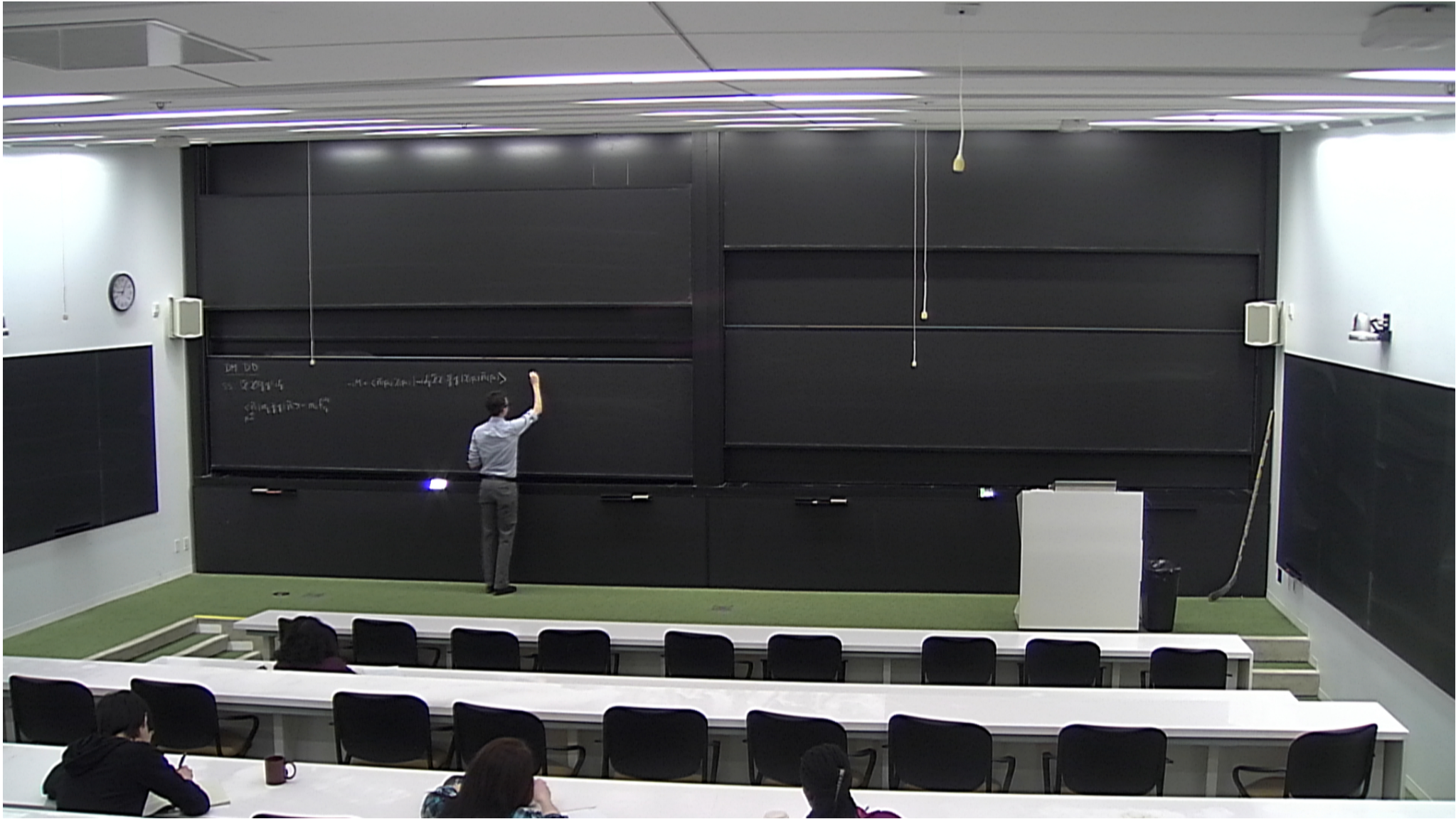


Title: Explorations in Particle Theory - Lecture 8

Date: Apr 12, 2012 09:00 AM

URL: <http://pirsa.org/12040010>

Abstract:

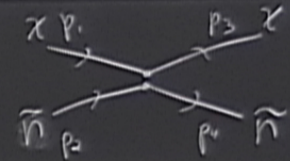


DM DD

$$SS: (\bar{\chi} \chi)(\bar{q} q) d_g$$

$$\langle \hat{n} | m_g \bar{q} q \rangle_{p, \hat{n}} = m_{\hat{n}} f_{\tau_s}^{(\hat{n})}$$

$$-iM = \langle \bar{n}(p_4) \chi(p_3) | -i d_g \bar{\chi} \chi \bar{q} q | \chi(p_1) \bar{n}(p_2) \rangle$$

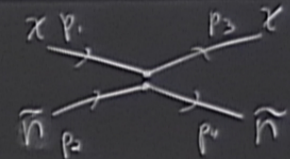


DM DD

$$SS: (\bar{\chi} \chi)(\bar{q} q) d_g$$

$$\langle \hat{n} | m_g \bar{q} q \rangle = m_{\bar{n}} f_{\tau_s}^{(m)}$$

$$\begin{aligned} -iM &= \langle \bar{n}(p_1) \chi(p_2) | -i d_g \bar{\chi} \chi \bar{q} q | \chi(p_1) \bar{n}(p_2) \rangle \\ &= -i d_g \langle \chi(p_2) | \bar{\chi} \chi | \chi(p_1) \rangle \langle \bar{n}(p_1) | \bar{q} q | \bar{n}(p_2) \rangle \end{aligned}$$

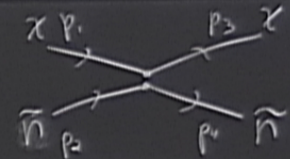


DM DD

$$SS: (\bar{\chi} \chi)(\bar{q} q) d_g$$

$$\langle \bar{\chi} | m_g \bar{q} q | \tilde{\chi} \rangle = m_{\tilde{\chi}} f_{T_g}^{(\tilde{\chi})}$$

$$\begin{aligned} -iM &= \langle \bar{\tilde{\chi}}(p_4) \chi(p_2) | -i d_g \bar{\chi} \chi \bar{q} q | \chi(p_1) \bar{\tilde{\chi}}(p_3) \rangle \\ &= -i d_g \langle \chi(p_3) | \bar{\chi} \chi | \chi(p_1) \rangle \langle \bar{\tilde{\chi}}(p_4) | \bar{q} q | \bar{\tilde{\chi}}(p_2) \rangle \\ &= -i d_g \bar{U}(p_3) U(p_1) \frac{m_{\tilde{\chi}}}{m_g} f_{T_g}^{(\tilde{\chi})} \bar{U}(p_4) U(p_2) \end{aligned}$$

