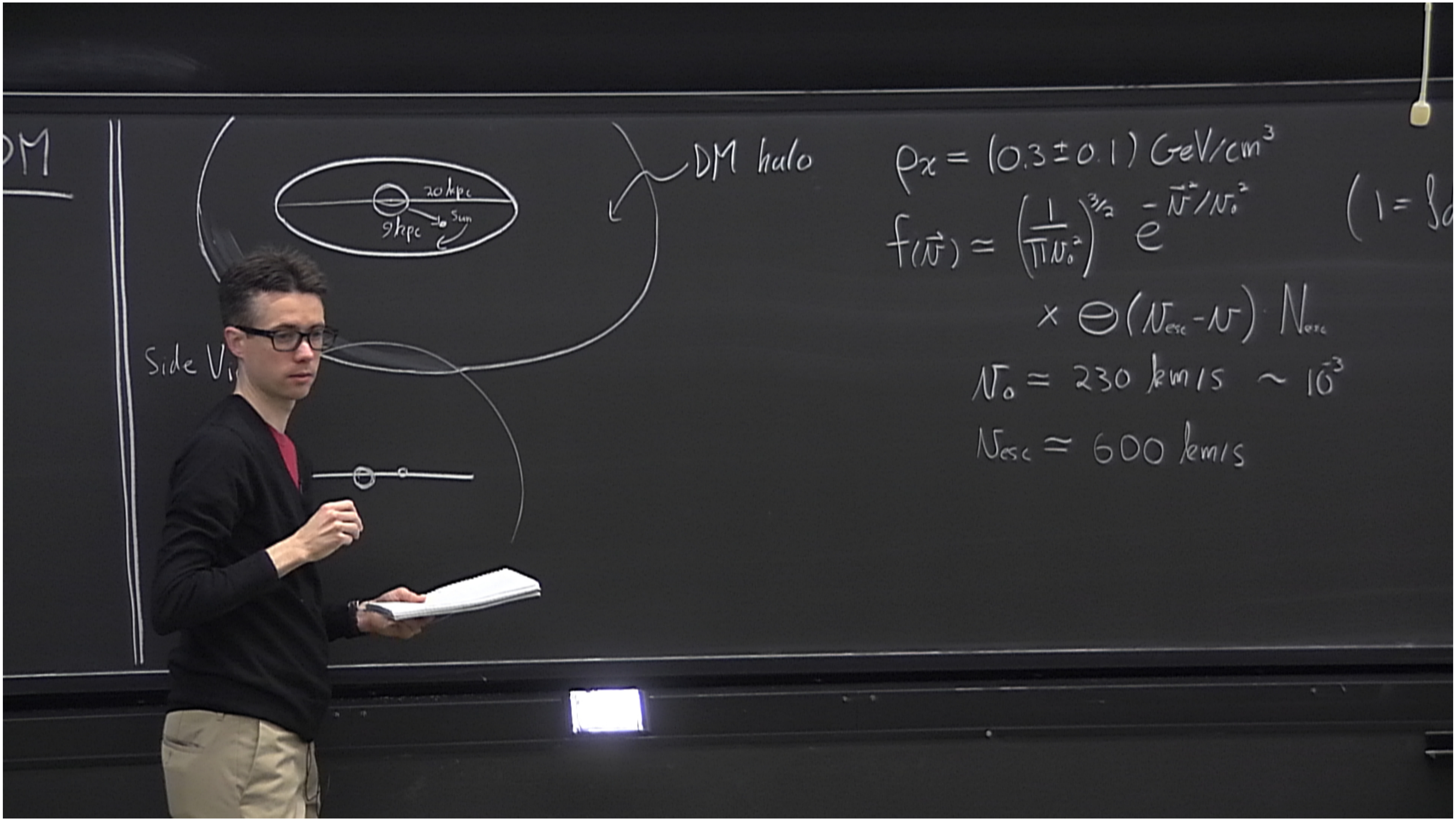
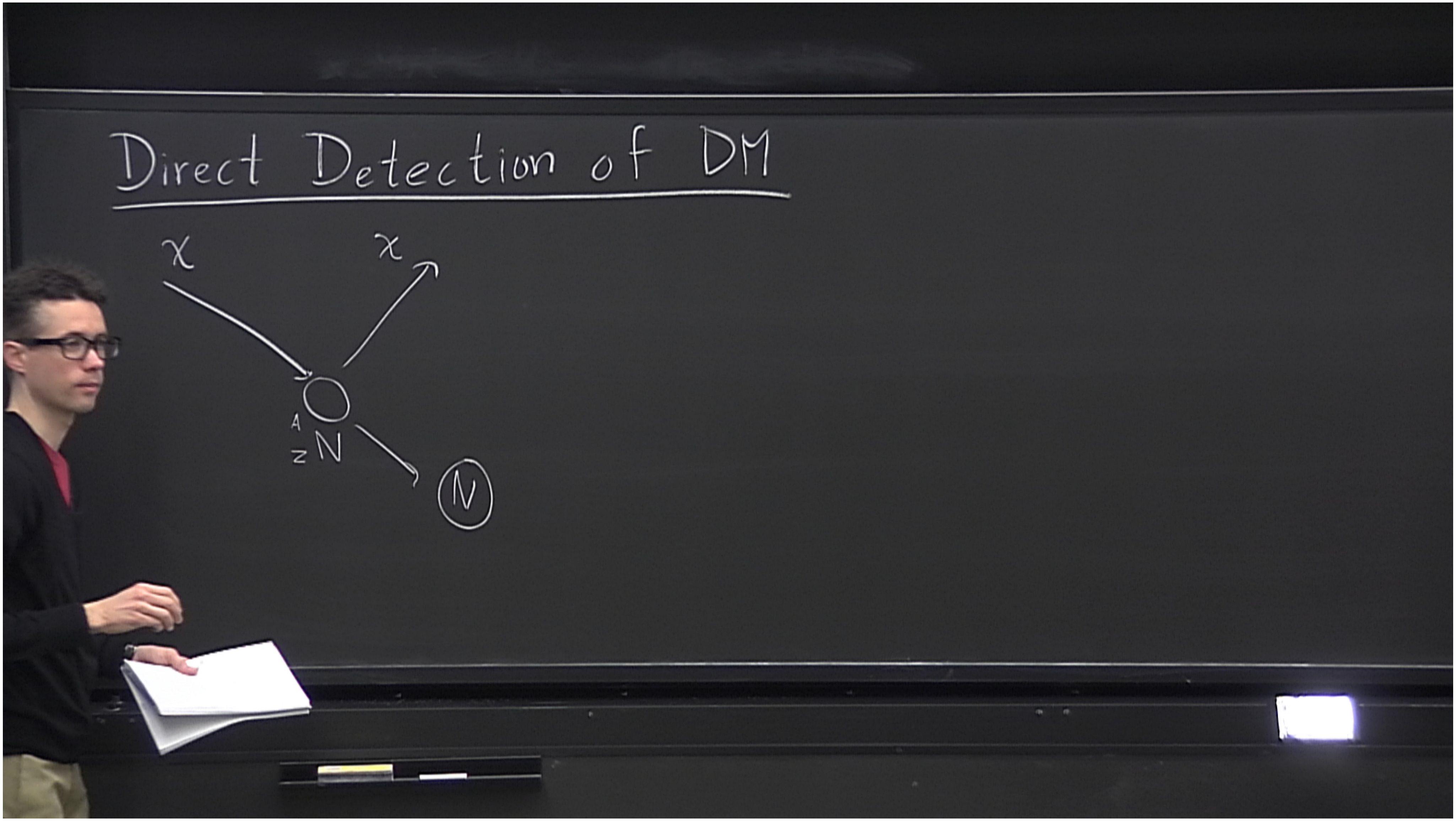
Title: Explorations in Particle Theory - Lecture 7

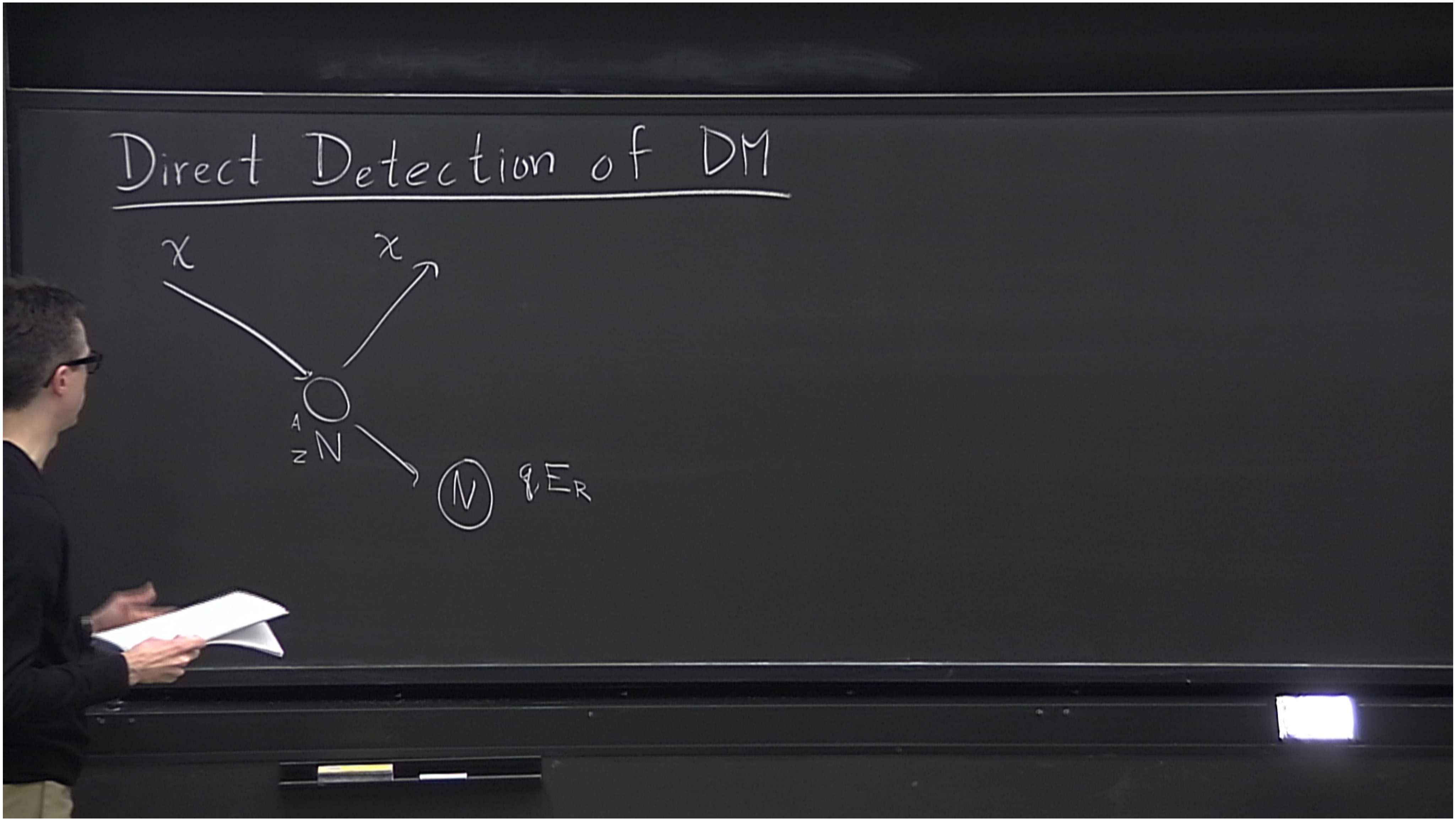
Date: Apr 11, 2012 09:00 AM

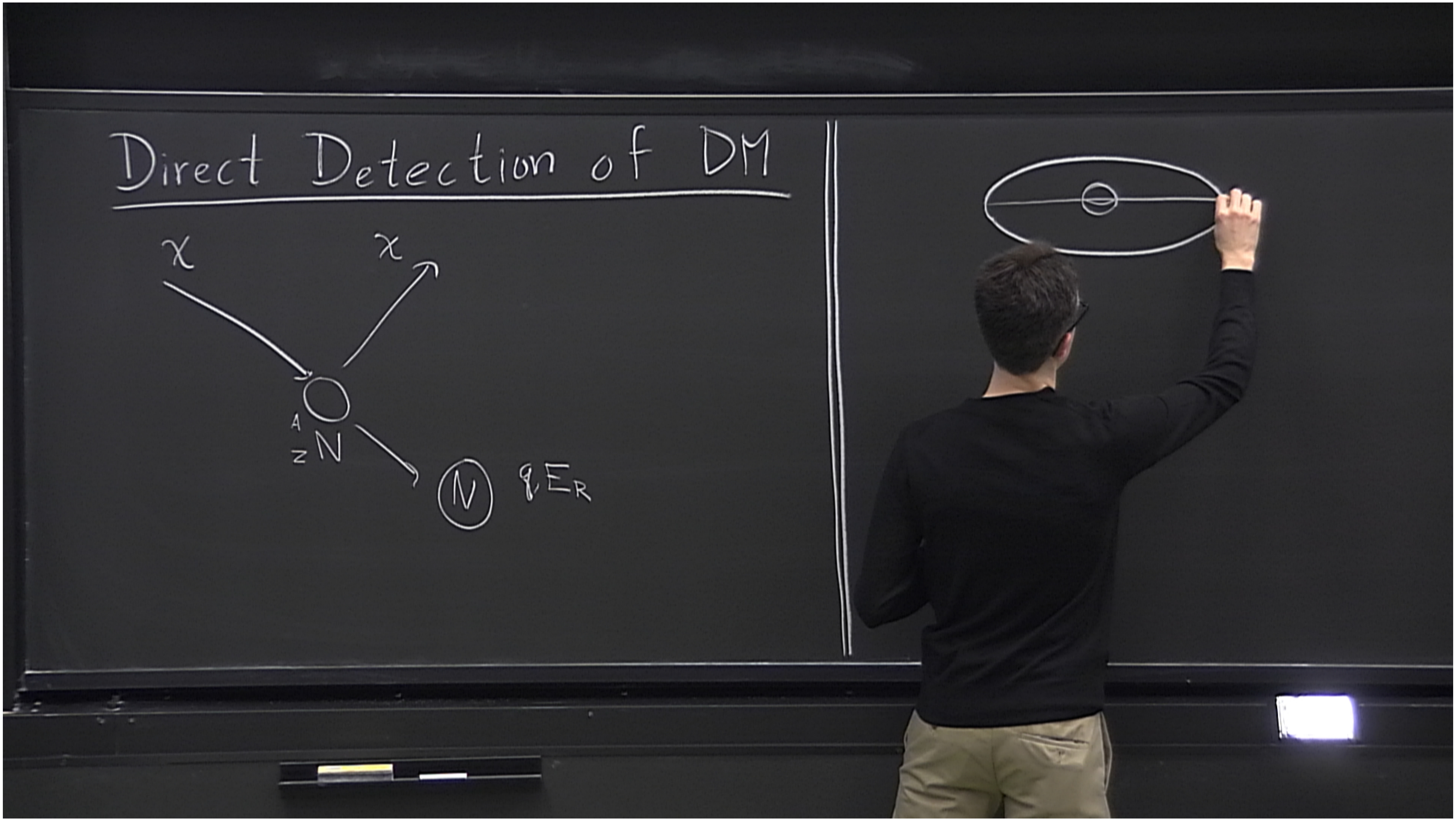
URL: http://pirsa.org/12040009

Abstract:


DM
20 kpc
9 kpc
Sun
DM halo
Side Vi
$\rho_\chi = (0.3 \pm 0.1)\ \mathrm{GeV/cm^3}$
$f(\vec{v}) \simeq \left(\frac{1}{\pi v_0^2}\right)^{3/2} e^{-\vec{v}^2/v_0^2}$
$\times\ \Theta(v_{esc} - v) \cdot N_{esc}$
$v_0 = 230\ \mathrm{km/s} \sim 10^{-3}$
$v_{esc} \simeq 600\ \mathrm{km/s}$


Direct Detection of DM
χ
χ
A
Z N
N


Direct Detection of DM
χ
χ
$^{A}_{Z}N$
N
q E_R


Direct Detection of DM
χ
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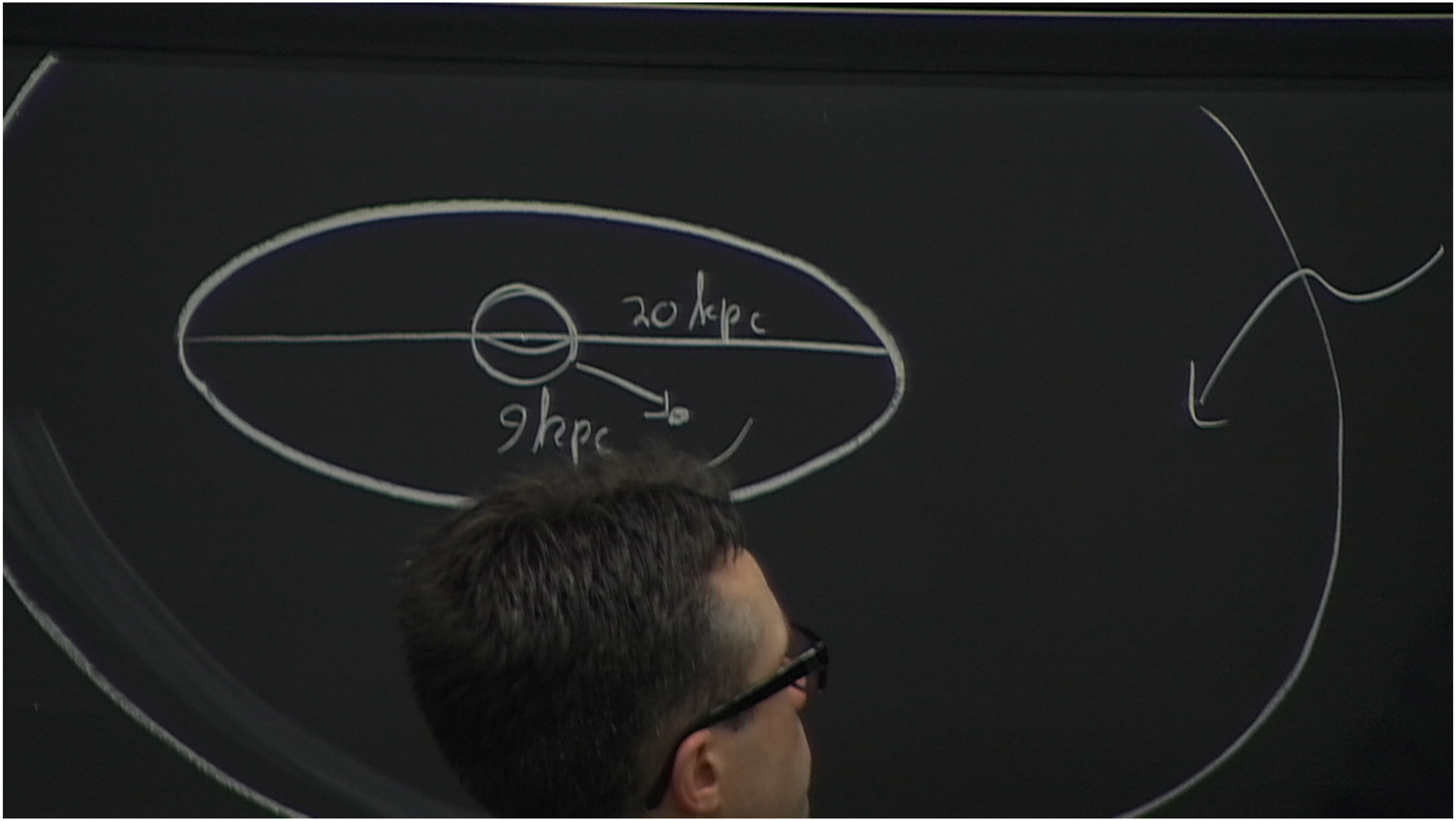
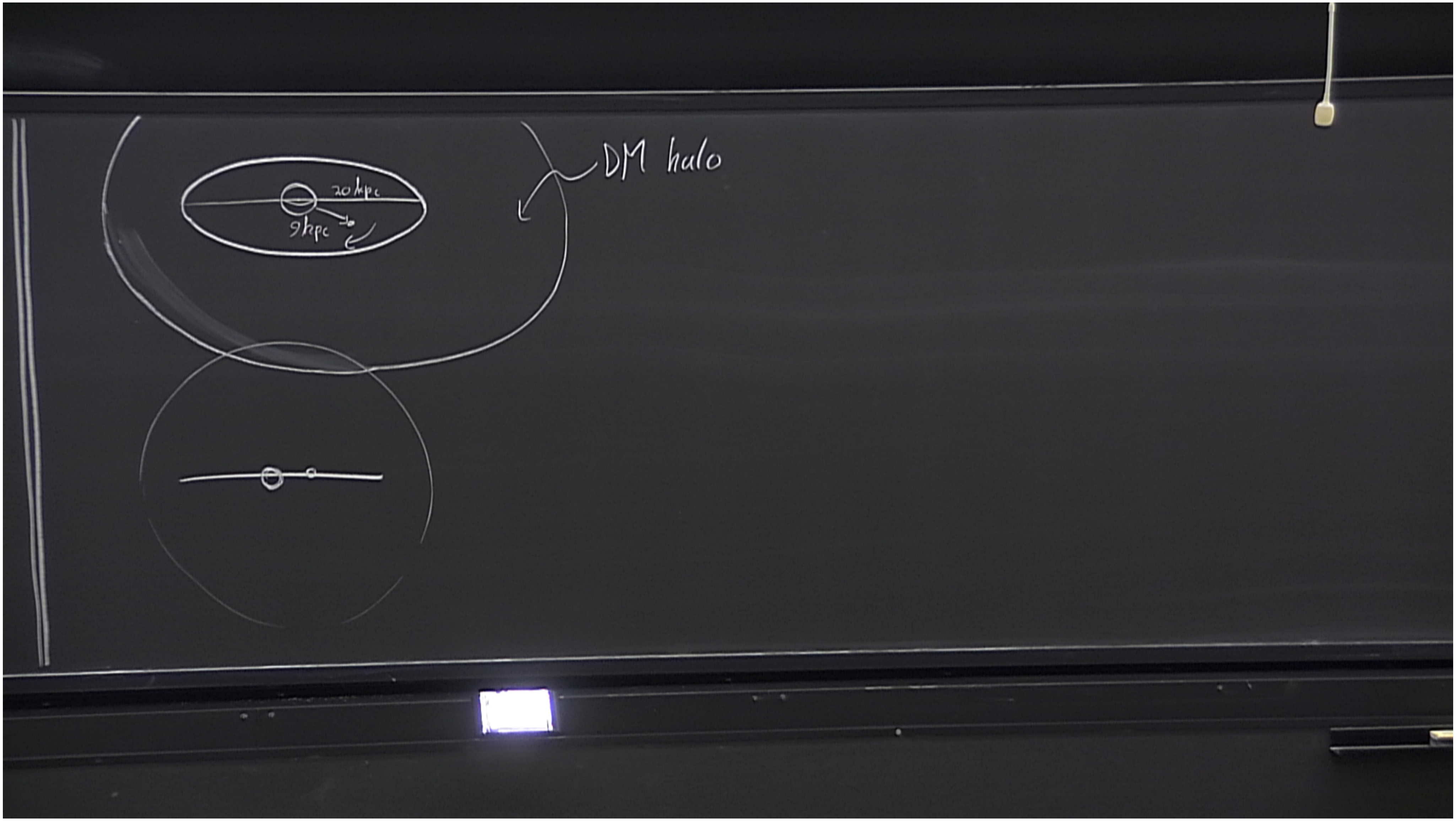


