Title: Negative Quasi-Probability Representation is a Necessary Resource for Quantum Computation

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Abstract: Abstract The magic state model of quantum computation gives a recipe for universal quantum computation using perfect Clifford operations and repeat preparations of a noisy ancilla state. It is an open problem to determine which ancilla states enable universal quantum computation in this model. Here we show that for systems of odd dimension a necessary condition for a state to enable universal quantum computation is that it have negative representation in a particular quasi-probability representation which is a discrete analogue to the Wigner function. This condition implies the existence of a large class of bound states for magic state distillation: states which cannot be prepared using Clifford operations but do not enable universal quantum computation.


CAUTION

Negative Quasi-Probability Representation is a Necessary Resource for Magic State Distillation

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Result

Big Picture Question

What are necessary and sufficient conditions for quantum computational speedup?

Result

Negative Gross-Wigner representation is a necessary resource for computational speedup in the magic state model of quantum computation

Corollaries

- Bound states for magic state distillation
- Efficient experimental verification

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Quantum Resources

Circuit model

Entanglement is a necessary resource for computational speedup in the circuit model of quantum computation (Vidal)

Magic state model

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Magic State Computing

Clifford Operations are Perfect

- Stabilizer State Preparations
- Clifford Group Transformations
- Projective Stabilizer Measurements

Additional Power

- Prepare a mixed ancilla state
- Consume many copies to distill high purity resources
- *Which states enable quantum computation?*

Quasi-Probability Representation

Probability Representation

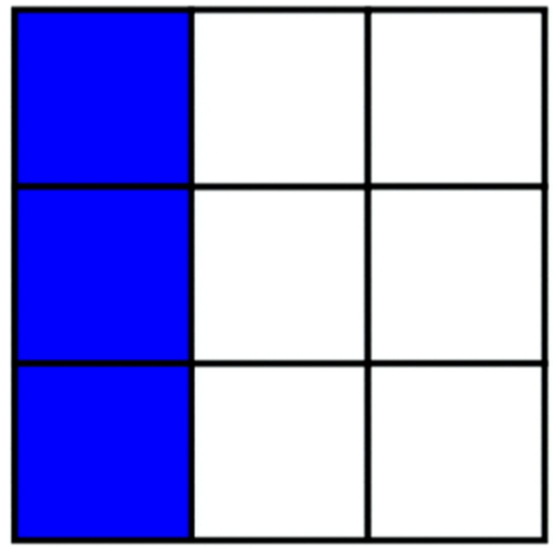
Map a *subtheory* of quantum theory to a classical probability representation

Negativity

Some states/measurements must be *negative*

Non-Contextuality

Quasi-probability distribution for state ρ at point $\boldsymbol{u}$ is given by $Q_\rho(\boldsymbol{u}) = \mathrm{Tr}(\rho F_{\boldsymbol{u}})$



Positive Qutrit State

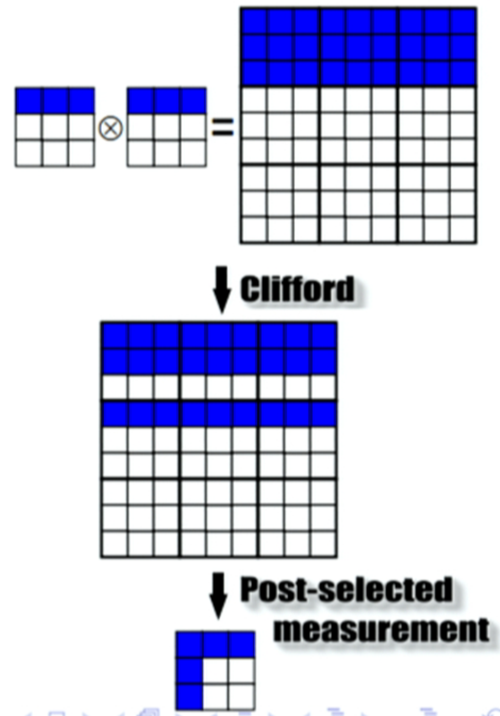
Gross-Wigner Representation for Odd Dimension

