Title: Mass gap, topological molecules, and strong gauge dynamics

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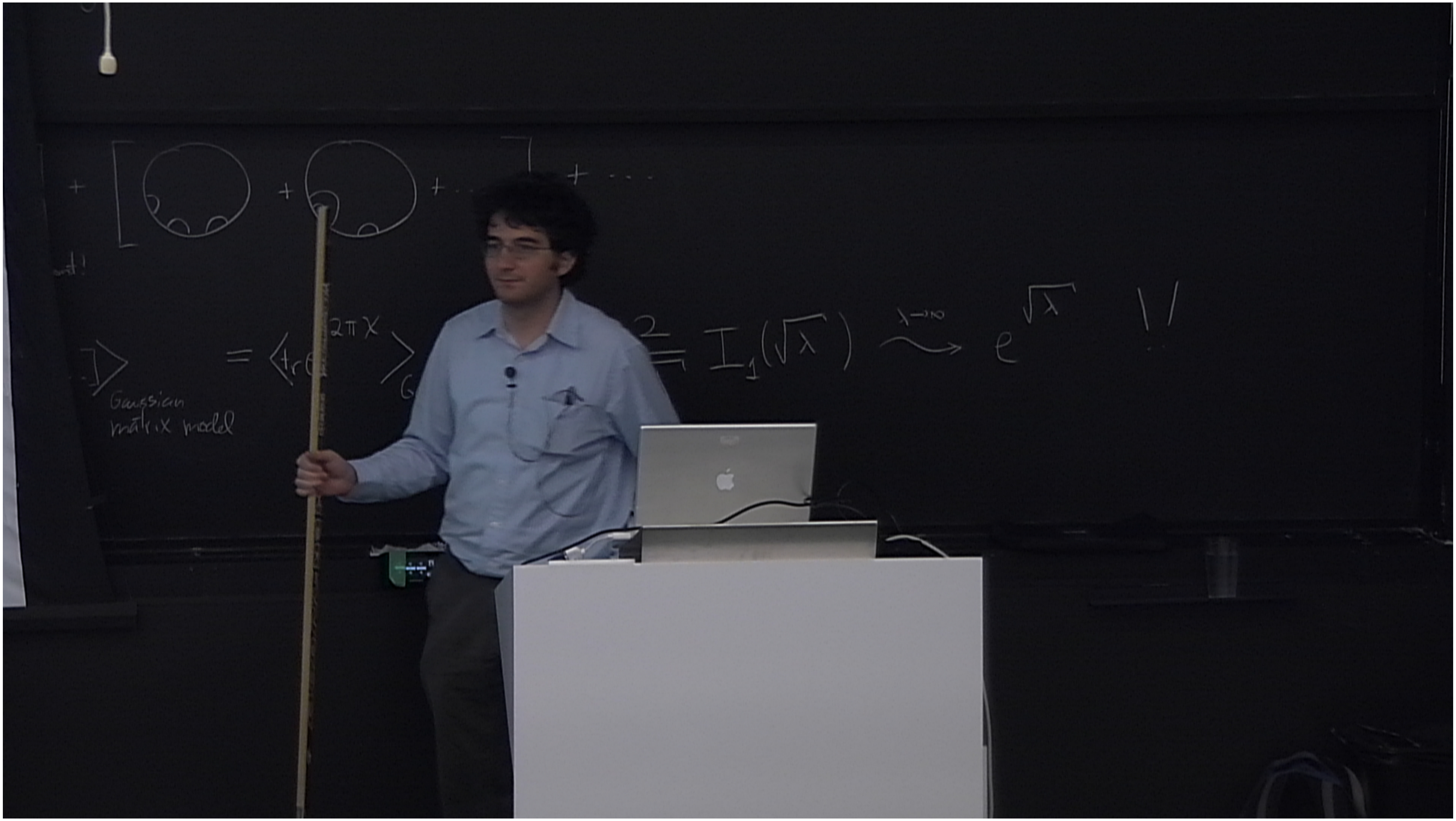
Abstract: Mass, a concept familiar to all of us, is also
one of the deepest
mysteries in nature. Almost all of the mass in the
visible universe,
you, me and any other stuff that we see around us, emerges from a
quantum field theory, called QCD, which has a completely negligible
microscopic mass content. How does QCD and the family of
gauge
theories it belongs to generate a mass?

This class of non-perturbative problems remained largely elusive despite much
effort over the years. Recently, new ideas based on compactification have been
shown useful to address some of these. Two such inter-related ideas are circle
compactifications, which avoid phase transitions and large-N volume
independence. Through the first one, we realized the existence of a
large-class of "topological molecules", e.g. magnetic bions,
which
generate mass gap in a class
of compactified gauge theories. The inception of the
second, the idea of large-N volume independence is old. The new
progress is the realization of its first working examples. This property
allows us to
map a four dimensional gauge theory (including pure Yang-Mills) to a
quantum

mechanics
at large-N.


Gaussian
matrix model

Mass

- This talk is about a concept familiar to all of us: "Mass"
- In classical mechanics, we first encounter mass through Newton's second law (F=ma), as the resistance that a body shows to accelerate in the presence of an applied force.
- In some quantum field theories, mass is an emergent concept. By this, I mean: a quantum theory with no mass at all may induce a mass. This is roughly what happens in nature.

Outline

- Experimental origins and statement of the mass gap problem
- Microscopic origin of mass in gauge theories and quantum chromodynamics
- New techniques
- Broad-brush overview of my contributions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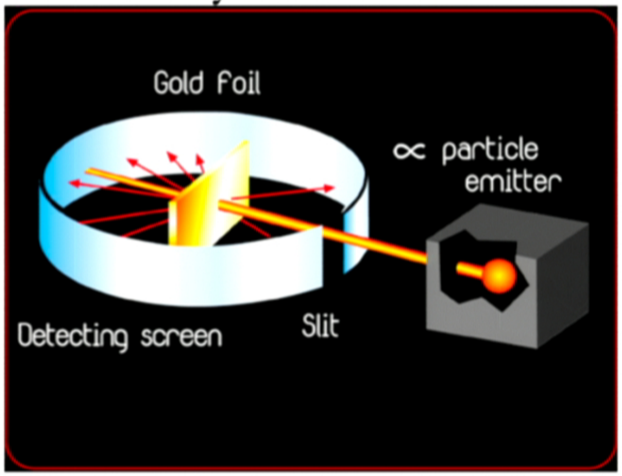
Gold foil experiment, 1910

(Geiger–Marsden or Rutherford experiment)

- Discovery of atomic nuclei



few meters

Rutherford and Geiger in the lab in Manchester

Rutherford: "It was quite the most incredible event that has ever happened to me in my life. It was almost as incredible **as if you fired a 15-inch shell at a piece of tissue paper and it came back and hit you.** On consideration, I realized that this scattering backward must be the result of a single collision, and when I made calculations, I saw that it was impossible to get anything of that order of magnitude unless you took a system in which the greater part of the **mass of the atom was concentrated in a minute nucleus.** It was then that I had the idea of an atom with a minute massive center, carrying a charge."

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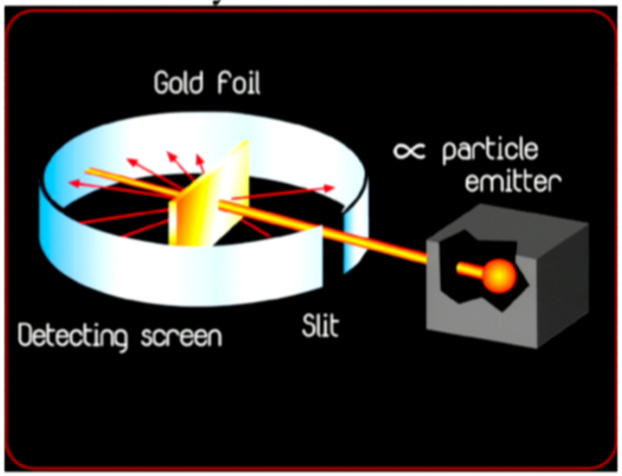

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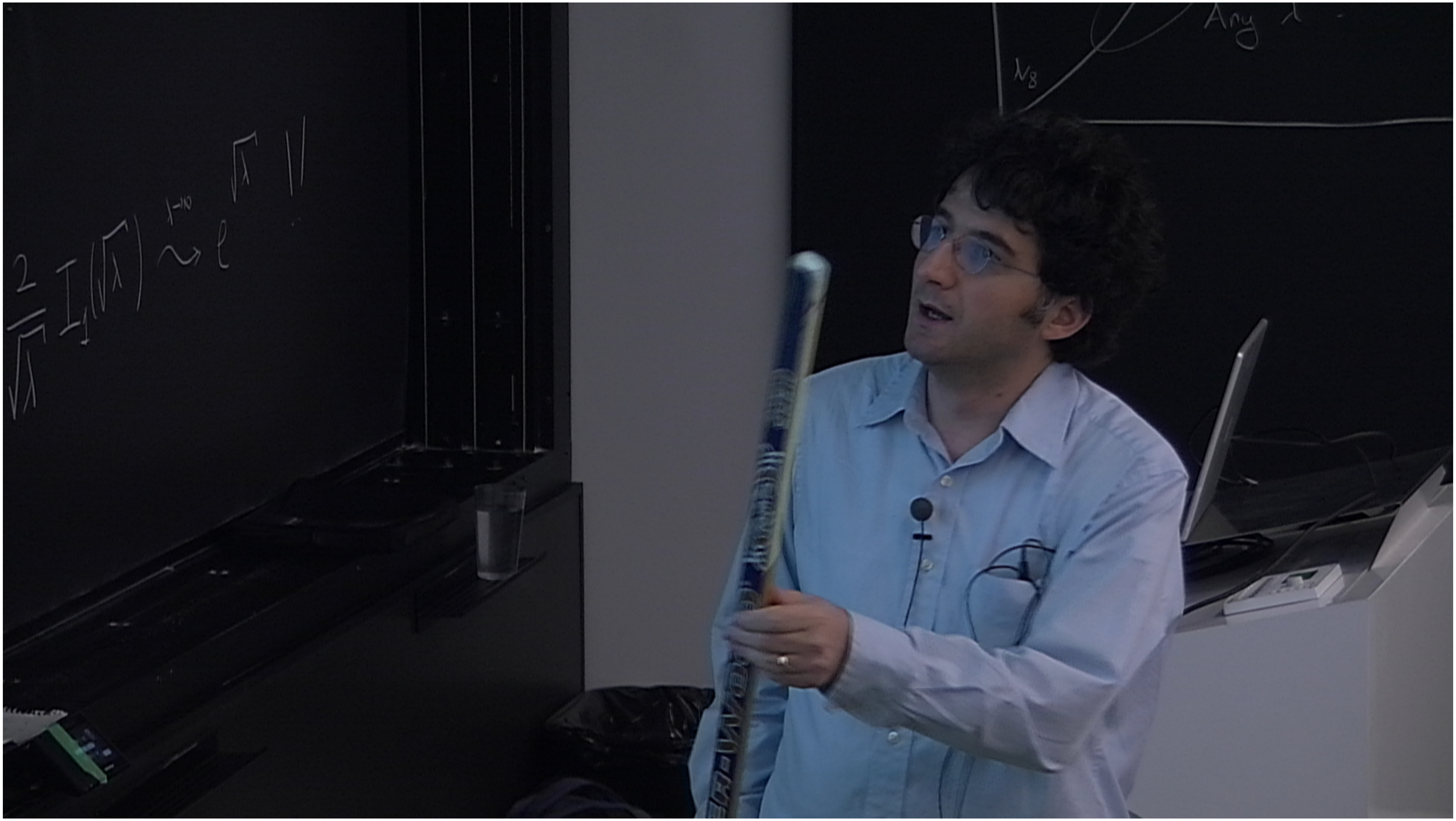
Fast forward in time: Late 60's-Early 70's at SLAC

The deep inelastic scattering experiments: Conceptually similar, in methods and results to the Rutherford experiment.