20 kpc
9 kpc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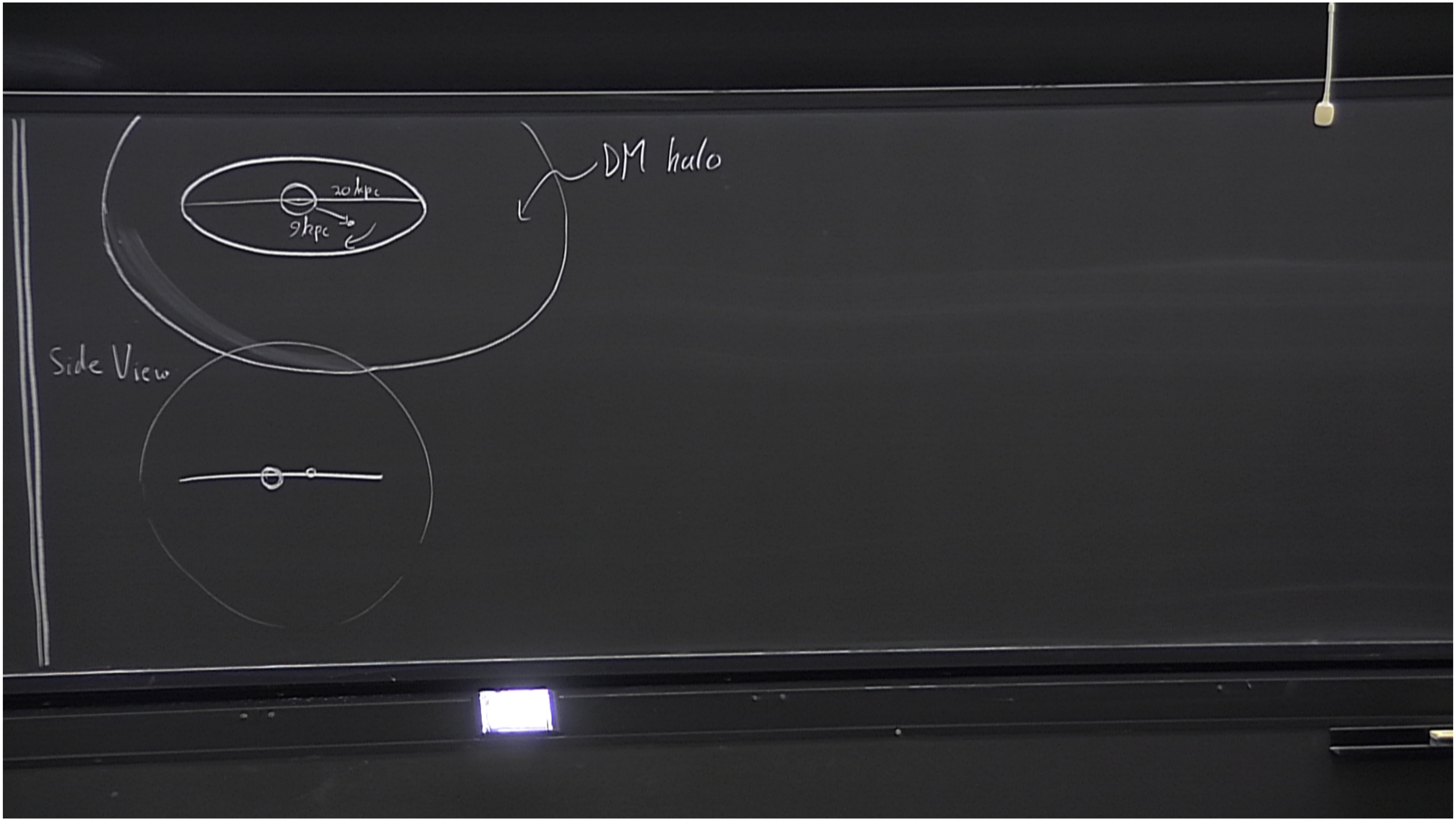
DM halo
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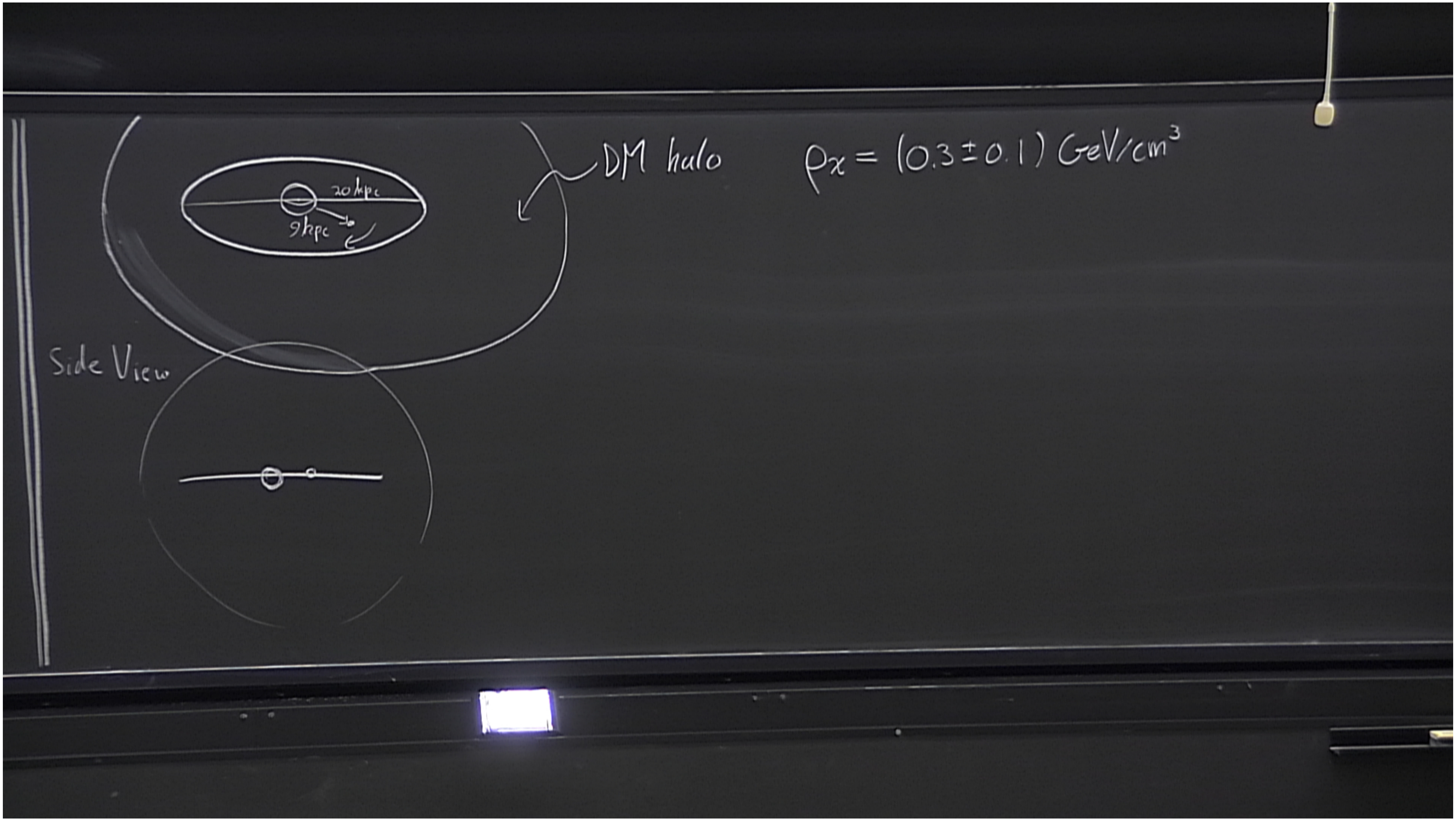
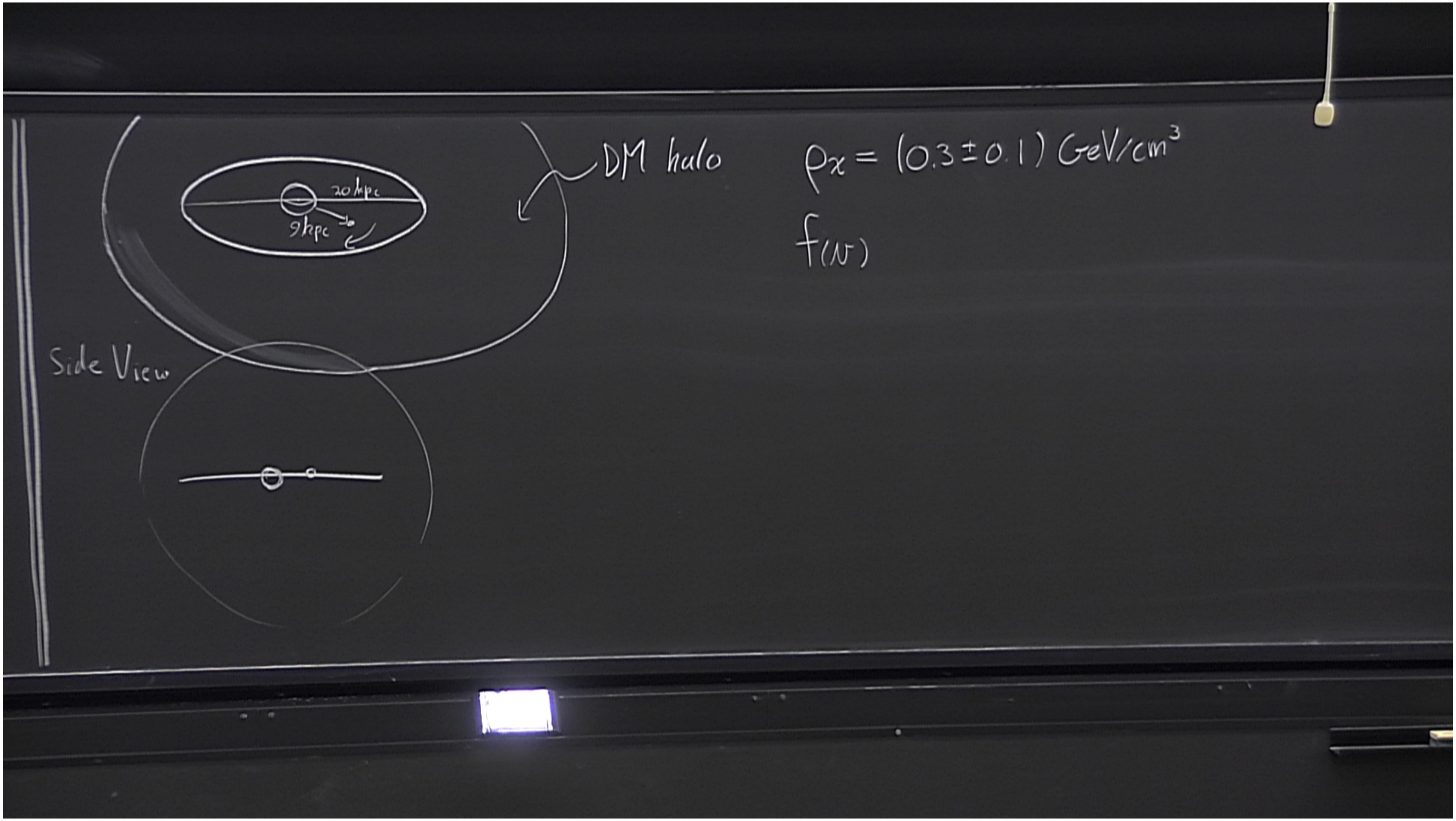
9 kpc
Side View


DM halo
$\rho_\chi = (0.3 \pm 0.1)$ GeV/cm^3
20 kpc
9 kpc
Side View


DM halo
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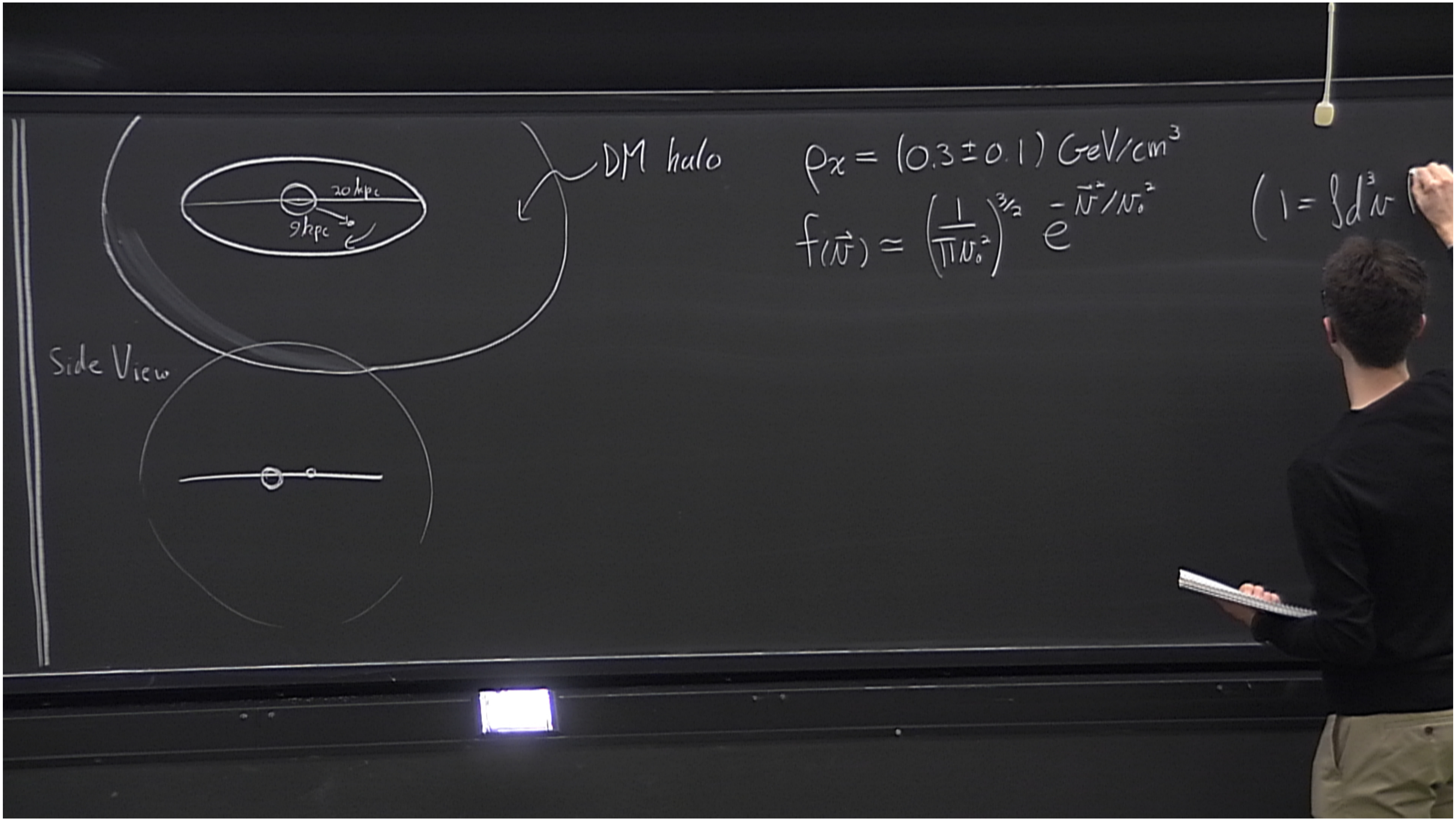
9 kpc
Side View
$\rho_\chi = (0.3 \pm 0.1)\ \mathrm{GeV/cm^3}$
$f(v)$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$f(\vec{v}) \simeq \left(\frac{1}{\pi v_0^2}\right)^{3/2} e^{-\vec{v}^2/v_0^2}$
$(1 = \int d^3v$

halo

$$\rho_\chi = (0.3 \pm 0.1)\ \mathrm{GeV/cm^3}$$

$$f(\vec{v}) \simeq \left(\frac{1}{\pi v_0^2}\right)^{3/2} e^{-\vec{v}^2/v_0^2} \qquad \left(1 = \int d^3v\, f(\vec{v})\right)$$

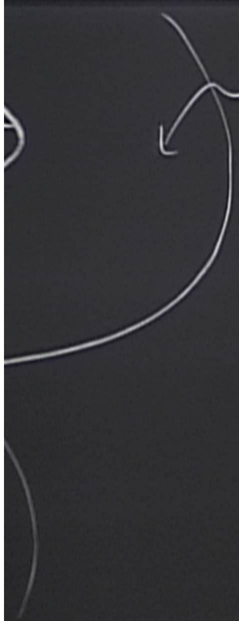
$$\times\ \Theta(v_{esc} - v)$$

halo

$$\rho_\chi = (0.3 \pm 0.1)\ \mathrm{GeV/cm^3}$$

$$f(\vec{v}) \simeq \left(\frac{1}{\pi v_0^2}\right)^{3/2} e^{-\vec{v}^2/v_0^2} \qquad \left(1 = \int d^3v\, f(\vec{v})\right)$$

$$\times\ \Theta(v_{esc} - v) \cdot N_{esc}$$



DM halo

$$\rho_\chi = (0.3 \pm 0.1)\ \mathrm{GeV/cm^3}$$

$$f(\vec{v}) \simeq \left(\frac{1}{\pi v_0^2}\right)^{3/2} e^{-\vec{v}^2/v_0^2} \qquad \left(1 = \int d^3v\, f(\vec{v})\right)$$

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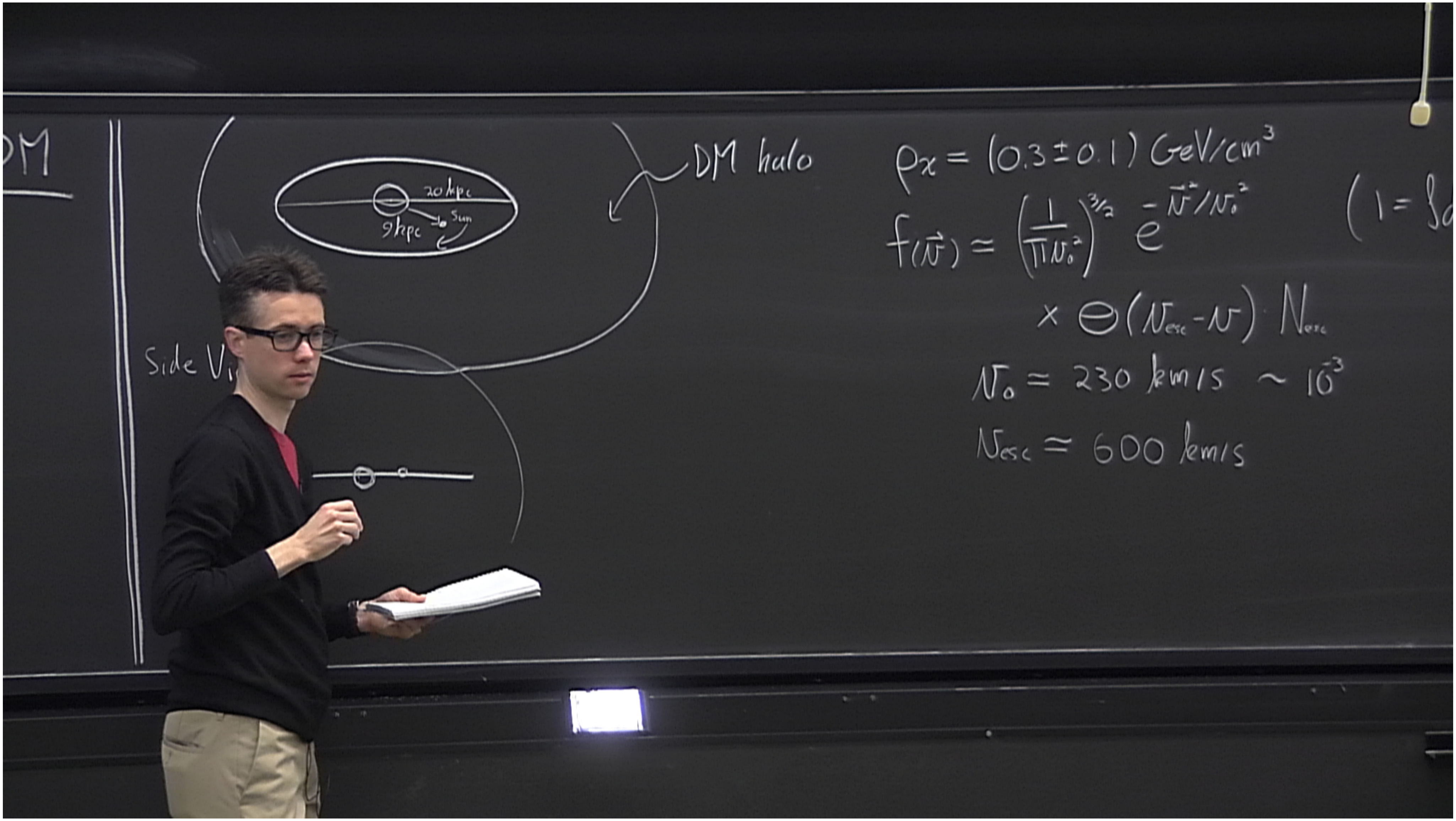
DM halo
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9 kpc
Sun
$\rho_\chi = (0.3 \pm 0.1)\ \mathrm{GeV/cm^3}$
$f(\vec{v}) \simeq \left(\frac{1}{\pi v_0^2}\right)^{3/2} e^{-\vec{v}^2/v_0^2}$
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$v_0 = 230\ \mathrm{km/s} \sim 10^{-3}$
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DM halo

$$\rho_\chi = (0.3 \pm 0.1)\ \mathrm{GeV/cm^3}$$

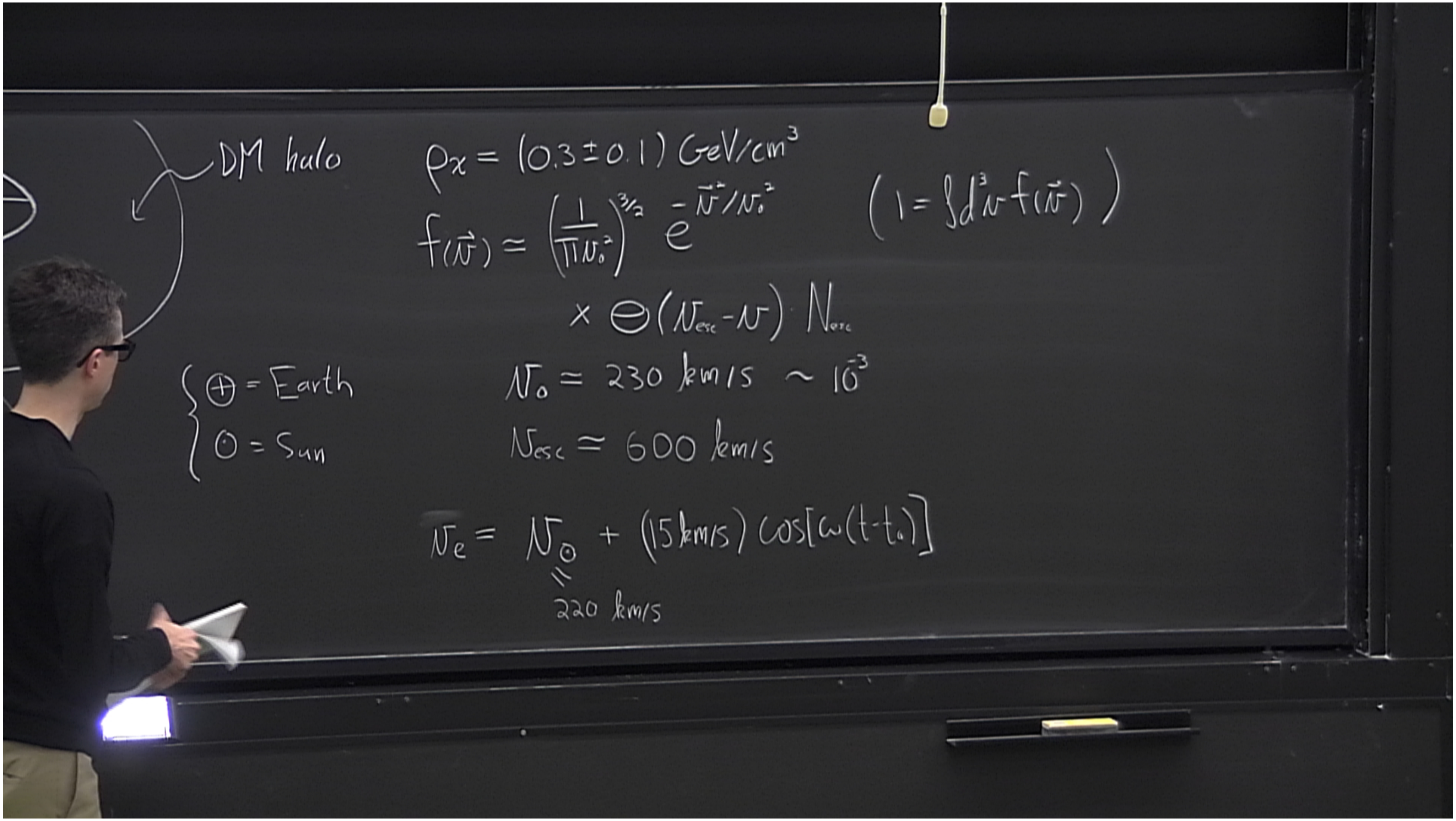
$$f(\vec{v}) \simeq \left(\frac{1}{\pi v_0^2}\right)^{3/2} e^{-\vec{v}^2/v_0^2} \qquad \left(1 = \int d^3v\, f(\vec{v})\right)$$

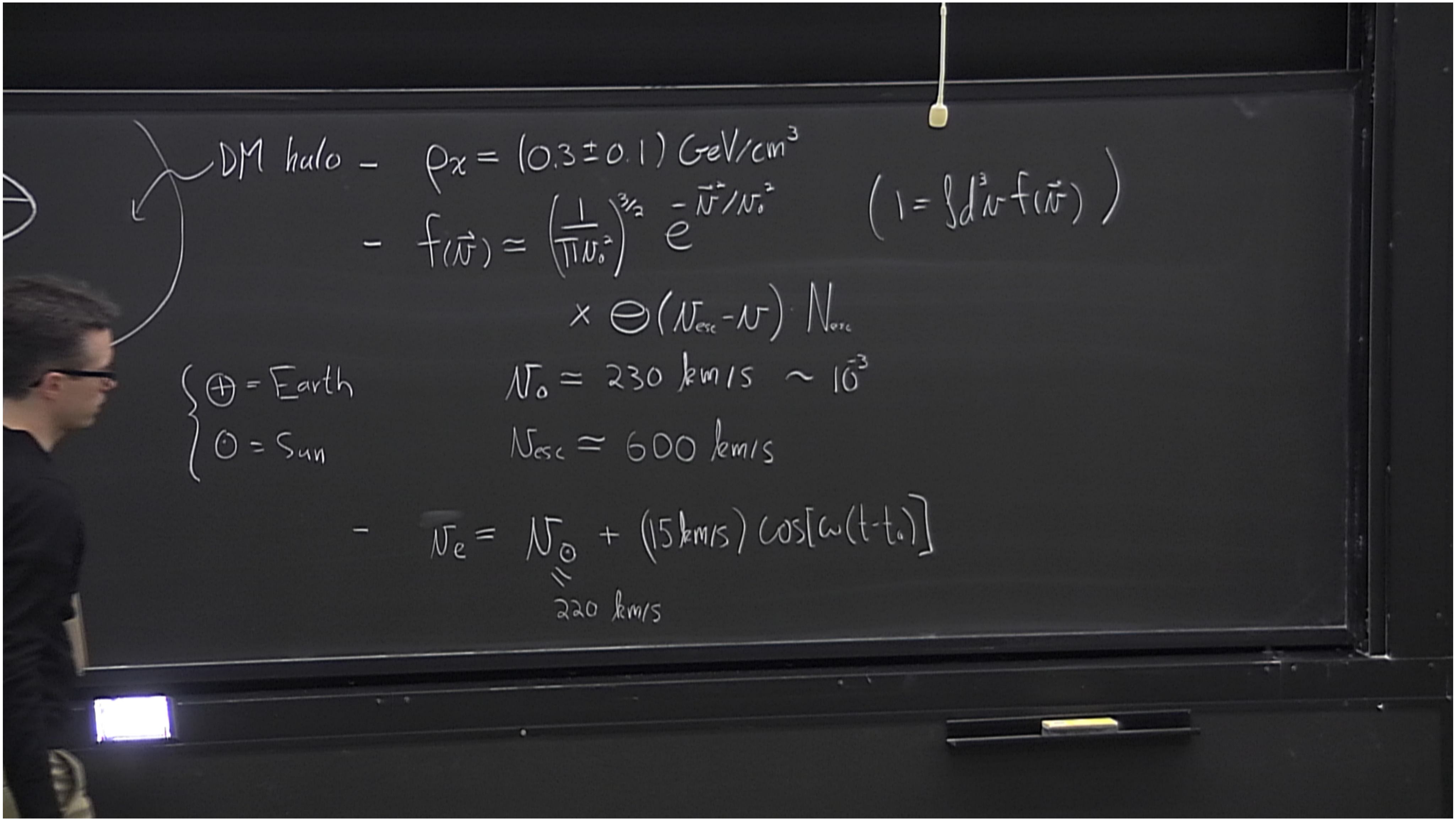
$$\times\ \Theta(v_{esc} - v)\cdot N_{esc}$$

$$v_0 \simeq 230\ \mathrm{km/s} \sim 10^{-3}$$

$$v_{esc} \simeq 600\ \mathrm{km/s}$$

$$v_e = \underset{\substack{\shortparallel \\ 220\ \mathrm{km/s}}}{v_\odot} + (15\ \mathrm{km/s})\cos[\omega(t-t_0)]$$


DM halo
$\rho_\chi = (0.3 \pm 0.1)\ \mathrm{GeV/cm^3}$
$f(\vec{v}) \simeq \left(\frac{1}{\pi v_0^2}\right)^{3/2} e^{-\vec{v}^2/v_0^2}$
$\left(1 = \int d^3v\, f(\vec{v})\right)$
$\times\ \Theta(v_{esc} - v) \cdot N_{esc}$
$\oplus$ = Earth
$\odot$ = Sun
$v_0 \simeq 230$ km/s $\sim 10^{-3}$
$v_{esc} \simeq 600$ km/s
$v_e = v_\odot + (15\,\mathrm{km/s}) \cos[\omega(t - t_0)]$
$\simeq$ 220 km/s


DM halo – $\rho_\chi = (0.3 \pm 0.1)\ \mathrm{GeV/cm^3}$
– $f(\vec{v}) \simeq \left(\frac{1}{\pi v_0^2}\right)^{3/2} e^{-\vec{v}^2/v_0^2}$
$\left(1 = \int d^3v\, f(\vec{v})\right)$
$\times\, \Theta(v_{esc} - v) \cdot N_{esc}$
$\oplus$ = Earth
$\odot$ = Sun
$v_0 \simeq 230\ \mathrm{km/s} \sim 10^{-3}$
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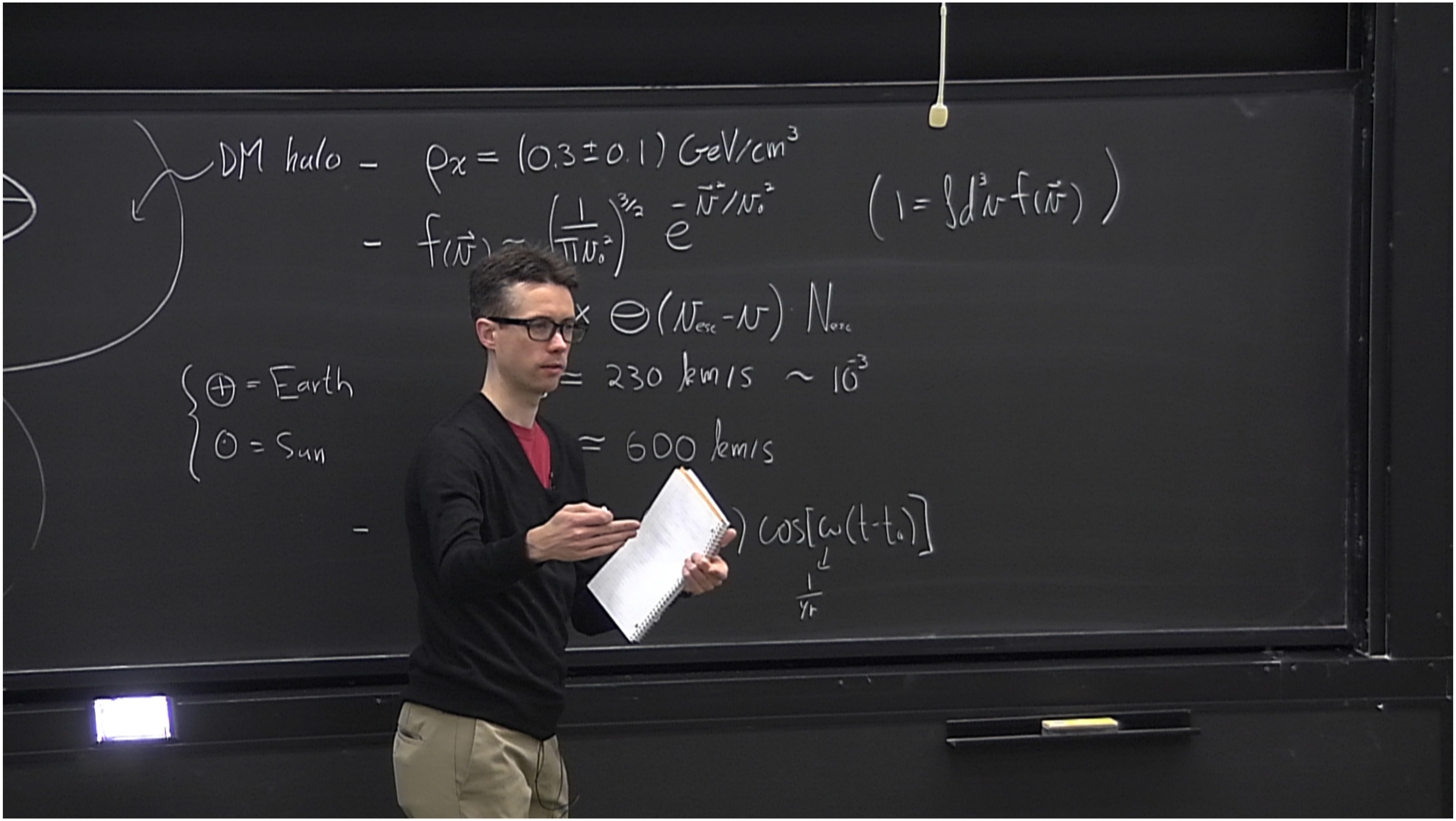
– $v_e = v_\odot + (15\ \mathrm{km/s}) \cos[\omega(t - t_0)]$
$220\ \mathrm{km/s}$


