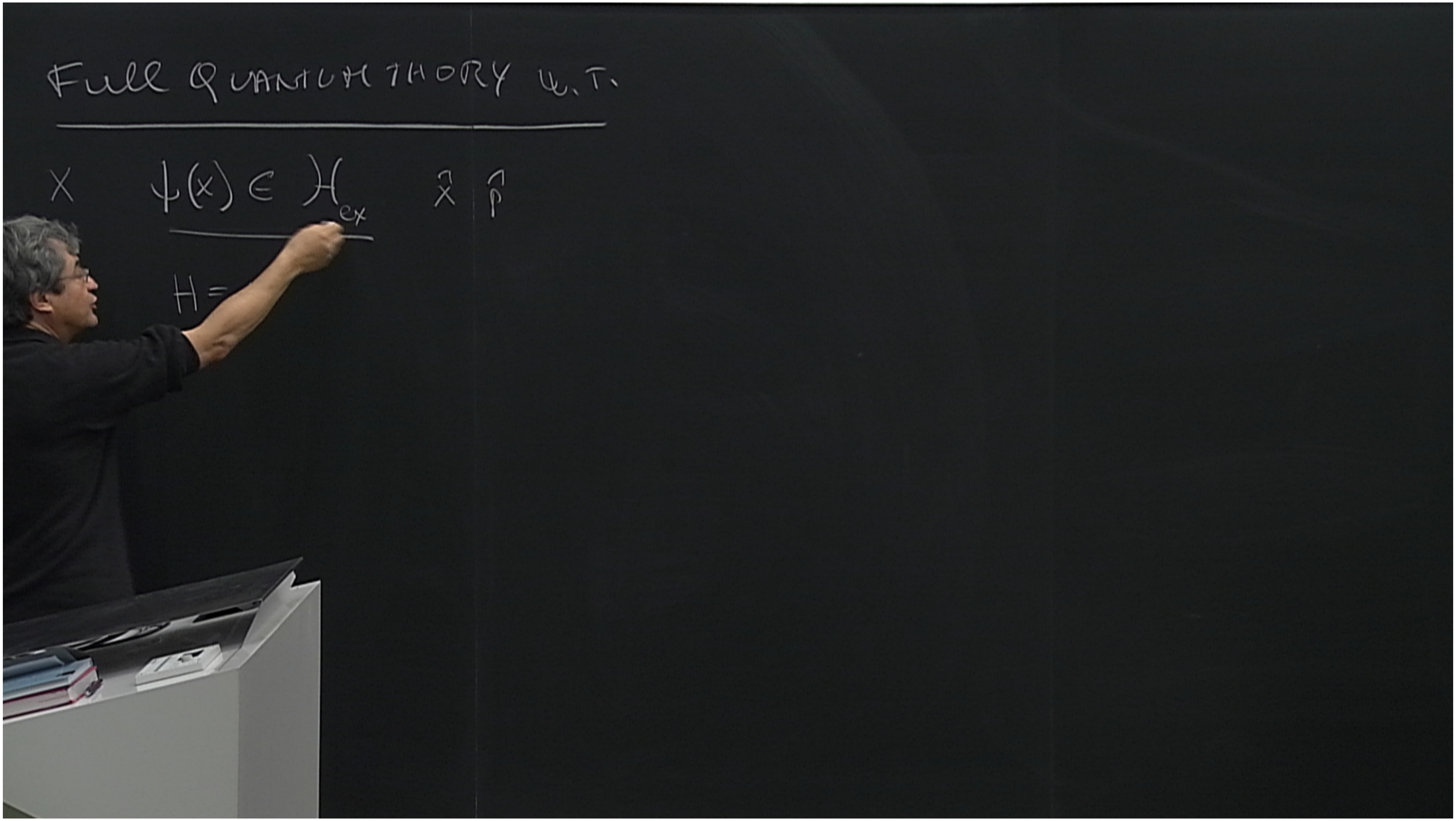

$S = \int d\tau \left(\frac{m}{2} \frac{\dot{q}^2}{\dot{t}} - \dot{t} V(q) \right)$
$S_N = \sum_{n=1}^{N} \varepsilon \left(\frac{m}{2} \frac{\frac{(q_{n+1}-q_n)^2}{\varepsilon^2}}{\left(\frac{t_{n+1}-t_n}{\varepsilon}\right)} - \frac{t_{n+1}-t_n}{\varepsilon} V(q_n) \right)$