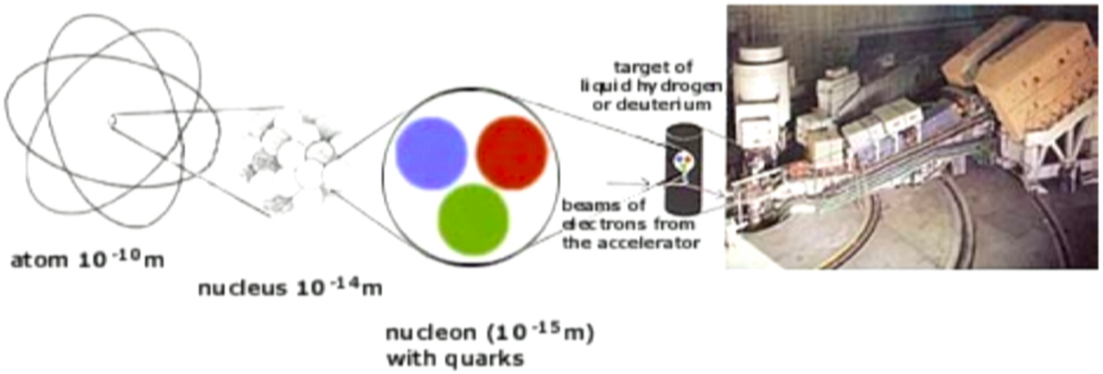
To probe shorter distances (or higher energies), one needs a bigger "microscope".

Result: More electrons bouncing back with high energy at large angles than could be explained if protons and neutrons were uniform spheres of matter.

Friedman, Kendall and Taylor: The Nobel Prize in Physics 1990


Any λ

Results of the deep inelastic scattering experiments



- Hadrons have sub-structure.
- Three points of deflection, much smaller than proton size.
- Quarks as point charges, as if they are (almost) non-interacting!
- The experiments also suggested the existence of gluons, an electrically neutral "glue" binding the quarks together.

Three deep riddles

- Almost free parton (quark) picture, very successful. How does it reconcile with the behavior of the force, which, under other circumstances, is extremely strong?

- The absence of any massless particles in the spectrum.

- The absence of observation of isolated quarks and gluons.

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- The absence of any massless particles in the spectrum.

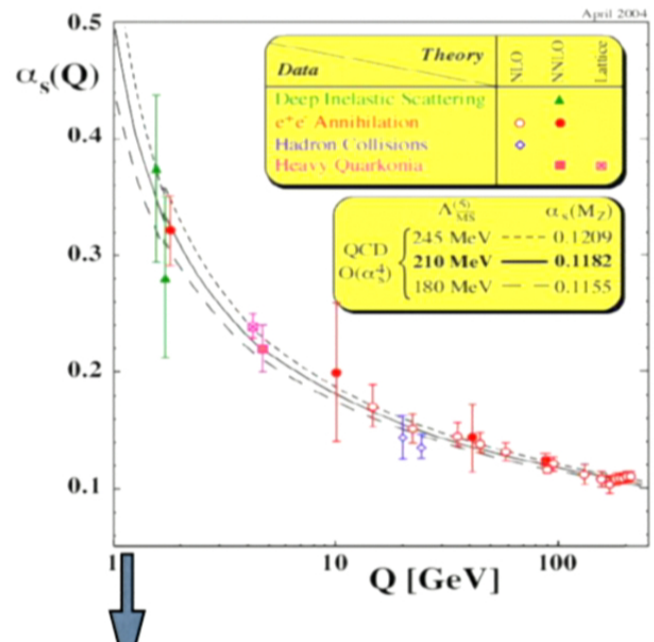
- The absence of observation of isolated quarks and gluons.

The last two are among most important unsolved problems. We are often told by "big boys" that these are impossible problems which do not admit any good approximation. Ignorance does not lead to progress though. Perhaps, instead, we should think different...

QCD and Asymptotic freedom

- Dynamics of quarks and gluons is described by a quantum field theory (QFT) called quantum chromodynamics (QCD).

- Asymptotic freedom: Shorter distance, weaker coupling. Longer distance, stronger coupling.



from: S. Bethke, hep-ex/0407021

1 GeV

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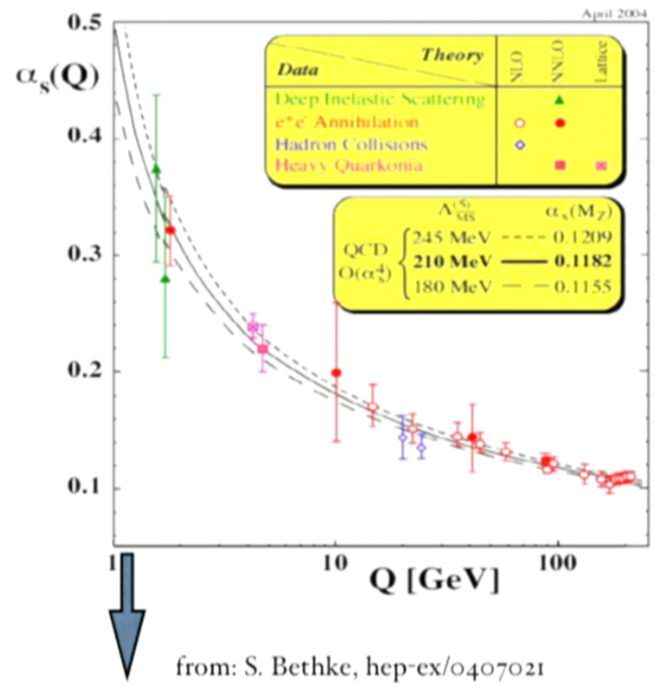

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A "simple" sign flip

- The flow of the coupling constant with energy (or length) is encoded into something called the "beta function", this is one of the things that we can calculate in a QFT.

- The Gross-Wilczek-Politzer calculation is the first example of QFT with a negative beta function, which meant asymptotic freedom.

- Until 1973, all calculations in QFTs had positive beta functions, which meant very strong coupling at short distance. Technically called, "Landau pole". This led most people to believe that QFTs were ill-defined at short distances.

- A flip in sign changed our perception of QFT.

Inputs of QCD

- Almost massless u, d quarks (u: 1.5 to 3.3 MeV, d: 3.5 to 6.0 MeV)

  uud: 6.5 MeV to 12.6MeV
- Exactly massless gluons (like photons)

Outputs of QCD

- Massive hadrons (P: 938MeV, N:939MeV ~ 1 GeV)
- Light pions (135MeV, 140MeV)

For comparison, electron mass is 0.511MeV.

What is the origin of mass in the visible universe?

M(uud) / M(proton) ~ 1/100,
M(electron)/M(proton) ~1/2000.

- Strong interactions generate approximately 99% of the mass of all the matter on the earth, including this computer, our human bodies, rocks and any other "stuff" that we see around us every day.
- In the quarkless theory (called Yang-Mills), there are still particles with masses comparable to proton mass, although microscopic theory only has exactly massless gluons.
- Either form of this problem is the famous mass gap problem, which is, literally, the masses of everything that we see around us!

J. W. Waterhouse: *Pandora*, 1896

The formal statement of problem

www.claymath.org/millennium/

Yang–Mills Existence and Mass Gap. *Prove that for any compact simple gauge group G, a non-trivial quantum Yang–Mills theory exists on $\mathbb{R}^4$ and has a mass gap $\Delta > 0$. Existence includes establishing axiomatic properties at least as strong as those cited in* [45, 35].

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A serious problem, with literally a million dollar price....
I am happy with the pursuit of knowledge for its own sake.

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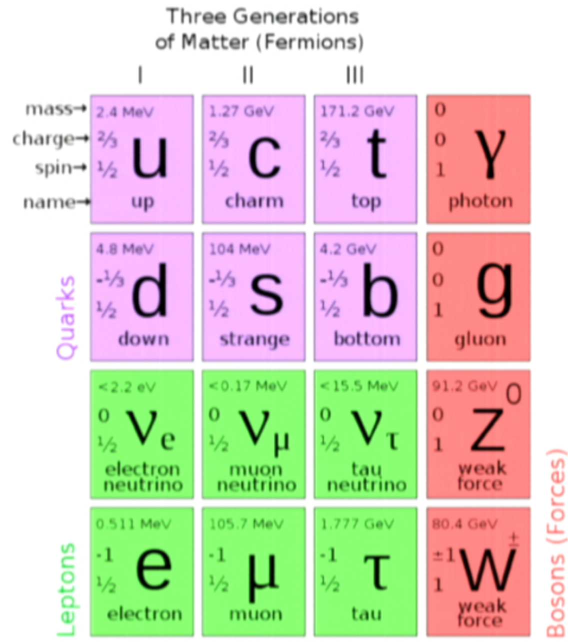
The following solution does not count.

$$\frac{\text{[image]}}{\text{number of protons and neutrons in my body}} \approx \Delta = \text{mass gap}$$