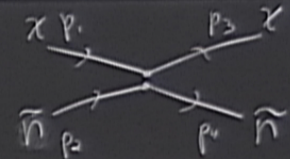


DM DD

$$SS: (\bar{\chi} \chi)(\bar{q} q) d_g$$

$$|m_g \bar{q} q| \tilde{h} \rangle = m_{\tilde{h}} f_{T_g}^{(\tilde{h})}$$

$$\begin{aligned} -iM &= \langle \tilde{h}(p_4) \chi(p_3) | -i d_g \bar{\chi} \chi \bar{q} q | \chi(p_1) \tilde{h}(p_2) \rangle \\ &= -i d_g \langle \chi(p_3) | \bar{\chi} \chi | \chi(p_1) \rangle \langle \tilde{h}(p_4) | \bar{q} q | \tilde{h}(p_2) \rangle \\ &= -i d_g \bar{U}(p_3) U(p_1) \underbrace{\frac{m_{\tilde{h}}}{m_g} f_{T_g}^{(\tilde{h})}} \bar{U}(p_4) U(p_2) \end{aligned}$$



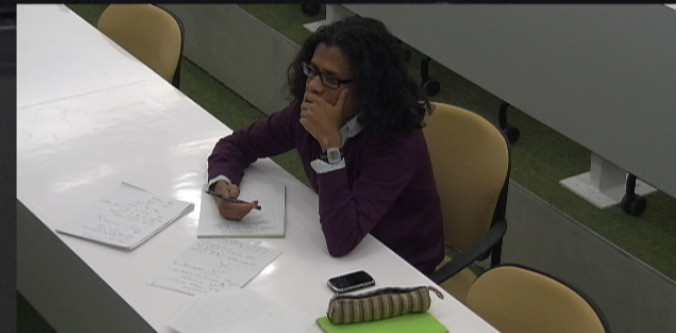
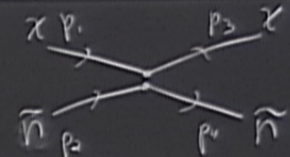
DM DD

SS: $(\bar{\chi} \chi)(\bar{q} q) d_g$

$|m_g \bar{q} q | \tilde{h} \rangle = m_{\tilde{h}} f_{T_g}^{(m)}$

$\langle \tilde{h} | \tilde{h} \tilde{h} | \tilde{h} \rangle = 1$

$$\begin{aligned}
 -iM &= \langle \tilde{h}(p_4) \chi(p_3) | -i d_g \bar{\chi} \chi \bar{q} q | \chi(p_1) \tilde{h}(p_2) \rangle \\
 &= -i d_g \langle \chi(p_3) | \bar{\chi} \chi | \chi(p_1) \rangle \langle \tilde{h}(p_4) | \bar{q} q | \tilde{h}(p_2) \rangle \\
 &= -i d_g \bar{U}_i(p_3) U_i(p_1) \underbrace{\frac{m_{\tilde{h}}}{m_g} f_{T_g}^{(m)}}_{\text{from lattice}} \underbrace{\bar{U}_i(p_4) U_i(p_2)}_{\text{polarization stuff}} \\
 -\mathcal{L}_{\text{eff}} &> d_g \frac{m_{\tilde{h}}}{m_g} f_{T_g}^{(m)} \bar{\chi} \chi \tilde{h} \tilde{h}
 \end{aligned}$$



DM DD

SS: $(\bar{\chi} \chi)(\bar{q} q) d_g$

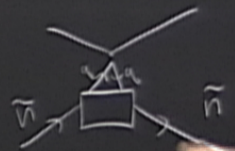
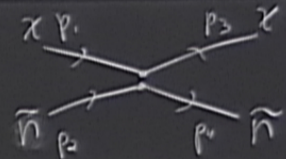
$$\langle \hat{n} | m_g \bar{q} q | \hat{n} \rangle = m_{\hat{n}} f_{T_g}^{(\hat{n})}$$

$\hat{n} \equiv p, \bar{n}$

$$\langle \hat{n} | \hat{n} \hat{n} | \hat{n} \rangle = 1$$

$$\begin{aligned}
 -iM &= \langle \hat{n}(p_4) \chi(p_2) | -i d_g \bar{\chi} \chi \bar{q} q | \chi(p_1) \bar{n}(p_3) \rangle \\
 &= -i d_g \langle \chi(p_3) | \bar{\chi} \chi | \chi(p_1) \rangle \langle \hat{n}(p_4) | \bar{q} q | \hat{n}(p_2) \rangle \\
 &= -i d_g \bar{U}_i(p_3) U_i(p_1) \underbrace{\frac{m_{\hat{n}}}{m_g} f_{T_g}^{(\hat{n})}}_{\text{from lattice}} \underbrace{\bar{U}_i(p_4) U_i(p_2)}_{\text{polarizat. stuff}}
 \end{aligned}$$

$$\alpha_{\text{eff}} > d_g \frac{m_{\hat{n}}}{m_g} f_{T_g}^{(\hat{n})} \bar{\chi} \chi \cdot \hat{n} \hat{n}$$



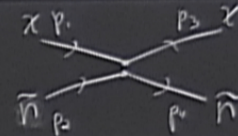
DM DD

SS: $(\bar{\chi} \chi)(\bar{q} q) d_4$

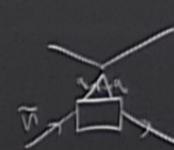
$$\langle \hat{n} | m_q \bar{q} q | \hat{n} \rangle = m_q f_{T_3}^{(n)}$$

$$\langle \hat{n} | \hat{n} \hat{n} | \hat{n} \rangle = 1$$

$$\begin{aligned}
 -iM &= \langle \hat{n}(p_4) \chi(p_2) | -i d_4 \bar{\chi} \chi \bar{q} q | \chi(p_1) \bar{n}(p_3) \rangle \\
 &= -i d_4 \langle \chi(p_3) | \bar{\chi} \chi | \chi(p_1) \rangle \langle \bar{n}(p_4) | \bar{q} q | \bar{n}(p_2) \rangle \\
 &= -i d_4 \underbrace{\bar{U}(p_3) U(p_1)}_{\text{from lattice}} \underbrace{\frac{m_q}{m_q} f_{T_3}^{(n)}}_{\text{polarization stuff}} \bar{U}(p_4) U(p_2) \\
 -\mathcal{L}_{\text{eff}} &> d_4 \frac{m_q}{m_q} f_{T_3}^{(n)} \bar{\chi} \chi \bar{n} \hat{n}
 \end{aligned}$$



$\bar{\chi} \chi G_n G^{nm}$



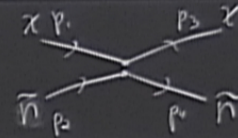
DM DD

SS: $(\bar{\chi} \chi)(\bar{q} q) d_4$

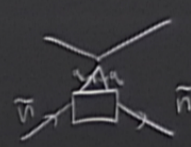
$\langle \hat{n} | m_q \bar{q} q | \hat{n} \rangle = m_q f_{T_3}^{(m)}$
 p, \hat{n}

