Title: Instanton - A Window into Physics of M5-branes

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Abstract: Instantons and W-bosons in 5d N=2 Yang-Mills theory arise from a circle compactification of the 6d (2,0) theory as Kaluza-Klein modes and winding self-dual strings, respectively. We study an index which counts BPS instantons with electric charges in Coulomb and symmetric phases. We first prove the existence of unique threshold bound state of U(1) instantons for any instanton number. By studying SU(N) self-dual strings in the Coulomb phase, we find novel momentum-carrying degrees on the worldsheet. The total number of these degrees equals the anomaly coefficient of SU(N) (2,0) theory. We finally propose that our index can be used to study the symmetric phase of this theory, and provide an interpretation as the superconformal index of the sigma model on instanton moduli space.

Instanton: a Window into M5s

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based on: arXiv:1110.2175 or JHEP 1112 (2011) **031**
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M5-branes

- **Theories on M5-branes:**

. 6d N=(2,0) superconformal FIELD theory for tensor supermultiplet

. ADE classification, but focus on A-type theories

. No Langrangian description so far…

. No tunable dimensionless coupling const.; strongly interacting theories

. Mysterious scaling behavior

$\Rightarrow$ AdS/CFT : **#** of degrees of freedom = N^3

anomaly inflow : **#** of degrees of freedom = N^3 - N

5d N=2 SYM

- **M5-branes on a circle:** M5s become D4s

. 5d G=U(N) Maximally SYM with $g_{\rm YM}^2 = 8\pi^2 R_{\rm M}$; non-renormalizable theory

. Massive modes in KK tower: D0s in D4s

$\Longrightarrow$ appear in 5d theory as instanton solitons

$$M_{\rm inst} = \frac{8\pi^2}{g_{\rm YM}^2} = \frac{1}{R_{\rm M}} = M_{\rm KK}$$

. Proposal: non-perturbative formulation of 5d N=2 SYM can define 6d N=(2,0) SCFT

. Study quantum degeneracy of BPS states, insensitive to UV details !

BPS Spectrum

- **SUSY algebra:**

instanton charge — U(1)^N charge

$\langle \phi_I \rangle = v_I \ (I = 1,2,3,4,5)$

$$\{Q^i_M, Q^j_N\} = P_\mu (\Gamma^\mu C)_{MN}\omega^{ij} + i\frac{8\pi^2}{g^2_{YM}} k \, C_{MN}\omega^{ij} - i\mathrm{tr}(Q v_I)(\Gamma^I \omega)^{ij} C_{MN}$$

M, N : SO(1,4) spinor index
i, j : Sp(4) R-symmetry index

. Symmetric phase:

½-BPS instanton (D0): massive tensor multiplet $(\mathbf{3},\mathbf{1},\mathbf{1}) \oplus (\mathbf{2},\mathbf{1},\mathbf{4}) \oplus (\mathbf{1},\mathbf{1},\mathbf{5})$ (SO(4), Sp(4))

. Coulomb phase: $U(N) \to U(1)^N$

½-BPS W-boson (F1): massive vector multiplet

¼-BPS W-boson + instanton (F1+D0); when D4s are parallel, $v_5 = \mathrm{diag}(v^1, v^2, ..., v^N)$

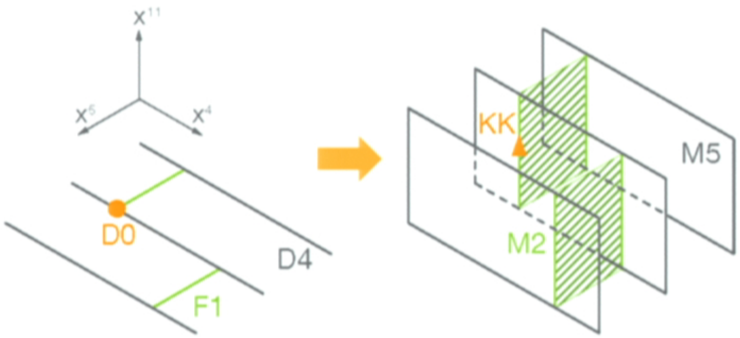
$Q = \mathrm{diag}(Q_1, Q_2, ..., Q_N)$

$$M_{1/4\text{-BPS}} = \frac{8\pi^2 |k|}{g^2_{\rm YM}} + \vec{Q}\cdot\vec{v}$$

$\Longrightarrow$ threshold bound states: subtle !