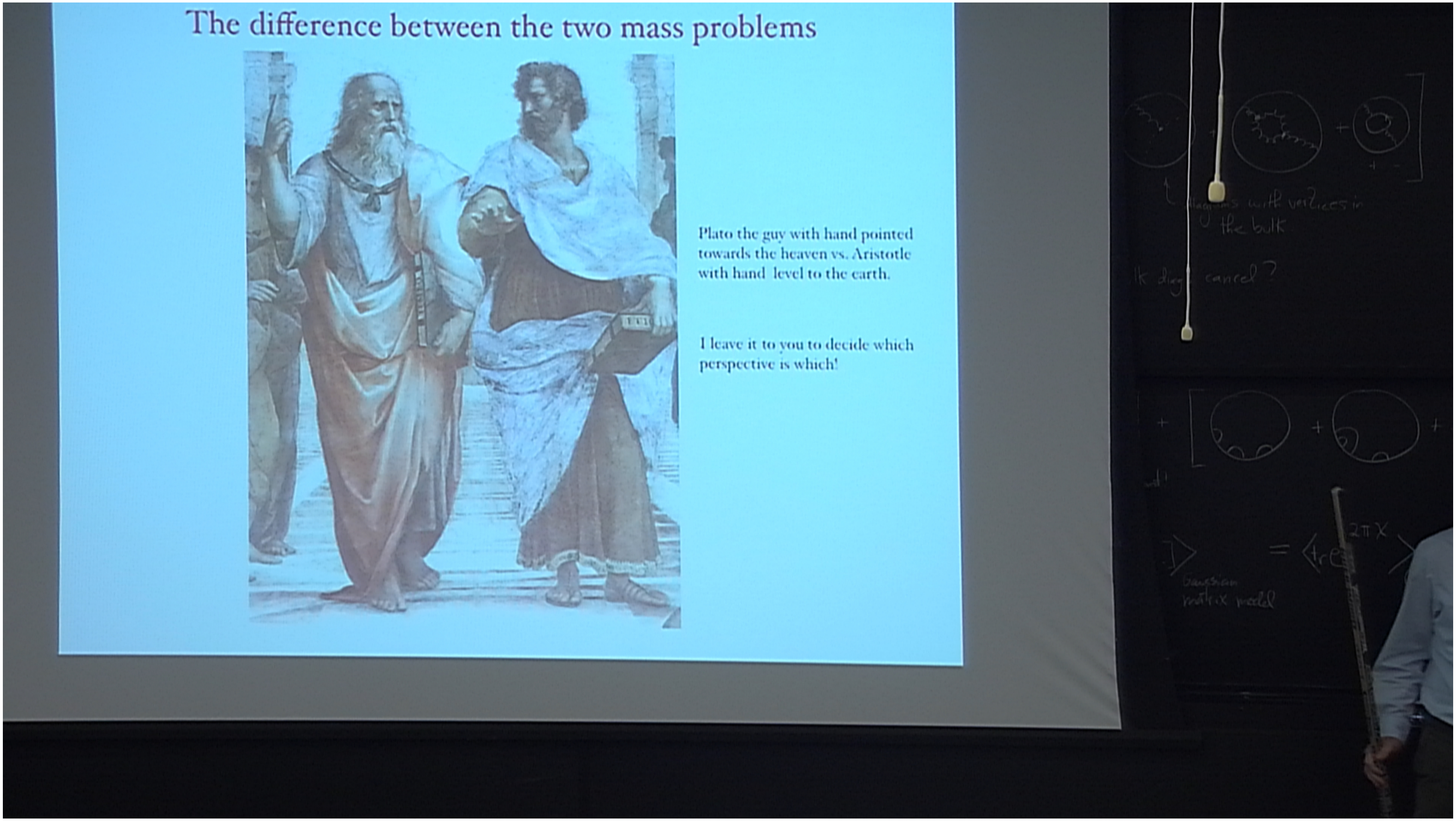
number of protons and neutrons in my body

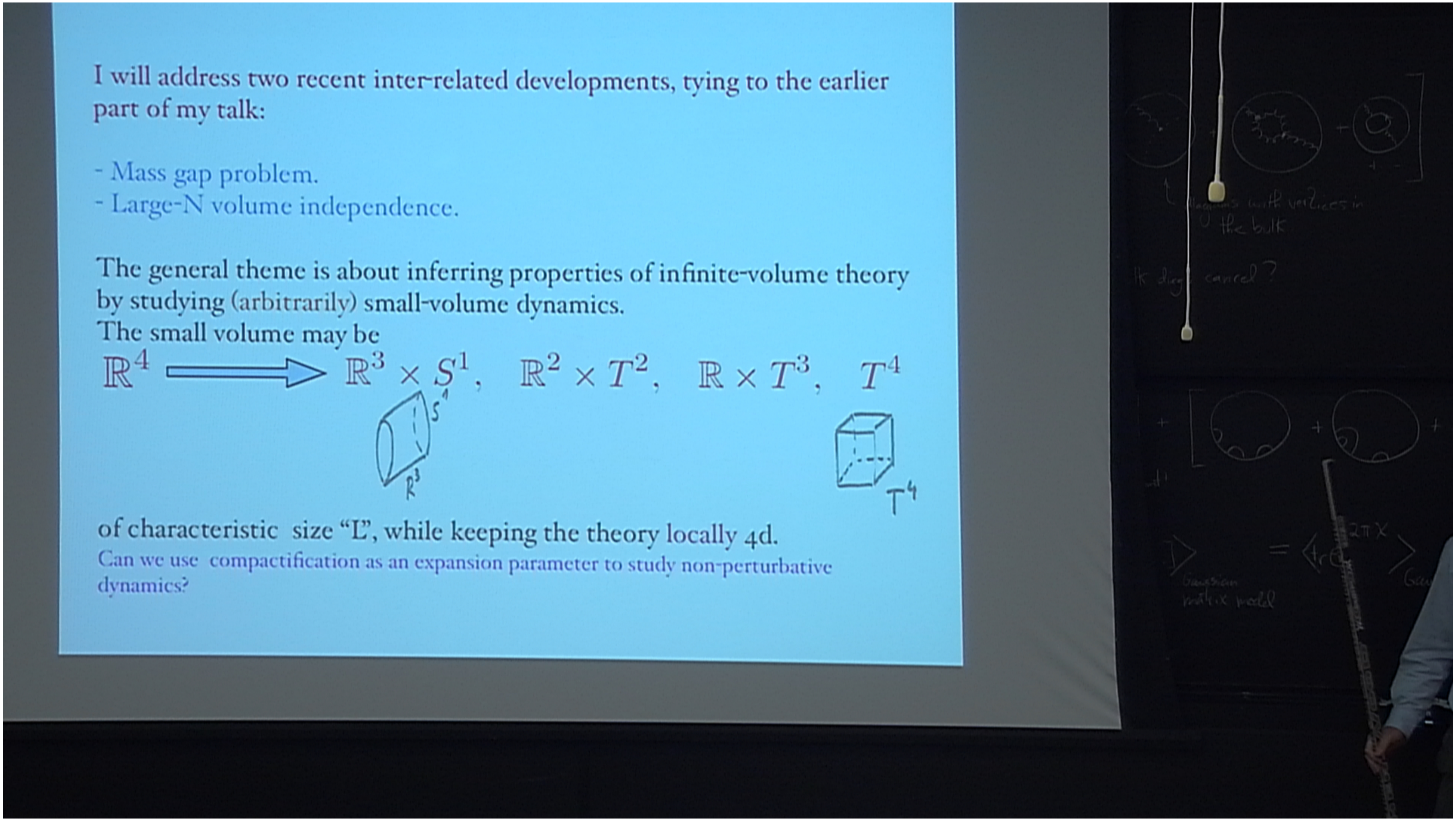
The other mass problem

- The problem of electro-weak symmetry breaking or Higgs mechanism being tested at LHC .
- **How do the elementary particles, quarks and leptons, acquire their own masses? (which accounts for the remaining 1-2% of our masses)**
- The masses span nine orders of magnitude. What could be the mechanism behind it? (Supersymmetric gauge theories, composite Higgs models, technicolor, new things we haven't imagined yet.)



This is also an important problem, but my curiosity is the former.


The difference between the two mass problems
Plato the guy with hand pointed towards the heaven vs. Aristotle with hand level to the earth.
I leave it to you to decide which perspective is which!

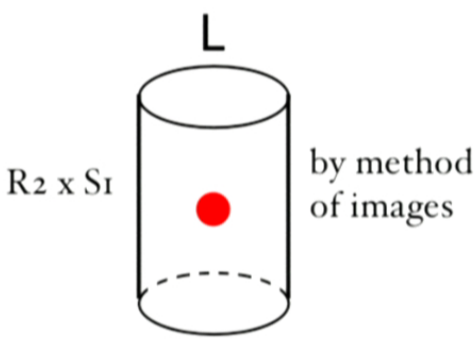

I will address two recent inter-related developments, tying to the earlier part of my talk:
- Mass gap problem.
- Large-N volume independence.
The general theme is about inferring properties of infinite-volume theory by studying (arbitrarily) small-volume dynamics.
The small volume may be
$\mathbb{R}^4 \Longrightarrow \mathbb{R}^3 \times S^1, \quad \mathbb{R}^2 \times T^2, \quad \mathbb{R} \times T^3, \quad T^4$
S^1
R^3
T^4
of characteristic size "L", while keeping the theory locally 4d.
Can we use compactification as an expansion parameter to study non-perturbative dynamics?

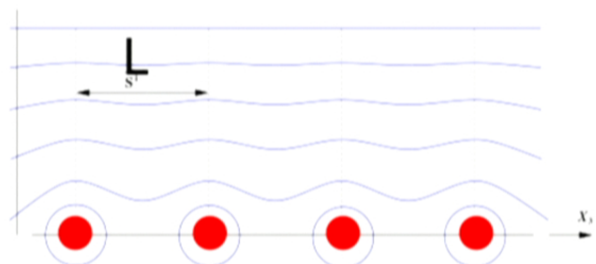
What is volume (in)dependence?

with electrostatic analogy.

- Consider a point charge in R3. Its potential is 1/r.
- Now, compactify one of the dimensions to a circle with size L. Space is R2 x S1.

r >> L → *ln r*.





r < L → *1/r*

- The characteristic length at which the potential (interaction between charges) changes from 3d behavior to 2d behavior is L. Very intuitive!

- By compactify more dimensions down to a space with size L, and using method of images, we obtain

The potential of a point charge in d-dimension: Gauss' law

Whereas volume independence demands

$$3d: \qquad \frac{1}{r} = \frac{1}{\sqrt{x_1^2 + x_2^2 + x_3^2}}$$

$$2d: \qquad \log r = \log \sqrt{x_1^2 + x_2^2}$$

$$1d: \qquad |r| = |x_1|$$

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Sounds outrageous. Almost like a joke!

Certainly wrong in electrodynamics, (or U(1) gauge theory), where our intuition is based on.

What is volume independence? or

"Eguchi-Kawai reduction" or "large-N reduction"

As strange as it may sound, large-N volume independence is one of the few exact properties of non-abelian gauge theories.

In 1982, Eguchi and Kawai proved that some of the most interesting aspects of the gauge theory (including spectrum of the theory) remain invariant

provided

there are no phase transitions as the volume of the space is shrunk.

The only problem was that no-one was able to find any example of gauge theory in which **"provided"** holds.

Why is it important?

If true: Consider the Schrödinger equation for QFT in infinite space and in the theory where the space is reduced to a single point, i.e. ordinary quantum mechanics.

$$H^{\text{YM}}_{\mathbb{R}^3}|\Psi_n\rangle = E_{\mathbb{R}^3}(n)|\Psi_n\rangle$$

$$H^{\text{YM}}_{\bullet}|\Psi_n\rangle = E_{\bullet}(n)|\Psi_n\rangle$$

$$E(1) - E(0) = \Delta = \text{mass gap}$$

PROMISE: Spectrum of large-N gauge theory = Spectrum of quantum mechanics of large matrices.

