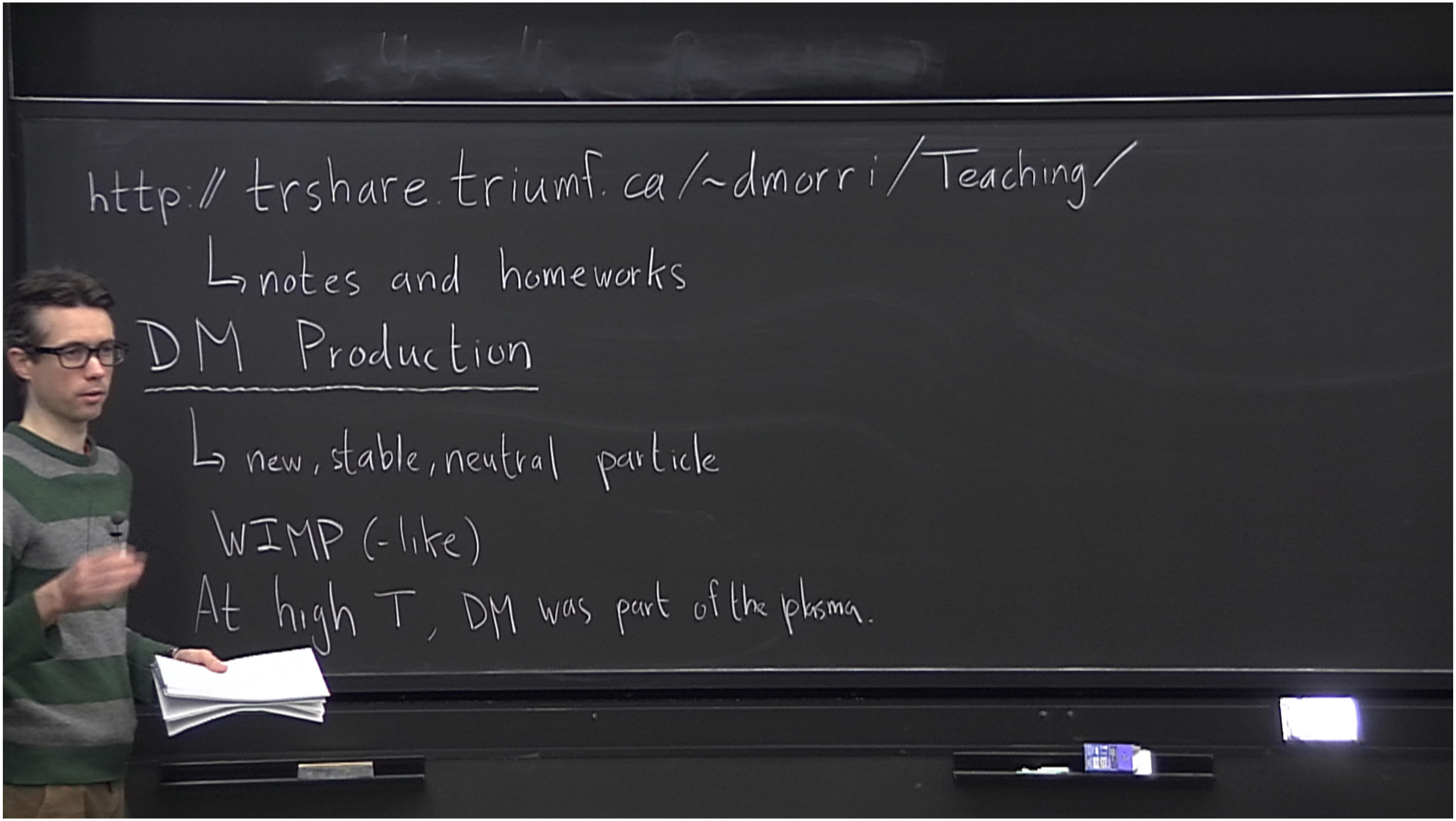
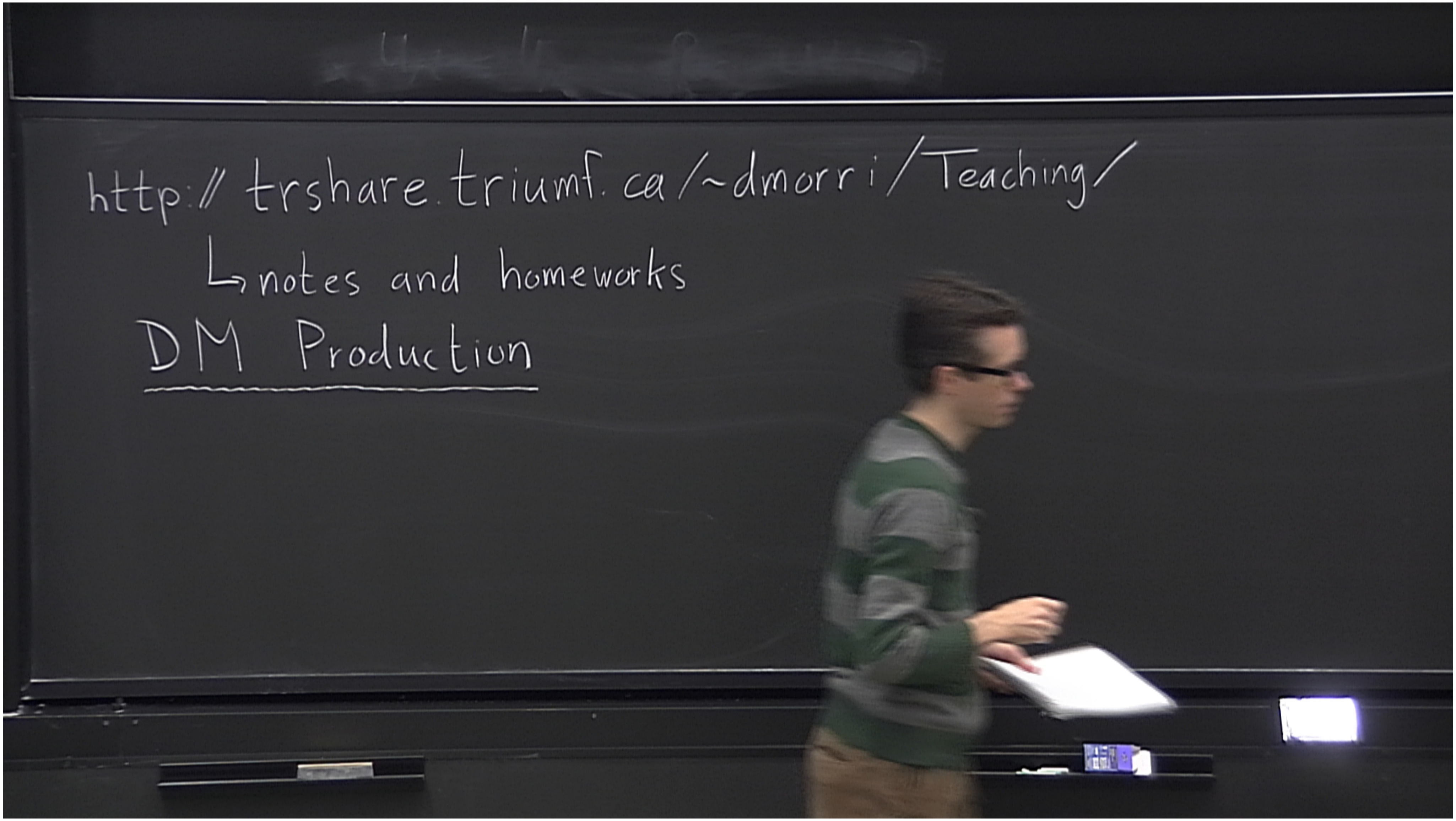
Title: Explorations in Particle Theory - Lecture 3

Date: Apr 04, 2012 09:00 AM

URL: http://pirsa.org/12040002

Abstract:


http://trshare.triumf.ca/~dmorri/Teaching/
↳ notes and homeworks
DM Production
↳ new, stable, neutral particle
WIMP (-like)
At high T, DM was part of the plasma.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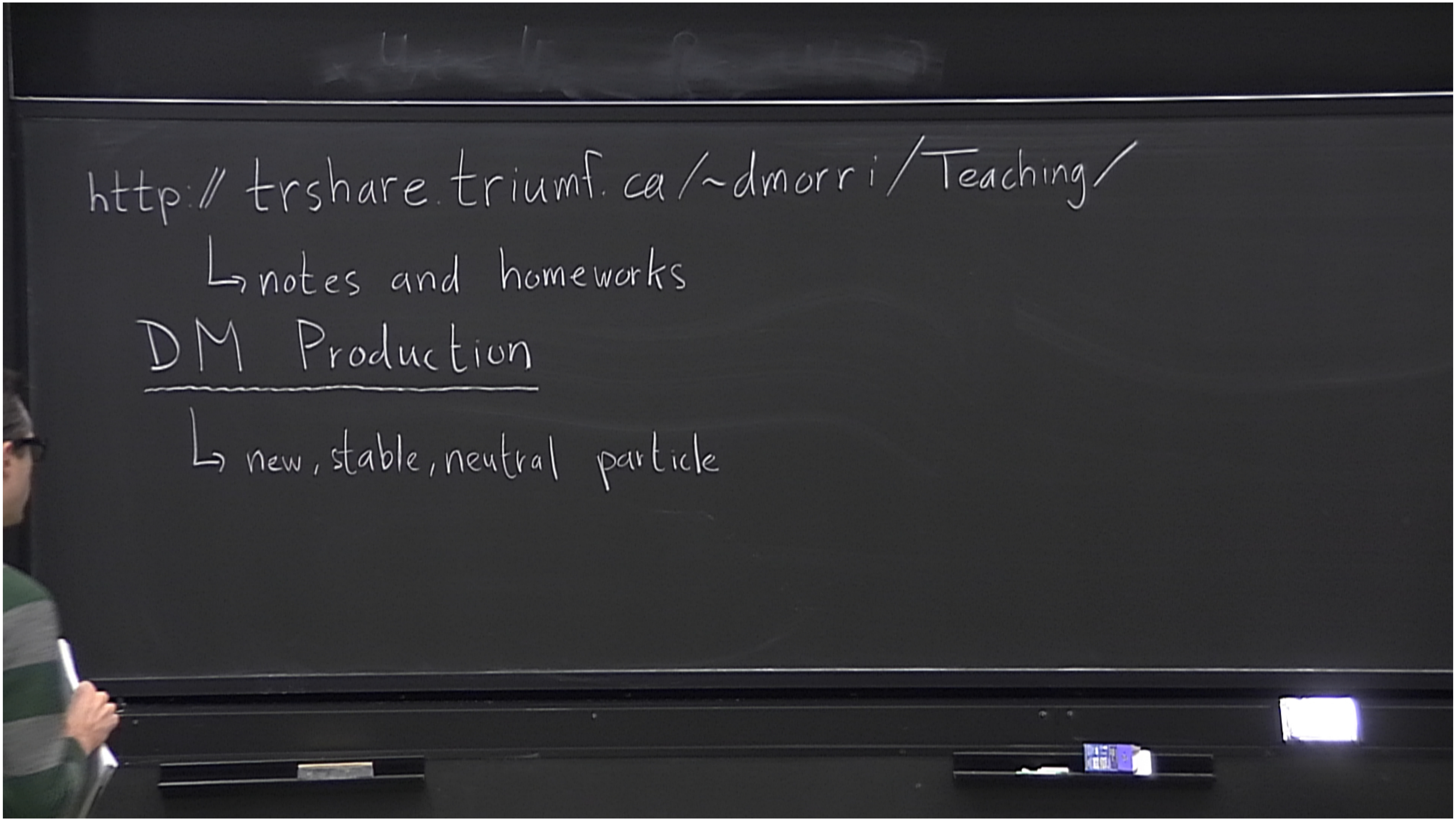
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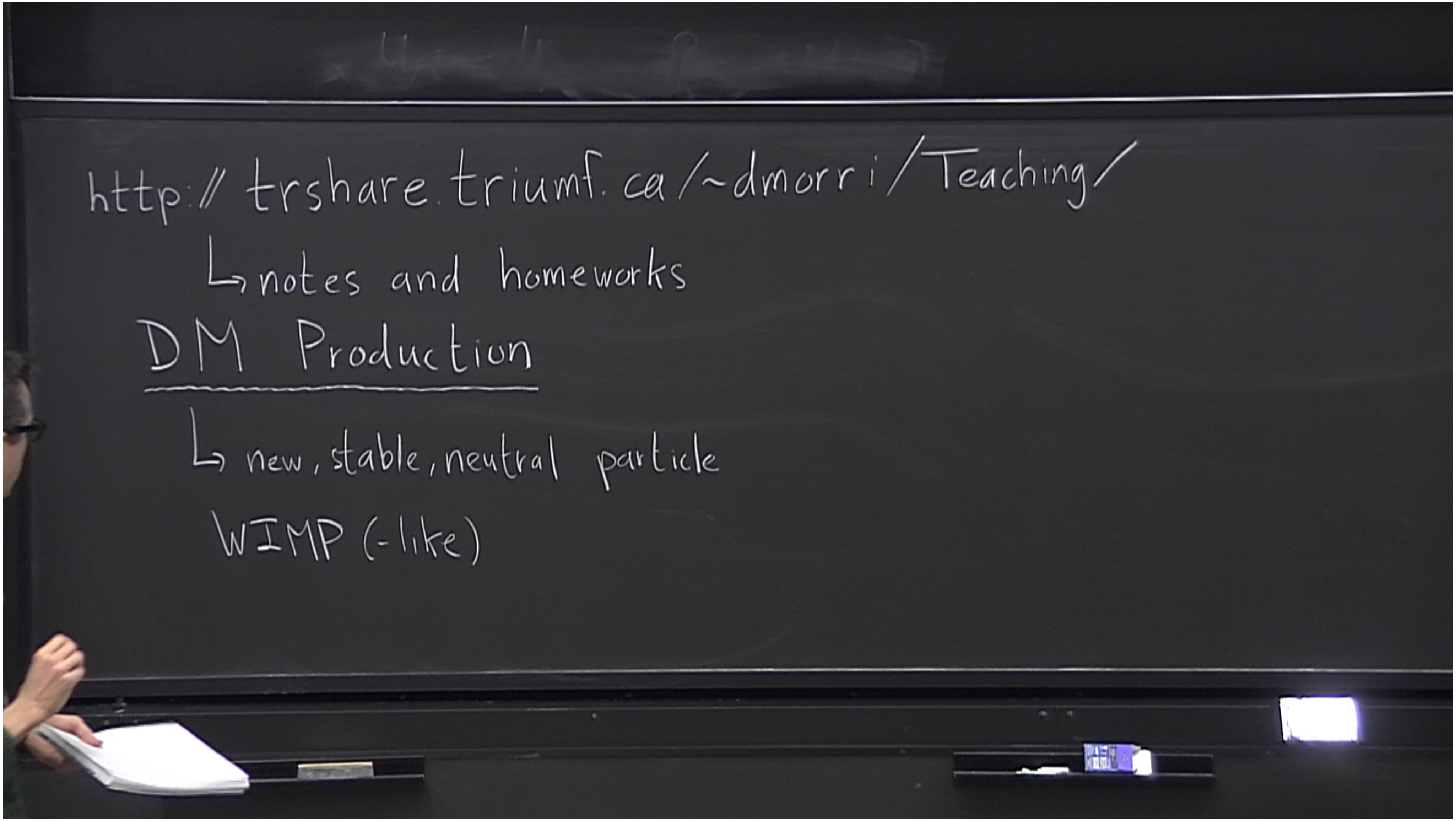
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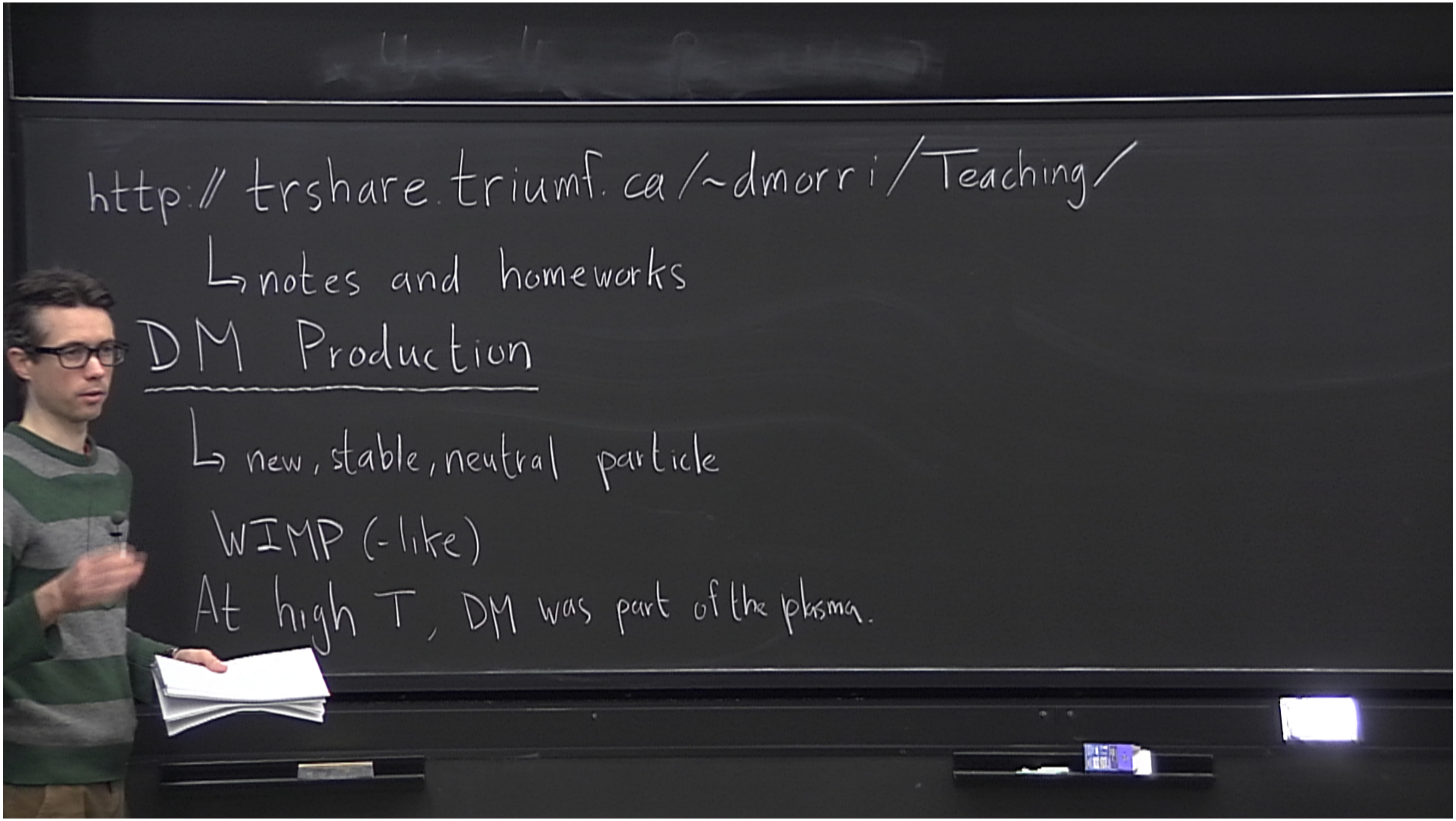
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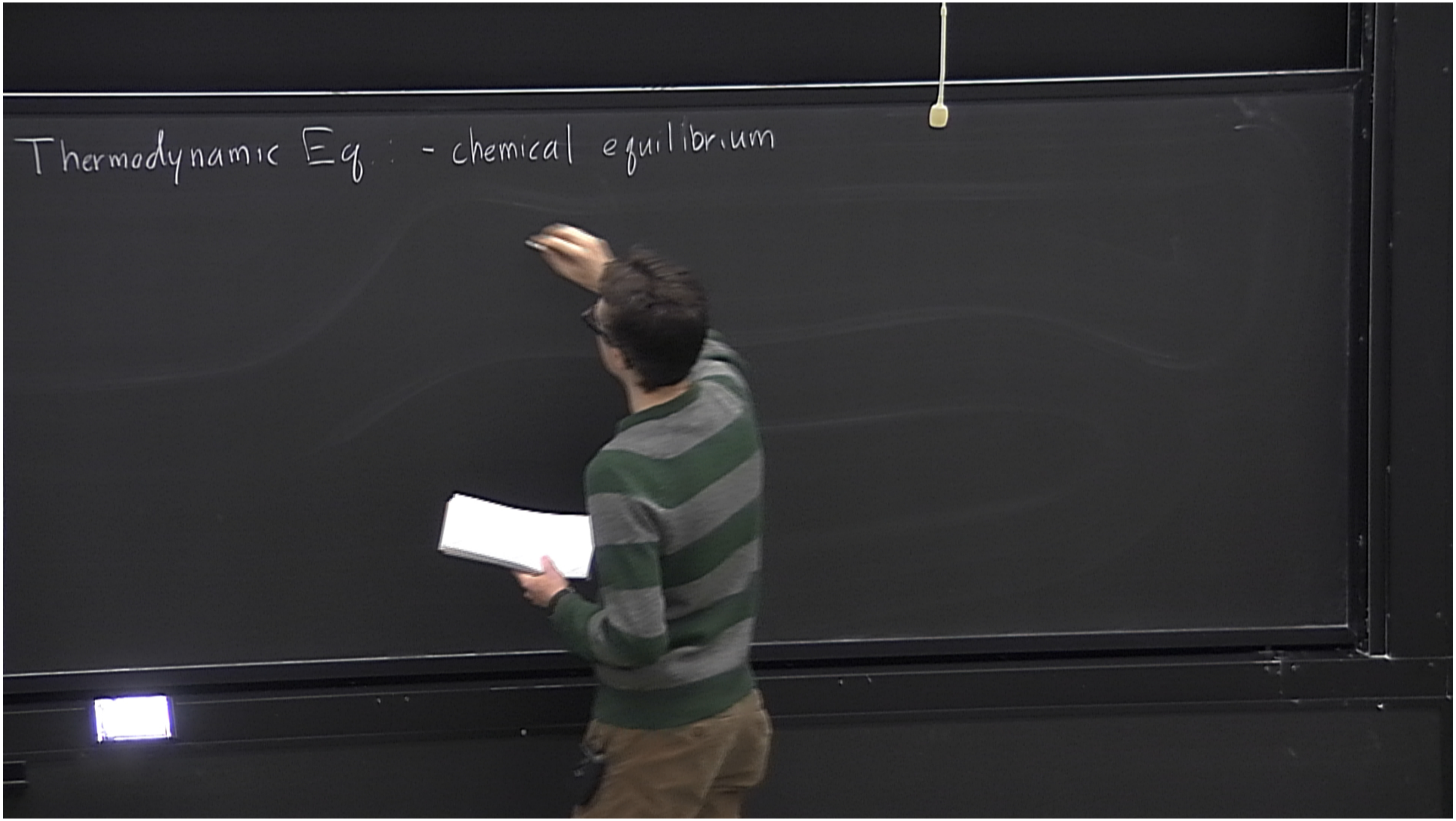
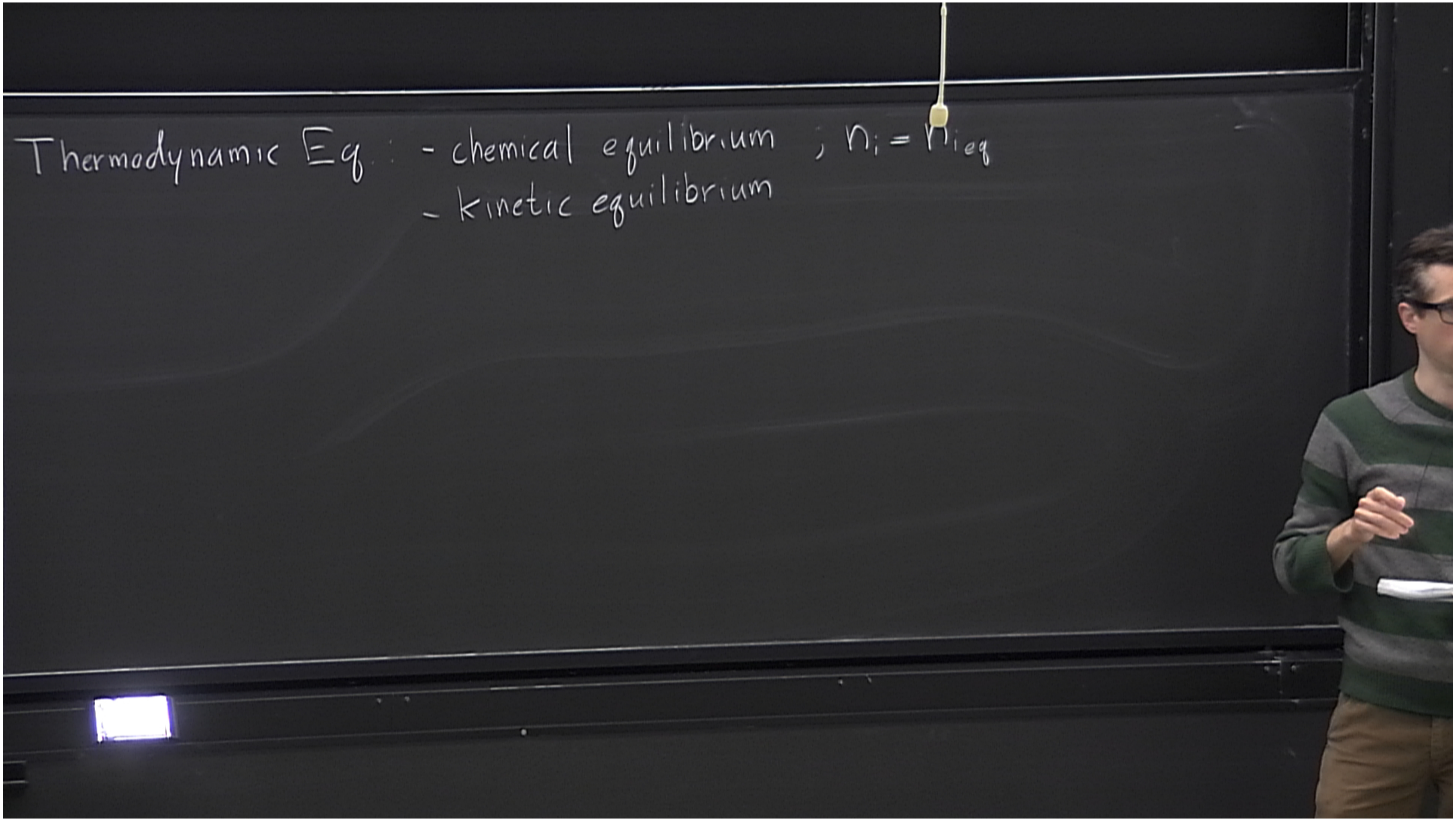
Thermodynam


Thermodynamic Eq : - chemical equilibrium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- kinetic equilibrium

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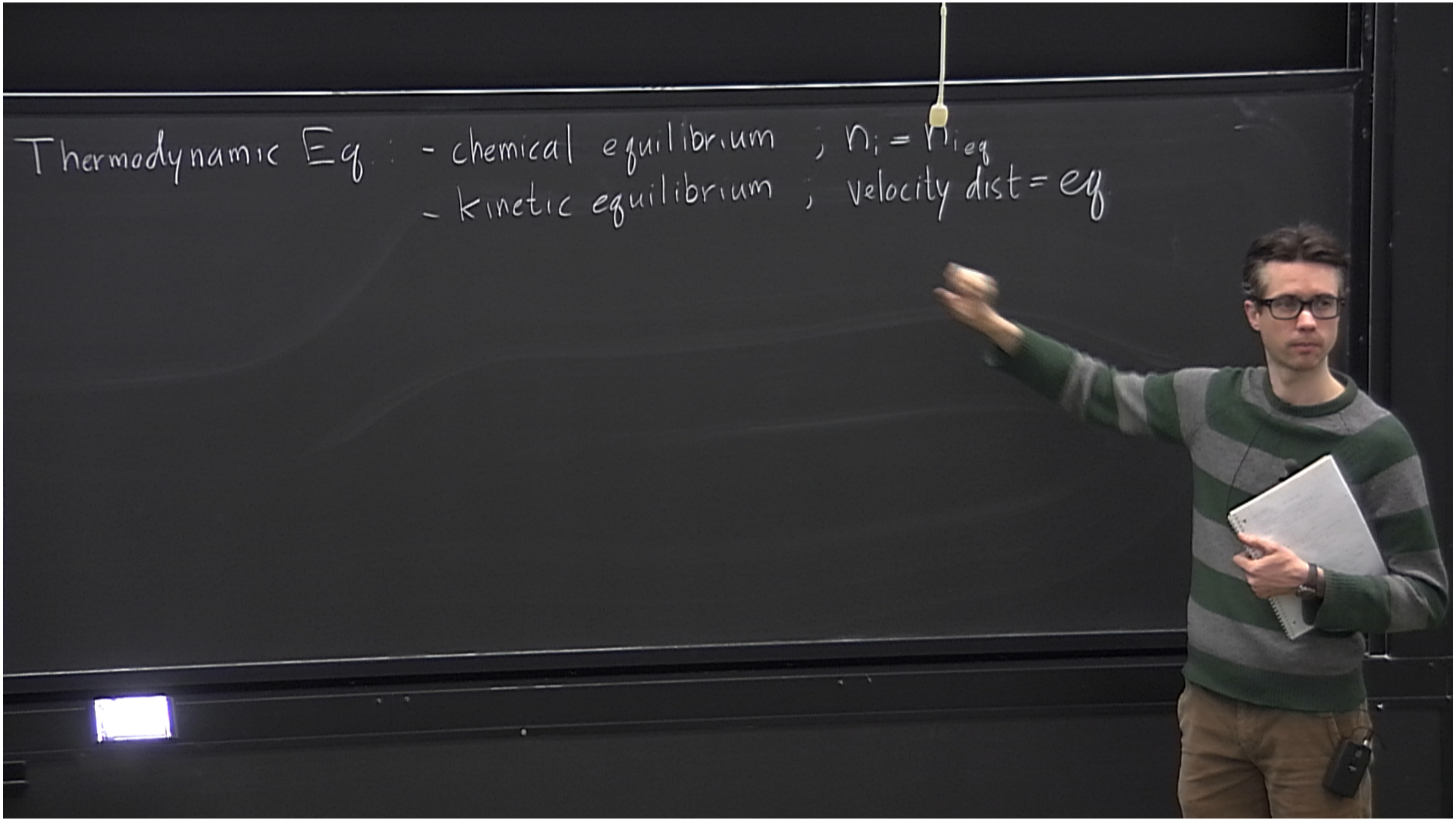
- kinetic equilibrium ; velocity dist = eq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DM = χ is stable due to a parity symmetry

$[\chi] = \text{odd}$, $[SM] = \text{even}$

$\Rightarrow$ DM can only be created or destroyed pairs.