DM halo – $\rho_\chi = (0.3 \pm 0.1)\ \mathrm{GeV/cm^3}$
$(1 = \int d^3v\, f(\vec{v}))$
$\oplus$ = Earth
$\odot$ = Sun
$\simeq 230$ km/s $\sim 10^{-3}$
$\simeq 600$ km/s
$\cos[\omega(t-t_0)]$

DM halo – $\rho_\chi = (0.3 \pm 0.1)\ \mathrm{GeV/cm^3}$

– $f(\vec{v}) \simeq \left(\frac{1}{\pi v_0^2}\right)^{3/2} e^{-\vec{v}^2/v_0^2}$ $\qquad \left(1 = \int d^3v\, f(\vec{v})\right)$

$$\times\ \Theta(v_{esc} - v) \cdot N_{esc}$$

$\left\{\begin{array}{l} \oplus = \text{Earth} \\ \odot = \text{Sun} \end{array}\right.$

$v_0 \simeq 230\ \mathrm{km/s} \sim 10^{-3}$

$v_{esc} \simeq 600\ \mathrm{km/s}$

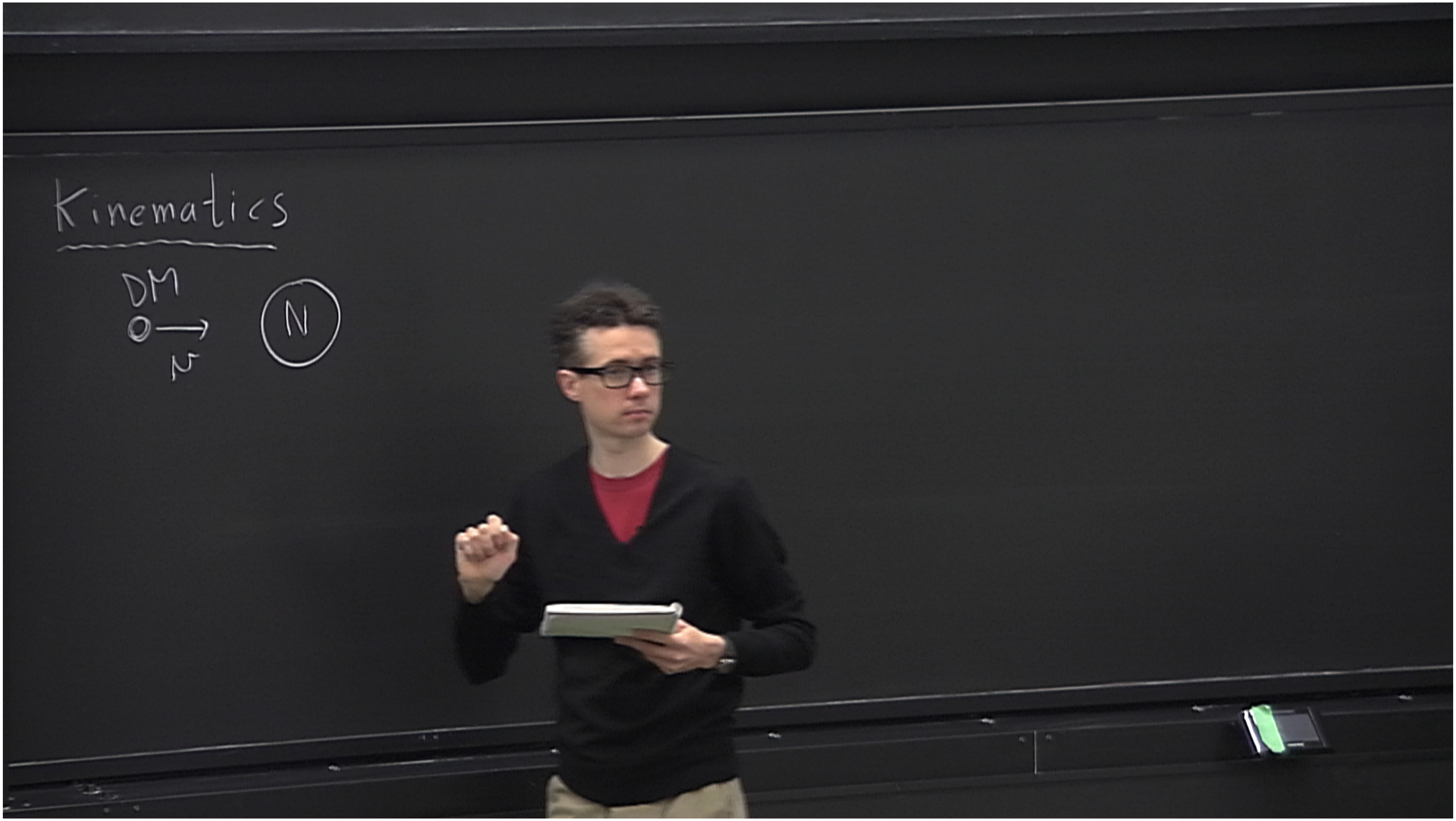
– $v_e = \underbrace{v_\odot}_{220\ \mathrm{km/s}} + (15\ \mathrm{km/s}) \cos[\underbrace{\omega}_{\frac{1}{yr}}(t - \underbrace{t_0}_{\text{June 2}})]$

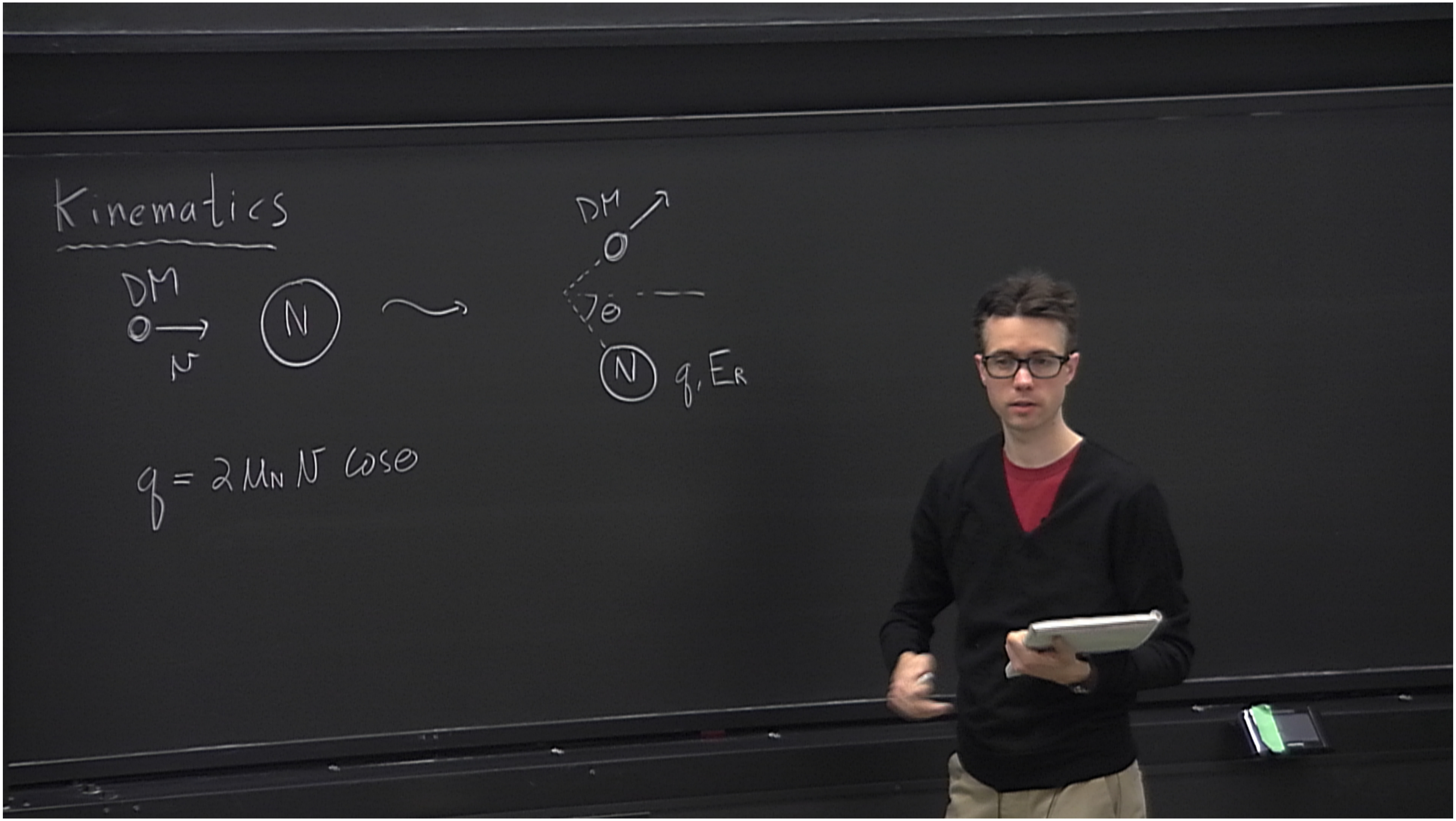
$$\times\ \Theta(v_{esc} - v) \cdot N_{esc}$$

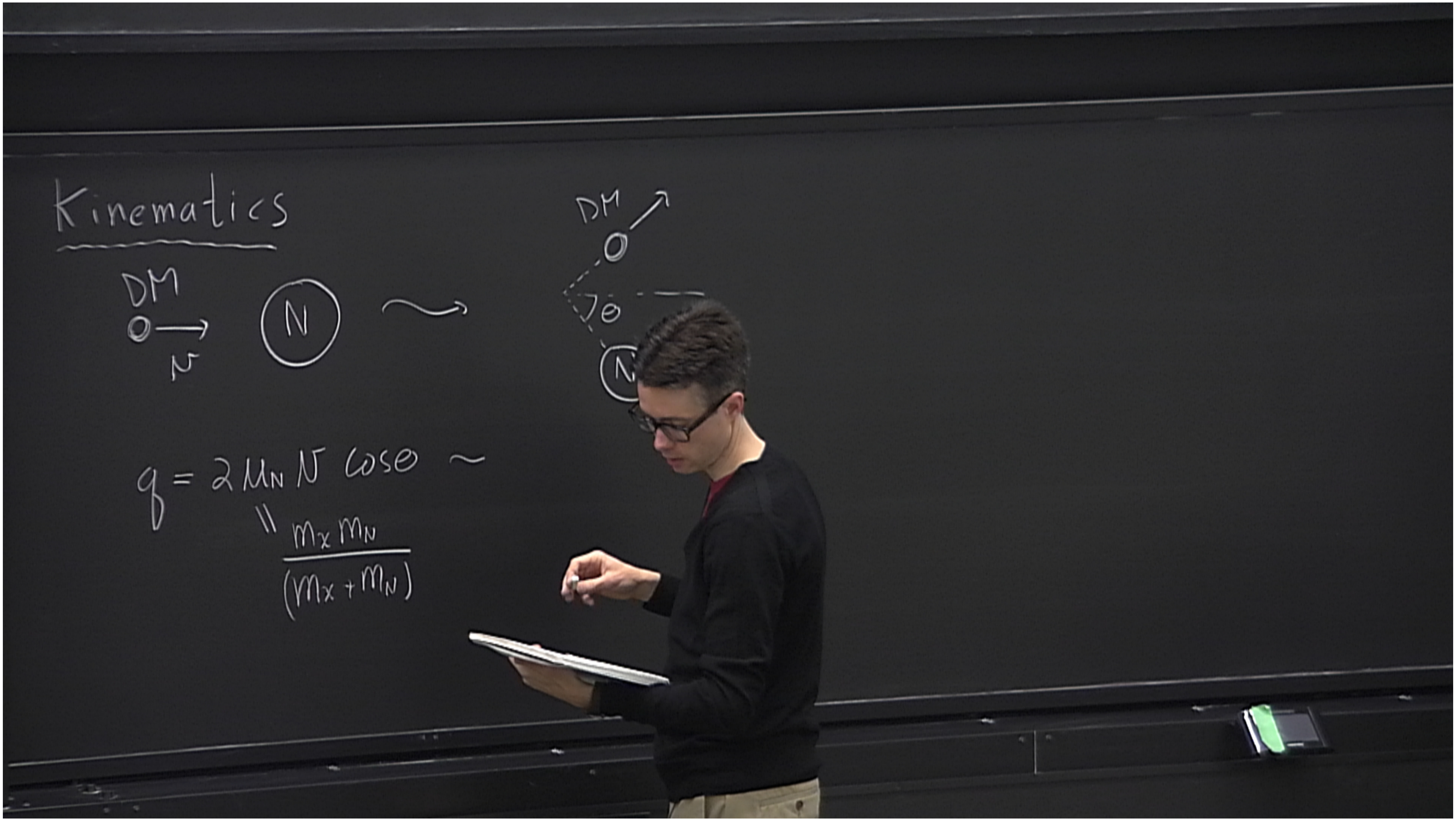
$$v_0 \simeq 230\ \text{km/s} \sim 10^{-3}$$

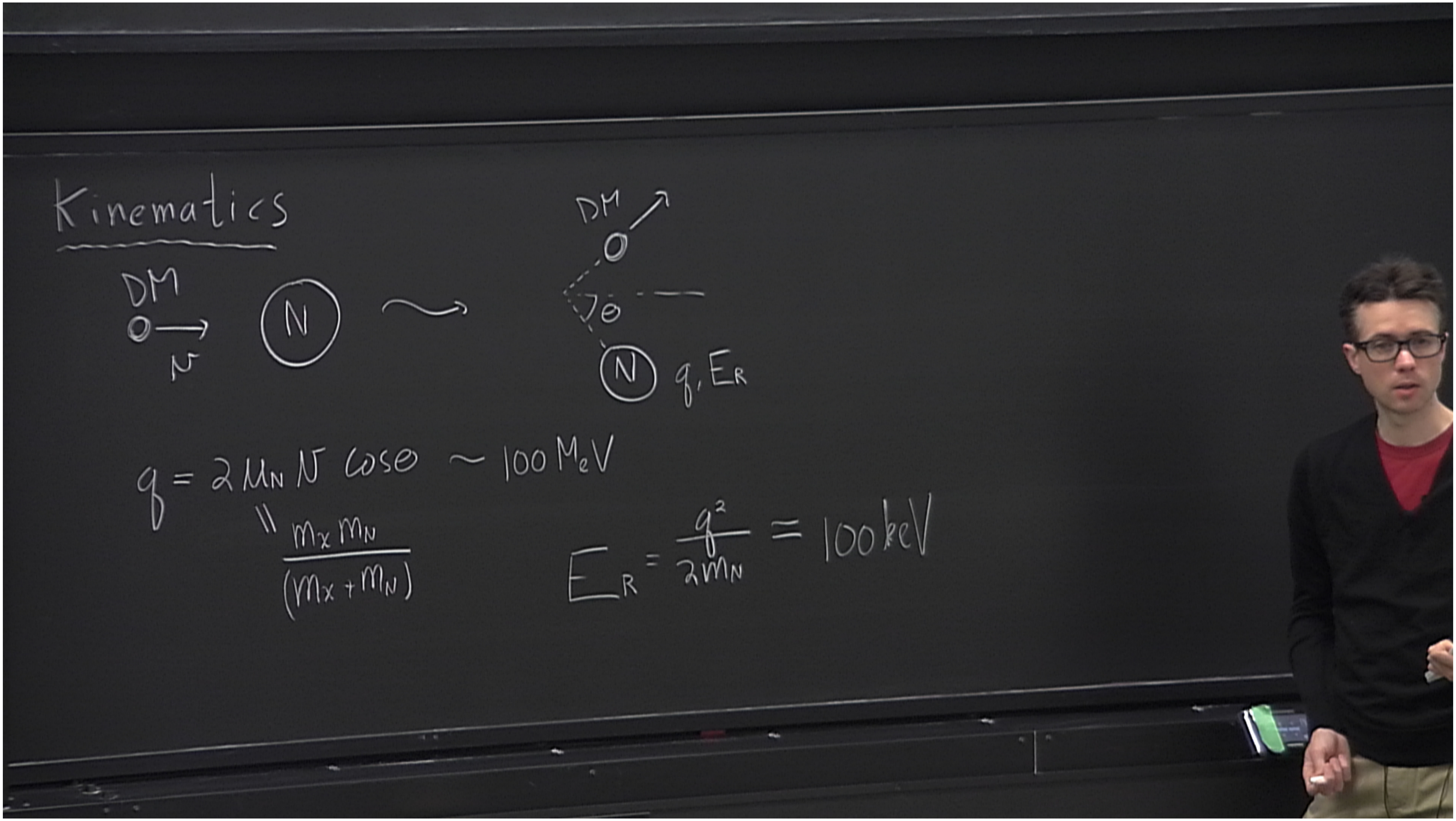
$$v_{esc} \simeq 600\ \text{km/s}$$

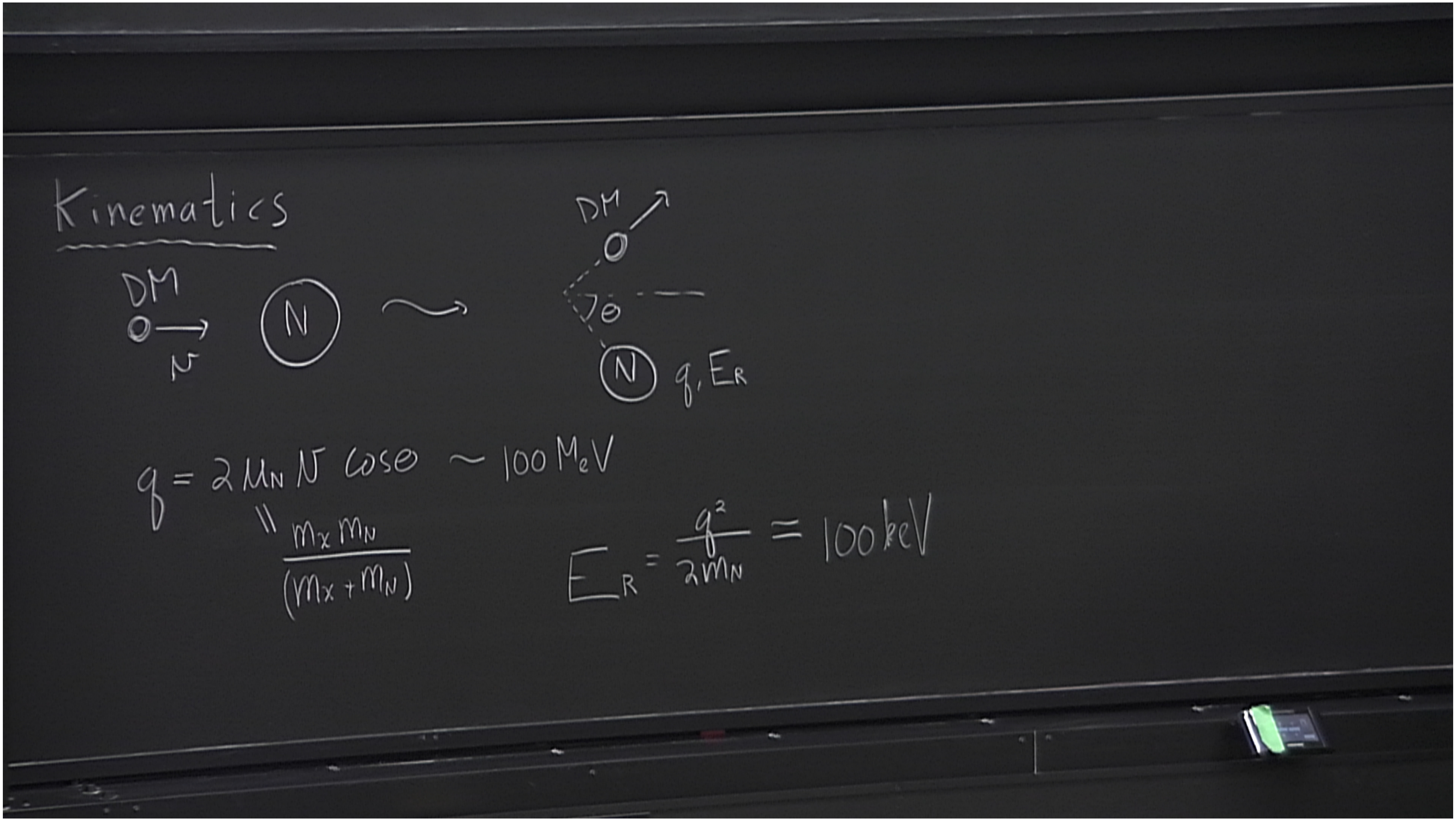
$$v_e = \underbrace{v_{\odot}}_{=\,220\ \text{km/s}} + (15\ \text{km/s}) \cos[\underbrace{\omega}_{\frac{1}{yr}}(t - \underbrace{t_0}_{\to\ \text{June } 2.})]$$

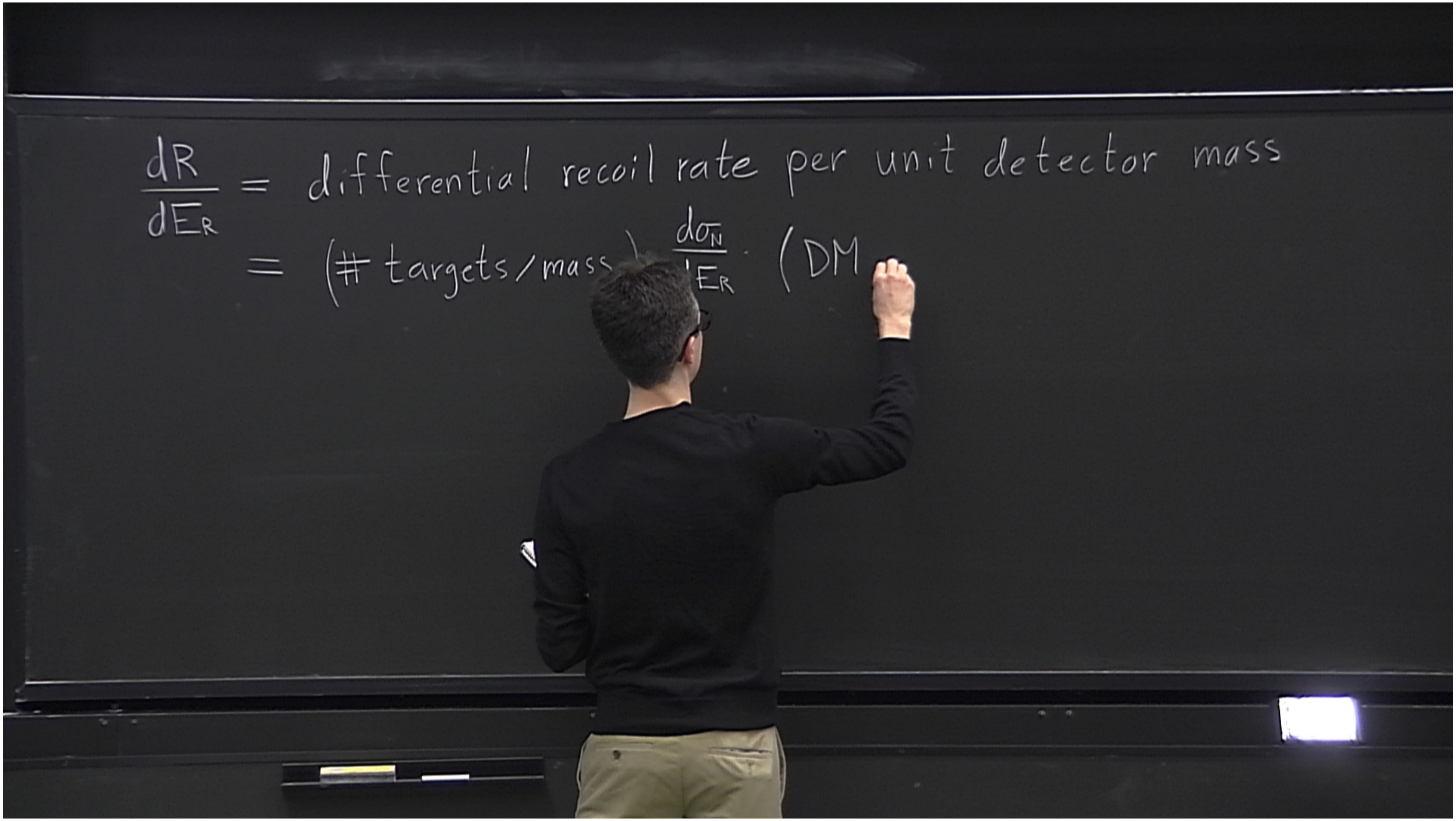

Kinematics
DM
N


Kinematics
DM
N
v
DM
θ
N
q, E_R
q = 2 μ_N v cos θ


Kinematics
DM
v
N
DM
θ
N
q = 2μN v cosθ ~
mχ mN / (mχ + mN)