- If we really understand how large-N volume independence works, we will most likely develop a new way to think about gauge theory.
- **large-N QFT = large-N matrix quantum mechanics**

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Basic intuition behind volume independence

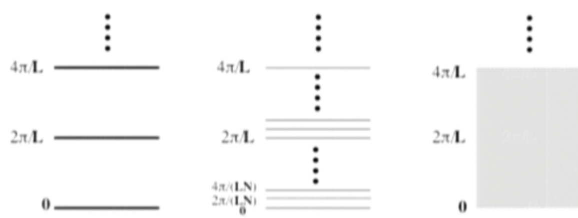
Consider the momentum modes in perturbation theory Unsal, Yaffe 2010

Infinite space: Continuum
on circle with **size L**: Discrete $P = \frac{2\pi}{L}k, \quad k \in \mathbb{Z}$
on circle with **size L** if **"provided"** holds: Discrete, but much-finer $P = \frac{2\pi}{LN}k, \quad k \in \mathbb{Z}$

Decompactification: • discrete spectrum $\Rightarrow$ continuum

• $L \to \infty$, N fixed (intuitive.)

if **"provided"** holds: • $N \to \infty$, L fixed (**surprising!**)



$L_{\text{eff}} = NL$

- The characteristic length at which the potential (interaction between charges) changes from 3d behavior to 2d behavior is LN. And LN$\to \infty$ as N $\to \infty$. Interactions never become 2-dimensional. Counter-intuitive, but correct!

$$r < NL \Longrightarrow V(r) \sim 1/r$$
$$r \gg NL \Longrightarrow V(r) \sim \log r$$

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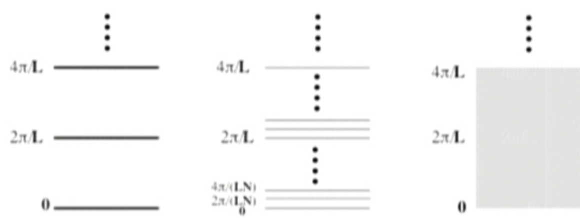

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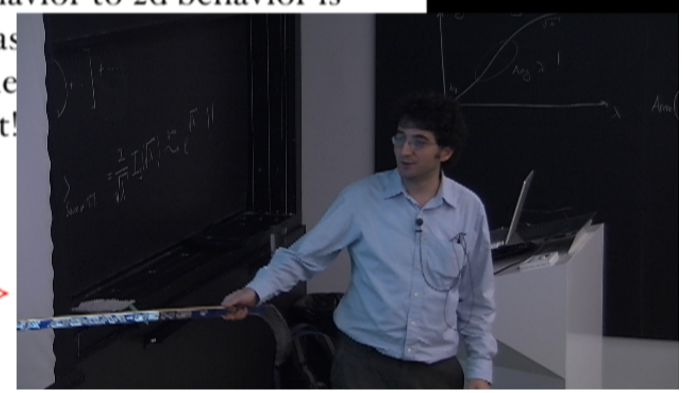

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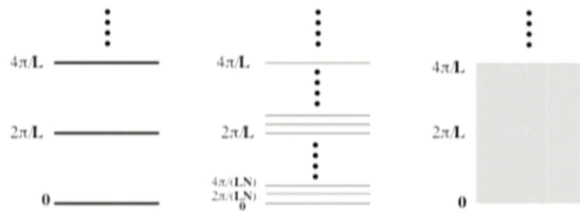

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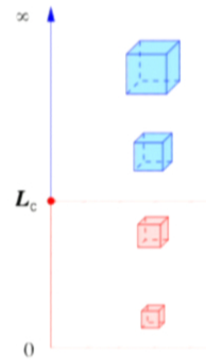
Stumbling block

- Because of the attractiveness of the idea, much effort has been devoted.
- However, there was always a phase transition when the space shrunk to small volume.
- Technically, an effective potential calculation in terms of Wilson lines (used to determine the phase of the small volume theory) gave a **negative** sign for **all** gauge theories. And we needed a positive sign!



Instability

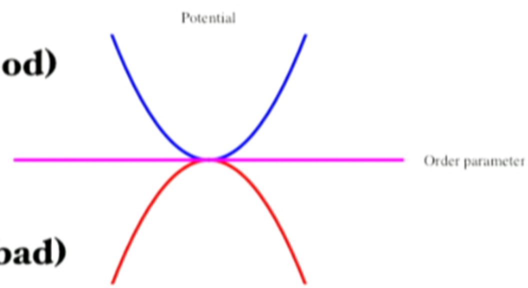
80's: EK, QEK, TEK.
Eguchi, Kawai, EK, Fails
Gonzalez-Arroyo, Okawa, TEK, Fails
Bhanot, Heller, Neuberger, QEK, Fails
Gross, Kitazawa,
Yaffe,
Migdal, Kazakov,
Parisi et.al.
Das, Wadia, Kogut,
+ 500 papers...., but no working example!

3rd ref in the list.
Teper, Vairinhos
Bringoltz, Sharpe

Stability (good)

Marginal

Instability (bad)



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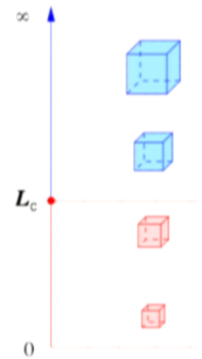

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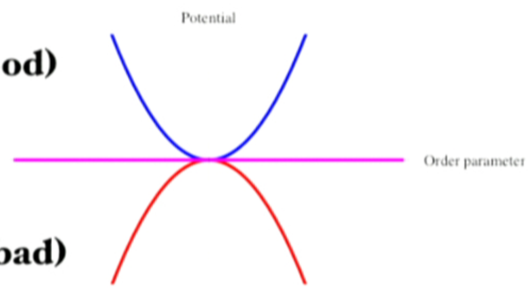


Evading the stumbling block

In 2006, I realized that the analog of the effective potential calculation in a supersymmetric gauge theory gave zero. At the heart of the cancelation was following identity:

$$-1+1=0$$

More precisely,

$$-1 \times (\text{stuff}) + 1 \times (\text{same stuff}) = 0$$

The crucial point: +1 appears due to the boundary conditions, and not supersymmetry!

Immediately, we deduce:

$$-1 + N_f > 0 \qquad \text{for } N_f > 1$$

The other "simple" sign flip

$N_f \geq 1$ massless adjoint rep. fermions
periodic boundary conditions ➡ stabilized center symmetry

$$\widetilde{Z}(L) = \mathrm{tr}[e^{-LH}(-1)^F]$$

Susy-theory: Supersymmetric (Witten) Index. **Witten, 1980**
Non-susy: **Twisted partition function (not an index, utility understood recently)**

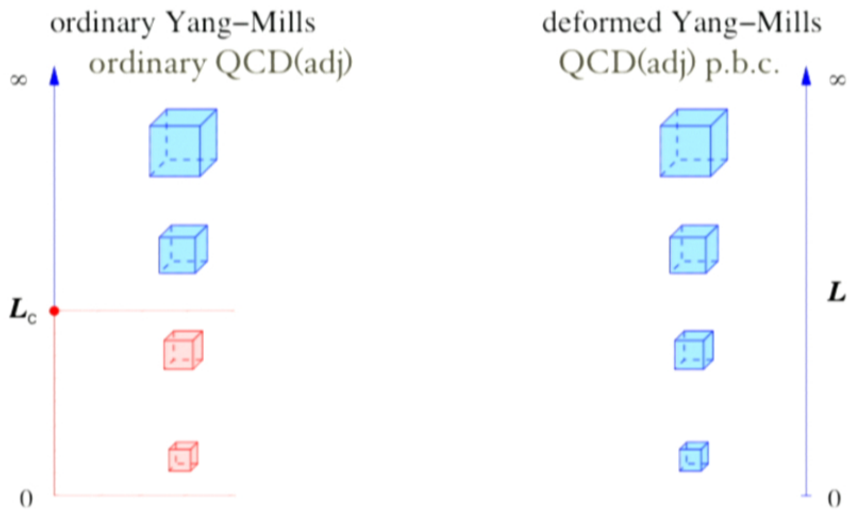
$$V_{1-\mathrm{loop}}[\Omega] = \frac{2}{\pi^2 L^4} \sum_{n=1}^{\infty} \underbrace{\frac{1}{n^4}(-1+N_f)}_{m_n^2} |\mathrm{tr}\,\Omega^n|^2 \qquad \Omega = e^{\int_{S^1} A}$$

$m_n^2 < 0$ instability, calculations between 1980-2007

$m_n^2 = 0$ Supersymmetric case, $N_f = 1$, marginal

$m_n^2 > 0$ QCD(adj), $N_f > 1$, stability Kovtun, Unsal, Yaffe, 07

Large-N Volume Independence



Two solutions:

SYM/QCD(adj): Kovtun, MU, Yaffe (2007)
Deformed YM: MU, Yaffe (2008)

Lattice tests:
Bringoltz, Sharpe,
Hanada, Azeyanagi, MU, Yacoby,
Narayanan, Hietanen,
Catterall, Galvez, MU,
Vairinhos, ...

Related works:
Ogilvie, Myers, Meisinger,
Bedaque, Cherman, Buchoff,
D'elia, Cossu,
Poppitz, MU,
Shifman,MU,
Veneziano, Wosiek.

First working examples, 25 years after the beautiful idea of Eguchi and Kawai

Can we now use quantum mechanics to solve 4d gauge theory?

Numerically, yes. **Analytic attempts give one many reason to be modest.**
Work in progress. Needs next good idea!

Mass gap and confinement

The theories with no phase transitions in the sense of confinement provide a new window to 4d dynamics.

$\mathbb{R}^3$ $\mathbb{R}^3 \times S^1$ $\mathbb{R}^4$

A smooth journey

A complementary regime to that of volume independence - a calculable shadow of the dynamics of the 4 dimensional "real thing".

fix-N, take L-small: Semiclassical studies of confinement
Many new surprising phenomena
New composite topological excitations--**Topological Molecules**

MU 2007; QCD(adj)
--for vectorlike or chiral theories + classification + thermal QCD
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The essence of mass gap in Polyakov-model in 3d

Polyakov 1977

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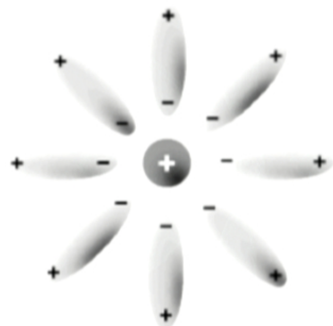
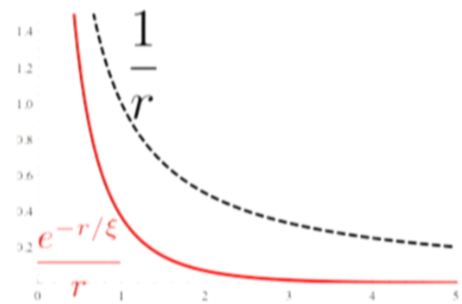
Proliferation of monopole-instantons generates mass gap for gauge fluctuations.