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- kinetic equilibrium ; velocity dist $=$ eq

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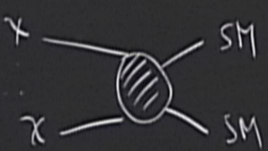
Chem Eq:



Thermodynamic Eq : - chemical equilibrium ; $n_i = n_{i\,eq}$

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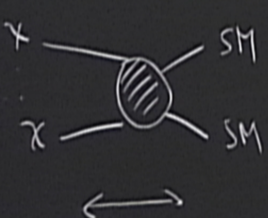
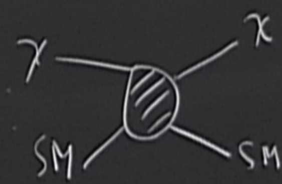
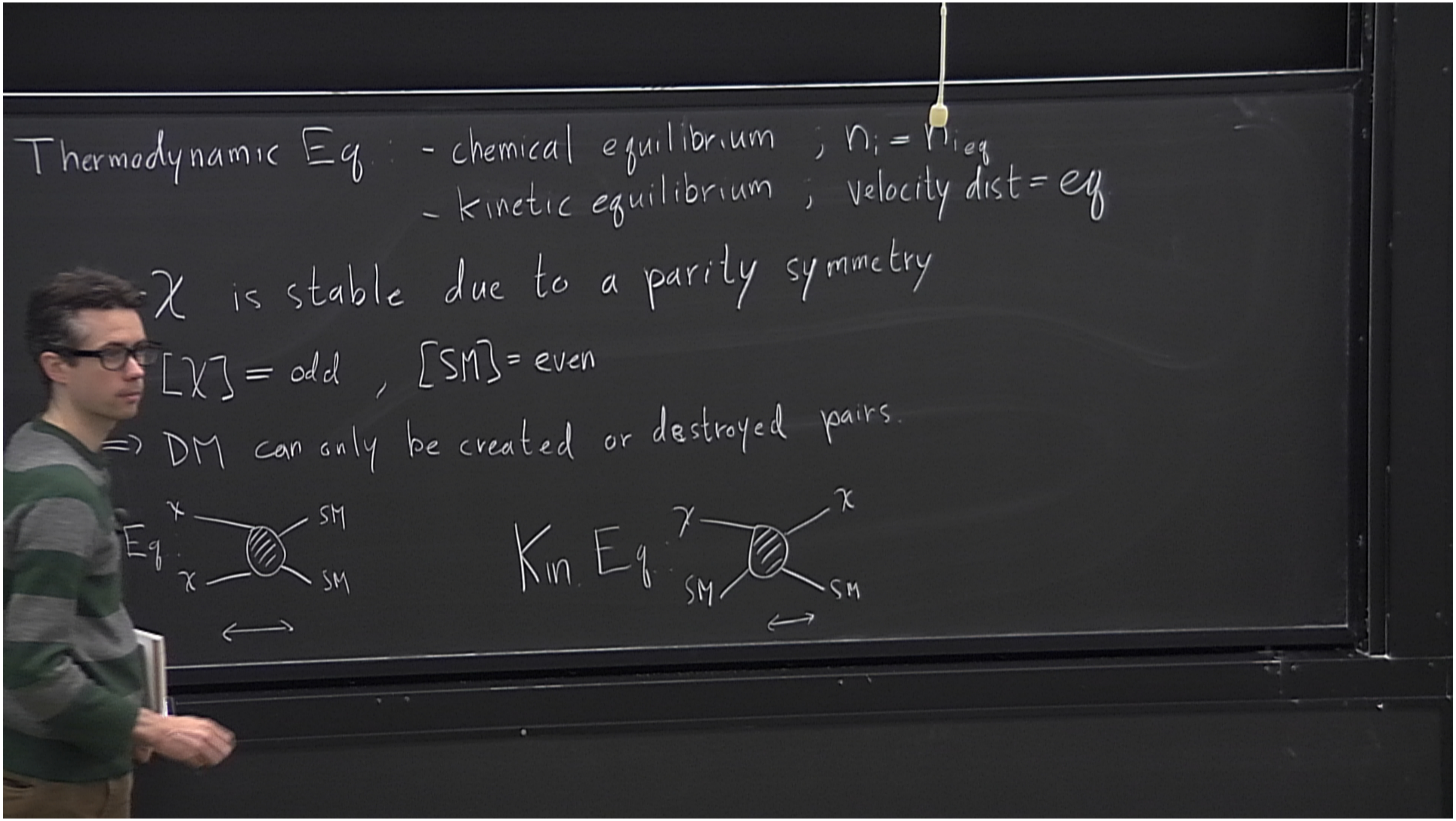
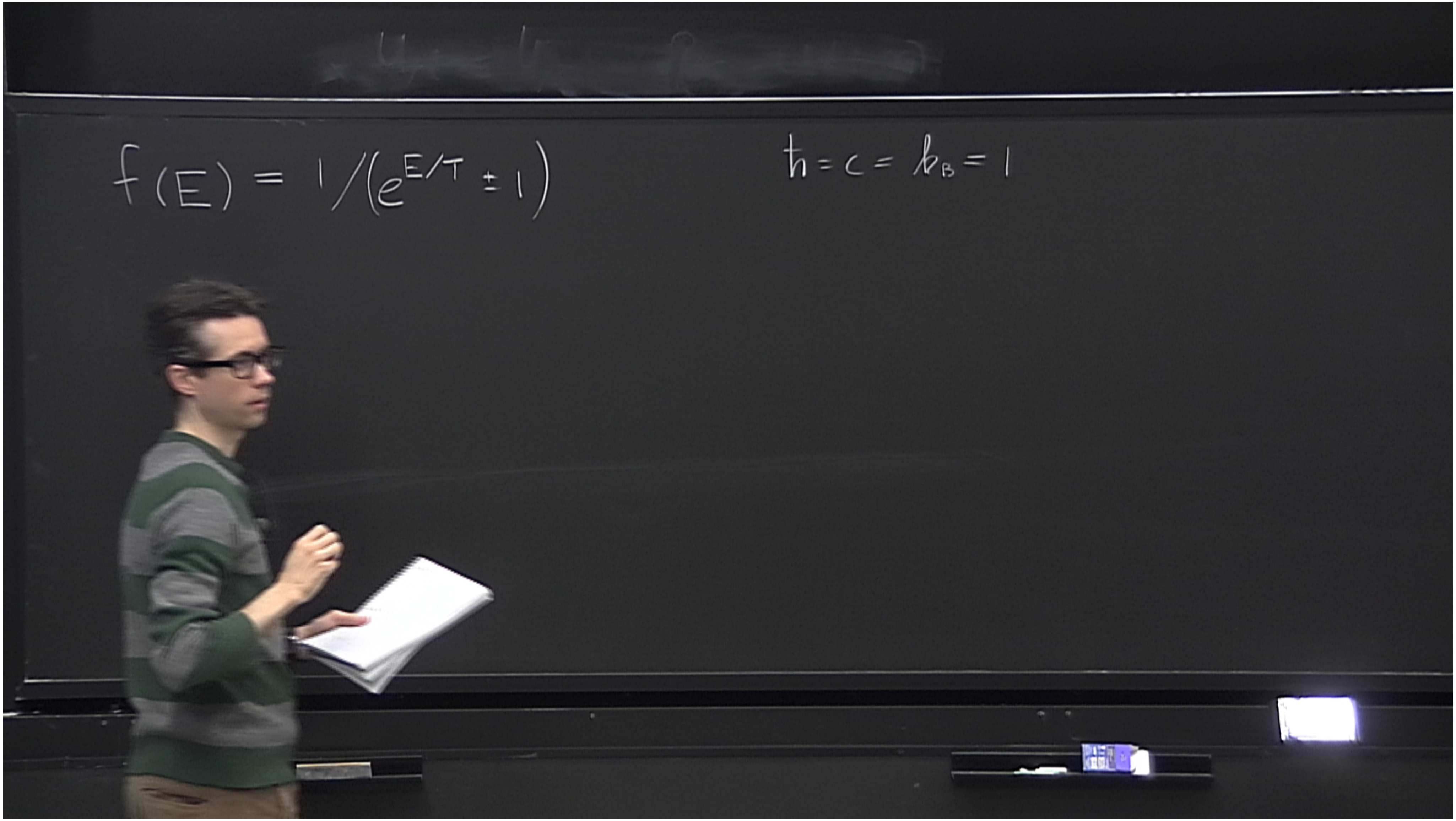
Chem. Eq :

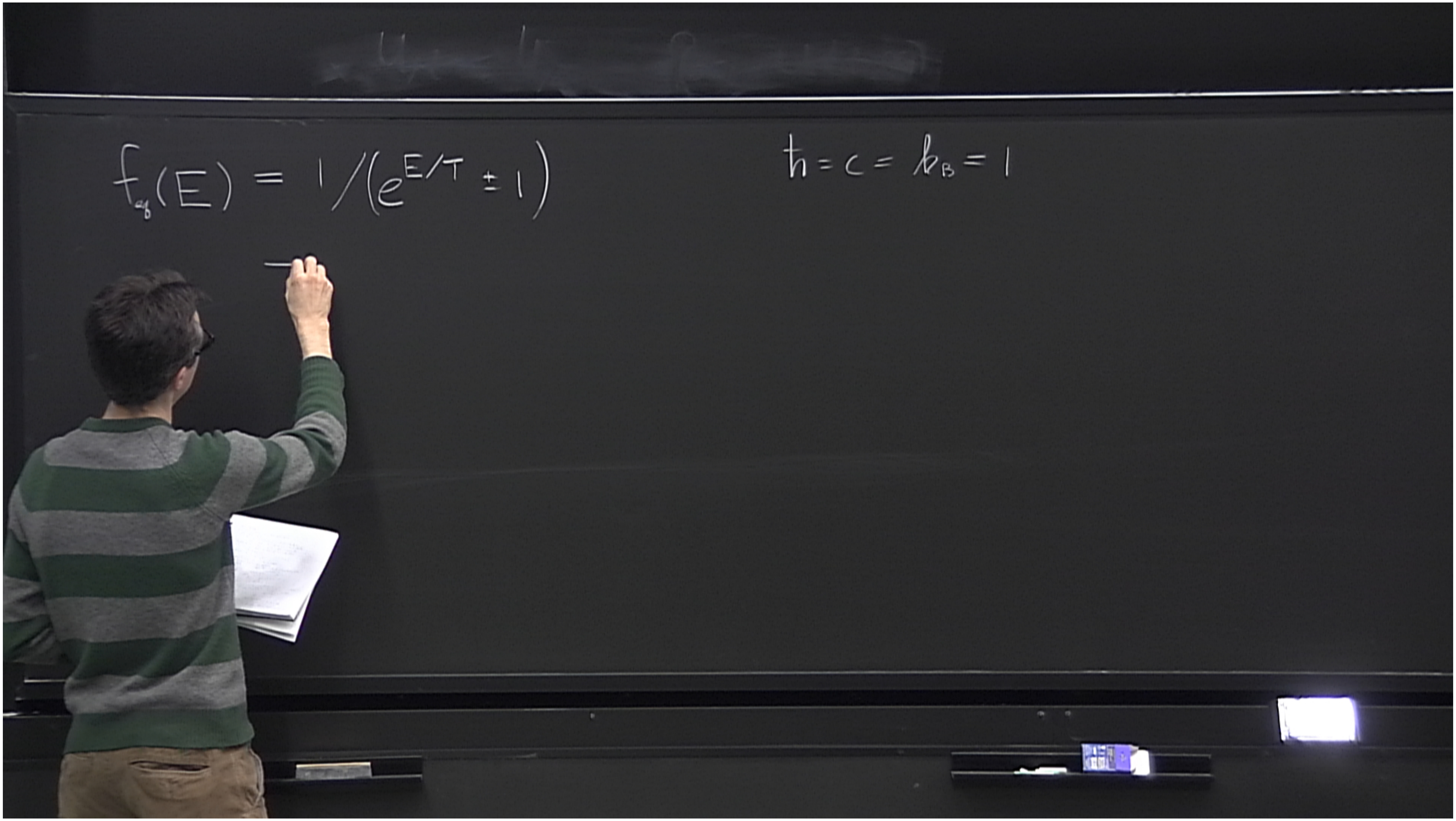


Kin. Eq :




Thermodynamic Eq : - chemical equilibrium ; n_i = n_ieq
- kinetic equilibrium ; velocity dist = eq
χ is stable due to a parity symmetry
[χ] = odd , [SM] = even
=> DM can only be created or destroyed pairs.
Eq:
χ
χ
SM
SM
Kin Eq:
χ
SM
χ
SM


$f(E) = 1/(e^{E/T} \pm 1)$
$\hbar = c = k_B = 1$


$f_{eq}(E) = 1/(e^{E/T} \pm 1)$
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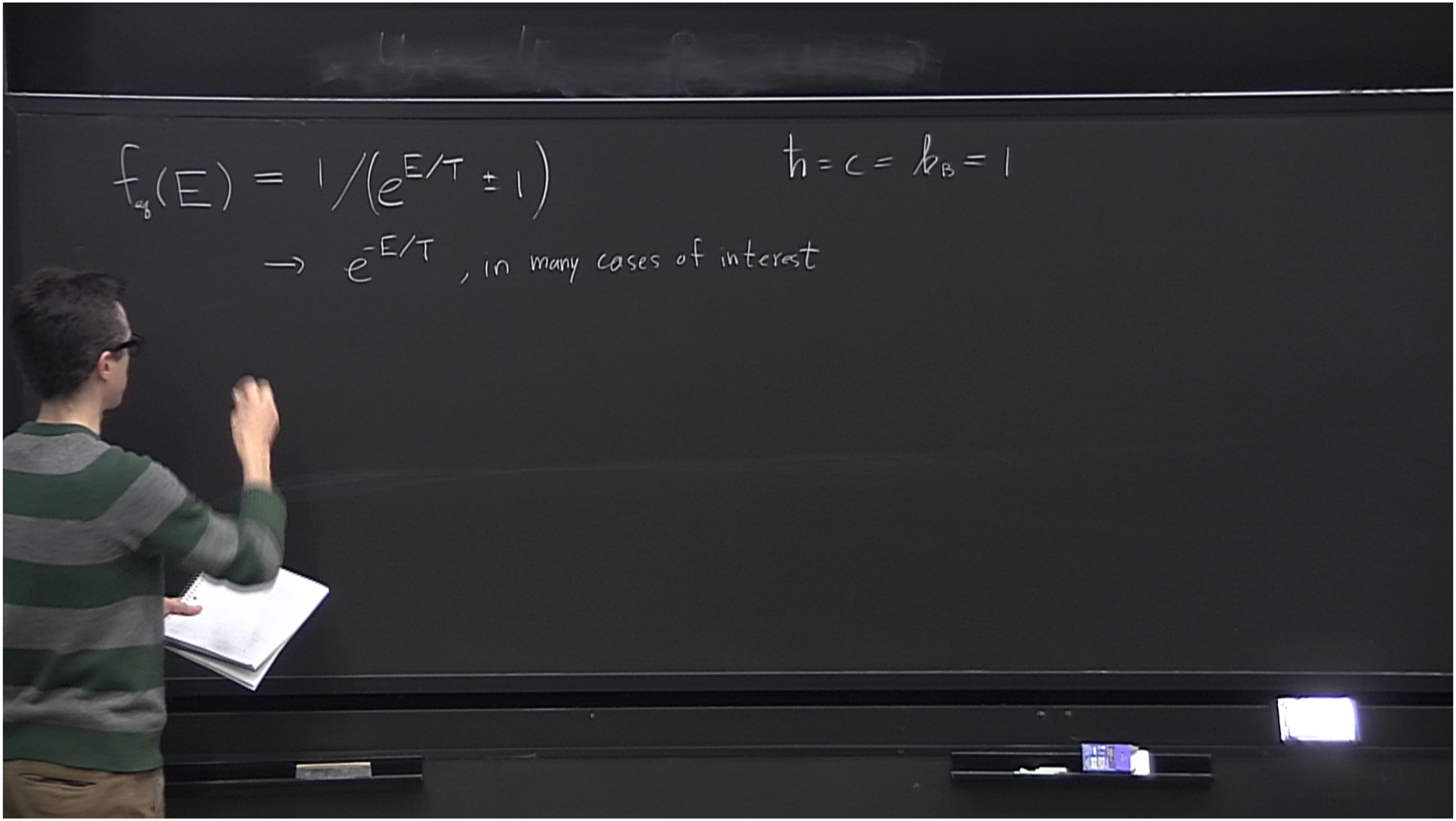
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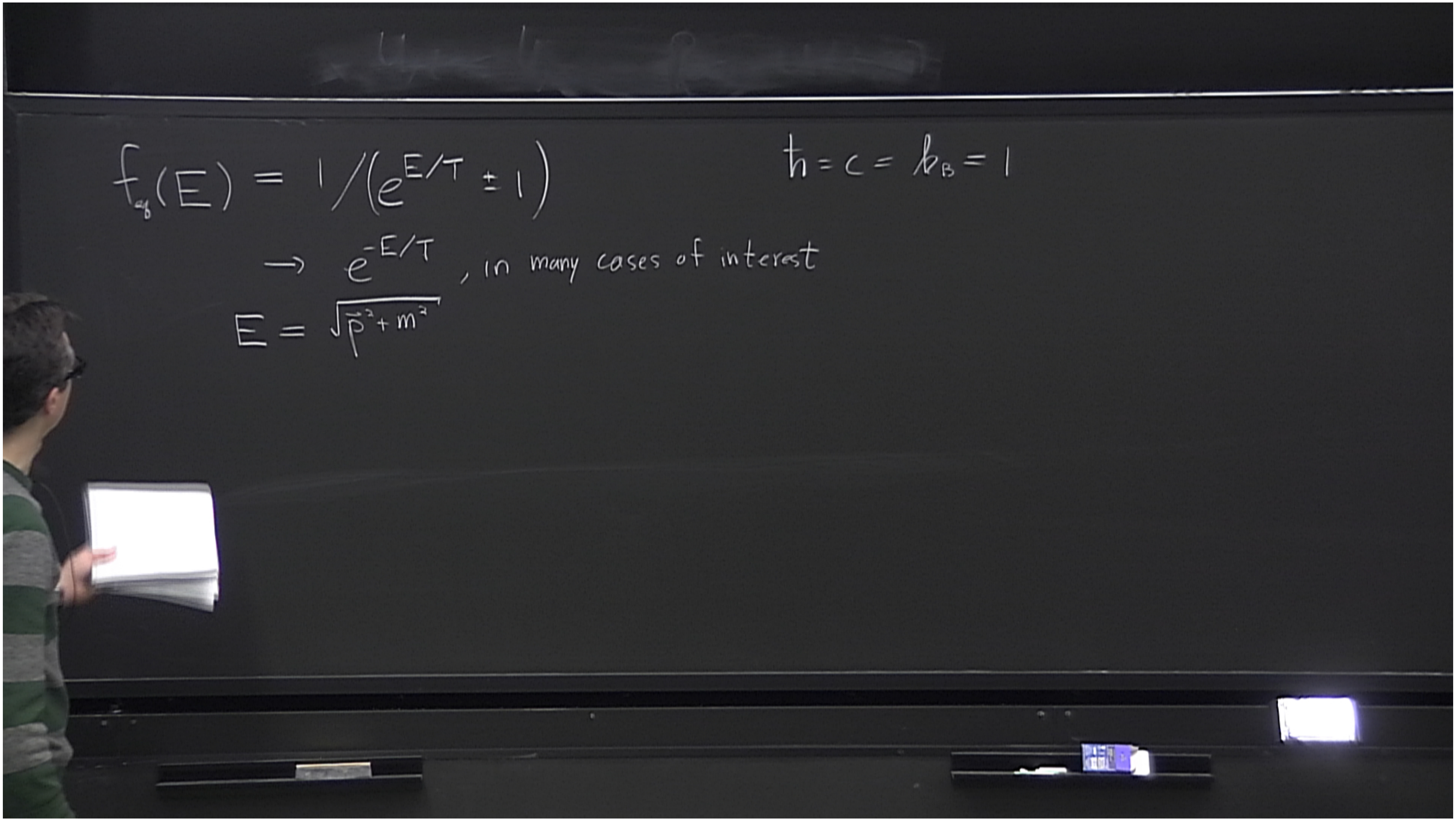
$\rightarrow e^{-E/T}$, in many cases of interest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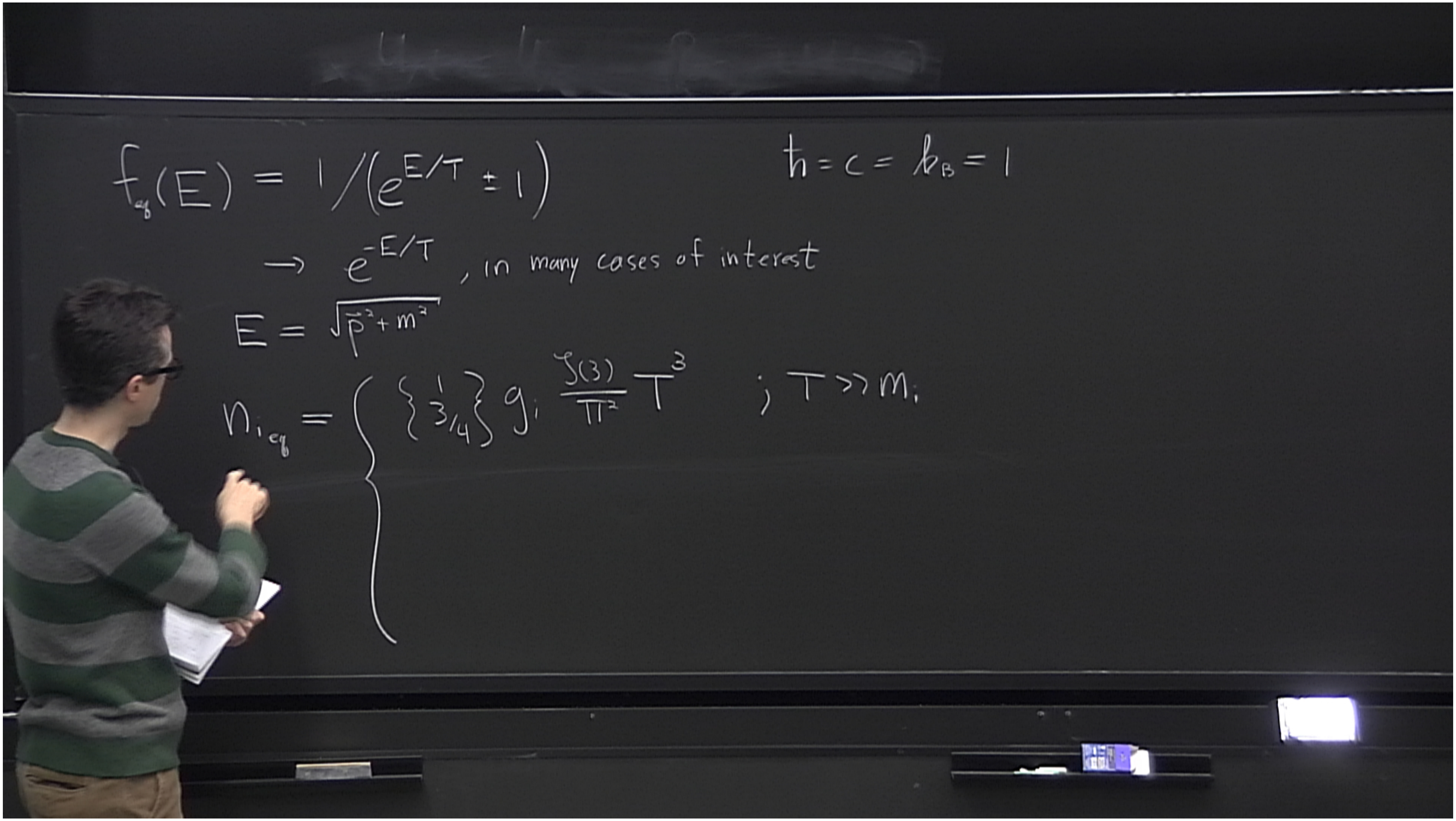
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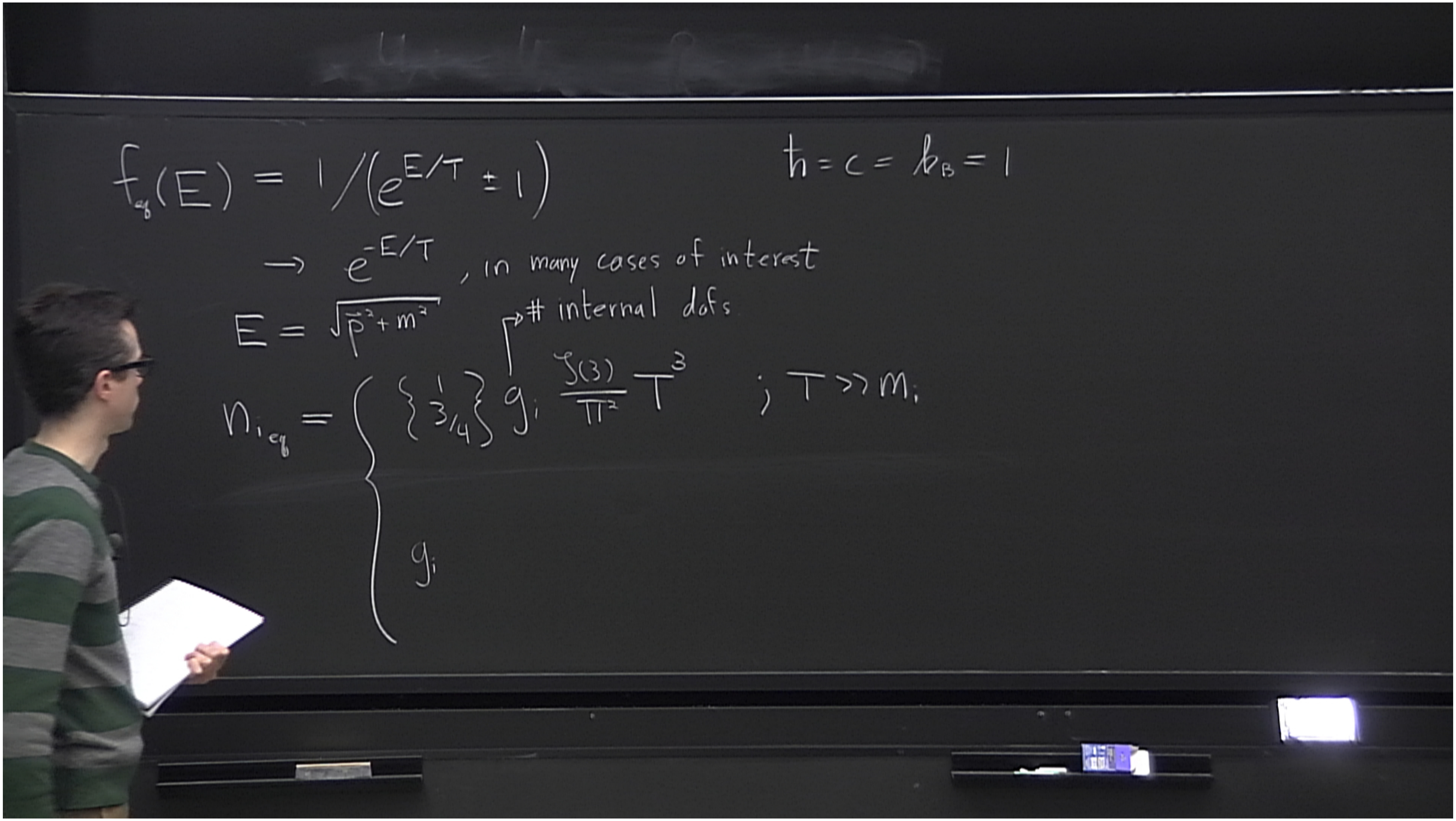
$n_{i_{eq}} = \left\{ \left\{ \begin{matrix} 1 \\ 3/4 \end{matrix} \right\} g_i \frac{\zeta(3)}{\pi^2} T^3 \quad ; T \gg m_i \right.$