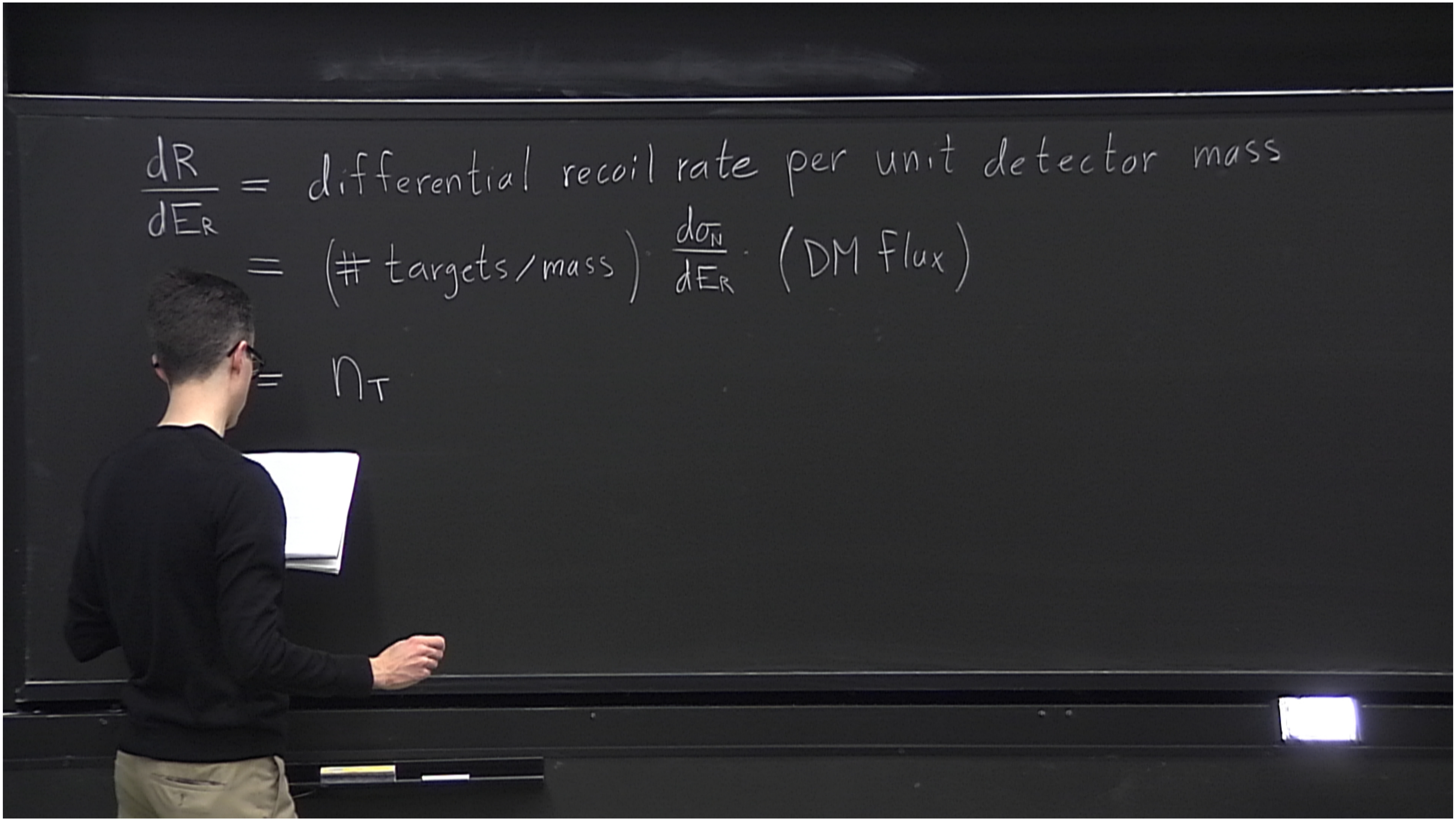

Kinematics
DM
v
N
DM
θ
N
q, E_R
q = 2 μ_N v cosθ ~ 100 MeV
μ_N = m_χ m_N / (m_χ + m_N)
E_R = q² / 2m_N ≃ 100 keV

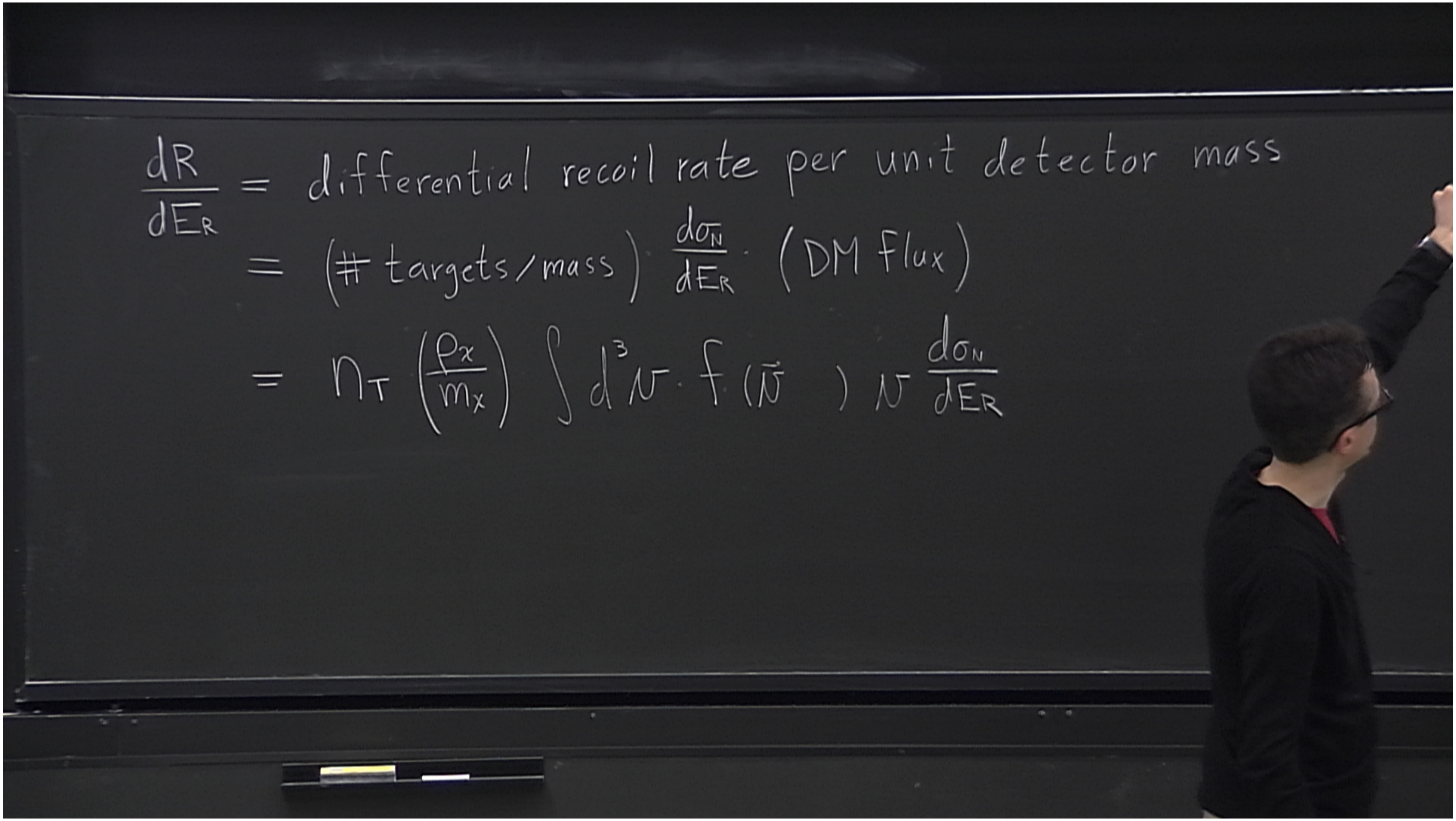

Kinematics
DM
v
N
DM
θ
N
q, E_R
$q = 2\mu_N v \cos\theta \sim 100\ \text{MeV}$
$\mu_N = \frac{m_\chi m_N}{(m_\chi + m_N)}$
$E_R = \frac{q^2}{2m_N} \simeq 100\ \text{keV}$

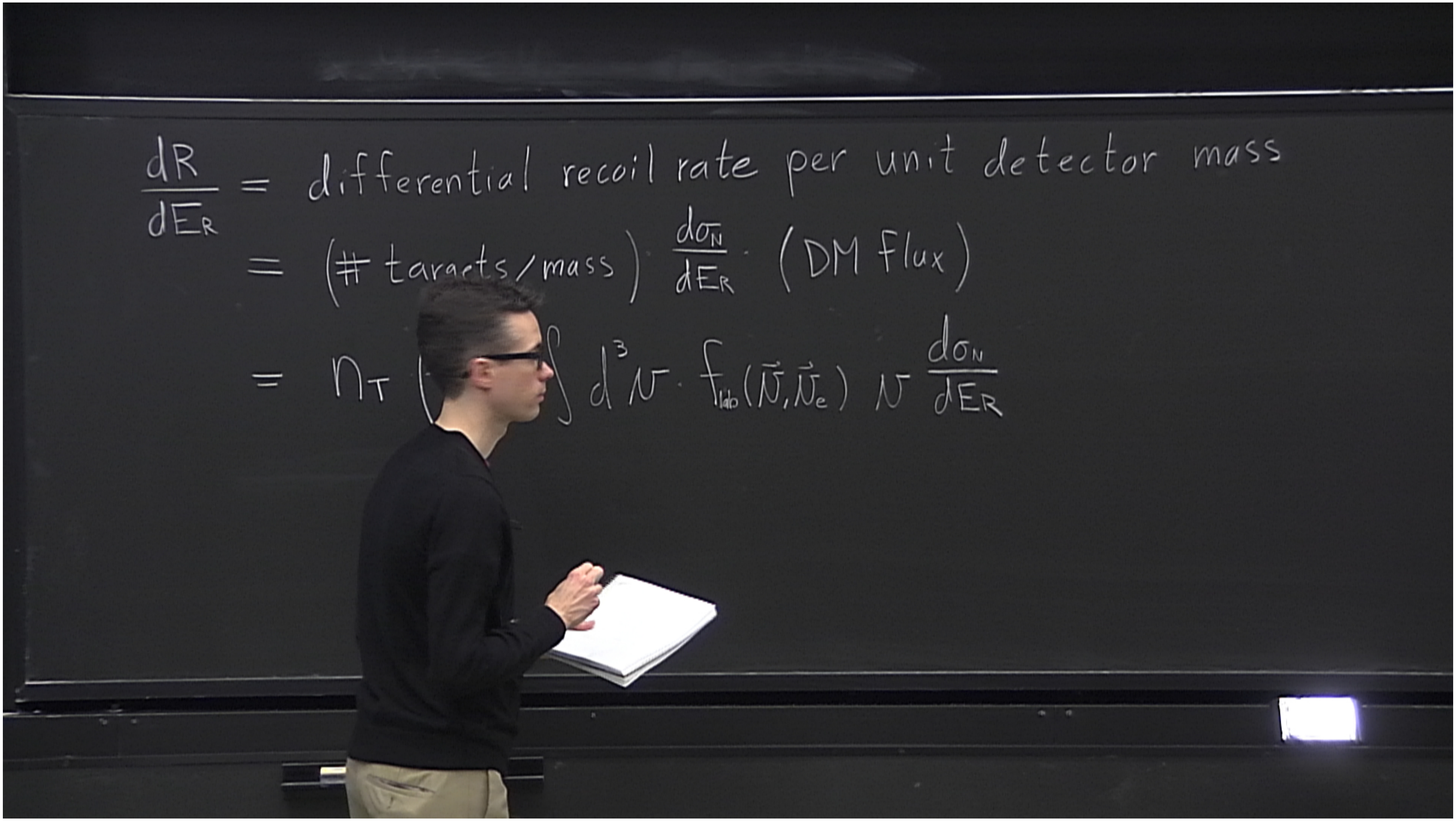

dR/dER = differential recoil rate per unit detector mass
= (# targets/mass
dσN
ER
(DM

$$\frac{dR}{dE_R} = \text{differential recoil rate per unit detector mass}$$

$$= (\#\ \text{targets}/\text{mass}) \cdot \frac{d\sigma_N}{dE_R} \cdot (\text{DM flux})$$


$\frac{dR}{dE_R}$ = differential recoil rate per unit detector mass
$= (\#\text{ targets/mass}) \cdot \frac{d\sigma_N}{dE_R} \cdot (\text{DM flux})$
$= n_T$


$\frac{dR}{dE_R}$ = differential recoil rate per unit detector mass
$= (\#\ \text{targets/mass}) \frac{d\sigma_N}{dE_R} \cdot (\text{DM flux})$
$= n_T \left(\frac{\rho_\chi}{m_\chi}\right) \int d^3v \cdot f(\vec{v}\ \ )\, v \frac{d\sigma_N}{dE_R}$



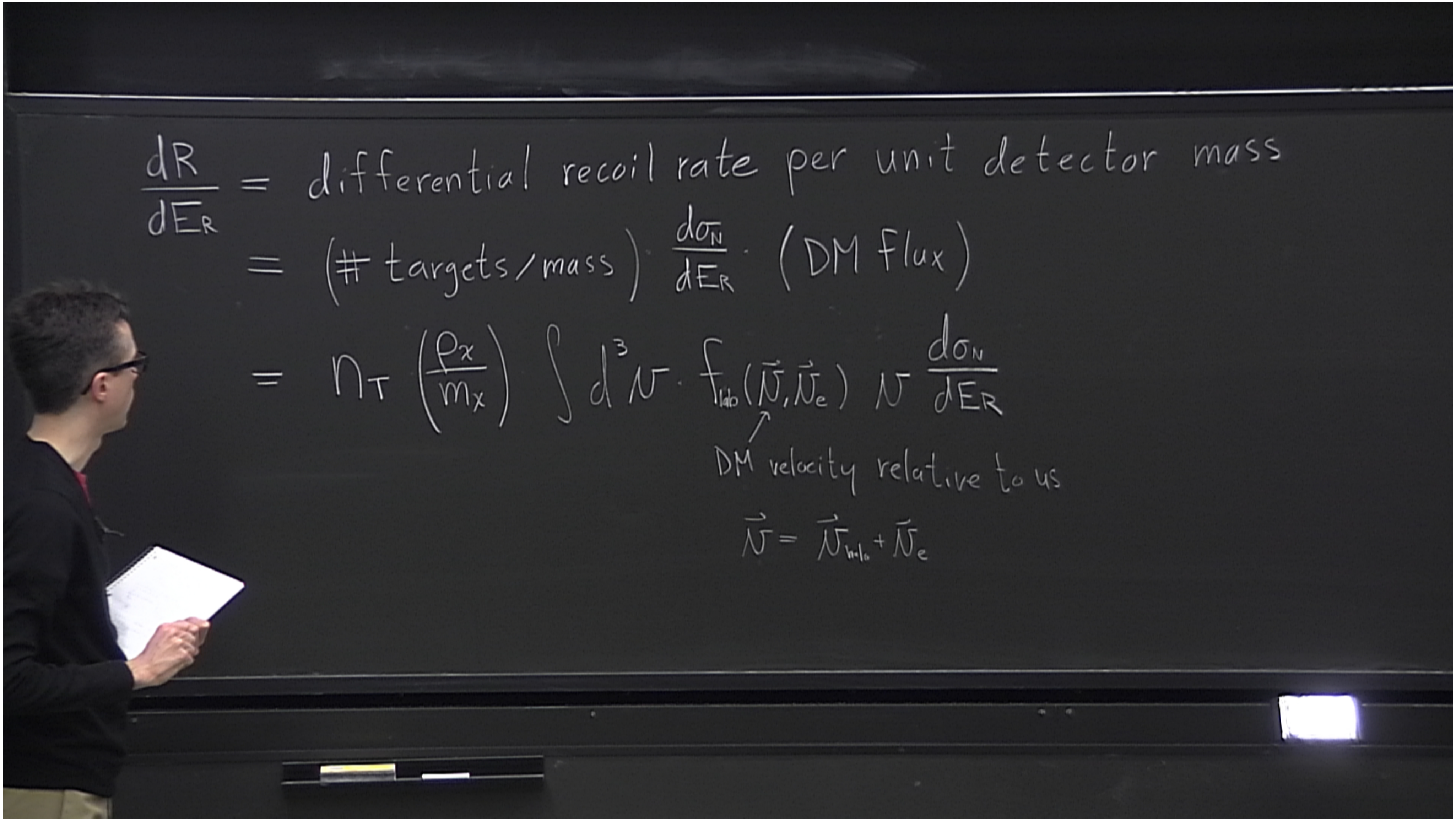
$\frac{dR}{dE_R}$ = differential recoil rate per unit detector mass
$= (\#\ \text{targets/mass}) \cdot \frac{d\sigma_N}{dE_R} \cdot (\text{DM flux})$
$= n_T \Big(\quad \int d^3v \cdot f_{lab}(\vec{v}, \vec{v}_e)\ v \frac{d\sigma_N}{dE_R}$

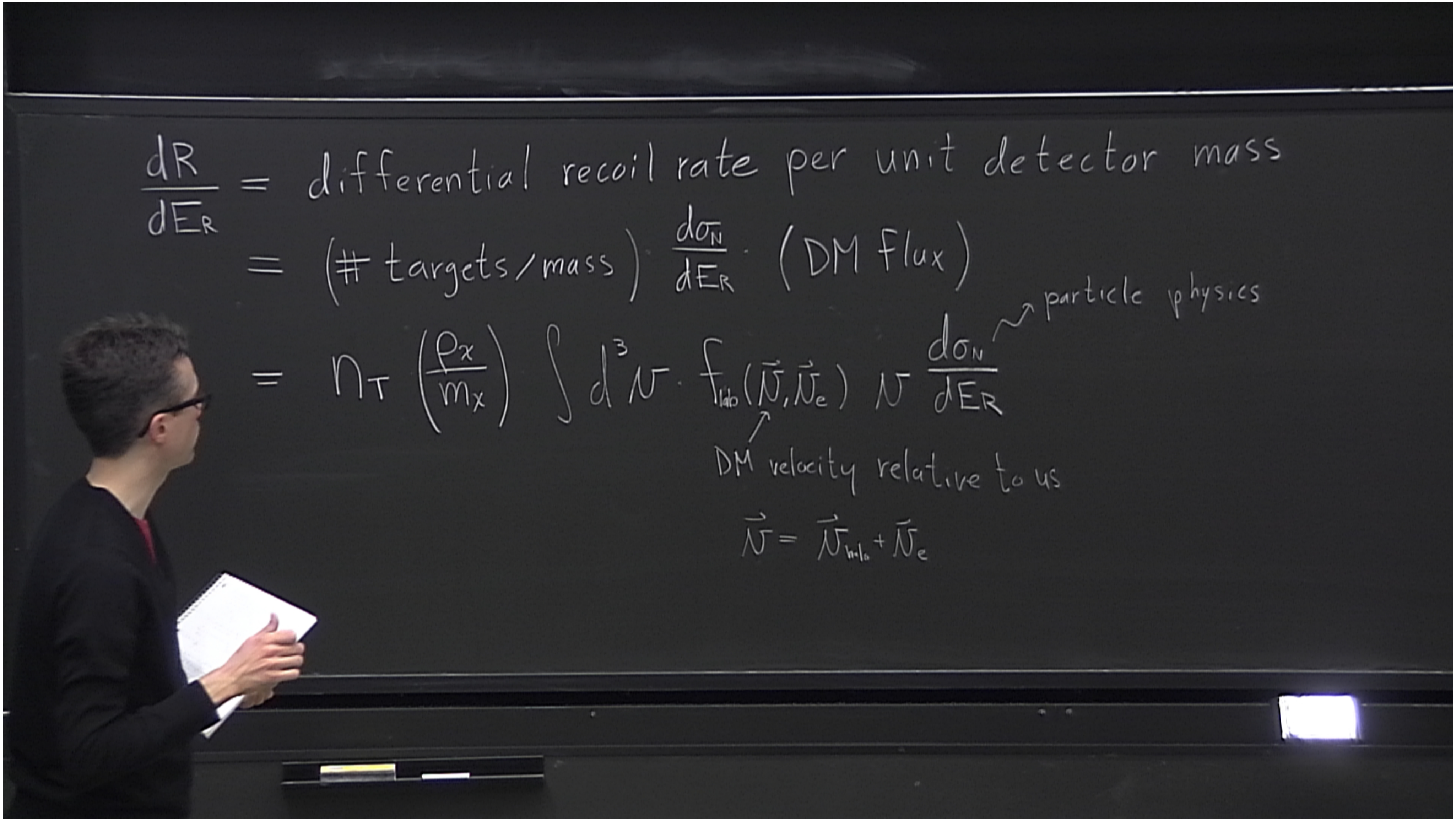
$$/\text{mass}\Big) \cdot \frac{d\sigma_N}{dE_R} \cdot \Big(\text{DM flux}\Big)$$

$$\int d^3 v \cdot f_{lab}(\vec{v}, \vec{v}_e)\, v \,\frac{d\sigma_N}{dE_R}$$

DM velocity relative to us

$$\vec{v} = \vec{v}_{halo} + \vec{v}_e$$


$\frac{dR}{dE_R}$ = differential recoil rate per unit detector mass
$= (\#\text{ targets/mass}) \cdot \frac{d\sigma_N}{dE_R} \cdot (\text{DM flux})$
$= n_T \left(\frac{\rho_\chi}{m_\chi}\right) \int d^3v \cdot f_{lab}(\vec{v}, \vec{v}_e)\, v \frac{d\sigma_N}{dE_R}$
DM velocity relative to us
$\vec{v} = \vec{v}_{halo} + \vec{v}_e$


$\frac{dR}{dE_R}$ = differential recoil rate per unit detector mass
$= (\#\ \text{targets/mass}) \cdot \frac{d\sigma_N}{dE_R} \cdot (\text{DM flux})$
particle physics
$= n_T \left(\frac{\rho_\chi}{m_\chi}\right) \int d^3v \cdot f_{lab}(\vec{v}, \vec{v}_e)\, v \frac{d\sigma_N}{dE_R}$
DM velocity relative to us
$\vec{v} = \vec{v}_{halo} + \vec{v}_e$

mass

ysics

${}^{A}_{Z}N$ = target nucleus

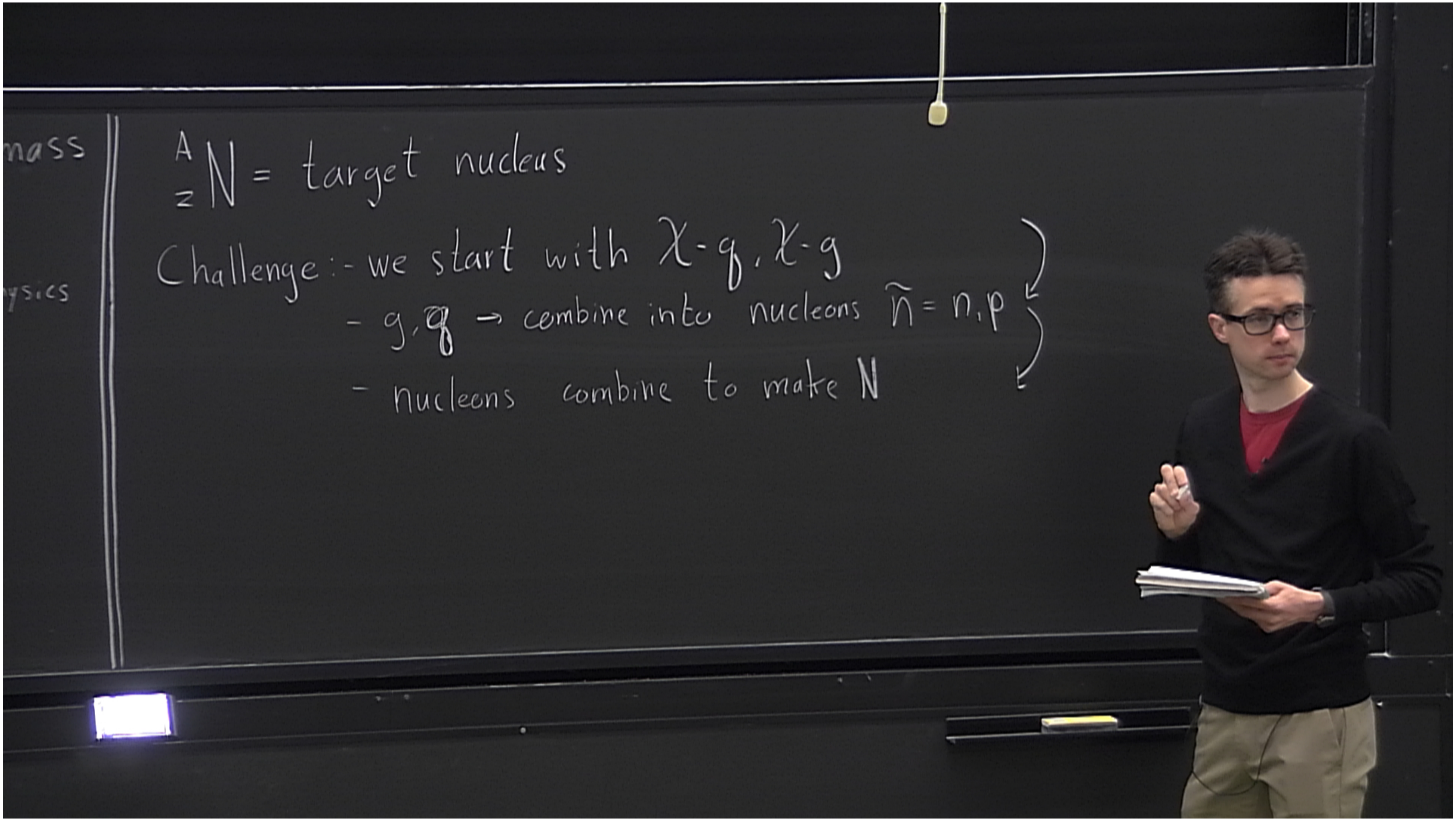
mass

ysics

$^{A}_{Z}N$ = target nucleus

Challenge: - we start with χ-q, χ-g

- g, q $\rightarrow$ combine into nucleons $\tilde{n} = n, p$
- nucleons combine to make N