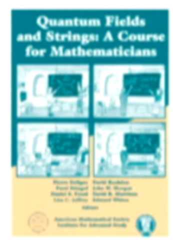


Debye screening

$$\frac{1}{r} \longrightarrow \frac{e^{-r/\xi}}{r}$$



Finite magnetic screening length
=mass for gauge fluctuations for U(1) photon
=Confinement of electric charge, flux tubes, see,



excellent QFT lectures, by Ed Witten

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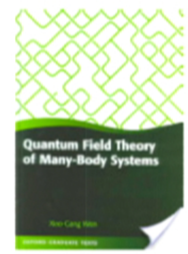

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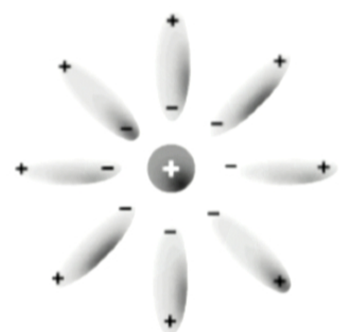

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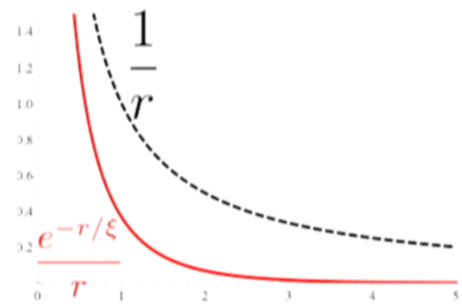

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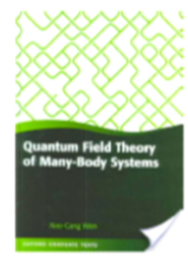

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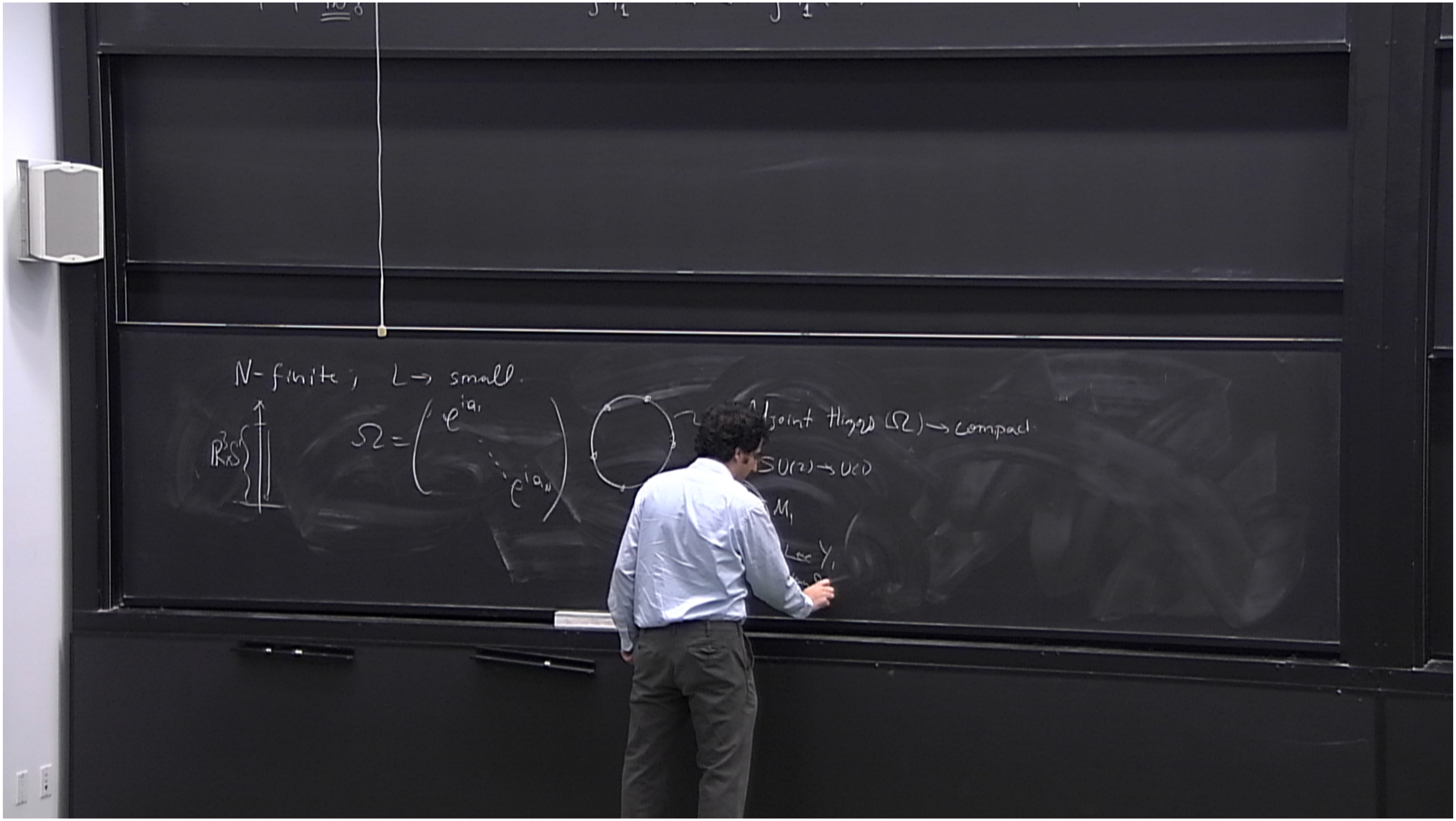
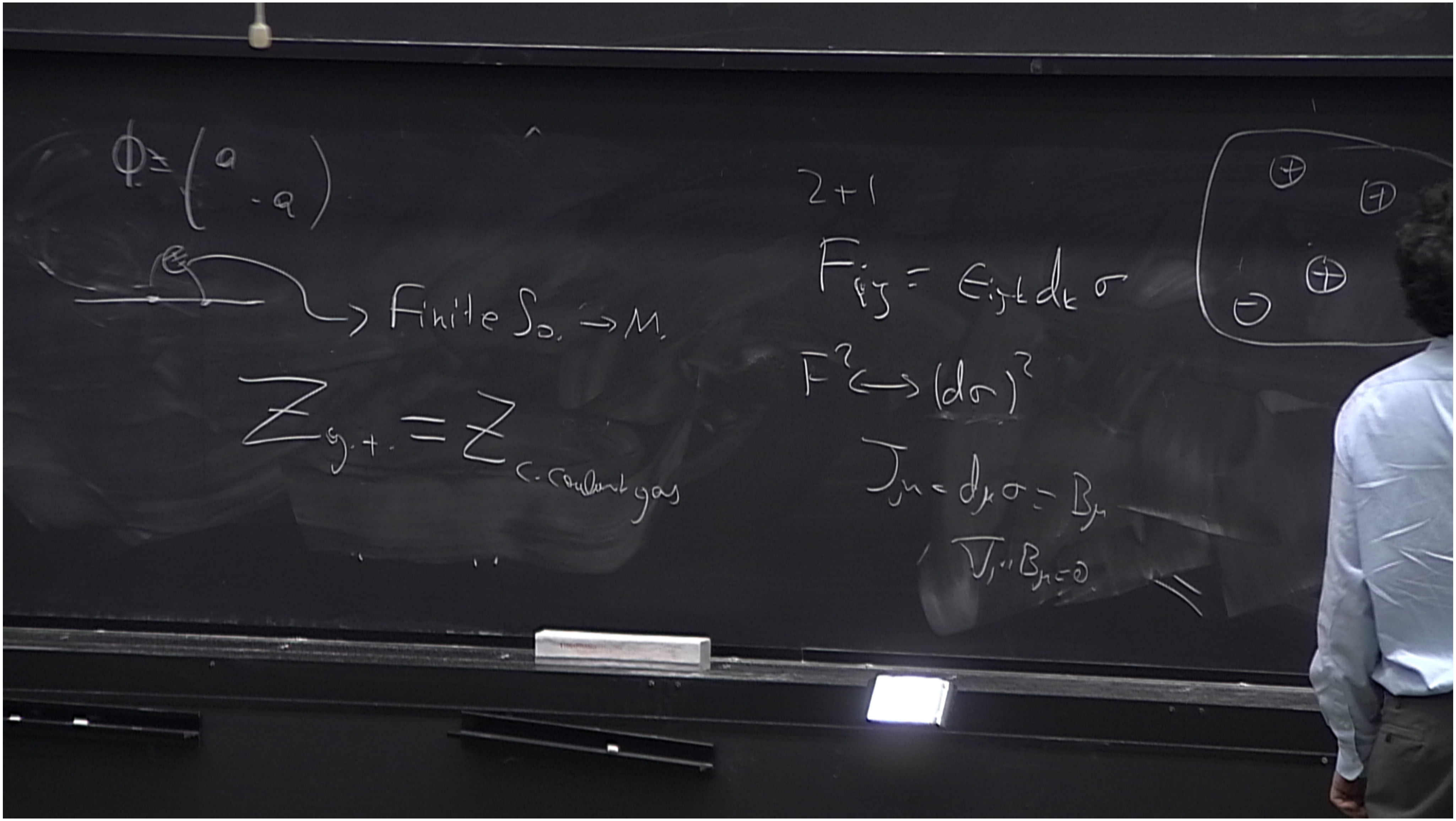
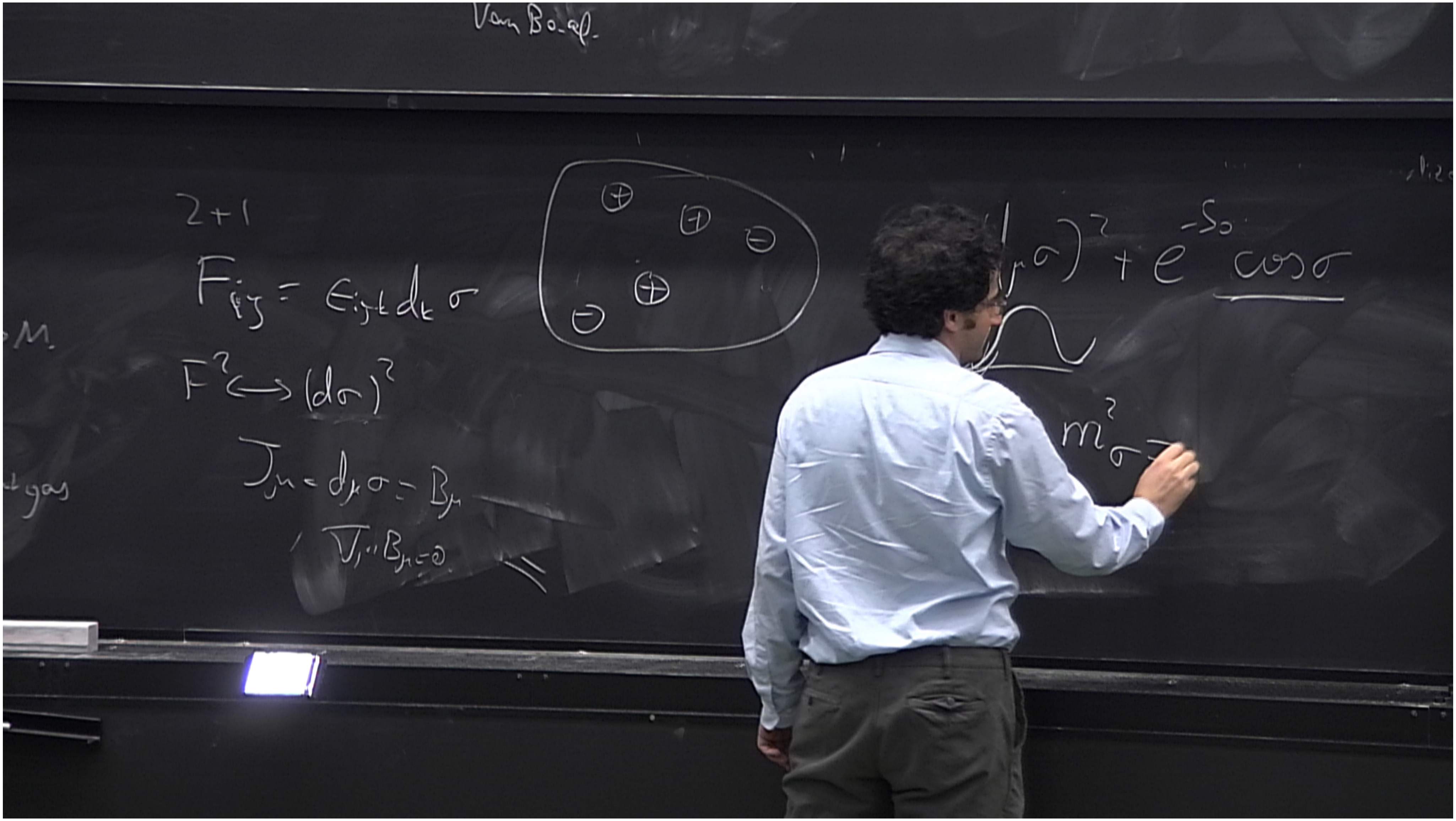
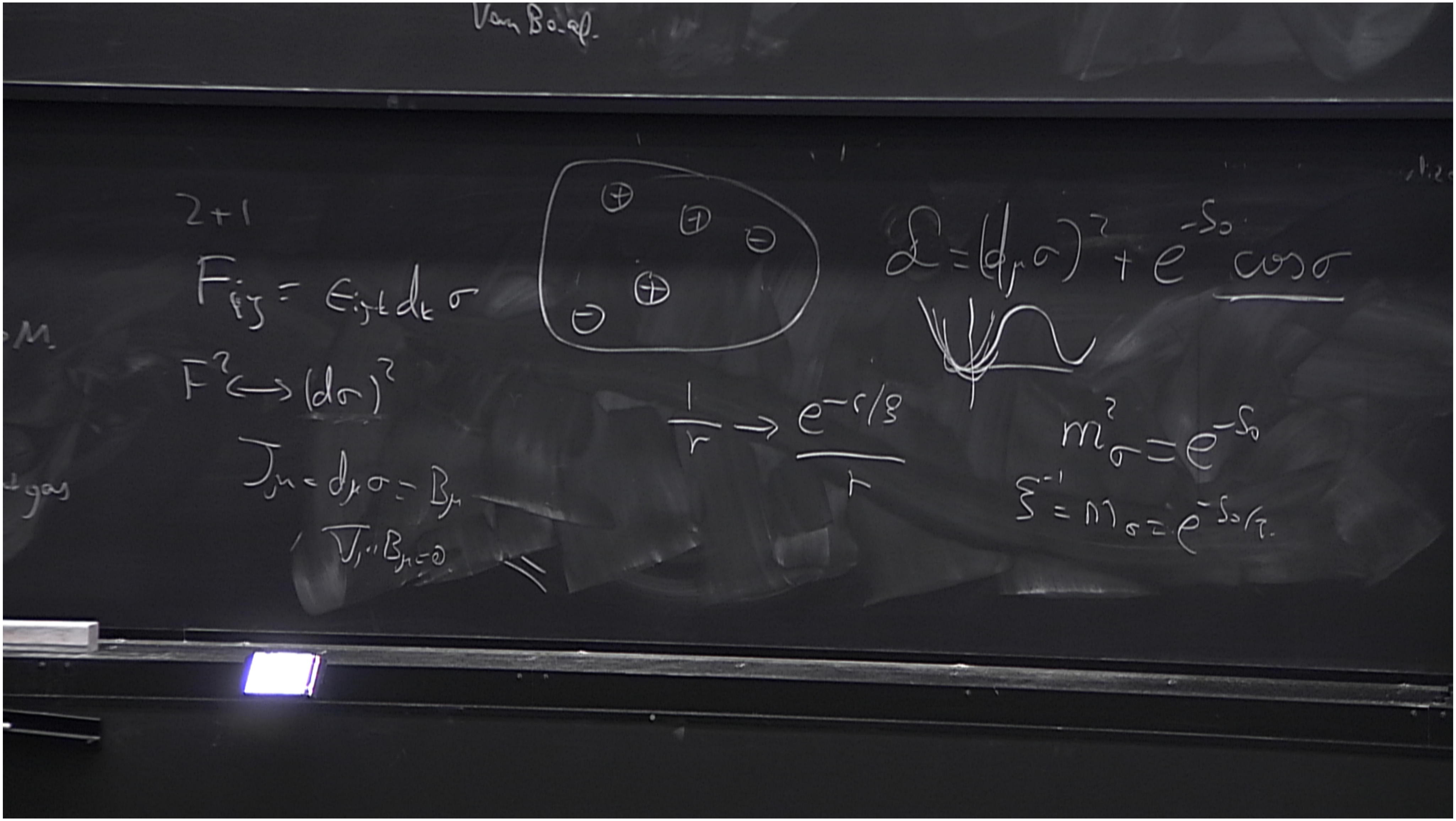
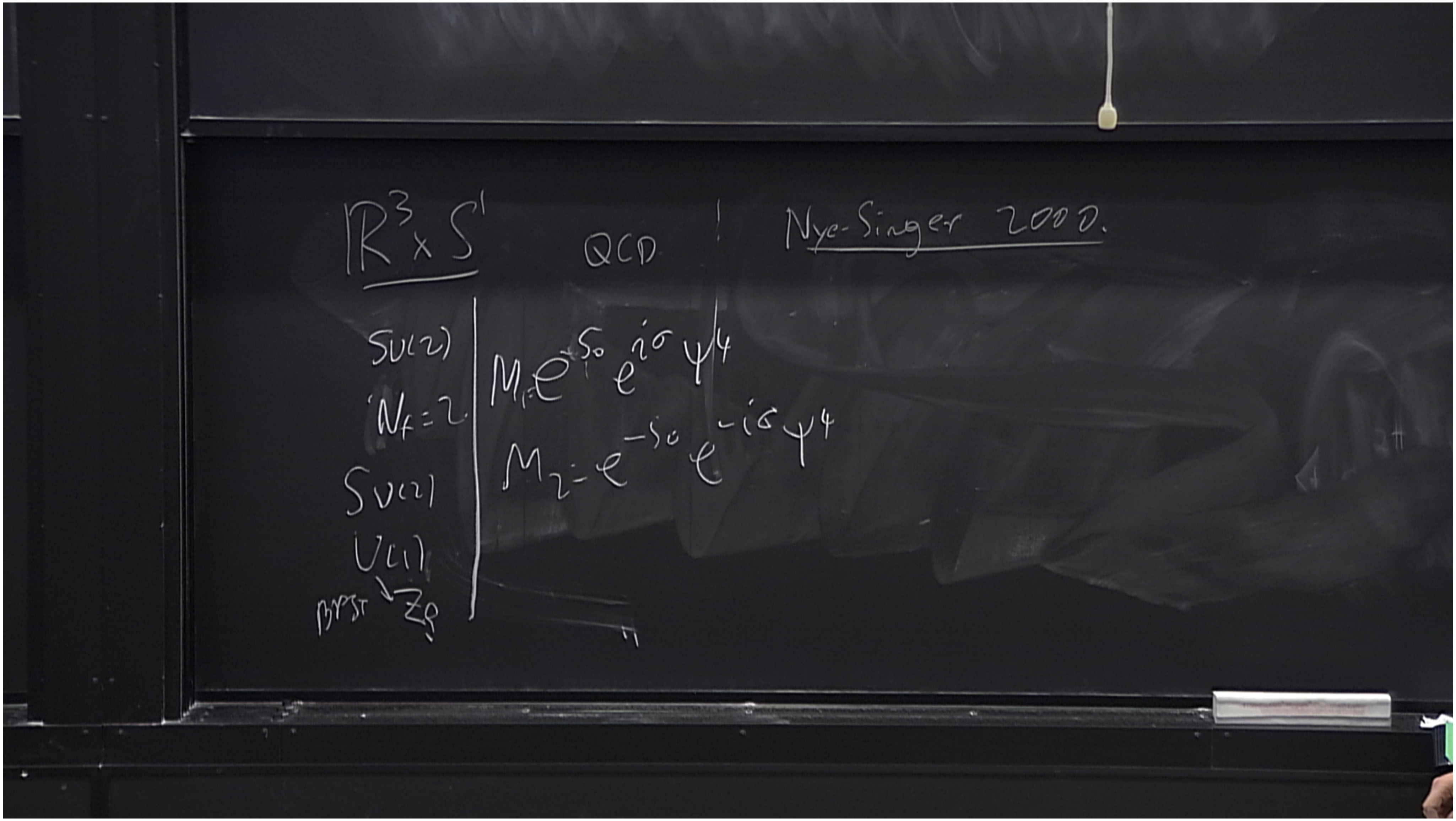