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internal dofs
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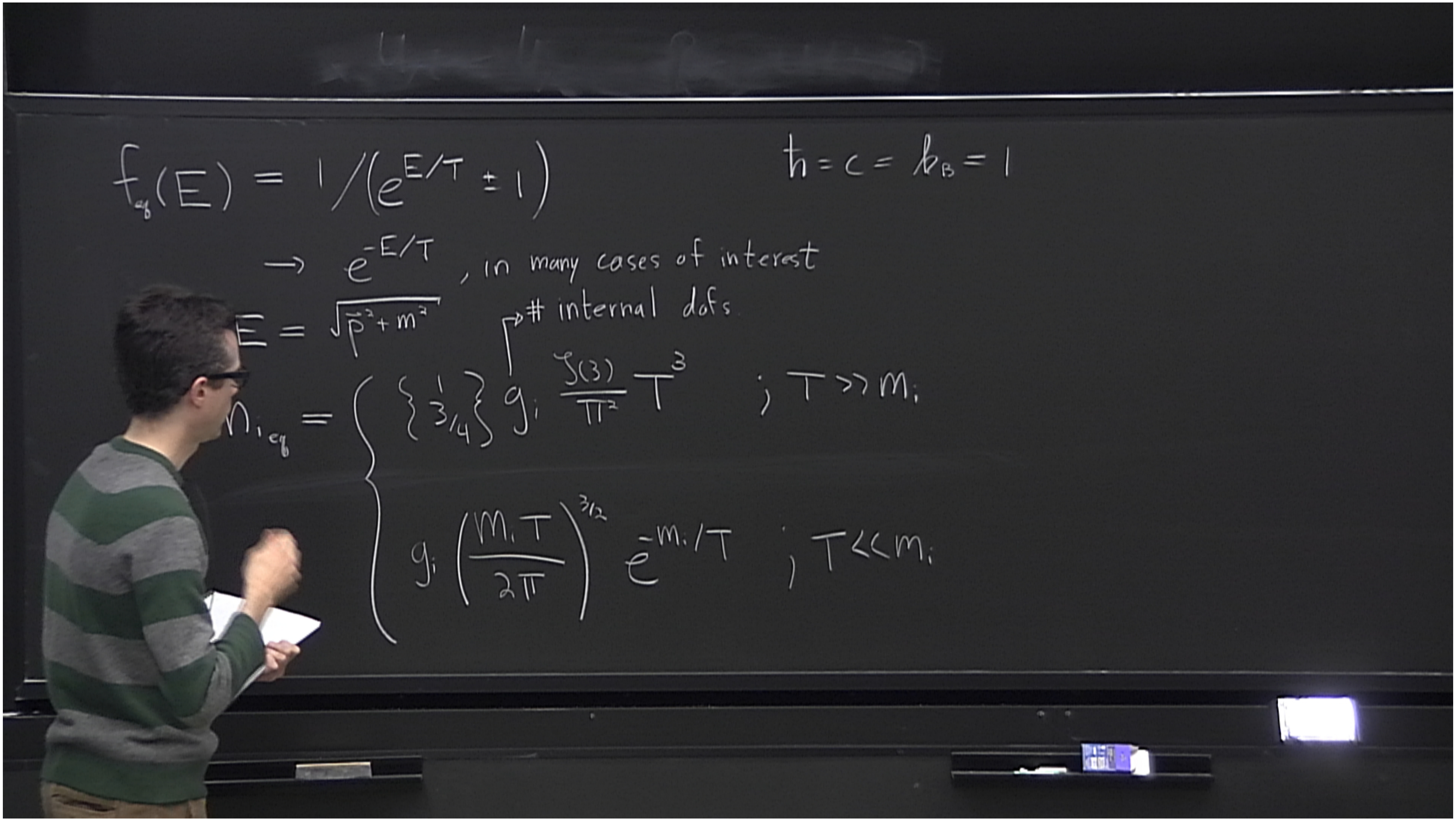
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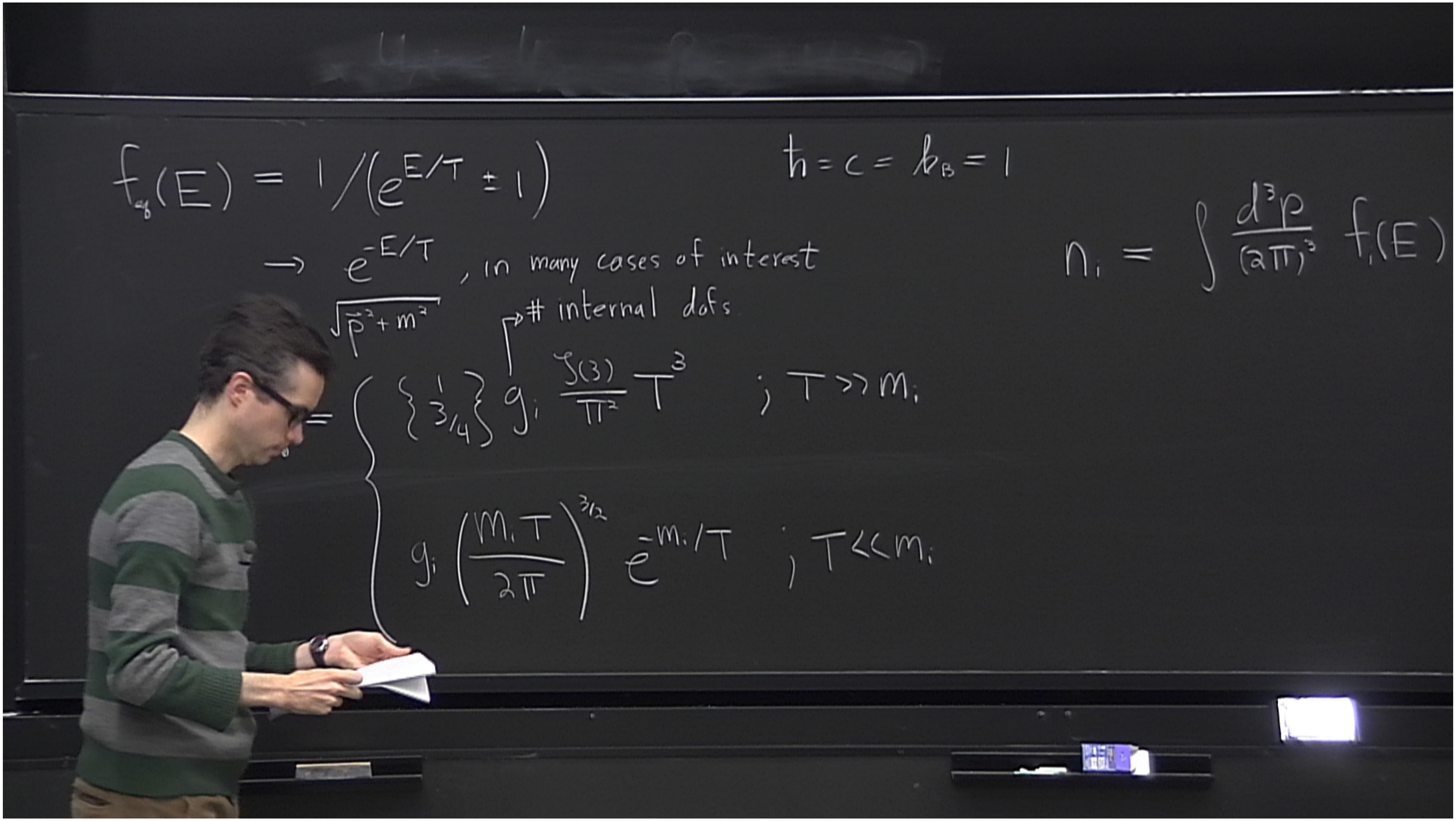
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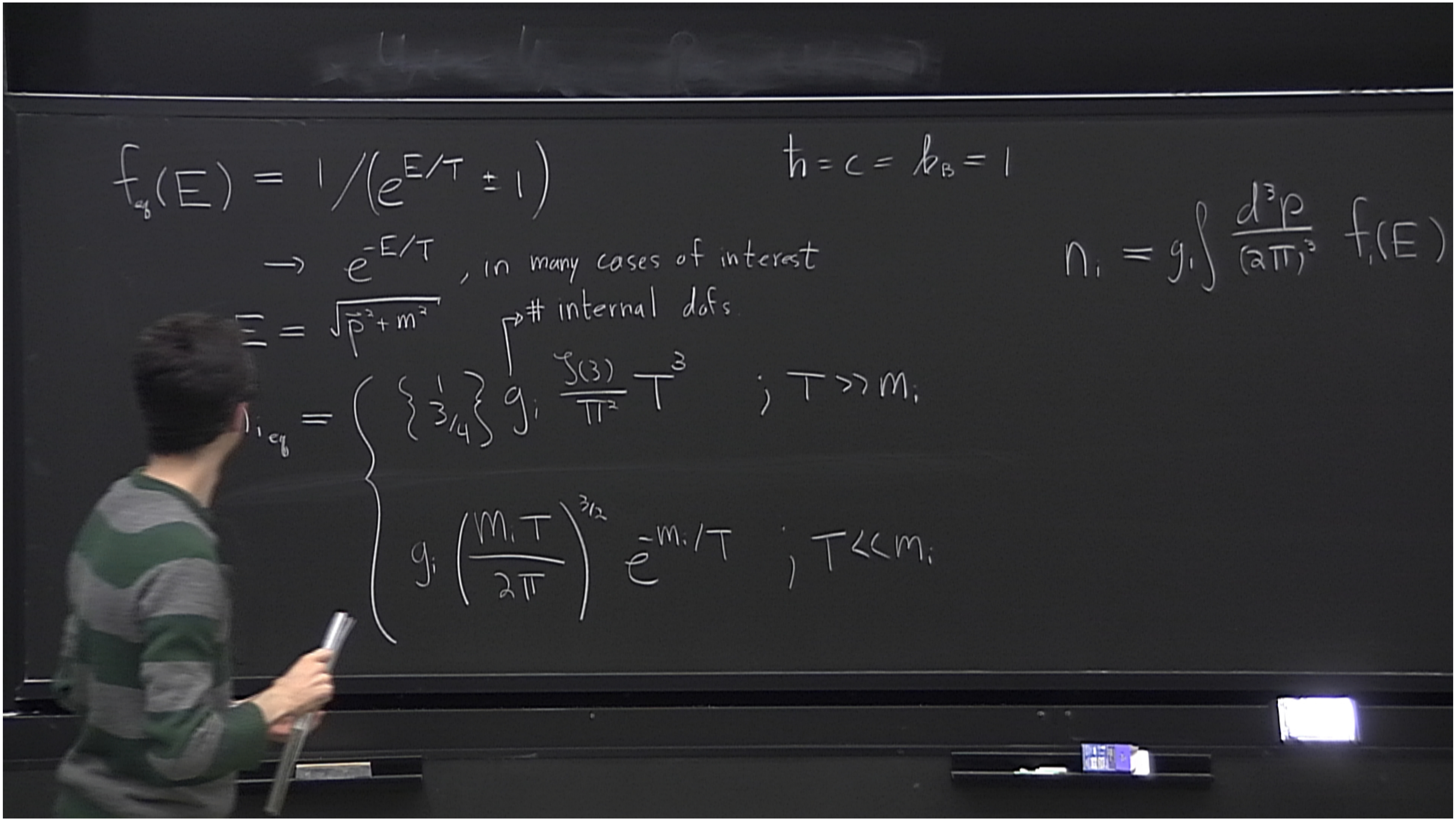
$\sqrt{\vec{p}^2 + m^2}$
$n_i = \int \frac{d^3p}{(2\pi)^3} f_i(E)$
internal dofs
$= \begin{cases} \left\{ \begin{matrix} 1 \\ 3/4 \end{matrix} \right\} g_i \frac{\zeta(3)}{\pi^2} T^3 & ; T \gg m_i \\ g_i \left(\frac{m_i T}{2\pi} \right)^{3/2} e^{-m_i/T} & ; T \ll m_i \end{cases}$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$n_i = g_i \int \frac{d^3p}{(2\pi)^3} f_i(E)$
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$n_{i\,eq} = \begin{cases} \left\{ {1 \atop 3/4} \right\} g_i \frac{\zeta(3)}{\pi^2} T^3 & ; T \gg m_i \\ g_i \left(\frac{m_i T}{2\pi} \right)^{3/2} e^{-m_i/T} & ; T \ll m_i \end{cases}$

$$(E) = 1/\left(e^{E/T} \pm 1\right)$$

$$\hbar = c = k_B = 1$$

$$\rightarrow e^{-E/T}$$, in many cases of interest

$$E = \sqrt{\vec{p}^2 + m^2}$$

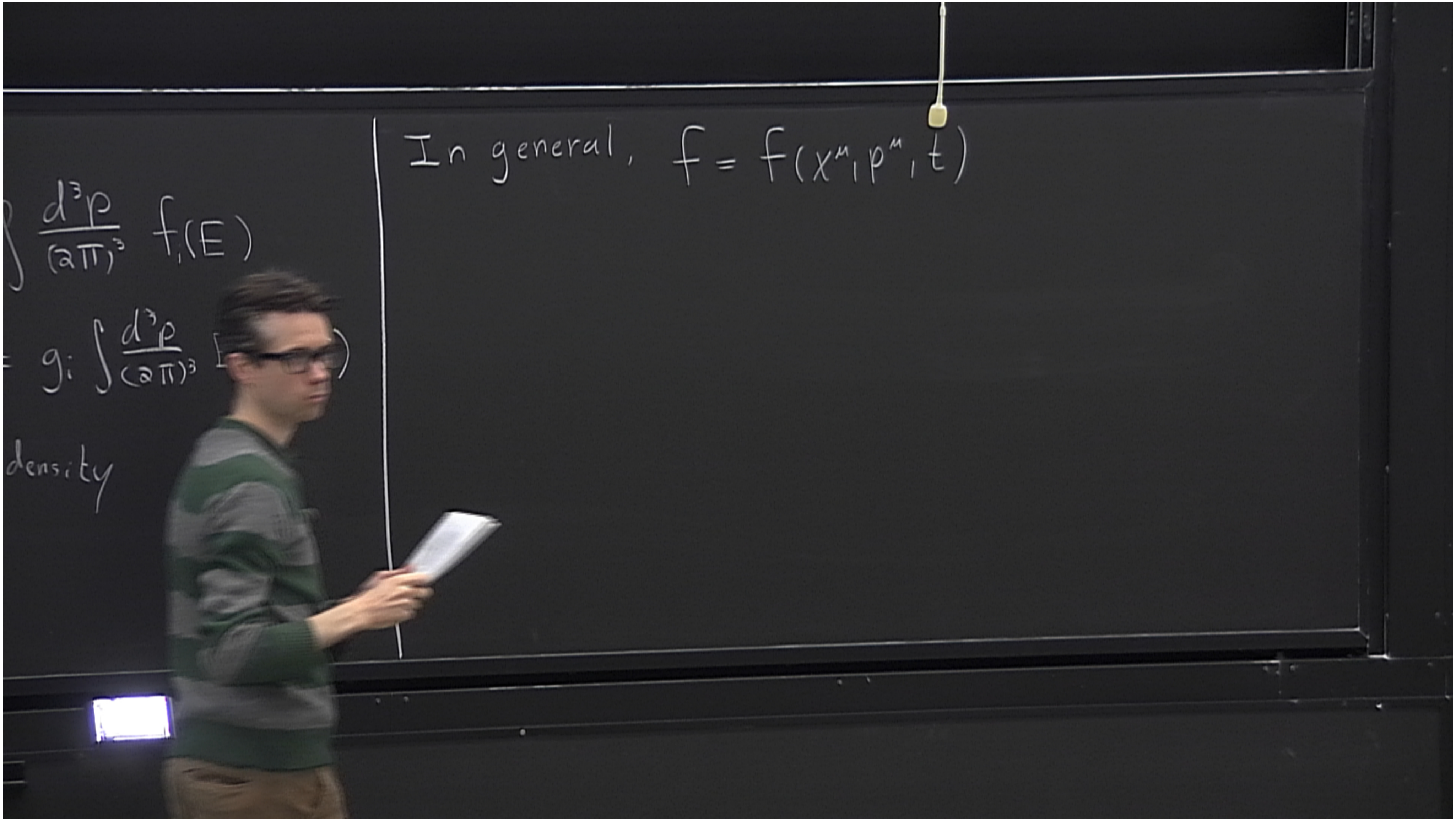
$\rightarrow$ # internal dofs

$$n_{i_{eq}} = \begin{cases} \left\{ \begin{matrix} 1 \\ 3/4 \end{matrix} \right\} g_i \dfrac{\zeta(3)}{\pi^2} T^3 & ; T \gg m_i \\[2ex] g_i \left(\dfrac{m_i T}{2\pi} \right)^{3/2} e^{-m_i/T} & ; T \ll m_i \end{cases}$$

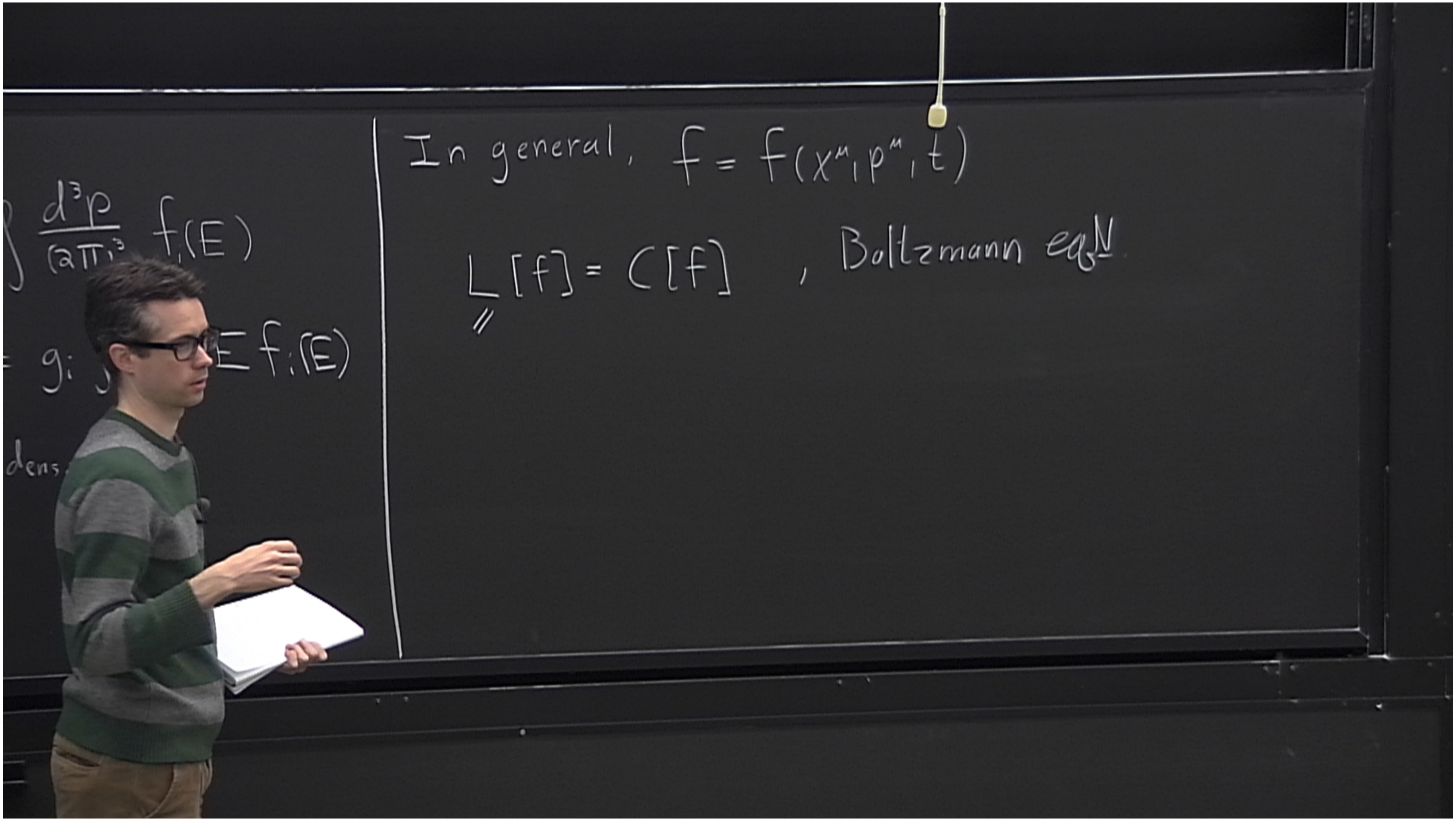
$$n_i = g_i \int \frac{d^3p}{(2\pi)^3} f_i(E)$$

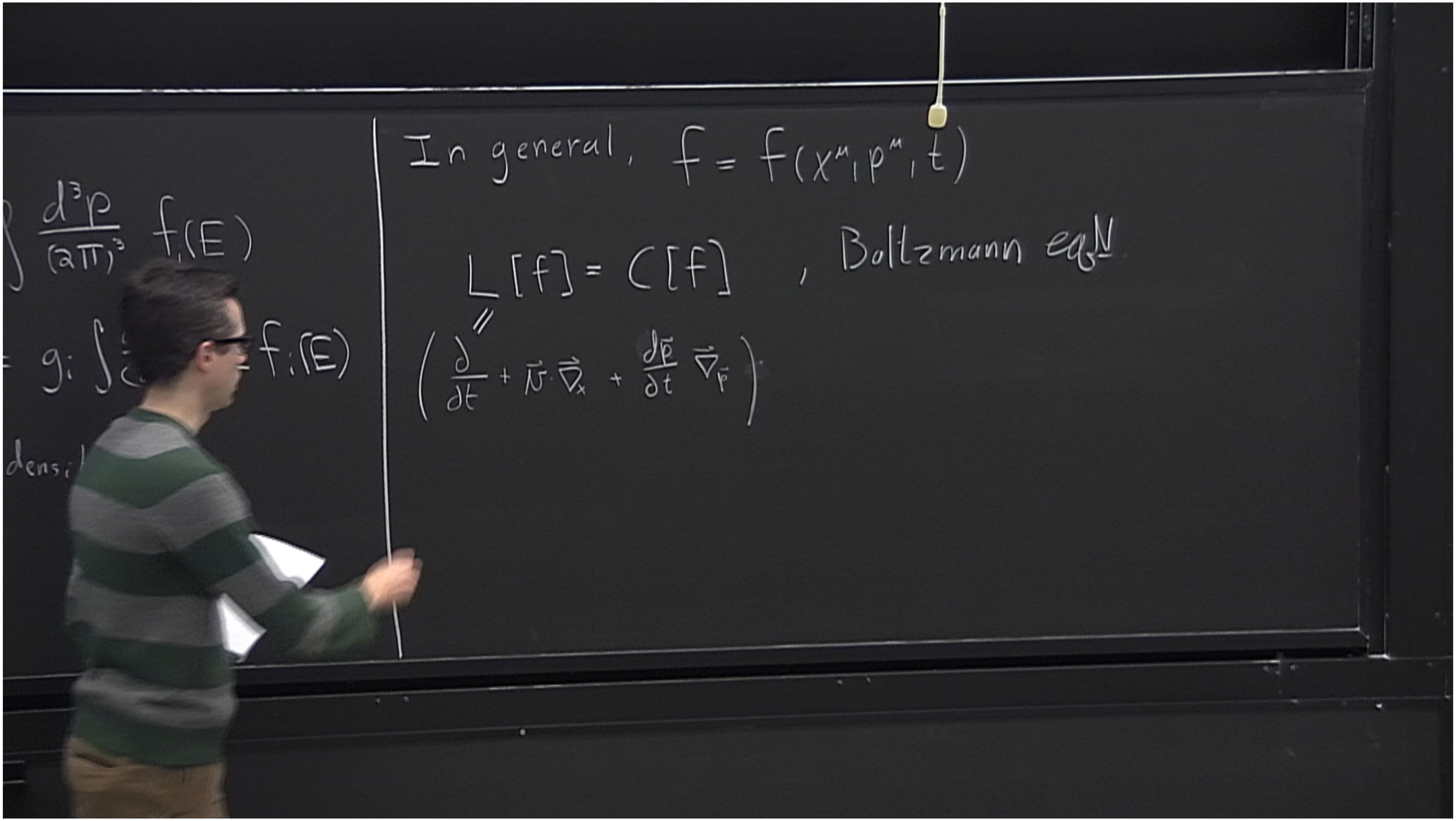
$$\rho_i = g_i \int \frac{d^3p}{(2\pi)^3} E f_i(E)$$