Probability Representation

Clifford operations have non-negative representation

Negativity

Ancilla preparation may be negatively represented



Gross-Wigner Representation More Formally

Gross-Wigner Representation

- Map to $d \times d$ "phase space" grid
- $W_\rho(u,v) = \frac{1}{d}\mathrm{Tr}(A_{(u,v)}\rho)$
- $A_{(0,0)} = \sum_{\boldsymbol{u}} T_{\boldsymbol{u}}$, $A_{\boldsymbol{u}} = T_{\boldsymbol{u}} A_{(0,0)} T_{\boldsymbol{u}}^\dagger$ where $T_{\boldsymbol{u}}$ are Heisenberg-Weyl operators
- For composite system $H_d \otimes H_d$ we have $A_{(u_1,v_1)\oplus(u_2,v_2)} = A_{(u_1,v_1)} \otimes A_{(u_2,v_2)}$

Salient Features

1. (Discrete Hudson's theorem) $\mathrm{Tr}(|S\rangle\langle S|A_{\boldsymbol{u}}) \geq 0 \ \forall \boldsymbol{u}$ if and only if $|S\rangle$ is a stabilizer state
2. Clifford operators are stochastic, symplectic linear transformations on phase space: $U_F A_{\boldsymbol{u}} U_F^\dagger = A_{F\boldsymbol{u}}$

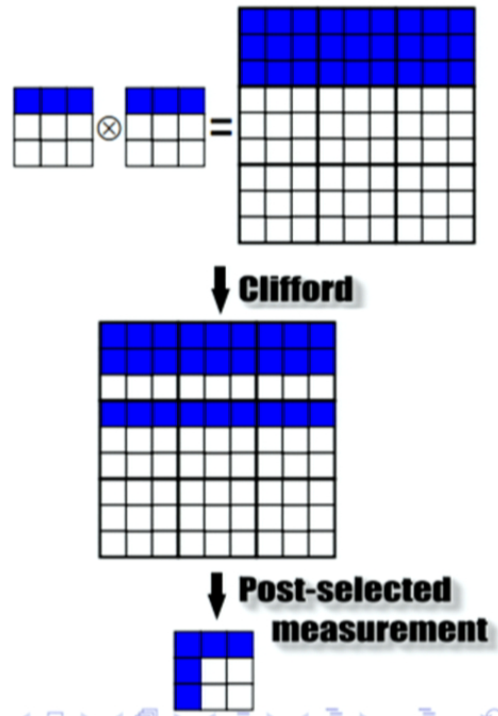
Negativity is Necessary for Magic State Distillation

Gross-Wigner Representation

- Clifford operations correspond to stochastic transformations
- Post selected stabilizer measurement preserves non-negativity

Unreachable Pure States

Pure states with negative representation cannot be reached even approximately



Negativity is Necessary for Clifford Accessible Scenario

Setup

- Have preparation ρ with non-negative representation
- Evolve by Clifford operation U_F corresponding to symplectic F
- Perform measurement $\{E_k\}$ with non-negative representation

Simulation Protocol

- Sample phase space point (u, v) according to distribution $W_\rho(u, v)$
- Evolve phase space point according to $(u, v) \rightarrow F(u, v)$
- Sample from measurement outcome according to $\tilde{W}_{\{E_k\}}(u, v)$

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$$T_u T_v T_u^\dagger = \omega^{\langle u, v \rangle} T_v$$

So What?