N- finite ; L→ small.
→ compact


2+1
Finite


2+1
F_ij = ε_ijk ∂_k σ
F² ⟷ (∂σ)²
J_μ = ∂_μ σ = B_μ
∇_μ B_μ = 0
+ e^{-S_0} cos σ

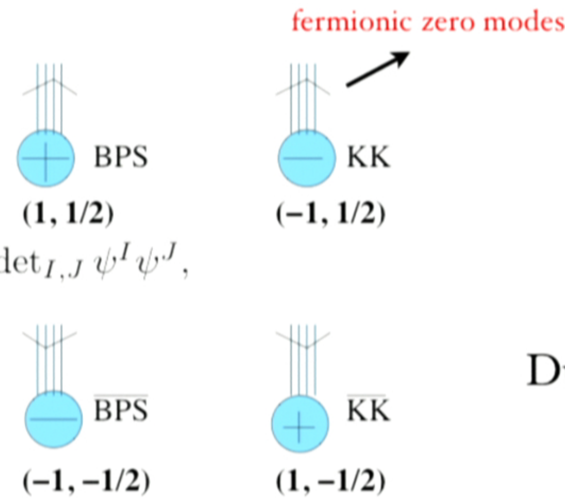

Van Bo-al.
2+1
$\mathcal{L} = (\partial_\mu \sigma)^2 + e^{-S_0} \cos\sigma$
$m_\sigma^2 = e^{-S_0}$
$m_\sigma = e^{-S_0/2}$


Van Baal
2+1
F_ij = ε_ijk ∂_k σ
F² ⟷ (∂σ)²
J_μ = ∂_μ σ = B_μ
∇_μ B_μ = 0
1/r → e^{-r/ξ}/r
ℒ = (∂_μ σ)² + e^{-S_0} cos σ
m_σ² = e^{-S_0}
ξ^{-1} = m_σ = e^{-S_0/2}


R³ x S¹
QCD
Nye-Singer 2000.
SU(2)
N_f = 2
M₁ = e^{-S₀} e^{iσ} ψ⁴
M₂ = e^{-S₀} e^{-iσ} ψ⁴
SU(2)
U(1)
BPST

Topological excitations in QCD(adj), SU(2), Nf=2

Magnetic Monopoles (3d instantons + a twisted 3d-instanton) with two topological quantum numbers: $\left(\int_{S^2} F,\ \int_{R^3\times S^1} F\tilde{F}\right)$



$e^{-S_0} e^{i\sigma} \det_{I,J} \psi^I \psi^J,$

$e^{-S_0} e^{i\sigma} \det_{I,J} \bar{\psi}^I \bar{\psi}^J$

Duality + Index thm + Symmetry allow

$$e^{-2S_0}(e^{2i\sigma} + e^{-2i\sigma}) \quad (\mathbb{Z}_2)_*$$

which generate a mass gap. What is the topological defect which leads to this?

