

Title: Explorations in Particle Theory - Lecture 14

Date: Apr 20, 2012 09:00 AM

URL: <http://pirsa.org/12040018>

Abstract:

Axioms



Axions

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Axion $\Rightarrow \bar{\theta} \rightarrow 0$ dynamically

$$\begin{aligned}
 -\mathcal{L} = & V(|\Phi|^2) + \lambda \left(\Phi \overline{\Psi}_L \overline{\Psi}_R + \Phi^* \underbrace{\overline{\Psi}_R \overline{\Psi}_L}_{\text{new "quark", with } Q_{em} = 0} \right) \\
 & \text{"complex scalar"} \\
 & -m^2 |\Phi|^2 + \dots
 \end{aligned}$$

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 & \underbrace{-\mu^2 |\Phi|^2 + \frac{\lambda}{2} |\Phi|^4}_{\text{"complex scalar"}}
 \end{aligned}$$

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$\underbrace{\hspace{10em}}_{\text{complex scalar}} \quad \underbrace{\hspace{10em}}_{\text{new "quark", with } Q_{em} = 0}$

$$-m^2 |\Phi|^2 + \frac{\lambda}{2} |\Phi|^4$$

$$\Rightarrow \Phi(x) = \frac{1}{\sqrt{2}} \left[f + h(x) \right] e^{i\alpha(x)/f}$$

$\underbrace{\hspace{10em}}_{\text{VEV of } \Phi}$

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$\hookrightarrow (\partial_\mu a)^2 + \dots$

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$\underbrace{\qquad\qquad\qquad}_{\text{VEV of } \Phi}$

$f \gg m_W \Rightarrow h, \bar{\Psi}_{L,R}$ get heavy

\hookrightarrow get rid of heavy stuff

$$\mathcal{L} \supset |\partial_\mu \Phi|^2$$

$\hookrightarrow (\partial_\mu a)^2 + \dots$

U(1) symmetry

$$\left\{ \begin{array}{l} \Psi_L \rightarrow e^{i\delta/2} \Psi_L \\ \Psi_R \rightarrow e^{-i\delta/2} \Psi_R \\ \phi \rightarrow e^{i\delta} \phi \end{array} \right.$$

U(1) "symmetry" $\left\{ \begin{array}{l} \Psi_L \rightarrow e^{i\delta/2} \Psi_L \\ \Psi_R \rightarrow e^{-i\delta/2} \Psi_R \\ \phi \rightarrow e^{i\delta} \phi \end{array} \right.$

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\Rightarrow full Lagrangian isn't invariant!

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(Would have a symmetry if $\bar{\psi} \rightarrow \bar{\psi} - \gamma$)

"symmetry"

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(Would have a symmetry if $\bar{\theta} \rightarrow \bar{\theta} - \gamma$)
 \hookrightarrow "spurious symmetry"

$$-\mathcal{L} \supset V(|\phi|^2) + \lambda (f/\mu^2) (e^{i\alpha/f} \bar{\Psi}_L \Psi_R + \text{h.c.})$$

↳ only depends on h

not invariant!

as

if $\bar{\theta} \rightarrow \bar{\theta} - \gamma$
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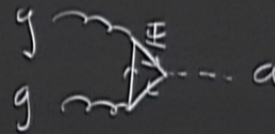
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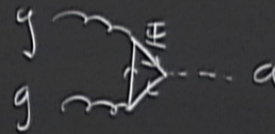
$$-\mathcal{L} \supset \sqrt{|\Phi|^2} + \lambda \left(\frac{f}{\sqrt{2}} \right) \left(e^{i\alpha/f} \bar{\Phi}_L \Phi_R + \text{h.c.} \right) + (\text{h-stuff})$$

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$$\begin{cases} \bar{\Psi}_L = e^{i\alpha/2f} \bar{\Psi}'_L \\ \bar{\Psi}_R = e^{-i\alpha/2f} \bar{\Psi}'_R \end{cases}$$