Counting ¼-BPS States

- **Why ¼-BPS states?:**



	0	1	2	3	4	5	6	7	8	9	11
M5	×	×	×	×	×						×
M2	×					×					×
KK	×										

$\Rightarrow$ Implication to quantum spectrum of 6d N=(2,0) theory & self-dual string

- **How to count them ? :** Witten index of (deformed) D0-D4 QM

Counting ¼-BPS States

- **SQM for k D0s in N D4s :** SUSY U(k) gauged QM with

. Global symmetry:
- $SO(4)_{1234}$ X $SO(4)_{6789}$
- 8 supercharges $\bar{Q}^i_{\dot{\alpha}}$: $(\mathbf{1},\mathbf{2},\mathbf{2},\mathbf{1}) \oplus (\mathbf{1},\mathbf{2},\mathbf{1},\mathbf{2})$
- flavor symmetry: $U(N) \rightarrow U(1)^N$

. Field contents & Reps: 1 vector- + 1 adj. hyper- (0-0 string) + N fund. hyper- multiplets (0-4 string)

$(\varphi_I, A_t, \bar{\lambda}^{i\dot{\alpha}})$ $(a_{\alpha\dot{\alpha}}, \lambda^i_\alpha)$ $(q_{\dot{\alpha}}, \psi^i)$

ADHM data:

$a_{\alpha\dot{\alpha}}$: $(\mathbf{2},\mathbf{2},\mathbf{1},\mathbf{1})$, motion of D0s inside D4s

$q_{\dot{\alpha}}$: $(\mathbf{1},\mathbf{2},\mathbf{1},\mathbf{1})$, scale modulus of instanton

φ_I : $(\mathbf{1},\mathbf{1},\mathbf{2},\mathbf{2}) \oplus (\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1})$, motion of D0s away from D4

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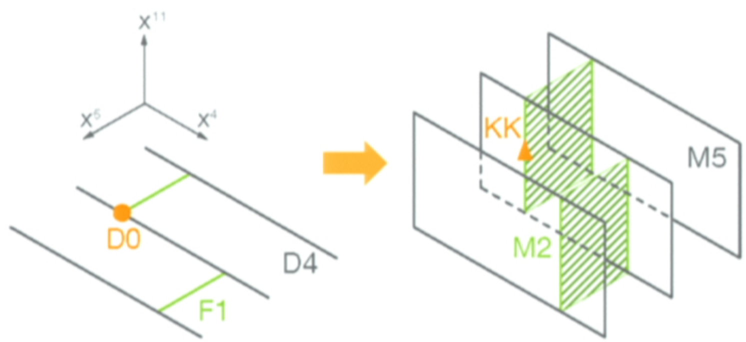

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- $\vec{v} = 0$: ½-BPS instanton = SUSY ground state in QM
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$$\mathcal{E} \geq \mathrm{tr}\big(v_5 Q\big) \qquad Q = i\big(q_{\dot{\alpha}} D_0 \bar{q}^{\dot{\alpha}} - D_0 q_{\dot{\alpha}} \bar{q}^{\dot{\alpha}}\big)$$

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- **How to count ½-BPS states in QM ?:**

. Define a supersymmetric index as

$$\mathcal{Q} \equiv \bar{Q}_{\dot{\alpha}\dot{a}}\epsilon^{\dot{\alpha}\dot{a}}$$

$$I_k = \mathrm{tr}\Big[(-1)^F e^{-\beta \mathcal{Q}^2}\Big] \qquad \boxed{\mathcal{Q}^2 = H - \vec{Q}\cdot\vec{v}}$$

$\Longrightarrow$ Since QM is too SUSY, Witten index becomes trivial !