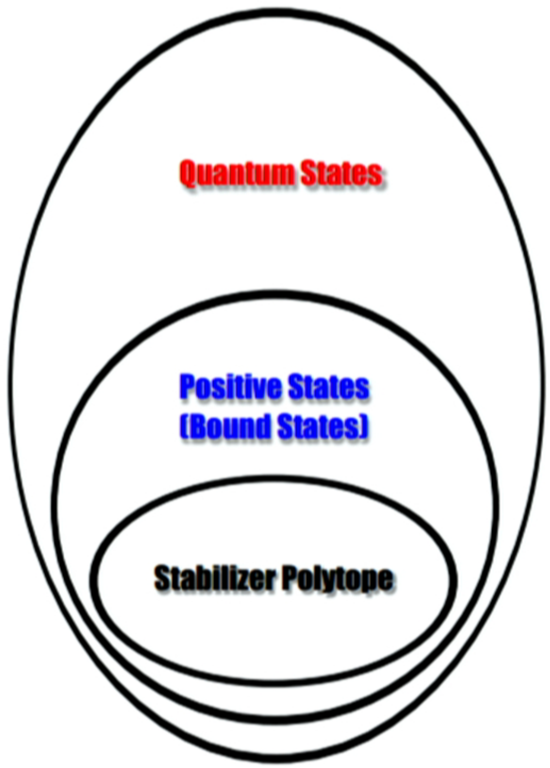
Result

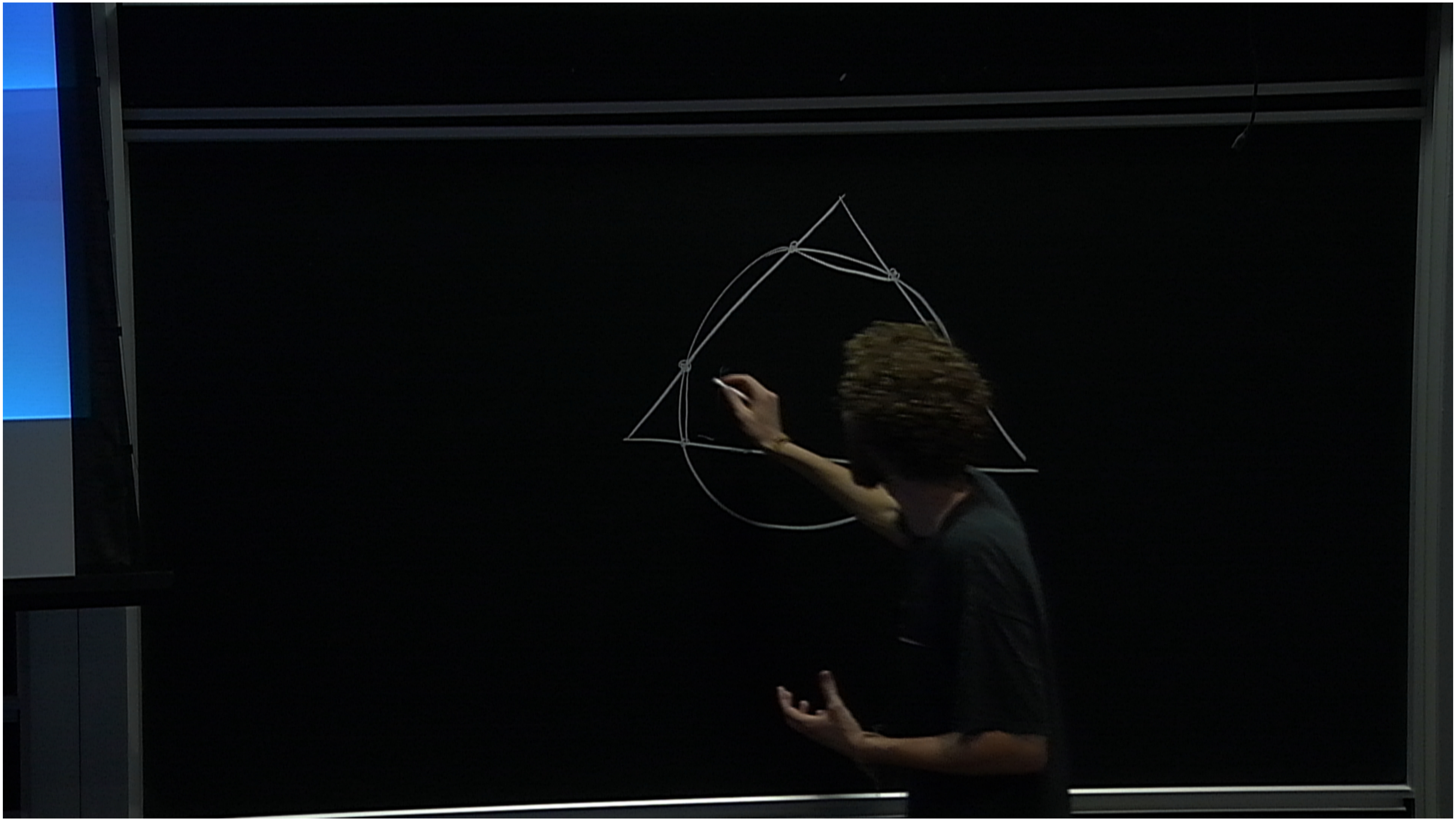
Stochastic representation for magic state distillation with non-negative input states