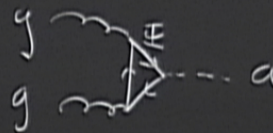
not inva
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$$-\mathcal{L} \supset \sqrt{|\mathcal{Q}|^2} + \lambda(f/\Omega) (e^{ia/f} \bar{\Psi}_L \bar{\Psi}_R + \text{h.c.}) + (\text{h-stuff})$$

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$$\begin{cases} \bar{\Psi}_L = e^{ia/2f} \bar{\Psi}'_L \\ \bar{\Psi}_R = e^{-ia/2f} \bar{\Psi}'_R \end{cases} \Rightarrow \left(\bar{\psi} + \frac{a}{f} \right) \frac{\gamma_5}{8\pi} G_{\mu\nu}^a \tilde{G}^{a\mu\nu}$$

$$-\mathcal{L} \supset \sqrt{|g|} + \lambda \left(\frac{1}{\Omega} \right) \left(e^{i a/f} \bar{\Psi}_L \Psi_R + \text{h.c.} \right) + (\text{h-stuff})$$

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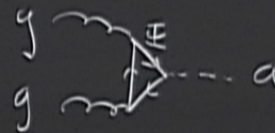
as

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$$\Psi_L = e^{i a/2f} \bar{\Psi}'_L$$

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$$\Rightarrow \left(\bar{\theta} + \frac{a}{f} \right) \frac{1}{8\pi} G_{\mu\nu}^a \tilde{G}^{a\mu\nu}$$

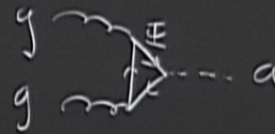
+ stuff that depends on (∂μ a)

not invariant!

if $\bar{\theta} \rightarrow \bar{\theta} - \gamma$
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$$-\mathcal{L} \supset \sqrt{|\Phi|^2} + \lambda(f/\sqrt{2}) (e^{i\alpha/f} \bar{\Psi}_L \Psi_R + \text{h.c.}) + (\text{h-stuff})$$

↳ only depends on h ↳ mass term



$$\begin{cases} \bar{\Psi}_L = e^{i\alpha/2f} \bar{\Psi}'_L \\ \Psi_R = e^{-i\alpha/2f} \Psi'_R \end{cases} \Rightarrow \left(\bar{\theta} + \frac{a}{f} \right) \frac{\sqrt{s}}{8\pi} G_{\mu\nu}^a \tilde{G}^{a\mu\nu}$$

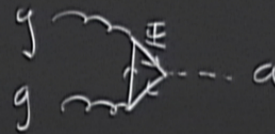
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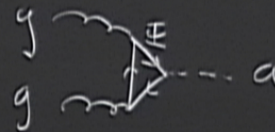
$$\mathcal{L}_{\text{eff}} = (\mathcal{Q}(D)) + (\partial_\mu a) + \left(\bar{\theta} + \frac{a}{f} \right) \frac{\alpha_s}{8\pi} G \tilde{G}$$

invariant!

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QCD becomes strong at $\Lambda_{\text{QCD}} \sim 200 \text{ MeV}$

$$\mathcal{L}_{\text{eff}} = \frac{\alpha}{8\pi} C_{\text{adr}} \frac{a}{f} \underbrace{F_{\mu\nu} \bar{F}^{\mu\nu}}_{\text{photon}} + i C_{\text{ace}} \frac{1}{f} (\partial_\mu a) \bar{e} \gamma^\mu \gamma^5 e + i \bar{\tilde{n}} \gamma^\mu \gamma^5 \tilde{n} \cdot C_{\text{ann}} \frac{1}{f} (\partial_\mu a)$$