$\langle \hat{n} | \hat{n} \hat{n} | \hat{n} \rangle = 1$

$$\begin{aligned}
 -iM &= \langle \hat{n}(p_4) \chi(p_2) | -i d_4 \bar{\chi} \chi \bar{q} q | \chi(p_1) \hat{n}(p_3) \rangle \\
 &= -i d_4 \langle \chi(p_3) | \bar{\chi} \chi | \chi(p_1) \rangle \langle \hat{n} | \bar{q} q | \hat{n}(p_2) \rangle \\
 &= -i d_4 \bar{U}(p_3) U(p_1) \underbrace{\frac{m_q}{m_q} f_{T_3}^{(m)}}_{\text{from lattice}} \underbrace{\bar{U}(p_4) U(p_2)}_{\text{polarization stuff}} \\
 -\mathcal{L}_{\text{eff}} &> d_4 \frac{m_q}{m_q} f_{T_3}^{(m)} \bar{\chi} \chi \hat{n} \hat{n}
 \end{aligned}$$



$\bar{\chi} \chi G_{\mu}^{-} G^{+\mu}$



DM DD

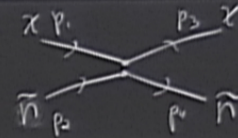
SS: $(\bar{\chi} \chi)(\bar{q} q) d_4$

$\langle \hat{n} | m_q \bar{q} q | \hat{n} \rangle = m_q f_{T_q}^{(m)}$ (...)

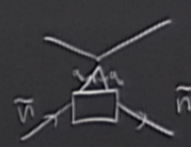
$\langle \hat{n} | \bar{\chi} \chi | \hat{n} \rangle = 1$ (...)

$$\begin{aligned}
 -iM &= \langle \hat{n}(p_4) \chi(p_2) | -i d_4 \bar{\chi} \chi \bar{q} q | \chi(p_1) \bar{n}(p_3) \rangle \\
 &= -i d_4 \langle \chi(p_3) | \bar{\chi} \chi | \chi(p_1) \rangle \langle \bar{n}(p_4) | \bar{q} q | \bar{n}(p_2) \rangle \\
 &= -i d_4 \bar{U}(p_3) U(p_1) \underbrace{\frac{m_q}{m_q} f_{T_q}^{(m)}}_{\text{From lattice}} \underbrace{\bar{U}(p_4) U(p_2)}_{\text{polarization stuff}}
 \end{aligned}$$

$\mathcal{L}_{\text{eff}} > d_4 \frac{m_q}{m_q} f_{T_q}^{(m)} \bar{\chi} \chi \bar{n} n$



$\bar{\chi} \chi G_n G^{nm}$



$$SS: (\bar{\chi} \chi)(\bar{g} g) d_1$$

$$\langle \bar{\tilde{h}} | m_g \bar{g} g | \tilde{h} \rangle = m_g f_{T_3}^{(m)} \quad (\dots)$$

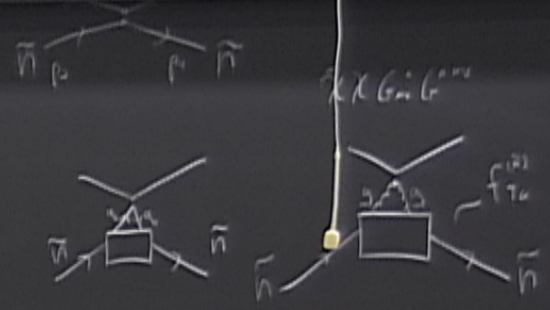
$$\langle \bar{\tilde{h}} | \bar{\tilde{h}} \tilde{h} \tilde{h} | \tilde{h} \rangle = 1 \quad (\dots)$$

$$-iM = \langle \bar{\tilde{h}}(p_1) \chi(p_2) | -i d_1 \chi \chi \bar{g} g | \chi(p_1) \tilde{h}(p_2) \rangle$$

$$= -i d_1 \langle \chi(p_1) \bar{\chi} \chi | \chi(p_1) \rangle \langle \bar{\tilde{h}} | \bar{g} g | \tilde{h}(p_2) \rangle$$

$$= -i d_1 \bar{u}(p_1) u(p_1) \underbrace{\frac{m_g}{m_g} f_{T_3}^{(m)}}_{\text{from lattice}} \underbrace{\bar{u}(p_2) u(p_2)}_{\text{polarizat. stuff}}$$

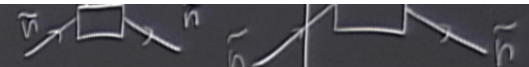
$$\alpha_{\text{eff}} = d_1 \frac{m_g}{m_g} f_{T_3}^{(m)} \bar{\chi} \chi \bar{\tilde{h}} \tilde{h}$$



$$\langle n | \hat{n} | n \rangle = 1 \dots$$

$$- \mathcal{L}_{\text{eff}} \supset \int d^4x \frac{m_N}{m_q} F_{T_q}^{(2)} \bar{\chi} \chi \cdot \bar{\tilde{n}} \tilde{n}$$

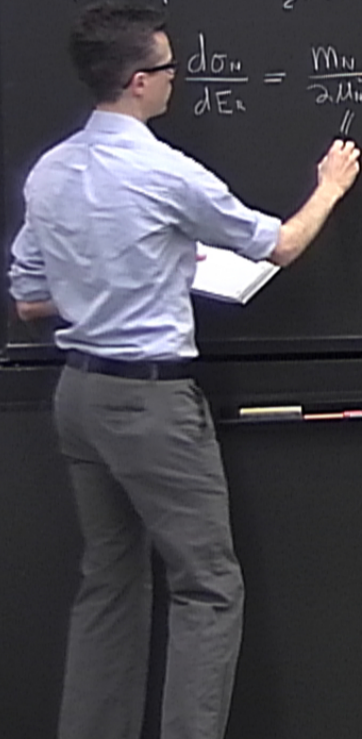
from lattice perturbative stuff



Nucleon \rightarrow Nucleus

$$\tilde{n} = n, p \quad \begin{matrix} A \\ z \end{matrix} N$$

$$\frac{d\sigma_{\text{N}}}{dE_{\text{R}}} = \frac{m_{\text{N}}}{2M_{\text{N}}^2} \tilde{\sigma}_{\text{N}} F^2(E_{\text{R}})$$



$$\langle n | n \rangle = 1$$

$$-\mathcal{L}_{\text{eff}} \supset d_q \frac{m_N}{m_q} F_q^{(2)} \bar{\chi} \chi \cdot \vec{n} \vec{n}$$

from lattice potential stuff



Nucleon \rightarrow Nucleus
 $\vec{n} = n, p$ $\begin{matrix} A \\ z \end{matrix} N$

$$\frac{d\sigma_H}{dE_R} = \frac{m_N}{2M_H N} \tilde{\sigma}_N F^2(E_R)$$

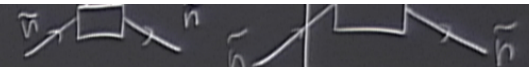
$\left(\frac{m_n m_\chi}{m_b + m_\chi} \right)_{\text{eff cs}} f_{\text{eff}}$