. Introduce chemical potentials w.r.t. charges, satisfying $\underline{[\text{global charges}, \mathcal{Q}] = 0}$

$$I_k(\vec{\mu}, \gamma_L, \tilde{\gamma}_L, \gamma_R) = \mathrm{tr}\Big[(-1)^F e^{-\beta \mathcal{Q}^2} e^{-\vec{\mu}\cdot\boxed{\vec{Q}}} e^{-2i\gamma_L \boxed{J_L}} e^{-2i\tilde{\gamma}_L \boxed{\tilde{J}_L}} e^{-2i\gamma_R \boxed{J_R^D}}\Big]$$

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Index = Nekrasov Inst. Partition Fn.

- **Index:**

. Define a generating ftn with a chemical potential to the instanton charge

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once the parameters in both quantities are indentified as follows

$q=e^{-\beta\frac{8\pi^2}{g^2_{\mathrm{YM}}}}$

$\vec{\mu}=\underbrace{2\vec{a}}$ scalar VEV

$i(\gamma_R-\gamma_L)=2\epsilon_1$, $i(\gamma_R+\gamma_L)=\underbrace{2\epsilon_2}$ Omega def. parameters

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- **Result:** [Nekrasov]

. Index can be computed by Gaussian path-integrals over a set of saddle-points (solutions of deformed ADHM), characterized by N-colored Young diagrams

$$\vec{Y} = \{Y_1, Y_2, ..Y_N\} \ :$$

$$\Rightarrow \qquad I(\vec{\mu}, \gamma, q) = \sum_{\vec{Y}=\{Y_1,Y_2,..Y_N\}} q^{|\vec{Y}|} I_{\{Y_1,Y_2,..Y_N\}}$$

where $$I_{\{Y_1,Y_2,\cdots,Y_N\}} = \prod_{i,j=1}^{N} \prod_{s\in Y_i} \frac{\sinh \frac{E_{ij}-i(\gamma_2+\gamma_R)}{2} \sinh \frac{E_{ij}+i(\gamma_2-\gamma_R)}{2}}{\sinh \frac{E_{ij}}{2} \sinh \frac{E_{ij}-2i\gamma_R}{2}}$$

$$E_{ij} = \mu_i - \mu_j + i(\gamma_1 - \gamma_R)h_j(s) + i(\gamma_1 + \gamma_R)(v_i(s) + 1)$$

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$$\Rightarrow \qquad I(\vec{\mu}, \gamma, q) = \sum_{\vec{Y}=\{Y_1,Y_2,..Y_N\}} q^{|\vec{Y}|} I_{\{Y_1,Y_2,..Y_N\}}$$

where $$I_{\{Y_1,Y_2,\cdots,Y_N\}} = \prod_{i,j=1}^{N} \prod_{s\in Y_i} \frac{\sinh\frac{E_{ij}-i(\gamma_2+\gamma_R)}{2}\sinh\frac{E_{ij}+i(\gamma_2-\gamma_R)}{2}}{\sinh\frac{E_{ij}}{2}\sinh\frac{E_{ij}-2i\gamma_R}{2}}$$

$$E_{ij} = \mu_i - \mu_j + i(\gamma_1 - \gamma_R)h_j(s) + i(\gamma_1 + \gamma_R)(v_i(s) + 1)$$

Window into Physics of M5s

- **Applications to physics of M5-branes**

$\Rightarrow$ [1] uniqueness of U(1) instantons (a single M5)

[2] world-sheet spectrum of self-dual strings & partons

[3] anomalies of self-dual strings

[4] superconformal index of 6d N=(2,0) A_N theories

Uniqueness of U(1) Instantons

- **Conjecture from M-theory:** KK mode in a M5 are in the massive tensor multiplets

  Unique 1-particle state for all instanton number k in 5d N=2 U(1) SYM
  or, equivalently unique threshold bound state of k U(1) instantons

  . Earlier attempts: count # of normalizable harmonic forms on the instanton moduli space. [K.Lee, Tong, S. Yi]

  . Use the instanton index to prove the conjecture.

- **Single-particle Index:**

  . Index counts all single-particle and multi-particle BPS states.