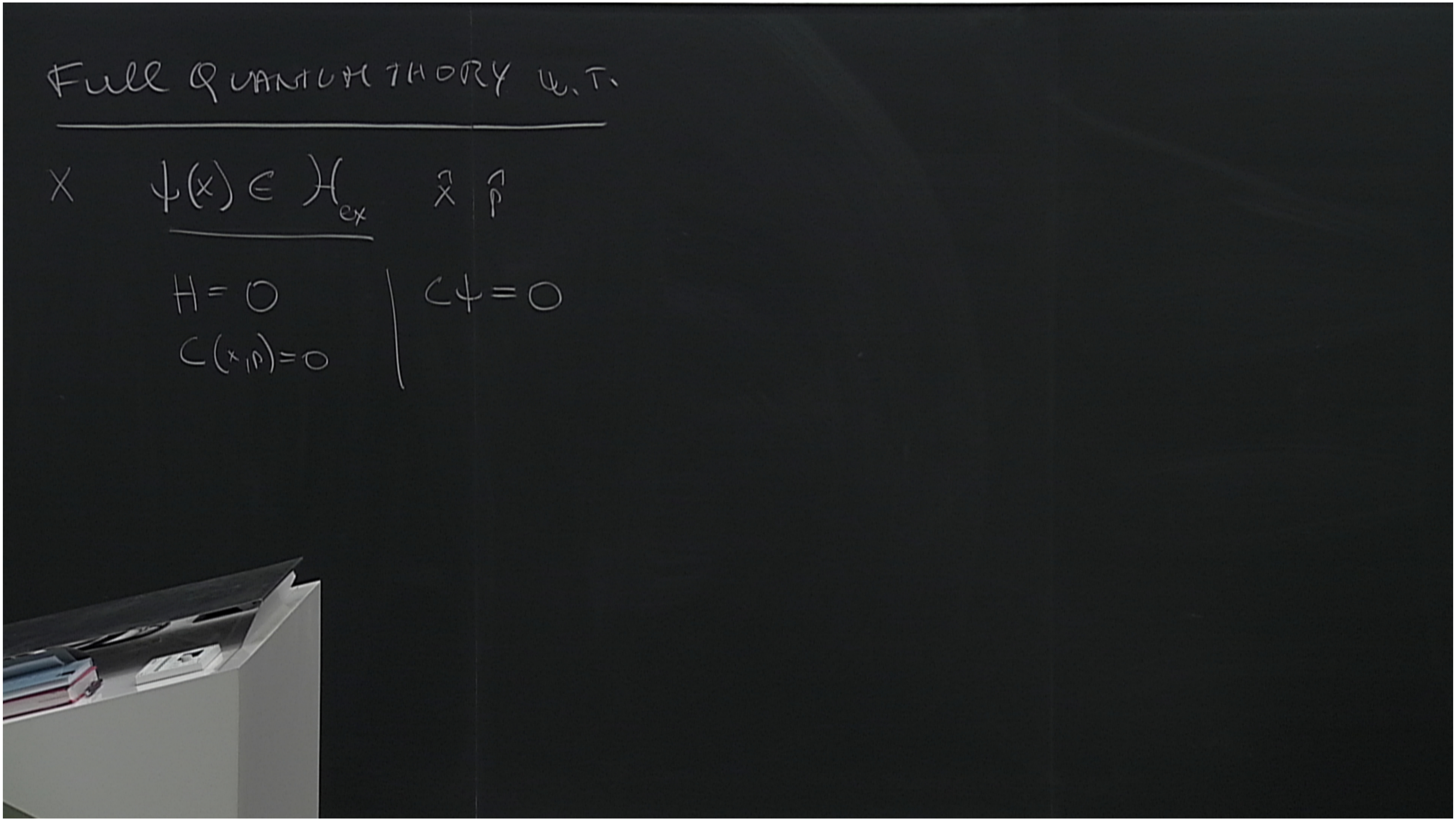

$S = \int d\tau \left(\frac{m}{2} \frac{\dot{q}^2}{\dot{t}} - \dot{t} V(q) \right)$
$S_N = \sum_{n=1}^{N} \epsilon \left(\frac{m}{2} \frac{\frac{(q_{n+1} - q_n)^2}{\epsilon^2}}{\left(\frac{t_{n+1} - t_n}{\epsilon}\right)} - \frac{t_{n+1} - t_n}{\epsilon} V(q_n) \right)$

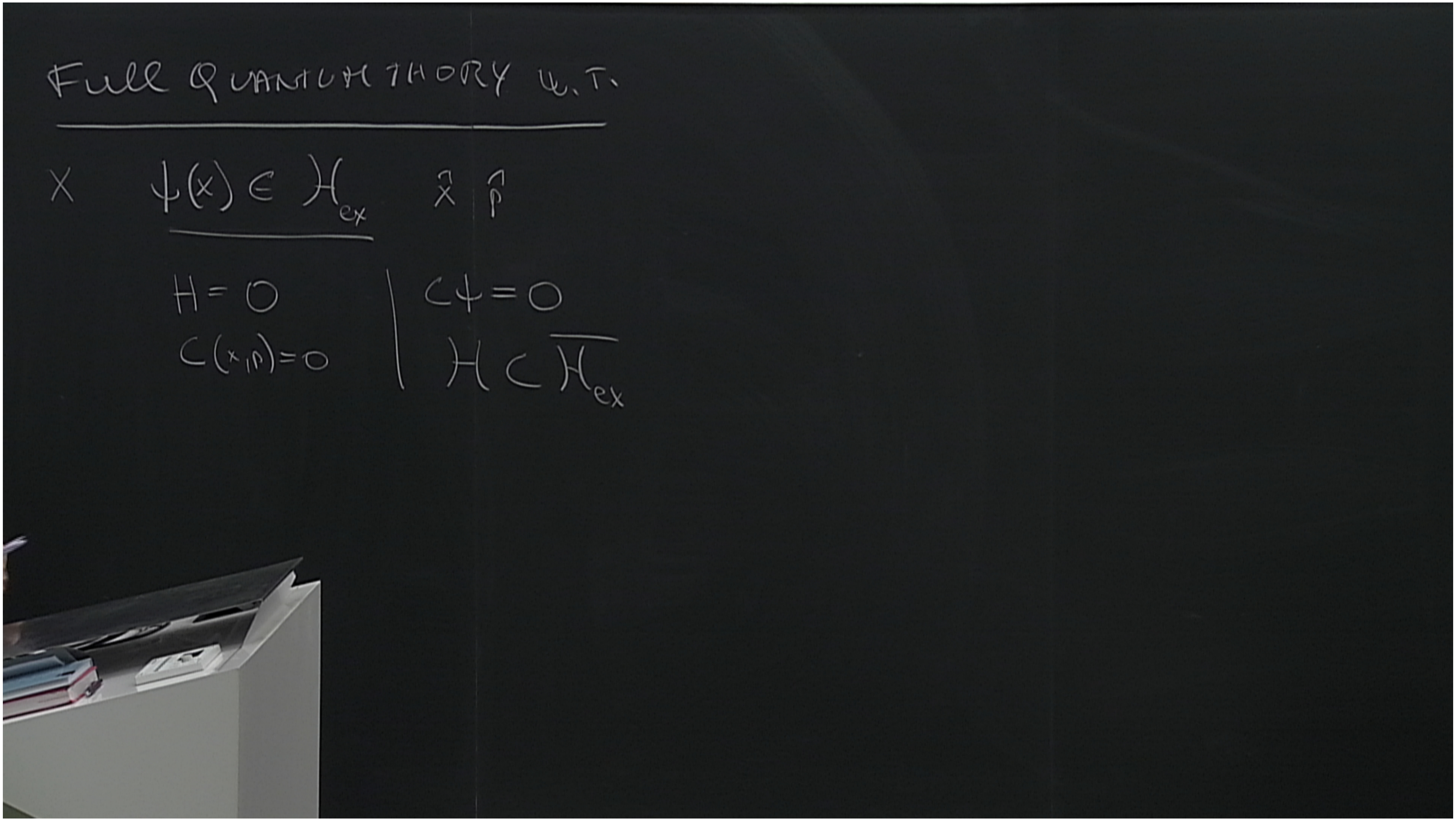
$$S = \int d\tau \left(\frac{m}{2} \frac{\dot{q}^2}{\dot{t}} - \dot{t}\, V(q) \right)$$