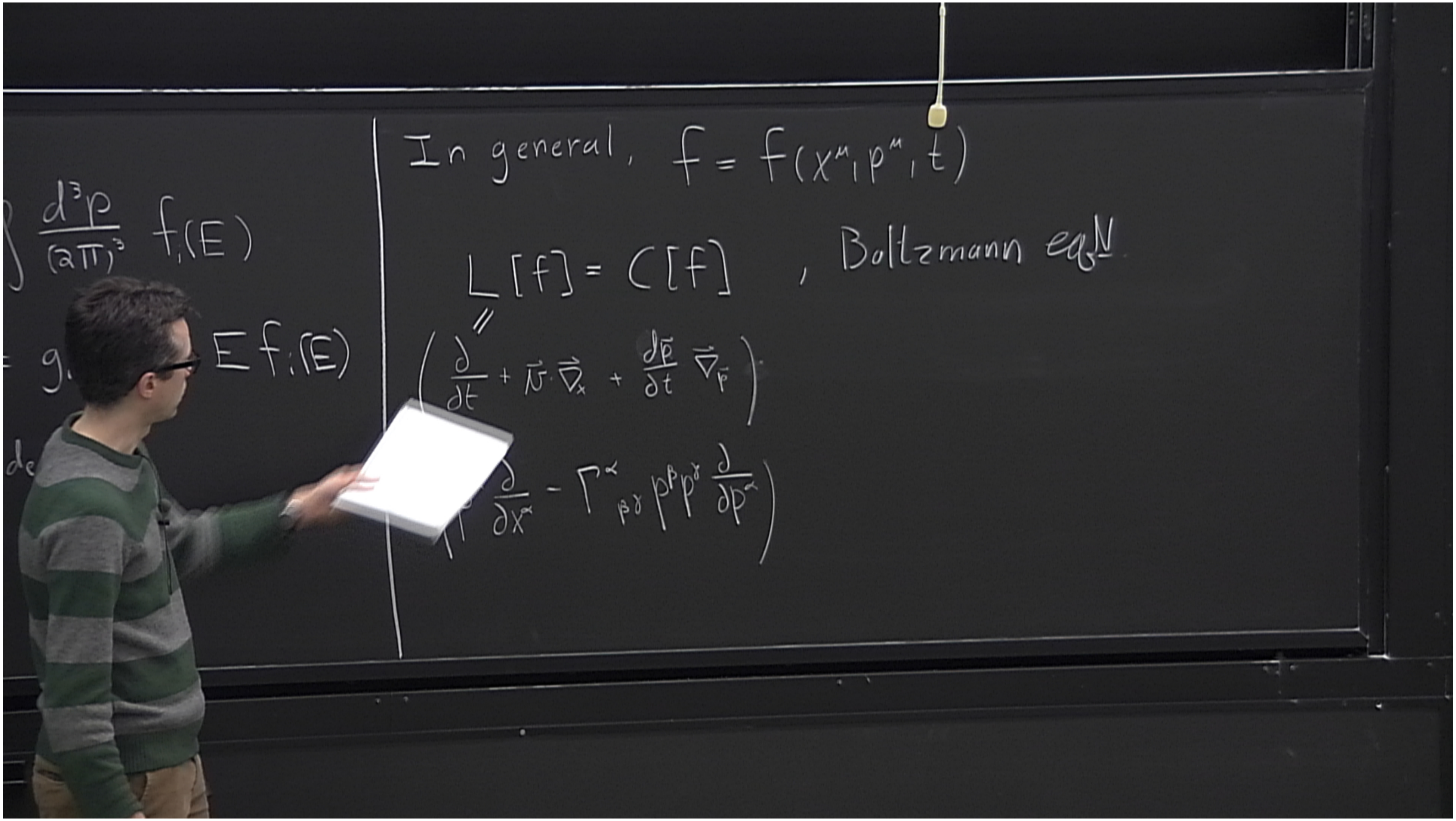
" energy density

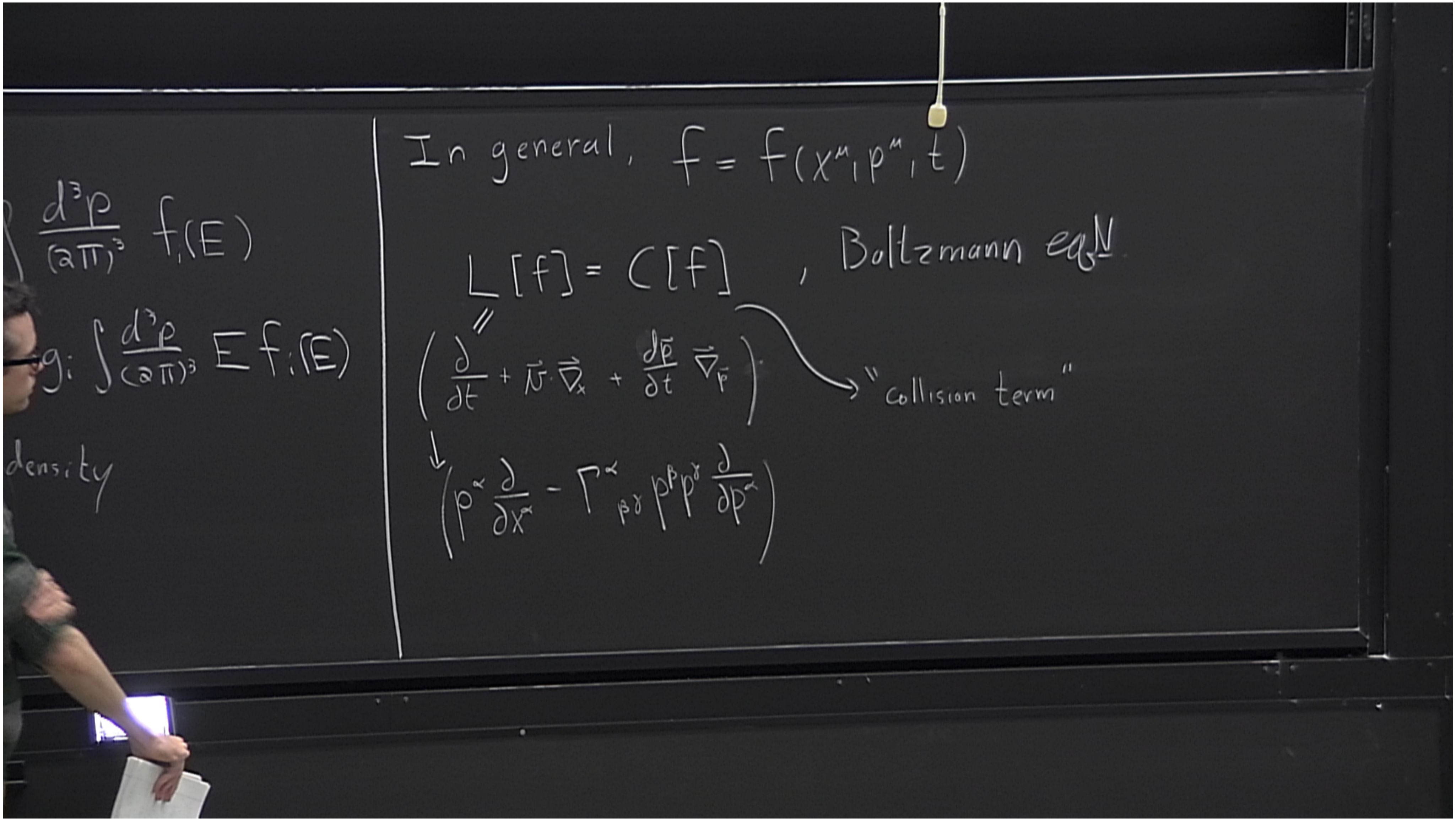


In general, $f = f(x^\mu, p^\mu, t)$
$\int \frac{d^3p}{(2\pi)^3} f_i(E)$
$= g_i \int \frac{d^3p}{(2\pi)^3}$
density


In general, f = f(x^μ, p^μ, t)
L[f] = C[f] , Boltzmann eqN
d³p/(2π)³ f_i(E)
g_i
E f_i(E)
dens


In general, $f = f(x^\mu, p^\mu, t)$
$L[f] = C[f]$, Boltzmann eq$^{\underline{N}}$
$\left(\frac{\partial}{\partial t} + \vec{v}\cdot\vec{\nabla}_x + \frac{d\vec{p}}{dt}\vec{\nabla}_{\vec{p}} \right)$
$\int \frac{d^3p}{(2\pi)^3} f_i(E)$
$= g_i \int$
$f_i(E)$


In general, $f = f(x^\mu, p^\mu, t)$
$L[f] = C[f]$, Boltzmann eqN
$\left(\frac{\partial}{\partial t} + \vec{v}\cdot\vec{\nabla}_x + \frac{d\vec{p}}{dt}\vec{\nabla}_{\vec{p}}\right)$
$\frac{\partial}{\partial x^\alpha} - \Gamma^\alpha_{\beta\gamma} p^\beta p^\gamma \frac{\partial}{\partial p^\alpha}$
$\frac{d^3p}{(2\pi)^3} f_i(E)$
$E f_i(E)$



$\int \frac{d^3p}{(2\pi)^3} f_i(E)$
$g_i \int \frac{d^3p}{(2\pi)^3} E f_i(E)$
density
In general, $f = f(x^\mu, p^\mu, t)$
$L[f] = C[f]$, Boltzmann eq$\underline{N}$
$\left(\frac{\partial}{\partial t} + \vec{v}\cdot\vec{\nabla}_x + \frac{d\vec{p}}{dt}\vec{\nabla}_{\vec{p}}\right)$
"collision term"
$\left(p^\alpha \frac{\partial}{\partial x^\alpha} - \Gamma^\alpha_{\beta\gamma} p^\beta p^\gamma \frac{\partial}{\partial p^\alpha}\right)$

$$\int \frac{d^3p}{(2\pi)^3} f_i(E)$$

$$= g_i \int \frac{d^3p}{(2\pi)^3} E f_i(E)$$

density

In general, $f = f(x^\mu, p^\mu, t)$

$$L[f] = C[f]$$, Boltzmann eq$^{\underline{N}}$

$$L = \left(\frac{\partial}{\partial t} + \vec{v}\cdot\vec{\nabla}_x + \frac{d\vec{p}}{dt}\vec{\nabla}_{\vec{p}}\right)$$

$$\downarrow$$

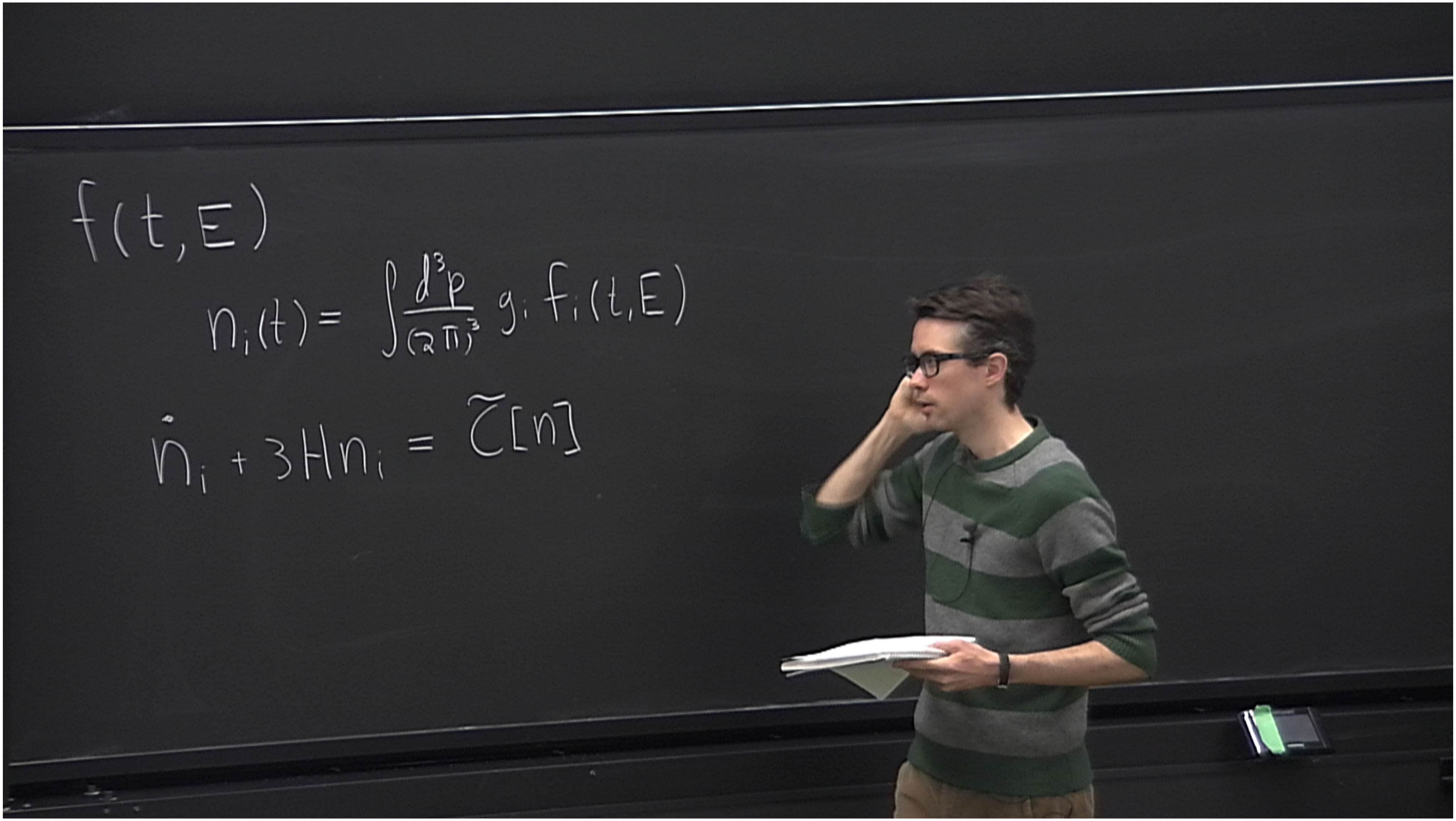
$$\left(p^\alpha \frac{\partial}{\partial x^\alpha} - \Gamma^\alpha{}_{\beta\gamma} p^\beta p^\gamma \frac{\partial}{\partial p^\alpha}\right)$$

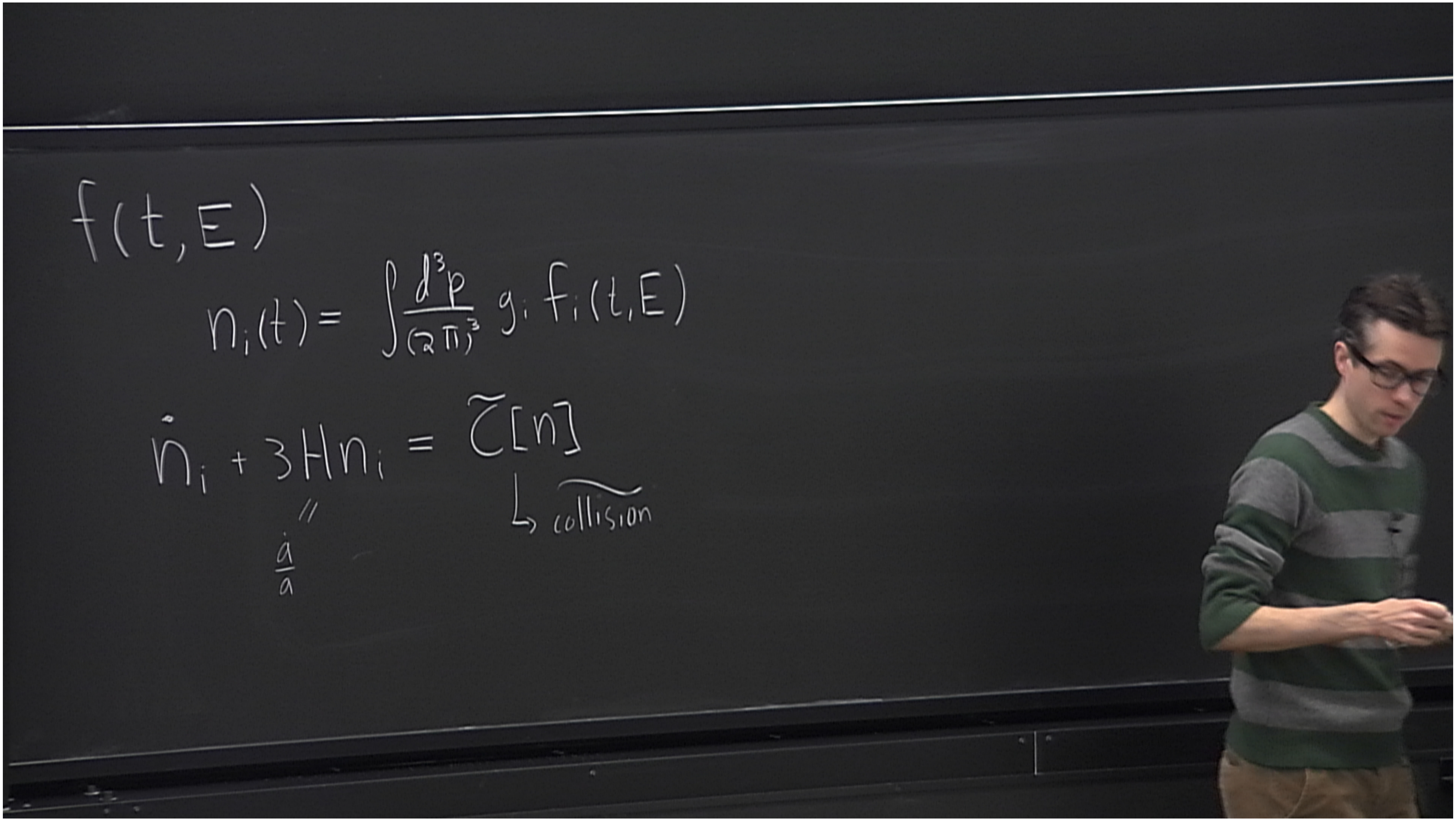
"collision term"

$f(t, E)$

$$f(t,E)$$

$$n_i(t) = \int \frac{d^3p}{(2\pi)^3} g_i f_i(t,E)$$


$f(t,E)$
$n_i(t) = \int \frac{d^3p}{(2\pi)^3} g_i f_i(t,E)$
$\dot{n}_i + 3Hn_i = \tilde{C}[n]$



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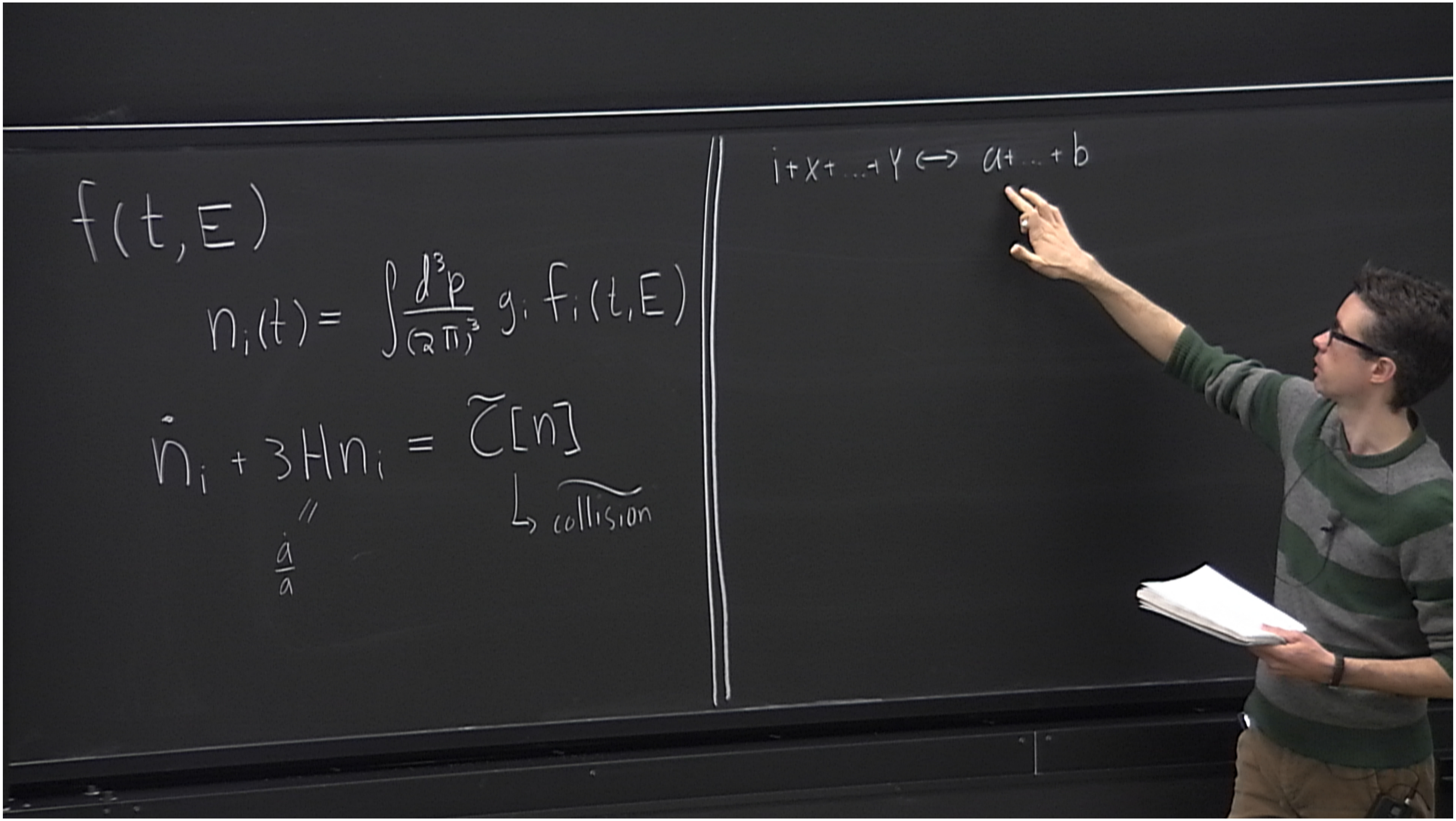
$\frac{\dot{a}}{a}$
collision



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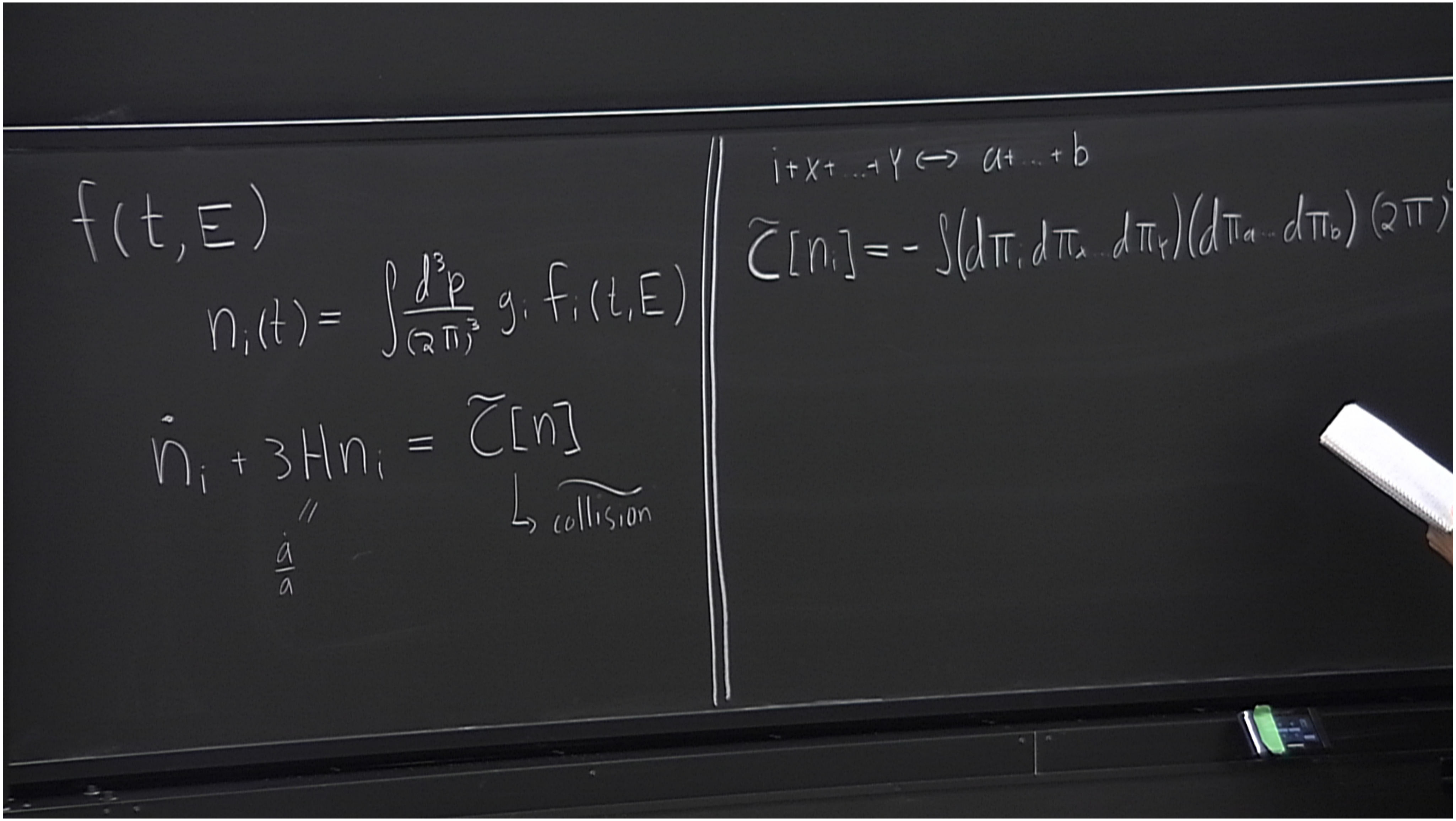
$\frac{\dot{a}}{a}$
collision
$i + X + \ldots + Y \longleftrightarrow a + \ldots + b$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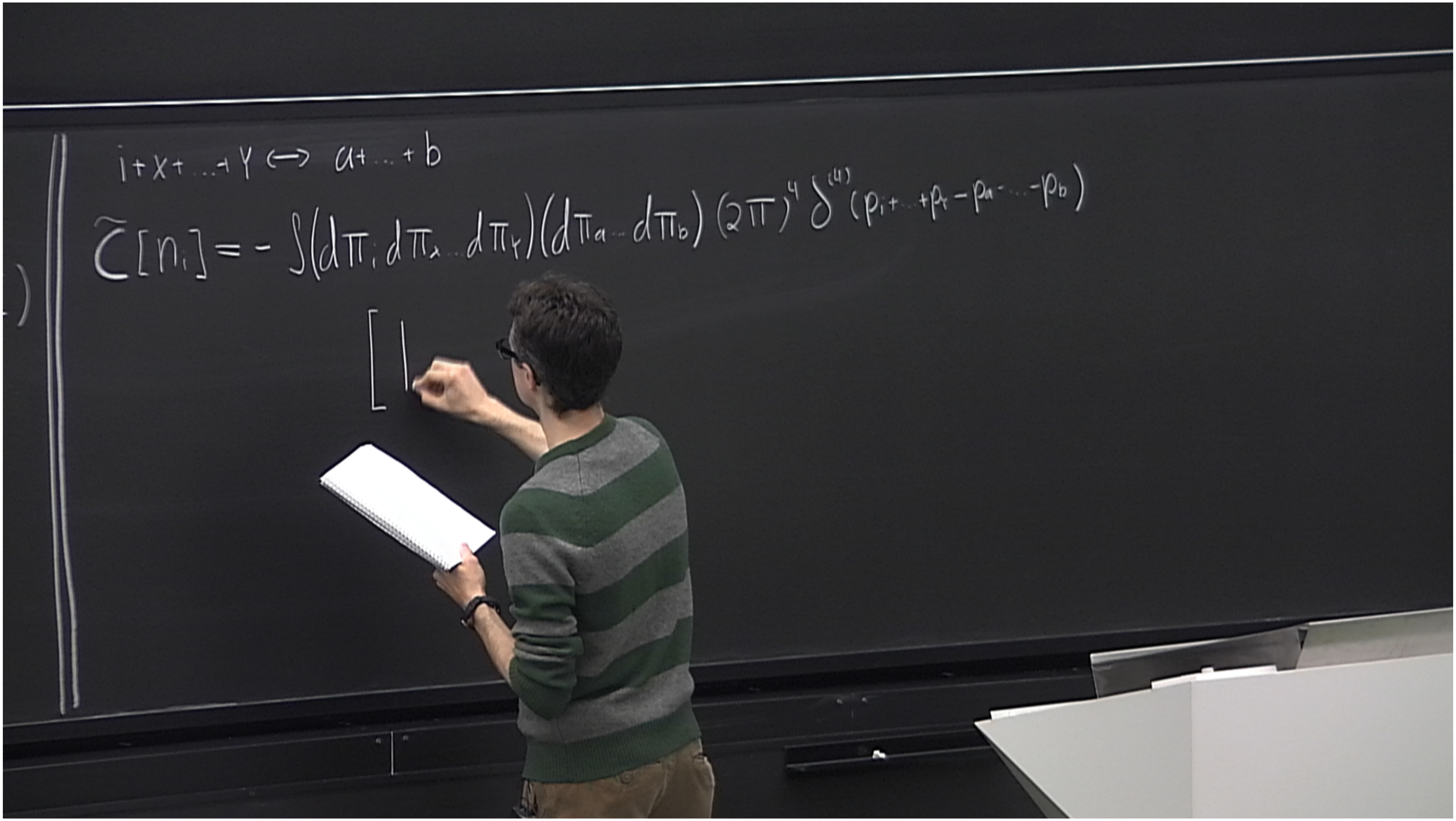
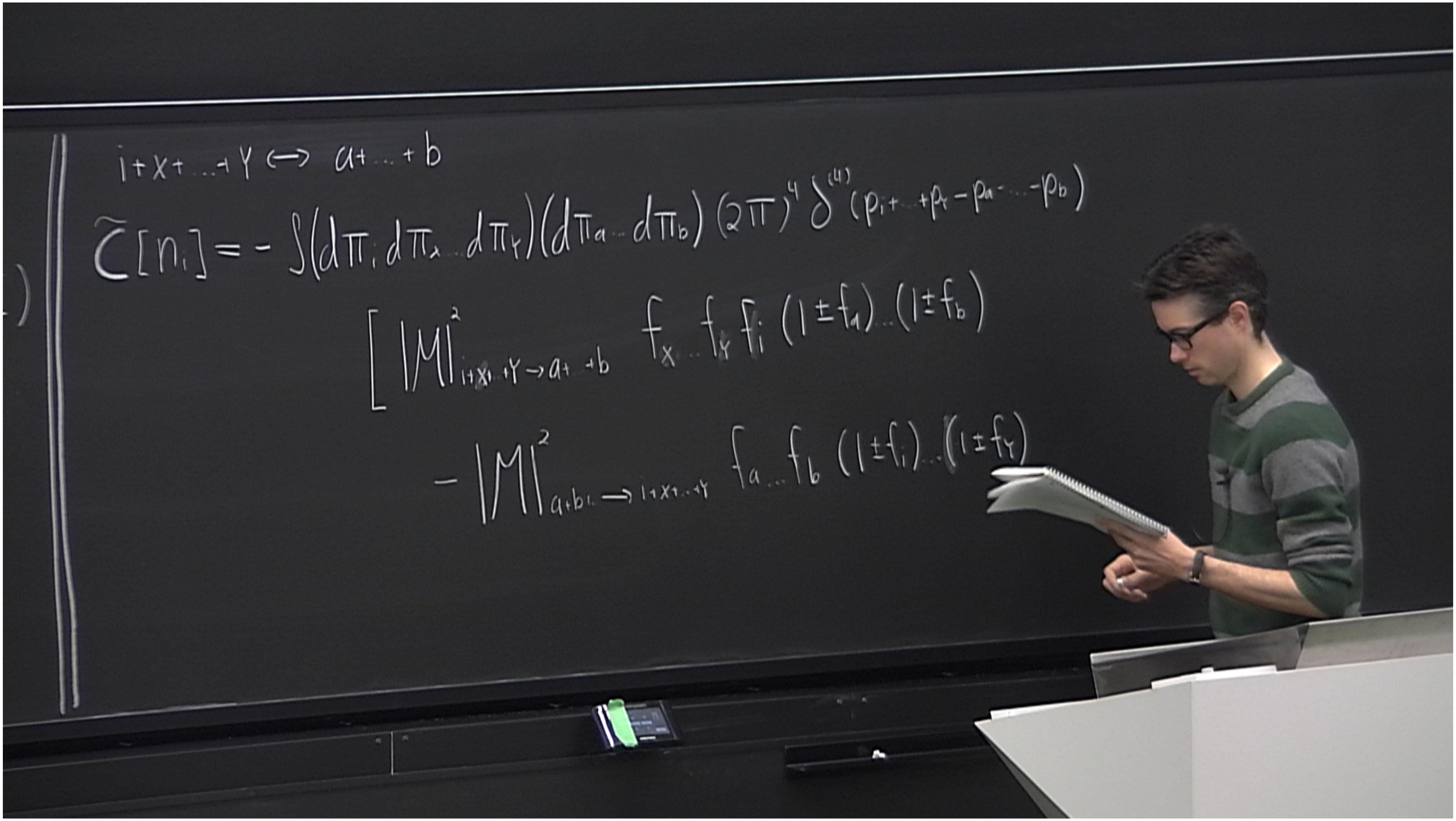
$\dot{n}_i + 3Hn_i = \tilde{C}[n]$
$\frac{\dot{a}}{a}$
collision
$i + X + \ldots + Y \leftrightarrow a + \ldots + b$
$\tilde{C}[n_i] = -\int (d\pi_i \, d\pi_X \ldots d\pi_Y)(d\pi_a \ldots d\pi_b)(2\pi)^4$