  $\Longrightarrow$ need to separate out single-particle contribution, which is in general very diffiucult !

  . BUT, D0-particles are mutually non-interacting !

  . Use plethystic exponential $I(\vec{\mu}, \gamma) = \text{Exp}\Big[\sum_{n=1}^{\infty} \frac{1}{n} i_{\text{sngl}}(n\vec{\mu}, n\gamma, q^n)\Big]$

Uniqueness of U(1) Instantons

- **Proof:**

. Expand $i_{sngl}(q,\gamma) = \sum_{k=1}^{\infty} q^k i_k(\gamma)$, the conjecture implies $i_k(\gamma) = i_{k=1}(\gamma)$

$\Rightarrow i_{\rm sngl}(q,\gamma) = \frac{q}{1-q} i_{k=1}(\gamma)$, or $I(\gamma;q) = {\rm Exp}\Big[\sum_{n=1}^{\infty} \frac{1}{n}\frac{q^n}{1-q^n} i_{k=1}(n\gamma)\Big]$

. Use the refined topological vertex techniques [Iqbal,Kozcaz,Shabbir]

$$Z = Z_{\rm pert}\cdot Z_{\rm inst}$$

$$Z = \prod_{k=1}^{\infty}\left[(1-Q_\bullet^k)^{-1}\prod_{i,j=1}^{\infty}\frac{(1-Q_\bullet^k Q_m^{-1} w^{i-\frac12} t^{j-\frac12})(1-Q_\bullet^k Q^{-1} w^{i-\frac12} t^{j-\frac12})}{(1-Q_\bullet^k w^{i-1} t^j)(1-Q_\bullet^k w^i t^{j-1})}\right]$$

$$Z_{\rm pert} = \prod_{i,j=1}^{\infty}(1-Q_m t^{i-\frac12} w^{j-\frac12})$$

$Q_\bullet \to q$ $\qquad t = e^{-i(\gamma_L-\gamma_R)}$

$Q_m \to e^{-2i\tilde\gamma_L}$ $\qquad w = e^{-i(\gamma_L+\gamma_R)}$

$(\mathbf{3},\mathbf{1},\mathbf{1}) \oplus (\mathbf{2},\mathbf{1},\mathbf{4}) \oplus (\mathbf{1},\mathbf{1},\mathbf{5})$

contribution from massive tensor multiplet

$I(\gamma;q) = {\rm Exp}\Big[\sum_{n=1}^{\infty}\frac{1}{n}\frac{q^n}{1-q^n} i_{k=1}(n\gamma)\Big]$ with $i_{k=1}(\gamma) = \frac{\sin\frac{\gamma_L+\tilde\gamma_L}{2}\sin\frac{\gamma_L-\tilde\gamma_L}{2}}{\sin\frac{\gamma_L+\gamma_R}{2}\sin\frac{\gamma_L-\gamma_R}{2}}$

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Spectrum of Self-Dual String

- **Self-dual string:** two M5s in Coulomb phase…

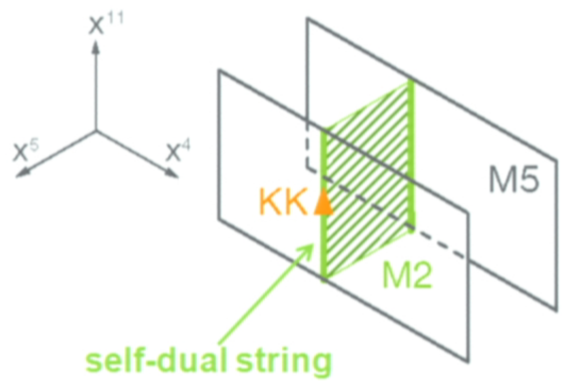
. ∃ massive string excitation, charged under the two-form gauge field $B_{\mu\nu}$ ($*_6 dB = dB$) both electrically and magnetically.

. Self-dual string = M2 stretched bet'n M5s

. world-sheet mom. mode of the self-dual string

= mom. along M-circle on M2

= bound state bet'n D0s and F1



⟹ Index provides non-trivial results for the world-sheet spectrum of self-dual strings !