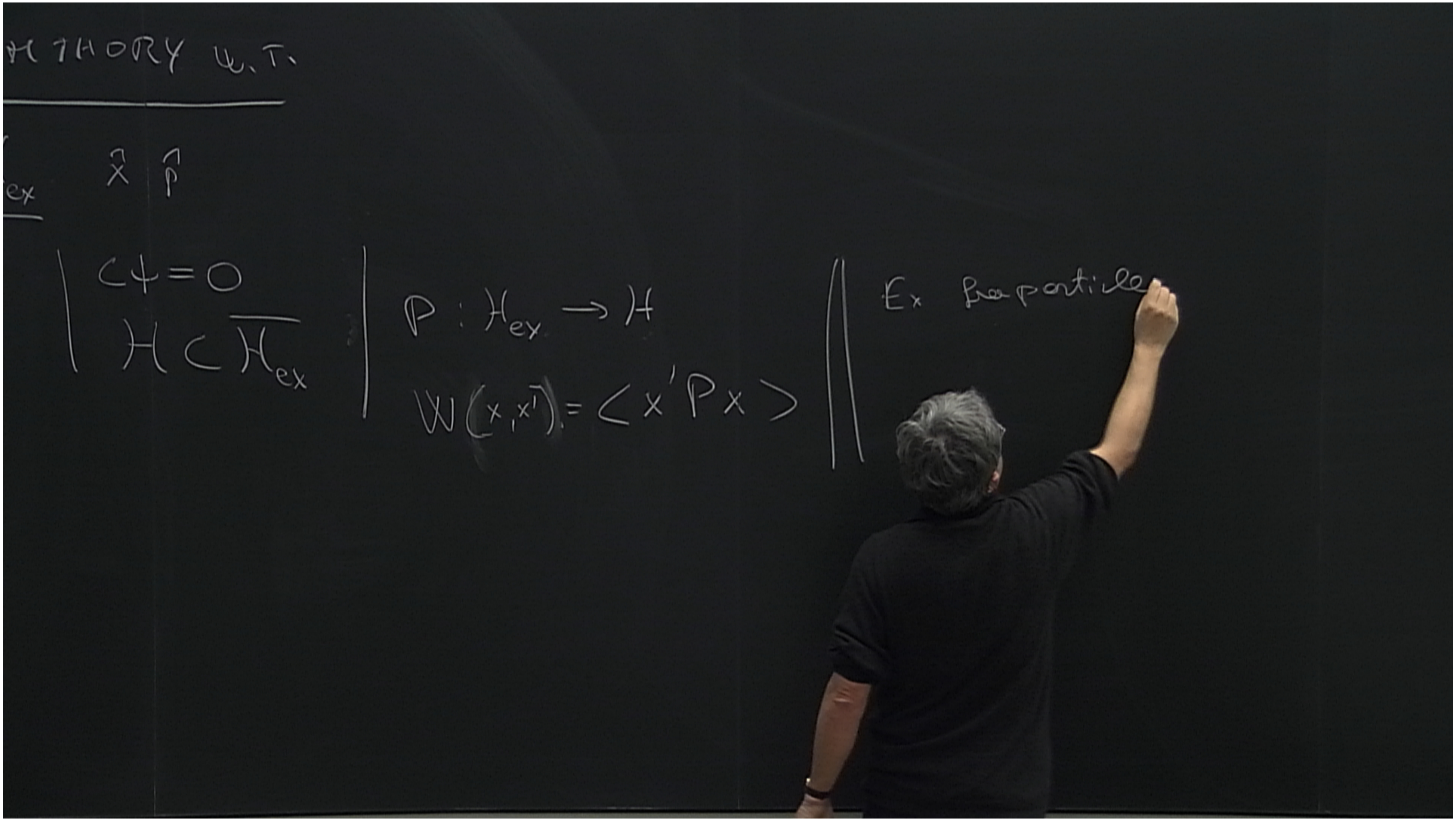
$$S_N = \sum_{n=1}^{N} \epsilon \left(\frac{m}{2} \frac{\frac{(q_{n+1}-q_n)^2}{\epsilon^2}}{\left(\frac{t_{n+1}-t_n}{\epsilon}\right)} - \frac{t_{n+1}-t_n}{\epsilon} V(q_n) \right)$$

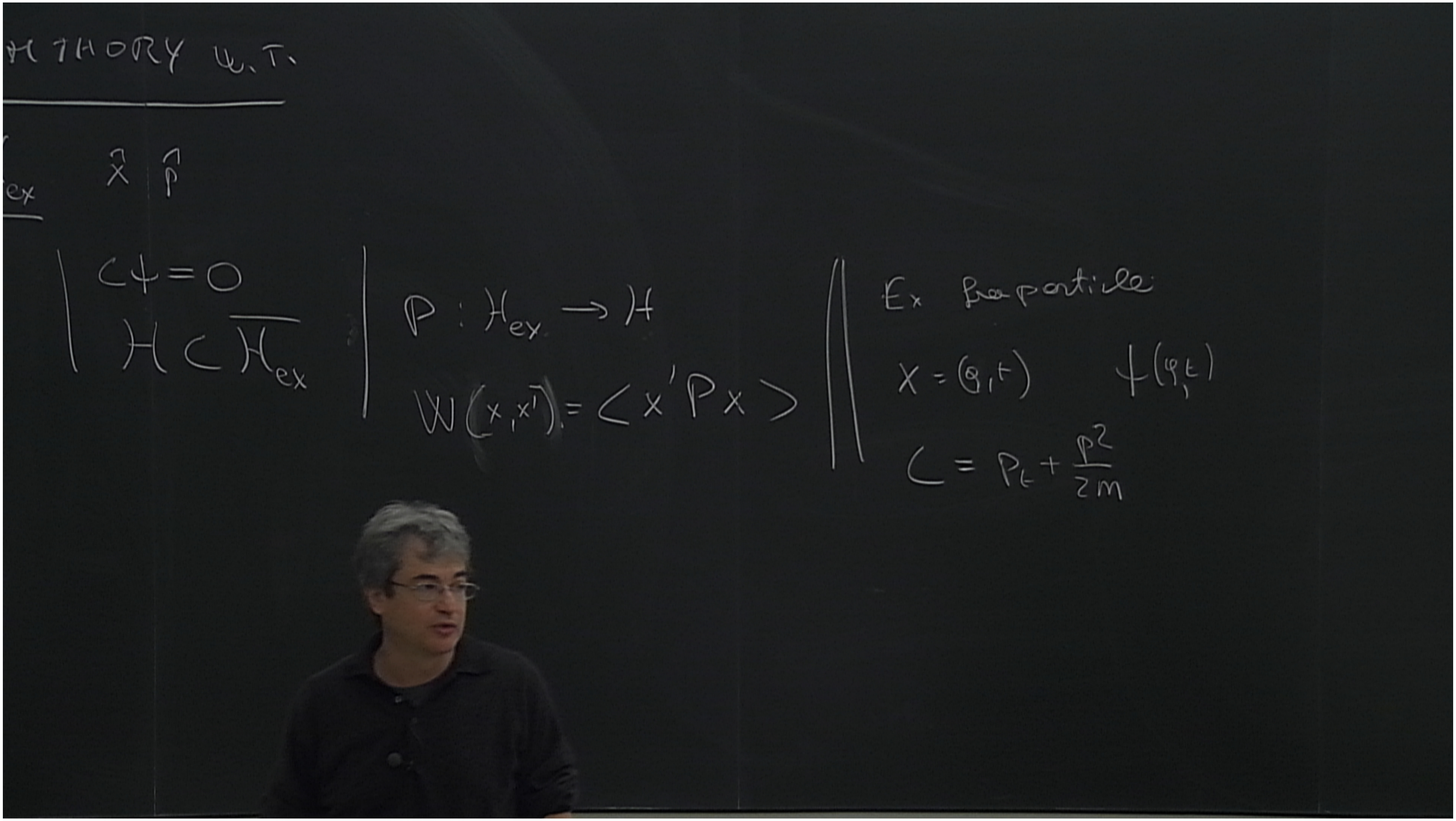
$$= \sum_{n=1}^{N} \left(\frac{m}{2} \frac{(q_{n+1}-q_n)^2}{(t_{n+1}-t_n)} - (t_{n+1}-t_n) V(q_n) \right)$$