$$\langle n | \hat{n} | n \rangle = 1 \dots$$

$$-\mathcal{L}_{\text{eff}} \supset \int d^4x \frac{m_N}{m_q} F_{T_4}^{(n)} \bar{\chi} \chi \cdot \bar{\hat{n}} \hat{n}$$

from lattice perturbative stuff

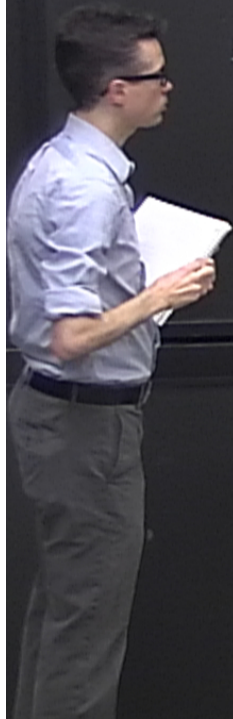


Nucleon \rightarrow Nucleus
 $\hat{n} = n, p$ $\frac{d\sigma_n}{dE_R}(E_R \rightarrow 0)$

$$\bar{\sigma}_N = \int_0^{2M_N M^*/m^*} dE_R \frac{d\sigma_n}{dE_R}(E_R \rightarrow 0)$$

$$\frac{d\sigma_n}{dE_R} = \underbrace{\frac{m_N}{2M_N M^*}}_{\left(\frac{m_n m_\chi}{m_b + m_\chi}\right)} \bar{\sigma}_N \underbrace{F^2(E_R)}_{\text{form factor}}$$

eff cs



Nucleon \rightarrow Nucleus
 $\tilde{n} = n, p$

$${}^A_Z N \quad \frac{d\sigma_{\tilde{n}}}{dE_R}(E_R \rightarrow 0)$$

$$= \frac{M_N}{2M_N N^2} \tilde{\sigma}_N F^2(E_R)$$

$\left(\frac{m_n m_X}{m_n + m_X} \right)$ eff cs form factor

$$\tilde{\sigma}_N = \int_0^{2M_N N^2/m_n} dE_R \cdot \frac{d\sigma_{\tilde{n}}}{dE_R}(E_R \rightarrow 0)$$

$F(E_R)$ contains momentum ($q^2 = 2m_n E_R$) dependence.

Nucleon \rightarrow Nucleus
 $\tilde{n} = n, p$

$$\frac{d\sigma_N}{dE_R} = \frac{m_N}{2M_N} F^2(E_R)$$

$\underbrace{\hspace{10em}}_{\text{eff. cross-section factor}}$

$$\tilde{\sigma}_N = \int_0^{2M_N N^2/m_N} dE_R \cdot \frac{d\sigma_N}{dE_R}(E_R \rightarrow 0)$$

$F(E_R)$ contains momentum ($q^2 = 2m_N E_R$) dependence.
 $\hookrightarrow F(E_R=0) = 1$

$$\tilde{\sigma}_N =$$

Nucleon \rightarrow Nucleus
 $\tilde{n} = n, p$

$$\frac{d\sigma_N}{dE_R} = \frac{m_N}{2\mu_N N^2} \tilde{\sigma}_N F^2$$

$\left(\frac{m_n m_\chi}{m_n + m_\chi} \right)$ eff cs factor

$$\tilde{\sigma}_N = \int_0^{2\mu_N N^2/m_N} dE_R \cdot \frac{d\tilde{\sigma}_N}{dE_R}(E_R \rightarrow 0)$$

$F(E_R)$ contains momentum ($q^2 = 2m_N E_R$) dependence.
 $\hookrightarrow F(E_R=0) = 1$

$$\tilde{\sigma}_N = \left(\frac{\mu_N}{M_P} \right)^2 \sigma_P \frac{[Z f_p + (A-Z) f_n]^2}{f_p}$$

Nucleon \rightarrow Nucleus
 $\tilde{n} = n, p$ ${}^A_Z N$ $\frac{d\sigma_n}{dE_R}(E_R \rightarrow 0)$

$$\frac{d\sigma_n}{dE_R} = \frac{m_N}{2\mu n N} \tilde{\sigma}_N F^2(E_R)$$

$\left(\frac{m_n m_x}{m_n + m_x} \right)$ eff cs. form factor

$$\tilde{\sigma}_N = \int_0^{2\mu n N / m_n} dE_R \frac{d\sigma_n}{dE_R}(E_R \rightarrow 0)$$

$F(E_R)$ contains momentum ($q^2 = 2\mu n E_R$) dependence.

$$\hookrightarrow F(E_R=0) = 1$$

$$\tilde{\sigma}_N = \left(\frac{\mu_n}{\mu_p} \right)^2 \sigma_p \frac{[Z f_p + (A-Z) f_n]^2}{f_p^2}$$

$$f_p = \left[\sum_{q \text{ odd}} \frac{f_{T_q}^{(p)}}{m_q} dq + \frac{2}{27} \sum_{q \text{ even}} \frac{f_{T_q}^{(p)}}{m_q} dq \right] m_p$$

Nucleon \rightarrow Nucleus
 $\tilde{n} = n, p$ ${}^A_Z N$ $\frac{d\sigma_H}{dE_R}(E_R \rightarrow 0)$

$$\frac{d\sigma_H}{dE_R} = \frac{m_N}{2\mu_N N} \tilde{\sigma}_N F^2(E_R)$$

$\left(\frac{m_n m_x}{m_n + m_x} \right)$ eff cs. form factor

$$\tilde{\sigma}_N = \int_0^{2\mu_N N/m_N} dE_R \frac{d\sigma_H}{dE_R}(E_R \rightarrow 0)$$

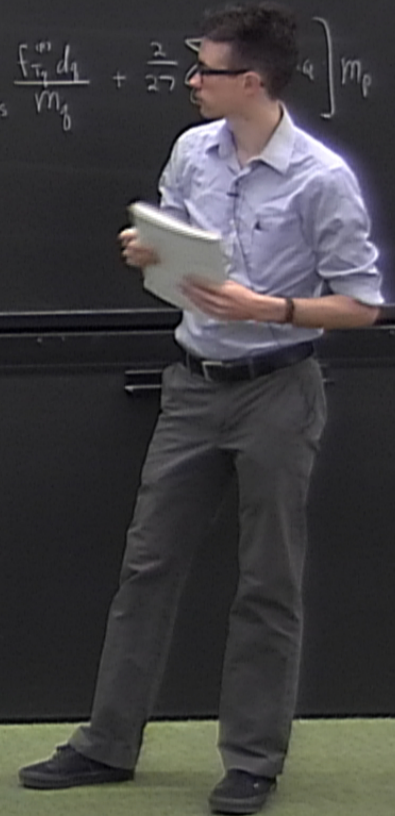
$F(E_R)$ contains momentum ($q^2 = 2m_N E_R$) dependence.