Spectrum of Self-Dual String

- **How to read off the spectrum from index ?:**

. Separate out the single-particle index: no long-range force bet'n ¼-BPS states

$$I(\vec{\mu},\gamma) = \mathrm{Exp}\Big[\sum_{n=1}^{\infty} \frac{1}{n} i_{\rm sngl}(n\vec{\mu}, n\gamma, q^n)\Big]$$

. Expand the 1-particle index in terms of chemical potentials for $U(1)^N$ charges,

$$i_{\rm sngl}(\mu,\gamma,q) = \underbrace{i_{\vec{e}_1-\vec{e}_3}(q,\gamma)}e^{-(\mu_1-\mu_3)} + \underbrace{i_{\vec{e}_2-\vec{e}_5}(q,\gamma)}e^{-(\mu_2-\mu_5)} + \cdots$$

it mesures # of bound states of D0s and F1 bet'n 1st and 3rd D4s

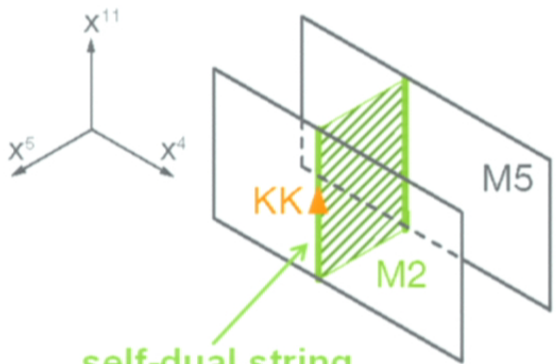
it mesures # of bound states of D0s and F1 bet'n 2nd and 5th D4s

$(\vec{e}_i)_j = \delta_{ij}$: basis of $U(1)^N$-charge vectors $(i = 1, 2, .., N)$

Spectrum of Self-Dual String

• **SU(2) self-dual string:** or, ¼-BPS state carrying electric charge $\vec{e}_1 - \vec{e}_2$

. set $\tilde{\gamma}_L = \pi$ which almost kills the effect of (-1)^F & take the limit $\gamma_L = \gamma_R \to 0$



$$\Rightarrow \quad i_{\vec{e}_1-\vec{e}_2}(q) = 1 + 8q + 40q^2 + 160q^3 + 552q^4 + \cdots$$

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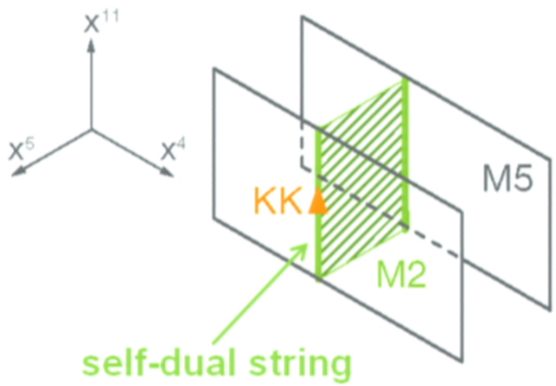


$$\Rightarrow \quad i_{\vec{e}_1 - \vec{e}_2}(q) = 1 + 8q + 40q^2 + 160q^3 + 552q^4 + \cdots$$

$$= \prod_{n=1}^{\infty} \left(\frac{1+q^n}{1-q^n} \right)^4$$

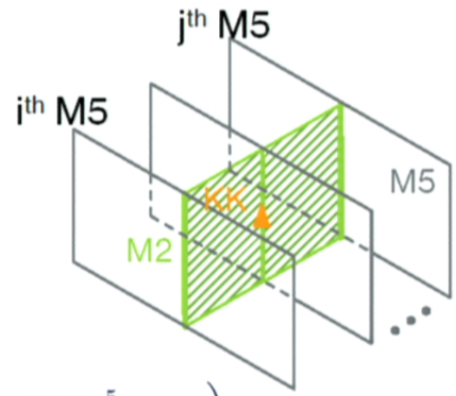
Partition function of 4 free bosons + 4 free fermions

c.o.m of self-dual string

. M-theory states are VISIBLE in 5d QFT.