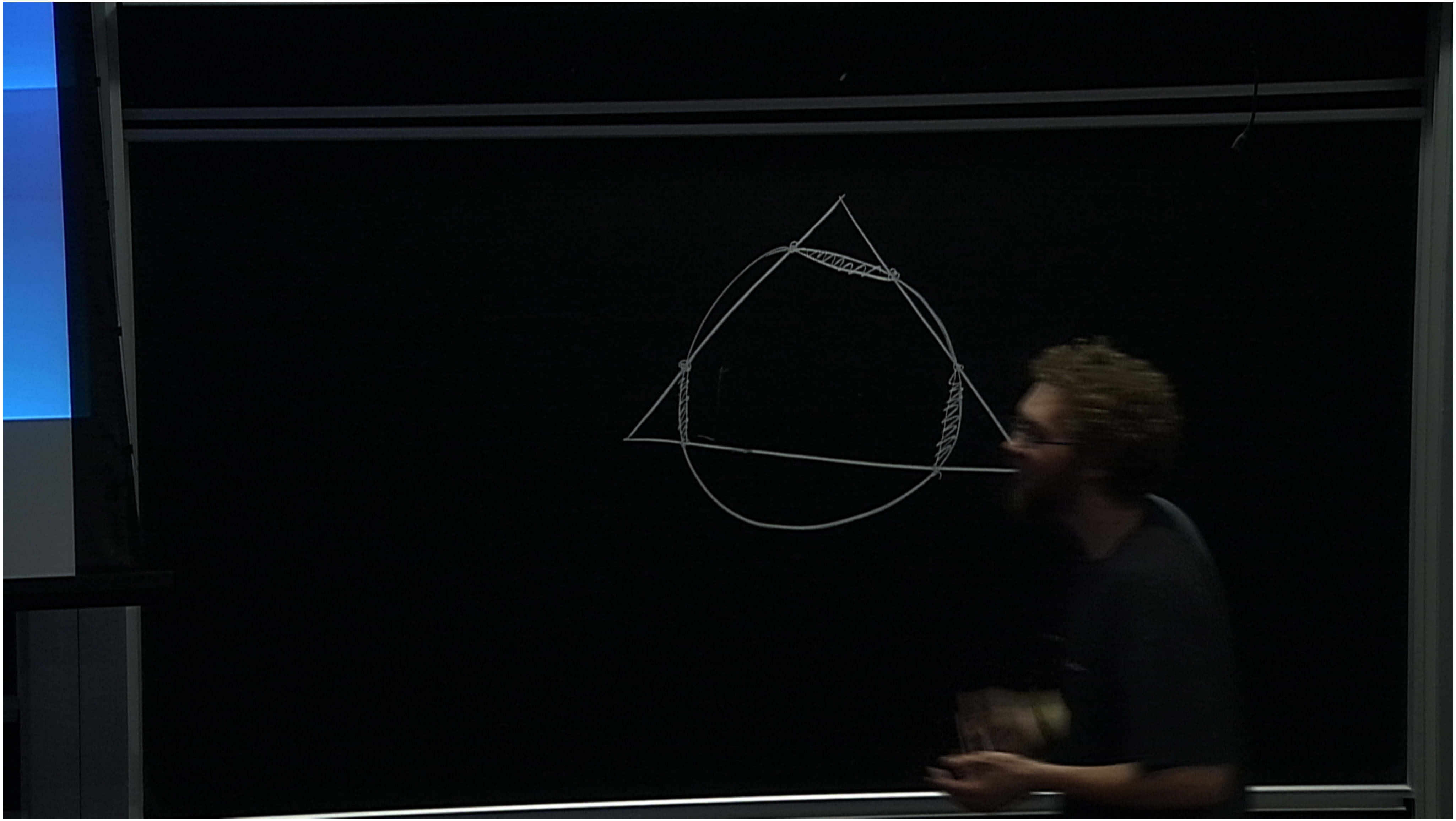
Distillable States?