$^{A}_{Z}N$ = target nucleus
Challenge: - we start with χ-q, χ-g
- g, q → combine into nucleons $\tilde{n}$ = n, p
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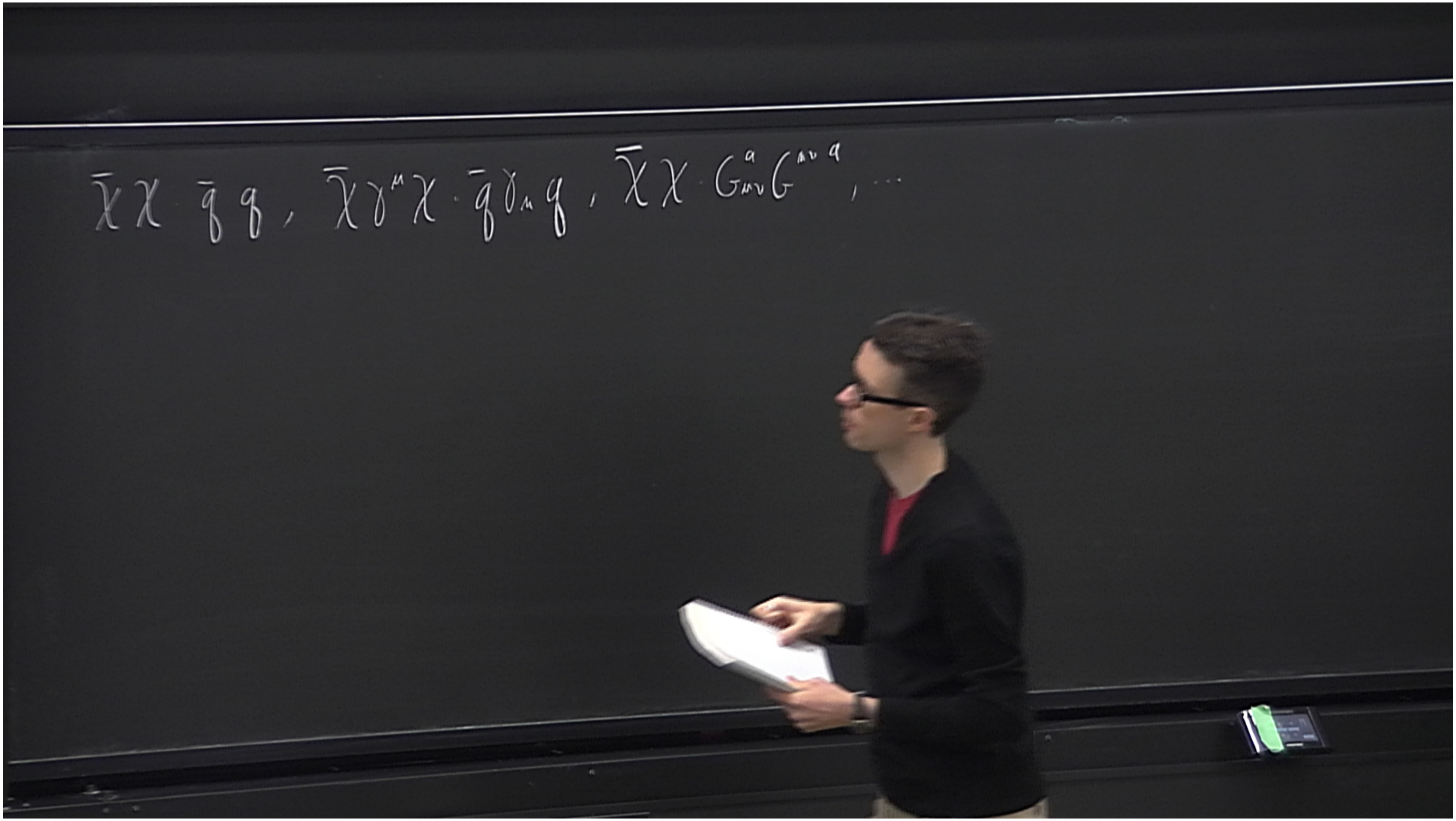
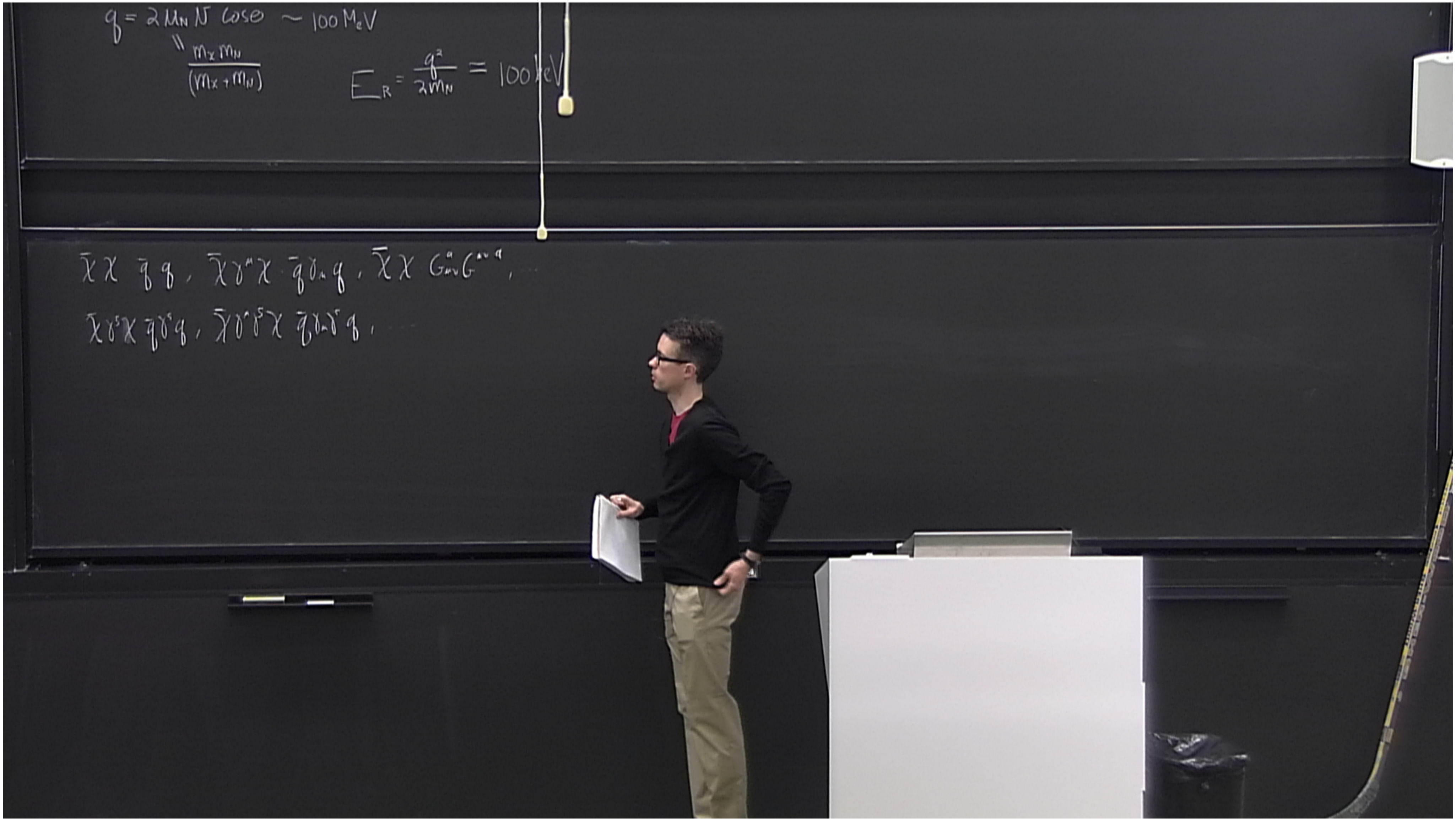
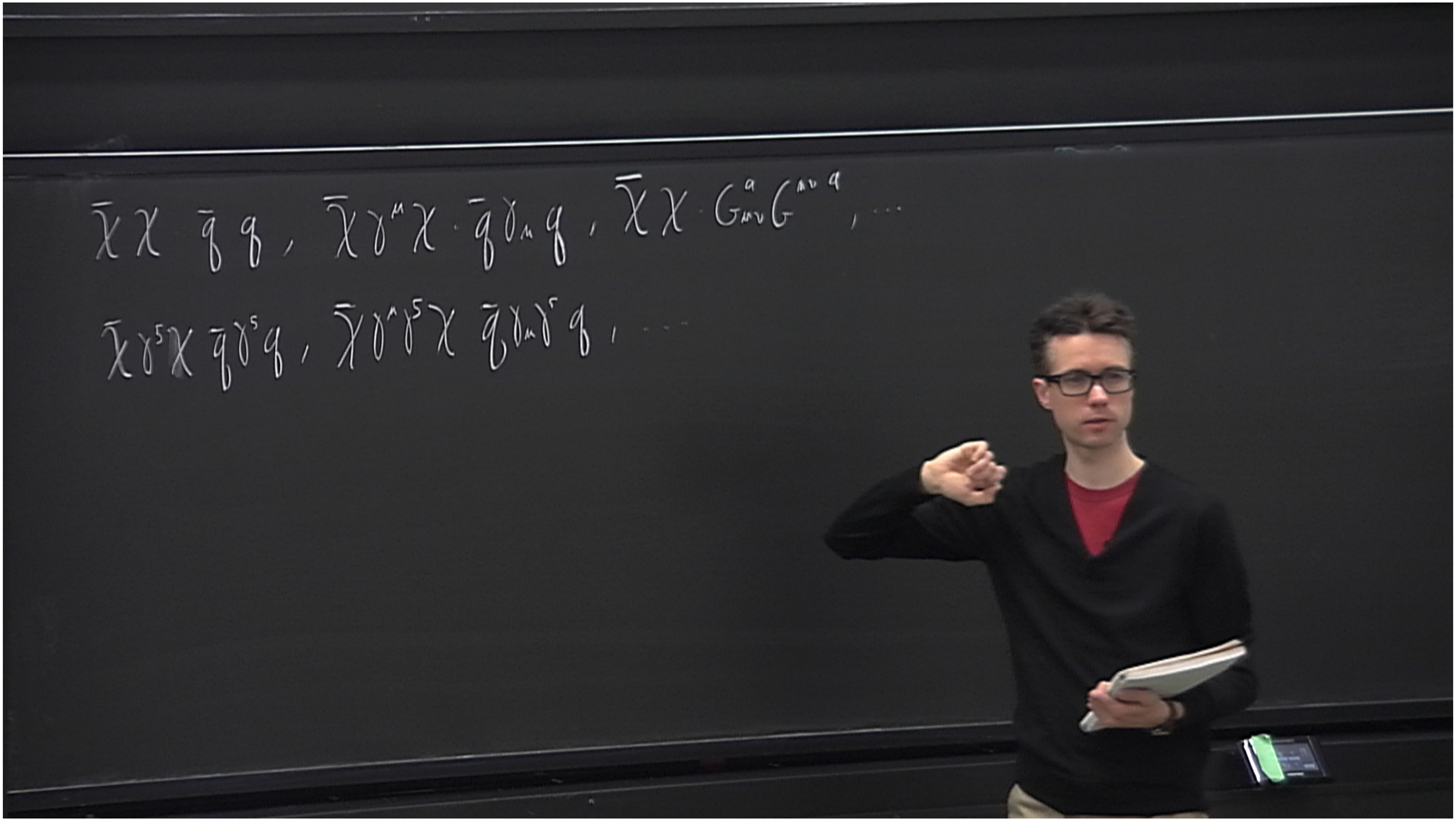
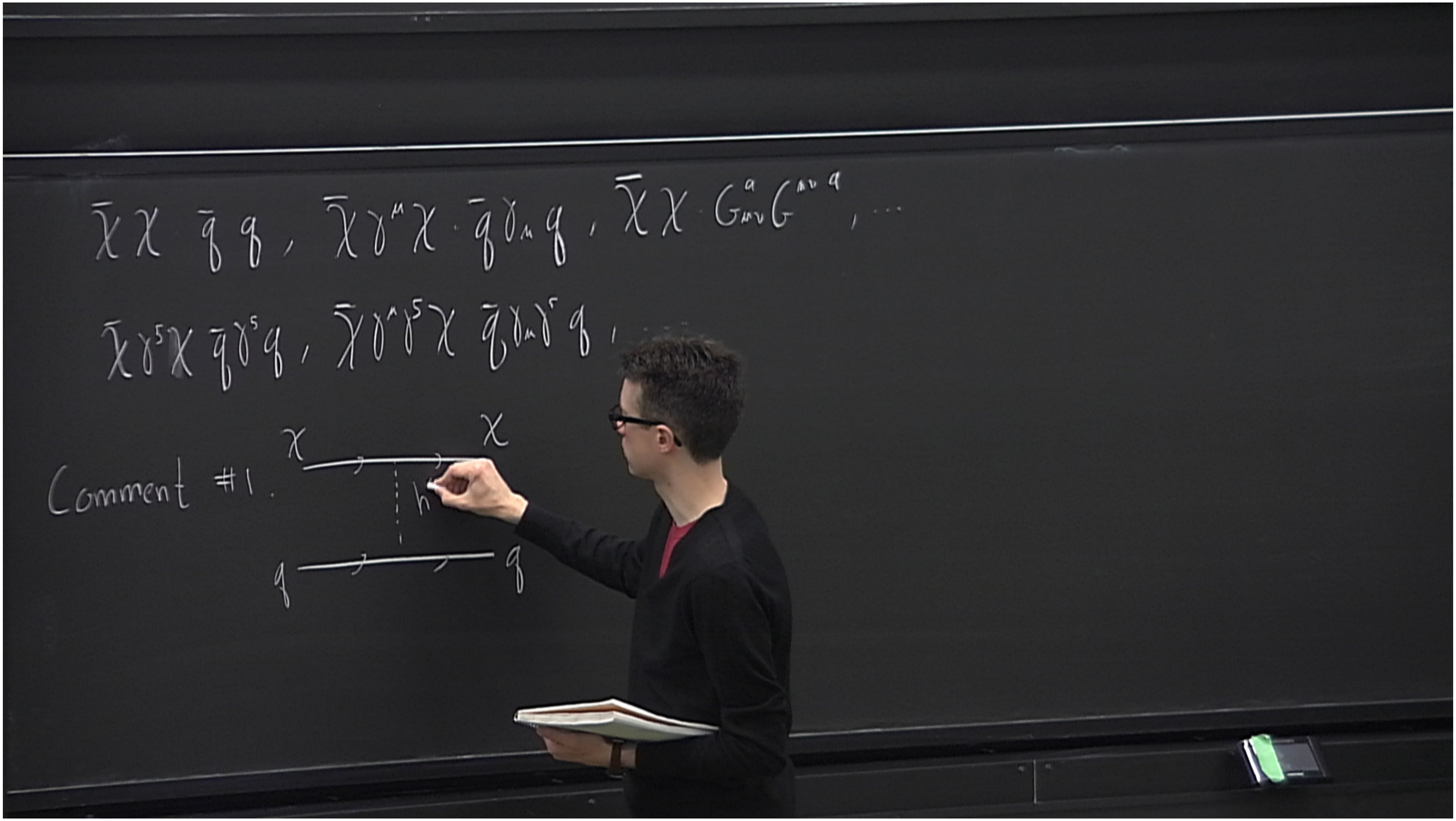

$\bar{\chi}\chi \cdot \bar{q}q, \quad \bar{\chi}\gamma^{\mu}\chi \cdot \bar{q}\gamma_{\mu}q, \quad \bar{\chi}\chi \cdot G^{a}_{\mu\nu}G^{\mu\nu\, a}, \ \ldots$

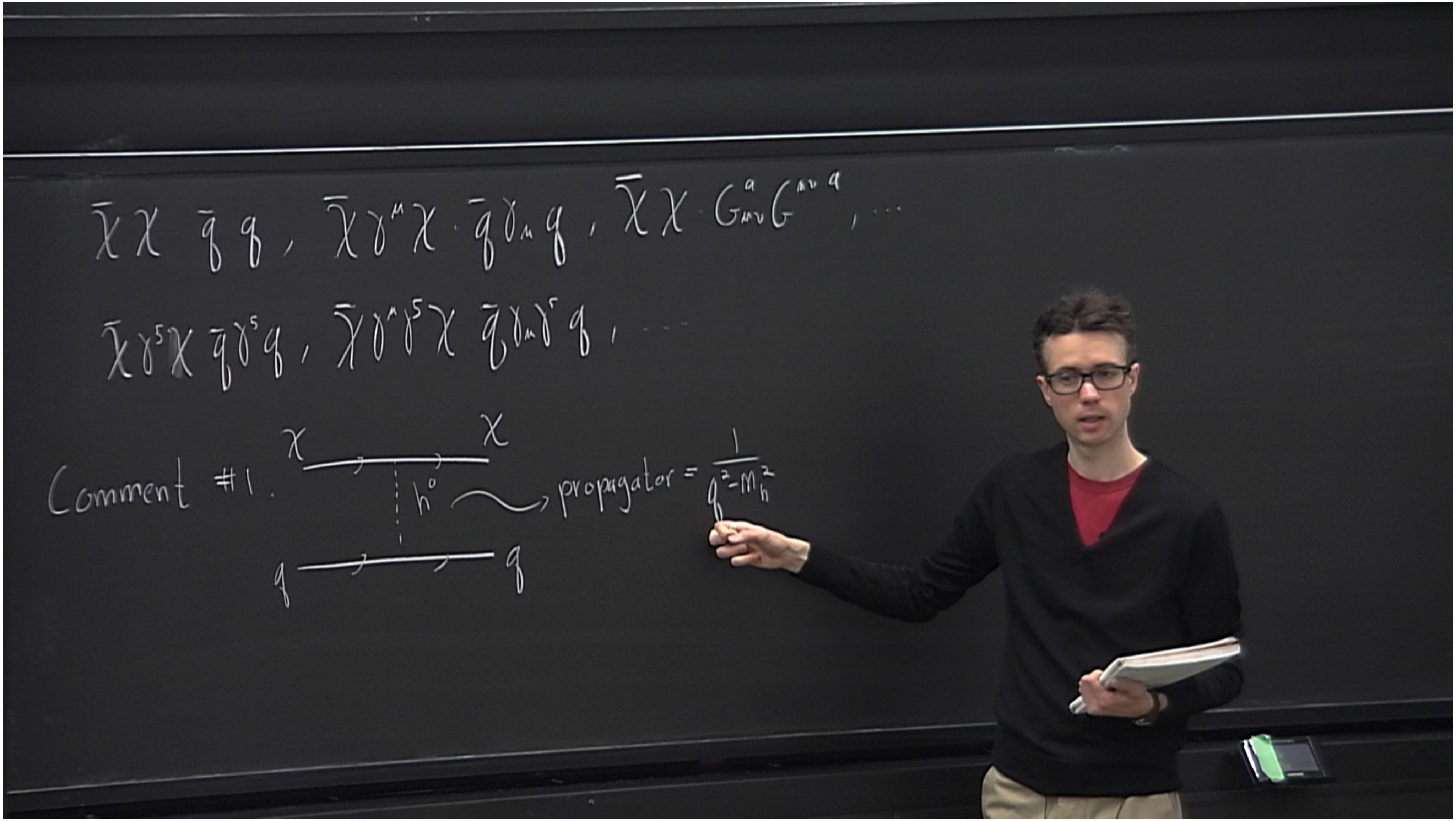

q = 2 μN v cosθ ~ 100 MeV
mχ mN / (mχ + mN)
ER = q²/2mN = 100 keV
χ̄χ q̄q , χ̄γ^μχ q̄γ_μq , χ̄χ G^a_μν G^μν a , ...
χ̄γ^5χ q̄γ^5q , χ̄γ^μγ^5χ q̄γ_μγ^5q , ...

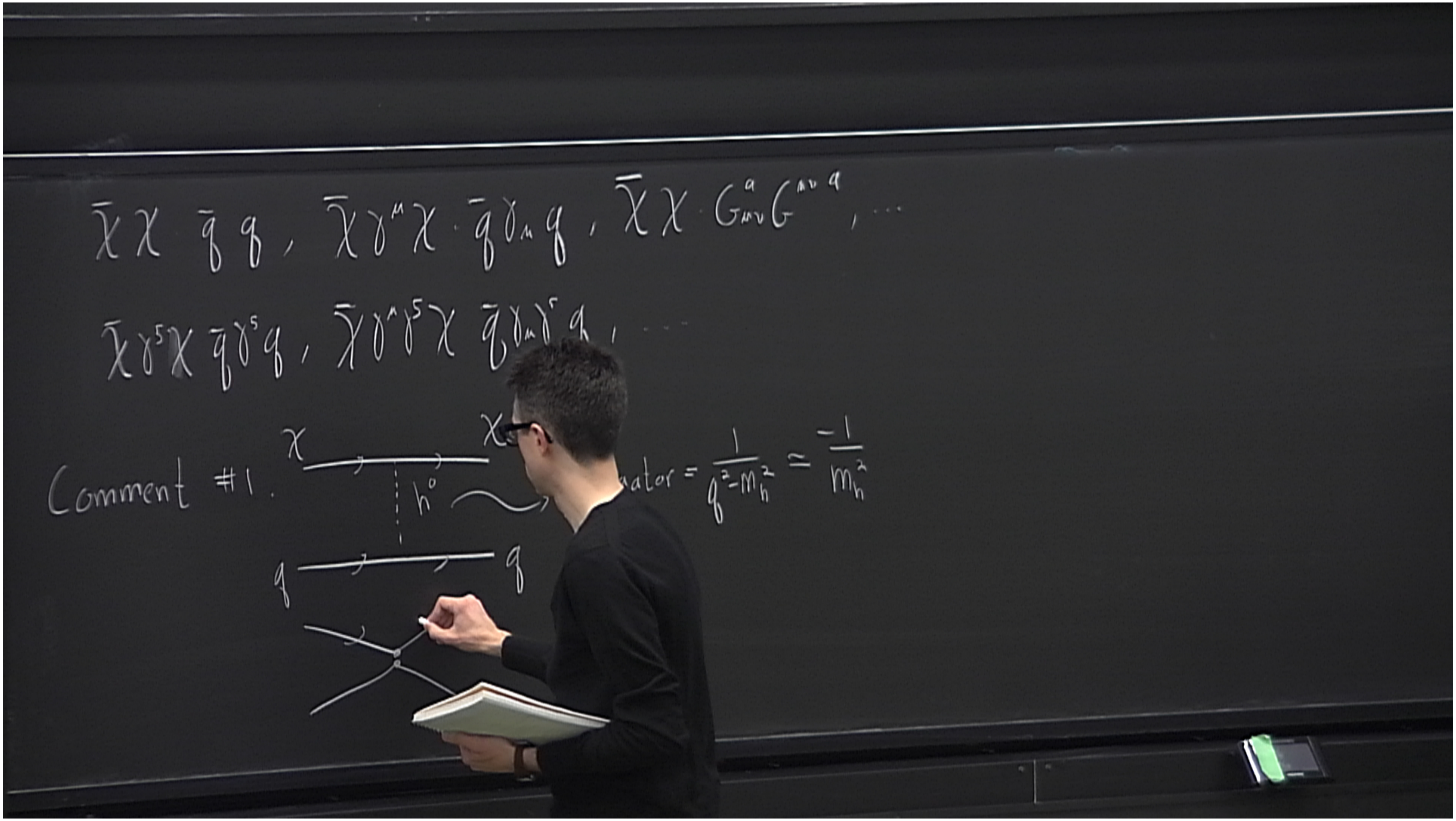
$$\gamma^\mu \chi \cdot \bar{q}\gamma_\mu q \,, \quad \bar{\chi}\chi \cdot G^a_{\mu\nu}G^{\mu\nu\, a} \,, \; \dots$$

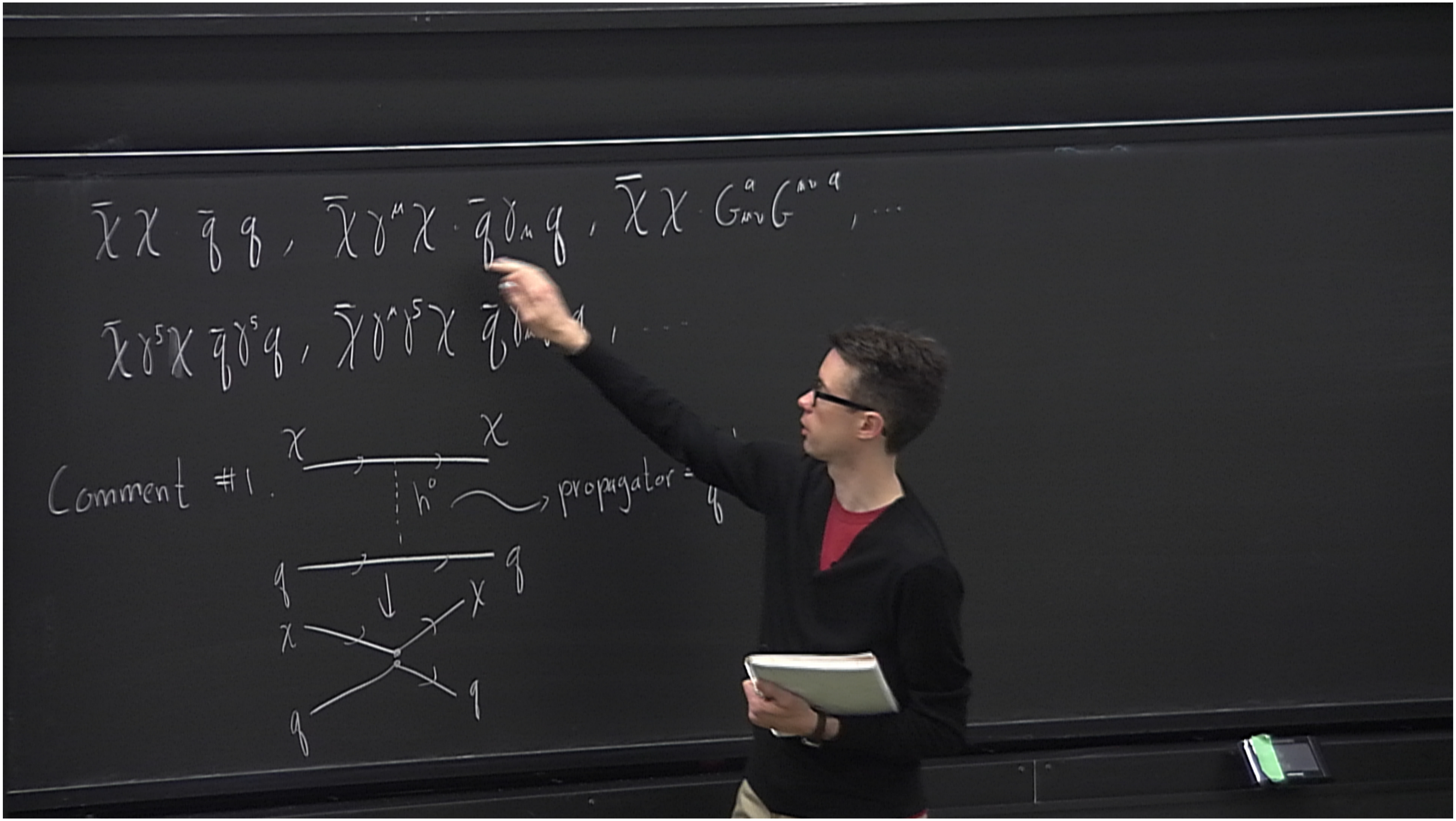
$$\gamma^\mu \gamma^5 \chi \;\; \bar{q}\gamma_\mu \gamma^5 q \,, \; \dots$$


$\bar\chi\chi\,\bar q q$, $\bar\chi\gamma^\mu\chi\,\bar q\gamma_\mu q$, $\bar\chi\chi\,G^a_{\mu\nu}G^{\mu\nu a}$, ...
$\bar\chi\gamma^5\chi\,\bar q\gamma^5 q$, $\bar\chi\gamma^\mu\gamma^5\chi\,\bar q\gamma_\mu\gamma^5 q$, ...