$\hookrightarrow F(E_R=0) = 1$

$$\tilde{\sigma}_N = \left(\frac{\mu_N}{M_p} \right)^2 \sigma_p \frac{[Z f_p + (A-Z) f_n]^2}{f_p^2}$$

\hookrightarrow cs for χ -p scattering

$$f_p = \left[\sum_{j=1}^4 \frac{f_{T_j}^{(p)}}{m_j} + \frac{2}{27} \right] m_p$$



Nucleon \rightarrow Nucleus
 $\tilde{n} = n, p$ ${}^A_Z N$

$$\frac{d\sigma_H}{dE_R}(E_R \rightarrow 0) = \left(\frac{\mu_{\alpha} \mu_X}{\mu_{\alpha} + \mu_X} \right)^2 \tilde{\sigma}_N F^2(E_R)$$

eff cs. form factor

$$\tilde{\sigma}_N = \int_0^{2M_N N/m_{\alpha}} dE_R \frac{d\sigma_H}{dE_R}(E_R \rightarrow 0)$$

$F(E_R)$ contains momentum ($q^2 = 2m_{\alpha} E_R$) dependence.

$$\hookrightarrow F(E_R=0) = 1$$

$$\tilde{\sigma}_N = \left(\frac{\mu_{\alpha}}{\mu_p} \right)^2 \sigma_p \frac{[Z f_p + (A-Z) f_n]^2}{f_p^2}$$

\hookrightarrow cs for χ -p scattering

$$f_p = \left[\sum_{q \in \text{protons}} \frac{f_{T_q}^{(p)}}{m_q} + \frac{2}{27} \sum_{q \in \text{neutrons}} \frac{f_{T_q}^{(p)}}{m_q} \right] m_p$$

Nucleon \rightarrow Nucleus
 $\tilde{n} = n, p$ ${}^A_Z N$ $\frac{d\sigma_H}{dE_R}(E_R \rightarrow 0)$

$$\frac{d\sigma_H}{dE_R} = \frac{m_N}{2\mu n N} \tilde{\sigma}_N F^2(E_R)$$

$\left(\frac{m_n m_x}{m_n + m_x} \right)$ eff cs. form factor

$$\tilde{\sigma}_N = \int_0^{2\mu n N / m_n} dE_R \frac{d\sigma_H}{dE_R}(E_R \rightarrow 0)$$

$F(E_R)$ contains momentum ($q^2 = 2m_n E_R$) dependence.

$\hookrightarrow F(E_R=0) = 1$

$$\tilde{\sigma}_N = \left(\frac{\mu_n}{M_p} \right)^2 \sigma_p \frac{[Z f_p + (A-Z) f_n]^2}{f_p^2}$$

\hookrightarrow cs for χ -p scattering

$$f_p = \left[\sum_{q \text{ nucleons}} \frac{f_{T_q}^{(p)}}{m_q} + \frac{2}{27} \sum_{q \text{ nucleons}} \frac{f_{T_q}^{(p)}}{m_q} \right] m_p$$

\rightarrow coherent scattering off all nucleons.

$$\bar{\sigma}_N = \int_0^{2M_N N^2 / m_N} dE_R \cdot \frac{d\sigma_i}{dE_R}(E_R \rightarrow 0)$$

$F(E_R)$ contains momentum ($q^2 = 2m_N E_R$) dependence.

$$\rightarrow F(E_R=0) = 1$$

$$f_p = \left[\sum_{q=u,d,s} \frac{f_{T_q}^{(p)}}{m_q} + \frac{2}{27} \sum_{Q=c,b} \frac{f_{T_Q}^{(p)}}{m_Q} \right] m_p$$

$$\bar{\sigma}_N = \left(\frac{M_N}{M_p} \right)^2 \cdot \sigma_p \frac{[Z f_p + (A-Z) f_n]^2}{f_p^2}$$

→ coherent scattering off all nucleons.

→ cs. for χ -p scattering

$$\bar{\sigma}_N = \int_0^{2\mu_N v^2/m_N} dE_R \frac{d\sigma_N}{dE_R}(E_R \rightarrow 0)$$

$F(E_R)$ contains momentum ($q^2 = 2m_N E_R$) dependence.

$$\hookrightarrow F(E_R=0) = 1$$

$$f_p = \left[\sum_{q=u,d,s} \frac{f_{T_q}^{(p)}}{m_q} + \frac{2}{27} \sum_{Q=c,b,b} \frac{f_{T_Q}^{(p)}}{m_Q} \right] m_p$$

$$\bar{\sigma}_N = \left(\frac{\mu_N}{m_p} \right)^2 \sigma_p \frac{[Z f_p + (A-Z) f_n]^2}{f_p^2}$$

Factor

\hookrightarrow cs for χ -p scattering

\rightarrow coherent scattering off all nucleons.

\hookrightarrow spin-independent (SI)

$F(E_R)$ = form factor describing loss of coherence

$$\approx \left[\frac{j_1(qR_1)}{qR_1} \right]^2 e^{-q^2 s^2}$$

$$q = \sqrt{2m_0 E_R}$$

$$\left\{ \begin{array}{l} R_1 = (R^2 - 5s^2)^{\frac{1}{2}} \\ R = (1.2 \text{ fm}) A^{\frac{1}{3}} \\ s = 1 \text{ fm} \end{array} \right.$$

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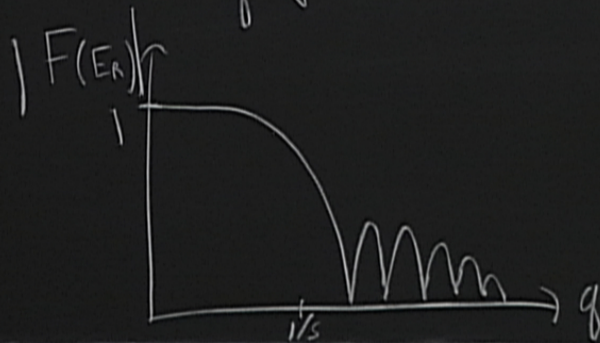
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$\frac{d\sigma_N}{dE_R}(E_R \rightarrow 0)$
 $F^2(E_R)$
 eff c.s. form factor

$$\tilde{\sigma}_N = \int_0^{2M_N N^2/m_N} dE_R \frac{d\sigma_N}{dE_R}(E_R \rightarrow 0)$$

$F(E_R)$ contains momentum ($q^2 = 2m_N E_R$) dependence.

$\hookrightarrow F(E_R=0) = 1$

$$\tilde{\sigma}_N = \left(\frac{M_N}{M_p}\right)^2 \sigma_p \frac{[Z f_p + (A-Z) f_n]^2}{f_p^2}$$

\hookrightarrow cs for χ -p scattering

$$f_p = \left[\sum_{q=u,d,s} \frac{f_{T_q}^{(p)}}{m_q} + \frac{2}{27} \sum_{Q=c,b} \frac{f_{T_Q}^{(p)}}{m_Q} \right] m_p$$

\rightarrow coherent scattering off all nucleons.