$i + x + \ldots + y \leftrightarrow a + \ldots + b$
$\tilde{C}[n_i] = -\int (d\Pi_i \, d\Pi_x \ldots d\Pi_y)(d\Pi_a \ldots d\Pi_b)(2\pi)^4 \delta^{(4)}(p_i + \ldots + p_y - p_a - \ldots - p_b)$


$i + x + \ldots + y \longleftrightarrow a + \ldots + b$
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$\Big[|M|^2_{i+x+\ldots+y \to a+\ldots+b} \, f_x \ldots f_y f_i (1 \pm f_a) \ldots (1 \pm f_b)$
$- |M|^2_{a+b\ldots \to i+x+\ldots+y} \, f_a \ldots f_b (1 \pm f_i) \ldots (1 \pm f_y)$

$$i + x + \dots + y \longleftrightarrow a + \dots + b$$

$$\tilde{C}[n_i] = -\int (d\Pi_i \, d\Pi_x \dots d\Pi_y)(d\Pi_a \dots d\Pi_b)\,(2\pi)^4 \delta^{(4)}(p_i + \dots + p_y - p_a - \dots - p_b)$$

$$\Big[\, |M|^2_{i+x+\dots+y \to a+\dots+b} \; f_x \dots f_y f_i (1 \pm f_a) \dots (1 \pm f_b)$$

$$- |M|^2_{a+b\dots \to i+x+\dots+y} \; f_a \dots f_b (1 \pm f_i) \dots (1 \pm f_y) \,\Big] \frac{1}{S}$$

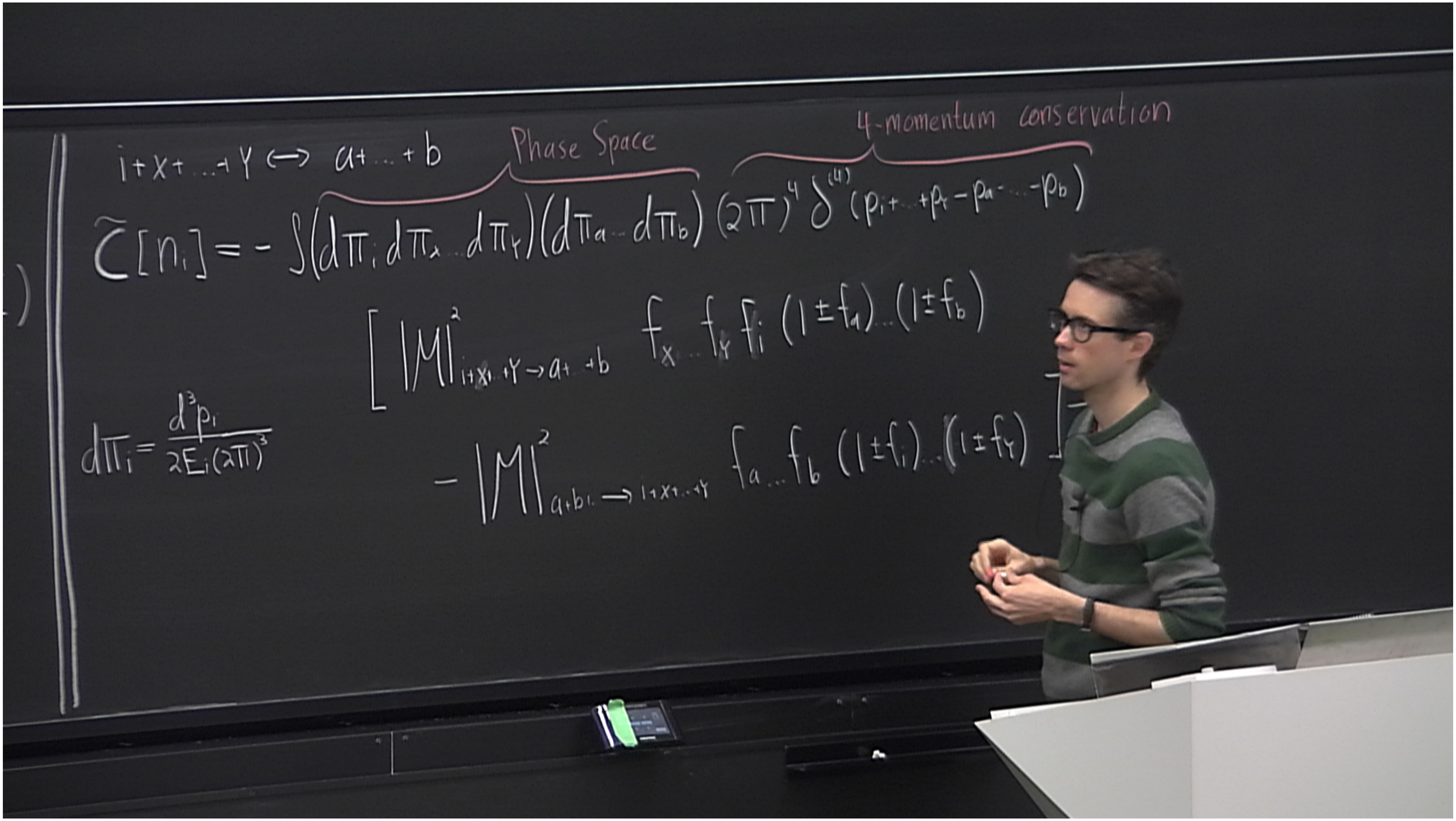
$$i + x + \dots + y \longleftrightarrow a + \dots + b$$

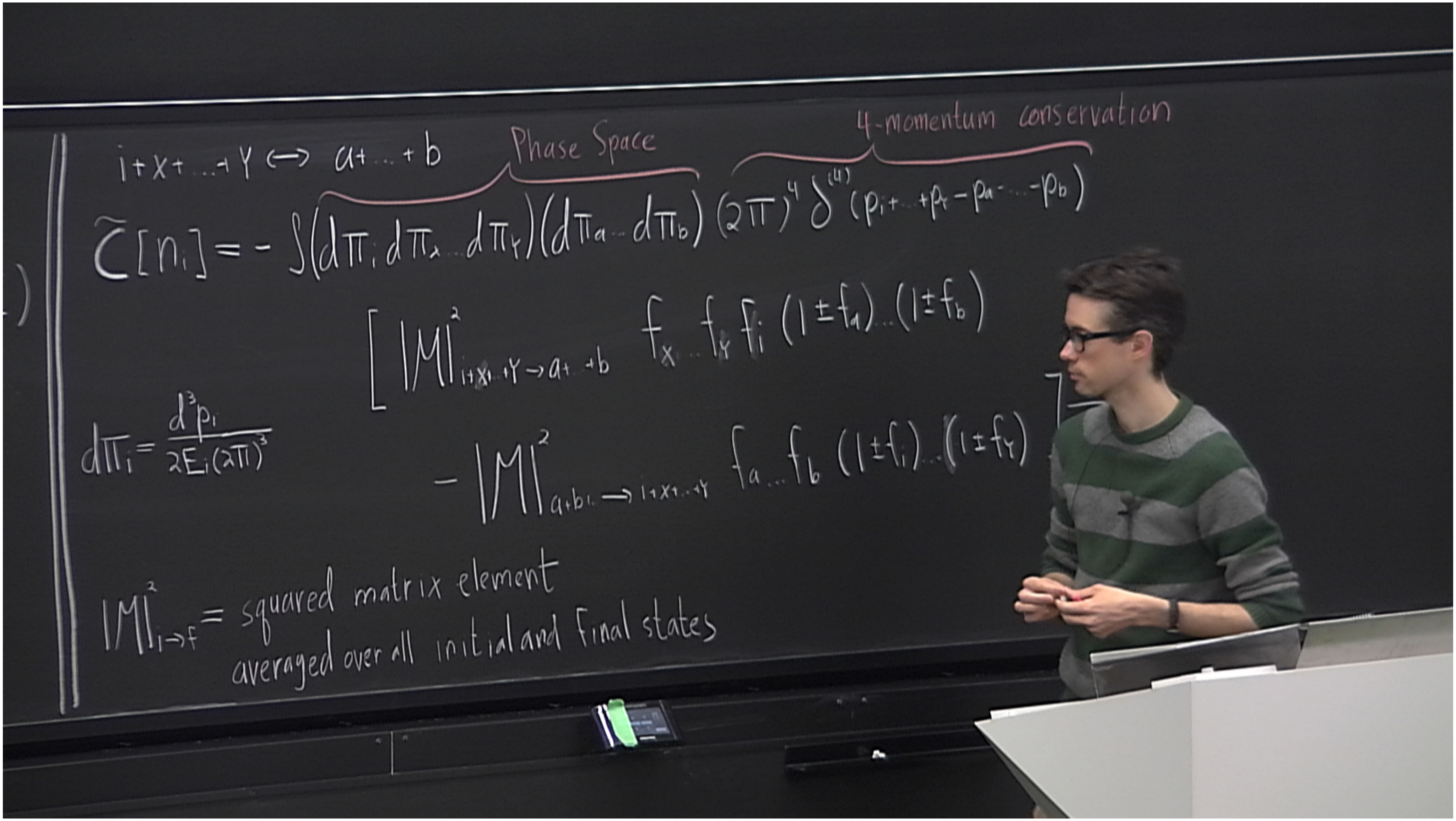
$$\tilde{C}[n_i] = -\int (d\Pi_i \, d\Pi_x \dots d\Pi_y)(d\Pi_a \dots d\Pi_b)\,(2\pi)^4\, \delta^{(4)}(p_i + \dots + p_y - p_a - \dots - p_b)$$

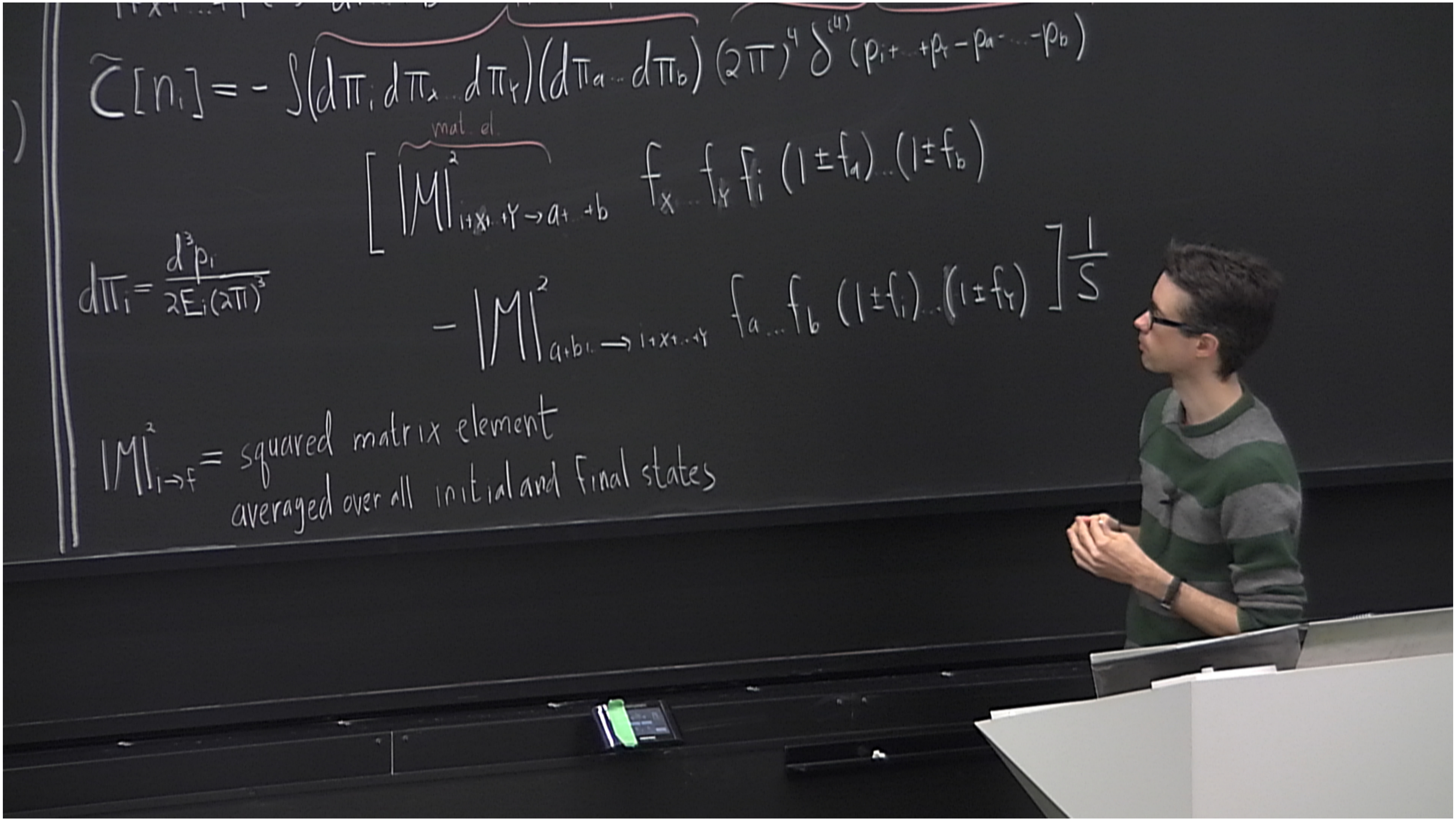
$$\Big[\, |M|^2_{i+x+\dots+y \to a+\dots+b}\, f_x \dots f_y f_i\, (1 \pm f_a) \dots (1 \pm f_b)$$

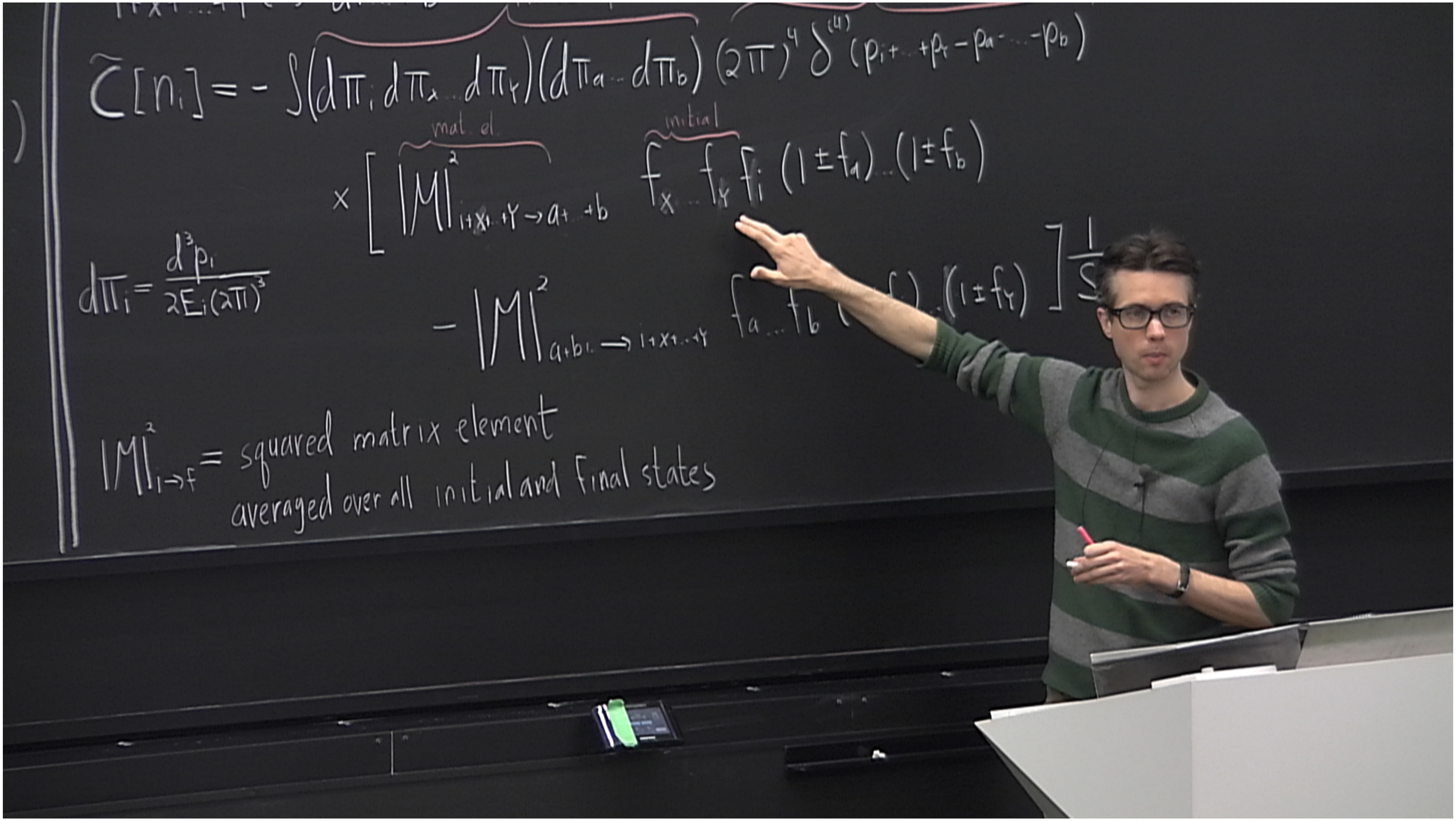
$$- |M|^2_{a+b+\dots \longrightarrow i+x+\dots+y}\, f_a \dots f_b\, (1 \pm f_i) \dots (1 \pm f_y) \,\Big] \frac{1}{S}$$

$$d\Pi_i = \frac{d^3 p_i}{2E_i (2\pi)^3}$$

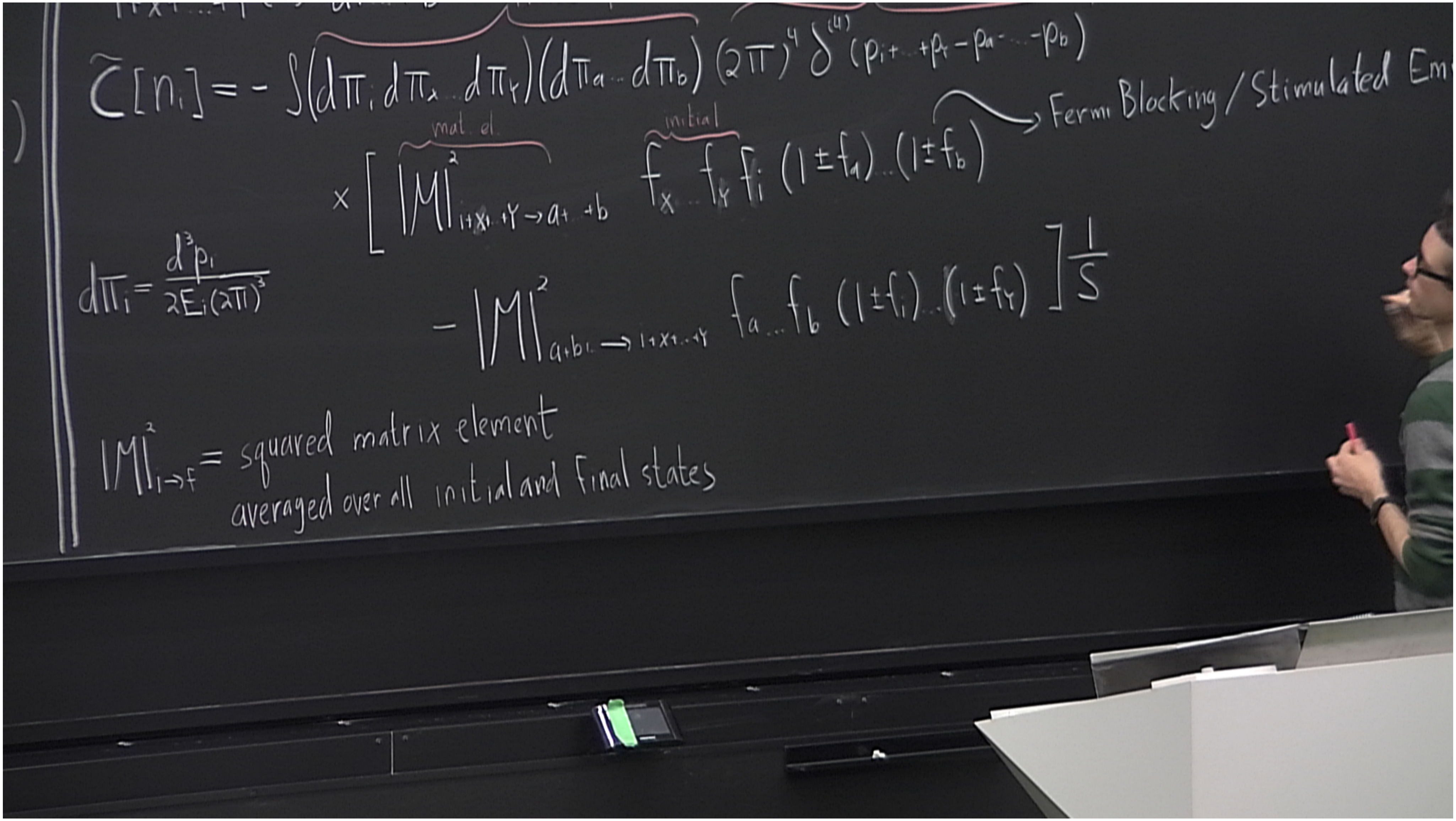

Phase Space
4-momentum conservation
$i+x+\dots+y \longleftrightarrow a+\dots+b$
$\tilde{C}[n_i] = -\int (d\Pi_i\, d\Pi_x \dots d\Pi_y)(d\Pi_a \dots d\Pi_b)\,(2\pi)^4\,\delta^{(4)}(p_i+\dots+p_y-p_a-\dots-p_b)$
$\Big[\,|M|^2_{i+x+\dots+y\to a+\dots+b}\; f_x \dots f_y f_i (1\pm f_a)\dots(1\pm f_b)$
$-|M|^2_{a+b\dots\to i+x+\dots+y}\; f_a\dots f_b (1\pm f_i)\dots(1\pm f_y)\,\Big]$
$d\Pi_i = \frac{d^3p_i}{2E_i(2\pi)^3}$

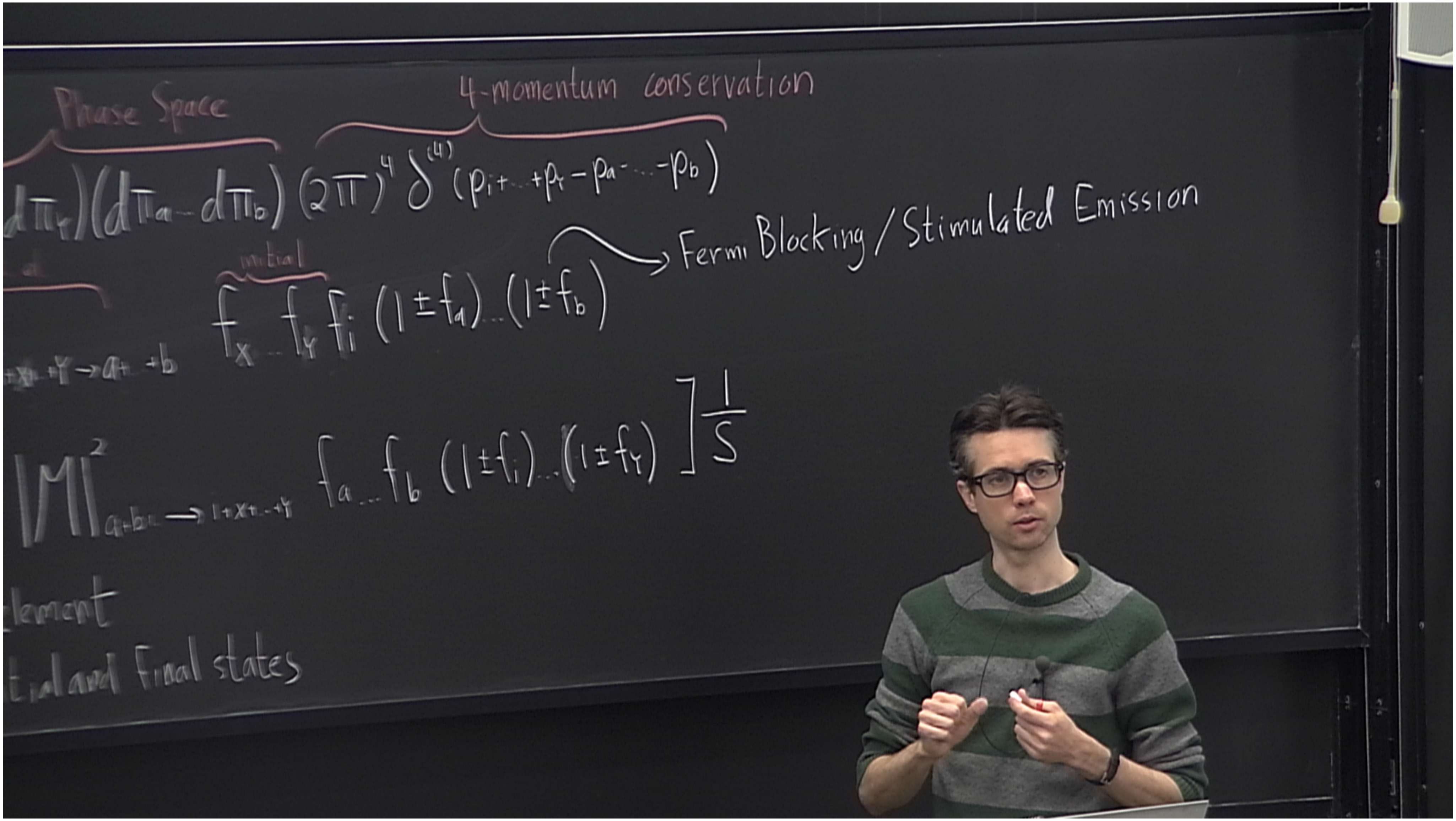

Phase Space
4-momentum conservation
i + X + ... + Y ⟷ a + ... + b
C̃[n_i] = - ∫(dπ_i dπ_x ... dπ_Y)(dπ_a ... dπ_b) (2π)^4 δ^(4)(p_i + ... + p_Y - p_a - ... - p_b)
[|M|²_{i+X+...+Y → a+...+b} f_X ... f_Y f_i (1 ± f_a)...(1 ± f_b)
- |M|²_{a+b... → i+X+...+Y} f_a ... f_b (1 ± f_i)...(1 ± f_Y)]
dπ_i = d³p_i / 2E_i(2π)³
|M|²_{i→f} = squared matrix element
averaged over all initial and final states