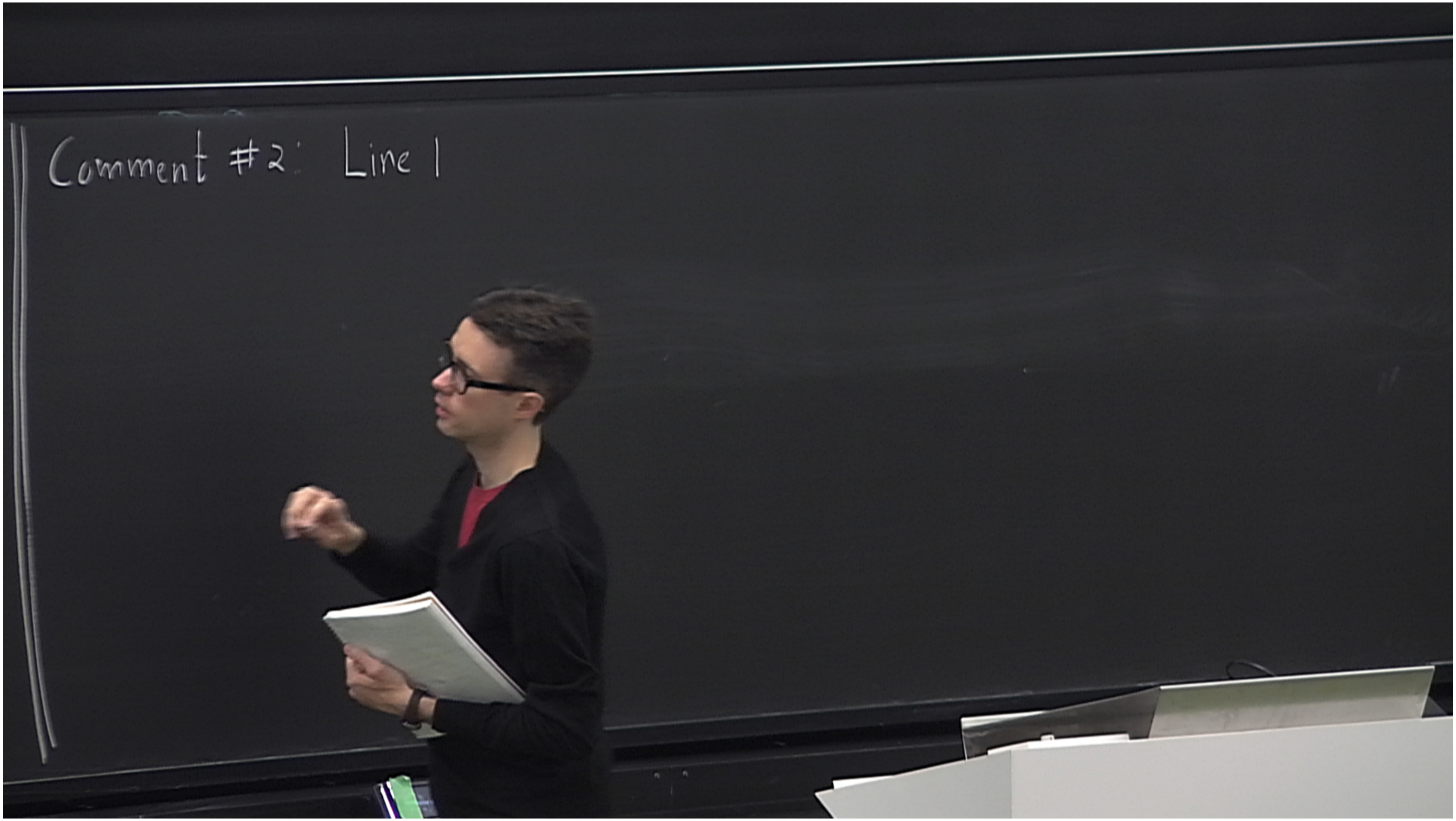

$\bar\chi\chi\,\bar q q$, $\bar\chi\gamma^\mu\chi\cdot\bar q\gamma_\mu q$, $\bar\chi\chi\cdot G^a_{\mu\nu}G^{\mu\nu a}$, ...
$\bar\chi\gamma^5\chi\,\bar q\gamma^5 q$, $\bar\chi\gamma^\mu\gamma^5\chi\,\bar q\gamma_\mu\gamma^5 q$, ...
Comment #1.
χ
χ
h
q
q

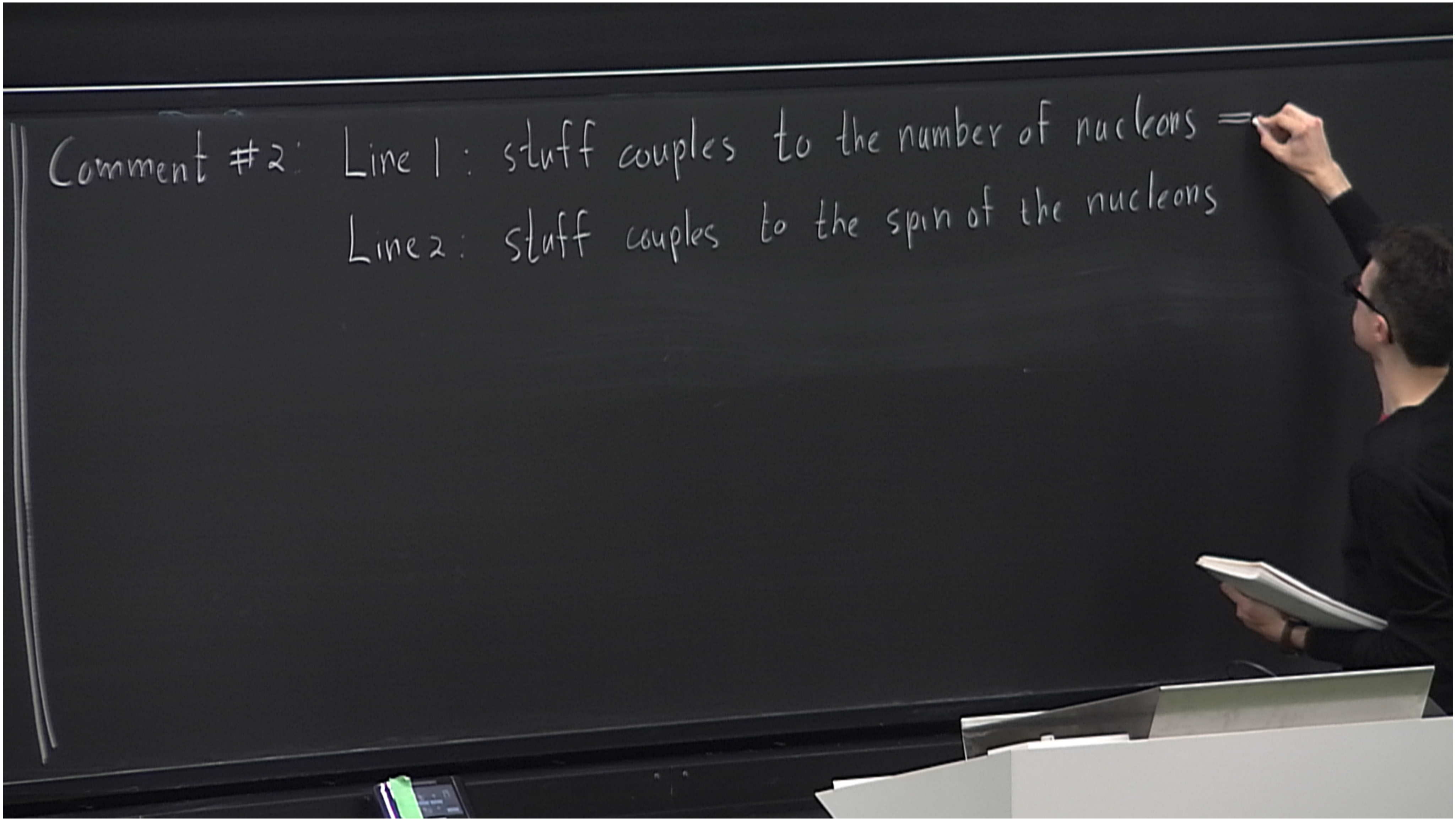

$\bar{\chi}\chi \cdot \bar{q}q$, $\bar{\chi}\gamma^{\mu}\chi \cdot \bar{q}\gamma_{\mu}q$, $\bar{\chi}\chi \cdot G^{a}_{\mu\nu}G^{\mu\nu a}$, ...
$\bar{\chi}\gamma^{5}\chi\,\bar{q}\gamma^{5}q$, $\bar{\chi}\gamma^{\mu}\gamma^{5}\chi\,\bar{q}\gamma_{\mu}\gamma^{5}q$, ...
Comment #1.
χ
χ
h^{0}
q
q
propagator $= \frac{1}{q^{2}-m_{h}^{2}}$

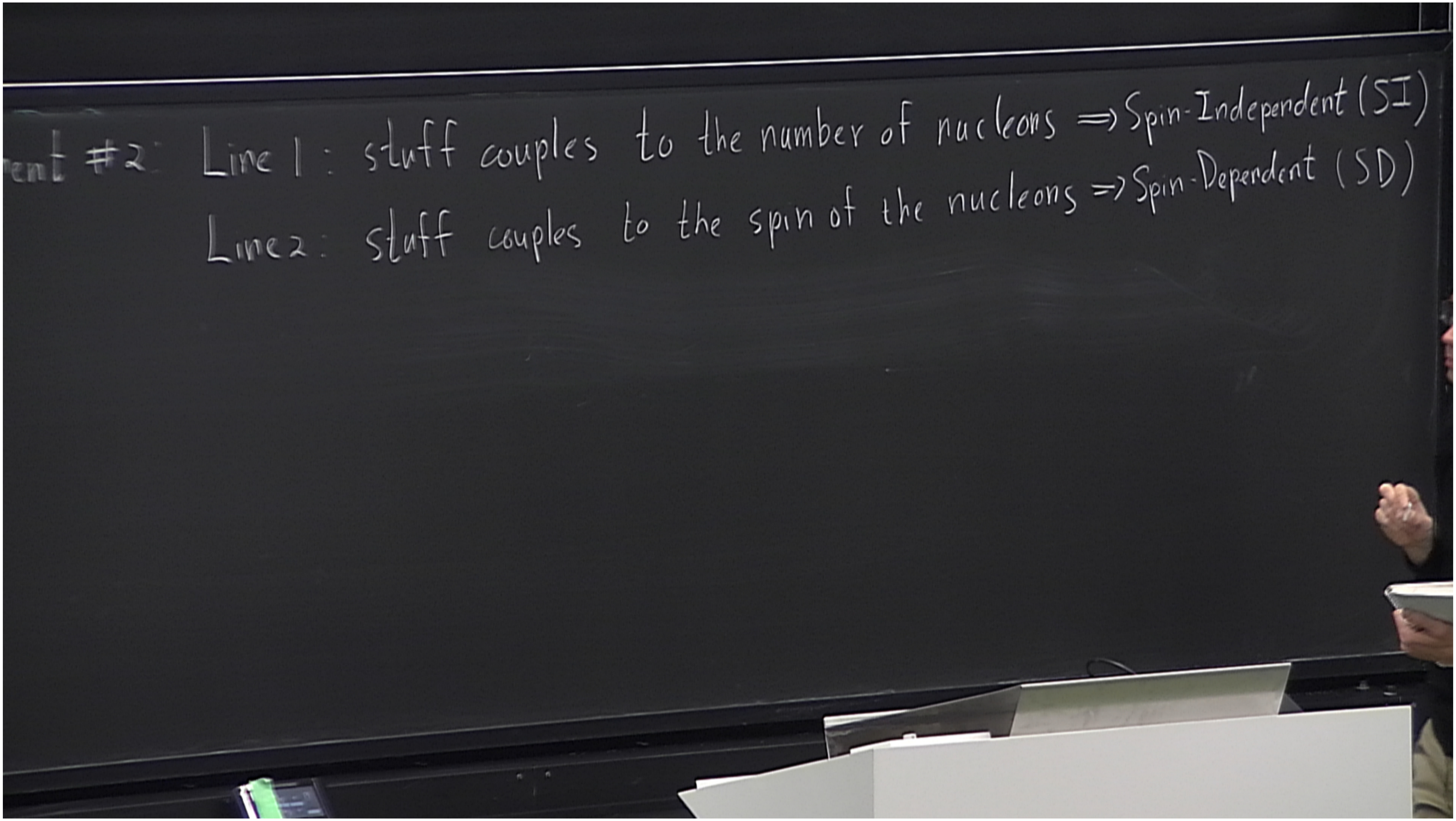

$\bar{\chi}\chi \cdot \bar{q}q \, , \; \bar{\chi}\gamma^{\mu}\chi \cdot \bar{q}\gamma_{\mu} q \, , \; \bar{\chi}\chi \cdot G^{a}_{\mu\nu}G^{\mu\nu a} \, , \ldots$
$\bar{\chi}\gamma^{5}\chi \, \bar{q}\gamma^{5} q \, , \; \bar{\chi}\gamma^{\mu}\gamma^{5}\chi \, \bar{q}\gamma_{\mu}\gamma^{5} q \, , \ldots$
Comment #1.
χ
χ
h^{0}
q
q
ator $= \frac{1}{q^{2}-m_{h}^{2}} \simeq \frac{-1}{m_{h}^{2}}$

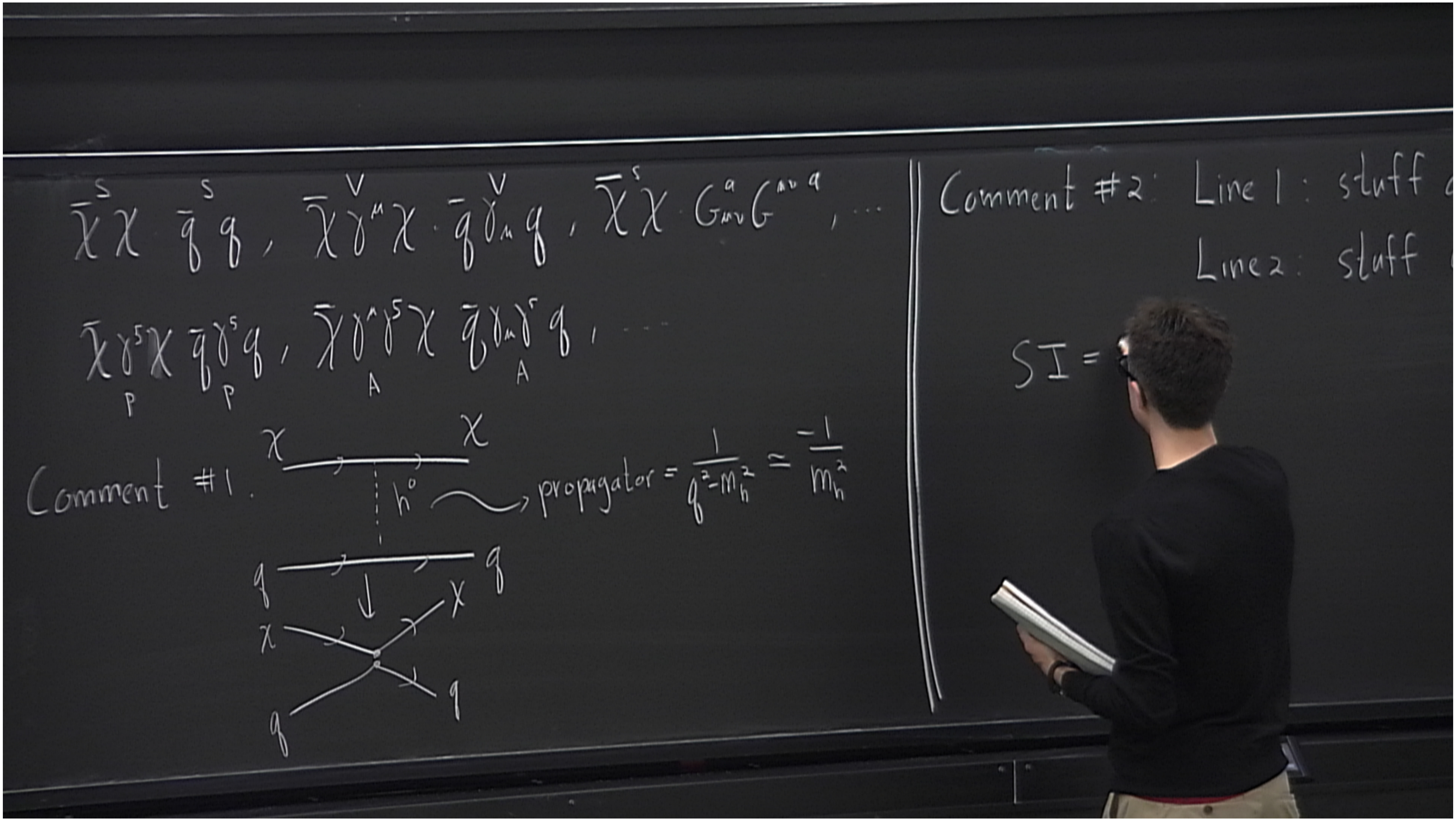


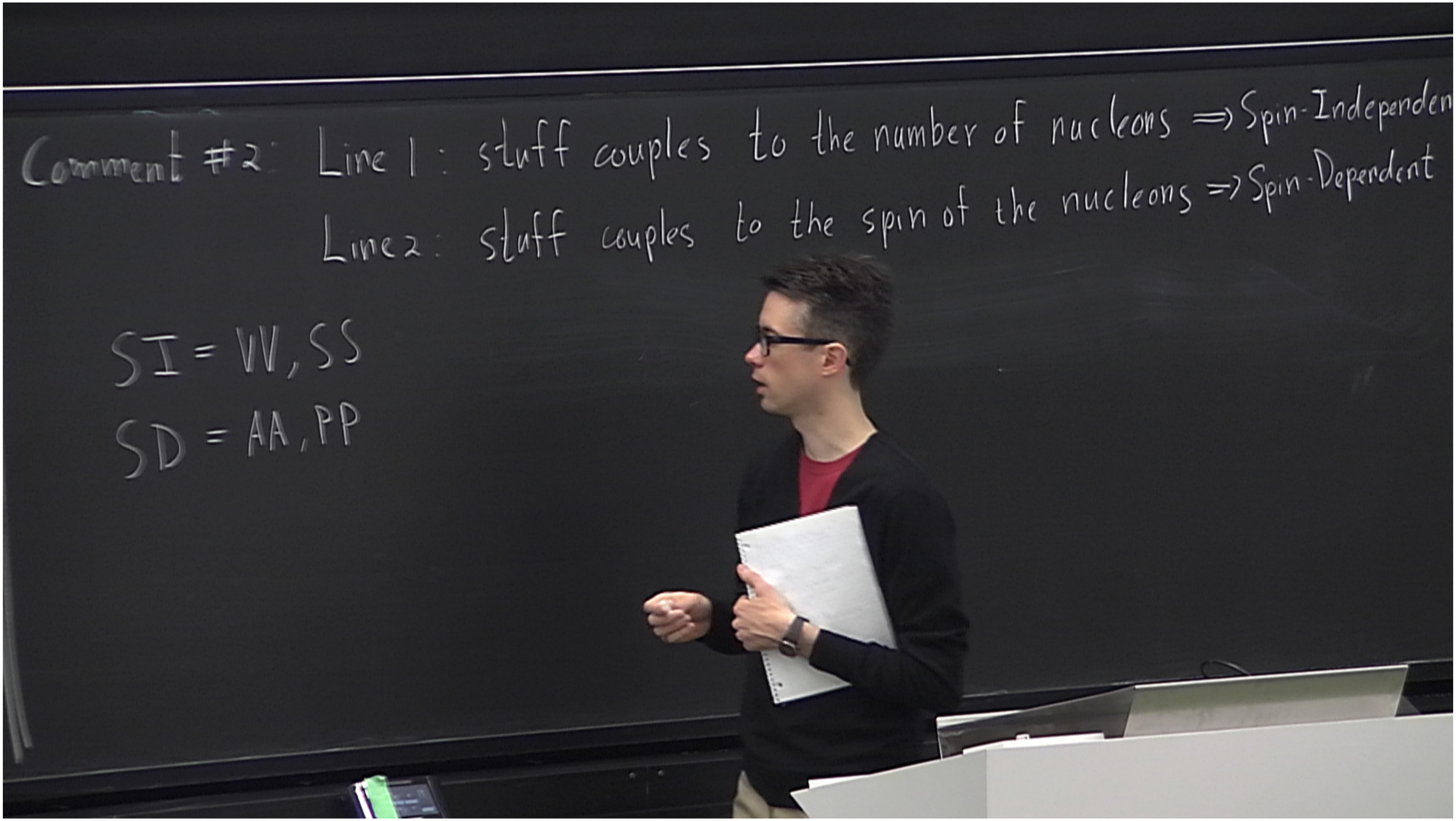
$\bar{\chi}\chi\,\bar{q}q,\ \bar{\chi}\gamma^{\mu}\chi\,\bar{q}\gamma_{\mu}q,\ \bar{\chi}\chi\,G^{a}_{\mu\nu}G^{\mu\nu a},\ \dots$
$\bar{\chi}\gamma^{5}\chi\,\bar{q}\gamma^{5}q,\ \bar{\chi}\gamma^{\mu}\gamma^{5}\chi\,\bar{q}\gamma_{\mu}\dots$
Comment #1.
χ
h^{0}
propagator
q

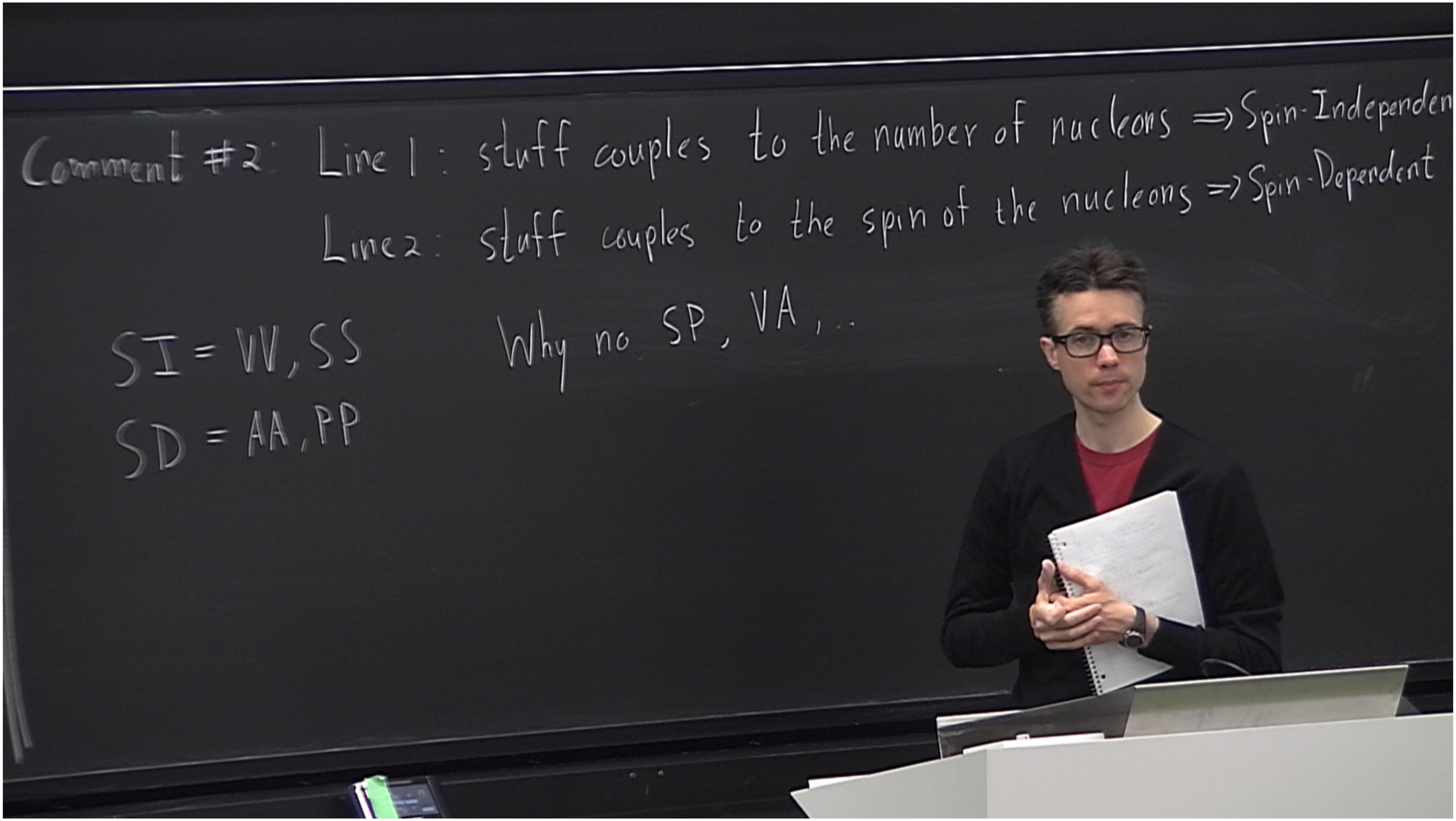

Comment #2: Line 1


Comment #2: Line 1: stuff couples to the number of nucleons =
Line 2: stuff couples to the spin of the nucleons


ent #2: Line 1: stuff couples to the number of nucleons ⇒ Spin-Independent (SI)
Line 2: stuff couples to the spin of the nucleons ⇒ Spin-Dependent (SD)


$\bar{\chi}\chi \; \bar{q}q$, $\bar{\chi}\gamma^\mu\chi \; \bar{q}\gamma_\mu q$, $\bar{\chi}\chi \; G^a_{\mu\nu}G^{\mu\nu a}$, ...
S S V V S
$\bar{\chi}\gamma^5\chi \; \bar{q}\gamma^5 q$, $\bar{\chi}\gamma^\mu\gamma^5\chi \; \bar{q}\gamma_\mu\gamma^5 q$, ...
P P A A
Comment #1.
χ χ
h^0
propagator = $\frac{1}{q^2-m_h^2} \simeq \frac{-1}{m_h^2}$
q q
χ χ
q q
Comment #2: Line 1: stuff
Line 2: stuff
SI =


Comment #2: Line 1: stuff couples to the number of nucleons ⟹ Spin-Independen
Line 2: stuff couples to the spin of the nucleons ⟹ Spin-Dependent
SI = VV, SS
SD = AA, PP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SI = VV, SS
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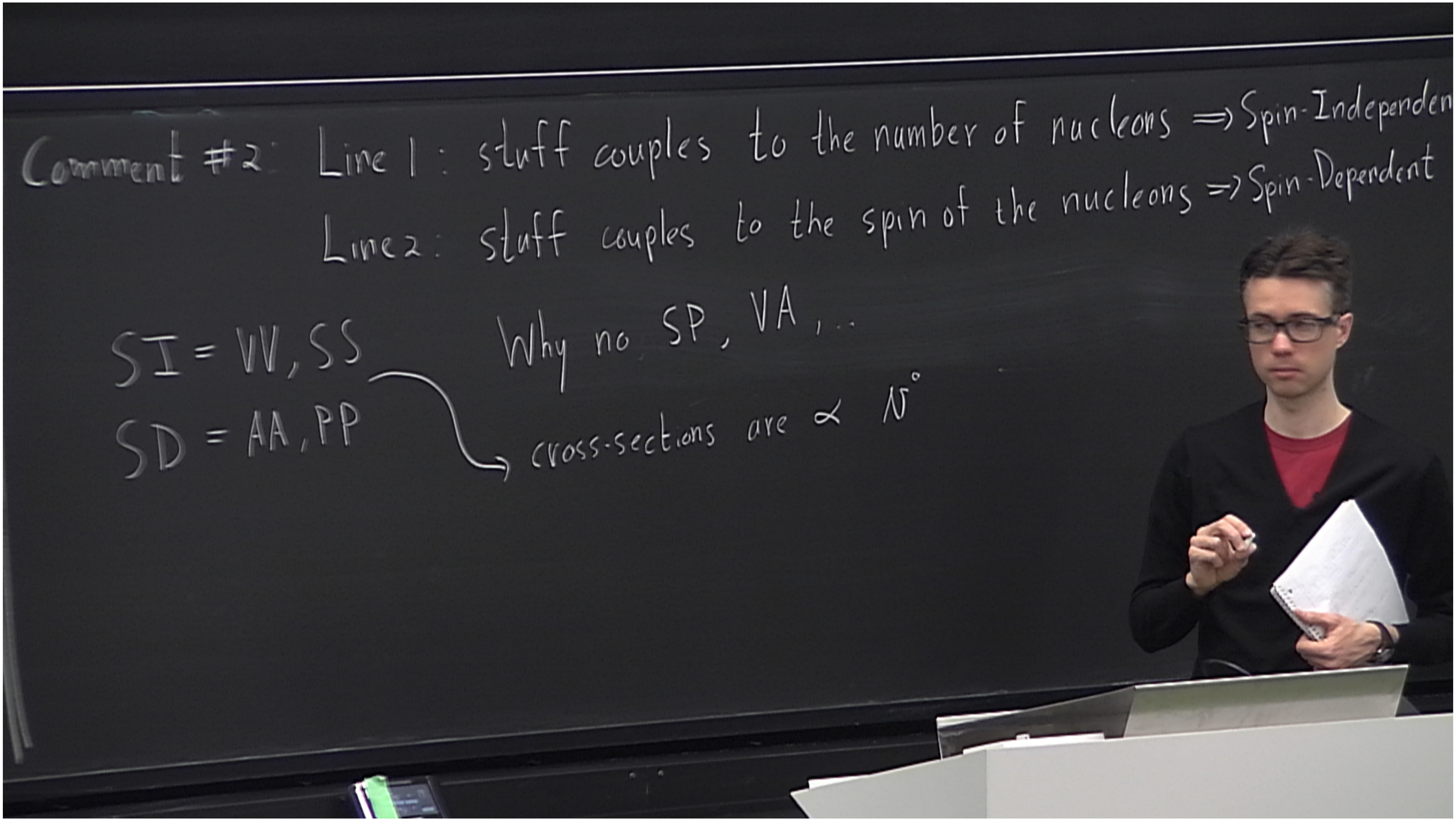
Why no SP, VA, ..