FULL QUANTUM THORY Q.T.
X ψ(x) ∈ H_ex x̂ p̂
H =

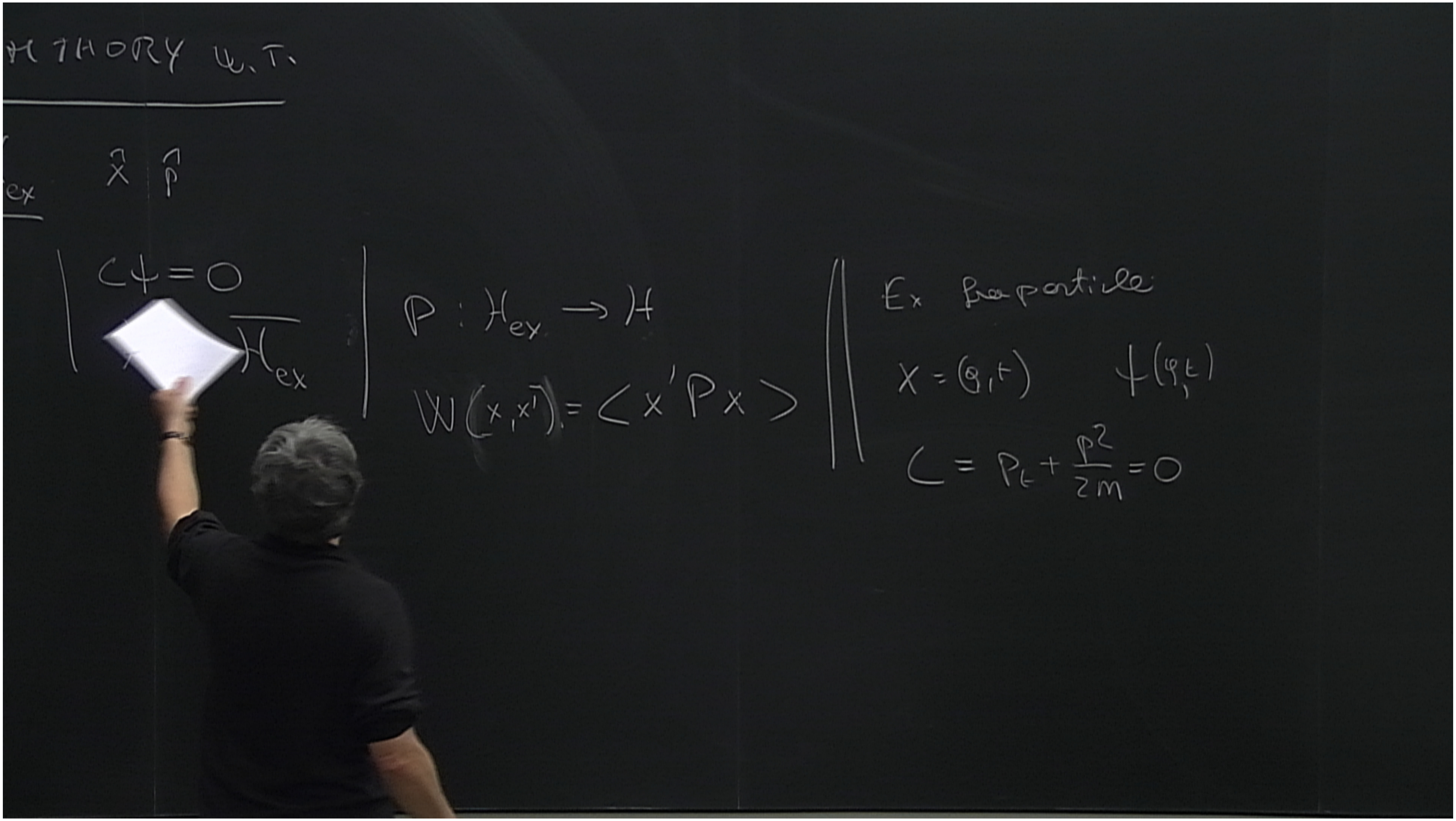

FULL QUANTUM THORY Q.T.
X $\psi(x) \in \mathcal{H}_{ex}$ $\hat{x}$ $\hat{p}$
$H=0$
$C(x,p)=0$
$C\psi=0$


FULL QUANTUM THEORY Q.T.
$X \quad \psi(x) \in \mathcal{H}_{ex} \quad \hat{x} \quad \hat{p}$
$H = 0$
$C(x,p) = 0$
$C\psi = 0$
$\mathcal{H} \subset \overline{\mathcal{H}_{ex}}$


Cψ = 0
H ⊂ H̄ex
P : Hex → H
W(x, x') = <x'Px>
Ex free particle



H THORY Q.T.
Ex free particle


Cψ = 0
P : H_ex → H
W(x, x') = ⟨x' P x⟩
Ex free particle
x = (q, t)   ψ(q, t)
C = p_t + p²/2m = 0