Realization of existence of KK-monopole, van Baal et.al. and Lu, Yi, 97

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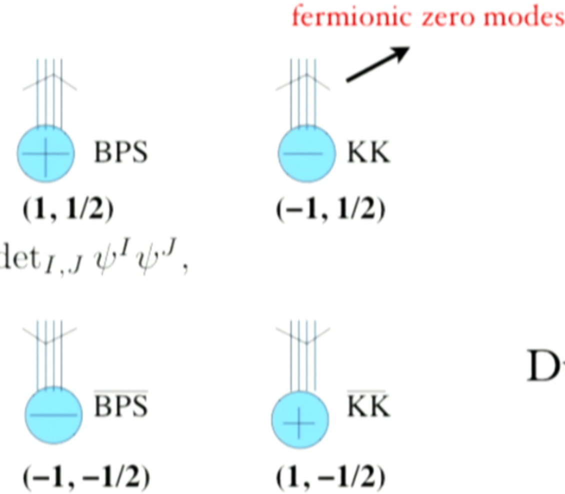

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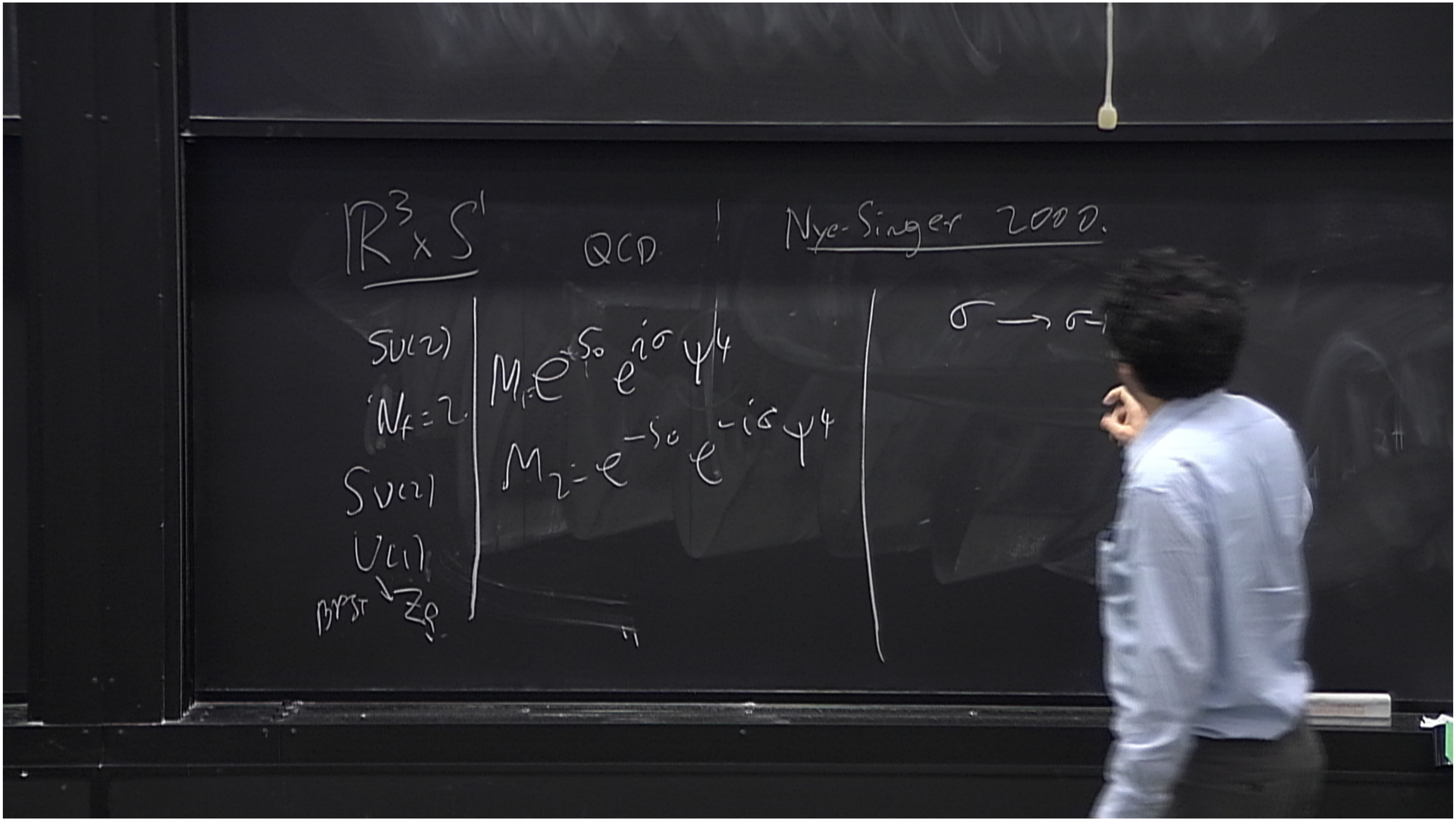
R^3 x S^1
QCD
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N_f = 2.
SU(2)
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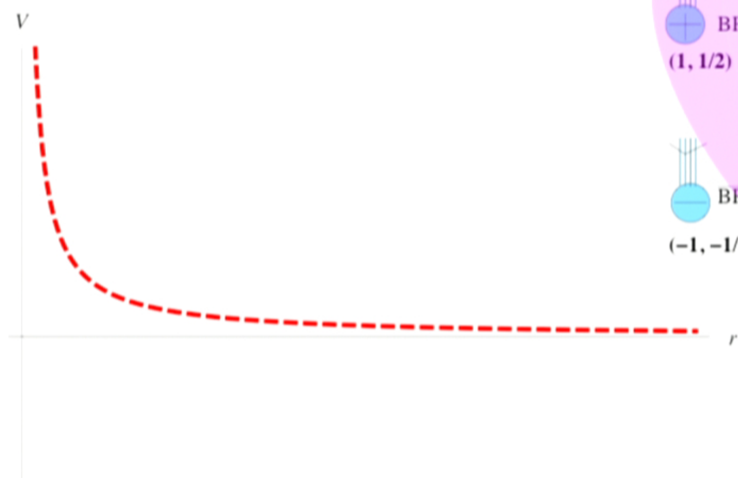
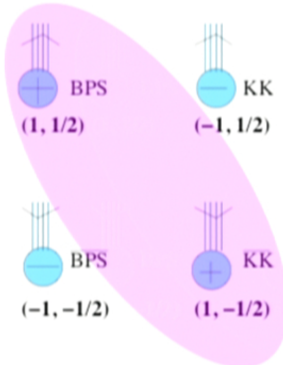
Topological molecules

The quantum numbers associated with $e^{-2S_0}(e^{2i\sigma} + e^{-2i\sigma})$ are (2, 0) and (-2,0). Since (2,0) = (1, 1/2) + (1,-1/2), we may think of it as a molecule. We refer to it as magnetic bion.

How is a stable molecule possible? Same sign magnetic charge objects should repel each other due to Coulomb law.





Coulomb law: $1/r$ repulsion

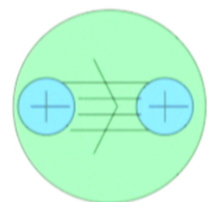
QCD(adj) vacuum is a plasma of magnetic bions

$$L^{\mathrm{dQCD}} = \frac{1}{2}(\partial\sigma)^2 - b\, e^{-2S_0} \cos 2\sigma + i\bar{\psi}^I \gamma_\mu \partial_\mu \psi_I + c\, e^{-S_0} \cos\sigma (\det_{I,J} \psi^I \psi^J + \mathrm{c.c.})$$

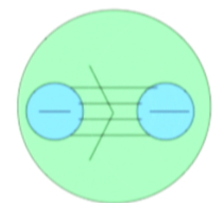
magnetic bions lead to mass gap!

magnetic monopoles

Alice with Tweedledum and Tweedledee, Through the Looking-Glass and what Alice found there (1871).



(2,0)



(−2, 0)

No net topological charge!!

Strongly-correlated pairs.

Three mechanisms of mass gap ?

- On R3, monopole-instantons, Polyakov mechanism. Polyakov **1977**
- On R4, another mechanism is monopole or dyon condensation, which takes place in a supersymmetric theory. Seiberg, Witten **1994**
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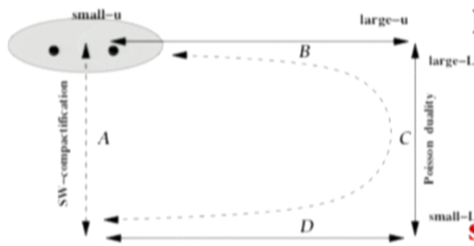
Three mechanisms are continuously connected.

Poppitz & MU (2011): Compactify SW-theory on R3 x S1.