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$$\mathcal{L} \rightarrow \frac{\alpha}{8\pi} C_{\text{ph}} \frac{g}{f} \underbrace{F_{\mu\nu} \bar{F}^{\mu\nu}}_{\text{photon}} + i C_{\text{ee}} \frac{1}{f} (\partial_\mu a) \bar{e} \gamma^\mu \gamma^5 e + i \bar{\tilde{h}} \gamma^\mu \gamma^5 \tilde{h} \cdot C_{\text{hh}} \frac{1}{f} (\partial_\mu a)$$

$$C_{\text{hh}} \sim 1$$

$$\mathcal{L}_{\text{eff}} = (Q(D)) + (\partial_\mu a) + \left(\bar{\theta} + \frac{a}{f}\right) \frac{\alpha_s}{8\pi} G^2$$

$$\bar{\psi} \gamma^\mu \gamma^5 \tilde{\eta} \cdot \text{canon } \frac{1}{f} (\partial_\mu a), \quad (a_{ii} \sim 1)$$

$$\frac{\lambda}{8\pi} C_{\text{arr}} \frac{a}{f} F \bar{F} + \frac{1}{f} C_{\text{arr}} (d_{\text{na}}) \bar{e} \delta^{n3} e + \dots, C_{\text{arr}} \sim 1.$$

$$\frac{\lambda}{8\pi} \cos \frac{a}{f} F \bar{F} + \frac{1}{f} \cos (2na) e + \dots, \cos \sim 1.$$

$$V(a) = m_a^2 f_a \left[1 - \cos \left(\bar{\theta} + \frac{a}{f} \right) \right]$$

$$\rightarrow m_a = \frac{\sqrt{m_u/m_d}}{1 + m_u/m_d} (\#) \frac{m_\pi f}{f}$$

$$\frac{\alpha}{8\pi} C_{\text{arr}} \frac{a}{f} F \bar{F} + \frac{1}{f} C_{\text{arr}} (m_a) \bar{e} \gamma^{\mu} \vec{v} e + \dots, C_{\text{arr}} \sim 1.$$

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$$\hookrightarrow m_a = \frac{\sqrt{m_u/m_d}}{1+m_u/m_d} (\#) \frac{m_{\pi} f_{\pi}}{f} \approx (0.6 \text{ eV}) \left(\frac{10^7 \text{ GeV}}{f} \right)$$

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$$V(a) = m_a^2 f \left[1 - \cos\left(\bar{\theta} + \frac{a}{f}\right) \right] = \text{const.} + \frac{1}{2} m_a^2 a^2 + \dots$$

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$$\mathcal{L}_{\text{eff}} = (\mathcal{Q}(D)) + (\partial_{\mu} a) + \left(\bar{\theta} + \frac{a}{f} \right) \frac{\alpha_s}{8\pi} G \tilde{G}$$

\Rightarrow minimum at $\frac{a}{f} = -\bar{\theta}$

$$\mathcal{L}_{\text{eff}} = (Q(D)) + (\partial_{\mu} a) + \left(\bar{\theta} + \frac{a}{f}\right) \frac{\alpha_s}{8\pi} G \tilde{G}$$

$$a = -f\bar{\theta} + \tilde{a}(x)$$

\Rightarrow minimum at $\frac{a}{f} = -\bar{\theta}$

$$\frac{\alpha}{8\pi} C_{\text{arr}} \frac{a}{f} F \bar{F} + \frac{1}{f} C_{\text{arr}} (m_u) \bar{e} \gamma^5 e + \dots, C_{\text{arr}} \sim 1.$$

$$= m_a^2 f \left[1 - \cos \left(\bar{\theta} + \frac{a}{f} \right) \right] = \text{const.} + \frac{1}{2} m_a^2 \bar{a}^2 + \dots$$

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\Rightarrow minimum

\Rightarrow strong CP

$$\mathcal{L}_{\text{eff}} = (Q(D)) + (\partial_\mu a) + \left(\bar{\theta} + \frac{a}{f}\right) \frac{\alpha_s}{8\pi} G^2$$