Comment #2: Line 1: stuff couples to the number of nucleons ⟹ Spin-Independen
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SI = VV, SS
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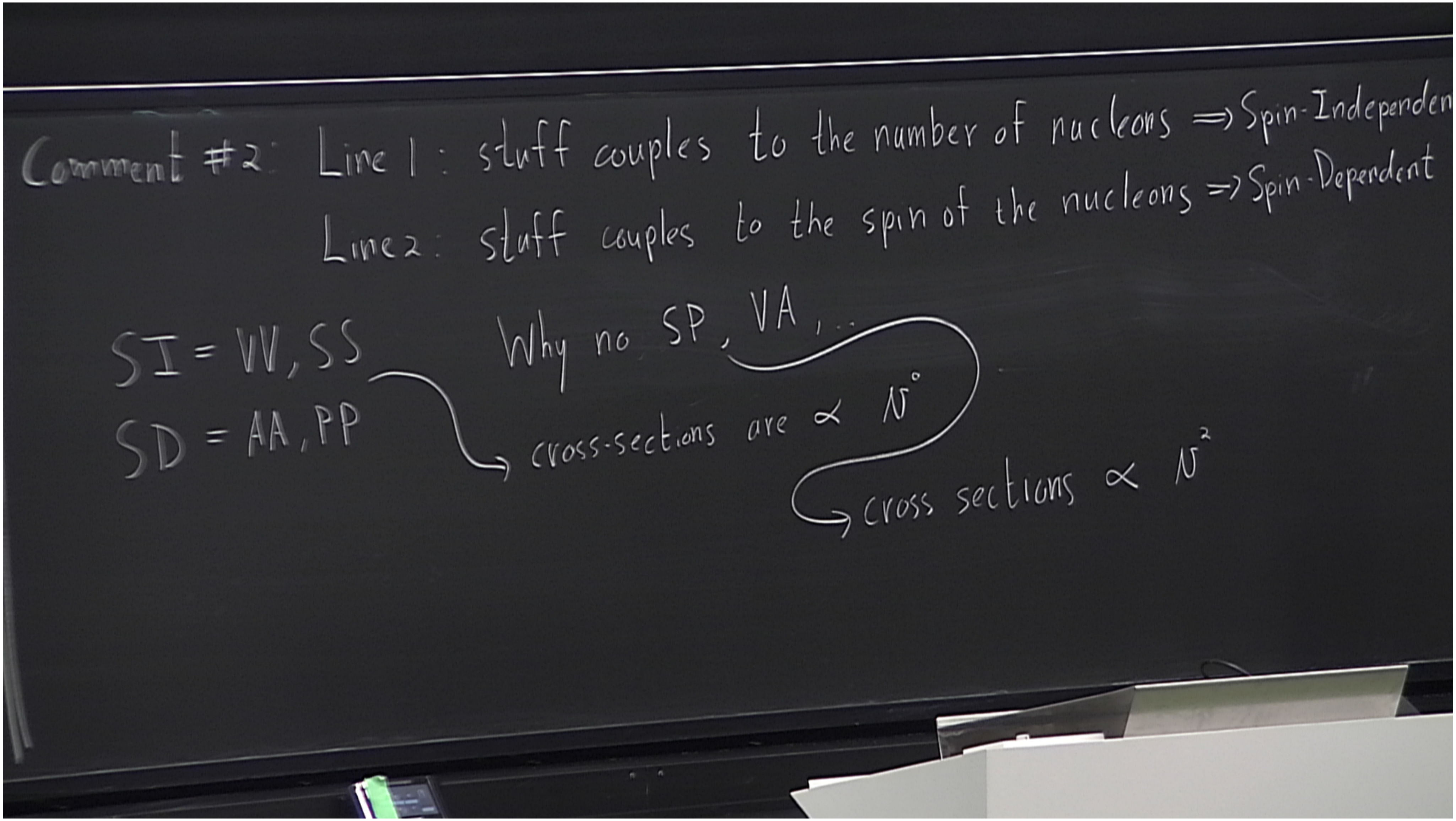
Why no SP, VA, ..
cross-sections are ∝ N°


Comment #2: Line 1: stuff couples to the number of nucleons ⇒ Spin-Independen
Line 2: stuff couples to the spin of the nucleons ⇒ Spin-Dependent
SI = VV, SS
SD = AA, PP
Why no SP, VA, ..
cross-sections are ∝ v^0
cross sections ∝ v^2

Comment #2: Line 1: stuff couples to the number of nucleons $\Longrightarrow$ Spin-Independen

Line 2: stuff couples to the spin of the nucleons $\Longrightarrow$ Spin-Dependent

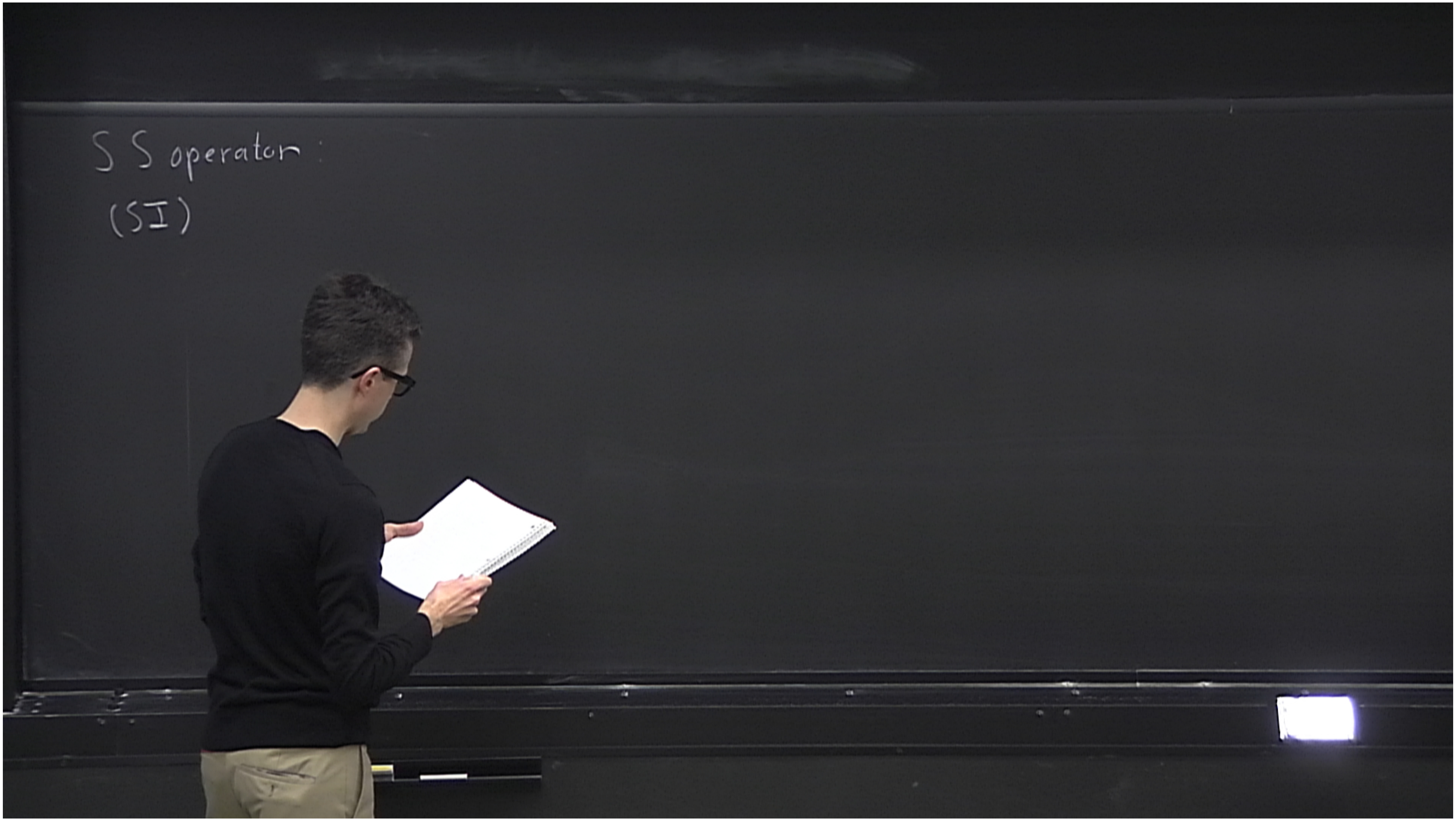
SI = VV, SS

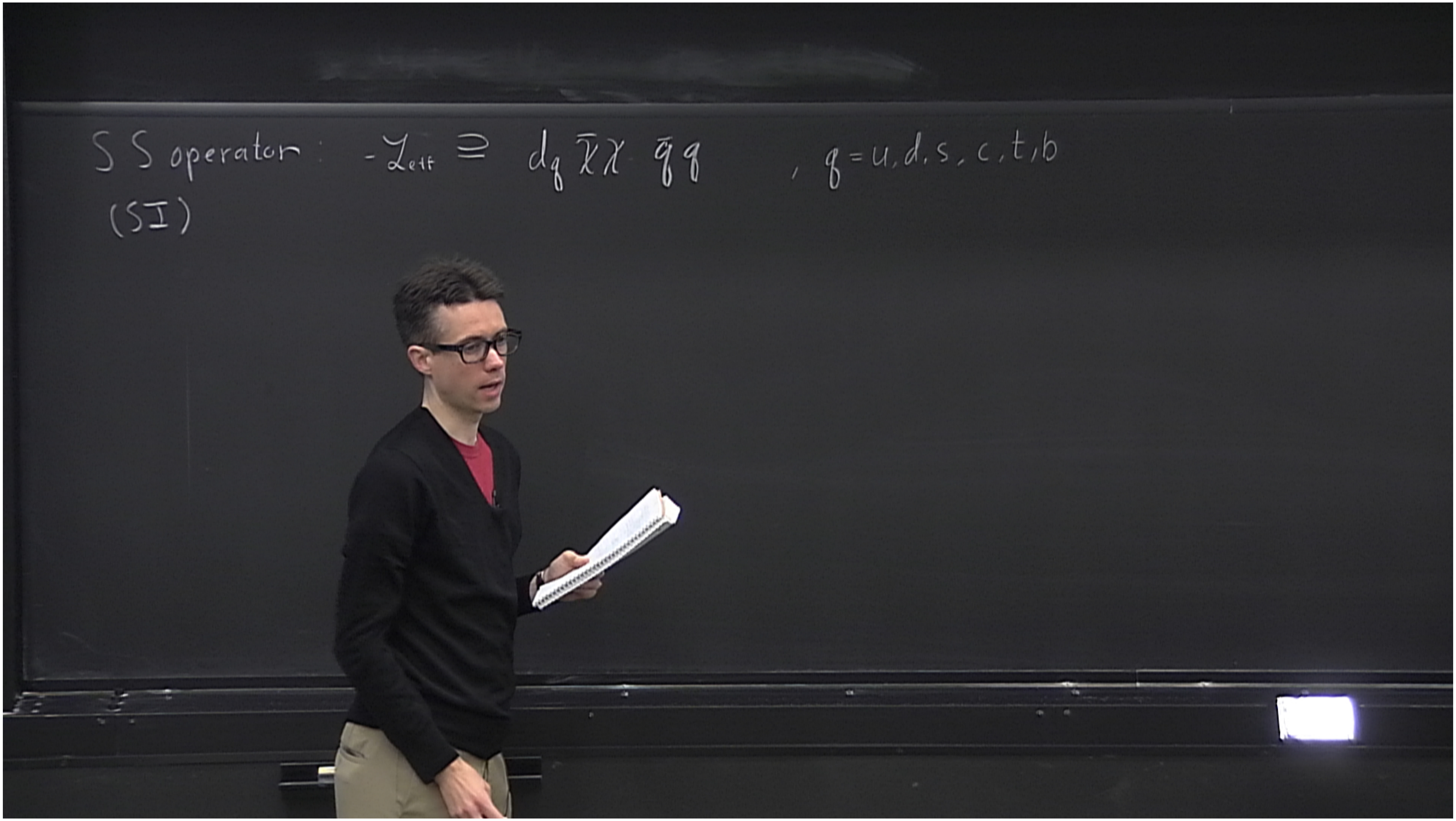
SD = AA, PP

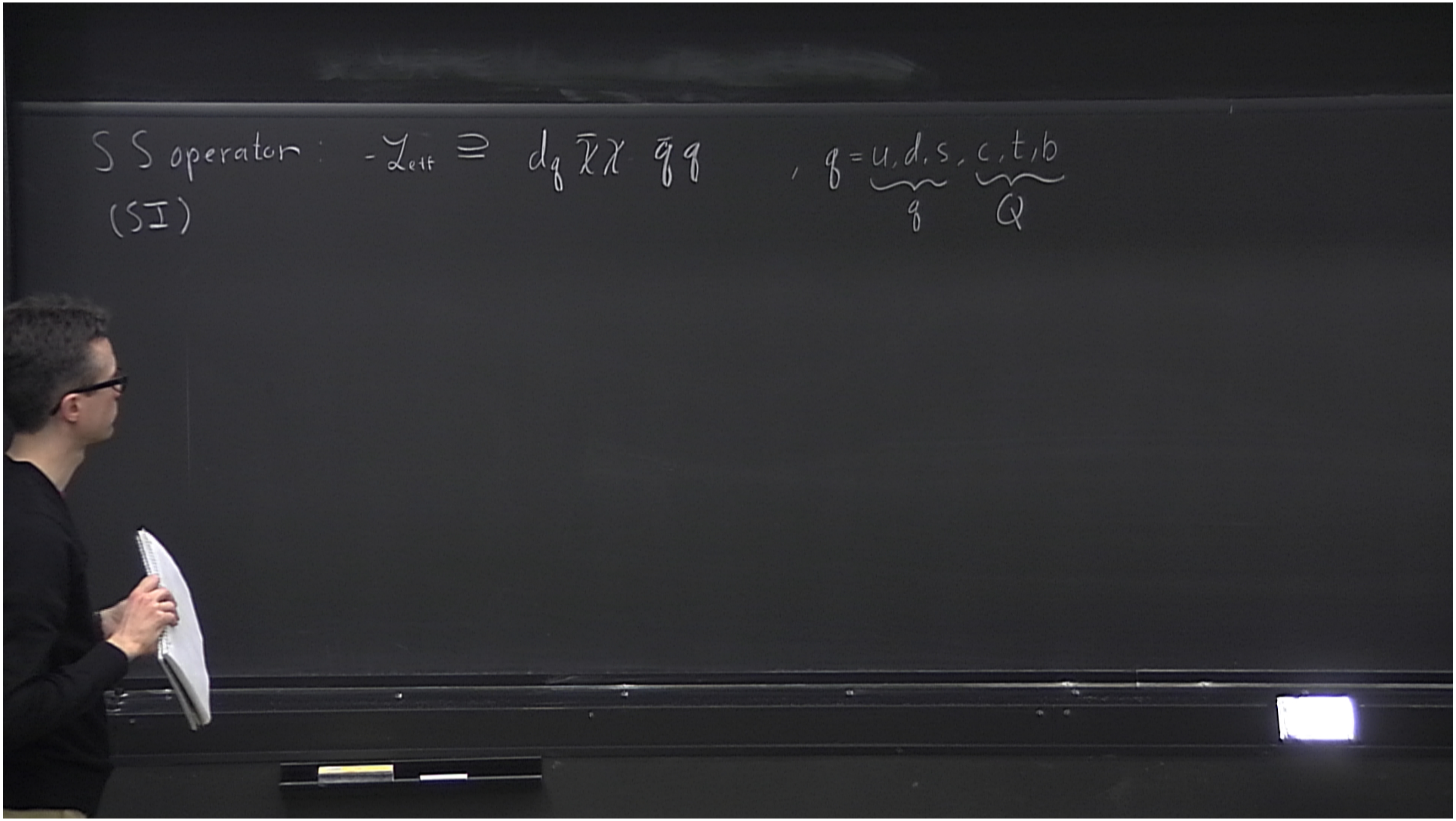
Why no SP, VA, ...

cross-sections are $\propto v^0$

cross sections $\propto v^2 \sim 10^{-6}$


S S operator :
(SI)


SS operator : $-\mathcal{L}_{eff} \supseteq d_q \bar{\chi}\chi \, \bar{q}q$, $q = u, d, s, c, t, b$
(SI)


SS operator : $-\mathcal{L}_{eff} \supseteq d_q \bar{\chi}\chi \, \bar{q}q$, $q = \underbrace{u, d, s}_{q}, \underbrace{c, t, b}_{Q}$
(SI)

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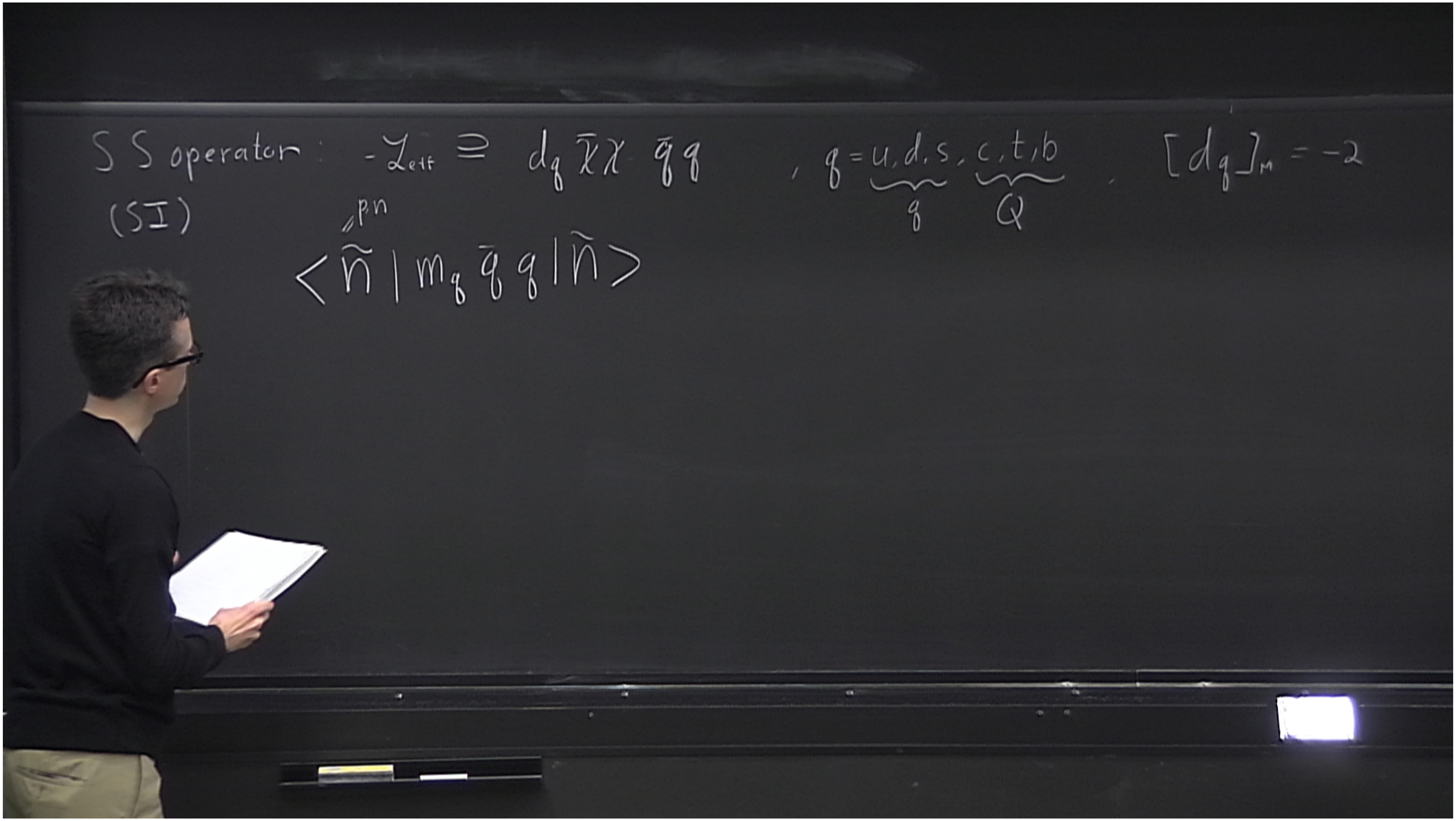
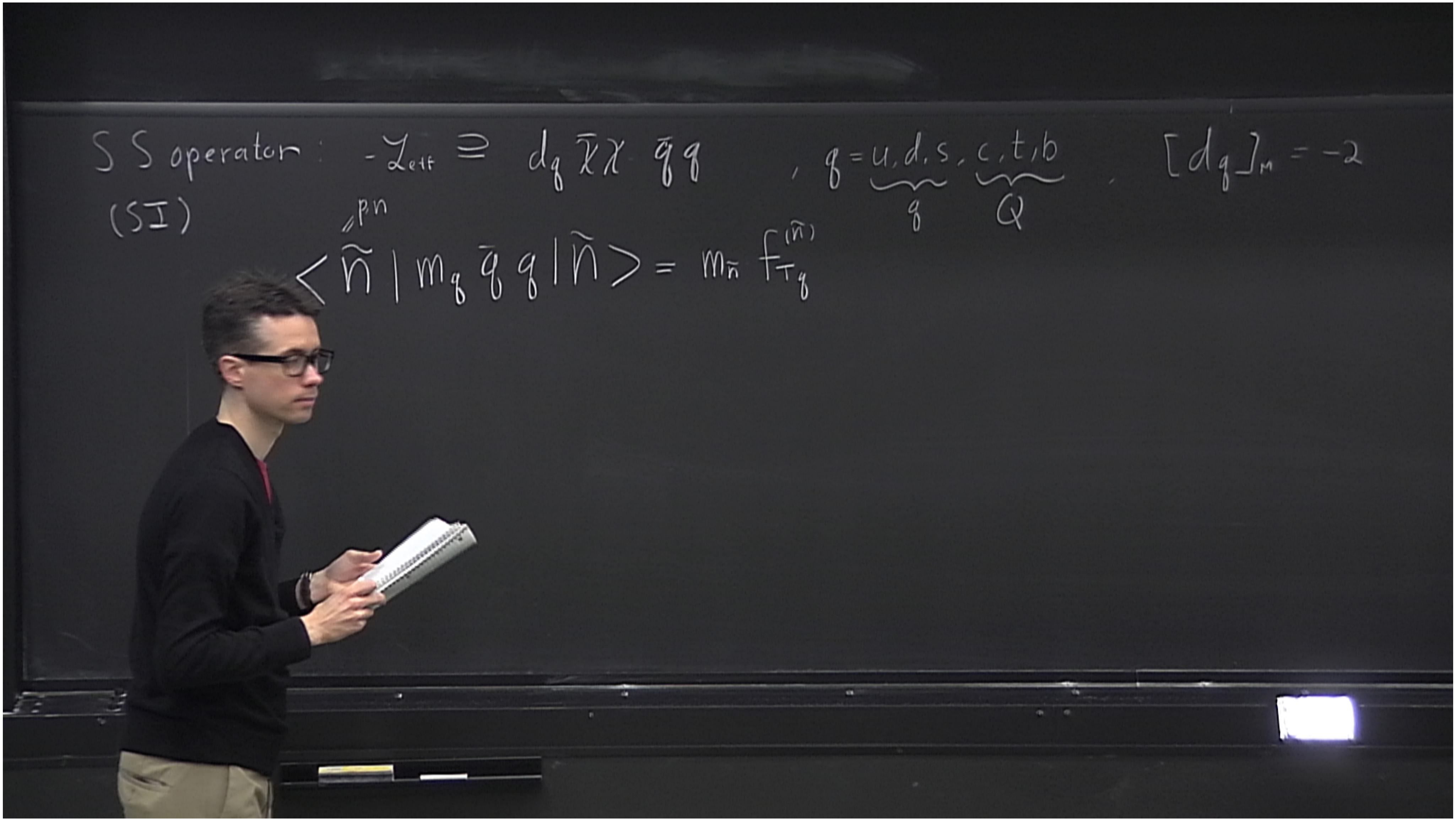
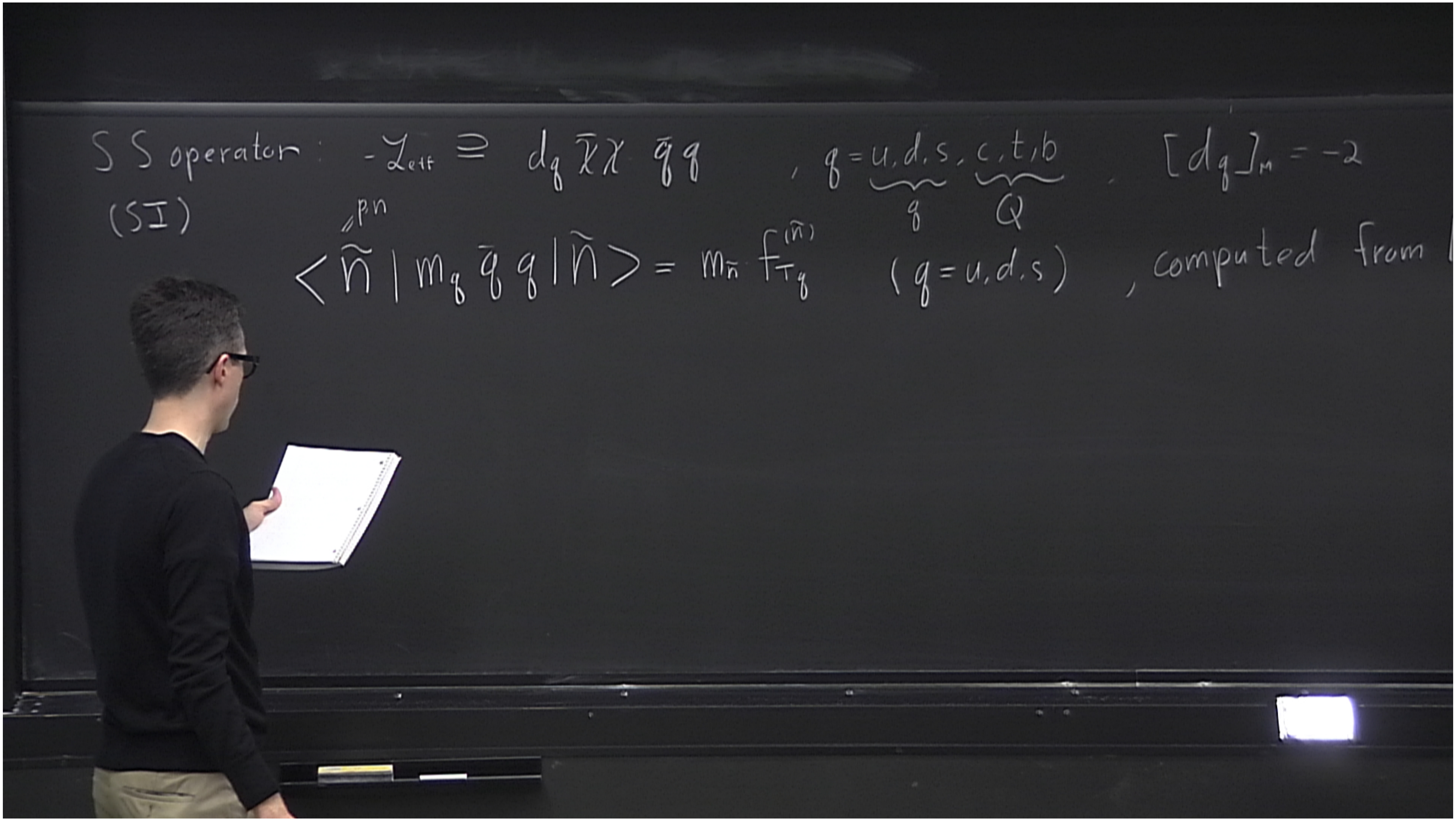
(SI)