Does this give new bounds on distillability?

Bound States



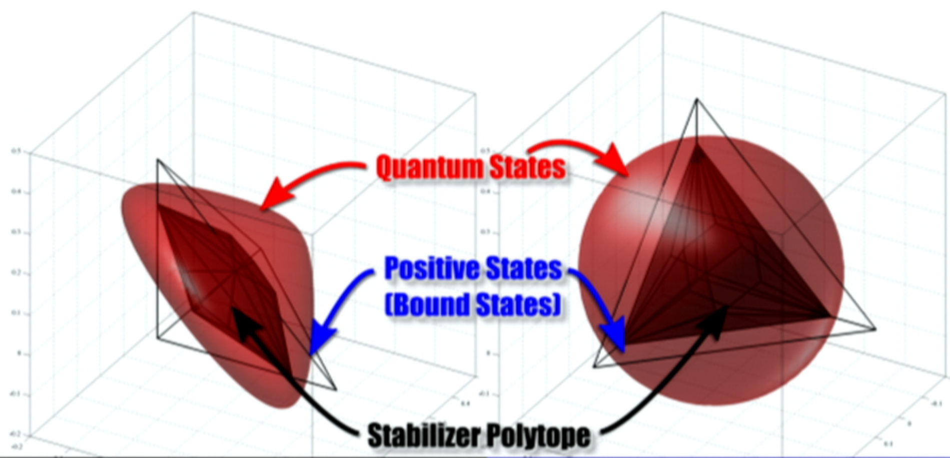




Experimental Verification

Is my state non-negative?

- Measure value of Gross-Wigner function at one point
- This is expectation of one Hermitian operator, $W_\rho(\boldsymbol{u}) = \mathrm{Tr}(\rho A_{\boldsymbol{u}})$
- Measurement is $A_{\boldsymbol{u}} = A_{\boldsymbol{u}_1} \otimes A_{\boldsymbol{u}_2} \cdots \otimes A_{\boldsymbol{u}_n}$ (easy to measure)



Qubits?

Can this be extended to qubits?

- Classical representation of stabilizers is impossible
- Can bound states exist anyways?

Sufficiency?

Is negativity sufficient?

- Representation used here is *unique* in prime dimension
- This would unify negative representation and computational speedup!

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- Negativity necessary for UQC in the magic state model
- Bound states for magic state distillation
- Experimental verification

Open Problems

- Can this be extended to qubits?
- Is negativity sufficient?

Paper Reference

arXiv:1201.1256


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Quasi-Probability for Magic States

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