\hookrightarrow spin-independent (SI)

$$\frac{d\sigma_N}{dE_R}(E_R \rightarrow 0)$$

$$\frac{m_N}{2\mu_{in} N^2} \tilde{\sigma}_N F^2(E_R)$$

$\frac{m_N m_X}{m_N + m_X}$ eff c.s. for

$$\tilde{\sigma}_N = \int_0^{2M_N N^2/m_N} dE_R \frac{d\sigma_N}{dE_R}(E_R \rightarrow 0)$$

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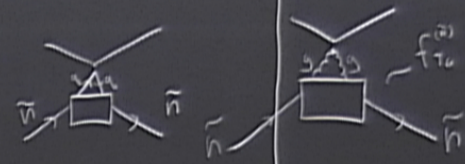


$$\langle \tilde{n} | m_y \tilde{q} | \tilde{n} \rangle = m_y f_{T_y}^{(m)} (\dots)$$

$$\langle \tilde{n} | \tilde{n} \tilde{n} | \tilde{n} \rangle = 1 (\dots)$$

$$= -id_y \bar{U}(p_2) U(p_1) \underbrace{\frac{m_y}{m_q} f_{T_y}^{(m)}}_{\text{from lattice}} \underbrace{\bar{U}(p_2) U(p_1)}_{\text{polarizat. stuff}}$$

$$\sigma_{\text{eff}} \approx d_y \frac{m_y}{m_q} f_{T_y}^{(2)} \bar{\chi} \chi \tilde{n} \tilde{n}$$



Nucleon \rightarrow Nucleus
 $\tilde{n} = n, p$ $\xrightarrow{A} N$ $\frac{d\sigma_n}{dE_R}(E_R \rightarrow 0)$

$$\frac{d\sigma_n}{dE_R} = \frac{m_N}{2\mu_{nN}} \tilde{\sigma}_N F^2(E_R)$$

$\left(\frac{m_n m_p}{m_n + m_p} \right)$ \leftarrow eff cs. form factor

$$\tilde{\sigma}_N = \int_0^{2\mu_{nN}/m_N} dE_R \frac{d\sigma}{dE_R}(E_R \rightarrow 0)$$

$F(E_R)$ contains momentum ($q^2 = 2m_N E_R$) dependence

$$\rightarrow F(E_R=0) = 1$$

$$\tilde{\sigma}_N = \left(\frac{\mu_N}{m_p} \right)^2 \sigma_p^{\text{cs}} \frac{[Z f_p + (A-Z) f_n]^2}{f_p^2}$$

\leftarrow cs for χ -p scattering

$$f_p = \left[\sum_{q=u,d,s} \frac{f_{T_q}^{(p)}}{m_q} d_q + \frac{2}{27} \sum_{q=c,b,t} \frac{f_{T_q}^{(p)}}{m_q} d_q \right] m_p$$

\rightarrow coherent scattering off all nucleons

\rightarrow spin independent (SI)

$$\int_{i/s} \dots \rightarrow q$$

$$VV: b_q \cdot \underbrace{\bar{\chi} \gamma^M \chi}_{\text{Majorana}} \cdot \bar{q} \gamma_m q$$

$L_{\rightarrow} = 0$ for Majorana fermions.

$$j_{em}^M = \sum_f Q_f^{em} \cdot \underbrace{\bar{f} \gamma^M f}_{\text{Dirac}}$$

$$M=0: (\#f - \#\bar{f})$$



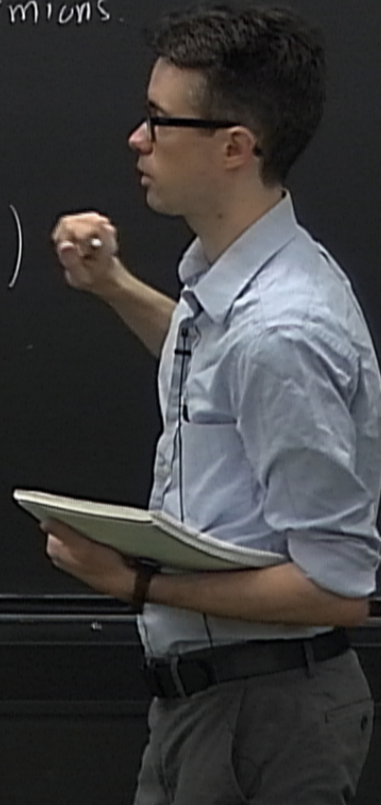
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$$\bar{\chi} \gamma^m \chi \quad \bar{\tilde{\eta}} \gamma_n \tilde{\eta} \quad b_{\bar{n}}$$

$$b_p = 2b_u + b_d$$

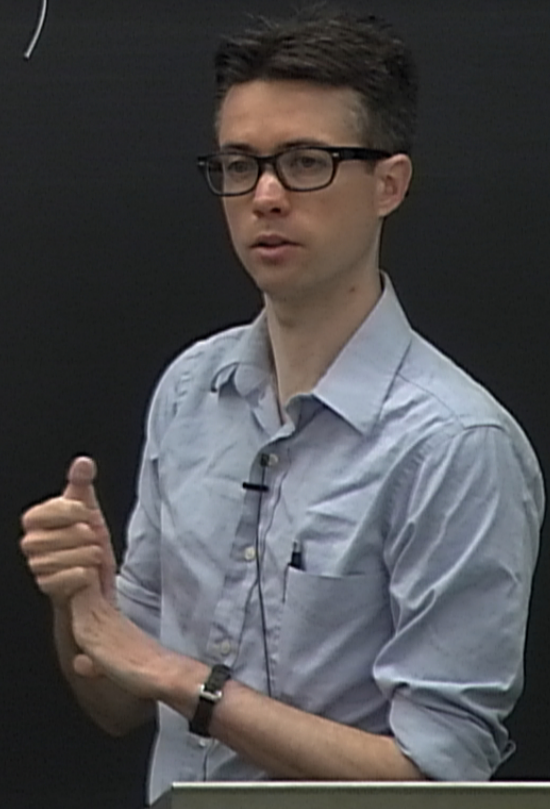
$$b_n = 2b_d + b_u$$

$$\bar{\chi} \gamma^{\mu} \chi \quad \bar{\tilde{\eta}} \gamma_{\mu} \tilde{\eta} \quad b_{\bar{n}}$$

$$b_p = 2b_u + b_d$$

$$b_n = 2b_d + b_u$$

$$(p = uud + \bar{q}q + G)$$



$$\bar{x} \gamma^m x \quad \bar{\tilde{n}} \gamma_n \tilde{n} \quad b_{\bar{n}}$$

$$b_p = z b_u + b_d \quad (p = u+d + \bar{q}q + G)$$

$$b_n = z b_d + b_u$$

$$b_N = z \cdot b_p + (A-z) b_n$$



$$= \int_0^{2M_N N^2 / M_N} dE_R \cdot \frac{d\sigma_N}{dE_R} (E_R \rightarrow 0)$$

$F(E_R)$ contains momentum ($q^2 = 2M_N E_R$) dependence.

$$\rightarrow F(E_R=0) = 1$$

$$f_p = \left[\sum_{q=u,d,s} \frac{f_{Tq}^{(p)}}{m_q} + \frac{2}{27} \sum_{q=c,b,t} \dots \right]$$

$$N = \left(\frac{M_N}{M_p} \right)^2 \cdot \sigma_p \cdot \frac{[Z f_p + (A-Z) f_n]^2}{f_p^2}$$

\rightarrow cs. for χ -p scattering

\rightarrow coherent scattering off all nucleons.