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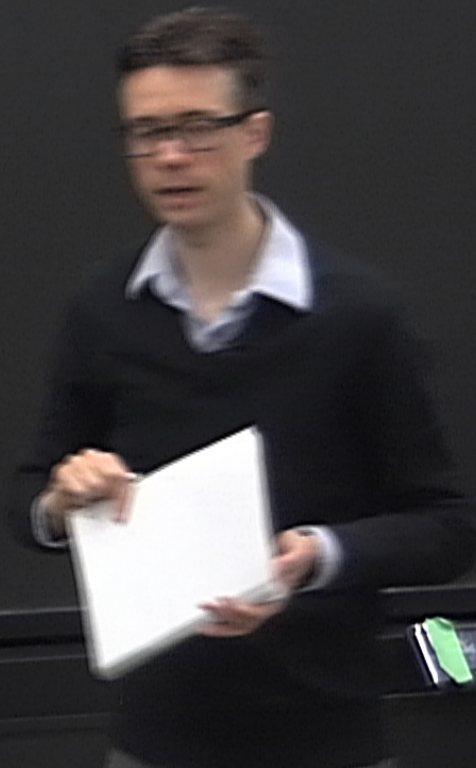
\Rightarrow minimum at $\frac{a}{f} = -\bar{\theta}$

\Rightarrow strong CP problem solved!

$$K^+ \rightarrow a + \pi^+, \dots$$



$$K^+ \rightarrow a + \pi^+, \dots \Rightarrow m_a \leq \text{keV} \quad (f \approx 10^3 \text{ GeV})$$



$$\frac{\alpha}{8\pi} C_{\text{arr}} \frac{g}{f} F \bar{F} + \frac{1}{f} C_{\text{arr}} (m_a) \bar{e} \gamma^{\mu 5} e + \dots, C_{\text{arr}} \sim 1.$$

$$m_a^2 f \left[1 - \cos\left(\bar{\theta} + \frac{g}{f}\right) \right] = \text{const.} + \frac{1}{2} m_a^2 \vec{a}^2 + \dots$$

$$\hookrightarrow m_a = \frac{\sqrt{m_u/m_d}}{1 + m_u/m_d} (\#) \frac{m_\pi f_\pi}{f} \simeq (0.6 \text{ eV}) \left(\frac{10^7 \text{ GeV}}{f} \right)$$

$$\mathcal{L}_{\text{eff}} = \frac{\alpha}{8\pi} C_{\text{arr}} \frac{a}{f} \overbrace{F\bar{F}}^{\partial_\mu K^\mu} + \frac{1}{f} C_{\text{arr}} (\partial_\mu a) \bar{e} \gamma^\mu \vec{\tau} e + \dots, C_{\text{arr}} \sim 1.$$

$$V(a) = m_a^2 f \left[1 - \cos\left(\bar{\theta} + \frac{a}{f}\right) \right] = \text{const.} + \frac{1}{2} m_a^2 \vec{a}^2 + \dots$$

$$\hookrightarrow m_a = \frac{\sqrt{m_u/m_d}}{1 + m_u/m_d} (\#) \frac{m_\pi f_\pi}{f} \approx (0.6 \text{ eV}) \left(\frac{10^3 \text{ GeV}}{f} \right)$$

$$K^+ \rightarrow a + \pi^+, \dots \Rightarrow m_a \leq \text{keV} \quad (f \approx 10^3 \text{ GeV})$$

$$T \gg \Lambda_{\text{QCD}} \\ m_a(T) = (0.1) m_a(T=0) \left(\frac{\Lambda_{\text{QCD}}}{200 \text{ MeV}} \right)^{3.7}$$

$$\mathcal{L}_{\text{eff}} = \underbrace{\frac{\alpha}{8\pi} C_{\text{arr}}}_{g_{\text{arr}}} \frac{a}{f} \underbrace{F\bar{F}}_{\partial_\mu K^\mu} + \frac{1}{f} C_{\text{arr}} (\partial_\mu a) \bar{e} \gamma^\mu \vec{e} + \dots, \quad C_{\text{arr}} \sim 1.$$