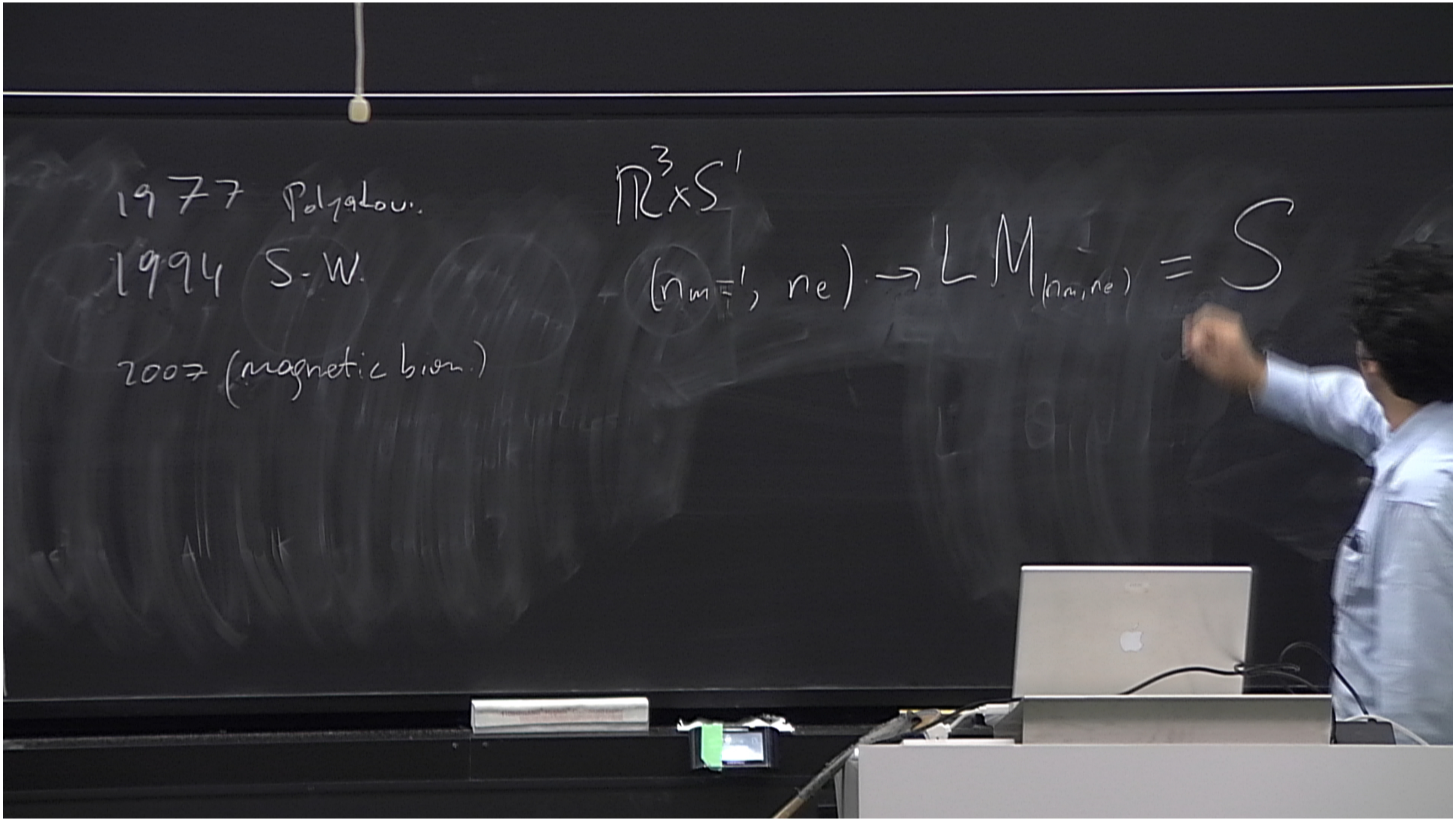


large-L: SW, 4d monopoles and their dyon towers

small-L: tower of 3d instantons

Two class of topological defects look a priori unrelated. The sums over the defects at first glance looks unrelated.

Yet, they are equal due to Poisson resummation. This is duality on R3 x S1.


1977 Polyakov
1994 S-W.
2007 (magnetic bion)
R³ x S¹
(nm=1, ne) → LM(nm, ne) = S

Can we define a gauge theory in continuum?

Argyres, MU, 2012

However, I feel deeply unsatisfied with one aspect of both Polyakov's work and the magnetic bion solution.

The problem is, in these theories, perturbation theory is always asymptotic, this is another way of saying that it is divergent. (Even when you regularize and renormalize it.), but if you truncate it wisely (by a prescription due to Poincare), the result will be a good approximation. (This does not change the fact that p.t. is divergent in general. You cannot hide an elephant under the rug.

Can we give a meaning to p.t.?
There is a way to give a meaning to p.t in some cases. It is called Borel resummation. But gauge theory as well as many simple quantum mechanical systems are non-Borel resummable.

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Argyres and I think that we found new topological objects which also lead multi-fold ambiguity in their amplitudes, exactly opposite in sign with respect to ambiguities of p.t.

Our conjecture is that p.t. + semi-classical expansion may provide a meaning to a continuum field theory...

This is the current work in progress.

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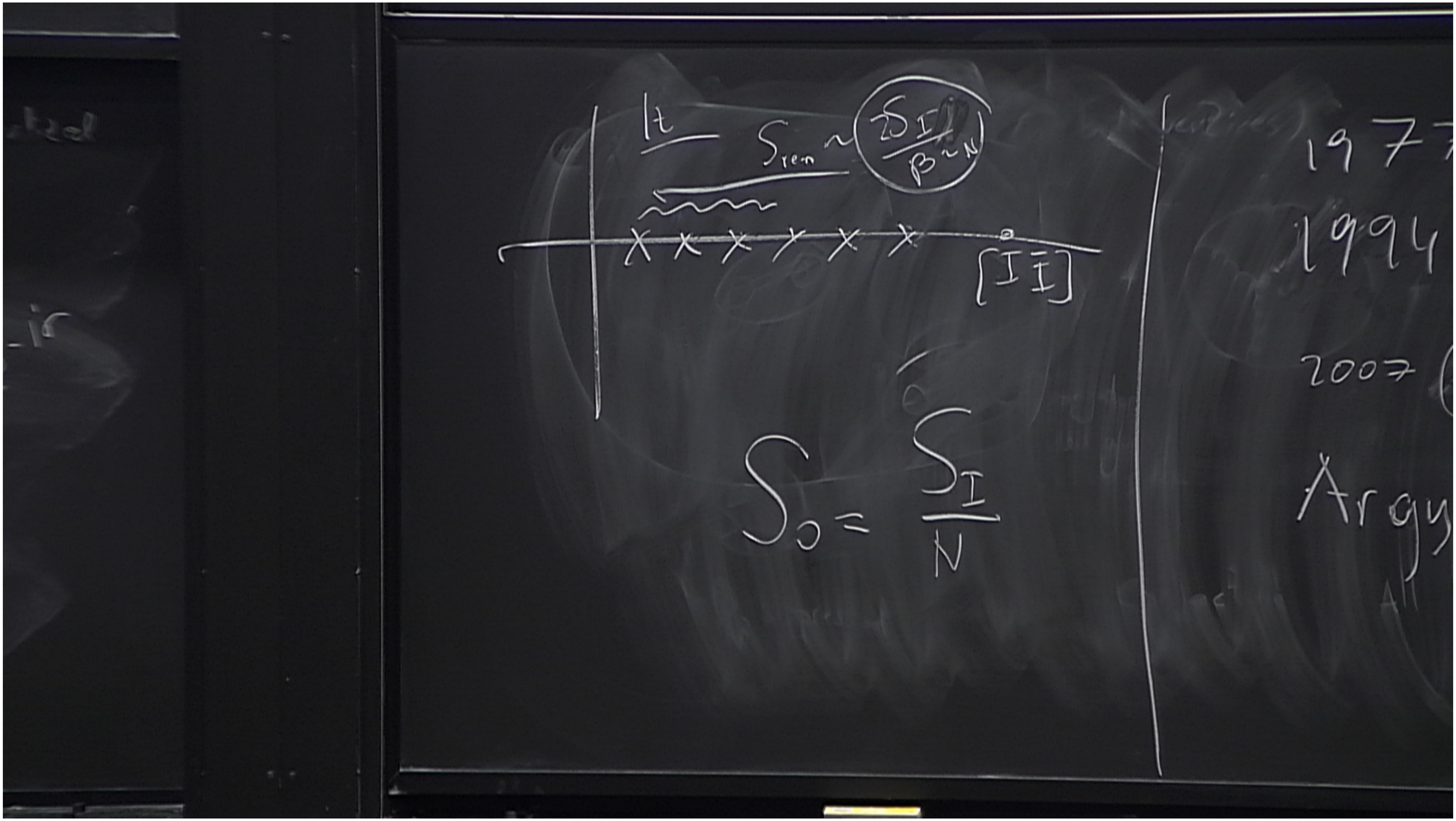
lt
1977
1994
2007
Argy

Conclusions and prospects

- There are new ways to study non-perturbative aspects of 4d gauge theories by using circle compactifications.
- Large-N volume independence and its semi-classical shadow provide new insights, both numeric and analytic. Useful for both supersymmetric and ordinary gauge theories.
- New topological molecules, new mechanism of confinement and mass gap. Magnetic bions are responsible for confinement at small circle for many gauge theories. Can we extend it to large circle and $\mathbb{R}^4$? (Yes in Seiberg-Witten theory. Recent work with Poppitz.)
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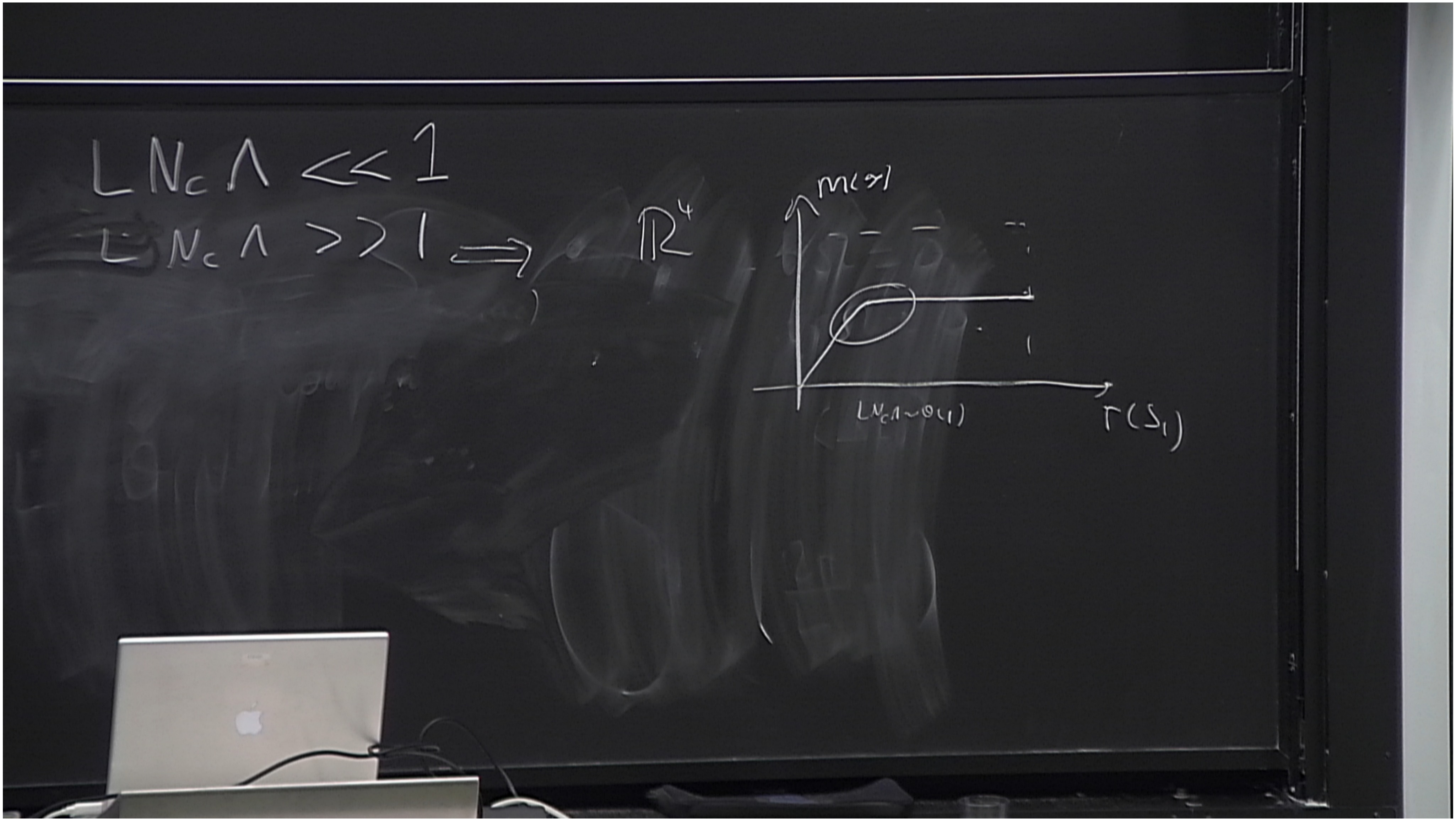
LNcΛ << 1
LNcΛ >> 1 ⟹ R4
m(σ)
LNcΛ ~ O(1)
r(S1)