	0	1	2	3	4	5	6	7	8	9	11
M5	$\times$	$\times$	$\times$	$\times$	$\times$						$\times$
M2	$\times$					$\times$					$\times$
KK	$\times$										

Spectrum of Self-Dual String



• **SU(N) self-dual string:**

[1] $i_{\vec{e}_1-\vec{e}_3} = 1 + 24q + 264q^2 + 2016q^3 + 12264q^4 + 63504q^5 + \cdots$

$$= \prod_{n=1}^{\infty}\left(\frac{1+q^n}{1-q^n}\right)^4 \times \underbrace{\left(1 + 16q + 96q^2 + 448q^3 + 1728q^4 + 5856q^5 + \cdots\right)}_{\text{EXTRA modes appear !}}$$

[2] $i_{\vec{e}_1-\vec{e}_4} = 1 + 40q + 744q^2 + 8992q^3 + 82344q^4 + 618864q^5 + \cdots$

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$$\Rightarrow \quad i_{\vec{e}_i-\vec{e}_j}(q) = \prod_{n=1}^{\infty}\left(\frac{1+q^n}{1-q^n}\right)^4 \times \left[\oint \frac{dz}{2\pi i z} \prod_{n=1}^{\infty}\left(\frac{(1+q^{\frac{2n-1}{2}}z)(1+q^{\frac{2n-1}{2}}z^{-1})}{(1-q^{\frac{2n-1}{2}}z)(1-q^{\frac{2n-1}{2}}z^{-1})}\right)^2\right]^{j-i-1}$$

N^3 - N & Partons

- **Anomaly coefficient of N M5s: N^3 - N**

. $\frac{N(N-1)}{2} \simeq N^2$ self-dual strings ending on two different M5s.

. In low-momentum sector, $\exists$ N^2 d.o.f. associated c.o.m of self-dual string.

. In high-momentum limit (6d limit), many extra degrees appear.

. How many then ?: take the limit $q \to 1^-$ then $i_{\vec{e}_i - \vec{e}_j}(q) \to \mathrm{Exp}\left[\frac{\pi^2}{6(1-q)} \cdot \underbrace{6(j-i)}\right]$

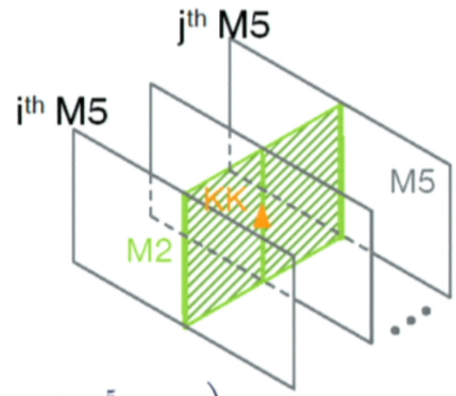
degeneracy of high mom, mode

$\Longrightarrow$ Collect d.o.f on all single-self-dual strings, then we get …

$$\#_{\rm tot} = \sum_{j=1}^{N} \sum_{i=1}^{j-1} 6(j-i) = N^3 - N$$

, which coincides with the anomaly coeff. Of N M5-branes !

Spectrum of Self-Dual String



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. Look at the closed-form for extra degrees of freedom

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$\Rightarrow$
- **[1]** emergent U(1) gauge symmetry
- [2] charged particles, fit into SUSY,
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In the 6d limit (high-momentum), or high-temp. limit, these partons can LIBERATE and move freely, providing extra 6(j-i-1)-modes !