$$\mathcal{H}_{ex} \longrightarrow \mathcal{H}$$

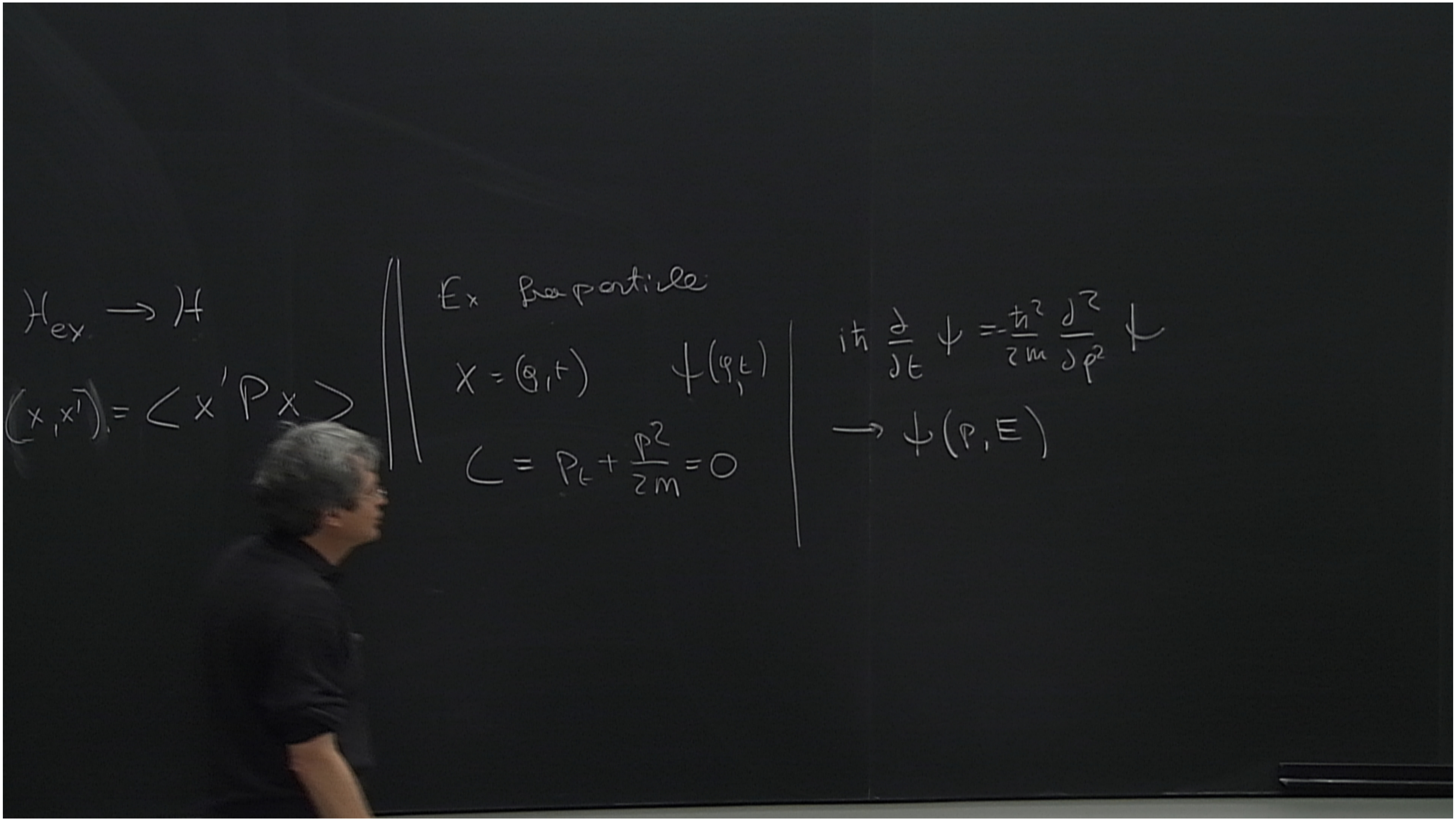
$$(x, x') = \langle x' P x \rangle$$

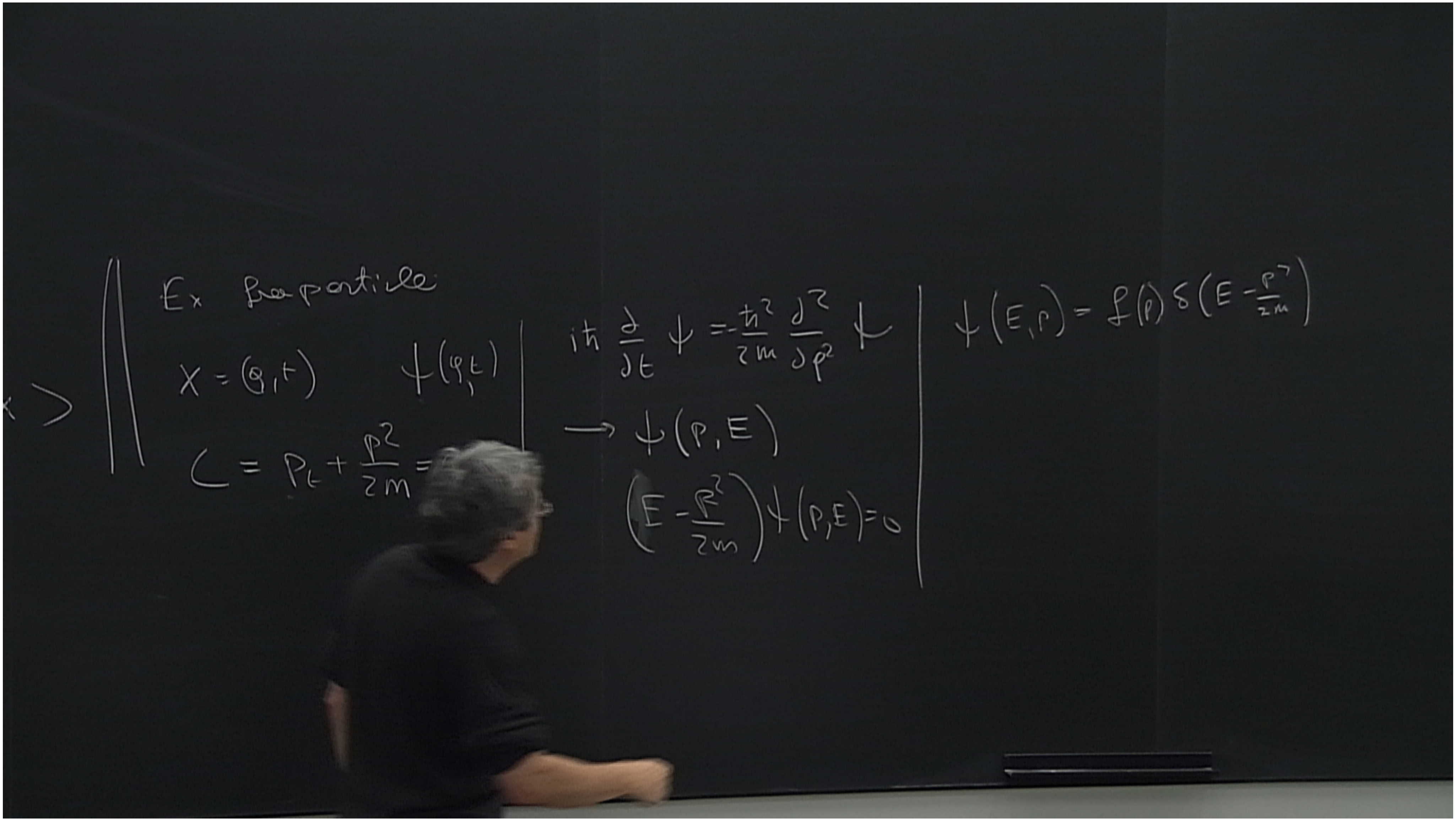
Ex free particle

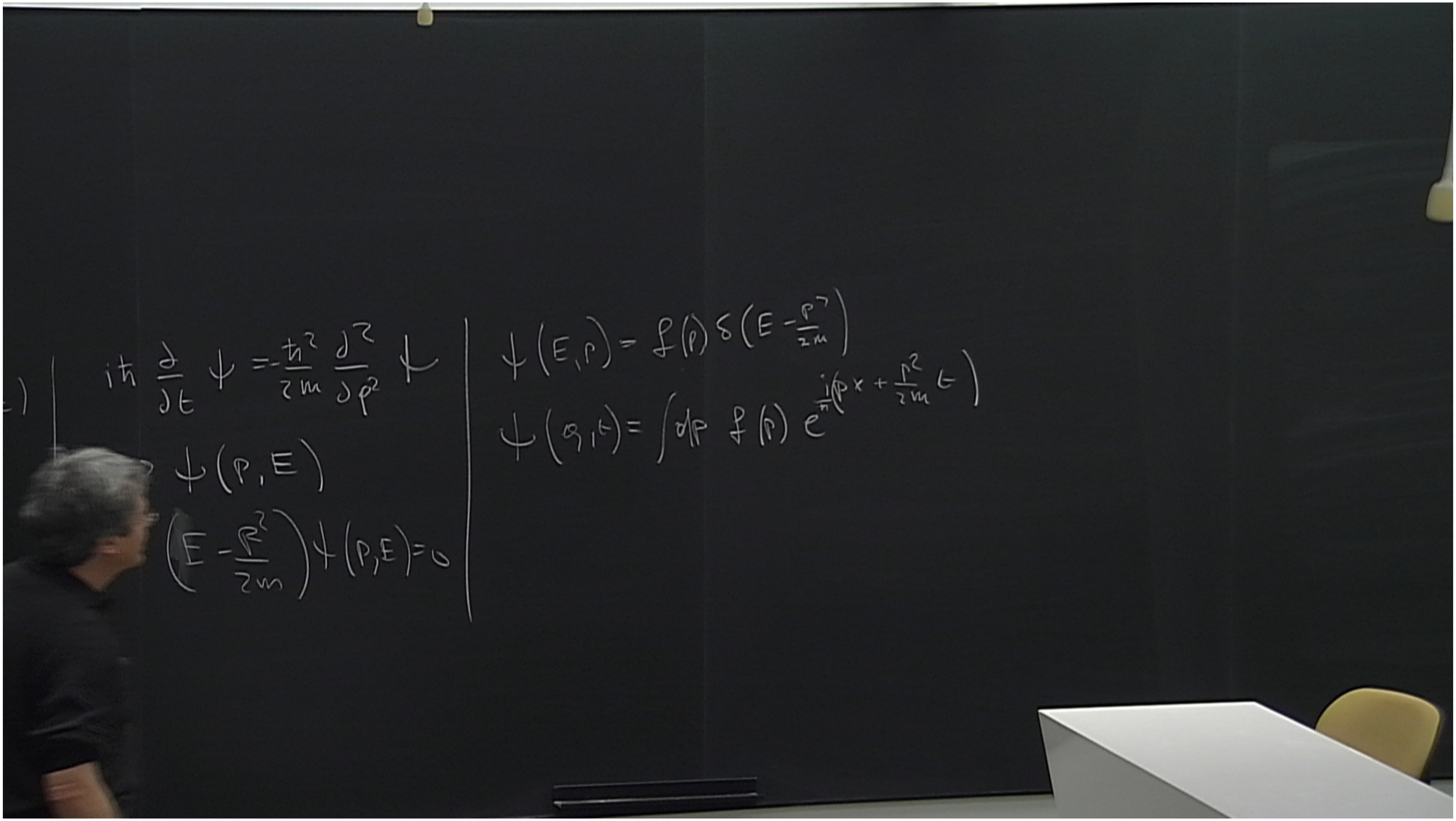
$$X = (q, t) \qquad \psi(q, t)$$

$$C = p_t + \frac{p^2}{2m} = 0$$

$$i\hbar \frac{\partial}{\partial t} \psi = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial q^2} \psi$$


$H_{ex} \to H$
$(x, x') = \langle x' P x \rangle$
Ex free particle
$X = (q, t)$
$\psi(q, t)$
$C = p_t + \frac{p^2}{2m} = 0$
$i\hbar \frac{\partial}{\partial t}\psi = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial q^2}\psi$
$\to \psi(p, E)$


Ex free particle
X = (q, t)
ψ(q, t)
C = p_t + p²/2m =
iħ ∂/∂t ψ = -ħ²/2m ∂²/∂q² ψ
→ ψ(p, E)
(E - p²/2m) ψ(p, E) = 0
ψ(E, p) = f(p) δ(E - p²/2m)


iħ ∂/∂t ψ = -ħ²/2m ∂²/∂p² ψ
ψ(p,E)
(E - p²/2m) ψ(p,E) = 0
ψ(E,p) = f(p) δ(E - p²/2m)
ψ(q,t) = ∫dp f(p) e^(i/ħ(px + p²/2m t))

$$\psi(E,p) = f(p)\,\delta\!\left(E - \frac{p^2}{2m}\right)$$

$$\psi(q,t) = \int dp\; f(p)\, e^{\frac{i}{\hbar}\left(px + \frac{p^2}{2m}t\right)}$$

$$(p,E) = 0$$

$$\hbar \frac{\partial}{\partial t}\psi = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial p^2}\psi$$