\rightarrow spin-independent

$$\bar{\chi} \gamma^{\mu} \chi \quad \bar{\tilde{\eta}} \gamma_{\mu} \tilde{\eta} \quad b_{\tilde{\eta}}$$

$$b_p = z b_u + b_d \quad (p = u, d + \bar{q} q + G)$$

$$b_n = z b_d + b_u$$

VV is also SI

$$b_N = z \cdot b_p + (A - z) b_n$$

$$\sigma_{SI} = \left(\frac{M_P}{M_N} \right)^2 \frac{1}{A^2} \tilde{\sigma}_N$$

o SI

$$\bar{\sigma}_N = \int_0^{2M_N N^2 / m_N} dE_R \frac{d\sigma_i}{dE_R}(E_R \rightarrow 0)$$

$F(E_R)$ contains momentum ($q^2 = 2m_N E_R$) dependence.

$$\hookrightarrow F(E_R=0) = 1$$

$$\bar{\sigma}_N = \left(\frac{m_N}{m_p}\right)^2 \cdot \sigma_p^{\text{SI}} \frac{[Z f_p + (A-Z) f_n]^2}{f_p^2}$$

\hookrightarrow cs for X-p scattering

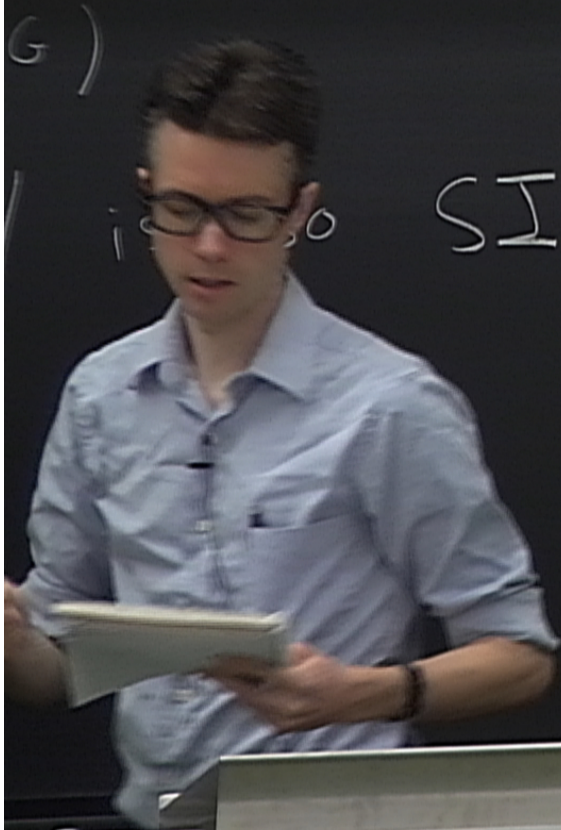
$$f_p = \left[\sum_{q=u,d,s} \frac{f_{T_q}^{(p)}}{m_q} + \frac{2}{27} \sum_{Q=c,t,b} \frac{f_{T_Q}^{(p)}}{m_Q} \right] m_p$$

\rightarrow coherent scattering off all nucleons.

\hookrightarrow spin-independent (SI)

$$\sigma_{SI} = \left(\frac{\mu_p}{\mu_n}\right)^2 \frac{1}{A^2} \tilde{\sigma}_N$$

↳ experiments quote limits on this.



DM DD

AA: $\underbrace{\bar{\chi} \gamma^m \gamma^5 \chi}_{\neq 0 \text{ for Majorana}} \bar{q} \gamma_m \gamma^5 q \cdot d_q$

$\neq 0$ for Majorana

$$\langle \tilde{n} | \bar{q} \gamma_m \gamma^5 q | \tilde{n} \rangle = 2 S_m^{(\tilde{n})} \Delta q^{(\tilde{n})}$$

// nucleon spin = get from data

pol stuff
(...)

DM DD

$$AA: \underbrace{\bar{\chi} \gamma^{\mu} \gamma^5 \chi}_{\neq 0 \text{ for Majorana}} \bar{q} \gamma_{\mu} \gamma^5 q \cdot d_q$$

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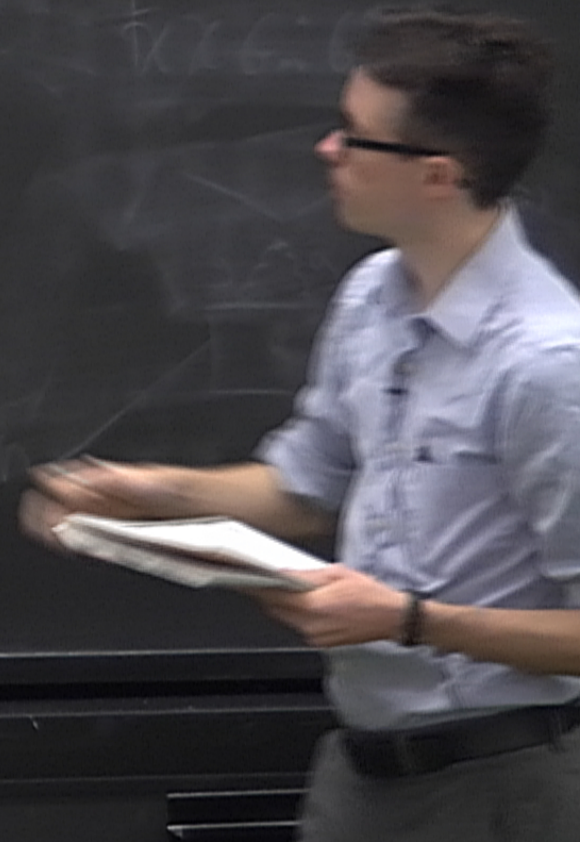
$$\langle \tilde{n} | \bar{q} \gamma_{\mu} \gamma^5 q | \tilde{n} \rangle = 2 S_M^{(\tilde{n})} \Delta q^{(\tilde{n})}$$

// nucleon spin = get from data (pol. stuff)