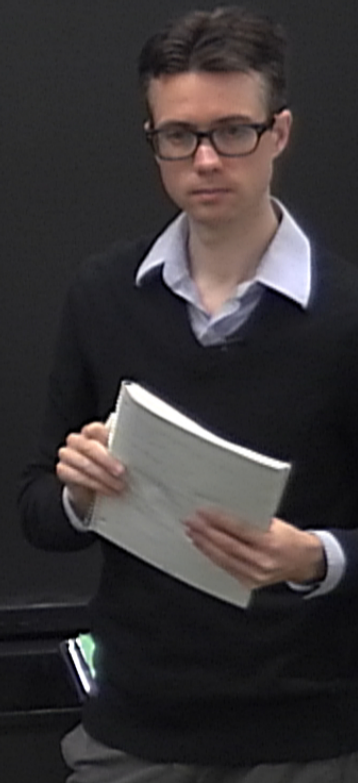
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$$\Gamma_{a \rightarrow \gamma\gamma} = \frac{g_{a\gamma\gamma}^2 m_a^3}{64\pi} \approx (1 \times 10^{-24} \text{ s}^{-1}) \left(\text{GeV} \right)^2 \left(\frac{m_a}{1 \text{ eV}} \right)^5$$



$$\mathcal{L}_{\text{eff}} = \underbrace{\frac{\alpha}{8\pi} C_{\text{arr}}}_{g_{\text{arr}}} \underbrace{\frac{a}{f} \bar{F} F}_{\partial_\mu K^\mu} = a \bar{F} F g_{\text{arr}} + \frac{1}{f} C_{\text{arr}} (\partial_\mu a) \bar{e} \gamma^\mu \vec{e} + \dots, C_{\text{arr}} \sim 1.$$

$$V(a) = m_a^2 f \left[1 - \cos\left(\bar{\theta} + \frac{a}{f}\right) \right] = \text{const.} + \frac{1}{2} m_a^2 \vec{a}^2 + \dots$$

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$$\Gamma_{a \rightarrow \gamma\gamma} = \frac{g_{a\gamma\gamma}^2 m_a^3}{64\pi} \approx (1 \times 10^{-24} \text{ s}^{-1}) \left(\text{const} \left(\frac{m_a}{\text{keV}} \right)^5 \right) = \tau_a^{-1}$$

$$\tau_a = 10^{10} \text{ yrs} = 10^{10} (\pi \times 10^3 \text{ s}) \sim 10^{13} \text{ s}$$

$T_{\text{star}} \sim \text{keV} - \text{MeV}$



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$$m_a \lesssim 10 \text{ meV}, f \gtrsim 10^9 \text{ GeV}$$



6411

$$t_c = 10^{10} \text{ yrs} = 10^{10} (\pi \times 10^7 \text{ s}) \sim 10^{17} \text{ s}$$

$$\ddot{\tilde{a}} + \underbrace{3H}_{\text{damping}} \dot{\tilde{a}} + \underbrace{m_a(T)}_{\text{oscillation}} \tilde{a} = 0, \quad \tilde{a} = a - \bar{a}/f$$

$H \gg m_a(T) \Rightarrow \tilde{a}$ is trapped



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oscillation

trapped

$$\leadsto \bar{a}(t_{\text{initial}}) \neq 0.$$

\bar{a}



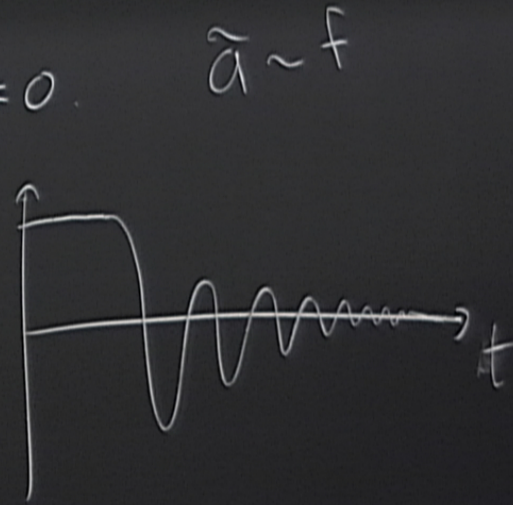
illates

6411

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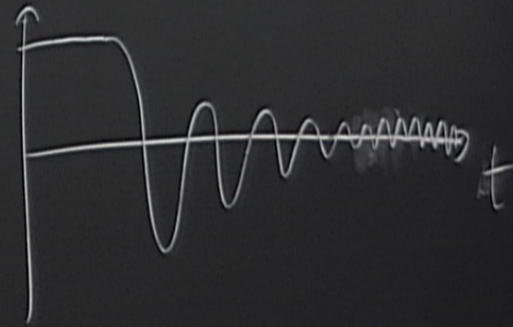