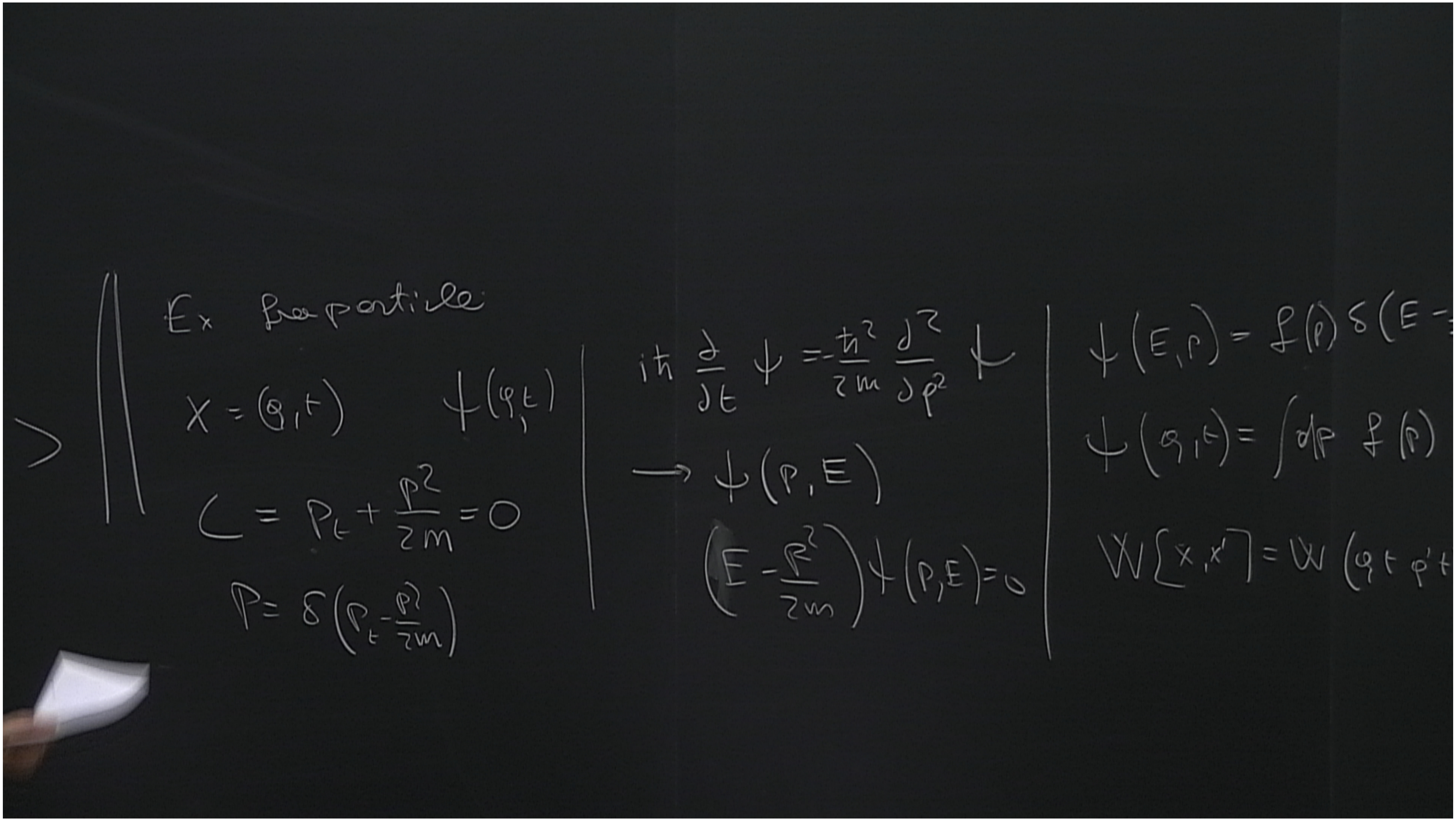
$$\longrightarrow \psi(p, E)$$

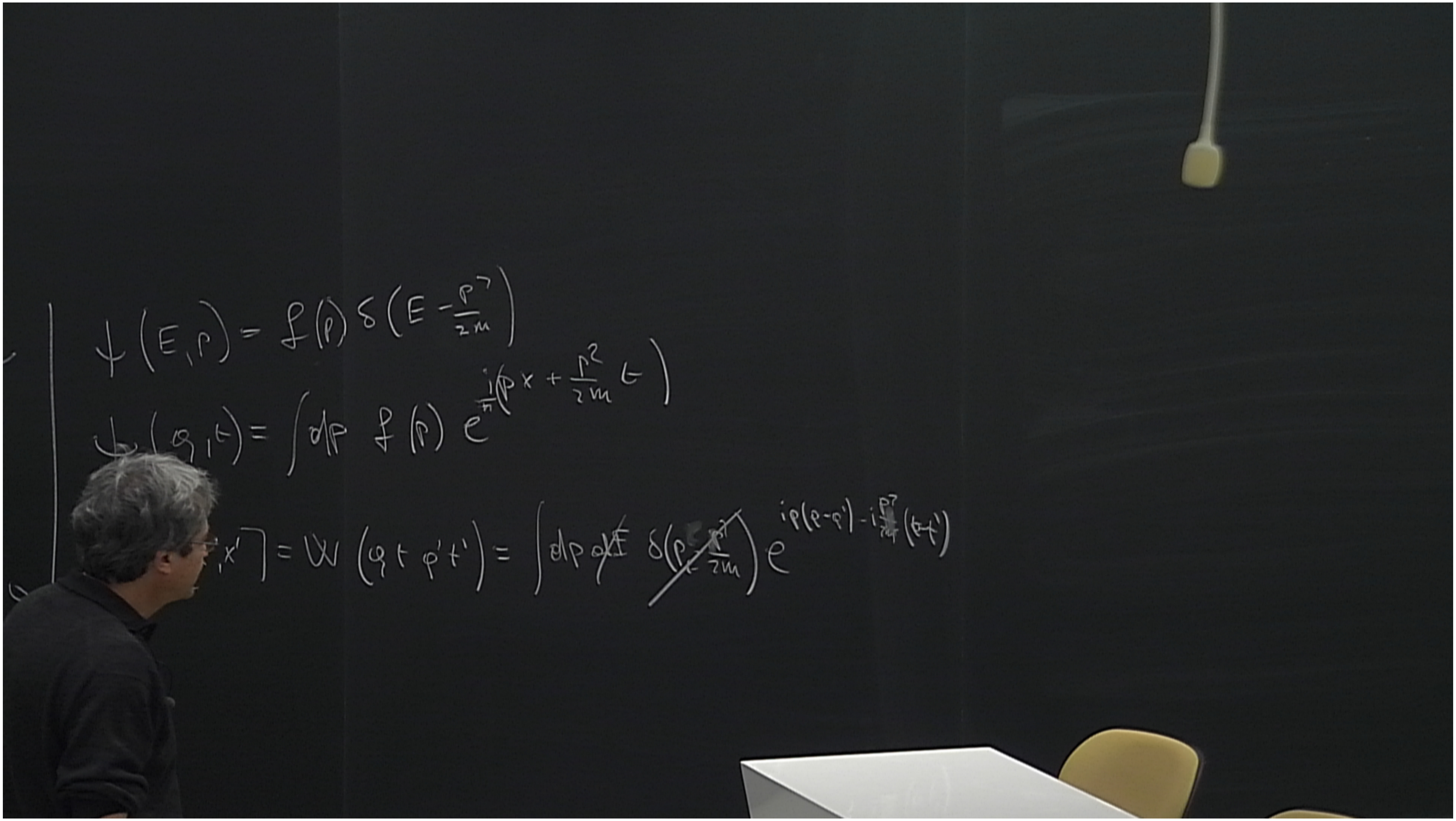
$$\left(E - \frac{p^2}{2m}\right)\psi(p, E) = 0$$