- \mathcal{L}_{eff} -



$$\begin{aligned}
 -\mathcal{L}_{\text{eff}} &= \bar{\chi} \gamma^{\mu} \gamma_5 \chi \left(\sum_{q=u,d,s} 2d_q \Delta q^{(\tilde{n})} \right) \bar{\tilde{n}} S_{\mu} \tilde{n} \\
 &\quad \underbrace{\hspace{10em}}_{2\sqrt{2} G_F a_{\tilde{n}}}
 \end{aligned}$$



DM DD

$$AA: \underbrace{\bar{\chi} \gamma^m \gamma^5 \chi}_{\neq 0 \text{ for Majorana}} \bar{q} \gamma_m \gamma^5 q \cdot d_q$$

$\neq 0$ for Majorana

$$\langle \tilde{n} | \bar{q} \gamma_m \gamma^5 q | \tilde{n} \rangle = 2 S_m^{(\tilde{n})} \Delta g^{(\tilde{n})}$$

// nucleon spin = get from data

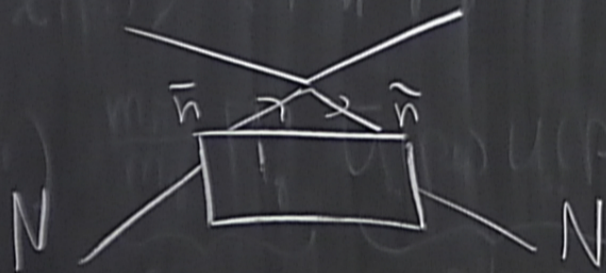
pol. stuff

(...)



$$- \mathcal{L}_{\text{eff}} = \bar{\chi} \gamma^{\mu} \gamma_5 \chi \left(\sum_{q=u,d,s} 2d_q \Delta q^{(\tilde{n})} \right) \bar{\tilde{n}} S_{\mu} \tilde{n}$$

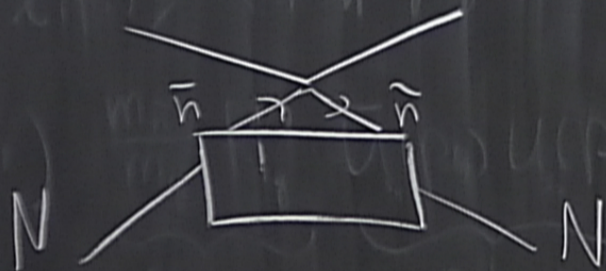
$$2\sqrt{2} G_F a_{\tilde{n}}$$



$$\langle N | \bar{\tilde{n}} S_{\mu}^{(\tilde{n})} \tilde{n} | N \rangle$$

$$- \mathcal{L}_{\text{eff}} = \bar{\chi} \gamma^{\mu} \gamma_5 \chi \left(\sum_{q=u,d,s} 2d_q \Delta q^{(\tilde{n})} \right) \bar{\tilde{n}} S_{\mu} \tilde{n}$$

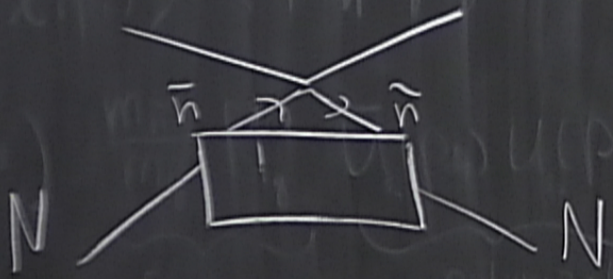
$$2\sqrt{2} G_F a_{\tilde{n}}$$



$$\langle N | \underbrace{\bar{\tilde{n}} S_{\mu} \tilde{n}}_{q^2=0} | N \rangle =$$

$$- \mathcal{L}_{\text{eff}} = \bar{\chi} \gamma^{\mu} \gamma_5 \chi \left(\sum_{q=u,d,s} 2d_q \Delta q^{(\tilde{n})} \right) \bar{\tilde{n}} S_{\mu} \tilde{n}$$

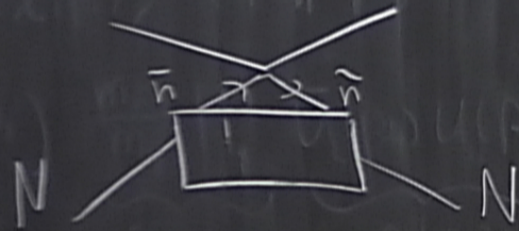
$$2\sqrt{2} G_F a_{\tilde{n}}$$



$$\langle N | \underbrace{\bar{\tilde{n}} S_{\mu}^{(\tilde{n})} \tilde{n}}_{q^2=0} | N \rangle = \langle S_{\tilde{n}}^{\mu} \rangle$$

$$- \mathcal{L}_{\text{eff}} = \bar{\chi} \gamma^\mu (1 - \gamma_5) \chi \underbrace{\left(\sum_{q=u,d,s} 2d_q \Delta q^{(\bar{n})} \right)}_{2\sqrt{2} G_F a_{\bar{n}}} \bar{\bar{n}} s_\mu \bar{\bar{n}}$$

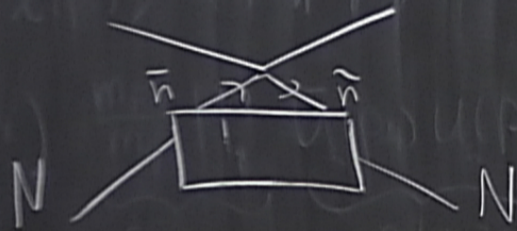
dropped polarization stuff



$$\langle N | \underbrace{\bar{\bar{n}} s_\mu^{(\bar{n})} \bar{\bar{n}}}_{q^2=0} | N \rangle = \langle S_{\bar{n}}^\mu \rangle_N = \text{spin of the nucleus from nucleon type } \bar{n}$$

$$- \mathcal{L}_{\text{eff}} = \bar{\chi} \gamma^\mu (1 - \gamma_5) \chi \underbrace{\left(\sum_{q=u,d,s} 2d_q \Delta q^{(\bar{n})} \right)}_{2\sqrt{2} G_F a_{\bar{n}}} \bar{n} s_\mu \tilde{n}$$

dropped polarization stuff



$$\langle N | \underbrace{\bar{n} s_\mu^{(\bar{n})} \tilde{n}}_{q^2=0} | N \rangle = \langle S_{\bar{n}}^\mu \rangle_N = \text{spin of the nucleus from nucleon type } \bar{n}$$

$$\int_{i/s} \gamma \gamma \gamma \gamma \gamma \rightarrow g$$

$$a_p \langle S_p^m \rangle + a_n \langle S_n^m \rangle = \Lambda_N \langle N | S^m | N \rangle J$$

"Nucleus spin operator"

$$-iM = \langle \chi_3 | \bar{\chi} \gamma^m \gamma^5 \chi | \chi_1 \rangle \langle N_1 | S^m | N_2 \rangle (2\sqrt{2} G_F) \Lambda_N$$