6411

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$H \gg m_a(T) \Rightarrow \tilde{a}$ is trapped $\sim \tilde{a}(t_{\text{initial}}) \neq 0$. $\tilde{a} \sim f$
 $m_a > H \Rightarrow \tilde{a}$ oscillates



$$\dot{\rho}_a + \left(\frac{\dot{m}_a}{m_a} + 3H \right) \rho_a = 0$$

//
oscillation energy density

$\sim f$



$$\ddot{\rho}_a + \left(\frac{\dot{m}_a}{m_a} + 3H \right) \rho_a = 0$$

// oscillation energy density \Rightarrow redshifts like matter for $m_a = 0$.

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$\sim f$
 \Rightarrow axion oscillations act like matter \Rightarrow DM!

$$\Omega_a h^2 = 0.1 \left(\frac{\bar{a}_i}{f} \right)^2 \left(\frac{m_a}{10 \text{ eV}} \right)^{-1.18}$$

↓
initial displacement in units of f.



$$\Omega_a h^2 = 0.1 \left(\frac{\bar{a}_i}{f} \right)^2 \left(\frac{m_a}{10 \text{ eV}} \right)^{-1.18} \rightarrow f = 10^{12} \text{ GeV}$$

\downarrow
 initial displacement in units of f .

A window: $10^9 \text{ GeV} \leq f \leq 10^{12} \text{ GeV}$

\uparrow stellar stuff \neq \uparrow not making too much DM

$$\Omega_a h^2 = 0.1 \left(\frac{\bar{a}_i}{f} \right)^2 \left(\frac{m_a}{10 \text{ eV}} \right)^{-1.18}$$

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Axion window: $10^9 \text{ GeV} \leq f \leq 10^{12} \text{ GeV}$

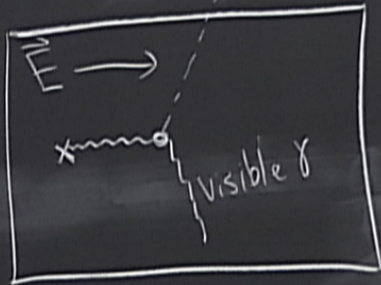
\uparrow stellar stuff $m_a \neq$

\nwarrow not making

garr a $F\bar{F}$
 $\underbrace{\quad\quad\quad}_{4\vec{E}\cdot\vec{B}}$

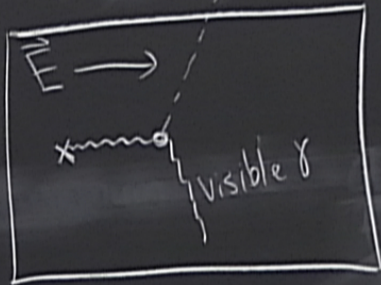
DM

$g_{\text{add}} \cdot a \vec{F} \vec{F}$
 $\approx 4 \vec{E} \cdot \vec{B} \quad a$



DM

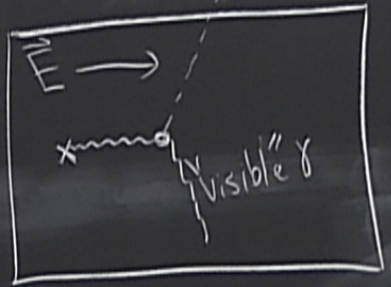
$g_{\text{add}} \propto a \vec{F} \vec{F}$
 $\approx 4 \vec{E} \cdot \vec{B} \quad a$



\Rightarrow Axion DD
ADMX experiment

DM

$g_{a\gamma\gamma} \propto \frac{1}{f_a^2}$
 $\propto 4\vec{E} \cdot \vec{B}$



\Rightarrow Axion DD
ADMX experiment