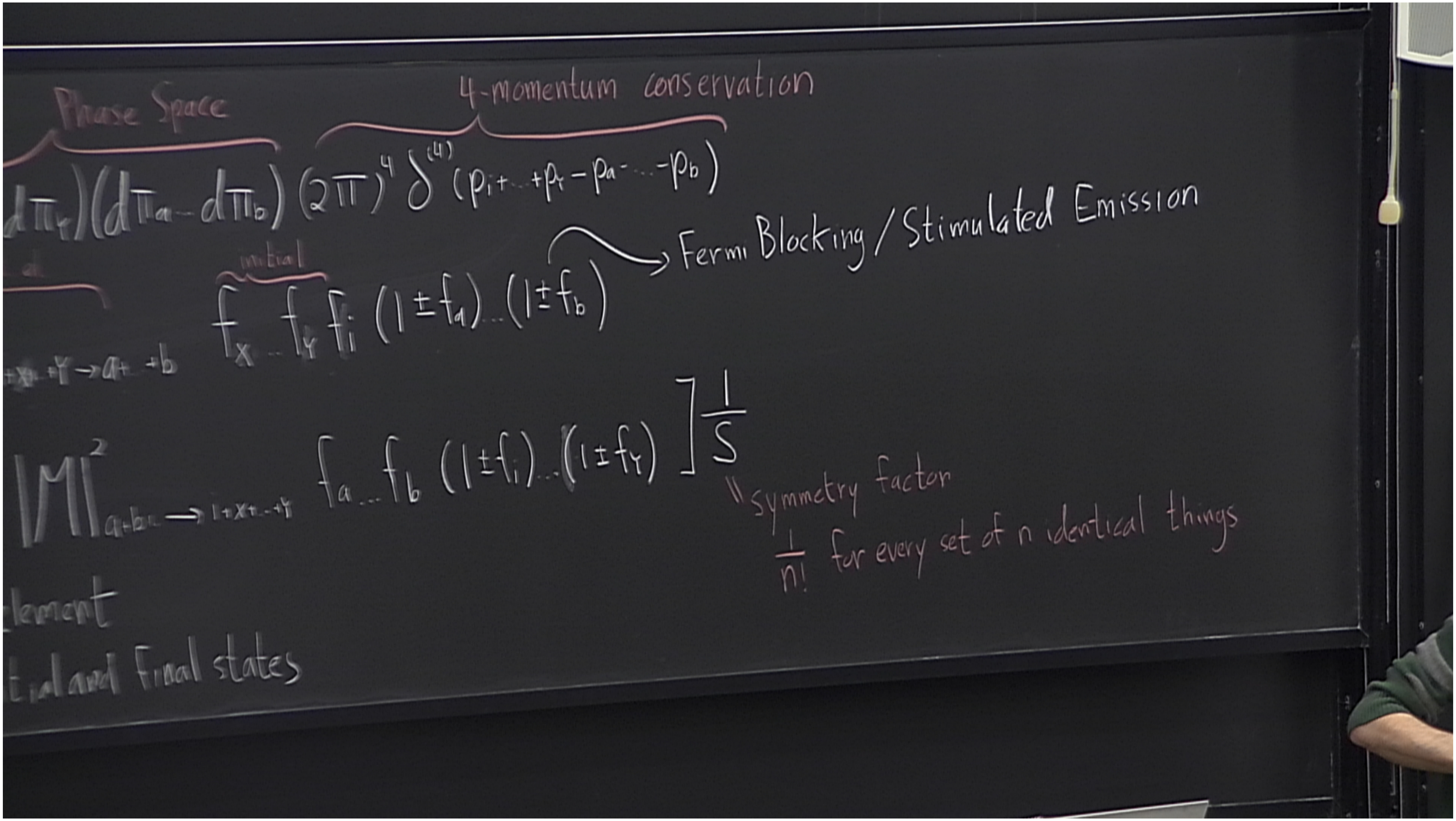

$\tilde{C}[n_1] = -\int (d\Pi_1 \, d\Pi_x \ldots d\Pi_Y)(d\Pi_a \ldots d\Pi_b)\,(2\pi)^4 \delta^{(4)}(p_1 + \ldots + p_Y - p_a - \ldots - p_b)$
mat. el.
$\Big[\, |M|^2_{1+X+\ldots+Y \to a+\ldots+b} \; f_X \ldots f_Y \, f_1 \, (1 \pm f_a) \ldots (1 \pm f_b)$
$- |M|^2_{a+b+\ldots \to 1+X+\ldots+Y} \; f_a \ldots f_b \, (1 \pm f_1) \ldots (1 \pm f_Y) \Big] \frac{1}{S}$
$d\Pi_i = \frac{d^3 p_i}{2E_i (2\pi)^3}$
$|M|^2_{i \to f}$ = squared matrix element
averaged over all initial and final states

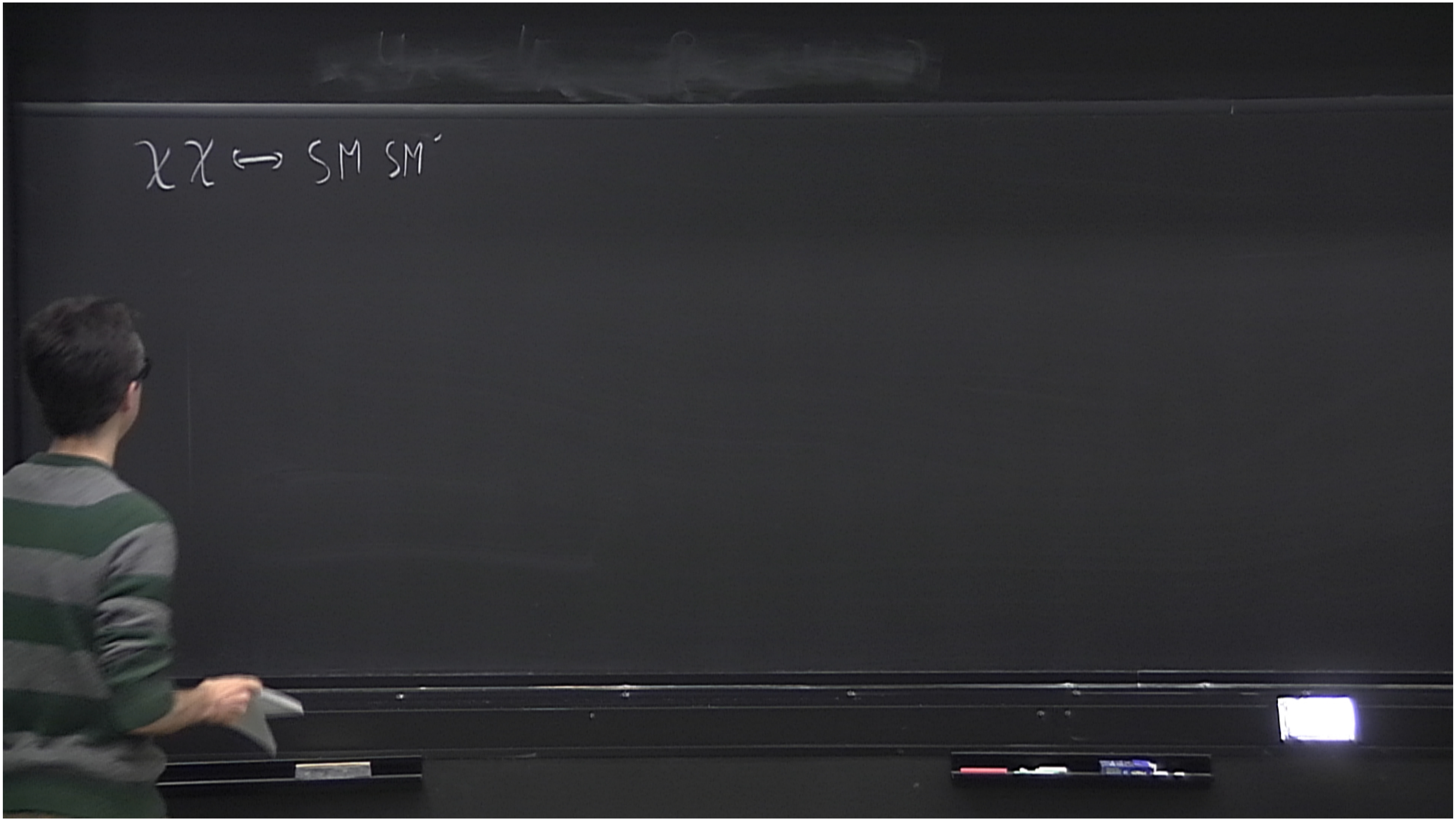


C̃[n_i] = − ∫(dΠ_i dΠ_X … dΠ_Y)(dΠ_a … dΠ_b) (2π)^4 δ^(4)(p_i + … + p_Y − p_a − … − p_b)
mat. el.
initial
× [|M|^2_(1+X+…+Y → a+…+b) f_X … f_Y f_i (1 ± f_a) … (1 ± f_b)
− |M|^2_(a+b… → 1+X+…+Y) f_a … f_b … (1 ± f_Y)] 1/S
dΠ_i = d^3p_i / 2E_i(2π)^3
|M|^2_(i→f) = squared matrix element
averaged over all initial and final states


$\tilde{C}[n_i] = -\int (d\Pi_i\, d\Pi_x \dots d\Pi_Y)(d\Pi_a \dots d\Pi_b)\,(2\pi)^4\, \delta^{(4)}(p_i + \dots + p_Y - p_a - \dots - p_b)$
mat. el.
initial
$\times \Big[\, |M|^2_{1+X+\dots+Y \to a+\dots+b}\; f_X \dots f_Y f_i\, (1 \pm f_a) \dots (1 \pm f_b)$
Fermi Blocking / Stimulated Em
$- |M|^2_{a+b \dots \to 1+X+\dots+Y}\; f_a \dots f_b\, (1 \pm f_i) \dots (1 \pm f_Y) \Big] \frac{1}{S}$
$d\Pi_i = \frac{d^3 p_i}{2E_i (2\pi)^3}$
$|M|^2_{i \to f}$ = squared matrix element
averaged over all initial and final states

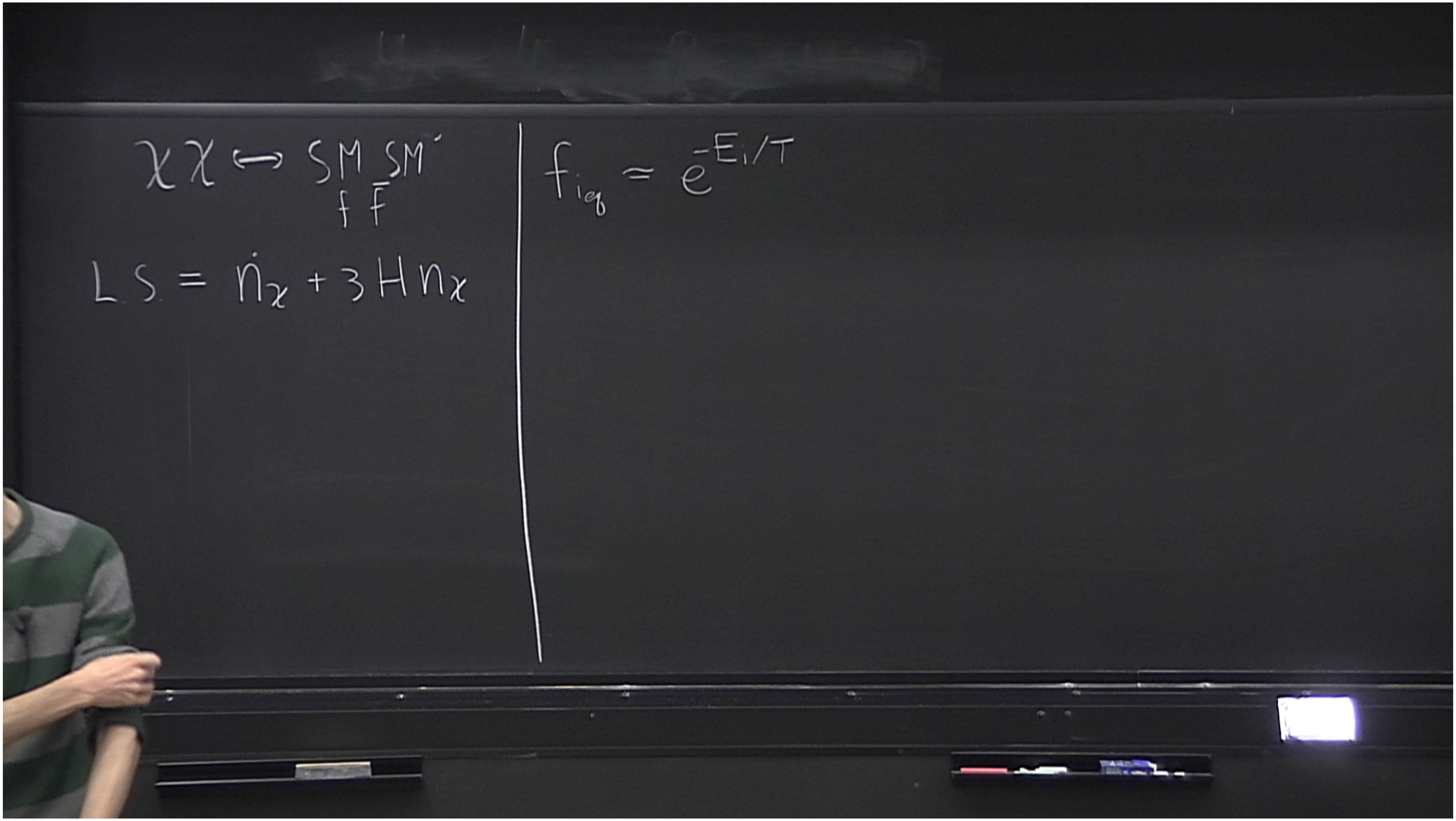

Phase Space
4-momentum conservation
Fermi Blocking / Stimulated Emission
initial
final states


Phase Space
4-momentum conservation
$d\Pi_Y)(d\Pi_a \ldots d\Pi_b)(2\pi)^4\delta^{(4)}(p_i+\ldots+p_Y-p_a-\ldots-p_b)$
Fermi Blocking / Stimulated Emission
initial
$f_X \ldots f_Y f_i (1\pm f_a)\ldots(1\pm f_b)$
$+X+\ldots+Y\to a+\ldots+b$
$|M|^2_{a+b\ldots\to 1+X+\ldots+Y}$
$f_a\ldots f_b(1\pm f_i)\ldots(1\pm f_Y)]\frac{1}{S}$
symmetry factor
$\frac{1}{n!}$ for every set of n identical things
element
tial and final states

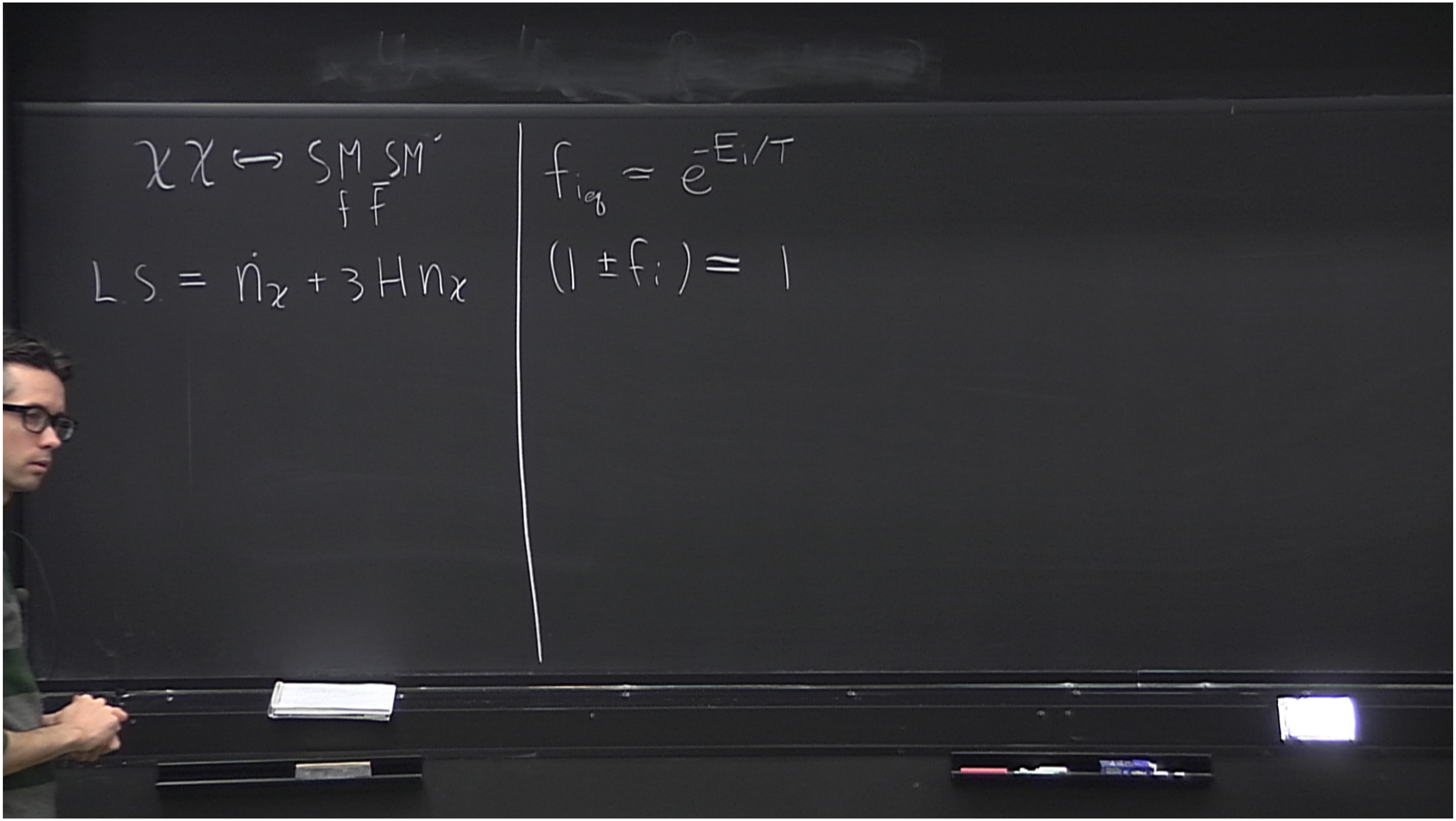

$\chi\chi \leftrightarrow SM\ SM'$

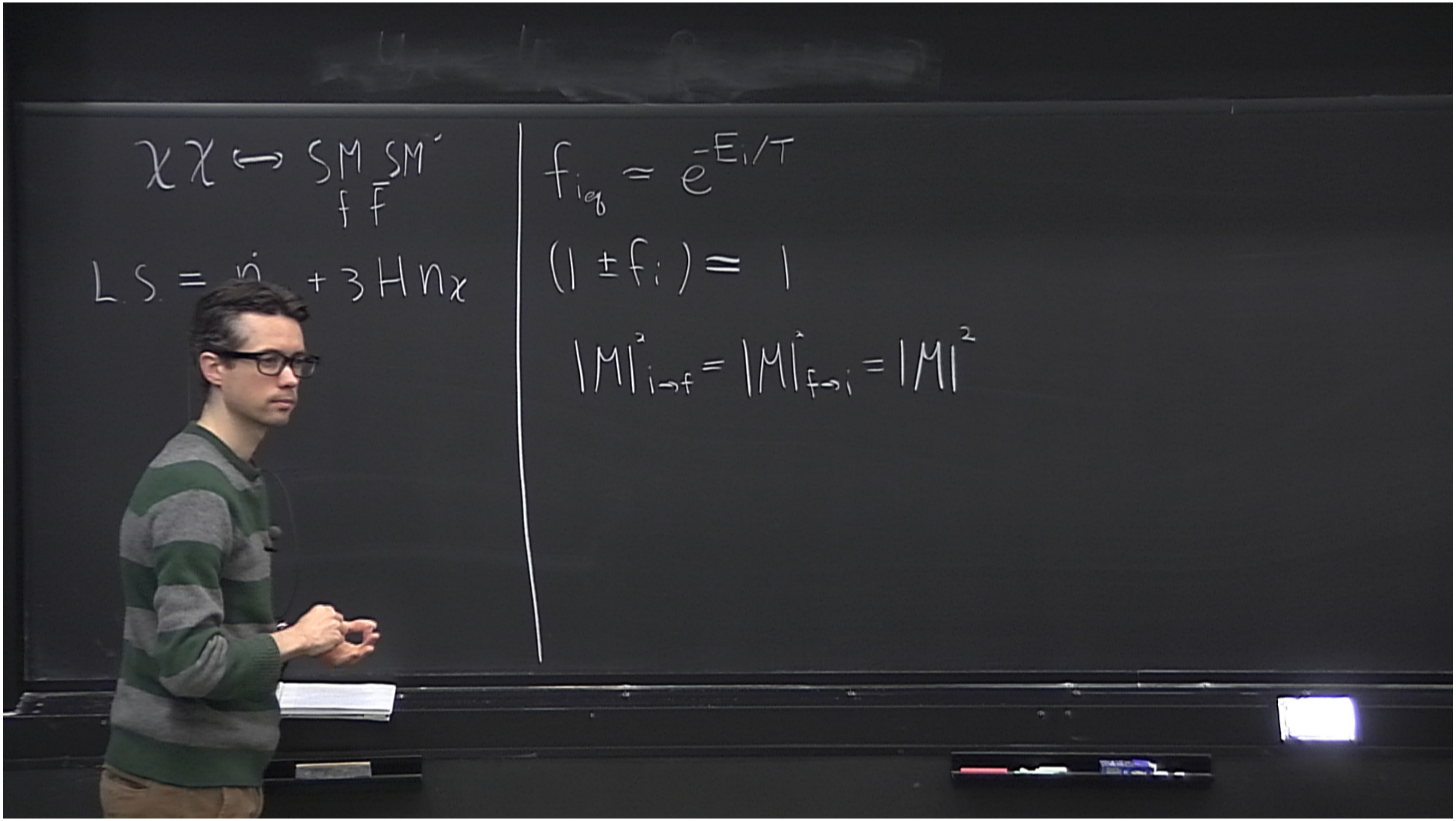
$$\chi\chi \leftrightarrow \underset{f}{SM}\ \underset{\bar{f}}{SM}$$

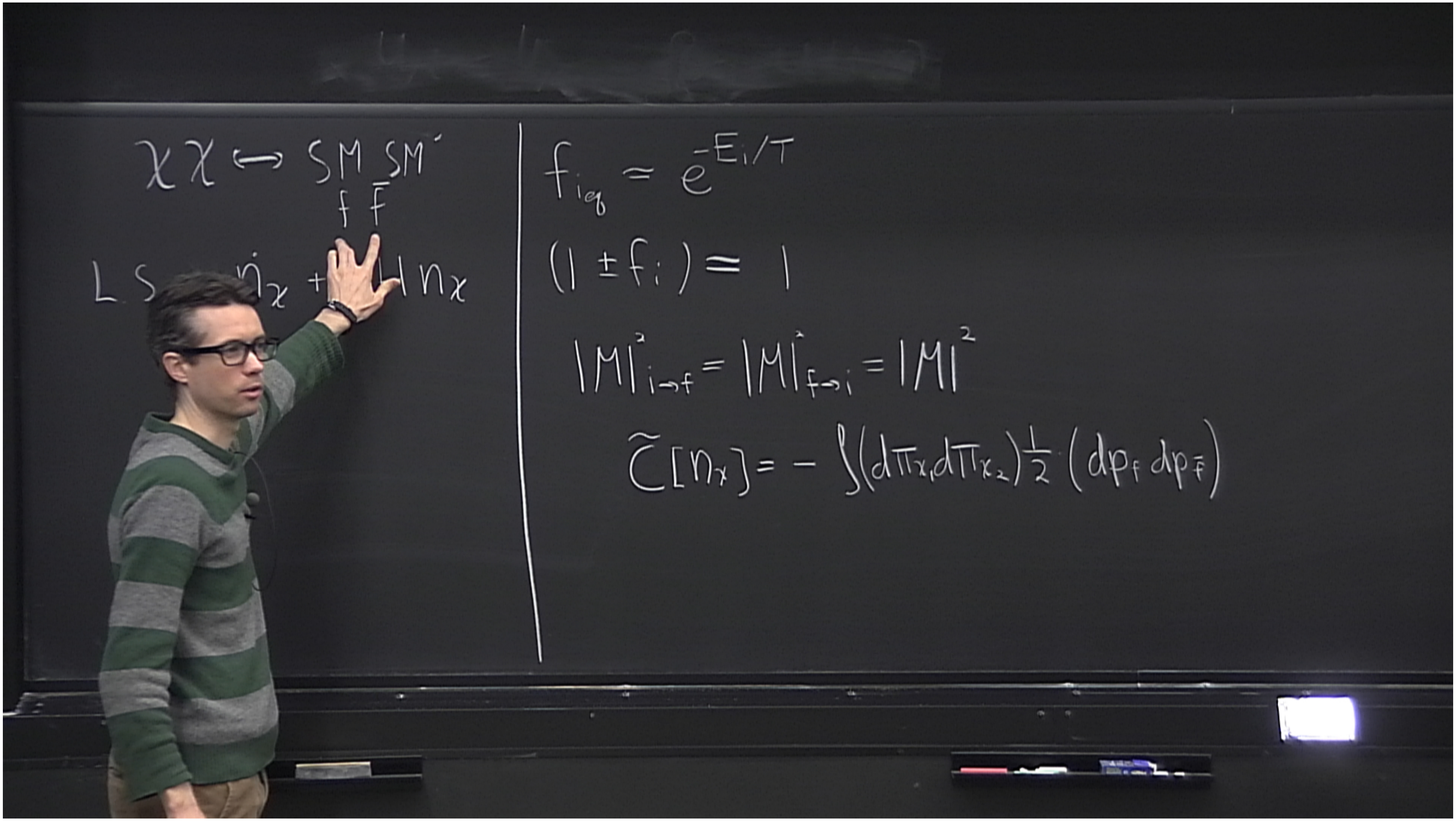
$$L.S. = \dot{n}_\chi + 3 H n_\chi$$



$\chi\chi \leftrightarrow SM\ SM$
$f\ \bar{f}$
$L.S. = \dot{n}_\chi + 3Hn_\chi$
$f_{ieq} \simeq e^{-E_i/T}$


$\chi\chi \leftrightarrow SM\ SM$
$f\ \bar{f}$
$L.S. = \dot{n}_\chi + 3Hn_\chi$
$f_{i_{eq}} \simeq e^{-E_i/T}$
$(1 \pm f_i) \simeq 1$


$\chi\chi \leftrightarrow SM\ SM$
$f\ \bar{f}$
$L.S. = \dot{n} \quad + 3Hn_\chi$
$f_{i_{eq}} \simeq e^{-E_i/T}$
$(1 \pm f_i) \simeq 1$
$|M|^2_{i\to f} = |M|^2_{f\to i} = |M|^2$