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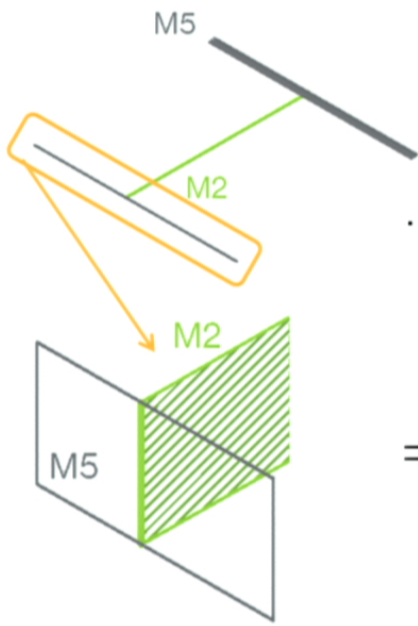
Self-Dual String Anomaly

- **Anomaly** : start with N M5-branes …

. Single M5 brane far away from the rests, i.e., $G = SU(N) \ \rightarrow \ H \times U(1)$



	0	1	2	3	4	5	6	7	8	9	11
		$SO(4)_{1234}$					$SO(4)_{6789}$				
M5	×	×	×	×	×						×
M2	×					×					×

. Anomaly cancellation: $\delta_{SO(4)\times SO(4)}\Big(S_{\rm string} + S_{\rm coupling} + S_{\rm M5}\Big) = 0$

$$S_{\rm M5} = \cdots + S_{\rm WZW}\Big(= \kappa \int_{\Sigma_6} H^{(3)} \wedge \Omega(\phi, A_{SO(4)_{6789}})\Big)$$

$$\kappa = |G| - |H| - 1$$ [Intriligator]

$\Longrightarrow$ $\delta_{SO(4)_{6789}} S_{\rm WZW} \neq 0$ must be cancelled by $SO(4)_{6789}$ anomaly of $S_{\rm string}$

\# of fermion zero-modes is $\frac{1}{2}(|G| - |H| - 1)$ [Berman,Harvey]

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- **Microscopic (or, less-macroscopic) derivation:**

. When the symmetry SU(N) is completely broken, i.e., $H = U(1)^{N-2}$

$$\# = \tfrac{1}{2}\,(N^2 - 1 - (N - 2) - 1) = \tfrac{1}{2}\,(N^2 - N)$$

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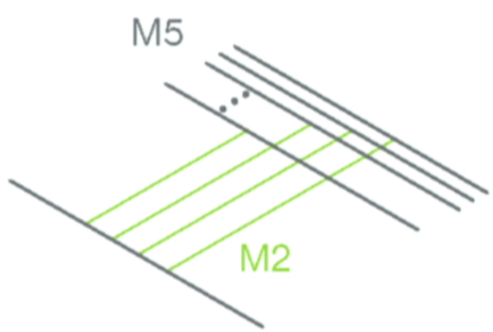

$\Rightarrow$ Sum over all world-sheet partons on N-1 different self-dual strings of our interest will give us

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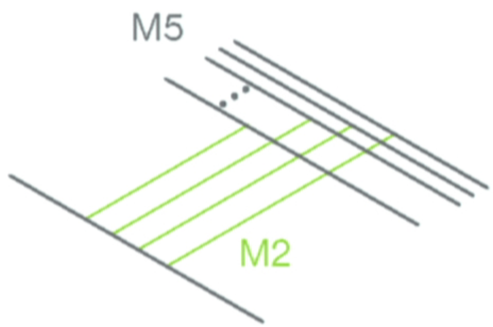

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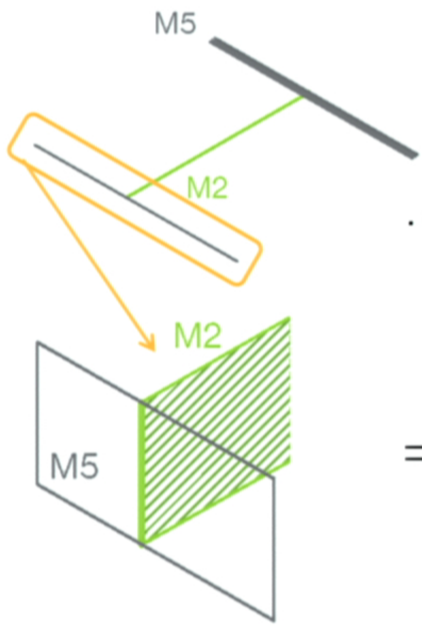