$$\psi(E, p) = f(p)\,\delta\left(E - \frac{p^2}{2m}\right)$$

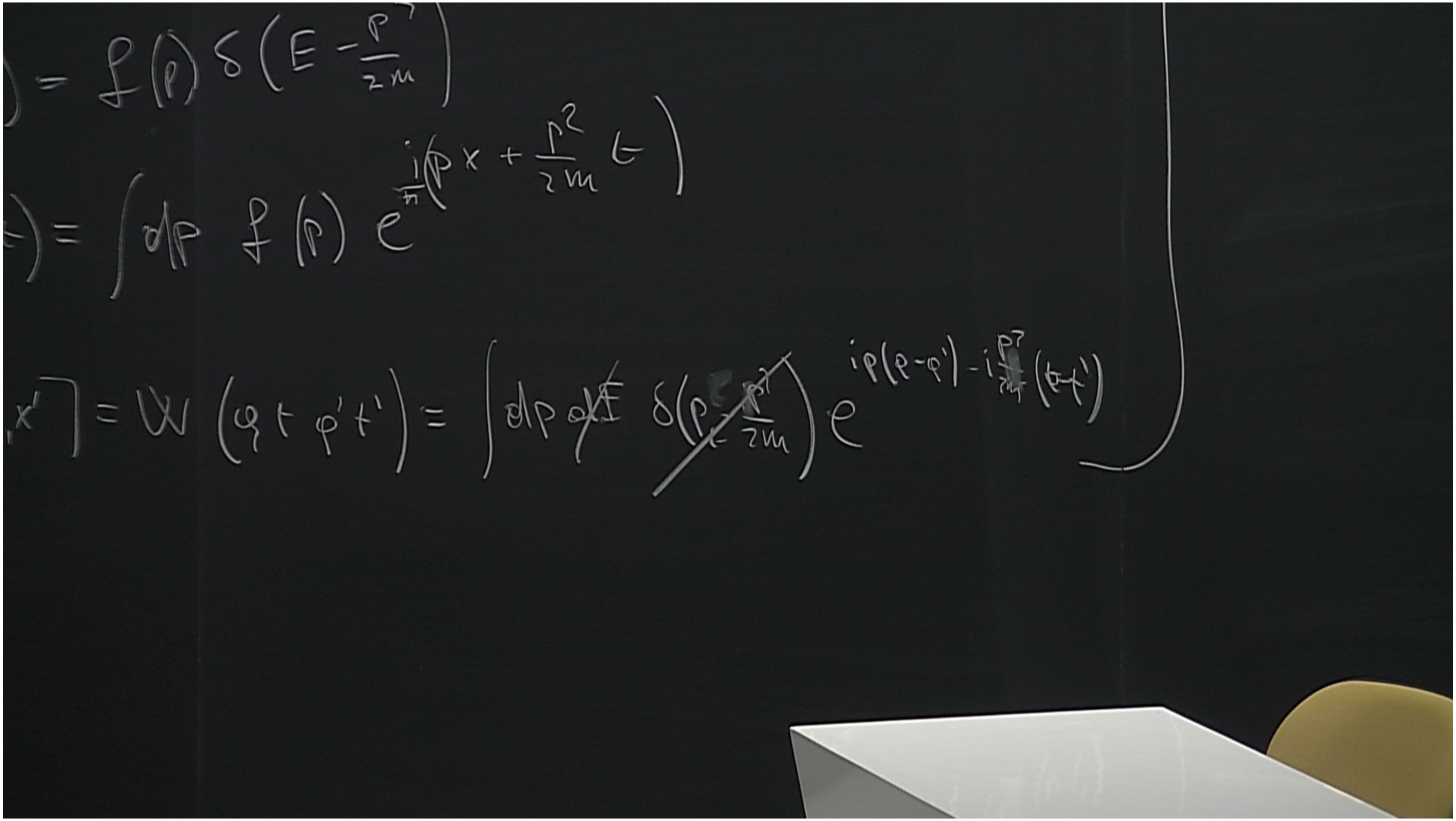
$$\psi(q, t) = \int dp\, f(p)\, e^{\frac{i}{\hbar}\left(px + \frac{p^2}{2m}t\right)}$$

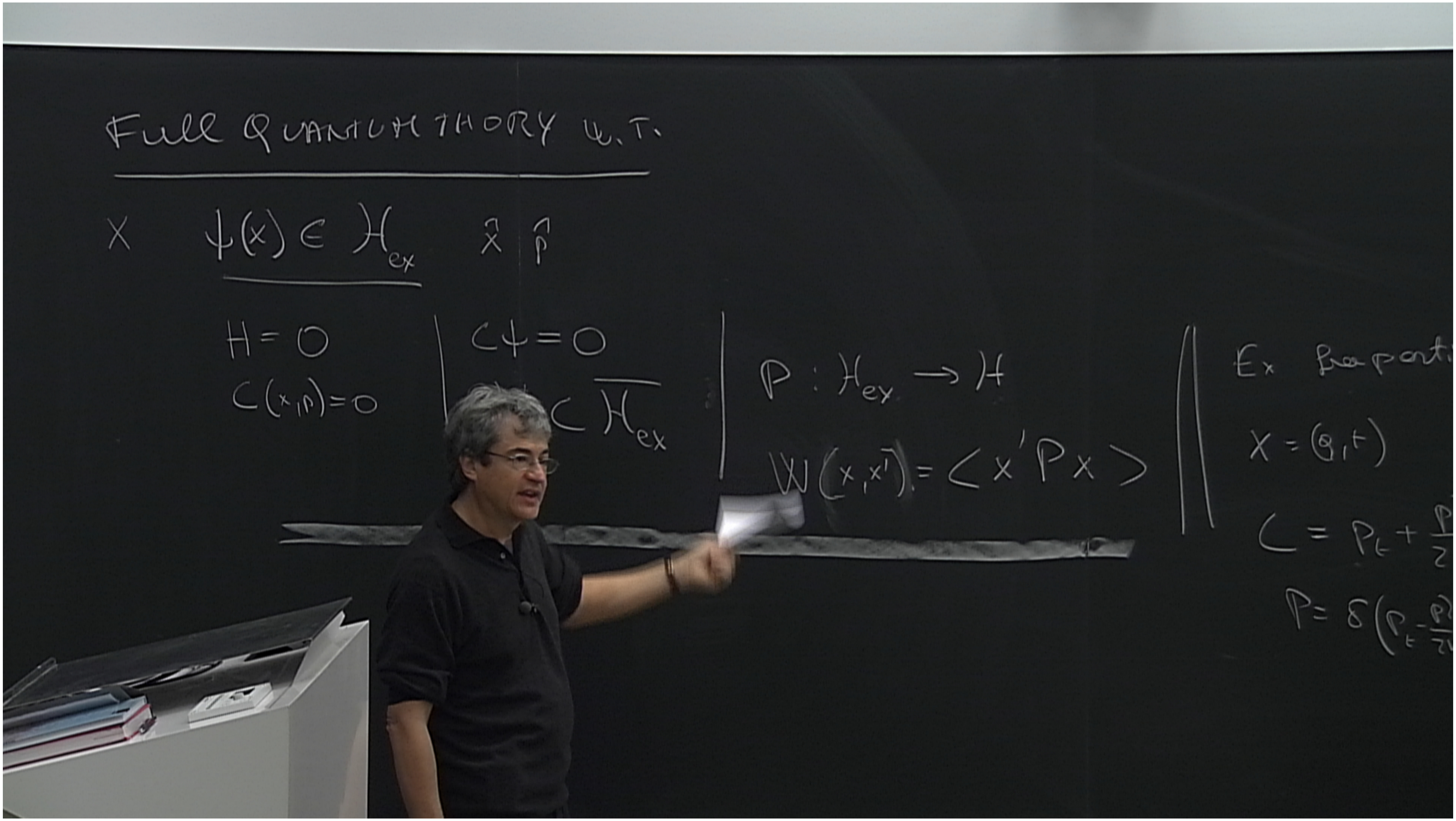
$$W[x, x'] = W(q\,t\,p'\,t') =$$


Ex free particle
X = (q, t)   ψ(q, t)
C = p_t + p²/2m = 0
P = δ(p_t − p²/2m)
iħ ∂/∂t ψ = −ħ²/2m ∂²/∂q² ψ
→ ψ(p, E)
(E − p²/2m) ψ(p, E) = 0
ψ(E, p) = f(p) δ(E −
ψ(q, t) = ∫dp f(p)
W[x, x′] = W(q t q′ t



$$\sqrt{\frac{\cdots}{\cdots}}\; e^{\frac{i}{\hbar}\frac{m(\varphi'-\varphi)^2}{2(t'-t)}}$$




X ψ(x) ∈ H_ex x̂ p̂
H = 0
C(x,p) = 0
Cψ = 0
P : H_ex → H
W(x,x') = <x' P x>
X = (q,t)