$\chi\chi \leftrightarrow SM\ \overline{SM}$
$f\ \bar{f}$
$L.S.$ $\dot{n}_\chi +$ $H n_\chi$
$f_{i_{eq}} \simeq e^{-E_i/T}$
$(1 \pm f_i) = 1$
$|M|^2_{i\to f} = |M|^2_{f\to i} = |M|^2$
$\tilde{C}[n_\chi] = -\int (d\Pi_{\chi_1} d\Pi_{\chi_2}) \frac{1}{2} (dp_f\, dp_{\bar{f}})$

$$\chi\chi \leftrightarrow \underset{f}{SM}\ \underset{\bar f}{SM}$$

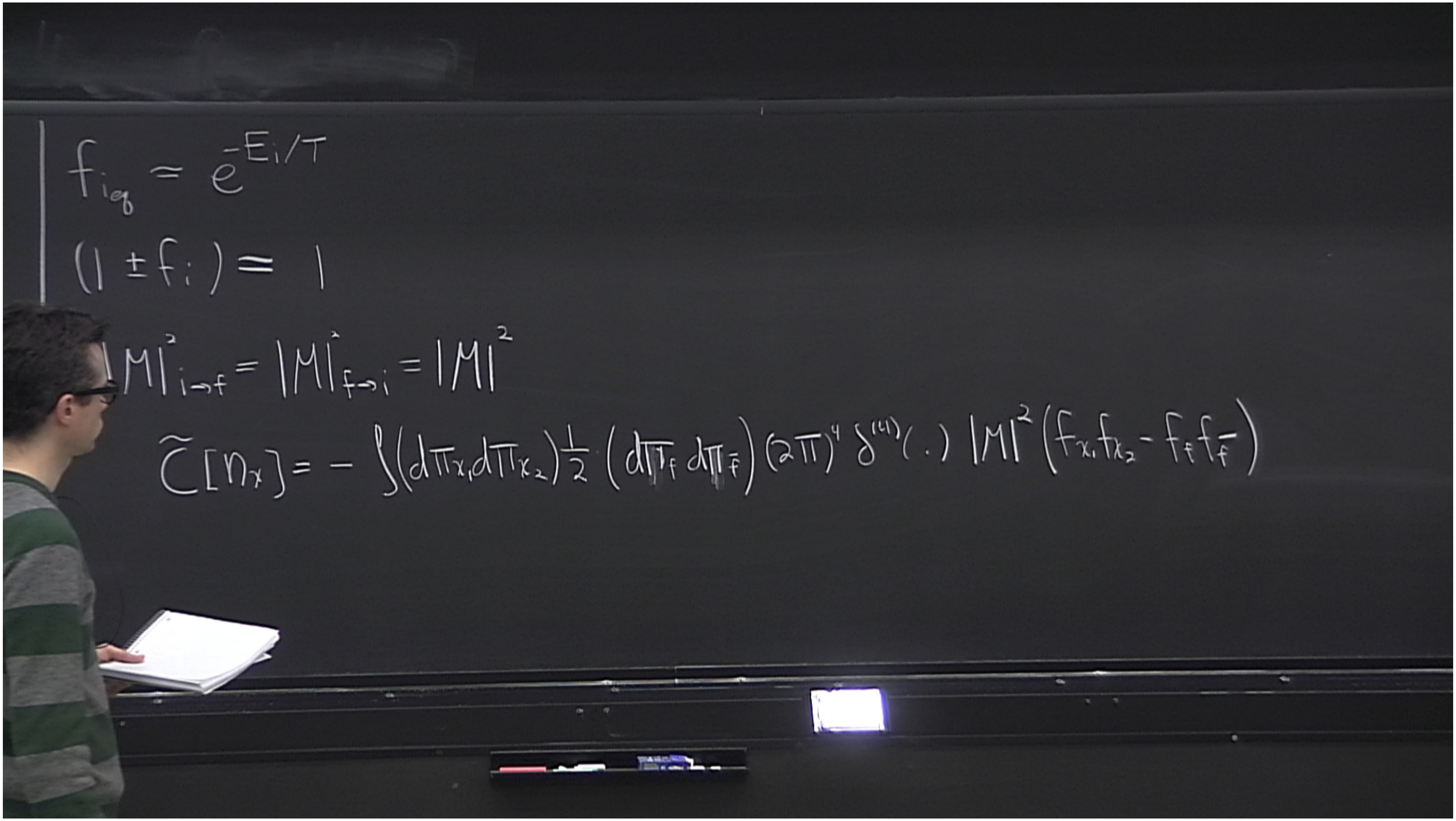
$$L.S. = \dot n_\chi + 3 H n_\chi$$

$$f_{i_{eq}} \simeq e^{-E_i/T}$$

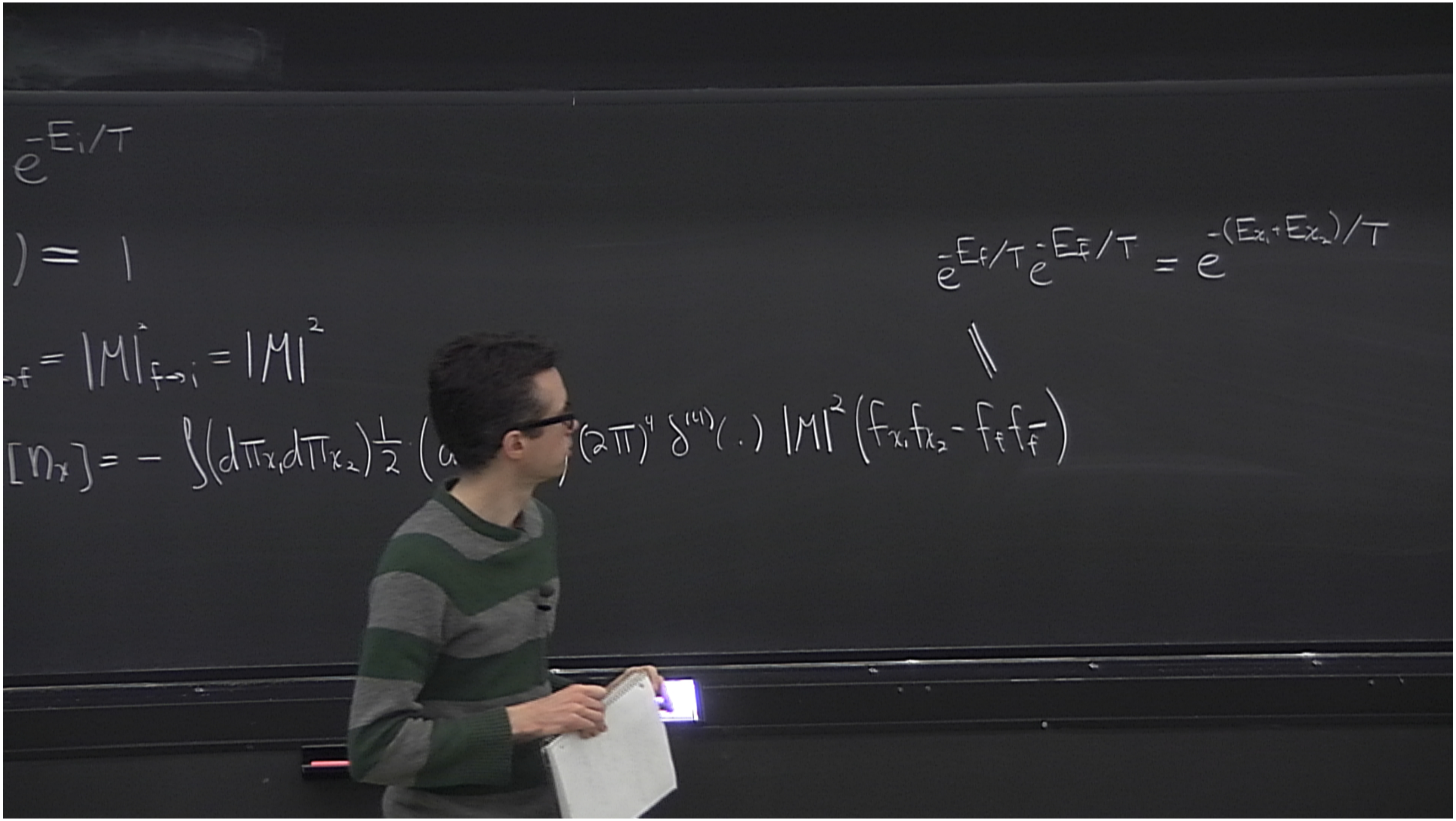
$$(1 \pm f_i) \simeq 1$$

$$|M|^2_{i\to f} = |M|^2_{f\to i} = |M|^2$$

$$\tilde C[n_\chi] = -\int (d\Pi_{\chi_1} d\Pi_{\chi_2}) \frac{1}{2} (d\Pi_f\, d\Pi_{\bar f})$$



$f_{i_{eq}} \simeq e^{-E_i/T}$
$(1 \pm f_i) \simeq 1$
$|M|^2_{i\to f} = |M|^2_{f\to i} = |M|^2$
$\tilde{C}[n_\chi] = -\int (d\Pi_{\chi_1} d\Pi_{\chi_2}) \frac{1}{2} (d\Pi_f \, d\Pi_{\bar{f}}) (2\pi)^4 \delta^{(4)}(.) \, |M|^2 (f_{\chi_1} f_{\chi_2} - f_f f_{\bar{f}})$


$e^{-E_i/T}$
$) = 1$
$= |M|^2_{f\to i} = |M|^2$
$e^{-E_f/T} e^{-E_{\bar{f}}/T} = e^{-(E_{\chi_1}+E_{\chi_2})/T}$
$[n_\chi] = -\int (d\Pi_{\chi_1} d\Pi_{\chi_2}) \frac{1}{2} (\;) (2\pi)^4 \delta^{(4)}(\,.\,) |M|^2 (f_{\chi_1} f_{\chi_2} - f_f f_{\bar{f}})$

$$e^{-E_i/T}$$

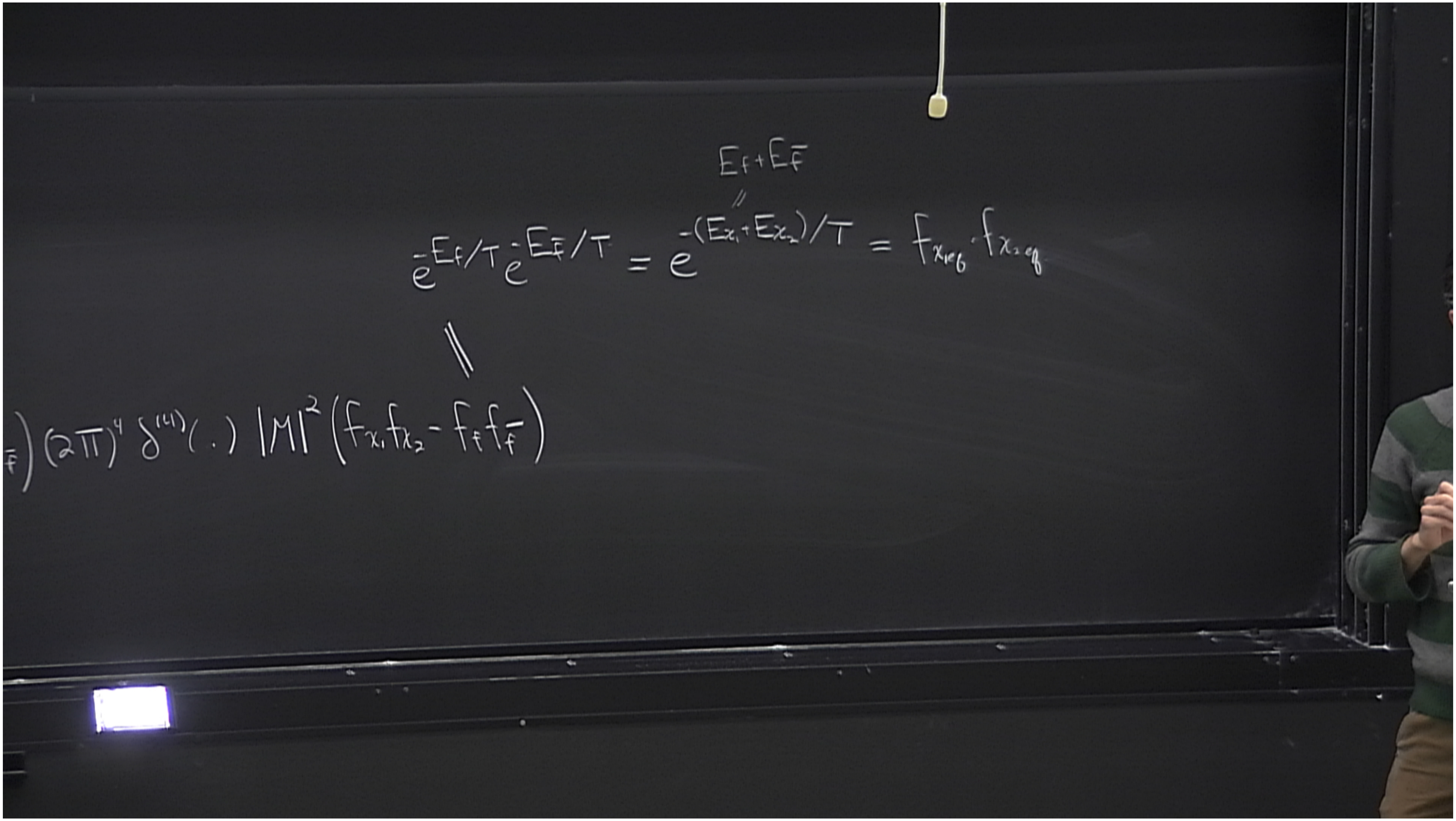
$$) \simeq 1$$

$$_{\to f} = |M|^2_{f\to i} = |M|^2$$

$$e^{-E_f/T} e^{-E_{\bar f}/T} = e^{-(E_{\chi_1}+E_{\chi_2})/T}$$

$$E_{\chi_1}+E_{\chi_2} = E_f + E_{\bar f}$$

$$[n_\chi] = -\int (d\Pi_{\chi_1} d\Pi_{\chi_2}) \frac{1}{2} (d\Pi_f\, d\Pi_{\bar f}) (2\pi)^4 \delta^{(4)}(\,.\,)\, |M|^2 \left(f_{\chi_1} f_{\chi_2} - f_f f_{\bar f}\right)$$



$E_f + E_{\bar{f}}$
$e^{-E_f/T} e^{-E_{\bar{f}}/T} = e^{-(E_{\chi_1}+E_{\chi_2})/T} = f_{\chi_1 eq} \cdot f_{\chi_2 eq}$
$\bar{f}) (2\pi)^4 \delta^{(4)}(.) |M|^2 (f_{\chi_1} f_{\chi_2} - f_f f_{\bar{f}})$

$$\overbrace{E_f + E_{\bar f}}^{}$$

$$e^{-E_f/T}\, e^{-E_{\bar f}/T} = e^{-(E_{\chi_1}+E_{\chi_2})/T} = f_{\chi_1 eq} \cdot f_{\chi_2 eq}$$

$$\|$$

$$\bar f)\, (2\pi)^4\, \delta^{(4)}(\,.\,)\, |M|^2 \Big(\underbrace{f_{\chi_1} f_{\chi_2} - f_f f_{\bar f}}_{\left(f_{\chi_1} f_{\chi_2} - f_{\chi_{1 eq}} f_{\chi_{2 eq}}\right)}\Big)$$

$$\left(g_i \left(\frac{m_i T}{2\pi}\right)^{3/2} e^{-m_i/T} \right. \quad ; T \ll m_i$$

$$\left(p^\alpha \frac{\partial}{\partial x^\alpha} - \Gamma^\alpha_{\beta\gamma} p^\beta p^\gamma \frac{\partial}{\partial p^\alpha} \right)$$

$\chi\chi \leftrightarrow SM\ SM^-$

$\qquad\quad f\ \bar{f}$

$$LS = \dot{n}_\chi + 3Hn_\chi$$

$$f_{i_{eq}} = e^{-E_i/T}$$

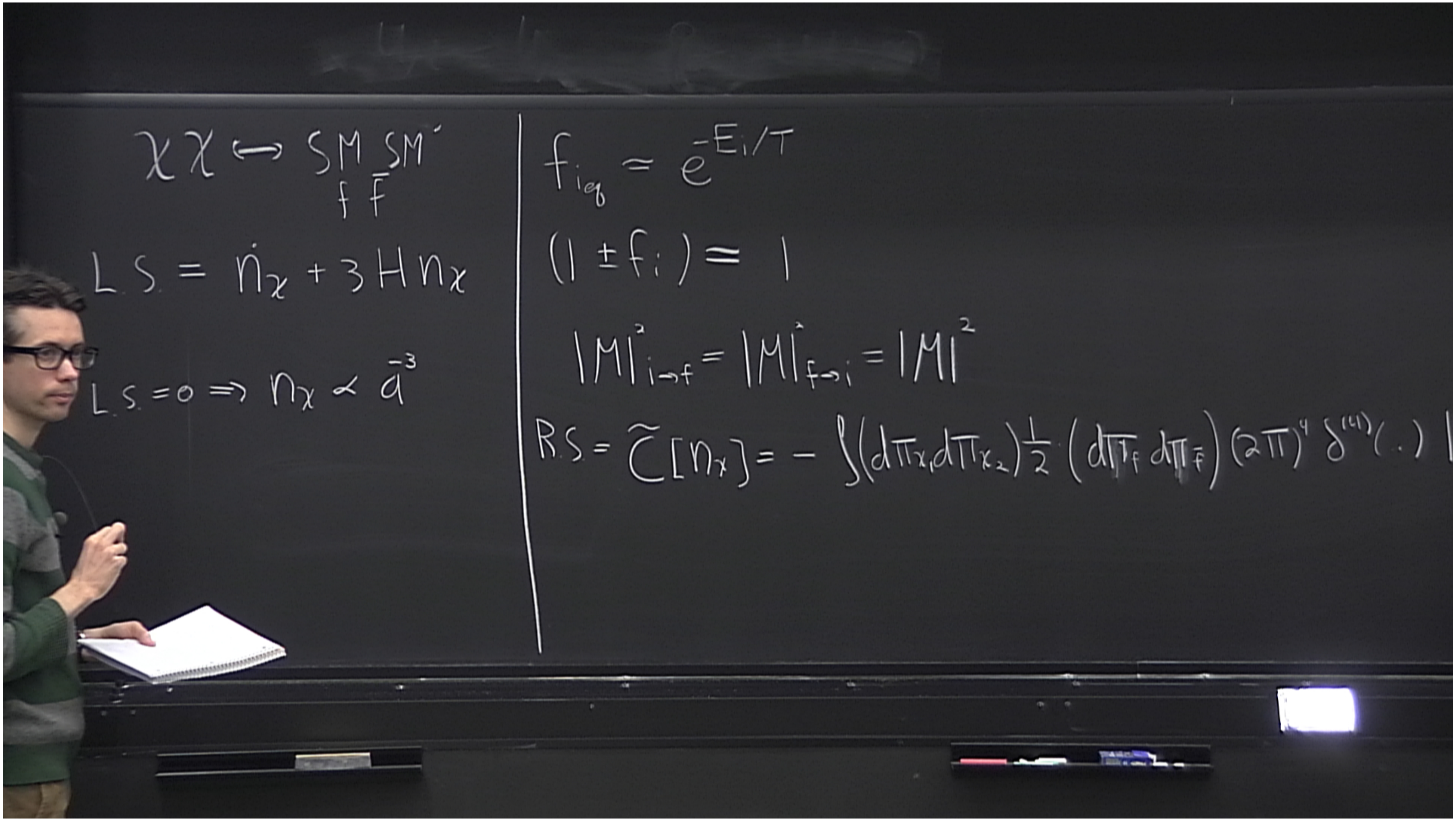
$$(1 \pm f_i) = 1$$

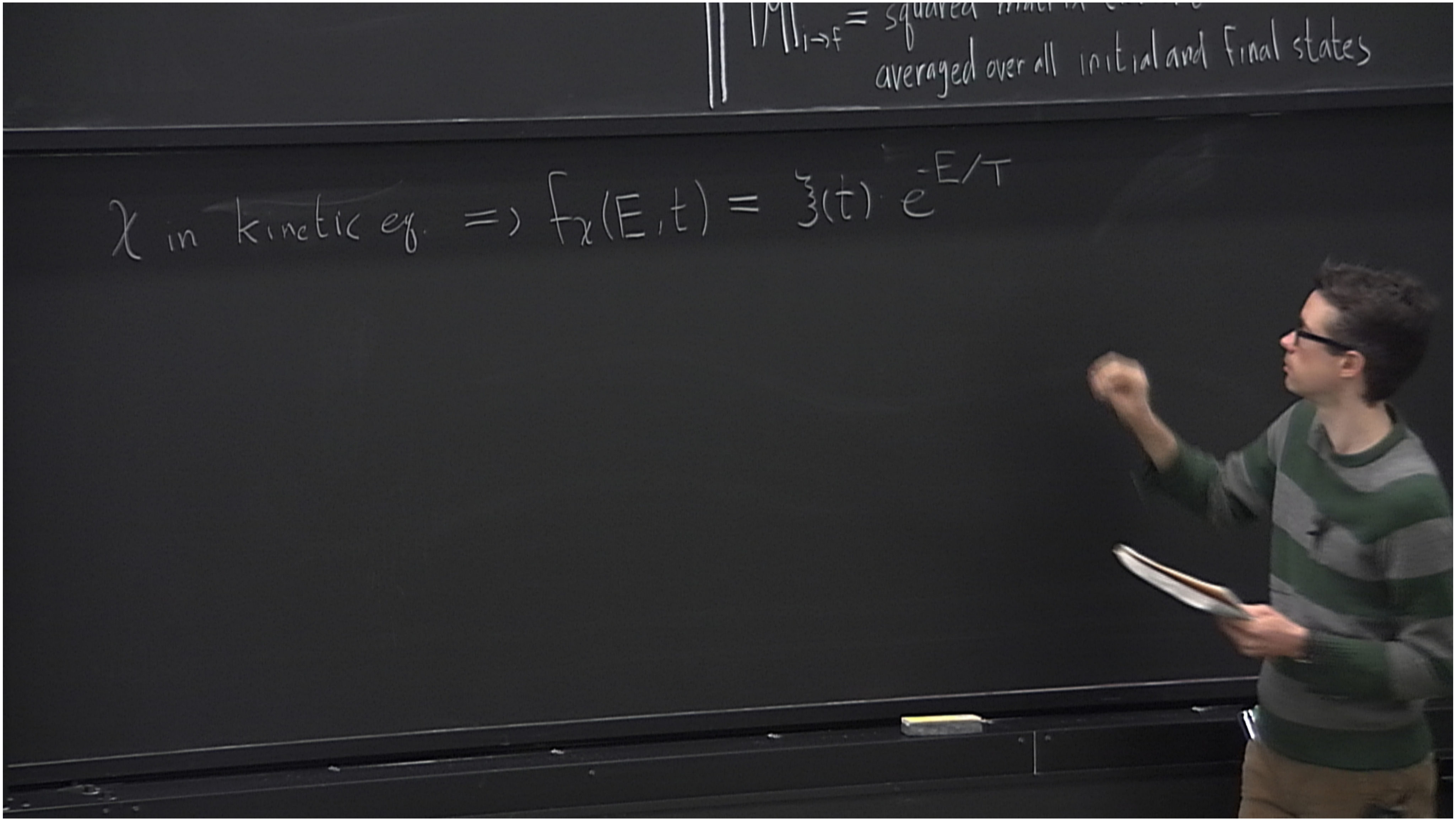
$$|M|^2_{i\to f} = |M|^2_{f\to i} = |M|^2$$

$$RS = \tilde{C}[n_\chi] = -\int (d\Pi_{\chi_1} d\Pi_{\chi_2}) \frac{1}{2} (d\Pi_f\, d\Pi_{\bar{f}}) (2\pi)^4 \delta^{(4)}(\ ) |M|^2 \underbrace{(f_{\chi_1} f_{\chi_2} - f_f f_{\bar{f}})}_{(f_{\chi_1} f_{\chi_2} - f_{\chi_{1eq}} f_{\chi_{2eq}})}$$

$$e^{-E_f/T} e^{-E_{\bar{f}}/T} = e^{-(E_{\chi_1} + E_{\chi_2})/T} = f_{\chi_{1eq}} \cdot f_{\chi_{2eq}}$$

$E_f + E_{\bar{f}} = E_{\chi_1} + E_{\chi_2}$


$\chi\chi \leftrightarrow SM\ SM$
$f\ \bar{f}$
$L.S. = \dot{n}_\chi + 3Hn_\chi$
$L.S. = 0 \Rightarrow n_\chi \propto a^{-3}$
$f_{i_{eq}} \simeq e^{-E_i/T}$
$(1 \pm f_i) \simeq 1$
$|M|^2_{i\to f} = |M|^2_{f\to i} = |M|^2$
$R.S. = \tilde{C}[n_\chi] = -\int (d\Pi_{\chi_1} d\Pi_{\chi_2}) \frac{1}{2} (d\Pi_f\, d\Pi_{\bar{f}}) (2\pi)^4 \delta^{(4)}(\ .\ )$


averaged over all initial and final states
χ in kinetic eq. $\Rightarrow f_\chi(E,t) = \xi(t)\cdot e^{-E/T}$

$|\mathcal{M}|_{i\to f}$ = squared matrix
averaged over all initial and final states

χ in kinetic eq. $\Rightarrow f_\chi(E,t) = \xi(t)\cdot e^{-E/T} = \xi(t)\, f_{\chi_{eq}}(E)$

$$\chi\chi \longleftrightarrow \underset{f}{SM}\ \underset{\bar{f}}{SM}$$

$$L.S. = \dot{n}_\chi + 3Hn_\chi$$

$$L.S. = 0 \Rightarrow n_\chi \propto a^{-3}$$

$$f_{ieq} \simeq e^{-E_i/T}$$

$$(1 \pm f_i) \simeq 1$$

$$|M|^2_{i\to f} = |M|^2_{f\to i} = |M|^2$$

$$R.S. = \tilde{C}[n_\chi] = -\int (d\Pi_{\chi_1} d\Pi_{\chi_2}) \frac{1}{2} (d\Pi_f \, d\Pi_{\bar{f}}) (2\pi)^4 \delta^{(4)}$$

$$= -\int (d\Pi_{\chi_1} \, d\Pi_{\chi_2})$$

$$e^{-E_f/T}\,e^{-E_{\bar f}/T} = e^{-(\overset{E_f + E_{\bar f}}{\overset{\parallel}{E_{\chi_1}+E_{\chi_2}}})/T} = f_{\chi_1 eq} \cdot f_{\chi_2 eq}$$

$$\parallel$$

$$\bar f)\,(2\pi)^4\,\delta^{(4)}(\,.\,)\,|M|^2 \underbrace{\left(f_{\chi_1} f_{\chi_2} - f_f f_{\bar f}\right)}_{\left(f_{\chi_1} f_{\chi_2} - f_{\chi_{1 eq}} f_{\chi_{2 eq}}\right)}{}^{2}$$

$$f_{i_{eq}} \simeq e^{-E_i/T}$$

$$(1 \pm f_i) \simeq 1$$

$$|M|^2_{i\to f} = |M|^2_{f\to i} = |M|^2$$

$$e^{-E_f/T} e^{-E_{\bar f}/T} = e^{-(E$$

$$RS = \tilde{C}[n_\chi] = -\int (d\Pi_{\chi_1} d\Pi_{\chi_2}) \frac{1}{2} (d\Pi_f \, d\Pi_{\bar f}) (2\pi)^4 \delta^{(4)}(\,.\,) |M|^2 \underbrace{(f_{\chi_1} f_{\chi_2} - f_f f_{\bar f})}_{(f_{\chi_1} f_{\chi_2} - f_{\chi_{1_{eq}}} f_{\chi_{2_{eq}}})} 2$$

$$= - \int (d\Pi_{\chi_1} \, d\Pi_{\chi_2}) (f_{\chi_1} f_{\chi_2} - f_{\chi_{1_{eq}}} f_{\chi_{2_{eq}}}) \int (d\Pi_f \, d\Pi_{\bar f})$$

$$\left(g_i \left(\frac{m_i T}{2\pi}\right)^{3/2} e^{-m_i/T} \quad ; T \ll m_i \right.$$

$$\left(p^\alpha \frac{\partial}{\partial x^\alpha} - \Gamma^\alpha_{\beta\gamma} p^\beta p^\gamma \frac{\partial}{\partial p^\alpha} \right)$$

$$\chi\chi \leftrightarrow SM\ SM^* \quad f\ \bar{f}$$

$$LS = \dot{n}_\chi + 3 H n_\chi$$

$$LS = 0 \Rightarrow n_\chi \propto a^{-3}$$

$$f_{i_{eq}} \simeq e^{-E_i/T}$$

$$(1 \pm f_i) = 1$$

$$|M|^2_{i\to f} = |M|^2_{f\to i} = |M|^2$$

$$RS = \tilde{C}[n_\chi] = -\int (d\Pi_{\chi_1} d\Pi_{\chi_2}) \frac{1}{2} (d\Pi_f\, d\Pi_{\bar{f}}) (2\pi)^4 \delta^{(4)}(\ ) |M|^2 \underbrace{(f_{\chi_1} f_{\chi_2} - f_f f_{\bar{f}})}_{(f_{\chi_1} f_{\chi_2} - f_{\chi_{1eq}} f_{\chi_{2eq}})} 2$$

$$= -\int (d\Pi_{\chi_1} d\Pi_{\chi_2}) (f_{\chi_1} f_{\chi_2} - f_{\chi_{1eq}} f_{\chi_{2eq}}) \int (d\Pi_f d\Pi_{\bar{f}})$$

$$e^{-E_f/T} e^{-E_{\bar{f}}/T} = e^{-(E_{\chi_1} + E_{\chi_2})/T} = f_{\chi_{1eq}} \cdot f_{\chi_{2eq}}$$

$$E_f + E_{\bar{f}} = E_{\chi_1} + E_{\chi_2}$$

$$f_{i_{eq}} \simeq e^{-E_i/T}$$

$$(1 \pm f_i) \simeq 1$$

$$|M|^2_{i\to f} = |M|^2_{f\to i} = |M|^2$$

$$e^{-E_f/T} e^{-E_{\bar f}/T} = e^{-(E}$$

$$\|$$

$$R.S = \tilde{C}[n_\chi] = -\int (d\Pi_{\chi_1} d\Pi_{\chi_2}) \frac{1}{2} (d\Pi_f \, d\Pi_{\bar f}) (2\pi)^4 \delta^{(4)}(\,.\,) |M|^2 (f_{\chi_1} f_{\chi_2} - f_f f_{\bar f}) 2$$

$$= -\int (d\Pi_{\chi_1} \, d\Pi_{\chi_2}) (f_{\chi_1} f_{\chi_2} - f_{\chi_{1_{eq}}} f_{\chi_{2_{eq}}}) \left[\int (d\Pi_f \, d\Pi_{\bar f}) (2\pi)^4 \delta^{(4)}(\,) |M|^2 \right]$$