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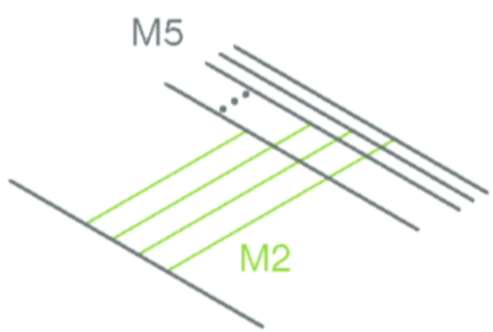

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Outlook

- **Summary:**

  . The instanton index is very useful to have a number of (quantitative) hints on mysterious dynamics of 6d N=(2,0) theory and self-dual strings.

- **Symmetric phase index:**

  . DLCQ of 6d N=(2,0) theory can be described by the instanton sigma model, NR SCFT.
  [Aharony,Berkooz,Kachru,Silverstein,Seiberg]

  . PROPOSAL: Index in symmetric phase is the superconformal index of (2,0) theory

  $\Leftarrow$ Observed that the index counts BPS eigenstates of NR dilatation operator

$$\{\hat{\mathcal{Q}}, \hat{\mathcal{S}}\} = \underbrace{i\hat{D}}_{\text{NR dilatation operator}} - 2(2J_R + \tilde{J}_R) \rightarrow H \text{ when } \beta \rightarrow 0$$

  . Index for k=1 with large N matches with SUGRA index ($AdS_7 \times S^4$) of k=1 sector

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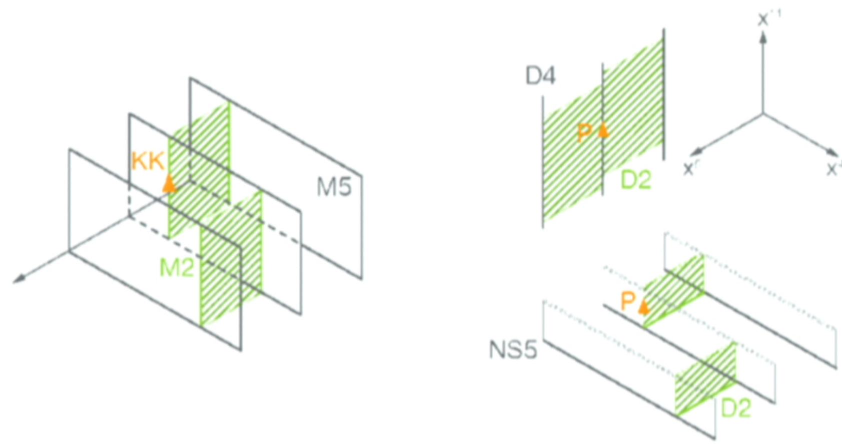

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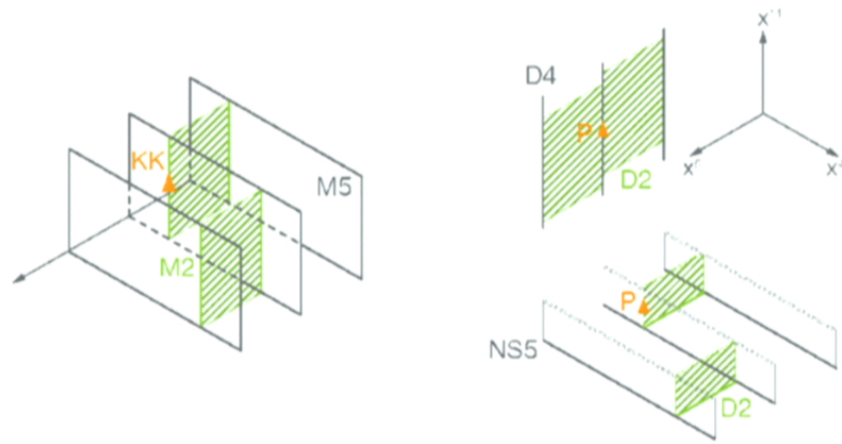

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