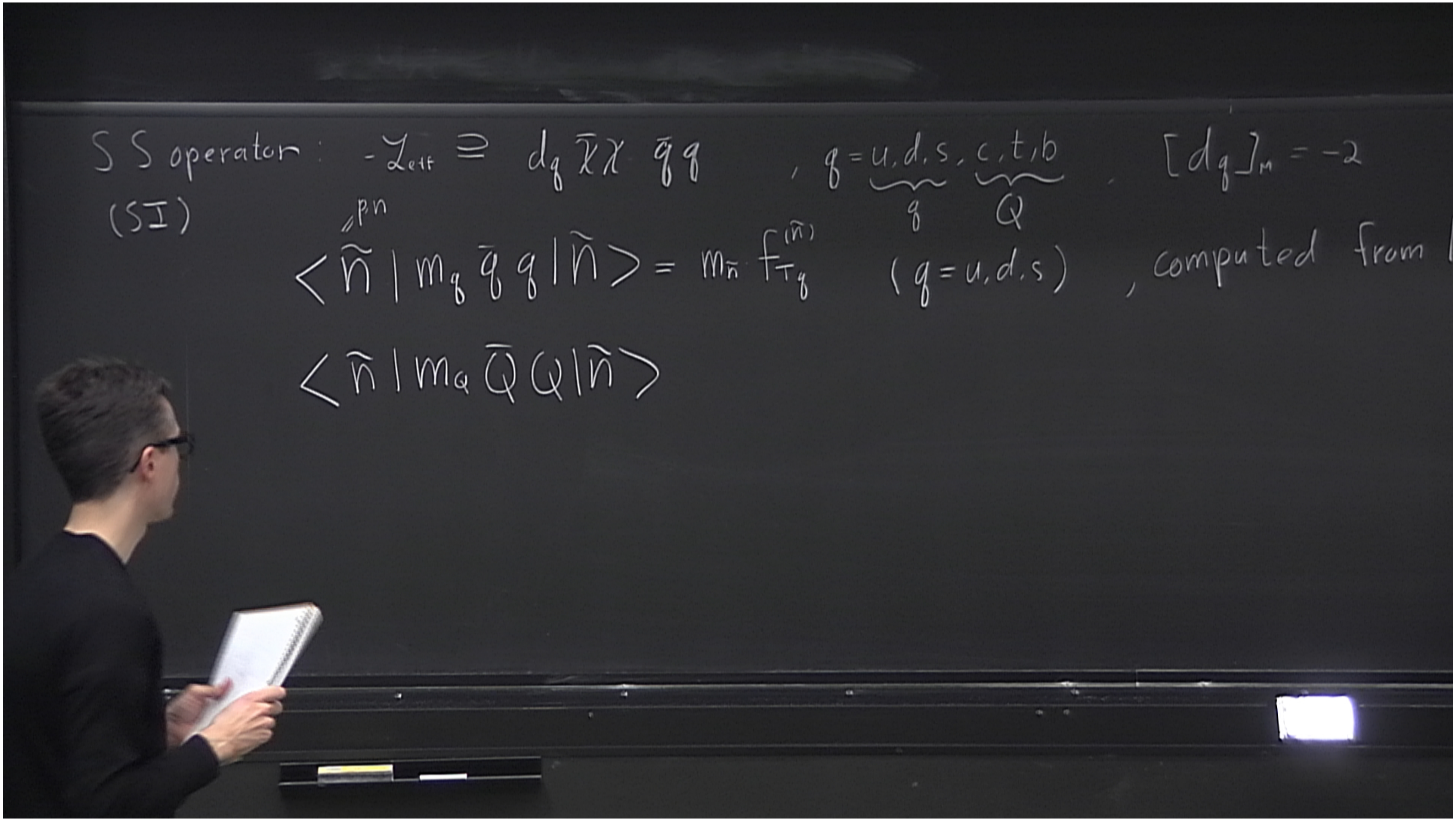

SS operator : $-\mathcal{L}_{eff} \supseteq d_q \bar{\chi}\chi\, \bar{q} q$, $q = \underbrace{u, d, s}_{q}, \underbrace{c, t, b}_{Q}$, $[d_q]_M = -2$
(SI)
$\langle \tilde{n}^{p,n} | m_q \bar{q} q | \tilde{n} \rangle$

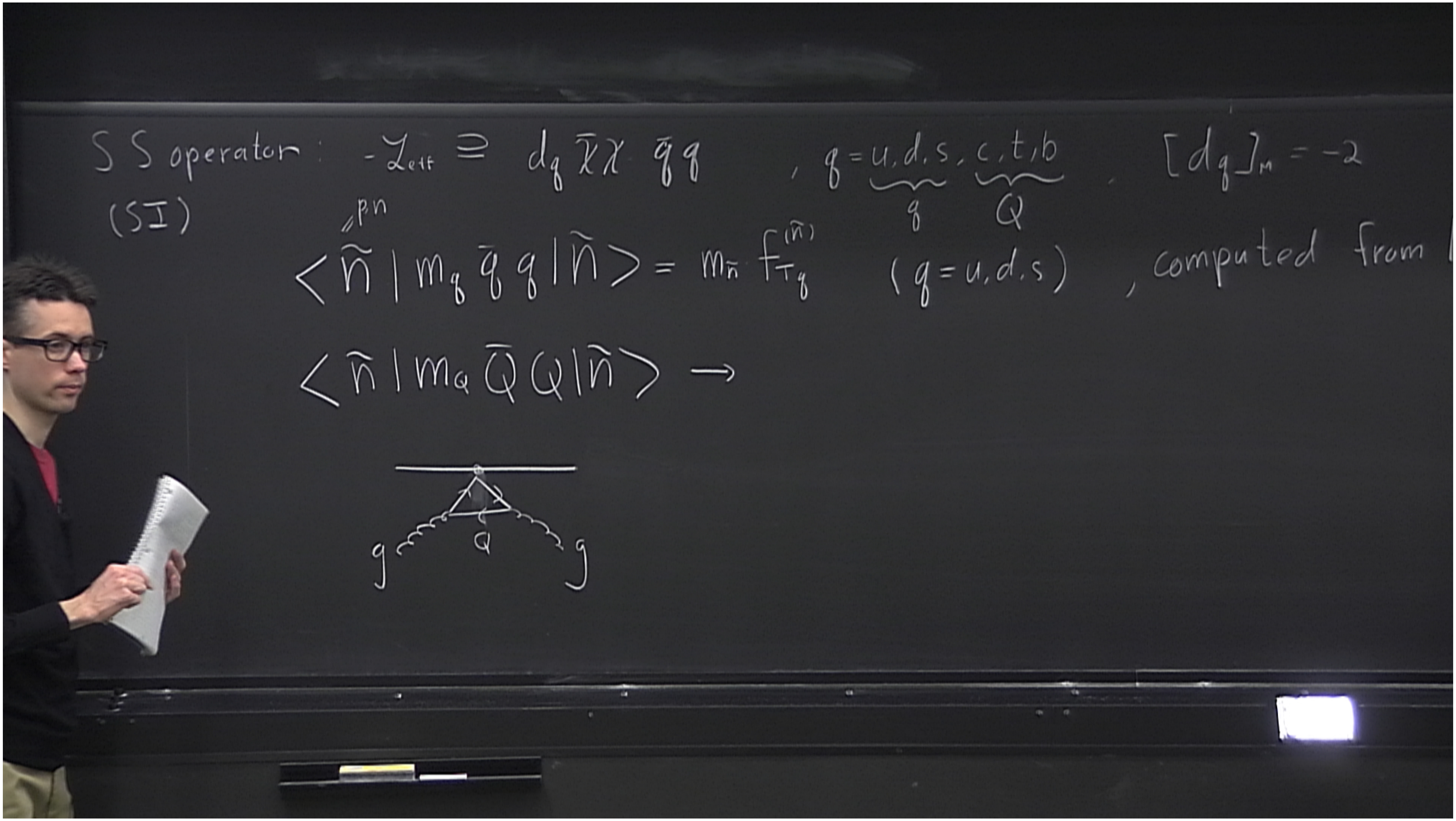

SS operator : $-\mathcal{L}_{eff} \supseteq d_q \bar{\chi}\chi \, \bar{q} q$
$, \; q = \underbrace{u, d, s}_{q}, \underbrace{c, t, b}_{Q}$
$[d_q]_M = -2$
(SI)
$\langle \tilde{n} | m_q \bar{q} q | \tilde{n} \rangle = m_{\tilde{n}} f^{(\tilde{n})}_{T_q}$
$\tilde{n} = p, n$

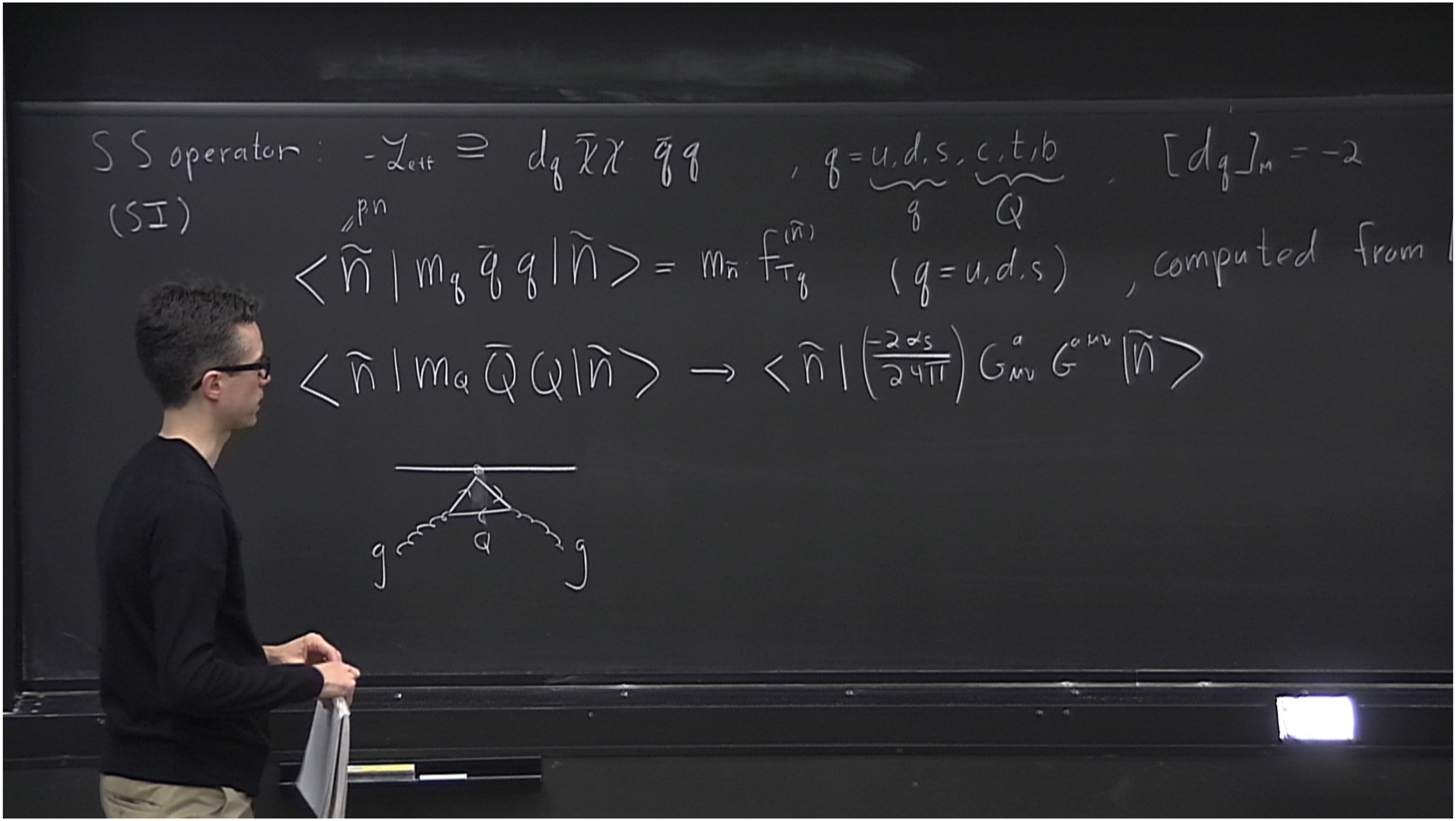
$d_q]_M = -2$

mputed from lattice QCD ; $f^{(n)}_{T_s} = f^{(p)}_{T_s} \gg f^{(\tilde{n})}_{T_{u,d}}$


SS operator : $-\mathcal{L}_{eff} \supseteq d_q \bar{\chi}\chi\, \bar{q}q$, $q = \underbrace{u,d,s}_{q}, \underbrace{c,t,b}_{Q}$, $[d_q]_M = -2$
(SI)
p,n
$\langle \tilde{n} | m_q \bar{q} q | \tilde{n} \rangle = m_{\tilde{n}} f^{(\tilde{n})}_{T_q}$ $(q = u,d,s)$, computed from


SS operator: $-\mathcal{L}_{eff} \supseteq d_q \bar{\chi}\chi\, \bar{q} q$, $q = \underbrace{u, d, s}_{q}, \underbrace{c, t, b}_{Q}$ $[d_q]_M = -2$
(SI)
p,n
$\langle \tilde{n} | m_q \bar{q} q | \tilde{n} \rangle = m_{\tilde{n}} f_{T_q}^{(\tilde{n})}$ $(q = u, d, s)$, computed from
$\langle \tilde{n} | m_Q \bar{Q} Q | \tilde{n} \rangle$

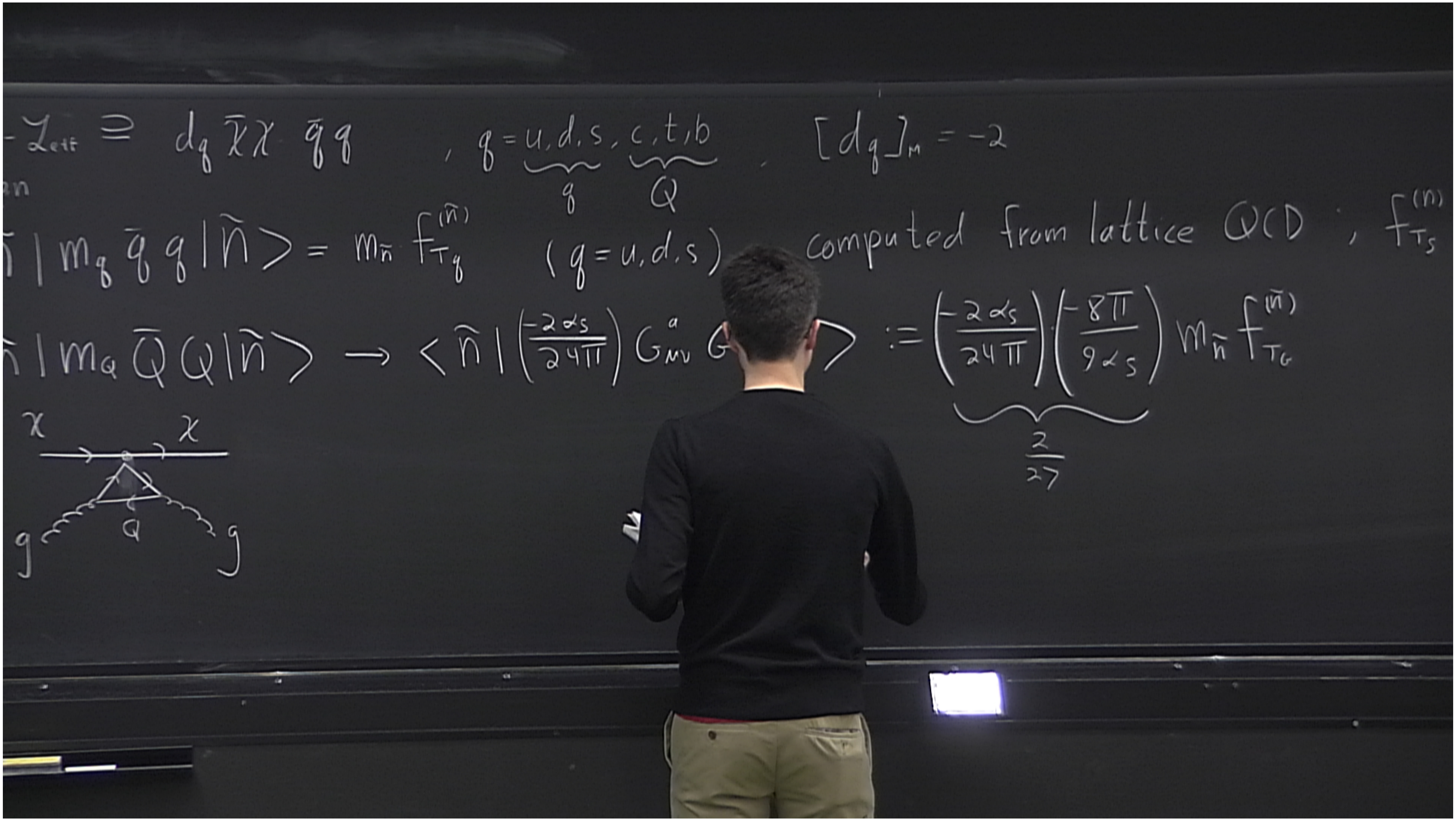

SS operator : $-\mathcal{L}_{eff} \supseteq d_q \bar{\chi}\chi \, \bar{q}q$, $q = \underbrace{u,d,s}_{q}, \underbrace{c,t,b}_{Q}$, $[d_q]_M = -2$
(SI)
$\langle \tilde{n} | m_q \bar{q} q | \tilde{n} \rangle = m_{\tilde{n}} f^{(\tilde{n})}_{T_q}$ ($q = u,d,s$) , computed from
p,n
$\langle \tilde{n} | m_Q \bar{Q} Q | \tilde{n} \rangle \rightarrow$
Q
g
g


SS operator : $-\mathcal{L}_{eff} \supseteq d_q \bar{\chi}\chi \, \bar{q} q$, $q = \underbrace{u,d,s}_{q}, \underbrace{c,t,b}_{Q}$; $[d_q]_M = -2$
(SI)
p,n
$\langle \tilde{n} | m_q \bar{q} q | \tilde{n} \rangle = m_{\tilde{n}} f_{T_q}^{(\tilde{n})}$ $(q = u,d,s)$, computed from
$\langle \tilde{n} | m_Q \bar{Q} Q | \tilde{n} \rangle \to \langle \tilde{n} | \left(\frac{-2\alpha_s}{24\pi} \right) G^a_{\mu\nu} G^{a\mu\nu} | \tilde{n} \rangle$
g
Q
g

b $\quad [d_q]_M = -2$

s) , computed from lattice QCD ; $f_{T_s}^{(n)} = f_{T_s}^{(p)} \gg f_{T_{u,d}}^{(\tilde{n})}$

$G^{a\mu\nu} |\tilde{n}\rangle := \left(-\frac{2\alpha_s}{24\pi}\right)\left(-\frac{8\pi}{9\alpha_s}\right) m_{\tilde{n}} f_{T_G}^{(\tilde{n})}$

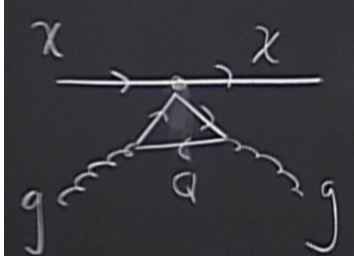

$-\mathcal{L}_{eff} \supseteq d_q \bar{\chi}\chi \, \bar{q}q$, $q = \underbrace{u, d, s}_{q}, \underbrace{c, t, b}_{Q}$, $[d_q]_M = -2$
$\langle \tilde{n} | m_q \bar{q} q | \tilde{n} \rangle = m_{\tilde{n}} f_{T_q}^{(\tilde{n})}$ $(q = u, d, s)$ computed from lattice QCD ; $f_{T_s}^{(n)}$
$\langle \tilde{n} | m_Q \bar{Q} Q | \tilde{n} \rangle \longrightarrow \langle \tilde{n} | \left(\frac{-2\alpha_s}{24\pi}\right) G^a_{\mu\nu} G$
$\rangle := \underbrace{\left(\frac{-2\alpha_s}{24\pi}\right)\left(\frac{-8\pi}{9\alpha_s}\right)}_{\frac{2}{27}} m_{\tilde{n}} f_{T_G}^{(\tilde{n})}$
χ χ Q g g

$$-\mathcal{L}_{eff} \supseteq d_q \bar{\chi}\chi\, \bar{q} q \quad , \quad q = \underbrace{u, d, s}_{q}, \underbrace{c, t, b}_{Q} \quad , \quad [d_q]_M = -2$$

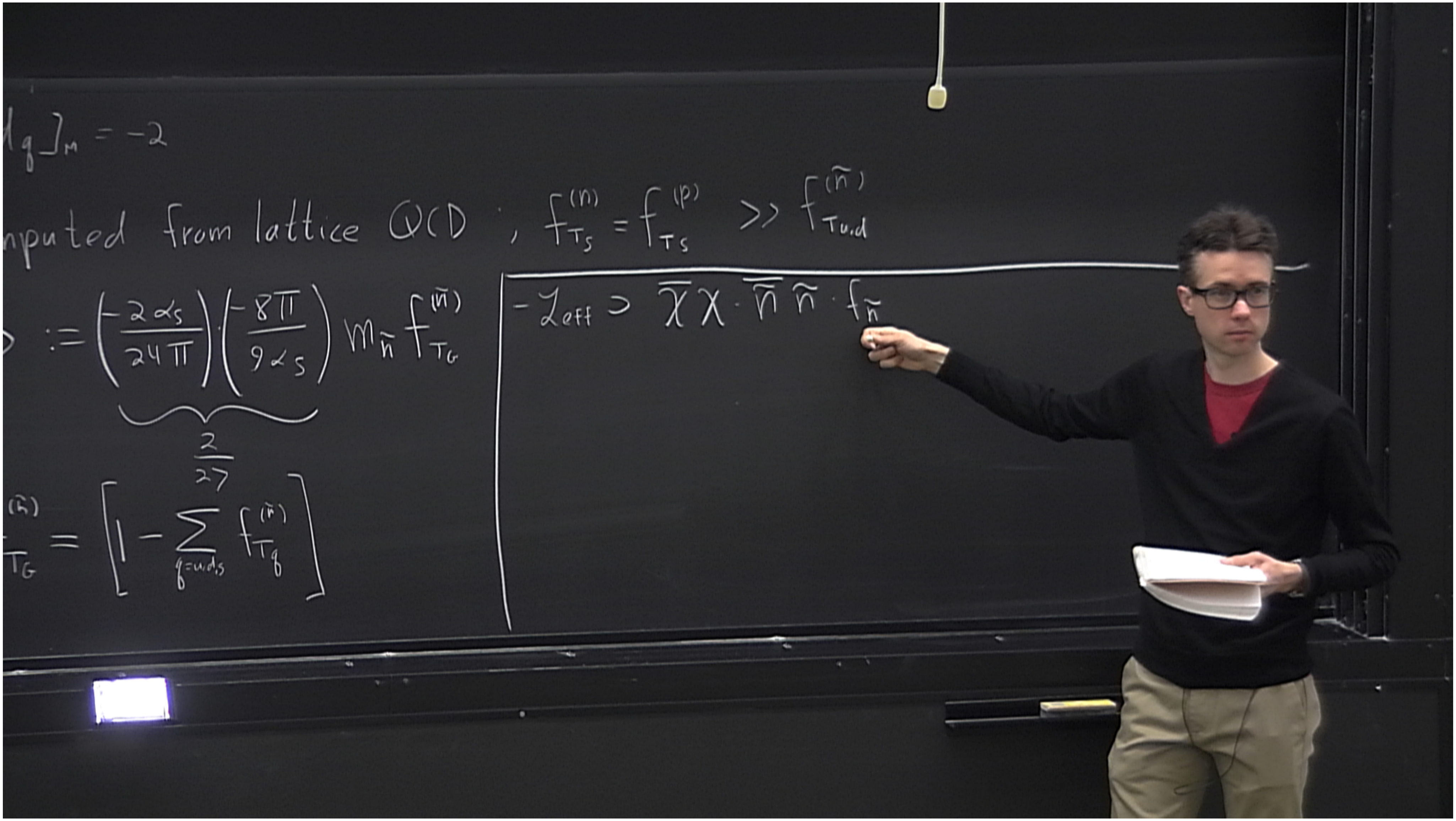
$$\langle \tilde{n} | m_q \bar{q} q | \tilde{n} \rangle = m_{\tilde{n}} f^{(\tilde{n})}_{T_q} \quad (q = u, d, s)$$

, computed from lattice QCD ; $f^{(n)}_{T_s}$

$$\langle \tilde{n} | m_Q \bar{Q} Q | \tilde{n} \rangle \longrightarrow \langle \tilde{n} | \left(\frac{-2\alpha_s}{24\pi}\right) G^a_{\mu\nu} G^{a\mu\nu} | \tilde{n} \rangle := \underbrace{\left(\frac{-2\alpha_s}{24\pi}\right)\left(\frac{-8\pi}{9\alpha_s}\right)}_{\frac{2}{27}} m_{\tilde{n}} f^{(\tilde{n})}_{T_G}$$



$$f^{(\tilde{n})}_{T_G} = \left[1 - \sum_{q=u,d,s} f^{(\tilde{n})}_{T_q}\right]$$


$\ldots d_q]_M = -2$
...mputed from lattice QCD ; $f^{(n)}_{T_s} = f^{(p)}_{T_s} \gg f^{(\tilde{n})}_{T_{u,d}}$
$:= \left(-\frac{2\alpha_s}{24\pi}\right)\left(-\frac{8\pi}{9\alpha_s}\right) m_{\tilde{n}} f^{(\tilde{n})}_{T_G}$
$\frac{2}{27}$
$f^{(\tilde{n})}_{T_G} = \left[1 - \sum_{q=u,d,s} f^{(\tilde{n})}_{T_q}\right]$
$-\mathcal{L}_{eff} \supset \bar{\chi}\chi \cdot \bar{\tilde{n}}\tilde{n} \cdot f_{\tilde{n}}$

$d_q]_M = -2$

...mputed from lattice QCD ; $f_{T_s}^{(n)} = f_{T_s}^{(p)} >> f_{T_{u,d}}^{(\tilde{n})}$

$$\ldots := \underbrace{\left(-\frac{2\alpha_s}{24\pi}\right)\left(-\frac{8\pi}{9\alpha_s}\right)}_{\frac{2}{27}} m_{\tilde{n}} f_{T_G}^{(\tilde{n})}$$

$$f_{T_G}^{(\tilde{n})} = \left[1 - \sum_{q=u,d,s} f_{T_q}^{(\tilde{n})}\right]$$

$$-\mathcal{L}_{eff} \supset \bar{\chi}\chi \cdot \bar{\tilde{n}}\tilde{n} \cdot f_{\tilde{n}}$$

$$\frac{f_p}{m_p} = \sum_{q=u,d,s} \frac{f_{T_q}^{(p)}}{m_q} \cdot d_q + \frac{2}{27} \sum_{Q=c,t,b} \frac{d_Q}{m_Q} f_{T_G}^{(p)}$$

$d_q]_M = -2$

...mputed from lattice QCD ; $f^{(n)}_{T_s} = f^{(p)}_{T_s} \gg f^{(\tilde{n})}_{T_{u,d}}$

$$:= \underbrace{\left(-\frac{2\alpha_s}{24\pi}\right)\left(-\frac{8\pi}{9\alpha_s}\right)}_{\frac{2}{27}} m_{\tilde{n}} f^{(\tilde{n})}_{T_G}$$

$$f^{(\tilde{n})}_{T_G} = \left[1 - \sum_{q=u,d,s} f^{(\tilde{n})}_{T_q}\right]$$

$$-\mathcal{L}_{eff} \supset \bar{\chi}\chi \cdot \bar{\tilde{n}}\tilde{n} \cdot f_{\tilde{n}}$$

$$f^{(n)}_{T_G} \;//$$

$$\frac{f_p}{m_p} = \sum_{q=u,d,s} \frac{f^{(p)}_{T_q}}{m_q} \cdot d_q + \frac{2}{27} \sum_{Q=c,t,b} \frac{d_Q}{m_Q} \cdot f^{(p)}_{T_G}$$

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$$f^{(\tilde{n})}_{T_G} = \left[1 - \sum_{q=u,d,s} f^{(\tilde{n})}_{T_q}\right]$$

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$$\frac{f_p}{m_p} = \sum_{q=u,d,s} \frac{f^{(p)}_{T_q}}{m_q} \cdot d_q + \frac{2}{27} \sum_{Q=c,t,b} \frac{d_Q}{m_Q} \cdot f^{(p)}_{T_G}$$

$$\simeq \frac{f_n}{m_n}$$

$d_q]_M = -2$

...mputed from lattice QCD ; $f^{(n)}_{T_s} = f^{(p)}_{T_s} \gg f^{(\tilde{n})}_{T_{u,d}}$

$$\ldots := \underbrace{\left(-\frac{2\alpha_s}{24\pi}\right)\left(-\frac{8\pi}{9\alpha_s}\right)}_{\frac{2}{27}} m_{\tilde{n}} f^{(\tilde{n})}_{T_G}$$

$$f^{(\tilde{n})}_{T_G} = \left[1 - \sum_{q=u,d,s} f^{(\tilde{n})}_{T_q}\right]$$

$$-\mathcal{L}_{eff} \supset \bar{\chi}\chi \cdot \bar{\tilde{n}}\tilde{n} \cdot f_{\tilde{n}}$$

$\parallel$ $f^{(n)}_{T_G}$

$$\frac{f_p}{m_p} = \sum_{q=u,d,s} \frac{f^{(p)}_{T_q}}{m_q} \cdot d_q + \frac{2}{27} \sum_{Q=c,t,b} \frac{d_Q}{m_Q} \cdot f^{(p)}_{T_G}$$

$$\simeq \frac{f_n}{m_n}$$