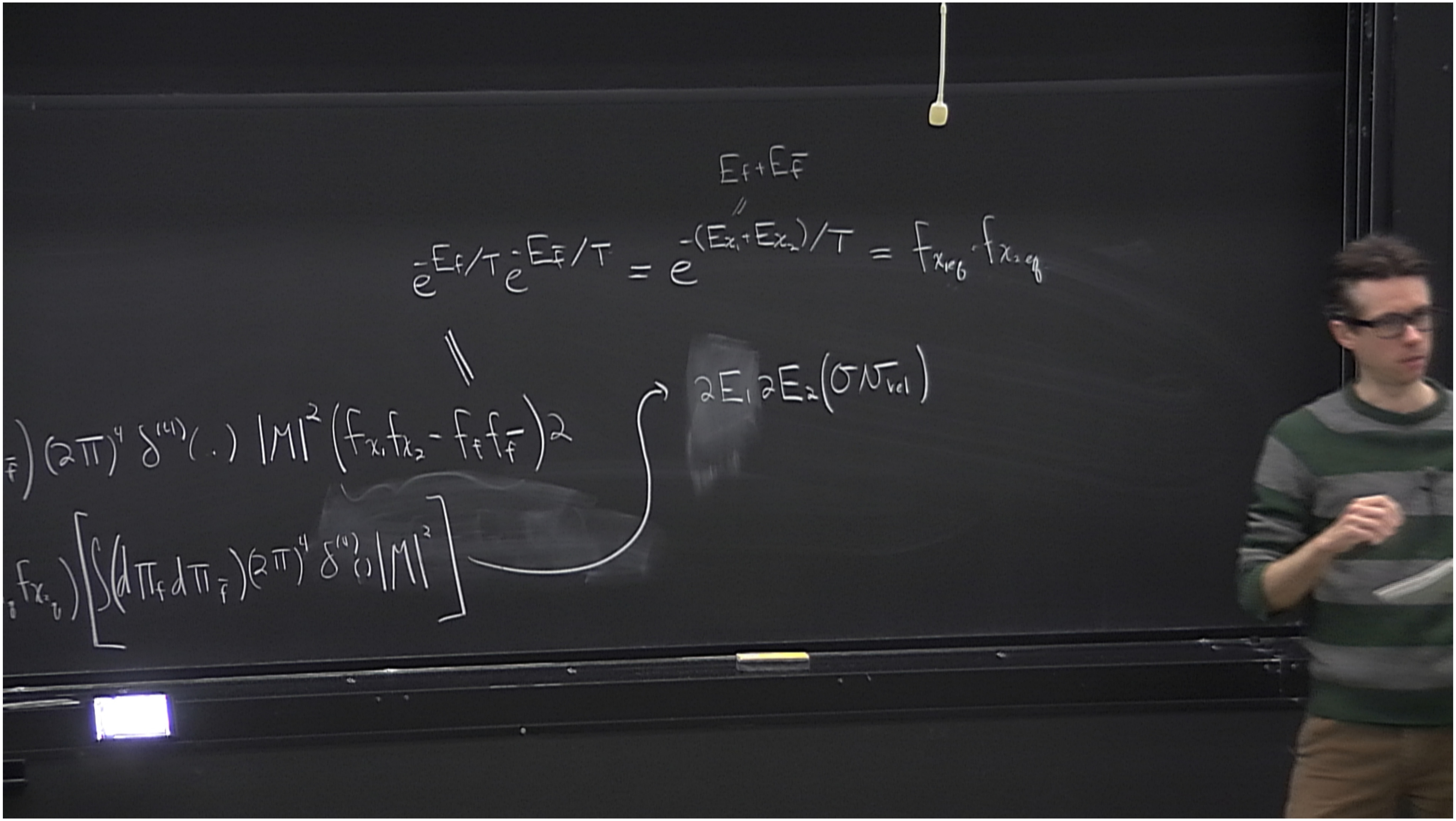
$$e^{-E_f/T}\, e^{-E_{\bar f}/T} = e^{-(E_{\chi_1}+E_{\chi_2})/T} = f_{\chi_1 eq} \cdot f_{\chi_2 eq}$$

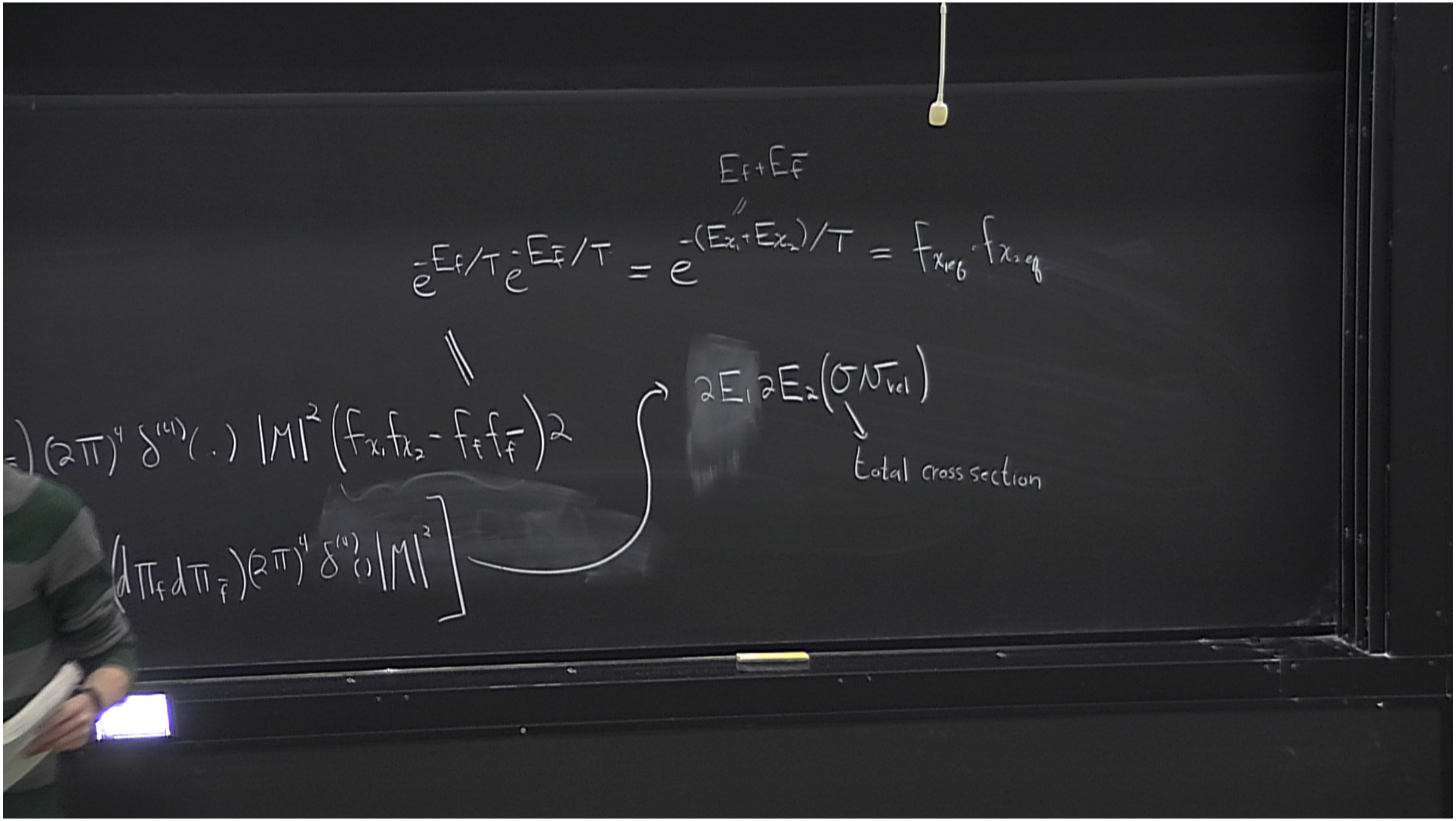
$$E_f + E_{\bar f} = E_{\chi_1}+E_{\chi_2}$$

$$\|$$

$$\ldots_{\bar f}) (2\pi)^4 \delta^{(4)}(\,.\,)\, |M|^2 \left(f_{\chi_1} f_{\chi_2} - f_f f_{\bar f}\right) 2$$

$$\ldots f_{\chi_2 eq}) \left[\int (d\Pi_f\, d\Pi_{\bar f}) (2\pi)^4 \delta^{(4)}(\,)\, |M|^2 \right] \to g_1 g_2$$


E_f + E_f̄
e^{-E_f/T} e^{-E_f̄/T} = e^{-(E_χ1 + E_χ2)/T} = f_χ1eq · f_χ2eq
2E_1 2E_2 (σ v_rel)
) (2π)^4 δ^(4)(.) |M|^2 (f_χ1 f_χ2 - f_f f_f̄) 2
f_χ2) [∫(dΠ_f dΠ_f̄)(2π)^4 δ^(4)( ) |M|^2]



$E_f + E_{\bar{f}}$
$e^{-E_f/T} e^{-E_{\bar{f}}/T} = e^{-(E_{\chi_1}+E_{\chi_2})/T} = f_{\chi_1 eq} \cdot f_{\chi_2 eq}$
$) (2\pi)^4 \delta^{(4)}(\,.\,) |M|^2 (f_{\chi_1} f_{\chi_2} - f_f f_{\bar{f}}) 2$
$2E_1 2E_2 (\sigma v_{rel})$
total cross section
$(d\Pi_f d\Pi_{\bar{f}})(2\pi)^4 \delta^{(4)}(\,) |M|^2]$

$$E_f + E_{\bar f}$$

$$e^{-E_f/T}\, e^{-E_{\bar f}/T} = e^{-(E_{\chi_1}+E_{\chi_2})/T} = f_{\chi_1 eq} \cdot f_{\chi_2 eq}$$

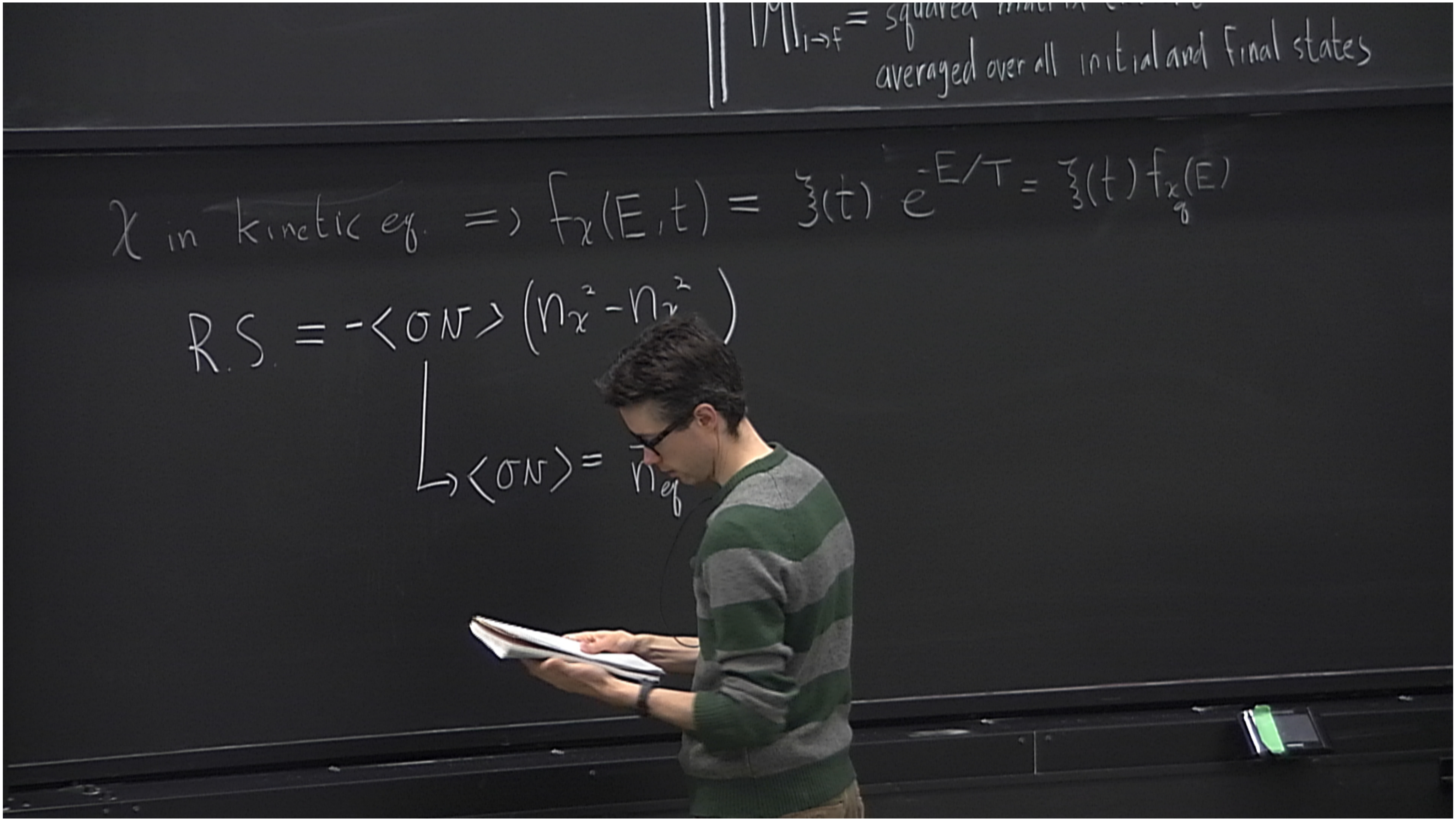
$$\parallel$$

$$(2\pi)^4\, \delta^{(4)}(\,.\,)\, |M|^2 \left(f_{\chi_1} f_{\chi_2} - f_f f_{\bar f}\right) 2$$

$$f_{\chi_2})\left[\int \left(d\Pi_f\, d\Pi_{\bar f}\right)(2\pi)^4\, \delta^{(4)}(\,)\, |M|^2\right]$$

$$2E_1\, 2E_2 \left(\sigma v_{rel}\right) = [\ .\ ]$$

total cross section


averaged over all initial and final states
χ in kinetic eq. => fχ(E,t) = ξ(t) e^-E/T = ξ(t) fχeq(E)
R.S. = -<σv>(nχ² - nχ²)
<σv> = neq

$\overline{|M|}_{i\to f}$ = squared
averaged over all initial and final states

χ in kinetic eq. $\Rightarrow f_\chi(E,t) = \xi(t)\cdot e^{-E/T} = \xi(t)\, f_{\chi_{eq}}(E)$

$$R.S. = -\langle \sigma v \rangle \left(n_\chi^2 - n_{\chi_{eq}}^2\right)$$

$$\hookrightarrow \langle \sigma v \rangle = \frac{1}{n_{eq}^2} \int \frac{d^3p_1}{(2\pi)^3} \frac{d^3p_2}{(2\pi)^3} f_{1_{eq}} f_{2_{eq}} \cdot \sigma v$$

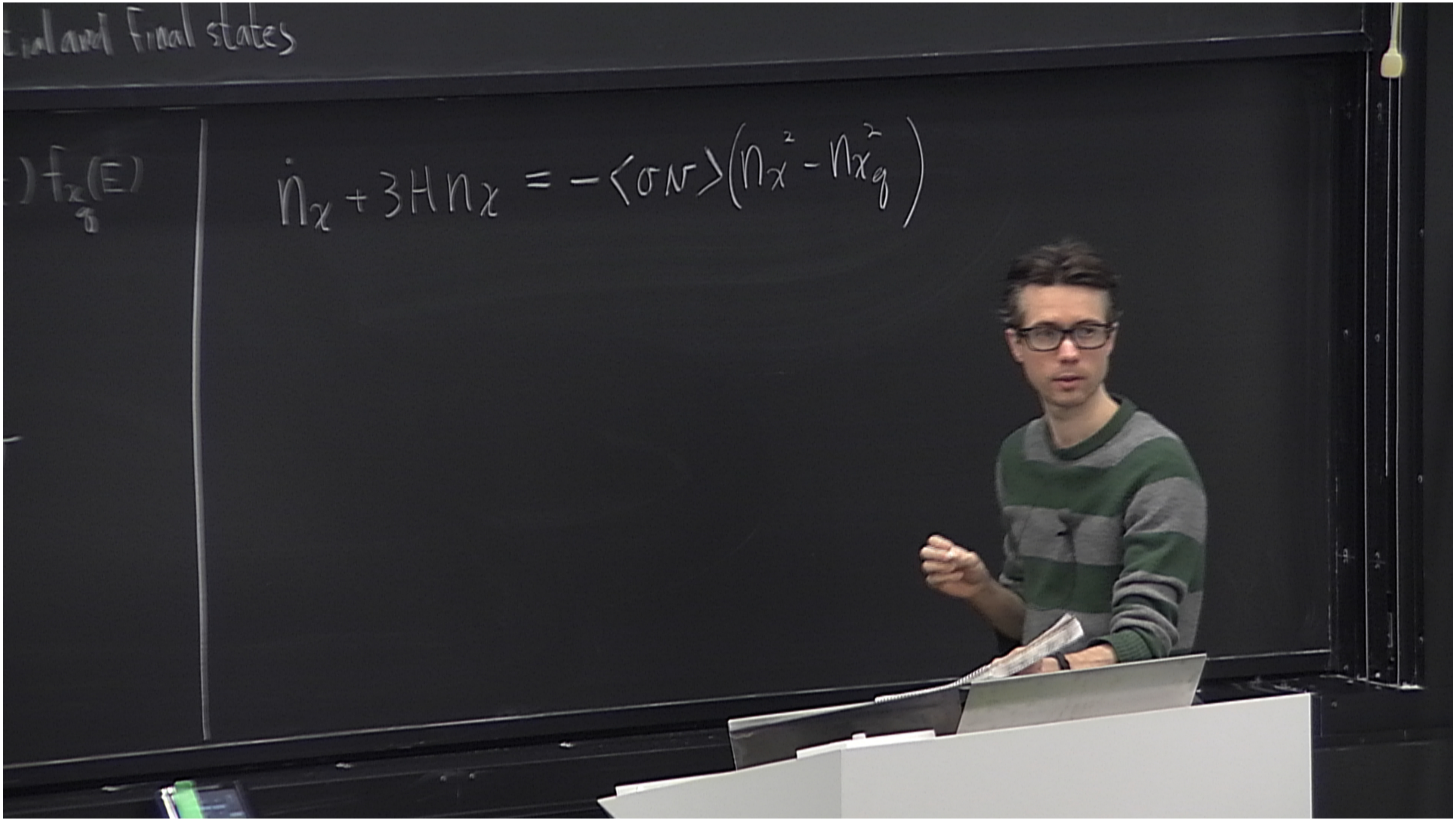
$|\mathcal{M}|_{i\to f}$ = squared matrix ...
averaged over all initial and final states

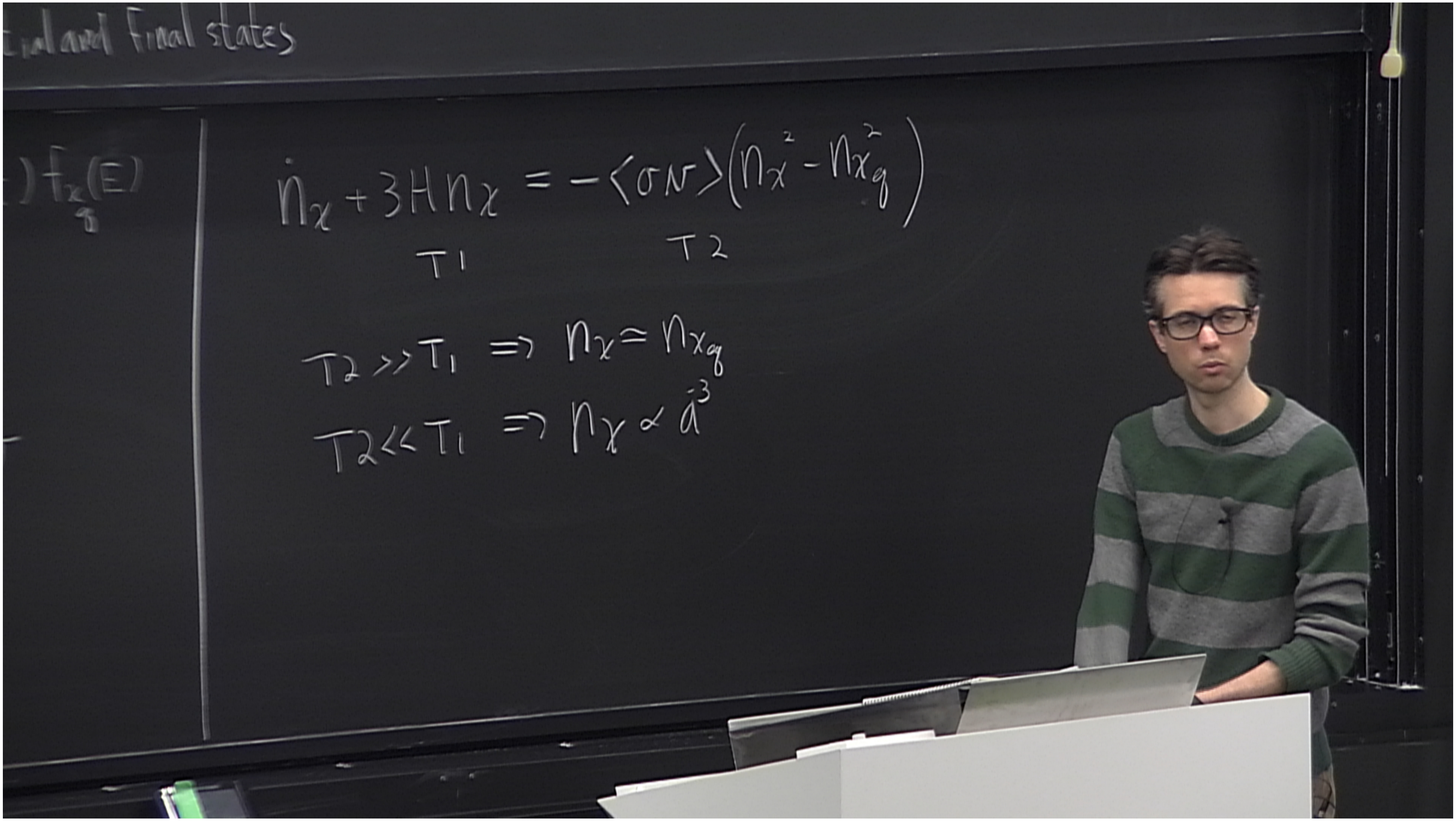
χ in kinetic eq. $\Rightarrow f_\chi(E,t) = \xi(t)\, e^{-E/T} = \xi(t)\, f_{\chi_{eq}}(E)$

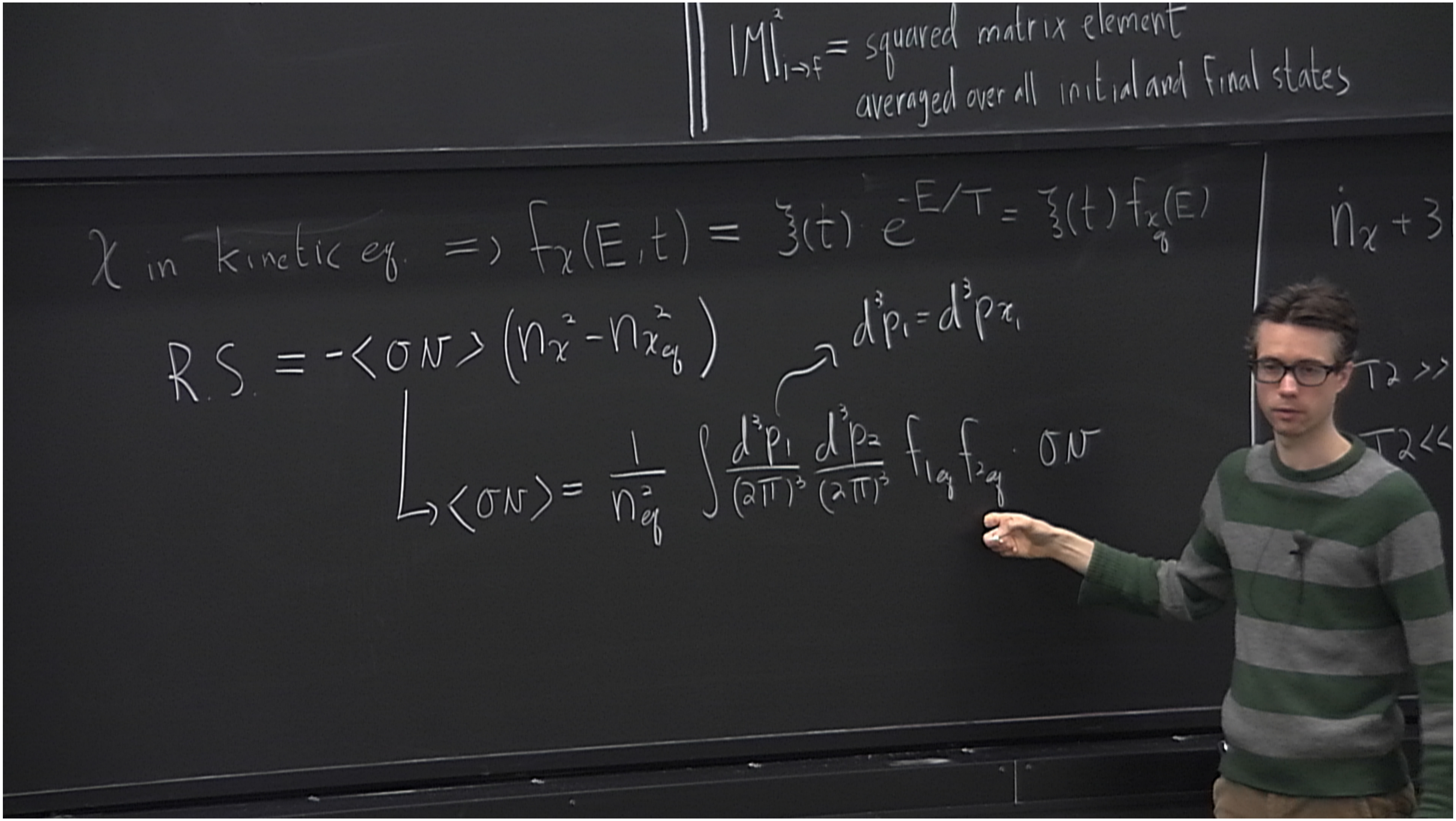
$$R.S. = -\langle \sigma v \rangle \left(n_\chi^2 - n_{\chi_{eq}}^2 \right)$$

$$\hookrightarrow \langle \sigma v \rangle = \frac{1}{n_{eq}^2} \int \frac{d^3p_1}{(2\pi)^3} \frac{d^3p_2}{(2\pi)^3} f_{1_{eq}} f_{2_{eq}} \cdot \sigma v$$

$\dot{n}_\chi$


and final states
$\dot{n}_\chi + 3Hn_\chi = -\langle\sigma v\rangle\left(n_\chi^2 - n_{\chi_{eq}}^2\right)$


...al and final states
$\dot{n}_\chi + 3Hn_\chi = -\langle\sigma v\rangle(n_\chi^2 - n_{\chi_{eq}}^2)$
T1
T2
$T2 \gg T_1 \Rightarrow n_\chi \simeq n_{\chi_{eq}}$
$T2 \ll T_1 \Rightarrow n_\chi \propto a^{-3}$


$|M|^2_{i\to f}$ = squared matrix element
averaged over all initial and final states
χ in kinetic eq. $\Rightarrow f_\chi(E,t) = \xi(t)\cdot e^{-E/T} = \xi(t) f_{\chi_{eq}}(E)$
$\dot{n}_\chi + 3$
$d^3p_1 = d^3p_{\chi_1}$
R.S. $= -\langle\sigma v\rangle (n_\chi^2 - n_{\chi_{eq}}^2)$
$\langle\sigma v\rangle = \frac{1}{n_{eq}^2}\int \frac{d^3p_1}{(2\pi)^3}\frac{d^3p_2}{(2\pi)^3} f_{1_{eq}} f_{2_{eq}} \cdot \sigma v$